Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/35-1.2.1.4-d+e-x-^m-f+g-x-^n-a+b-x+c-x^2-^p

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 5:29pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [958]. This is test number [35].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (958)	0.00 (0)
Mathematica	97.81 (937)	2.19 (21)
Maple	76.10 (729)	23.90 (229)
Fricas	68.27 (654)	31.73 (304)
Giac	41.96 (402)	58.04 (556)
Maxima	34.55 (331)	65.45 (627)
Mupad	28.81 (276)	71.19 (682)
Sympy	27.24 (261)	72.76 (697)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

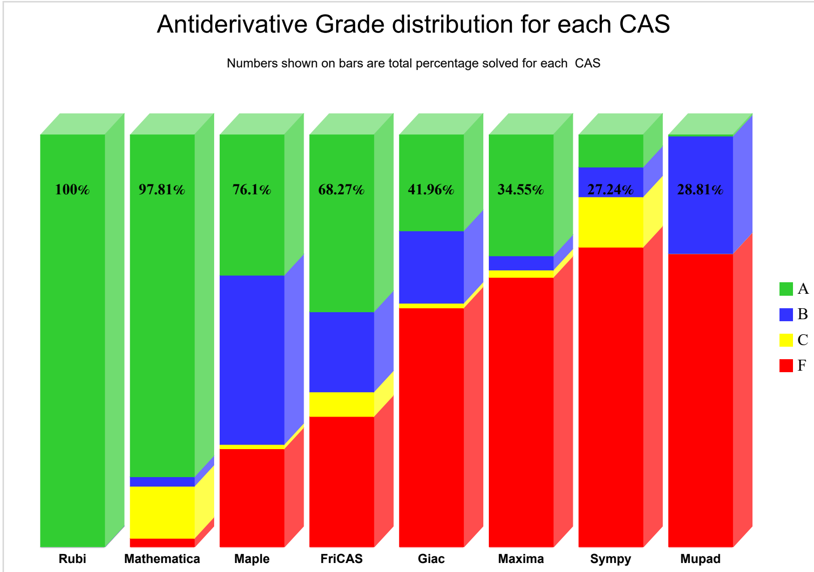
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

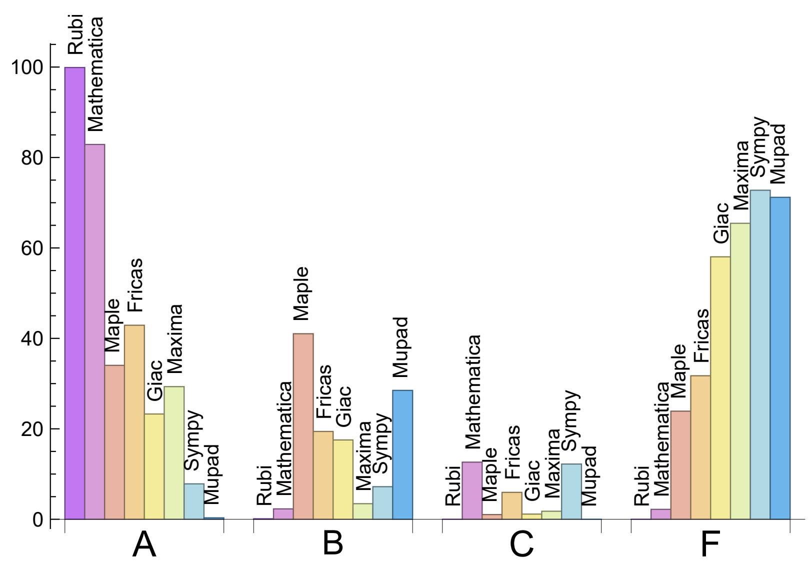
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.90	0.10	0.00	0.00
Mathematica	82.88	2.30	12.63	2.19
Fricas	42.90	19.42	5.95	31.73
Maple	34.03	41.02	1.04	23.90
Maxima	29.33	3.44	1.77	65.45
Giac	23.28	17.54	1.15	58.04
Sympy	7.83	7.20	12.21	72.76
Mupad	N/A	28.50	0.00	71.19

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	21	100.00 %	0.00 %	0.00 %
Maple	229	100.00 %	0.00 %	0.00 %
Fricas	304	75.99 %	24.01 %	0.00 %
Giac	556	72.48 %	17.09 %	10.43 %
Maxima	627	86.44 %	0.48 %	13.08 %
Sympy	697	70.16 %	24.53 %	5.31 %
Mupad	682	98.97 %	1.03 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

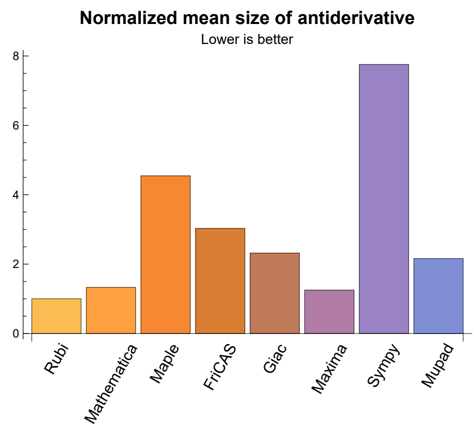
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

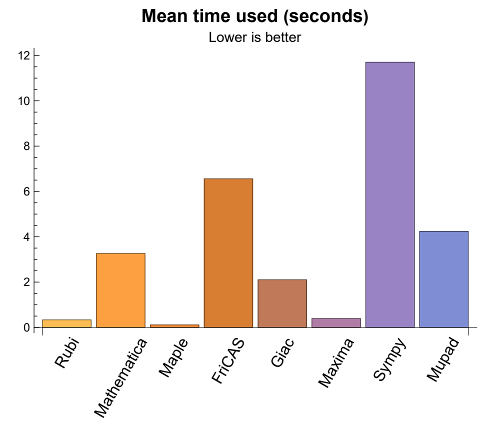
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.33	234.03	1.00	175.00	1.00
Mathematica	3.26	579.39	1.33	151.00	0.93
Maple	0.11	2073.37	4.55	337.00	1.92
Maxima	0.39	190.90	1.25	144.00	1.12
Fricas	6.56	767.48	3.03	244.50	1.62
Sympy	11.70	1658.44	7.75	376.00	3.21
Giac	2.10	549.44	2.32	223.50	1.62
Mupad	4.24	438.67	2.16	161.00	1.28

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{948, 952, 957\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{919\}$

Mathematica $\{267, 275, 276, 314, 542, 656, 802, 892, 899, 906, 907, 915, 917, 918\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

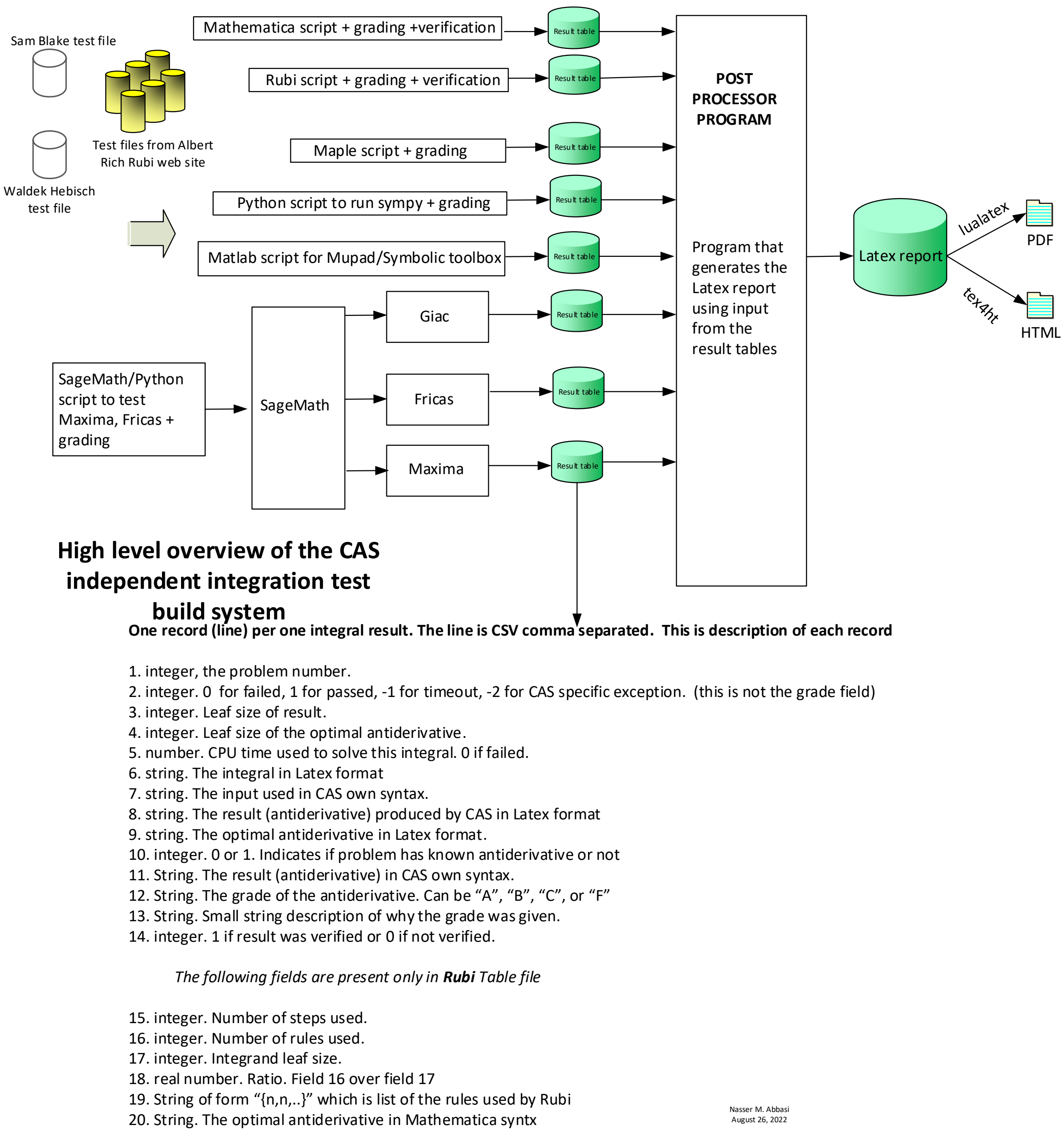
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958 }

B grade: { 833 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 269, 270, 271, 272, 273, 278, 279, 280, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299, 300, 302, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 493, 497, 500, 504, 507, 511, 514, 518, 521, 524, 526, 527, 530, 531, 532, 535, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 607, 611, 614, 619, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 801, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 919, 920, 921, 922, 924, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 947, 948, 951, 952, 956, 957 }

B grade: { 263, 268, 274, 277, 284, 285, 294, 295, 301, 304, 305, 360, 620, 621, 656, 802, 805, 851, 907, 918, 925, 926 }

C grade: { 267, 275, 276, 315, 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 525, 528, 529, 533, 534, 536, 537, 538, 539, 540, 587, 605, 606, 608, 609, 610, 612, 613, 615, 616, 617, 618, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 800, 813, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 927, 953, 958 }

F grade: { 379, 380, 408, 416, 434, 435, 436, 541, 803, 804, 810, 811, 812, 923, 928, 945, 946, 949, 950, 954, 955 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 69, 70, 96, 117, 120, 121, 122, 123, 124, 125, 126, 151, 152, 153, 154, 155, 156, 175, 184, 193, 218, 219, 224, 315, 316, 326, 327, 328, 332, 333, 341, 350, 352, 353, 439, 440, 448, 449, 459, 460, 471, 472, 473, 474, 475, 476, 481, 482, 483, 484, 489, 490, 491, 493, 494, 496, 497, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 511, 512, 514, 515, 516, 518, 521, 524, 525, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 588, 589, 590, 591, 592, 593, 594, 595, 597, 598, 599, 600, 601, 602, 604, 619, 640, 648, 649, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 679, 680, 681, 682, 683, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 696, 697, 700, 701, 702, 703, 704, 708, 712, 713, 714, 715, 716, 717, 718, 719, 721, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 745, 746, 747, 748, 749, 750, 753, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 788, 808, 814, 815, 816, 817, 819, 820, 821, 822, 823, 824, 825, 827, 828, 829, 830, 831, 832, 840, 841, 848, 849, 854, 855, 862, 863, 864, 870, 871, 872, 873, 875, 876, 912, 913, 918, 919, 948, 952, 957, 958 }

B grade: { 9, 10, 11, 18, 22, 23, 35, 44, 45, 46, 47, 48, 49, 51, 52, 53, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 225, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 351, 355, 356, 357, 359, 360, 361, 437, 438, 441, 442, 443, 444, 445, 446, 447, 450, 451, 452, 453, 454, 455, 456, 457, 458, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 477, 478, 479, 480, 485, 486, 487, 488, 492, 495, 503, 509, 510, 513, 517, 519, 520, 522, 523, 526, 579, 580, 581, 582, 583, 584, 585, 586, 587, 596, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 641, 642, 643, 644, 645, 646, 647, 650, 651, 652, 653, 654, 678, 688, 698, 699, 705, 706, 707, 709, 710, 711, 720, 727, 733, 742, 751, 752, 754, 755, 756, 789, 790, 791, 805, 806, 807, 818, 826, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 856, 857, 858, 859, 860, 861, 865, 866, 867, 868, 869, 874, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 914, 915, 916, 917, 920, 921, 925, 926 }

C grade: { 792, 793, 794, 795, 796, 797, 798, 799, 800, 801 }

F grade: { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 541, 542, 543, 544, 545, 546, 547, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 802, 803, 804, 809, 810, 811, 812, 813, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 136, 137, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 184, 202, 203, 220, 224, 316, 317, 318, 319, 320, 321, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 342, 343, 344, 345, 346, 347, 351, 352, 353, 355, 356, 357, 490, 497, 504, 511, 518, 524, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 589, 590, 591, 592, 596, 597, 598, 599, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 768, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 792, 793, 794, 797, 798, 799, 807, 808, 814, 815, 816, 819, 820, 821, 822, 826, 827, 828, 829, 948, 952, 957 }

B grade: { 19, 20, 21, 22, 44, 45, 84, 85, 138, 192, 193, 211, 212, 213, 214, 215, 222, 359, 360, 361, 449, 460, 579, 580, 581, 582, 583, 805, 806, 920, 921, 925, 926 }

C grade: { 102, 103, 104, 105, 106, 107, 157, 158, 159, 160, 161, 162, 198, 199, 200, 201, 604 }

F grade: { 98, 99, 100, 101, 124, 125, 126, 133, 134, 135, 144, 145, 146, 147, 154, 155, 156, 176, 177, 178, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 204, 205, 206, 207, 208, 209, 210, 216, 217, 218, 219, 221, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282,

283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 322, 323, 324, 325, 331, 332, 333, 339, 340, 341, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 585, 586, 587, 588, 593, 594, 595, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 661, 662, 663, 664, 669, 670, 671, 676, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 788, 789, 790, 791, 795, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 817, 818, 823, 824, 825, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958
}

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 28, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 141, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 224, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 341, 348, 349, 350, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 470, 471, 472, 473, 474, 475, 476, 490, 493, 497, 500, 504, 507, 511, 514, 518, 521, 524, 548, 549, 550, 551, 552, 553, 557, 558, 559, 560, 561, 562, 563, 564, 569, 570, 571, 572, 573, 576, 582, 583, 584, 589, 590, 591, 592, 593, 596, 597, 598, 599, 603, 604, 620, 621, 656, 657, 658, 659, 660, 661, 665, 666, 667, 668, 672, 673, 674, 679, 680, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 700, 701, 702, 703, 705, 706, 707, 712, 713, 714, 715, 716, 720, 721, 722, 727, 728, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 769, 770, 771, 780, 781, 783, 784, 785, 786, 787, 788, 792, 793, 794, 797, 798, 814, 815, 816, 819, 820, 821, 822, 823, 826, 827, 828, 829, 834, 835, 836, 837, 840, 842, 843, 844, 845, 857, 948, 952, 957 }

B grade: { 18, 21, 22, 23, 24, 25, 26, 27, 30, 31, 131, 132, 138, 139, 140, 142, 143, 144, 148, 149, 221, 223, 225, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 351, 352, 353, 355, 356, 357, 359, 360, 361, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 554, 555, 556, 565, 566, 567, 568, 574, 575, 577, 578, 579, 580, 581, 585, 586, 587, 588, 594, 595, 600, 601, 602, 607, 608, 611, 612, 613, 615, 616, 618, 619, 662, 663, 664, 669, 670, 671, 675, 676, 677, 678, 685, 686, 687, 688, 696, 697, 698, 699, 704, 708, 709, 710, 711, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 789, 790, 791, 795, 796, 799, 800, 805, 806, 807, 808, 824, 825, 830, 831, 832, 833, 838, 839, 841, 846, 847, 848, 849, 851, 873, 874, 875, 880, 881, 882, 920, 921, 925, 926 }

C grade: { 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 622, 623, 624, 625, 629, 630, 631, 632, 636, 637, 638, 639, 645, 646, 647, 648, 886, 887, 888, 889, 893, 894, 895, 896, 900, 901, 902, 903, 909, 910, 911, 912 }

F grade: { 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 469, 540, 541, 542, 543, 544, 545, 546, 547, 605, 606, 609, 610, 614, 617, 626, 627, 628, 633, 634, 635, 640, 641, 642, 643, 644, 649, 650, 651, 652, 653, 654, 655, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 801, 802, 803, 804, 809, 810, 811, 812, 813, 817, 818, 850, 852, 853, 854, 855, 856, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 876, 877, 878, 879, 883, 884, 885, 890, 891, 892, 897, 898, 899, 904, 905, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

2.1.6 Sympy

A grade: { 3, 5, 6, 25, 32, 34, 36, 37, 54, 55, 56, 57, 58, 62, 69, 104, 106, 117, 157, 159, 161, 222, 244, 245, 253, 254, 255, 262, 264, 265, 315, 385, 386, 394, 395, 396, 401, 403, 405, 406, 524, 548, 549, 550, 551, 557, 558, 559, 560, 561, 562, 566, 569, 570, 571, 572, 573, 576, 577, 578, 588, 593, 596, 597, 598, 599, 600, 823, 826, 827, 828, 829, 830, 948, 952 }

B grade: { 19, 21, 23, 240, 241, 242, 243, 249, 250, 251, 252, 258, 259, 260, 261, 263, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 381, 382, 383, 384, 390, 391, 392, 393, 399, 400, 402, 404, 552, 553, 554, 555, 556, 563, 564, 565, 567, 568, 574, 575, 589, 590, 591, 592, 805, 806, 807, 808, 814, 819, 820, 821, 822, 920, 921, 925, 926 }

C grade: { 1, 2, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 33, 35, 38, 39, 40, 41, 42, 43, 59, 60, 61, 63, 64, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 102, 103, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 220, 224, 226, 227, 228, 229, 230, 231, 235, 236, 246, 247, 248, 256, 257, 266, 268, 269, 270, 271, 272, 273, 274, 306, 307, 308, 309, 310, 313, 377, 378, 387, 388, 389, 397, 398, 407, 430, 431, 432, 433 }

F grade: { 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 65, 66, 67, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 225, 232, 233, 234, 237, 238, 239, 267, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 311, 312, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 379, 380, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 579, 580, 581, 582, 583, 584, 585, 586, 587, 594, 595, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 815, 816, 817, 818, 824, 825, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 957, 958 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 16, 17, 32, 33, 34, 35, 36, 37, 38, 39, 54, 55, 56, 57, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 108, 109, 117, 119, 120, 121, 122, 123, 124, 151, 152, 153, 154, 157, 158, 159, 161, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 219, 315, 316, 317, 318, 319, 320, 322, 323, 324, 326, 327, 328, 329, 330, 332, 333, 336, 337, 340, 341, 437, 438, 439, 440, 442, 446, 447, 448, 449, 451, 457, 458, 459, 460, 462, 470, 471, 504, 524, 529, 539, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 584, 588, 589, 590, 591, 592, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 604, 621, 658, 659, 660, 661, 666, 667, 668, 673, 674, 675, 683, 771, 780, 786, 787, 792, 793, 794, 814, 815, 816, 817, 820, 821, 822, 823, 824, 825, 828, 829, 830, 831, 832, 833, 834, 836, 837, 838, 845, 874, 948, 952, 957 }

B grade: { 9, 10, 11, 12, 13, 14, 15, 40, 41, 42, 43, 60, 61, 62, 63, 64, 75, 76, 77, 78, 79, 80, 81, 82, 90, 91, 98, 99, 100, 101, 110, 111, 112, 113, 114, 115, 116, 118, 125, 126, 156, 160, 162, 186, 187, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 325, 334, 335, 338, 348, 351, 352, 353, 355, 356, 357, 359, 360, 361, 443, 444, 445, 452, 453, 454, 455, 456, 463, 464, 465, 466, 467, 468, 469, 490, 497, 525, 526, 527, 528, 530, 532, 533, 534, 535, 536, 537, 538, 540, 579, 580, 581, 582, 583, 585, 587, 596, 618, 619, 620, 657, 665, 669, 672, 676, 679, 680, 681, 682, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 768, 769, 770, 783, 784, 785, 795, 797, 798, 799, 805, 806, 807, 808, 818, 819, 826, 827, 835, 839, 840, 841, 843, 844, 846, 847, 848, 849, 860, 861, 877, 880, 881, 882, 885, 920, 921, 925, 926 }

C grade: { 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178 }

F grade: { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 163, 164, 165, 166, 171, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 321, 331, 339, 342, 343, 344, 345, 346, 347, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 441, 450, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 531, 541, 542, 543, 544, 545, 546, 547, 586, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 662, 663, 664, 670, 671, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 788, 789, 790, 791, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 842, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 875, 876, 878, 879, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 }

2.1.8 Mupad

A grade: { 948, 952, 957 }

B grade: { 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 46, 47, 48, 49, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 117, 118, 123, 130, 131, 132, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 172, 173, 174, 175, 182, 183, 184, 192, 193, 210, 211, 212, 213, 214, 215, 220, 222, 224, 245, 315, 351, 352, 353, 355, 356, 357, 359, 360, 361, 386, 473, 480, 481, 482, 487, 488, 490, 497, 504, 511, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 582, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 620, 621, 657, 658, 659, 660, 665, 666, 667, 668, 672, 673, 674, 675, 679, 680, 681, 682, 683, 689, 690, 691, 692, 693, 700, 701, 702, 703, 704, 716, 717, 718, 719, 723, 724, 725, 726, 729, 730, 731, 732, 738, 739, 740, 741, 747, 748, 749, 750, 757, 758, 759, 760, 768, 769, 770, 771, 780, 783, 784, 785, 786, 787, 792, 793, 794, 795, 805, 806, 807, 808, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 836, 837, 840, 841, 844, 845, 848, 849, 920, 921, 925, 926 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 10, 11, 19, 20, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 50, 51, 52, 53, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 176, 177, 178, 179, 180, 181, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 216, 217, 218, 219, 221, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 541, 542, 543, 544, 545, 546, 547, 579, 580, 581, 585, 586, 587, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 622, 623, 624, 625, 626,

$627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 661, 662, 663, 664, 669, 670, 671, 676, 677, 678, 684, 685, 686, 687, 688, 694, 695, 696, 697, 698, 699, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 720, 721, 722, 727, 728, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 751, 752, 753, 754, 755, 756, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 788, 789, 790, 791, 796, 797, 798, 799, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 835, 838, 839, 842, 843, 846, 847, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958 \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	111	130	96	89	279	74	-1
N.S.	1	1.00	0.84	0.98	0.73	0.67	2.11	0.56	-0.01
time (sec)	N/A	0.048	0.147	0.091	0.489	2.519	2.900	1.629	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	158	215	163	128	830	117	-1
N.S.	1	1.00	0.79	1.07	0.81	0.64	4.13	0.58	-0.00
time (sec)	N/A	0.096	0.223	0.073	0.510	2.584	25.940	2.406	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	147	184	140	118	775	106	-1
N.S.	1	1.00	0.85	1.07	0.81	0.69	4.51	0.62	-0.01
time (sec)	N/A	0.066	0.209	0.092	0.496	2.324	25.547	1.933	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	136	153	117	108	653	96	-1
N.S.	1	1.00	0.86	0.96	0.74	0.68	4.11	0.60	-0.01
time (sec)	N/A	0.061	0.220	0.059	0.472	2.171	8.134	1.532	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	125	126	94	98	580	84	-1
N.S.	1	1.00	1.08	1.09	0.81	0.84	5.00	0.72	-0.01
time (sec)	N/A	0.021	0.202	0.050	0.478	2.445	8.117	1.614	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	125	126	94	98	580	84	-1
N.S.	1	1.00	1.08	1.09	0.81	0.84	5.00	0.72	-0.01
time (sec)	N/A	0.021	0.002	0.000	0.476	1.903	8.099	2.517	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	140	157	122	102	469	99	107
N.S.	1	1.00	1.24	1.39	1.08	0.90	4.15	0.88	0.95
time (sec)	N/A	0.060	0.318	0.063	0.494	2.109	10.469	2.789	2.904

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	143	188	128	122	386	157	114
N.S.	1	1.00	1.22	1.61	1.09	1.04	3.30	1.34	0.97
time (sec)	N/A	0.058	0.295	0.085	0.487	1.864	3.690	1.425	3.513

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	141	219	151	125	461	211	120
N.S.	1	1.00	1.17	1.81	1.25	1.03	3.81	1.74	0.99
time (sec)	N/A	0.059	0.420	0.079	0.504	2.302	4.262	1.602	3.735

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	139	250	173	121	457	256	-1
N.S.	1	1.00	1.16	2.08	1.44	1.01	3.81	2.13	-0.01
time (sec)	N/A	0.058	0.379	0.073	0.488	2.568	3.772	1.021	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	141	281	197	112	541	288	-1
N.S.	1	1.00	1.19	2.38	1.67	0.95	4.58	2.44	-0.01
time (sec)	N/A	0.059	0.395	0.066	0.517	2.165	4.954	0.858	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	151	166	146	93	774	366	93
N.S.	1	1.00	1.40	1.54	1.35	0.86	7.17	3.39	0.86
time (sec)	N/A	0.040	0.408	0.077	0.483	2.946	4.808	0.926	4.264

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	198	169	103	918	425	118
N.S.	1	1.00	0.88	1.38	1.18	0.72	6.42	2.97	0.83
time (sec)	N/A	0.060	0.415	0.064	0.483	2.357	8.963	1.012	4.660

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	132	225	192	113	1037	492	192
N.S.	1	1.00	0.77	1.31	1.12	0.66	6.03	2.86	1.12
time (sec)	N/A	0.077	0.495	0.085	0.495	2.499	9.762	1.073	5.334

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	148	256	215	123	1159	425	212
N.S.	1	1.00	0.74	1.27	1.07	0.61	5.77	2.11	1.05
time (sec)	N/A	0.099	0.518	0.083	0.495	2.992	26.955	1.024	6.044

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	92	107	75	68	177	54	112
N.S.	1	1.00	0.89	1.04	0.73	0.66	1.72	0.52	1.09
time (sec)	N/A	0.034	0.137	0.062	0.494	2.349	2.114	1.283	3.138

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	79	103	72	85	163	68	87
N.S.	1	1.00	1.08	1.41	0.99	1.16	2.23	0.93	1.19
time (sec)	N/A	0.026	0.203	0.063	0.485	2.341	4.061	1.075	2.956

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	120	80	98	231	0	55
N.S.	1	1.00	1.02	2.07	1.38	1.69	3.98	0.00	0.95
time (sec)	N/A	0.016	0.234	0.061	0.269	1.955	3.749	0.000	2.590

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	149	251	286	259	2004	0	-1
N.S.	1	1.00	0.93	1.56	1.78	1.61	12.45	0.00	-0.01
time (sec)	N/A	0.089	0.443	0.109	0.490	2.384	26.015	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	136	220	255	245	1821	0	-1
N.S.	1	1.00	0.93	1.50	1.73	1.67	12.39	0.00	-0.01
time (sec)	N/A	0.075	0.397	0.082	0.555	2.466	22.823	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	124	189	230	230	1739	0	-1
N.S.	1	1.00	1.02	1.55	1.89	1.89	14.25	0.00	-0.01
time (sec)	N/A	0.055	0.384	0.062	0.498	1.981	26.932	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	208	145	159	418	0	78
N.S.	1	1.00	0.98	2.48	1.73	1.89	4.98	0.00	0.93
time (sec)	N/A	0.031	0.305	0.053	0.267	2.198	23.407	0.000	2.701

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	82	178	122	160	337	0	78
N.S.	1	1.00	0.91	1.98	1.36	1.78	3.74	0.00	0.87
time (sec)	N/A	0.026	0.294	0.068	0.278	3.890	7.449	0.000	2.659

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	82	147	102	161	513	0	78
N.S.	1	1.00	0.87	1.56	1.09	1.71	5.46	0.00	0.83
time (sec)	N/A	0.028	0.313	0.066	0.274	4.242	7.487	0.000	2.615

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	120	79	160	432	0	78
N.S.	1	1.00	0.99	1.45	0.95	1.93	5.20	0.00	0.94
time (sec)	N/A	0.015	0.286	0.061	0.269	3.040	7.095	0.000	2.621

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	90	75	161	604	0	78
N.S.	1	1.00	1.02	1.12	0.94	2.01	7.55	0.00	0.98
time (sec)	N/A	0.012	0.008	0.054	0.270	2.947	8.106	0.000	2.584

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	118	182	153	231	2378	0	127
N.S.	1	1.00	1.01	1.56	1.31	1.97	20.32	0.00	1.09
time (sec)	N/A	0.064	0.447	0.056	0.277	2.965	15.357	0.000	3.079

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	134	213	182	254	2404	0	141
N.S.	1	1.00	0.88	1.39	1.19	1.66	15.71	0.00	0.92
time (sec)	N/A	0.083	0.421	0.079	0.270	2.722	12.669	0.000	3.305

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	147	244	206	270	2691	0	181
N.S.	1	1.00	0.80	1.33	1.12	1.47	14.62	0.00	0.98
time (sec)	N/A	0.103	0.480	0.092	0.271	2.381	15.771	0.000	3.428

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	104	173	123	221	903	0	164
N.S.	1	1.00	0.86	1.43	1.02	1.83	7.46	0.00	1.36
time (sec)	N/A	0.034	0.392	0.071	0.282	3.224	7.367	0.000	2.690

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	126	199	144	281	1401	0	202
N.S.	1	1.00	0.85	1.34	0.97	1.90	9.47	0.00	1.36
time (sec)	N/A	0.040	0.484	0.058	0.358	4.176	15.585	0.000	2.737

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	71	90	63	66	119	70	84
N.S.	1	1.00	1.31	1.67	1.17	1.22	2.20	1.30	1.56
time (sec)	N/A	0.022	0.197	0.111	0.507	2.041	3.530	0.760	0.088

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	125	295	142	98	558	84	-1
N.S.	1	1.00	0.72	1.71	0.82	0.57	3.23	0.49	-0.01
time (sec)	N/A	0.145	0.225	0.065	0.487	2.701	11.071	0.639	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	114	222	119	88	357	73	-1
N.S.	1	1.00	0.79	1.54	0.83	0.61	2.48	0.51	-0.01
time (sec)	N/A	0.115	0.205	0.075	0.479	1.865	3.850	1.155	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	103	202	96	78	386	63	-1
N.S.	1	1.00	0.90	1.76	0.83	0.68	3.36	0.55	-0.01
time (sec)	N/A	0.090	0.205	0.057	0.489	3.111	4.592	0.937	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	89	133	72	67	218	49	-1
N.S.	1	1.00	1.07	1.60	0.87	0.81	2.63	0.59	-0.01
time (sec)	N/A	0.054	0.163	0.056	0.485	3.220	2.416	1.143	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	113	50	57	269	40	-1
N.S.	1	1.00	0.94	1.36	0.60	0.69	3.24	0.48	-0.01
time (sec)	N/A	0.017	0.028	0.068	0.471	3.400	2.149	1.050	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	102	91	61	69	184	65	-1
N.S.	1	1.00	1.55	1.38	0.92	1.05	2.79	0.98	-0.02
time (sec)	N/A	0.073	0.192	0.067	0.480	3.220	2.884	0.812	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	102	93	65	77	207	107	-1
N.S.	1	1.00	1.50	1.37	0.96	1.13	3.04	1.57	-0.01
time (sec)	N/A	0.073	0.207	0.075	0.479	3.208	1.789	1.050	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	116	139	80	61	214	166	-1
N.S.	1	1.00	1.45	1.74	1.00	0.76	2.68	2.08	-0.01
time (sec)	N/A	0.072	0.253	0.066	0.483	2.547	2.835	0.805	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	86	151	103	71	303	237	-1
N.S.	1	1.00	0.80	1.41	0.96	0.66	2.83	2.21	-0.01
time (sec)	N/A	0.086	0.259	0.068	0.481	1.792	2.622	0.832	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	104	229	126	83	449	299	-1
N.S.	1	1.00	0.74	1.64	0.90	0.59	3.21	2.14	-0.01
time (sec)	N/A	0.105	0.294	0.069	0.486	2.064	4.868	1.483	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	115	240	149	93	510	363	-1
N.S.	1	1.00	0.68	1.42	0.88	0.55	3.02	2.15	-0.01
time (sec)	N/A	0.122	0.342	0.087	0.480	1.819	4.443	1.570	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	303	255	177	0	0	-1
N.S.	1	1.00	0.88	2.12	1.78	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.407	0.078	0.502	2.049	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	113	324	273	162	0	0	-1
N.S.	1	1.00	0.93	2.68	2.26	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.400	0.064	0.491	2.106	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	70	261	142	109	0	0	66
N.S.	1	1.00	0.72	2.69	1.46	1.12	0.00	0.00	0.68
time (sec)	N/A	0.112	0.323	0.070	0.275	2.061	0.000	0.000	2.893

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	70	282	120	110	0	0	67
N.S.	1	1.00	0.80	3.24	1.38	1.26	0.00	0.00	0.77
time (sec)	N/A	0.074	0.339	0.057	0.303	1.687	0.000	0.000	2.865

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	69	173	100	110	0	0	65
N.S.	1	1.00	0.78	1.94	1.12	1.24	0.00	0.00	0.73
time (sec)	N/A	0.021	0.303	0.058	0.273	2.346	0.000	0.000	2.862

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	70	193	73	111	0	0	66
N.S.	1	1.00	0.91	2.51	0.95	1.44	0.00	0.00	0.86
time (sec)	N/A	0.013	0.011	0.058	0.272	1.662	0.000	0.000	2.814

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	107	202	150	163	0	0	-1
N.S.	1	1.00	0.91	1.73	1.28	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.465	0.070	0.271	2.331	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	123	288	180	186	0	0	-1
N.S.	1	1.00	0.85	1.99	1.24	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.455	0.087	0.273	2.084	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	136	361	203	202	0	0	-1
N.S.	1	1.00	0.75	1.98	1.12	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.482	0.085	0.268	1.671	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	147	381	226	212	0	0	-1
N.S.	1	1.00	0.70	1.82	1.08	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.297	0.536	0.093	0.274	1.864	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	58	71	70	50	73	34	36
N.S.	1	1.00	0.72	0.88	0.86	0.62	0.90	0.42	0.44
time (sec)	N/A	0.057	0.126	0.112	0.481	1.181	0.257	1.121	2.501

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	57	56	45	60	30	31
N.S.	1	1.00	0.84	0.90	0.89	0.71	0.95	0.48	0.49
time (sec)	N/A	0.048	0.119	0.079	0.485	1.771	0.174	1.034	0.029

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	46	41	40	38	37	21	22
N.S.	1	1.00	1.12	1.00	0.98	0.93	0.90	0.51	0.54
time (sec)	N/A	0.029	0.089	0.063	0.487	1.861	0.110	0.912	0.029

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	29	28	33	27	19	21
N.S.	1	1.00	1.02	0.72	0.70	0.82	0.68	0.48	0.52
time (sec)	N/A	0.007	0.101	0.070	0.481	1.594	0.076	0.765	0.030

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	52	29	41	46	31	34	32
N.S.	1	1.00	1.62	0.91	1.28	1.44	0.97	1.06	1.00
time (sec)	N/A	0.037	0.087	0.122	0.477	1.955	3.020	0.882	0.047

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	57	30	42	53	51	55	35
N.S.	1	1.00	1.73	0.91	1.27	1.61	1.55	1.67	1.06
time (sec)	N/A	0.037	0.087	0.120	0.486	1.730	2.247	0.802	0.079

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	48	42	54	43	116	91	47
N.S.	1	1.00	0.94	0.82	1.06	0.84	2.27	1.78	0.92
time (sec)	N/A	0.039	0.087	0.100	0.485	2.008	3.561	1.368	2.487

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	47	56	68	48	128	125	67
N.S.	1	1.00	0.70	0.84	1.01	0.72	1.91	1.87	1.00
time (sec)	N/A	0.047	0.090	0.089	0.479	1.878	4.147	1.731	0.032

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	58	70	82	53	223	163	77
N.S.	1	1.00	0.65	0.79	0.92	0.60	2.51	1.83	0.87
time (sec)	N/A	0.053	0.105	0.082	0.479	2.742	6.423	1.701	0.032

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	63	84	96	58	201	199	90
N.S.	1	1.00	0.59	0.79	0.90	0.54	1.88	1.86	0.84
time (sec)	N/A	0.066	0.106	0.078	0.477	2.815	7.268	1.052	0.036

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	127	324	150	105	544	287	-1
N.S.	1	1.00	0.95	2.42	1.12	0.78	4.06	2.14	-0.01
time (sec)	N/A	0.134	0.443	0.071	0.517	1.899	5.220	1.080	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	210	610	251	179	0	170	-1
N.S.	1	1.00	0.68	1.97	0.81	0.58	0.00	0.55	-0.00
time (sec)	N/A	0.296	0.544	0.091	0.495	2.003	0.000	1.219	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	199	548	228	169	0	160	-1
N.S.	1	1.00	0.71	1.95	0.81	0.60	0.00	0.57	-0.00
time (sec)	N/A	0.250	0.426	0.064	0.505	2.418	0.000	1.197	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	188	486	205	159	0	149	-1
N.S.	1	1.00	0.75	1.93	0.81	0.63	0.00	0.59	-0.00
time (sec)	N/A	0.222	0.440	0.069	0.505	3.705	0.000	1.252	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	177	424	182	149	1681	139	-1
N.S.	1	1.00	0.79	1.90	0.82	0.67	7.54	0.62	-0.00
time (sec)	N/A	0.190	0.391	0.069	0.493	1.403	138.751	1.349	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	166	366	159	139	1554	128	-1
N.S.	1	1.00	0.72	1.59	0.69	0.60	6.76	0.56	-0.00
time (sec)	N/A	0.075	0.412	0.062	0.491	1.859	137.531	1.061	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	155	304	130	129	1284	117	-1
N.S.	1	1.00	0.82	1.62	0.69	0.69	6.83	0.62	-0.01
time (sec)	N/A	0.049	0.367	0.071	0.493	1.180	29.382	1.329	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	184	353	200	142	1263	143	-1
N.S.	1	1.00	0.97	1.86	1.05	0.75	6.65	0.75	-0.01
time (sec)	N/A	0.192	0.552	0.084	0.479	1.392	39.162	1.173	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	187	357	212	161	1057	199	-1
N.S.	1	1.00	0.97	1.85	1.10	0.83	5.48	1.03	-0.01
time (sec)	N/A	0.195	0.534	0.097	0.492	2.317	11.820	1.361	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	189	474	214	167	1059	256	-1
N.S.	1	1.00	0.91	2.29	1.03	0.81	5.12	1.24	-0.00
time (sec)	N/A	0.193	0.627	0.068	0.493	2.931	12.362	1.534	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	189	536	211	167	911	314	-1
N.S.	1	1.00	0.90	2.55	1.00	0.80	4.34	1.50	-0.00
time (sec)	N/A	0.195	0.597	0.091	0.485	1.940	7.738	1.310	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	187	598	233	168	1028	365	-1
N.S.	1	1.00	0.89	2.86	1.11	0.80	4.92	1.75	-0.00
time (sec)	N/A	0.191	0.654	0.069	0.558	1.476	9.473	1.382	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	189	660	259	168	1178	425	-1
N.S.	1	1.00	0.88	3.06	1.20	0.78	5.45	1.97	-0.00
time (sec)	N/A	0.186	0.694	0.072	0.483	1.640	8.129	1.629	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	192	722	282	167	1397	476	-1
N.S.	1	1.00	0.90	3.37	1.32	0.78	6.53	2.22	-0.00
time (sec)	N/A	0.187	0.714	0.074	0.485	1.600	12.696	1.114	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	188	580	303	161	1513	505	-1
N.S.	1	1.00	0.91	2.82	1.47	0.78	7.34	2.45	-0.00
time (sec)	N/A	0.188	0.726	0.076	0.522	1.800	12.947	0.663	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	185	640	327	152	1719	529	-1
N.S.	1	1.00	0.91	3.14	1.60	0.75	8.43	2.59	-0.00
time (sec)	N/A	0.184	0.801	0.082	0.499	3.652	31.668	0.740	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	156	504	230	133	1889	618	-1
N.S.	1	1.00	0.83	2.70	1.23	0.71	10.10	3.30	-0.01
time (sec)	N/A	0.159	0.709	0.104	0.492	2.543	34.480	0.712	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	167	568	253	143	2159	677	-1
N.S.	1	1.00	0.74	2.52	1.12	0.64	9.60	3.01	-0.00
time (sec)	N/A	0.178	0.785	0.132	0.492	3.457	143.030	0.577	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	175	626	276	153	2397	744	-1
N.S.	1	1.00	0.69	2.46	1.09	0.60	9.44	2.93	-0.00
time (sec)	N/A	0.204	0.837	0.177	0.486	2.640	154.534	0.567	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	123	450	284	181	0	214	-1
N.S.	1	1.00	0.71	2.59	1.63	1.04	0.00	1.23	-0.01
time (sec)	N/A	0.242	0.470	0.078	0.499	3.054	0.000	0.785	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	108	438	299	167	0	193	-1
N.S.	1	1.00	0.76	3.08	2.11	1.18	0.00	1.36	-0.01
time (sec)	N/A	0.194	0.385	0.072	0.485	3.044	0.000	0.690	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	95	377	273	152	0	170	-1
N.S.	1	1.00	0.81	3.19	2.31	1.29	0.00	1.44	-0.01
time (sec)	N/A	0.134	0.389	0.077	0.496	2.978	0.000	0.710	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	53	365	143	100	0	98	49
N.S.	1	1.00	0.57	3.92	1.54	1.08	0.00	1.05	0.53
time (sec)	N/A	0.078	0.318	0.058	0.287	3.630	0.000	0.687	2.690

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	50	308	119	98	0	128	46
N.S.	1	1.00	0.58	3.58	1.38	1.14	0.00	1.49	0.53
time (sec)	N/A	0.022	0.299	0.068	0.266	3.428	0.000	1.223	2.656

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	53	246	96	102	0	158	49
N.S.	1	1.00	0.51	2.39	0.93	0.99	0.00	1.53	0.48
time (sec)	N/A	0.029	0.009	0.065	0.268	2.513	0.000	0.988	2.657

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	89	305	148	153	0	195	-1
N.S.	1	1.00	0.78	2.68	1.30	1.34	0.00	1.71	-0.01
time (sec)	N/A	0.102	0.474	0.070	0.264	2.765	0.000	0.851	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	105	309	177	176	0	287	-1
N.S.	1	1.00	0.72	2.13	1.22	1.21	0.00	1.98	-0.01
time (sec)	N/A	0.182	0.434	0.084	0.280	3.413	0.000	0.721	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	118	436	201	192	0	353	-1
N.S.	1	1.00	0.65	2.40	1.10	1.05	0.00	1.94	-0.01
time (sec)	N/A	0.224	0.486	0.078	0.276	2.335	0.000	0.713	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	111	296	115	89	0	77	-1
N.S.	1	1.00	0.76	2.01	0.78	0.61	0.00	0.52	-0.01
time (sec)	N/A	0.088	0.161	0.072	0.552	2.803	0.000	0.669	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	100	243	93	78	0	66	-1
N.S.	1	1.00	0.85	2.06	0.79	0.66	0.00	0.56	-0.01
time (sec)	N/A	0.062	0.157	0.082	0.494	2.466	0.000	0.659	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	89	157	71	69	0	54	-1
N.S.	1	1.00	1.03	1.83	0.83	0.80	0.00	0.63	-0.01
time (sec)	N/A	0.071	0.134	0.066	0.474	5.373	0.000	1.368	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	77	135	52	57	0	43	-1
N.S.	1	1.00	1.24	2.18	0.84	0.92	0.00	0.69	-0.02
time (sec)	N/A	0.023	0.125	0.060	0.478	5.397	0.000	1.493	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	63	78	29	48	0	31	-1
N.S.	1	1.00	1.37	1.70	0.63	1.04	0.00	0.67	-0.02
time (sec)	N/A	0.010	0.111	0.069	0.491	4.628	0.000	2.337	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	81	141	55	51	0	48	-1
N.S.	1	1.00	1.76	3.07	1.20	1.11	0.00	1.04	-0.02
time (sec)	N/A	0.035	0.123	0.068	0.480	2.141	0.000	1.777	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	98	228	0	49	0	102	-1
N.S.	1	1.00	1.92	4.47	0.00	0.96	0.00	2.00	-0.02
time (sec)	N/A	0.035	0.170	0.061	0.000	2.338	0.000	1.334	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	75	326	0	61	0	172	-1
N.S.	1	1.00	0.91	3.98	0.00	0.74	0.00	2.10	-0.01
time (sec)	N/A	0.048	0.173	0.068	0.000	3.537	0.000	1.319	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	131	351	0	72	0	239	-1
N.S.	1	1.00	1.15	3.08	0.00	0.63	0.00	2.10	-0.01
time (sec)	N/A	0.068	0.246	0.082	0.000	4.270	0.000	1.197	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	101	477	0	82	0	299	-1
N.S.	1	1.00	0.71	3.34	0.00	0.57	0.00	2.09	-0.01
time (sec)	N/A	0.086	0.264	0.070	0.000	3.707	0.000	1.426	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	238	164	88	279	75	-1
N.S.	1	1.00	0.98	2.11	1.45	0.78	2.47	0.66	-0.01
time (sec)	N/A	0.082	0.141	0.070	0.485	4.540	3.251	1.250	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	155	502	230	129	830	119	-1
N.S.	1	1.00	0.77	2.50	1.14	0.64	4.13	0.59	-0.00
time (sec)	N/A	0.100	0.232	0.071	0.509	3.314	26.377	1.365	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	144	449	207	118	775	108	-1
N.S.	1	1.00	0.84	2.61	1.20	0.69	4.51	0.63	-0.01
time (sec)	N/A	0.076	0.227	0.059	0.487	3.271	26.006	0.910	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	133	317	186	109	653	96	-1
N.S.	1	1.00	0.95	2.26	1.33	0.78	4.66	0.69	-0.01
time (sec)	N/A	0.091	0.227	0.064	0.489	2.936	8.579	1.374	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	122	295	166	98	580	86	-1
N.S.	1	1.00	1.05	2.54	1.43	0.84	5.00	0.74	-0.01
time (sec)	N/A	0.036	0.223	0.084	0.485	2.843	8.452	1.294	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	111	192	105	89	435	74	-1
N.S.	1	1.00	1.11	1.92	1.05	0.89	4.35	0.74	-0.01
time (sec)	N/A	0.019	0.239	0.068	0.477	2.905	4.253	0.968	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	140	297	122	102	469	100	-1
N.S.	1	1.00	1.24	2.63	1.08	0.90	4.15	0.88	-0.01
time (sec)	N/A	0.073	0.343	0.067	0.480	3.346	9.619	1.188	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	143	430	130	121	386	156	-1
N.S.	1	1.00	1.24	3.74	1.13	1.05	3.36	1.36	-0.01
time (sec)	N/A	0.070	0.346	0.076	0.497	2.255	4.177	2.161	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	142	570	131	127	461	211	-1
N.S.	1	1.00	1.17	4.71	1.08	1.05	3.81	1.74	-0.01
time (sec)	N/A	0.069	0.507	0.076	0.487	1.832	4.865	2.795	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	139	735	125	122	457	255	-1
N.S.	1	1.00	1.16	6.12	1.04	1.02	3.81	2.12	-0.01
time (sec)	N/A	0.072	0.468	0.075	0.478	2.088	4.456	1.468	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	137	904	150	112	541	289	-1
N.S.	1	1.00	1.15	7.60	1.26	0.94	4.55	2.43	-0.01
time (sec)	N/A	0.072	0.510	0.079	0.477	3.063	5.810	1.266	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	151	1100	144	92	774	366	-1
N.S.	1	1.00	1.40	10.19	1.33	0.85	7.17	3.39	-0.01
time (sec)	N/A	0.058	0.492	0.069	0.474	2.575	5.619	1.079	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	123	1300	167	102	918	425	-1
N.S.	1	1.00	0.86	9.09	1.17	0.71	6.42	2.97	-0.01
time (sec)	N/A	0.076	0.523	0.076	0.479	3.841	9.832	1.112	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	134	1325	190	112	1037	492	-1
N.S.	1	1.00	0.78	7.70	1.10	0.65	6.03	2.86	-0.01
time (sec)	N/A	0.099	0.572	0.083	0.481	2.934	10.636	1.092	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	145	1555	213	122	1159	425	-1
N.S.	1	1.00	0.72	7.74	1.06	0.61	5.77	2.11	-0.00
time (sec)	N/A	0.119	0.615	0.078	0.484	3.110	28.049	1.787	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	34	28	31	29	19	20
N.S.	1	1.00	1.37	1.26	1.04	1.15	1.07	0.70	0.74
time (sec)	N/A	0.010	0.080	0.078	0.479	2.240	1.333	1.333	0.039

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	95	253	68	74	170	125	74
N.S.	1	1.00	1.86	4.96	1.33	1.45	3.33	2.45	1.45
time (sec)	N/A	0.044	0.192	0.079	0.484	1.921	2.662	1.058	0.051

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	109	215	104	108	0	92	-1
N.S.	1	1.00	0.92	1.82	0.88	0.92	0.00	0.78	-0.01
time (sec)	N/A	0.063	0.261	0.069	0.479	2.029	0.000	1.504	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	95	159	79	98	0	81	-1
N.S.	1	1.00	1.04	1.75	0.87	1.08	0.00	0.89	-0.01
time (sec)	N/A	0.042	0.212	0.090	0.493	2.242	0.000	1.052	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	97	58	83	0	69	-1
N.S.	1	1.00	1.01	1.26	0.75	1.08	0.00	0.90	-0.01
time (sec)	N/A	0.063	0.206	0.095	0.480	2.403	0.000	1.174	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	71	74	37	70	0	49	-1
N.S.	1	1.00	1.37	1.42	0.71	1.35	0.00	0.94	-0.02
time (sec)	N/A	0.014	0.165	0.072	0.480	2.774	0.000	1.337	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	46	29	35	0	38	29
N.S.	1	1.00	1.00	1.48	0.94	1.13	0.00	1.23	0.94
time (sec)	N/A	0.006	0.005	0.065	0.476	2.830	0.000	1.450	2.643

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	66	88	0	63	0	75	-1
N.S.	1	1.00	1.22	1.63	0.00	1.17	0.00	1.39	-0.02
time (sec)	N/A	0.029	0.229	0.059	0.000	2.733	0.000	0.942	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	108	0	88	0	164	-1
N.S.	1	1.00	0.95	1.33	0.00	1.09	0.00	2.02	-0.01
time (sec)	N/A	0.041	0.231	0.067	0.000	1.426	0.000	0.793	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	93	183	0	108	0	233	-1
N.S.	1	1.00	0.82	1.62	0.00	0.96	0.00	2.06	-0.01
time (sec)	N/A	0.057	0.269	0.076	0.000	1.449	0.000	1.256	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	129	350	139	179	0	0	-1
N.S.	1	1.00	1.01	2.73	1.09	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.355	0.087	0.484	1.990	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	257	114	165	0	0	-1
N.S.	1	1.00	1.01	2.27	1.01	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.321	0.078	0.485	2.335	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	102	204	91	148	0	0	-1
N.S.	1	1.00	1.15	2.29	1.02	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.287	0.066	0.504	1.634	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	149	78	97	0	0	56
N.S.	1	1.00	1.00	2.48	1.30	1.62	0.00	0.00	0.93
time (sec)	N/A	0.021	0.247	0.073	0.273	2.360	0.000	0.000	2.712

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	129	61	95	0	0	52
N.S.	1	1.00	0.97	2.22	1.05	1.64	0.00	0.00	0.90
time (sec)	N/A	0.014	0.231	0.069	0.271	1.565	0.000	0.000	2.712

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	104	62	98	0	0	56
N.S.	1	1.00	1.03	1.79	1.07	1.69	0.00	0.00	0.97
time (sec)	N/A	0.010	0.240	0.059	0.271	4.100	0.000	0.000	2.714

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	95	171	0	150	0	0	-1
N.S.	1	1.00	1.08	1.94	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.347	0.060	0.000	4.034	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	112	223	0	173	0	0	-1
N.S.	1	1.00	0.93	1.86	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.334	0.081	0.000	2.436	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	124	325	0	188	0	0	-1
N.S.	1	1.00	0.82	2.14	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.354	0.091	0.000	2.632	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	148	651	265	255	0	0	-1
N.S.	1	1.00	0.91	4.02	1.64	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.517	0.090	0.504	3.340	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	136	533	237	240	0	0	-1
N.S.	1	1.00	0.92	3.60	1.60	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.478	0.085	0.497	3.254	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	124	450	214	224	0	0	-1
N.S.	1	1.00	1.02	3.69	1.75	1.84	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.432	0.077	0.488	2.685	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	363	122	156	0	0	78
N.S.	1	1.00	0.96	4.27	1.44	1.84	0.00	0.00	0.92
time (sec)	N/A	0.048	0.342	0.065	0.275	2.149	0.000	0.000	2.952

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	311	100	159	0	0	78
N.S.	1	1.00	0.90	3.42	1.10	1.75	0.00	0.00	0.86
time (sec)	N/A	0.042	0.313	0.066	0.270	2.184	0.000	0.000	2.835

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	82	234	100	158	0	0	78
N.S.	1	1.00	0.86	2.46	1.05	1.66	0.00	0.00	0.82
time (sec)	N/A	0.031	0.341	0.056	0.273	2.012	0.000	0.000	2.788

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	212	82	159	0	0	78
N.S.	1	1.00	0.96	2.49	0.96	1.87	0.00	0.00	0.92
time (sec)	N/A	0.018	0.303	0.074	0.284	2.497	0.000	0.000	2.784

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	164	81	158	0	0	78
N.S.	1	1.00	1.00	2.00	0.99	1.93	0.00	0.00	0.95
time (sec)	N/A	0.013	0.015	0.066	0.277	2.222	0.000	0.000	2.759

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	255	0	224	0	0	-1
N.S.	1	1.00	0.99	2.14	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.462	0.059	0.000	1.939	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	133	333	0	249	0	0	-1
N.S.	1	1.00	0.86	2.16	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.461	0.086	0.000	2.324	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	146	459	0	265	0	0	-1
N.S.	1	1.00	0.78	2.47	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.528	0.113	0.000	2.459	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	157	569	0	275	0	0	-1
N.S.	1	1.00	0.73	2.65	0.00	1.28	0.00	0.00	-0.00
time (sec)	N/A	0.131	0.587	0.098	0.000	2.386	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	422	121	221	0	0	161
N.S.	1	1.00	0.88	3.58	1.03	1.87	0.00	0.00	1.36
time (sec)	N/A	0.047	0.394	0.069	0.282	2.325	0.000	0.000	2.948

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	104	319	121	220	0	0	161
N.S.	1	1.00	0.85	2.59	0.98	1.79	0.00	0.00	1.31
time (sec)	N/A	0.039	0.408	0.069	0.279	2.292	0.000	0.000	2.883

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	83	134	68	75	0	0	116
N.S.	1	1.00	1.26	2.03	1.03	1.14	0.00	0.00	1.76
time (sec)	N/A	0.034	0.188	0.074	0.487	1.990	0.000	0.000	0.071

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	71	84	52	66	0	70	84
N.S.	1	1.00	1.29	1.53	0.95	1.20	0.00	1.27	1.53
time (sec)	N/A	0.054	0.184	0.061	0.484	2.541	0.000	0.819	0.067

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	67	65	33	58	0	52	57
N.S.	1	1.00	1.97	1.91	0.97	1.71	0.00	1.53	1.68
time (sec)	N/A	0.011	0.130	0.055	0.499	2.405	0.000	0.785	2.605

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	36	23	28	0	34	23
N.S.	1	1.00	1.00	1.38	0.88	1.08	0.00	1.31	0.88
time (sec)	N/A	0.006	0.115	0.065	0.482	2.244	0.000	0.930	2.588

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	58	0	52	0	74	58
N.S.	1	1.00	1.34	1.41	0.00	1.27	0.00	1.80	1.41
time (sec)	N/A	0.026	0.164	0.063	0.000	2.232	0.000	1.016	2.652

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	73	0	76	0	0	81
N.S.	1	1.00	1.00	1.14	0.00	1.19	0.00	0.00	1.27
time (sec)	N/A	0.035	0.159	0.070	0.000	2.475	0.000	0.000	2.592

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	94	0	97	0	213	105
N.S.	1	1.00	0.84	1.04	0.00	1.08	0.00	2.37	1.17
time (sec)	N/A	0.048	0.189	0.071	0.000	1.673	0.000	0.758	2.611

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	155	750	278	128	571	332	-1
N.S.	1	1.00	0.68	3.28	1.21	0.56	2.49	1.45	-0.00
time (sec)	N/A	0.203	0.274	0.088	0.526	1.964	21.925	0.739	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	144	696	256	119	690	301	-1
N.S.	1	1.00	0.72	3.48	1.28	0.60	3.45	1.50	-0.00
time (sec)	N/A	0.173	0.250	0.087	0.544	2.420	26.660	0.738	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	133	565	234	108	450	270	-1
N.S.	1	1.00	0.78	3.30	1.37	0.63	2.63	1.58	-0.01
time (sec)	N/A	0.136	0.257	0.067	0.561	2.865	7.317	0.727	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	122	541	215	99	541	239	-1
N.S.	1	1.00	0.86	3.81	1.51	0.70	3.81	1.68	-0.01
time (sec)	N/A	0.111	0.217	0.065	0.499	2.227	8.775	0.716	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	111	438	156	88	321	208	-1
N.S.	1	1.00	0.82	3.22	1.15	0.65	2.36	1.53	-0.01
time (sec)	N/A	0.036	0.180	0.066	0.501	2.130	4.010	1.324	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	100	244	115	79	350	177	-1
N.S.	1	1.00	0.93	2.26	1.06	0.73	3.24	1.64	-0.01
time (sec)	N/A	0.026	0.215	0.058	0.481	2.336	4.465	1.629	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	125	544	101	91	267	0	-1
N.S.	1	1.00	1.30	5.67	1.05	0.95	2.78	0.00	-0.01
time (sec)	N/A	0.099	0.308	0.060	0.497	2.533	6.097	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	128	674	111	110	347	0	-1
N.S.	1	1.00	1.22	6.42	1.06	1.05	3.30	0.00	-0.01
time (sec)	N/A	0.102	0.313	0.079	0.493	2.650	4.462	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	124	816	106	112	347	0	-1
N.S.	1	1.00	1.13	7.42	0.96	1.02	3.15	0.00	-0.01
time (sec)	N/A	0.101	0.325	0.069	0.479	2.760	4.620	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	121	983	127	100	338	0	-1
N.S.	1	1.00	1.19	9.64	1.25	0.98	3.31	0.00	-0.01
time (sec)	N/A	0.105	0.345	0.069	0.512	3.369	4.179	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	140	1153	123	82	422	237	-1
N.S.	1	1.00	1.30	10.68	1.14	0.76	3.91	2.19	-0.01
time (sec)	N/A	0.094	0.407	0.079	0.489	3.423	5.636	1.502	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	112	1349	146	92	660	265	-1
N.S.	1	1.00	0.80	9.64	1.04	0.66	4.71	1.89	-0.01
time (sec)	N/A	0.107	0.404	0.073	0.477	4.023	5.399	1.353	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	123	1550	169	102	808	293	-1
N.S.	1	1.00	0.73	9.17	1.00	0.60	4.78	1.73	-0.01
time (sec)	N/A	0.127	0.424	0.073	0.484	3.372	10.190	1.500	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	134	1575	192	112	835	321	-1
N.S.	1	1.00	0.68	7.95	0.97	0.57	4.22	1.62	-0.01
time (sec)	N/A	0.149	0.503	0.079	0.495	4.225	9.387	1.104	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	113	363	156	161	0	0	-1
N.S.	1	1.00	0.92	2.95	1.27	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.414	0.076	0.493	3.633	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	309	143	109	0	171	66
N.S.	1	1.00	0.71	3.12	1.44	1.10	0.00	1.73	0.67
time (sec)	N/A	0.125	0.329	0.062	0.277	3.231	0.000	1.248	2.970

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	287	124	111	0	171	66
N.S.	1	1.00	0.79	3.22	1.39	1.25	0.00	1.92	0.74
time (sec)	N/A	0.089	0.343	0.077	0.287	1.802	0.000	1.053	2.896

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	69	262	126	109	0	160	65
N.S.	1	1.00	0.76	2.88	1.38	1.20	0.00	1.76	0.71
time (sec)	N/A	0.023	0.298	0.073	0.284	2.354	0.000	1.032	2.879

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	70	156	129	110	0	173	66
N.S.	1	1.00	0.77	1.71	1.42	1.21	0.00	1.90	0.73
time (sec)	N/A	0.020	0.331	0.065	0.274	1.682	0.000	1.099	2.847

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	107	330	0	162	0	254	-1
N.S.	1	1.00	0.91	2.80	0.00	1.37	0.00	2.15	-0.01
time (sec)	N/A	0.114	0.467	0.072	0.000	2.781	0.000	1.387	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	123	379	0	185	0	297	-1
N.S.	1	1.00	0.84	2.60	0.00	1.27	0.00	2.03	-0.01
time (sec)	N/A	0.187	0.464	0.086	0.000	1.959	0.000	3.170	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	136	483	0	201	0	330	-1
N.S.	1	1.00	0.74	2.64	0.00	1.10	0.00	1.80	-0.01
time (sec)	N/A	0.240	0.493	0.084	0.000	1.821	0.000	2.412	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	118	406	169	179	0	214	-1
N.S.	1	1.00	0.67	2.29	0.95	1.01	0.00	1.21	-0.01
time (sec)	N/A	0.274	0.443	0.087	0.480	2.506	0.000	3.127	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	106	340	146	164	0	194	-1
N.S.	1	1.00	0.73	2.33	1.00	1.12	0.00	1.33	-0.01
time (sec)	N/A	0.223	0.403	0.089	0.486	2.742	0.000	2.121	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	93	319	124	148	0	169	-1
N.S.	1	1.00	0.78	2.66	1.03	1.23	0.00	1.41	-0.01
time (sec)	N/A	0.163	0.351	0.057	0.486	2.361	0.000	1.944	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	52	288	113	98	0	98	48
N.S.	1	1.00	0.55	3.03	1.19	1.03	0.00	1.03	0.51
time (sec)	N/A	0.078	0.301	0.077	0.499	2.626	0.000	1.888	2.762

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	49	240	117	94	0	128	45
N.S.	1	1.00	0.51	2.47	1.21	0.97	0.00	1.32	0.46
time (sec)	N/A	0.027	0.277	0.066	0.472	2.845	0.000	1.903	2.590

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	145	122	100	0	158	48
N.S.	1	1.00	0.52	1.45	1.22	1.00	0.00	1.58	0.48
time (sec)	N/A	0.025	0.007	0.066	0.485	2.891	0.000	2.726	2.619

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	332	0	148	0	195	-1
N.S.	1	1.00	0.77	2.89	0.00	1.29	0.00	1.70	-0.01
time (sec)	N/A	0.112	0.452	0.067	0.000	2.669	0.000	1.827	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	103	349	0	173	0	287	-1
N.S.	1	1.00	0.71	2.39	0.00	1.18	0.00	1.97	-0.01
time (sec)	N/A	0.188	0.433	0.103	0.000	2.438	0.000	2.854	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	116	424	0	189	0	353	-1
N.S.	1	1.00	0.63	2.32	0.00	1.03	0.00	1.93	-0.01
time (sec)	N/A	0.235	0.480	0.087	0.000	2.223	0.000	1.506	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	130	432	0	188	0	224	-1
N.S.	1	1.00	0.64	2.12	0.00	0.92	0.00	1.10	-0.00
time (sec)	N/A	0.364	0.415	0.079	0.000	2.101	0.000	1.636	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	121	408	0	179	0	214	-1
N.S.	1	1.00	0.76	2.55	0.00	1.12	0.00	1.34	-0.01
time (sec)	N/A	0.261	0.408	0.073	0.000	1.811	0.000	1.452	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	106	349	0	164	0	193	-1
N.S.	1	1.00	0.72	2.36	0.00	1.11	0.00	1.30	-0.01
time (sec)	N/A	0.155	0.353	0.085	0.000	1.943	0.000	1.507	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	91	268	0	148	0	170	-1
N.S.	1	1.00	0.79	2.33	0.00	1.29	0.00	1.48	-0.01
time (sec)	N/A	0.095	0.350	0.084	0.000	1.638	0.000	1.046	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	52	141	113	96	0	128	46
N.S.	1	1.00	0.81	2.20	1.77	1.50	0.00	2.00	0.72
time (sec)	N/A	0.018	0.272	0.060	0.418	1.830	0.000	1.202	2.904

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	93	117	100	0	158	47
N.S.	1	1.00	0.76	1.39	1.75	1.49	0.00	2.36	0.70
time (sec)	N/A	0.014	0.368	0.056	0.364	1.776	0.000	1.549	2.776

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	88	415	0	148	0	195	-1
N.S.	1	1.00	0.80	3.77	0.00	1.35	0.00	1.77	-0.01
time (sec)	N/A	0.141	0.503	0.071	0.000	1.866	0.000	1.654	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	103	499	0	173	0	287	-1
N.S.	1	1.00	0.72	3.49	0.00	1.21	0.00	2.01	-0.01
time (sec)	N/A	0.191	0.511	0.086	0.000	2.462	0.000	2.086	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	116	598	0	189	0	353	-1
N.S.	1	1.00	0.63	3.27	0.00	1.03	0.00	1.93	-0.01
time (sec)	N/A	0.242	0.539	0.075	0.000	2.338	0.000	1.493	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	127	620	0	199	0	420	-1
N.S.	1	1.00	0.60	2.95	0.00	0.95	0.00	2.00	-0.00
time (sec)	N/A	0.309	0.619	0.089	0.000	2.420	0.000	1.456	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	151	1213	440	148	0	135	-1
N.S.	1	1.00	0.60	4.81	1.75	0.59	0.00	0.54	-0.00
time (sec)	N/A	0.418	0.365	0.079	0.536	1.710	0.000	1.380	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	143	1189	420	139	0	124	-1
N.S.	1	1.00	0.64	5.31	1.88	0.62	0.00	0.55	-0.00
time (sec)	N/A	0.330	0.397	0.073	0.524	1.514	0.000	4.654	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	129	1086	375	128	0	113	-1
N.S.	1	1.00	0.67	5.66	1.95	0.67	0.00	0.59	-0.01
time (sec)	N/A	0.264	0.319	0.075	0.525	2.088	0.000	1.686	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	121	890	335	119	0	102	-1
N.S.	1	1.00	0.66	4.89	1.84	0.65	0.00	0.56	-0.01
time (sec)	N/A	0.134	0.358	0.071	0.569	1.859	0.000	1.993	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	107	644	214	107	0	91	-1
N.S.	1	1.00	0.82	4.95	1.65	0.82	0.00	0.70	-0.01
time (sec)	N/A	0.039	0.293	0.076	0.514	1.857	0.000	1.499	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	347	128	98	0	80	-1
N.S.	1	1.00	0.84	3.07	1.13	0.87	0.00	0.71	-0.01
time (sec)	N/A	0.029	0.326	0.079	0.517	2.653	0.000	1.904	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	115	1192	0	112	0	97	-1
N.S.	1	1.00	1.29	13.39	0.00	1.26	0.00	1.09	-0.01
time (sec)	N/A	0.130	0.399	0.079	0.000	2.131	0.000	1.785	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	114	1322	0	125	0	170	-1
N.S.	1	1.00	1.21	14.06	0.00	1.33	0.00	1.81	-0.01
time (sec)	N/A	0.204	0.381	0.086	0.000	2.494	0.000	1.730	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	130	1461	0	107	0	229	-1
N.S.	1	1.00	1.18	13.28	0.00	0.97	0.00	2.08	-0.01
time (sec)	N/A	0.262	0.418	0.072	0.000	2.517	0.000	1.432	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	103	1626	0	117	0	300	-1
N.S.	1	1.00	0.75	11.87	0.00	0.85	0.00	2.19	-0.01
time (sec)	N/A	0.396	0.463	0.102	0.000	3.462	0.000	1.335	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	116	1799	0	129	0	360	-1
N.S.	1	1.00	0.68	10.58	0.00	0.76	0.00	2.12	-0.01
time (sec)	N/A	0.502	0.472	0.089	0.000	2.028	0.000	1.880	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	127	1997	0	139	0	426	-1
N.S.	1	1.00	0.65	10.19	0.00	0.71	0.00	2.17	-0.01
time (sec)	N/A	0.477	0.578	0.107	0.000	2.276	0.000	1.278	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	82	245	0	126	0	0	220
N.S.	1	1.00	0.86	2.58	0.00	1.33	0.00	0.00	2.32
time (sec)	N/A	0.092	0.358	0.068	0.000	1.605	0.000	0.000	2.703

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	50	258	153	102	0	0	287
N.S.	1	1.00	0.57	2.93	1.74	1.16	0.00	0.00	3.26
time (sec)	N/A	0.078	0.416	0.073	0.292	1.835	0.000	0.000	0.061

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	137	1212	365	291	0	0	252
N.S.	1	1.00	0.66	5.80	1.75	1.39	0.00	0.00	1.21
time (sec)	N/A	0.198	0.618	0.071	0.307	4.409	0.000	0.000	3.217

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	137	985	367	292	0	0	252
N.S.	1	1.00	0.66	4.71	1.76	1.40	0.00	0.00	1.21
time (sec)	N/A	0.130	0.592	0.068	0.314	2.226	0.000	0.000	3.189

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	137	708	371	291	0	0	252
N.S.	1	1.00	0.65	3.36	1.76	1.38	0.00	0.00	1.19
time (sec)	N/A	0.068	0.582	0.068	0.308	2.856	0.000	0.000	3.194

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	137	379	370	291	0	0	242
N.S.	1	1.00	0.67	1.85	1.80	1.42	0.00	0.00	1.18
time (sec)	N/A	0.061	0.018	0.067	0.319	5.288	0.000	0.000	3.118

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	173	1328	0	402	0	0	-1
N.S.	1	1.00	0.74	5.68	0.00	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.957	0.110	0.000	5.623	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	193	1429	0	425	0	0	-1
N.S.	1	1.00	0.71	5.27	0.00	1.57	0.00	0.00	-0.00
time (sec)	N/A	0.429	0.932	0.158	0.000	3.670	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	81	95	0	217	0	0	-1
N.S.	1	1.00	0.79	0.93	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.303	0.087	0.000	2.105	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	58	0	110	0	57	-1
N.S.	1	1.00	1.21	1.49	0.00	2.82	0.00	1.46	-0.03
time (sec)	N/A	0.025	0.218	0.067	0.000	2.390	0.000	1.380	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	52	62	48	92	82	0	38
N.S.	1	1.00	1.49	1.77	1.37	2.63	2.34	0.00	1.09
time (sec)	N/A	0.006	0.063	0.086	0.510	2.171	0.793	0.000	2.985

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	76	0	199	0	0	-1
N.S.	1	1.00	1.00	2.17	0.00	5.69	0.00	0.00	-0.03
time (sec)	N/A	0.018	1.509	0.071	0.000	2.705	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	46	57	68	90	29	0	36
N.S.	1	1.00	1.35	1.68	2.00	2.65	0.85	0.00	1.06
time (sec)	N/A	0.005	0.048	0.078	0.494	3.078	0.798	0.000	3.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	86	0	208	0	0	-1
N.S.	1	1.00	1.00	2.53	0.00	6.12	0.00	0.00	-0.03
time (sec)	N/A	0.018	1.546	0.077	0.000	2.569	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	64	84	82	111	148	0	54
N.S.	1	1.00	1.02	1.33	1.30	1.76	2.35	0.00	0.86
time (sec)	N/A	0.010	0.082	0.075	0.537	1.966	1.773	0.000	2.597

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	92	0	221	0	0	-1
N.S.	1	1.00	0.78	1.46	0.00	3.51	0.00	0.00	-0.02
time (sec)	N/A	0.021	1.469	0.073	0.000	2.479	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	199	0	0	0	513	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	2.05	0.00	-0.00
time (sec)	N/A	0.248	0.852	0.010	0.000	0.000	21.186	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	174	0	0	0	442	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	2.15	0.00	-0.00
time (sec)	N/A	0.135	0.739	0.010	0.000	0.000	15.518	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	121	0	0	0	374	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	2.31	0.00	-0.01
time (sec)	N/A	0.056	0.630	0.007	0.000	0.000	11.341	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	61	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.76	0.00	-0.01
time (sec)	N/A	0.016	0.381	0.009	0.000	0.000	5.180	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	248	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	1.52	0.00	-0.01
time (sec)	N/A	0.090	0.631	0.012	0.000	0.000	10.842	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	173	0	0	0	185	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.91	0.00	-0.00
time (sec)	N/A	0.136	0.640	0.010	0.000	0.000	33.842	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	245	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.716	0.011	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	199	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	1.081	0.010	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	174	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	0.884	0.009	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	162	121	0	0	0	117	0	-1
N.S.	1	1.31	0.98	0.00	0.00	0.00	0.94	0.00	-0.01
time (sec)	N/A	0.056	0.789	0.007	0.000	0.000	26.867	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	60	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.016	0.551	0.007	0.000	0.000	5.499	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	122	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	1.067	0.012	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	176	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	1.254	0.012	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	200	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	1.507	0.010	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	972	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	6.57	0.00	-0.01
time (sec)	N/A	0.066	0.223	0.019	0.000	0.000	2.356	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	129	0	0	0	972	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	6.61	0.00	-0.01
time (sec)	N/A	0.063	0.201	0.016	0.000	0.000	2.170	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	0	0	0	382	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	3.18	0.00	-0.01
time (sec)	N/A	0.047	0.186	0.014	0.000	0.000	1.840	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	103	0	0	0	382	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.21	0.00	-0.01
time (sec)	N/A	0.042	0.179	0.013	0.000	0.000	1.705	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	85	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.96	0.00	-0.01
time (sec)	N/A	0.021	0.134	0.013	0.000	0.000	1.451	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	0	82	0	78
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.99	0.00	0.94
time (sec)	N/A	0.015	0.148	0.012	0.000	0.000	1.477	0.000	4.345

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	78	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.035	0.149	0.007	0.000	0.000	3.610	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	0	0	0	82	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	-0.01
time (sec)	N/A	0.037	0.177	0.008	0.000	0.000	2.007	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	106	0	0	0	85	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.77	0.00	-0.01
time (sec)	N/A	0.039	0.181	0.013	0.000	0.000	2.073	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	159	0	0	0	2924	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	16.43	0.00	-0.01
time (sec)	N/A	0.099	0.276	0.022	0.000	0.000	3.565	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	186	0	0	0	1015	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	5.49	0.00	-0.01
time (sec)	N/A	0.106	0.294	0.019	0.000	0.000	2.906	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	138	0	0	0	1328	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	8.91	0.00	-0.01
time (sec)	N/A	0.086	0.236	0.018	0.000	0.000	2.617	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	168	0	0	0	425	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	2.74	0.00	-0.01
time (sec)	N/A	0.091	0.257	0.017	0.000	0.000	2.325	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	110	0	0	0	440	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	3.73	0.00	-0.01
time (sec)	N/A	0.061	0.178	0.015	0.000	0.000	1.969	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	134	0	0	0	124	0	-1
N.S.	1	1.00	1.89	0.00	0.00	0.00	1.75	0.00	-0.01
time (sec)	N/A	0.021	0.206	0.014	0.000	0.000	1.802	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	103	0	0	0	136	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	1.06	0.00	-0.01
time (sec)	N/A	0.060	0.187	0.011	0.000	0.000	3.610	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	153	0	0	0	116	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.91	0.00	-0.01
time (sec)	N/A	0.075	0.241	0.010	0.000	0.000	2.506	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	131	0	0	0	139	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.081	0.223	0.017	0.000	0.000	2.796	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	205	0	0	0	2966	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	13.36	0.00	-0.00
time (sec)	N/A	0.123	0.429	0.012	0.000	0.000	4.577	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	219	0	0	0	2966	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	13.61	0.00	-0.00
time (sec)	N/A	0.120	0.414	0.023	0.000	0.000	4.171	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	187	0	0	0	1370	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	7.10	0.00	-0.01
time (sec)	N/A	0.110	0.344	0.021	0.000	0.000	3.409	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	187	0	0	0	1370	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	7.25	0.00	-0.01
time (sec)	N/A	0.104	0.333	0.019	0.000	0.000	3.144	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	159	0	0	0	479	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	4.13	0.00	-0.01
time (sec)	N/A	0.048	0.360	0.021	0.000	0.000	2.582	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	155	0	0	0	476	0	-1
N.S.	1	1.00	2.12	0.00	0.00	0.00	6.52	0.00	-0.01
time (sec)	N/A	0.019	0.292	0.018	0.000	0.000	2.341	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	169	0	0	0	178	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.04	0.00	-0.01
time (sec)	N/A	0.081	0.287	0.016	0.000	0.000	4.668	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	158	0	0	0	177	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.11	0.00	-0.01
time (sec)	N/A	0.116	0.276	0.013	0.000	0.000	2.869	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	182	0	0	0	177	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	1.07	0.00	-0.01
time (sec)	N/A	0.137	0.290	0.016	0.000	0.000	3.335	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	66	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.273	0.027	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	245	0	0	0	5090	0	-1
N.S.	1	1.00	2.02	0.00	0.00	0.00	42.07	0.00	-0.01
time (sec)	N/A	0.059	0.395	0.023	0.000	0.000	65.345	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	198	0	0	0	4124	0	-1
N.S.	1	1.00	1.66	0.00	0.00	0.00	34.66	0.00	-0.01
time (sec)	N/A	0.056	0.336	0.018	0.000	0.000	12.623	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	147	0	0	0	427	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	4.74	0.00	-0.01
time (sec)	N/A	0.038	0.217	0.018	0.000	0.000	4.241	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	321	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	4.40	0.00	-0.01
time (sec)	N/A	0.023	0.173	0.018	0.000	0.000	2.865	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	151	0	0	0	355	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	3.41	0.00	-0.01
time (sec)	N/A	0.045	0.236	0.013	0.000	0.000	3.036	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	167	0	0	0	450	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	4.25	0.00	-0.01
time (sec)	N/A	0.052	0.305	0.018	0.000	0.000	4.552	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	219	0	0	0	498	0	-1
N.S.	1	1.00	2.03	0.00	0.00	0.00	4.61	0.00	-0.01
time (sec)	N/A	0.053	0.586	0.023	0.000	0.000	13.659	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	66	0	0	0	0	0	-1
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.340	0.029	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	66	0	0	0	0	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.323	0.029	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	332	0	0	0	0	0	-1
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.400	0.023	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	177	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.315	0.021	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	102	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.212	0.021	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.205	0.019	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	201	0	0	0	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.284	0.012	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	223	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.458	0.019	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	283	0	0	0	0	0	-1
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.532	0.023	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	334	0	0	0	0	0	-1
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.543	0.028	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	389	0	0	0	0	0	-1
N.S.	1	1.00	2.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.603	0.033	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	245	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.498	0.030	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	202	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.417	0.029	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	130	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.301	0.030	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.249	0.026	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.227	0.026	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	328	0	0	0	0	0	-1
N.S.	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.378	0.013	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	280	0	0	0	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.531	0.028	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	341	0	0	0	0	0	-1
N.S.	1	1.00	1.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.694	0.030	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	393	0	0	0	0	0	-1
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.639	0.037	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	446	0	0	0	0	0	-1
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.733	0.012	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	231	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.539	0.036	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	156	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.420	0.036	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	130	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.356	0.037	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.288	0.034	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.254	0.034	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	417	0	0	0	0	0	-1
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.445	0.013	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	337	0	0	0	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.596	0.035	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	399	0	0	0	0	0	-1
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	0.848	0.037	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	452	0	0	0	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.803	0.012	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	505	0	0	0	0	0	-1
N.S.	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.263	0.914	0.011	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	194	0	0	0	262	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.99	0.00	-0.00
time (sec)	N/A	0.242	0.210	0.013	0.000	0.000	11.271	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	169	0	0	0	192	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.93	0.00	-0.00
time (sec)	N/A	0.110	0.146	0.013	0.000	0.000	7.162	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	116	0	0	0	122	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.80	0.00	-0.01
time (sec)	N/A	0.045	0.102	0.010	0.000	0.000	4.289	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	61	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.013	0.067	0.024	0.000	0.000	1.360	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	124	0	0	0	337	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	2.07	0.00	-0.01
time (sec)	N/A	0.090	0.125	0.015	0.000	0.000	5.040	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	180	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.162	0.015	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	206	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.222	0.014	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	308	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.46	0.00	-0.01
time (sec)	N/A	0.036	0.114	0.014	0.000	0.000	4.449	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	90	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.153	0.027	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	68	167	0	307	68	167	201
N.S.	1	1.00	0.32	0.78	0.00	1.43	0.32	0.78	0.94
time (sec)	N/A	0.163	0.083	0.513	0.000	2.716	4.066	1.191	0.111

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	230	428	240	1045	0	252	-1
N.S.	1	1.00	0.90	1.68	0.94	4.10	0.00	0.99	-0.00
time (sec)	N/A	0.386	0.625	0.089	0.339	13.042	0.000	1.155	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	199	387	200	920	0	201	-1
N.S.	1	1.00	0.94	1.83	0.95	4.36	0.00	0.95	-0.00
time (sec)	N/A	0.244	0.476	0.089	0.316	10.539	0.000	0.982	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	161	323	140	745	0	157	-1
N.S.	1	1.00	1.05	2.11	0.92	4.87	0.00	1.03	-0.01
time (sec)	N/A	0.134	0.404	0.082	0.301	3.971	0.000	0.856	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	133	303	119	669	0	135	-1
N.S.	1	1.00	1.05	2.39	0.94	5.27	0.00	1.06	-0.01
time (sec)	N/A	0.067	0.322	0.080	0.305	3.948	0.000	1.919	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	110	261	83	565	0	109	-1
N.S.	1	1.00	1.07	2.53	0.81	5.49	0.00	1.06	-0.01
time (sec)	N/A	0.044	0.018	0.069	0.286	3.440	0.000	1.805	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	137	305	103	1299	0	0	-1
N.S.	1	1.00	1.18	2.63	0.89	11.20	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.232	0.075	0.296	3.513	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	124	370	0	596	0	145	-1
N.S.	1	1.00	1.18	3.52	0.00	5.68	0.00	1.38	-0.01
time (sec)	N/A	0.109	0.261	0.102	0.000	2.618	0.000	1.314	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	144	442	0	737	0	230	-1
N.S.	1	1.00	0.90	2.76	0.00	4.61	0.00	1.44	-0.01
time (sec)	N/A	0.138	0.441	0.092	0.000	2.605	0.000	1.011	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	173	465	0	829	0	309	-1
N.S.	1	1.00	0.91	2.43	0.00	4.34	0.00	1.62	-0.01
time (sec)	N/A	0.153	0.656	0.075	0.000	2.396	0.000	1.390	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	215	558	0	1016	0	596	-1
N.S.	1	1.00	0.78	2.04	0.00	3.71	0.00	2.18	-0.00
time (sec)	N/A	0.200	0.799	0.086	0.000	2.371	0.000	1.533	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	161	258	166	1021	0	163	-1
N.S.	1	1.00	0.83	1.32	0.85	5.24	0.00	0.84	-0.01
time (sec)	N/A	0.312	0.551	0.077	0.306	6.808	0.000	1.417	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	137	216	127	881	0	129	-1
N.S.	1	1.00	0.90	1.42	0.84	5.80	0.00	0.85	-0.01
time (sec)	N/A	0.183	0.403	0.098	0.298	6.429	0.000	1.195	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	116	172	89	721	0	105	-1
N.S.	1	1.00	1.06	1.58	0.82	6.61	0.00	0.96	-0.01
time (sec)	N/A	0.091	0.336	0.073	0.290	2.620	0.000	1.538	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	98	151	71	620	0	88	-1
N.S.	1	1.00	1.14	1.76	0.83	7.21	0.00	1.02	-0.01
time (sec)	N/A	0.031	0.253	0.062	0.307	2.706	0.000	1.106	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	64	127	53	205	0	59	-1
N.S.	1	1.00	1.19	2.35	0.98	3.80	0.00	1.09	-0.02
time (sec)	N/A	0.011	0.005	0.071	0.283	3.177	0.000	1.477	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	104	158	0	625	0	0	-1
N.S.	1	1.00	1.21	1.84	0.00	7.27	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.254	0.079	0.000	3.749	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	128	180	0	747	0	142	-1
N.S.	1	1.00	1.15	1.62	0.00	6.73	0.00	1.28	-0.01
time (sec)	N/A	0.069	0.345	0.066	0.000	3.022	0.000	1.447	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	198	235	0	969	0	239	-1
N.S.	1	1.00	1.18	1.40	0.00	5.77	0.00	1.42	-0.01
time (sec)	N/A	0.099	0.508	0.080	0.000	3.568	0.000	1.979	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	198	438	242	1493	0	299	-1
N.S.	1	1.00	1.36	3.00	1.66	10.23	0.00	2.05	-0.01
time (sec)	N/A	0.200	0.721	0.096	0.344	9.197	0.000	1.054	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	137	397	204	1304	0	219	-1
N.S.	1	1.00	1.11	3.23	1.66	10.60	0.00	1.78	-0.01
time (sec)	N/A	0.106	0.569	0.069	0.327	9.955	0.000	0.861	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	106	355	168	445	0	174	-1
N.S.	1	1.00	1.12	3.74	1.77	4.68	0.00	1.83	-0.01
time (sec)	N/A	0.073	0.437	0.074	0.313	2.385	0.000	1.263	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	99	335	147	416	0	162	-1
N.S.	1	1.00	1.12	3.81	1.67	4.73	0.00	1.84	-0.01
time (sec)	N/A	0.033	0.403	0.076	0.300	2.744	0.000	0.791	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	104	315	123	432	0	172	-1
N.S.	1	1.00	1.11	3.35	1.31	4.60	0.00	1.83	-0.01
time (sec)	N/A	0.031	0.050	0.068	0.294	3.017	0.000	1.180	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	144	363	0	1284	0	0	-1
N.S.	1	1.00	0.98	2.47	0.00	8.73	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.751	0.062	0.000	2.934	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	178	403	0	1527	0	266	-1
N.S.	1	1.00	0.92	2.08	0.00	7.87	0.00	1.37	-0.01
time (sec)	N/A	0.114	0.714	0.102	0.000	3.823	0.000	1.690	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	217	479	0	1961	0	358	-1
N.S.	1	1.00	0.79	1.74	0.00	7.11	0.00	1.30	-0.00
time (sec)	N/A	0.152	0.812	0.098	0.000	3.321	0.000	1.778	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	251	477	266	1960	0	0	-1
N.S.	1	1.00	1.03	1.95	1.09	8.03	0.00	0.00	-0.00
time (sec)	N/A	0.562	1.244	0.100	0.328	69.122	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	210	435	227	1723	0	0	-1
N.S.	1	1.00	1.03	2.13	1.11	8.45	0.00	0.00	-0.00
time (sec)	N/A	0.335	1.029	0.088	0.315	78.125	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	173	390	189	1376	0	0	-1
N.S.	1	1.00	1.08	2.44	1.18	8.60	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.861	0.079	0.308	11.063	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	146	369	170	1208	0	0	-1
N.S.	1	1.00	1.07	2.69	1.24	8.82	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.687	0.072	0.306	10.625	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	100	344	149	368	0	0	-1
N.S.	1	1.00	1.11	3.82	1.66	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.439	0.067	0.330	1.852	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	101	215	93	373	0	0	-1
N.S.	1	1.00	1.11	2.36	1.02	4.10	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.040	0.089	0.306	1.693	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	153	376	0	1213	0	600	-1
N.S.	1	1.00	0.85	2.10	0.00	6.78	0.00	3.35	-0.01
time (sec)	N/A	0.097	0.699	0.073	0.000	3.190	0.000	1.234	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	183	395	0	1479	0	0	-1
N.S.	1	1.00	0.86	1.86	0.00	6.98	0.00	0.00	-0.00
time (sec)	N/A	0.122	1.015	0.080	0.000	4.176	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	224	453	0	1910	0	0	-1
N.S.	1	1.00	0.84	1.69	0.00	7.13	0.00	0.00	-0.00
time (sec)	N/A	0.157	1.227	0.114	0.000	5.028	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	114	320	210	368	4134	624	363
N.S.	1	1.00	0.84	2.37	1.56	2.73	30.62	4.62	2.69
time (sec)	N/A	0.057	0.100	0.080	0.283	2.010	1.333	1.855	2.816

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	195	146	250	2181	410	255
N.S.	1	1.00	1.07	1.91	1.43	2.45	21.38	4.02	2.50
time (sec)	N/A	0.036	0.096	0.056	0.292	2.330	0.826	2.295	2.701

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	100	89	148	952	237	163
N.S.	1	1.00	0.93	1.43	1.27	2.11	13.60	3.39	2.33
time (sec)	N/A	0.022	0.060	0.071	0.295	2.358	0.481	1.394	2.629

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	0	0	0	345	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	4.48	0.00	-0.01
time (sec)	N/A	0.033	0.063	0.009	0.000	0.000	2.584	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	199	754	447	1027	14317	1750	932
N.S.	1	1.00	0.86	3.25	1.93	4.43	61.71	7.54	4.02
time (sec)	N/A	0.096	0.171	0.084	0.292	2.202	4.610	3.517	3.119

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	279	601	335	757	8940	1266	723
N.S.	1	1.00	1.51	3.25	1.81	4.09	48.32	6.84	3.91
time (sec)	N/A	0.069	0.193	0.080	0.288	2.025	2.670	2.763	3.051

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	184	420	235	519	5097	851	496
N.S.	1	1.00	1.31	3.00	1.68	3.71	36.41	6.08	3.54
time (sec)	N/A	0.048	0.117	0.066	0.284	2.129	1.521	1.790	2.837

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	132	0	0	0	1678	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	11.34	0.00	-0.01
time (sec)	N/A	0.140	0.138	0.011	0.000	0.000	3.710	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	302	2232	795	2165	35984	3713	1796
N.S.	1	1.00	0.88	6.51	2.32	6.31	104.91	10.83	5.24
time (sec)	N/A	0.143	0.226	0.095	0.319	1.941	12.136	1.242	3.809

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	578	1214	625	1675	24687	2851	1459
N.S.	1	1.00	2.05	4.30	2.22	5.94	87.54	10.11	5.17
time (sec)	N/A	0.114	0.379	0.094	0.300	3.308	7.109	1.600	3.478

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	413	958	472	1244	15990	2085	1144
N.S.	1	1.00	1.85	4.30	2.12	5.58	71.70	9.35	5.13
time (sec)	N/A	0.083	0.238	0.091	0.305	2.322	4.452	1.252	3.164

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	226	0	0	0	5692	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	23.14	0.00	-0.00
time (sec)	N/A	0.226	0.190	0.013	0.000	0.000	5.667	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	217	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.411	0.027	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	168	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.204	0.023	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	170	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.144	0.022	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	151	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.083	0.019	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	145	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.087	0.018	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	189	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.125	0.188	0.013	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	167	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.152	0.229	0.021	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	413	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.669	0.028	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	247	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.710	0.028	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	403	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.585	0.027	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	230	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.286	0.026	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	253	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	0.303	0.024	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	391	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	0.682	0.013	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	437	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.487	0.530	0.026	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	377	275	0	0	0	131	0	-1
N.S.	1	0.94	0.69	0.00	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.501	0.128	0.016	0.000	0.000	18.965	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	150	113	0	0	0	82	0	-1
N.S.	1	0.91	0.69	0.00	0.00	0.00	0.50	0.00	-0.01
time (sec)	N/A	0.090	0.070	0.010	0.000	0.000	7.101	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.106	0.015	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.178	0.015	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	112	0	0	0	950	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	7.60	0.00	-0.01
time (sec)	N/A	0.062	0.140	0.018	0.000	0.000	13.623	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	112	0	0	0	950	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	7.60	0.00	-0.01
time (sec)	N/A	0.061	0.121	0.014	0.000	0.000	9.272	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	0	364	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.64	0.00	-0.01
time (sec)	N/A	0.041	0.118	0.015	0.000	0.000	7.512	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	0	364	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.64	0.00	-0.01
time (sec)	N/A	0.042	0.116	0.012	0.000	0.000	5.005	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	65	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.87	0.00	-0.01
time (sec)	N/A	0.022	0.073	0.011	0.000	0.000	4.159	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	98	0	0	0	61	0	65
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.87	0.00	0.93
time (sec)	N/A	0.014	0.109	0.009	0.000	0.000	2.784	0.000	3.363

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	65	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.74	0.00	-0.01
time (sec)	N/A	0.034	0.102	0.008	0.000	0.000	4.034	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	68	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.035	0.104	0.007	0.000	0.000	4.146	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	71	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.77	0.00	-0.01
time (sec)	N/A	0.037	0.102	0.013	0.000	0.000	5.811	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	205	0	0	0	2883	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	15.34	0.00	-0.01
time (sec)	N/A	0.121	0.269	0.023	0.000	0.000	19.154	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	169	156	0	0	0	986	0	-1
N.S.	1	0.95	0.88	0.00	0.00	0.00	5.57	0.00	-0.01
time (sec)	N/A	0.107	0.276	0.020	0.000	0.000	18.005	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	152	0	0	0	1294	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	8.68	0.00	-0.01
time (sec)	N/A	0.097	0.194	0.019	0.000	0.000	10.343	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	144	139	0	0	0	400	0	-1
N.S.	1	0.95	0.91	0.00	0.00	0.00	2.63	0.00	-0.01
time (sec)	N/A	0.093	0.235	0.017	0.000	0.000	9.646	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	184	0	0	0	408	0	-1
N.S.	1	1.00	1.63	0.00	0.00	0.00	3.61	0.00	-0.01
time (sec)	N/A	0.066	0.196	0.015	0.000	0.000	5.473	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	125	133	0	0	0	97	0	-1
N.S.	1	0.94	1.00	0.00	0.00	0.00	0.73	0.00	-0.01
time (sec)	N/A	0.054	0.184	0.016	0.000	0.000	5.277	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	101	0	0	0	109	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.92	0.00	-0.01
time (sec)	N/A	0.063	0.163	0.013	0.000	0.000	5.078	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	134	0	0	0	95	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.083	0.210	0.011	0.000	0.000	5.392	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	119	0	0	0	119	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.94	0.00	-0.01
time (sec)	N/A	0.084	0.166	0.018	0.000	0.000	7.188	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	241	249	0	0	0	2919	0	-1
N.S.	1	0.98	1.01	0.00	0.00	0.00	11.82	0.00	-0.00
time (sec)	N/A	0.160	0.381	0.012	0.000	0.000	34.489	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	241	249	0	0	0	2919	0	-1
N.S.	1	0.97	1.00	0.00	0.00	0.00	11.72	0.00	-0.00
time (sec)	N/A	0.158	0.346	0.023	0.000	0.000	23.604	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	201	196	0	0	0	1329	0	-1
N.S.	1	0.97	0.95	0.00	0.00	0.00	6.42	0.00	-0.00
time (sec)	N/A	0.135	0.291	0.020	0.000	0.000	19.070	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	202	196	0	0	0	1329	0	-1
N.S.	1	0.96	0.93	0.00	0.00	0.00	6.33	0.00	-0.00
time (sec)	N/A	0.128	0.256	0.018	0.000	0.000	12.566	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	159	228	0	0	0	440	0	-1
N.S.	1	0.95	1.37	0.00	0.00	0.00	2.63	0.00	-0.01
time (sec)	N/A	0.101	0.283	0.018	0.000	0.000	10.253	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	169	223	0	0	0	437	0	-1
N.S.	1	0.96	1.27	0.00	0.00	0.00	2.48	0.00	-0.01
time (sec)	N/A	0.099	0.280	0.016	0.000	0.000	6.715	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	165	170	0	0	0	144	0	-1
N.S.	1	0.96	0.99	0.00	0.00	0.00	0.84	0.00	-0.01
time (sec)	N/A	0.088	0.259	0.012	0.000	0.000	7.368	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	154	0	0	0	143	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.90	0.00	-0.01
time (sec)	N/A	0.128	0.271	0.013	0.000	0.000	5.993	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	174	0	0	0	150	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.148	0.279	0.017	0.000	0.000	8.799	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.569	0.027	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	260	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.338	0.023	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	227	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.273	0.020	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	172	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.240	0.015	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	131	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.146	0.018	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	170	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.249	0.012	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	214	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.387	0.017	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	256	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.345	0.023	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.578	0.710	0.026	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	343	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.354	0.624	0.025	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	277	300	0	0	0	0	0	-1
N.S.	1	0.99	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.424	0.021	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	223	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.218	0.021	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	191	141	0	0	0	0	0	-1
N.S.	1	0.78	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.218	0.022	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	303	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.292	0.390	0.013	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	342	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.530	0.017	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	462	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.615	0.926	0.037	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	436	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.385	0.739	0.032	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	290	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	0.417	0.029	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	229	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.314	0.026	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	142	0	0	0	0	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.278	0.027	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	700	700	434	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	0.720	0.013	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	754	478	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.585	0.778	0.029	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	254	186	0	0	0	235	0	-1
N.S.	1	0.92	0.67	0.00	0.00	0.00	0.85	0.00	-0.00
time (sec)	N/A	0.316	0.183	0.014	0.000	0.000	121.875	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	194	162	0	0	0	172	0	-1
N.S.	1	0.95	0.79	0.00	0.00	0.00	0.84	0.00	-0.00
time (sec)	N/A	0.126	0.123	0.015	0.000	0.000	78.480	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	106	0	0	0	109	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.043	0.052	0.010	0.000	0.000	40.475	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	-0.02
time (sec)	N/A	0.012	0.022	0.022	0.000	0.000	10.691	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.106	0.015	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.100	0.016	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.188	0.016	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	275	955	0	653	0	292	-1
N.S.	1	1.00	0.80	2.77	0.00	1.89	0.00	0.85	-0.00
time (sec)	N/A	0.311	1.113	0.102	0.000	3.753	0.000	2.476	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	218	512	0	517	0	217	-1
N.S.	1	1.00	0.87	2.04	0.00	2.06	0.00	0.86	-0.00
time (sec)	N/A	0.170	0.780	0.084	0.000	3.246	0.000	1.397	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	161	291	0	409	0	160	-1
N.S.	1	1.00	0.78	1.41	0.00	1.98	0.00	0.77	-0.00
time (sec)	N/A	0.126	0.494	0.079	0.000	3.033	0.000	1.264	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	107	131	0	334	0	128	-1
N.S.	1	1.00	0.82	1.00	0.00	2.55	0.00	0.98	-0.01
time (sec)	N/A	0.041	0.129	0.103	0.000	1.386	0.000	2.285	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	167	308	0	935	0	0	-1
N.S.	1	1.00	0.99	1.83	0.00	5.57	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.369	0.074	0.000	2.536	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	141	708	0	356	0	229	-1
N.S.	1	1.00	1.03	5.17	0.00	2.60	0.00	1.67	-0.01
time (sec)	N/A	0.095	0.659	0.105	0.000	2.615	0.000	1.151	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	198	1353	0	445	0	517	-1
N.S.	1	1.00	0.98	6.70	0.00	2.20	0.00	2.56	-0.00
time (sec)	N/A	0.170	1.429	0.086	0.000	2.370	0.000	1.269	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	210	2059	0	565	0	925	-1
N.S.	1	1.00	0.73	7.20	0.00	1.98	0.00	3.23	-0.00
time (sec)	N/A	0.252	10.195	0.070	0.000	6.443	0.000	1.490	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	273	3471	0	719	0	1501	-1
N.S.	1	1.00	0.70	8.92	0.00	1.85	0.00	3.86	-0.00
time (sec)	N/A	0.376	10.274	0.102	0.000	13.762	0.000	1.768	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	385	1426	0	1013	0	486	-1
N.S.	1	1.00	0.86	3.18	0.00	2.26	0.00	1.08	-0.00
time (sec)	N/A	0.355	1.328	0.086	0.000	3.356	0.000	2.749	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	303	797	0	821	0	386	-1
N.S.	1	1.00	0.86	2.26	0.00	2.33	0.00	1.10	-0.00
time (sec)	N/A	0.208	0.768	0.085	0.000	2.898	0.000	1.957	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	236	483	0	653	0	298	-1
N.S.	1	1.00	0.80	1.64	0.00	2.21	0.00	1.01	-0.00
time (sec)	N/A	0.172	0.551	0.076	0.000	2.582	0.000	2.503	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	180	230	451	511	0	223	-1
N.S.	1	1.00	0.90	1.14	2.24	2.54	0.00	1.11	-0.00
time (sec)	N/A	0.074	0.139	0.087	0.301	2.706	0.000	2.458	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	232	607	0	1303	0	0	-1
N.S.	1	1.00	0.92	2.42	0.00	5.19	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.552	0.084	0.000	8.886	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	213	1300	0	1212	0	373	-1
N.S.	1	1.00	0.89	5.42	0.00	5.05	0.00	1.55	-0.00
time (sec)	N/A	0.175	0.428	0.082	0.000	3.681	0.000	1.371	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	243	2438	0	1379	0	647	-1
N.S.	1	1.00	0.95	9.52	0.00	5.39	0.00	2.53	-0.00
time (sec)	N/A	0.178	0.570	0.092	0.000	5.264	0.000	1.809	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	201	4330	0	565	0	1022	-1
N.S.	1	1.00	0.95	20.52	0.00	2.68	0.00	4.84	-0.00
time (sec)	N/A	0.151	0.366	0.082	0.000	2.852	0.000	1.402	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	247	7421	0	721	0	1640	-1
N.S.	1	1.00	0.84	25.16	0.00	2.44	0.00	5.56	-0.00
time (sec)	N/A	0.236	0.573	0.083	0.000	12.106	0.000	1.748	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	317	10575	0	907	0	2381	-1
N.S.	1	1.00	0.80	26.77	0.00	2.30	0.00	6.03	-0.00
time (sec)	N/A	0.319	0.839	0.084	0.000	27.835	0.000	1.518	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	402	16883	0	1117	0	3289	-1
N.S.	1	1.00	0.81	33.90	0.00	2.24	0.00	6.60	-0.00
time (sec)	N/A	0.446	1.166	0.088	0.000	90.620	0.000	3.618	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	549	1895	0	1485	0	738	-1
N.S.	1	1.00	0.96	3.30	0.00	2.59	0.00	1.29	-0.00
time (sec)	N/A	0.432	1.493	0.069	0.000	2.231	0.000	3.207	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	479	1080	0	1235	0	612	-1
N.S.	1	1.00	1.06	2.39	0.00	2.73	0.00	1.35	-0.00
time (sec)	N/A	0.268	1.243	0.075	0.000	2.365	0.000	1.135	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	388	673	0	1015	0	499	-1
N.S.	1	1.00	1.02	1.77	0.00	2.66	0.00	1.31	-0.00
time (sec)	N/A	0.234	0.947	0.075	0.000	2.676	0.000	2.123	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	267	327	896	815	0	396	-1
N.S.	1	1.00	0.97	1.19	3.27	2.97	0.00	1.45	-0.00
time (sec)	N/A	0.109	0.327	0.089	0.354	2.761	0.000	1.750	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	342	997	0	1817	0	0	-1
N.S.	1	1.00	0.87	2.53	0.00	4.61	0.00	0.00	-0.00
time (sec)	N/A	0.284	1.025	0.076	0.000	54.807	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	309	2076	0	1697	0	504	-1
N.S.	1	1.00	0.88	5.90	0.00	4.82	0.00	1.43	-0.00
time (sec)	N/A	0.270	1.035	0.089	0.000	19.068	0.000	2.115	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	287	3893	0	1581	0	785	-1
N.S.	1	1.00	0.85	11.48	0.00	4.66	0.00	2.32	-0.00
time (sec)	N/A	0.250	0.925	0.089	0.000	8.841	0.000	1.777	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	316	6850	0	1765	0	1237	-1
N.S.	1	1.00	0.85	18.46	0.00	4.76	0.00	3.33	-0.00
time (sec)	N/A	0.290	1.062	0.070	0.000	10.655	0.000	1.559	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	358	11685	0	1945	0	1783	-1
N.S.	1	1.00	0.89	28.92	0.00	4.81	0.00	4.41	-0.00
time (sec)	N/A	0.292	1.107	0.085	0.000	26.424	0.000	1.598	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	271	19539	0	901	0	2480	-1
N.S.	1	1.00	0.94	67.61	0.00	3.12	0.00	8.58	-0.00
time (sec)	N/A	0.214	0.725	0.084	0.000	21.728	0.000	1.551	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	404	32291	0	1115	0	3430	-1
N.S.	1	1.00	1.05	83.66	0.00	2.89	0.00	8.89	-0.00
time (sec)	N/A	0.322	0.972	0.072	0.000	85.937	0.000	1.905	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	497	45106	0	1355	0	4505	-1
N.S.	1	1.00	0.99	90.21	0.00	2.71	0.00	9.01	-0.00
time (sec)	N/A	0.411	1.298	0.081	0.000	176.659	0.000	1.814	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	572	70736	0	0	0	5747	-1
N.S.	1	1.00	0.91	112.64	0.00	0.00	0.00	9.15	-0.00
time (sec)	N/A	0.568	1.674	0.079	0.000	0.000	0.000	2.246	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	298	277	514	0	729	0	249	-1
N.S.	1	1.10	1.02	1.90	0.00	2.69	0.00	0.92	-0.00
time (sec)	N/A	0.219	0.475	0.086	0.000	3.700	0.000	1.166	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	201	259	0	563	0	202	-1
N.S.	1	1.00	1.03	1.33	0.00	2.89	0.00	1.04	-0.01
time (sec)	N/A	0.222	0.290	0.092	0.000	4.029	0.000	1.731	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	130	131	0	430	0	0	-1
N.S.	1	1.00	0.94	0.94	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.198	0.092	0.000	4.101	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	65	0	57	0	0	50
N.S.	1	1.00	0.81	1.25	0.00	1.10	0.00	0.00	0.96
time (sec)	N/A	0.015	0.011	0.079	0.000	3.914	0.000	0.000	2.645

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	131	136	0	443	0	0	-1
N.S.	1	1.00	0.92	0.95	0.00	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.191	0.088	0.000	4.978	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	201	270	0	599	0	0	-1
N.S.	1	1.00	0.88	1.18	0.00	2.62	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.269	0.089	0.000	9.244	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	283	545	0	799	0	0	-1
N.S.	1	1.00	0.86	1.66	0.00	2.43	0.00	0.00	-0.00
time (sec)	N/A	0.327	0.454	0.078	0.000	8.394	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	388	1923	0	2161	0	0	-1
N.S.	1	1.00	0.75	3.73	0.00	4.20	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.829	0.106	0.000	16.737	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	300	1112	0	1817	0	0	-1
N.S.	1	1.00	0.68	2.54	0.00	4.15	0.00	0.00	-0.00
time (sec)	N/A	0.364	0.550	0.087	0.000	7.732	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	218	639	0	1471	0	0	-1
N.S.	1	1.00	0.73	2.15	0.00	4.95	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.337	0.083	0.000	6.314	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	365	0	313	0	0	1071
N.S.	1	1.00	0.75	2.90	0.00	2.48	0.00	0.00	8.50
time (sec)	N/A	0.072	0.145	0.097	0.000	6.505	0.000	0.000	3.603

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	100	226	0	318	0	0	499
N.S.	1	1.00	0.72	1.64	0.00	2.30	0.00	0.00	3.62
time (sec)	N/A	0.063	0.133	0.095	0.000	7.417	0.000	0.000	3.319

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	97	146	0	308	0	0	120
N.S.	1	1.00	0.80	1.21	0.00	2.55	0.00	0.00	0.99
time (sec)	N/A	0.028	0.020	0.093	0.000	8.471	0.000	0.000	2.885

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	217	360	0	1483	0	0	-1
N.S.	1	1.00	0.80	1.33	0.00	5.47	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.346	0.088	0.000	8.951	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	303	716	0	1889	0	0	-1
N.S.	1	1.00	0.77	1.82	0.00	4.79	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.549	0.096	0.000	32.922	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	390	1354	0	2291	0	0	-1
N.S.	1	1.00	0.75	2.59	0.00	4.39	0.00	0.00	-0.00
time (sec)	N/A	0.502	0.795	0.079	0.000	73.547	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	664	493	2409	0	2695	0	0	-1
N.S.	1	1.00	0.74	3.63	0.00	4.06	0.00	0.00	-0.00
time (sec)	N/A	0.714	1.019	0.102	0.000	156.131	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	235	621	0	841	0	0	3099
N.S.	1	1.00	0.91	2.40	0.00	3.25	0.00	0.00	11.97
time (sec)	N/A	0.150	0.221	0.090	0.000	53.611	0.000	0.000	4.327

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	440	933	0	1540	0	0	2500
N.S.	1	1.00	1.29	2.74	0.00	4.52	0.00	0.00	7.33
time (sec)	N/A	0.182	0.344	0.092	0.000	99.678	0.000	0.000	7.725

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	221	257	0	33	0	0	-1
N.S.	1	1.00	1.30	1.51	0.00	0.19	0.00	0.00	-0.01
time (sec)	N/A	0.044	40.632	0.239	0.000	0.546	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	22	22	0	67	22
N.S.	1	1.00	1.00	1.00	0.96	0.96	0.00	2.91	0.96
time (sec)	N/A	0.012	10.036	0.102	0.855	3.112	0.000	0.991	2.620

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	347	361	0	30	0	0	-1
N.S.	1	1.00	1.18	1.23	0.00	0.10	0.00	0.00	-0.00
time (sec)	N/A	0.068	20.340	0.125	0.000	0.293	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	169	252	0	25	0	0	-1
N.S.	1	1.00	1.17	1.75	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.024	17.020	0.107	0.000	0.750	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	43	0	60	0	0	-1
N.S.	1	1.00	0.73	0.65	0.00	0.91	0.00	0.00	-0.02
time (sec)	N/A	0.021	15.328	0.109	0.000	2.007	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	349	363	0	32	0	0	-1
N.S.	1	1.00	1.22	1.26	0.00	0.11	0.00	0.00	-0.00
time (sec)	N/A	0.064	10.260	0.115	0.000	0.612	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	185	259	0	32	0	0	-1
N.S.	1	1.00	1.27	1.77	0.00	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.031	20.299	0.115	0.000	0.401	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	235	262	0	38	0	0	-1
N.S.	1	1.00	1.17	1.30	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.050	30.590	0.108	0.000	0.499	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	28	27	27	0	173	25
N.S.	1	1.00	1.00	1.22	1.17	1.17	0.00	7.52	1.09
time (sec)	N/A	0.013	10.045	0.112	0.503	2.170	0.000	1.115	0.117

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	244	366	0	38	0	0	-1
N.S.	1	1.00	0.75	1.13	0.00	0.12	0.00	0.00	-0.00
time (sec)	N/A	0.074	10.334	0.102	0.000	0.363	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	176	257	0	33	0	0	-1
N.S.	1	1.00	1.02	1.49	0.00	0.19	0.00	0.00	-0.01
time (sec)	N/A	0.050	20.440	0.112	0.000	0.441	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	53	57	0	65	0	0	-1
N.S.	1	1.00	0.56	0.61	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.059	10.053	0.095	0.000	2.685	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	244	368	0	39	0	0	-1
N.S.	1	1.00	0.76	1.14	0.00	0.12	0.00	0.00	-0.00
time (sec)	N/A	0.195	10.343	0.116	0.000	0.556	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	192	264	0	38	0	0	-1
N.S.	1	1.00	1.10	1.51	0.00	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.041	10.270	0.113	0.000	0.346	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	169	248	0	25	0	0	-1
N.S.	1	1.00	1.19	1.75	0.00	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.033	30.464	0.109	0.000	0.408	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	22	17	0	18	9
N.S.	1	1.00	1.00	0.78	0.96	0.74	0.00	0.78	0.39
time (sec)	N/A	0.012	10.049	0.101	0.501	1.229	0.000	1.243	0.150

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	375	275	0	9	0	0	-1
N.S.	1	1.00	1.48	1.09	0.00	0.04	0.00	0.00	-0.00
time (sec)	N/A	0.047	20.660	0.105	0.000	0.367	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	137	0	6	0	0	-1
N.S.	1	1.00	1.35	1.25	0.00	0.05	0.00	0.00	-0.01
time (sec)	N/A	0.015	20.112	0.104	0.000	0.486	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	33	0	43	0	0	-1
N.S.	1	1.00	0.69	0.79	0.00	1.02	0.00	0.00	-0.02
time (sec)	N/A	0.018	5.309	0.097	0.000	1.761	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	400	363	0	31	0	0	-1
N.S.	1	1.00	1.42	1.29	0.00	0.11	0.00	0.00	-0.00
time (sec)	N/A	0.066	10.378	0.113	0.000	0.315	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	144	171	259	0	30	0	0	-1
N.S.	1	0.99	1.17	1.77	0.00	0.21	0.00	0.00	-0.01
time (sec)	N/A	0.031	20.424	0.109	0.000	0.497	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	161	245	0	38	0	0	-1
N.S.	1	1.00	1.18	1.79	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.033	30.408	0.111	0.000	0.299	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	25	17	24	0	0	17
N.S.	1	1.00	1.00	1.09	0.74	1.04	0.00	0.00	0.74
time (sec)	N/A	0.014	10.029	0.101	0.481	2.126	0.000	0.000	2.692

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	402	356	0	42	0	0	-1
N.S.	1	1.00	1.43	1.26	0.00	0.15	0.00	0.00	-0.00
time (sec)	N/A	0.060	10.424	0.115	0.000	0.311	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	216	247	0	37	0	0	-1
N.S.	1	1.00	1.58	1.80	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.023	20.244	0.102	0.000	0.420	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	81	43	0	78	0	0	-1
N.S.	1	1.00	1.23	0.65	0.00	1.18	0.00	0.00	-0.02
time (sec)	N/A	0.024	11.213	0.126	0.000	1.457	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	409	363	0	47	0	0	-1
N.S.	1	1.00	1.29	1.15	0.00	0.15	0.00	0.00	-0.00
time (sec)	N/A	0.083	10.431	0.097	0.000	0.329	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	170	259	0	48	0	0	-1
N.S.	1	1.00	1.00	1.52	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.043	10.252	0.107	0.000	0.224	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	467	0	56	0	0	-1
N.S.	1	1.00	1.06	2.78	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.041	30.315	0.104	0.000	0.228	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	25	24	29	0	0	82
N.S.	1	1.00	1.00	1.09	1.04	1.26	0.00	0.00	3.57
time (sec)	N/A	0.014	10.028	0.115	0.483	1.015	0.000	0.000	2.875

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	409	688	0	61	0	0	-1
N.S.	1	1.00	1.29	2.16	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.075	10.487	0.112	0.000	0.189	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	469	0	56	0	0	-1
N.S.	1	1.00	1.06	2.79	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.031	20.326	0.123	0.000	0.400	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	95	69	0	101	0	0	-1
N.S.	1	1.00	0.99	0.72	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.028	10.090	0.113	0.000	1.407	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	414	695	0	62	0	0	-1
N.S.	1	1.00	1.19	1.99	0.00	0.18	0.00	0.00	-0.00
time (sec)	N/A	0.093	10.504	0.118	0.000	0.302	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	183	481	0	63	0	0	-1
N.S.	1	1.00	0.90	2.37	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.053	10.426	0.128	0.000	0.152	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	75	134	88	71	84
N.S.	1	1.00	0.80	0.70	0.77	1.38	0.91	0.73	0.87
time (sec)	N/A	0.050	0.035	0.195	0.497	1.142	0.095	1.532	0.129

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	627	708	0	5512	0	1171	2500
N.S.	1	1.00	1.28	1.44	0.00	11.25	0.00	2.39	5.10
time (sec)	N/A	12.345	1.887	0.247	0.000	2.601	0.000	1.700	4.857

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	397	465	559	0	4258	0	1045	2500
N.S.	1	1.22	1.43	1.71	0.00	13.06	0.00	3.21	7.67
time (sec)	N/A	4.959	1.469	0.148	0.000	1.581	0.000	4.053	4.366

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	375	430	0	2992	0	868	2500
N.S.	1	1.00	1.19	1.36	0.00	9.47	0.00	2.75	7.91
time (sec)	N/A	2.193	1.104	0.156	0.000	2.703	0.000	2.214	3.911

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	341	331	0	1742	0	753	2500
N.S.	1	1.00	1.19	1.15	0.00	6.07	0.00	2.62	8.71
time (sec)	N/A	2.197	0.964	0.148	0.000	2.376	0.000	1.332	3.820

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	252	251	0	683	0	223	709
N.S.	1	1.00	1.27	1.27	0.00	3.45	0.00	1.13	3.58
time (sec)	N/A	0.175	0.100	0.140	0.000	2.347	0.000	1.836	2.992

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	266	291	0	2489	0	712	2500
N.S.	1	1.00	0.97	1.06	0.00	9.05	0.00	2.59	9.09
time (sec)	N/A	0.743	0.843	0.122	0.000	2.727	0.000	1.982	7.410

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	356	337	373	0	4909	0	0	2500
N.S.	1	0.97	0.92	1.01	0.00	13.34	0.00	0.00	6.79
time (sec)	N/A	2.435	1.332	0.141	0.000	14.123	0.000	0.000	6.814

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	433	507	0	7479	0	1041	2500
N.S.	1	1.00	0.82	0.95	0.00	14.08	0.00	1.96	4.71
time (sec)	N/A	2.427	1.881	0.148	0.000	168.195	0.000	1.380	8.089

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	901	1094	0	14252	0	1577	2500
N.S.	1	1.00	1.39	1.68	0.00	21.93	0.00	2.43	3.85
time (sec)	N/A	1.856	3.032	0.173	0.000	25.086	0.000	1.515	7.969

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	755	893	0	11377	0	1362	2500
N.S.	1	1.00	1.30	1.54	0.00	19.58	0.00	2.34	4.30
time (sec)	N/A	13.804	2.460	0.160	0.000	21.025	0.000	1.092	7.139

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	537	674	0	8462	0	1160	2500
N.S.	1	1.00	1.22	1.53	0.00	19.19	0.00	2.63	5.67
time (sec)	N/A	1.518	1.839	0.141	0.000	7.602	0.000	1.086	5.724

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	493	521	0	5507	0	978	2500
N.S.	1	1.00	1.09	1.15	0.00	12.16	0.00	2.16	5.52
time (sec)	N/A	2.936	1.729	0.170	0.000	3.896	0.000	2.150	4.723

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	364	353	0	2688	0	783	2500
N.S.	1	1.00	1.13	1.10	0.00	8.35	0.00	2.43	7.76
time (sec)	N/A	0.800	0.267	0.143	0.000	2.475	0.000	0.897	4.435

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	371	370	0	5074	0	822	2500
N.S.	1	1.00	1.09	1.09	0.00	14.92	0.00	2.42	7.35
time (sec)	N/A	0.991	1.308	0.139	0.000	13.127	0.000	0.988	8.163

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	402	416	409	0	8526	0	425	2500
N.S.	1	1.00	1.03	1.01	0.00	21.16	0.00	1.05	6.20
time (sec)	N/A	2.031	1.599	0.158	0.000	54.106	0.000	0.782	7.365

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	560	601	0	0	0	1121	2500
N.S.	1	1.00	0.92	0.99	0.00	0.00	0.00	1.85	4.12
time (sec)	N/A	3.195	2.646	0.168	0.000	0.000	0.000	1.150	8.194

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	0.133	0.054	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	353	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.525	0.596	0.068	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	346	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.777	0.064	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	183	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.367	0.061	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	245	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.236	0.062	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	331	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	1.172	0.053	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	431	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	1.388	0.065	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	134	179	167	168	150	177	351
N.S.	1	1.00	0.95	1.27	1.18	1.19	1.06	1.26	2.49
time (sec)	N/A	0.122	0.055	0.078	0.289	3.784	0.271	1.073	0.111

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	103	149	133	131	109	139	197
N.S.	1	1.00	0.94	1.37	1.22	1.20	1.00	1.28	1.81
time (sec)	N/A	0.091	0.035	0.084	0.282	2.310	0.215	1.121	2.586

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	73	95	95	94	70	100	127
N.S.	1	1.00	1.12	1.46	1.46	1.45	1.08	1.54	1.95
time (sec)	N/A	0.038	0.025	0.074	0.297	2.561	0.172	0.805	0.071

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	59	64	62	46	64	65
N.S.	1	1.00	0.86	1.18	1.28	1.24	0.92	1.28	1.30
time (sec)	N/A	0.021	0.014	0.078	0.280	2.060	0.113	1.347	2.605

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	84	81	87	112	81	81
N.S.	1	1.00	0.89	1.35	1.31	1.40	1.81	1.31	1.31
time (sec)	N/A	0.053	0.017	0.087	0.284	1.803	0.294	0.918	0.154

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	112	112	164	182	116	109
N.S.	1	1.00	0.95	1.30	1.30	1.91	2.12	1.35	1.27
time (sec)	N/A	0.057	0.035	0.082	0.281	1.725	0.507	1.007	2.696

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	148	145	261	185	151	100
N.S.	1	1.00	1.00	1.70	1.67	3.00	2.13	1.74	1.15
time (sec)	N/A	0.061	0.054	0.088	0.288	2.928	0.492	1.284	0.129

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	122	184	197	397	248	181	152
N.S.	1	1.00	1.08	1.63	1.74	3.51	2.19	1.60	1.35
time (sec)	N/A	0.076	0.043	0.089	0.292	2.292	0.635	1.392	2.650

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	142	220	228	505	282	206	180
N.S.	1	1.00	1.02	1.58	1.64	3.63	2.03	1.48	1.29
time (sec)	N/A	0.093	0.063	0.095	0.302	3.303	0.740	1.251	0.145

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	226	259	245	317	250	257	1029
N.S.	1	1.00	1.04	1.19	1.12	1.45	1.15	1.18	4.72
time (sec)	N/A	0.183	0.086	0.082	0.279	2.587	0.541	1.276	2.641

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	185	217	208	280	199	218	565
N.S.	1	1.00	1.05	1.23	1.18	1.58	1.12	1.23	3.19
time (sec)	N/A	0.154	0.081	0.074	0.294	2.210	0.474	1.220	2.615

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	154	177	175	243	162	181	316
N.S.	1	1.00	1.05	1.21	1.20	1.66	1.11	1.24	2.16
time (sec)	N/A	0.117	0.062	0.093	0.285	3.629	0.406	1.924	0.094

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	115	133	137	201	119	141	185
N.S.	1	1.00	1.07	1.24	1.28	1.88	1.11	1.32	1.73
time (sec)	N/A	0.092	0.059	0.073	0.283	2.765	0.333	1.375	0.070

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	93	103	155	94	104	116
N.S.	1	1.00	1.06	1.19	1.32	1.99	1.21	1.33	1.49
time (sec)	N/A	0.065	0.042	0.089	0.280	1.769	0.277	1.997	2.535

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	63	67	96	61	68	72
N.S.	1	1.00	0.92	1.26	1.34	1.92	1.22	1.36	1.44
time (sec)	N/A	0.039	0.030	0.072	0.290	1.918	0.183	1.592	2.561

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	91	112	113	167	182	118	111
N.S.	1	1.00	1.06	1.30	1.31	1.94	2.12	1.37	1.29
time (sec)	N/A	0.054	0.033	0.103	0.300	2.996	0.509	2.111	2.644

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	85	136	106	126	156	101	115
N.S.	1	1.00	1.15	1.84	1.43	1.70	2.11	1.36	1.55
time (sec)	N/A	0.019	0.028	0.076	0.289	1.579	0.349	2.047	2.606

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	139	180	200	399	279	194	198
N.S.	1	1.00	1.15	1.49	1.65	3.30	2.31	1.60	1.64
time (sec)	N/A	0.088	0.077	0.089	0.291	2.069	0.634	1.991	0.146

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	171	189	189	331	241	227	148
N.S.	1	1.00	1.17	1.29	1.29	2.27	1.65	1.55	1.01
time (sec)	N/A	0.104	0.067	0.097	0.299	4.617	0.679	1.643	2.632

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	195	241	281	638	376	254	274
N.S.	1	1.00	1.10	1.35	1.58	3.58	2.11	1.43	1.54
time (sec)	N/A	0.132	0.100	0.102	0.307	3.531	0.912	2.219	2.700

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	229	278	323	685	427	296	314
N.S.	1	1.00	1.09	1.32	1.54	3.26	2.03	1.41	1.50
time (sec)	N/A	0.155	0.124	0.106	0.314	4.151	1.056	2.001	2.718

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	193	216	218	322	219	215	375
N.S.	1	1.00	1.08	1.21	1.22	1.80	1.22	1.20	2.09
time (sec)	N/A	0.164	0.064	0.089	0.287	4.083	0.747	1.797	0.141

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	157	174	182	282	178	177	240
N.S.	1	1.00	1.05	1.17	1.22	1.89	1.19	1.19	1.61
time (sec)	N/A	0.125	0.057	0.089	0.288	9.630	0.644	2.227	0.105

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	133	146	231	151	138	161
N.S.	1	1.00	1.00	1.13	1.24	1.96	1.28	1.17	1.36
time (sec)	N/A	0.095	0.063	0.070	0.291	3.505	0.566	2.022	2.597

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	93	101	102	156	102	94	107
N.S.	1	1.00	1.15	1.25	1.26	1.93	1.26	1.16	1.32
time (sec)	N/A	0.066	0.028	0.082	0.292	3.089	0.443	2.972	2.598

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	74	79	99	83	74	80
N.S.	1	1.00	0.80	1.21	1.30	1.62	1.36	1.21	1.31
time (sec)	N/A	0.039	0.019	0.079	0.283	3.225	0.255	1.461	0.069

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	152	146	261	185	142	103
N.S.	1	1.00	1.02	1.73	1.66	2.97	2.10	1.61	1.17
time (sec)	N/A	0.065	0.056	0.085	0.284	3.317	0.508	1.739	0.134

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	140	183	201	399	277	195	198
N.S.	1	1.00	1.15	1.50	1.65	3.27	2.27	1.60	1.62
time (sec)	N/A	0.077	0.071	0.092	0.294	2.940	0.644	1.610	2.638

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	110	216	140	188	144	127	114
N.S.	1	1.00	0.87	1.70	1.10	1.48	1.13	1.00	0.90
time (sec)	N/A	0.038	0.031	0.079	0.292	2.488	0.480	2.882	0.103

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	197	245	286	639	321	262	249
N.S.	1	1.00	1.05	1.30	1.52	3.40	1.71	1.39	1.32
time (sec)	N/A	0.138	0.108	0.117	0.301	2.862	0.899	2.033	2.679

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	244	297	332	758	372	332	296
N.S.	1	1.00	1.04	1.26	1.41	3.23	1.58	1.41	1.26
time (sec)	N/A	0.175	0.120	0.125	0.324	2.037	1.006	1.782	2.639

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	311	1029	1489	781	0	934	-1
N.S.	1	1.00	1.16	3.83	5.54	2.90	0.00	3.47	-0.00
time (sec)	N/A	0.598	1.455	0.142	0.520	3.079	0.000	1.333	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	240	842	1107	603	0	729	-1
N.S.	1	1.00	1.12	3.92	5.15	2.80	0.00	3.39	-0.00
time (sec)	N/A	0.410	1.288	0.106	0.555	2.651	0.000	2.172	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	165	688	834	442	0	537	-1
N.S.	1	1.00	0.90	3.76	4.56	2.42	0.00	2.93	-0.01
time (sec)	N/A	0.251	0.859	0.099	0.512	2.761	0.000	3.449	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	105	532	544	266	0	356	125
N.S.	1	1.00	0.72	3.67	3.75	1.83	0.00	2.46	0.86
time (sec)	N/A	0.137	0.612	0.091	0.325	1.837	0.000	1.993	2.871

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	77	409	346	174	0	264	79
N.S.	1	1.00	0.66	3.50	2.96	1.49	0.00	2.26	0.68
time (sec)	N/A	0.037	0.480	0.072	0.292	2.615	0.000	1.440	2.791

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	53	246	96	102	0	158	49
N.S.	1	1.00	0.51	2.39	0.93	0.99	0.00	1.53	0.48
time (sec)	N/A	0.032	0.008	0.083	0.285	2.237	0.000	1.865	2.701

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	225	1667	0	1742	0	607	-1
N.S.	1	1.00	0.93	6.89	0.00	7.20	0.00	2.51	-0.00
time (sec)	N/A	0.398	10.294	0.103	0.000	3.750	0.000	1.999	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	341	3289	0	3252	0	0	-1
N.S.	1	1.00	1.10	10.58	0.00	10.46	0.00	0.00	-0.00
time (sec)	N/A	1.013	10.440	0.099	0.000	5.305	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	387	6396	0	5268	0	1314	-1
N.S.	1	1.00	0.97	16.07	0.00	13.24	0.00	3.30	-0.00
time (sec)	N/A	2.839	10.959	0.090	0.000	27.972	0.000	1.409	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	118	114	0	493	107	116	124
N.S.	1	1.00	1.05	1.02	0.00	4.40	0.96	1.04	1.11
time (sec)	N/A	0.147	0.254	0.096	0.000	3.340	16.962	0.988	0.234

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	282	243	314	323	1040	378	222
N.S.	1	1.00	1.18	1.01	1.31	1.35	4.33	1.58	0.92
time (sec)	N/A	0.226	0.192	0.083	0.302	2.632	50.126	1.595	0.119

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	177	174	195	196	673	243	159
N.S.	1	1.00	1.01	0.99	1.11	1.12	3.85	1.39	0.91
time (sec)	N/A	0.156	0.117	0.073	0.317	3.296	31.540	1.008	2.580

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	94	105	110	99	374	134	100
N.S.	1	1.00	0.83	0.93	0.97	0.88	3.31	1.19	0.88
time (sec)	N/A	0.048	0.066	0.057	0.287	2.847	16.628	1.651	0.073

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	52	53	40	150	53	44
N.S.	1	1.00	0.72	0.85	0.87	0.66	2.46	0.87	0.72
time (sec)	N/A	0.017	0.031	0.070	0.282	3.278	3.341	1.244	2.558

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	96	0	292	100	107	107
N.S.	1	1.00	0.88	0.92	0.00	2.81	0.96	1.03	1.03
time (sec)	N/A	0.086	0.206	0.075	0.000	3.100	10.030	1.210	0.107

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	126	133	139	0	525	0	148	128
N.S.	1	1.03	1.09	1.14	0.00	4.30	0.00	1.21	1.05
time (sec)	N/A	0.132	0.425	0.089	0.000	2.487	0.000	1.071	2.682

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	182	166	220	0	870	0	278	224
N.S.	1	1.02	0.93	1.24	0.00	4.89	0.00	1.56	1.26
time (sec)	N/A	0.189	0.717	0.105	0.000	3.343	0.000	1.361	2.909

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	278	438	322	330	328	453	292
N.S.	1	1.00	1.17	1.84	1.35	1.39	1.38	1.90	1.23
time (sec)	N/A	0.166	0.211	0.097	0.304	3.152	25.729	1.022	0.091

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	177	256	203	204	204	275	199
N.S.	1	1.00	1.02	1.48	1.17	1.18	1.18	1.59	1.15
time (sec)	N/A	0.128	0.151	0.087	0.313	3.970	15.171	1.705	2.658

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	92	120	118	109	112	143	111
N.S.	1	1.00	0.83	1.08	1.06	0.98	1.01	1.29	1.00
time (sec)	N/A	0.041	0.070	0.101	0.283	3.173	8.333	1.469	0.075

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	48	54	49	58	56	44
N.S.	1	1.00	0.73	0.81	0.92	0.83	0.98	0.95	0.75
time (sec)	N/A	0.017	0.033	0.064	0.280	2.840	3.542	1.390	0.054

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	114	112	0	475	104	101	141
N.S.	1	1.00	1.02	1.00	0.00	4.24	0.93	0.90	1.26
time (sec)	N/A	0.120	0.328	0.079	0.000	3.332	13.715	1.091	0.136

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	148	152	0	890	0	225	187
N.S.	1	1.00	1.03	1.06	0.00	6.18	0.00	1.56	1.30
time (sec)	N/A	0.178	0.529	0.073	0.000	3.037	0.000	1.372	3.287

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	230	230	0	1507	0	361	310
N.S.	1	1.00	1.07	1.07	0.00	7.04	0.00	1.69	1.45
time (sec)	N/A	0.305	1.007	0.093	0.000	1.843	0.000	1.111	3.367

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	123	306	0	338	0	160	569
N.S.	1	1.00	0.84	2.08	0.00	2.30	0.00	1.09	3.87
time (sec)	N/A	0.091	0.335	0.079	0.000	2.768	0.000	1.159	20.128

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	9	12	0	12	16
N.S.	1	1.00	1.00	0.81	0.56	0.75	0.00	0.75	1.00
time (sec)	N/A	0.006	0.038	0.091	0.277	2.480	0.000	1.309	2.798

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	438	2385	0	0	0	0	-1
N.S.	1	1.00	1.07	5.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.580	1.330	0.116	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	363	1495	0	0	0	0	-1
N.S.	1	1.00	1.06	4.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.263	0.909	0.090	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	1387	0	1865	0	0	-1
N.S.	1	1.00	0.95	5.78	0.00	7.77	0.00	0.00	-0.00
time (sec)	N/A	0.223	10.328	0.106	0.000	19.256	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	361	5383	0	5816	0	0	-1
N.S.	1	1.00	1.03	15.34	0.00	16.57	0.00	0.00	-0.00
time (sec)	N/A	1.360	1.230	0.101	0.000	30.939	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	613	613	422	14861	0	0	0	0	-1
N.S.	1	1.00	0.69	24.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.993	1.681	0.081	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	363	2336	0	0	0	0	-1
N.S.	1	1.00	1.08	6.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.628	1.021	0.091	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	1387	0	1929	0	0	-1
N.S.	1	1.00	0.95	5.78	0.00	8.04	0.00	0.00	-0.00
time (sec)	N/A	0.212	10.264	0.071	0.000	19.959	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	286	1415	0	4509	0	0	-1
N.S.	1	1.00	1.24	6.15	0.00	19.60	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.858	0.098	0.000	61.510	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	383	10977	0	11846	0	0	-1
N.S.	1	1.00	1.08	31.01	0.00	33.46	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.956	0.089	0.000	50.320	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	336	8264	0	0	0	0	-1
N.S.	1	1.00	0.54	13.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.591	10.546	0.083	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	361	5383	0	5929	0	0	-1
N.S.	1	1.00	1.03	15.34	0.00	16.89	0.00	0.00	-0.00
time (sec)	N/A	1.160	1.237	0.085	0.000	121.106	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	383	10977	0	12761	0	0	-1
N.S.	1	1.00	1.08	31.01	0.00	36.05	0.00	0.00	-0.00
time (sec)	N/A	0.465	0.906	0.080	0.000	191.770	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	543	477	30648	0	0	0	0	-1
N.S.	1	0.99	0.87	55.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.944	1.918	0.141	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	305	0	744	0	375	1610
N.S.	1	1.00	0.91	4.69	0.00	11.45	0.00	5.77	24.77
time (sec)	N/A	0.031	0.067	0.181	0.000	2.716	0.000	4.031	8.490

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	125	0	193	0	266	164
N.S.	1	1.00	1.08	1.56	0.00	2.41	0.00	3.32	2.05
time (sec)	N/A	0.089	0.353	0.093	0.000	2.416	0.000	3.383	2.955

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	501	866	0	318	0	208	148
N.S.	1	1.00	4.68	8.09	0.00	2.97	0.00	1.94	1.38
time (sec)	N/A	0.155	1.793	0.101	0.000	3.343	0.000	3.975	0.286

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	501	242	0	318	0	131	148
N.S.	1	1.00	4.68	2.26	0.00	2.97	0.00	1.22	1.38
time (sec)	N/A	0.115	1.144	0.104	0.000	4.411	0.000	3.900	0.125

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	851	851	1034	6457	0	753	0	0	-1
N.S.	1	1.00	1.22	7.59	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	2.501	26.975	0.194	0.000	0.884	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	809	4351	0	503	0	0	-1
N.S.	1	1.00	1.27	6.85	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	1.067	24.782	0.100	0.000	0.739	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	610	2549	0	346	0	0	-1
N.S.	1	1.00	1.41	5.87	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.319	23.132	0.097	0.000	0.379	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	536	1162	0	229	0	0	-1
N.S.	1	1.00	1.48	3.21	0.00	0.63	0.00	0.00	-0.00
time (sec)	N/A	0.207	21.875	0.098	0.000	0.835	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	683	683	1216	2496	0	0	0	0	-1
N.S.	1	1.00	1.78	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.397	28.075	0.122	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	1331	6044	0	0	0	0	-1
N.S.	1	1.00	2.05	9.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.084	24.781	0.102	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1205	1205	2526	19181	0	0	0	0	-1
N.S.	1	1.00	2.10	15.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.756	27.495	0.105	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	864	5079	0	568	0	0	-1
N.S.	1	1.00	1.30	7.63	0.00	0.85	0.00	0.00	-0.00
time (sec)	N/A	1.040	25.135	0.120	0.000	0.601	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	503	712	3278	0	407	0	0	-1
N.S.	1	0.99	1.40	6.45	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.639	23.542	0.113	0.000	0.724	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	545	1828	0	276	0	0	-1
N.S.	1	1.00	1.50	5.02	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.244	23.178	0.135	0.000	0.343	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	456	688	0	208	0	0	-1
N.S.	1	1.00	1.42	2.14	0.00	0.65	0.00	0.00	-0.00
time (sec)	N/A	0.137	21.284	0.119	0.000	0.754	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1096	1216	0	0	0	0	-1
N.S.	1	1.00	2.32	2.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	23.038	0.125	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	1336	6034	0	0	0	0	-1
N.S.	1	1.00	1.93	8.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.102	24.479	0.142	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1241	1241	2197	19170	0	0	0	0	-1
N.S.	1	1.00	1.77	15.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.720	27.295	0.134	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	527	747	3922	0	436	0	0	-1
N.S.	1	0.99	1.41	7.39	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.738	24.114	0.128	0.000	0.381	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	596	2470	0	300	0	0	-1
N.S.	1	1.00	1.45	6.02	0.00	0.73	0.00	0.00	-0.00
time (sec)	N/A	0.356	22.914	0.125	0.000	0.575	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	464	1286	0	231	0	0	-1
N.S.	1	1.00	1.40	3.89	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.175	22.212	0.102	0.000	0.596	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	294	396	0	173	0	0	-1
N.S.	1	1.00	2.16	2.91	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.038	20.385	0.087	0.000	0.279	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	300	439	0	0	0	0	-1
N.S.	1	1.00	0.94	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	20.510	0.088	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	698	698	1330	5743	0	0	0	0	-1
N.S.	1	1.00	1.91	8.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.294	24.091	0.108	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1246	1246	2450	20359	0	0	0	0	-1
N.S.	1	1.00	1.97	16.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.869	27.315	0.109	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	808	1212	3164	0	0	0	0	-1
N.S.	1	1.35	2.02	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.611	27.825	0.122	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	927	959	0	0	0	0	-1
N.S.	1	1.00	1.98	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	10.751	0.104	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	625	2949	0	315	0	0	-1
N.S.	1	1.00	1.37	6.45	0.00	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.404	22.848	0.098	0.000	0.529	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	473	1769	0	245	0	0	-1
N.S.	1	1.00	1.33	4.97	0.00	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.263	22.315	0.103	0.000	0.825	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	439	520	0	184	0	0	-1
N.S.	1	1.00	1.52	1.81	0.00	0.64	0.00	0.00	-0.00
time (sec)	N/A	0.109	21.054	0.095	0.000	0.291	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	186	200	0	66	0	0	-1
N.S.	1	1.00	1.37	1.47	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.041	20.186	0.082	0.000	0.253	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	311	235	0	0	0	0	-1
N.S.	1	1.00	1.86	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	20.680	0.087	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	1349	5738	0	0	0	0	-1
N.S.	1	1.00	1.81	7.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.353	24.549	0.099	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1257	1257	2491	20366	0	0	0	0	-1
N.S.	1	1.00	1.98	16.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.872	27.951	0.118	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	468	2011	0	0	0	0	-1
N.S.	1	1.00	1.21	5.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	22.146	0.128	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	1917	9415	0	0	0	0	-1
N.S.	1	1.00	2.34	11.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.671	25.901	0.134	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	261	215	0	0	0	0	-1
N.S.	1	1.00	2.37	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	20.653	0.127	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	344	396	0	0	0	0	-1
N.S.	1	1.00	0.76	0.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	23.342	0.165	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	107	58	0	3	0	0	-1
N.S.	1	1.00	2.06	1.12	0.00	0.06	0.00	0.00	-0.02
time (sec)	N/A	0.028	33.712	0.125	0.000	0.917	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	136	170	219	194	0	605	218
N.S.	1	1.00	0.51	0.63	0.81	0.72	0.00	2.25	0.81
time (sec)	N/A	0.263	0.110	0.143	0.332	4.854	0.000	1.362	3.658

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	89	98	135	124	0	344	142
N.S.	1	1.00	0.44	0.49	0.68	0.62	0.00	1.72	0.71
time (sec)	N/A	0.147	0.068	0.135	0.326	3.341	0.000	1.160	3.404

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	53	49	67	74	0	160	88
N.S.	1	1.00	0.42	0.39	0.54	0.59	0.00	1.28	0.70
time (sec)	N/A	0.062	0.042	0.141	0.310	4.717	0.000	1.435	3.226

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	32	19	51	0	62	54
N.S.	1	1.00	0.76	0.70	0.41	1.11	0.00	1.35	1.17
time (sec)	N/A	0.014	0.011	0.137	0.302	2.560	0.000	1.601	3.196

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	77	0	259	0	120	-1
N.S.	1	1.00	1.16	0.96	0.00	3.24	0.00	1.50	-0.01
time (sec)	N/A	0.087	0.069	0.146	0.000	2.710	0.000	1.486	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	136	158	0	727	0	0	-1
N.S.	1	1.00	0.97	1.13	0.00	5.19	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.268	0.151	0.000	3.937	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	163	275	0	1325	0	0	-1
N.S.	1	1.00	0.77	1.29	0.00	6.22	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.337	0.166	0.000	3.646	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	191	440	0	2119	0	0	-1
N.S.	1	1.00	0.68	1.57	0.00	7.57	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.512	0.162	0.000	3.319	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	134	179	169	216	0	496	252
N.S.	1	1.00	0.52	0.70	0.66	0.84	0.00	1.93	0.98
time (sec)	N/A	0.213	0.099	0.130	0.332	2.782	0.000	2.363	3.614

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	88	108	102	147	0	285	178
N.S.	1	1.00	0.49	0.60	0.56	0.81	0.00	1.57	0.98
time (sec)	N/A	0.119	0.067	0.138	0.334	2.320	0.000	1.978	3.433

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	51	58	51	98	0	127	118
N.S.	1	1.00	0.34	0.39	0.34	0.65	0.00	0.85	0.79
time (sec)	N/A	0.096	0.048	0.143	0.314	1.875	0.000	1.527	3.368

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	42	19	75	0	62	82
N.S.	1	1.00	0.76	0.91	0.41	1.63	0.00	1.35	1.78
time (sec)	N/A	0.014	0.009	0.146	0.310	2.260	0.000	1.998	3.265

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	109	118	0	555	0	276	-1
N.S.	1	1.00	0.82	0.89	0.00	4.17	0.00	2.08	-0.01
time (sec)	N/A	0.111	0.110	0.139	0.000	2.743	0.000	2.419	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	141	215	0	1097	0	0	-1
N.S.	1	1.00	0.70	1.06	0.00	5.43	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.386	0.158	0.000	1.822	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	185	369	0	1941	0	0	-1
N.S.	1	1.00	0.68	1.35	0.00	7.08	0.00	0.00	-0.00
time (sec)	N/A	0.244	0.570	0.144	0.000	3.002	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	131	179	227	247	0	509	278
N.S.	1	1.00	0.55	0.75	0.95	1.03	0.00	2.13	1.16
time (sec)	N/A	0.178	0.114	0.153	0.360	1.848	0.000	1.366	3.773

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	87	108	145	181	0	290	206
N.S.	1	1.00	0.41	0.51	0.69	0.86	0.00	1.37	0.98
time (sec)	N/A	0.143	0.075	0.128	0.347	2.796	0.000	4.023	3.609

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	52	58	78	130	0	153	149
N.S.	1	1.00	0.34	0.38	0.51	0.84	0.00	0.99	0.97
time (sec)	N/A	0.089	0.055	0.135	0.317	2.396	0.000	2.378	3.504

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	30	107	0	88	110
N.S.	1	1.00	0.77	0.88	0.62	2.23	0.00	1.83	2.29
time (sec)	N/A	0.015	0.021	0.152	0.300	3.508	0.000	1.397	3.316

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	129	209	0	1029	0	661	-1
N.S.	1	1.00	0.69	1.11	0.00	5.47	0.00	3.52	-0.01
time (sec)	N/A	0.175	0.224	0.148	0.000	4.665	0.000	1.914	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	180	414	0	1969	0	0	-1
N.S.	1	1.00	0.67	1.54	0.00	7.35	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.444	0.138	0.000	3.624	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	240	660	0	3043	0	0	-1
N.S.	1	1.00	0.70	1.93	0.00	8.90	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.798	0.141	0.000	6.265	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	195	273	319	374	0	1088	347
N.S.	1	1.00	0.58	0.81	0.95	1.11	0.00	3.24	1.03
time (sec)	N/A	0.388	0.154	0.137	0.363	2.750	0.000	1.005	3.599

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	136	178	219	263	0	747	242
N.S.	1	1.00	0.51	0.66	0.81	0.98	0.00	2.78	0.90
time (sec)	N/A	0.250	0.112	0.126	0.340	1.303	0.000	3.889	3.371

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	90	106	135	174	0	465	157
N.S.	1	1.00	0.45	0.53	0.68	0.87	0.00	2.32	0.78
time (sec)	N/A	0.147	0.073	0.136	0.328	1.060	0.000	2.405	3.255

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	54	57	67	104	0	246	93
N.S.	1	1.00	0.43	0.46	0.54	0.83	0.00	1.97	0.74
time (sec)	N/A	0.063	0.046	0.148	0.322	1.419	0.000	1.742	3.131

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	40	19	60	0	88	49
N.S.	1	1.00	0.77	0.83	0.40	1.25	0.00	1.83	1.02
time (sec)	N/A	0.014	0.016	0.138	0.298	1.173	0.000	2.454	3.047

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	114	143	0	329	0	0	-1
N.S.	1	1.00	0.92	1.15	0.00	2.65	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.119	0.167	0.000	1.319	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	151	0	578	0	0	-1
N.S.	1	1.00	0.83	1.14	0.00	4.38	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.264	0.138	0.000	1.172	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	165	275	0	1095	0	0	-1
N.S.	1	1.00	0.80	1.33	0.00	5.29	0.00	0.00	-0.00
time (sec)	N/A	0.186	0.537	0.139	0.000	1.610	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	202	443	0	1801	0	0	-1
N.S.	1	1.00	0.73	1.60	0.00	6.50	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.812	0.138	0.000	1.320	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	234	686	0	2715	0	0	-1
N.S.	1	1.00	0.67	1.98	0.00	7.82	0.00	0.00	-0.00
time (sec)	N/A	0.321	1.200	0.138	0.000	3.521	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	195	275	408	469	0	2489	445
N.S.	1	1.00	0.58	0.82	1.21	1.40	0.00	7.41	1.32
time (sec)	N/A	0.388	0.198	0.138	0.360	1.815	0.000	3.678	3.800

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	137	180	292	339	0	1742	310
N.S.	1	1.00	0.51	0.67	1.09	1.26	0.00	6.48	1.15
time (sec)	N/A	0.257	0.142	0.157	0.352	1.621	0.000	2.608	3.646

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	90	108	192	229	0	1119	206
N.S.	1	1.00	0.45	0.54	0.96	1.14	0.00	5.60	1.03
time (sec)	N/A	0.153	0.100	0.149	0.317	2.049	0.000	3.113	3.426

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	54	59	108	139	0	616	109
N.S.	1	1.00	0.43	0.47	0.86	1.11	0.00	4.93	0.87
time (sec)	N/A	0.066	0.067	0.123	0.314	1.615	0.000	1.368	3.250

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	44	76	0	244	62
N.S.	1	1.00	0.77	0.88	0.92	1.58	0.00	5.08	1.29
time (sec)	N/A	0.015	0.022	0.139	0.310	1.846	0.000	3.118	3.077

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	132	253	0	419	0	0	-1
N.S.	1	1.00	0.74	1.41	0.00	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.208	0.149	0.000	1.711	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	144	296	0	457	0	0	-1
N.S.	1	1.00	0.81	1.66	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.369	0.161	0.000	2.076	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	135	266	0	864	0	0	-1
N.S.	1	1.00	0.69	1.36	0.00	4.43	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.510	0.139	0.000	2.118	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	201	443	0	1493	0	0	-1
N.S.	1	1.00	0.76	1.67	0.00	5.63	0.00	0.00	-0.00
time (sec)	N/A	0.236	0.828	0.137	0.000	2.053	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	240	655	0	2317	0	0	-1
N.S.	1	1.00	0.72	1.96	0.00	6.92	0.00	0.00	-0.00
time (sec)	N/A	0.295	1.247	0.139	0.000	3.282	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	302	945	0	3345	0	0	-1
N.S.	1	1.00	0.75	2.33	0.00	8.26	0.00	0.00	-0.00
time (sec)	N/A	0.407	2.388	0.133	0.000	7.487	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	205	275	488	562	0	4231	523
N.S.	1	1.00	0.61	0.82	1.45	1.67	0.00	12.59	1.56
time (sec)	N/A	0.386	0.183	0.143	0.377	1.803	0.000	6.417	4.086

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	147	180	356	412	0	3013	379
N.S.	1	1.00	0.55	0.67	1.32	1.53	0.00	11.20	1.41
time (sec)	N/A	0.260	0.144	0.161	0.338	1.771	0.000	6.102	3.805

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	100	108	240	282	0	1976	259
N.S.	1	1.00	0.50	0.54	1.20	1.41	0.00	9.88	1.30
time (sec)	N/A	0.156	0.099	0.126	0.352	1.282	0.000	4.915	3.562

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	64	59	140	173	0	1124	134
N.S.	1	1.00	0.51	0.47	1.12	1.38	0.00	8.99	1.07
time (sec)	N/A	0.066	0.068	0.131	0.302	1.511	0.000	6.084	3.373

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	42	60	92	0	469	79
N.S.	1	1.00	0.77	0.88	1.25	1.92	0.00	9.77	1.65
time (sec)	N/A	0.015	0.030	0.129	0.304	1.641	0.000	6.040	3.160

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	168	421	0	596	0	0	-1
N.S.	1	1.00	0.71	1.78	0.00	2.53	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.227	0.147	0.000	1.677	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	183	513	0	681	0	0	-1
N.S.	1	1.00	0.78	2.18	0.00	2.90	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.495	0.141	0.000	2.014	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	189	516	0	696	0	0	-1
N.S.	1	1.00	0.77	2.10	0.00	2.83	0.00	0.00	-0.00
time (sec)	N/A	0.230	0.563	0.141	0.000	2.076	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	171	431	0	1176	0	0	-1
N.S.	1	1.00	0.68	1.70	0.00	4.65	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.824	0.146	0.000	1.596	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	244	655	0	1929	0	0	-1
N.S.	1	1.00	0.76	2.03	0.00	5.97	0.00	0.00	-0.00
time (sec)	N/A	0.326	1.361	0.157	0.000	2.651	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	301	914	0	2865	0	0	-1
N.S.	1	1.00	0.77	2.33	0.00	7.29	0.00	0.00	-0.00
time (sec)	N/A	0.357	1.904	0.145	0.000	5.699	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	370	1251	0	4027	0	0	-1
N.S.	1	1.00	0.80	2.70	0.00	8.70	0.00	0.00	-0.00
time (sec)	N/A	0.482	2.994	0.144	0.000	23.518	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	207	501	0	845	0	0	-1
N.S.	1	1.00	0.66	1.60	0.00	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.361	0.333	0.146	0.000	3.366	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	179	318	0	669	0	0	-1
N.S.	1	1.00	0.73	1.30	0.00	2.74	0.00	0.00	-0.00
time (sec)	N/A	0.241	0.284	0.151	0.000	3.402	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	129	191	0	531	0	0	-1
N.S.	1	1.00	0.76	1.13	0.00	3.14	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.341	0.142	0.000	2.037	0.000	0.000	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	94	102	0	349	0	0	-1
N.S.	1	1.00	0.90	0.97	0.00	3.32	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.097	0.144	0.000	2.624	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	45	0	115	0	0	100
N.S.	1	1.00	0.82	0.74	0.00	1.89	0.00	0.00	1.64
time (sec)	N/A	0.043	0.051	0.140	0.000	3.034	0.000	0.000	4.638

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	61	0	302	0	0	147
N.S.	1	1.00	0.53	0.47	0.00	2.34	0.00	0.00	1.14
time (sec)	N/A	0.095	0.119	0.155	0.000	3.206	0.000	0.000	4.898

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	105	111	0	599	0	0	242
N.S.	1	1.00	0.53	0.56	0.00	3.03	0.00	0.00	1.22
time (sec)	N/A	0.149	0.145	0.143	0.000	5.081	0.000	0.000	5.170

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	152	183	0	1002	0	0	357
N.S.	1	1.00	0.57	0.69	0.00	3.75	0.00	0.00	1.34
time (sec)	N/A	0.212	0.172	0.138	0.000	4.104	0.000	0.000	5.511

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	183	638	0	979	0	0	-1
N.S.	1	1.00	0.61	2.12	0.00	3.25	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.378	0.151	0.000	9.017	0.000	0.000	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	133	386	0	727	0	0	-1
N.S.	1	1.00	0.59	1.70	0.00	3.20	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.582	0.145	0.000	5.821	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	117	200	0	577	0	0	-1
N.S.	1	1.00	0.73	1.24	0.00	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.120	0.142	0.000	4.830	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	55	0	122	0	0	147
N.S.	1	1.00	0.82	0.90	0.00	2.00	0.00	0.00	2.41
time (sec)	N/A	0.047	0.048	0.141	0.000	4.933	0.000	0.000	4.678

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	64	70	0	336	0	0	151
N.S.	1	1.00	0.52	0.56	0.00	2.71	0.00	0.00	1.22
time (sec)	N/A	0.097	0.130	0.145	0.000	2.057	0.000	0.000	4.976

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	105	120	0	679	0	0	268
N.S.	1	1.00	0.55	0.62	0.00	3.54	0.00	0.00	1.40
time (sec)	N/A	0.152	0.151	0.144	0.000	3.226	0.000	0.000	5.333

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	139	192	0	1114	0	0	414
N.S.	1	1.00	0.53	0.73	0.00	4.25	0.00	0.00	1.58
time (sec)	N/A	0.215	0.190	0.144	0.000	3.763	0.000	0.000	5.701

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	179	642	0	1061	0	0	-1
N.S.	1	1.00	0.62	2.22	0.00	3.67	0.00	0.00	-0.00
time (sec)	N/A	0.278	1.082	0.146	0.000	3.949	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	134	333	0	765	0	0	-1
N.S.	1	1.00	0.61	1.52	0.00	3.49	0.00	0.00	-0.00
time (sec)	N/A	0.196	0.210	0.148	0.000	4.548	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	55	0	188	0	0	169
N.S.	1	1.00	0.83	0.87	0.00	2.98	0.00	0.00	2.68
time (sec)	N/A	0.046	0.053	0.151	0.000	4.626	0.000	0.000	4.322

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	68	72	0	320	0	0	246
N.S.	1	1.00	0.53	0.56	0.00	2.50	0.00	0.00	1.92
time (sec)	N/A	0.100	0.119	0.144	0.000	8.180	0.000	0.000	5.059

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	102	121	0	690	0	0	255
N.S.	1	1.00	0.53	0.62	0.00	3.56	0.00	0.00	1.31
time (sec)	N/A	0.155	0.164	0.143	0.000	6.896	0.000	0.000	5.282

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	138	191	0	1108	0	0	416
N.S.	1	1.00	0.53	0.73	0.00	4.26	0.00	0.00	1.60
time (sec)	N/A	0.212	0.197	0.131	0.000	6.494	0.000	0.000	5.859

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	227	732	0	1075	0	0	-1
N.S.	1	1.00	0.59	1.90	0.00	2.79	0.00	0.00	-0.00
time (sec)	N/A	0.452	0.484	0.160	0.000	6.203	0.000	0.000	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	200	504	0	853	0	0	-1
N.S.	1	1.00	0.64	1.61	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.372	0.138	0.000	5.197	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	162	319	0	671	0	0	-1
N.S.	1	1.00	0.67	1.32	0.00	2.78	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.401	0.135	0.000	4.749	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	127	188	0	526	0	0	-1
N.S.	1	1.00	0.76	1.13	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.308	0.137	0.000	4.524	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	133	187	0	533	0	0	-1
N.S.	1	1.00	0.84	1.18	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.144	0.141	0.000	3.809	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	53	0	176	0	0	136
N.S.	1	1.00	0.83	0.84	0.00	2.79	0.00	0.00	2.16
time (sec)	N/A	0.045	0.055	0.141	0.000	2.002	0.000	0.000	3.923

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	70	0	424	0	0	187
N.S.	1	1.00	0.53	0.54	0.00	3.29	0.00	0.00	1.45
time (sec)	N/A	0.093	0.123	0.154	0.000	1.270	0.000	0.000	4.084

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	105	119	0	783	0	0	289
N.S.	1	1.00	0.53	0.60	0.00	3.95	0.00	0.00	1.46
time (sec)	N/A	0.149	0.144	0.139	0.000	1.514	0.000	0.000	4.290

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	152	191	0	1243	0	0	409
N.S.	1	1.00	0.57	0.72	0.00	4.66	0.00	0.00	1.53
time (sec)	N/A	0.209	0.166	0.141	0.000	5.173	0.000	0.000	4.501

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	238	732	0	1073	0	0	-1
N.S.	1	1.00	0.62	1.92	0.00	2.81	0.00	0.00	-0.00
time (sec)	N/A	0.448	0.635	0.146	0.000	6.104	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	201	504	0	855	0	0	-1
N.S.	1	1.00	0.65	1.63	0.00	2.76	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.381	0.157	0.000	4.349	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	161	315	0	665	0	0	-1
N.S.	1	1.00	0.68	1.32	0.00	2.79	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.280	0.145	0.000	4.067	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	148	373	0	675	0	0	-1
N.S.	1	1.00	0.67	1.68	0.00	3.04	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.563	0.158	0.000	2.189	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	152	321	0	699	0	0	-1
N.S.	1	1.00	0.71	1.50	0.00	3.27	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.267	0.151	0.000	1.480	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	55	0	244	0	0	232
N.S.	1	1.00	0.83	0.87	0.00	3.87	0.00	0.00	3.68
time (sec)	N/A	0.046	0.086	0.158	0.000	0.858	0.000	0.000	4.067

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	100	0	553	0	0	247
N.S.	1	1.00	0.53	0.78	0.00	4.29	0.00	0.00	1.91
time (sec)	N/A	0.097	0.164	0.142	0.000	0.724	0.000	0.000	4.305

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	113	172	0	968	0	0	377
N.S.	1	1.00	0.57	0.87	0.00	4.89	0.00	0.00	1.90
time (sec)	N/A	0.155	0.192	0.145	0.000	1.051	0.000	0.000	4.478

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	141	267	0	1496	0	0	519
N.S.	1	1.00	0.53	1.00	0.00	5.60	0.00	0.00	1.94
time (sec)	N/A	0.215	0.243	0.137	0.000	1.015	0.000	0.000	4.833

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	265	1005	0	1331	0	0	-1
N.S.	1	1.00	0.59	2.24	0.00	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.581	0.923	0.145	0.000	6.331	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	235	732	0	1073	0	0	-1
N.S.	1	1.00	0.62	1.95	0.00	2.85	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.605	0.145	0.000	2.432	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	191	498	0	843	0	0	-1
N.S.	1	1.00	0.63	1.64	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.351	0.135	0.000	1.965	0.000	0.000	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	199	625	0	931	0	0	-1
N.S.	1	1.00	0.68	2.13	0.00	3.17	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.377	0.156	0.000	1.879	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	189	628	0	993	0	0	-1
N.S.	1	1.00	0.67	2.21	0.00	3.50	0.00	0.00	-0.00
time (sec)	N/A	0.263	0.955	0.151	0.000	1.784	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	166	501	0	949	0	0	-1
N.S.	1	1.00	0.61	1.83	0.00	3.46	0.00	0.00	-0.00
time (sec)	N/A	0.248	0.267	0.138	0.000	1.404	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	78	0	314	0	0	325
N.S.	1	1.00	0.83	1.24	0.00	4.98	0.00	0.00	5.16
time (sec)	N/A	0.050	0.104	0.150	0.000	0.779	0.000	0.000	4.343

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	79	136	0	677	0	0	315
N.S.	1	1.00	0.61	1.05	0.00	5.25	0.00	0.00	2.44
time (sec)	N/A	0.103	0.169	0.145	0.000	0.808	0.000	0.000	4.543

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	113	231	0	1159	0	0	465
N.S.	1	1.00	0.57	1.17	0.00	5.85	0.00	0.00	2.35
time (sec)	N/A	0.158	0.218	0.153	0.000	0.927	0.000	0.000	4.822

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	141	349	0	1741	0	0	627
N.S.	1	1.00	0.53	1.31	0.00	6.52	0.00	0.00	2.35
time (sec)	N/A	0.214	0.281	0.148	0.000	0.947	0.000	0.000	5.119

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	-1
N.S.	1	1.17	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.291	0.048	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	110	98	0	0	0	0	0	-1
N.S.	1	1.06	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.206	0.047	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	118	98	0	0	0	0	0	-1
N.S.	1	1.13	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.140	0.048	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	120	100	0	0	0	0	0	-1
N.S.	1	1.15	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.139	0.050	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	-1
N.S.	1	1.17	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.205	0.046	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	110	0	0	0	0	0	-1
N.S.	1	1.17	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.193	0.049	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	95	0	0	0	0	0	-1
N.S.	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.066	0.063	0.000	0.000	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	134	527	333	699	0	2024	615
N.S.	1	1.00	0.39	1.54	0.97	2.04	0.00	5.90	1.79
time (sec)	N/A	0.428	0.270	0.166	0.315	0.824	0.000	3.340	3.753

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	131	235	196	348	0	981	327
N.S.	1	1.00	0.53	0.96	0.80	1.41	0.00	3.99	1.33
time (sec)	N/A	0.261	0.191	0.152	0.306	0.992	0.000	6.873	3.518

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	67	89	97	144	0	369	139
N.S.	1	1.00	0.45	0.59	0.65	0.96	0.00	2.46	0.93
time (sec)	N/A	0.098	0.107	0.153	0.326	0.796	0.000	3.124	3.362

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	35	59	0	87	57
N.S.	1	1.00	0.78	1.06	0.65	1.09	0.00	1.61	1.06
time (sec)	N/A	0.019	0.017	0.136	0.320	1.334	0.000	3.978	3.248

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.064	0.049	0.000	0.000	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	84	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.066	0.050	0.000	0.000	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.072	0.051	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.169	0.053	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.117	0.050	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.132	0.056	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.187	0.050	0.000	0.000	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.250	0.050	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	64	52	71	0	114	63
N.S.	1	1.00	0.82	0.98	0.80	1.09	0.00	1.75	0.97
time (sec)	N/A	0.028	0.036	0.166	0.292	4.095	0.000	4.289	3.543

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	32	34	0	0	-1
N.S.	1	1.00	0.82	0.00	0.41	0.44	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.025	0.120	0.301	2.899	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	222	145	0	0	0	0	0	-1
N.S.	1	1.04	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.251	0.049	0.000	0.000	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	380	623	678	608	0	1786	653
N.S.	1	1.00	0.76	1.24	1.35	1.21	0.00	3.56	1.30
time (sec)	N/A	0.554	0.304	0.122	0.366	3.854	0.000	3.951	4.088

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	264	407	476	411	0	1181	438
N.S.	1	1.00	0.64	0.99	1.16	1.00	0.00	2.87	1.06
time (sec)	N/A	0.396	0.198	0.131	0.340	4.906	0.000	3.179	3.864

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	169	237	306	254	0	700	279
N.S.	1	1.00	0.53	0.74	0.95	0.79	0.00	2.18	0.87
time (sec)	N/A	0.269	0.136	0.135	0.356	1.527	0.000	3.543	3.711

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	96	113	168	137	0	342	152
N.S.	1	1.00	0.46	0.54	0.80	0.66	0.00	1.64	0.73
time (sec)	N/A	0.126	0.078	0.130	0.310	2.925	0.000	6.253	3.484

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	54	51	66	75	0	135	85
N.S.	1	1.00	0.50	0.47	0.61	0.69	0.00	1.24	0.78
time (sec)	N/A	0.039	0.020	0.136	0.323	3.024	0.000	5.060	3.364

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	140	153	0	520	0	0	-1
N.S.	1	1.00	1.01	1.10	0.00	3.74	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.161	0.144	0.000	2.891	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	337	0	921	0	0	-1
N.S.	1	1.00	0.91	1.98	0.00	5.42	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.489	0.138	0.000	2.649	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	200	663	0	1761	0	0	-1
N.S.	1	1.00	0.77	2.54	0.00	6.75	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.923	0.142	0.000	2.277	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	279	1132	0	2867	0	0	-1
N.S.	1	1.00	0.79	3.23	0.00	8.17	0.00	0.00	-0.00
time (sec)	N/A	0.363	1.192	0.144	0.000	3.736	0.000	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	264	602	365	251	0	353	1768
N.S.	1	1.00	0.81	1.86	1.13	0.77	0.00	1.09	5.46
time (sec)	N/A	0.577	0.726	0.166	0.498	1.717	0.000	5.791	31.330

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	151	291	171	134	0	159	897
N.S.	1	1.00	0.91	1.75	1.03	0.81	0.00	0.96	5.40
time (sec)	N/A	0.212	0.455	0.145	0.492	2.402	0.000	6.260	13.854

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	82	117	57	67	0	60	232
N.S.	1	1.00	1.30	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.040	0.236	0.133	0.493	2.199	0.000	5.603	7.760

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	455	1759	0	4313	0	681	2500
N.S.	1	1.00	1.61	6.24	0.00	15.29	0.00	2.41	8.87
time (sec)	N/A	0.343	10.422	0.216	0.000	2.599	0.000	5.457	82.367

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	800	41834	0	35403	0	0	-1
N.S.	1	1.00	1.40	73.26	0.00	62.00	0.00	0.00	-0.00
time (sec)	N/A	4.037	11.060	0.600	0.000	81.338	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	261	755	371	376	0	736	-1
N.S.	1	1.00	0.95	2.74	1.34	1.36	0.00	2.67	-0.00
time (sec)	N/A	0.371	0.910	0.180	0.513	1.888	0.000	5.436	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	148	381	176	204	0	391	-1
N.S.	1	1.00	1.10	2.82	1.30	1.51	0.00	2.90	-0.01
time (sec)	N/A	0.119	0.577	0.159	0.501	1.636	0.000	3.520	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	70	151	61	101	0	186	-1
N.S.	1	1.00	1.75	3.78	1.52	2.52	0.00	4.65	-0.02
time (sec)	N/A	0.031	0.369	0.116	0.482	5.946	0.000	2.731	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	1546	11142	0	21628	0	0	-1
N.S.	1	1.00	3.49	25.15	0.00	48.82	0.00	0.00	-0.00
time (sec)	N/A	0.965	2.675	0.154	0.000	28.511	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	939	938	1302	108974	0	0	0	0	-1
N.S.	1	1.00	1.39	116.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.232	14.693	2.559	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	167	0	0	0	0	0	-1
N.S.	1	1.00	3.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.229	0.038	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.101	0.041	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.057	0.042	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	577	1786	811	2095	24206	3760	1943
N.S.	1	1.00	2.10	6.49	2.95	7.62	88.02	13.67	7.07
time (sec)	N/A	0.169	0.578	0.135	0.335	2.917	4.967	4.409	3.901

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	348	1029	512	1188	11946	2114	1133
N.S.	1	1.00	1.67	4.95	2.46	5.71	57.43	10.16	5.45
time (sec)	N/A	0.115	0.308	0.109	0.310	1.890	2.499	3.906	3.522

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	190	449	289	577	4952	1018	572
N.S.	1	1.00	1.30	3.08	1.98	3.95	33.92	6.97	3.92
time (sec)	N/A	0.070	0.202	0.109	0.308	4.016	1.222	6.460	3.288

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	147	135	224	1489	373	211
N.S.	1	1.00	1.10	1.75	1.61	2.67	17.73	4.44	2.51
time (sec)	N/A	0.037	0.104	0.105	0.315	4.926	0.567	3.810	3.072

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	125	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.336	0.029	0.000	0.000	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.176	0.029	0.000	0.000	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.209	0.039	0.000	0.000	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.246	0.052	0.000	0.000	0.000	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	227	190	0	0	0	0	0	-1
N.S.	1	0.98	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.323	0.022	0.000	0.000	0.000	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	84	86	96	420	88	84
N.S.	1	1.00	1.02	1.01	1.04	1.16	5.06	1.06	1.01
time (sec)	N/A	0.065	0.035	0.098	0.294	1.222	74.151	4.681	3.420

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	177	280	253	306	0	281	266
N.S.	1	1.00	0.96	1.52	1.38	1.66	0.00	1.53	1.45
time (sec)	N/A	0.191	0.102	0.148	0.274	1.379	0.000	5.771	3.505

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	476	940	710	722	0	907	794
N.S.	1	1.00	0.90	1.77	1.34	1.36	0.00	1.71	1.50
time (sec)	N/A	0.619	0.290	0.145	0.299	5.022	0.000	3.532	4.195

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	246	244	0	0	0	392	2500
N.S.	1	1.00	1.00	0.99	0.00	0.00	0.00	1.59	10.16
time (sec)	N/A	0.296	0.217	0.283	0.000	0.000	0.000	4.427	19.247

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	710	2228	0	0	0	3315	2500
N.S.	1	1.00	1.10	3.46	0.00	0.00	0.00	5.15	3.88
time (sec)	N/A	1.259	1.661	0.936	0.000	0.000	0.000	4.824	32.634

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	412	285	411	458	1544	565	283
N.S.	1	1.00	1.44	0.99	1.43	1.60	5.38	1.97	0.99
time (sec)	N/A	0.317	0.317	0.080	0.316	2.194	73.950	4.317	0.150

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	256	205	258	278	1001	363	204
N.S.	1	1.00	1.21	0.97	1.22	1.31	4.72	1.71	0.96
time (sec)	N/A	0.208	0.198	0.100	0.306	2.784	46.645	3.039	3.169

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	125	138	140	549	199	125
N.S.	1	1.00	0.96	0.91	1.01	1.02	4.01	1.45	0.91
time (sec)	N/A	0.069	0.111	0.075	0.283	8.783	25.051	4.353	0.077

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	75	77	54	223	77	58
N.S.	1	1.00	0.74	1.03	1.05	0.74	3.05	1.05	0.79
time (sec)	N/A	0.026	0.041	0.083	0.276	2.450	4.763	3.956	3.119

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	104	115	0	334	112	128	117
N.S.	1	1.00	0.90	0.99	0.00	2.88	0.97	1.10	1.01
time (sec)	N/A	0.109	0.159	0.109	0.000	1.421	14.416	5.444	0.142

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	144	150	156	0	617	0	175	146
N.S.	1	1.03	1.07	1.11	0.00	4.41	0.00	1.25	1.04
time (sec)	N/A	0.183	0.478	0.126	0.000	1.114	0.000	5.113	0.233

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	210	203	260	0	1062	0	373	270
N.S.	1	1.02	0.99	1.26	0.00	5.16	0.00	1.81	1.31
time (sec)	N/A	0.240	0.830	0.114	0.000	1.761	0.000	4.006	0.281

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	406	650	419	466	452	669	394
N.S.	1	1.00	1.42	2.28	1.47	1.64	1.59	2.35	1.38
time (sec)	N/A	0.262	0.364	0.110	0.285	1.163	47.049	2.970	0.116

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	252	379	266	287	272	404	270
N.S.	1	1.00	1.20	1.80	1.27	1.37	1.30	1.92	1.29
time (sec)	N/A	0.188	0.213	0.092	0.277	2.134	25.837	2.728	3.127

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	128	173	146	150	141	204	147
N.S.	1	1.00	0.95	1.28	1.08	1.11	1.04	1.51	1.09
time (sec)	N/A	0.061	0.114	0.078	0.304	3.759	12.559	3.999	3.134

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	63	66	63	70	74	58
N.S.	1	1.00	0.76	0.89	0.93	0.89	0.99	1.04	0.82
time (sec)	N/A	0.024	0.046	0.071	0.301	4.685	5.222	3.036	0.060

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	124	122	0	543	116	112	162
N.S.	1	1.00	1.02	1.00	0.00	4.45	0.95	0.92	1.33
time (sec)	N/A	0.143	0.229	0.102	0.000	4.806	19.694	3.438	3.209

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	176	175	0	1110	0	282	218
N.S.	1	1.00	1.07	1.06	0.00	6.73	0.00	1.71	1.32
time (sec)	N/A	0.233	0.654	0.091	0.000	12.158	0.000	3.485	0.300

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	297	294	0	1888	0	462	363
N.S.	1	1.00	1.20	1.19	0.00	7.61	0.00	1.86	1.46
time (sec)	N/A	0.401	1.310	0.095	0.000	6.474	0.000	5.430	3.409

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	191	102	231	0	214	0	16	916
N.S.	1	2.10	1.12	2.54	0.00	2.35	0.00	0.18	10.07
time (sec)	N/A	0.093	0.239	0.143	0.000	2.801	0.000	5.151	5.018

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	141	425	0	378	0	184	833
N.S.	1	1.00	0.86	2.59	0.00	2.30	0.00	1.12	5.08
time (sec)	N/A	0.118	0.340	0.084	0.000	3.315	0.000	5.017	22.379

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	289	1207	0	844	0	1048	-1
N.S.	1	1.00	0.87	3.62	0.00	2.53	0.00	3.15	-0.00
time (sec)	N/A	0.223	0.816	0.072	0.000	5.555	0.000	4.668	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	199	763	0	564	0	436	1832
N.S.	1	1.00	0.81	3.10	0.00	2.29	0.00	1.77	7.45
time (sec)	N/A	0.162	0.731	0.081	0.000	5.514	0.000	2.877	74.336

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	141	425	0	378	0	184	833
N.S.	1	1.00	0.86	2.59	0.00	2.30	0.00	1.12	5.08
time (sec)	N/A	0.086	0.055	0.000	0.000	6.439	0.000	4.144	0.002

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	131	697	0	564	0	218	-1
N.S.	1	1.00	1.02	5.40	0.00	4.37	0.00	1.69	-0.01
time (sec)	N/A	0.089	0.340	0.083	0.000	5.183	0.000	4.099	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	145	773	0	770	0	286	-1
N.S.	1	1.00	0.91	4.83	0.00	4.81	0.00	1.79	-0.01
time (sec)	N/A	0.118	0.229	0.088	0.000	7.884	0.000	4.274	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	177	210	0	360	0	452	260
N.S.	1	1.00	0.89	1.06	0.00	1.82	0.00	2.28	1.31
time (sec)	N/A	0.135	0.177	0.106	0.000	19.885	0.000	6.146	4.304

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	301	427	0	658	0	752	452
N.S.	1	1.00	1.07	1.52	0.00	2.34	0.00	2.68	1.61
time (sec)	N/A	0.180	0.352	0.082	0.000	84.763	0.000	2.514	4.654

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	167	834	0	573	0	0	-1
N.S.	1	1.00	0.67	3.35	0.00	2.30	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.443	0.090	0.000	4.287	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	189	571	0	514	0	717	-1
N.S.	1	1.00	0.79	2.38	0.00	2.14	0.00	2.99	-0.00
time (sec)	N/A	0.144	0.431	0.083	0.000	3.133	0.000	3.583	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	152	392	0	398	0	441	1797
N.S.	1	1.00	0.86	2.23	0.00	2.26	0.00	2.51	10.21
time (sec)	N/A	0.106	0.441	0.090	0.000	2.482	0.000	4.345	73.154

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	103	247	0	308	0	145	893
N.S.	1	1.00	0.84	2.02	0.00	2.52	0.00	1.19	7.32
time (sec)	N/A	0.076	0.266	0.082	0.000	4.911	0.000	2.992	20.635

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	122	438	0	457	0	193	-1
N.S.	1	1.00	1.13	4.06	0.00	4.23	0.00	1.79	-0.01
time (sec)	N/A	0.073	0.255	0.074	0.000	4.591	0.000	3.987	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	99	601	0	651	0	218	-1
N.S.	1	1.00	0.85	5.18	0.00	5.61	0.00	1.88	-0.01
time (sec)	N/A	0.077	0.171	0.077	0.000	6.034	0.000	3.378	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	115	122	0	286	0	337	268
N.S.	1	1.00	0.86	0.92	0.00	2.15	0.00	2.53	2.02
time (sec)	N/A	0.078	0.129	0.077	0.000	5.380	0.000	3.548	4.301

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	185	207	0	482	0	524	389
N.S.	1	1.00	0.98	1.10	0.00	2.55	0.00	2.77	2.06
time (sec)	N/A	0.118	0.161	0.094	0.000	8.212	0.000	3.453	4.506

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	473	11688	0	0	0	0	-1
N.S.	1	1.00	1.13	28.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.029	4.278	0.183	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	925	5482	0	4495	0	0	-1
N.S.	1	1.00	3.25	19.24	0.00	15.77	0.00	0.00	-0.00
time (sec)	N/A	0.363	10.903	0.136	0.000	100.483	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	410	5507	0	0	0	0	-1
N.S.	1	1.00	1.43	19.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	3.511	0.158	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	543	47351	0	0	0	0	-1
N.S.	1	1.00	1.27	110.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.910	3.754	0.231	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	513	879	0	0	0	0	-1
N.S.	1	1.00	0.96	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.051	3.280	0.147	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	316	585	0	0	0	0	-1
N.S.	1	1.00	0.97	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	1.565	0.145	0.000	0.000	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	213	407	0	0	0	0	-1
N.S.	1	1.00	0.97	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.884	0.123	0.000	0.000	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	140	329	0	990	0	0	-1
N.S.	1	1.00	0.92	2.16	0.00	6.51	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.082	0.128	0.000	5.279	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	212	673	0	0	0	0	-1
N.S.	1	1.00	0.93	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.655	0.130	0.000	0.000	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	228	1359	0	0	0	0	-1
N.S.	1	1.00	0.47	2.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.448	1.007	0.115	0.000	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	609	2512	0	0	0	1844	-1
N.S.	1	1.00	0.90	3.73	0.00	0.00	0.00	2.74	-0.00
time (sec)	N/A	0.556	11.029	0.153	0.000	0.000	0.000	6.856	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	933	933	858	3777	0	0	0	8035	-1
N.S.	1	1.00	0.92	4.05	0.00	0.00	0.00	8.61	-0.00
time (sec)	N/A	0.809	12.741	0.123	0.000	0.000	0.000	15.268	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1098	1098	743	1409	0	0	0	0	-1
N.S.	1	1.00	0.68	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.326	11.644	0.170	0.000	0.000	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	662	662	536	998	0	0	0	0	-1
N.S.	1	1.00	0.81	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.978	10.911	0.172	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	427	742	0	0	0	0	-1
N.S.	1	1.00	0.97	1.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	2.696	0.135	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	625	0	0	0	0	-1
N.S.	1	1.00	0.96	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.102	0.128	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	323	1265	0	0	0	0	-1
N.S.	1	1.00	0.66	2.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	10.748	0.191	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	787	357	2372	0	0	0	0	-1
N.S.	1	1.00	0.45	3.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.881	10.984	0.167	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1066	1066	1036	4242	0	0	0	0	-1
N.S.	1	1.00	0.97	3.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.086	12.175	0.138	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	886	886	647	2107	0	0	0	0	-1
N.S.	1	1.00	0.73	2.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.094	11.713	0.219	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	380	755	0	0	0	0	-1
N.S.	1	1.00	0.88	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.859	2.167	0.167	0.000	0.000	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	247	480	0	0	0	0	-1
N.S.	1	1.00	0.91	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.448	1.252	0.132	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	183	301	0	0	0	0	-1
N.S.	1	1.00	1.04	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.717	0.126	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	142	199	0	1073	0	0	-1
N.S.	1	1.00	1.08	1.52	0.00	8.19	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.578	0.124	0.000	196.387	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	92	157	0	340	0	72	-1
N.S.	1	1.00	1.16	1.99	0.00	4.30	0.00	0.91	-0.01
time (sec)	N/A	0.025	0.018	0.120	0.000	3.321	0.000	3.206	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	207	327	0	1952	0	0	-1
N.S.	1	1.00	1.14	1.80	0.00	10.73	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.770	0.133	0.000	70.441	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	256	609	0	0	0	0	-1
N.S.	1	1.00	0.75	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	10.692	0.132	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	549	1185	0	0	0	2256	-1
N.S.	1	1.00	0.94	2.02	0.00	0.00	0.00	3.84	-0.00
time (sec)	N/A	0.563	11.508	0.128	0.000	0.000	0.000	4.861	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	587	1026	0	0	0	0	-1
N.S.	1	1.00	1.18	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.724	11.493	0.159	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	354	740	0	0	0	0	-1
N.S.	1	1.00	0.99	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	4.029	0.137	0.000	0.000	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	244	555	0	2032	0	757	-1
N.S.	1	1.00	1.02	2.31	0.00	8.47	0.00	3.15	-0.00
time (sec)	N/A	0.188	1.197	0.132	0.000	12.230	0.000	6.140	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	198	445	0	1630	0	568	-1
N.S.	1	1.00	1.06	2.38	0.00	8.72	0.00	3.04	-0.01
time (sec)	N/A	0.088	0.825	0.133	0.000	14.438	0.000	4.497	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	187	400	0	1306	0	447	-1
N.S.	1	1.00	1.21	2.58	0.00	8.43	0.00	2.88	-0.01
time (sec)	N/A	0.066	0.146	0.114	0.000	6.987	0.000	4.349	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	347	815	0	0	0	0	-1
N.S.	1	1.00	0.99	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.286	3.123	0.165	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	623	1481	0	0	0	0	-1
N.S.	1	1.00	0.97	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	13.235	0.157	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	1013	2685	0	0	0	14731	-1
N.S.	1	1.00	0.95	2.52	0.00	0.00	0.00	13.84	-0.00
time (sec)	N/A	1.186	15.278	0.150	0.000	0.000	0.000	12.059	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1551	1551	26600	32647	0	1782	0	0	-1
N.S.	1	1.00	17.15	21.05	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	5.336	35.026	0.283	0.000	0.703	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1015	1015	15781	20224	0	1158	0	0	-1
N.S.	1	1.00	15.55	19.93	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	2.622	33.952	0.155	0.000	0.430	0.000	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	8432	10711	0	740	0	0	-1
N.S.	1	1.00	12.93	16.43	0.00	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.705	33.058	0.129	0.000	0.761	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	1086	4356	0	481	0	0	-1
N.S.	1	1.00	2.12	8.49	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.402	28.195	0.156	0.000	0.470	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	764	969	35245	6812	0	0	0	0	-1
N.S.	1	1.27	46.13	8.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.730	34.175	0.174	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	743	934	16573	16696	0	0	0	0	-1
N.S.	1	1.26	22.31	22.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.226	33.308	0.141	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1034	1705	33765	55360	0	0	0	0	-1
N.S.	1	1.65	32.65	53.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.413	34.715	0.214	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1098	1098	17771	22215	0	1267	0	0	-1
N.S.	1	1.00	16.18	20.23	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	4.406	34.282	0.158	0.000	0.622	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	755	755	10030	12922	0	840	0	0	-1
N.S.	1	1.00	13.28	17.12	0.00	1.11	0.00	0.00	-0.00
time (sec)	N/A	1.321	33.317	0.150	0.000	0.361	0.000	0.000	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	792	6207	0	567	0	0	-1
N.S.	1	1.00	1.53	11.96	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.382	28.165	0.138	0.000	0.785	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	936	1854	0	417	0	0	-1
N.S.	1	1.00	2.11	4.18	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.256	25.434	0.132	0.000	0.932	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	700	700	1261	3126	0	0	0	0	-1
N.S.	1	1.00	1.80	4.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.335	29.923	0.138	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	736	957	1471	13872	0	0	0	0	-1
N.S.	1	1.30	2.00	18.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.214	32.260	0.152	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1049	1747	36617	57841	0	0	0	0	-1
N.S.	1	1.67	34.91	55.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.536	34.740	0.208	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	1402	14978	0	883	0	0	-1
N.S.	1	1.00	1.81	19.35	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	1.391	32.253	0.143	0.000	0.848	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	567	567	1002	8248	0	613	0	0	-1
N.S.	1	1.00	1.77	14.55	0.00	1.08	0.00	0.00	-0.00
time (sec)	N/A	0.656	29.281	0.154	0.000	0.550	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	638	3805	0	453	0	0	-1
N.S.	1	1.00	1.41	8.42	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	24.733	0.125	0.000	0.610	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	365	747	0	359	0	0	-1
N.S.	1	1.00	1.94	3.97	0.00	1.91	0.00	0.00	-0.01
time (sec)	N/A	0.049	20.563	0.117	0.000	0.388	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	379	834	0	0	0	0	-1
N.S.	1	1.00	0.81	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.041	20.670	0.134	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	994	994	1502	13017	0	0	0	0	-1
N.S.	1	1.00	1.51	13.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.484	32.651	0.124	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1786	1786	36634	59522	0	0	0	0	-1
N.S.	1	1.00	20.51	33.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.675	34.862	0.233	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	1385	1879	0	0	0	0	-1
N.S.	1	1.00	2.05	2.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.138	12.485	0.156	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1138	1138	37137	7464	0	0	0	0	-1
N.S.	1	1.00	32.63	6.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.474	34.337	0.156	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	855	8755	0	626	0	0	-1
N.S.	1	1.00	1.35	13.87	0.00	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.736	32.684	0.141	0.000	0.668	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	981	4295	0	467	0	0	-1
N.S.	1	1.00	2.05	8.97	0.00	0.97	0.00	0.00	-0.00
time (sec)	N/A	0.416	28.963	0.129	0.000	0.327	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	814	1014	0	369	0	0	-1
N.S.	1	1.00	2.07	2.58	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.166	23.688	0.116	0.000	0.292	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	308	287	0	128	0	0	-1
N.S.	1	1.00	1.63	1.52	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.068	20.462	0.116	0.000	0.438	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	499	330	0	0	0	0	-1
N.S.	1	1.00	1.78	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.817	21.157	0.139	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1037	1037	842	14048	0	0	0	0	-1
N.S.	1	1.00	0.81	13.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.387	33.027	0.134	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1114	1762	40396	64947	0	0	0	0	-1
N.S.	1	1.58	36.26	58.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.421	35.348	0.288	0.000	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	950	4757	0	0	0	0	-1
N.S.	1	1.00	1.72	8.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.115	24.037	0.144	0.000	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1125	1125	14759	27597	0	0	0	0	-1
N.S.	1	1.00	13.12	24.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.606	34.029	0.199	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	475	475	1118	590	0	0	0	0	-1
N.S.	1	1.00	2.35	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	26.599	0.208	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	375	550	0	0	0	0	-1
N.S.	1	1.00	0.64	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.683	24.772	0.140	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	440	1249	690	1222	15757	2740	1354
N.S.	1	1.00	2.00	5.68	3.14	5.55	71.62	12.45	6.15
time (sec)	N/A	0.134	0.445	0.086	0.363	2.229	3.224	1.728	3.945

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	180	503	358	485	5930	1162	602
N.S.	1	1.00	1.25	3.49	2.49	3.37	41.18	8.07	4.18
time (sec)	N/A	0.073	0.306	0.087	0.325	3.424	1.380	2.184	3.594

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	166	0	0	0	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.550	0.026	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.215	0.029	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	243	185	0	0	0	0	0	-1
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.663	0.042	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	1263	4653	2031	4356	74400	10489	2500
N.S.	1	1.00	2.41	8.86	3.87	8.30	141.71	19.98	4.76
time (sec)	N/A	0.381	1.858	0.161	0.339	2.885	13.840	1.917	5.383

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	655	2149	1121	2084	32864	4940	2307
N.S.	1	1.00	2.11	6.91	3.60	6.70	105.67	15.88	7.42
time (sec)	N/A	0.235	1.140	0.129	0.342	3.221	6.383	3.583	4.380

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	254	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.547	0.585	0.062	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.726	0.989	0.063	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	281	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.942	2.117	0.072	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	151	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.291	0.069	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	117	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.142	0.066	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	91	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.107	0.062	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.100	0.059	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	94	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.110	0.057	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	233	0	0	0	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.568	0.064	0.000	0.000	0.000	0.000	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	152	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.177	0.071	0.000	0.000	0.000	0.000	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	251	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.193	0.346	0.082	0.000	0.000	0.000	0.000	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	252	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.331	0.083	0.000	0.000	0.000	0.000	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	156	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.332	0.082	0.000	0.000	0.000	0.000	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	149	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.215	0.077	0.000	0.000	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.180	0.076	0.000	0.000	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	274	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.572	0.086	0.000	0.000	0.000	0.000	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	287	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	0.500	0.087	0.000	0.000	0.000	0.000	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	230	171	0	0	0	0	0	-1
N.S.	1	0.97	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.230	0.523	0.020	0.000	0.000	0.000	0.000	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	509	506	0	0	0	0	0	0	-1
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	1.425	0.059	0.000	0.000	0.000	0.000	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.884	0.050	0.000	0.000	0.000	0.000	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.015	0.051	0.000	0.000	0.000	0.000	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.501	0.057	0.000	0.000	0.000	0.000	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	502	500	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	1.381	0.057	0.000	0.000	0.000	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.941	0.053	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.019	0.052	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.598	0.062	0.000	0.000	0.000	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	263	189	0	0	0	0	0	-1
N.S.	1	0.99	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.335	0.021	0.000	0.000	0.000	0.000	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	525	523	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	1.528	0.060	0.000	0.000	0.000	0.000	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.887	0.053	0.000	0.000	0.000	0.000	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	205	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.077	0.086	0.000	0.000	0.000	0.000	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.227	0.064	0.000	0.000	0.000	0.000	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	188	148	0	0	0	0	-1
N.S.	1	1.00	2.11	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	20.484	0.105	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [781] had the largest ratio of [73]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	5	1.00	25	0.200
2	A	8	5	1.00	25	0.200
3	A	7	5	1.00	25	0.200
4	A	12	5	1.00	25	0.200
5	A	5	4	1.00	23	0.174
6	A	5	4	1.00	23	0.174
7	A	8	7	1.00	25	0.280
8	A	8	8	1.00	25	0.320
9	A	8	7	1.00	25	0.280
10	A	8	8	1.00	25	0.320
11	A	8	7	1.00	25	0.280
12	A	6	5	1.00	25	0.200
13	A	7	6	1.00	25	0.240
14	A	8	6	1.00	25	0.240
15	A	9	6	1.00	25	0.240
16	A	8	5	1.00	25	0.200
17	A	5	5	1.00	25	0.200
18	A	3	3	1.00	25	0.120
19	A	6	4	1.00	25	0.160
20	A	6	4	1.00	25	0.160
21	A	5	4	1.00	25	0.160
22	A	4	3	1.00	25	0.120
23	A	3	3	1.00	25	0.120
24	A	3	3	1.00	25	0.120
25	A	3	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	3	1.00	22	0.136
27	A	7	5	1.00	25	0.200
28	A	7	5	1.00	25	0.200
29	A	8	6	1.00	25	0.240
30	A	4	4	1.00	25	0.160
31	A	5	4	1.00	25	0.160
32	A	4	4	1.00	24	0.167
33	A	7	5	1.00	27	0.185
34	A	6	5	1.00	27	0.185
35	A	5	5	1.00	27	0.185
36	A	4	4	1.00	25	0.160
37	A	4	4	1.00	24	0.167
38	A	7	7	1.00	27	0.259
39	A	7	7	1.00	27	0.259
40	A	5	5	1.00	27	0.185
41	A	6	6	1.00	27	0.222
42	A	7	6	1.00	27	0.222
43	A	8	6	1.00	27	0.222
44	A	6	5	1.00	27	0.185
45	A	6	5	1.00	27	0.185
46	A	3	2	1.00	27	0.074
47	A	3	3	1.00	27	0.111
48	A	3	3	1.00	25	0.120
49	A	3	3	1.00	24	0.125
50	A	7	6	1.00	27	0.222
51	A	7	5	1.00	27	0.185
52	A	8	6	1.00	27	0.222
53	A	9	6	1.00	27	0.222
54	A	5	4	1.00	20	0.200
55	A	4	4	1.00	20	0.200
56	A	3	3	1.00	18	0.167
57	A	3	3	1.00	17	0.176
58	A	6	6	1.00	20	0.300
59	A	6	6	1.00	20	0.300
60	A	5	5	1.00	20	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	6	6	1.00	20	0.300
62	A	7	6	1.00	20	0.300
63	A	8	6	1.00	20	0.300
64	A	9	8	1.00	27	0.296
65	A	12	6	1.00	27	0.222
66	A	11	6	1.00	27	0.222
67	A	10	6	1.00	27	0.222
68	A	9	6	1.00	27	0.222
69	A	9	6	1.00	25	0.240
70	A	8	5	1.00	24	0.208
71	A	11	8	1.00	27	0.296
72	A	11	9	1.00	27	0.333
73	A	11	8	1.00	27	0.296
74	A	11	9	1.00	27	0.333
75	A	11	9	1.00	27	0.333
76	A	11	8	1.00	27	0.296
77	A	11	9	1.00	27	0.333
78	A	11	9	1.00	27	0.333
79	A	11	8	1.00	27	0.296
80	A	9	6	1.00	27	0.222
81	A	10	7	1.00	27	0.259
82	A	11	7	1.00	27	0.259
83	A	7	5	1.00	27	0.185
84	A	6	4	1.00	27	0.148
85	A	5	4	1.00	27	0.148
86	A	3	3	1.00	27	0.111
87	A	3	3	1.00	25	0.120
88	A	4	3	1.00	24	0.125
89	A	7	6	1.00	27	0.222
90	A	7	5	1.00	27	0.185
91	A	8	6	1.00	27	0.222
92	A	7	5	1.00	27	0.185
93	A	6	5	1.00	27	0.185
94	A	6	6	1.00	27	0.222
95	A	4	4	1.00	25	0.160

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	24	0.125
97	A	7	7	1.00	27	0.259
98	A	5	5	1.00	27	0.185
99	A	6	6	1.00	27	0.222
100	A	7	6	1.00	27	0.222
101	A	8	6	1.00	27	0.222
102	A	7	7	1.00	27	0.259
103	A	9	6	1.00	27	0.222
104	A	8	6	1.00	27	0.222
105	A	8	7	1.00	27	0.259
106	A	6	5	1.00	25	0.200
107	A	5	4	1.00	24	0.167
108	A	9	8	1.00	27	0.296
109	A	9	9	1.00	27	0.333
110	A	9	8	1.00	27	0.296
111	A	9	9	1.00	27	0.333
112	A	9	8	1.00	27	0.296
113	A	7	6	1.00	27	0.222
114	A	8	7	1.00	27	0.259
115	A	9	7	1.00	27	0.259
116	A	10	7	1.00	27	0.259
117	A	3	3	1.00	18	0.167
118	A	7	7	1.00	26	0.269
119	A	6	6	1.00	27	0.222
120	A	5	5	1.00	27	0.185
121	A	5	5	1.00	27	0.185
122	A	3	3	1.00	25	0.120
123	A	1	1	1.00	24	0.042
124	A	5	5	1.00	27	0.185
125	A	5	5	1.00	27	0.185
126	A	6	6	1.00	27	0.222
127	A	6	5	1.00	27	0.185
128	A	6	5	1.00	27	0.185
129	A	5	5	1.00	27	0.185
130	A	3	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	25	0.080
132	A	2	2	1.00	24	0.083
133	A	6	6	1.00	27	0.222
134	A	6	6	1.00	27	0.222
135	A	7	7	1.00	27	0.259
136	A	7	5	1.00	27	0.185
137	A	7	5	1.00	27	0.185
138	A	6	5	1.00	27	0.185
139	A	5	4	1.00	27	0.148
140	A	4	4	1.00	27	0.148
141	A	3	3	1.00	27	0.111
142	A	3	3	1.00	25	0.120
143	A	3	3	1.00	24	0.125
144	A	7	6	1.00	27	0.222
145	A	7	6	1.00	27	0.222
146	A	8	7	1.00	27	0.259
147	A	9	7	1.00	27	0.259
148	A	5	5	1.00	27	0.185
149	A	4	4	1.00	27	0.148
150	A	4	4	1.00	25	0.160
151	A	4	4	1.00	25	0.160
152	A	2	2	1.00	23	0.087
153	A	1	1	1.00	22	0.045
154	A	5	5	1.00	26	0.192
155	A	5	5	1.00	26	0.192
156	A	6	6	1.00	26	0.231
157	A	10	7	1.00	27	0.259
158	A	9	7	1.00	27	0.259
159	A	8	7	1.00	27	0.259
160	A	7	7	1.00	27	0.259
161	A	6	5	1.00	25	0.200
162	A	6	6	1.00	24	0.250
163	A	9	9	1.00	27	0.333
164	A	9	9	1.00	27	0.333
165	A	9	9	1.00	27	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	9	9	1.00	27	0.333
167	A	7	7	1.00	27	0.259
168	A	8	8	1.00	27	0.296
169	A	9	8	1.00	27	0.296
170	A	10	8	1.00	27	0.296
171	A	7	6	1.00	27	0.222
172	A	4	3	1.00	27	0.111
173	A	4	4	1.00	27	0.148
174	A	3	3	1.00	25	0.120
175	A	3	2	1.00	24	0.083
176	A	8	7	1.00	27	0.259
177	A	8	6	1.00	27	0.222
178	A	9	7	1.00	27	0.259
179	A	8	6	1.00	27	0.222
180	A	7	5	1.00	27	0.185
181	A	6	5	1.00	27	0.185
182	A	4	4	1.00	27	0.148
183	A	3	3	1.00	25	0.120
184	A	3	2	1.00	24	0.083
185	A	8	7	1.00	27	0.259
186	A	8	6	1.00	27	0.222
187	A	9	7	1.00	27	0.259
188	A	9	6	1.00	27	0.222
189	A	7	5	1.00	27	0.185
190	A	9	7	1.00	27	0.259
191	A	8	6	1.00	27	0.222
192	A	2	2	1.00	25	0.080
193	A	2	2	1.00	24	0.083
194	A	8	7	1.00	27	0.259
195	A	8	6	1.00	27	0.222
196	A	9	7	1.00	27	0.259
197	A	10	7	1.00	27	0.259
198	A	11	6	1.00	27	0.222
199	A	10	6	1.00	27	0.222
200	A	9	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	8	7	1.00	27	0.259
202	A	6	6	1.00	25	0.240
203	A	5	4	1.00	24	0.167
204	A	9	9	1.00	27	0.333
205	A	9	9	1.00	27	0.333
206	A	7	7	1.00	27	0.259
207	A	8	7	1.00	27	0.259
208	A	9	7	1.00	27	0.259
209	A	10	7	1.00	27	0.259
210	A	7	5	1.00	26	0.192
211	A	4	4	1.00	26	0.154
212	A	9	5	1.00	27	0.185
213	A	8	5	1.00	27	0.185
214	A	7	4	1.00	25	0.160
215	A	7	3	1.00	24	0.125
216	A	12	7	1.00	27	0.259
217	A	12	6	1.00	27	0.222
218	A	4	4	1.00	29	0.138
219	A	2	2	1.00	29	0.069
220	A	3	3	1.00	16	0.188
221	A	4	4	1.00	29	0.138
222	A	3	3	1.00	15	0.200
223	A	4	4	1.00	30	0.133
224	A	4	3	1.00	16	0.188
225	A	5	4	1.00	29	0.138
226	A	7	4	1.00	29	0.138
227	A	6	4	1.00	29	0.138
228	A	5	3	1.00	27	0.111
229	A	2	2	1.00	22	0.091
230	A	8	5	1.00	29	0.172
231	A	7	5	1.00	29	0.172
232	A	8	5	1.00	29	0.172
233	A	6	4	1.00	29	0.138
234	A	6	4	1.00	29	0.138
235	A	5	3	1.31	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	2	1.00	22	0.091
237	A	8	5	1.00	29	0.172
238	A	7	5	1.00	29	0.172
239	A	7	5	1.00	29	0.172
240	A	6	5	1.00	23	0.217
241	A	6	5	1.00	23	0.217
242	A	6	5	1.00	23	0.217
243	A	6	5	1.00	23	0.217
244	A	4	4	1.00	21	0.190
245	A	3	3	1.00	20	0.150
246	A	5	5	1.00	23	0.217
247	A	5	5	1.00	23	0.217
248	A	5	5	1.00	23	0.217
249	A	7	6	1.00	25	0.240
250	A	8	7	1.00	25	0.280
251	A	7	6	1.00	25	0.240
252	A	8	7	1.00	25	0.280
253	A	7	6	1.00	23	0.261
254	A	2	2	1.00	22	0.091
255	A	7	7	1.00	25	0.280
256	A	6	6	1.00	25	0.240
257	A	6	6	1.00	25	0.240
258	A	7	6	1.00	25	0.240
259	A	7	6	1.00	25	0.240
260	A	7	6	1.00	25	0.240
261	A	7	6	1.00	25	0.240
262	A	3	3	1.00	23	0.130
263	A	2	2	1.00	22	0.091
264	A	7	7	1.00	25	0.280
265	A	8	8	1.00	25	0.320
266	A	7	6	1.00	25	0.240
267	A	7	6	1.00	25	0.240
268	A	7	6	1.00	25	0.240
269	A	7	6	1.00	25	0.240
270	A	5	5	1.00	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	2	2	1.00	22	0.091
272	A	6	6	1.00	25	0.240
273	A	6	6	1.00	25	0.240
274	A	6	6	1.00	25	0.240
275	A	8	7	1.00	25	0.280
276	A	9	8	1.00	25	0.320
277	A	8	7	1.00	25	0.280
278	A	9	8	1.00	25	0.320
279	A	3	3	1.00	23	0.130
280	A	2	2	1.00	22	0.091
281	A	8	8	1.00	25	0.320
282	A	7	7	1.00	25	0.280
283	A	7	7	1.00	25	0.280
284	A	7	7	1.00	25	0.280
285	A	7	7	1.00	25	0.280
286	A	8	7	1.00	25	0.280
287	A	8	7	1.00	25	0.280
288	A	4	4	1.00	25	0.160
289	A	3	3	1.00	23	0.130
290	A	2	2	1.00	22	0.091
291	A	8	8	1.00	25	0.320
292	A	9	9	1.00	25	0.360
293	A	8	7	1.00	25	0.280
294	A	8	7	1.00	25	0.280
295	A	8	7	1.00	25	0.280
296	A	9	8	1.00	25	0.320
297	A	5	4	1.00	25	0.160
298	A	4	4	1.00	25	0.160
299	A	3	3	1.00	23	0.130
300	A	2	2	1.00	22	0.091
301	A	9	9	1.00	25	0.360
302	A	9	9	1.00	25	0.360
303	A	10	9	1.00	25	0.360
304	A	9	7	1.00	25	0.280
305	A	9	7	1.00	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	7	4	1.00	27	0.148
307	A	6	4	1.00	27	0.148
308	A	5	3	1.00	25	0.120
309	A	2	2	1.00	20	0.100
310	A	8	5	1.00	27	0.185
311	A	7	5	1.00	27	0.185
312	A	8	5	1.00	27	0.185
313	A	6	4	1.00	25	0.160
314	A	4	3	1.00	27	0.111
315	A	11	7	1.00	16	0.438
316	A	9	6	1.00	22	0.273
317	A	8	6	1.00	22	0.273
318	A	8	7	1.00	22	0.318
319	A	6	5	1.00	20	0.250
320	A	6	5	1.00	19	0.263
321	A	9	8	1.00	22	0.364
322	A	15	11	1.00	22	0.500
323	A	19	12	1.00	22	0.546
324	A	20	13	1.00	22	0.591
325	A	25	14	1.00	22	0.636
326	A	8	5	1.00	22	0.227
327	A	7	5	1.00	22	0.227
328	A	7	6	1.00	22	0.273
329	A	5	4	1.00	20	0.200
330	A	2	2	1.00	19	0.105
331	A	7	6	1.00	22	0.273
332	A	8	7	1.00	22	0.318
333	A	12	8	1.00	22	0.364
334	A	7	6	1.00	22	0.273
335	A	6	5	1.00	22	0.227
336	A	4	4	1.00	22	0.182
337	A	4	4	1.00	20	0.200
338	A	4	4	1.00	19	0.210
339	A	10	9	1.00	22	0.409
340	A	12	11	1.00	22	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	17	12	1.00	22	0.546
342	A	9	6	1.00	22	0.273
343	A	8	6	1.00	22	0.273
344	A	7	6	1.00	22	0.273
345	A	6	5	1.00	22	0.227
346	A	3	3	1.00	20	0.150
347	A	3	3	1.00	19	0.158
348	A	10	7	1.00	22	0.318
349	A	11	8	1.00	22	0.364
350	A	15	9	1.00	22	0.409
351	A	2	1	1.00	18	0.056
352	A	2	1	1.00	16	0.062
353	A	2	1	1.00	15	0.067
354	A	3	3	1.00	18	0.167
355	A	2	1	1.00	20	0.050
356	A	2	1	1.00	18	0.056
357	A	2	1	1.00	17	0.059
358	A	4	3	1.00	20	0.150
359	A	2	1	1.00	20	0.050
360	A	2	1	1.00	18	0.056
361	A	2	1	1.00	17	0.059
362	A	4	3	1.00	20	0.150
363	A	6	3	1.00	20	0.150
364	A	6	3	1.00	20	0.150
365	A	6	3	1.00	20	0.150
366	A	4	2	1.00	18	0.111
367	A	4	2	1.00	17	0.118
368	A	7	4	1.00	20	0.200
369	A	7	4	1.00	20	0.200
370	A	5	3	1.00	20	0.150
371	A	5	3	1.00	20	0.150
372	A	5	3	1.00	20	0.150
373	A	5	3	1.00	18	0.167
374	A	5	3	1.00	17	0.176
375	A	12	5	1.00	20	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	12	6	1.00	20	0.300
377	A	6	5	0.94	22	0.227
378	A	4	4	0.91	20	0.200
379	A	6	3	1.00	22	0.136
380	A	12	4	1.00	22	0.182
381	A	6	5	1.00	18	0.278
382	A	6	5	1.00	18	0.278
383	A	6	5	1.00	18	0.278
384	A	6	5	1.00	18	0.278
385	A	4	4	1.00	16	0.250
386	A	3	3	1.00	15	0.200
387	A	5	5	1.00	18	0.278
388	A	5	5	1.00	18	0.278
389	A	5	5	1.00	18	0.278
390	A	7	6	1.00	20	0.300
391	A	8	7	0.95	20	0.350
392	A	7	6	1.00	20	0.300
393	A	8	7	0.95	20	0.350
394	A	7	6	1.00	18	0.333
395	A	4	4	0.94	17	0.235
396	A	7	7	1.00	20	0.350
397	A	6	6	1.00	20	0.300
398	A	6	6	1.00	20	0.300
399	A	7	6	0.98	20	0.300
400	A	7	6	0.97	20	0.300
401	A	7	6	0.97	20	0.300
402	A	7	6	0.96	20	0.300
403	A	7	6	0.95	18	0.333
404	A	4	4	0.96	17	0.235
405	A	7	7	0.96	20	0.350
406	A	8	8	1.00	20	0.400
407	A	7	6	1.00	20	0.300
408	A	7	6	1.00	20	0.300
409	A	6	6	1.00	20	0.300
410	A	6	6	1.00	20	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	9	8	1.00	18	0.444
412	A	6	5	1.00	17	0.294
413	A	7	7	1.00	20	0.350
414	A	7	7	1.00	20	0.350
415	A	8	8	1.00	20	0.400
416	A	12	10	1.00	20	0.500
417	A	11	10	1.00	20	0.500
418	A	10	9	0.99	20	0.450
419	A	10	9	1.00	18	0.500
420	A	8	7	0.78	17	0.412
421	A	18	10	1.00	20	0.500
422	A	20	12	1.00	20	0.600
423	A	12	10	1.00	20	0.500
424	A	11	9	1.00	20	0.450
425	A	11	10	1.00	20	0.500
426	A	11	9	1.00	18	0.500
427	A	11	9	1.00	17	0.529
428	A	29	12	1.00	20	0.600
429	A	31	14	1.00	20	0.700
430	A	7	4	0.92	22	0.182
431	A	6	4	0.95	22	0.182
432	A	5	3	1.00	20	0.150
433	A	2	2	1.00	15	0.133
434	A	5	3	1.00	22	0.136
435	A	8	3	1.00	22	0.136
436	A	10	3	1.00	22	0.136
437	A	6	5	1.00	40	0.125
438	A	5	5	1.00	40	0.125
439	A	4	4	1.00	38	0.105
440	A	3	3	1.00	37	0.081
441	A	6	5	1.00	40	0.125
442	A	4	4	1.00	40	0.100
443	A	5	5	1.00	40	0.125
444	A	6	5	1.00	40	0.125
445	A	7	5	1.00	40	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	7	6	1.00	40	0.150
447	A	6	6	1.00	40	0.150
448	A	5	5	1.00	38	0.132
449	A	4	4	1.00	37	0.108
450	A	7	6	1.00	40	0.150
451	A	7	6	1.00	40	0.150
452	A	7	6	1.00	40	0.150
453	A	5	5	1.00	40	0.125
454	A	6	6	1.00	40	0.150
455	A	7	6	1.00	40	0.150
456	A	8	6	1.00	40	0.150
457	A	8	6	1.00	40	0.150
458	A	7	6	1.00	40	0.150
459	A	6	5	1.00	38	0.132
460	A	5	4	1.00	37	0.108
461	A	8	6	1.00	40	0.150
462	A	8	7	1.00	40	0.175
463	A	8	6	1.00	40	0.150
464	A	8	7	1.00	40	0.175
465	A	8	6	1.00	40	0.150
466	A	6	5	1.00	40	0.125
467	A	7	6	1.00	40	0.150
468	A	8	6	1.00	40	0.150
469	A	9	6	1.00	40	0.150
470	A	5	5	1.10	40	0.125
471	A	4	4	1.00	40	0.100
472	A	3	3	1.00	38	0.079
473	A	1	1	1.00	37	0.027
474	A	5	5	1.00	40	0.125
475	A	5	5	1.00	40	0.125
476	A	6	6	1.00	40	0.150
477	A	6	5	1.00	40	0.125
478	A	6	5	1.00	40	0.125
479	A	5	5	1.00	40	0.125
480	A	3	3	1.00	40	0.075

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	2	2	1.00	38	0.053
482	A	2	2	1.00	37	0.054
483	A	6	5	1.00	40	0.125
484	A	6	5	1.00	40	0.125
485	A	7	6	1.00	40	0.150
486	A	8	6	1.00	40	0.150
487	A	3	3	1.00	40	0.075
488	A	4	4	1.00	40	0.100
489	A	4	4	1.00	23	0.174
490	A	1	1	1.00	23	0.043
491	A	5	5	1.00	21	0.238
492	A	3	3	1.00	20	0.150
493	A	5	5	1.00	23	0.217
494	A	5	5	1.00	23	0.217
495	A	3	3	1.00	23	0.130
496	A	5	4	1.00	23	0.174
497	A	1	1	1.00	23	0.043
498	A	6	5	1.00	21	0.238
499	A	4	3	1.00	20	0.150
500	A	6	5	1.00	23	0.217
501	A	6	6	1.00	23	0.261
502	A	4	4	1.00	23	0.174
503	A	3	3	1.00	23	0.130
504	A	1	1	1.00	23	0.043
505	A	4	4	1.00	21	0.190
506	A	2	2	1.00	20	0.100
507	A	4	4	1.00	23	0.174
508	A	5	5	1.00	23	0.217
509	A	3	3	0.99	23	0.130
510	A	3	3	1.00	23	0.130
511	A	1	1	1.00	23	0.043
512	A	5	5	1.00	21	0.238
513	A	3	3	1.00	20	0.150
514	A	5	5	1.00	23	0.217
515	A	6	6	1.00	23	0.261

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	4	4	1.00	23	0.174
517	A	4	4	1.00	23	0.174
518	A	1	1	1.00	23	0.043
519	A	6	5	1.00	21	0.238
520	A	4	3	1.00	20	0.150
521	A	6	5	1.00	23	0.217
522	A	7	6	1.00	23	0.261
523	A	5	4	1.00	23	0.174
524	A	7	6	1.00	19	0.316
525	A	6	4	1.00	25	0.160
526	A	6	4	1.22	25	0.160
527	A	6	4	1.00	25	0.160
528	A	5	4	1.00	23	0.174
529	A	4	3	1.00	22	0.136
530	A	7	5	1.00	25	0.200
531	A	9	6	0.97	25	0.240
532	A	12	6	1.00	25	0.240
533	A	6	4	1.00	25	0.160
534	A	6	4	1.00	25	0.160
535	A	6	4	1.00	25	0.160
536	A	6	4	1.00	23	0.174
537	A	5	4	1.00	22	0.182
538	A	7	5	1.00	25	0.200
539	A	9	6	1.00	25	0.240
540	A	12	6	1.00	25	0.240
541	A	6	3	1.00	23	0.130
542	A	4	2	1.00	23	0.087
543	A	4	2	1.00	23	0.087
544	A	4	2	1.00	21	0.095
545	A	4	2	1.00	20	0.100
546	A	7	4	1.00	23	0.174
547	A	8	4	1.00	23	0.174
548	A	3	2	1.00	29	0.069
549	A	3	2	1.00	29	0.069
550	A	3	2	1.00	29	0.069

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	3	2	1.00	27	0.074
552	A	5	3	1.00	22	0.136
553	A	3	2	1.00	29	0.069
554	A	4	3	1.00	29	0.103
555	A	4	3	1.00	29	0.103
556	A	4	3	1.00	29	0.103
557	A	3	2	1.00	29	0.069
558	A	3	2	1.00	29	0.069
559	A	3	2	1.00	29	0.069
560	A	3	2	1.00	29	0.069
561	A	3	2	1.00	29	0.069
562	A	3	2	1.00	29	0.069
563	A	3	2	1.00	27	0.074
564	A	2	2	1.00	22	0.091
565	A	4	3	1.00	29	0.103
566	A	4	3	1.00	29	0.103
567	A	4	3	1.00	29	0.103
568	A	4	3	1.00	29	0.103
569	A	3	2	1.00	29	0.069
570	A	3	2	1.00	29	0.069
571	A	3	2	1.00	29	0.069
572	A	3	2	1.00	29	0.069
573	A	3	2	1.00	29	0.069
574	A	4	3	1.00	29	0.103
575	A	4	3	1.00	27	0.111
576	A	3	3	1.00	22	0.136
577	A	4	3	1.00	29	0.103
578	A	4	3	1.00	29	0.103
579	A	7	5	1.00	31	0.161
580	A	6	4	1.00	31	0.129
581	A	5	4	1.00	31	0.129
582	A	3	3	1.00	31	0.097
583	A	3	3	1.00	29	0.103
584	A	4	3	1.00	24	0.125
585	A	6	5	1.00	31	0.161

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	6	4	1.00	31	0.129
587	A	7	5	1.00	31	0.161
588	A	4	3	1.00	24	0.125
589	A	3	2	1.00	24	0.083
590	A	3	2	1.00	24	0.083
591	A	2	1	1.00	22	0.045
592	A	2	1	1.00	17	0.059
593	A	4	3	1.00	24	0.125
594	A	4	4	1.03	24	0.167
595	A	4	4	1.02	24	0.167
596	A	3	2	1.00	24	0.083
597	A	3	2	1.00	24	0.083
598	A	2	1	1.00	22	0.045
599	A	2	1	1.00	17	0.059
600	A	4	3	1.00	24	0.125
601	A	4	4	1.00	24	0.167
602	A	5	5	1.00	24	0.208
603	A	5	5	1.00	26	0.192
604	A	1	1	1.00	22	0.045
605	A	11	8	1.00	28	0.286
606	A	10	7	1.00	28	0.250
607	A	6	3	1.00	28	0.107
608	A	8	5	1.00	28	0.179
609	A	11	7	1.00	28	0.250
610	A	11	7	1.00	28	0.250
611	A	6	3	1.00	28	0.107
612	A	6	3	1.00	28	0.107
613	A	8	4	1.00	28	0.143
614	A	21	10	1.00	28	0.357
615	A	8	5	1.00	28	0.179
616	A	8	4	1.00	28	0.143
617	A	12	6	0.99	28	0.214
618	A	6	3	1.00	20	0.150
619	A	5	5	1.00	26	0.192
620	A	6	6	1.00	30	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	5	5	1.00	29	0.172
622	A	10	6	1.00	28	0.214
623	A	9	6	1.00	28	0.214
624	A	7	6	1.00	26	0.231
625	A	7	6	1.00	21	0.286
626	A	14	10	1.00	28	0.357
627	A	14	10	1.00	28	0.357
628	A	23	11	1.00	28	0.393
629	A	9	6	1.00	28	0.214
630	A	8	6	0.99	28	0.214
631	A	6	5	1.00	26	0.192
632	A	6	5	1.00	21	0.238
633	A	10	9	1.00	28	0.321
634	A	14	10	1.00	28	0.357
635	A	23	11	1.00	28	0.393
636	A	8	6	0.99	28	0.214
637	A	7	6	1.00	28	0.214
638	A	6	5	1.00	26	0.192
639	A	2	2	1.00	21	0.095
640	A	7	7	1.00	28	0.250
641	A	14	10	1.00	28	0.357
642	A	23	11	1.00	28	0.393
643	A	16	10	1.35	28	0.357
644	A	10	8	1.00	28	0.286
645	A	7	6	1.00	28	0.214
646	A	7	6	1.00	28	0.214
647	A	5	4	1.00	26	0.154
648	A	2	2	1.00	21	0.095
649	A	4	4	1.00	28	0.143
650	A	14	10	1.00	28	0.357
651	A	23	10	1.00	28	0.357
652	A	10	9	1.00	28	0.321
653	A	17	12	1.00	28	0.429
654	A	4	4	1.00	28	0.143
655	A	2	2	1.00	30	0.067

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	4	3	1.00	26	0.115
657	A	4	3	1.00	46	0.065
658	A	3	3	1.00	46	0.065
659	A	2	2	1.00	44	0.045
660	A	1	1	1.00	39	0.026
661	A	2	2	1.00	46	0.043
662	A	3	3	1.00	46	0.065
663	A	4	3	1.00	46	0.065
664	A	5	3	1.00	46	0.065
665	A	4	4	1.00	46	0.087
666	A	3	3	1.00	46	0.065
667	A	2	2	1.00	44	0.045
668	A	1	1	1.00	39	0.026
669	A	3	3	1.00	46	0.065
670	A	4	4	1.00	46	0.087
671	A	5	4	1.00	46	0.087
672	A	4	3	1.00	46	0.065
673	A	3	3	1.00	46	0.065
674	A	2	2	1.00	44	0.045
675	A	1	1	1.00	39	0.026
676	A	4	3	1.00	46	0.065
677	A	5	4	1.00	46	0.087
678	A	6	4	1.00	46	0.087
679	A	5	3	1.00	46	0.065
680	A	4	3	1.00	46	0.065
681	A	3	3	1.00	46	0.065
682	A	2	2	1.00	44	0.045
683	A	1	1	1.00	39	0.026
684	A	3	3	1.00	46	0.065
685	A	3	3	1.00	46	0.065
686	A	4	4	1.00	46	0.087
687	A	5	4	1.00	46	0.087
688	A	6	4	1.00	46	0.087
689	A	5	3	1.00	46	0.065
690	A	4	3	1.00	46	0.065

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	3	3	1.00	46	0.065
692	A	2	2	1.00	44	0.045
693	A	1	1	1.00	39	0.026
694	A	4	3	1.00	46	0.065
695	A	4	4	1.00	46	0.087
696	A	4	3	1.00	46	0.065
697	A	5	4	1.00	46	0.087
698	A	6	4	1.00	46	0.087
699	A	7	4	1.00	46	0.087
700	A	5	3	1.00	46	0.065
701	A	4	3	1.00	46	0.065
702	A	3	3	1.00	46	0.065
703	A	2	2	1.00	44	0.045
704	A	1	1	1.00	39	0.026
705	A	5	3	1.00	46	0.065
706	A	5	4	1.00	46	0.087
707	A	5	4	1.00	46	0.087
708	A	5	3	1.00	46	0.065
709	A	6	4	1.00	46	0.087
710	A	7	4	1.00	46	0.087
711	A	8	4	1.00	46	0.087
712	A	7	5	1.00	48	0.104
713	A	6	5	1.00	48	0.104
714	A	5	5	1.00	48	0.104
715	A	4	4	1.00	48	0.083
716	A	1	1	1.00	48	0.021
717	A	2	2	1.00	48	0.042
718	A	3	2	1.00	48	0.042
719	A	4	2	1.00	48	0.042
720	A	7	6	1.00	48	0.125
721	A	6	6	1.00	48	0.125
722	A	5	5	1.00	48	0.104
723	A	1	1	1.00	48	0.021
724	A	2	2	1.00	48	0.042
725	A	3	3	1.00	48	0.062

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	4	3	1.00	48	0.062
727	A	7	6	1.00	48	0.125
728	A	6	5	1.00	48	0.104
729	A	1	1	1.00	48	0.021
730	A	2	2	1.00	48	0.042
731	A	3	2	1.00	48	0.042
732	A	4	3	1.00	48	0.062
733	A	8	6	1.00	48	0.125
734	A	7	6	1.00	48	0.125
735	A	6	6	1.00	48	0.125
736	A	5	5	1.00	48	0.104
737	A	5	5	1.00	48	0.104
738	A	1	1	1.00	48	0.021
739	A	2	2	1.00	48	0.042
740	A	3	2	1.00	48	0.042
741	A	4	2	1.00	48	0.042
742	A	8	6	1.00	48	0.125
743	A	7	6	1.00	48	0.125
744	A	6	5	1.00	48	0.104
745	A	6	6	1.00	48	0.125
746	A	6	5	1.00	48	0.104
747	A	1	1	1.00	48	0.021
748	A	2	2	1.00	48	0.042
749	A	3	2	1.00	48	0.042
750	A	4	2	1.00	48	0.042
751	A	9	6	1.00	48	0.125
752	A	8	6	1.00	48	0.125
753	A	7	5	1.00	48	0.104
754	A	7	6	1.00	48	0.125
755	A	7	6	1.00	48	0.125
756	A	7	5	1.00	48	0.104
757	A	1	1	1.00	48	0.021
758	A	2	2	1.00	48	0.042
759	A	3	2	1.00	48	0.042
760	A	4	2	1.00	48	0.042

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
761	A	3	3	1.17	46	0.065
762	A	3	3	1.06	46	0.065
763	A	3	3	1.13	46	0.065
764	A	3	3	1.15	46	0.065
765	A	3	3	1.17	46	0.065
766	A	3	3	1.17	46	0.065
767	A	3	3	1.04	44	0.068
768	A	4	3	1.00	44	0.068
769	A	3	3	1.00	44	0.068
770	A	2	2	1.00	42	0.048
771	A	1	1	1.00	37	0.027
772	A	2	2	1.00	44	0.045
773	A	2	2	1.00	44	0.045
774	A	2	2	1.00	44	0.045
775	A	3	3	1.00	46	0.065
776	A	3	3	1.00	46	0.065
777	A	3	3	1.00	46	0.065
778	A	3	3	1.00	46	0.065
779	A	3	3	1.00	46	0.065
780	A	1	1	1.00	47	0.021
781	A	3	3	1.00	73	0.041
782	A	4	4	1.04	46	0.087
783	A	6	4	1.00	46	0.087
784	A	5	4	1.00	46	0.087
785	A	4	4	1.00	46	0.087
786	A	3	3	1.00	44	0.068
787	A	2	2	1.00	39	0.051
788	A	3	3	1.00	46	0.065
789	A	3	3	1.00	46	0.065
790	A	4	4	1.00	46	0.087
791	A	5	4	1.00	46	0.087
792	A	8	4	1.00	32	0.125
793	A	6	4	1.00	32	0.125
794	A	4	4	1.00	30	0.133
795	A	6	4	1.00	32	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	7	5	1.00	32	0.156
797	A	7	5	1.00	32	0.156
798	A	5	5	1.00	32	0.156
799	A	4	4	1.00	30	0.133
800	A	7	5	1.00	32	0.156
801	A	8	6	1.00	32	0.188
802	A	3	3	1.00	25	0.120
803	A	4	3	1.00	25	0.120
804	A	4	3	1.00	28	0.107
805	A	2	1	1.00	28	0.036
806	A	2	1	1.00	28	0.036
807	A	2	1	1.00	26	0.038
808	A	2	1	1.00	21	0.048
809	A	3	3	1.00	28	0.107
810	A	3	2	1.00	28	0.071
811	A	3	3	1.00	28	0.107
812	A	3	3	1.00	28	0.107
813	A	4	4	0.98	28	0.143
814	A	2	1	1.00	25	0.040
815	A	2	1	1.00	27	0.037
816	A	2	1	1.00	27	0.037
817	A	6	5	1.00	27	0.185
818	A	9	6	1.00	27	0.222
819	A	3	2	1.00	27	0.074
820	A	3	2	1.00	27	0.074
821	A	2	1	1.00	25	0.040
822	A	2	1	1.00	20	0.050
823	A	4	3	1.00	27	0.111
824	A	4	4	1.03	27	0.148
825	A	4	4	1.02	27	0.148
826	A	3	2	1.00	27	0.074
827	A	3	2	1.00	27	0.074
828	A	2	1	1.00	25	0.040
829	A	2	1	1.00	20	0.050
830	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
831	A	4	4	1.00	27	0.148
832	A	5	5	1.00	27	0.185
833	B	9	7	2.10	25	0.280
834	A	5	5	1.00	29	0.172
835	A	7	6	1.00	29	0.207
836	A	6	6	1.00	29	0.207
837	A	5	5	1.00	29	0.172
838	A	5	5	1.00	29	0.172
839	A	5	5	1.00	29	0.172
840	A	3	3	1.00	29	0.103
841	A	4	4	1.00	29	0.138
842	A	6	6	1.00	29	0.207
843	A	7	6	1.00	38	0.158
844	A	6	6	1.00	38	0.158
845	A	5	5	1.00	38	0.132
846	A	5	5	1.00	38	0.132
847	A	5	5	1.00	38	0.132
848	A	3	3	1.00	38	0.079
849	A	4	4	1.00	38	0.105
850	A	11	7	1.00	31	0.226
851	A	6	3	1.00	31	0.097
852	A	6	3	1.00	31	0.097
853	A	8	4	1.00	31	0.129
854	A	8	6	1.00	29	0.207
855	A	7	6	1.00	29	0.207
856	A	6	5	1.00	27	0.185
857	A	6	5	1.00	22	0.227
858	A	8	5	1.00	29	0.172
859	A	20	7	1.00	29	0.241
860	A	23	8	1.00	29	0.276
861	A	27	9	1.00	29	0.310
862	A	9	6	1.00	29	0.207
863	A	8	6	1.00	29	0.207
864	A	7	5	1.00	27	0.185
865	A	7	6	1.00	22	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	13	7	1.00	29	0.241
867	A	23	8	1.00	29	0.276
868	A	30	9	1.00	29	0.310
869	A	15	7	1.00	29	0.241
870	A	8	5	1.00	29	0.172
871	A	7	5	1.00	29	0.172
872	A	6	5	1.00	29	0.172
873	A	5	4	1.00	27	0.148
874	A	2	2	1.00	22	0.091
875	A	6	3	1.00	29	0.103
876	A	9	4	1.00	29	0.138
877	A	13	6	1.00	29	0.207
878	A	7	6	1.00	29	0.207
879	A	6	5	1.00	29	0.172
880	A	4	4	1.00	29	0.138
881	A	4	4	1.00	27	0.148
882	A	4	4	1.00	22	0.182
883	A	10	5	1.00	29	0.172
884	A	14	6	1.00	29	0.207
885	A	19	7	1.00	29	0.241
886	A	10	6	1.00	31	0.194
887	A	9	6	1.00	31	0.194
888	A	7	6	1.00	29	0.207
889	A	7	6	1.00	24	0.250
890	A	15	10	1.27	31	0.323
891	A	15	10	1.26	31	0.323
892	A	25	11	1.65	31	0.355
893	A	9	6	1.00	31	0.194
894	A	8	6	1.00	31	0.194
895	A	6	5	1.00	29	0.172
896	A	6	5	1.00	24	0.208
897	A	11	9	1.00	31	0.290
898	A	15	10	1.30	31	0.323
899	A	25	11	1.67	31	0.355
900	A	8	6	1.00	31	0.194

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	7	6	1.00	31	0.194
902	A	6	5	1.00	29	0.172
903	A	2	2	1.00	24	0.083
904	A	8	7	1.00	31	0.226
905	A	15	10	1.00	31	0.323
906	A	25	11	1.00	31	0.355
907	A	11	8	1.00	31	0.258
908	A	17	10	1.00	31	0.323
909	A	7	6	1.00	31	0.194
910	A	7	6	1.00	31	0.194
911	A	5	4	1.00	29	0.138
912	A	2	2	1.00	24	0.083
913	A	5	4	1.00	31	0.129
914	A	15	10	1.00	31	0.323
915	A	25	10	1.58	31	0.323
916	A	11	9	1.00	31	0.290
917	A	18	12	1.00	31	0.387
918	A	1	1	1.00	33	0.030
919	A	2	2	1.00	33	0.061
920	A	2	1	1.00	25	0.040
921	A	2	1	1.00	23	0.043
922	A	3	3	1.00	25	0.120
923	A	3	3	1.00	25	0.120
924	A	3	3	0.99	25	0.120
925	A	2	1	1.00	27	0.037
926	A	2	1	1.00	25	0.040
927	A	4	3	1.00	27	0.111
928	A	4	3	1.00	27	0.111
929	A	5	5	1.00	27	0.185
930	A	4	2	1.00	27	0.074
931	A	4	2	1.00	27	0.074
932	A	4	2	1.00	27	0.074
933	A	4	2	1.00	25	0.080
934	A	4	2	1.00	20	0.100
935	A	7	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	8	3	1.00	27	0.111
937	A	5	3	1.00	27	0.111
938	A	5	3	1.00	27	0.111
939	A	5	3	1.00	27	0.111
940	A	5	3	1.00	25	0.120
941	A	5	3	1.00	20	0.150
942	A	12	4	1.00	27	0.148
943	A	13	4	1.00	27	0.148
944	A	4	4	0.97	27	0.148
945	A	6	4	0.99	29	0.138
946	A	5	3	1.00	27	0.111
947	A	2	2	1.00	22	0.091
948	A	0	0	0.00	0	0.000
949	A	6	4	1.00	29	0.138
950	A	5	3	1.00	27	0.111
951	A	2	2	1.00	22	0.091
952	A	0	0	0.00	0	0.000
953	A	4	4	0.99	25	0.160
954	A	6	4	1.00	27	0.148
955	A	5	3	1.00	25	0.120
956	A	2	2	1.00	20	0.100
957	A	0	0	0.00	0	0.000
958	A	5	5	1.00	27	0.185

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=132

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] -1/3*d^2*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*d*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/5*(-e^
2*x^2+d^2)^(5/2)/e^3+1/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d^3*x
*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {811, 655, 201, 223, 209}

$$\frac{d^5\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - (d^2*(d^2 - e^2*x^2)^(3/2))/(3*e^3) -
 (d*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int x^2(d+ex)\sqrt{d^2-e^2x^2}\,dx = -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}+\frac{d^2\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{d\int (d^2-e^2x^2)^{3/2}\,dx}{e^2}+\frac{d^3\int \sqrt{d^2-}}{e^2}$$

$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}-$$

$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}-$$

$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+$$

$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{d^2(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{dx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{(d^2-e^2x^2)^{5/2}}{5e^3}+$$

Mathematica [A]
time = 0.15, size = 111, normalized size = 0.84

$$\frac{e\sqrt{d^2-e^2x^2}\left(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4\right)+15d^5\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{120e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-16*d^4 - 15*d^3*e*x - 8*d^2*e^2*x^2 + 30*d*e^3*x^3
 + 24*e^4*x^4) + 15*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2
]])/(120*e^4)
```

Maple [A]
time = 0.09, size = 130, normalized size = 0.98

method	result
risch	$-\frac{\left(-24e^{4}x^{4}-30d\,e^{3}x^{3}+8d^{2}x^{2}e^{2}+15d^{3}ex+16d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{120e^{3}}+\frac{d^{5}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8e^{2}\sqrt{e^{2}}}$
default	$e\left(-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{5e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{15e^{4}}\right)+d\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4e^{2}}+\frac{d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+\right.}}\right.}{2\sqrt{e^{2}}}\right.}{4e^{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/5*x^2*(-e^2*x^2+d^2)^(3/2)/e^2-2/15*d^2/e^4*(-e^2*x^2+d^2)^(3/2))+d*(
-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d^2/e^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2
*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))
```

Maxima [A]
time = 0.49, size = 96, normalized size = 0.73

$$\frac{1}{8}\,d^{5}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+\frac{1}{8}\,\sqrt{-x^{2}e^{2}+d^{2}}\,d^{3}xe^{(-2)}-\frac{1}{5}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}x^{2}e^{(-1)}-\frac{1}{4}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}dxe^{(-2)}-\frac{2}{15}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{2}e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*d^5*arcsin(x*e/d)*e^(-3) + 1/8*sqrt(-x^2*e^2 + d^2)*d^3*x*e^(-2) - 1/5*
(-x^2*e^2 + d^2)^(3/2)*x^2*e^(-1) - 1/4*(-x^2*e^2 + d^2)^(3/2)*d*x*e^(-2) -
 2/15*(-x^2*e^2 + d^2)^(3/2)*d^2*e^(-3)
```

Fricas [A]
time = 2.52, size = 89, normalized size = 0.67

$$-\frac{1}{120}\left(30\,d^{5}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)-\left(24\,x^{4}e^{4}+30\,dx^{3}e^{3}-8\,d^{2}x^{2}e^{2}-15\,d^{3}xe-16\,d^{4}\right)\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (24*x^4*e^4 +
 30*d*x^3*e^3 - 8*d^2*x^2*e^2 - 15*d^3*x*e - 16*d^4)*sqrt(-x^2*e^2 + d^2))*
e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.90, size = 279, normalized size = 2.11

$$d\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8
*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e*
*2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e*Piece
wise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2
*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**
2)/4, True))
```

Giac [A]
time = 1.63, size = 74, normalized size = 0.56

$$\frac{1}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d) - \frac{1}{120}\left(16\,d^4e^{(-3)} + \left(15\,d^3e^{(-2)} + 2\left(4\,d^2e^{(-1)} - 3\left(4\,xe + 5\,d\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^5*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/120*(16*d^4*e^(-3) + (15*d^3*e^(-2)
 + 2*(4*d^2*e^(-1) - 3*(4*x*e + 5*d)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x),x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x), x)
```

3.2 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=201

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4}-\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3}-\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}-\frac{x^4(d^2-e^2x^2)^{5/2}}{9e}-\frac{d^3(128d}{}$$

```
[Out] 1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4-4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-1
/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2-1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e-1/5040*d^3
*(315*e*x+128*d)*(-e^2*x^2+d^2)^(5/2)/e^5+3/128*d^9*arctan(e*x/(-e^2*x^2+d^
2)^(1/2))/e^5+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A]
time = 0.10, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 794, 201, 223, 209}

$$\frac{3d^9\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}-\frac{x^4(d^2-e^2x^2)^{5/2}}{9e}-\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}-\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3}+\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4}-\frac{d^3(128d+315ex)\left(d^2-e^2x^2\right)^{5/2}}{5040e^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) - (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) - (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d^3*(128*d + 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{\int x^3(-4d^2e-9de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{9e^2}$$

$$= -\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{\int x^2(27d^3e^2+32d^2e^3x)\,(d^2 -}{72e^4}$$

$$= -\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{\int x(-6}{}$$

$$= -\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{d^3(128}{}$$

$$= \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2 -}{9}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2 -}{8}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2 -}{8}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2 -}{8}$$

Mathematica [A]
time = 0.22, size = 158, normalized size = 0.79

$$\frac{\sqrt{d^2-e^2x^2}\,(-1024d^8-945d^7ex-512d^6e^2x^2-630d^5e^3x^3-384d^4e^4x^4+7560d^3e^5x^5+6400d^2e^6x^6-5040de^7x^7-4480e^8x^8)}{40320e^5}+\frac{3d^9\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{128e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-1024*d^8 - 945*d^7*e*x - 512*d^6*e^2*x^2 - 630*d^5*e
^3*x^3 - 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 + 6400*d^2*e^6*x^6 - 5040*d*e^7
*x^7 - 4480*e^8*x^8))/(40320*e^5) + (3*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
 Sqrt[d^2 - e^2*x^2]])/(128*e^6)
```

Maple [A]
time = 0.07, size = 215, normalized size = 1.07

method	result
risch	$-\frac{\left(4480e^8x^8+5040d\,e^7x^7-6400d^2e^6x^6-7560d^3e^5x^5+384d^4x^4e^4+630d^5e^3x^3+512d^6e^2x^2+945d^7ex+1024d^8\right)\sqrt{-e^2x^2+d^2}}{40320e^5}+$

default	$e\left(-\frac{x^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{9e^{2}}+\frac{4d^{2}\left(-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{7e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{35e^{4}}\right)}{9e^{2}}\right)+d\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{8e^{2}}+\frac{3d^{2}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right.}{6e^{2}}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e^2+4/9*d^2/e^2*(-1/7*x^2*(-e^2*x^2+d^2)^(
5/2)/e^2-2/35*d^2/e^4*(-e^2*x^2+d^2)^(5/2)))+d*(-1/8*x^3*(-e^2*x^2+d^2)^(5/
2)/e^2+3/8*d^2/e^2*(-1/6*x*(-e^2*x^2+d^2)^(5/2)/e^2+1/6*d^2/e^2*(1/4*x*(-e^
2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))
```

Maxima [A]
time = 0.51, size = 163, normalized size = 0.81

$$\frac{3}{128}\,d^{9}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}+\frac{3}{128}\,\sqrt{-x^{2}e^{2}+d^{2}}\,d^{7}xe^{(-4)}+\frac{1}{64}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{5}xe^{(-4)}-\frac{1}{9}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}x^{4}e^{(-1)}-\frac{1}{8}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}dx^{3}e^{(-2)}-\frac{4}{63}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}x^{2}e^{(-3)}-\frac{1}{16}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}xe^{(-4)}-\frac{8}{315}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*d^9*arcsin(x*e/d)*e^(-5) + 3/128*sqrt(-x^2*e^2 + d^2)*d^7*x*e^(-4) +
1/64*(-x^2*e^2 + d^2)^(3/2)*d^5*x*e^(-4) - 1/9*(-x^2*e^2 + d^2)^(5/2)*x^4*e
^(-1) - 1/8*(-x^2*e^2 + d^2)^(5/2)*d*x^3*e^(-2) - 4/63*(-x^2*e^2 + d^2)^(5/
2)*d^2*x^2*e^(-3) - 1/16*(-x^2*e^2 + d^2)^(5/2)*d^3*x*e^(-4) - 8/315*(-x^2*
e^2 + d^2)^(5/2)*d^4*e^(-5)
```

Fricas [A]
time = 2.58, size = 128, normalized size = 0.64

$$-\frac{1}{40320}\left(1890\,d^9\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(4480\,x^8e^8+5040\,dx^7e^7-6400\,d^2x^6e^6-7560\,d^3x^5e^5+384\,d^4x^4e^4+630\,d^5x^3e^3+512\,d^6x^2e^2+945\,d^7xe+1024\,d^8\right)\sqrt{-x^2e^2+d^2}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (4480*x^8
*e^8 + 5040*d*x^7*e^7 - 6400*d^2*x^6*e^6 - 7560*d^3*x^5*e^5 + 384*d^4*x^4*e
^4 + 630*d^5*x^3*e^3 + 512*d^6*x^2*e^2 + 945*d^7*x*e + 1024*d^8)*sqrt(-x^2*
e^2 + d^2))*e^(-5)
```

Sympy [C] Result contains complex when optimal does not.
time = 25.94, size = 830, normalized size = 4.13

$$d^3\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for } e\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7}+\frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5id^5x^3}{384e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^5}{192e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{7idx^7}{48\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^9}{8d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7}-\frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5d^5x^3}{384e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^5}{192e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{7dx^7}{48\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^9}{8d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{16d^8\sqrt{d^2-e^2x^2}}{315e^8}-\frac{8d^6x^2\sqrt{d^2-e^2x^2}}{315e^6}-\frac{2d^4x^4\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^6\sqrt{d^2-e^2x^2}}{63e^2}+\frac{x^8\sqrt{d^2-e^2x^2}}{9} & \text{for } e\neq 0\\ \frac{x^8\sqrt{d^2}}{8} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) + d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(1
05*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d
**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**
6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7
) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e*
*4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d
**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e*
*7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*
sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) +
 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2
```

```
/d**2)), True)) - e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8
) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 -
 e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*
sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [A]
time = 2.41, size = 117, normalized size = 0.58

$$\frac{3}{128}\,d^{9}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{1}{40320}\left(1024\,d^{8}e^{(-5)}+\left(945\,d^{7}e^{(-4)}+2\left(256\,d^{6}e^{(-3)}+\left(315\,d^{5}e^{(-2)}+4\left(48\,d^{4}e^{(-1)}-5\left(189\,d^{3}+2\left(80\,d^{2}e-7\left(8\,xe^{3}+9\,de^{2}\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^9*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/40320*(1024*d^8*e^(-5) + (945*d^7
*e^(-4) + 2*(256*d^6*e^(-3) + (315*d^5*e^(-2) + 4*(48*d^4*e^(-1) - 5*(189*d
^3 + 2*(80*d^2*e - 7*(8*x*e^3 + 9*d*e^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x^4*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.3 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}-\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}+$$

```
[Out] 1/64*d^4*x*(-e^2*x^2+d^2)^(3/2)/e^3-1/7*d*x^2*(-e^2*x^2+d^2)^(5/2)/e^2-1/8*
x^3*(-e^2*x^2+d^2)^(5/2)/e-1/560*d^2*(35*e*x+32*d)*(-e^2*x^2+d^2)^(5/2)/e^4
+3/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+3/128*d^6*x*(-e^2*x^2+d^2)^
(1/2)/e^3
```

Rubi [A]
time = 0.07, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {847, 794, 201, 223, 209}

$$\frac{3d^8\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}-\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}+\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}+\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) + (d^4*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) - (x^3*(d^2 - e^2*x^2)^(5/2))
/(8*e) - (d^2*(32*d + 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) + (3*d^8*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\int x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{\int x^2(-3d^2e-8de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$

$$= -\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} + \frac{\int x(16d^3e^2+21d^2e^3x)\left(d^2-e\right.}{56e^4}$$

$$= -\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4}$$

$$= \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+3}{}$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2}{8e}$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2}{8e}$$

$$= \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2}{8e}$$

Mathematica [A]
time = 0.21, size = 147, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\,(-256d^7-105d^6ex-128d^5e^2x^2-70d^4e^3x^3+1024d^3e^4x^4+840d^2e^5x^5-640de^6x^6-560e^7x^7)}{4480e^4} + \frac{3d^8\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{128e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^7 - 105*d^6*e*x - 128*d^5*e^2*x^2 - 70*d^4*e^3
*x^3 + 1024*d^3*e^4*x^4 + 840*d^2*e^5*x^5 - 640*d*e^6*x^6 - 560*e^7*x^7))/(
4480*e^4) + (3*d^8*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(
128*e^5)
```

Maple [A]
time = 0.09, size = 184, normalized size = 1.07

method	result
risch	$-\frac{\left(560e^7x^7+640d\,e^6x^6-840d^2e^5x^5-1024d^3e^4x^4+70d^4e^3x^3+128d^5e^2x^2+105d^6ex+256d^7\right)\sqrt{-e^2x^2+d^2}}{4480e^4}+\frac{3d^8\arctan\left(\frac{\ldots}{\sqrt{-\ldots}}\right.}{128e^3\ldots}$ …
default	$e\left(-\frac{x^3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{8e^2}+\frac{3d^2\left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6e^2}+\frac{d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right.\ldots}{2\sqrt{e^2}}\ldots\right.}{4}\ldots\right.}{6e^2}\ldots\right.}{8e^2}\ldots\right.$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/8*x^3*(-e^2*x^2+d^2)^(5/2)/e^2+3/8*d^2/e^2*(-1/6*x*(-e^2*x^2+d^2)^(5/
2)/e^2+1/6*d^2/e^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2
)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+
d*(-1/7*x^2*(-e^2*x^2+d^2)^(5/2)/e^2-2/35*d^2/e^4*(-e^2*x^2+d^2)^(5/2))
```

Maxima [A]
time = 0.50, size = 140, normalized size = 0.81

$$\frac{3}{128}\,d^8\arcsin\left(\frac{xe}{d}\right)e^{(-4)}+\frac{3}{128}\,\sqrt{-x^2e^2+d^2}\,d^6xe^{(-3)}+\frac{1}{64}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^4xe^{(-3)}-\frac{1}{8}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}x^3e^{(-1)}-\frac{1}{7}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}dx^2e^{(-2)}-\frac{1}{16}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^2xe^{(-3)}-\frac{2}{35}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^3e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*d^8*arcsin(x*e/d)*e^(-4) + 3/128*sqrt(-x^2*e^2 + d^2)*d^6*x*e^(-3) +
1/64*(-x^2*e^2 + d^2)^(3/2)*d^4*x*e^(-3) - 1/8*(-x^2*e^2 + d^2)^(5/2)*x^3*e
^(-1) - 1/7*(-x^2*e^2 + d^2)^(5/2)*d*x^2*e^(-2) - 1/16*(-x^2*e^2 + d^2)^(5/
2)*d^2*x*e^(-3) - 2/35*(-x^2*e^2 + d^2)^(5/2)*d^3*e^(-4)
```

Fricas [A]
time = 2.32, size = 118, normalized size = 0.69

$$-\frac{1}{4480}\left(210\,d^8\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(560\,x^7e^7+640\,dx^6e^6-840\,d^2x^5e^5-1024\,d^3x^4e^4+70\,d^4x^3e^3+128\,d^5x^2e^2+105\,d^6xe+256\,d^7\right)\sqrt{-x^2e^2+d^2}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4480*(210*d^8*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (560*x^7*e^
7 + 640*d*x^6*e^6 - 840*d^2*x^5*e^5 - 1024*d^3*x^4*e^4 + 70*d^4*x^3*e^3 + 1
28*d^5*x^2*e^2 + 105*d^6*x*e + 256*d^7)*sqrt(-x^2*e^2 + d^2))*e^(-4)
```

Sympy [A]
time = 25.55, size = 775, normalized size = 4.51

$$d^3\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\: e \neq 0\\\frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right|>1\\\frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\: e \neq 0\\\frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7}+\frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5id^5x^3}{384e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^5}{192e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{7idx^7}{48\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^9}{8d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right|>1\\\frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7}-\frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5d^5x^3}{384e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^5}{192e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{7dx^7}{48\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^9}{8d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + d**2*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e
```

```
*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48
*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) -
e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((-8*d**
6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(1
05*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e*
*2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - e**3*Piecewise((-5*I*d*
*8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)
) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e
**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2))
+ I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5
*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2))
 + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sq
rt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**
9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 1.93, size = 106, normalized size = 0.62

$$\frac{3}{128}\,d^8\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{4480}\left(256\,d^7e^{(-4)}+\left(105\,d^6e^{(-3)}+2\left(64\,d^5e^{(-2)}+\left(35\,d^4e^{(-1)}-4\left(128\,d^3+5\left(21\,d^2e-2\left(7\,xe^3+8\,de^2\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/128*d^8*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/4480*(256*d^7*e^(-4) + (105*d^6*e
^(-3) + 2*(64*d^5*e^(-2) + (35*d^4*e^(-1) - 4*(128*d^3 + 5*(21*d^2*e - 2*(7
*x*e^3 + 8*d*e^2)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(d^2-e^2\,x^2\right)^{3/2}\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.4 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}-\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3}-\frac{dx(d^2-e^2x^2)^{5/2}}{6e^2}+\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] 1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*d^2*(-e^2*x^2+d^2)^(5/2)/e^3-1/6*d*
x*(-e^2*x^2+d^2)^(5/2)/e^2+1/7*(-e^2*x^2+d^2)^(7/2)/e^3+1/16*d^7*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.06, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {811, 655, 201, 223, 209}

$$\frac{d^7\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}-\frac{dx(d^2-e^2x^2)^{5/2}}{6e^2}-\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3}+\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) - (d^2*(d^2 - e^2*x^2)^(5/2))/(5*e^3) - (d*x*(d^2 - e^2*x^2)^(5/2))/(6*e
^2) + (d^2 - e^2*x^2)^(7/2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(16*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{\int (d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx}{e^2} + \frac{d^2\int (d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx}{e^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} + \frac{(d^2-e^2x^2)^{7/2}}{7e^3} - \frac{d\int (d^2-e^2x^2)^{5/2}\,dx}{e^2} + \frac{d^3\int (d^2-$$

$$= \frac{d^3x(d^2-e^2x^2)^{3/2}}{4e^2} - \frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2-e^2x^2)^{5/2}}{6e^2} + \frac{(d^2-e^2x^2)^{7/2}}{7e^3}$$

$$= \frac{3d^5x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} - \frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2-e^2x^2}{6e^2}$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} - \frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2-e^2x^2)}{6e^2}$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} - \frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2-e^2x^2)}{6e^2}$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} - \frac{d^2(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2-e^2x^2)}{6e^2}$$

Mathematica [A]
time = 0.22, size = 136, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\,(-96d^6-105d^5ex-48d^4e^2x^2+490d^3e^3x^3+384d^2e^4x^4-280de^5x^5-240e^6x^6)}{1680e^3} + \frac{d^7\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{16e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

[Out] (Sqrt[d^2 - e^2*x^2]*(-96*d^6 - 105*d^5*e*x - 48*d^4*e^2*x^2 + 490*d^3*e^3*x^3 + 384*d^2*e^4*x^4 - 280*d*e^5*x^5 - 240*e^6*x^6))/(1680*e^3) + (d^7*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^4)

Maple [A]
time = 0.06, size = 153, normalized size = 0.96

method	result
risch	$-\frac{\left(240 e^{6} x^{6}+280 d\,e^{5} x^{5}-384 d^{2} e^{4} x^{4}-490 d^{3} x^{3} e^{3}+48 d^{4} e^{2} x^{2}+105 d^{5} e x +96 d^{6}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{1680 e^{3}}+\frac{d^{7} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 e^{2} \sqrt{e^{2}}}$
default	$e \left(-\frac{x^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{7 e^{2}}-\frac{2 d^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{35 e^{4}}\right)+d \left(-\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6 e^{2}}+\frac{d^{2} \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2} \left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\cdots\right)}{\cdots}\right)}{6 e^{2}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] e*(-1/7*x^2*(-e^2*x^2+d^2)^(5/2)/e^2-2/35*d^2/e^4*(-e^2*x^2+d^2)^(5/2))+d*(-1/6*x*(-e^2*x^2+d^2)^(5/2)/e^2+1/6*d^2/e^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))

Maxima [A]
time = 0.47, size = 117, normalized size = 0.74

$$\frac{1}{16}\,d^{7}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+\frac{1}{16}\,\sqrt{-x^{2}e^{2}+d^{2}}\,d^{5}xe^{(-2)}+\frac{1}{24}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{3}xe^{(-2)}-\frac{1}{7}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}x^{2}e^{(-1)}-\frac{1}{6}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}dxe^{(-2)}-\frac{2}{35}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")

[Out] 1/16*d^7*arcsin(x*e/d)*e^(-3) + 1/16*sqrt(-x^2*e^2 + d^2)*d^5*x*e^(-2) + 1/24*(-x^2*e^2 + d^2)^(3/2)*d^3*x*e^(-2) - 1/7*(-x^2*e^2 + d^2)^(5/2)*x^2*e^(

```
-1) - 1/6*(-x^2*e^2 + d^2)^(5/2)*d*x*e^(-2) - 2/35*(-x^2*e^2 + d^2)^(5/2)*d
^2*e^(-3)
```

<u>Fricas</u> [A]
time = 2.17, size = 108, normalized size = 0.68

$$-\frac{1}{1680}\left(210\,d^7\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(240\,x^6e^6+280\,dx^5e^5-384\,d^2x^4e^4-490\,d^3x^3e^3+48\,d^4x^2e^2+105\,d^5xe+96\,d^6\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (240*x^6*e^
6 + 280*d*x^5*e^5 - 384*d^2*x^4*e^4 - 490*d^3*x^3*e^3 + 48*d^4*x^2*e^2 + 10
5*d^5*x*e + 96*d^6)*sqrt(-x^2*e^2 + d^2))*e^(-3)
```

<u>Sympy</u> [C] Result contains complex when optimal does not.
time = 8.13, size = 653, normalized size = 4.11

$$d^3\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2
*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**
2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*
sqrt(d**2)/4, True)) - d*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I
*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
+ e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**
7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x
/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e
**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e*
*2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - e**3*Piecewise((-8*d**6*sq
rt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e
**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x
**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

<u>Giac</u> [A]
time = 1.53, size = 96, normalized size = 0.60

$$\frac{1}{16}\,d^7\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d)-\frac{1}{1680}\left(96\,d^6e^{(-3)}+\left(105\,d^5e^{(-2)}+2\left(24\,d^4e^{(-1)}-\left(245\,d^3+4\left(48\,d^2e-5\left(6\,xe^3+7\,de^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^7*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/1680*(96*d^6*e^(-3) + (105*d^5*e^(
-2) + 2*(24*d^4*e^(-1) - (245*d^3 + 4*(48*d^2*e - 5*(6*x*e^3 + 7*d*e^2)*x)*
x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.5 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] 1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e^2
+1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+1/16*d^4*x*(-e^2*x^2+d^2)^(1
/2)/e
```

Rubi [A]
time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {794, 201, 223, 209}

$$\frac{d^6\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx=-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}$$
$$=\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int}{}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\mathrm{Su}}{}$$
$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.20, size = 125, normalized size = 1.08

$$\frac{\sqrt{d^2-e^2x^2}\left(-48d^5-15d^4ex+96d^3e^2x^2+70d^2e^3x^3-48de^4x^4-40e^5x^5\right)}{240e^2}+\frac{d^6\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 - 15*d^4*e*x + 96*d^3*e^2*x^2 + 70*d^2*e^3*x^
3 - 48*d*e^4*x^4 - 40*e^5*x^5))/(240*e^2) + (d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2
]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Maple [A]
time = 0.05, size = 126, normalized size = 1.09

method	result

risch	$-\frac{(40e^5x^5+48d\,e^4x^4-70d^2e^3x^3-96x^2d^3e^2+15d^4xe+48d^5)\sqrt{-e^2x^2+d^2}}{240e^2}+\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e\sqrt{e^2}}$
default	$e\left(-\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6e^2}\right)-\frac{d(-e^2x^2+d}{5e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/6*x*(-e^2*x^2+d^2)^(5/2)/e^2+1/6*d^2/e^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+
3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*
x/(-e^2*x^2+d^2)^(1/2)))))-1/5*d*(-e^2*x^2+d^2)^(5/2)/e^2
```

Maxima [A]
time = 0.48, size = 94, normalized size = 0.81

$$\frac{1}{16}d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}+\frac{1}{16}\sqrt{-x^2e^2+d^2}\,d^4xe^{(-1)}+\frac{1}{24}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^2xe^{(-1)}-\frac{1}{6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}xe^{(-1)}-\frac{1}{5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}de^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2) + 1/16*sqrt(-x^2*e^2 + d^2)*d^4*x*e^(-1) + 1/
24*(-x^2*e^2 + d^2)^(3/2)*d^2*x*e^(-1) - 1/6*(-x^2*e^2 + d^2)^(5/2)*x*e^(-1
) - 1/5*(-x^2*e^2 + d^2)^(5/2)*d*e^(-2)
```

Fricas [A]
time = 2.45, size = 98, normalized size = 0.84

$$-\frac{1}{240}\left(30\,d^6\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(40\,x^5e^5+48\,dx^4e^4-70\,d^2x^3e^3-96\,d^3x^2e^2+15\,d^4xe+48\,d^5\right)\sqrt{-x^2e^2+d^2}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (40*x^5*e^5 +
 48*d*x^4*e^4 - 70*d^2*x^3*e^3 - 96*d^3*x^2*e^2 + 15*d^4*x*e + 48*d^5)*sqrt
(-x^2*e^2 + d^2))*e^(-2)
```

Sympy [A]
time = 8.12, size = 580, normalized size = 5.00

$$d^3\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)+d^2e\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)-de^2\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e\neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)-e^3\left(\begin{cases} -\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 1.61, size = 84, normalized size = 0.72

$$\frac{1}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{1}{240}\left(48\,d^5e^{(-2)}+\left(15\,d^4e^{(-1)}-2\left(48\,d^3+\left(35\,d^2e-4\left(5\,xe^3+6\,de^2\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) + (15*d^4*e^(-1
) - 2*(48*d^3 + (35*d^2*e - 4*(5*x*e^3 + 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
[Out] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.6 $\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] 1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e^2
+1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+1/16*d^4*x*(-e^2*x^2+d^2)^(1
/2)/e
```

Rubi [A]
time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {794, 201, 223, 209}

$$\frac{d^6\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e) -
 ((6*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) + (d^6*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/(16*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int x(d+ex)\left(d^2-e^2x^2\right)^{3/2}\,dx = -\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^2\int\left(d^2-e^2x^2\right)^{3/2}\,dx}{6e}$$

$$=\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^4\int\sqrt{d^2-e^2x^2}\,dx}{8e}$$

$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\int -}{}$$

$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\text{Sub}}{}$$

$$=\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}+\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e^2}+\frac{d^6\tan}{}$$

Mathematica [A]
time = 0.00, size = 125, normalized size = 1.08

$$\frac{\sqrt{d^2-e^2x^2}\,(-48d^5-15d^4ex+96d^3e^2x^2+70d^2e^3x^3-48de^4x^4-40e^5x^5)}{240e^2}+\frac{d^6\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{16e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 - 15*d^4*e*x + 96*d^3*e^2*x^2 + 70*d^2*e^3*x^
3 - 48*d*e^4*x^4 - 40*e^5*x^5))/(240*e^2) + (d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2
]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Maple [A]
time = 0.00, size = 126, normalized size = 1.09

method	result

risch	$-\frac{(40e^5x^5+48d\,e^4x^4-70d^2e^3x^3-96x^2d^3e^2+15d^4xe+48d^5)\sqrt{-e^2x^2+d^2}}{240e^2}+\frac{d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e\sqrt{e^2}}$
default	$e\left(-\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6e^2}\right)-\frac{d(-e^2x^2+}{5e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/6*x*(-e^2*x^2+d^2)^(5/2)/e^2+1/6*d^2/e^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+
3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*
x/(-e^2*x^2+d^2)^(1/2)))))-1/5*d*(-e^2*x^2+d^2)^(5/2)/e^2
```

Maxima [A]
time = 0.48, size = 94, normalized size = 0.81

$$\frac{1}{16}d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}+\frac{1}{16}\sqrt{-x^2e^2+d^2}\,d^4xe^{(-1)}+\frac{1}{24}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^2xe^{(-1)}-\frac{1}{6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}xe^{(-1)}-\frac{1}{5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}de^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2) + 1/16*sqrt(-x^2*e^2 + d^2)*d^4*x*e^(-1) + 1/
24*(-x^2*e^2 + d^2)^(3/2)*d^2*x*e^(-1) - 1/6*(-x^2*e^2 + d^2)^(5/2)*x*e^(-1
) - 1/5*(-x^2*e^2 + d^2)^(5/2)*d*e^(-2)
```

Fricas [A]
time = 1.90, size = 98, normalized size = 0.84

$$-\frac{1}{240}\left(30\,d^6\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(40\,x^5e^5+48\,dx^4e^4-70\,d^2x^3e^3-96\,d^3x^2e^2+15\,d^4xe+48\,d^5\right)\sqrt{-x^2e^2+d^2}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(30*d^6*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (40*x^5*e^5 +
 48*d*x^4*e^4 - 70*d^2*x^3*e^3 - 96*d^3*x^2*e^2 + 15*d^4*x*e + 48*d^5)*sqrt
(-x^2*e^2 + d^2))*e^(-2)
```

Sympy [A]
time = 8.10, size = 580, normalized size = 5.00

$$d^{3}\left(\begin{cases}\frac{x^{2}\sqrt{d^{2}}}{2} & \text{for}\: e^{2} = 0\\-\frac{\left(d^{2} - e^{2}x^{2}\right)^{\frac{3}{2}}}{3e^{2}} & \text{otherwise}\end{cases}\right) + d^{2}e\left(\begin{cases}-\frac{id^{4}\operatorname{acosh}{\left(\frac{ex}{d}\right)}}{8e^{3}} + \frac{id^{3}x}{8e^{2}\sqrt{-1 + \frac{e^{2}x^{2}}{d^{2}}}} - \frac{3idx^{3}}{8\sqrt{-1 + \frac{e^{2}x^{2}}{d^{2}}}} + \frac{ie^{2}x^{5}}{4d\sqrt{-1 + \frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2}x^{2}}{d^{2}}}\right| > 1\\\frac{d^{4}\operatorname{asin}{\left(\frac{ex}{d}\right)}}{8e^{3}} - \frac{d^{3}x}{8e^{2}\sqrt{1 - \frac{e^{2}x^{2}}{d^{2}}}} + \frac{3dx^{3}}{8\sqrt{1 - \frac{e^{2}x^{2}}{d^{2}}}} - \frac{e^{2}x^{5}}{4d\sqrt{1 - \frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right) - de^{2}\left(\begin{cases}-\frac{2d^{4}\sqrt{d^{2} - e^{2}x^{2}}}{15e^{4}} - \frac{d^{2}x^{2}\sqrt{d^{2} - e^{2}x^{2}}}{15e^{2}} + \frac{x^{4}\sqrt{d^{2} - e^{2}x^{2}}}{5} & \text{for}\: e \neq 0\\\frac{x^{4}\sqrt{d^{2}}}{4} & \text{otherwise}\end{cases}\right) - e^{3}\left(\begin{cases}-\frac{id^{6}\operatorname{acosh}{\left(\frac{ex}{d}\right)}}{16e^{5}} + \frac{id^{5}x}{16e^{4}\sqrt{-1 + \frac{e^{2}x^{2}}{d^{2}}}} - \frac{id^{3}x^{3}}{48e^{2}\sqrt{-1 + \frac{e^{2}x^{2}}{d^{2}}}} - \frac{5idx^{5}}{24\sqrt{-1 + \frac{e^{2}x^{2}}{d^{2}}}} + \frac{ie^{2}x^{7}}{6d\sqrt{-1 + \frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2}x^{2}}{d^{2}}}\right| > 1\\\frac{d^{6}\operatorname{asin}{\left(\frac{ex}{d}\right)}}{16e^{5}} - \frac{d^{5}x}{16e^{4}\sqrt{1 - \frac{e^{2}x^{2}}{d^{2}}}} + \frac{d^{3}x^{3}}{48e^{2}\sqrt{1 - \frac{e^{2}x^{2}}{d^{2}}}} + \frac{5dx^{5}}{24\sqrt{1 - \frac{e^{2}x^{2}}{d^{2}}}} - \frac{e^{2}x^{7}}{6d\sqrt{1 - \frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 2.52, size = 84, normalized size = 0.72

$$\frac{1}{16}\,d^{6}\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d) - \frac{1}{240}\,\left(48\,d^{5}e^{(-2)} + \left(15\,d^{4}e^{(-1)} - 2\,\left(48\,d^{3} + \left(35\,d^{2}e - 4\,\left(5\,xe^{3} + 6\,de^{2}\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2} + d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) + (15*d^4*e^(-1
) - 2*(48*d^3 + (35*d^2*e - 4*(5*x*e^3 + 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x),x)
[Out] int(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x), x)
```

3.7 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=113

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/12*(3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)+3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/
2))-d^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*(3*e*x+8*d)*(-e^2*x^2+d^2)^
(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {829, 858, 223, 209, 272, 65, 214}

$$\frac{3}{8}d^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2}-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (d^2*(8*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d + 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 + (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x}\,dx = \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{\left(-4d^3e^2-3d^2e^3x\right)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4+3d^4e^5x}{x\sqrt{d^2-e^2x^2}}}{8e^4}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + d^5\int \frac{1}{x\sqrt{d^2-e}}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^5\text{Subst}\Bigg(\int \frac{}{x}$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\tan^{-1}\left(\frac{}{\sqrt{d}}\right.$$

$$= \frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{3}{8}d^4\tan^{-1}\left(\frac{}{\sqrt{d}}\right.$$

Mathematica [A]
time = 0.32, size = 140, normalized size = 1.24

$$\frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3+15d^2ex-8de^2x^2-6e^3x^3\right)+2d^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{3d^4e\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{8\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 + 15*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))/24 +
2*d^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^4*e*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*Sqrt[-e^2])
```

Maple [A]
time = 0.06, size = 157, normalized size = 1.39

method	result
default	$e\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)+d\left(\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(
e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+d*(1/3*(-e^2*x^2+d^
2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x)))
```

Maxima [A]
time = 0.49, size = 122, normalized size = 1.08

$$\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)-d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)+\frac{3}{8}\,\sqrt{-x^2e^2+d^2}\,d^2xe+\sqrt{-x^2e^2+d^2}\,d^3+\frac{1}{4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}xe+\frac{1}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*d^4*arcsin(x*e/d) - d^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs
(x)) + 3/8*sqrt(-x^2*e^2 + d^2)*d^2*x*e + sqrt(-x^2*e^2 + d^2)*d^3 + 1/4*(-
x^2*e^2 + d^2)^(3/2)*x*e + 1/3*(-x^2*e^2 + d^2)^(3/2)*d
```

Fricas [A]
time = 2.11, size = 102, normalized size = 0.90

$$-\frac{3}{4}\,d^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+d^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-\frac{1}{24}\left(6\,x^3e^3+8\,dx^2e^2-15\,d^2xe-32\,d^3\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -3/4*d^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + d^4*log(-(d - sqrt(
-x^2*e^2 + d^2))/x) - 1/24*(6*x^3*e^3 + 8*d*x^2*e^2 - 15*d^2*x*e - 32*d^3)*
sqrt(-x^2*e^2 + d^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.47, size = 469, normalized size = 4.15

$$d^3\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x,x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
```

```
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*
sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), A
bs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d
**2)/2, True)) - d*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2
 - e**2*x**2)**(3/2)/(3*e**2), True)) - e**3*Piecewise((-I*d**4*acosh(e*x/d
)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sq
rt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 -
 e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*
sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 2.79, size = 99, normalized size = 0.88

$$\frac{3}{8}\,d^4 \arcsin\left(\frac{xe}{d}\right)\mathrm{sgn}(d) - d^4 \log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) + \frac{1}{24}\left(32\,d^3 + \left(15\,d^2e - 2\left(3\,xe^3 + 4\,de^2\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 3/8*d^4*arcsin(x*e/d)*sgn(d) - d^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d
^2)*e)*e^(-2)/abs(x)) + 1/24*(32*d^3 + (15*d^2*e - 2*(3*x*e^3 + 4*d*e^2)*x)
*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [B]
time = 2.90, size = 107, normalized size = 0.95

$$\frac{d\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{3} - d^4\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right) + d^3\,\sqrt{d^2-e^2\,x^2} + \frac{e\,x\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{{\left(1-\frac{e^2\,x^2}{d^2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x,x)
```

```
[Out] (d*(d^2 - e^2*x^2)^(3/2))/3 - d^4*atanh((d^2 - e^2*x^2)^(1/2)/d) + d^3*(d^2
 - e^2*x^2)^(1/2) + (e*x*(d^2 - e^2*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2,
(e^2*x^2)/d^2))/(1 - (e^2*x^2)/d^2)^(3/2)
```

3.8 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=117

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x-3/2*d^3*e*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))-d^3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*d*e*(-3*e*x+2*d)*(-e^2*x^2
+d^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {827, 829, 858, 223, 209, 272, 65, 214}

$$-\frac{3}{2}d^3e\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2}-\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
```

```
[Out] (d*e*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/2 - ((3*d - e*x)*(d^2 - e^2*x^2)^(3
/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - d^3*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^2}\,dx = -\frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{\left(-2d^2e+6de^2x\right)\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \frac{\int \frac{4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,d}{4e^2}$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \left(d^4e\right)\int \frac{1}{x\sqrt{d^2-}}$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} + \frac{1}{2}\left(d^4e\right)\operatorname{Subst}\left(\int\right.$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.$$

$$= \frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)\left(d^2-e^2x^2\right)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.$$

Mathematica [A]
time = 0.30, size = 143, normalized size = 1.22

$$\frac{\sqrt{d^2-e^2x^2}\left(-6d^3+8d^2ex-3de^2x^2-2e^3x^3\right)}{6x} + 2d^3e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{3}{2}d^3\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^2,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 3*d*e^2*x^2 - 2*e^3*x^3))/(6*x)
+ 2*d^3*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^3*Sqrt[-
e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

Maple [A]
time = 0.08, size = 188, normalized size = 1.61

method	result
risch	$-\frac{d^3\sqrt{-e^2x^2+d^2}}{x} - \frac{e^3x^2\sqrt{-e^2x^2+d^2}}{3} + \frac{4e\,d^2\sqrt{-e^2x^2+d^2}}{3} - \frac{e^2dx\sqrt{-e^2x^2+d^2}}{2} - \frac{3e^2d^3\arctan\left(\right.}{2}$

default	$d\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{d^{2}x}-\frac{4e^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{d^{2}}\right)+e\left(\frac{\left(-e^{2}x^{2}\right.}{3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/d^2/x*(-e^2*x^2+d^2)^(5/2)-4*e^2/d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*
d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2)))))+e*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2
)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))
```

Maxima [A]
time = 0.49, size = 128, normalized size = 1.09

$$-\frac{3}{2}\,d^{3}\arcsin\left(\frac{xe}{d}\right)e-d^{3}e\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)-\frac{3}{2}\,\sqrt{-x^{2}e^{2}+d^{2}}\,dxe^{2}+\sqrt{-x^{2}e^{2}+d^{2}}\,d^{2}e+\frac{1}{3}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}e-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*d^3*arcsin(x*e/d)*e - d^3*e*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*
d/abs(x)) - 3/2*sqrt(-x^2*e^2 + d^2)*d*x*e^2 + sqrt(-x^2*e^2 + d^2)*d^2*e +
 1/3*(-x^2*e^2 + d^2)^(3/2)*e - (-x^2*e^2 + d^2)^(3/2)*d/x
```

Fricas [A]
time = 1.86, size = 122, normalized size = 1.04

$$\frac{18\,d^{3}x\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)e+6\,d^{3}xe\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+8\,d^{3}xe-\left(2\,x^{3}e^{3}+3\,dx^{2}e^{2}-8\,d^{2}xe+6\,d^{3}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*x*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e + 6*d^3*x*e*lo
g(-(d - sqrt(-x^2*e^2 + d^2))/x) + 8*d^3*x*e - (2*x^3*e^3 + 3*d*x^2*e^2 - 8
*d^2*x*e + 6*d^3)*sqrt(-x^2*e^2 + d^2))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 3.69, size = 386, normalized size = 3.30

$$d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0\\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**2,x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) + d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e)
- I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 -
e**2*x**2/d**2)/2, True)) - e**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0))
, (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

Giac [A]
time = 1.42, size = 157, normalized size = 1.34

$$-\frac{3}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)\operatorname{esgn}(d)-d^3e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{d^3xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3e^{(-1)}}{2\,x}+\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\,\left(8\,d^2e-\left(2\,xe^3+3\,de^2\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/2*d^3*arcsin(x*e/d)*e*sgn(d) - d^3*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^
2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^3*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) -
 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^(-1)/x + 1/6*sqrt(-x^2*e^2 + d^2)
*(8*d^2*e - (2*x*e^3 + 3*d*e^2)*x)
```

Mupad [B]
time = 3.51, size = 114, normalized size = 0.97

$$\frac{e\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{3}+d^2\,e\,\sqrt{d^2-e^2\,x^2}-d^3\,e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)-\frac{d^3\,\sqrt{d^2-e^2\,x^2}\;{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2\,x^2}{d^2}\right)}{x\,\sqrt{1-\frac{e^2\,x^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^2,x)
```

```
[Out] (e*(d^2 - e^2*x^2)^(3/2))/3 + d^2*e*(d^2 - e^2*x^2)^(1/2) - d^3*e*atanh((d^
2 - e^2*x^2)^(1/2)/d) - (d^3*(d^2 - e^2*x^2)^(1/2)*hypergeom([-3/2, -1/2],
1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(1/2))
```

3.9 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=121

$$-\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}-\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-}}{d}\right.$$

```
[Out] -1/2*(-e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2-3/2*d^2*e^2*arctan(e*x/(-e^2*x^2+d^2
)^(1/2))+3/2*d^2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)-3/2*d*e*(e*x+d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A]
time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {827, 858, 223, 209, 272, 65, 214}

$$-\frac{3}{2}d^2e^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d - e*x)*(d^2 - e^2*x^2)^(
3/2))/(2*x^2) - (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^
2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^3}\,dx = -\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{\left(-4d^2e+4de^2x\right)\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{-8d^3e^2-8d}{x\sqrt{d^2-e^2x}}$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{2}\left(3d^3e^2\right)\int \frac{}{x\sqrt{d^2}}$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{1}{4}\left(3d^3e^2\right)\text{Subst}\left(\int\right.$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.$$

$$= -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2} - \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.$$

Mathematica [A]
time = 0.42, size = 141, normalized size = 1.17

$$-\frac{\sqrt{d^2-e^2x^2}\,(d^3+2d^2ex+2de^2x^2+e^3x^3)}{2x^2} - 3d^2e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) - \frac{3}{2}d^2e\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^3,x]
```

```
[Out] -1/2*(Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/x^2 -
3*d^2*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] - (3*d^2*e*Sqrt[-
e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(105) = 210$.
time = 0.08, size = 219, normalized size = 1.81

method	result
risch	$-\frac{d^2\sqrt{-e^2x^2+d^2}\,(2ex+d)}{2x^2} - \frac{e^3x\sqrt{-e^2x^2+d^2}}{2} - \frac{3e^3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}} - e^2d\sqrt{-e^2x^2+d^2}$

default	$d\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{2d^{2}x^{2}}-\frac{3e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)}{2d^{2}}\right)+e\Bigg(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(5/2)-3/2*e^2/d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+
d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x))))+e*(-1/d^2/x*(-e^2*x^2+d^2)^(5/2)-4*e^2/d^2*(1/4*x*(-e^2*
x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))
```

Maxima [A]
time = 0.50, size = 151, normalized size = 1.25

$$-\frac{3}{2}\,d^{2}\arcsin\left(\frac{xe}{d}\right)e^{2}+\frac{3}{2}\,d^{2}e^{2}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)-\frac{3}{2}\,\sqrt{-x^{2}e^{2}+d^{2}}\,xe^{3}-\frac{3}{2}\,\sqrt{-x^{2}e^{2}+d^{2}}\,de^{2}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{2\,d}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}e}{x}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}{2\,dx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*d^2*arcsin(x*e/d)*e^2 + 3/2*d^2*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2
 + d^2)*d/abs(x)) - 3/2*sqrt(-x^2*e^2 + d^2)*x*e^3 - 3/2*sqrt(-x^2*e^2 + d^
2)*d*e^2 - 1/2*(-x^2*e^2 + d^2)^(3/2)*e^2/d - (-x^2*e^2 + d^2)^(3/2)*e/x -
1/2*(-x^2*e^2 + d^2)^(5/2)/(d*x^2)
```

Fricas [A]
time = 2.30, size = 125, normalized size = 1.03

$$\frac{6\,d^{2}x^{2}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)e^{2}-3\,d^{2}x^{2}e^{2}\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)-2\,d^{2}x^{2}e^{2}-\left(x^{3}e^{3}+2\,dx^{2}e^{2}+2\,d^{2}xe+d^{3}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*x^2*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^2 - 3*d^2*x^2
*e^2*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - 2*d^2*x^2*e^2 - (x^3*e^3 + 2*d*x^
2*e^2 + 2*d^2*x*e + d^3)*sqrt(-x^2*e^2 + d^2))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 4.26, size = 461, normalized size = 3.81

$$d^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**3,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d),
 True)) + d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x
**2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)
) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)
) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*
e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - e**3*Piecewise((-I*d**2*acosh(e*x
/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1
+ e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x
*sqrt(1 - e**2*x**2/d**2)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(104) = 208.
time = 1.60, size = 211, normalized size = 1.74

$$-\frac{3}{2}d^2\arcsin\left(\frac{xe}{d}\right)e^2\operatorname{sgn}(d)+\frac{3}{2}d^2e^2\log\left(\frac{\left|-2de-2\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2|x|}\right)-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^{(-2)}}{8x^2}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2}{2x}+\frac{\left(d^2e^2+\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2}{x}\right)x^2e^4}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}-\frac{1}{2}\sqrt{-x^2e^2+d^2}\left(xe^3+2de^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/2*d^2*arcsin(x*e/d)*e^2*sgn(d) + 3/2*d^2*e^2*log(1/2*abs(-2*d*e - 2*sqrt
(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d
^2*e^(-2)/x^2 - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2/x + 1/8*(d^2*e^2 + 4
*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2/x)*x^2*e^4/(d*e + sqrt(-x^2*e^2 + d^2)*
e)^2 - 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^3 + 2*d*e^2)
```

Mupad [B]
time = 3.74, size = 120, normalized size = 0.99

$$\frac{3\,d^2\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2}-\frac{d^3\,\sqrt{d^2-e^2\,x^2}}{2\,x^2}-d\,e^2\,\sqrt{d^2-e^2\,x^2}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ \frac{e^2\,x^2}{d^2}\right)}{x\,{\left(1-\frac{e^2\,x^2}{d^2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^3,x)
```

```
[Out] (3*d^2*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/2 - (d^3*(d^2 - e^2*x^2)^(1/2))/
(2*x^2) - d*e^2*(d^2 - e^2*x^2)^(1/2) - (e*(d^2 - e^2*x^2)^(3/2)*hypergeom(
[-3/2, -1/2], 1/2, (e^2*x^2)/d^2))/(x*(1 - (e^2*x^2)/d^2)^(3/2))
```

3.10 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/6*(3*e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x^3+d*e^3*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))+3/2*d*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e^2*(-3*e*x+2*d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A]
time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {825, 827, 858, 223, 209, 272, 65, 214}

$$de^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] (e^2*(2*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d + 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^4}\,dx = -\frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{\int \frac{(4d^3e^2+6d^2e^3x)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + \frac{\int \frac{-12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,d}{8d^2}$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{1}{2}\left(3d^2e^3\right)\int \frac{}{x\sqrt{d^2}}$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} - \frac{1}{4}\left(3d^2e^3\right)\text{Subst}\Bigg($$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{e}{\sqrt{d^2-}}\right.$$

$$= \frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{e}{\sqrt{d^2-}}\right.$$

Mathematica [A]
time = 0.38, size = 139, normalized size = 1.16

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^3-3d^2ex+8de^2x^2-6e^3x^3\right)}{6x^3} - 3de^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) - d\left(-e^2\right)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 - 3*d^2*e*x + 8*d*e^2*x^2 - 6*e^3*x^3))/(6*x^3
) - 3*d*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - d*(-e^2)^(3
/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(106) = 212$.
time = 0.07, size = 250, normalized size = 2.08

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,d\left(-8e^2x^2+3dex+2d^2\right)}{6x^3} - e^3\sqrt{-e^2x^2+d^2} + \frac{e^4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{3e^3d^2\ln\left(\frac{2d^2+2}{}\right.}{}$

default	$e\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{2d^{2}x^{2}}-\frac{3e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}\right)\right)}{2d^{2}}\right)+d\left(-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(5/2)-3/2*e^2/d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+
d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x))))+d*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(5/2)-2/3*e^2/d^2*(-1/d^2
/x*(-e^2*x^2+d^2)^(5/2)-4*e^2/d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*
x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d
^2)^(1/2))))))
```

Maxima [A]
time = 0.49, size = 173, normalized size = 1.44

$$d\arcsin\left(\frac{xe}{d}\right)e^{3}+\frac{3}{2}\,de^{3}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)+\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,xe^{4}}{d}-\frac{3}{2}\,\sqrt{-x^{2}e^{2}+d^{2}}\,e^{3}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}{2\,d^{2}}+\frac{2\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}e^{2}}{3\,dx}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}e}{2\,d^{2}x^{2}}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}{3\,dx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] d*arcsin(x*e/d)*e^3 + 3/2*d*e^3*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d
/abs(x)) + sqrt(-x^2*e^2 + d^2)*x*e^4/d - 3/2*sqrt(-x^2*e^2 + d^2)*e^3 - 1/
2*(-x^2*e^2 + d^2)^(3/2)*e^3/d^2 + 2/3*(-x^2*e^2 + d^2)^(3/2)*e^2/(d*x) - 1
/2*(-x^2*e^2 + d^2)^(5/2)*e/(d^2*x^2) - 1/3*(-x^2*e^2 + d^2)^(5/2)/(d*x^3)
```

Fricas [A]
time = 2.57, size = 121, normalized size = 1.01

$$-\frac{12\,dx^3\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^3+9\,dx^3e^3\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+6\,dx^3e^3+\left(6\,x^3e^3-8\,dx^2e^2+3\,d^2xe+2\,d^3\right)\sqrt{-x^2e^2+d^2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*x^3*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^3 + 9*d*x^3*e
^3*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + 6*d*x^3*e^3 + (6*x^3*e^3 - 8*d*x^2*
e^2 + 3*d^2*x*e + 2*d^3)*sqrt(-x^2*e^2 + d^2))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 3.77, size = 457, normalized size = 3.81

$$d^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**4,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) +
 d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x)
)/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*
d), True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acos
h(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
, (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**
2*x**2/d**2)), True)) - e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1
)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2
)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I
*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(102) = 204.
time = 1.02, size = 256, normalized size = 2.13

$$d\arcsin\left(\frac{xe}{d}\right)e^3\operatorname{sgn}(d)+\frac{3}{2}\,de^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(de^3+\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de}{x}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{(-1)}}{x^2}\right)x^3e^6}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de}{8\,x}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{(-1)}}{8\,x^2}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3de^{(-3)}}{24\,x^3}-\sqrt{-x^2e^2+d^2}\,e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] d*arcsin(x*e/d)*e^3*sgn(d) + 3/2*d*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/24*(d*e^3 + 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d*e/x - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^(-1)/x^2)*x^3*e^6/(d*e + sq
rt(-x^2*e^2 + d^2)*e)^3 + 5/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e/x - 1/8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^(-1)/x^2 - 1/24*(d*e + sqrt(-x^2*e^2 + d
^2)*e)^3*d*e^(-3)/x^3 - sqrt(-x^2*e^2 + d^2)*e^3
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^4, x)
```

3.11 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^5} dx$

Optimal. Leaf size=118

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\,(d^2-e^2x^2)^{3/2}}{12x^4}+e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/12*(4*e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x^4+e^4*arctan(e*x/(-e^2*x^2+d^2)^(1
/2))-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*e^2*(8*e*x+3*d)*(-e^2*x^2+
d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {825, 858, 223, 209, 272, 65, 214}

$$e^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d+4ex)\,(d^2-e^2x^2)^{3/2}}{12x^4}-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] (e^2*(3*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d + 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) + e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx = -\frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} - \frac{\int \frac{(6d^3e^2+8d^2e^3x)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{\int \frac{12d^5e^4+32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{8}\left(3de^4\right)\int \frac{}{x\sqrt{d^2}}$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + \frac{1}{16}\left(3de^4\right)\text{Subst}\Bigg($$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-}}\right.$$

$$= \frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4} + e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-}}\right.$$

Mathematica [A]
time = 0.40, size = 141, normalized size = 1.19

$$\frac{\sqrt{d^2-e^2x^2}\left(-6d^3-8d^2ex+15de^2x^2+32e^3x^3\right)}{24x^4} + \frac{3}{4}e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) + e^3\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x + \sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 8*d^2*e*x + 15*d*e^2*x^2 + 32*e^3*x^3))/(24*
x^4) + (3*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/4 + e^3*Sq
rt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(104) = 208.
time = 0.07, size = 281, normalized size = 2.38

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,\left(-32e^3x^3-15d\,e^2x^2+8d^2ex+6d^3\right)}{24x^4} + \frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{3e^4d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x}}{x}\right.}{8\sqrt{d^2}}$

default	$d\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{4d^{2}x^{4}}-\frac{e^{2}\left(-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{2d^{2}x^{2}}-\frac{3e^{2}\left(\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2}\left(\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right.}{\sqrt{d^{2}}}\right.\right.}{2d^{2}}\right.}{4d^{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(5/2)-1/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^
2)^(5/2)-3/2*e^2/d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^
2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+e*(-1/3/
d^2/x^3*(-e^2*x^2+d^2)^(5/2)-2/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(5/2)-4*e
^2/d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*
d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))
```

Maxima [A]
time = 0.52, size = 197, normalized size = 1.67

$$\arcsin\left(\frac{xe}{d}\right)e^{4}-\frac{3}{8}e^{4}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)+\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,xe^{5}}{d^{2}}+\frac{3\sqrt{-x^{2}e^{2}+d^{2}}\,e^{4}}{8\,d}+\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}e^{4}}{8\,d^{3}}+\frac{2\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}e^{3}}{3\,d^{2}x}+\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}e^{2}}{8\,d^{3}x^{2}}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}e}{3\,d^{2}x^{3}}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}{4\,dx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="maxima")

[Out] arcsin(x*e/d)*e^4 - 3/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x)) + sqrt(-x^2*e^2 + d^2)*x*e^5/d^2 + 3/8*sqrt(-x^2*e^2 + d^2)*e^4/d + 1/8*(-x^2*e^2 + d^2)^(3/2)*e^4/d^3 + 2/3*(-x^2*e^2 + d^2)^(3/2)*e^3/(d^2*x) + 1/8*(-x^2*e^2 + d^2)^(5/2)*e^2/(d^3*x^2) - 1/3*(-x^2*e^2 + d^2)^(5/2)*e/(d^2*x^3) - 1/4*(-x^2*e^2 + d^2)^(5/2)/(d*x^4)

Fricas [A]
time = 2.16, size = 112, normalized size = 0.95

$$-\frac{48\,x^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^4-9\,x^4e^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-\left(32\,x^3e^3+15\,dx^2e^2-8\,d^2xe-6\,d^3\right)\sqrt{-x^2e^2+d^2}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="fricas")

[Out] -1/24*(48*x^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^4 - 9*x^4*e^4*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (32*x^3*e^3 + 15*d*x^2*e^2 - 8*d^2*x*e - 6*d^3)*sqrt(-x^2*e^2 + d^2))/x^4

Sympy [C] Result contains complex when optimal does not.
time = 4.95, size = 541, normalized size = 4.58

$$d^3\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**5,x)

[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) - e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(100) = 200.
time = 0.86, size = 288, normalized size = 2.44

$$\arcsin\left(\frac{xe}{d}\right)e^4\mathrm{sgn}(d) + \frac{x^4\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^2}{x} - \frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-2)}}{x^3} - \frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}{x^2} + 3\,e^4\right)e^8}{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4} - \frac{3}{8}e^4\log\left(\frac{\left|-2\,de-2\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) + \frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^2}{8\,x} - \frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-2)}}{24\,x^3} - \frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-4)}}{64\,x^4} + \frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e^4*sgn(d) + 1/192*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^2/
x - 120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-2)/x^3 - 24*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^2/x^2 + 3*e^4)*e^8/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 - 3/8*e^4*
log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 5/8*(d*e +
sqrt(-x^2*e^2 + d^2)*e)*e^2/x - 1/24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-2
)/x^3 - 1/64*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-4)/x^4 + 1/8*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^2/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^5, x)
```

3.12 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=108

$$\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] -1/4*e*(-e^2*x^2+d^2)^(3/2)/x^4-1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5-3/8*e^5*arct
anh((-e^2*x^2+d^2)^(1/2)/d)/d+3/8*e^3*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}-\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}+\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] (3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) - (
d^2 - e^2*x^2)^(5/2)/(5*d*x^5) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*
d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx &= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+e\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx\\
&= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{1}{2}e\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,x,x^2\right.\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}+\frac{1}{16}\left(3e^5\right)\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{1}{8}\left(3e^3\right)\text{Subst}\left(\int \frac{}{\frac{d^2}{e^2}-}\right.\\
&= \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}-\frac{e\left(d^2-e^2x^2\right)^{3/2}}{4x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5dx^5}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e}}{d}\right.}{8d}
\end{aligned}$$

Mathematica [A]
time = 0.41, size = 151, normalized size = 1.40

$$\frac{\sqrt{d^2-e^2x^2}\left(-8d^4-10d^3ex+16d^2e^2x^2+25de^3x^3-8e^4x^4\right)+15e^5x^5\log\left(d\left(-d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)-15e^5x^5\log\left(d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{40dx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 10*d^3*e*x + 16*d^2*e^2*x^2 + 25*d*e^3*x^3 -
 8*e^4*x^4) + 15*e^5*x^5*Log[d*(-d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2])] -
 15*e^5*x^5*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]])/(40*d*x^5)
```

Maple [A]
time = 0.08, size = 166, normalized size = 1.54

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(8 e^{4} x^{4}-25 d\, e^{3} x^{3}-16 d^{2} x^{2} e^{2}+10 d^{3} e x +8 d^{4}\right)}{40 x^{5} d}-\frac{3 e^{5} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{8 \sqrt{d^{2}}}$
default	$-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{5 d \,x^{5}}+e \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{4 d^{2} x^{4}}-\frac{e^{2} \left(-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{2 d^{2} x^{2}}-\frac{3 e^{2} \left(\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{3}+d^{2} \left(\sqrt{-e^{2} x^{2}+d^{2}}-\frac{d^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{\cdots}}{\cdots}\right)}{\cdots}\right.\right.}{2 d^{2}}\right.}{4 d^{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5+e*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(5/2)-1/4*e^
2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(5/2)-3/2*e^2/d^2*(1/3*(-e^2*x^2+d^2)^(3
/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x)))))
```

Maxima [A]
time = 0.48, size = 146, normalized size = 1.35

$$-\frac{3\,e^{5}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)}{8\,d}+\frac{3\,\sqrt{-x^{2}e^{2}+d^{2}}\,e^{5}}{8\,d^{2}}+\frac{(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}e^{5}}{8\,d^{4}}+\frac{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e^{3}}{8\,d^{4}x^{2}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e}{4\,d^{2}x^{4}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}{5\,dx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d + 3/8*sqrt(-
x^2*e^2 + d^2)*e^5/d^2 + 1/8*(-x^2*e^2 + d^2)^(3/2)*e^5/d^4 + 1/8*(-x^2*e^2
 + d^2)^(5/2)*e^3/(d^4*x^2) - 1/4*(-x^2*e^2 + d^2)^(5/2)*e/(d^2*x^4) - 1/5*
(-x^2*e^2 + d^2)^(5/2)/(d*x^5)
```

Fricas [A]
time = 2.95, size = 93, normalized size = 0.86

$$\frac{15\,x^5e^5\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(8\,x^4e^4-25\,dx^3e^3-16\,d^2x^2e^2+10\,d^3xe+8\,d^4)\sqrt{-x^2e^2+d^2}}{40\,dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(15*x^5*e^5*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (8*x^4*e^4 - 25*d*x^3
*e^3 - 16*d^2*x^2*e^2 + 10*d^3*x*e + 8*d^4)*sqrt(-x^2*e^2 + d^2))/(d*x^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.81, size = 774, normalized size = 7.17

$$d^3\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**6,x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) + d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**
2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sq
rt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - e**3*Piecewise((-e*sqrt(d**2/(
e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(87) = 174.
time = 0.93, size = 366, normalized size = 3.39

$$\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^3}{x}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e}{x^2}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-1)}}{x^3}+\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-3)}}{x^4}+2\,e^5\right)e^{10}}{320\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d}-\frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{8\,d}-\frac{\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4e^3}{x}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^4e}{x^2}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^4e^{(-1)}}{x^3}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^4e^{(-3)}}{x^4}+\frac{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^4e^{(-5)}}{x^5}}{320\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/320*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^3/x - 10*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*e/x^2 - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-1)/x^3 + 20*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-3)/x^4 + 2*e^5)*e^10/((d*e + sqrt(-x^2*e
^2 + d^2)*e)^5*d) - 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*
e^(-2)/abs(x))/d - 1/320*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^3/x - 40*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e/x^2 - 10*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d^4*e^(-1)/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^(-3)/x^4 +
2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^4*e^(-5)/x^5)/d^5
```

Mupad [B]
time = 4.26, size = 93, normalized size = 0.86

$$\frac{3\,d^2\,e\,\sqrt{d^2-e^2\,x^2}}{8\,x^4}-\frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{5\,d\,x^5}-\frac{3\,e^5\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{8\,d}-\frac{5\,e\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^6,x)
```

```
[Out] (3*d^2*e*(d^2 - e^2*x^2)^(1/2))/(8*x^4) - (d^2 - e^2*x^2)^(5/2)/(5*d*x^5) -
 (3*e^5*atanh((d^2 - e^2*x^2)^(1/2)/d))/(8*d) - (5*e*(d^2 - e^2*x^2)^(3/2))
/(8*x^4)
```

3.13 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7} \, dx$

Optimal. Leaf size=143

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}-\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

```
[Out] -1/24*e^2*(-e^2*x^2+d^2)^(3/2)/d/x^4-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6-1/5*e*(
-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/16*e^6*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/
16*e^4*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {849, 821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}-\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}+\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^7}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{\int \frac{(-6d^2e-de^2x)\left(d^2-e^2x^2\right)^{3/2}}{x^6}\,dx}{6d^2}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^2\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{x^5}\,dx}{6d}$$

$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^2\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d}$$

$$= -\frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^4\text{Subst}\left(\int \frac{\sqrt{d^2-}}{x^2}\right.}{16d}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} + \frac{e^6\text{S}}{}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^4\text{S}}{}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2\left(d^2-e^2x^2\right)^{3/2}}{24dx^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{6dx^6} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{5d^2x^5} - \frac{e^6\text{t}}{}$$

Mathematica [A]
time = 0.41, size = 126, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\,(-40d^5-48d^4ex+70d^3e^2x^2+96d^2e^3x^3-15de^4x^4-48e^5x^5)}{240d^2x^6} + \frac{e^6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^7,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 - 48*d^4*e*x + 70*d^3*e^2*x^2 + 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 - 48*e^5*x^5))/(240*d^2*x^6) + (e^6*ArcTanh[(Sqrt[-e^2]*x)
/d - Sqrt[d^2 - e^2*x^2]/d])/(8*d^2)
```

Maple [A]
time = 0.06, size = 198, normalized size = 1.38

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(48e^5x^5+15d\,e^4x^4-96d^2e^3x^3-70x^2d^3e^2+48d^4xe+40d^5)}{240x^6d^2} - \frac{e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16d\sqrt{d^2}}$

default	$-\frac{e(-e^2x^2+d^2)^{\frac{5}{2}}}{5d^2x^5}+d\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{6d^2x^6}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2 \ldots}\right)\right)}{\ldots}\right)}{4d^2 \ldots}\right)}{6d^2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*e*(-e^2*x^2+d^2)^(5/2)/d^2/x^5+d*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(5/2)+1/
6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(5/2)-1/4*e^2/d^2*(-1/2/d^2/x^2*(-e^
2*x^2+d^2)^(5/2)-3/2*e^2/d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^
(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))
```

Maxima [A]

time = 0.48, size = 169, normalized size = 1.18

$$-\frac{e^6\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{16\,d^2}+\frac{\sqrt{-x^2e^2+d^2}\,e^6}{16\,d^3}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^6}{48\,d^5}+\frac{(-x^2e^2+d^2)^{\frac{5}{2}}e^4}{48\,d^5x^2}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}e^2}{24\,d^3x^4}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}e}{5\,d^2x^5}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}}{6\,dx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="maxima")

[Out] -1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^2 + 1/16*sqrt(-x^2*e^2 + d^2)*e^6/d^3 + 1/48*(-x^2*e^2 + d^2)^(3/2)*e^6/d^5 + 1/48*(-x^2*e^2 + d^2)^(5/2)*e^4/(d^5*x^2) - 1/24*(-x^2*e^2 + d^2)^(5/2)*e^2/(d^3*x^4) - 1/5*(-x^2*e^2 + d^2)^(5/2)*e/(d^2*x^5) - 1/6*(-x^2*e^2 + d^2)^(5/2)/(d*x^6)

Fricas [A]
time = 2.36, size = 103, normalized size = 0.72

$$\frac{15\,x^6e^6\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-\left(48\,x^5e^5+15\,dx^4e^4-96\,d^2x^3e^3-70\,d^3x^2e^2+48\,d^4xe+40\,d^5\right)\sqrt{-x^2e^2+d^2}}{240\,d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="fricas")

[Out] 1/240*(15*x^6*e^6*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (48*x^5*e^5 + 15*d*x^4*e^4 - 96*d^2*x^3*e^3 - 70*d^3*x^2*e^2 + 48*d^4*x*e + 40*d^5)*sqrt(-x^2*e^2 + d^2))/(d^2*x^6)

Sympy [C] Result contains complex when optimal does not.
time = 8.96, size = 918, normalized size = 6.42

$$d^3\left(\begin{cases}-\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**7,x)

[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**2*e*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - d*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +

```
e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5
*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1))
+ I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d
**3), True)) - e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))
```

<u>Giac</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(116) = 232.

time = 1.01, size = 425, normalized size = 2.97

$$\frac{x^6\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^4}{x}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^2}{x^2}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-2)}}{x^4}+\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-4)}}{x^5}-\frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3}{x^3}+5\,e^6\right)e^{12}}{1920\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6d^2}-\frac{e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{16\,d^2}-\frac{\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{10}e^4}{x}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^{10}e^2}{x^2}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^{10}e^{(-2)}}{x^4}+\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^{10}e^{(-4)}}{x^5}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6d^{10}e^{(-6)}}{x^6}-\frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{10}}{x^3}}{1920\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/1920*x^6*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x - 15*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^2*e^2/x^2 - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-2)/x^4 + 1
20*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-4)/x^5 - 60*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^3/x^3 + 5*e^6)*e^12/((d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2) - 1/16*e
^6*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 1/19
20*(120*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^10*e^4/x - 15*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*d^10*e^2/x^2 - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^10*e^(-2)
/x^4 + 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^10*e^(-4)/x^5 + 5*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^6*d^10*e^(-6)/x^6 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*
d^10/x^3)/d^12
```

<u>Mupad</u> [**B**]

time = 4.66, size = 118, normalized size = 0.83

$$\frac{d^3\,\sqrt{d^2-e^2\,x^2}}{16\,x^6}-\frac{d\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{6\,x^6}-\frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{16\,d\,x^6}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{5\,d^2\,x^5}+\frac{e^6\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1{}\mathrm{i}}{d}\right)\,1{}\mathrm{i}}{16\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^7,x)
```

```
[Out] (d^3*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (d*(d^2 - e^2*x^2)^(3/2))/(6*x^6) -
(d^2 - e^2*x^2)^(5/2)/(16*d*x^6) + (e^6*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*
1i)/(16*d^2) - (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5)
```

3.14 $\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=172

$$\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}-\frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6}-\frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e}}{d}\right.}{16d^3}$$

```
[Out] -1/24*e^3*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7-1/6*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^6-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5-1/16*e^
7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3+1/16*e^5*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {849, 821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}-\frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6}+\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4}-\frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (e^3*(d^2 - e^2*x^2)^(3/2))/(24*d^
2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) - (e*(d^2 - e^2*x^2)^(5/2))/(6*d^2
*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) - (e^7*ArcTanh[Sqrt[d^2
- e^2*x^2]/d])/(16*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^8}\,dx = -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{\int \frac{(-7d^2e-2de^2x)(d^2-e^2x^2)^{3/2}}{x^7}dx}{7d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} + \frac{\int \frac{(12d^3e^2+7d^2e^3x)(d^2-e^2x^2)^{3/2}}{x^6}dx}{42d^4}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^3\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}dx}{6d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^3\text{Subst}\left(\int \frac{(d^2-e^2x}{x^3}\right.}{12d^2}$$

$$= -\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} -$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - 2e^2$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - 2e^2$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - 2e^2$$

Mathematica [A]
time = 0.50, size = 132, normalized size = 0.77

$$\frac{-\frac{\sqrt{d^2-e^2x^2}\,(240d^6+280d^5ex-384d^4e^2x^2-490d^3e^3x^3+48d^2e^4x^4+105de^5x^5+96e^6x^6)}{x^7}+210e^7\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{1680d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^8,x]
```

```
[Out] (-((Sqrt[d^2 - e^2*x^2]*(240*d^6 + 280*d^5*e*x - 384*d^4*e^2*x^2 - 490*d^3*
e^3*x^3 + 48*d^2*e^4*x^4 + 105*d*e^5*x^5 + 96*e^6*x^6))/x^7) + 210*e^7*ArcT
anh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(1680*d^3)
```

Maple [A]
time = 0.08, size = 225, normalized size = 1.31

method	result

risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(96e^6x^6+105d\,e^5x^5+48d^2e^4x^4-490d^3x^3e^3-384d^4e^2x^2+280d^5ex+240d^6)}{1680x^7d^3}-\frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x}}{x}\right.}{16d^2\sqrt{d^2}}$
default	$e\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{6d^2x^6}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{}}{}\right.}{}\right.\right.}{2d^2}\right.}{4d^2}\right.}{6d^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(5/2)+1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^
2)^(5/2)-1/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(5/2)-3/2*e^2/d^2*(1/3*(-
e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d
^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))+d*(-1/7/d^2/x^7*(-e^2*x^2+d^2)^(5/2
)-2/35*e^2/d^4/x^5*(-e^2*x^2+d^2)^(5/2))
```

Maxima [A]

time = 0.49, size = 192, normalized size = 1.12

$$-\frac{e^{7}\log\left(\frac{2d^{2}}{|x|}+\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{3}}+\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,e^{7}}{16\,d^{4}}+\frac{(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}e^{7}}{48\,d^{6}}+\frac{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e^{5}}{48\,d^{6}x^{2}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e^{3}}{24\,d^{4}x^{4}}-\frac{2\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e^{2}}{35\,d^{3}x^{5}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e}{6\,d^{2}x^{6}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}{7\,dx^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^3 + 1/16*sq
rt(-x^2*e^2 + d^2)*e^7/d^4 + 1/48*(-x^2*e^2 + d^2)^(3/2)*e^7/d^6 + 1/48*(-x
^2*e^2 + d^2)^(5/2)*e^5/(d^6*x^2) - 1/24*(-x^2*e^2 + d^2)^(5/2)*e^3/(d^4*x^
4) - 2/35*(-x^2*e^2 + d^2)^(5/2)*e^2/(d^3*x^5) - 1/6*(-x^2*e^2 + d^2)^(5/2)
*e/(d^2*x^6) - 1/7*(-x^2*e^2 + d^2)^(5/2)/(d*x^7)
```

Fricas [A]
time = 2.50, size = 113, normalized size = 0.66

$$\frac{105\,x^{7}e^{7}\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)-(96\,x^{6}e^{6}+105\,dx^{5}e^{5}+48\,d^{2}x^{4}e^{4}-490\,d^{3}x^{3}e^{3}-384\,d^{4}x^{2}e^{2}+280\,d^{5}xe+240\,d^{6})\sqrt{-x^{2}e^{2}+d^{2}}}{1680\,d^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/1680*(105*x^7*e^7*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (96*x^6*e^6 + 105*
d*x^5*e^5 + 48*d^2*x^4*e^4 - 490*d^3*x^3*e^3 - 384*d^4*x^2*e^2 + 280*d^5*x*
e + 240*d^6)*sqrt(-x^2*e^2 + d^2))/(d^3*x^7)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.76, size = 1037, normalized size = 6.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**8,x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e*P
iecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e*
*5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-
```

```
d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I
*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*d**3*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1)
, (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**
2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**
6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*
sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - e**3*Pi
ecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*aco
sh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d
**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3
/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(139) = 278$.
time = 1.07, size = 492, normalized size = 2.86

$$\frac{x^{7}\left(\frac{35\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)e^{5}}{x}-\frac{21\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{2}e^{3}}{x^{2}}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{3}e}{x^{3}}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{4}e^{(-1)}}{x^{4}}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{5}e^{(-3)}}{x^{5}}+\frac{315\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{6}e^{(-5)}}{x^{6}}+15e^{7}\right)e^{14}}{13440\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{7}d^{3}}-\frac{e^{7}\log\left(\frac{\left|-2de-2\sqrt{-x^{2}e^{2}+d^{2}}e\right|e^{(-2)}}{2|x|}\right)}{16d^{3}}-\frac{\frac{315\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)d^{18}e^{5}}{x}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{2}d^{18}e^{3}}{x^{2}}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{3}d^{18}e}{x^{3}}-\frac{105\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{4}d^{18}e^{(-1)}}{x^{4}}-\frac{21\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{5}d^{18}e^{(-3)}}{x^{5}}+\frac{35\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{6}d^{18}e^{(-5)}}{x^{6}}+\frac{15\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{7}d^{18}e^{(-7)}}{x^{7}}}{13440d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/13440*x^7*(35*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^5/x - 21*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^2*e^3/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e/x^3 - 105*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-1)/x^4 - 105*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^5*e^(-3)/x^5 + 315*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-5)/x^6 + 15*e
^7)*e^14/((d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^3) - 1/16*e^7*log(1/2*abs(-2*d
*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/13440*(315*(d*e + sqr
t(-x^2*e^2 + d^2)*e)*d^18*e^5/x - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^18
*e^3/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e/x^3 - 105*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^4*d^18*e^(-1)/x^4 - 21*(d*e + sqrt(-x^2*e^2 + d^2)*e)^
5*d^18*e^(-3)/x^5 + 35*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^18*e^(-5)/x^6 + 1
5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^18*e^(-7)/x^7)/d^21
```

Mupad [B]
time = 5.33, size = 192, normalized size = 1.12

$$\frac{8\,d\,e^2\,\sqrt{d^2-e^2\,x^2}}{35\,x^5}-\frac{d^3\,\sqrt{d^2-e^2\,x^2}}{7\,x^7}-\frac{e^4\,\sqrt{d^2-e^2\,x^2}}{35\,d\,x^3}-\frac{2\,e^6\,\sqrt{d^2-e^2\,x^2}}{35\,d^3\,x}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{6\,x^6}+\frac{d^2\,e\,\sqrt{d^2-e^2\,x^2}}{16\,x^6}-\frac{e\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{16\,d^2\,x^6}+\frac{e^7\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1{}\mathrm{i}}{d}\right)\,1{}\mathrm{i}}{16\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^8,x)
```

```
[Out] (e^7*atan(((d^2 - e^2*x^2)^(1/2)*1i)/d)*1i)/(16*d^3) - (d^3*(d^2 - e^2*x^2)
^(1/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(3/2))/(6*x^6) - (e^4*(d^2 - e^2*x^2)^
(1/2))/(35*d*x^3) - (2*e^6*(d^2 - e^2*x^2)^(1/2))/(35*d^3*x) + (8*d*e^2*(d^
2 - e^2*x^2)^(1/2))/(35*x^5) + (d^2*e*(d^2 - e^2*x^2)^(1/2))/(16*x^6) - (e*
(d^2 - e^2*x^2)^(5/2))/(16*d^2*x^6)
```

3.15 $\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=201

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}-\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-$$

```
[Out] -1/64*e^4*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8-1/7*e
*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6-2/35*e^
3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5-3/128*e^8*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^
4+3/128*e^6*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {849, 821, 272, 43, 65, 214}

$$-\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8}-\frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7}-\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}-\frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}-\frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6}+\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64
*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) - (e*(d^2 - e^2*x^2)^(5/2))/(7*
d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) - (2*e^3*(d^2 - e^2*x^2
)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{x^9}\,dx = -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{(-8d^2e-3de^2x)\left(d^2-e^2x^2\right)^{3/2}}{x^8}dx}{8d^2}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} + \frac{\int \frac{(21d^3e^2+16d^2e^3x)\left(d^2-e^2x^2\right)^{3/2}}{x^7}dx}{56d^4}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{\int \frac{(-96d^4e^3-21d^3e^4x)}{x^6}}{336d}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}$$
$$= -\frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} - \frac{2e^3\left(d^2-e^2x^2\right)^{5/2}}{35d^4x^5}$$
$$= -\frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} - \frac{e^2\left(d^2-e^2x^2\right)^{5/2}}{16d^3x^6} -$$
$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} -$$
$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} -$$
$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4\left(d^2-e^2x^2\right)^{3/2}}{64d^3x^4} - \frac{\left(d^2-e^2x^2\right)^{5/2}}{8dx^8} - \frac{e\left(d^2-e^2x^2\right)^{5/2}}{7d^2x^7} -$$

Mathematica [A]
time = 0.52, size = 148, normalized size = 0.74

$$\frac{\sqrt{d^2-e^2x^2}\,(-560d^7-640d^6ex+840d^5e^2x^2+1024d^4e^3x^3-70d^3e^4x^4-128d^2e^5x^5-105de^6x^6-256e^7x^7)}{4480d^4x^8} + \frac{3e^8\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{64d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 - 640*d^6*e*x + 840*d^5*e^2*x^2 + 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 - 128*d^2*e^5*x^5 - 105*d*e^6*x^6 - 256*e^7*x^7))/(
4480*d^4*x^8) + (3*e^8*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(
64*d^4)
```

Maple [A]
time = 0.08, size = 256, normalized size = 1.27

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(256e^7x^7+105d\,e^6x^6+128d^2e^5x^5+70d^3e^4x^4-1024d^4e^3x^3-840d^5e^2x^2+640d^6ex+560d^7)}{4480x^8d^4}-\frac{3e^8\ln\left(\frac{2d^2+2\sqrt{}}{}\right.}{12}$

default		$d\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{8d^2x^8}+\frac{3e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{6d^2x^6}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{4d^2x^4}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{2d^2x^2}-\frac{3e^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2}\right.\right.}{}\right)}{4d^2}\right)}{6d^2}\right)}{8d^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/8/d^2/x^8*(-e^2*x^2+d^2)^(5/2)+3/8*e^2/d^2*(-1/6/d^2/x^6*(-e^2*x^2+d^
2)^(5/2)+1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(5/2)-1/4*e^2/d^2*(-1/2/d
^2/x^2*(-e^2*x^2+d^2)^(5/2)-3/2*e^2/d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^
2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/
2))/x)))))))+e*(-1/7/d^2/x^7*(-e^2*x^2+d^2)^(5/2)-2/35*e^2/d^4/x^5*(-e^2*x^
2+d^2)^(5/2))
```

Maxima [A]
time = 0.49, size = 215, normalized size = 1.07

$$-\frac{3\,e^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{128\,d^4}+\frac{3\,\sqrt{-x^2e^2+d^2}\,e^8}{128\,d^5}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^8}{128\,d^7}+\frac{(-x^2e^2+d^2)^{\frac{5}{2}}e^6}{128\,d^7x^2}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}e^4}{64\,d^5x^4}-\frac{2\,(-x^2e^2+d^2)^{\frac{5}{2}}e^3}{35\,d^4x^5}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}e^2}{16\,d^3x^6}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}e}{7\,d^2x^7}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}}{8\,dx^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^4 + 3/128*
sqrt(-x^2*e^2 + d^2)*e^8/d^5 + 1/128*(-x^2*e^2 + d^2)^(3/2)*e^8/d^7 + 1/128
*(-x^2*e^2 + d^2)^(5/2)*e^6/(d^7*x^2) - 1/64*(-x^2*e^2 + d^2)^(5/2)*e^4/(d^
5*x^4) - 2/35*(-x^2*e^2 + d^2)^(5/2)*e^3/(d^4*x^5) - 1/16*(-x^2*e^2 + d^2)^
(5/2)*e^2/(d^3*x^6) - 1/7*(-x^2*e^2 + d^2)^(5/2)*e/(d^2*x^7) - 1/8*(-x^2*e^
2 + d^2)^(5/2)/(d*x^8)
```

Fricas [A]
time = 2.99, size = 123, normalized size = 0.61

$$\frac{105\,x^8e^8\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(256\,x^7e^7+105\,dx^6e^6+128\,d^2x^5e^5+70\,d^3x^4e^4-1024\,d^4x^3e^3-840\,d^5x^2e^2+640\,d^6xe+560\,d^7)\sqrt{-x^2e^2+d^2}}{4480\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/4480*(105*x^8*e^8*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (256*x^7*e^7 + 105
*d*x^6*e^6 + 128*d^2*x^5*e^5 + 70*d^3*x^4*e^4 - 1024*d^4*x^3*e^3 - 840*d^5*
x^2*e^2 + 640*d^6*x*e + 560*d^7)*sqrt(-x^2*e^2 + d^2))/(d^4*x^8)
```

Sympy [C] Result contains complex when optimal does not.
time = 26.95, size = 1159, normalized size = 5.77

$$d^3\left(\begin{cases}-\frac{d^2}{8ex^9\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{7e}{48x^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{192d^2x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e^5}{384d^4x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{5e^7}{128d^6x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e^8\operatorname{acosh}\left(\frac{d}{ex}\right)}{128d^7} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{8ex^9\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{7ie}{48x^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{192d^2x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie^5}{384d^4x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{5ie^7}{128d^6x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie^8\operatorname{asin}\left(\frac{d}{ex}\right)}{128d^7} & \text{otherwise}\end{cases}\right)+d^2e\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{7x^6}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{35d^2x^4}+\frac{4e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^4x^2}+\frac{8e^7\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^6} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{7x^6}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{35d^2x^4}+\frac{4ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^4x^2}+\frac{8ie^7\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^6} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise}\end{cases}\right)-e^3\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**9,x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) + d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) +
 e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x*
*2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6)
, True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) +
5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
*2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d
/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - e**3*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(162) = 324.
time = 1.02, size = 425, normalized size = 2.11

$$\frac{x^8\left(\frac{80\left(de+\sqrt{-x^2e^2+d^2}e\right)e^6}{x}-\frac{112\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^2}{x^3}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^{(-2)}}{x^5}+\frac{1680\left(de+\sqrt{-x^2e^2+d^2}e\right)^7e^{(-6)}}{x^7}-\frac{280\left(de+\sqrt{-x^2e^2+d^2}e\right)^4}{x^4}+35\,e^8\right)e^{16}}{71680\left(de+\sqrt{-x^2e^2+d^2}e\right)^8d^4}-\frac{3\,e^8\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2\,|x|}\right)}{128\,d^4}-\frac{\frac{1680\left(de+\sqrt{-x^2e^2+d^2}e\right)d^{28}e^6}{x}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^{28}e^2}{x^3}-\frac{112\left(de+\sqrt{-x^2e^2+d^2}e\right)^5d^{28}e^{(-2)}}{x^5}+\frac{80\left(de+\sqrt{-x^2e^2+d^2}e\right)^7d^{28}e^{(-6)}}{x^7}+\frac{35\left(de+\sqrt{-x^2e^2+d^2}e\right)^8d^{28}e^{(-8)}}{x^8}-\frac{280\left(de+\sqrt{-x^2e^2+d^2}e\right)^4d^{28}}{x^4}}{71680\,d^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/71680*x^8*(80*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^6/x - 112*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^3*e^2/x^3 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-2)/x^5
+ 1680*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^(-6)/x^7 - 280*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^4/x^4 + 35*e^8)*e^16/((d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^4) -
```

```
3/128*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4
 - 1/71680*(1680*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^28*e^6/x - 560*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^3*d^28*e^2/x^3 - 112*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*
d^28*e^(-2)/x^5 + 80*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^28*e^(-6)/x^7 + 35*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^28*e^(-8)/x^8 - 280*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^4*d^28/x^4)/d^32
```

Mupad [B]
time = 6.04, size = 212, normalized size = 1.05

$$\frac{3\,d^3\,\sqrt{d^2-e^2\,x^2}}{128\,x^8}-\frac{11\,d\,{\left(d^2-e^2\,x^2\right)}^{3/2}}{128\,x^8}-\frac{11\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{128\,d\,x^8}+\frac{3\,{\left(d^2-e^2\,x^2\right)}^{7/2}}{128\,d^3\,x^8}+\frac{8\,e^3\,\sqrt{d^2-e^2\,x^2}}{35\,x^5}-\frac{e^5\,\sqrt{d^2-e^2\,x^2}}{35\,d^2\,x^3}-\frac{2\,e^7\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,x}-\frac{d^2\,e\,\sqrt{d^2-e^2\,x^2}}{7\,x^7}+\frac{e^8\,\mathrm{atan}\left(\frac{\sqrt{d^2-e^2\,x^2}\,1{}\mathrm{i}}{d}\right)\,3{}\mathrm{i}}{128\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(3/2)*(d + e*x))/x^9,x)
```

```
[Out] (3*d^3*(d^2 - e^2*x^2)^(1/2))/(128*x^8) - (11*d*(d^2 - e^2*x^2)^(3/2))/(128
*x^8) - (11*(d^2 - e^2*x^2)^(5/2))/(128*d*x^8) + (3*(d^2 - e^2*x^2)^(7/2))/
(128*d^3*x^8) + (8*e^3*(d^2 - e^2*x^2)^(1/2))/(35*x^5) + (e^8*atan(((d^2 -
e^2*x^2)^(1/2)*1i)/d)*3i)/(128*d^4) - (e^5*(d^2 - e^2*x^2)^(1/2))/(35*d^2*x
^3) - (2*e^7*(d^2 - e^2*x^2)^(1/2))/(35*d^4*x) - (d^2*e*(d^2 - e^2*x^2)^(1/
2))/(7*x^7)
```

3.16 $\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] 1/3*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-d
^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*d*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]

time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {811, 655, 201, 223, 209}

$$\frac{d^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((d^2*Sqrt[d^2 - e^2*x^2])/e^3) - (d*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (d^2
 - e^2*x^2)^(3/2)/(3*e^3) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\int (d+ex)\sqrt{d^2-e^2x^2}\,dx}{e^2}+\frac{d^2\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\ &= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{d\int \sqrt{d^2-e^2x^2}\,dx}{e^2}+\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\ &= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}+\frac{d^3\text{Subs}}{}\\ &= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{d^3}{}\\ &= -\frac{d^2\sqrt{d^2-e^2x^2}}{e^3}-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 92, normalized size = 0.89

$$\frac{(-4d^2-3dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{6e^3}+\frac{d^3\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((-4*d^2 - 3*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(6*e^3) + (d^3*Sqrt[-e
^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Maple [A]
time = 0.06, size = 107, normalized size = 1.04

method	result
risch	$-\frac{(2e^2x^2+3dex+4d^2)\sqrt{-e^2x^2+d^2}}{6e^3}+\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$
default	$e\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)+d\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/3*x^2/e^2*(-e^2*x^2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+d^2)^(1/2))+d*(-
1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [A]
time = 0.49, size = 75, normalized size = 0.73

$$\frac{1}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{3}\sqrt{-x^2e^2+d^2}\,x^2e^{(-1)}-\frac{1}{2}\sqrt{-x^2e^2+d^2}\,dxe^{(-2)}-\frac{2}{3}\sqrt{-x^2e^2+d^2}\,d^2e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3) - 1/3*sqrt(-x^2*e^2 + d^2)*x^2*e^(-1) - 1/2*sq
rt(-x^2*e^2 + d^2)*d*x*e^(-2) - 2/3*sqrt(-x^2*e^2 + d^2)*d^2*e^(-3)
```

Fricas [A]
time = 2.35, size = 68, normalized size = 0.66

$$-\frac{1}{6}\left(6\,d^3\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(2\,x^2e^2+3\,dxe+4\,d^2\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (2*x^2*e^2 + 3*d
*x*e + 4*d^2)*sqrt(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.11, size = 177, normalized size = 1.72

$$d\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**2*x
**2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2), Tru
e)) + e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2
 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

Giac [A]
time = 1.28, size = 54, normalized size = 0.52

$$\frac{1}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\,\left(4\,d^2e^{(-3)}+\left(2\,xe^{(-1)}+3\,de^{(-2)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3
) + (2*x*e^(-1) + 3*d*e^(-2))*x)
```

Mupad [B]
time = 3.14, size = 112, normalized size = 1.09

$$\begin{cases} \frac{d\,x^3}{3\,\sqrt{d^2}} & \text{if}\ \ e=0\\ -\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3}-\frac{d^3\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}}-\frac{d\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2} & \text{if}\ \ e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (d*x^3)/(3*(d^2)^(1/2)), e ~= 0, - ((d^2 - e^2*x^2)^(1/2)
*(2*d^2 + e^2*x^2))/(3*e^3) - (d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)
^(1/2)))/(2*(-e^2)^(3/2)) - (d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

3.17 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+d*(e*x+d)/e^3/(-e^2*x^2+d^2)^(1/2)+
(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {811, 655, 223, 209, 651}

$$-\frac{d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
```

```
[Out] (d*(d + e*x))/(e^3*Sqrt[d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e^3 - (d*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx &= -\frac{\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}+\frac{d^2\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx}{e^2}\\ &= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\ &= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\ &= \frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 79, normalized size = 1.08

$$\frac{(-2d+ex)\sqrt{d^2-e^2x^2}}{e^3(-d+ex)}-\frac{d\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{(-e^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x]
[Out] ((-2*d + e*x)*Sqrt[d^2 - e^2*x^2])/(e^3*(-d + e*x)) - (d*Log[-(Sqrt[-e^2]*x
) + Sqrt[d^2 - e^2*x^2]])/(-e^2)^(3/2)
```

Maple [A]
time = 0.06, size = 103, normalized size = 1.41

method	result	siz

risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^3}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}-\frac{d\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)e}}{e^4\left(x-\frac{d}{e}\right)}$	9
default	$e\left(-\frac{x^2}{e^2\sqrt{-e^2x^2+d^2}}+\frac{2d^2}{e^4\sqrt{-e^2x^2+d^2}}\right)+d\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-x^2/e^2/(-e^2*x^2+d^2)^(1/2)+2*d^2/e^4/(-e^2*x^2+d^2)^(1/2))+d*(x/e^2/(
-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2)))
```

Maxima [A]
time = 0.48, size = 72, normalized size = 0.99

$$-d\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{x^2e^{(-1)}}{\sqrt{-x^2e^2+d^2}}+\frac{dxe^{(-2)}}{\sqrt{-x^2e^2+d^2}}+\frac{2\,d^2e^{(-3)}}{\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -d*arcsin(x*e/d)*e^(-3) - x^2*e^(-1)/sqrt(-x^2*e^2 + d^2) + d*x*e^(-2)/sqrt
(-x^2*e^2 + d^2) + 2*d^2*e^(-3)/sqrt(-x^2*e^2 + d^2)
```

Fricas [A]
time = 2.34, size = 85, normalized size = 1.16

$$\frac{2\,dxe-2\,d^2+2\left(dxe-d^2\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\sqrt{-x^2e^2+d^2}\,(xe-2\,d)}{xe^4-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*d*x*e - 2*d^2 + 2*(d*x*e - d^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1
)/x) + sqrt(-x^2*e^2 + d^2)*(x*e - 2*d))/(x*e^4 - d*e^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.06, size = 163, normalized size = 2.23

$$d\left(\begin{cases}\frac{i\operatorname{acosh}\left(\frac{ex}{d}\right)}{e^3}-\frac{ix}{de^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{\operatorname{asin}\left(\frac{ex}{d}\right)}{e^3}+\frac{x}{de^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}\tilde{\infty}x^4 & \text{for}\ d=0\wedge e=0\\ \frac{x^4}{4\left(d^2\right)^{\frac{3}{2}}} & \text{for}\ e=0\\ \tilde{\infty}x^4 & \text{for}\ d=-\sqrt{e^2x^2}\vee d=\sqrt{e^2x^2}\\ \frac{2d^2}{e^4\sqrt{d^2-e^2x^2}}-\frac{x^2}{e^2\sqrt{d^2-e^2x^2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d*Piecewise((I*acosh(e*x/d)/e**3 - I*x/(d*e**2*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2/d**2) > 1), (-asin(e*x/d)/e**3 + x/(d*e**2*sqrt(1 - e**2*x**2
/d**2)), True)) + e*Piecewise((zoo*x**4, Eq(d, 0) & Eq(e, 0)), (x**4/(4*(d*
*2)**(3/2)), Eq(e, 0)), (zoo*x**4, Eq(d, sqrt(e**2*x**2)) | Eq(d, -sqrt(e**
2*x**2))), (2*d**2/(e**4*sqrt(d**2 - e**2*x**2)) - x**2/(e**2*sqrt(d**2 - e
**2*x**2)), True))
```

Giac [A]
time = 1.08, size = 68, normalized size = 0.93

$$-d\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\sqrt{-x^2e^2+d^2}\,e^{(-3)}+\frac{2\,de^{(-3)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -d*arcsin(x*e/d)*e^(-3)*sgn(d) + sqrt(-x^2*e^2 + d^2)*e^(-3) + 2*d*e^(-3)/(
(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)
```

Mupad [B]
time = 2.96, size = 87, normalized size = 1.19

$$\frac{2\,d^2-e^2\,x^2}{e^3\,\sqrt{d^2-e^2\,x^2}}+\frac{d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{{\left(-e^2\right)}^{3/2}}+\frac{d\,x}{e^2\,\sqrt{d^2-e^2\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(3/2),x)
```

```
[Out] (2*d^2 - e^2*x^2)/(e^3*(d^2 - e^2*x^2)^(1/2)) + (d*log(x*(-e^2)^(1/2) + (d^
2 - e^2*x^2)^(1/2)))/(-e^2)^(3/2) + (d*x)/(e^2*(d^2 - e^2*x^2)^(1/2))
```

3.18 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/3*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(3/2)-2/3/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {810, 12, 267}

$$\frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}}-\frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (x^2*(d + e*x))/(3*d*e*(d^2 - e^2*x^2)^(3/2)) - 2/(3*e^3*Sqrt[d^2 - e^2*x^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 810

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[x^2*(a*g - c*f*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}} - \frac{\int \frac{2d^2ex}{(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2}$$
$$= \frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}} - \frac{2\int \frac{x}{(d^2-e^2x^2)^{3/2}}\,dx}{3e}$$
$$= \frac{x^2(d+ex)}{3de\,(d^2-e^2x^2)^{3/2}} - \frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

Mathematica [**A**]
time = 0.23, size = 59, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\,(-2d^2+2dex+e^2x^2)}{3de^3(d-ex)^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^2 + 2*d*e*x + e^2*x^2))/(3*d*e^3*(d - e*x)^2*(d
+ e*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(50) = 100.
time = 0.06, size = 120, normalized size = 2.07

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2(-e^2x^2-2dex+2d^2)}{3d\,e^3(-e^2x^2+d^2)^{\frac{5}{2}}}$
trager	$-\frac{(-e^2x^2-2dex+2d^2)\sqrt{-e^2x^2+d^2}}{3e^3d(-ex+d)^2(ex+d)}$
default	$e\left(\frac{x^2}{e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{2d^2}{3e^4(-e^2x^2+d^2)^{\frac{3}{2}}}\right)+d\left(\frac{x}{2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{d^2\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{2e^2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(x^2/e^2/(-e^2*x^2+d^2)^(3/2)-2/3*d^2/e^4/(-e^2*x^2+d^2)^(3/2))+d*(1/2*x/
e^2/(-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/
d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [A]
time = 0.27, size = 80, normalized size = 1.38

$$\frac{x^2 e^{(-1)}}{(-x^2e^2 + d^2)^{\frac{3}{2}}} + \frac{dx e^{(-2)}}{3\,(-x^2e^2 + d^2)^{\frac{3}{2}}} - \frac{2\,d^2 e^{(-3)}}{3\,(-x^2e^2 + d^2)^{\frac{3}{2}}} - \frac{x e^{(-2)}}{3\,\sqrt{-x^2e^2 + d^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] x^2*e^(-1)/(-x^2*e^2 + d^2)^(3/2) + 1/3*d*x*e^(-2)/(-x^2*e^2 + d^2)^(3/2) -
 2/3*d^2*e^(-3)/(-x^2*e^2 + d^2)^(3/2) - 1/3*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)
*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(47) = 94$.
time = 1.96, size = 98, normalized size = 1.69

$$-\frac{2\,x^3e^3 - 2\,dx^2e^2 - 2\,d^2xe + 2\,d^3 - (x^2e^2 + 2\,dxe - 2\,d^2)\sqrt{-x^2e^2 + d^2}}{3\,(dx^3e^6 - d^2x^2e^5 - d^3xe^4 + d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3*e^3 - 2*d*x^2*e^2 - 2*d^2*x*e + 2*d^3 - (x^2*e^2 + 2*d*x*e - 2*
d^2)*sqrt(-x^2*e^2 + d^2))/(d*x^3*e^6 - d^2*x^2*e^5 - d^3*x*e^4 + d^4*e^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.75, size = 231, normalized size = 3.98

$$d\left(\begin{cases} \frac{ix^3}{-3d^5\sqrt{-1 + \frac{e^2x^2}{d^2}} + 3d^3e^2x^2\sqrt{-1 + \frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{x^3}{-3d^5\sqrt{1 - \frac{e^2x^2}{d^2}} + 3d^3e^2x^2\sqrt{1 - \frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \frac{2d^2}{-3d^2e^4\sqrt{d^2 - e^2x^2} + 3e^6x^2\sqrt{d^2 - e^2x^2}} - \frac{3e^2x^2}{-3d^2e^4\sqrt{d^2 - e^2x^2} + 3e^6x^2\sqrt{d^2 - e^2x^2}} & \text{for}\ e \neq 0 \\ \frac{x^4}{4(d^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d*Piecewise((I*x**3/(-3*d**5*sqrt(-1 + e**2*x**2/d**2) + 3*d**3*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-x**3/(-3*d**5*sqrt(1
 - e**2*x**2/d**2) + 3*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), True)) + e
*Piecewise((2*d**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e**6*x**2*sqrt(
d**2 - e**2*x**2)) - 3*e**2*x**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2) + 3*e
**6*x**2*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(5/2)), True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*x^2/(-x^2*e^2 + d^2)^(5/2), x)
```

Mupad [B]
time = 2.59, size = 55, normalized size = 0.95

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^2+2\,d\,e\,x+e^2\,x^2\right)}{3\,d\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(5/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 2*d^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)*(d -
 e*x)^2)
```

3.19 $\int \frac{x^7(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=161

$$\frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}-\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-}}\right.}{2e^8}$$

```
[Out] 1/5*x^6*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^4*(7*e*x+6*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-7/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^8+1/15*x^2*(35*e*x+2
4*d)/e^6/(-e^2*x^2+d^2)^(1/2)+1/10*(35*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)/e^8
```

Rubi [A]
time = 0.09, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {833, 794, 223, 209}

$$-\frac{7d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}+\frac{x^6(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^4(6d+7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^6*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d + 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d + 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d + 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) - (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{x^5(6d^3+7d^2ex)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{x^3(24d^5+35d^4ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x(48d^7+105d^6ex)}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^{2}}}{10e^8}$$

$$= \frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^{2}}}{10e^8}$$

$$= \frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d+35ex)\sqrt{d^2-e^{2}}}{10e^8}$$

Mathematica [A]
time = 0.44, size = 149, normalized size = 0.93

$$\frac{\frac{e\sqrt{d^2-e^2x^2}\,(-96d^6-9d^5ex+249d^4e^2x^2-4d^3e^3x^3-176d^2e^4x^4+15de^5x^5+15e^6x^6)}{(-d+ex)^3(d+ex)^2} - 105d^2\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{30e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(-96*d^6 - 9*d^5*e*x + 249*d^4*e^2*x^2 - 4*d^3*e^3*
x^3 - 176*d^2*e^4*x^4 + 15*d*e^5*x^5 + 15*e^6*x^6))/((-d + e*x)^3*(d + e*x)
^2) - 105*d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(30*e^
9)
```

Maple [A]
time = 0.11, size = 251, normalized size = 1.56

method	result
default	$e\left(-\frac{x^{7}}{2e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{7d^{2}\left(\frac{x^{5}}{5e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{\frac{x^{3}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{2}\sqrt{e^{2}}}}{e^{2}}}{e^{2}}\right)}{2e^{2}}\right.$
risch	$\frac{(ex+2d)\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{8}}-\frac{7d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{7}\sqrt{e^{2}}}-\frac{31d^{2}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{48e^{9}\left(x+\frac{d}{e}\right)}-\frac{773d^{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/2*x^7/e^2/(-e^2*x^2+d^2)^(5/2)+7/2*d^2/e^2*(1/5*x^5/e^2/(-e^2*x^2+d^2
)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)
^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+d*(
-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2
/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(136) = 272$.
time = 0.49, size = 286, normalized size = 1.78

$$-\frac{x^{7}e^{(-1)}}{2\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{dx^{6}e^{(-2)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6\,d^{3}x^{4}e^{(-4)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{8\,d^{5}x^{2}e^{(-6)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{16\,d^{7}e^{(-8)}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{7}{30}\left(\frac{15\,x^{4}e^{(-2)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{20\,d^{2}x^{2}e^{(-4)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{8\,d^{4}e^{(-6)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}\right)d^{2}xe^{(-1)}-\frac{7}{6}\left(\frac{3\,x^{2}e^{(-2)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{2\,d^{2}e^{(-4)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}\right)d^{2}xe^{(-3)}+\frac{14\,d^{4}xe^{(-7)}}{15\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{7}{2}\,d^{2}\arcsin\left(\frac{xe}{d}\right)e^{(-8)}-\frac{49\,d^{2}xe^{(-7)}}{30\sqrt{-x^{2}e^{2}+d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^7*e^(-1)/(-x^2*e^2 + d^2)^(5/2) - d*x^6*e^(-2)/(-x^2*e^2 + d^2)^(5/2
) + 6*d^3*x^4*e^(-4)/(-x^2*e^2 + d^2)^(5/2) - 8*d^5*x^2*e^(-6)/(-x^2*e^2 +
d^2)^(5/2) + 16/5*d^7*e^(-8)/(-x^2*e^2 + d^2)^(5/2) + 7/30*(15*x^4*e^(-2)/(
-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8*d^4*e^
(-6)/(-x^2*e^2 + d^2)^(5/2))*d^2*x*e^(-1) - 7/6*(3*x^2*e^(-2)/(-x^2*e^2 + d
^2)^(3/2) - 2*d^2*e^(-4)/(-x^2*e^2 + d^2)^(3/2))*d^2*x*e^(-3) + 14/15*d^4*x
*e^(-7)/(-x^2*e^2 + d^2)^(3/2) - 7/2*d^2*arcsin(x*e/d)*e^(-8) - 49/30*d^2*x
*e^(-7)/sqrt(-x^2*e^2 + d^2)
```

Fricas [A]
time = 2.38, size = 259, normalized size = 1.61

$$\frac{96\,d^{2}x^{5}e^{5}-96\,d^{3}x^{4}e^{4}-192\,d^{4}x^{3}e^{3}+192\,d^{5}x^{2}e^{2}+96\,d^{6}xe-96\,d^{7}+210\,(d^{2}x^{5}e^{5}-d^{3}x^{4}e^{4}-2\,d^{4}x^{3}e^{3}+2\,d^{5}x^{2}e^{2}+d^{6}xe-d^{7})\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)+(15\,x^{6}e^{6}+15\,dx^{5}e^{5}-176\,d^{2}x^{4}e^{4}-4\,d^{3}x^{3}e^{3}+249\,d^{4}x^{2}e^{2}-9\,d^{5}xe-96\,d^{6})\sqrt{-x^{2}e^{2}+d^{2}}}{30\,(x^{5}e^{13}-dx^{4}e^{12}-2\,d^{2}x^{3}e^{11}+2\,d^{3}x^{2}e^{10}+d^{4}xe^{9}-d^{5}e^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*x^5*e^5 - 96*d^3*x^4*e^4 - 192*d^4*x^3*e^3 + 192*d^5*x^2*e^2 +
 96*d^6*x*e - 96*d^7 + 210*(d^2*x^5*e^5 - d^3*x^4*e^4 - 2*d^4*x^3*e^3 + 2*d
^5*x^2*e^2 + d^6*x*e - d^7)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) +
(15*x^6*e^6 + 15*d*x^5*e^5 - 176*d^2*x^4*e^4 - 4*d^3*x^3*e^3 + 249*d^4*x^2*
e^2 - 9*d^5*x*e - 96*d^6)*sqrt(-x^2*e^2 + d^2))/(x^5*e^13 - d*x^4*e^12 - 2*
d^2*x^3*e^11 + 2*d^3*x^2*e^10 + d^4*x*e^9 - d^5*e^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(144) = 288$.
time = 26.02, size = 2004, normalized size = 12.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x*
*2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 40*d**4*
e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**
2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*d**2*e**4*x**4/(
5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x*
*2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5*d**4*e**8*sqrt(
d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**
4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2)), True)) + e*P
iecewise((210*I*d**7*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9*s
qrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) +
60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 105*pi*d**7*sqrt(-1 + e**2*x**
```

```
2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(
-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 210*I*d
**6*e*x/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(
-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 420*I*d
**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9*sqrt(-1
+ e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e*
*13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 210*pi*d**5*e**2*x**2*sqrt(-1 + e**2*
x**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sq
rt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 490*
I*d**4*e**3*x**3/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x
**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2))
+ 210*I*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(60*d**5*e**9
*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2)
+ 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) - 105*pi*d**3*e**4*x**4*sqrt(-
1 + e**2*x**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**1
1*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2
)) - 322*I*d**2*e**5*x**5/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**
3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**
2/d**2)) + 30*I*e**7*x**7/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**
3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (-105*d**7*sqrt(1 - e**2*x**2/d**2)*asi
n(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1
 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 105*d**6*e
*x/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**
2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 210*d**5*e**2*x*
*2*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d*
*2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1
- e**2*x**2/d**2)) - 245*d**4*e**3*x**3/(30*d**5*e**9*sqrt(1 - e**2*x**2/d*
*2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1
- e**2*x**2/d**2)) - 105*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d
)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2
*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) + 161*d**2*e**5*x**
5/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2
*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*e**7*x**7/(30*
d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/
d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((x*e + d)*x^7/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\,(d+e\,x)}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^7*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.20 $\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}} \, dx$

Optimal. Leaf size=147

$$\frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

```
[Out] 1/5*x^5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^3*(6*e*x+5*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7+1/5*x*(8*e*x+5*d)/e^6/(-
e^2*x^2+d^2)^(1/2)+16/5*(-e^2*x^2+d^2)^(1/2)/e^7
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {833, 655, 223, 209}

$$-\frac{d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}+\frac{x^5(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{16\sqrt{d^2-e^2x^2}}{5e^7}+\frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{x^3(5d+6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^5*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d + 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d + 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) + (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{x^4(5d^3+6d^2ex)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{x^2(15d^5+24d^4ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^7+48d^6ex}{\sqrt{d^2-e^2x^2}}\,dx}{15d^6e^6}$$

$$= \frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\int \cdots}{\cdots}$$

$$= \frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\text{Sub}\cdots}{\cdots}$$

$$= \frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d\tan\cdots}{\cdots}$$

Mathematica [A]
time = 0.40, size = 136, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2x^2}\,(-48d^5+33d^4ex+87d^3e^2x^2-52d^2e^3x^3-38de^4x^4+15e^5x^5)}{15e^7(-d+ex)^3(d+ex)^2} + \frac{d\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^6\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 + 33*d^4*e*x + 87*d^3*e^2*x^2 - 52*d^2*e^3*x^
3 - 38*d*e^4*x^4 + 15*e^5*x^5))/(15*e^7*(-d + e*x)^3*(d + e*x)^2) + (d*Log[
-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(e^6*Sqrt[-e^2])
```

Maple [A]
time = 0.08, size = 220, normalized size = 1.50

method	result
default	$e\left(-\frac{x^{6}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6d^{2}\left(\frac{x^{4}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{4d^{2}\left(\frac{x^{2}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2d^{2}}{15e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)}{e^{2}}\right)}{e^{2}}\right)+d\left(\frac{x^{5}}{5e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right.$
risch	$\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{e^{7}}-\frac{d\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{6}\sqrt{e^{2}}}+\frac{25d\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{48e^{8}\left(x+\frac{d}{e}\right)}-\frac{493d\sqrt{-\left(x-\right.}}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*
d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)
)))+d*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(
3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(123) = 246$.
time = 0.55, size = 255, normalized size = 1.73

$$-\frac{x^{6}e^{(-1)}}{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}+\frac{6\,d^{2}x^{4}e^{(-3)}}{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}-\frac{8\,d^{4}x^{2}e^{(-5)}}{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}+\frac{16\,d^{6}e^{(-7)}}{5\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}-\frac{1}{3}\left(\frac{3\,x^{2}e^{(-2)}}{(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}}-\frac{2\,d^{2}e^{(-4)}}{(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}}\right)dxe^{(-2)}+\frac{4\,d^{3}xe^{(-6)}}{15\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}}+\frac{1}{15}\left(\frac{15\,x^{4}e^{(-2)}}{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}-\frac{20\,d^{2}x^{2}e^{(-4)}}{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}+\frac{8\,d^{4}e^{(-6)}}{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}}\right)dx-d\arcsin\left(\frac{xe}{d}\right)e^{(-7)}-\frac{7\,dxe^{(-6)}}{15\,\sqrt{-x^{2}e^{2}+d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -x^6*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 6*d^2*x^4*e^(-3)/(-x^2*e^2 + d^2)^(5/2
) - 8*d^4*x^2*e^(-5)/(-x^2*e^2 + d^2)^(5/2) + 16/5*d^6*e^(-7)/(-x^2*e^2 + d
^2)^(5/2) - 1/3*(3*x^2*e^(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d^2*e^(-4)/(-x^2*e
^2 + d^2)^(3/2))*d*x*e^(-2) + 4/15*d^3*x*e^(-6)/(-x^2*e^2 + d^2)^(3/2) + 1/
15*(15*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2*e^(-4)/(-x^2*e^2 + d^
2)^(5/2) + 8*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2))*d*x - d*arcsin(x*e/d)*e^(-7
) - 7/15*d*x*e^(-6)/sqrt(-x^2*e^2 + d^2)
```

Fricas [A]
time = 2.47, size = 245, normalized size = 1.67

$$\frac{48\,dx^5e^5-48\,d^2x^4e^4-96\,d^3x^3e^3+96\,d^4x^2e^2+48\,d^5xe-48\,d^6+30\,(dx^5e^5-d^2x^4e^4-2\,d^3x^3e^3+2\,d^4x^2e^2+d^5xe-d^6)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(15\,x^5e^5-38\,dx^4e^4-52\,d^2x^3e^3+87\,d^3x^2e^2+33\,d^4xe-48\,d^5)\sqrt{-x^2e^2+d^2}}{15\,(x^5e^{12}-dx^4e^{11}-2\,d^2x^3e^{10}+2\,d^3x^2e^9+d^4xe^8-d^5e^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(48*d*x^5*e^5 - 48*d^2*x^4*e^4 - 96*d^3*x^3*e^3 + 96*d^4*x^2*e^2 + 48*
d^5*x*e - 48*d^6 + 30*(d*x^5*e^5 - d^2*x^4*e^4 - 2*d^3*x^3*e^3 + 2*d^4*x^2*
e^2 + d^5*x*e - d^6)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (15*x^5
*e^5 - 38*d*x^4*e^4 - 52*d^2*x^3*e^3 + 87*d^3*x^2*e^2 + 33*d^4*x*e - 48*d^5
)*sqrt(-x^2*e^2 + d^2))/(x^5*e^12 - d*x^4*e^11 - 2*d^2*x^3*e^10 + 2*d^3*x^2
*e^9 + d^4*x*e^8 - d^5*e^7)
```

Sympy [C] Result contains complex when optimal does not.
time = 22.82, size = 1821, normalized size = 12.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7
*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) +
30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d**5*sqrt(-1 + e**2*x**2
/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1
+ e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 30*I*d**4*
e*x/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 60*I*d**3*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1 + e**2*
x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*
sqrt(-1 + e**2*x**2/d**2)) + 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)
/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2
*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 70*I*d**2*e**3*x
**3/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*I*d*e**4*x
**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1 + e**2*x**
2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqr
t(-1 + e**2*x**2/d**2)) - 15*pi*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(30*d
**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/
d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 46*I*e**5*x**5/(30*d**
5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d*
*2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2
```

```
*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*
sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*sqrt(1 - e**2*x**2/d*
*2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 -
 e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/
(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x*
*2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 35*d**2*e**3*x**3/(1
5*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2
/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*d*e**4*x**4*sqrt(1
- e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d
**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2
/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**
9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((16*d**6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d*
*2*e**10*x**2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2))
 - 40*d**4*e**2*x**2/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x*
*2*sqrt(d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) + 30*d**2*
e**4*x**4/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**
2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x**6/(5*d**4
*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**2 - e**2*x**2) +
5*e**12*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**8/(8*(d**2)**(7/2)), T
rue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((x*e + d)*x^6/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,\left(d+e\,x\right)}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^6*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.21 $\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=122

$$\frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] 1/5*x^4*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^2*(5*e*x+4*d)/e^4/(-e^2*x^2
+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(15*e*x+8*d)/e^6/(-e^
2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {833, 792, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}+\frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d + 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d + 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) - Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{x^3\left(4d^3+5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\
&= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{x\left(8d^5+15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}\\
&= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^5}\\
&= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x\right.}{e^5}\\
&= \frac{x^4(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x^2(4d+5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 124, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\left(-8d^4-7d^3ex+27d^2e^2x^2+8de^3x^3-23e^4x^4\right)}{15e^6(-d+ex)^3(d+ex)^2}+\frac{\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^5\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 7*d^3*e*x + 27*d^2*e^2*x^2 + 8*d*e^3*x^3 - 2
3*e^4*x^4))/(15*e^6*(-d + e*x)^3*(d + e*x)^2) + Log[-(Sqrt[-e^2]*x) + Sqrt[
d^2 - e^2*x^2]]/(e^5*Sqrt[-e^2])
```

Maple [A]
time = 0.06, size = 189, normalized size = 1.55

method	result
default	$e\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)+d\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)
-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(
-e^2*x^2+d^2)^(1/2)))))+d*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/
e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(104) = 208$.
time = 0.50, size = 230, normalized size = 1.89

$$\frac{dx^4e^{(-2)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{4\,d^3x^2e^{(-4)}}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{8\,d^5e^{(-6)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{1}{15}\left(\frac{15\,x^4e^{(-2)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{20\,d^2x^2e^{(-4)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{8\,d^4e^{(-6)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}\right)xe-\frac{1}{3}\left(\frac{3\,x^2e^{(-2)}}{(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{2\,d^2e^{(-4)}}{(-x^2e^2+d^2)^{\frac{3}{2}}}\right)xe^{(-1)}+\frac{4\,d^2xe^{(-5)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}}-\arcsin\left(\frac{xe}{d}\right)e^{(-6)}-\frac{7\,xe^{(-5)}}{15\,\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] d*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 4/3*d^3*x^2*e^(-4)/(-x^2*e^2 + d^2)^(
5/2) + 8/15*d^5*e^(-6)/(-x^2*e^2 + d^2)^(5/2) + 1/15*(15*x^4*e^(-2)/(-x^2*e
^2 + d^2)^(5/2) - 20*d^2*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8*d^4*e^(-6)/(
-x^2*e^2 + d^2)^(5/2))*x*e - 1/3*(3*x^2*e^(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d
^2*e^(-4)/(-x^2*e^2 + d^2)^(3/2))*x*e^(-1) + 4/15*d^2*x*e^(-5)/(-x^2*e^2 +
d^2)^(3/2) - arcsin(x*e/d)*e^(-6) - 7/15*x*e^(-5)/sqrt(-x^2*e^2 + d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(104) = 208$.
time = 1.98, size = 230, normalized size = 1.89

$$\frac{8\,x^5e^5-8\,dx^4e^4-16\,d^2x^3e^3+16\,d^3x^2e^2+8\,d^4xe-8\,d^5+30\,(x^5e^5-dx^4e^4-2\,d^2x^3e^3+2\,d^3x^2e^2+d^4xe-d^5)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(23\,x^4e^4-8\,dx^3e^3-27\,d^2x^2e^2+7\,d^3xe+8\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(x^5e^{11}-dx^4e^{10}-2\,d^2x^3e^9+2\,d^3x^2e^8+d^4xe^7-d^5e^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*x^5*e^5 - 8*d*x^4*e^4 - 16*d^2*x^3*e^3 + 16*d^3*x^2*e^2 + 8*d^4*x*e
 - 8*d^5 + 30*(x^5*e^5 - d*x^4*e^4 - 2*d^2*x^3*e^3 + 2*d^3*x^2*e^2 + d^4*x*
e - d^5)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (23*x^4*e^4 - 8*d*x
^3*e^3 - 27*d^2*x^2*e^2 + 7*d^3*x*e + 8*d^4)*sqrt(-x^2*e^2 + d^2))/(x^5*e^1
1 - d*x^4*e^10 - 2*d^2*x^3*e^9 + 2*d^3*x^2*e^8 + d^4*x*e^7 - d^5*e^6)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(104) = 208$.
time = 26.93, size = 1739, normalized size = 14.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**
2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) - 20*d**2*
e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**
2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e**4*x**4/(15*d
**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2 - e**2*x**2)
+ 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(d**2)**(7/2))
, True)) + e*Piecewise((30*I*d**5*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(3
0*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x*
*2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d**5*sqrt(-1
+ e**2*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x*
*2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) -
 30*I*d**4*e*x/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*
sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 60
*I*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt
(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*
e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*pi*d**3*e**2*x**2*sqrt(-1 + e**2
*x**2/d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqr
t(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 70*I*
d**2*e**3*x**3/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*
sqrt(-1 + e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30
*I*d*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)*acosh(e*x/d)/(30*d**5*e**7*sqrt(-1
 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e**2*x**2/d**2) + 30*d*e**
11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 15*pi*d*e**4*x**4*sqrt(-1 + e**2*x**2/
d**2)/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 +
 e**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)) - 46*I*e**5*x
**5/(30*d**5*e**7*sqrt(-1 + e**2*x**2/d**2) - 60*d**3*e**9*x**2*sqrt(-1 + e
**2*x**2/d**2) + 30*d*e**11*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/
d**2) > 1), (-15*d**5*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sq
rt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*
```

```
e**11*x**4*sqrt(1 - e**2*x**2/d**2)) + 15*d**4*e*x/(15*d**5*e**7*sqrt(1 - e
**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x*
*4*sqrt(1 - e**2*x**2/d**2)) + 30*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*a
sin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(
1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 35*d**2*e
**3*x**3/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) - 30*d**3*e**9*x**2*sqrt(1
- e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*d*e**4*x
**4*sqrt(1 - e**2*x**2/d**2)*asin(e*x/d)/(15*d**5*e**7*sqrt(1 - e**2*x**2/d
**2) - 30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1
- e**2*x**2/d**2)) + 23*e**5*x**5/(15*d**5*e**7*sqrt(1 - e**2*x**2/d**2) -
30*d**3*e**9*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d*e**11*x**4*sqrt(1 - e**2*
x**2/d**2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((x*e + d)*x^5/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(d+e\,x\right)}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.22 $\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$

Optimal. Leaf size=84

$$\frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}}-\frac{4d^2}{15e^5\,(d^2-e^2x^2)^{3/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^4*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)-4/15*d^2/e^5/(-e^2*x^2+d^2)^(3/2)+
4/5/e^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {819, 272, 45}

$$\frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}}+\frac{4}{5e^5\sqrt{d^2-e^2x^2}}-\frac{4d^2}{15e^5\,(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^4*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (4*d^2)/(15*e^5*(d^2 - e^2*
x^2)^(3/2)) + 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{4\int \frac{x^3}{(d^2-e^2x^2)^{5/2}}\,dx}{5e}$$
$$= \frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{2\text{Subst}\left(\int \frac{x}{(d^2-e^2x)^{5/2}}\,dx, x, x^2\right)}{5e}$$
$$= \frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{2\text{Subst}\left(\int \left(\frac{d^2}{e^2(d^2-e^2x)^{5/2}} - \frac{1}{e^2(d^2-e^2x)^{3/2}}\right)dx, x, x^2\right)}{5e}$$
$$= \frac{x^4(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{4d^2}{15e^5\,(d^2-e^2x^2)^{3/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.30, size = 82, normalized size = 0.98

$$\frac{\sqrt{d^2-e^2x^2}\,(8d^4-8d^3ex-12d^2e^2x^2+12de^3x^3+3e^4x^4)}{15de^5(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 - 8*d^3*e*x - 12*d^2*e^2*x^2 + 12*d*e^3*x^3 + 3
*e^4*x^4))/(15*d*e^5*(d - e*x)^3*(d + e*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(72) = 144$.
time = 0.05, size = 208, normalized size = 2.48

method	result
gosper	$\frac{(-ex+d)(ex+d)^2(3e^4x^4+12d\,e^3x^3-12d^2x^2e^2-8d^3ex+8d^4)}{15d\,e^5(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(3e^4x^4+12d\,e^3x^3-12d^2x^2e^2-8d^3ex+8d^4)\sqrt{-e^2x^2+d^2}}{15e^5d(-ex+d)^3(ex+d)^2}$

default	$e\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)+d\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)
-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+d*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/
2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+
d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)
^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(67) = 134$.
time = 0.27, size = 145, normalized size = 1.73

$$\frac{x^4e^{(-1)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{dx^3e^{(-2)}}{2\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{4\,d^2x^2e^{(-3)}}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{3\,d^3xe^{(-4)}}{10\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{8\,d^4e^{(-5)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{dxe^{(-4)}}{10\,(-x^2e^2+d^2)^{\frac{3}{2}}}+\frac{xe^{(-4)}}{5\,\sqrt{-x^2e^2+d^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1/2*d*x^3*e^(-2)/(-x^2*e^2 + d^2)^(5/2)
 - 4/3*d^2*x^2*e^(-3)/(-x^2*e^2 + d^2)^(5/2) - 3/10*d^3*x*e^(-4)/(-x^2*e^2
+ d^2)^(5/2) + 8/15*d^4*e^(-5)/(-x^2*e^2 + d^2)^(5/2) + 1/10*d*x*e^(-4)/(-x
^2*e^2 + d^2)^(3/2) + 1/5*x*e^(-4)/(sqrt(-x^2*e^2 + d^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(67) = 134$.
time = 2.20, size = 159, normalized size = 1.89

$$\frac{8\,x^5e^5-8\,dx^4e^4-16\,d^2x^3e^3+16\,d^3x^2e^2+8\,d^4xe-8\,d^5-(3\,x^4e^4+12\,dx^3e^3-12\,d^2x^2e^2-8\,d^3xe+8\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(dx^5e^{10}-d^2x^4e^9-2\,d^3x^3e^8+2\,d^4x^2e^7+d^5xe^6-d^6e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*x^5*e^5 - 8*d*x^4*e^4 - 16*d^2*x^3*e^3 + 16*d^3*x^2*e^2 + 8*d^4*x*e
 - 8*d^5 - (3*x^4*e^4 + 12*d*x^3*e^3 - 12*d^2*x^2*e^2 - 8*d^3*x*e + 8*d^4)*
sqrt(-x^2*e^2 + d^2))/(d*x^5*e^10 - d^2*x^4*e^9 - 2*d^3*x^3*e^8 + 2*d^4*x^2
*e^7 + d^5*x*e^6 - d^6*e^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 23.41, size = 418, normalized size = 4.98

$$d\left(\begin{cases} -\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)+e\left(\begin{cases} \frac{8d^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}}-\frac{20d^2e^2x^2}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}}+\frac{15e^4x^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0 \\ \frac{x^6}{6(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) - 10*d**5*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2/d**2) - 10*d**5*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2
)), True)) + e*Piecewise((8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*
d**2*e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2
)) - 20*d**2*e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*
x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) + 15*e*
*4*x**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*sqrt(d**2
- e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**6/(6*(
d**2)**(7/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*x^4/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [B]
time = 2.70, size = 78, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(8\,d^4-8\,d^3\,e\,x-12\,d^2\,e^2\,x^2+12\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d\,e^5\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 + 12*d*e^3*x^3 - 12*d^2*e^2*x^2 -
 8*d^3*e*x))/(15*d*e^5*(d + e*x)^2*(d - e*x)^3)
```

3.23 $\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=90

$$\frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)+1/15*(-3*e*x-2*d)/e^4/(-e^2*x^2+d^
2)^(3/2)+1/5*x/d^2/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {833, 792, 197}

$$\frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d + 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) + x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{x\left(2d^3+3d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}\\ &= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3}\\ &= \frac{x^2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d+3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 82, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}\,\left(-2d^4+2d^3ex+3d^2e^2x^2-3de^3x^3+3e^4x^4\right)}{15d^2e^4(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^3*(d + e*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(78) = 156.
time = 0.07, size = 178, normalized size = 1.98

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2\left(-3e^4x^4+3d\,e^3x^3-3d^2x^2e^2-2d^3ex+2d^4\right)}{15d^2e^4\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
trager	$-\frac{\left(-3e^4x^4+3d\,e^3x^3-3d^2x^2e^2-2d^3ex+2d^4\right)\sqrt{-e^2x^2+d^2}}{15d^2e^4(-ex+d)^3(ex+d)^2}$

default	$e\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)}{2e^2}\right)+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(
5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x
^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+d*(1/3*x^2/e^2/(-e^2*x^2+d
^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))
```

Maxima [A]
time = 0.28, size = 122, normalized size = 1.36

$$\frac{x^3e^{(-1)}}{2\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}+\frac{dx^2e^{(-2)}}{3\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}-\frac{3\,d^2xe^{(-3)}}{10\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}-\frac{2\,d^3e^{(-4)}}{15\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}+\frac{xe^{(-3)}}{10\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}+\frac{xe^{(-3)}}{5\sqrt{-x^2e^2+d^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1/3*d*x^2*e^(-2)/(-x^2*e^2 + d^2)^(
5/2) - 3/10*d^2*x*e^(-3)/(-x^2*e^2 + d^2)^(5/2) - 2/15*d^3*e^(-4)/(-x^2*e^2
 + d^2)^(5/2) + 1/10*x*e^(-3)/(-x^2*e^2 + d^2)^(3/2) + 1/5*x*e^(-3)/(sqrt(-
x^2*e^2 + d^2)*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(74) = 148.
time = 3.89, size = 160, normalized size = 1.78

$$-\frac{2\,x^5e^5-2\,dx^4e^4-4\,d^2x^3e^3+4\,d^3x^2e^2+2\,d^4xe-2\,d^5+\left(3\,x^4e^4-3\,dx^3e^3+3\,d^2x^2e^2+2\,d^3xe-2\,d^4\right)\sqrt{-x^2e^2+d^2}}{15\left(d^2x^5e^9-d^3x^4e^8-2\,d^4x^3e^7+2\,d^5x^2e^6+d^6xe^5-d^7e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*x^5*e^5 - 2*d*x^4*e^4 - 4*d^2*x^3*e^3 + 4*d^3*x^2*e^2 + 2*d^4*x*e
- 2*d^5 + (3*x^4*e^4 - 3*d*x^3*e^3 + 3*d^2*x^2*e^2 + 2*d^3*x*e - 2*d^4)*sqr
```

```
t(-x^2*e^2 + d^2))/(d^2*x^5*e^9 - d^3*x^4*e^8 - 2*d^4*x^3*e^7 + 2*d^5*x^2*e
^6 + d^6*x*e^5 - d^7*e^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(78) = 156$.
time = 7.45, size = 337, normalized size = 3.74

$$d\left(\begin{cases} -\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} + \frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e \neq 0 \\ \frac{x^4}{4(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} -\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x*
*2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)) + 5*e**2*x
**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*sqrt(d**2 - e*
*2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)
**(7/2)), True)) + e*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) -
 10*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2
/d**2) - 10*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt
(1 - e**2*x**2/d**2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*x^3/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [B]
time = 2.66, size = 78, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2-3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 - 3*d*e^3*x^3 + 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^2*e^4*(d + e*x)^2*(d - e*x)^3)
```

3.24 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)-2/15*(-e*x+d)/d/e^3/(-e^2*x^2+d^2)
^(3/2)-2/15*x/d^3/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {810, 792, 197}

$$\frac{x^2(d+ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d-ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x^2*(d + e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d - e*x))/(15*d*e^3*(d^
2 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 810

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[x^2*(a*g - c*f*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x^2(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{x(2d^2e-2de^2x)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$
$$= \frac{x^2(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{2(d-ex)}{15de^3\,(d^2-e^2x^2)^{3/2}} - \frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15de^2}$$
$$= \frac{x^2(d+ex)}{5de\,(d^2-e^2x^2)^{5/2}} - \frac{2(d-ex)}{15de^3\,(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.31, size = 82, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2x^2}\,(-2d^4+2d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4)}{15d^3e^3(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 + 2*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*
e^4*x^4))/(15*d^3*e^3*(d - e*x)^3*(d + e*x)^2)
```

Maple [A]
time = 0.07, size = 147, normalized size = 1.56

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2(2e^4x^4-2d\,e^3x^3-3d^2x^2e^2-2d^3ex+2d^4)}{15d^3e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(2e^4x^4-2d\,e^3x^3-3d^2x^2e^2-2d^3ex+2d^4)\sqrt{-e^2x^2+d^2}}{15d^3e^3(-ex+d)^3(ex+d)^2}$
default	$e\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d^2}{15e^4(-e^2x^2+d^2)^{\frac{5}{2}}}\right)+d\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{}{15d^4\sqrt{-}}}{d^2}\right.}{4e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+d*(1
/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4
/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))
```

Maxima [A]
time = 0.27, size = 102, normalized size = 1.09

$$\frac{x^2e^{(-1)}}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{dxe^{(-2)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{2\,d^2e^{(-3)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{xe^{(-2)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d}-\frac{2\,xe^{(-2)}}{15\,\sqrt{-x^2e^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1/5*d*x*e^(-2)/(-x^2*e^2 + d^2)^(5/
2) - 2/15*d^2*e^(-3)/(-x^2*e^2 + d^2)^(5/2) - 1/15*x*e^(-2)/((-x^2*e^2 + d^
2)^(3/2)*d) - 2/15*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(79) = 158$.
time = 4.24, size = 161, normalized size = 1.71

$$-\frac{2\,x^5e^5-2\,dx^4e^4-4\,d^2x^3e^3+4\,d^3x^2e^2+2\,d^4xe-2\,d^5-(2\,x^4e^4-2\,dx^3e^3-3\,d^2x^2e^2-2\,d^3xe+2\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(d^3x^5e^8-d^4x^4e^7-2\,d^5x^3e^6+2\,d^6x^2e^5+d^7xe^4-d^8e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*x^5*e^5 - 2*d*x^4*e^4 - 4*d^2*x^3*e^3 + 4*d^3*x^2*e^2 + 2*d^4*x*e
- 2*d^5 - (2*x^4*e^4 - 2*d*x^3*e^3 - 3*d^2*x^2*e^2 - 2*d^3*x*e + 2*d^4)*sqr
t(-x^2*e^2 + d^2))/(d^3*x^5*e^8 - d^4*x^4*e^7 - 2*d^5*x^3*e^6 + 2*d^6*x^2*e
^5 + d^7*x*e^4 - d^8*e^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.49, size = 513, normalized size = 5.46

$$d\left(\begin{cases}-\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}}+\frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e\neq 0\\ \frac{x^4}{4(d^2)^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e**2*x**
2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**9*sqrt(1 - e**2*x**2/d**2) -
 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e*
*2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**
2)), True)) + e*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 3
```

```
0*d**2*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**
2)) + 5*e**2*x**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*
sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (
x**4/(4*(d**2)**(7/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*x^2/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [B]
time = 2.61, size = 78, normalized size = 0.83

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4+2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d*e^3*x^3 - 2*e^4*x^4 - 2*d^4 + 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^3*e^3*(d + e*x)^2*(d - e*x)^3)
```

3.25 $\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=83

$$\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x/d^2/e/(-e^2*x^2+d^2)^(3/2)-2/15
*x/d^4/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {792, 198, 197}

$$-\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{d+ex}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)
) - (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{1}{(d^2-e^2x^2)^{5/2}}\,dx}{5e}$$
$$= \frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} - \frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2e}$$
$$= \frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.29, size = 82, normalized size = 0.99

$$\frac{\sqrt{d^2-e^2x^2}\,(3d^4-3d^3ex+3d^2e^2x^2+2de^3x^3-2e^4x^4)}{15d^4e^2(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^3*(d + e*x)^2)
```

Maple [A]
time = 0.06, size = 120, normalized size = 1.45

method	result	size
gosper	$\frac{(-ex+d)(ex+d)^2(-2e^4x^4+2d\,e^3x^3+3d^2x^2e^2-3d^3ex+3d^4)}{15d^4e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$	77
trager	$\frac{(-2e^4x^4+2d\,e^3x^3+3d^2x^2e^2-3d^3ex+3d^4)\sqrt{-e^2x^2+d^2}}{15d^4(-ex+d)^3(ex+d)^2e^2}$	79
default	$e\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)+\frac{d}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/
2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
)+1/5*d/e^2/(-e^2*x^2+d^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.95

$$\frac{xe^{(-1)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{de^{(-2)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{xe^{(-1)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}-\frac{2\,xe^{(-1)}}{15\,\sqrt{-x^2e^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1/5*d*e^(-2)/(-x^2*e^2 + d^2)^(5/2) -
 1/15*x*e^(-1)/((-x^2*e^2 + d^2)^(3/2)*d^2) - 2/15*x*e^(-1)/(sqrt(-x^2*e^2
+ d^2)*d^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(66) = 132.
time = 3.04, size = 160, normalized size = 1.93

$$\frac{3\,x^5e^5-3\,dx^4e^4-6\,d^2x^3e^3+6\,d^3x^2e^2+3\,d^4xe-3\,d^5+\left(2\,x^4e^4-2\,dx^3e^3-3\,d^2x^2e^2+3\,d^3xe-3\,d^4\right)\sqrt{-x^2e^2+d^2}}{15\,\left(d^4x^5e^7-d^5x^4e^6-2\,d^6x^3e^5+2\,d^7x^2e^4+d^8xe^3-d^9e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^5*e^5 - 3*d*x^4*e^4 - 6*d^2*x^3*e^3 + 6*d^3*x^2*e^2 + 3*d^4*x*e -
 3*d^5 + (2*x^4*e^4 - 2*d*x^3*e^3 - 3*d^2*x^2*e^2 + 3*d^3*x*e - 3*d^4)*sqrt
(-x^2*e^2 + d^2))/(d^4*x^5*e^7 - d^5*x^4*e^6 - 2*d^6*x^3*e^5 + 2*d^7*x^2*e^
4 + d^8*x*e^3 - d^9*e^2)
```

Sympy [A]
time = 7.09, size = 432, normalized size = 5.20

$$d\left(\begin{cases}\frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e\neq 0\\ \frac{x^2}{2(d^2)^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)+e\left(\begin{cases}-\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**4*x**2*sqrt
(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**2/
(2*(d**2)**(7/2)), True)) + e*Piecewise((-5*I*d**2*x**3/(15*d**9*sqrt(-1 +
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**
4*x**4*sqrt(-1 + e**2*x**2/d**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x
**2/d**2) - 30*d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(15*d**
9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1
5*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 2*e**2*x**5/(15*d**9*sqrt(1 -
```

```
e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**5*e**4
*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*x/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [B]
time = 2.62, size = 78, normalized size = 0.94

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4-3\,d^3\,e\,x+3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^4\,e^2\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 + 2*d*e^3*x^3 + 3*d^2*e^2*x^2 - 3
*d^3*e*x))/(15*d^4*e^2*(d + e*x)^2*(d - e*x)^3)
```

3.26 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=80

$$\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)+8/15*x
/d^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {653, 198, 197}

$$\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/
2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d+ex}{5de\,(d^2-e^2x^2)^{5/2}} + \frac{4\int \frac{1}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$
$$= \frac{d+ex}{5de\,(d^2-e^2x^2)^{5/2}} + \frac{4x}{15d^3\,(d^2-e^2x^2)^{3/2}} + \frac{8\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^3}$$
$$= \frac{d+ex}{5de\,(d^2-e^2x^2)^{5/2}} + \frac{4x}{15d^3\,(d^2-e^2x^2)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\,(3d^4+12d^3ex-12d^2e^2x^2-8de^3x^3+8e^4x^4)}{15d^5e(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 + 8
*e^4*x^4))/(15*d^5*e*(d - e*x)^3*(d + e*x)^2)
```

Maple [A]
time = 0.05, size = 90, normalized size = 1.12

method	result	size
gosper	$\frac{(-ex+d)(ex+d)^2(8e^4x^4-8d\,e^3x^3-12d^2x^2e^2+12d^3ex+3d^4)}{15d^5e(-e^2x^2+d^2)^{\frac{7}{2}}}$	77
trager	$\frac{(8e^4x^4-8d\,e^3x^3-12d^2x^2e^2+12d^3ex+3d^4)\sqrt{-e^2x^2+d^2}}{15d^5(-ex+d)^3(ex+d)^2e}$	79
default	$\frac{1}{5e(-e^2x^2+d^2)^{\frac{5}{2}}}+d\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/e/(-e^2*x^2+d^2)^(5/2)+d*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x
/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [A]
time = 0.27, size = 75, normalized size = 0.94

$$\frac{e^{(-1)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}} + \frac{x}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d} + \frac{4\,x}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3} + \frac{8\,x}{15\,\sqrt{-x^2e^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1/5*x/((-x^2*e^2 + d^2)^(5/2)*d) + 4/15
*x/((-x^2*e^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-x^2*e^2 + d^2)*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(65) = 130$.
time = 2.95, size = 161, normalized size = 2.01

$$\frac{3\,x^5e^5 - 3\,dx^4e^4 - 6\,d^2x^3e^3 + 6\,d^3x^2e^2 + 3\,d^4xe - 3\,d^5 - (8\,x^4e^4 - 8\,dx^3e^3 - 12\,d^2x^2e^2 + 12\,d^3xe + 3\,d^4)\sqrt{-x^2e^2 + d^2}}{15\,(d^5x^5e^6 - d^6x^4e^5 - 2\,d^7x^3e^4 + 2\,d^8x^2e^3 + d^9xe^2 - d^{10}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^5*e^5 - 3*d*x^4*e^4 - 6*d^2*x^3*e^3 + 6*d^3*x^2*e^2 + 3*d^4*x*e -
 3*d^5 - (8*x^4*e^4 - 8*d*x^3*e^3 - 12*d^2*x^2*e^2 + 12*d^3*x*e + 3*d^4)*sq
rt(-x^2*e^2 + d^2))/(d^5*x^5*e^6 - d^6*x^4*e^5 - 2*d^7*x^3*e^4 + 2*d^8*x^2*
e^3 + d^9*x*e^2 - d^10*e)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.11, size = 604, normalized size = 7.55

$$d\left(\begin{cases} -\frac{15id^4x}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}} - 30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}} + 15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{20id^2e^2x^3}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}} - 30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}} + 15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{8ie^4x^5}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}} - 30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}} + 15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{15d^4x}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}} - 30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}} + 15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{20d^2e^2x^3}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}} - 30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}} + 15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{8e^4x^5}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}} - 30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}} + 15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2} - 10d^2e^4x^2\sqrt{d^2-e^2x^2} + 5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for}\ e \neq 0 \\ \frac{x^2}{2(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**
2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d
**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2))
, Abs(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d**2) -
 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e*
*2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*
d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x
**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*
x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**
4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e,
0)), (x**2/(2*(d**2)**(7/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [B]
time = 2.58, size = 78, normalized size = 0.98

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+12\,d^3\,e\,x-12\,d^2\,e^2\,x^2-8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 +
12*d^3*e*x))/(15*d^5*e*(d + e*x)^2*(d - e*x)^3)
```

3.27 $\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] 1/5*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*(4*e*x+5*d)/d^4/(-e^2*x^2+d^2)^(3
/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*(8*e*x+15*d)/d^6/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {837, 12, 272, 65, 214}

$$\frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (5*d + 4*e*x)/(15*d^4*(d^2 - e^2*
x^2)^(3/2)) + (15*d + 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^
2 - e^2*x^2]/d]/d^6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$\int \frac{d+ex}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{5d^3e^2+4d^2e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^5e^4+8d^4e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{15d^7e^6}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^5}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2}}\right.}{2d^5}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,\right.}{d^5e^2}$$

$$= \frac{d+ex}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x}}{d}\right.}{d^6}$$

Mathematica [A]

time = 0.45, size = 118, normalized size = 1.01

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(23d^4-8d^3ex-27d^2e^2x^2+7de^3x^3+8e^4x^4)}{(d-ex)^3(d+ex)^2}+30\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(23*d^4 - 8*d^3*e*x - 27*d^2*e^2*x^2 + 7*d*e^3*x^3 +
8*e^4*x^4))/((d - e*x)^3*(d + e*x)^2) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2
 - e^2*x^2])/d])/(15*d^6)
```

Maple [A]
time = 0.06, size = 182, normalized size = 1.56

method	result
default	$e\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+d\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\cdots}{d^2\sqrt{\cdots}}}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2
/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+d*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/
d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2
)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.28, size = 153, normalized size = 1.31

$$\frac{xe}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2}+\frac{1}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d}+\frac{4\,xe}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^4}+\frac{1}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}+\frac{8\,xe}{15\,\sqrt{-x^2e^2+d^2}\,d^6}-\frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d^6}+\frac{1}{\sqrt{-x^2e^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x*e/((-x^2*e^2 + d^2)^(5/2)*d^2) + 1/5/((-x^2*e^2 + d^2)^(5/2)*d) + 4/1
5*x*e/((-x^2*e^2 + d^2)^(3/2)*d^4) + 1/3/((-x^2*e^2 + d^2)^(3/2)*d^3) + 8/1
```

```
5*x*e/(sqrt(-x^2*e^2 + d^2)*d^6) - log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2
)*d/abs(x))/d^6 + 1/(sqrt(-x^2*e^2 + d^2)*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(102) = 204$.
time = 2.97, size = 231, normalized size = 1.97

$$\frac{23\,x^{5}e^{5}-23\,dx^{4}e^{4}-46\,d^{2}x^{3}e^{3}+46\,d^{3}x^{2}e^{2}+23\,d^{4}xe-23\,d^{5}+15\,\left(x^{5}e^{5}-dx^{4}e^{4}-2\,d^{2}x^{3}e^{3}+2\,d^{3}x^{2}e^{2}+d^{4}xe-d^{5}\right)\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)-\left(8\,x^{4}e^{4}+7\,dx^{3}e^{3}-27\,d^{2}x^{2}e^{2}-8\,d^{3}xe+23\,d^{4}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{15\,\left(d^{6}x^{5}e^{5}-d^{7}x^{4}e^{4}-2\,d^{8}x^{3}e^{3}+2\,d^{9}x^{2}e^{2}+d^{10}xe-d^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*x^5*e^5 - 23*d*x^4*e^4 - 46*d^2*x^3*e^3 + 46*d^3*x^2*e^2 + 23*d^4*
x*e - 23*d^5 + 15*(x^5*e^5 - d*x^4*e^4 - 2*d^2*x^3*e^3 + 2*d^3*x^2*e^2 + d^
4*x*e - d^5)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (8*x^4*e^4 + 7*d*x^3*e^3
- 27*d^2*x^2*e^2 - 8*d^3*x*e + 23*d^4)*sqrt(-x^2*e^2 + d^2))/(d^6*x^5*e^5 -
 d^7*x^4*e^4 - 2*d^8*x^3*e^3 + 2*d^9*x^2*e^2 + d^10*x*e - d^11)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.36, size = 2378, normalized size = 20.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((-46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d**
2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6)
 + 30*d**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) - 30*I*d**6*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x*
*2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*I*d**4*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d
**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**1
1*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**2*x**2*lo
g(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6
*x**6) + 90*I*d**4*e**2*x**2*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x**2
- 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**2*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7
*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e
**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e**4*x**4*log(e
*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x*
*6) - 90*I*d**2*e**4*x**4*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**2*x**2 - 9
0*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*log(e**2*x**2/d**2)/(-
30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30
```

```
*e**6*x**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) + 30*I*e**6*x**6*asin(d/(e*x))/(-30*d**13 + 90*d**11*e*
*2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6), Abs(e**2*x**2/d**2) > 1),
 (-46*d**6*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d*
*9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d**2)/(-30*d**13
+ 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 30*d**6*log
(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e*
*4*x**4 + 30*d**7*e**6*x**6) - 15*I*pi*d**6/(-30*d**13 + 90*d**11*e**2*x**2
 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*d**4*e**2*x**2*sqrt(1 - e**2
*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e
**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**2*x**2*log(sqr
t(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x
**4 + 30*d**7*e**6*x**6) + 45*I*pi*d**4*e**2*x**2/(-30*d**13 + 90*d**11*e**
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*d**2*e**4*x**4*sqrt(1
- e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*
d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**
11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e**4*x**4*l
og(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*
e**4*x**4 + 30*d**7*e**6*x**6) - 45*I*pi*d**2*e**4*x**4/(-30*d**13 + 90*d**
11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*log(e*
*2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7
*e**6*x**6) - 30*e**6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 9
0*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*I*pi*e**6*x
**6/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6
), True)) + e*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) -
 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 +
e**2*x**2/d**2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2)
- 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*
d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2
*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*
x**2/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4
*sqrt(1 - e**2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2
/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqr
t(1 - e**2*x**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 3
0*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2
*x**2/d**2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)/((-x^2*e^2 + d^2)^(7/2)*x), x)
```

Mupad [B]
time = 3.08, size = 127, normalized size = 1.09

$$\frac{\frac{d^2-e^2\,x^2}{3\,d^3}+\frac{{\left(d^2-e^2\,x^2\right)}^2}{d^5}+\frac{1}{5\,d}}{{\left(d^2-e^2\,x^2\right)}^{5/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{d^6}+\frac{e\,x\,\left(15\,d^4-20\,d^2\,e^2\,x^2+8\,e^4\,x^4\right)}{15\,d^6\,{\left(d^2-e^2\,x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] ((d^2 - e^2*x^2)/(3*d^3) + (d^2 - e^2*x^2)^2/d^5 + 1/(5*d))/(d^2 - e^2*x^2)
^(5/2) - atanh((d^2 - e^2*x^2)^(1/2)/d)/d^6 + (e*x*(15*d^4 + 8*e^4*x^4 - 20
*d^2*e^2*x^2))/(15*d^6*(d^2 - e^2*x^2)^(5/2))
```

3.28 $\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=153

$$\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

```
[Out] 1/5*(e*x+d)/d^2/x/(-e^2*x^2+d^2)^(5/2)+1/15*(5*e*x+6*d)/d^4/x/(-e^2*x^2+d^2
)^(3/2)-e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+1/5*(5*e*x+8*d)/d^6/x/(-e^2*x
^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/d^7/x
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {837, 821, 272, 65, 214}

$$\frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}-\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x*(d^2 - e^2*x^2)^(5/2)) + (6*d + 5*e*x)/(15*d^4*x*(d^2 -
e^2*x^2)^(3/2)) + (8*d + 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[d^
2 - e^2*x^2])/(5*d^7*x) - (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$\int \frac{d+ex}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{6d^3e^2+5d^2e^3x}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{24d^5e^4+15d^4e^5x}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} + \frac{\int \frac{48d^7e^6+15d^6e^7}{x^2\sqrt{d^2-e^2x}}}{15d^{12}e^6}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x}$$

$$= \frac{d+ex}{5d^2x\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d+5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x}$$

Mathematica [A]
time = 0.42, size = 134, normalized size = 0.88

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(15d^5-38d^4ex-52d^3e^2x^2+87d^2e^3x^3+33de^4x^4-48e^5x^5\right)}{x(-d+ex)^3(d+ex)^2}+30e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^5 - 38*d^4*e*x - 52*d^3*e^2*x^2 + 87*d^2*e^3*x^
3 + 33*d*e^4*x^4 - 48*e^5*x^5))/(x*(-d + e*x)^3*(d + e*x)^2) + 30*e*ArcTanh
[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(15*d^7)
```

Maple [A]
time = 0.08, size = 213, normalized size = 1.39

method	result

default	$d\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6e^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{d^2}\right)+e\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}\right.$
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^7x}-\frac{23\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{48d^7\left(x+\frac{d}{e}\right)}-\frac{413\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)e}}{240d^7\left(x-\frac{d}{e}\right)}-$ $e\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+
4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+e
*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1
/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x
^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.27, size = 182, normalized size = 1.19

$$\frac{6\,xe^2}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d^3}+\frac{e}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2}-\frac{1}{(-x^2e^2+d^2)^{\frac{5}{2}}dx}+\frac{8\,xe^2}{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^5}+\frac{e}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}d^4}-\frac{e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d^7}+\frac{16\,xe^2}{5\,\sqrt{-x^2e^2+d^2}\,d^7}+\frac{e}{\sqrt{-x^2e^2+d^2}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 6/5*x*e^2/((-x^2*e^2 + d^2)^(5/2)*d^3) + 1/5*e/((-x^2*e^2 + d^2)^(5/2)*d^2)
 - 1/((-x^2*e^2 + d^2)^(5/2)*d*x) + 8/5*x*e^2/((-x^2*e^2 + d^2)^(3/2)*d^5)
+ 1/3*e/((-x^2*e^2 + d^2)^(3/2)*d^4) - e*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2
 + d^2)*d/abs(x))/d^7 + 16/5*x*e^2/(sqrt(-x^2*e^2 + d^2)*d^7) + e/(sqrt(-x^
2*e^2 + d^2)*d^6)
```

Fricas [A]
time = 2.72, size = 254, normalized size = 1.66

$$\frac{23\,x^6e^6-23\,dx^5e^5-46\,d^2x^4e^4+46\,d^3x^3e^3+23\,d^4x^2e^2-23\,d^5xe+15\,(x^6e^6-dx^5e^5-2\,d^2x^4e^4+2\,d^3x^3e^3+d^4x^2e^2-d^5xe)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(48\,x^5e^5-33\,dx^4e^4-87\,d^2x^3e^3+52\,d^3x^2e^2+38\,d^4xe-15\,d^5)\sqrt{-x^2e^2+d^2}}{15\,(d^7x^6e^5-d^8x^5e^4-2\,d^9x^4e^3+2\,d^{10}x^3e^2+d^{11}x^2e-d^{12}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*x^6*e^6 - 23*d*x^5*e^5 - 46*d^2*x^4*e^4 + 46*d^3*x^3*e^3 + 23*d^4*
x^2*e^2 - 23*d^5*x*e + 15*(x^6*e^6 - d*x^5*e^5 - 2*d^2*x^4*e^4 + 2*d^3*x^3*
e^3 + d^4*x^2*e^2 - d^5*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (48*x^5*e
^5 - 33*d*x^4*e^4 - 87*d^2*x^3*e^3 + 52*d^3*x^2*e^2 + 38*d^4*x*e - 15*d^5)*
sqrt(-x^2*e^2 + d^2))/(d^7*x^6*e^5 - d^8*x^5*e^4 - 2*d^9*x^4*e^3 + 2*d^10*x
^3*e^2 + d^11*x^2*e - d^12*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.67, size = 2404, normalized size = 15.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*
x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2
/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d
**8*e**6*x**6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 1
5*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*s
qrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x*
*4 + 5*d**8*e**6*x**6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/
(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d*
*8*e**6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 +
 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*I*d**2*e*
*5*x**4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**
10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) +
1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6),
 True)) + e*Piecewise((-46*I*d**6*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90
*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**
2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*
e**6*x**6) + 30*d**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e
**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**6*asin(d/(e*x))/(-30*d**13 + 90*d**
11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*I*d**4*e**2*x**2
*sqrt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x
**4 + 30*d**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13
 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e*
*2*x**2*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30
*d**7*e**6*x**6) + 90*I*d**4*e**2*x**2*asin(d/(e*x))/(-30*d**13 + 90*d**11*
e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*I*d**2*e**4*x**4*sq
rt(-1 + e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4
 + 30*d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**13 +
90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e**4*
x**4*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d*
*7*e**6*x**6) - 90*I*d**2*e**4*x**4*asin(d/(e*x))/(-30*d**13 + 90*d**11*e**
```

```
2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x**6*log(e**2*x**
2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*
x**6) - 30*e**6*x**6*log(e*x/d)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e
**4*x**4 + 30*d**7*e**6*x**6) + 30*I*e**6*x**6*asin(d/(e*x))/(-30*d**13 + 9
0*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6), Abs(e**2*x**2/d
**2) > 1), (-46*d**6*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x*
*2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*d**6*log(e**2*x**2/d**2)/(
-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 3
0*d**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 -
90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 15*I*pi*d**6/(-30*d**13 + 90*d**11
*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 70*d**4*e**2*x**2*sqr
t(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 +
 30*d**7*e**6*x**6) + 45*d**4*e**2*x**2*log(e**2*x**2/d**2)/(-30*d**13 + 90
*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 90*d**4*e**2*x*
*2*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d
**9*e**4*x**4 + 30*d**7*e**6*x**6) + 45*I*pi*d**4*e**2*x**2/(-30*d**13 + 90
*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 30*d**2*e**4*x*
*4*sqrt(1 - e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*
x**4 + 30*d**7*e**6*x**6) - 45*d**2*e**4*x**4*log(e**2*x**2/d**2)/(-30*d**1
3 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 90*d**2*e
**4*x**4*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30*d**13 + 90*d**11*e**2*x**2
- 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) - 45*I*pi*d**2*e**4*x**4/(-30*d**1
3 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*e**6*x
**6*log(e**2*x**2/d**2)/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4
 + 30*d**7*e**6*x**6) - 30*e**6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-30
*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7*e**6*x**6) + 15*I
*pi*e**6*x**6/(-30*d**13 + 90*d**11*e**2*x**2 - 90*d**9*e**4*x**4 + 30*d**7
*e**6*x**6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)/((-x^2*e^2 + d^2)^(7/2)*x^2), x)
```

Mupad [B]
time = 3.31, size = 141, normalized size = 0.92

$$\frac{\frac{e}{5\,d^2}+\frac{e\,{\left(d^2-e^2\,x^2\right)}^2}{d^6}+\frac{e\,\left(d^2-e^2\,x^2\right)}{3\,d^4}}{{\left(d^2-e^2\,x^2\right)}^{5/2}}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{d^7}-\frac{d^6-6\,d^4\,e^2\,x^2+8\,d^2\,e^4\,x^4-\frac{16\,e^6\,x^6}{5}}{d^7\,x\,{\left(d^2-e^2\,x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] (e/(5*d^2) + (e*(d^2 - e^2*x^2)^2)/d^6 + (e*(d^2 - e^2*x^2))/(3*d^4))/(d^2
- e^2*x^2)^(5/2) - (e*atanh((d^2 - e^2*x^2)^(1/2)/d))/d^7 - (d^6 - (16*e^6*
x^6)/5 - 6*d^4*e^2*x^2 + 8*d^2*e^4*x^4)/(d^7*x*(d^2 - e^2*x^2)^(5/2))
```

3.29 $\int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=184

$$\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}-\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7e^2\tan}{}$$

```
[Out] 1/5*(e*x+d)/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+1/15*(6*e*x+7*d)/d^4/x^2/(-e^2*x^2
+d^2)^(3/2)-7/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+1/15*(24*e*x+35*d)/
d^6/x^2/(-e^2*x^2+d^2)^(1/2)-7/2*(-e^2*x^2+d^2)^(1/2)/d^7/x^2-16/5*e*(-e^2*
x^2+d^2)^(1/2)/d^8/x
```

Rubi [A]

time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {837, 849, 821, 272, 65, 214}

$$\frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (d + e*x)/(5*d^2*x^2*(d^2 - e^2*x^2)^(5/2)) + (7*d + 6*e*x)/(15*d^4*x^2*(d^
2 - e^2*x^2)^(3/2)) + (35*d + 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) - (7
*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) - (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x) -
(7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{d+ex}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{7d^3e^2+6d^2e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^4e^2}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{35d^5e^4+24d^4e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^8e^4}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{\int \frac{105d^7e^6+48\ldots}{x^3\sqrt{d^2-\ldots}}}{15d^{12}\ldots}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x\ldots}}{2d^7x^2}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x\ldots}}{2d^7x^2}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x\ldots}}{2d^7x^2}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x\ldots}}{2d^7x^2}$$

$$= \frac{d+ex}{5d^2x^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{7d+6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} - \frac{7\sqrt{d^2-e^2x\ldots}}{2d^7x^2}$$

Mathematica [A]
time = 0.48, size = 147, normalized size = 0.80

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(15d^6+15d^5ex-176d^4e^2x^2-4d^3e^3x^3+249d^2e^4x^4-9de^5x^5-96e^6x^6\right)}{x^2(-d+ex)^3(d+ex)^2}+210e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{30d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^6 + 15*d^5*e*x - 176*d^4*e^2*x^2 - 4*d^3*e^3*x^
3 + 249*d^2*e^4*x^4 - 9*d*e^5*x^5 - 96*e^6*x^6))/(x^2*(-d + e*x)^3*(d + e*x
)^2) + 210*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(30*d^8)
```

Maple [A]
time = 0.09, size = 244, normalized size = 1.33

method	result
default	$d\left(-\frac{1}{2d^{2}x^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+\frac{7e^{2}\left(\frac{1}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{1}{3d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\sqrt{-e^{2}x^{2}}}{x}\right)}{d^{2}\sqrt{d^{2}}}}{d^{2}}}{d^{2}}\right)}{2d^{2}}\right)$
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,(2ex+d)}{2d^{8}x^{2}}+\frac{29e\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{48d^{8}\left(x+\frac{d}{e}\right)}-\frac{673e\sqrt{-\left(x-\frac{d}{e}\right)^{2}e^{2}-2d\left(x-\frac{d}{e}\right)}}{240d^{8}\left(x-\frac{d}{e}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/2/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+7/2*e^2/d^2*(1/5/d^2/(-e^2*x^2+d^2)^(5
/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1
/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+e*(-1
/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d
^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))
```

Maxima [A]
time = 0.27, size = 206, normalized size = 1.12

$$\frac{6\,xe^{3}}{5\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}d^{4}}+\frac{7\,e^{2}}{10\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}d^{3}}-\frac{e}{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}d^{2}x}-\frac{1}{2\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}dx^{2}}+\frac{8\,xe^{3}}{5\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}d^{6}}+\frac{7\,e^{2}}{6\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}d^{5}}-\frac{7\,e^{2}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)}{2\,d^{8}}+\frac{16\,xe^{3}}{5\,\sqrt{-x^{2}e^{2}+d^{2}}\,d^{8}}+\frac{7\,e^{2}}{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 6/5*x*e^3/((-x^2*e^2 + d^2)^(5/2)*d^4) + 7/10*e^2/((-x^2*e^2 + d^2)^(5/2)*d
^3) - e/((-x^2*e^2 + d^2)^(5/2)*d^2*x) - 1/2/((-x^2*e^2 + d^2)^(5/2)*d*x^2)
 + 8/5*x*e^3/((-x^2*e^2 + d^2)^(3/2)*d^6) + 7/6*e^2/((-x^2*e^2 + d^2)^(3/2)
*d^5) - 7/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^8 + 1
6/5*x*e^3/(sqrt(-x^2*e^2 + d^2)*d^8) + 7/2*e^2/(sqrt(-x^2*e^2 + d^2)*d^7)
```

Fricas [A]
time = 2.38, size = 270, normalized size = 1.47

$$\frac{116\,x^7e^7-116\,dx^6e^6-232\,d^2x^5e^5+232\,d^3x^4e^4+116\,d^4x^3e^3-116\,d^5x^2e^2+105\,(x^7e^7-dx^6e^6-2\,d^2x^5e^5+2\,d^3x^4e^4+d^4x^3e^3-d^5x^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(96\,x^6e^6+9\,dx^5e^5-249\,d^2x^4e^4+4\,d^3x^3e^3+176\,d^4x^2e^2-15\,d^5xe-15\,d^6)\sqrt{-x^2e^2+d^2}}{30\,(d^8x^7e^5-d^9x^6e^4-2\,d^{10}x^5e^3+2\,d^{11}x^4e^2+d^{12}x^3e-d^{13}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*x^7*e^7 - 116*d*x^6*e^6 - 232*d^2*x^5*e^5 + 232*d^3*x^4*e^4 + 116
*d^4*x^3*e^3 - 116*d^5*x^2*e^2 + 105*(x^7*e^7 - d*x^6*e^6 - 2*d^2*x^5*e^5 +
 2*d^3*x^4*e^4 + d^4*x^3*e^3 - d^5*x^2*e^2)*log(-(d - sqrt(-x^2*e^2 + d^2))
/x) - (96*x^6*e^6 + 9*d*x^5*e^5 - 249*d^2*x^4*e^4 + 4*d^3*x^3*e^3 + 176*d^4
*x^2*e^2 - 15*d^5*x*e - 15*d^6)*sqrt(-x^2*e^2 + d^2))/(d^8*x^7*e^5 - d^9*x^
6*e^4 - 2*d^10*x^5*e^3 + 2*d^11*x^4*e^2 + d^12*x^3*e - d^13*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.77, size = 2691, normalized size = 14.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((30*I*d**8*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**1
3*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 322*I*d**6*e**2*x*
*2*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d*
*11*e**4*x**6 + 60*d**9*e**6*x**8) - 105*d**6*e**2*x**2*log(e**2*x**2/d**2)
/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6
*x**8) + 210*d**6*e**2*x**2*log(e*x/d)/(-60*d**15*x**2 + 180*d**13*e**2*x**
4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*I*d**6*e**2*x**2*asin(d/
(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**
9*e**6*x**8) + 490*I*d**4*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-60*d**15*x*
*2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 315*d
**4*e**4*x**4*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 1
80*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 630*d**4*e**4*x**4*log(e*x/d)/(-6
0*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**
8) + 630*I*d**4*e**4*x**4*asin(d/(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x*
*4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*I*d**2*e**6*x**6*sqrt(-
1 + e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*
x**6 + 60*d**9*e**6*x**8) - 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(-60*d**
15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) +
630*d**2*e**6*x**6*log(e*x/d)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d
**11*e**4*x**6 + 60*d**9*e**6*x**8) - 630*I*d**2*e**6*x**6*asin(d/(e*x))/(-
60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x*
*8) + 105*e**8*x**8*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x*
```

```
*4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*e**8*x**8*log(e*x/d)/(-
60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x*
*8) + 210*I*e**8*x**8*asin(d/(e*x))/(-60*d**15*x**2 + 180*d**13*e**2*x**4 -
 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), Abs(e**2*x**2/d**2) > 1), (30*d*
*8*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**
11*e**4*x**6 + 60*d**9*e**6*x**8) - 322*d**6*e**2*x**2*sqrt(1 - e**2*x**2/d
**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*
e**6*x**8) - 105*d**6*e**2*x**2*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d
**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 210*d**6*e**2*x
**2*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4
 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 105*I*pi*d**6*e**2*x**2/(-60*
d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8)
 + 490*d**4*e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*
e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 315*d**4*e**4*x**4*l
og(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x
**6 + 60*d**9*e**6*x**8) - 630*d**4*e**4*x**4*log(sqrt(1 - e**2*x**2/d**2)
+ 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*
e**6*x**8) + 315*I*pi*d**4*e**4*x**4/(-60*d**15*x**2 + 180*d**13*e**2*x**4
- 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*d**2*e**6*x**6*sqrt(1 - e*
*2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 +
 60*d**9*e**6*x**8) - 315*d**2*e**6*x**6*log(e**2*x**2/d**2)/(-60*d**15*x**
2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 630*d*
*2*e**6*x**6*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*
e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 315*I*pi*d**2*e**6*x
**6/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e
**6*x**8) + 105*e**8*x**8*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e
**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*e**8*x**8*log(sqr
t(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**1
1*e**4*x**6 + 60*d**9*e**6*x**8) + 105*I*pi*e**8*x**8/(-60*d**15*x**2 + 180
*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), True)) + e*Pie
cewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2
- 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3*x**2*sqrt(d**2/(e**
2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e
**6*x**6) + 40*d**2*e**5*x**4*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**
12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*sqrt(d
**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 +
5*d**8*e**6*x**6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/(e**2
*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e*
*6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d
**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*I*d**2*e**5*x*
*4*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e*
*4*x**4 + 5*d**8*e**6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) + 1)/(-
5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6), True
))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((x*e + d)/((-x^2*e^2 + d^2)^(7/2)*x^3), x)
```

Mupad [**B**]
time = 3.43, size = 181, normalized size = 0.98

$$\frac{161\,e^2}{30\,d^3\,(d^2-e^2\,x^2)^{5/2}}-\frac{1}{2\,d\,x^2\,(d^2-e^2\,x^2)^{5/2}}-\frac{7\,e^2\,\mathrm{atanh}\left(\frac{\sqrt{d^2-e^2\,x^2}}{d}\right)}{2\,d^8}-\frac{49\,e^4\,x^2}{6\,d^5\,(d^2-e^2\,x^2)^{5/2}}+\frac{7\,e^6\,x^4}{2\,d^7\,(d^2-e^2\,x^2)^{5/2}}-\frac{e\,(5\,d^6-30\,d^4\,e^2\,x^2+40\,d^2\,e^4\,x^4-16\,e^6\,x^6)}{5\,d^8\,x\,(d^2-e^2\,x^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
[Out] (161*e^2)/(30*d^3*(d^2 - e^2*x^2)^(5/2)) - 1/(2*d*x^2*(d^2 - e^2*x^2)^(5/2)
) - (7*e^2*atanh((d^2 - e^2*x^2)^(1/2)/d))/(2*d^8) - (49*e^4*x^2)/(6*d^5*(d
^2 - e^2*x^2)^(5/2)) + (7*e^6*x^4)/(2*d^7*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d^
6 - 16*e^6*x^6 - 30*d^4*e^2*x^2 + 40*d^2*e^4*x^4))/(5*d^8*x*(d^2 - e^2*x^2)
^(5/2))
```

3.30 $\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx$

Optimal. Leaf size=121

$$\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/7*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(7/2)-2/35*(-2*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(5/2)-4/105*x/d^3/e^2/(-e^2*x^2+d^2)^(3/2)-8/105*x/d^5/e^2/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {810, 792, 198, 197}

$$\frac{x^2(d+ex)}{7de\left(d^2-e^2x^2\right)^{7/2}}-\frac{2(d-2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
```

```
[Out] (x^2*(d + e*x))/(7*d*e*(d^2 - e^2*x^2)^(7/2)) - (2*(d - 2*e*x))/(35*d*e^3*(
d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(
105*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 810

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[x^2*(a*g - c*f*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}}\,dx = \frac{x^2(d+ex)}{7de\,(d^2-e^2x^2)^{7/2}} - \frac{\int \frac{x(2d^2e-4de^2x)}{(d^2-e^2x^2)^{7/2}}\,dx}{7d^2e^2}$$
$$= \frac{x^2(d+ex)}{7de\,(d^2-e^2x^2)^{7/2}} - \frac{2(d-2ex)}{35de^3\,(d^2-e^2x^2)^{5/2}} - \frac{4\int \frac{1}{(d^2-e^2x^2)^{5/2}}\,dx}{35de^2}$$
$$= \frac{x^2(d+ex)}{7de\,(d^2-e^2x^2)^{7/2}} - \frac{2(d-2ex)}{35de^3\,(d^2-e^2x^2)^{5/2}} - \frac{4x}{105d^3e^2\,(d^2-e^2x^2)^{3/2}} - \frac{8\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,d}{105d^3e^2}$$
$$= \frac{x^2(d+ex)}{7de\,(d^2-e^2x^2)^{7/2}} - \frac{2(d-2ex)}{35de^3\,(d^2-e^2x^2)^{5/2}} - \frac{4x}{105d^3e^2\,(d^2-e^2x^2)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-}}$$

Mathematica [A]
time = 0.39, size = 104, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\,(-6d^6+6d^5ex+15d^4e^2x^2+20d^3e^3x^3-20d^2e^4x^4-8de^5x^5+8e^6x^6)}{105d^5e^3(d-ex)^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^6 + 6*d^5*e*x + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3
- 20*d^2*e^4*x^4 - 8*d*e^5*x^5 + 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^4*(d +
e*x)^3)
```

Maple [A]
time = 0.07, size = 173, normalized size = 1.43

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2(-8e^6x^6+8d\,e^5x^5+20d^2e^4x^4-20d^3x^3e^3-15d^4e^2x^2-6d^5ex+6d^6)}{105d^5e^3(-e^2x^2+d^2)^{\frac{9}{2}}}$
trager	$-\frac{(-8e^6x^6+8d\,e^5x^5+20d^2e^4x^4-20d^3x^3e^3-15d^4e^2x^2-6d^5ex+6d^6)\sqrt{-e^2x^2+d^2}}{105d^5(-ex+d)^4(ex+d)^3e^3}$

default	$e\left(\frac{x^2}{5e^2(-e^2x^2+d^2)^{\frac{7}{2}}}-\frac{2d^2}{35e^4(-e^2x^2+d^2)^{\frac{7}{2}}}\right)+d\left(\frac{x}{6e^2(-e^2x^2+d^2)^{\frac{7}{2}}}-\frac{d^2\left(\frac{x}{7d^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{6\left(\frac{\cdots}{15d^2\cdots}\right.}{\cdots}}{\cdots}\right.}{6e^2\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/5*x^2/e^2/(-e^2*x^2+d^2)^(7/2)-2/35*d^2/e^4/(-e^2*x^2+d^2)^(7/2))+d*(1
/6*x/e^2/(-e^2*x^2+d^2)^(7/2)-1/6*d^2/e^2*(1/7*x/d^2/(-e^2*x^2+d^2)^(7/2)+6
/7/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3
/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))
```

Maxima [A]
time = 0.28, size = 123, normalized size = 1.02

$$\frac{x^2e^{(-1)}}{5\,(-x^2e^2+d^2)^{\frac{7}{2}}}+\frac{dxe^{(-2)}}{7\,(-x^2e^2+d^2)^{\frac{7}{2}}}-\frac{2\,d^2e^{(-3)}}{35\,(-x^2e^2+d^2)^{\frac{7}{2}}}-\frac{xe^{(-2)}}{35\,(-x^2e^2+d^2)^{\frac{5}{2}}d}-\frac{4\,xe^{(-2)}}{105\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}-\frac{8\,xe^{(-2)}}{105\,\sqrt{-x^2e^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^2*e^(-1)/(-x^2*e^2 + d^2)^(7/2) + 1/7*d*x*e^(-2)/(-x^2*e^2 + d^2)^(7/
2) - 2/35*d^2*e^(-3)/(-x^2*e^2 + d^2)^(7/2) - 1/35*x*e^(-2)/((-x^2*e^2 + d^
2)^(5/2)*d) - 4/105*x*e^(-2)/((-x^2*e^2 + d^2)^(3/2)*d^3) - 8/105*x*e^(-2)/
(sqrt(-x^2*e^2 + d^2)*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(101) = 202$.
time = 3.22, size = 221, normalized size = 1.83

$$-\frac{6\,x^7e^7-6\,dx^6e^6-18\,d^2x^5e^5+18\,d^3x^4e^4+18\,d^4x^3e^3-18\,d^5x^2e^2-6\,d^6xe+6\,d^7-(8\,x^6e^6-8\,dx^5e^5-20\,d^2x^4e^4+20\,d^3x^3e^3+15\,d^4x^2e^2+6\,d^5xe-6\,d^6)\sqrt{-x^2e^2+d^2}}{105\,(d^5x^7e^{10}-d^6x^6e^9-3\,d^7x^5e^8+3\,d^8x^4e^7+3\,d^9x^3e^6-3\,d^{10}x^2e^5-d^{11}xe^4+d^{12}e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(6*x^7*e^7 - 6*d*x^6*e^6 - 18*d^2*x^5*e^5 + 18*d^3*x^4*e^4 + 18*d^4*
x^3*e^3 - 18*d^5*x^2*e^2 - 6*d^6*x*e + 6*d^7 - (8*x^6*e^6 - 8*d*x^5*e^5 - 2
0*d^2*x^4*e^4 + 20*d^3*x^3*e^3 + 15*d^4*x^2*e^2 + 6*d^5*x*e - 6*d^6)*sqrt(-
x^2*e^2 + d^2))/(d^5*x^7*e^10 - d^6*x^6*e^9 - 3*d^7*x^5*e^8 + 3*d^8*x^4*e^7
 + 3*d^9*x^3*e^6 - 3*d^10*x^2*e^5 - d^11*x*e^4 + d^12*e^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.37, size = 903, normalized size = 7.46

$$d\left(\begin{cases} \frac{35id^4x^3}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{28id^2e^2x^5}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{8ie^4x^7}{-105d^{13}\sqrt{-1+\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ -\frac{35d^4x^3}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{28d^2e^2x^5}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{8e^4x^7}{-105d^{13}\sqrt{1-\frac{e^2x^2}{d^2}}+315d^{11}e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}-315d^9e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}+105d^7e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \frac{2d^2}{-35d^6e^4\sqrt{d^2-e^2x^2}+105d^4e^6x^2\sqrt{d^2-e^2x^2}-105d^2e^8x^4\sqrt{d^2-e^2x^2}+35e^{10}x^6\sqrt{d^2-e^2x^2}} - \frac{7e^2x^2}{-35d^6e^4\sqrt{d^2-e^2x^2}+105d^4e^6x^2\sqrt{d^2-e^2x^2}-105d^2e^8x^4\sqrt{d^2-e^2x^2}+35e^{10}x^6\sqrt{d^2-e^2x^2}} & \text{for}\ e\neq 0 \\ \frac{x^4}{4(d^2)^{\frac{9}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(9/2),x)
```

```
[Out] d*Piecewise((35*I*d**4*x**3/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**
11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*
x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) - 28*I*d**2*e**2
*x**5/(-105*d**13*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 +
 e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*
e**6*x**6*sqrt(-1 + e**2*x**2/d**2)) + 8*I*e**4*x**7/(-105*d**13*sqrt(-1 +
e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) - 315*d**9*
e**4*x**4*sqrt(-1 + e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (-35*d**4*x**3/(-105*d**13*sqrt(1 - e**
2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4
*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2
)) + 28*d**2*e**2*x**5/(-105*d**13*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**
2*x**2*sqrt(1 - e**2*x**2/d**2) - 315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**
2) + 105*d**7*e**6*x**6*sqrt(1 - e**2*x**2/d**2)) - 8*e**4*x**7/(-105*d**13
*sqrt(1 - e**2*x**2/d**2) + 315*d**11*e**2*x**2*sqrt(1 - e**2*x**2/d**2) -
315*d**9*e**4*x**4*sqrt(1 - e**2*x**2/d**2) + 105*d**7*e**6*x**6*sqrt(1 - e
**2*x**2/d**2)), True)) + e*Piecewise((2*d**2/(-35*d**6*e**4*sqrt(d**2 - e*
*2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**2*x**2) - 105*d**2*e**8*x**4*s
qrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d**2 - e**2*x**2)) - 7*e**2*x**2
/(-35*d**6*e**4*sqrt(d**2 - e**2*x**2) + 105*d**4*e**6*x**2*sqrt(d**2 - e**
2*x**2) - 105*d**2*e**8*x**4*sqrt(d**2 - e**2*x**2) + 35*e**10*x**6*sqrt(d*
*2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(9/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*x^2/(-x^2*e^2 + d^2)^(9/2), x)
```

Mupad [B]
time = 2.69, size = 164, normalized size = 1.36

$$\frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,{\left(d-e\,x\right)}^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}-\frac{3\,x}{70\,d\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}+\frac{4\,x}{105\,d^3\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(9/2),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d - e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(2/
(35*e^3) - (3*x)/(70*d*e^2)))/((d + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^
(1/2)*(1/(56*d^2*e^3) + (4*x)/(105*d^3*e^2)))/((d + e*x)^2*(d - e*x)^2) - (
8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

3.31 $\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{16x}{315d^7e^2\sqrt{d^2-e^2}}$$

```
[Out] 1/9*x^2*(e*x+d)/d/e/(-e^2*x^2+d^2)^(9/2)-2/63*(-3*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(7/2)-2/105*x/d^3/e^2/(-e^2*x^2+d^2)^(5/2)-8/315*x/d^5/e^2/(-e^2*x^2+d^2
)^(3/2)-16/315*x/d^7/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {810, 792, 198, 197}

$$\frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}-\frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}-\frac{8x}{315d^5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (x^2*(d + e*x))/(9*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d - 3*e*x))/(63*d*e^3*(
d^2 - e^2*x^2)^(7/2)) - (2*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(5/2)) - (8*x)/(
315*d^5*e^2*(d^2 - e^2*x^2)^(3/2)) - (16*x)/(315*d^7*e^2*Sqrt[d^2 - e^2*x^2
])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 810

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[x^2*(a*g - c*f*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Dist[1/(2*a
*c*(p + 1)), Int[x*Simp[2*a*g - c*f*(2*p + 5)*x, x]*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, f, g}, x] && EqQ[a*g^2 + f^2*c, 0] && LtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d+ex)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx &= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{\int \frac{x(2d^2e-6de^2x)}{(d^2-e^2x^2)^{9/2}}\,dx}{9d^2e^2}\\ &= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}-\frac{2\int \frac{1}{(d^2-e^2x^2)^{7/2}}\,dx}{21de^2}\\ &= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{8\int \frac{1}{(d^2-e^2x^2)^{5/}}}{105d^3e^2}\\ &= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{315d^5e^2(d^2-}\\ &= \frac{x^2(d+ex)}{9de\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d-3ex)}{63de^3\left(d^2-e^2x^2\right)^{7/2}}-\frac{2x}{105d^3e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{315d^5e^2(d^2-}\end{aligned}$$

Mathematica [A]
time = 0.48, size = 126, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(-10d^8+10d^7ex+35d^6e^2x^2+70d^5e^3x^3-70d^4e^4x^4-56d^3e^5x^5+56d^2e^6x^6+16de^7x^7-16e^8x^8\right)}{315d^7e^3(d-ex)^5(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-10*d^8 + 10*d^7*e*x + 35*d^6*e^2*x^2 + 70*d^5*e^3*x^
3 - 70*d^4*e^4*x^4 - 56*d^3*e^5*x^5 + 56*d^2*e^6*x^6 + 16*d*e^7*x^7 - 16*e^
8*x^8))/(315*d^7*e^3*(d - e*x)^5*(d + e*x)^4)
```

Maple [A]
time = 0.06, size = 199, normalized size = 1.34

method	result
gosper	$-\frac{(-ex+d)(ex+d)^2\left(16e^8x^8-16e^7x^7d-56d^2e^6x^6+56d^3e^5x^5+70d^4x^4e^4-70x^3d^5e^3-35d^6e^2x^2-10d^7ex+10d^8\right)}{315d^7e^3\left(-e^2x^2+d^2\right)^{\frac{11}{2}}}$

trager	$-\frac{\left(16e^8x^8-16e^7x^7d-56d^2e^6x^6+56d^3e^5x^5+70d^4x^4e^4-70x^3d^5e^3-35d^6e^2x^2-10d^7ex+10d^8\right)\sqrt{-e^2x^2+d^2}}{315d^7(-ex+d)^5(ex+d)^4e^3}$
default	$e\left(\frac{x^2}{7e^2(-e^2x^2+d^2)^{\frac{9}{2}}}-\frac{2d^2}{63e^4(-e^2x^2+d^2)^{\frac{9}{2}}}\right)+d\left(\frac{x}{8e^2(-e^2x^2+d^2)^{\frac{9}{2}}}-\frac{d^2\left(\frac{x}{9d^2\left(-e^2x^2+d^2\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}+\frac{8\left(\frac{\cdots}{35d^2(\cdots}\right.}{\cdots}}{\cdots}\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/7*x^2/e^2/(-e^2*x^2+d^2)^(9/2)-2/63*d^2/e^4/(-e^2*x^2+d^2)^(9/2))+d*(1
/8*x/e^2/(-e^2*x^2+d^2)^(9/2)-1/8*d^2/e^2*(1/9*x/d^2/(-e^2*x^2+d^2)^(9/2)+8
/9/d^2*(1/7*x/d^2/(-e^2*x^2+d^2)^(7/2)+6/7/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5
/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))
))))
```

Maxima [A]
time = 0.36, size = 144, normalized size = 0.97

$$\frac{x^2e^{(-1)}}{7\left(-x^2e^2+d^2\right)^{\frac{9}{2}}}+\frac{dxe^{(-2)}}{9\left(-x^2e^2+d^2\right)^{\frac{9}{2}}}-\frac{2\,d^2e^{(-3)}}{63\left(-x^2e^2+d^2\right)^{\frac{9}{2}}}-\frac{xe^{(-2)}}{63\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d}-\frac{2\,xe^{(-2)}}{105\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^3}-\frac{8\,xe^{(-2)}}{315\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^5}-\frac{16\,xe^{(-2)}}{315\sqrt{-x^2e^2+d^2}\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/7*x^2*e^(-1)/(-x^2*e^2 + d^2)^(9/2) + 1/9*d*x*e^(-2)/(-x^2*e^2 + d^2)^(9/
2) - 2/63*d^2*e^(-3)/(-x^2*e^2 + d^2)^(9/2) - 1/63*x*e^(-2)/((-x^2*e^2 + d^
2)^(7/2)*d) - 2/105*x*e^(-2)/((-x^2*e^2 + d^2)^(5/2)*d^3) - 8/315*x*e^(-2)/
((-x^2*e^2 + d^2)^(3/2)*d^5) - 16/315*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(122) = 244$.
time = 4.18, size = 281, normalized size = 1.90

$$-\frac{10\,x^{9}e^{9}-10\,dx^{8}e^{8}-40\,d^{2}x^{7}e^{7}+40\,d^{3}x^{6}e^{6}+60\,d^{4}x^{5}e^{5}-60\,d^{5}x^{4}e^{4}-40\,d^{6}x^{3}e^{3}+40\,d^{7}x^{2}e^{2}+10\,d^{8}xe-10\,d^{9}-\left(16\,x^{8}e^{8}-16\,dx^{7}e^{7}-56\,d^{2}x^{6}e^{6}+56\,d^{3}x^{5}e^{5}+70\,d^{4}x^{4}e^{4}-70\,d^{5}x^{3}e^{3}-35\,d^{6}x^{2}e^{2}-10\,d^{7}xe+10\,d^{8}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{315\,\left(d^{7}x^{9}e^{12}-d^{8}x^{8}e^{11}-4\,d^{9}x^{7}e^{10}+4\,d^{10}x^{6}e^{9}+6\,d^{11}x^{5}e^{8}-6\,d^{12}x^{4}e^{7}-4\,d^{13}x^{3}e^{6}+4\,d^{14}x^{2}e^{5}+d^{15}xe^{4}-d^{16}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/315*(10*x^9*e^9 - 10*d*x^8*e^8 - 40*d^2*x^7*e^7 + 40*d^3*x^6*e^6 + 60*d^
4*x^5*e^5 - 60*d^5*x^4*e^4 - 40*d^6*x^3*e^3 + 40*d^7*x^2*e^2 + 10*d^8*x*e -
 10*d^9 - (16*x^8*e^8 - 16*d*x^7*e^7 - 56*d^2*x^6*e^6 + 56*d^3*x^5*e^5 + 70
*d^4*x^4*e^4 - 70*d^5*x^3*e^3 - 35*d^6*x^2*e^2 - 10*d^7*x*e + 10*d^8)*sqrt(
-x^2*e^2 + d^2))/(d^7*x^9*e^12 - d^8*x^8*e^11 - 4*d^9*x^7*e^10 + 4*d^10*x^6
*e^9 + 6*d^11*x^5*e^8 - 6*d^12*x^4*e^7 - 4*d^13*x^3*e^6 + 4*d^14*x^2*e^5 +
d^15*x*e^4 - d^16*e^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.58, size = 1401, normalized size = 9.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(11/2),x)
```

```
[Out] d*Piecewise((-105*I*d**6*x**3/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d
**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*
e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) + 126*I*d**4*e**2*x**5/(315*d**17*sqrt
(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 18
90*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1
 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) - 72*I*d
**2*e**4*x**7/(315*d**17*sqrt(-1 + e**2*x**2/d**2) - 1260*d**15*e**2*x**2*s
qrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) -
 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(-
1 + e**2*x**2/d**2)) + 16*I*e**6*x**9/(315*d**17*sqrt(-1 + e**2*x**2/d**2)
- 1260*d**15*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 3
15*d**9*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (10
5*d**6*x**3/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt
(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260
*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**
2*x**2/d**2)) - 126*d**4*e**2*x**5/(315*d**17*sqrt(1 - e**2*x**2/d**2) - 12
60*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 -
 e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9
```

```
*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) + 72*d**2*e**4*x**7/(315*d**17*sqrt(1
- e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1890*d*
*13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x**6*sqrt(1 - e**2
*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2)) - 16*e**6*x**9/(
315*d**17*sqrt(1 - e**2*x**2/d**2) - 1260*d**15*e**2*x**2*sqrt(1 - e**2*x**
2/d**2) + 1890*d**13*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**11*e**6*x
**6*sqrt(1 - e**2*x**2/d**2) + 315*d**9*e**8*x**8*sqrt(1 - e**2*x**2/d**2))
, True)) + e*Piecewise((-2*d**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) - 252*
d**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 - e**2
*x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqrt(d*
*2 - e**2*x**2)) + 9*e**2*x**2/(63*d**8*e**4*sqrt(d**2 - e**2*x**2) - 252*d
**6*e**6*x**2*sqrt(d**2 - e**2*x**2) + 378*d**4*e**8*x**4*sqrt(d**2 - e**2*
x**2) - 252*d**2*e**10*x**6*sqrt(d**2 - e**2*x**2) + 63*e**12*x**8*sqrt(d**
2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(11/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*x^2/(-x^2*e^2 + d^2)^(11/2), x)
```

Mupad [B]
time = 2.74, size = 202, normalized size = 1.36

$$\frac{\sqrt{d^2-e^2\,x^2}}{144\,d^3\,e^3\,{\left(d-e\,x\right)}^5}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{252\,e^3}-\frac{17\,x}{252\,d\,e^2}\right)}{{\left(d+e\,x\right)}^4\,{\left(d-e\,x\right)}^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{5}{144\,d^2\,e^3}+\frac{131\,x}{5040\,d^3\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{315\,d^5\,e^2\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{16\,x\,\sqrt{d^2-e^2\,x^2}}{315\,d^7\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x))/(d^2 - e^2*x^2)^(11/2),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(144*d^3*e^3*(d - e*x)^5) - ((d^2 - e^2*x^2)^(1/2)*(1
/(252*e^3) - (17*x)/(252*d*e^2)))/((d + e*x)^4*(d - e*x)^4) - ((d^2 - e^2*x
^2)^(1/2)*(5/(144*d^2*e^3) + (131*x)/(5040*d^3*e^2)))/((d + e*x)^3*(d - e*x
)^3) - (8*x*(d^2 - e^2*x^2)^(1/2))/(315*d^5*e^2*(d + e*x)^2*(d - e*x)^2) -
(16*x*(d^2 - e^2*x^2)^(1/2))/(315*d^7*e^2*(d + e*x)*(d - e*x))
```

3.32 $\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sin^{-1}(ax)}{a^3}$$

```
[Out] -arcsin(a*x)/a^3+(a*x-1)/a^3/(-a^2*x^2+1)^(1/2)-(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {811, 655, 222, 651}

$$-\frac{\text{ArcSin}(ax)}{a^3}-\frac{1-ax}{a^3\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -((1 - a*x)/(a^3*Sqrt[1 - a^2*x^2])) - Sqrt[1 - a^2*x^2]/a^3 - ArcSin[a*x]/
a^3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rubi steps

$$\int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{1-ax}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{1-ax}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -\frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$
$$= -\frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3} - \frac{\sin^{-1}(ax)}{a^3}$$

Mathematica [A]
time = 0.20, size = 71, normalized size = 1.31

$$\frac{(-2-ax)\sqrt{1-a^2x^2}}{a^3(1+ax)} - \frac{\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{(-a^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - a*x))/(1 - a^2*x^2)^(3/2),x]
[Out] ((-2 - a*x)*Sqrt[1 - a^2*x^2])/(a^3*(1 + a*x)) - Log[-(Sqrt[-a^2]*x) + Sqrt
[1 - a^2*x^2]]/(-a^2)^(3/2)
```

Maple [A]
time = 0.11, size = 90, normalized size = 1.67

method	result	size
default	$-a\left(-\frac{x^2}{a^2\sqrt{-a^2x^2+1}}+\frac{2}{a^4\sqrt{-a^2x^2+1}}\right)+\frac{x}{a^2\sqrt{-a^2x^2+1}}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^2\sqrt{a^2}}$	90
risch	$\frac{a^2x^2-1}{a^3\sqrt{-a^2x^2+1}}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^2\sqrt{a^2}}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{a^4\left(x+\frac{1}{a}\right)}$	92
meijerg	$-\frac{-2\sqrt{\pi}+\frac{\sqrt{\pi}\,\left(-4a^2x^2+8\right)}{4\sqrt{-a^2x^2+1}}}{a^3\sqrt{\pi}}-\frac{\frac{\sqrt{\pi}\,x\left(-a^2\right)^{\frac{3}{2}}}{a^2\sqrt{-a^2x^2+1}}-\frac{\sqrt{\pi}\,\left(-a^2\right)^{\frac{3}{2}}\arcsin(ax)}{a^3}}{a^2\sqrt{\pi}\,\sqrt{-a^2}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -a*(-x^2/a^2/(-a^2*x^2+1)^(1/2)+2/a^4/(-a^2*x^2+1)^(1/2))+x/a^2/(-a^2*x^2+1
)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A]
time = 0.51, size = 63, normalized size = 1.17

$$\frac{x^2}{\sqrt{-a^2x^2+1}\,a}+\frac{x}{\sqrt{-a^2x^2+1}\,a^2}-\frac{\arcsin(ax)}{a^3}-\frac{2}{\sqrt{-a^2x^2+1}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(-a^2*x^2 + 1)*a) + x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a*x)/a^3 -
 2/(sqrt(-a^2*x^2 + 1)*a^3)
```

Fricas [A]
time = 2.04, size = 66, normalized size = 1.22

$$-\frac{2\,ax-2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,(ax+2)+2}{a^4x+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

Sympy [A]
time = 3.53, size = 119, normalized size = 2.20

$$-a\left(\begin{cases}\frac{a^2x^2\sqrt{-a^2x^2+1}}{a^6x^2-a^4}-\frac{2\sqrt{-a^2x^2+1}}{a^6x^2-a^4} & \text{for } a\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ix}{a^2\sqrt{a^2x^2-1}}+\frac{i\operatorname{acosh}(ax)}{a^3} & \text{for } |a^2x^2|>1\\ \frac{x}{a^2\sqrt{-a^2x^2+1}}-\frac{\operatorname{asin}(ax)}{a^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*x+1)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] -a*Piecewise((a**2*x**2*sqrt(-a**2*x**2 + 1)/(a**6*x**2 - a**4) - 2*sqrt(-a
**2*x**2 + 1)/(a**6*x**2 - a**4), Ne(a, 0)), (x**4/4, True)) + Piecewise((-
I*x/(a**2*sqrt(a**2*x**2 - 1)) + I*acosh(a*x)/a**3, Abs(a**2*x**2) > 1), (x
/(a**2*sqrt(-a**2*x**2 + 1)) - asin(a*x)/a**3, True))
```

Giac [A]
time = 0.76, size = 70, normalized size = 1.30

$$-\frac{\arcsin(ax)\operatorname{sgn}(a)}{a^2|a|}-\frac{\sqrt{-a^2x^2+1}}{a^3}+\frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

Mupad [B]
time = 0.09, size = 84, normalized size = 1.56

$$\frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(a*x - 1))/(1 - a^2*x^2)^(3/2),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

3.33 $\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=173

$$-\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}$$

```
[Out] 11/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-8/15*d^3*x^2*(-e^2*x^2+d^2)^
(1/2)/e^3-11/24*d^2*x^3*(-e^2*x^2+d^2)^(1/2)/e^2-2/5*d*x^4*(-e^2*x^2+d^2)^(
1/2)/e-1/6*x^5*(-e^2*x^2+d^2)^(1/2)-1/240*d^4*(165*e*x+256*d)*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A]
time = 0.14, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1823, 847, 794, 223, 209}

$$\frac{11d^6\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}-\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e}-\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5}-\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-8*d^3*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (11*d^2*x^3*Sqrt[d^2 - e^2*x^2]
)/(24*e^2) - (2*d*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) - (x^5*Sqrt[d^2 - e^2*x^2]
)/6 - (d^4*(256*d + 165*e*x)*Sqrt[d^2 - e^2*x^2])/(240*e^5) + (11*d^6*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^4(-11d^2e^2-12de^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{6e^2}$$

$$= -\frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{x^3(48d^3e^3+55d^2e^4x)}{\sqrt{d^2-e^2x^2}}\,dx}{30e^4}$$

$$= -\frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^2(-165d^4e^4-192d^3e^5x)}{\sqrt{d^2-e^2x^2}}\,d}{120e^6}$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} +$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} -$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} -$$

$$= -\frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{1}{6}x^5\sqrt{d^2-e^2x^2} -$$

Mathematica [A]
time = 0.23, size = 125, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(-256d^5-165d^4ex-128d^3e^2x^2-110d^2e^3x^3-96de^4x^4-40e^5x^5\right)}{240e^5}+\frac{11d^6\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{16e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-256*d^5 - 165*d^4*e*x - 128*d^3*e^2*x^2 - 110*d^2*e^
3*x^3 - 96*d*e^4*x^4 - 40*e^5*x^5))/(240*e^5) + (11*d^6*Sqrt[-e^2]*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e^6)
```

Maple [A]
time = 0.06, size = 295, normalized size = 1.71

method	result
risch	$-\frac{\left(40e^5x^5+96d\,e^4x^4+110d^2e^3x^3+128x^2d^3e^2+165d^4xe+256d^5\right)\sqrt{-e^2x^2+d^2}}{240e^5}+\frac{11d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^4\sqrt{e^2}}$
default	$e^2\left(-\frac{x^5\sqrt{-e^2x^2+d^2}}{6e^2}+\frac{5d^2\left(-\frac{x^3\sqrt{-e^2x^2+d^2}}{4e^2}+\frac{3d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)}{4e^2}\right)}{6e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(-1/6*x^5/e^2*(-e^2*x^2+d^2)^(1/2)+5/6*d^2/e^2*(-1/4*x^3/e^2*(-e^2*x^2+
d^2)^(1/2)+3/4*d^2/e^2*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+2*d*e*(-1/5*x^4/e^2*(-e^
2*x^2+d^2)^(1/2)+4/5*d^2/e^2*(-1/3*x^2/e^2*(-e^2*x^2+d^2)^(1/2)-2/3*d^2/e^4
*(-e^2*x^2+d^2)^(1/2)))+d^2*(-1/4*x^3/e^2*(-e^2*x^2+d^2)^(1/2)+3/4*d^2/e^2*
(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))))
```

Maxima [A]
time = 0.49, size = 142, normalized size = 0.82

$$\frac{11}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-5)}-\frac{2}{5}\,\sqrt{-x^2e^2+d^2}\,dx^4e^{(-1)}-\frac{11}{24}\,\sqrt{-x^2e^2+d^2}\,d^2x^3e^{(-2)}-\frac{8}{15}\,\sqrt{-x^2e^2+d^2}\,d^3x^2e^{(-3)}-\frac{11}{16}\,\sqrt{-x^2e^2+d^2}\,d^4xe^{(-4)}-\frac{16}{15}\,\sqrt{-x^2e^2+d^2}\,d^5e^{(-5)}-\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 11/16*d^6*arcsin(x*e/d)*e^(-5) - 2/5*sqrt(-x^2*e^2 + d^2)*d*x^4*e^(-1) - 11
/24*sqrt(-x^2*e^2 + d^2)*d^2*x^3*e^(-2) - 8/15*sqrt(-x^2*e^2 + d^2)*d^3*x^2
*e^(-3) - 11/16*sqrt(-x^2*e^2 + d^2)*d^4*x*e^(-4) - 16/15*sqrt(-x^2*e^2 + d
^2)*d^5*e^(-5) - 1/6*sqrt(-x^2*e^2 + d^2)*x^5
```

Fricas [A]
time = 2.70, size = 98, normalized size = 0.57

$$-\frac{1}{240}\left(330\,d^6\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(40\,x^5e^5+96\,dx^4e^4+110\,d^2x^3e^3+128\,d^3x^2e^2+165\,d^4xe+256\,d^5\right)\sqrt{-x^2e^2+d^2}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/240*(330*d^6*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (40*x^5*e^5
+ 96*d*x^4*e^4 + 110*d^2*x^3*e^3 + 128*d^3*x^2*e^2 + 165*d^4*x*e + 256*d^5)
*sqrt(-x^2*e^2 + d^2))*e^(-5)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.07, size = 558, normalized size = 3.23

$$d^2\left(\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{8d^4\sqrt{d^2-e^2x^2}}{15e^6}-\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^4}-\frac{x^4\sqrt{d^2-e^2x^2}}{5e^2} & \text{for}\ e\neq 0\\ \frac{x^6}{6\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{5id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^7}+\frac{5id^5x}{16e^6\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5id^3x^3}{48e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^5}{24e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{5d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^7}-\frac{5d^5x}{16e^6\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5d^3x^3}{48e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^5}{24e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5
/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*
x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2
*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) +
2*d*e*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*sqr
t(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(e,
 0)), (x**6/(6*sqrt(d**2)), True)) + e**2*Piecewise((-5*I*d**6*acosh(e*x/d)
/(16*e**7) + 5*I*d**5*x/(16*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**3*x**3
/(48*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**5/(24*e**2*sqrt(-1 + e**2*x**
2/d**2)) - I*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
```

```
, (5*d**6*asin(e*x/d)/(16*e**7) - 5*d**5*x/(16*e**6*sqrt(1 - e**2*x**2/d**2
)) + 5*d**3*x**3/(48*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**5/(24*e**2*sqrt(
1 - e**2*x**2/d**2)) + x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]

time = 0.64, size = 84, normalized size = 0.49

$$\frac{11}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{1}{240}\left(256\,d^5e^{(-5)}+\left(165\,d^4e^{(-4)}+2\left(64\,d^3e^{(-3)}+\left(55\,d^2e^{(-2)}+4\left(12\,de^{(-1)}+5\,x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 11/16*d^6*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/240*(256*d^5*e^(-5) + (165*d^4*e^
(-4) + 2*(64*d^3*e^(-3) + (55*d^2*e^(-2) + 4*(12*d*e^(-1) + 5*x)*x)*x)*x)*x
)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.34 $\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}$$

```
[Out] 3/4*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-3/5*d^2*x^2*(-e^2*x^2+d^2)^(1/
2)/e^2-1/2*d*x^3*(-e^2*x^2+d^2)^(1/2)/e-1/5*x^4*(-e^2*x^2+d^2)^(1/2)-3/20*d
^3*(5*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A]
time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1823, 847, 794, 223, 209}

$$\frac{3d^5\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4}-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(2*e
) - (x^4*Sqrt[d^2 - e^2*x^2])/5 - (3*d^3*(8*d + 5*e*x)*Sqrt[d^2 - e^2*x^2])
/(20*e^4) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{\int \frac{x^3\left(-9d^2e^2-10de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2}$$

$$= -\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}+\frac{\int \frac{x^2\left(30d^3e^3+36d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{\int \frac{x\left(-72d^4e^4-90d^3e^5x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2}}{20e^4}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2}}{20e^4}$$

$$= -\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{2e}-\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{3d^3(8d+5ex)\sqrt{d^2-e^2}}{20e^4}$$

Mathematica [A]

time = 0.21, size = 114, normalized size = 0.79

$$\frac{\sqrt{d^2-e^2x^2}\left(-24d^4-15d^3ex-12d^2e^2x^2-10de^3x^3-4e^4x^4\right)}{20e^4}+\frac{3d^5\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{4e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^4 - 15*d^3*e*x - 12*d^2*e^2*x^2 - 10*d*e^3*x^3
- 4*e^4*x^4))/(20*e^4) + (3*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
 e^2*x^2]])/(4*e^5)
```

Maple [A]
time = 0.08, size = 222, normalized size = 1.54

method	result
risch	$-\frac{\left(4e^4x^4+10d\,e^3x^3+12d^2x^2e^2+15d^3ex+24d^4\right)\sqrt{-e^2x^2+d^2}}{20e^4}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4e^3\sqrt{e^2}}$
default	$e^2\left(-\frac{x^4\sqrt{-e^2x^2+d^2}}{5e^2}+\frac{4d^2\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)}{5e^2}\right)+2de\left(-\frac{x^3\sqrt{-e^2x^2+d^2}}{4e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(-1/5*x^4/e^2*(-e^2*x^2+d^2)^(1/2)+4/5*d^2/e^2*(-1/3*x^2/e^2*(-e^2*x^2+
d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+d^2)^(1/2)))+2*d*e*(-1/4*x^3/e^2*(-e^2*x^2
+d^2)^(1/2)+3/4*d^2/e^2*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+d^2*(-1/3*x^2/e^2*(-e^2*
x^2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+d^2)^(1/2))
```

Maxima [A]
time = 0.48, size = 119, normalized size = 0.83

$$\frac{3}{4}d^5\arcsin\left(\frac{xe}{d}\right)e^{(-4)}-\frac{1}{2}\sqrt{-x^2e^2+d^2}\,dx^3e^{(-1)}-\frac{3}{5}\sqrt{-x^2e^2+d^2}\,d^2x^2e^{(-2)}-\frac{3}{4}\sqrt{-x^2e^2+d^2}\,d^3xe^{(-3)}-\frac{6}{5}\sqrt{-x^2e^2+d^2}\,d^4e^{(-4)}-\frac{1}{5}\sqrt{-x^2e^2+d^2}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*d^5*arcsin(x*e/d)*e^(-4) - 1/2*sqrt(-x^2*e^2 + d^2)*d*x^3*e^(-1) - 3/5*
sqrt(-x^2*e^2 + d^2)*d^2*x^2*e^(-2) - 3/4*sqrt(-x^2*e^2 + d^2)*d^3*x*e^(-3)
 - 6/5*sqrt(-x^2*e^2 + d^2)*d^4*e^(-4) - 1/5*sqrt(-x^2*e^2 + d^2)*x^4
```

Fricas [A]
time = 1.86, size = 88, normalized size = 0.61

$$-\frac{1}{20}\left(30\,d^5\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(4\,x^4e^4+10\,dx^3e^3+12\,d^2x^2e^2+15\,d^3xe+24\,d^4\right)\sqrt{-x^2e^2+d^2}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(30*d^5*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (4*x^4*e^4 + 1
0*d*x^3*e^3 + 12*d^2*x^2*e^2 + 15*d^3*x*e + 24*d^4)*sqrt(-x^2*e^2 + d^2))*e
^(-4)
```

Sympy [A]
time = 3.85, size = 357, normalized size = 2.48

$$d^2\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{8d^4\sqrt{d^2-e^2x^2}}{15e^6}-\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^4}-\frac{x^4\sqrt{d^2-e^2x^2}}{5e^2} & \text{for}\ e\neq 0\\ \frac{x^6}{6\sqrt{d^2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 -
e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + 2*d*e*Piecew
ise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1 + e**2*x*
*2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(4*d*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*x/d)/(8*e**
5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*sqrt(1 - e
**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piecew
ise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**6) - 4*d**2*x**2*sqrt(d**2 - e**
2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x**2)/(5*e**2), Ne(e, 0)), (x**6/
(6*sqrt(d**2)), True))
```

Giac [A]
time = 1.15, size = 73, normalized size = 0.51

$$\frac{3}{4}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\operatorname{sgn}(d)-\frac{1}{20}\left(24\,d^4e^{(-4)}+\left(15\,d^3e^{(-3)}+2\left(6\,d^2e^{(-2)}+\left(5\,de^{(-1)}+2\,x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^5*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/20*(24*d^4*e^(-4) + (15*d^3*e^(-3)
+ 2*(6*d^2*e^(-2) + (5*d*e^(-1) + 2*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(d+e\,x)^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
[Out] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.35 $\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}+\frac{7d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] 7/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2/3*d*x^2*(-e^2*x^2+d^2)^(1/2)
/e-1/4*x^3*(-e^2*x^2+d^2)^(1/2)-1/24*d^2*(21*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)
/e^3
```

Rubi [A]

time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1823, 847, 794, 223, 209}

$$\frac{7d^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}-\frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-2*d*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) - (x^3*Sqrt[d^2 - e^2*x^2])/4 - (d^2*(
32*d + 21*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) + (7*d^4*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(8*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{\int \frac{x^2\left(-7d^2e^2-8de^3x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} + \frac{\int \frac{x\left(16d^3e^3+21d^2e^4x\right)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4}$$

$$= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{(7d^4)\int \frac{}{\sqrt{d^2 -}}}{8e^2}$$

$$= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{(7d^4)\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{7d^4\tan^{-1}\Big(}{8e}$$

Mathematica [A]
time = 0.21, size = 103, normalized size = 0.90

$$\frac{\sqrt{d^2-e^2x^2}\left(-32d^3-21d^2ex-16de^2x^2-6e^3x^3\right)}{24e^3} + \frac{7d^4\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-32*d^3 - 21*d^2*e*x - 16*d*e^2*x^2 - 6*e^3*x^3))/(24
*e^3) + (7*d^4*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*e^
4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(99) = 198$.
time = 0.06, size = 202, normalized size = 1.76

method	result
risch	$-\frac{(6e^3x^3+16d\,e^2x^2+21d^2ex+32d^3)\sqrt{-e^2x^2+d^2}}{24e^3}+\frac{7d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}$
default	$e^2\left(-\frac{x^3\sqrt{-e^2x^2+d^2}}{4e^2}+\frac{3d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)}{4e^2}\right)+2de\left(-\frac{x^2\sqrt{-e^2x^2}}{3e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(-1/4*x^3/e^2*(-e^2*x^2+d^2)^(1/2)+3/4*d^2/e^2*(-1/2*x/e^2*(-e^2*x^2+d^
2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
))+2*d*e*(-1/3*x^2/e^2*(-e^2*x^2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+d^2)^(1/2
))+d^2*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2
)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [A]
time = 0.49, size = 96, normalized size = 0.83

$$\frac{7}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{2}{3}\,\sqrt{-x^2e^2+d^2}\,dx^2e^{(-1)}-\frac{7}{8}\,\sqrt{-x^2e^2+d^2}\,d^2xe^{(-2)}-\frac{4}{3}\,\sqrt{-x^2e^2+d^2}\,d^3e^{(-3)}-\frac{1}{4}\,\sqrt{-x^2e^2+d^2}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 7/8*d^4*arcsin(x*e/d)*e^(-3) - 2/3*sqrt(-x^2*e^2 + d^2)*d*x^2*e^(-1) - 7/8*
sqrt(-x^2*e^2 + d^2)*d^2*x*e^(-2) - 4/3*sqrt(-x^2*e^2 + d^2)*d^3*e^(-3) - 1
/4*sqrt(-x^2*e^2 + d^2)*x^3
```

Fricas [A]
time = 3.11, size = 78, normalized size = 0.68

$$-\frac{1}{24}\left(42\,d^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(6\,x^3e^3+16\,dx^2e^2+21\,d^2xe+32\,d^3\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(42*d^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (6*x^3*e^3 + 1
6*d*x^2*e^2 + 21*d^2*x*e + 32*d^3)*sqrt(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.59, size = 386, normalized size = 3.36

$$d^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\ e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**
2*x**2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2),
True)) + 2*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sq
rt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + e*
*2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*asin(e*x/
d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(8*e**2*s
qrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 0.94, size = 63, normalized size = 0.55

$$\frac{7}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{24}\left(32\,d^3e^{(-3)}+\left(21\,d^2e^{(-2)}+2\left(8\,de^{(-1)}+3\,x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 7/8*d^4*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24*(32*d^3*e^(-3) + (21*d^2*e^(-2)
+ 2*(8*d*e^(-1) + 3*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
[Out] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.36 $\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/3*x^2*(-e^2*x^2+d^2)^(1/2)-1/3*d
*(3*e*x+5*d)*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1823, 794, 223, 209}

$$\frac{d^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}-\frac{1}{3}x^2\sqrt{d^2-e^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/3*(x^2*Sqrt[d^2 - e^2*x^2]) - (d*(5*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(3*e
^2) + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2}-\frac{\int \frac{x(-5d^2e^2-6de^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2}\\ &= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2}-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}+\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}\\ &= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2}-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}+\frac{d^3\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}\\ &= -\frac{1}{3}x^2\sqrt{d^2-e^2x^2}-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 89, normalized size = 1.07

$$\frac{\sqrt{d^2-e^2x^2}\,(-5d^2-3dex-e^2x^2)}{3e^2}+\frac{d^3\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^2 - 3*d*e*x - e^2*x^2))/(3*e^2) + (d^3*Sqrt[-e^2
]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^3
```

Maple [A]
time = 0.06, size = 133, normalized size = 1.60

method	result
risch	$-\frac{(e^2x^2+3dex+5d^2)\sqrt{-e^2x^2+d^2}}{3e^2}+\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}$

default	$e^{2}\left(-\frac{x^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3e^{2}}-\frac{2d^{2}\sqrt{-e^{2}x^{2}+d^{2}}}{3e^{4}}\right)+2de\left(-\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{2}}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{2}\sqrt{e^{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(-1/3*x^2/e^2*(-e^2*x^2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+d^2)^(1/2))+2*
d*e*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2)))-d^2/e^2*(-e^2*x^2+d^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 72, normalized size = 0.87

$$d^{3}\arcsin\left(\frac{xe}{d}\right)e^{(-2)}-\sqrt{-x^{2}e^{2}+d^{2}}\,dxe^{(-1)}-\frac{5}{3}\sqrt{-x^{2}e^{2}+d^{2}}\,d^{2}e^{(-2)}-\frac{1}{3}\sqrt{-x^{2}e^{2}+d^{2}}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] d^3*arcsin(x*e/d)*e^(-2) - sqrt(-x^2*e^2 + d^2)*d*x*e^(-1) - 5/3*sqrt(-x^2*
e^2 + d^2)*d^2*e^(-2) - 1/3*sqrt(-x^2*e^2 + d^2)*x^2
```

Fricas [A]
time = 3.22, size = 67, normalized size = 0.81

$$-\frac{1}{3}\left(6\,d^{3}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)+\left(x^{2}e^{2}+3\,dxe+5\,d^{2}\right)\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*d^3*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (x^2*e^2 + 3*d*x
*e + 5*d^2)*sqrt(-x^2*e^2 + d^2))*e^(-2)
```

Sympy [A]
time = 2.42, size = 218, normalized size = 2.63

$$d^{2}\left(\begin{cases}\frac{x^{2}}{2\sqrt{d^{2}}} & \text{for}\ e^{2}=0\\-\frac{\sqrt{d^{2}-e^{2}x^{2}}}{e^{2}} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{id^{2}\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^{3}}+\frac{idx}{2e^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{ix^{3}}{2d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\\frac{d^{2}\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^{3}}-\frac{dx\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{2e^{2}} & \text{otherwise}\end{cases}\right)+e^{2}\left(\begin{cases}-\frac{2d^{2}\sqrt{d^{2}-e^{2}x^{2}}}{3e^{4}}-\frac{x^{2}\sqrt{d^{2}-e^{2}x^{2}}}{3e^{2}} & \text{for}\ e\neq 0\\\frac{x^{4}}{4\sqrt{d^{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)
/e**2, True)) + 2*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e
**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2
/d**2)/(2*e**2), True)) + e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3
*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**
2)), True))
```

Giac [A]
time = 1.14, size = 49, normalized size = 0.59

$$d^3 \arcsin\left(\frac{xe}{d}\right) e^{(-2)}\mathrm{sgn}(d) - \frac{1}{3}\sqrt{-x^2e^2 + d^2}\left(5\,d^2e^{(-2)} + \left(3\,de^{(-1)} + x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] d^3*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/3*sqrt(-x^2*e^2 + d^2)*(5*d^2*e^(-2) +
(3*d*e^(-1) + x)*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d+e\,x\right)}^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(1/2), x)
```

3.37 $\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] 3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e-3/2*d*(-e^2*x^2+d^2)^(1/2)/e-1/2
*(e*x+d)*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {685, 655, 223, 209}

$$\frac{3d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}-\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-3*d*Sqrt[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) +
(3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
```

```
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{1}{2}(3d)\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx\\ &= -\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{1}{2}(3d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\ &= -\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{1}{2}(3d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-\cdots}}\right)\\ &= -\frac{3d\sqrt{d^2-e^2x^2}}{2e}-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.94

$$\frac{(-4d-ex)\sqrt{d^2-e^2x^2}}{2e}-\frac{3d^2\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((-4*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e) - (3*d^2*Log[-(Sqrt[-e^2]*x) + Sqr
t[d^2 - e^2*x^2]])/(2*Sqrt[-e^2])
```

Maple [A]
time = 0.07, size = 113, normalized size = 1.36

method	result
risch	$-\frac{(ex+4d)\sqrt{-e^2x^2+d^2}}{2e}+\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}$
default	$e^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)-\frac{2d\sqrt{-e^2x^2+d^2}}{e}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2)))-2*d*(-e^2*x^2+d^2)^(1/2)/e+d^2/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A]
time = 0.47, size = 50, normalized size = 0.60

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}-2\,\sqrt{-x^2e^2+d^2}\,de^{(-1)}-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-1) - 2*sqrt(-x^2*e^2 + d^2)*d*e^(-1) - 1/2*sqrt(-
x^2*e^2 + d^2)*x
```

Fricas [A]
time = 3.40, size = 57, normalized size = 0.69

$$-\frac{1}{2}\left(6\,d^2\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\sqrt{-x^2e^2+d^2}\,(xe+4\,d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + sqrt(-x^2*e^2 +
d^2)*(x*e + 4*d))*e^(-1)
```

Sympy [A]
time = 2.15, size = 269, normalized size = 3.24

$$d^2\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2<0\wedge e^2<0\end{cases}\right)+2de\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\ e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2),
(d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d
**2), (d**2 < 0) & (e**2 < 0))) + 2*d*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(
e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + e**2*Piecewise((-I*d**2*
acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**3/(
2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)
/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2), True))
```

Giac [**A**]
time = 1.05, size = 40, normalized size = 0.48

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d)-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(4\,de^{(-1)}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2)*(4*d*e^(-1)
+ x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2), x)
```

3.38 $\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 2*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d*arctanh((-e^2*x^2+d^2)^(1/2)/d)-(-e^
2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1823, 858, 223, 209, 272, 65, 214}

$$2d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\sqrt{d^2-e^2x^2}-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sq
rt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}}\,dx &= -\sqrt{d^2-e^2x^2}-\frac{\int \frac{-d^2e^2-2de^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\sqrt{d^2-e^2x^2}+d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+(2de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\sqrt{d^2-e^2x^2}+\frac{1}{2}d^2\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)+(2de)\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,d\right.\\
&= -\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d^2\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2}\\
&= -\sqrt{d^2-e^2x^2}+2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A]

time = 0.19, size = 102, normalized size = 1.55

$$-\sqrt{d^2-e^2x^2}+2d\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{2de\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -Sqrt[d^2 - e^2*x^2] + 2*d*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d
] - (2*d*e*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/Sqrt[-e^2]
```

Maple [A]
time = 0.07, size = 91, normalized size = 1.38

method	result	size
default	$-\sqrt{-e^2x^2+d^2}+\frac{2ed\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)+2*e*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [A]
time = 0.48, size = 61, normalized size = 0.92

$$2\,d\arcsin\left(\frac{xe}{d}\right)-d\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*d*arcsin(x*e/d) - d*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x)) -
 sqrt(-x^2*e^2 + d^2)
```

Fricas [A]
time = 3.22, size = 69, normalized size = 1.05

$$-4\,d\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+d\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*d*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + d*log(-(d - sqrt(-x^2*e
^2 + d^2))/x) - sqrt(-x^2*e^2 + d^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.88, size = 184, normalized size = 2.79

$$d^2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+e^2\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e
*x))/d, True)) + 2*d*e*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/s
qrt(d**2), (d**2 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d
**2))/sqrt(d**2), (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e
**2/d**2))/sqrt(-d**2), (d**2 < 0) & (e**2 < 0))) + e**2*Piecewise((x**2/(2
*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True))
```

Giac [A]
time = 0.81, size = 65, normalized size = 0.98

$$2\,d\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d)-d\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d*arcsin(x*e/d)*sgn(d) - d*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x)) - sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(d+e\,x\right)}^2}{x\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(1/2)), x)
```

3.39 $\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-(-e^
2*x^2+d^2)^(1/2)/x
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1821, 858, 223, 209, 272, 65, 214}

$$e\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{\sqrt{d^2-e^2x^2}}{x}-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - 2*e*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{x^2\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{x}-\frac{\int \frac{-2d^3e-d^2e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+(2de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx+e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+(de)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)+e^2\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx\right.\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(2d)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right.}{e}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{x}+e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}
$$

Mathematica [A]

time = 0.21, size = 102, normalized size = 1.50

$$-\frac{\sqrt{d^2-e^2x^2}}{x}+4e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/x) + 4*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^
2]/d] + Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [A]
time = 0.08, size = 93, normalized size = 1.37

method	result	size
default	$\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}}{x}-\frac{2de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$	93
risch	$\frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{\sqrt{-e^2x^2+d^2}}{x}-\frac{2de\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-(-e^2*x^2+d^2)^(
1/2)/x-2*d*e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [A]
time = 0.48, size = 65, normalized size = 0.96

$$\arcsin\left(\frac{xe}{d}\right)e-2\,e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\frac{\sqrt{-x^2e^2+d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x*e/d)*e - 2*e*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x)) -
 sqrt(-x^2*e^2 + d^2)/x
```

Fricas [A]
time = 3.21, size = 77, normalized size = 1.13

$$-\frac{2\,x\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e-2\,xe\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\sqrt{-x^2e^2+d^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*x*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e - 2*x*e*log(-(d - sqrt
(-x^2*e^2 + d^2))/x) + sqrt(-x^2*e^2 + d^2))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 1.79, size = 207, normalized size = 3.04

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise} \end{cases}\right)+2de\left(\begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise} \end{cases}\right)+e^2\left(\begin{cases} \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0 \\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0 \\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, Abs(d**2/(e**2*x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) + 2*d*e*Piecewise((-aco
sh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(e*x))/d, True)) + e**
2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0)
 & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d*
*2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2
), (d**2 < 0) & (e**2 < 0)))
```

Giac [A]
time = 1.05, size = 107, normalized size = 1.57

$$\arcsin\left(\frac{xe}{d}\right)e\operatorname{sgn}(d)-2e\log\left(\frac{\left|-2de-2\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2|x|}\right)+\frac{xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-1)}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e*sgn(d) - 2*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x)) + 1/2*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) - 1/2*(d*e + sqr
t(-x^2*e^2 + d^2)*e)*e^(-1)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{e^2\ln\left(x\sqrt{-e^2}+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-\frac{2de\ln\left(\frac{\sqrt{d^2}+\sqrt{d^2-e^2x^2}}{x}\right)}{\sqrt{d^2}} & \text{if } e^2<0 \\ \int\frac{e^2}{\sqrt{d^2-e^2x^2}}+\frac{d^2}{x^2\sqrt{d^2-e^2x^2}}+\frac{2de}{x\sqrt{d^2-e^2x^2}}\,dx & \text{if } \neg e^2<0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] piecewise(e^2 < 0, - (d^2 - e^2*x^2)^(1/2)/x + (e^2*log(x*(-e^2)^(1/2) + (d
^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (2*d*e*log(((d^2)^(1/2) + (d^2 - e^2*x
^2)^(1/2))/x))/(d^2)^(1/2), ~e^2 < 0, int(e^2/(d^2 - e^2*x^2)^(1/2) + d^2/(
x^2*(d^2 - e^2*x^2)^(1/2)) + (2*d*e)/(x*(d^2 - e^2*x^2)^(1/2)), x))
```

3.40 $\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

```
[Out] -3/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-1/2*(-e^2*x^2+d^2)^(1/2)/x^2-2*e
*(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1821, 821, 272, 65, 214}

$$-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/2*Sqrt[d^2 - e^2*x^2]/x^2 - (2*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (3*e^2*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{x^3\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{\int \frac{-4d^3e-3d^2e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{2}\left(3e^2\right)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}+\frac{1}{4}\left(3e^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 116, normalized size = 1.45

$$
-\frac{(d+4ex)\sqrt{d^2-e^2x^2}-3e^2x^2\log\left(d\left(-d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)+3e^2x^2\log\left(d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2dx^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/2*((d + 4*e*x)*Sqrt[d^2 - e^2*x^2] - 3*e^2*x^2*Log[d*(-d - Sqrt[-e^2]*x
+ Sqrt[d^2 - e^2*x^2])] + 3*e^2*x^2*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x
^2]])/(d*x^2)
```

Maple [A]
time = 0.07, size = 139, normalized size = 1.74

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(4ex+d)}{2d\,x^2}-\frac{3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}$
default	$d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}\right)-\frac{2e\sqrt{-e^2x^2+d^2}}{dx}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{\ldots}}{\ldots}\right)}{\sqrt{d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(1/2)-1/2*e^2/d^2/(d^2)^(1/2)*ln((2*d^2+2*
(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))-2*e*(-e^2*x^2+d^2)^(1/2)/d/x-e^2/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [A]
time = 0.48, size = 80, normalized size = 1.00

$$-\frac{3\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{2\,d}-\frac{2\,\sqrt{-x^2e^2+d^2}\,e}{dx}-\frac{\sqrt{-x^2e^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d - 2*sqrt(-x^
2*e^2 + d^2)*e/(d*x) - 1/2*sqrt(-x^2*e^2 + d^2)/x^2
```

Fricas [A]
time = 2.55, size = 61, normalized size = 0.76

$$\frac{3\,x^2e^2\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-\sqrt{-x^2e^2+d^2}\,(4\,xe+d)}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*x^2*e^2*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - sqrt(-x^2*e^2 + d^2)*(4
*x*e + d))/(d*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.84, size = 214, normalized size = 2.68

$$d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x}-\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i\operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) - e**2*acosh(d/(e*
x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**2*asin(d/(e*x)
)/(2*d**3), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/d**2, A
bs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/d**2, True)) +
 e**2*Piecewise((-acosh(d/(e*x))/d, Abs(d**2/(e**2*x**2)) > 1), (I*asin(d/(
e*x))/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(67) = 134.
time = 0.80, size = 166, normalized size = 2.08

$$\frac{x^2\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}{x}+e^2\right)e^4}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d}-\frac{3\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d}-\frac{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{(-2)}}{x^2}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d}{x}}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*x^2*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)/x + e^2)*e^4/((d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*d) - 3/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e
^(-2)/abs(x))/d - 1/8*((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^(-2)/x^2 + 8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)*d/x)/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^3\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(1/2)), x)
```

3.41 $\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] -e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-1/3*(-e^2*x^2+d^2)^(1/2)/x^3-e*(-e
^2*x^2+d^2)^(1/2)/d/x^2-5/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1821, 849, 821, 272, 65, 214}

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}-\frac{e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/3*Sqrt[d^2 - e^2*x^2]/x^3 - (e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (5*e^2*Sqr
t[d^2 - e^2*x^2])/(3*d^2*x) - (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{\int \frac{-6d^3e-5d^2e^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} + \frac{\int \frac{10d^4e^2+6d^3e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{e^3\text{Subst}\Big(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x}{2d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{e\text{Subst}\Big(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-}}{d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Mathematica [A]
time = 0.26, size = 86, normalized size = 0.80

$$\frac{-\frac{\sqrt{d^2-e^2x^2}\,(d^2+3dex+5e^2x^2)}{x^3} + 6e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-((Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + 5*e^2*x^2))/x^3) + 6*e^3*ArcTanh[(
Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(3*d^2)
```

Maple [A]
time = 0.07, size = 151, normalized size = 1.41

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(5e^2x^2+3dex+d^2)}{3x^3d^2} - \frac{e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{d\sqrt{d^2}}$

default	$2de\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}\right)-\frac{e^2\sqrt{-e^2x^2+d^2}}{d^2x}+d^2\left(-\frac{\sqrt{-e^2x^2+}}{3d^2x^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*e*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(1/2)-1/2*e^2/d^2/(d^2)^(1/2)*ln((2*d^2+
2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))-e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x+d^2*
(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(1/2)-2/3*e^2/d^4/x*(-e^2*x^2+d^2)^(1/2))
```

Maxima [A]
time = 0.48, size = 103, normalized size = 0.96

$$-\frac{e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d^2}-\frac{5\sqrt{-x^2e^2+d^2}\,e^2}{3\,d^2x}-\frac{\sqrt{-x^2e^2+d^2}\,e}{dx^2}-\frac{\sqrt{-x^2e^2+d^2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -e^3*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^2 - 5/3*sqrt(-x^
2*e^2 + d^2)*e^2/(d^2*x) - sqrt(-x^2*e^2 + d^2)*e/(d*x^2) - 1/3*sqrt(-x^2*e
^2 + d^2)/x^3
```

Fricas [A]
time = 1.79, size = 71, normalized size = 0.66

$$\frac{3\,x^3e^3\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(5\,x^2e^2+3\,dxe+d^2)\sqrt{-x^2e^2+d^2}}{3\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*e^3*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (5*x^2*e^2 + 3*d*x*e +
d^2)*sqrt(-x^2*e^2 + d^2))/(d^2*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.62, size = 303, normalized size = 2.83

$$d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2}-\frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2}-\frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise}\end{cases}\right)+2de\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x}-\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2
/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d
**4), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*d**2*x) -
e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1), (I/(2*e*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*e
**2*asin(d/(e*x))/(2*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**
2) - 1)/d**2, Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)
/d**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(90) = 180.
time = 0.83, size = 237, normalized size = 2.21

$$\frac{x^3\left(\frac{6\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e}{x}+\frac{21\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-1)}}{x^2}+e^3\right)e^6}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2}-\frac{e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^2}-\frac{\frac{21\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4e}{x}+\frac{6\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^4e^{(-1)}}{x^2}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^4e^{(-3)}}{x^3}}{24\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*x^3*(6*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e/x + 21*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^2*e^(-1)/x^2 + e^3)*e^6/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2) - e^
3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 1/24*
(21*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e/x + 6*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^2*d^4*e^(-1)/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^(-3)/x^3)/d^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^4\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(1/2)), x)
```

3.42 $\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}$$

```
[Out] -7/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/4*(-e^2*x^2+d^2)^(1/2)/x^4-2
/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-7/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-4/3*e^
3*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A]
time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1821, 849, 821, 272, 65, 214}

$$-\frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}-\frac{2e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{7e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}-\frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/4*Sqrt[d^2 - e^2*x^2]/x^4 - (2*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (7*e^2
*Sqrt[d^2 - e^2*x^2])/(8*d^2*x^2) - (4*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) -
 (7*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^5\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{\int \frac{-8d^3e-7d^2e^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{\int \frac{21d^4e^2+16d^3e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{\int \frac{-32d^5e^3-21d^4e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{(7e^4)\int \frac{}{x\sqrt{}}}{}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{(7e^4)\,\text{Subs}}{}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{(7e^2)\,\text{Subs}}{}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4x^4} - \frac{2e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{7e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} - \frac{4e^3\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{7e^4\tanh^{-1}}{}$$

Mathematica [A]
time = 0.29, size = 104, normalized size = 0.74

$$\frac{\sqrt{d^2-e^2x^2}\,(-6d^3-16d^2ex-21de^2x^2-32e^3x^3)}{24d^3x^4} + \frac{7e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^5*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 16*d^2*e*x - 21*d*e^2*x^2 - 32*e^3*x^3))/(24
*d^3*x^4) + (7*e^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/(4*d^
3)
```

Maple [A]
time = 0.07, size = 229, normalized size = 1.64

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(32e^3x^3+21d\,e^2x^2+16d^2ex+6d^3)}{24d^3x^4} - \frac{7e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8d^2\sqrt{d^2}}$

default	$d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{4d^2x^4}+\frac{3e^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}\right)}{4d^2}\right)+e^2\left(-\frac{\sqrt{-e^2x}}{2d^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(1/2)+3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+
d^2)^(1/2)-1/2*e^2/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x)))+e^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(1/2)-1/2*e^2/d^2/(d^2)^(1/2)*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))+2*d*e*(-1/3/d^2/x^3*(-e^2*
x^2+d^2)^(1/2)-2/3*e^2/d^4/x*(-e^2*x^2+d^2)^(1/2))
```

Maxima [A]
time = 0.49, size = 126, normalized size = 0.90

$$-\frac{7\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{8\,d^3}-\frac{4\,\sqrt{-x^2e^2+d^2}\,e^3}{3\,d^3x}-\frac{7\,\sqrt{-x^2e^2+d^2}\,e^2}{8\,d^2x^2}-\frac{2\,\sqrt{-x^2e^2+d^2}\,e}{3\,dx^3}-\frac{\sqrt{-x^2e^2+d^2}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -7/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^3 - 4/3*sqrt
(-x^2*e^2 + d^2)*e^3/(d^3*x) - 7/8*sqrt(-x^2*e^2 + d^2)*e^2/(d^2*x^2) - 2/3
*sqrt(-x^2*e^2 + d^2)*e/(d*x^3) - 1/4*sqrt(-x^2*e^2 + d^2)/x^4
```

Fricas [A]
time = 2.06, size = 83, normalized size = 0.59

$$\frac{21\,x^4e^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(32\,x^3e^3+21\,dx^2e^2+16\,d^2xe+6\,d^3)\sqrt{-x^2e^2+d^2}}{24\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*x^4*e^4*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (32*x^3*e^3 + 21*d*x^
2*e^2 + 16*d^2*x*e + 6*d^3)*sqrt(-x^2*e^2 + d^2))/(d^3*x^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.87, size = 449, normalized size = 3.21

$$d^2\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**5/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*s
qrt(d**2/(e**2*x**2) - 1)) + 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) -
 3*e**4*acosh(d/(e*x))/(8*d**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1))
 - 3*I*e**3/(8*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))
/(8*d**5), True)) + 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*
x**2) - 2*e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) >
 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/
(e**2*x**2) + 1)/(3*d**4), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2
) - 1)/(2*d**2*x) - e**2*acosh(d/(e*x))/(2*d**3), Abs(d**2/(e**2*x**2)) > 1
), (I/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*d**2*x*sqrt(-d**2/(e*
*2*x**2) + 1)) + I*e**2*asin(d/(e*x))/(2*d**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(113) = 226.
time = 1.48, size = 299, normalized size = 2.14

$$\frac{x^4\left(\frac{16\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^2}{x} + \frac{144\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-2)}}{x^3} + \frac{48\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}{x^2} + 3e^4\right)e^8}{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3} - \frac{7e^4\log\left(\frac{\left|-2de-2\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2|x|}\right)}{8d^3} - \frac{\frac{144\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^9e^2}{x} + \frac{16\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^9e^{(-2)}}{x^3} + \frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^9e^{(-4)}}{x^4} + \frac{48\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^9}{x^2}}{192d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^2/x + 144*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^3*e^(-2)/x^3 + 48*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2/x^2 + 3*e^4
)*e^8/((d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3) - 7/8*e^4*log(1/2*abs(-2*d*e -
 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/192*(144*(d*e + sqrt(-x^2
*e^2 + d^2)*e)*d^9*e^2/x + 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^9*e^(-2)/x
^3 + 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^9*e^(-4)/x^4 + 48*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^2*d^9/x^2)/d^12
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^2}{x^5\,\sqrt{d^2-e^2\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)),x)
[Out] int((d + e*x)^2/(x^5*(d^2 - e^2*x^2)^(1/2)), x)
```

3.43 $\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=169

$$-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}$$

```
[Out] -3/4*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/5*(-e^2*x^2+d^2)^(1/2)/x^5-1
/2*e*(-e^2*x^2+d^2)^(1/2)/d/x^4-3/5*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-3/4*e^
3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-6/5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A]
time = 0.12, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1821, 849, 821, 272, 65, 214}

$$-\frac{\sqrt{d^2-e^2x^2}}{5x^5}-\frac{e\sqrt{d^2-e^2x^2}}{2dx^4}-\frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^4}-\frac{6e^4\sqrt{d^2-e^2x^2}}{5d^4x}-\frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*Sqrt[d^2 - e^2*x^2]/x^5 - (e*Sqrt[d^2 - e^2*x^2])/(2*d*x^4) - (3*e^2*S
qrt[d^2 - e^2*x^2])/(5*d^2*x^3) - (3*e^3*Sqrt[d^2 - e^2*x^2])/(4*d^3*x^2) -
 (6*e^4*Sqrt[d^2 - e^2*x^2])/(5*d^4*x) - (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(4*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^6\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{\int \frac{-10d^3e-9d^2e^2x}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} + \frac{\int \frac{36d^4e^2+30d^3e^3x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{20d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{\int \frac{-90d^5e^3-72d^4e^4x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{60d^6}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} + \frac{\int \frac{144d^6e^4+90}{x^2\sqrt{d^2-}}}{120d}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e}}{5d^4x}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e}}{5d^4x}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e}}{5d^4x}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{5x^5} - \frac{e\sqrt{d^2-e^2x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2-e^2x^2}}{4d^3x^2} - \frac{6e^4\sqrt{d^2-e}}{5d^4x}$$

Mathematica [A]
time = 0.34, size = 115, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2x^2}\,(-4d^4-10d^3ex-12d^2e^2x^2-15de^3x^3-24e^4x^4)}{20d^4x^5} + \frac{3e^5\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^6*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^4 - 10*d^3*e*x - 12*d^2*e^2*x^2 - 15*d*e^3*x^3 -
 24*e^4*x^4))/(20*d^4*x^5) + (3*e^5*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e
^2*x^2]/d])/(2*d^4)
```

Maple [A]
time = 0.09, size = 240, normalized size = 1.42

method	result

risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(24e^4x^4+15d\,e^3x^3+12d^2x^2e^2+10d^3ex+4d^4)}{20x^5d^4}-\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4d^3\sqrt{d^2}}$
default	$d^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{5d^2x^5}+\frac{4e^2\left(-\frac{\sqrt{-e^2x^2+d^2}}{3d^2x^3}-\frac{2e^2\sqrt{-e^2x^2+d^2}}{3d^4x}\right)}{5d^2}\right)+2de\left(-\frac{\sqrt{-e^2x^2+d^2}}{4d^2x^4}+\frac{3e^2 \cdots}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/5/d^2/x^5*(-e^2*x^2+d^2)^(1/2)+4/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+
d^2)^(1/2)-2/3*e^2/d^4/x*(-e^2*x^2+d^2)^(1/2)))+2*d*e*(-1/4/d^2/x^4*(-e^2*x
^2+d^2)^(1/2)+3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(1/2)-1/2*e^2/d^2/(d
^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))+e^2*(-1/3/d^2/
x^3*(-e^2*x^2+d^2)^(1/2)-2/3*e^2/d^4/x*(-e^2*x^2+d^2)^(1/2))
```

Maxima [A]
time = 0.48, size = 149, normalized size = 0.88

$$-\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{4\,d^4}-\frac{6\sqrt{-x^2e^2+d^2}\,e^4}{5\,d^4x}-\frac{3\sqrt{-x^2e^2+d^2}\,e^3}{4\,d^3x^2}-\frac{3\sqrt{-x^2e^2+d^2}\,e^2}{5\,d^2x^3}-\frac{\sqrt{-x^2e^2+d^2}\,e}{2\,dx^4}-\frac{\sqrt{-x^2e^2+d^2}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^4 - 6/5*sqrt
(-x^2*e^2 + d^2)*e^4/(d^4*x) - 3/4*sqrt(-x^2*e^2 + d^2)*e^3/(d^3*x^2) - 3/5
*sqrt(-x^2*e^2 + d^2)*e^2/(d^2*x^3) - 1/2*sqrt(-x^2*e^2 + d^2)*e/(d*x^4) -
1/5*sqrt(-x^2*e^2 + d^2)/x^5
```

Fricas [A]
time = 1.82, size = 93, normalized size = 0.55

$$\frac{15\,x^5e^5\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(24\,x^4e^4+15\,dx^3e^3+12\,d^2x^2e^2+10\,d^3xe+4\,d^4)\sqrt{-x^2e^2+d^2}}{20\,d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*x^5*e^5*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (24*x^4*e^4 + 15*d*x^
3*e^3 + 12*d^2*x^2*e^2 + 10*d^3*x*e + 4*d^4)*sqrt(-x^2*e^2 + d^2))/(d^4*x^5
)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.44, size = 510, normalized size = 3.02

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{5d^2x^4}-\frac{4e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^4x^2}-\frac{8e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^6} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{5d^2x^4}-\frac{4ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^4x^2}-\frac{8ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^6} & \text{otherwise} \end{cases}\right)+2de\left(\begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases}\right)+e^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2}-\frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2}-\frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**6/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(5*d**2*x**4) - 4*e**3*sqrt(d
**2/(e**2*x**2) - 1)/(15*d**4*x**2) - 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(15
*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(5*d*
*2*x**4) - 4*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(15*d**4*x**2) - 8*I*e**5*s
qrt(-d**2/(e**2*x**2) + 1)/(15*d**6), True)) + 2*d*e*Piecewise((-1/(4*e*x**
5*sqrt(d**2/(e**2*x**2) - 1)) - e/(8*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1))
+ 3*e**3/(8*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) - 3*e**4*acosh(d/(e*x))/(8*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1))
 + I*e/(8*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e**3/(8*d**4*x*sqrt(
-d**2/(e**2*x**2) + 1)) + 3*I*e**4*asin(d/(e*x))/(8*d**5), True)) + e**2*Pi
ecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2*x**2) - 2*e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**4), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*d**2*x**2) - 2*I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**4), T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(136) = 272.
time = 1.57, size = 363, normalized size = 2.15

$$\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^3}{x}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e}{x^2}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-1)}}{x^3}+\frac{110\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-3)}}{x^4}+e^5\right)e^{10}}{160\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^4}-\frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{4\,d^4}-\frac{\frac{110\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{16}e^3}{x}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^{16}e}{x^2}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{16}e^{(-1)}}{x^3}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^{16}e^{(-3)}}{x^4}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^{16}e^{(-5)}}{x^5}}{160\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/160*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^3/x + 15*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*e/x^2 + 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-1)/x^3 + 110*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-3)/x^4 + e^5)*e^10/((d*e + sqrt(-x^2*e^
2 + d^2)*e)^5*d^4) - 3/4*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x))/d^4 - 1/160*(110*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^16*e^3/x
+ 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^16*e/x^2 + 15*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^3*d^16*e^(-1)/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^16*e^(-3
)/x^4 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^16*e^(-5)/x^5)/d^20
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^6\,\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)),x)
[Out] int((d + e*x)^2/(x^6*(d^2 - e^2*x^2)^(1/2)), x)
```

3.44 $\int \frac{x^5(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=143

$$\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}-\frac{2d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] 1/5*d^4*(e*x+d)^2/e^6/(-e^2*x^2+d^2)^(5/2)-22/15*d^3*(e*x+d)/e^6/(-e^2*x^2+
d^2)^(3/2)-2*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+2/15*d*(23*e*x+30*d)/e^
6/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e^6
```

Rubi [A]
time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1649, 1828, 655, 223, 209}

$$-\frac{2d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}+\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^6}+\frac{d^4(d+ex)^2}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{22d^3(d+ex)}{15e^6\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^2)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (22*d^3*(d + e*x))/(15*e^
6*(d^2 - e^2*x^2)^(3/2)) + (2*d*(30*d + 23*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + Sqrt[d^2 - e^2*x^2]/e^6 - (2*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{\frac{16d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{30d^5}{e^5}+\frac{15d^4x}{e^4}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)}{}$$

$$= \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{(2d)}{}$$

$$= \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{2d\,\mathrm{ta}}{}$$

Mathematica [**A**]

time = 0.41, size = 126, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\,(-56d^4+82d^3ex+32d^2e^2x^2-76de^3x^3+15e^4x^4)}{15e^6(-d+ex)^3(d+ex)}+\frac{2d(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-56*d^4 + 82*d^3*e*x + 32*d^2*e^2*x^2 - 76*d*e^3*x^3
+ 15*e^4*x^4))/(15*e^6*(-d + e*x)^3*(d + e*x)) + (2*d*(-e^2)^(3/2)*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^9
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(127) = 254.
time = 0.08, size = 303, normalized size = 2.12

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^6}-\frac{2d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{8e^7\left(x+\frac{d}{e}\right)}-\frac{383d\sqrt{-\left(x-\frac{d}{e}\right)}}{120e}$
default	$e^2\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)}{e^2}\right)+2ed\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-
4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/
2))))+2*e*d*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+
d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+d^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e
^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(121) = 242.
time = 0.50, size = 255, normalized size = 1.78

$$\frac{7\,d^2x^4e^{(-2)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{28\,d^4x^2e^{(-4)}}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{56\,d^6e^{(-6)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{2}{15}\left(\frac{15\,x^4e^{(-2)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{20\,d^2x^2e^{(-4)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{8\,d^4e^{(-6)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}\right)dxe-\frac{2}{3}\left(\frac{3\,x^2e^{(-2)}}{(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{2\,d^2e^{(-4)}}{(-x^2e^2+d^2)^{\frac{3}{2}}}\right)dxe^{(-1)}-\frac{x^6}{(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{8\,d^3xe^{(-5)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}}-2\,d\arcsin\left(\frac{xe}{d}\right)e^{(-6)}-\frac{14\,dxe^{(-5)}}{15\,\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 7*d^2*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 28/3*d^4*x^2*e^(-4)/(-x^2*e^2 + d
^2)^(5/2) + 56/15*d^6*e^(-6)/(-x^2*e^2 + d^2)^(5/2) + 2/15*(15*x^4*e^(-2)/(
-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8*d^4*e^
(-6)/(-x^2*e^2 + d^2)^(5/2))*d*x*e - 2/3*(3*x^2*e^(-2)/(-x^2*e^2 + d^2)^(3/
2) - 2*d^2*e^(-4)/(-x^2*e^2 + d^2)^(3/2))*d*x*e^(-1) - x^6/(-x^2*e^2 + d^2)
^(5/2) + 8/15*d^3*x*e^(-5)/(-x^2*e^2 + d^2)^(3/2) - 2*d*arcsin(x*e/d)*e^(-6
) - 14/15*d*x*e^(-5)/sqrt(-x^2*e^2 + d^2)
```

Fricas [A]
time = 2.05, size = 177, normalized size = 1.24

$$\frac{56\,dx^4e^4 - 112\,d^2x^3e^3 + 112\,d^4xe - 56\,d^5 + 60\,(dx^4e^4 - 2\,d^2x^3e^3 + 2\,d^4xe - d^5)\arctan\left(-\frac{\left(d - \sqrt{-x^2e^2 + d^2}\right)e^{(-1)}}{x}\right) + (15\,x^4e^4 - 76\,dx^3e^3 + 32\,d^2x^2e^2 + 82\,d^3xe - 56\,d^4)\sqrt{-x^2e^2 + d^2}}{15\,(x^4e^{10} - 2\,dx^3e^9 + 2\,d^3xe^7 - d^4e^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(56*d*x^4*e^4 - 112*d^2*x^3*e^3 + 112*d^4*x*e - 56*d^5 + 60*(d*x^4*e^4
 - 2*d^2*x^3*e^3 + 2*d^4*x*e - d^5)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-
1)/x) + (15*x^4*e^4 - 76*d*x^3*e^3 + 32*d^2*x^2*e^2 + 82*d^3*x*e - 56*d^4)*
sqrt(-x^2*e^2 + d^2))/(x^4*e^10 - 2*d*x^3*e^9 + 2*d^3*x*e^7 - d^4*e^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**5*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*x^5/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,(d+e\,x)^2}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^5*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.45 $\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=121

$$\frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}-\frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

[Out] 1/5*d^3*(e*x+d)^2/e^5/(-e^2*x^2+d^2)^(5/2)-17/15*d^2*(e*x+d)/e^5/(-e^2*x^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+2/15*(13*e*x+15*d)/e^5/(-e^2*x^2+d^2)^(1/2)

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1649, 1828, 12, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}}+\frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}}+\frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]

[Out] (d^3*(d + e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (17*d^2*(d + e*x))/(15*e^5*(d^2 - e^2*x^2)^(3/2)) + (2*(15*d + 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^4}{e^4\sqrt{d^2-e^2x^2}}\,dx}{15d^4}$$

$$= \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x,\right.}{e^4}$$

$$= \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Mathematica [A]

time = 0.40, size = 113, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2x^2}\,(-16d^3+17d^2ex+22de^2x^2-26e^3x^3)}{15e^5(-d+ex)^3(d+ex)}+\frac{\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^4\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 + 17*d^2*e*x + 22*d*e^2*x^2 - 26*e^3*x^3))/(1
5*e^5*(-d + e*x)^3*(d + e*x)) + Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]/
(e^4*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(107) = 214$.
time = 0.06, size = 324, normalized size = 2.68

method	result
default	$e^2\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)+2ed\left(\frac{x^4}{e^2(-e^2x^2+d^2)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/
2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2)))))+2*e*d*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/
3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+d^2*(1/2
*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1
/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2
)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(103) = 206$.
time = 0.49, size = 273, normalized size = 2.26

$$\frac{2\,dx^4e^{(-1)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{d^2x^3e^{(-2)}}{2\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{8\,d^3x^2e^{(-3)}}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{3\,d^4xe^{(-4)}}{10\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{16\,d^5e^{(-5)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{1}{15}\left(\frac{15\,x^4e^{(-2)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{20\,d^2x^2e^{(-4)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{8\,d^4e^{(-6)}}{(-x^2e^2+d^2)^{\frac{5}{2}}}\right)xe^2+\frac{11\,d^2xe^{(-4)}}{30\,(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{1}{3}\left(\frac{3\,x^2e^{(-2)}}{(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{2\,d^2e^{(-4)}}{(-x^2e^2+d^2)^{\frac{3}{2}}}\right)x-\arcsin\left(\frac{xe}{d}\right)e^{(-5)}-\frac{4\,xe^{(-4)}}{15\sqrt{-x^2e^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2*d*x^4*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1/2*d^2*x^3*e^(-2)/(-x^2*e^2 + d^2)
^(5/2) - 8/3*d^3*x^2*e^(-3)/(-x^2*e^2 + d^2)^(5/2) - 3/10*d^4*x*e^(-4)/(-x^
2*e^2 + d^2)^(5/2) + 16/15*d^5*e^(-5)/(-x^2*e^2 + d^2)^(5/2) + 1/15*(15*x^4
*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) +
 8*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2))*x*e^2 + 11/30*d^2*x*e^(-4)/(-x^2*e^2
+ d^2)^(3/2) - 1/3*(3*x^2*e^(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d^2*e^(-4)/(-x^
2*e^2 + d^2)^(3/2))*x - arcsin(x*e/d)*e^(-5) - 4/15*x*e^(-4)/sqrt(-x^2*e^2
+ d^2)
```

Fricas [A]
time = 2.11, size = 162, normalized size = 1.34

$$\frac{16\,x^4e^4 - 32\,dx^3e^3 + 32\,d^3xe - 16\,d^4 + 30\,(x^4e^4 - 2\,dx^3e^3 + 2\,d^3xe - d^4)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right) - (26\,x^3e^3 - 22\,dx^2e^2 - 17\,d^2xe + 16\,d^3)\sqrt{-x^2e^2+d^2}}{15\,(x^4e^9 - 2\,dx^3e^8 + 2\,d^3xe^6 - d^4e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(16*x^4*e^4 - 32*d*x^3*e^3 + 32*d^3*x*e - 16*d^4 + 30*(x^4*e^4 - 2*d*x
^3*e^3 + 2*d^3*x*e - d^4)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (2
6*x^3*e^3 - 22*d*x^2*e^2 - 17*d^2*x*e + 16*d^3)*sqrt(-x^2*e^2 + d^2))/(x^4*
e^9 - 2*d*x^3*e^8 + 2*d^3*x*e^6 - d^4*e^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*x^4/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int((x^4*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.46 $\int \frac{x^3(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d^2*(e*x+d)^2/e^4/(-e^2*x^2+d^2)^(5/2)-4/5*d*(e*x+d)/e^4/(-e^2*x^2+d^2)
^(3/2)+1/5*(2*e*x+5*d)/d/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1649, 651}

$$\frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d + e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d + 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)\left(\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{(d^2-e^2x^2)^{3/2}}dx}{15d^2}$$

$$= \frac{d^2(d+ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d+ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.32, size = 70, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\,(2d^3-4d^2ex+de^2x^2+2e^3x^3)}{5de^4(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 2*e^3*x^3))/(5*d*e^4*
(d - e*x)^3*(d + e*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(85) = 170$.
time = 0.07, size = 261, normalized size = 2.69

method	result
gosper	$\frac{(-ex+d)(ex+d)^3\left(2e^3x^3+d\,e^2x^2-4d^2ex+2d^3\right)}{5e^4d\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
trager	$\frac{\left(2e^3x^3+d\,e^2x^2-4d^2ex+2d^3\right)\sqrt{-e^2x^2+d^2}}{5e^4d(-ex+d)^3(ex+d)}$

default	$e^{2}\left(\frac{x^{4}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{4d^{2}\left(\frac{x^{2}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2d^{2}}{15e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)}{e^{2}}\right)+2ed\left(\frac{x^{3}}{2e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{3d^{2}\left(\frac{x}{4e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/
2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+2*e*d*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5
/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^
2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^
2+d^2)^(1/2)))))+d^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x
^2+d^2)^(5/2))
```

Maxima [A]
time = 0.27, size = 142, normalized size = 1.46

$$\frac{dx^{3}e^{(-1)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{d^{2}x^{2}e^{(-2)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{3\,d^{3}xe^{(-3)}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{2\,d^{4}e^{(-4)}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{x^{4}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{dxe^{(-3)}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{2\,xe^{(-3)}}{5\sqrt{-x^{2}e^{2}+d^{2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] d*x^3*e^(-1)/(-x^2*e^2 + d^2)^(5/2) - d^2*x^2*e^(-2)/(-x^2*e^2 + d^2)^(5/2)
 - 3/5*d^3*x*e^(-3)/(-x^2*e^2 + d^2)^(5/2) + 2/5*d^4*e^(-4)/(-x^2*e^2 + d^2
)^(5/2) + x^4/(-x^2*e^2 + d^2)^(5/2) + 1/5*d*x*e^(-3)/(-x^2*e^2 + d^2)^(3/2
) + 2/5*x*e^(-3)/(sqrt(-x^2*e^2 + d^2)*d)
```

Fricas [A]
time = 2.06, size = 109, normalized size = 1.12

$$\frac{2\,x^{4}e^{4}-4\,dx^{3}e^{3}+4\,d^{3}xe-2\,d^{4}-\left(2\,x^{3}e^{3}+dx^{2}e^{2}-4\,d^{2}xe+2\,d^{3}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{5\left(dx^{4}e^{8}-2\,d^{2}x^{3}e^{7}+2\,d^{4}xe^{5}-d^{5}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

[Out] 1/5*(2*x^4*e^4 - 4*d*x^3*e^3 + 4*d^3*x*e - 2*d^4 - (2*x^3*e^3 + d*x^2*e^2 - 4*d^2*x*e + 2*d^3)*sqrt(-x^2*e^2 + d^2))/(d*x^4*e^8 - 2*d^2*x^3*e^7 + 2*d^4*x*e^5 - d^5*e^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)

[Out] Integral(x**3*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")

[Out] integrate((x*e + d)^2*x^3/(-x^2*e^2 + d^2)^(7/2), x)

Mupad [B]
time = 2.89, size = 66, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3-4\,d^2\,e\,x+d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d\,e^4\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)

[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 + d*e^2*x^2 - 4*d^2*e*x))/(5*d*e^4*(d + e*x)*(d - e*x)^3)

3.47 $\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=87

$$\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d*(e*x+d)^2/e^3/(-e^2*x^2+d^2)^(5/2)-7/15*(e*x+d)/e^3/(-e^2*x^2+d^2)^(3
/2)+1/15*x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1649, 792, 197}

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}+\frac{d(d+ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7(d+ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (7*(d + e*x))/(15*e^3*(d^2
- e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d(d+ex)^2}{5e^3\,(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{\left(\frac{2d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$
$$= \frac{d(d+ex)^2}{5e^3\,(d^2-e^2x^2)^{5/2}} - \frac{7(d+ex)}{15e^3\,(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15e^2}$$
$$= \frac{d(d+ex)^2}{5e^3\,(d^2-e^2x^2)^{5/2}} - \frac{7(d+ex)}{15e^3\,(d^2-e^2x^2)^{3/2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Mathematica [**A**]
time = 0.34, size = 70, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\,(-4d^3+8d^2ex-2de^2x^2+e^3x^3)}{15d^2e^3(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*
e^3*(d - e*x)^3*(d + e*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(75) = 150$.
time = 0.06, size = 282, normalized size = 3.24

method	result
gosper	$-\frac{(-ex+d)(ex+d)^3(-e^3x^3+2d\,e^2x^2-8d^2ex+4d^3)}{15d^2e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(-e^3x^3+2d\,e^2x^2-8d^2ex+4d^3)\sqrt{-e^2x^2+d^2}}{15e^3d^2(-ex+d)^3(ex+d)}$

default	$e^2\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)}{2e^2}\right)+$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)
^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2
*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+2*e*d*(1/3*x^2/e^2/(-e^2
*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+d^2*(1/4*x/e^2/(-e^2*x^2
+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/
(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))
```

Maxima [A]
time = 0.30, size = 120, normalized size = 1.38

$$\frac{2\,dx^2e^{(-1)}}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{d^2xe^{(-2)}}{10\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{4\,d^3e^{(-3)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{x^3}{2\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{xe^{(-2)}}{30\,(-x^2e^2+d^2)^{\frac{3}{2}}}+\frac{xe^{(-2)}}{15\,\sqrt{-x^2e^2+d^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*d*x^2*e^(-1)/(-x^2*e^2 + d^2)^(5/2) - 1/10*d^2*x*e^(-2)/(-x^2*e^2 + d^2
)^(5/2) - 4/15*d^3*e^(-3)/(-x^2*e^2 + d^2)^(5/2) + 1/2*x^3/(-x^2*e^2 + d^2)
^(5/2) + 1/30*x*e^(-2)/(-x^2*e^2 + d^2)^(3/2) + 1/15*x*e^(-2)/(sqrt(-x^2*e^
2 + d^2)*d^2)
```

Fricas [A]
time = 1.69, size = 110, normalized size = 1.26

$$-\frac{4\,x^4e^4-8\,dx^3e^3+8\,d^3xe-4\,d^4+(x^3e^3-2\,dx^2e^2+8\,d^2xe-4\,d^3)\sqrt{-x^2e^2+d^2}}{15\,(d^2x^4e^7-2\,d^3x^3e^6+2\,d^5xe^4-d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(4*x^4*e^4 - 8*d*x^3*e^3 + 8*d^3*x*e - 4*d^4 + (x^3*e^3 - 2*d*x^2*e^2
 + 8*d^2*x*e - 4*d^3)*sqrt(-x^2*e^2 + d^2))/(d^2*x^4*e^7 - 2*d^3*x^3*e^6 +
2*d^5*x*e^4 - d^6*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*x^2/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [B]
time = 2.87, size = 67, normalized size = 0.77

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3-8\,d^2\,e\,x+2\,d\,e^2\,x^2-e^3\,x^3\right)}{15\,d^2\,e^3\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(4*d^3 - e^3*x^3 + 2*d*e^2*x^2 - 8*d^2*e*x))/(15*d^
2*e^3*(d + e*x)*(d - e*x)^3)
```

3.48 $\int \frac{x(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=89

$$\frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*(e*x+d)/d/e^2/(-e^2*x^2+d^2)^(3
/2)-4/15*x/d^3/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {803, 653, 197}

$$\frac{(d+ex)^2}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(15*d*e^2*(d^2 -
e^2*x^2)^(3/2)) - (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}\,dx}{5e}$$
$$= \frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{4\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{15de}$$
$$= \frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.30, size = 69, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}\,(d^3-2d^2ex+8de^2x^2-4e^3x^3)}{15d^3e^2(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 - 2*d^2*e*x + 8*d*e^2*x^2 - 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)^3*(d + e*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(77) = 154$.
time = 0.06, size = 173, normalized size = 1.94

method	result
gosper	$\frac{(-ex+d)(ex+d)^3(-4e^3x^3+8d\,e^2x^2-2d^2ex+d^3)}{15d^3e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(-4e^3x^3+8d\,e^2x^2-2d^2ex+d^3)\sqrt{-e^2x^2+d^2}}{15d^3(-ex+d)^3e^2(ex+d)}$
default	$e^2\left(\frac{x^2}{3e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{2d^2}{15e^4(-e^2x^2+d^2)^{\frac{5}{2}}}\right)+2ed\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{}{15d}}{d^2}\right.}{4e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+2*
e*d*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(
```

```
5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)
)))+1/5*d^2/e^2/(-e^2*x^2+d^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 100, normalized size = 1.12

$$\frac{2\,dxe^{(-1)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{d^2e^{(-2)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{x^2}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{2\,xe^{(-1)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d}-\frac{4\,xe^{(-1)}}{15\,\sqrt{-x^2e^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*d*x*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1/15*d^2*e^(-2)/(-x^2*e^2 + d^2)^(5
/2) + 1/3*x^2/(-x^2*e^2 + d^2)^(5/2) - 2/15*x*e^(-1)/((-x^2*e^2 + d^2)^(3/2
)*d) - 4/15*x*e^(-1)/(sqrt(-x^2*e^2 + d^2)*d^3)
```

Fricas [A]
time = 2.35, size = 110, normalized size = 1.24

$$\frac{x^4e^4-2\,dx^3e^3+2\,d^3xe-d^4+(4\,x^3e^3-8\,dx^2e^2+2\,d^2xe-d^3)\sqrt{-x^2e^2+d^2}}{15\,(d^3x^4e^6-2\,d^4x^3e^5+2\,d^6xe^3-d^7e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(x^4*e^4 - 2*d*x^3*e^3 + 2*d^3*x*e - d^4 + (4*x^3*e^3 - 8*d*x^2*e^2 +
2*d^2*x*e - d^3)*sqrt(-x^2*e^2 + d^2))/(d^3*x^4*e^6 - 2*d^4*x^3*e^5 + 2*d^6
*x*e^3 - d^7*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(d+ex)^2}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((x*e + d)^2*x/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [B]
time = 2.86, size = 65, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^3-2\,d^2\,e\,x+8\,d\,e^2\,x^2-4\,e^3\,x^3\right)}{15\,d^3\,e^2\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^3 - 4*e^3*x^3 + 8*d*e^2*x^2 - 2*d^2*e*x))/(15*d^3
*e^2*(d + e*x)*(d - e*x)^3)
```

3.49 $\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/5*(e*x+d)/e/(-e^2*x^2+d^2)^(5/2)+1/5*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/5*x/d^4
/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {667, 198, 197}

$$\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + x/(5*d^2*(d^2 - e^2*x^2)^(3/2))
 + (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}} + \frac{3}{5}\int \frac{1}{(d^2-e^2x^2)^{5/2}}\,dx$$

$$= \frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}} + \frac{x}{5d^2\,(d^2-e^2x^2)^{3/2}} + \frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{5d^2}$$

$$= \frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}} + \frac{x}{5d^2\,(d^2-e^2x^2)^{3/2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}\,(2d^3+d^2ex-4de^2x^2+2e^3x^3)}{5d^4e(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e*
(d - e*x)^3*(d + e*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(65) = 130.
time = 0.06, size = 193, normalized size = 2.51

method	result
gosper	$\frac{(-ex+d)(ex+d)^3(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)}{5d^4e(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)\sqrt{-e^2x^2+d^2}}{5d^4(-ex+d)^3e(ex+d)}$
default	$e^2\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)+\frac{2d}{5e(-e^2x^2+d^2)^{\frac{5}{2}}}+d^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(
5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)
```

```
)))+2/5/e*d/(-e^2*x^2+d^2)^(5/2)+d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^
2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.95

$$\frac{2\,de^{(-1)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{2\,x}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{x}{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}+\frac{2\,x}{5\,\sqrt{-x^2e^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*d*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 2/5*x/(-x^2*e^2 + d^2)^(5/2) + 1/5*x/
((-x^2*e^2 + d^2)^(3/2)*d^2) + 2/5*x/(sqrt(-x^2*e^2 + d^2)*d^4)
```

Fricas [A]
time = 1.66, size = 111, normalized size = 1.44

$$\frac{2\,x^4e^4-4\,dx^3e^3+4\,d^3xe-2\,d^4-\left(2\,x^3e^3-4\,dx^2e^2+d^2xe+2\,d^3\right)\sqrt{-x^2e^2+d^2}}{5\,(d^4x^4e^5-2\,d^5x^3e^4+2\,d^7xe^2-d^8e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*x^4*e^4 - 4*d*x^3*e^3 + 4*d^3*x*e - 2*d^4 - (2*x^3*e^3 - 4*d*x^2*e^2
 + d^2*x*e + 2*d^3)*sqrt(-x^2*e^2 + d^2))/(d^4*x^4*e^5 - 2*d^5*x^3*e^4 + 2*
d^7*x*e^2 - d^8*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [B]
time = 2.81, size = 66, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3+d^2\,e\,x-4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,\left(d+e\,x\right)\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 - 4*d*e^2*x^2 + d^2*e*x))/(5*d^4*
e*(d + e*x)*(d - e*x)^3)
```

3.50 $\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] 2/5*(e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/15*(8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(3/2
)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*(16*e*x+15*d)/d^5/(-e^2*x^2+d^2)
^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1819, 837, 12, 272, 65, 214}

$$\frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d + 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d + 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-8dex}{x(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^4e^2-16d^3e^3x}{x(d^2-e^2x^2)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^6e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\right.}{2d^4}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x\right.}{d^4e^2}$$

$$= \frac{2(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right.}{d^5}$$

Mathematica [A]
time = 0.47, size = 107, normalized size = 0.91

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(26d^3-22d^2ex-17de^2x^2+16e^3x^3)}{(d-ex)^3(d+ex)} + 30\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(26*d^3 - 22*d^2*e*x - 17*d*e^2*x^2 + 16*e^3*x^3))/((
d - e*x)^3*(d + e*x)) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])
/(15*d^5)
```

Maple [A]
time = 0.07, size = 202, normalized size = 1.73

method	result

default	$\frac{1}{5(-e^2x^2+d^2)^{\frac{5}{2}}}+2ed\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+d^2\left(\frac{1}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(-e^2*x^2+d^2)^(5/2)+2*e*d*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3
*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+d^2*(1/5/d^2/(
-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*
x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/
2))/x))))
```

Maxima [A]

time = 0.27, size = 150, normalized size = 1.28

$$\frac{2\,xe}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d}+\frac{2}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{8\,xe}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}+\frac{1}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}+\frac{16\,xe}{15\,\sqrt{-x^2e^2+d^2}\,d^5}-\frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d^5}+\frac{1}{\sqrt{-x^2e^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*x*e/((-x^2*e^2 + d^2)^(5/2)*d) + 2/5/(-x^2*e^2 + d^2)^(5/2) + 8/15*x*e/
((-x^2*e^2 + d^2)^(3/2)*d^3) + 1/3/((-x^2*e^2 + d^2)^(3/2)*d^2) + 16/15*x*e
/(sqrt(-x^2*e^2 + d^2)*d^5) - log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/a
bs(x))/d^5 + 1/(sqrt(-x^2*e^2 + d^2)*d^4)
```

Fricas [A]

time = 2.33, size = 163, normalized size = 1.39

$$\frac{26\,x^4e^4-52\,dx^3e^3+52\,d^3xe-26\,d^4+15\,(x^4e^4-2\,dx^3e^3+2\,d^3xe-d^4)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(16\,x^3e^3-17\,dx^2e^2-22\,d^2xe+26\,d^3)\sqrt{-x^2e^2+d^2}}{15\,(d^5x^4e^4-2\,d^6x^3e^3+2\,d^8xe-d^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*x^4*e^4 - 52*d*x^3*e^3 + 52*d^3*x*e - 26*d^4 + 15*(x^4*e^4 - 2*d*x
^3*e^3 + 2*d^3*x*e - d^4)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (16*x^3*e^3
- 17*d*x^2*e^2 - 22*d^2*x*e + 26*d^3)*sqrt(-x^2*e^2 + d^2))/(d^5*x^4*e^4 -
2*d^6*x^3*e^3 + 2*d^8*x*e - d^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((d + e*x)**2/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((x*e + d)^2/((-x^2*e^2 + d^2)^(7/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^2/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

3.51 $\int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] 2/5*e*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e*(13*e*x+10*d)/d^4/(-e^2*x^2+d
^2)^(3/2)-2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e*(41*e*x+30*d)/d^6/
(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A]
time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1819, 821, 272, 65, 214}

$$\frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e*(10*d + 13*e*x))/(15*d^4
*(d^2 - e^2*x^2)^(3/2)) + (e*(30*d + 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
- Sqrt[d^2 - e^2*x^2]/(d^6*x) - (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2-10dex-8e^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2+30dex+26e^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{(2e\ldots}{\ldots}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{e\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2\mathrm{S}\ldots}{\ldots}$$

$$= \frac{2e(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(10d+13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} - \frac{2e\ldots}{\ldots}$$

Mathematica [A]
time = 0.46, size = 123, normalized size = 0.85

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(15d^4-76d^3ex+32d^2e^2x^2+82de^3x^3-56e^4x^4)}{x(-d+ex)^3(d+ex)}+60e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^4 - 76*d^3*e*x + 32*d^2*e^2*x^2 + 82*d*e^3*x^3
- 56*e^4*x^4))/(x*(-d + e*x)^3*(d + e*x)) + 60*e*ArcTanh[(Sqrt[-e^2]*x - Sq
rt[d^2 - e^2*x^2])/d])/(15*d^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(129) = 258$.
time = 0.09, size = 288, normalized size = 1.99

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^6x}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{8d^6\left(x+\frac{d}{e}\right)}-\frac{313\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)e}}{120d^6\left(x-\frac{d}{e}\right)}-\frac{2e\ln}{}$
default	$e^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+d^2\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6e^2\left(\frac{x}{5d^2(-e^2x^2+d^2}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)
+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+d^2*(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/
d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)
+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+2*e*d*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^
2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^
2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.27, size = 180, normalized size = 1.24

$$\frac{7\,xe^2}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2}+\frac{2\,e}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d}-\frac{1}{(-x^2e^2+d^2)^{\frac{5}{2}}x}+\frac{28\,xe^2}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^4}+\frac{2\,e}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}-\frac{2\,e\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d^6}+\frac{56\,xe^2}{15\,\sqrt{-x^2e^2+d^2}\,d^6}+\frac{2\,e}{\sqrt{-x^2e^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 7/5*x*e^2/((-x^2*e^2 + d^2)^(5/2)*d^2) + 2/5*e/((-x^2*e^2 + d^2)^(5/2)*d) -
 1/((-x^2*e^2 + d^2)^(5/2)*x) + 28/15*x*e^2/((-x^2*e^2 + d^2)^(3/2)*d^4) +
2/3*e/((-x^2*e^2 + d^2)^(3/2)*d^3) - 2*e*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2
 + d^2)*d/abs(x))/d^6 + 56/15*x*e^2/(sqrt(-x^2*e^2 + d^2)*d^6) + 2*e/(sqrt(
-x^2*e^2 + d^2)*d^5)
```

Fricas [**A**]
time = 2.08, size = 186, normalized size = 1.28

$$\frac{46\,x^5e^5-92\,dx^4e^4+92\,d^3x^2e^2-46\,d^4xe+30\,(x^5e^5-2\,dx^4e^4+2\,d^3x^2e^2-d^4xe)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(56\,x^4e^4-82\,dx^3e^3-32\,d^2x^2e^2+76\,d^3xe-15\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(d^6x^5e^4-2\,d^7x^4e^3+2\,d^9x^2e-d^{10}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(46*x^5*e^5 - 92*d*x^4*e^4 + 92*d^3*x^2*e^2 - 46*d^4*x*e + 30*(x^5*e^5
 - 2*d*x^4*e^4 + 2*d^3*x^2*e^2 - d^4*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x
) - (56*x^4*e^4 - 82*d*x^3*e^3 - 32*d^2*x^2*e^2 + 76*d^3*x*e - 15*d^4)*sqrt
(-x^2*e^2 + d^2))/(d^6*x^5*e^4 - 2*d^7*x^4*e^3 + 2*d^9*x^2*e - d^10*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x^2\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2/((-x^2*e^2 + d^2)^(7/2)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^2\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^2/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.52 $\int \frac{(d+ex)^2}{x^3(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{2e^2(d+ex)}{5d^3(d^2-e^2x^2)^{5/2}}+\frac{e^2(5d+6ex)}{5d^5(d^2-e^2x^2)^{3/2}}+\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}-\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

```
[Out] 2/5*e^2*(e*x+d)/d^3/(-e^2*x^2+d^2)^(5/2)+1/5*e^2*(6*e*x+5*d)/d^5/(-e^2*x^2+
d^2)^(3/2)-9/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+2/5*e^2*(11*e*x+10*d
)/d^7/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^6/x^2-2*e*(-e^2*x^2+d
^2)^(1/2)/d^7/x
```

Rubi [A]
time = 0.23, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1819, 1821, 821, 272, 65, 214}

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}}-\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{e^2(5d+6ex)}{5d^5(d^2-e^2x^2)^{3/2}}+\frac{2e^2(d+ex)}{5d^3(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^2*(d + e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d + 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d + 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) - (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^3 (d^2-e^2x^2)^{7/2}}\,dx = \frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{8e^3x^3}{d}}{x^3(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5 (d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{36e^3x^3}{d}}{x^3(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5 (d^2-e^2x^2)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,d}{15d^6}$$

$$= \frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5 (d^2-e^2x^2)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{\int \frac{60}{x^2}}{}$$

$$= \frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5 (d^2-e^2x^2)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{}}{}$$

$$= \frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5 (d^2-e^2x^2)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{}}{}$$

$$= \frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5 (d^2-e^2x^2)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{}}{}$$

$$= \frac{2e^2(d+ex)}{5d^3 (d^2-e^2x^2)^{5/2}} + \frac{e^2(5d+6ex)}{5d^5 (d^2-e^2x^2)^{3/2}} + \frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{2e\sqrt{}}{}$$

Mathematica [A]
time = 0.48, size = 136, normalized size = 0.75

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(5d^5+10d^4ex-94d^3e^2x^2+58d^2e^3x^3+83de^4x^4-64e^5x^5)}{x^2(-d+ex)^3(d+ex)} + 90e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{10d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^5 + 10*d^4*e*x - 94*d^3*e^2*x^2 + 58*d^2*e^3*x^3
 + 83*d*e^4*x^4 - 64*e^5*x^5))/(x^2*(-d + e*x)^3*(d + e*x)) + 90*e^2*ArcTan
h[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(10*d^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(160) = 320$.
time = 0.08, size = 361, normalized size = 1.98

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(4ex+d)}{2d^7x^2}+\frac{e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{8d^7\left(x+\frac{d}{e}\right)}-\frac{181e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)e}}{40d^7\left(x-\frac{d}{e}\right)}$
default	$d^2\left(-\frac{1}{2d^2x^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7e^2\left(\frac{1}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}}-\frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2}\right)}{2d^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/2/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+7/2*e^2/d^2*(1/5/d^2/(-e^2*x^2+d^2)^
(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)
-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+2*d
*e*(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)
+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+
e^2*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2
*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^
2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]

time = 0.27, size = 203, normalized size = 1.12

$$\frac{12\,xe^3}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d^3}+\frac{9\,e^2}{10\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2}-\frac{2\,e}{(-x^2e^2+d^2)^{\frac{5}{2}}dx}-\frac{1}{2\,(-x^2e^2+d^2)^{\frac{5}{2}}x^2}+\frac{16\,xe^3}{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^5}+\frac{3\,e^2}{2\,(-x^2e^2+d^2)^{\frac{3}{2}}d^4}-\frac{9\,e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{2\,d^7}+\frac{32\,xe^3}{5\,\sqrt{-x^2e^2+d^2}\,d^7}+\frac{9\,e^2}{2\,\sqrt{-x^2e^2+d^2}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 12/5*x*e^3/((-x^2*e^2 + d^2)^(5/2)*d^3) + 9/10*e^2/((-x^2*e^2 + d^2)^(5/2)*
d^2) - 2*e/((-x^2*e^2 + d^2)^(5/2)*d*x) - 1/2/((-x^2*e^2 + d^2)^(5/2)*x^2)
```

```
+ 16/5*x*e^3/((-x^2*e^2 + d^2)^(3/2)*d^5) + 3/2*e^2/((-x^2*e^2 + d^2)^(3/2)
*d^4) - 9/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^7 + 3
2/5*x*e^3/(sqrt(-x^2*e^2 + d^2)*d^7) + 9/2*e^2/(sqrt(-x^2*e^2 + d^2)*d^6)
```

Fricas [A]
time = 1.67, size = 202, normalized size = 1.11

$$\frac{54\,x^6e^6-108\,dx^5e^5+108\,d^3x^3e^3-54\,d^4x^2e^2+45\,(x^6e^6-2\,dx^5e^5+2\,d^3x^3e^3-d^4x^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(64\,x^5e^5-83\,dx^4e^4-58\,d^2x^3e^3+94\,d^3x^2e^2-10\,d^4xe-5\,d^5)\sqrt{-x^2e^2+d^2}}{10\,(d^7x^6e^4-2\,d^8x^5e^3+2\,d^{10}x^3e-d^{11}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*x^6*e^6 - 108*d*x^5*e^5 + 108*d^3*x^3*e^3 - 54*d^4*x^2*e^2 + 45*(x
^6*e^6 - 2*d*x^5*e^5 + 2*d^3*x^3*e^3 - d^4*x^2*e^2)*log(-(d - sqrt(-x^2*e^2
 + d^2))/x) - (64*x^5*e^5 - 83*d*x^4*e^4 - 58*d^2*x^3*e^3 + 94*d^3*x^2*e^2
- 10*d^4*x*e - 5*d^5)*sqrt(-x^2*e^2 + d^2))/(d^7*x^6*e^4 - 2*d^8*x^5*e^3 +
2*d^10*x^3*e - d^11*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{x^3\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2/((-x^2*e^2 + d^2)^(7/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2}{x^3\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^2/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.53 $\int \frac{(d+ex)^2}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}$$

```
[Out] 2/5*e^3*(e*x+d)/d^4/(-e^2*x^2+d^2)^(5/2)+1/15*e^3*(23*e*x+20*d)/d^6/(-e^2*x
^2+d^2)^(3/2)-7*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+2/15*e^3*(53*e*x+45
*d)/d^8/(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/d^6/x^3-e*(-e^2*x^2+d
^2)^(1/2)/d^7/x^2-14/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^8/x
```

Rubi [A]
time = 0.30, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1819, 1821, 821, 272, 65, 214}

$$-\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x}+\frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}}-\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8}-\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^6x^3}+\frac{e^3(20d+23ex)}{15d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{2e^3(d+ex)}{5d^4\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*e^3*(d + e*x))/(5*d^4*(d^2 - e^2*x^2)^(5/2)) + (e^3*(20*d + 23*e*x))/(15
*d^6*(d^2 - e^2*x^2)^(3/2)) + (2*e^3*(45*d + 53*e*x))/(15*d^8*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^6*x^3) - (e*Sqrt[d^2 - e^2*x^2])/(d^7*x^
2) - (14*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^8*x) - (7*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2}{x^4 (d^2-e^2x^2)^{7/2}}\,dx = \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^2-10dex-10e^2x^2-\frac{10e^3x^3}{d}-\frac{8e^4x^4}{d^2}}{x^4(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6 (d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^2+30dex+45e^2x^2+\frac{60e^3x^3}{d}+\frac{46e^4x^4}{d^2}}{x^4(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6 (d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2-30dex-60e^2}{x^4\sqrt{d^2-e}}}{15d^6}$$

$$= \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6 (d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} + \frac{\int}{}$$

$$= \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6 (d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e}{}$$

$$= \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6 (d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e}{}$$

$$= \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6 (d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e}{}$$

$$= \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6 (d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e}{}$$

$$= \frac{2e^3(d+ex)}{5d^4 (d^2-e^2x^2)^{5/2}} + \frac{e^3(20d+23ex)}{15d^6 (d^2-e^2x^2)^{3/2}} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} - \frac{e}{}$$

<u>Mathematica</u> [A]
time = 0.54, size = 147, normalized size = 0.70

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(5d^6+5d^5ex+40d^4e^2x^2-246d^3e^3x^3+122d^2e^4x^4+247de^5x^5-176e^6x^6)}{x^3(-d+ex)^3(d+ex)} + 210e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^6 + 5*d^5*e*x + 40*d^4*e^2*x^2 - 246*d^3*e^3*x^3
 + 122*d^2*e^4*x^4 + 247*d*e^5*x^5 - 176*e^6*x^6))/(x^3*(-d + e*x)^3*(d + e
*x)) + 210*e^3*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(15*d^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(185) = 370$.

time = 0.09, size = 381, normalized size = 1.82

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(14e^2x^2+3dex+d^2)}{3d^8x^3} - \frac{e^2\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}}{8d^8\left(x+\frac{d}{e}\right)} - \frac{833e^2\sqrt{-\left(x-\frac{d}{e}\right)^2 e^2-2d \cdots}}{120d^8\left(x-\frac{d}{e}\right)} \cdots$
default	$2de\left(-\frac{1}{2d^2x^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{7e^2\left(\frac{1}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{\frac{1}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{\frac{1}{d^2\sqrt{-e^2x^2+d^2}} - \frac{\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2 \cdots}}{x}\right)}{d^2\sqrt{d^2}}}{d^2}}{d^2} \cdots\right)}{2d^2} \cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*e*(-1/2/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+7/2*e^2/d^2*(1/5/d^2/(-e^2*x^2+d^2
)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/
2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+e
^2*(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)
+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+
d^2*(-1/3/d^2/x^3/(-e^2*x^2+d^2)^(5/2)+8/3*e^2/d^2*(-1/d^2/x/(-e^2*x^2+d^2)
^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x
^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))
```

Maxima [A]

time = 0.27, size = 226, normalized size = 1.08

$$\frac{22\,xe^4}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d^4} + \frac{7\,e^3}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d^3} - \frac{11\,e^2}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2x} - \frac{e}{(-x^2e^2+d^2)^{\frac{5}{2}}dx^2} - \frac{1}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}x^3} + \frac{88\,xe^4}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^6} + \frac{7\,e^3}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}d^5} - \frac{7\,e^3\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d^8} + \frac{176\,xe^4}{15\,\sqrt{-x^2e^2+d^2}\,d^8} + \frac{7\,e^3}{\sqrt{-x^2e^2+d^2}\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 22/5*x*e^4/((-x^2*e^2 + d^2)^(5/2)*d^4) + 7/5*e^3/((-x^2*e^2 + d^2)^(5/2)*d
^3) - 11/3*e^2/((-x^2*e^2 + d^2)^(5/2)*d^2*x) - e/((-x^2*e^2 + d^2)^(5/2)*d
*x^2) - 1/3/((-x^2*e^2 + d^2)^(5/2)*x^3) + 88/15*x*e^4/((-x^2*e^2 + d^2)^(3
/2)*d^6) + 7/3*e^3/((-x^2*e^2 + d^2)^(3/2)*d^5) - 7*e^3*log(2*d^2/abs(x) +
2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^8 + 176/15*x*e^4/(sqrt(-x^2*e^2 + d^2)*d
^8) + 7*e^3/(sqrt(-x^2*e^2 + d^2)*d^7)
```

Fricas [A]
time = 1.86, size = 212, normalized size = 1.01

$$\frac{116\,x^{7}e^{7}-232\,dx^{6}e^{6}+232\,d^{3}x^{4}e^{4}-116\,d^{4}x^{3}e^{3}+105\,(x^{7}e^{7}-2\,dx^{6}e^{6}+2\,d^{3}x^{4}e^{4}-d^{4}x^{3}e^{3})\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)-(176\,x^{6}e^{6}-247\,dx^{5}e^{5}-122\,d^{2}x^{4}e^{4}+246\,d^{3}x^{3}e^{3}-40\,d^{4}x^{2}e^{2}-5\,d^{5}xe-5\,d^{6})\sqrt{-x^{2}e^{2}+d^{2}}}{15\,(d^{8}x^{7}e^{4}-2\,d^{9}x^{6}e^{3}+2\,d^{11}x^{4}e-d^{12}x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(116*x^7*e^7 - 232*d*x^6*e^6 + 232*d^3*x^4*e^4 - 116*d^4*x^3*e^3 + 105
*(x^7*e^7 - 2*d*x^6*e^6 + 2*d^3*x^4*e^4 - d^4*x^3*e^3)*log(-(d - sqrt(-x^2*
e^2 + d^2))/x) - (176*x^6*e^6 - 247*d*x^5*e^5 - 122*d^2*x^4*e^4 + 246*d^3*x
^3*e^3 - 40*d^4*x^2*e^2 - 5*d^5*x*e - 5*d^6)*sqrt(-x^2*e^2 + d^2))/(d^8*x^7
*e^4 - 2*d^9*x^6*e^3 + 2*d^11*x^4*e - d^12*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{2}}{x^{4}\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**4*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2/((-x^2*e^2 + d^2)^(7/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^2}{x^4\,{\left(d^2-e^2\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^2/(x^4*(d^2 - e^2*x^2)^(7/2)), x)
```

3.54 $\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=81

$$-\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3}{4}\sin^{-1}(x)$$

```
[Out] 3/4*arcsin(x)-3/5*x^2*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)-1/5*x^4*(-x^2+1
)^(1/2)-3/20*(8+5*x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1823, 847, 794, 222}

$$\frac{3\text{ArcSin}(x)}{4}-\frac{3}{5}\sqrt{1-x^2}\,x^2-\frac{3}{20}(5x+8)\sqrt{1-x^2}-\frac{1}{5}\sqrt{1-x^2}\,x^4-\frac{1}{2}\sqrt{1-x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2*Sqrt[1 - x^2])/5 - (x^3*Sqrt[1 - x^2])/2 - (x^4*Sqrt[1 - x^2])/5 -
(3*(8 + 5*x)*Sqrt[1 - x^2])/20 + (3*ArcSin[x])/4
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{5}x^4\sqrt{1-x^2}-\frac{1}{5}\int \frac{(-9-10x)x^3}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}+\frac{1}{20}\int \frac{x^2(30+36x)}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{1}{60}\int \frac{(-72-90x)x}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3}{4}\int \frac{1}{\sqrt{1-x^2}}\\ &= -\frac{3}{5}x^2\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{5}x^4\sqrt{1-x^2}-\frac{3}{20}(8+5x)\sqrt{1-x^2}+\frac{3}{4}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.13, size = 58, normalized size = 0.72

$$\frac{1}{20}\sqrt{1-x^2}\left(-24-15x-12x^2-10x^3-4x^4\right)+\frac{3}{2}\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] (Sqrt[1 - x^2]*(-24 - 15*x - 12*x^2 - 10*x^3 - 4*x^4))/20 + (3*ArcTan[x/(-1
 + Sqrt[1 - x^2])])/2
```

Maple [A]
time = 0.11, size = 71, normalized size = 0.88

method	result
risch	$\frac{(4x^4+10x^3+12x^2+15x+24)(x^2-1)}{20\sqrt{-x^2+1}}+\frac{3\arcsin(x)}{4}$

trager	$\left(-\frac{1}{5}x^4-\frac{1}{2}x^3-\frac{3}{5}x^2-\frac{3}{4}x-\frac{6}{5}\right)\sqrt{-x^2+1}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{4}$
default	$-\frac{x^4\sqrt{-x^2+1}}{5}-\frac{3x^2\sqrt{-x^2+1}}{5}-\frac{6\sqrt{-x^2+1}}{5}-\frac{x^3\sqrt{-x^2+1}}{2}-\frac{3x\sqrt{-x^2+1}}{4}+\frac{3\arcsin(x)}{4}$
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(4x^2+8)\sqrt{-x^2+1}}{6}}{2\sqrt{\pi}}-\frac{i\left(-\frac{i\sqrt{\pi}\,x\left(10x^2+15\right)\sqrt{-x^2+1}}{20}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{4}\right)}{\sqrt{\pi}}-\frac{-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,(6x}{}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*x^4*(-x^2+1)^(1/2)-3/5*x^2*(-x^2+1)^(1/2)-6/5*(-x^2+1)^(1/2)-1/2*x^3*(
-x^2+1)^(1/2)-3/4*x*(-x^2+1)^(1/2)+3/4*arcsin(x)
```

Maxima [A]
time = 0.48, size = 70, normalized size = 0.86

$$-\frac{1}{5}\sqrt{-x^2+1}\,x^4-\frac{1}{2}\sqrt{-x^2+1}\,x^3-\frac{3}{5}\sqrt{-x^2+1}\,x^2-\frac{3}{4}\sqrt{-x^2+1}\,x-\frac{6}{5}\sqrt{-x^2+1}+\frac{3}{4}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)*x^4 - 1/2*sqrt(-x^2 + 1)*x^3 - 3/5*sqrt(-x^2 + 1)*x^2 -
 3/4*sqrt(-x^2 + 1)*x - 6/5*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

Fricas [A]
time = 1.18, size = 50, normalized size = 0.62

$$-\frac{1}{20}\left(4\,x^4+10\,x^3+12\,x^2+15\,x+24\right)\sqrt{-x^2+1}-\frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*(4*x^4 + 10*x^3 + 12*x^2 + 15*x + 24)*sqrt(-x^2 + 1) - 3/2*arctan((sq
rt(-x^2 + 1) - 1)/x)
```

Sympy [A]
time = 0.26, size = 73, normalized size = 0.90

$$-\frac{x^4\sqrt{1-x^2}}{5}-\frac{x^3\sqrt{1-x^2}}{2}-\frac{3x^2\sqrt{1-x^2}}{5}-\frac{3x\sqrt{1-x^2}}{4}-\frac{6\sqrt{1-x^2}}{5}+\frac{3\,\mathrm{asin}\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**4*sqrt(1 - x**2)/5 - x**3*sqrt(1 - x**2)/2 - 3*x**2*sqrt(1 - x**2)/5 -
3*x*sqrt(1 - x**2)/4 - 6*sqrt(1 - x**2)/5 + 3*asin(x)/4
```

Giac [A]
time = 1.12, size = 34, normalized size = 0.42

$$-\frac{1}{20}\left(\left(2\left(\left(2x+5\right)x+6\right)x+15\right)x+24\right)\sqrt{-x^2+1}+\frac{3}{4}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/20*((2*((2*x + 5)*x + 6)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

Mupad [B]
time = 2.50, size = 36, normalized size = 0.44

$$\frac{3\,\mathrm{asin}(x)}{4}-\sqrt{1-x^2}\,\left(\frac{x^4}{5}+\frac{x^3}{2}+\frac{3\,x^2}{5}+\frac{3\,x}{4}+\frac{6}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

```
[Out] (3*asin(x))/4 - (1 - x^2)^(1/2)*((3*x)/4 + (3*x^2)/5 + x^3/2 + x^4/5 + 6/5)
```

3.55 $\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{24}(32+21x)\sqrt{1-x^2}+\frac{7}{8}\sin^{-1}(x)$$

```
[Out] 7/8*arcsin(x)-2/3*x^2*(-x^2+1)^(1/2)-1/4*x^3*(-x^2+1)^(1/2)-1/24*(32+21*x)*
(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1823, 847, 794, 222}

$$\frac{7\text{ArcSin}(x)}{8}-\frac{2}{3}\sqrt{1-x^2}\,x^2-\frac{1}{24}(21x+32)\sqrt{1-x^2}-\frac{1}{4}\sqrt{1-x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] (-2*x^2*Sqrt[1 - x^2])/3 - (x^3*Sqrt[1 - x^2])/4 - ((32 + 21*x)*Sqrt[1 - x^
2])/24 + (7*ArcSin[x])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(1+x)^2}{\sqrt{1-x^2}}\,dx &= -\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{4}\int \frac{(-7-8x)x^2}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}+\frac{1}{12}\int \frac{x(16+21x)}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{24}(32+21x)\sqrt{1-x^2}+\frac{7}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2}{3}x^2\sqrt{1-x^2}-\frac{1}{4}x^3\sqrt{1-x^2}-\frac{1}{24}(32+21x)\sqrt{1-x^2}+\frac{7}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 53, normalized size = 0.84

$$\frac{1}{24}\sqrt{1-x^2}\left(-32-21x-16x^2-6x^3\right)+\frac{7}{4}\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] (Sqrt[1 - x^2]*(-32 - 21*x - 16*x^2 - 6*x^3))/24 + (7*ArcTan[x/(-1 + Sqrt[1
 - x^2])])/4
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.90

method	result
risch	$\frac{(6x^3+16x^2+21x+32)(x^2-1)}{24\sqrt{-x^2+1}}+\frac{7\arcsin(x)}{8}$
trager	$\left(-\frac{1}{4}x^3-\frac{2}{3}x^2-\frac{7}{8}x-\frac{4}{3}\right)\sqrt{-x^2+1}+\frac{7\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{8}$

default	$-\frac{x^3\sqrt{-x^2+1}}{4}-\frac{7x\sqrt{-x^2+1}}{8}+\frac{7\arcsin(x)}{8}-\frac{2x^2\sqrt{-x^2+1}}{3}-\frac{4\sqrt{-x^2+1}}{3}$
meijerg	$\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}+\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(4x^2+8)\sqrt{-x^2+1}}{6}}{\sqrt{\pi}}-\frac{i\left(-\frac{i\sqrt{\pi}\,x(10x^2+15)\sqrt{-x^2+}}{20}\right.}{2\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^3*(-x^2+1)^(1/2)-7/8*x*(-x^2+1)^(1/2)+7/8*arcsin(x)-2/3*x^2*(-x^2+1)
^(1/2)-4/3*(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 56, normalized size = 0.89

$$-\frac{1}{4}\sqrt{-x^2+1}\,x^3-\frac{2}{3}\sqrt{-x^2+1}\,x^2-\frac{7}{8}\sqrt{-x^2+1}\,x-\frac{4}{3}\sqrt{-x^2+1}+\frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 - 2/3*sqrt(-x^2 + 1)*x^2 - 7/8*sqrt(-x^2 + 1)*x - 4
/3*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

Fricas [A]
time = 1.77, size = 45, normalized size = 0.71

$$-\frac{1}{24}\left(6\,x^3+16\,x^2+21\,x+32\right)\sqrt{-x^2+1}-\frac{7}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*x^3 + 16*x^2 + 21*x + 32)*sqrt(-x^2 + 1) - 7/4*arctan((sqrt(-x^2 +
 1) - 1)/x)
```

Sympy [A]
time = 0.17, size = 60, normalized size = 0.95

$$-\frac{x^3\sqrt{1-x^2}}{4}-\frac{2x^2\sqrt{1-x^2}}{3}-\frac{7x\sqrt{1-x^2}}{8}-\frac{4\sqrt{1-x^2}}{3}+\frac{7\operatorname{asin}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -x**3*sqrt(1 - x**2)/4 - 2*x**2*sqrt(1 - x**2)/3 - 7*x*sqrt(1 - x**2)/8 - 4
*sqrt(1 - x**2)/3 + 7*asin(x)/8
```

Giac [A]
time = 1.03, size = 30, normalized size = 0.48

$$-\frac{1}{24}\left((2(3x+8)x+21)x+32\right)\sqrt{-x^2+1}+\frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x + 8)*x + 21)*x + 32)*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

Mupad [B]
time = 0.03, size = 31, normalized size = 0.49

$$\frac{7\operatorname{asin}(x)}{8}-\sqrt{1-x^2}\left(\frac{x^3}{4}+\frac{2x^2}{3}+\frac{7x}{8}+\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 1)^2)/(1 - x^2)^(1/2),x)
```

```
[Out] (7*asin(x))/8 - (1 - x^2)^(1/2)*((7*x)/8 + (2*x^2)/3 + x^3/4 + 4/3)
```

3.56 $\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}(5+3x)\sqrt{1-x^2}+\sin^{-1}(x)$$

```
[Out] arcsin(x)-1/3*x^2*(-x^2+1)^(1/2)-1/3*(5+3*x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1823, 794, 222}

$$\text{ArcSin}(x)-\frac{1}{3}\sqrt{1-x^2}\,x^2-\frac{1}{3}(3x+5)\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
```

```
[Out] -1/3*(x^2*Sqrt[1 - x^2]) - ((5 + 3*x)*Sqrt[1 - x^2])/3 + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}x^2\sqrt{1-x^2} - \frac{1}{3}\int \frac{(-5-6x)x}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{1}{3}x^2\sqrt{1-x^2} - \frac{1}{3}(5+3x)\sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{1}{3}x^2\sqrt{1-x^2} - \frac{1}{3}(5+3x)\sqrt{1-x^2} + \sin^{-1}(x)$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{1-x^2}\left(-5-3x-x^2\right)+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x)^2)/Sqrt[1 - x^2],x]
[Out] (Sqrt[1 - x^2]*(-5 - 3*x - x^2))/3 + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A]
time = 0.06, size = 41, normalized size = 1.00

method	result
risch	$\frac{(x^2+3x+5)(x^2-1)}{3\sqrt{-x^2+1}}+\arcsin(x)$
default	$-\frac{x^2\sqrt{-x^2+1}}{3}-\frac{5\sqrt{-x^2+1}}{3}-x\sqrt{-x^2+1}+\arcsin(x)$
trager	$\left(-\frac{1}{3}x^2-x-\frac{5}{3}\right)\sqrt{-x^2+1}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{-x^2+1}}{2\sqrt{\pi}}+\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{\sqrt{\pi}}+\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(4x^2+8)\sqrt{-x^2+1}}{6}}{2\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*x^2*(-x^2+1)^(1/2)-5/3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)+arcsin(x)
```

Maxima [A]
time = 0.49, size = 40, normalized size = 0.98

$$-\frac{1}{3}\sqrt{-x^2+1}\,x^2-\sqrt{-x^2+1}\,x-\frac{5}{3}\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - sqrt(-x^2 + 1)*x - 5/3*sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [A]
time = 1.86, size = 38, normalized size = 0.93

$$-\frac{1}{3}\left(x^{2}+3\,x+5\right)\sqrt{-x^{2}+1}-2\,\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*(x^2 + 3*x + 5)*sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A]
time = 0.11, size = 37, normalized size = 0.90

$$-\frac{x^{2}\sqrt{1-x^{2}}}{3}-x\sqrt{1-x^{2}}-\frac{5\sqrt{1-x^{2}}}{3}+\operatorname{asin}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**2/(-x**2+1)**(1/2),x)
[Out] -x**2*sqrt(1 - x**2)/3 - x*sqrt(1 - x**2) - 5*sqrt(1 - x**2)/3 + asin(x)
```

Giac [A]
time = 0.91, size = 21, normalized size = 0.51

$$-\frac{1}{3}\left((x+3)x+5\right)\sqrt{-x^{2}+1}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/3*((x + 3)*x + 5)*sqrt(-x^2 + 1) + arcsin(x)
```

Mupad [B]
time = 0.03, size = 22, normalized size = 0.54

$$\operatorname{asin}(x)-\sqrt{1-x^{2}}\,\left(\frac{x^{2}}{3}+x+\frac{5}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^2)/(1 - x^2)^(1/2),x)
[Out] asin(x) - (1 - x^2)^(1/2)*(x + x^2/3 + 5/3)
```

3.57 $\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=40

$$-\frac{3}{2}\sqrt{1-x^2}-\frac{1}{2}(1+x)\sqrt{1-x^2}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] 3/2*arcsin(x)-3/2*(-x^2+1)^(1/2)-1/2*(1+x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {685, 655, 222}

$$\frac{3\text{ArcSin}(x)}{2}-\frac{1}{2}\sqrt{1-x^2}\,(x+1)-\frac{3\sqrt{1-x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/Sqrt[1 - x^2],x]
```

```
[Out] (-3*Sqrt[1 - x^2])/2 - ((1 + x)*Sqrt[1 - x^2])/2 + (3*ArcSin[x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(1+x)^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}(1+x)\sqrt{1-x^2} + \frac{3}{2}\int \frac{1+x}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x^2} - \frac{1}{2}(1+x)\sqrt{1-x^2} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x^2} - \frac{1}{2}(1+x)\sqrt{1-x^2} + \frac{3}{2}\sin^{-1}(x)$$

Mathematica [A]
time = 0.10, size = 41, normalized size = 1.02

$$\frac{1}{2}(-4-x)\sqrt{1-x^2} - 3\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/Sqrt[1 - x^2],x]
[Out] ((-4 - x)*Sqrt[1 - x^2])/2 - 3*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.07, size = 29, normalized size = 0.72

method	result	size
risch	$\frac{(x+4)(x^2-1)}{2\sqrt{-x^2+1}} + \frac{3\arcsin(x)}{2}$	25
default	$-\frac{x\sqrt{-x^2+1}}{2} + \frac{3\arcsin(x)}{2} - 2\sqrt{-x^2+1}$	29
trager	$\left(-\frac{x}{2}-2\right)\sqrt{-x^2+1} + \frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	44
meijerg	$\arcsin(x) - \frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{\sqrt{\pi}} + \frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*x*(-x^2+1)^(1/2)+3/2*arcsin(x)-2*(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.48, size = 28, normalized size = 0.70

$$-\frac{1}{2}\sqrt{-x^2+1}\,x - 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A]
time = 1.59, size = 33, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2+1}\,(x+4) - 3\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) - 3*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.68

$$-\frac{x\sqrt{1-x^2}}{2} - 2\sqrt{1-x^2} + \frac{3\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**2+1)**(1/2),x)
[Out] -x*sqrt(1 - x**2)/2 - 2*sqrt(1 - x**2) + 3*asin(x)/2
```

Giac [A]
time = 0.77, size = 19, normalized size = 0.48

$$-\frac{1}{2}\sqrt{-x^2+1}\,(x+4) + \frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 + 1)*(x + 4) + 3/2*arcsin(x)
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.52

$$\frac{3\operatorname{asin}(x)}{2} - \left(\frac{x}{2}+2\right)\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(1 - x^2)^(1/2),x)
[Out] (3*asin(x))/2 - (x/2 + 2)*(1 - x^2)^(1/2)
```

3.58 $\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=32

$$-\sqrt{1-x^2}+2\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] 2*arcsin(x)-arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {1823, 858, 222, 272, 65, 212}

$$2\text{ArcSin}(x)-\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
```

```
[Out] -Sqrt[1 - x^2] + 2*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x\sqrt{1-x^2}}\,dx &= -\sqrt{1-x^2}-\int \frac{-1-2x}{x\sqrt{1-x^2}}\,dx \\
&= -\sqrt{1-x^2}+2\int \frac{1}{\sqrt{1-x^2}}\,dx+\int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\sqrt{1-x^2}+2\sin^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right) \\
&= -\sqrt{1-x^2}+2\sin^{-1}(x)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right) \\
&= -\sqrt{1-x^2}+2\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 1.62

$$-\sqrt{1-x^2}+4\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)-\log(x)+\log\left(-1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x*Sqrt[1 - x^2]),x]
[Out] -Sqrt[1 - x^2] + 4*ArcTan[x/(-1 + Sqrt[1 - x^2])] - Log[x] + Log[-1 + Sqrt[
1 - x^2]]
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.91

method	result
default	$-\sqrt{-x^2+1}+2\arcsin\left(x\right)-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$
trager	$-\sqrt{-x^2+1}+\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(x\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2+1}\right.$
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)+(-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2\sqrt{\pi}}+2\arcsin\left(x\right)-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+1)^(1/2)+2*arcsin(x)-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A]
time = 0.48, size = 41, normalized size = 1.28

$$-\sqrt{-x^2+1}+2\,\arcsin\left(x\right)-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A]
time = 1.96, size = 46, normalized size = 1.44

$$-\sqrt{-x^2+1}-4\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1) - 4*arctan((sqrt(-x^2 + 1) - 1)/x) + log((sqrt(-x^2 + 1) -
1)/x)
```

Sympy [A]
time = 3.02, size = 31, normalized size = 0.97

$$-\sqrt{1-x^2}+\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}+2\operatorname{asin}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), T
rue)) + 2*asin(x)
```

Giac [A]
time = 0.88, size = 34, normalized size = 1.06

$$-\sqrt{-x^2+1}+2\arcsin(x)+\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1) + 2*arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

Mupad [B]
time = 0.05, size = 32, normalized size = 1.00

$$2\,\mathrm{asin}(x)+\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x*(1 - x^2)^(1/2)),x)
```

```
[Out] 2*asin(x) + log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)
```

3.59 $\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\sqrt{1-x^2}}{x}+\sin^{-1}(x)-2\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] arcsin(x)-2*arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {1821, 858, 222, 272, 65, 212}

$$\text{ArcSin}(x)-\frac{\sqrt{1-x^2}}{x}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2*ArcTanh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^2\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{x}-\int \frac{-2-x}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{x}+2\int \frac{1}{x\sqrt{1-x^2}}\,dx+\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{x}+\sin^{-1}(x)+\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}}{x}+\sin^{-1}(x)-2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sqrt{1-x^2}}{x}+\sin^{-1}(x)-2\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 1.73

$$-\frac{\sqrt{1-x^2}}{x}+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)-2\log(x)+2\log\left(-1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])] - 2*Log[x] + 2*Log[-1
 + Sqrt[1 - x^2]]
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.91

method	result
default	$\arcsin\left(x\right)-\frac{\sqrt{-x^{2}+1}}{x}-2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$
risch	$\frac{x^{2}-1}{x\sqrt{-x^{2}+1}}+\arcsin\left(x\right)-2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)$
meijerg	$-\frac{\sqrt{-x^{2}+1}}{x}+\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{2}+1}}{2}\right)+\left(-2\ln\left(2\right)+2\ln\left(x\right)+i\pi\right)\sqrt{\pi}}{\sqrt{\pi}}+\arcsin\left(x\right)$
trager	$-\frac{\sqrt{-x^{2}+1}}{x}-\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(x\,\mathrm{RootOf}\left(\_Z^{2}+1\right)+\sqrt{-x^{2}+1}\right)-2\ln\left(\frac{\sqrt{-x^{2}+1}+}{x}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(x)-(-x^2+1)^(1/2)/x-2*arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A]
time = 0.49, size = 42, normalized size = 1.27

$$-\frac{\sqrt{-x^{2}+1}}{x}+\arcsin\left(x\right)-2\,\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/x + arcsin(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A]
time = 1.73, size = 53, normalized size = 1.61

$$-\frac{2\,x\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-2\,x\log\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)+\sqrt{-x^{2}+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*x*arctan((sqrt(-x^2 + 1) - 1)/x) - 2*x*log((sqrt(-x^2 + 1) - 1)/x) + sq
rt(-x^2 + 1))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 2.25, size = 51, normalized size = 1.55

$$\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2| > 1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases} + 2\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}\right) + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**2/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
+ 2*Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) + asin(x
)
```

Giac [A]
time = 0.80, size = 55, normalized size = 1.67

$$\frac{x}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\sqrt{-x^2+1}-1}{2x} + \arcsin(x) + 2\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x + arcsin(x) + 2*log
(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

Mupad [B]
time = 0.08, size = 35, normalized size = 1.06

$$\operatorname{asin}(x) + 2\ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right) - \frac{\sqrt{1-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^2*(1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) + 2*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)/x
```

3.60 $\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -3/2*arctanh((-x^2+1)^(1/2))-1/2*(-x^2+1)^(1/2)/x^2-2*(-x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1821, 821, 272, 65, 212}

$$-\frac{2\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2x^2}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
```

```
[Out] -1/2*Sqrt[1 - x^2]/x^2 - (2*Sqrt[1 - x^2])/x - (3*ArcTanh[Sqrt[1 - x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^2}{x^3\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{1}{2}\int \frac{-4-3x}{x^2\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}+\frac{3}{2}\int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right) \\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right) \\
&= -\frac{\sqrt{1-x^2}}{2x^2}-\frac{2\sqrt{1-x^2}}{x}-\frac{3}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 48, normalized size = 0.94

$$\frac{(-1-4x)\sqrt{1-x^2}}{2x^2}-\frac{3\log(x)}{2}+\frac{3}{2}\log\left(-1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^3*Sqrt[1 - x^2]),x]
[Out] ((-1 - 4*x)*Sqrt[1 - x^2])/(2*x^2) - (3*Log[x])/2 + (3*Log[-1 + Sqrt[1 - x^
2]])/2
```

Maple [**A**]
time = 0.10, size = 42, normalized size = 0.82

method	result
trager	$-\frac{(1+4x)\sqrt{-x^2+1}}{2x^2}-\frac{3\ln\left(\frac{\sqrt{-x^2+1}+1}{x}\right)}{2}$
risch	$\frac{4x^3+x^2-4x-1}{2x^2\sqrt{-x^2+1}}-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}$
default	$-\frac{\sqrt{-x^2+1}}{2x^2}-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{2}-\frac{2\sqrt{-x^2+1}}{x}$
meijerg	$-\frac{-\frac{\sqrt{\pi}\,(-4x^2+8)}{8x^2}+\frac{\sqrt{\pi}\,\sqrt{-x^2+1}}{x^2}+\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)-\frac{(1-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2}+\frac{\sqrt{\pi}}{x^2}}{2\sqrt{\pi}}-\frac{2\sqrt{-x^2}}{x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^3/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-x^2+1)^(1/2)/x^2-3/2*arctanh(1/(-x^2+1)^(1/2))-2*(-x^2+1)^(1/2)/x
```

Maxima [A]
time = 0.48, size = 54, normalized size = 1.06

$$-\frac{2\sqrt{-x^2+1}}{x}-\frac{\sqrt{-x^2+1}}{2\,x^2}-\frac{3}{2}\,\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/x - 1/2*sqrt(-x^2 + 1)/x^2 - 3/2*log(2*sqrt(-x^2 + 1)/abs
(x) + 2/abs(x))
```

Fricas [A]
time = 2.01, size = 43, normalized size = 0.84

$$\frac{3\,x^2\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}\,(4\,x+1)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*x^2*log((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*(4*x + 1))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 3.56, size = 116, normalized size = 2.27

$$2\left(\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2|>1\\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}+\frac{1}{2x\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|}>1\\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise}\end{cases}+\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**3/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True)
) + Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*x**3*sqrt(
-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/x**2)/(2*x)
, True)) + Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(41) = 82.
time = 1.37, size = 91, normalized size = 1.78

$$\frac{x^2\left(\frac{8\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{8\left(\sqrt{-x^2+1}-1\right)^2}-\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{8\,x^2}+\frac{3}{2}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*x^2*(8*(sqrt(-x^2 + 1) - 1)/x - 1)/(sqrt(-x^2 + 1) - 1)^2 - (sqrt(-x^2
+ 1) - 1)/x + 1/8*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3/2*log(-(sqrt(-x^2 + 1) - 1
)/abs(x))
```

Mupad [B]
time = 2.49, size = 47, normalized size = 0.92

$$\frac{3\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{2}-\frac{2\,\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^3*(1 - x^2)^(1/2)),x)
```

```
[Out] (3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/2 - (2*(1 - x^2)^(1/2))/x - (1 -
 x^2)^(1/2)/(2*x^2)
```

3.61 $\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -arctanh((-x^2+1)^(1/2))-1/3*(-x^2+1)^(1/2)/x^3-(-x^2+1)^(1/2)/x^2-5/3*(-x^
2+1)^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1821, 849, 821, 272, 65, 212}

$$-\frac{5\sqrt{1-x^2}}{3x}-\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] -1/3*Sqrt[1 - x^2]/x^3 - Sqrt[1 - x^2]/x^2 - (5*Sqrt[1 - x^2])/(3*x) - ArcT
anh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^2}{x^4\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{1}{3}\int \frac{-6-5x}{x^3\sqrt{1-x^2}}\,dx\\ &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{6}\int \frac{10+6x}{x^2\sqrt{1-x^2}}\,dx\\ &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\ &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\ &= -\frac{\sqrt{1-x^2}}{3x^3}-\frac{\sqrt{1-x^2}}{x^2}-\frac{5\sqrt{1-x^2}}{3x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [A]

time = 0.09, size = 47, normalized size = 0.70

$$\frac{(-1-3x-5x^2)\sqrt{1-x^2}}{3x^3}-\log(x)+\log\left(-1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] ((-1 - 3*x - 5*x^2)*Sqrt[1 - x^2])/(3*x^3) - Log[x] + Log[-1 + Sqrt[1 - x^2
]]
```

Maple [A]
time = 0.09, size = 56, normalized size = 0.84

method	result
trager	$-\frac{(5x^2+3x+1)\sqrt{-x^2+1}}{3x^3}+\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$
risch	$\frac{5x^4+3x^3-4x^2-3x-1}{3x^3\sqrt{-x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$
default	$-\frac{\sqrt{-x^2+1}}{x^2}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\frac{5\sqrt{-x^2+1}}{3x}-\frac{\sqrt{-x^2+1}}{3x^3}$
meijerg	$-\frac{(2x^2+1)\sqrt{-x^2+1}}{3x^3}-\frac{-\frac{\sqrt{\pi}\,(-4x^2+8)}{8x^2}+\frac{\sqrt{\pi}\,\sqrt{-x^2+1}}{x^2}+\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)-\frac{(1-2\ln(2)+2\ln(x)+i\pi)}{2}}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^4/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+1)^(1/2)/x^2-arctanh(1/(-x^2+1)^(1/2))-5/3*(-x^2+1)^(1/2)/x-1/3*(-x^
2+1)^(1/2)/x^3
```

Maxima [A]
time = 0.48, size = 68, normalized size = 1.01

$$-\frac{5\sqrt{-x^2+1}}{3\,x}-\frac{\sqrt{-x^2+1}}{x^2}-\frac{\sqrt{-x^2+1}}{3\,x^3}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/3*sqrt(-x^2 + 1)/x - sqrt(-x^2 + 1)/x^2 - 1/3*sqrt(-x^2 + 1)/x^3 - log(2
*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A]
time = 1.88, size = 48, normalized size = 0.72

$$\frac{3\,x^3\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(5\,x^2+3\,x+1)\sqrt{-x^2+1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log((sqrt(-x^2 + 1) - 1)/x) - (5*x^2 + 3*x + 1)*sqrt(-x^2 + 1))/
x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 4.15, size = 128, normalized size = 1.91

$$\begin{cases}-\frac{\sqrt{1-x^2}}{x}-\frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x>-1\wedge x<1\end{cases}+\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2|>1\\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}+2\left(\begin{cases}-\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}+\frac{1}{2x\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|}>1\\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**4/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x,
True)) + 2*Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*x**
3*sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/x**2
)/(2*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(55) = 110.
time = 1.73, size = 125, normalized size = 1.87

$$-\frac{x^3\left(\frac{6\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{21\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{24\left(\sqrt{-x^2+1}-1\right)^3}-\frac{7\left(\sqrt{-x^2+1}-1\right)}{8\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{24\,x^3}+\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*x^3*(6*(sqrt(-x^2 + 1) - 1)/x - 21*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(s
qrt(-x^2 + 1) - 1)^3 - 7/8*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1
)^2/x^2 - 1/24*(sqrt(-x^2 + 1) - 1)^3/x^3 + log(-(sqrt(-x^2 + 1) - 1)/abs(x
))
```

Mupad [B]
time = 0.03, size = 67, normalized size = 1.00

$$\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\sqrt{1-x^2}\,\left(\frac{2}{3\,x}+\frac{1}{3\,x^3}\right)-\frac{\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^4*(1 - x^2)^(1/2)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - (1 - x^2)^(1/2)*(2/(3*x) + 1/(3*x^
3)) - (1 - x^2)^(1/2)/x - (1 - x^2)^(1/2)/x^2
```

3.62 $\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -7/8*arctanh((-x^2+1)^(1/2))-1/4*(-x^2+1)^(1/2)/x^4-2/3*(-x^2+1)^(1/2)/x^3-
7/8*(-x^2+1)^(1/2)/x^2-4/3*(-x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {1821, 849, 821, 272, 65, 212}

$$-\frac{4\sqrt{1-x^2}}{3x}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] -1/4*Sqrt[1 - x^2]/x^4 - (2*Sqrt[1 - x^2])/(3*x^3) - (7*Sqrt[1 - x^2])/(8*x
^2) - (4*Sqrt[1 - x^2])/(3*x) - (7*ArcTanh[Sqrt[1 - x^2]])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^2}{x^5\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{1}{4}\int \frac{-8-7x}{x^4\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}+\frac{1}{12}\int \frac{21+16x}{x^3\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{1}{24}\int \frac{-32-21x}{x^2\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}+\frac{7}{8}\int \frac{1}{x\sqrt{1-x^2}}\,dx \\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}+\frac{7}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,\right. \\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1}\right. \\
&= -\frac{\sqrt{1-x^2}}{4x^4}-\frac{2\sqrt{1-x^2}}{3x^3}-\frac{7\sqrt{1-x^2}}{8x^2}-\frac{4\sqrt{1-x^2}}{3x}-\frac{7}{8}\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 58, normalized size = 0.65

$$\frac{\sqrt{1-x^2}\left(-6-16x-21x^2-32x^3\right)}{24x^4}-\frac{7\log(x)}{8}+\frac{7}{8}\log\left(-1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^5*Sqrt[1 - x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*(-6 - 16*x - 21*x^2 - 32*x^3))/(24*x^4) - (7*Log[x])/8 + (7*
Log[-1 + Sqrt[1 - x^2]])/8
```

Maple [A]
time = 0.08, size = 70, normalized size = 0.79

method	result
trager	$-\frac{\left(32x^3+21x^2+16x+6\right)\sqrt{-x^2+1}}{24x^4}-\frac{7\ln\left(\frac{\sqrt{-x^2+1}+1}{x}\right)}{8}$
risch	$\frac{32x^5+21x^4-16x^3-15x^2-16x-6}{24x^4\sqrt{-x^2+1}}-\frac{7\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{8}$
default	$-\frac{\sqrt{-x^2+1}}{4x^4}-\frac{7\sqrt{-x^2+1}}{8x^2}-\frac{7\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{8}-\frac{2\sqrt{-x^2+1}}{3x^3}-\frac{4\sqrt{-x^2+1}}{3x}$
meijerg	$\frac{\frac{\sqrt{\pi}\left(-7x^4+8x^2+8\right)}{16x^4}-\frac{\sqrt{\pi}\left(12x^2+8\right)\sqrt{-x^2+1}}{16x^4}-\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)}{4}+\frac{3\left(\frac{7}{6}-2\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}}{8}-\frac{\sqrt{\pi}}{2x^4}-\frac{\sqrt{}}{2}}{2\sqrt{\pi}}$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^5/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-x^2+1)^(1/2)/x^4-7/8*(-x^2+1)^(1/2)/x^2-7/8*arctanh(1/(-x^2+1)^(1/2)
)-2/3*(-x^2+1)^(1/2)/x^3-4/3*(-x^2+1)^(1/2)/x
```

Maxima [A]
time = 0.48, size = 82, normalized size = 0.92

$$-\frac{4\sqrt{-x^2+1}}{3x}-\frac{7\sqrt{-x^2+1}}{8x^2}-\frac{2\sqrt{-x^2+1}}{3x^3}-\frac{\sqrt{-x^2+1}}{4x^4}-\frac{7}{8}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(-x^2 + 1)/x - 7/8*sqrt(-x^2 + 1)/x^2 - 2/3*sqrt(-x^2 + 1)/x^3 - 1
/4*sqrt(-x^2 + 1)/x^4 - 7/8*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A]
time = 2.74, size = 53, normalized size = 0.60

$$\frac{21\,x^4\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(32\,x^3+21\,x^2+16\,x+6)\sqrt{-x^2+1}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(21*x^4*log((sqrt(-x^2 + 1) - 1)/x) - (32*x^3 + 21*x^2 + 16*x + 6)*sqr
t(-x^2 + 1))/x^4
```

Sympy [A]
time = 6.42, size = 223, normalized size = 2.51

$$2\left(\begin{cases}-\frac{\sqrt{1-x^2}}{x}-\frac{(1-x^2)^{\frac{3}{2}}}{3x^3} & \text{for } x>-1\wedge x<1\end{cases}\right)+\begin{cases}-\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2}+\frac{1}{2x\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{2x^3\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|}>1\\ \frac{i\operatorname{asin}\left(\frac{1}{x}\right)}{2}-\frac{i\sqrt{1-\frac{1}{x^2}}}{2x} & \text{otherwise}\end{cases}+\begin{cases}-\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8}+\frac{3}{8x\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|}>1\\ \frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8}-\frac{3i}{8x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}}+\frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**5/(-x**2+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x
< 1))) + Piecewise((-acosh(1/x)/2 + 1/(2*x*sqrt(-1 + x**(-2))) - 1/(2*x**3*
sqrt(-1 + x**(-2))), 1/Abs(x**2) > 1), (I*asin(1/x)/2 - I*sqrt(1 - 1/x**2)/
(2*x), True)) + Piecewise((-3*acosh(1/x)/8 + 3/(8*x*sqrt(-1 + x**(-2))) - 1
/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqrt(-1 + x**(-2))), 1/Abs(x**2) >
 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/x**2)) + I/(8*x**3*sqrt(1 - 1/x
**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(69) = 138.
time = 1.70, size = 163, normalized size = 1.83

$$\frac{x^4\left(\frac{16\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{48\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{144\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-3\right)}{192\left(\sqrt{-x^2+1}-1\right)^4}-\frac{3\left(\sqrt{-x^2+1}-1\right)}{4\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-\frac{\left(\sqrt{-x^2+1}-1\right)^3}{12\,x^3}+\frac{\left(\sqrt{-x^2+1}-1\right)^4}{64\,x^4}+\frac{7}{8}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^5/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(sqrt(-x^2 + 1) - 1)/x - 48*(sqrt(-x^2 + 1) - 1)^2/x^2 + 144*
(sqrt(-x^2 + 1) - 1)^3/x^3 - 3)/(sqrt(-x^2 + 1) - 1)^4 - 3/4*(sqrt(-x^2 + 1
) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1/12*(sqrt(-x^2 + 1) - 1)^3/x^3
 + 1/64*(sqrt(-x^2 + 1) - 1)^4/x^4 + 7/8*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

Mupad [B]
time = 0.03, size = 77, normalized size = 0.87

$$\frac{7\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{8}-\sqrt{1-x^2}\left(\frac{4}{3x}+\frac{2}{3x^3}\right)-\sqrt{1-x^2}\left(\frac{3}{8x^2}+\frac{1}{4x^4}\right)-\frac{\sqrt{1-x^2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^5*(1 - x^2)^(1/2)),x)
```

```
[Out] (7*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/8 - (1 - x^2)^(1/2)*(4/(3*x) + 2
/(3*x^3)) - (1 - x^2)^(1/2)*(3/(8*x^2) + 1/(4*x^4)) - (1 - x^2)^(1/2)/(2*x^
2)
```

3.63 $\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{6\sqrt{1-x^2}}{5x}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -3/4*arctanh((-x^2+1)^(1/2))-1/5*(-x^2+1)^(1/2)/x^5-1/2*(-x^2+1)^(1/2)/x^4-
3/5*(-x^2+1)^(1/2)/x^3-3/4*(-x^2+1)^(1/2)/x^2-6/5*(-x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1821, 849, 821, 272, 65, 212}

$$-\frac{6\sqrt{1-x^2}}{5x}-\frac{3\sqrt{1-x^2}}{4x^2}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{5x^5}-\frac{\sqrt{1-x^2}}{2x^4}-\frac{3\sqrt{1-x^2}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] -1/5*Sqrt[1 - x^2]/x^5 - Sqrt[1 - x^2]/(2*x^4) - (3*Sqrt[1 - x^2])/(5*x^3)
- (3*Sqrt[1 - x^2])/(4*x^2) - (6*Sqrt[1 - x^2])/(5*x) - (3*ArcTanh[Sqrt[1 -
 x^2]])/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(1+x)^2}{x^6\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}}{5x^5} - \frac{1}{5}\int \frac{-10-9x}{x^5\sqrt{1-x^2}}\,dx$$
$$= -\frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} + \frac{1}{20}\int \frac{36+30x}{x^4\sqrt{1-x^2}}\,dx$$
$$= -\frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} - \frac{3\sqrt{1-x^2}}{5x^3} - \frac{1}{60}\int \frac{-90-72x}{x^3\sqrt{1-x^2}}\,dx$$
$$= -\frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} - \frac{3\sqrt{1-x^2}}{5x^3} - \frac{3\sqrt{1-x^2}}{4x^2} + \frac{1}{120}\int \frac{144+90x}{x^2\sqrt{1-x^2}}\,dx$$
$$= -\frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} - \frac{3\sqrt{1-x^2}}{5x^3} - \frac{3\sqrt{1-x^2}}{4x^2} - \frac{6\sqrt{1-x^2}}{5x} + \frac{3}{4}\int \frac{1}{x\sqrt{1-x^2}}$$
$$= -\frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} - \frac{3\sqrt{1-x^2}}{5x^3} - \frac{3\sqrt{1-x^2}}{4x^2} - \frac{6\sqrt{1-x^2}}{5x} + \frac{3}{8}\text{Subst}\Big(\int \frac{}{\sqrt{1-}}$$
$$= -\frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} - \frac{3\sqrt{1-x^2}}{5x^3} - \frac{3\sqrt{1-x^2}}{4x^2} - \frac{6\sqrt{1-x^2}}{5x} - \frac{3}{4}\text{Subst}\Big(\int \frac{1}{1-}$$
$$= -\frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} - \frac{3\sqrt{1-x^2}}{5x^3} - \frac{3\sqrt{1-x^2}}{4x^2} - \frac{6\sqrt{1-x^2}}{5x} - \frac{3}{4}\tanh^{-1}\Big(\sqrt{1-}$$

Mathematica [A]
time = 0.11, size = 63, normalized size = 0.59

$$\frac{\sqrt{1-x^2}\,(-4-10x-12x^2-15x^3-24x^4)}{20x^5} - \frac{3\log(x)}{4} + \frac{3}{4}\log\left(-1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/(x^6*Sqrt[1 - x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*(-4 - 10*x - 12*x^2 - 15*x^3 - 24*x^4))/(20*x^5) - (3*Log[x]
)/4 + (3*Log[-1 + Sqrt[1 - x^2]])/4
```

Maple [A]
time = 0.08, size = 84, normalized size = 0.79

method	result
trager	$-\frac{(24x^4+15x^3+12x^2+10x+4)\sqrt{-x^2+1}}{20x^5} + \frac{3\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{4}$
risch	$\frac{24x^6+15x^5-12x^4-5x^3-8x^2-10x-4}{20x^5\sqrt{-x^2+1}} - \frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)}{4}$

default	$-\frac{\sqrt{-x^{2}+1}}{5x^{5}}-\frac{3\sqrt{-x^{2}+1}}{5x^{3}}-\frac{6\sqrt{-x^{2}+1}}{5x}-\frac{\sqrt{-x^{2}+1}}{2x^{4}}-\frac{3\sqrt{-x^{2}+1}}{4x^{2}}-\frac{3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)}{4}$
meijerg	$-\frac{\left(\frac{8}{3}x^{4}+\frac{4}{3}x^{2}+1\right)\sqrt{-x^{2}+1}}{5x^{5}}+\frac{\frac{\sqrt{\pi}\,\left(-7x^{4}+8x^{2}+8\right)}{16x^{4}}-\frac{\sqrt{\pi}\,\left(12x^{2}+8\right)\sqrt{-x^{2}+1}}{16x^{4}}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{2}+1}}{2}\right)}{4}+\frac{3\left(\frac{7}{6}\right.}{}}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/x^6/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-x^2+1)^(1/2)/x^5-3/5*(-x^2+1)^(1/2)/x^3-6/5*(-x^2+1)^(1/2)/x-1/2*(-x
^2+1)^(1/2)/x^4-3/4*(-x^2+1)^(1/2)/x^2-3/4*arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A]
time = 0.48, size = 96, normalized size = 0.90

$$-\frac{6\,\sqrt{-x^{2}+1}}{5\,x}-\frac{3\,\sqrt{-x^{2}+1}}{4\,x^{2}}-\frac{3\,\sqrt{-x^{2}+1}}{5\,x^{3}}-\frac{\sqrt{-x^{2}+1}}{2\,x^{4}}-\frac{\sqrt{-x^{2}+1}}{5\,x^{5}}-\frac{3}{4}\,\log\left(\frac{2\,\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -6/5*sqrt(-x^2 + 1)/x - 3/4*sqrt(-x^2 + 1)/x^2 - 3/5*sqrt(-x^2 + 1)/x^3 - 1
/2*sqrt(-x^2 + 1)/x^4 - 1/5*sqrt(-x^2 + 1)/x^5 - 3/4*log(2*sqrt(-x^2 + 1)/a
bs(x) + 2/abs(x))
```

Fricas [A]
time = 2.82, size = 58, normalized size = 0.54

$$\frac{15\,x^{5}\log\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\left(24\,x^{4}+15\,x^{3}+12\,x^{2}+10\,x+4\right)\sqrt{-x^{2}+1}}{20\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(15*x^5*log((sqrt(-x^2 + 1) - 1)/x) - (24*x^4 + 15*x^3 + 12*x^2 + 10*x
 + 4)*sqrt(-x^2 + 1))/x^5
```

Sympy [C] Result contains complex when optimal does not.
time = 7.27, size = 201, normalized size = 1.88

$$\begin{cases}-\frac{\sqrt{1-x^{2}}}{x}-\frac{\left(1-x^{2}\right)^{\frac{3}{2}}}{3x^{3}} & \text{for}\: x>-1\wedge x<1\end{cases}+\begin{cases}-\frac{\sqrt{1-x^{2}}}{x}-\frac{2\left(1-x^{2}\right)^{\frac{3}{2}}}{3x^{3}}-\frac{\left(1-x^{2}\right)^{\frac{5}{2}}}{5x^{5}} & \text{for}\: x>-1\wedge x<1\end{cases}+2\left(\begin{cases}-\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8}+\frac{3}{8x\sqrt{-1+\frac{1}{x^{2}}}}-\frac{1}{8x^{3}\sqrt{-1+\frac{1}{x^{2}}}}-\frac{1}{4x^{5}\sqrt{-1+\frac{1}{x^{2}}}} & \text{for}\: \frac{1}{|x^{2}|}>1\\\frac{3i\operatorname{asin}\left(\frac{1}{x}\right)}{8}-\frac{3i}{8x\sqrt{1-\frac{1}{x^{2}}}}+\frac{i}{8x^{3}\sqrt{1-\frac{1}{x^{2}}}}+\frac{i}{4x^{5}\sqrt{1-\frac{1}{x^{2}}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/x**6/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1))) + Piecewise((-sqrt(1 - x**2)/x - 2*(1 - x**2)**(3/2)/(3*x**3) - (1 - x
**2)**(5/2)/(5*x**5), (x > -1) & (x < 1))) + 2*Piecewise((-3*acosh(1/x)/8 +
 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*x**3*sqrt(-1 + x**(-2))) - 1/(4*x**5*sqr
t(-1 + x**(-2))), 1/Abs(x**2) > 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/
x**2)) + I/(8*x**3*sqrt(1 - 1/x**2)) + I/(4*x**5*sqrt(1 - 1/x**2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(83) = 166$.
time = 1.05, size = 199, normalized size = 1.86

$$-\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{15\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{40\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-\frac{110\left(\sqrt{-x^2+1}-1\right)^4}{x^4}-1\right)}{160\left(\sqrt{-x^2+1}-1\right)^5}-\frac{11\left(\sqrt{-x^2+1}-1\right)}{16\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{4\,x^2}-\frac{3\left(\sqrt{-x^2+1}-1\right)^3}{32\,x^3}+\frac{\left(\sqrt{-x^2+1}-1\right)^4}{32\,x^4}-\frac{\left(\sqrt{-x^2+1}-1\right)^5}{160\,x^5}+\frac{3}{4}\log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/x^6/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/160*x^5*(5*(sqrt(-x^2 + 1) - 1)/x - 15*(sqrt(-x^2 + 1) - 1)^2/x^2 + 40*(
sqrt(-x^2 + 1) - 1)^3/x^3 - 110*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1)/(sqrt(-x^2
+ 1) - 1)^5 - 11/16*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2
 - 3/32*(sqrt(-x^2 + 1) - 1)^3/x^3 + 1/32*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1/16
0*(sqrt(-x^2 + 1) - 1)^5/x^5 + 3/4*log(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

Mupad [B]
time = 0.04, size = 90, normalized size = 0.84

$$\frac{3\,\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{4}-\sqrt{1-x^2}\,\left(\frac{2}{3\,x}+\frac{1}{3\,x^3}\right)-\sqrt{1-x^2}\,\left(\frac{3}{4\,x^2}+\frac{1}{2\,x^4}\right)-\sqrt{1-x^2}\,\left(\frac{8}{15\,x}+\frac{4}{15\,x^3}+\frac{1}{5\,x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/(x^6*(1 - x^2)^(1/2)),x)
```

```
[Out] (3*log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)))/4 - (1 - x^2)^(1/2)*(2/(3*x) + 1
/(3*x^3)) - (1 - x^2)^(1/2)*(3/(4*x^2) + 1/(2*x^4)) - (1 - x^2)^(1/2)*(8/(1
5*x) + 4/(15*x^3) + 1/(5*x^5))
```

3.64 $\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx$

Optimal. Leaf size=134

$$-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{e(d^2-e^2x^2)^{3/2}}{x^3}-e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{\ }}{\ }\right.$$

```
[Out] -1/4*d*(-e^2*x^2+d^2)^(3/2)/x^4-e*(-e^2*x^2+d^2)^(3/2)/x^3-e^4*arctan(e*x/(
-e^2*x^2+d^2)^(1/2))+13/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/8*e^2*(8*e*
x+13*d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.13, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1821, 825, 858, 223, 209, 272, 65, 214}

$$e^4\left(-\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{d(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{e(d^2-e^2x^2)^{3/2}}{x^3}+\frac{13}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -1/8*(e^2*(13*d + 8*e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (d*(d^2 - e^2*x^2)^(3/2
))/(4*x^4) - (e*(d^2 - e^2*x^2)^(3/2))/x^3 - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^
2*x^2]] + (13*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{x^5}\,dx = -\frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{\int \frac{\sqrt{d^2-e^2x^2}\,(-12d^4e-13d^3e^2x-4d^2e^3x^2)}{x^4}dx}{4d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} + \frac{\int \frac{(39d^5e^2+12d^4e^3x)\sqrt{d^2-e^2x^2}}{x^3}dx}{12d^4}$$

$$= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} - \frac{\int \frac{78d^7e^4+}{x\sqrt{d^2-}}}{4}$$

$$= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} - \frac{1}{8}(13de^4$$

$$= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} - \frac{1}{16}(13de$$

$$= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} - e^4\tan^{-1}$$

$$= -\frac{e^2(13d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{d(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{3/2}}{x^3} - e^4\tan^{-1}$$

Mathematica [**A**]
time = 0.44, size = 127, normalized size = 0.95

$$-\frac{d\sqrt{d^2-e^2x^2}\,(2d^2+8dex+11e^2x^2)}{8x^4} - \frac{13}{4}e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) + e(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/x^5,x]
```

```
[Out] -1/8*(d*Sqrt[d^2 - e^2*x^2]*(2*d^2 + 8*d*e*x + 11*e^2*x^2))/x^4 - (13*e^4*A
rcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/4 + e*(-e^2)^(3/2)*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(118) = 236$.
time = 0.07, size = 324, normalized size = 2.42

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(11e^2x^2+8dex+2d^2)d}{8x^4} - \frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{13e^4d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$

default	$d^3\left(-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{4d^2x^4}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{2d^2x^2}-\frac{e^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}\right)}{2d^2}\right)}{4d^2}\right)+3d$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)

[Out] d^3*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(3/2)+1/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(3/2)-1/2*e^2/d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))+3*d*e^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(3/2)-1/2*e^2/d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))+e^3*(-1/d^2/x*(-e^2*x^2+d^2)^(3/2)-2*e^2/d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-e*(-e^2*x^2+d^2)^(3/2)/x^3

Maxima [A]
time = 0.52, size = 150, normalized size = 1.12

$$-\arcsin\left(\frac{xe}{d}\right)e^4+\frac{13}{8}e^4\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\frac{13\sqrt{-x^2e^2+d^2}\,e^4}{8d}-\frac{\sqrt{-x^2e^2+d^2}\,e^3}{x}-\frac{13(-x^2e^2+d^2)^{\frac{3}{2}}e^2}{8dx^2}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{x^3}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}d}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="maxima")

[Out] -arcsin(x*e/d)*e^4 + 13/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x)) - 13/8*sqrt(-x^2*e^2 + d^2)*e^4/d - sqrt(-x^2*e^2 + d^2)*e^3/x - 13/8*(-x^2*e^2 + d^2)^(3/2)*e^2/(d*x^2) - (-x^2*e^2 + d^2)^(3/2)*e/x^3 - 1/4*(-x^2*e^2 + d^2)^(3/2)*d/x^4

Fricas [A]
time = 1.90, size = 105, normalized size = 0.78

$$\frac{16x^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^4-13x^4e^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(11dx^2e^2+8d^2xe+2d^3)\sqrt{-x^2e^2+d^2}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(16*x^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^4 - 13*x^4*e^4*l
og(-(d - sqrt(-x^2*e^2 + d^2))/x) - (11*d*x^2*e^2 + 8*d^2*x*e + 2*d^3)*sqrt
(-x^2*e^2 + d^2))/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 5.22, size = 544, normalized size = 4.06

$$d^{3}\left(\begin{cases}-\frac{d^{2}}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{3e}{8x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{3}}{8d^{2}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{4}\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{id^{2}}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{3ie}{8x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{3}}{8d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{4}\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{otherwise}\end{cases}\right)+3d^{2}e\left(\begin{cases}-\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3x^{2}}+\frac{e^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3d^{2}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3x^{2}}+\frac{ie^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3d^{2}} & \text{otherwise}\end{cases}\right)+3de^{2}\left(\begin{cases}-\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{2x}+\frac{e^{2}\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{id^{2}}{2ex^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie}{2x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{2}\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)+e^{3}\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^{2}x}{d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^{2}x}{d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(1/2)/x**5,x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + 3*d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(3*d**2), True)) + 3*d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x)
+ e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e*
*2*asin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2
/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e*
*2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2
*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(113) = 226.
time = 1.08, size = 287, normalized size = 2.14

$$-\arcsin\left(\frac{xe}{d}\right)e^{4}\operatorname{sgn}(d)+\frac{x^{4}\left(\frac{8\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)e^{2}}{x}+\frac{8\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{3}e^{(-2)}}{x^{3}}+\frac{24\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{2}}{x^{2}}+e^{4}\right)e^{8}}{64\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{4}}+\frac{13}{8}e^{4}\log\left(\frac{\left|-2\,de-2\sqrt{-x^{2}e^{2}+d^{2}}e\right|e^{(-2)}}{2\,|x|}\right)-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)e^{2}}{8\,x}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{3}e^{(-2)}}{8\,x^{3}}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{4}e^{(-4)}}{64\,x^{4}}-\frac{3\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{2}}{8\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^4*sgn(d) + 1/64*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^2/
x + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-2)/x^3 + 24*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^2/x^2 + e^4)*e^8/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 + 13/8*e^4*log
(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(d*e + sqr
```

```
t(-x^2*e^2 + d^2)*e)*e^2/x - 1/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-2)/x^
3 - 1/64*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-4)/x^4 - 3/8*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^2/x^2
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3)/x^5, x)
```

3.65 $\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx$

Optimal. Leaf size=310

$$\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5}+\frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5}-\frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4}-\frac{7d^4x^3(d^2-e^2x^2)^{7/}}{48e^3}$$

```
[Out] 35/3072*d^10*x*(-e^2*x^2+d^2)^(3/2)/e^5+7/768*d^8*x*(-e^2*x^2+d^2)^(5/2)/e^
5-124/1287*d^5*x^2*(-e^2*x^2+d^2)^(7/2)/e^4-7/48*d^4*x^3*(-e^2*x^2+d^2)^(7/
2)/e^3-31/143*d^3*x^4*(-e^2*x^2+d^2)^(7/2)/e^2-7/24*d^2*x^5*(-e^2*x^2+d^2)^
(7/2)/e-3/13*d*x^6*(-e^2*x^2+d^2)^(7/2)-1/14*e*x^7*(-e^2*x^2+d^2)^(7/2)-1/1
153152*d^6*(63063*e*x+31744*d)*(-e^2*x^2+d^2)^(7/2)/e^6+35/2048*d^14*arctan
(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+35/2048*d^12*x*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A]
time = 0.30, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1823, 847, 794, 201, 223, 209}

$$\frac{35d^{14}\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6}-\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2}-\frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e}+\frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5}+\frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5}+\frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5}-\frac{d^6(31744d+63063ex)\left(d^2-e^2x^2\right)^{7/2}}{1153152e^6}-\frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4}-\frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3}-\frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (35*d^12*x*Sqrt[d^2 - e^2*x^2])/(2048*e^5) + (35*d^10*x*(d^2 - e^2*x^2)^(3/
2))/(3072*e^5) + (7*d^8*x*(d^2 - e^2*x^2)^(5/2))/(768*e^5) - (124*d^5*x^2*(
d^2 - e^2*x^2)^(7/2))/(1287*e^4) - (7*d^4*x^3*(d^2 - e^2*x^2)^(7/2))/(48*e^
3) - (31*d^3*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e^2) - (7*d^2*x^5*(d^2 - e^2*x
^2)^(7/2))/(24*e) - (3*d*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (e*x^7*(d^2 - e^2*
x^2)^(7/2))/14 - (d^6*(31744*d + 63063*e*x)*(d^2 - e^2*x^2)^(7/2))/(1153152
*e^6) + (35*d^14*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2048*e^6)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^5(d^2-e^2x^2)^{5/2}\left(-14d^3e^2-49d^2e^3x-42de^4\right.}{14e^2}$$

$$= -\frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^5(434d^3e^4+637d^2e^5}{182e}$$

$$= -\frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{14}ex^7\left(d^2-e^2x^2\right)^{7/2} -$$

$$= -\frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} - \frac{3}{13}dx^6\left(d^2-e^2x^2\right)^{7/2} -$$

$$= -\frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} -$$

$$= -\frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} -$$

$$= -\frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2} -$$

$$= \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} -$$

$$= \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} -$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} -$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} -$$

$$= \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} -$$

Mathematica [A]
time = 0.54, size = 210, normalized size = 0.68

$$\frac{e\sqrt{d^2-e^2x^2}\left(-507904d^{13}-315315d^{12}ex-253952d^{11}e^2x^2-210210d^{10}e^3x^3-190464d^9e^4x^4-168168d^8e^5x^5+2916352d^7e^6x^6+7763184d^6e^7x^7+2551808d^5e^8x^8-9499776d^4e^9x^9-8773632d^3e^{10}x^{10}+1427712d^2e^{11}x^{11}+4257792de^{12}x^{12}+1317888e^{13}x^{13}\right)+315315d^{14}\sqrt{-e^2}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{18450432e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-507904*d^13 - 315315*d^12*e*x - 253952*d^11*e^2*x^
2 - 210210*d^10*e^3*x^3 - 190464*d^9*e^4*x^4 - 168168*d^8*e^5*x^5 + 2916352
*d^7*e^6*x^6 + 7763184*d^6*e^7*x^7 + 2551808*d^5*e^8*x^8 - 9499776*d^4*e^9*
x^9 - 8773632*d^3*e^10*x^10 + 1427712*d^2*e^11*x^11 + 4257792*d*e^12*x^12 +
```

```
1317888*e^13*x^13) + 315315*d^14*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2
- e^2*x^2]])/(18450432*e^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(266) = 532$.
time = 0.09, size = 610, normalized size = 1.97

method	result
risch	$-\frac{(-1317888e^{13}x^{13}-4257792d\,e^{12}x^{12}-1427712d^2e^{11}x^{11}+8773632d^3e^{10}x^{10}+9499776d^4e^9x^9-2551808d^5e^8x^8-7763184d^6e^7x^7-2}{184504}$

$$5d^2\left(\frac{x\left(\right.}{}\right.$$

$$d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\right.$$

$$3d^2\left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8e^2}+\right.$$

$$5d^2\left(-\frac{x^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{10e^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/14*x^7*(-e^2*x^2+d^2)^(7/2)/e^2+1/2*d^2/e^2*(-1/12*x^5*(-e^2*x^2+d^
2)^(7/2)/e^2+5/12*d^2/e^2*(-1/10*x^3*(-e^2*x^2+d^2)^(7/2)/e^2+3/10*d^2/e^2*
(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/
6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d
^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))))+3*e^2*d*(-
1/13*x^6*(-e^2*x^2+d^2)^(7/2)/e^2+6/13*d^2/e^2*(-1/11*x^4*(-e^2*x^2+d^2)^(7
/2)/e^2+4/11*d^2/e^2*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*(-e^2*
x^2+d^2)^(7/2))))+3*e*d^2*(-1/12*x^5*(-e^2*x^2+d^2)^(7/2)/e^2+5/12*d^2/e^2*
(-1/10*x^3*(-e^2*x^2+d^2)^(7/2)/e^2+3/10*d^2/e^2*(-1/8*x*(-e^2*x^2+d^2)^(7/
2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2
)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2
)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))+d^3*(-1/11*x^4*(-e^2*x^2+d^2)^(7/2)/e
^2+4/11*d^2/e^2*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*(-e^2*x^2+d
^2)^(7/2)))
```

Maxima [A]
time = 0.50, size = 251, normalized size = 0.81

$$\frac{35}{2048}\,d^{14}\arcsin\left(\frac{xe}{d}\right)e^{(-6)}+\frac{35}{2048}\,\sqrt{-x^2e^2+d^2}\,d^{12}xe^{(-5)}+\frac{35}{3072}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^{10}xe^{(-5)}+\frac{7}{768}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^8xe^{(-5)}-\frac{1}{14}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}x^7e-\frac{7}{24}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^2x^5e^{(-1)}-\frac{31}{143}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^3x^4e^{(-2)}-\frac{7}{48}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^4x^3e^{(-3)}-\frac{124}{1287}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^5x^2e^{(-4)}-\frac{7}{128}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^6xe^{(-5)}-\frac{248}{9009}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^7e^{(-6)}-\frac{3}{13}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}dx^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 35/2048*d^14*arcsin(x*e/d)*e^(-6) + 35/2048*sqrt(-x^2*e^2 + d^2)*d^12*x*e^(
-5) + 35/3072*(-x^2*e^2 + d^2)^(3/2)*d^10*x*e^(-5) + 7/768*(-x^2*e^2 + d^2)
^(5/2)*d^8*x*e^(-5) - 1/14*(-x^2*e^2 + d^2)^(7/2)*x^7*e - 7/24*(-x^2*e^2 +
d^2)^(7/2)*d^2*x^5*e^(-1) - 31/143*(-x^2*e^2 + d^2)^(7/2)*d^3*x^4*e^(-2) -
7/48*(-x^2*e^2 + d^2)^(7/2)*d^4*x^3*e^(-3) - 124/1287*(-x^2*e^2 + d^2)^(7/2
)*d^5*x^2*e^(-4) - 7/128*(-x^2*e^2 + d^2)^(7/2)*d^6*x*e^(-5) - 248/9009*(-x
^2*e^2 + d^2)^(7/2)*d^7*e^(-6) - 3/13*(-x^2*e^2 + d^2)^(7/2)*d*x^6
```

Fricas [A]
time = 2.00, size = 179, normalized size = 0.58

$$-\frac{1}{18450432}\left(630630\,d^{14}\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(1317888\,x^{13}e^{13}+4257792\,dx^{12}e^{12}+1427712\,d^2x^{11}e^{11}-8773632\,d^3x^{10}e^{10}-9499776\,d^4x^9e^9+2551808\,d^5x^8e^8+7763184\,d^6x^7e^7+2916352\,d^7x^6e^6-168168\,d^8x^5e^5-190464\,d^9x^4e^4-210210\,d^{10}x^3e^3-253952\,d^{11}x^2e^2-315315\,d^{12}xe-507904\,d^{13}\right)\sqrt{-x^2e^2+d^2}\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/18450432*(630630*d^14*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (13
17888*x^13*e^13 + 4257792*d*x^12*e^12 + 1427712*d^2*x^11*e^11 - 8773632*d^3
```

```
*x^10*e^10 - 9499776*d^4*x^9*e^9 + 2551808*d^5*x^8*e^8 + 7763184*d^6*x^7*e^
7 + 2916352*d^7*x^6*e^6 - 168168*d^8*x^5*e^5 - 190464*d^9*x^4*e^4 - 210210*
d^10*x^3*e^3 - 253952*d^11*x^2*e^2 - 315315*d^12*x*e - 507904*d^13)*sqrt(-x
^2*e^2 + d^2))*e^(-6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.22, size = 170, normalized size = 0.55

$$\frac{35}{2048}\,d^{14}\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d) - \frac{1}{18450432}\,\left(507904\,d^{13}e^{(-6)} + \left(315315\,d^{12}e^{(-5)} + 2\,\left(126976\,d^{11}e^{(-4)} + \left(105105\,d^{10}e^{(-3)} + 4\,\left(23808\,d^{9}e^{(-2)} + \left(21021\,d^{8}e^{(-1)} - 2\,\left(182272\,d^{7} + \left(485199\,d^{6}e + 8\,\left(19936\,d^{5}e^{2} - 3\,\left(24739\,d^{4}e^{3} + 2\,\left(11424\,d^{3}e^{4} - 11\,\left(169\,d^{2}e^{5} + 12\,\left(13\,xe^{7} + 42\,de^{6}\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2} + d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/2048*d^14*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/18450432*(507904*d^13*e^(-6) +
 (315315*d^12*e^(-5) + 2*(126976*d^11*e^(-4) + (105105*d^10*e^(-3) + 4*(238
08*d^9*e^(-2) + (21021*d^8*e^(-1) - 2*(182272*d^7 + (485199*d^6*e + 8*(1993
6*d^5*e^2 - 3*(24739*d^4*e^3 + 2*(11424*d^3*e^4 - 11*(169*d^2*e^5 + 12*(13*
x*e^7 + 42*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^5*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.66 $\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=281

$$\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}$$

```
[Out] 9/512*d^9*x*(-e^2*x^2+d^2)^(3/2)/e^4+9/640*d^7*x*(-e^2*x^2+d^2)^(5/2)/e^4-2
0/143*d^4*x^2*(-e^2*x^2+d^2)^(7/2)/e^3-9/40*d^3*x^3*(-e^2*x^2+d^2)^(7/2)/e^
2-45/143*d^2*x^4*(-e^2*x^2+d^2)^(7/2)/e-1/4*d*x^5*(-e^2*x^2+d^2)^(7/2)-1/13
*e*x^6*(-e^2*x^2+d^2)^(7/2)-1/320320*d^5*(27027*e*x+12800*d)*(-e^2*x^2+d^2)
^(7/2)/e^5+27/1024*d^13*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+27/1024*d^11*x
*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A]
time = 0.25, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1823, 847, 794, 201, 223, 209}

$$\frac{27d^{13}\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5}-\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2}-\frac{45d^2x^4\left(d^2-e^2x^2\right)^{7/2}}{143e}+\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x\left(d^2-e^2x^2\right)^{3/2}}{512e^4}+\frac{9d^7x\left(d^2-e^2x^2\right)^{5/2}}{640e^4}-\frac{d^5(12800d+27027ex)\left(d^2-e^2x^2\right)^{7/2}}{320320e^5}-\frac{20d^4x^2\left(d^2-e^2x^2\right)^{7/2}}{143e^3}-\frac{9d^3x^3\left(d^2-e^2x^2\right)^{7/2}}{40e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (27*d^11*x*Sqrt[d^2 - e^2*x^2])/(1024*e^4) + (9*d^9*x*(d^2 - e^2*x^2)^(3/2)
)/(512*e^4) + (9*d^7*x*(d^2 - e^2*x^2)^(5/2))/(640*e^4) - (20*d^4*x^2*(d^2
- e^2*x^2)^(7/2))/(143*e^3) - (9*d^3*x^3*(d^2 - e^2*x^2)^(7/2))/(40*e^2) -
(45*d^2*x^4*(d^2 - e^2*x^2)^(7/2))/(143*e) - (d*x^5*(d^2 - e^2*x^2)^(7/2))/
4 - (e*x^6*(d^2 - e^2*x^2)^(7/2))/13 - (d^5*(12800*d + 27027*e*x)*(d^2 - e^
2*x^2)^(7/2))/(320320*e^5) + (27*d^13*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1
024*e^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^4(d^2-e^2x^2)^{5/2}\left(-13d^3e^2-45d^2e^3x-39de\right.}{13e^2}$$

$$= -\frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^4(351d^3e^4+540d^2e^5}{156e}$$

$$= -\frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e} - \frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{13}ex^6\left(d^2-e^2x^2\right)^{7/2} -$$

$$= -\frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e} - \frac{1}{4}dx^5\left(d^2-e^2x^2\right)^{7/2} -$$

$$= -\frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}$$

$$= -\frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - \frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e}$$

$$= \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} - \frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - \frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2} - 45$$

$$= \frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4} + \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} - \frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3} - 9d^3$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4} + \frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4} + \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} - 20d^4$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4} + \frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4} + \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} - 20d^4$$

$$= \frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4} + \frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4} + \frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} - 20d^4$$

<u>Mathematica</u> [A]

time = 0.43, size = 199, normalized size = 0.71

$$\frac{e\sqrt{d^2-e^2x^2}\left(-204800d^{12}-135135d^{11}ex-102400d^{10}e^2x^2-90090d^9e^3x^3-76800d^8e^4x^4+952952d^7e^5x^5+2498560d^6e^6x^6+816816d^5e^7x^7-2938880d^4e^8x^8-2690688d^3e^9x^9+430080d^2e^{10}x^{10}+1281280de^{11}x^{11}+394240e^{12}x^{12}\right)+135135d^{13}\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{5125120e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-204800*d^12 - 135135*d^11*e*x - 102400*d^10*e^2*x^
2 - 90090*d^9*e^3*x^3 - 76800*d^8*e^4*x^4 + 952952*d^7*e^5*x^5 + 2498560*d^
6*e^6*x^6 + 816816*d^5*e^7*x^7 - 2938880*d^4*e^8*x^8 - 2690688*d^3*e^9*x^9
+ 430080*d^2*e^10*x^10 + 1281280*d*e^11*x^11 + 394240*e^12*x^12) + 135135*d
^13*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(5125120*e^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(241) = 482$.

time = 0.06, size = 548, normalized size = 1.95

method	result
risch	$-\frac{(-394240e^{12}x^{12}-1281280d\,e^{11}x^{11}-430080d^2e^{10}x^{10}+2690688d^3e^9x^9+2938880d^4e^8x^8-816816d^5e^7x^7-2498560d^6e^6x^6-952952d^{7}}{5125120e^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/13*x^6*(-e^2*x^2+d^2)^(7/2)/e^2+6/13*d^2/e^2*(-1/11*x^4*(-e^2*x^2+d
^2)^(7/2)/e^2+4/11*d^2/e^2*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*
(-e^2*x^2+d^2)^(7/2))))+3*e^2*d*(-1/12*x^5*(-e^2*x^2+d^2)^(7/2)/e^2+5/12*d^
2/e^2*(-1/10*x^3*(-e^2*x^2+d^2)^(7/2)/e^2+3/10*d^2/e^2*(-1/8*x*(-e^2*x^2+d^
2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x
^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arcta
n((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))+3*e*d^2*(-1/11*x^4*(-e^2*x^2+d^
2)^(7/2)/e^2+4/11*d^2/e^2*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*(
-e^2*x^2+d^2)^(7/2)))+d^3*(-1/10*x^3*(-e^2*x^2+d^2)^(7/2)/e^2+3/10*d^2/e^2*
(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/
6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d
^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))
```

Maxima [A]
time = 0.51, size = 228, normalized size = 0.81

$$\frac{27}{1024}\,d^{13}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}+\frac{27}{1024}\,\sqrt{-x^2e^2+d^2}\,d^{11}xe^{(-4)}+\frac{9}{512}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^9xe^{(-4)}+\frac{9}{640}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^7xe^{(-4)}-\frac{1}{13}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}x^6e-\frac{45}{143}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^2x^4e^{(-1)}-\frac{9}{40}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^3x^3e^{(-2)}-\frac{20}{143}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^4x^2e^{(-3)}-\frac{27}{320}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^5xe^{(-4)}-\frac{40}{1001}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^6e^{(-5)}-\frac{1}{4}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}dx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 27/1024*d^13*arcsin(x*e/d)*e^(-5) + 27/1024*sqrt(-x^2*e^2 + d^2)*d^11*x*e^(
-4) + 9/512*(-x^2*e^2 + d^2)^(3/2)*d^9*x*e^(-4) + 9/640*(-x^2*e^2 + d^2)^(5
/2)*d^7*x*e^(-4) - 1/13*(-x^2*e^2 + d^2)^(7/2)*x^6*e - 45/143*(-x^2*e^2 + d
^2)^(7/2)*d^2*x^4*e^(-1) - 9/40*(-x^2*e^2 + d^2)^(7/2)*d^3*x^3*e^(-2) - 20/
143*(-x^2*e^2 + d^2)^(7/2)*d^4*x^2*e^(-3) - 27/320*(-x^2*e^2 + d^2)^(7/2)*d
^5*x*e^(-4) - 40/1001*(-x^2*e^2 + d^2)^(7/2)*d^6*e^(-5) - 1/4*(-x^2*e^2 + d
^2)^(7/2)*d*x^5
```

Fricas [A]
time = 2.42, size = 169, normalized size = 0.60

$$-\frac{1}{5125120}\left(270270\,d^{13}\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(394240\,x^{12}e^{12}+1281280\,dx^{11}e^{11}+430080\,d^2x^{10}e^{10}-2690688\,d^3x^9e^9-2938880\,d^4x^8e^8+816816\,d^5x^7e^7+2498560\,d^6x^6e^6+952952\,d^7x^5e^5-76800\,d^8x^4e^4-90090\,d^9x^3e^3-102400\,d^{10}x^2e^2-135135\,d^{11}xe-204800\,d^{12}\right)\sqrt{-x^2e^2+d^2}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5125120*(270270*d^13*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (394
240*x^12*e^12 + 1281280*d*x^11*e^11 + 430080*d^2*x^10*e^10 - 2690688*d^3*x^
9*e^9 - 2938880*d^4*x^8*e^8 + 816816*d^5*x^7*e^7 + 2498560*d^6*x^6*e^6 + 95
```

```
2952*d^7*x^5*e^5 - 76800*d^8*x^4*e^4 - 90090*d^9*x^3*e^3 - 102400*d^10*x^2*
e^2 - 135135*d^11*x*e - 204800*d^12)*sqrt(-x^2*e^2 + d^2))*e^(-5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
[Out] Timed out
```

Giac [**A**]
time = 1.20, size = 160, normalized size = 0.57

$$\frac{27}{1024}\,d^{13}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\frac{1}{5125120}\left(204800\,d^{12}e^{(-5)}+\left(135135\,d^{11}e^{(-4)}+2\left(51200\,d^{10}e^{(-3)}+\left(45045\,d^{9}e^{(-2)}+4\left(9600\,d^{8}e^{(-1)}-\left(119119\,d^{7}+2\left(156160\,d^{6}e+7\left(7293\,d^{5}e^{2}-8\left(3280\,d^{4}e^{3}+\left(3003\,d^{3}e^{4}-10\left(48\,d^{2}e^{5}+11\left(4\,xe^{7}+13\,de^{6}\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] 27/1024*d^13*arcsin(x*e/d)*e^(-5)*sgn(d) - 1/5125120*(204800*d^12*e^(-5) +
(135135*d^11*e^(-4) + 2*(51200*d^10*e^(-3) + (45045*d^9*e^(-2) + 4*(9600*d^
8*e^(-1) - (119119*d^7 + 2*(156160*d^6*e + 7*(7293*d^5*e^2 - 8*(3280*d^4*e^
3 + (3003*d^3*e^4 - 10*(48*d^2*e^5 + 11*(4*x*e^7 + 13*d*e^6)*x)*x)*x)*x)*x)
*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
[Out] int(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.67 $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3}+\frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3}-\frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2}-\frac{41d^2x^3(d^2-e^2x^2)^{7/}}{120e}$$

```
[Out] 41/1536*d^8*x*(-e^2*x^2+d^2)^(3/2)/e^3+41/1920*d^6*x*(-e^2*x^2+d^2)^(5/2)/e
^3-23/99*d^3*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-41/120*d^2*x^3*(-e^2*x^2+d^2)^(7/
2)/e-3/11*d*x^4*(-e^2*x^2+d^2)^(7/2)-1/12*e*x^5*(-e^2*x^2+d^2)^(7/2)-1/2217
60*d^4*(28413*e*x+14720*d)*(-e^2*x^2+d^2)^(7/2)/e^4+41/1024*d^12*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^4+41/1024*d^10*x*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A]
time = 0.22, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1823, 847, 794, 201, 223, 209}

$$\frac{41d^{12}\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4}-\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2}-\frac{41d^2x^3\left(d^2-e^2x^2\right)^{7/2}}{120e}+\frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3}+\frac{41d^8x\left(d^2-e^2x^2\right)^{3/2}}{1536e^3}+\frac{41d^6x\left(d^2-e^2x^2\right)^{5/2}}{1920e^3}-\frac{d^4(14720d+28413ex)\left(d^2-e^2x^2\right)^{7/2}}{221760e^4}-\frac{23d^3x^2\left(d^2-e^2x^2\right)^{7/2}}{99e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (41*d^10*x*Sqrt[d^2 - e^2*x^2])/(1024*e^3) + (41*d^8*x*(d^2 - e^2*x^2)^(3/2
))/(1536*e^3) + (41*d^6*x*(d^2 - e^2*x^2)^(5/2))/(1920*e^3) - (23*d^3*x^2*(
d^2 - e^2*x^2)^(7/2))/(99*e^2) - (41*d^2*x^3*(d^2 - e^2*x^2)^(7/2))/(120*e)
 - (3*d*x^4*(d^2 - e^2*x^2)^(7/2))/11 - (e*x^5*(d^2 - e^2*x^2)^(7/2))/12 -
(d^4*(14720*d + 28413*e*x)*(d^2 - e^2*x^2)^(7/2))/(221760*e^4) + (41*d^12*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(1024*e^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^3(d^2-e^2x^2)^{5/2}\left(-12d^3e^2-41d^2e^3x-36de^4\right.}{12e^2}$$

$$= -\frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^3(276d^3e^4+451d^2e^5}{132e}$$

$$= -\frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{12}ex^5\left(d^2-e^2x^2\right)^{7/2} -$$

$$= -\frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} -$$

$$= -\frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e} - \frac{3}{11}dx^4\left(d^2-e^2x^2\right)^{7/2} -$$

$$= \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e} - \frac{3}{1}$$

$$= \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2} - \frac{41}{}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{23d}{}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{23d}{}$$

$$= \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{23d}{}$$

Mathematica [A]
time = 0.44, size = 188, normalized size = 0.75

$$\frac{e\sqrt{d^2-e^2x^2}\left(-235520d^{11}-142065d^{10}ex-117760d^9e^2x^2-94710d^8e^3x^3+798720d^7e^4x^4+2053128d^6e^5x^5+665600d^5e^6x^6-2295216d^4e^7x^7-2078720d^3e^8x^8+325248d^2e^9x^9+967680de^{10}x^{10}+295680e^{11}x^{11}\right)+142065d^{12}\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{3548160e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-235520*d^11 - 142065*d^10*e*x - 117760*d^9*e^2*x^2
 - 94710*d^8*e^3*x^3 + 798720*d^7*e^4*x^4 + 2053128*d^6*e^5*x^5 + 665600*d^
5*e^6*x^6 - 2295216*d^4*e^7*x^7 - 2078720*d^3*e^8*x^8 + 325248*d^2*e^9*x^9
+ 967680*d*e^10*x^10 + 295680*e^11*x^11) + 142065*d^12*Sqrt[-e^2]*Log[-(Sqr
t[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(3548160*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(216) = 432$.
time = 0.07, size = 486, normalized size = 1.93

method	result
risch	$-\frac{(-295680e^{11}x^{11}-967680d\,e^{10}x^{10}-325248d^2e^9x^9+2078720d^3e^8x^8+2295216d^4e^7x^7-665600d^5e^6x^6-2053128d^6e^5x^5-798720d^7e}{3548160e^4}$

$$3d^2 \left(\frac{x\sqrt{\cdots}}{\cdots} \right.$$

$$5d^2 \left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4} + \frac{\cdots}{\cdots} \right.$$

$$d^2 \left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6} + \frac{\cdots}{\cdots} \right.$$

$$3d^2 \left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8e^2} + \frac{\cdots}{\cdots} \right.$$

$$5d^2 \left(-\frac{x^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{10e^2} + \frac{\cdots}{10e^2} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/12*x^5*(-e^2*x^2+d^2)^(7/2)/e^2+5/12*d^2/e^2*(-1/10*x^3*(-e^2*x^2+d
^2)^(7/2)/e^2+3/10*d^2/e^2*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/
6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x
*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^
2)^(1/2)))))))+3*e^2*d*(-1/11*x^4*(-e^2*x^2+d^2)^(7/2)/e^2+4/11*d^2/e^2*(-
1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*(-e^2*x^2+d^2)^(7/2)))+3*e*d^
2*(-1/10*x^3*(-e^2*x^2+d^2)^(7/2)/e^2+3/10*d^2/e^2*(-1/8*x*(-e^2*x^2+d^2)^(
7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d
^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+d^3*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e
^2-2/63*d^2/e^4*(-e^2*x^2+d^2)^(7/2))
```

Maxima [A]
time = 0.51, size = 205, normalized size = 0.81

$$\frac{41}{1024}\,d^{12}\arcsin\left(\frac{xe}{d}\right)e^{(-4)}+\frac{41}{1024}\,\sqrt{-x^2e^2+d^2}\,d^{10}xe^{(-3)}+\frac{41}{1536}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^8xe^{(-3)}+\frac{41}{1920}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^6xe^{(-3)}-\frac{1}{12}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}x^5e-\frac{41}{120}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^2x^3e^{(-1)}-\frac{23}{99}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^3x^2e^{(-2)}-\frac{41}{320}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^4xe^{(-3)}-\frac{46}{693}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^5e^{(-4)}-\frac{3}{11}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 41/1024*d^12*arcsin(x*e/d)*e^(-4) + 41/1024*sqrt(-x^2*e^2 + d^2)*d^10*x*e^(
-3) + 41/1536*(-x^2*e^2 + d^2)^(3/2)*d^8*x*e^(-3) + 41/1920*(-x^2*e^2 + d^2
)^(5/2)*d^6*x*e^(-3) - 1/12*(-x^2*e^2 + d^2)^(7/2)*x^5*e - 41/120*(-x^2*e^2
 + d^2)^(7/2)*d^2*x^3*e^(-1) - 23/99*(-x^2*e^2 + d^2)^(7/2)*d^3*x^2*e^(-2)
- 41/320*(-x^2*e^2 + d^2)^(7/2)*d^4*x*e^(-3) - 46/693*(-x^2*e^2 + d^2)^(7/2
)*d^5*e^(-4) - 3/11*(-x^2*e^2 + d^2)^(7/2)*d*x^4
```

Fricas [A]
time = 3.70, size = 159, normalized size = 0.63

$$-\frac{1}{3548160}\left(284130\,d^{12}\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(295680\,x^{11}e^{11}+967680\,dx^{10}e^{10}+325248\,d^2x^9e^9-2078720\,d^3x^8e^8-2295216\,d^4x^7e^7+665600\,d^5x^6e^6+2053128\,d^6x^5e^5+798720\,d^7x^4e^4-94710\,d^8x^3e^3-117760\,d^9x^2e^2-142065\,d^{10}xe-235520\,d^{11}\right)\sqrt{-x^2e^2+d^2}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3548160*(284130*d^12*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (295
680*x^11*e^11 + 967680*d*x^10*e^10 + 325248*d^2*x^9*e^9 - 2078720*d^3*x^8*e
^8 - 2295216*d^4*x^7*e^7 + 665600*d^5*x^6*e^6 + 2053128*d^6*x^5*e^5 + 79872
0*d^7*x^4*e^4 - 94710*d^8*x^3*e^3 - 117760*d^9*x^2*e^2 - 142065*d^10*x*e -
235520*d^11)*sqrt(-x^2*e^2 + d^2))*e^(-4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.25, size = 149, normalized size = 0.59

$$\frac{41}{1024} \, d^{12} \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \mathrm{sgn}(d) - \frac{1}{3548160} \, {\left(235520 \, d^{11} e^{(-4)} + {\left(142065 \, d^{10} e^{(-3)} + 2 \, {\left(58880 \, d^{9} e^{(-2)} + {\left(47355 \, d^{8} e^{(-1)} - 4 \, {\left(99840 \, d^{7} + {\left(256641 \, d^{6} e + 2 \, {\left(41600 \, d^{5} e^{2} - 7 \, {\left(20493 \, d^{4} e^{3} + 8 \, {\left(2320 \, d^{3} e^{4} - 3 \, {\left(121 \, d^{2} e^{5} + 10 \, {\left(11 \, x e^{7} + 36 \, d e^{6}\right)} x\right)} x\right)} x\right)} x\right)} x\right)} x\right)} x\right)} x\right)} x\right)} x\right)} \sqrt{-x^{2} e^{2} + d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 41/1024*d^12*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/3548160*(235520*d^11*e^(-4) +
(142065*d^10*e^(-3) + 2*(58880*d^9*e^(-2) + (47355*d^8*e^(-1) - 4*(99840*d^
7 + (256641*d^6*e + 2*(41600*d^5*e^2 - 7*(20493*d^4*e^3 + 8*(2320*d^3*e^4 -
 3*(121*d^2*e^5 + 10*(11*x*e^7 + 36*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sq
rt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.68 $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=223

$$\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/}$$

```
[Out] 19/384*d^7*x*(-e^2*x^2+d^2)^(3/2)/e^2+19/480*d^5*x*(-e^2*x^2+d^2)^(5/2)/e^2
-37/99*d^2*x^2*(-e^2*x^2+d^2)^(7/2)/e-3/10*d*x^3*(-e^2*x^2+d^2)^(7/2)-1/11*
e*x^4*(-e^2*x^2+d^2)^(7/2)-1/55440*d^3*(13167*e*x+5920*d)*(-e^2*x^2+d^2)^(7
/2)/e^3+19/256*d^11*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+19/256*d^9*x*(-e^2
*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.19, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1823, 847, 794, 201, 223, 209}

$$\frac{19d^{11}\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3}-\frac{37d^2x^2\left(d^2-e^2x^2\right)^{7/2}}{99e}-\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}-\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}+\frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2}+\frac{19d^7x\left(d^2-e^2x^2\right)^{3/2}}{384e^2}+\frac{19d^5x\left(d^2-e^2x^2\right)^{5/2}}{480e^2}-\frac{d^3(5920d+13167ex)\left(d^2-e^2x^2\right)^{7/2}}{55440e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (19*d^9*x*Sqrt[d^2 - e^2*x^2])/(256*e^2) + (19*d^7*x*(d^2 - e^2*x^2)^(3/2))
/(384*e^2) + (19*d^5*x*(d^2 - e^2*x^2)^(5/2))/(480*e^2) - (37*d^2*x^2*(d^2
- e^2*x^2)^(7/2))/(99*e) - (3*d*x^3*(d^2 - e^2*x^2)^(7/2))/10 - (e*x^4*(d^2
 - e^2*x^2)^(7/2))/11 - (d^3*(5920*d + 13167*e*x)*(d^2 - e^2*x^2)^(7/2))/(5
5440*e^3) + (19*d^11*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx = -\frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} - \frac{\int x^2(d^2-e^2x^2)^{5/2}\left(-11d^3e^2-37d^2e^3x-33de\right.}{11e^2}$$

$$= -\frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2} + \frac{\int x^2(209d^3e^4+370d^2e}{110}$$

$$= -\frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$= -\frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2} - \frac{1}{11}ex^4\left(d^2-e^2x^2\right)^{7/2}$$

$$= \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} - \frac{3}{10}dx^3\left(d^2-e^2x^2\right)^{7/2}$$

$$= \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e}$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d}{}$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d}{}$$

$$= \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{37d}{}$$

Mathematica [**A**]
time = 0.39, size = 177, normalized size = 0.79

$$\frac{e\sqrt{d^2-e^2x^2}\left(-94720d^{10}-65835d^9ex-47360d^8e^2x^2+251790d^7e^3x^3+629760d^6e^4x^4+201432d^5e^5x^5-657920d^4e^6x^6-587664d^3e^7x^7+89600d^2e^8x^8+266112de^9x^9+80640e^{10}x^{10}\right)+65835d^{11}\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{887040e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-94720*d^10 - 65835*d^9*e*x - 47360*d^8*e^2*x^2 + 2
51790*d^7*e^3*x^3 + 629760*d^6*e^4*x^4 + 201432*d^5*e^5*x^5 - 657920*d^4*e^
6*x^6 - 587664*d^3*e^7*x^7 + 89600*d^2*e^8*x^8 + 266112*d*e^9*x^9 + 80640*e
^10*x^10) + 65835*d^11*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]
])/(887040*e^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(191) = 382$.
time = 0.07, size = 424, normalized size = 1.90

method	result
risch	$-\frac{(-80640e^{10}x^{10}-266112d\,e^{9}x^{9}-89600d^{2}e^{8}x^{8}+587664d^{3}e^{7}x^{7}+657920d^{4}e^{6}x^{6}-201432d^{5}e^{5}x^{5}-629760d^{6}e^{4}x^{4}-251790d^{7}e^{3}x^{3}+4736}{887040e^{3}}$

default $e^3\left(-\frac{x^4(-e^2x^2+d^2)^{\frac{7}{2}}}{11e^2}+\frac{4d^2\left(-\frac{x^2(-e^2x^2+d^2)^{\frac{7}{2}}}{9e^2}-\frac{2d^2(-e^2x^2+d^2)^{\frac{7}{2}}}{63e^4}\right)}{11e^2}\right)+3e^2d\left(-\frac{x^3(-e^2x^2+d^2)^{\frac{7}{2}}}{10e^2}+\frac{3d^2\left(-\frac{x(-e^2x\ldots}{8\ldots}\right.}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/11*x^4*(-e^2*x^2+d^2)^(7/2)/e^2+4/11*d^2/e^2*(-1/9*x^2*(-e^2*x^2+d^
2)^(7/2)/e^2-2/63*d^2/e^4*(-e^2*x^2+d^2)^(7/2)))+3*e^2*d*(-1/10*x^3*(-e^2*x
^2+d^2)^(7/2)/e^2+3/10*d^2/e^2*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2
*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1
/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2)))))))+3*e*d^2*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*
(-e^2*x^2+d^2)^(7/2))+d^3*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6
*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*
(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))))))
```

Maxima [A]
time = 0.49, size = 182, normalized size = 0.82

$$\frac{19}{256}\,d^{11}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+\frac{19}{256}\,\sqrt{-x^2e^2+d^2}\,d^9xe^{(-2)}+\frac{19}{384}\,(-x^2e^2+d^2)^{\frac{3}{2}}d^7xe^{(-2)}+\frac{19}{480}\,(-x^2e^2+d^2)^{\frac{5}{2}}d^5xe^{(-2)}-\frac{1}{11}\,(-x^2e^2+d^2)^{\frac{7}{2}}x^4e-\frac{37}{99}\,(-x^2e^2+d^2)^{\frac{7}{2}}d^2x^2e^{(-1)}-\frac{19}{80}\,(-x^2e^2+d^2)^{\frac{7}{2}}d^3xe^{(-2)}-\frac{74}{693}\,(-x^2e^2+d^2)^{\frac{7}{2}}d^4e^{(-3)}-\frac{3}{10}\,(-x^2e^2+d^2)^{\frac{7}{2}}dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 19/256*d^11*arcsin(x*e/d)*e^(-3) + 19/256*sqrt(-x^2*e^2 + d^2)*d^9*x*e^(-2)
 + 19/384*(-x^2*e^2 + d^2)^(3/2)*d^7*x*e^(-2) + 19/480*(-x^2*e^2 + d^2)^(5/
2)*d^5*x*e^(-2) - 1/11*(-x^2*e^2 + d^2)^(7/2)*x^4*e - 37/99*(-x^2*e^2 + d^2
)^(7/2)*d^2*x^2*e^(-1) - 19/80*(-x^2*e^2 + d^2)^(7/2)*d^3*x*e^(-2) - 74/693
*(-x^2*e^2 + d^2)^(7/2)*d^4*e^(-3) - 3/10*(-x^2*e^2 + d^2)^(7/2)*d*x^3
```

Fricas [A]
time = 1.40, size = 149, normalized size = 0.67

$$-\frac{1}{887040}\left(131670\,d^{11}\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(80640\,x^{10}e^{10}+266112\,dx^9e^9+89600\,d^2x^8e^8-587664\,d^3x^7e^7-657920\,d^4x^6e^6+201432\,d^5x^5e^5+629760\,d^6x^4e^4+251790\,d^7x^3e^3-47360\,d^8x^2e^2-65835\,d^9xe-94720\,d^{10}\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/887040*(131670*d^11*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (8064
0*x^10*e^10 + 266112*d*x^9*e^9 + 89600*d^2*x^8*e^8 - 587664*d^3*x^7*e^7 - 6
57920*d^4*x^6*e^6 + 201432*d^5*x^5*e^5 + 629760*d^6*x^4*e^4 + 251790*d^7*x^
3*e^3 - 47360*d^8*x^2*e^2 - 65835*d^9*x*e - 94720*d^10)*sqrt(-x^2*e^2 + d^2
))*e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 138.75, size = 1681, normalized size = 7.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*
*6*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) + d**5*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5
) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqr
t(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asi
n(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/
(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2))
 - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise
((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*
x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d
**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 5*d**3*e**4*Pie
cewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 +
e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d
**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e*
*2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e
**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x*
*5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d*
*2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewi
se((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e*
*2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*
x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e
, 0)), (x**8*sqrt(d**2)/8, True)) + 3*d*e**6*Piecewise((-7*I*d**10*acosh(e*
x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**
7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt
(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) -
 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9)
- 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt
(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) +
d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*
x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Pie
cewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*sqrt(
d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**2)/(1155*e
**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) - d**2*x**8*sqrt(d**2
```

```
- e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x**10
*sqrt(d**2)/10, True))
```

Giac [A]
time = 1.35, size = 139, normalized size = 0.62

$$\frac{19}{256}\,d^{11}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{887040}\,\left(94720\,d^{10}e^{(-3)}+\left(65835\,d^{9}e^{(-2)}+2\,\left(23680\,d^{8}e^{(-1)}-\left(125895\,d^{7}+4\,\left(78720\,d^{6}e+\left(25179\,d^{5}e^{2}-2\,\left(41120\,d^{4}e^{3}+7\,\left(5247\,d^{3}e^{4}-8\,\left(100\,d^{2}e^{5}+9\,\left(10\,xe^{7}+33\,de^{6}\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 19/256*d^11*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/887040*(94720*d^10*e^(-3) + (65
835*d^9*e^(-2) + 2*(23680*d^8*e^(-1) - (125895*d^7 + 4*(78720*d^6*e + (2517
9*d^5*e^2 - 2*(41120*d^4*e^3 + 7*(5247*d^3*e^4 - 8*(100*d^2*e^5 + 9*(10*x*e
^7 + 33*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.69 $\int x(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=230

$$\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}-\frac{11d^2(d+ex)\left(d^2-e^2\right.}{240e^2}$$

```
[Out] 11/128*d^6*x*(-e^2*x^2+d^2)^(3/2)/e+11/160*d^4*x*(-e^2*x^2+d^2)^(5/2)/e-33/
560*d^3*(-e^2*x^2+d^2)^(7/2)/e^2-11/240*d^2*(e*x+d)*(-e^2*x^2+d^2)^(7/2)/e^
2-1/30*d*(e*x+d)^2*(-e^2*x^2+d^2)^(7/2)/e^2-1/10*(e*x+d)^3*(-e^2*x^2+d^2)^(
7/2)/e^2+33/256*d^10*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+33/256*d^8*x*(-e^
2*x^2+d^2)^(1/2)/e
```

Rubi [A]
time = 0.07, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {809, 685, 655, 201, 223, 209}

$$\frac{33d^{10}\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2}-\frac{11d^2(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{240e^2}-\frac{d(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{30e^2}-\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}{10e^2}+\frac{33d^8x\sqrt{d^2-e^2x^2}}{256e}+\frac{11d^6x\left(d^2-e^2x^2\right)^{3/2}}{128e}+\frac{11d^4x\left(d^2-e^2x^2\right)^{5/2}}{160e}-\frac{33d^3\left(d^2-e^2x^2\right)^{7/2}}{560e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (33*d^8*x*Sqrt[d^2 - e^2*x^2])/(256*e) + (11*d^6*x*(d^2 - e^2*x^2)^(3/2))/(
128*e) + (11*d^4*x*(d^2 - e^2*x^2)^(5/2))/(160*e) - (33*d^3*(d^2 - e^2*x^2)
^(7/2))/(560*e^2) - (11*d^2*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(240*e^2) - (d
*(d + e*x)^2*(d^2 - e^2*x^2)^(7/2))/(30*e^2) - ((d + e*x)^3*(d^2 - e^2*x^2)
^(7/2))/(10*e^2) + (33*d^10*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(256*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\int x(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx = -\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2} + \frac{(3d)\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} dx}{10e}$$

$$= -\frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2} - \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}}{10e^2} + \frac{(11d^2)\int (d+e$$

$$= -\frac{11d^2(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{240e^2} - \frac{d(d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}}{30e^2} - \frac{(d+ex)^3 (}{1}$$

$$= -\frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2} - \frac{11d^2(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{240e^2} - \frac{d(d+ex)^2 \left(d^2-e\right.}{30e^2}$$

$$= \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2} - \frac{11d^2(d+ex) \left(d^2-e^2x^2\right)^{7/2}}{240e^2}$$

$$= \frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} + \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2} - \frac{11d^2}{}$$

$$= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e} + \frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} + \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3}{}$$

$$= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e} + \frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} + \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3}{}$$

$$= \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e} + \frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} + \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3}{}$$

Mathematica [A]
time = 0.41, size = 166, normalized size = 0.72

$$\frac{e\sqrt{d^2-e^2x^2}\,(-6400d^9-3465d^8ex+10240d^7e^2x^2+24570d^6e^3x^3+7680d^5e^4x^4-23352d^4e^5x^5-20480d^3e^6x^6+3024d^2e^7x^7+8960de^8x^8+2688e^9x^9)+3465d^{10}\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{26880e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-6400*d^9 - 3465*d^8*e*x + 10240*d^7*e^2*x^2 + 2457
0*d^6*e^3*x^3 + 7680*d^5*e^4*x^4 - 23352*d^4*e^5*x^5 - 20480*d^3*e^6*x^6 +
3024*d^2*e^7*x^7 + 8960*d*e^8*x^8 + 2688*e^9*x^9) + 3465*d^10*Sqrt[-e^2]*Lo
g[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(26880*e^3)
```

Maple [A]
time = 0.06, size = 366, normalized size = 1.59

method	result

risch	$-\frac{(-2688e^9x^9-8960d\,e^8x^8-3024d^2e^7x^7+20480d^3e^6x^6+23352d^4e^5x^5-7680d^5e^4x^4-24570d^6e^3x^3-10240x^2d^7e^2+3465d^8xe+6400d^9}{26880e^2}$

default | $e^3 \left(-\frac{x^3(-e^2x^2+d^2)^{\frac{7}{2}}}{10e^2} + \frac{3d^2 \left(-\frac{x(-e^2x^2+d^2)^{\frac{7}{2}}}{8e^2} + \frac{d^2 \left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6} + \frac{5d^2 \left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4} + \frac{3d^2 \left(\frac{x\sqrt{-e^2x^2+d^2}}{2} + \frac{d^2 \text{ ar}}{} \right.}{4} \right.}{6} \right.}{8e^2} \right.}{10e^2} \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/10*x^3*(-e^2*x^2+d^2)^(7/2)/e^2+3/10*d^2/e^2*(-1/8*x*(-e^2*x^2+d^2)
^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2
+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))+3*e^2*d*(-1/9*x^2*(-e^2*x^2+d^2)^(
7/2)/e^2-2/63*d^2/e^4*(-e^2*x^2+d^2)^(7/2))+3*e*d^2*(-1/8*x*(-e^2*x^2+d^2)^
(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+
d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((
e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-1/7*d^3*(-e^2*x^2+d^2)^(7/2)/e^2
```

Maxima [A]
time = 0.49, size = 159, normalized size = 0.69

$$\frac{33}{256}\,d^{10}\arcsin\left(\frac{xe}{d}\right)e^{(-2)}+\frac{33}{256}\,\sqrt{-x^2e^2+d^2}\,d^8xe^{(-1)}+\frac{11}{128}\,\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^6xe^{(-1)}+\frac{11}{160}\,\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^4xe^{(-1)}-\frac{1}{10}\,\left(-x^2e^2+d^2\right)^{\frac{7}{2}}x^3e-\frac{33}{80}\,\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^2xe^{(-1)}-\frac{5}{21}\,\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^3e^{(-2)}-\frac{1}{3}\,\left(-x^2e^2+d^2\right)^{\frac{7}{2}}dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 33/256*d^10*arcsin(x*e/d)*e^(-2) + 33/256*sqrt(-x^2*e^2 + d^2)*d^8*x*e^(-1)
 + 11/128*(-x^2*e^2 + d^2)^(3/2)*d^6*x*e^(-1) + 11/160*(-x^2*e^2 + d^2)^(5/
2)*d^4*x*e^(-1) - 1/10*(-x^2*e^2 + d^2)^(7/2)*x^3*e - 33/80*(-x^2*e^2 + d^2
)^(7/2)*d^2*x*e^(-1) - 5/21*(-x^2*e^2 + d^2)^(7/2)*d^3*e^(-2) - 1/3*(-x^2*e
^2 + d^2)^(7/2)*d*x^2
```

Fricas [A]
time = 1.86, size = 139, normalized size = 0.60

$$-\frac{1}{26880}\left(6930\,d^{10}\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(2688\,x^9e^9+8960\,dx^8e^8+3024\,d^2x^7e^7-20480\,d^3x^6e^6-23352\,d^4x^5e^5+7680\,d^5x^4e^4+24570\,d^6x^3e^3+10240\,d^7x^2e^2-3465\,d^8xe-6400\,d^9\right)\sqrt{-x^2e^2+d^2}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/26880*(6930*d^10*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (2688*x^
9*e^9 + 8960*d*x^8*e^8 + 3024*d^2*x^7*e^7 - 20480*d^3*x^6*e^6 - 23352*d^4*x
^5*e^5 + 7680*d^5*x^4*e^4 + 24570*d^6*x^3*e^3 + 10240*d^7*x^2*e^2 - 3465*d^
8*x*e - 6400*d^9)*sqrt(-x^2*e^2 + d^2))*e^(-2)
```

Sympy [A]
time = 137.53, size = 1554, normalized size = 6.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) + 3*d**6*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*
d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x*
*2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2
) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2
)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x
**2/d**2)), True)) + d**5*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(1
5*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**
2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 5*d**4*e**3*Piecewise((-
I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)
) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-
1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**
2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e
**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(
24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), Tr
ue)) - 5*d**3*e**4*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4
*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x
**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)
/6, True)) + d**2*e**5*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d
**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7
*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*
x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d
**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 -
e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7
/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)),
True)) + 3*d*e**6*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8
*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2
*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(
d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) + e**7*Piecewise
((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*
x**2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5
*x**5/(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1
 + e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x*
*11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*as
in(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7*d**
7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1
- e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x*
*9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2
)), True))
```

Giac [A]

time = 1.06, size = 128, normalized size = 0.56

$$\frac{33}{256}\,d^{10}\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{1}{26880}\left(6400\,d^9e^{(-2)}+\left(3465\,d^8e^{(-1)}-2\left(5120\,d^7+\left(12285\,d^6e+4\left(960\,d^5e^2-\left(2919\,d^4e^3+2\left(1280\,d^3e^4-7\left(27\,d^2e^5+8\left(3\,xe^7+10\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 33/256*d^10*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/26880*(6400*d^9*e^(-2) + (3465*
d^8*e^(-1) - 2*(5120*d^7 + (12285*d^6*e + 4*(960*d^5*e^2 - (2919*d^4*e^3 +
2*(1280*d^3*e^4 - 7*(27*d^2*e^5 + 8*(3*x*e^7 + 10*d*e^6)*x)*x)*x)*x)*x)*x)*
x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(d^2-e^2\,x^2\right)^{5/2}\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.70 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=188

$$\frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-\right.}{72e}$$

```
[Out] 55/192*d^5*x*(-e^2*x^2+d^2)^(3/2)+11/48*d^3*x*(-e^2*x^2+d^2)^(5/2)-11/56*d^
2*(-e^2*x^2+d^2)^(7/2)/e-11/72*d*(e*x+d)*(-e^2*x^2+d^2)^(7/2)/e-1/9*(e*x+d)
^2*(-e^2*x^2+d^2)^(7/2)/e+55/128*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+55/
128*d^7*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {685, 655, 201, 223, 209}

$$\frac{55d^9\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (55*d^7*x*Sqrt[d^2 - e^2*x^2])/128 + (55*d^5*x*(d^2 - e^2*x^2)^(3/2))/192 +
 (11*d^3*x*(d^2 - e^2*x^2)^(5/2))/48 - (11*d^2*(d^2 - e^2*x^2)^(7/2))/(56*e
) - (11*d*(d + e*x)*(d^2 - e^2*x^2)^(7/2))/(72*e) - ((d + e*x)^2*(d^2 - e^2
*x^2)^(7/2))/(9*e) + (55*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int (d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{1}{9}(11d)\int (d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx\\
&= -\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}{9e}+\frac{1}{8}\left(11d^2\right)\int (d+e\\
&= -\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{7}}{9e}\\
&= \frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(d+ex)\left(d^2-e^2x^2\right)^{7/2}}{72e}-\\
&= \frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2\left(d^2-e^2x^2\right)^{7/2}}{56e}-\frac{11d(}{}\\
&= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2}{}\\
&= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2}{}\\
&= \frac{55}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{55}{192}d^5x\left(d^2-e^2x^2\right)^{3/2}+\frac{11}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}-\frac{11d^2}{}
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 155, normalized size = 0.82

$$
\frac{\sqrt{d^2-e^2x^2}\left(-3712d^8+4599d^7ex+10240d^6e^2x^2+3066d^5e^3x^3-8448d^4e^4x^4-7224d^3e^5x^5+1024d^2e^6x^6+3024de^7x^7+896e^8x^8\right)}{8064e}-\frac{55d^9\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{128\sqrt{-e^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3712*d^8 + 4599*d^7*e*x + 10240*d^6*e^2*x^2 + 3066*d
^5*e^3*x^3 - 8448*d^4*e^4*x^4 - 7224*d^3*e^5*x^5 + 1024*d^2*e^6*x^6 + 3024*
d*e^7*x^7 + 896*e^8*x^8))/(8064*e) - (55*d^9*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2
 - e^2*x^2]])/(128*Sqrt[-e^2])
```

Maple [A]

time = 0.07, size = 304, normalized size = 1.62

method	result
risch	$-\frac{\left(-896 e^{8} x^{8}-3024 d\, e^{7} x^{7}-1024 d^{2} e^{6} x^{6}+7224 d^{3} e^{5} x^{5}+8448 d^{4} e^{4} x^{4}-3066 d^{5} e^{3} x^{3}-10240 d^{6} e^{2} x^{2}-4599 d^{7} e x+3712 d^{8}\right) \sqrt{-e^{2} x^{2}+}}{8064 e}$
default	$e^{3}\left(-\frac{x^{2}\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{9 e^{2}}-\frac{2 d^{2}\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{63 e^{4}}\right)+3 d\, e^{2}\left(-\frac{x\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{8 e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5 d^{2}\left(\frac{x\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*(-e^2*x^2+d^2)^(7/2))+3
*d*e^2*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(
5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2
)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-3/7*d^
2*(-e^2*x^2+d^2)^(7/2)/e+d^3*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e
^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))
```

Maxima [A]
time = 0.49, size = 130, normalized size = 0.69

$$\frac{55}{128}\,d^9\arcsin\left(\frac{xe}{d}\right)e^{(-1)}+\frac{55}{128}\,\sqrt{-x^2e^2+d^2}\,d^7x+\frac{55}{192}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^5x+\frac{11}{48}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^3x-\frac{1}{9}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}x^2e-\frac{29}{63}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^2e^{(-1)}-\frac{3}{8}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 55/128*d^9*arcsin(x*e/d)*e^(-1) + 55/128*sqrt(-x^2*e^2 + d^2)*d^7*x + 55/19
2*(-x^2*e^2 + d^2)^(3/2)*d^5*x + 11/48*(-x^2*e^2 + d^2)^(5/2)*d^3*x - 1/9*(
-x^2*e^2 + d^2)^(7/2)*x^2*e - 29/63*(-x^2*e^2 + d^2)^(7/2)*d^2*e^(-1) - 3/8
*(-x^2*e^2 + d^2)^(7/2)*d*x
```

Fricas [A]
time = 1.18, size = 129, normalized size = 0.69

$$-\frac{1}{8064}\left(6930\,d^9\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(896\,x^8e^8+3024\,dx^7e^7+1024\,d^2x^6e^6-7224\,d^3x^5e^5-8448\,d^4x^4e^4+3066\,d^5x^3e^3+10240\,d^6x^2e^2+4599\,d^7xe-3712\,d^8\right)\sqrt{-x^2e^2+d^2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(6930*d^9*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (896*x^8*e
^8 + 3024*d*x^7*e^7 + 1024*d^2*x^6*e^6 - 7224*d^3*x^5*e^5 - 8448*d^4*x^4*e^
4 + 3066*d^5*x^3*e^3 + 10240*d^6*x^2*e^2 + 4599*d^7*x*e - 3712*d^8)*sqrt(-x
^2*e^2 + d^2))*e^(-1)
```

Sympy [C] Result contains complex when optimal does not.
time = 29.38, size = 1284, normalized size = 6.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + 3*d*
*6*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
```

```
)/(3*e**2), True)) + d**5*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I
*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x
**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**
2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**
2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*
x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)
/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 -
e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) - 5*d**3*e**4*Piecewise
((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d*
*2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqr
t(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(
e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1
- e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**
5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)),
 True)) + d**2*e**5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) -
4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*
x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2
)/6, True)) + 3*d*e**6*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d
**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-
1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7
*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*
x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d
**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 -
e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7
/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)),
True)) + e**7*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**
6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**
2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2
 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [A]
time = 1.33, size = 117, normalized size = 0.62

$$\frac{55}{128}\,d^9\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d)-\frac{1}{8064}\left(3712\,d^8e^{(-1)}-\left(4599\,d^7+2\left(5120\,d^6e+\left(1533\,d^5e^2-4\left(1056\,d^4e^3+\left(903\,d^3e^4-2\left(64\,d^2e^5+7\left(8\,xe^7+27\,de^6\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] 55/128*d^9*arcsin(x*e/d)*e^(-1)*sgn(d) - 1/8064*(3712*d^8*e^(-1) - (4599*d^
7 + 2*(5120*d^6*e + (1533*d^5*e^2 - 4*(1056*d^4*e^3 + (903*d^3*e^4 - 2*(64*
d^2*e^5 + 7*(8*x*e^7 + 27*d*e^6)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3,x)
[Out] int((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3, x)
```

3.71 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=190

$$\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-$$

```
[Out] 1/192*d^4*(125*e*x+64*d)*(-e^2*x^2+d^2)^(3/2)+1/240*d^2*(125*e*x+48*d)*(-e^
2*x^2+d^2)^(5/2)-3/7*d*(-e^2*x^2+d^2)^(7/2)-1/8*e*x*(-e^2*x^2+d^2)^(7/2)+12
5/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d^8*arctanh((-e^2*x^2+d^2)^(1/2)
/d)+1/128*d^6*(125*e*x+128*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1823, 829, 858, 223, 209, 272, 65, 214}

$$\frac{125}{128}d^8\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{240}d^2(48d+125ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{3}{7}d\left(d^2-e^2x^2\right)^{7/2}-\frac{1}{8}ex\left(d^2-e^2x^2\right)^{7/2}-d^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2}+\frac{1}{192}d^4(64d+125ex)\left(d^2-e^2x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (d^6*(128*d + 125*e*x)*Sqrt[d^2 - e^2*x^2])/128 + (d^4*(64*d + 125*e*x)*(d^
2 - e^2*x^2)^(3/2))/192 + (d^2*(48*d + 125*e*x)*(d^2 - e^2*x^2)^(5/2))/240
- (3*d*(d^2 - e^2*x^2)^(7/2))/7 - (e*x*(d^2 - e^2*x^2)^(7/2))/8 + (125*d^8*
ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/128 - d^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x}\,dx = -\frac{1}{8}ex(d^2-e^2x^2)^{7/2} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-8d^3e^2-25d^2e^3x-24de^4x^2)}{x}\,dx}{8e^2}$$

$$= -\frac{3}{7}d(d^2-e^2x^2)^{7/2} - \frac{1}{8}ex(d^2-e^2x^2)^{7/2} + \frac{\int \frac{(56d^3e^4+175d^2e^5x)(d^2-e^2x^2)^{5/2}}{x}\,dx}{56e^4}$$

$$= \frac{1}{240}d^2(48d+125ex)(d^2-e^2x^2)^{5/2} - \frac{3}{7}d(d^2-e^2x^2)^{7/2} - \frac{1}{8}ex(d^2-e^2x^2)$$

$$= \frac{1}{192}d^4(64d+125ex)(d^2-e^2x^2)^{3/2} + \frac{1}{240}d^2(48d+125ex)(d^2-e^2x^2)^{5/2}$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)(d^2-e^2x^2)^{3/2} +$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)(d^2-e^2x^2)^{3/2} +$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)(d^2-e^2x^2)^{3/2} +$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)(d^2-e^2x^2)^{3/2} +$$

$$= \frac{1}{128}d^6(128d+125ex)\sqrt{d^2-e^2x^2} + \frac{1}{192}d^4(64d+125ex)(d^2-e^2x^2)^{3/2} +$$

Mathematica [A]
time = 0.55, size = 184, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\,(14848d^7+27195d^6ex+7424d^5e^2x^2-17710d^4e^3x^3-14592d^3e^4x^4+1960d^2e^5x^5+5760de^6x^6+1680e^7x^7)}{13440} + 2d^8\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{125d^8e\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{128\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(14848*d^7 + 27195*d^6*e*x + 7424*d^5*e^2*x^2 - 17710*
d^4*e^3*x^3 - 14592*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 + 5760*d*e^6*x^6 + 1680*
e^7*x^7))/13440 + 2*d^8*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] -
 (125*d^8*e*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(128*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(164) = 328$.
time = 0.08, size = 353, normalized size = 1.86

method	result
default	$e^3\left(-\frac{x(-e^2x^2+d^2)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{8e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2
)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1
/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-3/7*d*(-e
^2*x^2+d^2)^(7/2)+3*d^2*e*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*
x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+d^3*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2
*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]

time = 0.48, size = 200, normalized size = 1.05

$$\frac{125}{128}\,d^8\arcsin\left(\frac{xe}{d}\right)-d^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)+\frac{125}{128}\,\sqrt{-x^2e^2+d^2}\,d^6xe+\sqrt{-x^2e^2+d^2}\,d^7+\frac{125}{192}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^4xe+\frac{1}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^5+\frac{25}{48}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^2xe+\frac{1}{5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^3-\frac{1}{8}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}xe-\frac{3}{7}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 125/128*d^8*arcsin(x*e/d) - d^8*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d
/abs(x)) + 125/128*sqrt(-x^2*e^2 + d^2)*d^6*x*e + sqrt(-x^2*e^2 + d^2)*d^7
+ 125/192*(-x^2*e^2 + d^2)^(3/2)*d^4*x*e + 1/3*(-x^2*e^2 + d^2)^(3/2)*d^5 +
 25/48*(-x^2*e^2 + d^2)^(5/2)*d^2*x*e + 1/5*(-x^2*e^2 + d^2)^(5/2)*d^3 - 1/
8*(-x^2*e^2 + d^2)^(7/2)*x*e - 3/7*(-x^2*e^2 + d^2)^(7/2)*d
```

Fricas [A]
time = 1.39, size = 142, normalized size = 0.75

$$-\frac{125}{64}\,d^8\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+d^8\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\frac{1}{13440}\left(1680\,x^7e^7+5760\,dx^6e^6+1960\,d^2x^5e^5-14592\,d^3x^4e^4-17710\,d^4x^3e^3+7424\,d^5x^2e^2+27195\,d^6xe+14848\,d^7\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -125/64*d^8*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + d^8*log(-(d - sq
rt(-x^2*e^2 + d^2))/x) + 1/13440*(1680*x^7*e^7 + 5760*d*x^6*e^6 + 1960*d^2*
x^5*e^5 - 14592*d^3*x^4*e^4 - 17710*d^4*x^3*e^3 + 7424*d^5*x^2*e^2 + 27195*
d^6*x*e + 14848*d^7)*sqrt(-x^2*e^2 + d^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 39.16, size = 1263, normalized size = 6.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x,x)
```

```
[Out] d**7*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) + 3*d**6*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(
2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)),
 Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2
/d**2)/2, True)) + d**5*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-
(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) - 5*d**4*e**3*Piecewise((-I*d**4
*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*
d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2
/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e
```

```
**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**
2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewise((-2*d
**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15
*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True
)) + d**2*e**5*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**
4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**
2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1
+ e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) -
 d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e*
*2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sq
rt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewise((-8*d**6*sqrt(d**2 - e
**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2
*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(
e, 0)), (x**6*sqrt(d**2)/6, True)) + e**7*Piecewise((-5*I*d**8*acosh(e*x/d)
/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x*
*3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(
8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/
d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/
(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**
2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 -
 e**2*x**2/d**2)), True))
```

Giac [A]
time = 1.17, size = 143, normalized size = 0.75

$$\frac{125}{128}\,d^{8}\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d)-d^{8}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{1}{13440}\,\left(14848\,d^{7}+\left(27195\,d^{6}e+2\,\left(3712\,d^{5}e^{2}-\left(8855\,d^{4}e^{3}+4\,\left(1824\,d^{3}e^{4}-5\,\left(49\,d^{2}e^{5}+6\,\left(7\,xe^{7}+24\,de^{6}\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 125/128*d^8*arcsin(x*e/d)*sgn(d) - d^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/13440*(14848*d^7 + (27195*d^6*e + 2*(3712*d^5
*e^2 - (8855*d^4*e^3 + 4*(1824*d^3*e^4 - 5*(49*d^2*e^5 + 6*(7*x*e^7 + 24*d*
e^6)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x, x)
```

3.72 $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^2} \, dx$

Optimal. Leaf size=193

$$\frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right.$$

```
[Out] 1/8*d^3*e*(-5*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)+1/10*d*e*(-5*e*x+6*d)*(-e^2*x^2
+d^2)^(5/2)-1/7*e*(-e^2*x^2+d^2)^(7/2)-d*(-e^2*x^2+d^2)^(7/2)/x-15/16*d^7*e
*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-3*d^7*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)+3
/16*d^5*e*(-5*e*x+16*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1821, 1823, 829, 858, 223, 209, 272, 65, 214}

$$-\frac{15}{16}d^7e\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d(d^2-e^2x^2)^{7/2}}{x}+\frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2}-3d^7e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2}+\frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (3*d^5*e*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^3*e*(8*d - 5*e*x)*(d^2
 - e^2*x^2)^(3/2))/8 + (d*e*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/10 - (e*(d
^2 - e^2*x^2)^(7/2))/7 - (d*(d^2 - e^2*x^2)^(7/2))/x - (15*d^7*e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]])/16 - 3*d^7*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
```

```
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^2}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-3d^4e+3d^3e^2x-d^2e^3x^2\right)}{x}\,dx}{d^2}$$

$$= -\frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} + \frac{\int \frac{\left(21d^4e^3-21d^3e^4x\right)\left(d^2-e^2x^2\right)^{5/2}}{x}\,dx}{7d^2e^2}$$

$$= \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2-e^2x^2\right)^{7/2} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{x} - \int$$

$$= \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6d-5ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{1}{7}e\left(d^2 -\right.$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6$$

$$= \frac{3}{16}d^5e(16d-5ex)\sqrt{d^2-e^2x^2} + \frac{1}{8}d^3e(8d-5ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{10}de(6$$

Mathematica [A]
time = 0.53, size = 187, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(-560d^7+2496d^6ex+525d^5e^2x^2-992d^4e^3x^3-770d^3e^4x^4+96d^2e^5x^5+280de^6x^6+80e^7x^7\right)}{560x} + 6d^7e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{15}{16}d^7\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 2496*d^6*e*x + 525*d^5*e^2*x^2 - 992*d^4*e
^3*x^3 - 770*d^3*e^4*x^4 + 96*d^2*e^5*x^5 + 280*d*e^6*x^6 + 80*e^7*x^7))/(5
60*x) + 6*d^7*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (15*d^7
*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(169) = 338$.

time = 0.10, size = 357, normalized size = 1.85

method	result
risch	$-\frac{d^7\sqrt{-e^2x^2+d^2}}{x}+\frac{e^7x^6\sqrt{-e^2x^2+d^2}}{7}+\frac{6e^5d^2x^4\sqrt{-e^2x^2+d^2}}{35}-\frac{62e^3d^4x^2\sqrt{-e^2x^2+d^2}}{35}+\frac{156e\,d^6\sqrt{}}{}$
default	$-\frac{e(-e^2x^2+d^2)^{\frac{7}{2}}}{7}+3d\,e^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right.}{4}\right.}{6}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*e*(-e^2*x^2+d^2)^(7/2)+3*d*e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/
4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+d^3*(-1/d^2/x*(-e^2*x^2
```

```
+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+
d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((
e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+3*d^2*e*(1/5*(-e^2*x^2+d^2)^(5/2)+d^
2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [**A**]
time = 0.49, size = 212, normalized size = 1.10

$$-\frac{15}{16} d^7 \arcsin\left(\frac{xe}{d}\right) e - 3 d^7 e \log\left(\frac{2d^2}{|x|} + \frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right) - \frac{15}{16}\sqrt{-x^2e^2+d^2}\, d^5 x e^2 + 3\sqrt{-x^2e^2+d^2}\, d^6 e - \frac{5}{8}(-x^2e^2+d^2)^{\frac{3}{2}} d^3 x e^2 + (-x^2e^2+d^2)^{\frac{3}{2}} d^4 e + \frac{1}{2}(-x^2e^2+d^2)^{\frac{5}{2}} d x e^2 + \frac{3}{5}(-x^2e^2+d^2)^{\frac{5}{2}} d^2 e - \frac{(-x^2e^2+d^2)^{\frac{5}{2}} d^3}{x} - \frac{1}{7}(-x^2e^2+d^2)^{\frac{7}{2}} e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] -15/16*d^7*arcsin(x*e/d)*e - 3*d^7*e*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d
^2)*d/abs(x)) - 15/16*sqrt(-x^2*e^2 + d^2)*d^5*x*e^2 + 3*sqrt(-x^2*e^2 + d^
2)*d^6*e - 5/8*(-x^2*e^2 + d^2)^(3/2)*d^3*x*e^2 + (-x^2*e^2 + d^2)^(3/2)*d^
4*e + 1/2*(-x^2*e^2 + d^2)^(5/2)*d*x*e^2 + 3/5*(-x^2*e^2 + d^2)^(5/2)*d^2*e
 - (-x^2*e^2 + d^2)^(5/2)*d^3/x - 1/7*(-x^2*e^2 + d^2)^(7/2)*e
```

Fricas [**A**]
time = 2.32, size = 161, normalized size = 0.83

$$\frac{1050\, d^7 x \arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right) e + 1680\, d^7 x e \log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right) + 2496\, d^7 x e + \left(80\, x^7 e^7 + 280\, d x^6 e^6 + 96\, d^2 x^5 e^5 - 770\, d^3 x^4 e^4 - 992\, d^4 x^3 e^3 + 525\, d^5 x^2 e^2 + 2496\, d^6 x e - 560\, d^7\right)\sqrt{-x^2e^2+d^2}}{560\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/560*(1050*d^7*x*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e + 1680*d^7
*x*e*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + 2496*d^7*x*e + (80*x^7*e^7 + 280*
d*x^6*e^6 + 96*d^2*x^5*e^5 - 770*d^3*x^4*e^4 - 992*d^4*x^3*e^3 + 525*d^5*x^
2*e^2 + 2496*d^6*x*e - 560*d^7)*sqrt(-x^2*e^2 + d^2))/x
```

Sympy [**C**] Result contains complex when optimal does not.
time = 11.82, size = 1057, normalized size = 5.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**2,x)
```

```
[Out] d**7*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) + 3*d**6*e*Piecewise((d**2/(e*x**sqrt(d**2/(e**2*x**2) - 1)) - d*acos
```

```
h(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-
I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-
d**2/(e**2*x**2) + 1), True)) + d**5*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(
2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**
2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt
(1 - e**2*x**2/d**2)/2, True)) - 5*d**4*e**3*Piecewise((x**2*sqrt(d**2)/2,
Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) - 5*d**3*e**4*Pi
ecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x*
*2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqr
t(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**
3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x
**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*P
iecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt
(d**2)/4, True)) + 3*d*e**6*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d
**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/
(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d
)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**
2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2
*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((-8*d**6*sqrt
(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**
4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**
2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

Giac [A]
time = 1.36, size = 199, normalized size = 1.03

$$-\frac{15}{16}\,d^{7}\arcsin\left(\frac{xe}{d}\right)\mathrm{esgn}(d)-3\,d^{7}e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{d^{7}xe^{3}}{2\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d^{7}e^{(-1)}}{2\,x}+\frac{1}{560}\left(2496\,d^{6}e+\left(525\,d^{5}e^{2}-2\left(496\,d^{4}e^{3}+\left(385\,d^{3}e^{4}-4\left(12\,d^{2}e^{5}+5\left(2\,xe^{7}+7\,de^{6}\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*d^7*arcsin(x*e/d)*e*sgn(d) - 3*d^7*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^7*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*
e) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^7*e^(-1)/x + 1/560*(2496*d^6*e +
(525*d^5*e^2 - 2*(496*d^4*e^3 + (385*d^3*e^4 - 4*(12*d^2*e^5 + 5*(2*x*e^7 +
 7*d*e^6)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^2, x)
```

3.73 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=207

$$\frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{d(d^2-e^2}{2x^2}$$

```
[Out] 1/24*d^2*e^2*(-85*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)+1/30*e^2*(-85*e*x+3*d)*(-e^
2*x^2+d^2)^(5/2)-1/2*d*(-e^2*x^2+d^2)^(7/2)/x^2-3*e*(-e^2*x^2+d^2)^(7/2)/x-
85/16*d^6*e^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-1/2*d^6*e^2*arctanh((-e^2*x^
2+d^2)^(1/2)/d)+1/16*d^4*e^2*(-85*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1821, 829, 858, 223, 209, 272, 65, 214}

$$-\frac{85}{16}d^6e^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{x}+\frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2}-\frac{1}{2}d^6e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] (d^4*e^2*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/16 + (d^2*e^2*(4*d - 85*e*x)*(
d^2 - e^2*x^2)^(3/2))/24 + (e^2*(3*d - 85*e*x)*(d^2 - e^2*x^2)^(5/2))/30 -
(d*(d^2 - e^2*x^2)^(7/2))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(7/2))/x - (85*d^6
*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/16 - (d^6*e^2*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^3} \, dx = -\frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-6d^4e-d^3e^2x-2d^2e^3x^2)}{x^2} \, dx}{2d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{3e(d^2-e^2x^2)^{7/2}}{x} + \frac{\int \frac{(d^5e^2-34d^4e^3x)(d^2-e^2x^2)^{5/2}}{x} \, dx}{2d^4}$$

$$= \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d(d^2-e^2x^2)^{7/2}}{2x^2} - \frac{3e(d^2-e^2x^2)^{7/2}}{x} - \int$$

$$= \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2(3d-85ex)\left(d^2-e^2x^2\right)^{5/2} - \frac{d(d^2}{}$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2$$

$$= \frac{1}{16}d^4e^2(8d-85ex)\sqrt{d^2-e^2x^2} + \frac{1}{24}d^2e^2(4d-85ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{30}e^2$$

Mathematica [A]
time = 0.63, size = 189, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}\left(-120d^7-720d^6ex+544d^5e^2x^2-645d^4e^3x^3-448d^3e^4x^4+50d^2e^5x^5+144de^6x^6+40e^7x^7\right)}{240x^2} + d^6e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{85}{16}d^6e\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^3,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^7 - 720*d^6*e*x + 544*d^5*e^2*x^2 - 645*d^4*e^
3*x^3 - 448*d^3*e^4*x^4 + 50*d^2*e^5*x^5 + 144*d*e^6*x^6 + 40*e^7*x^7))/(24
0*x^2) + d^6*e^2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (85*d^
6*e*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(181) = 362$.
time = 0.07, size = 474, normalized size = 2.29

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(-40 e^{7} x^{7}-144 d\,e^{6} x^{6}-50 d^{2} e^{5} x^{5}+448 d^{3} e^{4} x^{4}+645 d^{4} e^{3} x^{3}-544 d^{5} e^{2} x^{2}+720 d^{6} e x+120 d^{7}\right)}{240 x^{2}}-\frac{85 d^{6} e^{3} \arctan\left(\right.}{1}$
default	$e^{3}\left(\frac{x\left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5 d^{2}\left(\frac{x\left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2}\left(\frac{x\sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)+d^{3}\left(-\frac{(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2
*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2
*x^2+d^2)^(1/2)))))+d^3*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5
*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/
2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+3*d
^2*e*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5
```

/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*
d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+3*d*e^2*(1/5
*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/
2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))

Maxima [A]
time = 0.49, size = 214, normalized size = 1.03

$$-\frac{85}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^2-\frac{1}{2}\,d^6e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\frac{85}{16}\,\sqrt{-x^2e^2+d^2}\,d^4xe^3+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,d^5e^2-\frac{85}{24}\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2xe^3+\frac{1}{6}\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3e^2+\frac{1}{6}\,(-x^2e^2+d^2)^{\frac{5}{2}}xe^3+\frac{1}{10}\,(-x^2e^2+d^2)^{\frac{5}{2}}de^2-\frac{3\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2e}{x}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -85/16*d^6*arcsin(x*e/d)*e^2 - 1/2*d^6*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e
^2 + d^2)*d/abs(x)) - 85/16*sqrt(-x^2*e^2 + d^2)*d^4*x*e^3 + 1/2*sqrt(-x^2*
e^2 + d^2)*d^5*e^2 - 85/24*(-x^2*e^2 + d^2)^(3/2)*d^2*x*e^3 + 1/6*(-x^2*e^2
 + d^2)^(3/2)*d^3*e^2 + 1/6*(-x^2*e^2 + d^2)^(5/2)*x*e^3 + 1/10*(-x^2*e^2 +
 d^2)^(5/2)*d*e^2 - 3*(-x^2*e^2 + d^2)^(5/2)*d^2*e/x - 1/2*(-x^2*e^2 + d^2)
^(7/2)*d/x^2
```

Fricas [A]
time = 2.93, size = 167, normalized size = 0.81

$$\frac{2550\,d^6x^2\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^2+120\,d^6x^2e^2\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+544\,d^6x^2e^2+\left(40\,x^7e^7+144\,dx^6e^6+50\,d^2x^5e^5-448\,d^3x^4e^4-645\,d^4x^3e^3+544\,d^5x^2e^2-720\,d^6xe-120\,d^7\right)\sqrt{-x^2e^2+d^2}}{240\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/240*(2550*d^6*x^2*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^2 + 120*
d^6*x^2*e^2*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + 544*d^6*x^2*e^2 + (40*x^7*
e^7 + 144*d*x^6*e^6 + 50*d^2*x^5*e^5 - 448*d^3*x^4*e^4 - 645*d^4*x^3*e^3 +
544*d^5*x^2*e^2 - 720*d^6*x*e - 120*d^7)*sqrt(-x^2*e^2 + d^2))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 12.36, size = 1059, normalized size = 5.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**3,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d),
 True)) + 3*d**6*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh
```

```
(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2
*x**2/d**2)), True)) + d**5*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2)
 - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*
x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x))
 + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**4*e**3*Piecewise((-I*d*
*2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(
2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)
/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - 5*d**3*e**4*Piecewise((x*
*2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
 + d**2*e**5*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sq
rt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asi
n(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*
sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)
) + 3*d*e**6*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**
2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0
)), (x**4*sqrt(d**2)/4, True)) + e**7*Piecewise((-I*d**6*acosh(e*x/d)/(16*e
**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*
sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*
e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*
asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x*
*3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**
2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 1.53, size = 256, normalized size = 1.24

$$-\frac{85}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^2\mathrm{sgn}(d)-\frac{1}{2}\,d^6e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6e^{(-2)}}{8\,x^2}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6}{2\,x}+\frac{\left(d^6e^2+\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6}{x}\right)x^2e^4}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}+\frac{1}{240}\left(544\,d^5e^2-\left(645\,d^4e^3+2\left(224\,d^3e^4-\left(25\,d^2e^5+4\left(5\,xe^7+18\,de^6\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -85/16*d^6*arcsin(x*e/d)*e^2*sgn(d) - 1/2*d^6*e^2*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2
*d^6*e^(-2)/x^2 - 3/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6/x + 1/8*(d^6*e^2 +
 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6/x)*x^2*e^4/(d*e + sqrt(-x^2*e^2 + d^
2)*e)^2 + 1/240*(544*d^5*e^2 - (645*d^4*e^3 + 2*(224*d^3*e^4 - (25*d^2*e^5
+ 4*(5*x*e^7 + 18*d*e^6)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^3, x)
```

3.74 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=210

$$-\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2}-\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x}-\frac{d(d^2-}{3}$$

```
[Out] -1/12*d*e^3*(25*e*x+26*d)*(-e^2*x^2+d^2)^(3/2)-1/30*e^2*(39*e*x+50*d)*(-e^2
*x^2+d^2)^(5/2)/x-1/3*d*(-e^2*x^2+d^2)^(7/2)/x^3-3/2*e*(-e^2*x^2+d^2)^(7/2)
/x^2-25/8*d^5*e^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+13/2*d^5*e^3*arctanh((-e
^2*x^2+d^2)^(1/2)/d)-1/8*d^3*e^3*(25*e*x+52*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1821, 827, 829, 858, 223, 209, 272, 65, 214}

$$-\frac{25}{8}d^5e^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{2x^2}-\frac{e^2(50d+39ex)\left(d^2-e^2x^2\right)^{5/2}}{30x}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{3x^3}-\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2}+\frac{13}{2}d^5e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]
```

```
[Out] -1/8*(d^3*e^3*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2]) - (d*e^3*(26*d + 25*e*x)
*(d^2 - e^2*x^2)^(3/2))/12 - (e^2*(50*d + 39*e*x)*(d^2 - e^2*x^2)^(5/2))/(3
0*x) - (d*(d^2 - e^2*x^2)^(7/2))/(3*x^3) - (3*e*(d^2 - e^2*x^2)^(7/2))/(2*x
^2) - (25*d^5*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (13*d^5*e^3*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^4}\,dx &= -\frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-9d^4e-5d^3e^2x-3d^2e^3x^2)}{x^3}\,dx}{3d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} + \frac{\int \frac{(10d^5e^2-39d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^2}\,dx}{6d^4} \\
&= -\frac{e^2(50d+39ex)(d^2-e^2x^2)^{5/2}}{30x} - \frac{d(d^2-e^2x^2)^{7/2}}{3x^3} - \frac{3e(d^2-e^2x^2)^{7/2}}{2x^2} - \int \\
&= -\frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{e^2(50d+39ex)(d^2-e^2x^2)^{5/2}}{30x} - d(\\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - e \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - e \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - e \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - e \\
&= -\frac{1}{8}d^3e^3(52d+25ex)\sqrt{d^2-e^2x^2} - \frac{1}{12}de^3(26d+25ex)\left(d^2-e^2x^2\right)^{3/2} - e
\end{aligned}$$

Mathematica [A]
time = 0.60, size = 189, normalized size = 0.90

$$\frac{\sqrt{d^2-e^2x^2}\,(-40d^7-180d^6ex-80d^5e^2x^2-656d^4e^3x^3-345d^3e^4x^4+32d^2e^5x^5+90de^6x^6+24e^7x^7)}{120x^3} - 13d^5e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{25}{8}d^5\left(-e^2\right)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^4,x]

[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^7 - 180*d^6*e*x - 80*d^5*e^2*x^2 - 656*d^4*e^3*
x^3 - 345*d^3*e^4*x^4 + 32*d^2*e^5*x^5 + 90*d*e^6*x^6 + 24*e^7*x^7))/(120*x
```

```
^3) - 13*d^5*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (25*d^
5*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(182) = 364$.
time = 0.09, size = 536, normalized size = 2.55

method	result
risch	$-\frac{d^5\sqrt{-e^2x^2+d^2}\,(4e^2x^2+9dex+2d^2)}{6x^3}+\frac{e^7x^4\sqrt{-e^2x^2+d^2}}{5}+\frac{4e^5d^2x^2\sqrt{-e^2x^2+d^2}}{15}-\frac{82e^3d^4\sqrt{-e^2x^2+}}{15}$

default	$3d^2e\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5}+d^2\left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3}+d^2\left(\sqrt{-e^2x^2+d^2}-\frac{d^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2}}{x}\right.}{\sqrt{d^2}}\right.\right.\right.}{2d^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3*d^2*e*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^
(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/
2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+3*d*e^2*(-1/d^2/x*(
-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-
e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*
arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+d^3*(-1/3/d^2/x^3*(-e^2*x^2+
d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^
2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x
^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
))))))+e^3*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e
^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x))))
```

Maxima [A]
time = 0.49, size = 211, normalized size = 1.00

$$-\frac{25}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^3+\frac{13}{2}\,d^5e^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\frac{25}{8}\,\sqrt{-x^2e^2+d^2}\,d^3xe^4-\frac{13}{2}\,\sqrt{-x^2e^2+d^2}\,d^4e^3-\frac{25}{12}\,(-x^2e^2+d^2)^{\frac{3}{2}}dxe^4-\frac{13}{6}\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2e^3-\frac{13}{10}\,(-x^2e^2+d^2)^{\frac{5}{2}}e^3-\frac{5\,(-x^2e^2+d^2)^{\frac{5}{2}}de^2}{3\,x}-\frac{3\,(-x^2e^2+d^2)^{\frac{7}{2}}e}{2\,x^2}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}d}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] -25/8*d^5*arcsin(x*e/d)*e^3 + 13/2*d^5*e^3*log(2*d^2/abs(x) + 2*sqrt(-x^2*e
^2 + d^2)*d/abs(x)) - 25/8*sqrt(-x^2*e^2 + d^2)*d^3*x*e^4 - 13/2*sqrt(-x^2*
e^2 + d^2)*d^4*e^3 - 25/12*(-x^2*e^2 + d^2)^(3/2)*d*x*e^4 - 13/6*(-x^2*e^2
+ d^2)^(3/2)*d^2*e^3 - 13/10*(-x^2*e^2 + d^2)^(5/2)*e^3 - 5/3*(-x^2*e^2 + d
^2)^(5/2)*d*e^2/x - 3/2*(-x^2*e^2 + d^2)^(7/2)*e/x^2 - 1/3*(-x^2*e^2 + d^2)
^(7/2)*d/x^3
```

Fricas [A]
time = 1.94, size = 167, normalized size = 0.80

$$\frac{750\,d^5x^3\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^3-780\,d^5x^3e^3\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-656\,d^5x^3e^3+(24\,x^7e^7+90\,dx^6e^6+32\,d^2x^5e^5-345\,d^3x^4e^4-656\,d^4x^3e^3-80\,d^5x^2e^2-180\,d^6xe-40\,d^7)\sqrt{-x^2e^2+d^2}}{120\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/120*(750*d^5*x^3*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^3 - 780*d
^5*x^3*e^3*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - 656*d^5*x^3*e^3 + (24*x^7*e
^7 + 90*d*x^6*e^6 + 32*d^2*x^5*e^5 - 345*d^3*x^4*e^4 - 656*d^4*x^3*e^3 - 80
*d^5*x^2*e^2 - 180*d^6*x*e - 40*d^7)*sqrt(-x^2*e^2 + d^2))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 7.74, size = 911, normalized size = 4.34

$$d^7\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right) + 3d^6e\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right) + d^5e^2\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) - 5d^4e^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) - 5d^3e^4\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right) + d^2e^5\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) + 3de^6\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^7\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**4,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) +
 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*
x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x
**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(
2*d), True)) + d**5*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e
*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1
- e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e*
*2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**
2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(
d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) - 5*d**3*e**4*Piecewis
e((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asi
n(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + d**2*e**5*Piecewi
se((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2),
True)) + 3*d*e**6*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e*
*2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) +
I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**
4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**
3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)),
True)) + e**7*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x*
*2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e,
0)), (x**4*sqrt(d**2)/4, True))
```

Giac [A]
time = 1.31, size = 314, normalized size = 1.50

$$-\frac{25}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^3\operatorname{sgn}(d) + \frac{13}{2}\,d^5e^3\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) - \frac{3\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5e}{8\,x} - \frac{3\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^5e^{(-1)}}{8\,x^2} - \frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^5e^{(-3)}}{24\,x^3} + \frac{\left(d^5e^3 + \frac{9\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5e}{x} + \frac{9\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^5e^{(-1)}}{x^2}\right)x^3e^6}{24\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3} - \frac{1}{120}\,\left(656\,d^4e^3 + \left(345\,d^3e^4 - 2\,\left(16\,d^2e^5 + 3\,\left(4\,xe^7 + 15\,de^6\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] -25/8*d^5*arcsin(x*e/d)*e^3*sgn(d) + 13/2*d^5*e^3*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 3/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d
^5*e/x - 3/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*e^(-1)/x^2 - 1/24*(d*e +
```

```
sqrt(-x^2*e^2 + d^2)*e)^3*d^5*e^(-3)/x^3 + 1/24*(d^5*e^3 + 9*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*d^5*e/x + 9*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*e^(-1)/x^
2)*x^3*e^6/(d*e + sqrt(-x^2*e^2 + d^2)*e)^3 - 1/120*(656*d^4*e^3 + (345*d^3
*e^4 - 2*(16*d^2*e^5 + 3*(4*x*e^7 + 15*d*e^6)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^4, x)
```

3.75 $\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x}-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{7/}}{4x^4}$$

```
[Out] 15/8*d*e^3*(-e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x-3/8*e^2*(2*e*x+3*d)*(-e^2*x^2+
d^2)^(5/2)/x^2-1/4*d*(-e^2*x^2+d^2)^(7/2)/x^4-e*(-e^2*x^2+d^2)^(7/2)/x^3+45
/8*d^4*e^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+45/8*d^4*e^4*arctanh((-e^2*x^2+
d^2)^(1/2)/d)-45/8*d^2*e^4*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1821, 827, 829, 858, 223, 209, 272, 65, 214}

$$\frac{45}{8}d^4e^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{x^3}-\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2}+\frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x}+\frac{45}{8}d^4e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] (-45*d^2*e^4*(d - e*x)*Sqrt[d^2 - e^2*x^2])/8 + (15*d*e^3*(2*d - e*x)*(d^2
- e^2*x^2)^(3/2))/(8*x) - (3*e^2*(3*d + 2*e*x)*(d^2 - e^2*x^2)^(5/2))/(8*x^
2) - (d*(d^2 - e^2*x^2)^(7/2))/(4*x^4) - (e*(d^2 - e^2*x^2)^(7/2))/x^3 + (4
5*d^4*e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 + (45*d^4*e^4*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^5}\,dx &= -\frac{d(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-12d^4e-9d^3e^2x-4d^2e^3x^2)}{x^4}\,dx}{4d^2} \\
&= -\frac{d(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{7/2}}{x^3} + \frac{\int \frac{(27d^5e^2-36d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^3}\,dx}{12d^4} \\
&= -\frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d(d^2-e^2x^2)^{7/2}}{4x^4} - \frac{e(d^2-e^2x^2)^{7/2}}{x^3} - 5\int \\
&= \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+2ex)\left(d^2-e^2x^2\right)^{5/2}}{8x^2} - \frac{d(d^2-e^2}{4x^4} \\
&= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+}{} \\
&= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+}{} \\
&= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+}{} \\
&= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+}{} \\
&= -\frac{45}{8}d^2e^4(d-ex)\sqrt{d^2-e^2x^2} + \frac{15de^3(2d-ex)\left(d^2-e^2x^2\right)^{3/2}}{8x} - \frac{3e^2(3d+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.65, size = 187, normalized size = 0.89

$$\frac{1}{8}\left(\frac{\sqrt{d^2-e^2x^2}\,(-2d^7-8d^6ex-3d^5e^2x^2+48d^4e^3x^3-48d^3e^4x^4+3d^2e^5x^5+8de^6x^6+2e^7x^7)}{x^4} - 90d^4e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) - 45d^4e(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^5,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-2*d^7 - 8*d^6*e*x - 3*d^5*e^2*x^2 + 48*d^4*e^3*x^3
- 48*d^3*e^4*x^4 + 3*d^2*e^5*x^5 + 8*d*e^6*x^6 + 2*e^7*x^7))/x^4 - 90*d^4*e
```

```
^4*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] - 45*d^4*e*(-e^2)^(3/2)*
Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(183) = 366$.
time = 0.07, size = 598, normalized size = 2.86

method	result
risch	$-\frac{d^4\sqrt{-e^2x^2+d^2}\,(-48e^3x^3+3d\,e^2x^2+8d^2ex+2d^3)}{8x^4}+\frac{e^7x^3\sqrt{-e^2x^2+d^2}}{4}+\frac{3e^5d^2x\sqrt{-e^2x^2+d^2}}{8}+\frac{45e^5d^4\,\mathrm{arc}}{}$

default	$d^3 \left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{4d^2x^4} - \frac{3e^2 \left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{2d^2x^2} - \frac{5e^2 \left(\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5} + d^2 \left(\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3} + d^2 \left(\sqrt{-e^2x^2+d^2} - \frac{d^2 \ln\left(\frac{2d^2+2\sqrt{\ldots}}{\ldots} \right.}{\ldots} \right. \right. \right.}{2d^2} \right.}{4d^2} \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+
d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3
/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x))))))+3*d*e^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2
/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2
+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x
)))))+e^3*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5
/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)
+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+3*d^2*e
*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7
/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/
2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2)))))))
```

Maxima [A]
time = 0.56, size = 233, normalized size = 1.11

$$\frac{45}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^4+\frac{45}{8}\,d^4e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)+\frac{45}{8}\,\sqrt{-x^2e^2+d^2}\,d^2xe^5-\frac{45}{8}\,\sqrt{-x^2e^2+d^2}\,d^3e^4+\frac{15}{4}\,(-x^2e^2+d^2)^{\frac{3}{2}}xe^5-\frac{15}{8}\,(-x^2e^2+d^2)^{\frac{3}{2}}de^4-\frac{9\,(-x^2e^2+d^2)^{\frac{5}{2}}e^4}{8\,d}+\frac{3\,(-x^2e^2+d^2)^{\frac{5}{2}}e^3}{x}-\frac{9\,(-x^2e^2+d^2)^{\frac{7}{2}}e^2}{8\,dx^2}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}e}{x^3}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}d}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 45/8*d^4*arcsin(x*e/d)*e^4 + 45/8*d^4*e^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^
2 + d^2)*d/abs(x)) + 45/8*sqrt(-x^2*e^2 + d^2)*d^2*x*e^5 - 45/8*sqrt(-x^2*e
^2 + d^2)*d^3*e^4 + 15/4*(-x^2*e^2 + d^2)^(3/2)*x*e^5 - 15/8*(-x^2*e^2 + d^
2)^(3/2)*d*e^4 - 9/8*(-x^2*e^2 + d^2)^(5/2)*e^4/d + 3*(-x^2*e^2 + d^2)^(5/2
)*e^3/x - 9/8*(-x^2*e^2 + d^2)^(7/2)*e^2/(d*x^2) - (-x^2*e^2 + d^2)^(7/2)*e
/x^3 - 1/4*(-x^2*e^2 + d^2)^(7/2)*d/x^4
```

Fricas [A]
time = 1.48, size = 168, normalized size = 0.80

$$-\frac{90\,d^4x^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^4+45\,d^4x^4e^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+48\,d^4x^4e^4-(2\,x^7e^7+8\,dx^6e^6+3\,d^2x^5e^5-48\,d^3x^4e^4+48\,d^4x^3e^3-3\,d^5x^2e^2-8\,d^6xe-2\,d^7)\sqrt{-x^2e^2+d^2}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(90*d^4*x^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^4 + 45*d^4*
x^4*e^4*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + 48*d^4*x^4*e^4 - (2*x^7*e^7 +
```

```
8*d*x^6*e^6 + 3*d^2*x^5*e^5 - 48*d^3*x^4*e^4 + 48*d^4*x^3*e^3 - 3*d^5*x^2*e
^2 - 8*d^6*x*e - 2*d^7)*sqrt(-x^2*e^2 + d^2))/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 9.47, size = 1028, normalized size = 4.92

$$d^{7}\left(\begin{cases} -\frac{d^{2}}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{3e}{8x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{3}}{8d^{2}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{4}\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{id^{2}}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{3ie}{8x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{3}}{8d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{4}\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{otherwise}\end{cases}\right)+3d^{6}e\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3x^{2}}+\frac{e^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3d^{2}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3x^{2}}+\frac{ie^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3d^{2}} & \text{otherwise}\end{cases}\right)+d^{5}e^{2}\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{2x}+\frac{e^{2}\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{id^{2}}{2ex^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie}{2x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{2}\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-5d^{4}e^{3}\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^{2}x}{d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^{2}x}{d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)-5d^{3}e^{4}\left(\begin{cases} \frac{d^{2}}{ex\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ -\frac{id^{2}}{ex\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} & \text{otherwise}\end{cases}\right)+d^{2}e^{5}\left(\begin{cases} -\frac{id^{2}\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\frac{ie^{2}x^{3}}{2d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\ \frac{d^{2}\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{2} & \text{otherwise}\end{cases}\right)+3de^{6}\left(\begin{cases} \frac{x^{2}\sqrt{d^{2}}}{2} & \text{for}\ e^{2}=0\\ -\frac{\left(d^{2}-e^{2}x^{2}\right)^{\frac{3}{2}}}{3e^{2}} & \text{otherwise}\end{cases}\right)+e^{7}\left(\begin{cases} -\frac{id^{4}\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^{3}}+\frac{id^{3}x}{8e^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{3idx^{3}}{8\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\frac{ie^{2}x^{5}}{4d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\ \frac{d^{4}\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^{3}}-\frac{d^{3}x}{8e^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{3dx^{3}}{8\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-\frac{e^{2}x^{5}}{4d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**5,x)
```

```
[Out] d**7*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) +
e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/
(3*d**2), True)) + d**5*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x)
 + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3
*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
**2*asin(d/(e*x))/(2*d), True)) - 5*d**4*e**3*Piecewise((I*d/(x*sqrt(-1 + e
**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2))
, Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d
) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Piecewise((d*
*2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**
2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x*
*2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) +
 d**2*e**5*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*
x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d*
*2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
+ 3*d*e**6*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)
**(3/2)/(3*e**2), True)) + e**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) +
I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*
x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d*
*2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2
*x**2/d**2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(176) = 352.
time = 1.38, size = 365, normalized size = 1.75

$$\frac{45}{8}\,d^{4}\arcsin\left(\frac{xe}{d}\right)e^{4}\mathrm{sgn}\left(d\right)+\frac{45}{8}\,d^{4}e^{4}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{\left(-2\right)}}{2\,\left|x\right|}\right)+\frac{23\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d^{4}e^{2}}{8\,x}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}d^{4}e^{\left(-2\right)}}{8\,x^{3}}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}d^{4}e^{\left(-4\right)}}{64\,x^{4}}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}d^{4}}{8\,x^{2}}+\frac{\left(d^{4}e^{4}+\frac{8\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d^{4}e^{2}}{x}-\frac{184\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}d^{4}e^{\left(-2\right)}}{x^{3}}+\frac{8\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}d^{4}}{x^{2}}\right)x^{4}e^{8}}{64\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}}-\frac{1}{8}\,\left(48\,d^{3}e^{4}-\left(3\,d^{2}e^{5}+2\,\left(xe^{7}+4\,de^{6}\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 45/8*d^4*arcsin(x*e/d)*e^4*sgn(d) + 45/8*d^4*e^4*log(1/2*abs(-2*d*e - 2*sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 23/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d
^4*e^2/x - 1/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^(-2)/x^3 - 1/64*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^(-4)/x^4 - 1/8*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^2*d^4/x^2 + 1/64*(d^4*e^4 + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^2/x
 - 184*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*e^(-2)/x^3 + 8*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^2*d^4/x^2)*x^4*e^8/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 - 1/8*(4
8*d^3*e^4 - (3*d^2*e^5 + 2*(x*e^7 + 4*d*e^6)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^5, x)
```

3.76 $\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=216

$$\frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x}+\frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2}-\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3}-\frac{d(d^2-e^2x}{5x^5}$$

```
[Out] 1/24*d*e^3*(-52*e*x+25*d)*(-e^2*x^2+d^2)^(3/2)/x^2-1/60*e^2*(25*e*x+52*d)*(
-e^2*x^2+d^2)^(5/2)/x^3-1/5*d*(-e^2*x^2+d^2)^(7/2)/x^5-3/4*e*(-e^2*x^2+d^2)
^(7/2)/x^4+13/2*d^3*e^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-25/8*d^3*e^5*arcta
nh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*e^4*(25*e*x+52*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A]
time = 0.19, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1821, 827, 858, 223, 209, 272, 65, 214}

$$\frac{13}{2}d^3e^5\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{4x^4}-\frac{e^2(52d+25ex)\left(d^2-e^2x^2\right)^{5/2}}{60x^3}+\frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x}+\frac{de^3(25d-52ex)\left(d^2-e^2x^2\right)^{3/2}}{24x^2}-\frac{25}{8}d^3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] (d^2*e^4*(52*d + 25*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x) + (d*e^3*(25*d - 52*e*x
)*(d^2 - e^2*x^2)^(3/2))/(24*x^2) - (e^2*(52*d + 25*e*x)*(d^2 - e^2*x^2)^(5
/2))/(60*x^3) - (d*(d^2 - e^2*x^2)^(7/2))/(5*x^5) - (3*e*(d^2 - e^2*x^2)^(7
/2))/(4*x^4) + (13*d^3*e^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (25*d^3*e
^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^6}\,dx = -\frac{d(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-15d^4e-13d^3e^2x-5d^2e^3x^2)}{x^5}\,dx}{5d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{3e(d^2-e^2x^2)^{7/2}}{4x^4} + \frac{\int \frac{(52d^5e^2-25d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^4}\,dx}{20d^4}$$

$$= -\frac{e^2(52d+25ex)(d^2-e^2x^2)^{5/2}}{60x^3} - \frac{d(d^2-e^2x^2)^{7/2}}{5x^5} - \frac{3e(d^2-e^2x^2)^{7/2}}{4x^4} - \int$$

$$= \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2} - \frac{e^2(52d+25ex)(d^2-e^2x^2)^{5/2}}{60x^3} - \frac{d(d^2-}{5}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2} - \frac{e^2(52d}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2} - \frac{e^2(52d}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2} - \frac{e^2(52d}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2} - \frac{e^2(52d}{}$$

$$= \frac{d^2e^4(52d+25ex)\sqrt{d^2-e^2x^2}}{8x} + \frac{de^3(25d-52ex)(d^2-e^2x^2)^{3/2}}{24x^2} - \frac{e^2(52d}{}$$

Mathematica [A]
time = 0.69, size = 189, normalized size = 0.88

$$\frac{1}{120}\left(\frac{\sqrt{d^2-e^2x^2}\,(-24d^7-90d^6ex-32d^5e^2x^2+345d^4e^3x^3+656d^3e^4x^4+80d^2e^5x^5+180de^6x^6+40e^7x^7)}{x^5}+750d^3e^5\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)+780d^3e^4\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^6,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-24*d^7 - 90*d^6*e*x - 32*d^5*e^2*x^2 + 345*d^4*e^3*
x^3 + 656*d^3*e^4*x^4 + 80*d^2*e^5*x^5 + 180*d*e^6*x^6 + 40*e^7*x^7))/x^5 +
 750*d^3*e^5*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] + 780*d^3*e^4*
Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/120
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(188) = 376$.
time = 0.07, size = 660, normalized size = 3.06

method	result
risch	$-\frac{d^{3} \sqrt{-e^{2} x^{2}+d^{2}}\, \left(-656 e^{4} x^{4}-345 d\, e^{3} x^{3}+32 d^{2} x^{2} e^{2}+90 d^{3} e x+24 d^{4}\right)}{120 x^{5}}+\frac{e^{7} x^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3}+\frac{2 e^{5} d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3}$

$$3d^2 \left(\frac{x\sqrt{-}}{} \right.$$

$$5d^2 \left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4} + \right.$$

$$6e^2 \left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6} + \right.$$

$$4e^2 \left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x} - \frac{}{d^2} \right.$$

$$2e^2 \left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^2x^3} - \frac{}{3d^2} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(-1/5/d^2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+
d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^
2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x
^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)))))))+3*d^2*e*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^
2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2
*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))+e^3*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)
-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((
-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^
(1/2))/x)))))+3*d*e^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^
2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4
*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(
1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))
```

Maxima [A]
time = 0.48, size = 259, normalized size = 1.20

$$\frac{13}{2}\,d^{3}\arcsin\left(\frac{xe}{d}\right)e^{5}-\frac{25}{8}\,d^{3}e^{5}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)+\frac{13}{2}\,\sqrt{-x^{2}e^{2}+d^{2}}\,dxe^{6}+\frac{25}{8}\,\sqrt{-x^{2}e^{2}+d^{2}}\,d^{2}e^{5}+\frac{13\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}xe^{6}}{3\,d}+\frac{25}{24}\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}e^{5}+\frac{5\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e^{5}}{8\,d^{2}}+\frac{52\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e^{4}}{15\,dx}+\frac{5\,(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e^{3}}{8\,d^{2}x^{2}}-\frac{13\,(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e^{2}}{15\,dx^{3}}-\frac{3\,(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e}{4\,x^{4}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}d}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 13/2*d^3*arcsin(x*e/d)*e^5 - 25/8*d^3*e^5*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^
2 + d^2)*d/abs(x)) + 13/2*sqrt(-x^2*e^2 + d^2)*d*x*e^6 + 25/8*sqrt(-x^2*e^2
 + d^2)*d^2*e^5 + 13/3*(-x^2*e^2 + d^2)^(3/2)*x*e^6/d + 25/24*(-x^2*e^2 + d
^2)^(3/2)*e^5 + 5/8*(-x^2*e^2 + d^2)^(5/2)*e^5/d^2 + 52/15*(-x^2*e^2 + d^2)
^(5/2)*e^4/(d*x) + 5/8*(-x^2*e^2 + d^2)^(7/2)*e^3/(d^2*x^2) - 13/15*(-x^2*e
^2 + d^2)^(7/2)*e^2/(d*x^3) - 3/4*(-x^2*e^2 + d^2)^(7/2)*e/x^4 - 1/5*(-x^2*
e^2 + d^2)^(7/2)*d/x^5
```

Fricas [A]
time = 1.64, size = 168, normalized size = 0.78

$$-\frac{1560\,d^{3}x^{5}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)e^{5}-375\,d^{3}x^{5}e^{5}\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)-80\,d^{3}x^{5}e^{5}-(40\,x^{7}e^{7}+180\,dx^{6}e^{6}+80\,d^{2}x^{5}e^{5}+656\,d^{3}x^{4}e^{4}+345\,d^{4}x^{3}e^{3}-32\,d^{5}x^{2}e^{2}-90\,d^{6}xe-24\,d^{7})\sqrt{-x^{2}e^{2}+d^{2}}}{120\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(1560*d^3*x^5*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^5 - 375
*d^3*x^5*e^5*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - 80*d^3*x^5*e^5 - (40*x^7*
e^7 + 180*d*x^6*e^6 + 80*d^2*x^5*e^5 + 656*d^3*x^4*e^4 + 345*d^4*x^3*e^3 -
32*d^5*x^2*e^2 - 90*d^6*x*e - 24*d^7)*sqrt(-x^2*e^2 + d^2))/x^5
```

Sympy [C] Result contains complex when optimal does not.
time = 8.13, size = 1178, normalized size = 5.45

$$d^{7}\left(\begin{cases} \frac{3 i d^{3} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}}{- 15 d^{2} x^{5} + 15 e^{2} x^{7}} - \frac{4 i d e^{2} x^{2} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}}{- 15 d^{2} x^{5} + 15 e^{2} x^{7}} + \frac{2 i e^{6} x^{6} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}}{- 15 d^{5} x^{5} + 15 d^{3} e^{2} x^{7}} - \frac{i e^{4} x^{4} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}}{- 15 d^{3} x^{5} + 15 d e^{2} x^{7}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\ \frac{3 d^{3} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}}{- 15 d^{2} x^{5} + 15 e^{2} x^{7}} - \frac{4 d e^{2} x^{2} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}}{- 15 d^{2} x^{5} + 15 e^{2} x^{7}} + \frac{2 e^{6} x^{6} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}}{- 15 d^{5} x^{5} + 15 d^{3} e^{2} x^{7}} - \frac{e^{4} x^{4} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}}{- 15 d^{3} x^{5} + 15 d e^{2} x^{7}} & \text{otherwise} \end{cases}\right) + 3 d^{6} e \left(\begin{cases} - \frac{d^{2}}{4 e x^{5} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{3 e}{8 x^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} - \frac{e^{3}}{8 d^{2} x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} + \frac{e^{4} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{8 d^{3}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\ \frac{i d^{2}}{4 e x^{5} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} - \frac{3 i e}{8 x^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} + \frac{i e^{3}}{8 d^{2} x \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} - \frac{i e^{4} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{8 d^{3}} & \text{otherwise} \end{cases}\right) + d^{5} e^{2} \left(\begin{cases} - \frac{e \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 x^{2}} + \frac{e^{3} \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{3 d^{2}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\ - \frac{i e \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 x^{2}} + \frac{i e^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}}{3 d^{2}} & \text{otherwise} \end{cases}\right) - 5 d^{4} e^{3} \left(\begin{cases} - \frac{e \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}}{2 x} + \frac{e^{2} \operatorname{acosh}{\left(\frac{d}{e x} \right)}}{2 d} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\ \frac{i d^{2}}{2 e x^{3} \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} - \frac{i e}{2 x \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} - \frac{i e^{2} \operatorname{asin}{\left(\frac{d}{e x} \right)}}{2 d} & \text{otherwise} \end{cases}\right) - 5 d^{3} e^{4} \left(\begin{cases} \frac{i d}{x \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + i e \operatorname{acosh}{\left(\frac{e x}{d} \right)} - \frac{i e^{2} x}{d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\ - \frac{d}{x \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - e \operatorname{asin}{\left(\frac{e x}{d} \right)} + \frac{e^{2} x}{d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right) + d^{2} e^{5} \left(\begin{cases} \frac{d^{2}}{e x \sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} - d \operatorname{acosh}{\left(\frac{d}{e x} \right)} - \frac{e x}{\sqrt{\frac{d^{2}}{e^{2} x^{2}} - 1}} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\ - \frac{i d^{2}}{e x \sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} + i d \operatorname{asin}{\left(\frac{d}{e x} \right)} + \frac{i e x}{\sqrt{- \frac{d^{2}}{e^{2} x^{2}} + 1}} & \text{otherwise} \end{cases}\right) + 3 d e^{6} \left(\begin{cases} - \frac{i d^{2} \operatorname{acosh}{\left(\frac{e x}{d} \right)}}{2 e} - \frac{i d x}{2 \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + \frac{i e^{2} x^{3}}{2 d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\ \frac{d^{2} \operatorname{asin}{\left(\frac{e x}{d} \right)}}{2 e} + \frac{d x \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}}{2} & \text{otherwise} \end{cases}\right) + e^{7} \left(\begin{cases} \frac{x^{2} \sqrt{d^{2}}}{2} & \text{for}\: e^{2} = 0 \\ - \frac{\left(d^{2} - e^{2} x^{2}\right)^{\frac{3}{2}}}{3 e^{2}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**6,x)
```

```
[Out] d**7*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) + 3*d**6*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e*
*2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*s
qrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x
**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)
) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**5*e**2*Piecewise((-e*sqrt(d*
*2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - 5*d**4*e**3*Piecewise((-e
*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e*
*2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*s
qrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) - 5*d**3*e
**4*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**
2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
- e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), T
rue)) + d**2*e**5*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acos
h(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-
I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-
d**2/(e**2*x**2) + 1), True)) + 3*d*e**6*Piecewise((-I*d**2*acosh(e*x/d)/(2
*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2
*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(
1 - e**2*x**2/d**2)/2, True)) + e**7*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2,
 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(181) = 362.

time = 1.63, size = 425, normalized size = 1.97

$$\frac{13}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^5\mathrm{sgn}(d)-\frac{25}{8}\,d^3e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{43\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3e^3}{16\,x}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^3e}{8\,x^2}-\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^3e^{(-1)}}{96\,x^3}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3e^{(-3)}}{64\,x^4}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^3e^{(-5)}}{160\,x^5}+\frac{\left(6\,d^3e^5+\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3e^3}{x}+\frac{50\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^3e}{x^2}-\frac{600\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^3e^{(-1)}}{x^3}-\frac{2580\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3e^{(-3)}}{x^4}\right)x^5e^{10}}{960\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5}+\frac{1}{6}\left(4\,d^2e^5+\left(2\,xe^7+9\,de^6\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 13/2*d^3*arcsin(x*e/d)*e^5*sgn(d) - 25/8*d^3*e^5*log(1/2*abs(-2*d*e - 2*sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 43/16*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d^3*e^3/x + 5/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*e/x^2 - 5/96*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^3*d^3*e^(-1)/x^3 - 3/64*(d*e + sqrt(-x^2*e^2 + d^2)*e
)^4*d^3*e^(-3)/x^4 - 1/160*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^3*e^(-5)/x^5
+ 1/960*(6*d^3*e^5 + 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^3/x + 50*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^2*d^3*e/x^2 - 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^
3*d^3*e^(-1)/x^3 - 2580*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3*e^(-3)/x^4)*x^
5*e^10/(d*e + sqrt(-x^2*e^2 + d^2)*e)^5 + 1/6*(4*d^2*e^5 + (2*x*e^7 + 9*d*e
^6)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^6, x)
```

3.77 $\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^7} \, dx$

Optimal. Leaf size=214

$$-\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3}-\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4}-\frac{d(d^2-e^2x^2}{6x^6}$$

```
[Out] 1/48*d*e^3*(85*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)/x^3-1/120*e^2*(12*e*x+85*d)*(-
e^2*x^2+d^2)^(5/2)/x^4-1/6*d*(-e^2*x^2+d^2)^(7/2)/x^6-3/5*e*(-e^2*x^2+d^2)^
(7/2)/x^5-1/2*d^2*e^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-85/16*d^2*e^6*arctan
h((-e^2*x^2+d^2)^(1/2)/d)-1/16*d*e^5*(-85*e*x+8*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A]
time = 0.19, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1821, 827, 825, 858, 223, 209, 272, 65, 214}

$$-\frac{1}{2}d^2e^6\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6}-\frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5}-\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4}-\frac{85}{16}d^2e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] -1/16*(d*e^5*(8*d - 85*e*x)*Sqrt[d^2 - e^2*x^2])/x + (d*e^3*(8*d + 85*e*x)*
(d^2 - e^2*x^2)^(3/2))/(48*x^3) - (e^2*(85*d + 12*e*x)*(d^2 - e^2*x^2)^(5/2
))/(120*x^4) - (d*(d^2 - e^2*x^2)^(7/2))/(6*x^6) - (3*e*(d^2 - e^2*x^2)^(7/
2))/(5*x^5) - (d^2*e^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 - (85*d^2*e^6*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/16
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^7} \, dx &= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-18d^4e-17d^3e^2x-6d^2e^3x^2\right)}{x^6} \, dx}{6d^2} \\
&= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} + \frac{\int \frac{\left(85d^5e^2-6d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^5} \, dx}{30d^4} \\
&= -\frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{6x^6} - \frac{3e\left(d^2-e^2x^2\right)^{7/2}}{5x^5} - \int \\
&= \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+12ex)\left(d^2-e^2x^2\right)^{5/2}}{120x^4} - \frac{d(d^2-e}{6x} \\
&= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+}{} \\
&= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+}{} \\
&= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+}{} \\
&= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+}{} \\
&= -\frac{de^5(8d-85ex)\sqrt{d^2-e^2x^2}}{16x} + \frac{de^3(8d+85ex)\left(d^2-e^2x^2\right)^{3/2}}{48x^3} - \frac{e^2(85d+}{}
\end{aligned}$$

Mathematica [A]
time = 0.71, size = 192, normalized size = 0.90

$$\frac{\sqrt{d^2-e^2x^2}\left(-40d^7-144d^6ex-50d^5e^2x^2+448d^4e^3x^3+645d^3e^4x^4-544d^2e^5x^5+720de^6x^6+120e^7x^7\right)}{240x^6}+\frac{85}{8}d^2e^6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{1}{2}d^2e\left(-e^2\right)^{5/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^7,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^7 - 144*d^6*e*x - 50*d^5*e^2*x^2 + 448*d^4*e^3*
x^3 + 645*d^3*e^4*x^4 - 544*d^2*e^5*x^5 + 720*d*e^6*x^6 + 120*e^7*x^7))/(24
```

```
0*x^6) + (85*d^2*e^6*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/8 -
 (d^2*e*(-e^2)^(5/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(186) = 372$.
time = 0.07, size = 722, normalized size = 3.37

method	result
risch	$-\frac{d^{2} \sqrt{-e^{2} x^{2}+d^{2}}\, \left(544 e^{5} x^{5}-645 d\, e^{4} x^{4}-448 d^{2} e^{3} x^{3}+50 x^{2} d^{3} e^{2}+144 d^{4} x e+40 d^{5}\right)}{240 x^{6}}+\frac{e^{7} x \sqrt{-e^{2} x^{2}+d^{2}}}{2}-\frac{e^{7} d^{2} \arctan\left(\right.}{}$

$$3d^2\left(\frac{x\sqrt{\ }}{\ }\right.$$

$$5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\right.$$

$$6e^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\right.$$

$$4e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x}-\right.$$

$$2e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^2x^3}-\frac{\quad}{3d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 3*d^2*e*(-1/5/d^2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*
x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*
(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e
^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))))))))+3*d*e^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^
2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(
-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(
d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))+e^3*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(
7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+
d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2
)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))
+d^3*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2
+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5
*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/
2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))))
```

Maxima [A]
time = 0.49, size = 282, normalized size = 1.32

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^6-\frac{85}{16}\,d^2e^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,xe^7+\frac{85}{16}\,\sqrt{-x^2e^2+d^2}\,de^6-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}xe^7}{3\,d^2}+\frac{85\,(-x^2e^2+d^2)^{\frac{3}{2}}e^6}{48\,d}+\frac{17\,(-x^2e^2+d^2)^{\frac{5}{2}}e^6}{16\,d^3}-\frac{4\,(-x^2e^2+d^2)^{\frac{5}{2}}e^5}{15\,d^2x}+\frac{17\,(-x^2e^2+d^2)^{\frac{7}{2}}e^4}{16\,d^3x^2}+\frac{(-x^2e^2+d^2)^{\frac{7}{2}}e^3}{15\,d^2x^3}-\frac{17\,(-x^2e^2+d^2)^{\frac{7}{2}}e^2}{24\,dx^4}-\frac{3\,(-x^2e^2+d^2)^{\frac{7}{2}}e}{5\,x^5}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}d}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^6 - 85/16*d^2*e^6*log(2*d^2/abs(x) + 2*sqrt(-x^2*e
^2 + d^2)*d/abs(x)) - 1/2*sqrt(-x^2*e^2 + d^2)*x*e^7 + 85/16*sqrt(-x^2*e^2
+ d^2)*d*e^6 - 1/3*(-x^2*e^2 + d^2)^(3/2)*x*e^7/d^2 + 85/48*(-x^2*e^2 + d^2
)^(3/2)*e^6/d + 17/16*(-x^2*e^2 + d^2)^(5/2)*e^6/d^3 - 4/15*(-x^2*e^2 + d^2
)^(5/2)*e^5/(d^2*x) + 17/16*(-x^2*e^2 + d^2)^(7/2)*e^4/(d^3*x^2) + 1/15*(-x
^2*e^2 + d^2)^(7/2)*e^3/(d^2*x^3) - 17/24*(-x^2*e^2 + d^2)^(7/2)*e^2/(d*x^4
) - 3/5*(-x^2*e^2 + d^2)^(7/2)*e/x^5 - 1/6*(-x^2*e^2 + d^2)^(7/2)*d/x^6
```

Fricas [A]
time = 1.60, size = 167, normalized size = 0.78

$$\frac{240\,d^2x^6\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^6+1275\,d^2x^6e^6\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+720\,d^2x^6e^6+\left(120\,x^7e^7+720\,dx^6e^6-544\,d^2x^5e^5+645\,d^3x^4e^4+448\,d^4x^3e^3-50\,d^5x^2e^2-144\,d^6xe-40\,d^7\right)\sqrt{-x^2e^2+d^2}}{240\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/240*(240*d^2*x^6*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^6 + 1275*
d^2*x^6*e^6*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + 720*d^2*x^6*e^6 + (120*x^7
*e^7 + 720*d*x^6*e^6 - 544*d^2*x^5*e^5 + 645*d^3*x^4*e^4 + 448*d^4*x^3*e^3
- 50*d^5*x^2*e^2 - 144*d^6*x*e - 40*d^7)*sqrt(-x^2*e^2 + d^2))/x^6
```

Sympy [C] Result contains complex when optimal does not.
time = 12.70, size = 1397, normalized size = 6.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**7,x)
```

```
[Out] d**7*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + 3*d**6*e*Piecewise((3*I*d**3*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*s
qrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d*
*2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) -
 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*
e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e*
*4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) +
 d**5*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*
x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1
)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e
*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))
/(8*d**3), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3
*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) >
1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x*
*2) + 1)/(3*d**2), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(d**2/(e**2*x**2)
 - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**
2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) + d**2*e**5*Piecewise((I*d/(x*sq
rt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x*
*2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*a
sin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d*e**6*Piecewi
se((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**
2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e
```

```
**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), Tr
ue)) + e**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2
*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d
**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(179) = 358$.
time = 1.11, size = 476, normalized size = 2.22

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^6\mathrm{sgn}(d)-\frac{85}{16}\,d^2e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^4}{16\,x}+\frac{81\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^2}{128\,x^2}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^2e^{(-2)}}{128\,x^4}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^2e^{(-4)}}{160\,x^5}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6d^2e^{(-6)}}{384\,x^6}+\frac{17\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2}{96\,x^3}+\frac{\left(5\,d^2e^6+\frac{36\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^4}{x}+\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^2}{x^2}-\frac{1215\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^2e^{(-2)}}{x^4}+\frac{1800\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^2e^{(-4)}}{x^5}-\frac{340\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2}{x^3}\right)x^6e^{12}}{1920\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6}+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,(xe^7+6\,de^6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^6*sgn(d) - 85/16*d^2*e^6*log(1/2*abs(-2*d*e - 2*sq
rt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 15/16*(d*e + sqrt(-x^2*e^2 + d^2)*e)
*d^2*e^4/x + 81/128*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^2/x^2 - 3/128*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^2*e^(-2)/x^4 - 3/160*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^5*d^2*e^(-4)/x^5 - 1/384*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2*e^(
-6)/x^6 + 17/96*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2/x^3 + 1/1920*(5*d^2*e^
6 + 36*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*e^4/x + 45*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*d^2*e^2/x^2 - 1215*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^2*e^(-2)/x
^4 + 1800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^2*e^(-4)/x^5 - 340*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^3*d^2/x^3)*x^6*e^12/(d*e + sqrt(-x^2*e^2 + d^2)*e)^6 +
1/2*sqrt(-x^2*e^2 + d^2)*(x*e^7 + 6*d*e^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^7, x)
```

3.78 $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^8} dx$

Optimal. Leaf size=206

$$-\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)(d^2-e^2x^2)^{3/2}}{16x^3}-\frac{e^2(24d+5ex)(d^2-e^2x^2)^{5/2}}{40x^5}-\frac{d(d^2-e^2x^2)^{7/}}{7x^7}$$

```
[Out] 1/16*e^4*(5*e*x+16*d)*(-e^2*x^2+d^2)^(3/2)/x^3-1/40*e^2*(5*e*x+24*d)*(-e^2*
x^2+d^2)^(5/2)/x^5-1/7*d*(-e^2*x^2+d^2)^(7/2)/x^7-1/2*e*(-e^2*x^2+d^2)^(7/2
)/x^6-3*d*e^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-15/16*d*e^7*arctanh((-e^2*x^
2+d^2)^(1/2)/d)-3/16*e^6*(-5*e*x+16*d)*(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A]

time = 0.19, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1821, 825, 827, 858, 223, 209, 272, 65, 214}

$$-3de^7\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{d(d^2-e^2x^2)^{7/2}}{7x^7}-\frac{e(d^2-e^2x^2)^{7/2}}{2x^6}-\frac{e^2(24d+5ex)(d^2-e^2x^2)^{5/2}}{40x^5}-\frac{15}{16}de^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)(d^2-e^2x^2)^{3/2}}{16x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] (-3*e^6*(16*d - 5*e*x)*Sqrt[d^2 - e^2*x^2])/(16*x) + (e^4*(16*d + 5*e*x)*(d
^2 - e^2*x^2)^(3/2))/(16*x^3) - (e^2*(24*d + 5*e*x)*(d^2 - e^2*x^2)^(5/2))/
(40*x^5) - (d*(d^2 - e^2*x^2)^(7/2))/(7*x^7) - (e*(d^2 - e^2*x^2)^(7/2))/(2
*x^6) - 3*d*e^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (15*d*e^7*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/16
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^8}\,dx &= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-21d^4e-21d^3e^2x-7d^2e^3x^2\right)}{x^7}\,dx}{7d^2}\\
&= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}+\frac{\int \frac{\left(126d^5e^2+21d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^6}\,dx}{42d^4}\\
&= -\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-\frac{d\left(d^2-e^2x^2\right)^{7/2}}{7x^7}-\frac{e\left(d^2-e^2x^2\right)^{7/2}}{2x^6}-\int\\
&= \frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+5ex)\left(d^2-e^2x^2\right)^{5/2}}{40x^5}-\frac{d\left(d^2-e^2x\right.}{7x^7}\\
&= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+}{}\\
&= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+}{}\\
&= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+}{}\\
&= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+}{}\\
&= -\frac{3e^6(16d-5ex)\sqrt{d^2-e^2x^2}}{16x}+\frac{e^4(16d+5ex)\left(d^2-e^2x^2\right)^{3/2}}{16x^3}-\frac{e^2(24d+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.73, size = 188, normalized size = 0.91

$$
\frac{\sqrt{d^2-e^2x^2}\left(-80d^7-280d^6ex-96d^5e^2x^2+770d^4e^3x^3+992d^3e^4x^4-525d^2e^5x^5-2496de^6x^6+560e^7x^7\right)}{560x^7}+\frac{15}{8}de^7\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+3de^4\left(-e^2\right)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^8,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-80*d^7 - 280*d^6*e*x - 96*d^5*e^2*x^2 + 770*d^4*e^3*
x^3 + 992*d^3*e^4*x^4 - 525*d^2*e^5*x^5 - 2496*d*e^6*x^6 + 560*e^7*x^7))/(5
```

```
60*x^7) + (15*d*e^7*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/8 +
3*d*e^4*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(180) = 360$.
time = 0.08, size = 580, normalized size = 2.82

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-560e^7x^7+2496d\,e^6x^6+525d^2e^5x^5-992d^3e^4x^4-770d^4e^3x^3+96d^5e^2x^2+280d^6ex+80d^7)}{560x^7}-\frac{3d\,e^8\arctan\left(\right.}{}$

$$3d^2\left(\frac{x\sqrt{}}{}\right.$$

$$5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\right.$$

$$6e^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\right.$$

$$4e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x}-\right.$$

$$2e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^2x^3}-\frac{}{3d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 3*d*e^2*(-1/5/d^2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*
x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*
(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e
^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))))))))+e^3*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^
2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2
*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))+3*d^2*e*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(
7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x
^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^
2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2
)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))))-1/7*d*(-e^2*x^2+d^2)^(7/2)/x^7
```

Maxima [A]
time = 0.52, size = 303, normalized size = 1.47

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{7}-\frac{15}{16}\,de^{7}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)-\frac{3\,\sqrt{-x^{2}e^{2}+d^{2}}\,xe^{8}}{d}+\frac{15}{16}\,\sqrt{-x^{2}e^{2}+d^{2}}\,e^{7}-\frac{2\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}xe^{8}}{d^{3}}+\frac{5\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}e^{7}}{16\,d^{2}}+\frac{3\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e^{7}}{16\,d^{4}}-\frac{8\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}e^{6}}{5\,d^{3}x}+\frac{3\,(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e^{5}}{16\,d^{4}x^{2}}+\frac{2\,(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e^{4}}{5\,d^{3}x^{3}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e^{3}}{8\,d^{2}x^{4}}-\frac{3\,(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e^{2}}{5\,dx^{5}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e}{2\,x^{6}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}d}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] -3*d*arcsin(x*e/d)*e^7 - 15/16*d*e^7*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d
^2)*d/abs(x)) - 3*sqrt(-x^2*e^2 + d^2)*x*e^8/d + 15/16*sqrt(-x^2*e^2 + d^2)
*e^7 - 2*(-x^2*e^2 + d^2)^(3/2)*x*e^8/d^3 + 5/16*(-x^2*e^2 + d^2)^(3/2)*e^7
/d^2 + 3/16*(-x^2*e^2 + d^2)^(5/2)*e^7/d^4 - 8/5*(-x^2*e^2 + d^2)^(5/2)*e^6
/(d^3*x) + 3/16*(-x^2*e^2 + d^2)^(7/2)*e^5/(d^4*x^2) + 2/5*(-x^2*e^2 + d^2)
^(7/2)*e^4/(d^3*x^3) - 1/8*(-x^2*e^2 + d^2)^(7/2)*e^3/(d^2*x^4) - 3/5*(-x^2
*e^2 + d^2)^(7/2)*e^2/(d*x^5) - 1/2*(-x^2*e^2 + d^2)^(7/2)*e/x^6 - 1/7*(-x^
2*e^2 + d^2)^(7/2)*d/x^7
```

Fricas [A]
time = 1.80, size = 161, normalized size = 0.78

$$\frac{3360\,dx^{7}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)e^{7}+525\,dx^{7}e^{7}\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+560\,dx^{7}e^{7}+(560\,x^{7}e^{7}-2496\,dx^{6}e^{6}-525\,d^{2}x^{5}e^{5}+992\,d^{3}x^{4}e^{4}+770\,d^{4}x^{3}e^{3}-96\,d^{5}x^{2}e^{2}-280\,d^{6}xe-80\,d^{7})\sqrt{-x^{2}e^{2}+d^{2}}}{560\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/560*(3360*d*x^7*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^7 + 525*d*
x^7*e^7*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + 560*d*x^7*e^7 + (560*x^7*e^7 -
```

```
 2496*d*x^6*e^6 - 525*d^2*x^5*e^5 + 992*d^3*x^4*e^4 + 770*d^4*x^3*e^3 - 96*
d^5*x^2*e^2 - 280*d^6*x*e - 80*d^7)*sqrt(-x^2*e^2 + d^2))/x^7
```

Sympy [C] Result contains complex when optimal does not.
time = 12.95, size = 1513, normalized size = 7.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**8,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + 3*d**6*e
*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt
(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) -
e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5)
, Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)
) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) -
 I*e**6*asin(d/(e*x))/(16*d**5), True)) + d**5*e**2*Piecewise((3*I*d**3*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2)
 > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*
d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**
6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*
x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - 5*
d**4*e**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x
**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)
) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*
x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) +
1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/
(8*d**3), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*
x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1
), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**
2) + 1)/(3*d**2), True)) + d**2*e**5*Piecewise((-e*sqrt(d**2/(e**2*x**2) -
1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(
2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**2*asin(d/(e*x))/(2*d), True)) + 3*d*e**6*Piecewise((I*d/(x*sqrt(-
1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d
```

```
**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(
e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((d**2
/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*
x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2
) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(173) = 346.
time = 0.66, size = 505, normalized size = 2.45

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{7}\mathrm{sgn}(d)-\frac{15}{16}\,de^{7}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{\left(5\,de^{7}+\frac{35\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)de^{5}}{x}+\frac{49\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}de^{3}}{x^{2}}-\frac{245\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}de}{x^{3}}-\frac{875\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}de^{(-1)}}{x^{4}}+\frac{455\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{5}de^{(-3)}}{x^{5}}+\frac{9065\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{6}de^{(-5)}}{x^{6}}\right)x^{7}e^{14}}{4480\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{7}}-\frac{259\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)de^{5}}{128\,x}-\frac{13\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}de^{3}}{128\,x^{2}}+\frac{25\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}de}{128\,x^{3}}+\frac{7\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}de^{(-1)}}{128\,x^{4}}-\frac{7\,\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{5}de^{(-3)}}{640\,x^{5}}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{6}de^{(-5)}}{128\,x^{6}}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{7}de^{(-7)}}{896\,x^{7}}+\sqrt{-x^{2}e^{2}+d^{2}}\,e^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^7*sgn(d) - 15/16*d*e^7*log(1/2*abs(-2*d*e - 2*sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/4480*(5*d*e^7 + 35*(d*e + sqrt(-x^2*e^2
+ d^2)*e)*d*e^5/x + 49*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^3/x^2 - 245*(d*
e + sqrt(-x^2*e^2 + d^2)*e)^3*d*e/x^3 - 875*(d*e + sqrt(-x^2*e^2 + d^2)*e)^
4*d*e^(-1)/x^4 + 455*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d*e^(-3)/x^5 + 9065*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d*e^(-5)/x^6)*x^7*e^14/(d*e + sqrt(-x^2*e^2
 + d^2)*e)^7 - 259/128*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^5/x - 13/128*(d*e
 + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^3/x^2 + 25/128*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d*e/x^3 + 7/128*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d*e^(-1)/x^4 - 7/640
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d*e^(-3)/x^5 - 1/128*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^6*d*e^(-5)/x^6 - 1/896*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d*e^(-7)
/x^7 + sqrt(-x^2*e^2 + d^2)*e^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^8, x)
```

3.79 $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\, dx$

Optimal. Leaf size=204

$$-\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}-\frac{d(d^2-}{8}$$

```
[Out] 1/192*e^4*(64*e*x+125*d)*(-e^2*x^2+d^2)^(3/2)/x^4-1/240*e^2*(48*e*x+125*d)*
(-e^2*x^2+d^2)^(5/2)/x^6-1/8*d*(-e^2*x^2+d^2)^(7/2)/x^8-3/7*e*(-e^2*x^2+d^2
)^(7/2)/x^7-e^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+125/128*e^8*arctanh((-e^2*
x^2+d^2)^(1/2)/d)-1/128*e^6*(128*e*x+125*d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.18, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1821, 825, 858, 223, 209, 272, 65, 214}

$$e^8\left(-\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)-\frac{d(d^2-e^2x^2)^{7/2}}{8x^8}-\frac{3e(d^2-e^2x^2)^{7/2}}{7x^7}-\frac{e^2(125d+48ex)\left(d^2-e^2x^2\right)^{5/2}}{240x^6}+\frac{125}{128}e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2}+\frac{e^4(125d+64ex)\left(d^2-e^2x^2\right)^{3/2}}{192x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] -1/128*(e^6*(125*d + 128*e*x)*Sqrt[d^2 - e^2*x^2])/x^2 + (e^4*(125*d + 64*e
*x)*(d^2 - e^2*x^2)^(3/2))/(192*x^4) - (e^2*(125*d + 48*e*x)*(d^2 - e^2*x^2
)^(5/2))/(240*x^6) - (d*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (3*e*(d^2 - e^2*x^
2)^(7/2))/(7*x^7) - e^8*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] + (125*e^8*ArcTan
h[Sqrt[d^2 - e^2*x^2]/d])/128
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^9}\,dx = -\frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-24d^4e-25d^3e^2x-8d^2e^3x^2)}{x^8}\,dx}{8d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} + \frac{\int \frac{(175d^5e^2+56d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{e^2(125d+48ex)(d^2-e^2x^2)^{5/2}}{240x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{3e(d^2-e^2x^2)^{7/2}}{7x^7} -$$

$$= \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{e^2(125d+48ex)(d^2-e^2x^2)^{5/2}}{240x^6} - d(d^2 -$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - e^2(12$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - e^2(12$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - e^2(12$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - e^2(12$$

$$= -\frac{e^6(125d+128ex)\sqrt{d^2-e^2x^2}}{128x^2} + \frac{e^4(125d+64ex)(d^2-e^2x^2)^{3/2}}{192x^4} - e^2(12$$

<u>**Mathematica**</u> [**A**]
time = 0.80, size = 185, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}\,(-1680d^7-5760d^6ex-1960d^5e^2x^2+14592d^4e^3x^3+17710d^3e^4x^4-7424d^2e^5x^5-27195de^6x^6-14848e^7x^7)}{13440x^8} - \frac{125}{64}e^8\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{e^9\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^9,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-1680*d^7 - 5760*d^6*e*x - 1960*d^5*e^2*x^2 + 14592*d
^4*e^3*x^3 + 17710*d^3*e^4*x^4 - 7424*d^2*e^5*x^5 - 27195*d*e^6*x^6 - 14848
*e^7*x^7))/(13440*x^8) - (125*e^8*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2
*x^2]/d])/64 + (e^9*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/Sqrt[-e^2]
```

<u>**Maple**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(178) = 356$.
time = 0.08, size = 640, normalized size = 3.14

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,\left(14848e^{7}x^{7}+27195d\,e^{6}x^{6}+7424d^{2}e^{5}x^{5}-17710d^{3}e^{4}x^{4}-14592d^{4}e^{3}x^{3}+1960d^{5}e^{2}x^{2}+5760d^{6}ex+1680d^{7}\right)}{13440x^{8}}-\frac{e^{9}}{}$

$$3d^2\left(\frac{x\sqrt{-}}{}\right.$$

$$5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\right.$$

$$6e^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\right.$$

$$4e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{d^2x}-\frac{}{d^2}\right.$$

$$2e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{3d^2x^3}-\frac{}{3d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/5/d^2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+
d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^
2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x
^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)))))))+d^3*(-1/8/d^2/x^8*(-e^2*x^2+d^2)^(7/2)+1/8*e^2/d^2*(-1/6/d^2/x^6*(-
e^2*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d
^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)
+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln
((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))))+3*d*e^2*(-1/6/d^2/x^6
*(-e^2*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^
2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5
/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)
*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))))-3/7*e*(-e^2*x^2+d^2
)^(7/2)/x^7
```

Maxima [A]
time = 0.50, size = 327, normalized size = 1.60

$$-\arcsin\left(\frac{xe}{d}\right)e^8+\frac{125}{128}e^8\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\frac{\sqrt{-x^2e^2+d^2}\,xe^9}{d^2}-\frac{125\sqrt{-x^2e^2+d^2}\,e^8}{128\,d}-\frac{2(-x^2e^2+d^2)^{\frac{3}{2}}xe^9}{3\,d^4}-\frac{125(-x^2e^2+d^2)^{\frac{3}{2}}e^8}{384\,d^3}-\frac{25(-x^2e^2+d^2)^{\frac{5}{2}}e^8}{128\,d^5}-\frac{8(-x^2e^2+d^2)^{\frac{5}{2}}e^7}{15\,d^4x}-\frac{25(-x^2e^2+d^2)^{\frac{7}{2}}e^6}{128\,d^5x^2}+\frac{2(-x^2e^2+d^2)^{\frac{7}{2}}e^5}{15\,d^4x^3}+\frac{25(-x^2e^2+d^2)^{\frac{7}{2}}e^4}{192\,d^3x^4}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}e^3}{5\,d^2x^5}-\frac{25(-x^2e^2+d^2)^{\frac{7}{2}}e^2}{48\,dx^6}-\frac{3(-x^2e^2+d^2)^{\frac{7}{2}}e}{7\,x^7}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}d}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] -arcsin(x*e/d)*e^8 + 125/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*
d/abs(x)) - sqrt(-x^2*e^2 + d^2)*x*e^9/d^2 - 125/128*sqrt(-x^2*e^2 + d^2)*e
^8/d - 2/3*(-x^2*e^2 + d^2)^(3/2)*x*e^9/d^4 - 125/384*(-x^2*e^2 + d^2)^(3/2
)*e^8/d^3 - 25/128*(-x^2*e^2 + d^2)^(5/2)*e^8/d^5 - 8/15*(-x^2*e^2 + d^2)^(
5/2)*e^7/(d^4*x) - 25/128*(-x^2*e^2 + d^2)^(7/2)*e^6/(d^5*x^2) + 2/15*(-x^2
*e^2 + d^2)^(7/2)*e^5/(d^4*x^3) + 25/192*(-x^2*e^2 + d^2)^(7/2)*e^4/(d^3*x^
4) - 1/5*(-x^2*e^2 + d^2)^(7/2)*e^3/(d^2*x^5) - 25/48*(-x^2*e^2 + d^2)^(7/2
)*e^2/(d*x^6) - 3/7*(-x^2*e^2 + d^2)^(7/2)*e/x^7 - 1/8*(-x^2*e^2 + d^2)^(7/
2)*d/x^8
```

Fricas [A]
time = 3.65, size = 152, normalized size = 0.75

$$\frac{26880\,x^8\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^8-13125\,x^8e^8\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(14848\,x^7e^7+27195\,dx^6e^6+7424\,d^2x^5e^5-17710\,d^3x^4e^4-14592\,d^4x^3e^3+1960\,d^5x^2e^2+5760\,d^6xe+1680\,d^7)\sqrt{-x^2e^2+d^2}}{13440\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/13440*(26880*x^8*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^8 - 13125
*x^8*e^8*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (14848*x^7*e^7 + 27195*d*x^6*
e^6 + 7424*d^2*x^5*e^5 - 17710*d^3*x^4*e^4 - 14592*d^4*x^3*e^3 + 1960*d^5*x
^2*e^2 + 5760*d^6*x*e + 1680*d^7)*sqrt(-x^2*e^2 + d^2))/x^8
```

Sympy [C] Result contains complex when optimal does not.
time = 31.67, size = 1719, normalized size = 8.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**9,x)
```

```
[Out] d**7*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6)
 + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*
x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6),
Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*
e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**
6), True)) + d**5*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*ac
osh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(
-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e
**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 5*d**4*e**3
*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**
7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**
7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*
x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
*7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x
**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15
*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d
*e**2*x**7), True)) - 5*d**3*e**4*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**
2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sq
rt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x*
```

```
*2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*s
qrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
 - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**2*e**5*Piecewise((-e*sqrt(d**
2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs
(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**
3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + 3*d*e**6*Piecewise((-e*sqr
t(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x
**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(
-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**7*Piecewi
se((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt
(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2
/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(171) = 342$.

time = 0.74, size = 529, normalized size = 2.59

$$-\arcsin\left(\frac{xe}{d}\right)e^8\mathrm{sgn}(d)+\frac{x^8\left(\frac{720\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^6}{x}+\frac{1120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^4}{x^2}-\frac{3696\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^2}{x^3}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-2)}}{x^5}+\frac{77280\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6e^{(-4)}}{x^6}+\frac{122640\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7e^{(-6)}}{x^7}-\frac{14280\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4}{x^4}+105\,e^8\right)e^{16}}{215040\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8}+\frac{125}{128}e^8\log\left(\frac{\left|-2\,de-2\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\frac{73\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^6}{128\,x}-\frac{23\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^4}{64\,x^2}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^2}{384\,x^3}+\frac{11\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-2)}}{640\,x^5}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6e^{(-4)}}{192\,x^6}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7e^{(-6)}}{896\,x^7}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8e^{(-8)}}{2048\,x^8}+\frac{17\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4}{256\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^8*sgn(d) + 1/215040*x^8*(720*(d*e + sqrt(-x^2*e^2 + d^2)*e
)*e^6/x + 1120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^4/x^2 - 3696*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^3*e^2/x^3 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-2)/
x^5 + 77280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-4)/x^6 + 122640*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^7*e^(-6)/x^7 - 14280*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4/
x^4 + 105*e^8)*e^16/(d*e + sqrt(-x^2*e^2 + d^2)*e)^8 + 125/128*e^8*log(1/2*
abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 73/128*(d*e + sqrt(
-x^2*e^2 + d^2)*e)*e^6/x - 23/64*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^4/x^2 +
 1/384*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^2/x^3 + 11/640*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^5*e^(-2)/x^5 - 1/192*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-4)/x
^6 - 3/896*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^(-6)/x^7 - 1/2048*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^8*e^(-8)/x^8 + 17/256*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4/
x^4
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^3}{x^9} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^9, x)
```

3.80 $\int \frac{(d+ex)^3(d^2-e^2x^2)^{5/2}}{x^{10}} dx$

Optimal. Leaf size=187

$$-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4}-\frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6}-\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}-\frac{3e(d^2-e^2x^2)^{7/2}}{8x^8}-\frac{29e^2(d^2 \cdots}{63 \cdots}$$

```
[Out] 55/192*e^5*(-e^2*x^2+d^2)^(3/2)/x^4-11/48*e^3*(-e^2*x^2+d^2)^(5/2)/x^6-1/9*
d*(-e^2*x^2+d^2)^(7/2)/x^9-3/8*e*(-e^2*x^2+d^2)^(7/2)/x^8-29/63*e^2*(-e^2*x
^2+d^2)^(7/2)/d/x^7+55/128*e^9*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-55/128*e^7
*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.16, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1821, 821, 272, 43, 65, 214}

$$-\frac{d(d^2-e^2x^2)^{7/2}}{9x^9}-\frac{3e(d^2-e^2x^2)^{7/2}}{8x^8}-\frac{29e^2(d^2-e^2x^2)^{7/2}}{63dx^7}+\frac{55e^9\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d}-\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2}+\frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4}-\frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] (-55*e^7*Sqrt[d^2 - e^2*x^2])/(128*x^2) + (55*e^5*(d^2 - e^2*x^2)^(3/2))/(1
92*x^4) - (11*e^3*(d^2 - e^2*x^2)^(5/2))/(48*x^6) - (d*(d^2 - e^2*x^2)^(7/2
))/(9*x^9) - (3*e*(d^2 - e^2*x^2)^(7/2))/(8*x^8) - (29*e^2*(d^2 - e^2*x^2)^
(7/2))/(63*d*x^7) + (55*e^9*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{10}} dx = -\frac{d(d^2-e^2x^2)^{7/2}}{9x^9} - \frac{\int \frac{(d^2-e^2x^2)^{5/2}(-27d^4e-29d^3e^2x-9d^2e^3x^2)}{x^9} dx}{9d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{9x^9} - \frac{3e(d^2-e^2x^2)^{7/2}}{8x^8} + \frac{\int \frac{(232d^5e^2+99d^4e^3x)(d^2-e^2x^2)^{5/2}}{x^8} dx}{72d^4}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{9x^9} - \frac{3e(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{29e^2(d^2-e^2x^2)^{7/2}}{63dx^7} + \frac{1}{8}(11e^3) \int \frac{(}{}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{9x^9} - \frac{3e(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{29e^2(d^2-e^2x^2)^{7/2}}{63dx^7} + \frac{1}{16}(11e^3)\,\mathrm{Su}$$

$$= -\frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{9x^9} - \frac{3e(d^2-e^2x^2)^{7/2}}{8x^8} - \frac{29e^2(d^2-e}{63dx^7}$$

$$= \frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{9x^9} - \frac{3e(d^2-e^2x}{8x^8}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6} - \frac{d(d^2-e}{9x}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6} - \frac{d(d^2-e}{9x}$$

$$= -\frac{55e^7\sqrt{d^2-e^2x^2}}{128x^2} + \frac{55e^5(d^2-e^2x^2)^{3/2}}{192x^4} - \frac{11e^3(d^2-e^2x^2)^{5/2}}{48x^6} - \frac{d(d^2-e}{9x}$$

Mathematica [A]
time = 0.71, size = 156, normalized size = 0.83

$$-\frac{\sqrt{d^2-e^2x^2}\,(896d^8+3024d^7ex+1024d^6e^2x^2-7224d^5e^3x^3-8448d^4e^4x^4+3066d^3e^5x^5+10240d^2e^6x^6+4599de^7x^7-3712e^8x^8)+6930e^9x^9\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{8064dx^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^10,x]
```

```
[Out] -1/8064*(Sqrt[d^2 - e^2*x^2]*(896*d^8 + 3024*d^7*e*x + 1024*d^6*e^2*x^2 - 7
224*d^5*e^3*x^3 - 8448*d^4*e^4*x^4 + 3066*d^3*e^5*x^5 + 10240*d^2*e^6*x^6 +
 4599*d*e^7*x^7 - 3712*e^8*x^8) + 6930*e^9*x^9*ArcTanh[(Sqrt[-e^2]*x - Sqrt
[d^2 - e^2*x^2])/d])/(d*x^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(159) = 318.
time = 0.10, size = 504, normalized size = 2.70

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-3712e^8x^8+4599d\,e^7x^7+10240d^2e^6x^6+3066d^3e^5x^5-8448d^4e^4x^4-7224d^5e^3x^3+1024d^6e^2x^2+3024d^7ex+896}{8064x^9d}$

default	$3d^2e\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{8d^2x^8}+\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{6d^2x^6}-\frac{e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{4d^2x^4}-\frac{3e^2\left(-\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{5}+d^2\left(\frac{(-e^2x^2+\cdots}{3}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{8d^2}\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 3*d^2*e*(-1/8/d^2/x^8*(-e^2*x^2+d^2)^(7/2)+1/8*e^2/d^2*(-1/6/d^2/x^6*(-e^2*
x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(
-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2
*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))))+d^3*(-1/9/d^2/x^9*(-e^2*x
^2+d^2)^(7/2)-2/63*e^2/d^4/x^7*(-e^2*x^2+d^2)^(7/2))+e^3*(-1/6/d^2/x^6*(-e^
2*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2
*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d
^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((
2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))))-3/7*e^2*(-e^2*x^2+d^2)^(
7/2)/d/x^7
```

Maxima [A]
time = 0.49, size = 230, normalized size = 1.23

$$\frac{55\,e^9\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{128\,d}-\frac{55\,\sqrt{-x^2e^2+d^2}\,e^9}{128\,d^2}-\frac{55\,(-x^2e^2+d^2)^{\frac{3}{2}}e^9}{384\,d^4}-\frac{11\,(-x^2e^2+d^2)^{\frac{5}{2}}e^9}{128\,d^6}-\frac{11\,(-x^2e^2+d^2)^{\frac{7}{2}}e^7}{128\,d^6x^2}+\frac{11\,(-x^2e^2+d^2)^{\frac{7}{2}}e^5}{192\,d^4x^4}-\frac{11\,(-x^2e^2+d^2)^{\frac{7}{2}}e^3}{48\,d^2x^6}-\frac{29\,(-x^2e^2+d^2)^{\frac{7}{2}}e^2}{63\,dx^7}-\frac{3\,(-x^2e^2+d^2)^{\frac{7}{2}}e}{8\,x^8}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}d}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 55/128*e^9*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d - 55/128*s
qrt(-x^2*e^2 + d^2)*e^9/d^2 - 55/384*(-x^2*e^2 + d^2)^(3/2)*e^9/d^4 - 11/12
8*(-x^2*e^2 + d^2)^(5/2)*e^9/d^6 - 11/128*(-x^2*e^2 + d^2)^(7/2)*e^7/(d^6*x
^2) + 11/192*(-x^2*e^2 + d^2)^(7/2)*e^5/(d^4*x^4) - 11/48*(-x^2*e^2 + d^2)^
(7/2)*e^3/(d^2*x^6) - 29/63*(-x^2*e^2 + d^2)^(7/2)*e^2/(d*x^7) - 3/8*(-x^2*
e^2 + d^2)^(7/2)*e/x^8 - 1/9*(-x^2*e^2 + d^2)^(7/2)*d/x^9
```

Fricas [A]
time = 2.54, size = 133, normalized size = 0.71

$$-\frac{3465\,x^9e^9\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(3712\,x^8e^8-4599\,dx^7e^7-10240\,d^2x^6e^6-3066\,d^3x^5e^5+8448\,d^4x^4e^4+7224\,d^5x^3e^3-1024\,d^6x^2e^2-3024\,d^7xe-896\,d^8)\sqrt{-x^2e^2+d^2}}{8064\,dx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/8064*(3465*x^9*e^9*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (3712*x^8*e^8 -
4599*d*x^7*e^7 - 10240*d^2*x^6*e^6 - 3066*d^3*x^5*e^5 + 8448*d^4*x^4*e^4 +
7224*d^5*x^3*e^3 - 1024*d^6*x^2*e^2 - 3024*d^7*x*e - 896*d^8)*sqrt(-x^2*e^2
 + d^2))/(d*x^9)
```

Sympy [C] Result contains complex when optimal does not.
time = 34.48, size = 1889, normalized size = 10.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**10,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d*
*2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**
2/(e**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x
**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt
(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2)
+ 1)/(315*d**8), True)) + 3*d**6*e*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e*
*2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*
x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x*
*2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d
/(e*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**
2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/
(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(
-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1))
 - 5*I*e**8*asin(d/(e*x))/(128*d**7), True)) + d**5*e**2*Piecewise((-e*sqrt
(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*
x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**
2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2
/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x*
*4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(
-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 5*d**4*e**3*Piecewise((-d**2/(6
*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) -
1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) +
 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x)
)/(16*d**5), True)) - 5*d**3*e**4*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d
**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d
**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**
2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d*
*2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqr
t(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1
 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e*
*2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2
*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + d**2*e**5*Piecewise(
```

(-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + 3*d*e**6*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(148) = 296$.
time = 0.71, size = 618, normalized size = 3.30

$$\frac{x^9\left(\frac{189\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^7}{x}+\frac{324\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^5}{x^2}-\frac{672\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^3}{x^3}-\frac{3024\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e}{x^4}-\frac{1512\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-1)}}{x^5}+\frac{9744\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6e^{(-3)}}{x^6}+\frac{18144\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7e^{(-5)}}{x^7}-\frac{16632\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8e^{(-7)}}{x^8}+28e^9\right)e^{18}}{129024\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^9d}+\frac{55e^9\log\left(\frac{\left|-2de-2\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2|x|}\right)}{128d}+\frac{\frac{16632\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^8e^7}{x}-\frac{18144\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^8e^5}{x^2}-\frac{9744\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^8e^3}{x^3}+\frac{1512\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^8e}{x^4}+\frac{3024\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^8e^{(-1)}}{x^5}+\frac{672\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6d^8e^{(-3)}}{x^6}-\frac{324\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7d^8e^{(-5)}}{x^7}-\frac{189\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8d^8e^{(-7)}}{x^8}-\frac{28\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^9d^8e^{(-9)}}{x^9}}{129024d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/129024*x^9*(189*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^7/x + 324*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^5/x^2 - 672*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^3/x^3 - 3024*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e/x^4 - 1512*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-1)/x^5 + 9744*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-3)/x^6 + 18144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^(-5)/x^7 - 16632*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*e^(-7)/x^8 + 28*e^9)*e^18/((d*e + sqrt(-x^2*e^2 + d^2)*e)^9*d) + 55/128*e^9*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d + 1/129024*(16632*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^8*e^7/x - 18144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^8*e^5/x^2 - 9744*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^8*e^3/x^3 + 1512*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^8*e/x^4 + 3024*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^8*e^(-1)/x^5 + 672*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^8*e^(-3)/x^6 - 324*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^8*e^(-5)/x^7 - 189*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^8*e^(-7)/x^8 - 28*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*d^8*e^(-9)/x^9)/d^9
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^10, x)
```

3.81 $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{11}} dx$

Optimal. Leaf size=225

$$-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4}-\frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6}-\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}}-\frac{e(d^2-e^2x^2)^{7/2}}{3x^9}-\frac{33e^2(d^2-$$

```
[Out] 11/128*e^6*(-e^2*x^2+d^2)^(3/2)/d/x^4-11/160*e^4*(-e^2*x^2+d^2)^(5/2)/d/x^6
-1/10*d*(-e^2*x^2+d^2)^(7/2)/x^10-1/3*e*(-e^2*x^2+d^2)^(7/2)/x^9-33/80*e^2*
(-e^2*x^2+d^2)^(7/2)/d/x^8-5/21*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^7+33/256*e^1
0*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-33/256*e^8*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A]
time = 0.18, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1821, 849, 821, 272, 43, 65, 214}

$$-\frac{d(d^2-e^2x^2)^{7/2}}{10x^{10}}-\frac{e(d^2-e^2x^2)^{7/2}}{3x^9}-\frac{33e^2(d^2-e^2x^2)^{7/2}}{80dx^8}+\frac{33e^{10}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^2}-\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2}+\frac{11e^6(d^2-e^2x^2)^{3/2}}{128dx^4}-\frac{11e^4(d^2-e^2x^2)^{5/2}}{160dx^6}-\frac{5e^3(d^2-e^2x^2)^{7/2}}{21d^2x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] (-33*e^8*Sqrt[d^2 - e^2*x^2])/(256*d*x^2) + (11*e^6*(d^2 - e^2*x^2)^(3/2))/
(128*d*x^4) - (11*e^4*(d^2 - e^2*x^2)^(5/2))/(160*d*x^6) - (d*(d^2 - e^2*x^
2)^(7/2))/(10*x^10) - (e*(d^2 - e^2*x^2)^(7/2))/(3*x^9) - (33*e^2*(d^2 - e^
2*x^2)^(7/2))/(80*d*x^8) - (5*e^3*(d^2 - e^2*x^2)^(7/2))/(21*d^2*x^7) + (33
*e^10*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(256*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}{x^{11}}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-30d^4e-33d^3e^2x-10d^2e^3x^2\right)}{x^{10}}\,dx}{10d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} + \frac{\int \frac{\left(297d^5e^2+150d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^9}\,dx}{90d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \int \frac{(-1200d^6e^3-2}{}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2-e^2x^2\right.}{21d^2x^7}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x^2\right)^{7/2}}{80dx^8} - \frac{5e^3\left(d^2-e^2x^2\right.}{21d^2x^7}$$

$$= -\frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right)^{7/2}}{3x^9} - \frac{33e^2\left(d^2-e^2x\right.}{80dx^8}$$

$$= \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e^2x^2\right)^{7/2}}{10x^{10}} - \frac{e\left(d^2-e^2x^2\right.}{3x^9}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e\right.}{10x}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e\right.}{10x}$$

$$= -\frac{33e^8\sqrt{d^2-e^2x^2}}{256dx^2} + \frac{11e^6\left(d^2-e^2x^2\right)^{3/2}}{128dx^4} - \frac{11e^4\left(d^2-e^2x^2\right)^{5/2}}{160dx^6} - \frac{d\left(d^2-e\right.}{10x}$$

Mathematica [A]
time = 0.79, size = 167, normalized size = 0.74

$$-\frac{\sqrt{d^2-e^2x^2}\left(2688d^9+8960d^8ex+3024d^7e^2x^2-20480d^6e^3x^3-23352d^5e^4x^4+7680d^4e^5x^5+24570d^3e^6x^6+10240d^2e^7x^7-3465de^8x^8-6400e^9x^9\right)+6930e^{10}x^{10}\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{26880d^2x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^11,x]
```

```
[Out] -1/26880*(Sqrt[d^2 - e^2*x^2]*(2688*d^9 + 8960*d^8*e*x + 3024*d^7*e^2*x^2 -
 20480*d^6*e^3*x^3 - 23352*d^5*e^4*x^4 + 7680*d^4*e^5*x^5 + 24570*d^3*e^6*x
^6 + 10240*d^2*e^7*x^7 - 3465*d*e^8*x^8 - 6400*e^9*x^9) + 6930*e^10*x^10*Ar
cTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(d^2*x^10)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(193) = 386$.

time = 0.13, size = 568, normalized size = 2.52

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(-6400 e^{9} x^{9}-3465 d\, e^{8} x^{8}+10240 d^{2} e^{7} x^{7}+24570 d^{3} e^{6} x^{6}+7680 d^{4} e^{5} x^{5}-23352 d^{5} e^{4} x^{4}-20480 d^{6} e^{3} x^{3}+3024 d^{7} e^{2} x^{2}\right.}{26880 x^{10} d^{2}}$

$$3e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8d^2x^8}+\frac{e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{6d^2x^6}-\frac{e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{4d^2x^4}-\frac{3e^2\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{2d^2x^2}-\frac{5e^2\left(\frac{\left(-e^2x^2-\right.}{5}\right.}{}\right.}{}\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(-1/10/d^2/x^10*(-e^2*x^2+d^2)^(7/2)+3/10*e^2/d^2*(-1/8/d^2/x^8*(-e^2*x
^2+d^2)^(7/2)+1/8*e^2/d^2*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-
1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(
7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^
2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d
^2)^(1/2))/x)))))))))+3*d*e^2*(-1/8/d^2/x^8*(-e^2*x^2+d^2)^(7/2)+1/8*e^2/d^
2*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^
2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-
e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-
d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))))+3*d
^2*e*(-1/9/d^2/x^9*(-e^2*x^2+d^2)^(7/2)-2/63*e^2/d^4/x^7*(-e^2*x^2+d^2)^(7/
2))-1/7*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^7
```

Maxima [A]
time = 0.49, size = 253, normalized size = 1.12

$$\frac{33\,e^{10}\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{256\,d^2}-\frac{33\,\sqrt{-x^2e^2+d^2}\,e^{10}}{256\,d^3}-\frac{11\,(-x^2e^2+d^2)^{\frac{3}{2}}e^{10}}{256\,d^5}-\frac{33\,(-x^2e^2+d^2)^{\frac{5}{2}}e^{10}}{1280\,d^7}-\frac{33\,(-x^2e^2+d^2)^{\frac{7}{2}}e^8}{1280\,d^7x^2}+\frac{11\,(-x^2e^2+d^2)^{\frac{7}{2}}e^6}{640\,d^5x^4}-\frac{11\,(-x^2e^2+d^2)^{\frac{7}{2}}e^4}{160\,d^3x^6}-\frac{5\,(-x^2e^2+d^2)^{\frac{7}{2}}e^3}{21\,d^2x^7}-\frac{33\,(-x^2e^2+d^2)^{\frac{7}{2}}e^2}{80\,dx^8}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}e}{3\,x^9}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}d}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 33/256*e^10*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^2 - 33/25
6*sqrt(-x^2*e^2 + d^2)*e^10/d^3 - 11/256*(-x^2*e^2 + d^2)^(3/2)*e^10/d^5 -
33/1280*(-x^2*e^2 + d^2)^(5/2)*e^10/d^7 - 33/1280*(-x^2*e^2 + d^2)^(7/2)*e^
8/(d^7*x^2) + 11/640*(-x^2*e^2 + d^2)^(7/2)*e^6/(d^5*x^4) - 11/160*(-x^2*e^
2 + d^2)^(7/2)*e^4/(d^3*x^6) - 5/21*(-x^2*e^2 + d^2)^(7/2)*e^3/(d^2*x^7) -
33/80*(-x^2*e^2 + d^2)^(7/2)*e^2/(d*x^8) - 1/3*(-x^2*e^2 + d^2)^(7/2)*e/x^9
 - 1/10*(-x^2*e^2 + d^2)^(7/2)*d/x^10
```

Fricas [A]
time = 3.46, size = 143, normalized size = 0.64

$$-\frac{3465\,x^{10}e^{10}\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(6400\,x^9e^9+3465\,dx^8e^8-10240\,d^2x^7e^7-24570\,d^3x^6e^6-7680\,d^4x^5e^5+23352\,d^5x^4e^4+20480\,d^6x^3e^3-3024\,d^7x^2e^2-8960\,d^8xe-2688\,d^9)\sqrt{-x^2e^2+d^2}}{26880\,d^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/26880*(3465*x^10*e^10*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (6400*x^9*e^9
 + 3465*d*x^8*e^8 - 10240*d^2*x^7*e^7 - 24570*d^3*x^6*e^6 - 7680*d^4*x^5*e^
```

```
5 + 23352*d^5*x^4*e^4 + 20480*d^6*x^3*e^3 - 3024*d^7*x^2*e^2 - 8960*d^8*x*e
 - 2688*d^9)*sqrt(-x^2*e^2 + d^2))/(d^2*x^10)
```

Sympy [C] Result contains complex when optimal does not.
time = 143.03, size = 2159, normalized size = 9.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**11,x)
```

```
[Out] d**7*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/(80*x**
9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*x**2) -
 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(768*d**
6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e**2*x**
2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqrt(-d**2
/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 7
*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(768*d**6*x
**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2/(e**2*x*
*2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + 3*d**6*e*Piecewise
((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**2) - 1)/
(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4) + 8*e**7
*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e**2*x**2)
 - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
+ 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) + 2*I*e**
5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2/(e**2*x
**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**8
), True)) + d**5*e**2*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)
) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**
2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5
*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*
d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2)
 + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**
5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x
**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*a
sin(d/(e*x))/(128*d**7), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2
*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e
**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**
2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2
) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e
**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2
*x**2) + 1)/(105*d**6), True)) - 5*d**3*e**4*Piecewise((-d**2/(6*e*x**7*sqr
t(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/
(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2
```

```
*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(
e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e*
*5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5)
, True)) + d**2*e**5*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3
*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x*
*2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/
d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)
/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(
-15*d**3*x**5 + 15*d*e**2*x**7), True)) + 3*d*e**6*Piecewise((-d**2/(4*e*x*
*5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) -
e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1))
- 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/
(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**7*Piecewise(
(-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(
3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x
**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(180) = 360$.
time = 0.58, size = 677, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/430080*x^10*(280*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^8/x + 525*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^2*e^6/x^2 - 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^4/x^3
- 3570*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^2/x^4 + 5880*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^6*e^(-2)/x^6 + 16800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^(-4)/x^7
 + 10500*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*e^(-6)/x^8 - 31920*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^9*e^(-8)/x^9 - 3360*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5/x^5 +
 42*e^10)*e^20/((d*e + sqrt(-x^2*e^2 + d^2)*e)^10*d^2) + 33/256*e^10*log(1/
2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 + 1/430080*(319
20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^18*e^8/x - 10500*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*d^18*e^6/x^2 - 16800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e^4/x
^3 - 5880*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^18*e^2/x^4 + 3570*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^6*d^18*e^(-2)/x^6 + 600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7
*d^18*e^(-4)/x^7 - 525*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^18*e^(-6)/x^8 - 2
80*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*d^18*e^(-8)/x^9 - 42*(d*e + sqrt(-x^2*e
```

```
^2 + d^2)*e)^10*d^18*e^(-10)/x^10 + 3360*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d
^18/x^5)/d^20
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d^2 - e^2 x^2)^{5/2} (d + e x)^3}{x^{11}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^11, x)
```

3.82 $\int \frac{(d+ex)^3 (d^2-e^2x^2)^{5/2}}{x^{12}} dx$

Optimal. Leaf size=254

$$-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4}-\frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6}-\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}}-\frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}}-\frac{37e^2(d^2-}{99d}$$

```
[Out] 19/384*e^7*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-19/480*e^5*(-e^2*x^2+d^2)^(5/2)/d^2
/x^6-1/11*d*(-e^2*x^2+d^2)^(7/2)/x^11-3/10*e*(-e^2*x^2+d^2)^(7/2)/x^10-37/9
9*e^2*(-e^2*x^2+d^2)^(7/2)/d/x^9-19/80*e^3*(-e^2*x^2+d^2)^(7/2)/d^2/x^8-74/
693*e^4*(-e^2*x^2+d^2)^(7/2)/d^3/x^7+19/256*e^11*arctanh((-e^2*x^2+d^2)^(1/
2)/d)/d^3-19/256*e^9*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A]
time = 0.20, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1821, 849, 821, 272, 43, 65, 214}

$$-\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}}-\frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}}-\frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9}-\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2}+\frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4}-\frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6}-\frac{19e^3(d^2-e^2x^2)^{7/2}}{80d^2x^8}+\frac{19e^{11}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{256d^3}-\frac{74e^4(d^2-e^2x^2)^{7/2}}{693d^3x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] (-19*e^9*Sqrt[d^2 - e^2*x^2])/(256*d^2*x^2) + (19*e^7*(d^2 - e^2*x^2)^(3/2)
)/(384*d^2*x^4) - (19*e^5*(d^2 - e^2*x^2)^(5/2))/(480*d^2*x^6) - (d*(d^2 -
e^2*x^2)^(7/2))/(11*x^11) - (3*e*(d^2 - e^2*x^2)^(7/2))/(10*x^10) - (37*e^2
*(d^2 - e^2*x^2)^(7/2))/(99*d*x^9) - (19*e^3*(d^2 - e^2*x^2)^(7/2))/(80*d^2
*x^8) - (74*e^4*(d^2 - e^2*x^2)^(7/2))/(693*d^3*x^7) + (19*e^11*ArcTanh[Sqr
t[d^2 - e^2*x^2]/d])/(256*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}}{x^{12}}\,dx = -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{5/2}\left(-33d^4e-37d^3e^2x-11d^2e^3x^2\right)}{x^{11}}\,dx}{11d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} + \frac{\int \frac{\left(370d^5e^2+209d^4e^3x\right)\left(d^2-e^2x^2\right)^{5/2}}{x^{10}}\,dx}{110d^4}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \int \frac{(-1881d^6e^3-7\cdots}{\cdots}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x\cdots}{80d^2x^8}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x\cdots}{80d^2x^8}$$

$$= -\frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x^2)^{7/2}}{99dx^9} - \frac{19e^3(d^2-e^2x\cdots}{80d^2x^8}$$

$$= -\frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2-e^2x\cdots}{99dx^9}$$

$$= \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2-e^2x^2)\cdots}{10x^{10}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2\cdots}{11x^{11}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2\cdots}{11x^{11}}$$

$$= -\frac{19e^9\sqrt{d^2-e^2x^2}}{256d^2x^2} + \frac{19e^7(d^2-e^2x^2)^{3/2}}{384d^2x^4} - \frac{19e^5(d^2-e^2x^2)^{5/2}}{480d^2x^6} - \frac{d(d^2-e^2\cdots}{11x^{11}}$$

Mathematica [A]
time = 0.84, size = 175, normalized size = 0.69

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-80640d^{10}-266112d^9ex-89600d^8e^2x^2+587664d^7e^3x^3+657920d^6e^4x^4-201432d^5e^5x^5-629760d^4e^6x^6-251790d^3e^7x^7+47360d^2e^8x^8+65835de^9x^9+94720e^{10}x^{10}\right)}{x^{11}} - 131670e^{11}\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{887040d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^(5/2))/x^12,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-80640*d^10 - 266112*d^9*e*x - 89600*d^8*e^2*x^2 + 5
87664*d^7*e^3*x^3 + 657920*d^6*e^4*x^4 - 201432*d^5*e^5*x^5 - 629760*d^4*e^
```

```
6*x^6 - 251790*d^3*e^7*x^7 + 47360*d^2*e^8*x^8 + 65835*d*e^9*x^9 + 94720*e^
10*x^10))/x^11 - 131670*e^11*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d
])/(887040*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(218) = 436$.
time = 0.18, size = 626, normalized size = 2.46

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-94720e^{10}x^{10}-65835d\,e^9x^9-47360d^2e^8x^8+251790d^3e^7x^7+629760d^4e^6x^6+201432d^5e^5x^5-657920d^6e^4x^4-}{887040x^{11}d^3}$

$$3e^2 \left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8d^2x^8} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(-1/11/d^2/x^11*(-e^2*x^2+d^2)^(7/2)+4/11*e^2/d^2*(-1/9/d^2/x^9*(-e^2*x
^2+d^2)^(7/2)-2/63*e^2/d^4/x^7*(-e^2*x^2+d^2)^(7/2)))+3*d^2*e*(-1/10/d^2/x^
10*(-e^2*x^2+d^2)^(7/2)+3/10*e^2/d^2*(-1/8/d^2/x^8*(-e^2*x^2+d^2)^(7/2)+1/8
*e^2/d^2*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2
*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*
(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)
^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))
))))+e^3*(-1/8/d^2/x^8*(-e^2*x^2+d^2)^(7/2)+1/8*e^2/d^2*(-1/6/d^2/x^6*(-e^2
*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*
(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^
2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2
*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))))+3*d*e^2*(-1/9/d^2/x^9*(-
e^2*x^2+d^2)^(7/2)-2/63*e^2/d^4/x^7*(-e^2*x^2+d^2)^(7/2))
```

Maxima [A]
time = 0.49, size = 276, normalized size = 1.09

$$\frac{19\,e^{11}\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{256\,d^3}-\frac{19\,\sqrt{-x^2e^2+d^2}\,e^{11}}{256\,d^4}-\frac{19\,(-x^2e^2+d^2)^{\frac{3}{2}}e^{11}}{768\,d^6}-\frac{19\,(-x^2e^2+d^2)^{\frac{5}{2}}e^{11}}{1280\,d^8}-\frac{19\,(-x^2e^2+d^2)^{\frac{7}{2}}e^9}{1280\,d^8x^2}+\frac{19\,(-x^2e^2+d^2)^{\frac{7}{2}}e^7}{1920\,d^6x^4}-\frac{19\,(-x^2e^2+d^2)^{\frac{7}{2}}e^5}{480\,d^4x^6}-\frac{74\,(-x^2e^2+d^2)^{\frac{7}{2}}e^4}{693\,d^3x^7}-\frac{19\,(-x^2e^2+d^2)^{\frac{7}{2}}e^3}{80\,d^2x^8}-\frac{37\,(-x^2e^2+d^2)^{\frac{7}{2}}e^2}{99\,dx^9}-\frac{3\,(-x^2e^2+d^2)^{\frac{7}{2}}e}{10\,x^{10}}-\frac{(-x^2e^2+d^2)^{\frac{7}{2}}d}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 19/256*e^11*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^3 - 19/25
6*sqrt(-x^2*e^2 + d^2)*e^11/d^4 - 19/768*(-x^2*e^2 + d^2)^(3/2)*e^11/d^6 -
19/1280*(-x^2*e^2 + d^2)^(5/2)*e^11/d^8 - 19/1280*(-x^2*e^2 + d^2)^(7/2)*e^
9/(d^8*x^2) + 19/1920*(-x^2*e^2 + d^2)^(7/2)*e^7/(d^6*x^4) - 19/480*(-x^2*e
^2 + d^2)^(7/2)*e^5/(d^4*x^6) - 74/693*(-x^2*e^2 + d^2)^(7/2)*e^4/(d^3*x^7)
 - 19/80*(-x^2*e^2 + d^2)^(7/2)*e^3/(d^2*x^8) - 37/99*(-x^2*e^2 + d^2)^(7/2
)*e^2/(d*x^9) - 3/10*(-x^2*e^2 + d^2)^(7/2)*e/x^10 - 1/11*(-x^2*e^2 + d^2)^
(7/2)*d/x^11
```

Fricas [A]
time = 2.64, size = 153, normalized size = 0.60

$$-\frac{65835\,x^{11}e^{11}\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(94720\,x^{10}e^{10}+65835\,dx^9e^9+47360\,d^2x^8e^8-251790\,d^3x^7e^7-629760\,d^4x^6e^6-201432\,d^5x^5e^5+657920\,d^6x^4e^4+587664\,d^7x^3e^3-89600\,d^8x^2e^2-266112\,d^9xe-80640\,d^{10})\sqrt{-x^2e^2+d^2}}{887040\,d^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/887040*(65835*x^11*e^11*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (94720*x^10
*e^10 + 65835*d*x^9*e^9 + 47360*d^2*x^8*e^8 - 251790*d^3*x^7*e^7 - 629760*d
```

```
^4*x^6*e^6 - 201432*d^5*x^5*e^5 + 657920*d^6*x^4*e^4 + 587664*d^7*x^3*e^3 -
 89600*d^8*x^2*e^2 - 266112*d^9*x*e - 80640*d^10)*sqrt(-x^2*e^2 + d^2))/(d^
3*x^11)
```

Sympy [C] Result contains complex when optimal does not.
time = 154.53, size = 2397, normalized size = 9.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**12,x)
```

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(11*x**10) + e**3*sqrt(d**2/(
e**2*x**2) - 1)/(99*d**2*x**8) + 8*e**5*sqrt(d**2/(e**2*x**2) - 1)/(693*d**
4*x**6) + 16*e**7*sqrt(d**2/(e**2*x**2) - 1)/(1155*d**6*x**4) + 64*e**9*sqr
t(d**2/(e**2*x**2) - 1)/(3465*d**8*x**2) + 128*e**11*sqrt(d**2/(e**2*x**2)
- 1)/(3465*d**10), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2)
 + 1)/(11*x**10) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(99*d**2*x**8) + 8*I*
e**5*sqrt(-d**2/(e**2*x**2) + 1)/(693*d**4*x**6) + 16*I*e**7*sqrt(-d**2/(e*
*2*x**2) + 1)/(1155*d**6*x**4) + 64*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(346
5*d**8*x**2) + 128*I*e**11*sqrt(-d**2/(e**2*x**2) + 1)/(3465*d**10), True))
 + 3*d**6*e*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9*e/
(80*x**9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sqrt(d**2/(e**2*
x**2) - 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**7/(
768*d**6*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 7*e**9/(256*d**8*x*sqrt(d**2/(e
**2*x**2) - 1)) + 7*e**10*acosh(d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2))
> 1), (I*d**2/(10*e*x**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqr
t(-d**2/(e**2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) +
1)) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**7/(768
*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d**8*x*sqrt(-d**2/(
e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256*d**9), True)) + d**5*e**2*P
iecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**
2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4)
+ 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e*
*2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) +
 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2
/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(
315*d**8), True)) - 5*d**4*e**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*
x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**
5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2)
 - 1)) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e
*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(
e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(19
2*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d*
```

```
*2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) -
5*I*e**8*asin(d/(e*x))/(128*d**7), True)) - 5*d**3*e**4*Piecewise((-e*sqrt(
d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x
**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2
/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/
(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**
4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-
d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e**5*Piecewise((-d**2/(6*e*
x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1))
 + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d*
*2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2))
 > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)
) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(
16*d**5), True)) + 3*d*e**6*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(
-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(
-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-1
5*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-
15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 -
e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**
2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**
2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/
d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e**7*Piecewise((-d**2/(4*e
*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1))
 - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3
), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1
)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d*
*2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 744 vs. 2(203) = 406.
time = 0.57, size = 744, normalized size = 2.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/14192640*x^11*(4158*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^9/x + 8470*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^2*e^7/x^2 - 3465*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^5
/x^3 - 40590*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^3/x^4 - 57750*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^5*e/x^5 + 6930*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-1)/x^
6 + 138600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^(-3)/x^7 + 244860*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^8*e^(-5)/x^8 + 152460*(d*e + sqrt(-x^2*e^2 + d^2)*e)^9*
e^(-7)/x^9 - 568260*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*e^(-9)/x^10 + 630*e^1
```

```
1)*e^22/((d*e + sqrt(-x^2*e^2 + d^2)*e)^11*d^3) + 19/256*e^11*log(1/2*abs(-
2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 + 1/14192640*(568260*(
d*e + sqrt(-x^2*e^2 + d^2)*e)*d^30*e^9/x - 152460*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^2*d^30*e^7/x^2 - 244860*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^30*e^5/x^3
 - 138600*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^30*e^3/x^4 - 6930*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^5*d^30*e/x^5 + 57750*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^
30*e^(-1)/x^6 + 40590*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^30*e^(-3)/x^7 + 34
65*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^30*e^(-5)/x^8 - 8470*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^9*d^30*e^(-7)/x^9 - 4158*(d*e + sqrt(-x^2*e^2 + d^2)*e)^10*d
^30*e^(-9)/x^10 - 630*(d*e + sqrt(-x^2*e^2 + d^2)*e)^11*d^30*e^(-11)/x^11)/
d^33
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)^3}{x^{12}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3)/x^12, x)
```

3.83 $\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}} \, dx$

Optimal. Leaf size=174

$$\frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}}+\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}+\frac{x\sqrt{d^2-e^2x^2}}{2e^5}-\frac{13d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] 1/5*d^4*(e*x+d)^3/e^6/(-e^2*x^2+d^2)^(5/2)-23/15*d^3*(e*x+d)^2/e^6/(-e^2*x^
2+d^2)^(3/2)-13/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+127/15*d^2*(e*x+
d)/e^6/(-e^2*x^2+d^2)^(1/2)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6+1/2*x*(-e^2*x^2+d^
2)^(1/2)/e^5
```

Rubi [A]
time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1649, 1829, 655, 223, 209}

$$-\frac{13d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}+\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}}-\frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^4*(d + e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d + e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d + e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 + (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}+\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}+\frac{5dx^4}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}+\frac{15d^3x^2}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{\int \frac{-}{\sqrt{}}}{}$$

$$= \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{}}{}$$

$$= \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{}}{}$$

$$= \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{}}{}$$

Mathematica [A]
time = 0.47, size = 123, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(-304d^4+717d^3ex-479d^2e^2x^2+45de^3x^3+15e^4x^4\right)}{30e^6(-d+ex)^3}+\frac{13d^2(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-304*d^4 + 717*d^3*e*x - 479*d^2*e^2*x^2 + 45*d*e^3*x
^3 + 15*e^4*x^4))/(30*e^6*(-d + e*x)^3) + (13*d^2*(-e^2)^(3/2)*Log[-(Sqrt[-
e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(152) = 304.
time = 0.08, size = 450, normalized size = 2.59

method	result
risch	$\frac{(ex+6d)\sqrt{-e^2x^2+d^2}}{2e^6}-\frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}}-\frac{127d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)e}}{15e^7\left(x-\frac{d}{e}\right)}-\frac{d^4\sqrt{}}{}$
default	$e^3\left(-\frac{x^7}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7d^2\left(\frac{x^5}{5e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e^2}\right)}{2e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/2*x^7/e^2/(-e^2*x^2+d^2)^(5/2)+7/2*d^2/e^2*(1/5*x^5/e^2/(-e^2*x^2+d
^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^
2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+3
*e^2*d*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/
```

```
2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^
(5/2))))+3*e*d^2*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2
*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+d^3*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*
d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)
))
```

Maxima [A]
time = 0.50, size = 284, normalized size = 1.63

$$-\frac{x^7 e}{2\left(-x^2 e^2 + d^2\right)^{\frac{5}{2}}} + \frac{19\, d^3 x^4 e^{(-2)}}{\left(-x^2 e^2 + d^2\right)^{\frac{5}{2}}} - \frac{76\, d^5 x^2 e^{(-4)}}{3\left(-x^2 e^2 + d^2\right)^{\frac{5}{2}}} + \frac{152\, d^7 e^{(-6)}}{15\left(-x^2 e^2 + d^2\right)^{\frac{5}{2}}} + \frac{13}{30}\left(\frac{15\, x^4 e^{(-2)}}{\left(-x^2 e^2 + d^2\right)^{\frac{5}{2}}} - \frac{20\, d^2 x^2 e^{(-4)}}{\left(-x^2 e^2 + d^2\right)^{\frac{5}{2}}} + \frac{8\, d^4 e^{(-6)}}{\left(-x^2 e^2 + d^2\right)^{\frac{5}{2}}}\right) d^2 x e - \frac{13}{6}\left(\frac{3\, x^2 e^{(-2)}}{\left(-x^2 e^2 + d^2\right)^{\frac{3}{2}}} - \frac{2\, d^2 e^{(-4)}}{\left(-x^2 e^2 + d^2\right)^{\frac{3}{2}}}\right) d^2 x e^{(-1)} - \frac{3\, d x^6}{\left(-x^2 e^2 + d^2\right)^{\frac{5}{2}}} + \frac{26\, d^4 x e^{(-5)}}{15\left(-x^2 e^2 + d^2\right)^{\frac{3}{2}}} - \frac{13}{2}\, d^2 \arcsin\left(\frac{x e}{d}\right) e^{(-6)} - \frac{91\, d^2 x e^{(-5)}}{30\sqrt{-x^2 e^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^7*e/(-x^2*e^2 + d^2)^(5/2) + 19*d^3*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2
) - 76/3*d^5*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 152/15*d^7*e^(-6)/(-x^2*e^
2 + d^2)^(5/2) + 13/30*(15*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2*e
^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2))*d^2*x*e
 - 13/6*(3*x^2*e^(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d^2*e^(-4)/(-x^2*e^2 + d^2
)^(3/2))*d^2*x*e^(-1) - 3*d*x^6/(-x^2*e^2 + d^2)^(5/2) + 26/15*d^4*x*e^(-5)
/(-x^2*e^2 + d^2)^(3/2) - 13/2*d^2*arcsin(x*e/d)*e^(-6) - 91/30*d^2*x*e^(-5
)/sqrt(-x^2*e^2 + d^2)
```

Fricas [A]
time = 3.05, size = 181, normalized size = 1.04

$$\frac{304\, d^2 x^3 e^3 - 912\, d^3 x^2 e^2 + 912\, d^4 x e - 304\, d^5 + 390\left(d^2 x^3 e^3 - 3\, d^3 x^2 e^2 + 3\, d^4 x e - d^5\right) \arctan\left(-\frac{\left(d - \sqrt{-x^2 e^2 + d^2}\right) e^{(-1)}}{x}\right) + \left(15\, x^4 e^4 + 45\, d x^3 e^3 - 479\, d^2 x^2 e^2 + 717\, d^3 x e - 304\, d^4\right)\sqrt{-x^2 e^2 + d^2}}{30\left(x^3 e^9 - 3\, d x^2 e^8 + 3\, d^2 x e^7 - d^3 e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*x^3*e^3 - 912*d^3*x^2*e^2 + 912*d^4*x*e - 304*d^5 + 390*(d^2*
x^3*e^3 - 3*d^3*x^2*e^2 + 3*d^4*x*e - d^5)*arctan(-(d - sqrt(-x^2*e^2 + d^2
))*e^(-1)/x) + (15*x^4*e^4 + 45*d*x^3*e^3 - 479*d^2*x^2*e^2 + 717*d^3*x*e -
 304*d^4)*sqrt(-x^2*e^2 + d^2))/(x^3*e^9 - 3*d*x^2*e^8 + 3*d^2*x*e^7 - d^3*
e^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (d + ex)^3}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**5*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A]
time = 0.79, size = 214, normalized size = 1.23

$$-\frac{13}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(xe^{(-5)}+6\,de^{(-6)}\right)-\frac{2\left(\frac{445\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^{(-2)}}{x}-\frac{665\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^{(-4)}}{x^2}+\frac{405\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2e^{(-6)}}{x^3}-\frac{90\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^2e^{(-8)}}{x^4}-107\,d^2\right)e^{(-6)}}{15\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -13/2*d^2*arcsin(x*e/d)*e^(-6)*sgn(d) + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^(-5)
+ 6*d*e^(-6)) - 2/15*(445*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*e^(-2)/x - 665
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^(-4)/x^2 + 405*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^3*d^2*e^(-6)/x^3 - 90*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^2*e^(-8
)/x^4 - 107*d^2)*e^(-6)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,{\left(d+e\,x\right)}^3}{{\left(d^2-e^2\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.84 $\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=142

$$\frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] 1/5*d^3*(e*x+d)^3/e^5/(-e^2*x^2+d^2)^(5/2)-6/5*d^2*(e*x+d)^2/e^5/(-e^2*x^2+
d^2)^(3/2)-3*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+24/5*d*(e*x+d)/e^5/(-e^
2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A]
time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1649, 655, 223, 209}

$$-\frac{3d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}-\frac{6d^2(d+ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e^5}+\frac{d^3(d+ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^3*(d + e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^(5/2)) - (6*d^2*(d + e*x)^2)/(5*e^
5*(d^2 - e^2*x^2)^(3/2)) + (24*d*(d + e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2]) + S
qrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^4}{e^4}+\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}+\frac{5dx^3}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{27d^4}{e^4}+\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}+\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x^2}}\,dx}{15d^3}$$

$$= \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\int}{}$$

$$= \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{(3d)\mathrm{Su}}{}$$

$$= \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d\tan}{}$$

Mathematica [A]
time = 0.38, size = 108, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\,(-24d^3+57d^2ex-39de^2x^2+5e^3x^3)}{5e^5(-d+ex)^3} + \frac{3d(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^3 + 57*d^2*e*x - 39*d*e^2*x^2 + 5*e^3*x^3))/(5*
e^5*(-d + e*x)^3) + (3*d*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*
x^2]])/e^8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(126) = 252$.
time = 0.07, size = 438, normalized size = 3.08

method	result
risch	$\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e^{5}}-\frac{3 d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{4} \sqrt{e^{2}}}-\frac{24 d \sqrt{-\left(x-\frac{d}{e}\right)^{2} e^{2}-2 d \left(x-\frac{d}{e}\right) e}}{5 e^{6} \left(x-\frac{d}{e}\right)}-\frac{d^{3} \sqrt{-\left(x-\frac{d}{e}\right)}}{5e}$
default	$e^{3} \left(-\frac{x^{6}}{e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6 d^{2} \left(\frac{x^{4}}{e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{4 d^{2} \left(\frac{x^{2}}{3 e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2 d^{2}}{15 e^{4} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)}{e^{2}}\right)}{e^{2}}\right)+3 d\, e^{2} \left(\frac{x^{5}}{5 e^{2} \left(-e^{2} x^{2}+d^{2}\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-
4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/
2))))+3*d*e^2*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^
2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+3*d^2*e*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4
*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2
)))+d^3*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+
d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(
-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(120) = 240$.
time = 0.48, size = 299, normalized size = 2.11

$$-\frac{x^{6} e}{(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}+\frac{9 d^{2} x^{4} e^{(-1)}}{(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}+\frac{d^{3} x^{3} e^{(-2)}}{2\,(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}-\frac{12 d^{4} x^{2} e^{(-3)}}{(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}-\frac{3 d^{5} x e^{(-4)}}{10\,(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}+\frac{24 d^{6} e^{(-5)}}{5\,(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}+\frac{1}{5}\left(\frac{15 x^{4} e^{(-2)}}{(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}-\frac{20 d^{2} x^{2} e^{(-4)}}{(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}+\frac{8 d^{4} e^{(-6)}}{(-x^{2} e^{2}+d^{2})^{\frac{5}{2}}}\right) d x e^{2}+\frac{9 d^{3} x e^{(-4)}}{10\,(-x^{2} e^{2}+d^{2})^{\frac{3}{2}}}-\left(\frac{3 x^{2} e^{(-2)}}{(-x^{2} e^{2}+d^{2})^{\frac{3}{2}}}-\frac{2 d^{2} e^{(-4)}}{(-x^{2} e^{2}+d^{2})^{\frac{3}{2}}}\right) d x-3 d \arcsin\left(\frac{x e}{d}\right) e^{(-5)}-\frac{6 d x e^{(-4)}}{5 \sqrt{-x^{2} e^{2}+d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -x^6*e/(-x^2*e^2 + d^2)^(5/2) + 9*d^2*x^4*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1
/2*d^3*x^3*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 12*d^4*x^2*e^(-3)/(-x^2*e^2 + d^
2)^(5/2) - 3/10*d^5*x*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 24/5*d^6*e^(-5)/(-x^2
*e^2 + d^2)^(5/2) + 1/5*(15*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2*
e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2))*d*x*e^
2 + 9/10*d^3*x*e^(-4)/(-x^2*e^2 + d^2)^(3/2) - (3*x^2*e^(-2)/(-x^2*e^2 + d^
2)^(3/2) - 2*d^2*e^(-4)/(-x^2*e^2 + d^2)^(3/2))*d*x - 3*d*arcsin(x*e/d)*e^(
-5) - 6/5*d*x*e^(-4)/sqrt(-x^2*e^2 + d^2)
```

Fricas [**A**]
time = 3.04, size = 167, normalized size = 1.18

$$\frac{24\,dx^3e^3-72\,d^2x^2e^2+72\,d^3xe-24\,d^4+30\,(dx^3e^3-3\,d^2x^2e^2+3\,d^3xe-d^4)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(5\,x^3e^3-39\,dx^2e^2+57\,d^2xe-24\,d^3)\sqrt{-x^2e^2+d^2}}{5\,(x^3e^8-3\,dx^2e^7+3\,d^2xe^6-d^3e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*d*x^3*e^3 - 72*d^2*x^2*e^2 + 72*d^3*x*e - 24*d^4 + 30*(d*x^3*e^3 -
3*d^2*x^2*e^2 + 3*d^3*x*e - d^4)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/
x) + (5*x^3*e^3 - 39*d*x^2*e^2 + 57*d^2*x*e - 24*d^3)*sqrt(-x^2*e^2 + d^2))
/(x^3*e^8 - 3*d*x^2*e^7 + 3*d^2*x*e^6 - d^3*e^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [**A**]
time = 0.69, size = 193, normalized size = 1.36

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)+\sqrt{-x^2e^2+d^2}\,e^{(-5)}-\frac{2\left(\frac{80\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^{(-2)}}{x}-\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{(-4)}}{x^2}+\frac{70\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3de^{(-6)}}{x^3}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4de^{(-8)}}{x^4}-19\,d\right)e^{(-5)}}{5\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^(-5)*sgn(d) + sqrt(-x^2*e^2 + d^2)*e^(-5) - 2/5*(80*(d
*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^(-2)/x - 120*(d*e + sqrt(-x^2*e^2 + d^2)*e
```

```
)^2*d*e^(-4)/x^2 + 70*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*e^(-6)/x^3 - 15*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^4*d*e^(-8)/x^4 - 19*d)*e^(-5)/((d*e + sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/x - 1)^5
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.85 $\int \frac{x^3(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(e*x+d)^3/e^4/(-e^2*x^2+d^2)^(5/2)-13/15*d*(e*x+d)^2/e^4/(-e^2*x^2+
d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+32/15*(e*x+d)/e^4/(-e^2*x^2
+d^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1649, 792, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}+\frac{d^2(d+ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d+ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d^2*(d + e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d + e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d + e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) - Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx &= \frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}+\frac{5dx^2}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d} \\
&= \frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{\left(\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d+ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2} \\
&= \frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3} \\
&= \frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \right.}{e^3} \\
&= \frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 95, normalized size = 0.81

$$\frac{(-22d^2+51dex-32e^2x^2)\sqrt{d^2-e^2x^2}}{15e^4(-d+ex)^3} + \frac{\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^3\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
[Out] ((-22*d^2 + 51*d*e*x - 32*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*e^4*(-d + e*x)^
3) + Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]/(e^3*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(104) = 208$.
time = 0.08, size = 377, normalized size = 3.19

method	result
default	$e^{3}\left(\frac{x^{5}}{5e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{\frac{x^{3}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{2}\sqrt{e^{2}}}}{e^{2}}}{e^{2}}\right)+3e^{2}d\left(\frac{x^{4}}{e^{2}\left(-e^{2}x^{2}+d\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/
2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-e^2*x^2+d^2)^(1/2)))))+3*e^2*d*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(
1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+3*e*d^
2*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(
5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x
^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+d^3*(1/3*x^2/e^2/(-e^2*x^2
+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(100) = 200$.
time = 0.50, size = 273, normalized size = 2.31

$$\frac{3\,d^{2}x^{3}e^{(-1)}}{2\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{11\,d^{3}x^{2}e^{(-2)}}{3\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{9\,d^{4}xe^{(-3)}}{10\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{22\,d^{5}e^{(-4)}}{15\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{1}{15}\left(\frac{15\,x^{4}e^{(-2)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{20\,d^{2}x^{2}e^{(-4)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{8\,d^{4}e^{(-6)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}\right)xe^{3}-\frac{1}{3}\left(\frac{3\,x^{2}e^{(-2)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{2\,d^{2}e^{(-4)}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}\right)xe+\frac{3\,dx^{4}}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{17\,d^{2}xe^{(-3)}}{30\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}-\arcsin\left(\frac{xe}{d}\right)e^{(-4)}+\frac{2\,xe^{(-3)}}{15\sqrt{-x^{2}e^{2}+d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*x^3*e^(-1)/(-x^2*e^2 + d^2)^(5/2) - 11/3*d^3*x^2*e^(-2)/(-x^2*e^2 +
 d^2)^(5/2) - 9/10*d^4*x*e^(-3)/(-x^2*e^2 + d^2)^(5/2) + 22/15*d^5*e^(-4)/(
-x^2*e^2 + d^2)^(5/2) + 1/15*(15*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 20*d^2
*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2))*x
*e^3 - 1/3*(3*x^2*e^(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d^2*e^(-4)/(-x^2*e^2 +
d^2)^(3/2))*x*e + 3*d*x^4/(-x^2*e^2 + d^2)^(5/2) + 17/30*d^2*x*e^(-3)/(-x^2
*e^2 + d^2)^(3/2) - arcsin(x*e/d)*e^(-4) + 2/15*x*e^(-3)/sqrt(-x^2*e^2 + d^
2)
```

Fricas [A]
time = 2.98, size = 152, normalized size = 1.29

$$\frac{22\,x^{3}e^{3}-66\,dx^{2}e^{2}+66\,d^{2}xe-22\,d^{3}+30\,(x^{3}e^{3}-3\,dx^{2}e^{2}+3\,d^{2}xe-d^{3})\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)-(32\,x^{2}e^{2}-51\,dxe+22\,d^{2})\sqrt{-x^{2}e^{2}+d^{2}}}{15\,(x^{3}e^{7}-3\,dx^{2}e^{6}+3\,d^{2}xe^{5}-d^{3}e^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*x^3*e^3 - 66*d*x^2*e^2 + 66*d^2*x*e - 22*d^3 + 30*(x^3*e^3 - 3*d*x
^2*e^2 + 3*d^2*x*e - d^3)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (3
2*x^2*e^2 - 51*d*x*e + 22*d^2)*sqrt(-x^2*e^2 + d^2))/(x^3*e^7 - 3*d*x^2*e^6
 + 3*d^2*x*e^5 - d^3*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}(d+ex)^{3}}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A]
time = 0.71, size = 170, normalized size = 1.44

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{2\left(\frac{95\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}-\frac{145\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}e^{(-4)}}{x^{2}}+\frac{75\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}e^{(-6)}}{x^{3}}-\frac{15\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}e^{(-8)}}{x^{4}}-22\right)e^{(-4)}}{15\left(\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-4)*sgn(d) - 2/15*(95*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/x - 145*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-4)/x^2 + 75*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^3*e^(-6)/x^3 - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x
^4 - 22)*e^(-4)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3}\,{\left(d+e\,x\right)}^{3}}{{\left(d^{2}-e^{2}\,x^{2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((x^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.86 $\int \frac{x^2(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=93

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d*(e*x+d)^3/e^3/(-e^2*x^2+d^2)^(5/2)-8/15*(e*x+d)^2/e^3/(-e^2*x^2+d^2)^
(3/2)+7/15*(e*x+d)/d/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1649, 803, 651}

$$\frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d*(d + e*x)^3)/(5*e^3*(d^2 - e^2*x^2)^(5/2)) - (8*(d + e*x)^2)/(15*e^3*(d^
2 - e^2*x^2)^(3/2)) + (7*(d + e*x))/(15*d*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

```
&& GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx &= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{\left(\frac{3d^2}{e^2}+\frac{5dx}{e}\right)(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx}{5d} \\
&= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{7\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx}{15e^2} \\
&= \frac{d(d+ex)^3}{5e^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{8(d+ex)^2}{15e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.32, size = 53, normalized size = 0.57

$$\frac{\sqrt{d^2-e^2x^2}\,(2d^2-6dex+7e^2x^2)}{15de^3(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 - 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d - e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(81) = 162$.
time = 0.06, size = 365, normalized size = 3.92

method	result
trager	$\frac{(7e^2x^2-6dex+2d^2)\sqrt{-e^2x^2+d^2}}{15d\,e^3(-ex+d)^3}$
gosper	$\frac{(-ex+d)(ex+d)^4(7e^2x^2-6dex+2d^2)}{15d\,e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$

default	$e^{3}\left(\frac{x^{4}}{e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{4d^{2}\left(\frac{x^{2}}{3e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2d^{2}}{15e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}\right)}{e^{2}}\right)+3e^{2}d\left(\frac{x^{3}}{2e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{3d^{2}\left(\frac{x}{4e^{2}\left(-e^{2}x^{2}+d^{2}\right.}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/
2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+3*e^2*d*(1/2*x^3/e^2/(-e^2*x^2+d^2)^
(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-
e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*
x^2+d^2)^(1/2)))))+3*e*d^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(
-e^2*x^2+d^2)^(5/2))+d^3*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x
/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4
/(-e^2*x^2+d^2)^(1/2))))
```

Maxima [A]
time = 0.29, size = 143, normalized size = 1.54

$$\frac{x^{4}e}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{d^{2}x^{2}e^{(-1)}}{3\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{7\,d^{3}xe^{(-2)}}{10\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{2\,d^{4}e^{(-3)}}{15\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{3\,dx^{3}}{2\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{7\,dxe^{(-2)}}{30\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{7\,xe^{(-2)}}{15\sqrt{-x^{2}e^{2}+d^{2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] x^4*e/(-x^2*e^2 + d^2)^(5/2) - 1/3*d^2*x^2*e^(-1)/(-x^2*e^2 + d^2)^(5/2) -
7/10*d^3*x*e^(-2)/(-x^2*e^2 + d^2)^(5/2) + 2/15*d^4*e^(-3)/(-x^2*e^2 + d^2)
^(5/2) + 3/2*d*x^3/(-x^2*e^2 + d^2)^(5/2) + 7/30*d*x*e^(-2)/(-x^2*e^2 + d^2
)^(3/2) + 7/15*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d)
```

Fricas [A]
time = 3.63, size = 100, normalized size = 1.08

$$\frac{2\,x^{3}e^{3}-6\,dx^{2}e^{2}+6\,d^{2}xe-2\,d^{3}-\left(7\,x^{2}e^{2}-6\,dxe+2\,d^{2}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{15\left(dx^{3}e^{6}-3\,d^{2}x^{2}e^{5}+3\,d^{3}xe^{4}-d^{4}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*x^3*e^3 - 6*d*x^2*e^2 + 6*d^2*x*e - 2*d^3 - (7*x^2*e^2 - 6*d*x*e +
2*d^2)*sqrt(-x^2*e^2 + d^2))/(d*x^3*e^6 - 3*d^2*x^2*e^5 + 3*d^3*x*e^4 - d^4
*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A]
time = 0.69, size = 98, normalized size = 1.05

$$-\frac{4\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}-1\right)e^{(-3)}}{15\,d\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -4/15*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 10*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^2*e^(-4)/x^2 - 1)*e^(-3)/(d*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2
)/x - 1)^5)
```

Mupad [B]
time = 2.69, size = 49, normalized size = 0.53

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2-6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 - 6*d*e*x))/(15*d*e^3*(d - e*x)^3
)
```

3.87 $\int \frac{x(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(e*x+d)^3/e^2/(-e^2*x^2+d^2)^(5/2)-2/5*(e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)
-1/5*x/d^2/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {803, 667, 197}

$$\frac{(d+ex)^3}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)}{5e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x))/(5*e^2*(d^2 - e^2
*x^2)^(3/2)) - x/(5*d^2*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{3\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx}{5e}$$
$$= \frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{5e^2(d^2-e^2x^2)^{3/2}} - \frac{\int \frac{1}{(d^2-e^2x^2)^{3/2}}\,dx}{5e}$$
$$= \frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{5e^2(d^2-e^2x^2)^{3/2}} - \frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Mathematica [**A**]
time = 0.30, size = 50, normalized size = 0.58

$$-\frac{\sqrt{d^2-e^2x^2}\,(d^2-3dex+e^2x^2)}{5d^2e^2(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(d^2 - 3*d*e*x + e^2*x^2))/(d^2*e^2*(d - e*x)^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(74) = 148.
time = 0.07, size = 308, normalized size = 3.58

method	result
trager	$-\frac{(e^2x^2-3dex+d^2)\sqrt{-e^2x^2+d^2}}{5d^2(-ex+d)^3e^2}$
gosper	$-\frac{(-ex+d)(ex+d)^4(e^2x^2-3dex+d^2)}{5d^2e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$
default	$e^3\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}\right)}{2e^2}\right)+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)
^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2
*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+3*e^2*d*(1/3*x^2/e^2/(-e
^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+3*e*d^2*(1/4*x/e^2/(-e
^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*
x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+1/5*d^3/e^2/(-
e^2*x^2+d^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 119, normalized size = 1.38

$$\frac{x^3e}{2\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}+\frac{3\,d^2xe^{(-1)}}{10\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}-\frac{d^3e^{(-2)}}{5\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}+\frac{dx^2}{\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}-\frac{xe^{(-1)}}{10\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}-\frac{xe^{(-1)}}{5\sqrt{-x^2e^2+d^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^3*e/(-x^2*e^2 + d^2)^(5/2) + 3/10*d^2*x*e^(-1)/(-x^2*e^2 + d^2)^(5/2)
 - 1/5*d^3*e^(-2)/(-x^2*e^2 + d^2)^(5/2) + d*x^2/(-x^2*e^2 + d^2)^(5/2) - 1
/10*x*e^(-1)/(-x^2*e^2 + d^2)^(3/2) - 1/5*x*e^(-1)/(sqrt(-x^2*e^2 + d^2)*d^
2)
```

Fricas [A]
time = 3.43, size = 98, normalized size = 1.14

$$-\frac{x^3e^3-3\,dx^2e^2+3\,d^2xe-d^3-\left(x^2e^2-3\,dxe+d^2\right)\sqrt{-x^2e^2+d^2}}{5\left(d^2x^3e^5-3\,d^3x^2e^4+3\,d^4xe^3-d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(x^3*e^3 - 3*d*x^2*e^2 + 3*d^2*x*e - d^3 - (x^2*e^2 - 3*d*x*e + d^2)*s
qrt(-x^2*e^2 + d^2))/(d^2*x^3*e^5 - 3*d^3*x^2*e^4 + 3*d^4*x*e^3 - d^5*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A]
time = 1.22, size = 128, normalized size = 1.49

$$\frac{2\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-2)}}{x}-\frac{5\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{(-4)}}{x^2}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{(-6)}}{x^3}-1\right)e^{(-2)}}{5\,d^2\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 2/5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 5*(d*e + sqrt(-x^2*e^2 + d
^2)*e)^2*e^(-4)/x^2 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 - 1)*e^
(-2)/(d^2*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [B]
time = 2.66, size = 46, normalized size = 0.53

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2-3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 - 3*d*e*x))/(5*d^2*e^2*(d - e*x)^3)
```

3.88 $\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(-e*x+d)^3+2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(-e
*x+d)^2+2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(-e*x+d)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {669, 673, 665}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d} \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2} \\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.51

$$\frac{\sqrt{d^2-e^2x^2}\,\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(91) = 182$.
time = 0.06, size = 246, normalized size = 2.39

method	result
trager	$\frac{\left(2e^2x^2-6dex+7d^2\right)\sqrt{-e^2x^2+d^2}}{15d^3\left(-ex+d\right)^3 e}$
gosper	$\frac{\left(-ex+d\right)\left(ex+d\right)^4\left(2e^2x^2-6dex+7d^2\right)}{15d^3e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
default	$e^3\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)+3d\,e^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\cdots}{d^2}\right)}{4e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+3*
d*e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)
^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/
2))))+3/5*d^2/e/(-e^2*x^2+d^2)^(5/2)+d^3*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/
5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [**A**]
time = 0.27, size = 96, normalized size = 0.93

$$\frac{x^2 e}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{7\,d^2e^{(-1)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{4\,dx}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{x}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d}+\frac{2\,x}{15\,\sqrt{-x^2e^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2*e/(-x^2*e^2 + d^2)^(5/2) + 7/15*d^2*e^(-1)/(-x^2*e^2 + d^2)^(5/2) +
 4/5*d*x/(-x^2*e^2 + d^2)^(5/2) + 1/15*x/((-x^2*e^2 + d^2)^(3/2)*d) + 2/15*
x/(sqrt(-x^2*e^2 + d^2)*d^3)
```

Fricas [**A**]
time = 2.51, size = 102, normalized size = 0.99

$$\frac{7\,x^3e^3-21\,dx^2e^2+21\,d^2xe-7\,d^3-(2\,x^2e^2-6\,dxe+7\,d^2)\sqrt{-x^2e^2+d^2}}{15\,(d^3x^3e^4-3\,d^4x^2e^3+3\,d^5xe^2-d^6e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*x^3*e^3 - 21*d*x^2*e^2 + 21*d^2*x*e - 7*d^3 - (2*x^2*e^2 - 6*d*x*e
+ 7*d^2)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3*e^4 - 3*d^4*x^2*e^3 + 3*d^5*x*e^2 -
 d^6*e)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [**A**]
time = 0.99, size = 158, normalized size = 1.53

$$-\frac{2\left(\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-8)}}{x^4}-7\right)e^{(-1)}}{15\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 40*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*e^(-4)/x^2 + 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 -
15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^4 - 7)*e^(-1)/(d^3*((d*e + sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [B]
time = 2.66, size = 49, normalized size = 0.48

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^3
)
```

3.89 $\int \frac{(d+ex)^3}{x(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=114

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] 4/5*(e*x+d)/(-e^2*x^2+d^2)^(5/2)+1/15*(11*e*x+5*d)/d^2/(-e^2*x^2+d^2)^(3/2)
-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/15*(22*e*x+15*d)/d^4/(-e^2*x^2+d^2)^
(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1819, 837, 12, 272, 65, 214}

$$\frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(d + e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d + 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d + 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-11d^2ex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2-22d^4e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^{10}e^4} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2 \cdots}}\right.}{2d^3} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x \cdots\right.}{d^3e^2} \\
&= \frac{4(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d+11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\cdots\right.}{d^4}
\end{aligned}
$$

Mathematica [A]
time = 0.47, size = 89, normalized size = 0.78

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\,(32d^2-51dex+22e^2x^2)}{(d-ex)^3} + 30\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(32*d^2 - 51*d*e*x + 22*e^2*x^2))/(d - e*x)^3 + 30*Ar
cTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(15*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(100) = 200$.
time = 0.07, size = 305, normalized size = 2.68

method	result

default	$e^{3}\left(\frac{x}{4e^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}-\frac{d^{2}\left(\frac{x}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{d^{2}}\right)}{4e^{2}}\right)+\frac{3d}{5(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+3e\,d^{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(
5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)
)))+3/5*d/(-e^2*x^2+d^2)^(5/2)+3*e*d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/
d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+d^3*(1
/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^
2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+
d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.26, size = 148, normalized size = 1.30

$$\frac{4\,xe}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{4\,d}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{11\,xe}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}+\frac{1}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}d}+\frac{22\,xe}{15\,\sqrt{-x^2e^2+d^2}\,d^4}-\frac{\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d^4}+\frac{1}{\sqrt{-x^2e^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 4/5*x*e/(-x^2*e^2 + d^2)^(5/2) + 4/5*d/(-x^2*e^2 + d^2)^(5/2) + 11/15*x*e/(
(-x^2*e^2 + d^2)^(3/2)*d^2) + 1/3/((-x^2*e^2 + d^2)^(3/2)*d) + 22/15*x*e/(s
qrt(-x^2*e^2 + d^2)*d^4) - log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(
x))/d^4 + 1/(sqrt(-x^2*e^2 + d^2)*d^3)
```

Fricas [A]
time = 2.76, size = 153, normalized size = 1.34

$$\frac{32\,x^3e^3-96\,dx^2e^2+96\,d^2xe-32\,d^3+15\,(x^3e^3-3\,dx^2e^2+3\,d^2xe-d^3)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(22\,x^2e^2-51\,dxe+32\,d^2)\sqrt{-x^2e^2+d^2}}{15\,(d^4x^3e^3-3\,d^5x^2e^2+3\,d^6xe-d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*x^3*e^3 - 96*d*x^2*e^2 + 96*d^2*x*e - 32*d^3 + 15*(x^3*e^3 - 3*d*x
^2*e^2 + 3*d^2*x*e - d^3)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - (22*x^2*e^2
```

```
- 51*d*x*e + 32*d^2)*sqrt(-x^2*e^2 + d^2))/(d^4*x^3*e^3 - 3*d^5*x^2*e^2 + 3
*d^6*x*e - d^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [A]
time = 0.85, size = 195, normalized size = 1.71

$$-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^4}-\frac{2\left(\frac{115\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-\frac{185\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{135\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}-\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-8)}}{x^4}-32\right)}{15\,d^4\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4 - 2/15*(
115*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 185*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^2*e^(-4)/x^2 + 135*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 - 45*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^4 - 32)/(d^4*((d*e + sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/(x*(d^2 - e^2*x^2)^(7/2)), x)
```

3.90 $\int \frac{(d+ex)^3}{x^2 \left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=145

$$\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] 4/5*e*(e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/5*e*(7*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(
3/2)-3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/5*e*(19*e*x+15*d)/d^5/(-e^2*
x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A]
time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1819, 821, 272, 65, 214}

$$\frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e*(d + e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (e*(5*d + 7*e*x))/(5*d^3*(d^2
 - e^2*x^2)^(3/2)) + (e*(15*d + 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqrt
[d^2 - e^2*x^2]/(d^5*x) - (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3-45d^2ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(3e)}{}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{(3e)\mathrm{S}}{}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{3\mathrm{Sub}}{}$$

$$= \frac{4e(d+ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{e(5d+7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} - \frac{3e\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.43, size = 105, normalized size = 0.72

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(5d^3-39d^2ex+57de^2x^2-24e^3x^3)}{x(-d+ex)^3}+30e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^3 - 39*d^2*e*x + 57*d*e^2*x^2 - 24*e^3*x^3))/(x*
(-d + e*x)^3) + 30*e*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(5*d^
5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(129) = 258.
time = 0.08, size = 309, normalized size = 2.13

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}-\frac{19\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)e}}{5d^5\left(x-\frac{d}{e}\right)}-\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}-\sqrt{-(x}$
default	$\frac{e}{5(-e^2x^2+d^2)^{\frac{5}{2}}}+3e^2d\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+d^3\left(-\frac{1}{d^2x(-e^2x^2+d^2)^{\frac{5}{2}}}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e/(-e^2*x^2+d^2)^(5/2)+3*e^2*d*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*
(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+d^3*(-1/d^
2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*
(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+3*e*d^2*(
1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d
^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2
+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.28, size = 177, normalized size = 1.22

$$\frac{9\,xe^2}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}d}+\frac{4\,e}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{d}{(-x^2e^2+d^2)^{\frac{5}{2}}x}+\frac{12\,xe^2}{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}+\frac{e}{(-x^2e^2+d^2)^{\frac{3}{2}}d^2}-\frac{3\,e\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{d^5}+\frac{24\,xe^2}{5\sqrt{-x^2e^2+d^2}\,d^5}+\frac{3\,e}{\sqrt{-x^2e^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 9/5*x*e^2/((-x^2*e^2 + d^2)^(5/2)*d) + 4/5*e/(-x^2*e^2 + d^2)^(5/2) - d/((-
x^2*e^2 + d^2)^(5/2)*x) + 12/5*x*e^2/((-x^2*e^2 + d^2)^(3/2)*d^3) + e/((-x^
2*e^2 + d^2)^(3/2)*d^2) - 3*e*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/a
bs(x))/d^5 + 24/5*x*e^2/(sqrt(-x^2*e^2 + d^2)*d^5) + 3*e/(sqrt(-x^2*e^2 + d
^2)*d^4)
```

Fricas [A]
time = 3.41, size = 176, normalized size = 1.21

$$\frac{24\,x^4e^4-72\,dx^3e^3+72\,d^2x^2e^2-24\,d^3xe+15\,(x^4e^4-3\,dx^3e^3+3\,d^2x^2e^2-d^3xe)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(24\,x^3e^3-57\,dx^2e^2+39\,d^2xe-5\,d^3)\sqrt{-x^2e^2+d^2}}{5\,(d^5x^4e^3-3\,d^6x^3e^2+3\,d^7x^2e-d^8x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(24*x^4*e^4 - 72*d*x^3*e^3 + 72*d^2*x^2*e^2 - 24*d^3*x*e + 15*(x^4*e^4
- 3*d*x^3*e^3 + 3*d^2*x^2*e^2 - d^3*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x)
 - (24*x^3*e^3 - 57*d*x^2*e^2 + 39*d^2*x*e - 5*d^3)*sqrt(-x^2*e^2 + d^2))/(
d^5*x^4*e^3 - 3*d^6*x^3*e^2 + 3*d^7*x^2*e - d^8*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x^2\,(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(131) = 262$.
time = 0.72, size = 287, normalized size = 1.98

$$-\frac{3\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^5}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,d^5x}+\frac{x\left(\frac{121\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{x}-\frac{410\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-3)}}{x^2}+\frac{610\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-5)}}{x^3}-\frac{425\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-7)}}{x^4}+\frac{125\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-9)}}{x^5}-5\,e\right)e^2}{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -3*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5 - 1/
2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/(d^5*x) + 1/10*x*(121*(d*e + sqrt(-
```

```
x^2*e^2 + d^2)*e)*e^(-1)/x - 410*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-3)/x^
2 + 610*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-5)/x^3 - 425*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^4*e^(-7)/x^4 + 125*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-9)/x^
5 - 5*e)*e^2/((d*e + sqrt(-x^2*e^2 + d^2)*e)*d^5*((d*e + sqrt(-x^2*e^2 + d^
2)*e)*e^(-2)/x - 1)^5)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x^2\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^3/(x^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.91 $\int \frac{(d+ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=182

$$\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\right.}{2d}$$

```
[Out] 4/5*e^2*(e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e^2*(31*e*x+25*d)/d^4/(-e^2*x
^2+d^2)^(3/2)-13/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e^2*(107*e*
x+90*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2-3*e*(-e^2
*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A]

time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1819, 1821, 821, 272, 65, 214}

$$\frac{4e^2(d+ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{e^2(25d+31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*e^2*(d + e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d + 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d + 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) - (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}}\,dx = \frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^3-15d^2ex-20de^2x^2-16e^3x^3}{x^3(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^3+45d^2ex+75de^2x^2+62e^3x^3}{x^3(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3-45d^2ex-90d}{x^3\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= \frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \int$$

$$= \frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - 3$$

$$= \frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - 3$$

$$= \frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - 3$$

$$= \frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - 3$$

Mathematica [**A**]
time = 0.49, size = 118, normalized size = 0.65

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(15d^4+45d^3ex-479d^2e^2x^2+717de^3x^3-304e^4x^4)}{x^2(-d+ex)^3} + 390e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{30d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^4 + 45*d^3*e*x - 479*d^2*e^2*x^2 + 717*d*e^3*x^
3 - 304*e^4*x^4))/(x^2*(-d + e*x)^3) + 390*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqrt
[d^2 - e^2*x^2])/d])/(30*d^6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(160) = 320.
time = 0.08, size = 436, normalized size = 2.40

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,(6ex+d)}{2d^{6}x^{2}}-\frac{107e\sqrt{-\left(x-\frac{d}{e}\right)^{2}e^{2}-2d\left(x-\frac{d}{e}\right)e}}{15d^{6}\left(x-\frac{d}{e}\right)}-\frac{13e^{2}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{2d^{5}\sqrt{d^{2}}}$
default	$e^{3}\left(\frac{x}{5d^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{d^{2}}\right)+d^{3}\left(-\frac{1}{2d^{2}x^{2}(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}+\frac{7e^{2}\left(\frac{1}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right.}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)
+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+d^3*(-1/2/d^2/x^2/(-e^2*x^2+d^2)^(5/2)+7/
2*e^2/d^2*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)
+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2
)*(-e^2*x^2+d^2)^(1/2))/x)))))+3*d^2*e*(-1/d^2/x/(-e^2*x^2+d^2)^(5/2)+6*e^2
/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2
)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+3*e^2*d*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)+1
/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/
(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]

time = 0.28, size = 201, normalized size = 1.10

$$\frac{19\,xe^{3}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}}+\frac{13\,e^{2}}{10\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d}-\frac{3\,e}{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}x}-\frac{d}{2\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}x^{2}}+\frac{76\,xe^{3}}{15\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{4}}+\frac{13\,e^{2}}{6\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{3}}-\frac{13\,e^{2}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)}{2\,d^{6}}+\frac{152\,xe^{3}}{15\sqrt{-x^{2}e^{2}+d^{2}}\,d^{6}}+\frac{13\,e^{2}}{2\sqrt{-x^{2}e^{2}+d^{2}}\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 19/5*x*e^3/((-x^2*e^2 + d^2)^(5/2)*d^2) + 13/10*e^2/((-x^2*e^2 + d^2)^(5/2)
*d) - 3*e/((-x^2*e^2 + d^2)^(5/2)*x) - 1/2*d/((-x^2*e^2 + d^2)^(5/2)*x^2) +
```

```
 76/15*x*e^3/((-x^2*e^2 + d^2)^(3/2)*d^4) + 13/6*e^2/((-x^2*e^2 + d^2)^(3/2
)*d^3) - 13/2*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^6 +
 152/15*x*e^3/(sqrt(-x^2*e^2 + d^2)*d^6) + 13/2*e^2/(sqrt(-x^2*e^2 + d^2)*d
^5)
```

Fricas [A]
time = 2.33, size = 192, normalized size = 1.05

$$\frac{254\,x^5e^5-762\,dx^4e^4+762\,d^2x^3e^3-254\,d^3x^2e^2+195\,(x^5e^5-3\,dx^4e^4+3\,d^2x^3e^3-d^3x^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(304\,x^4e^4-717\,dx^3e^3+479\,d^2x^2e^2-45\,d^3xe-15\,d^4)\sqrt{-x^2e^2+d^2}}{30\,(d^6x^5e^3-3\,d^7x^4e^2+3\,d^8x^3e-d^9x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*x^5*e^5 - 762*d*x^4*e^4 + 762*d^2*x^3*e^3 - 254*d^3*x^2*e^2 + 195
*(x^5*e^5 - 3*d*x^4*e^4 + 3*d^2*x^3*e^3 - d^3*x^2*e^2)*log(-(d - sqrt(-x^2*
e^2 + d^2))/x) - (304*x^4*e^4 - 717*d*x^3*e^3 + 479*d^2*x^2*e^2 - 45*d^3*x*
e - 15*d^4)*sqrt(-x^2*e^2 + d^2))/(d^6*x^5*e^3 - 3*d^7*x^4*e^2 + 3*d^8*x^3*
e - d^9*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{x^3\,(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/x**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(x**3*(-(-d + e*x)*(d + e*x))**(7/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(154) = 308.
time = 0.71, size = 353, normalized size = 1.94

$$-\frac{13\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^6}+\frac{x^2\left(\frac{2782\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-2)}}{x^2}-\frac{9410\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-4)}}{x^3}+\frac{13645\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-6)}}{x^4}-\frac{9285\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-8)}}{x^5}+\frac{2580\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6e^{(-10)}}{x^6}-\frac{105\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}{x}-15\,e^2\right)e^4}{120\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}-\frac{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6e^{(-2)}}{x^2}+\frac{12\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6}{x}}{8\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -13/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6
 + 1/120*x^2*(2782*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-2)/x^2 - 9410*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^3*e^(-4)/x^3 + 13645*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^4*e^(-6)/x^4 - 9285*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-8)/x^5 + 2580*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-10)/x^6 - 105*(d*e + sqrt(-x^2*e^2 + d^
2)*e)/x - 15*e^2)*e^4/((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*((d*e + sqrt(-x
```

```
^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5) - 1/8*((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d
^6*e^(-2)/x^2 + 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6/x)/d^12
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^3}{x^3\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^3/(x^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.92 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=147

$$\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5e}+\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2}}\right.}{8e^5}$$

```
[Out] 3/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+4/15*d^2*x^2*(-e^2*x^2+d^2)^(1
/2)/e^3-1/4*d*x^3*(-e^2*x^2+d^2)^(1/2)/e^2+1/5*x^4*(-e^2*x^2+d^2)^(1/2)/e+1
/120*d^3*(-45*e*x+64*d)*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 847, 794, 223, 209}

$$\frac{3d^5\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^5}+\frac{x^4\sqrt{d^2-e^2x^2}}{5e}-\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2}+\frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3}+\frac{d^3(64d-45ex)\sqrt{d^2-e^2x^2}}{120e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (4*d^2*x^2*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*x^3*Sqrt[d^2 - e^2*x^2])/(4*e
^2) + (x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (d^3*(64*d - 45*e*x)*Sqrt[d^2 - e^2
*x^2])/(120*e^5) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= \int \frac{x^4(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx \\
&= \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x^3(4d^2e-5de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{5e^2} \\
&= -\frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{\int \frac{x^2(15d^3e^2-16d^2e^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{20e^4} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} - \frac{\int \frac{x(32d^4e^3-45d^3e^4x)}{\sqrt{d^2-e^2x^2}}\,dx}{60e^6} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x}}{120e^5} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x}}{120e^5} \\
&= \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^3} - \frac{dx^3\sqrt{d^2-e^2x^2}}{4e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5e} + \frac{d^3(64d-45ex)\sqrt{d^2-e^2x}}{120e^5}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 111, normalized size = 0.76

$$\frac{e\sqrt{d^2-e^2x^2}\left(64d^4-45d^3ex+32d^2e^2x^2-30de^3x^3+24e^4x^4\right)+45d^5\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{120e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(64*d^4 - 45*d^3*e*x + 32*d^2*e^2*x^2 - 30*d*e^3*x^3
 + 24*e^4*x^4) + 45*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2
]])/(120*e^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(127) = 254$.
time = 0.07, size = 296, normalized size = 2.01

method	result
risch	$\frac{\left(24e^4x^4-30d\,e^3x^3+32d^2x^2e^2-45d^3ex+64d^4\right)\sqrt{-e^2x^2+d^2}}{120e^5}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^4\sqrt{e^2}}$
default	$\frac{-\frac{x^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{5e^2}-\frac{2d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{15e^4}}{e}-\frac{d\left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4e^2}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4e^2}\right)}{e^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/5*x^2*(-e^2*x^2+d^2)^(3/2)/e^2-2/15*d^2/e^4*(-e^2*x^2+d^2)^(3/2))-d
/e^2*(-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d^2/e^2*(1/2*x*(-e^2*x^2+d^2)^(1/
2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-1/3*d^2
/e^5*(-e^2*x^2+d^2)^(3/2)-d^3/e^4*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))+1/e^5*d^4*((-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.55, size = 115, normalized size = 0.78

$$\frac{3}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-5)}-\frac{5}{8}\,\sqrt{-x^2e^2+d^2}\,d^3xe^{(-4)}+\sqrt{-x^2e^2+d^2}\,d^4e^{(-5)}-\frac{1}{5}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}x^2e^{(-3)}+\frac{1}{4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}dxe^{(-4)}-\frac{7}{15}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^2e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*d^5*arcsin(x*e/d)*e^(-5) - 5/8*sqrt(-x^2*e^2 + d^2)*d^3*x*e^(-4) + sqrt
(-x^2*e^2 + d^2)*d^4*e^(-5) - 1/5*(-x^2*e^2 + d^2)^(3/2)*x^2*e^(-3) + 1/4*(
-x^2*e^2 + d^2)^(3/2)*d*x*e^(-4) - 7/15*(-x^2*e^2 + d^2)^(3/2)*d^2*e^(-5)
```

Fricas [A]
time = 2.80, size = 89, normalized size = 0.61

$$-\frac{1}{120}\left(90\,d^5\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(24\,x^4e^4-30\,dx^3e^3+32\,d^2x^2e^2-45\,d^3xe+64\,d^4\right)\sqrt{-x^2e^2+d^2}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(90*d^5*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (24*x^4*e^4 -
 30*d*x^3*e^3 + 32*d^2*x^2*e^2 - 45*d^3*x*e + 64*d^4)*sqrt(-x^2*e^2 + d^2))
*e^(-5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4\sqrt{-(-d+ex)(d+ex)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A]
time = 0.67, size = 77, normalized size = 0.52

$$\frac{3}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)+\frac{1}{120}\left(64\,d^4e^{(-5)}-\left(45\,d^3e^{(-4)}-2\left(16\,d^2e^{(-3)}+3\left(4\,xe^{(-1)}-5\,de^{(-2)}\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/8*d^5*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/120*(64*d^4*e^(-5) - (45*d^3*e^(-4)
 - 2*(16*d^2*e^(-3) + 3*(4*x*e^(-1) - 5*d*e^(-2))*x)*x)*x)*sqrt(-x^2*e^2 +
d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.93 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=118

$$-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}-\frac{3d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}$$

```
[Out] -3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-1/3*d*x^2*(-e^2*x^2+d^2)^(1/2
)/e^2+1/4*x^3*(-e^2*x^2+d^2)^(1/2)/e-1/24*d^2*(-9*e*x+16*d)*(-e^2*x^2+d^2)^
(1/2)/e^4
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 847, 794, 223, 209}

$$-\frac{3d^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}-\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2}+\frac{x^3\sqrt{d^2-e^2x^2}}{4e}-\frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -1/3*(d*x^2*Sqrt[d^2 - e^2*x^2])/e^2 + (x^3*Sqrt[d^2 - e^2*x^2])/(4*e) - (d
^2*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^4) - (3*d^4*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(8*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{x^3(d-ex)}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{\int \frac{x^2(3d^2e-4de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} + \frac{\int \frac{x(8d^3e^2-9d^2e^3x)}{\sqrt{d^2-e^2x^2}}\,dx}{12e^4}$$

$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{(3d^4)\int \frac{1}{\sqrt{d^2-}}}{8e^3}$$

$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{(3d^4)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{dx^2\sqrt{d^2-e^2x^2}}{3e^2} + \frac{x^3\sqrt{d^2-e^2x^2}}{4e} - \frac{d^2(16d-9ex)\sqrt{d^2-e^2x^2}}{24e^4} - \frac{3d^4\tan^{-1}\left(\frac{}{\sqrt{d}}\right.}{8e^4}$$

Mathematica [A]
time = 0.16, size = 100, normalized size = 0.85

$$\frac{e\sqrt{d^2-e^2x^2}\,(-16d^3+9d^2ex-8de^2x^2+6e^3x^3)-9d^4\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{24e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-16*d^3 + 9*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3) - 9*
d^4*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(24*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(102) = 204$.
time = 0.08, size = 243, normalized size = 2.06

method	result
risch	$-\frac{\left(-6e^3x^3+8d\,e^2x^2-9d^2ex+16d^3\right)\sqrt{-e^2x^2+d^2}}{24e^4}-\frac{3d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^3\sqrt{e^2}}$
default	$\frac{-\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4e^2}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4e^2}}{e}+\frac{d\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3e^4}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d^2/e^2*(1/2*x*(-e^2*x^2+d^2)^(1/2
)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+1/3*d/e^
4*(-e^2*x^2+d^2)^(3/2)+d^2/e^3*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1
/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))-d^3/e^4*((-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.49, size = 93, normalized size = 0.79

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-4)}+\frac{5}{8}\,\sqrt{-x^2e^2+d^2}\,d^2xe^{(-3)}-\sqrt{-x^2e^2+d^2}\,d^3e^{(-4)}-\frac{1}{4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}xe^{(-3)}+\frac{1}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}de^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*d^4*arcsin(x*e/d)*e^(-4) + 5/8*sqrt(-x^2*e^2 + d^2)*d^2*x*e^(-3) - sqr
t(-x^2*e^2 + d^2)*d^3*e^(-4) - 1/4*(-x^2*e^2 + d^2)^(3/2)*x*e^(-3) + 1/3*(-
x^2*e^2 + d^2)^(3/2)*d*e^(-4)
```

Fricas [A]
time = 2.47, size = 78, normalized size = 0.66

$$\frac{1}{24}\left(18\,d^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(6\,x^3e^3-8\,dx^2e^2+9\,d^2xe-16\,d^3\right)\sqrt{-x^2e^2+d^2}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(18*d^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (6*x^3*e^3 - 8*
d*x^2*e^2 + 9*d^2*x*e - 16*d^3)*sqrt(-x^2*e^2 + d^2))*e^(-4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A]
time = 0.66, size = 66, normalized size = 0.56

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{24}\left(16\,d^3e^{(-4)}-\left(9\,d^2e^{(-3)}+2\left(3\,xe^{(-1)}-4\,de^{(-2)}\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/8*d^4*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/24*(16*d^3*e^(-4) - (9*d^2*e^(-3)
+ 2*(3*x*e^(-1) - 4*d*e^(-2))*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.94 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=86

$$\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+
1/2*d*(-e*x+2*d)*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1653, 12, 799, 794, 223, 209}

$$\frac{d^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}+\frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{(d^2-e^2x^2)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (d^2 - e^2*x^2)^(3/2)/(3*e^3)
 + (d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= -\frac{(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{\int \frac{3de^3x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}\\
&= -\frac{(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{d\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{e}\\
&= -\frac{(d^2-e^2x^2)^{3/2}}{3e^3}-\frac{\int \frac{x(d^2e-de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}\\
&= \frac{d(2d-ex)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}
\end{aligned}
$$

Mathematica [A]

time = 0.13, size = 89, normalized size = 1.03

$$\frac{e\sqrt{d^2-e^2x^2}\,(4d^2-3dex+2e^2x^2)+3d^3\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(4*d^2 - 3*d*e*x + 2*e^2*x^2) + 3*d^3*Sqrt[-e^2]*Log
[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(6*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(74) = 148.
time = 0.07, size = 157, normalized size = 1.83

method	result
risch	$\frac{(2e^2x^2-3dex+4d^2)\sqrt{-e^2x^2+d^2}}{6e^3}+\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3e^3}-\frac{d\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^2}+\frac{d^2\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/e^3-d/e^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^
2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))+1/e^3*d^2*((-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.47, size = 71, normalized size = 0.83

$$\frac{1}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,dxe^{(-2)}+\sqrt{-x^2e^2+d^2}\,d^2e^{(-3)}-\frac{1}{3}\,(-x^2e^2+d^2)^{\frac{3}{2}}e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3) - 1/2*sqrt(-x^2*e^2 + d^2)*d*x*e^(-2) + sqrt(-
x^2*e^2 + d^2)*d^2*e^(-3) - 1/3*(-x^2*e^2 + d^2)^(3/2)*e^(-3)
```

Fricas [A]
time = 5.37, size = 69, normalized size = 0.80

$$-\frac{1}{6}\left(6\,d^{3}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)-\left(2\,x^{2}e^{2}-3\,dxe+4\,d^{2}\right)\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] -1/6*(6*d^3*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (2*x^2*e^2 - 3*d
*x*e + 4*d^2)*sqrt(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A]
time = 1.37, size = 54, normalized size = 0.63

$$\frac{1}{2}\,d^{3}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{6}\,\sqrt{-x^{2}e^{2}+d^{2}}\left(4\,d^{2}e^{(-3)}+\left(2\,xe^{(-1)}-3\,de^{(-2)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3
) + (2*x*e^(-1) - 3*d*e^(-2))*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{2}\,\sqrt{d^{2}-e^{2}\,x^{2}}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
[Out] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.95 $\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=62

$$-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}$$

```
[Out] -1/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/2*(-e*x+2*d)*(-e^2*x^2+d^2)
^(1/2)/e^2
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {799, 794, 223, 209}

$$-\frac{d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}-\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] -1/2*((2*d - e*x)*Sqrt[d^2 - e^2*x^2])/e^2 - (d^2*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(2*e^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{d^2-e^2x^2}}{d+ex}\,dx &= \frac{\int \frac{x(d^2e-de^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{de}\\
&= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e}\\
&= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e}\\
&= -\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{2e^2}-\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 77, normalized size = 1.24

$$\frac{e(-2d+ex)\sqrt{d^2-e^2x^2}-d^2\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
[Out] (e*(-2*d + e*x)*Sqrt[d^2 - e^2*x^2] - d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(54) = 108$.
time = 0.06, size = 135, normalized size = 2.18

method	result
risch	$-\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^2}-\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e\sqrt{e^2}}$

default	$\frac{\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}}{e}-\frac{d\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{}{\sqrt{-(x+}}\right.}{}\right.}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2)))-d/e^2*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.48, size = 52, normalized size = 0.84

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-2)}+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,xe^{(-1)}-\sqrt{-x^2e^2+d^2}\,de^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^(-2) + 1/2*sqrt(-x^2*e^2 + d^2)*x*e^(-1) - sqrt(-x
^2*e^2 + d^2)*d*e^(-2)
```

Fricas [A]
time = 5.40, size = 57, normalized size = 0.92

$$\frac{1}{2}\left(2\,d^2\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\sqrt{-x^2e^2+d^2}\,(xe-2\,d)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + sqrt(-x^2*e^2 + d
^2)*(x*e - 2*d))*e^(-2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A]
time = 1.49, size = 43, normalized size = 0.69

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(xe^{(-1)}-2\,de^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e^(-2)*sgn(d) + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^(-1) -
 2*d*e^(-2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x), x)
```

3.96 $\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

```
[Out] d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {679, 223, 209}

$$\frac{d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{\sqrt{d^2-e^2x^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/e + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}}{e} + d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{\sqrt{d^2-e^2x^2}}{e} + d\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)$$
$$= \frac{\sqrt{d^2-e^2x^2}}{e} + \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

Mathematica [A]
time = 0.11, size = 63, normalized size = 1.37

$$\frac{\sqrt{d^2-e^2x^2}}{e} - \frac{d\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x),x]
[Out] Sqrt[d^2 - e^2*x^2]/e - (d*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/Sqrt
[-e^2]
```

Maple [A]
time = 0.07, size = 78, normalized size = 1.70

method	result	size
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e}+\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}$	49
default	$\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}}{e}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
[Out] 1/e*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.49, size = 29, normalized size = 0.63

$$d\arcsin\left(\frac{xe}{d}\right)e^{(-1)} + \sqrt{-x^2e^2+d^2}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] d*arcsin(x*e/d)*e^(-1) + sqrt(-x^2*e^2 + d^2)*e^(-1)
```

Fricas [A]
time = 4.63, size = 48, normalized size = 1.04

$$-\left(2\,d\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] -(2*d*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - sqrt(-x^2*e^2 + d^2))*
e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)
```

Giac [A]
time = 2.34, size = 31, normalized size = 0.67

$$d\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d)+\sqrt{-x^2e^2+d^2}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] d*arcsin(x*e/d)*e^(-1)*sgn(d) + sqrt(-x^2*e^2 + d^2)*e^(-1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{d^2-e^2\,x^2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(d + e*x),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(d + e*x), x)
```

3.97 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=46

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -arctan(e*x/(-e^2*x^2+d^2)^(1/2))-arctanh((-e^2*x^2+d^2)^(1/2)/d)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {864, 858, 223, 209, 272, 65, 214}

$$-\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] -ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)}\,dx &= \int \frac{d-ex}{x\sqrt{d^2-e^2x^2}}\,dx\\
&= d\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{1}{2}d\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)\\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2}\\
&= -\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 1.76

$$2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{e\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)),x]
```

```
[Out] 2*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (e*Log[-(Sqrt[-e^2]*x
) + Sqrt[d^2 - e^2*x^2]])/Sqrt[-e^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(42) = 84$.
time = 0.07, size = 141, normalized size = 3.07

method	result
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2de\left(x+\frac{d}{e}\right)}+\frac{de \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^{2}}}}{d}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}-\frac{d^{2}\ln}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/d*((-e^2*x^2+d^2)^(1/2)-d^2/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [A]
time = 0.48, size = 55, normalized size = 1.20

$$-\frac{\left(d \arcsin\left(\frac{xe}{d}\right) e^{(-1)} + de^{(-1)} \log\left(\frac{2\,d^2}{|x|} + \frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)\right)e}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -(d*arcsin(x*e/d)*e^(-1) + d*e^(-1)*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^
2)*d/abs(x)))*e/d
```

Fricas [A]
time = 2.14, size = 51, normalized size = 1.11

$$2 \arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right) + \log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + log(-(d - sqrt(-x^2*e^2 +
d^2))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)), x)
```

Giac [A]
time = 1.78, size = 48, normalized size = 1.04

$$-\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d)-\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*sgn(d) - log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/abs(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.98 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{d^2-e^2x^2}}{dx}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d-(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 821, 272, 65, 214}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\frac{\sqrt{d^2-e^2x^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*x)) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)}\,dx &= \int \frac{d-ex}{x^2\sqrt{d^2-e^2x^2}}\,dx\\ &= -\frac{\sqrt{d^2-e^2x^2}}{dx} - e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx\\ &= -\frac{\sqrt{d^2-e^2x^2}}{dx} - \frac{1}{2}e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e}\\ &= -\frac{\sqrt{d^2-e^2x^2}}{dx} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 98, normalized size = 1.92

$$-\frac{\sqrt{d^2-e^2x^2}+ex\log\left(d\left(-d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)-ex\log\left(d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[d^2 - e^2*x^2] + e*x*Log[d*(-d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]
)] - e*x*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]])/(d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(47) = 94$.
time = 0.06, size = 228, normalized size = 4.47

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{dx}+\frac{e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$
default	$\frac{e\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d^2}+\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{d^2x}-\frac{2e^2(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e/d^2*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/d*(-1/d^2/x*(-e^2*x^2+d^2)^(
3/2)-2*e^2/d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-e/d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2
)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)/((x*e + d)*x^2), x)
```

Fricas [A]
time = 2.34, size = 49, normalized size = 0.96

$$-\frac{xe\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\sqrt{-x^2e^2+d^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] -(x*e*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + sqrt(-x^2*e^2 + d^2))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^{2}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(46) = 92.
time = 1.33, size = 102, normalized size = 2.00

$$\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d}+\frac{xe^{3}}{2\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-1)}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d + 1/2*x*e
^3/((d*e + sqrt(-x^2*e^2 + d^2)*e)*d) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
e^(-1)/(d*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d^{2}-e^{2}\,x^{2}}}{x^{2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.99 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

```
[Out] -1/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-1/2*(-e^2*x^2+d^2)^(1/2)/d/x^2
+e*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {864, 849, 821, 272, 65, 214}

$$\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*Sqrt[d^2 - e^2*x^2]/(d*x^2) + (e*Sqrt[d^2 - e^2*x^2])/(d^2*x) - (e^2*A
rcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)}\,dx &= \int \frac{d-ex}{x^3\sqrt{d^2-e^2x^2}}\,dx\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}-\frac{\int \frac{2d^2e-de^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}+\frac{e^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}+\frac{e^2\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{4d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{2d}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{2dx^2}+\frac{e\sqrt{d^2-e^2x^2}}{d^2x}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 75, normalized size = 0.91

$$\frac{-\frac{(d-2ex)\sqrt{d^2-e^2x^2}}{x^2}+2e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] (-(((d - 2*e*x)*Sqrt[d^2 - e^2*x^2])/x^2) + 2*e^2*ArcTanh[(Sqrt[-e^2]*x - S
qrt[d^2 - e^2*x^2])/d])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(72) = 144$.
time = 0.07, size = 326, normalized size = 3.98

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^2x^2}-\frac{e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)}{2d\sqrt{d^2}}$
default	$-\frac{e^2\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d^3}+\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2d^2x^2}-\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/d^3*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/d*(-1/2/d^2/x^2*(-e^2*x^2
+d^2)^(3/2)-1/2*e^2/d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(
d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))-e/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(3/2)-
2*e^2/d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-e^2*x^2+d^2)^(1/2))))+e^2/d^3*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*l
n((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)/((x*e + d)*x^3), x)
```

Fricas [A]
time = 3.54, size = 61, normalized size = 0.74

$$\frac{x^2e^2\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\sqrt{-x^2e^2+d^2}\,(2\,xe-d)}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2*e^2*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + sqrt(-x^2*e^2 + d^2)*(2*x
*e - d))/(d^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(-d+ex)(d+ex)}}{x^3(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(69) = 138.
time = 1.32, size = 172, normalized size = 2.10

$$-\frac{x^2\left(\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}{x}-e^2\right)e^4}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2}-\frac{e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^2}-\frac{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^{(-2)}}{x^2}-\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2}{x}}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/8*x^2*(4*(d*e + sqrt(-x^2*e^2 + d^2)*e)/x - e^2)*e^4/((d*e + sqrt(-x^2*e
^2 + d^2)*e)^2*d^2) - 1/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e
)*e^(-2)/abs(x))/d^2 - 1/8*((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^(-2)/x^2
 - 4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2/x)/d^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.100 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

```
[Out] 1/2*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/3*(-e^2*x^2+d^2)^(1/2)/d/x^3+
1/2*e*(-e^2*x^2+d^2)^(1/2)/d^2/x^2-2/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {864, 849, 821, 272, 65, 214}

$$\frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x}+\frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -1/3*Sqrt[d^2 - e^2*x^2]/(d*x^3) + (e*Sqrt[d^2 - e^2*x^2])/(2*d^2*x^2) - (2
*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) + (e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
(2*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)}\,dx = \int \frac{d-ex}{x^4\sqrt{d^2-e^2x^2}}\,dx$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{3d^2e-2de^2x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} + \frac{\int \frac{4d^3e^2-3d^2e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^4}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{e^3\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} - \frac{e^3\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x\right.}{4d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{e\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e\right.}{2d^2}$$

$$= -\frac{\sqrt{d^2-e^2x^2}}{3dx^3} + \frac{e\sqrt{d^2-e^2x^2}}{2d^2x^2} - \frac{2e^2\sqrt{d^2-e^2x^2}}{3d^3x} + \frac{e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}$$

Mathematica [A]
time = 0.25, size = 131, normalized size = 1.15

$$\frac{(-2d^2+3dex-4e^2x^2)\sqrt{d^2-e^2x^2} - 3e^3x^3\log\left(d^3\left(-d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right) + 3e^3x^3\log\left(d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{6d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] ((-2*d^2 + 3*d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2] - 3*e^3*x^3*Log[d^3*(-d
 - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2])] + 3*e^3*x^3*Log[d - Sqrt[-e^2]*x +
Sqrt[d^2 - e^2*x^2]])/(6*d^3*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(98) = 196.
time = 0.08, size = 351, normalized size = 3.08

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(4e^2x^2-3dex+2d^2)}{6d^3x^3} + \frac{e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^2\sqrt{d^2}}$

default	$\frac{e^3\left(\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d^4}-\frac{e\left(-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{2d^2x^2}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e^3/d^4*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^
(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-e/d^2*(-1/2/d^2/x^2*(-e^2*x^
2+d^2)^(3/2)-1/2*e^2/d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*
(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))+e^2/d^3*(-1/d^2/x*(-e^2*x^2+d^2)^(3/
2)-2*e^2/d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2))))-1/3/d^3/x^3*(-e^2*x^2+d^2)^(3/2)-e^3/d^4*((-
e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)/((x*e + d)*x^4), x)
```

Fricas [A]
time = 4.27, size = 72, normalized size = 0.63

$$-\frac{3\,x^3e^3\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\left(4\,x^2e^2-3\,dxe+2\,d^2\right)\sqrt{-x^2e^2+d^2}}{6\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(3*x^3*e^3*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (4*x^2*e^2 - 3*d*x*e +
 2*d^2)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^4\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(93) = 186.
time = 1.20, size = 239, normalized size = 2.10

$$-\frac{x^3\left(\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e}{x}-\frac{9\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-1)}}{x^2}-e^3\right)e^6}{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^3}+\frac{e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^3}-\frac{\frac{9\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6e}{x}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6e^{(-1)}}{x^2}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^6e^{(-3)}}{x^3}}{24\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/24*x^3*(3*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e/x - 9*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^2*e^(-1)/x^2 - e^3)*e^6/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3) + 1/
2*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1
/24*(9*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6*e/x - 3*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^2*d^6*e^(-1)/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^6*e^(-3)/x^3)/d
^9
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.101 $\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}$$

```
[Out] -3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/4*(-e^2*x^2+d^2)^(1/2)/d/x^4
+1/3*e*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-3/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^3/x^2+2/
3*e^3*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {864, 849, 821, 272, 65, 214}

$$-\frac{\sqrt{d^2-e^2x^2}}{4dx^4}+\frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{3e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}+\frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x}-\frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -1/4*Sqrt[d^2 - e^2*x^2]/(d*x^4) + (e*Sqrt[d^2 - e^2*x^2])/(3*d^2*x^3) - (3
*e^2*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) + (2*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^4*
x) - (3*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^5(d+ex)}\,dx = \int \frac{d-ex}{x^5\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} - \frac{\int \frac{4d^2e-3de^2x}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} + \frac{\int \frac{9d^3e^2-8d^2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^4}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{\int \frac{16d^4e^3-9d^3e^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{24d^6}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} + \frac{(3e^4)\int \frac{}{x\sqrt{d^2 \ldots}}}{8d^3 \ldots}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} + \frac{(3e^4)\,\text{Subst}\Big(\int \ldots}{\ldots}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} - \frac{(3e^2)\,\text{Subst}\Big(\int \ldots}{\ldots}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{4dx^4} + \frac{e\sqrt{d^2-e^2x^2}}{3d^2x^3} - \frac{3e^2\sqrt{d^2-e^2x^2}}{8d^3x^2} + \frac{2e^3\sqrt{d^2-e^2x^2}}{3d^4x} - \frac{3e^4\tanh^{-1}\Big(\ldots}{8d \ldots}$$

Mathematica [A]
time = 0.26, size = 101, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\,(-6d^3+8d^2ex-9de^2x^2+16e^3x^3)+18e^4x^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{24d^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x - 9*d*e^2*x^2 + 16*e^3*x^3) + 18*e
^4*x^4*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(24*d^4*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(123) = 246.
time = 0.07, size = 477, normalized size = 3.34

method	result

risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-16e^3x^3+9d\,e^2x^2-8d^2ex+6d^3)}{24d^4x^4}-\frac{3e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8d^3\sqrt{d^2}}$
default	$-\frac{e^4\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d^5}+\frac{-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4d^2x^4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^4/d^5*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/d*(-1/4/d^2/x^4*(-e^2*x^2
+d^2)^(3/2)+1/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(3/2)-1/2*e^2/d^2*((-e
^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x))))+e^2/d^3*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(3/2)-1/2*e^2/d^2*((-e^2*x^
2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x)))-e^3/d^4*(-1/d^2/x*(-e^2*x^2+d^2)^(3/2)-2*e^2/d^2*(1/2*x*(-e^2*x^2+d^2)
^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+1/3
*e/d^4/x^3*(-e^2*x^2+d^2)^(3/2)+e^4/d^5*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/
2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)/((x*e + d)*x^5), x)
```

Fricas [A]
time = 3.71, size = 82, normalized size = 0.57

$$\frac{9\,x^4e^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(16\,x^3e^3-9\,dx^2e^2+8\,d^2xe-6\,d^3)\sqrt{-x^2e^2+d^2}}{24\,d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(9*x^4*e^4*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (16*x^3*e^3 - 9*d*x^2*
e^2 + 8*d^2*x*e - 6*d^3)*sqrt(-x^2*e^2 + d^2))/(d^4*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^5\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**5*(d + e*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(116) = 232.
time = 1.43, size = 299, normalized size = 2.09

$$-\frac{x^4\left(\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^2}{x}+\frac{72\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-2)}}{x^3}-\frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}{x^2}-3e^4\right)e^8}{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^4}-\frac{3e^4\log\left(\frac{\left|-2de-2\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2|x|}\right)}{8d^4}+\frac{\frac{72\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{12}e^2}{x}+\frac{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{12}e^{(-2)}}{x^3}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^{12}e^{(-4)}}{x^4}-\frac{24\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^{12}}{x^2}}{192d^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/192*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^2/x + 72*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^3*e^(-2)/x^3 - 24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2/x^2 - 3*e^4)
*e^8/((d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4) - 3/8*e^4*log(1/2*abs(-2*d*e -
2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4 + 1/192*(72*(d*e + sqrt(-x^2*e
^2 + d^2)*e)*d^12*e^2/x + 8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^12*e^(-2)/x^
3 - 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^12*e^(-4)/x^4 - 24*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^2*d^12/x^2)/d^16
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.102 $\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=113

$$\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] 1/12*d*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)/e^3-1/5*(-e^2*x^2+d^2)^(5/2)/e^3+1
/8*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d^3*x*(-e^2*x^2+d^2)^(1/2)/
e^2
```

Rubi [A]
time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1653, 12, 799, 794, 201, 223, 209}

$$\frac{d^5\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}+\frac{d(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{12e^3}-\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e^3}+\frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (d^3*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) + (d*(4*d - 3*e*x)*(d^2 - e^2*x^2)^(3/2
))/(12*e^3) - (d^2 - e^2*x^2)^(5/2)/(5*e^3) + (d^5*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/(8*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d^2-e^2x^2)^{3/2}}{d+ex}dx = -\frac{(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{\int \frac{5de^3x(d^2-e^2x^2)^{3/2}}{d+ex}dx}{5e^4}$$
$$= -\frac{(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{d\int \frac{x(d^2-e^2x^2)^{3/2}}{d+ex}dx}{e}$$
$$= -\frac{(d^2-e^2x^2)^{5/2}}{5e^3} - \frac{\int x(d^2e-de^2x)\sqrt{d^2-e^2x^2}\,dx}{e^2}$$
$$= \frac{d(4d-3ex)(d^2-e^2x^2)^{3/2}}{12e^3} - \frac{(d^2-e^2x^2)^{5/2}}{5e^3} + \frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{4e^2}$$
$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d(4d-3ex)(d^2-e^2x^2)^{3/2}}{12e^3} - \frac{(d^2-e^2x^2)^{5/2}}{5e^3} + \frac{d^5\int \frac{1}{\sqrt{d^2-e^2x}}}{8e^2}$$
$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d(4d-3ex)(d^2-e^2x^2)^{3/2}}{12e^3} - \frac{(d^2-e^2x^2)^{5/2}}{5e^3} + \frac{d^5\text{Subst}\left(\int \frac{1}{1+e}\right.}{}$$
$$= \frac{d^3x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{d(4d-3ex)(d^2-e^2x^2)^{3/2}}{12e^3} - \frac{(d^2-e^2x^2)^{5/2}}{5e^3} + \frac{d^5\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.}{8e^3}$$

Mathematica [A]
time = 0.14, size = 111, normalized size = 0.98

$$\frac{e\sqrt{d^2-e^2x^2}\,(16d^4-15d^3ex+8d^2e^2x^2+30de^3x^3-24e^4x^4)+15d^5\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{120e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x]
[Out] (e*Sqrt[d^2 - e^2*x^2]*(16*d^4 - 15*d^3*e*x + 8*d^2*e^2*x^2 + 30*d*e^3*x^3
- 24*e^4*x^4) + 15*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]
])/(120*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(97) = 194$.
time = 0.07, size = 238, normalized size = 2.11

method	result
risch	$\frac{(-24e^4x^4+30de^3x^3+8d^2x^2e^2-15d^3ex+16d^4)\sqrt{-e^2x^2+d^2}}{120e^3} + \frac{d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^2\sqrt{e^2}}$

default	$-\frac{\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{5e^{3}}-\frac{d\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{e^{2}}+\frac{d^{2}\left(\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2d\right.}{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-e^2*x^2+d^2)^(5/2)/e^3-d/e^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/
2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))))+1/e^3*d^2*(1/3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+d*e*(-1/
4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(
e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 164, normalized size = 1.45

$$-\frac{1}{2}i\,d^{5}\arcsin\left(\frac{xe}{d}+2\right)e^{(-3)}-\frac{3}{8}\,d^{5}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+\frac{1}{2}\sqrt{x^{2}e^{2}+4\,dxe+3\,d^{2}}\,d^{3}xe^{(-2)}-\frac{3}{8}\sqrt{-x^{2}e^{2}+d^{2}}\,d^{3}xe^{(-2)}+\sqrt{x^{2}e^{2}+4\,dxe+3\,d^{2}}\,d^{4}e^{(-3)}-\frac{1}{4}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}dxe^{(-2)}+\frac{1}{3}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{2}e^{(-3)}-\frac{1}{5}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*I*d^5*arcsin(x*e/d + 2)*e^(-3) - 3/8*d^5*arcsin(x*e/d)*e^(-3) + 1/2*sq
rt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^3*x*e^(-2) - 3/8*sqrt(-x^2*e^2 + d^2)*d^3*x
*e^(-2) + sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^4*e^(-3) - 1/4*(-x^2*e^2 + d^2)
^(3/2)*d*x*e^(-2) + 1/3*(-x^2*e^2 + d^2)^(3/2)*d^2*e^(-3) - 1/5*(-x^2*e^2 +
 d^2)^(5/2)*e^(-3)
```

Fricas [A]
time = 4.54, size = 88, normalized size = 0.78

$$-\frac{1}{120}\left(30\,d^{5}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)+\left(24\,x^{4}e^{4}-30\,dx^{3}e^{3}-8\,d^{2}x^{2}e^{2}+15\,d^{3}xe-16\,d^{4}\right)\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/120*(30*d^5*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (24*x^4*e^4 -
 30*d*x^3*e^3 - 8*d^2*x^2*e^2 + 15*d^3*x*e - 16*d^4)*sqrt(-x^2*e^2 + d^2))*
e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.25, size = 279, normalized size = 2.47

$$d\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)-e\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e\neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(3/2)/(e*x+d),x)
```

```
[Out] d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**
2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8
*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e*
*2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - e*Piece
wise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2
*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**
2)/4, True))
```

Giac [A]
time = 1.25, size = 75, normalized size = 0.66

$$\frac{1}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{120}\left(16\,d^4e^{(-3)}-\left(15\,d^3e^{(-2)}-2\left(4\,d^2e^{(-1)}-3\left(4\,xe-5\,d\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/8*d^5*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/120*(16*d^4*e^(-3) - (15*d^3*e^(-2)
 - 2*(4*d^2*e^(-1) - 3*(4*x*e - 5*d)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(d^2-e^2\,x^2\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(3/2))/(d + e*x), x)
```

3.103 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=201

$$\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4}+\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3}-\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}+\frac{x^4(d^2-e^2x^2)^{5/2}}{9e}+\frac{d^3(128d-$$

```
[Out] 1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4+4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-1
/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2+1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e+1/5040*d^3
*(-315*e*x+128*d)*(-e^2*x^2+d^2)^(5/2)/e^5+3/128*d^9*arctan(e*x/(-e^2*x^2+d
^2)^(1/2))/e^5+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A]

time = 0.10, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {864, 847, 794, 201, 223, 209}

$$\frac{3d^9\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}+\frac{x^4(d^2-e^2x^2)^{5/2}}{9e}-\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}+\frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3}+\frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4}+\frac{d^3(128d-315ex)(d^2-e^2x^2)^{5/2}}{5040e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (3*d^7*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64
*e^4) + (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)
^(5/2))/(8*e^2) + (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) + (d^3*(128*d - 315*e*x
)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(128*e^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int x^4(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$$

$$= \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{\int x^3(4d^2e-9de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{9e^2}$$

$$= -\frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{\int x^2(27d^3e^2-32d^2e^3x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{72e^4}$$

$$= \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{\int x(64d^4e^3-189d^3}{5}$$

$$= \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} + \frac{d^3(128d-315ex)\,(}{5040e^5}$$

$$= \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} + \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} +$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}$$

$$= \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} + \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2}$$

Mathematica [A]
time = 0.23, size = 155, normalized size = 0.77

$$\frac{e\sqrt{d^2-e^2x^2}\,(1024d^8-945d^7ex+512d^6e^2x^2-630d^5e^3x^3+384d^4e^4x^4+7560d^3e^5x^5-6400d^2e^6x^6-5040de^7x^7+4480e^8x^8)+945d^9\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{40320e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(1024*d^8 - 945*d^7*e*x + 512*d^6*e^2*x^2 - 630*d^5*
e^3*x^3 + 384*d^4*e^4*x^4 + 7560*d^3*e^5*x^5 - 6400*d^2*e^6*x^6 - 5040*d*e^
7*x^7 + 4480*e^8*x^8) + 945*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
 e^2*x^2]])/(40320*e^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(173) = 346$.
time = 0.07, size = 502, normalized size = 2.50

method	result

risch	$\frac{(4480e^8x^8-5040d e^7x^7-6400d^2e^6x^6+7560d^3e^5x^5+384d^4e^4x^4-630d^5e^3x^3+512d^6e^2x^2-945d^7ex+1024d^8)\sqrt{-e^2x^2+d^2}}{40320e^5}+$
default	$\frac{-\frac{x^2(-e^2x^2+d^2)^{\frac{7}{2}}}{9e^2}-\frac{2d^2(-e^2x^2+d^2)^{\frac{7}{2}}}{63e^4}}{e}-\frac{d\left(-\frac{x(-e^2x^2+d^2)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+}}{2}\right.}{}\right.}{6}\right.}{8e^2}\right.}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*(-e^2*x^2+d^2)^(7/2))-d
/e^2*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/
2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+
1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-1/7*d^2/
e^5*(-e^2*x^2+d^2)^(7/2)-d^3/e^4*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x
*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+1/e^5*d^4*(1/5*(-(x+d/e)^2
```

```
*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 230, normalized size = 1.14

$$-\frac{3}{8}i\,d^9\arcsin\left(\frac{xe}{d}+2\right)e^{(-5)}-\frac{45}{128}d^9\arcsin\left(\frac{xe}{d}\right)e^{(-5)}+\frac{3}{8}\sqrt{x^2e^2+4dxe+3d^2}\,d^7xe^{(-4)}-\frac{45}{128}\sqrt{-x^2e^2+d^2}\,d^7xe^{(-4)}+\frac{3}{4}\sqrt{x^2e^2+4dxe+3d^2}\,d^8e^{(-5)}+\frac{1}{64}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^5xe^{(-4)}-\frac{3}{16}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^3xe^{(-4)}+\frac{1}{5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^4e^{(-5)}-\frac{1}{9}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}x^2e^{(-3)}+\frac{1}{8}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}dxe^{(-4)}-\frac{11}{63}\left(-x^2e^2+d^2\right)^{\frac{7}{2}}d^2e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^9*arcsin(x*e/d + 2)*e^(-5) - 45/128*d^9*arcsin(x*e/d)*e^(-5) + 3/8
*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^7*x*e^(-4) - 45/128*sqrt(-x^2*e^2 + d^2)
*d^7*x*e^(-4) + 3/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^8*e^(-5) + 1/64*(-x^2
*e^2 + d^2)^(3/2)*d^5*x*e^(-4) - 3/16*(-x^2*e^2 + d^2)^(5/2)*d^3*x*e^(-4) +
 1/5*(-x^2*e^2 + d^2)^(5/2)*d^4*e^(-5) - 1/9*(-x^2*e^2 + d^2)^(7/2)*x^2*e^(
-3) + 1/8*(-x^2*e^2 + d^2)^(7/2)*d*x*e^(-4) - 11/63*(-x^2*e^2 + d^2)^(7/2)*
d^2*e^(-5)
```

Fricas [A]
time = 3.31, size = 129, normalized size = 0.64

$$-\frac{1}{40320}\left(1890\,d^9\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(4480\,x^8e^8-5040\,dx^7e^7-6400\,d^2x^6e^6+7560\,d^3x^5e^5+384\,d^4x^4e^4-630\,d^5x^3e^3+512\,d^6x^2e^2-945\,d^7xe+1024\,d^8\right)\sqrt{-x^2e^2+d^2}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40320*(1890*d^9*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (4480*x^8
*e^8 - 5040*d*x^7*e^7 - 6400*d^2*x^6*e^6 + 7560*d^3*x^5*e^5 + 384*d^4*x^4*e
^4 - 630*d^5*x^3*e^3 + 512*d^6*x^2*e^2 - 945*d^7*x*e + 1024*d^8)*sqrt(-x^2*
e^2 + d^2))*e^(-5)
```

Sympy [C] Result contains complex when optimal does not.
time = 26.38, size = 830, normalized size = 4.13

$$d^3\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7}+\frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5id^5x^3}{384e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^5}{192e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{7idx^7}{48\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^9}{8d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7}-\frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5d^5x^3}{384e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^5}{192e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{7dx^7}{48\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^9}{8d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}-\frac{16d^8\sqrt{d^2-e^2x^2}}{315e^8}-\frac{8d^6x^2\sqrt{d^2-e^2x^2}}{315e^6}-\frac{2d^4x^4\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^6\sqrt{d^2-e^2x^2}}{63e^2}+\frac{x^8\sqrt{d^2-e^2x^2}}{9} & \text{for}\ e\neq 0\\ \frac{x^8\sqrt{d^2}}{8} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
```

```
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) - d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(1
05*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d
**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**
6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7
) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e*
*4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d
**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e*
*7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*
sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) +
 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2
/d**2)), True)) + e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8
) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 -
 e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*
sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [A]
time = 1.36, size = 119, normalized size = 0.59

$$\frac{3}{128}\, d^9 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \mathrm{sgn}(d) + \frac{1}{40320}\, \left(1024\, d^8 e^{(-5)} - \left(945\, d^7 e^{(-4)} - 2\, \left(256\, d^6 e^{(-3)} - \left(315\, d^5 e^{(-2)} - 4\, \left(48\, d^4 e^{(-1)} + 5\, \left(189\, d^3 - 2\, \left(80\, d^2 e - 7\, \left(8\, x e^3 - 9\, d e^2\right) x\right) x\right) x\right) x\right) x\right) x\right) x\right) \sqrt{-x^2 e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/128*d^9*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/40320*(1024*d^8*e^(-5) - (945*d^7
*e^(-4) - 2*(256*d^6*e^(-3) - (315*d^5*e^(-2) - 4*(48*d^4*e^(-1) + 5*(189*d
^3 - 2*(80*d^2*e - 7*(8*x*e^3 - 9*d*e^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2
 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.104 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=172

$$-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}-\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}+\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d-35ex)(d^2-e^2x^2)^{5/2}}{560e^4}$$

```
[Out] -1/64*d^4*x*(-e^2*x^2+d^2)^(3/2)/e^3-1/7*d*x^2*(-e^2*x^2+d^2)^(5/2)/e^2+1/8
*x^3*(-e^2*x^2+d^2)^(5/2)/e-1/560*d^2*(-35*e*x+32*d)*(-e^2*x^2+d^2)^(5/2)/e
^4-3/128*d^8*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-3/128*d^6*x*(-e^2*x^2+d^2
)^(1/2)/e^3
```

Rubi [A]
time = 0.08, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {864, 847, 794, 201, 223, 209}

$$-\frac{3d^8\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4}-\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2}+\frac{x^3(d^2-e^2x^2)^{5/2}}{8e}-\frac{d^2(32d-35ex)(d^2-e^2x^2)^{5/2}}{560e^4}-\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3}-\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^3) - (d^4*x*(d^2 - e^2*x^2)^(3/2))/(6
4*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) + (x^3*(d^2 - e^2*x^2)^(5/2)
)/(8*e) - (d^2*(32*d - 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) - (3*d^8*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(128*e^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = \int x^3(d-ex)\left(d^2-e^2x^2\right)^{3/2}\,dx$$

$$= \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{\int x^2(3d^2e-8de^2x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{8e^2}$$

$$= -\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} + \frac{\int x(16d^3e^2-21d^2e^3x)\left(d^2-e^2x^2\right)^{3/2}\,dx}{56e^4}$$

$$= -\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d-35ex)\left(d^2-e^2x^2\right)^{5/2}}{560e^4} - \frac{d^4\int (d^2}{}$$

$$= -\frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d-35ex)\,(d^2}{560e^4}$$

$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} -$$

$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} -$$

$$= -\frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} - \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3} - \frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} + \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} -$$

Mathematica [A]
time = 0.23, size = 144, normalized size = 0.84

$$\frac{e\sqrt{d^2-e^2x^2}\left(-256d^7+105d^6ex-128d^5e^2x^2+70d^4e^3x^3+1024d^3e^4x^4-840d^2e^5x^5-640de^6x^6+560e^7x^7\right)-105d^8\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{4480e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-256*d^7 + 105*d^6*e*x - 128*d^5*e^2*x^2 + 70*d^4*e
^3*x^3 + 1024*d^3*e^4*x^4 - 840*d^2*e^5*x^5 - 640*d*e^6*x^6 + 560*e^7*x^7)
- 105*d^8*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(4480*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(148) = 296$.
time = 0.06, size = 449, normalized size = 2.61

method	result
risch	$-\frac{\left(-560e^7x^7+640d\,e^6x^6+840d^2e^5x^5-1024d^3e^4x^4-70d^4e^3x^3+128d^5e^2x^2-105d^6ex+256d^7\right)\sqrt{-e^2x^2+d^2}}{4480e^4} - \frac{3d^8\arctan\left(\frac{}{\sqrt{}}\right.}{128e}$

default	$\frac{-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8e^{2}}+\frac{d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{8e^{2}}}{e}$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2
)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1
/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+1/7*d/e^4
*(-e^2*x^2+d^2)^(7/2)+d^2/e^3*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-
e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*
arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))-d^3/e^4*(1/5*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 207, normalized size = 1.20

$$\frac{3}{8}i\,d^{8}\arcsin\left(\frac{xe}{d}+2\right)e^{(-4)}+\frac{45}{128}d^{8}\arcsin\left(\frac{xe}{d}\right)e^{(-4)}-\frac{3}{8}\sqrt{x^{2}e^{2}+4\,dxe+3\,d^{2}}\,d^{6}xe^{(-3)}+\frac{45}{128}\sqrt{-x^{2}e^{2}+d^{2}}\,d^{6}xe^{(-3)}-\frac{3}{4}\sqrt{x^{2}e^{2}+4\,dxe+3\,d^{2}}\,d^{7}e^{(-4)}-\frac{1}{64}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{4}xe^{(-3)}+\frac{3}{16}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}xe^{(-3)}-\frac{1}{5}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{(-4)}-\frac{1}{8}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{7}{2}}xe^{(-3)}+\frac{1}{7}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{7}{2}}de^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*I*d^8*arcsin(x*e/d + 2)*e^(-4) + 45/128*d^8*arcsin(x*e/d)*e^(-4) - 3/8*
sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^6*x*e^(-3) + 45/128*sqrt(-x^2*e^2 + d^2)*
d^6*x*e^(-3) - 3/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^7*e^(-4) - 1/64*(-x^2*
e^2 + d^2)^(3/2)*d^4*x*e^(-3) + 3/16*(-x^2*e^2 + d^2)^(5/2)*d^2*x*e^(-3) -
1/5*(-x^2*e^2 + d^2)^(5/2)*d^3*e^(-4) - 1/8*(-x^2*e^2 + d^2)^(7/2)*x*e^(-3)
 + 1/7*(-x^2*e^2 + d^2)^(7/2)*d*e^(-4)
```

Fricas [A]
time = 3.27, size = 118, normalized size = 0.69

$$\frac{1}{4480}\left(210\,d^{8}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)+\left(560\,x^{7}e^{7}-640\,dx^{6}e^{6}-840\,d^{2}x^{5}e^{5}+1024\,d^{3}x^{4}e^{4}+70\,d^{4}x^{3}e^{3}-128\,d^{5}x^{2}e^{2}+105\,d^{6}xe-256\,d^{7}\right)\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(210*d^8*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (560*x^7*e^7
 - 640*d*x^6*e^6 - 840*d^2*x^5*e^5 + 1024*d^3*x^4*e^4 + 70*d^4*x^3*e^3 - 12
8*d^5*x^2*e^2 + 105*d^6*x*e - 256*d^7)*sqrt(-x^2*e^2 + d^2))*e^(-4)
```

Sympy [A]
time = 26.01, size = 775, normalized size = 4.51

$$d^{3}\left(\begin{cases}-\frac{2d^{4}\sqrt{d^{2}-e^{2}x^{2}}}{15e^{4}}-\frac{d^{2}x^{2}\sqrt{d^{2}-e^{2}x^{2}}}{15e^{2}}+\frac{x^{4}\sqrt{d^{2}-e^{2}x^{2}}}{5} & \text{for}\: e\neq 0\\ \frac{x^{4}\sqrt{d^{2}}}{4} & \text{otherwise}\end{cases}\right)-d^{2}e\left(\begin{cases}-\frac{id^{6}\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^{5}}+\frac{id^{5}x}{16e^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{id^{3}x^{3}}{48e^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{5idx^{5}}{24\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\frac{ie^{2}x^{7}}{6d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\: \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\ \frac{d^{6}\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^{5}}-\frac{d^{5}x}{16e^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{d^{3}x^{3}}{48e^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{5dx^{5}}{24\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-\frac{e^{2}x^{7}}{6d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)-de^{2}\left(\begin{cases}-\frac{8d^{6}\sqrt{d^{2}-e^{2}x^{2}}}{105e^{6}}-\frac{4d^{4}x^{2}\sqrt{d^{2}-e^{2}x^{2}}}{105e^{4}}-\frac{d^{2}x^{4}\sqrt{d^{2}-e^{2}x^{2}}}{35e^{2}}+\frac{x^{6}\sqrt{d^{2}-e^{2}x^{2}}}{7} & \text{for}\: e\neq 0\\ \frac{x^{6}\sqrt{d^{2}}}{6} & \text{otherwise}\end{cases}\right)+e^{3}\left(\begin{cases}-\frac{5id^{8}\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^{7}}+\frac{5id^{7}x}{128e^{6}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{5id^{5}x^{3}}{384e^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{id^{3}x^{5}}{192e^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}-\frac{7idx^{7}}{48\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+\frac{ie^{2}x^{9}}{8d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\: \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\ \frac{5d^{8}\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^{7}}-\frac{5d^{7}x}{128e^{6}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{5d^{5}x^{3}}{384e^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{d^{3}x^{5}}{192e^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}+\frac{7dx^{7}}{48\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-\frac{e^{2}x^{9}}{8d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) - d**2*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
 I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-
1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x
**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e
*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48
*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) -
e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - d*e**2*Piecewise((-8*d**
6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(1
05*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e*
*2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + e**3*Piecewise((-5*I*d*
*8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)
) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e
**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2))
+ I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5
*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2))
 + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sq
rt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**
9/(8*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 0.91, size = 108, normalized size = 0.63

$$-\frac{3}{128}\,d^{8}\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{1}{4480}\left(256\,d^{7}e^{(-4)}-\left(105\,d^{6}e^{(-3)}-2\left(64\,d^{5}e^{(-2)}-\left(35\,d^{4}e^{(-1)}+4\left(128\,d^{3}-5\left(21\,d^{2}e-2\left(7\,xe^{3}-8\,de^{2}\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/128*d^8*arcsin(x*e/d)*e^(-4)*sgn(d) - 1/4480*(256*d^7*e^(-4) - (105*d^6*
e^(-3) - 2*(64*d^5*e^(-2) - (35*d^4*e^(-1) + 4*(128*d^3 - 5*(21*d^2*e - 2*(
7*x*e^3 - 8*d*e^2)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.105 $\int \frac{x^2\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=140

$$\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}+\frac{d(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^3}-\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^7\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] 1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/30*d*(-5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2
)/e^3-1/7*(-e^2*x^2+d^2)^(7/2)/e^3+1/16*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2)
)/e^3+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1653, 12, 799, 794, 201, 223, 209}

$$\frac{d^7\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{d(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^3}-\frac{(d^2-e^2x^2)^{7/2}}{7e^3}+\frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^5*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (d^3*x*(d^2 - e^2*x^2)^(3/2))/(24*e^
2) + (d*(6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^3) - (d^2 - e^2*x^2)^(7/
2)/(7*e^3) + (d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{d+ex}\,dx = -\frac{(d^2-e^2x^2)^{7/2}}{7e^3} - \frac{\int \frac{7de^3x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx}{7e^4}$$

$$= -\frac{(d^2-e^2x^2)^{7/2}}{7e^3} - \frac{d\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx}{e}$$

$$= -\frac{(d^2-e^2x^2)^{7/2}}{7e^3} - \frac{\int x(d^2e-de^2x)(d^2-e^2x^2)^{3/2}\,dx}{e^2}$$

$$= \frac{d(6d-5ex)(d^2-e^2x^2)^{5/2}}{30e^3} - \frac{(d^2-e^2x^2)^{7/2}}{7e^3} + \frac{d^3\int (d^2-e^2x^2)^{3/2}\,dx}{6e^2}$$

$$= \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} + \frac{d(6d-5ex)(d^2-e^2x^2)^{5/2}}{30e^3} - \frac{(d^2-e^2x^2)^{7/2}}{7e^3} + \frac{d^5\int \sqrt{d^2-e^2x}}{8e^2}$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} + \frac{d(6d-5ex)(d^2-e^2x^2)^{5/2}}{30e^3} - \frac{(d^2-e^2x^2)^{7/}}{7e^3}$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} + \frac{d(6d-5ex)(d^2-e^2x^2)^{5/2}}{30e^3} - \frac{(d^2-e^2x^2)^{7/}}{7e^3}$$

$$= \frac{d^5x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{d^3x(d^2-e^2x^2)^{3/2}}{24e^2} + \frac{d(6d-5ex)(d^2-e^2x^2)^{5/2}}{30e^3} - \frac{(d^2-e^2x^2)^{7/}}{7e^3}$$

Mathematica [**A**]
time = 0.23, size = 133, normalized size = 0.95

$$\frac{e\sqrt{d^2-e^2x^2}\,(96d^6-105d^5ex+48d^4e^2x^2+490d^3e^3x^3-384d^2e^4x^4-280de^5x^5+240e^6x^6)+105d^7\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{1680e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(96*d^6 - 105*d^5*e*x + 48*d^4*e^2*x^2 + 490*d^3*e^3
*x^3 - 384*d^2*e^4*x^4 - 280*d*e^5*x^5 + 240*e^6*x^6) + 105*d^7*Sqrt[-e^2]*
Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(1680*e^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(120) = 240$.
time = 0.06, size = 317, normalized size = 2.26

method	result
risch	$\frac{(240e^6x^6-280d\,e^5x^5-384d^2e^4x^4+490d^3e^3x^3+48d^4e^2x^2-105e\,d^5x+96d^6)\sqrt{-e^2x^2+d^2}}{1680e^3} + \frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^2\sqrt{e^2}}$

default	$-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{7 e^{3}}-\frac{d \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5 d^{2} \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2} \left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*(-e^2*x^2+d^2)^(7/2)/e^3-d/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/
4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+1/e^3*d^2*(1/5*(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 186, normalized size = 1.33

$$-\frac{3}{8} i\, d^{7} \arcsin\left(\frac{xe}{d}+2\right) e^{(-3)}-\frac{5}{16} d^{7} \arcsin\left(\frac{xe}{d}\right) e^{(-3)}+\frac{3}{8} \sqrt{x^{2} e^{2}+4 dxe+3 d^{2}}\, d^{5} x e^{(-2)}-\frac{5}{16} \sqrt{-x^{2} e^{2}+d^{2}}\, d^{5} x e^{(-2)}+\frac{3}{4} \sqrt{x^{2} e^{2}+4 dxe+3 d^{2}}\, d^{6} e^{(-3)}+\frac{1}{24} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} d^{3} x e^{(-2)}-\frac{1}{6} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{5}{2}} dxe^{(-2)}+\frac{1}{5} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{5}{2}} d^{2} e^{(-3)}-\frac{1}{7} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{7}{2}} e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^7*arcsin(x*e/d + 2)*e^(-3) - 5/16*d^7*arcsin(x*e/d)*e^(-3) + 3/8*s
qrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^5*x*e^(-2) - 5/16*sqrt(-x^2*e^2 + d^2)*d^5
*x*e^(-2) + 3/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^6*e^(-3) + 1/24*(-x^2*e^2
 + d^2)^(3/2)*d^3*x*e^(-2) - 1/6*(-x^2*e^2 + d^2)^(5/2)*d*x*e^(-2) + 1/5*(-
x^2*e^2 + d^2)^(5/2)*d^2*e^(-3) - 1/7*(-x^2*e^2 + d^2)^(7/2)*e^(-3)
```

Fricas [A]
time = 2.94, size = 109, normalized size = 0.78

$$-\frac{1}{1680} \left(210 d^{7} \arctan\left(-\frac{\left(d-\sqrt{-x^{2} e^{2}+d^{2}}\right) e^{(-1)}}{x}\right)-\left(240 x^{6} e^{6}-280 dx^{5} e^{5}-384 d^{2} x^{4} e^{4}+490 d^{3} x^{3} e^{3}+48 d^{4} x^{2} e^{2}-105 d^{5} xe+96 d^{6}\right) \sqrt{-x^{2} e^{2}+d^{2}}\right) e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(210*d^7*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (240*x^6*e^
6 - 280*d*x^5*e^5 - 384*d^2*x^4*e^4 + 490*d^3*x^3*e^3 + 48*d^4*x^2*e^2 - 10
5*d^5*x*e + 96*d^6)*sqrt(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.58, size = 653, normalized size = 4.66

$$d^{3}\left(\begin{cases} - \frac{i d^{4} \operatorname{acosh}{\left(\frac{e x}{d} \right)}}{8 e^{3}} + \frac{i d^{3} x}{8 e^{2} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{3 i d x^{3}}{8 \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + \frac{i e^{2} x^{5}}{4 d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\\frac{d^{4} \operatorname{asin}{\left(\frac{e x}{d} \right)}}{8 e^{3}} - \frac{d^{3} x}{8 e^{2} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{3 d x^{3}}{8 \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - \frac{e^{2} x^{5}}{4 d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right) - d^{2} e \left(\begin{cases} - \frac{2 d^{4} \sqrt{d^{2} - e^{2} x^{2}}}{15 e^{4}} - \frac{d^{2} x^{2} \sqrt{d^{2} - e^{2} x^{2}}}{15 e^{2}} + \frac{x^{4} \sqrt{d^{2} - e^{2} x^{2}}}{5} & \text{for}\: e \neq 0 \\\frac{x^{4} \sqrt{d^{2}}}{4} & \text{otherwise} \end{cases}\right) - d e^{2} \left(\begin{cases} - \frac{i d^{6} \operatorname{acosh}{\left(\frac{e x}{d} \right)}}{16 e^{5}} + \frac{i d^{5} x}{16 e^{4} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{i d^{3} x^{3}}{48 e^{2} \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} - \frac{5 i d x^{5}}{24 \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} + \frac{i e^{2} x^{7}}{6 d \sqrt{-1 + \frac{e^{2} x^{2}}{d^{2}}}} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\\frac{d^{6} \operatorname{asin}{\left(\frac{e x}{d} \right)}}{16 e^{5}} - \frac{d^{5} x}{16 e^{4} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{d^{3} x^{3}}{48 e^{2} \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} + \frac{5 d x^{5}}{24 \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} - \frac{e^{2} x^{7}}{6 d \sqrt{1 - \frac{e^{2} x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right) + e^{3} \left(\begin{cases} - \frac{8 d^{6} \sqrt{d^{2} - e^{2} x^{2}}}{105 e^{6}} - \frac{4 d^{4} x^{2} \sqrt{d^{2} - e^{2} x^{2}}}{105 e^{4}} - \frac{d^{2} x^{4} \sqrt{d^{2} - e^{2} x^{2}}}{35 e^{2}} + \frac{x^{6} \sqrt{d^{2} - e^{2} x^{2}}}{7} & \text{for}\: e \neq 0 \\\frac{x^{6} \sqrt{d^{2}}}{6} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - d**2
*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**
2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*
sqrt(d**2)/4, True)) - d*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I
*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
+ e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**
7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x
/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e
**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e*
*2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**3*Piecewise((-8*d**6*sq
rt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e
**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x
**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

Giac [A]
time = 1.37, size = 96, normalized size = 0.69

$$\frac{1}{16}\,d^7\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{1680}\,\left(96\,d^6e^{(-3)}-\left(105\,d^5e^{(-2)}-2\,\left(24\,d^4e^{(-1)}+\left(245\,d^3-4\,\left(48\,d^2e-5\,\left(6\,xe^3-7\,de^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/16*d^7*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/1680*(96*d^6*e^(-3) - (105*d^5*e^(
-2) - 2*(24*d^4*e^(-1) + (245*d^3 - 4*(48*d^2*e - 5*(6*x*e^3 - 7*d*e^2)*x)*
x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(d^2-e^2\,x^2\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.106 $\int \frac{x\left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=116

$$-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}-\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2}-\frac{d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}$$

```
[Out] -1/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e-1/30*(-5*e*x+6*d)*(-e^2*x^2+d^2)^(5/2)/e
^2-1/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/16*d^4*x*(-e^2*x^2+d^2)^
(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {799, 794, 201, 223, 209}

$$-\frac{d^6\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^2}-\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e}-\frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2}-\frac{d^4x\sqrt{d^2-e^2x^2}}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -1/16*(d^4*x*Sqrt[d^2 - e^2*x^2])/e - (d^2*x*(d^2 - e^2*x^2)^(3/2))/(24*e)
- ((6*d - 5*e*x)*(d^2 - e^2*x^2)^(5/2))/(30*e^2) - (d^6*ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]])/(16*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] && !ILtQ[p - 1/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(d^2-e^2x^2)^{5/2}}{d+ex}\,dx &= \frac{\int x(d^2e-de^2x)\,(d^2-e^2x^2)^{3/2}\,dx}{de} \\
&= -\frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2} - \frac{d^2\int (d^2-e^2x^2)^{3/2}\,dx}{6e} \\
&= -\frac{d^2x(d^2-e^2x^2)^{3/2}}{24e} - \frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2} - \frac{d^4\int \sqrt{d^2-e^2x^2}\,dx}{8e} \\
&= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e} - \frac{d^2x(d^2-e^2x^2)^{3/2}}{24e} - \frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2} - \frac{d^6\int \frac{1}{\sqrt{d^2-}}}{16e} \\
&= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e} - \frac{d^2x(d^2-e^2x^2)^{3/2}}{24e} - \frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2} - \frac{d^6\text{Subst}\left(\int\right.}{} \\
&= -\frac{d^4x\sqrt{d^2-e^2x^2}}{16e} - \frac{d^2x(d^2-e^2x^2)^{3/2}}{24e} - \frac{(6d-5ex)\,(d^2-e^2x^2)^{5/2}}{30e^2} - \frac{d^6\tan^{-1}\left(\right.}{1}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 122, normalized size = 1.05

$$
\frac{e\sqrt{d^2-e^2x^2}\,(-48d^5+15d^4ex+96d^3e^2x^2-70d^2e^3x^3-48de^4x^4+40e^5x^5)-15d^6\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{240e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-48*d^5 + 15*d^4*e*x + 96*d^3*e^2*x^2 - 70*d^2*e^3*
x^3 - 48*d*e^4*x^4 + 40*e^5*x^5) - 15*d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
Sqrt[d^2 - e^2*x^2]])/(240*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(100) = 200$.
time = 0.08, size = 295, normalized size = 2.54

method	result
risch	$-\frac{\left(-40 e^{5} x^{5}+48 d\, e^{4} x^{4}+70 d^{2} e^{3} x^{3}-96 x^{2} d^{3} e^{2}-15 d^{4} x e+48 d^{5}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{240 e^{2}}-\frac{d^{6} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 e \sqrt{e^{2}}}$
default	$\frac{\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5 d^{2} \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2} \left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}\right)}{4}\right)}{6}}{e}-\frac{d \left(\frac{\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\right.\right.}{5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2
*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2
*x^2+d^2)^(1/2)))))-d/e^2*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1
/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*
(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d
^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.49, size = 166, normalized size = 1.43

$$\frac{3}{8} i\, d^{6} \arcsin\left(\frac{xe}{d}+2\right) e^{(-2)}+\frac{5}{16} d^{6} \arcsin\left(\frac{xe}{d}\right) e^{(-2)}-\frac{3}{8} \sqrt{x^{2} e^{2}+4\, dxe+3\, d^{2}}\, d^{4} x e^{(-1)}+\frac{5}{16} \sqrt{-x^{2} e^{2}+d^{2}}\, d^{4} x e^{(-1)}-\frac{3}{4} \sqrt{x^{2} e^{2}+4\, dxe+3\, d^{2}}\, d^{5} e^{(-2)}-\frac{1}{24} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} d^{2} x e^{(-1)}+\frac{1}{6} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{5}{2}} x e^{(-1)}-\frac{1}{5} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{5}{2}} d e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*I*d^6*arcsin(x*e/d + 2)*e^(-2) + 5/16*d^6*arcsin(x*e/d)*e^(-2) - 3/8*sq
rt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^4*x*e^(-1) + 5/16*sqrt(-x^2*e^2 + d^2)*d^4*
x*e^(-1) - 3/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^5*e^(-2) - 1/24*(-x^2*e^2
+ d^2)^(3/2)*d^2*x*e^(-1) + 1/6*(-x^2*e^2 + d^2)^(5/2)*x*e^(-1) - 1/5*(-x^2
*e^2 + d^2)^(5/2)*d*e^(-2)
```

Fricas [A]
time = 2.84, size = 98, normalized size = 0.84

$$\frac{1}{240}\left(30\,d^6\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(40\,x^5e^5-48\,dx^4e^4-70\,d^2x^3e^3+96\,d^3x^2e^2+15\,d^4xe-48\,d^5\right)\sqrt{-x^2e^2+d^2}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(30*d^6*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (40*x^5*e^5 -
48*d*x^4*e^4 - 70*d^2*x^3*e^3 + 96*d^3*x^2*e^2 + 15*d^4*x*e - 48*d^5)*sqrt(
-x^2*e^2 + d^2))*e^(-2)
```

Sympy [A]
time = 8.45, size = 580, normalized size = 5.00

$$d^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) - d**2*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d*
*3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
 + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) - d*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**
2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + e**3*Piecewise((-I*d**6*acosh
(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**
2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 -
e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 1.29, size = 86, normalized size = 0.74

$$-\frac{1}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\operatorname{sgn}(d)-\frac{1}{240}\left(48\,d^5e^{(-2)}-\left(15\,d^4e^{(-1)}+2\left(48\,d^3-\left(35\,d^2e-4\left(5\,xe^3-6\,de^2\right)x\right)x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/16*d^6*arcsin(x*e/d)*e^(-2)*sgn(d) - 1/240*(48*d^5*e^(-2) - (15*d^4*e^(-
1) + 2*(48*d^3 - (35*d^2*e - 4*(5*x*e^3 - 6*d*e^2)*x)*x)*x)*x)*sqrt(-x^2*e^
2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x)
```

3.107 $\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=100

$$\frac{3}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2}+\frac{(d^2-e^2x^2)^{5/2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] 1/4*d*x*(-e^2*x^2+d^2)^(3/2)+1/5*(-e^2*x^2+d^2)^(5/2)/e+3/8*d^5*arctan(e*x/
(-e^2*x^2+d^2)^(1/2))/e+3/8*d^3*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {679, 201, 223, 209}

$$\frac{3d^5\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}+\frac{1}{4}dx\left(d^2-e^2x^2\right)^{3/2}+\frac{(d^2-e^2x^2)^{5/2}}{5e}+\frac{3}{8}d^3x\sqrt{d^2-e^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*d^3*x*Sqrt[d^2 - e^2*x^2])/8 + (d*x*(d^2 - e^2*x^2)^(3/2))/4 + (d^2 - e^
2*x^2)^(5/2)/(5*e) + (3*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx &= \frac{(d^2-e^2x^2)^{5/2}}{5e}+d\int (d^2-e^2x^2)^{3/2}\,dx\\
&= \frac{1}{4}dx(d^2-e^2x^2)^{3/2}+\frac{(d^2-e^2x^2)^{5/2}}{5e}+\frac{1}{4}(3d^3)\int \sqrt{d^2-e^2x^2}\,dx\\
&= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{1}{4}dx(d^2-e^2x^2)^{3/2}+\frac{(d^2-e^2x^2)^{5/2}}{5e}+\frac{1}{8}(3d^5)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{1}{4}dx(d^2-e^2x^2)^{3/2}+\frac{(d^2-e^2x^2)^{5/2}}{5e}+\frac{1}{8}(3d^5)\,\text{Subst}\left(\int \frac{1}{1+e^2x^2}\right.\\
&= \frac{3}{8}d^3x\sqrt{d^2-e^2x^2}+\frac{1}{4}dx(d^2-e^2x^2)^{3/2}+\frac{(d^2-e^2x^2)^{5/2}}{5e}+\frac{3d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 111, normalized size = 1.11

$$\frac{\sqrt{d^2-e^2x^2}\,(8d^4+25d^3ex-16d^2e^2x^2-10de^3x^3+8e^4x^4)}{40e}-\frac{3d^5\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{8\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 + 25*d^3*e*x - 16*d^2*e^2*x^2 - 10*d*e^3*x^3 +
8*e^4*x^4))/(40*e) - (3*d^5*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*
Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(84) = 168$.
time = 0.07, size = 192, normalized size = 1.92

method	result

risch	$\frac{(8e^4x^4-10d\,e^3x^3-16d^2x^2e^2+25d^3ex+8d^4)\sqrt{-e^2x^2+d^2}}{40e}+\frac{3d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2 \cdots}}{4e^2}\cdots\right.}{\cdots}\right.}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d
*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+
2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 105, normalized size = 1.05

$$-\frac{3}{8}i\,d^5\arcsin\left(\frac{xe}{d}+2\right)e^{(-1)}+\frac{3}{4}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^4e^{(-1)}+\frac{3}{8}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^3x+\frac{1}{4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}dx+\frac{1}{5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*I*d^5*arcsin(x*e/d + 2)*e^(-1) + 3/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d
^4*e^(-1) + 3/8*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^3*x + 1/4*(-x^2*e^2 + d^2
)^(3/2)*d*x + 1/5*(-x^2*e^2 + d^2)^(5/2)*e^(-1)
```

Fricas [A]
time = 2.91, size = 89, normalized size = 0.89

$$-\frac{1}{40}\left(30\,d^5\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(8\,x^4e^4-10\,dx^3e^3-16\,d^2x^2e^2+25\,d^3xe+8\,d^4\right)\sqrt{-x^2e^2+d^2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(30*d^5*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (8*x^4*e^4 - 1
0*d*x^3*e^3 - 16*d^2*x^2*e^2 + 25*d^3*x*e + 8*d^4)*sqrt(-x^2*e^2 + d^2))*e^
(-1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.25, size = 435, normalized size = 4.35

$$d^3\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) - de^2\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^3\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - d**2
*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/
(3*e**2), True)) - d*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3
*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) +
 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/
d**2)), True)) + e**3*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) -
 d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5
, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

Giac [A]
time = 0.97, size = 74, normalized size = 0.74

$$\frac{3}{8}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d) + \frac{1}{40}\left(8\,d^4e^{(-1)} + \left(25\,d^3 - 2\left(8\,d^2e - \left(4\,xe^3 - 5\,de^2\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/8*d^5*arcsin(x*e/d)*e^(-1)*sgn(d) + 1/40*(8*d^4*e^(-1) + (25*d^3 - 2*(8*d
^2*e - (4*x*e^3 - 5*d*e^2)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\,x^2\right)^{5/2}}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x), x)
```

3.108 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=113

$$\frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}-\frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 1/12*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(3/2)-3/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1
/2))-d^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*d^2*(-3*e*x+8*d)*(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {864, 829, 858, 223, 209, 272, 65, 214}

$$-\frac{3}{8}d^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2}+\frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2}-d^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (d^2*(8*d - 3*e*x)*Sqrt[d^2 - e^2*x^2])/8 + ((4*d - 3*e*x)*(d^2 - e^2*x^2)^
(3/2))/12 - (3*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/8 - d^4*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)}\,dx = \int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x}\,dx$$
$$= \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{(-4d^3e^2+3d^2e^3x)\sqrt{d^2-e^2x^2}}{x}\,dx}{4e^2}$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{8d^5e^4-3d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{8e^4}$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + d^5\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^5\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-}\right.$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x}}\right.$$
$$= \frac{1}{8}d^2(8d-3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d-3ex)\left(d^2-e^2x^2\right)^{3/2} - \frac{3}{8}d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x}}\right.$$

Mathematica [A]
time = 0.34, size = 140, normalized size = 1.24

$$\frac{1}{24}\sqrt{d^2-e^2x^2}\left(32d^3-15d^2ex-8de^2x^2+6e^3x^3\right)+2d^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{3d^4e\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{8\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(32*d^3 - 15*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3))/24 +
2*d^4*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (3*d^4*e*Log[-(Sq
rt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(99) = 198.
time = 0.07, size = 297, normalized size = 2.63

method	result

default	$-\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+d e \left(-\frac{\left(-2 e^{2} \left(x+\frac{d}{e}\right)+2 d e \right) \left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8 e^{2}}+\frac{3 d^{2} \left(-\frac{\left(-2 e^{2} \left(x+\frac{d}{e}\right)+2 d e \right) \sqrt{-\left(x+\frac{d}{e}\right)^{2} e \cdots}}{4 e^{2}} \cdots\right)}{\cdots}\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*
d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d*(1/5*(-e^2*x^2+
d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]

time = 0.48, size = 122, normalized size = 1.08

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)-d^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\frac{3}{8}\,\sqrt{-x^2e^2+d^2}\,d^2xe+\sqrt{-x^2e^2+d^2}\,d^3-\frac{1}{4}\,(-x^2e^2+d^2)^{\frac{3}{2}}xe+\frac{1}{3}\,(-x^2e^2+d^2)^{\frac{3}{2}}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/8*d^4*arcsin(x*e/d) - d^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/ab
s(x)) - 3/8*sqrt(-x^2*e^2 + d^2)*d^2*x*e + sqrt(-x^2*e^2 + d^2)*d^3 - 1/4*(
-x^2*e^2 + d^2)^(3/2)*x*e + 1/3*(-x^2*e^2 + d^2)^(3/2)*d
```

Fricas [A]

time = 3.35, size = 102, normalized size = 0.90

$$\frac{3}{4}\,d^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+d^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\frac{1}{24}\left(6\,x^3e^3-8\,dx^2e^2-15\,d^2xe+32\,d^3\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] 3/4*d^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + d^4*log(-(d - sqrt(-
x^2*e^2 + d^2))/x) + 1/24*(6*x^3*e^3 - 8*d*x^2*e^2 - 15*d^2*x*e + 32*d^3)*s
qrt(-x^2*e^2 + d^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.62, size = 469, normalized size = 4.15

$$d^3\left(\begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d\operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id\operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases}\right) - d^2e\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right) - de^2\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) + e^3\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d),x)
```

```
[Out] d**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) - d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*
sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), A
bs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d
**2)/2, True)) - d*e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2
 - e**2*x**2)**(3/2)/(3*e**2), True)) + e**3*Piecewise((-I*d**4*acosh(e*x/d
)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sq
rt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 -
 e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*
sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 1.19, size = 100, normalized size = 0.88

$$-\frac{3}{8}\,d^4\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d) - d^4\log\left(\frac{\left|-2\,de - 2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right) + \frac{1}{24}\left(32\,d^3 - \left(15\,d^2e - 2\left(3\,xe^3 - 4\,de^2\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/8*d^4*arcsin(x*e/d)*sgn(d) - d^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 +
d^2)*e)*e^(-2)/abs(x)) + 1/24*(32*d^3 - (15*d^2*e - 2*(3*x*e^3 - 4*d*e^2)*x
)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)), x)
```

3.109 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}-\frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x-3/2*d^3*e*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))+d^3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/2*d*e*(3*e*x+2*d)*(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {864, 827, 829, 858, 223, 209, 272, 65, 214}

$$-\frac{3}{2}d^3e\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2}-\frac{(3d+ex)\left(d^2-e^2x^2\right)^{3/2}}{3x}+d^3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] -1/2*(d*e*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2]) - ((3*d + e*x)*(d^2 - e^2*x^2)
^(3/2))/(3*x) - (3*d^3*e*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + d^3*e*ArcTa
nh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
```

```
p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)}\,dx &= \int \frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{x^2}\,dx \\
&= -\frac{(3d+ex)\,(d^2-e^2x^2)^{3/2}}{3x} - \frac{1}{2}\int \frac{(2d^2e+6de^2x)\,\sqrt{d^2-e^2x^2}}{x}\,dx \\
&= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\,(d^2-e^2x^2)^{3/2}}{3x} + \frac{\int \frac{-4d^4e^3-6d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{4e^2} \\
&= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\,(d^2-e^2x^2)^{3/2}}{3x} - (d^4e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx \\
&= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\,(d^2-e^2x^2)^{3/2}}{3x} - \frac{1}{2}(d^4e)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-}} \right. \\
&= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\,(d^2-e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right. \\
&= -\frac{1}{2}de(2d+3ex)\sqrt{d^2-e^2x^2} - \frac{(3d+ex)\,(d^2-e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 143, normalized size = 1.24

$$
\frac{\sqrt{d^2-e^2x^2}\,(-6d^3-8d^2ex-3de^2x^2+2e^3x^3)}{6x} - 2d^3e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{3}{2}d^3\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 - 8*d^2*e*x - 3*d*e^2*x^2 + 2*e^3*x^3))/(6*x)
- 2*d^3*e*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - (3*d^3*Sqrt[-
e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(101) = 202$.
time = 0.08, size = 430, normalized size = 3.74

method	result

risch	$-\frac{d^3\sqrt{-e^2x^2+d^2}}{x}+\frac{e^3x^2\sqrt{-e^2x^2+d^2}}{3}-\frac{4e\,d^2\sqrt{-e^2x^2+d^2}}{3}-\frac{e^2dx\sqrt{-e^2x^2+d^2}}{2}-\frac{3e^2d^3\arctan\Big(}{2}$
default	$\frac{e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\sqrt{-\left(x+\frac{d}{e}\right)^2}}{4e^2}\right.}{}\right.\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] e/d^2*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-e/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))

Maxima [A]
time = 0.50, size = 130, normalized size = 1.13

$$-\frac{3}{2}d^3\arcsin\left(\frac{xe}{d}\right)e+d^3e\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)-\frac{1}{2}\sqrt{-x^2e^2+d^2}\,dxe^2-\sqrt{-x^2e^2+d^2}\,d^2e-\frac{\sqrt{-x^2e^2+d^2}\,d^3}{x}-\frac{1}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="maxima")

[Out] -3/2*d^3*arcsin(x*e/d)*e + d^3*e*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x)) - 1/2*sqrt(-x^2*e^2 + d^2)*d*x*e^2 - sqrt(-x^2*e^2 + d^2)*d^2*e - sqrt(-x^2*e^2 + d^2)*d^3/x - 1/3*(-x^2*e^2 + d^2)^(3/2)*e

Fricas [A]
time = 2.25, size = 121, normalized size = 1.05

$$\frac{18\,d^3x\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e-6\,d^3xe\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-8\,d^3xe+\left(2\,x^3e^3-3\,dx^2e^2-8\,d^2xe-6\,d^3\right)\sqrt{-x^2e^2+d^2}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/6*(18*d^3*x*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e - 6*d^3*x*e*lo
g(-(d - sqrt(-x^2*e^2 + d^2))/x) - 8*d^3*x*e + (2*x^3*e^3 - 3*d*x^2*e^2 - 8
*d^2*x*e - 6*d^3)*sqrt(-x^2*e^2 + d^2))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 4.18, size = 386, normalized size = 3.36

$$d^3\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d),x)
```

```
[Out] d**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) - d**2*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(
d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*
d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
*2/(e**2*x**2) + 1), True)) - d*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e)
- I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 -
e**2*x**2/d**2)/2, True)) + e**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0))
, (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True))
```

Giac [A]
time = 2.16, size = 156, normalized size = 1.36

$$-\frac{3}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)\operatorname{esgn}(d)+d^3e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{d^3xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3e^{(-1)}}{2\,x}-\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\left(8\,d^2e-\left(2\,xe^3-3\,de^2\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -3/2*d^3*arcsin(x*e/d)*e*sgn(d) + d^3*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^
2 + d^2)*e)*e^(-2)/abs(x)) + 1/2*d^3*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)*e) -
```

```
1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^3*e^(-1)/x - 1/6*sqrt(-x^2*e^2 + d^2)
*(8*d^2*e - (2*x*e^3 - 3*d*e^2)*x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^2 (d + e x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

3.110 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)} \, dx$

Optimal. Leaf size=121

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/2*(e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2+3/2*d^2*e^2*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))+3/2*d^2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)+3/2*d*e*(-e*x+d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A]
time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {864, 827, 858, 223, 209, 272, 65, 214}

$$\frac{3}{2}d^2e^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}{2x^2}+\frac{3}{2}d^2e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((d + e*x)*(d^2 - e^2*x^2)^(3
/2))/(2*x^2) + (3*d^2*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + (3*d^2*e^2
*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)}\,dx = \int \frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{x^3}\,dx$$

$$= -\frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{(4d^2e+4de^2x)\,\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{-8d^3e^2+8d^2e^3x}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{2x^2} - \frac{1}{2}(3d^3e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx +$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{2x^2} - \frac{1}{4}(3d^3e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2}}\right.$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) +$$

$$= \frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)\,(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) +$$

Mathematica [A]
time = 0.51, size = 142, normalized size = 1.17

$$\frac{1}{2}\left(\frac{\sqrt{d^2-e^2x^2}\,(-d^3+2d^2ex-2de^2x^2+e^3x^3)}{x^2} - 6d^2e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) + 3d^2e\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^3 + 2*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3))/x^2 - 6*d
^2*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] + 3*d^2*e*Sqrt[-e^2]
*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(105) = 210.
time = 0.08, size = 570, normalized size = 4.71

method	result
risch	$-\frac{d^2\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2x^2} + \frac{e^3x\sqrt{-e^2x^2+d^2}}{2} + \frac{3e^3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}} - e^2d\sqrt{-e^2x^2+d^2}$

default	$-\frac{e^2\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\sqrt{-\left(x+\right.}}{4e^2}\right.}{}\right.\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/d^3*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e
)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+
d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d*(-1/2/d^2/x
^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^
2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2
)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))-e/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6
*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/
4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2))))))+e^2/d^3*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*
x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^
(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.49, size = 131, normalized size = 1.08

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^2+\frac{3}{2}\,d^2e^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,xe^3-\frac{3}{2}\,\sqrt{-x^2e^2+d^2}\,de^2+\frac{\sqrt{-x^2e^2+d^2}\,d^2e}{x}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^2 + 3/2*d^2*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2
+ d^2)*d/abs(x)) + 1/2*sqrt(-x^2*e^2 + d^2)*x*e^3 - 3/2*sqrt(-x^2*e^2 + d^2
)*d*e^2 + sqrt(-x^2*e^2 + d^2)*d^2*e/x - 1/2*(-x^2*e^2 + d^2)^(3/2)*d/x^2
```

Fricas [A]
time = 1.83, size = 127, normalized size = 1.05

$$-\frac{6\,d^2x^2\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^2+3\,d^2x^2e^2\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+2\,d^2x^2e^2-(x^3e^3-2\,dx^2e^2+2\,d^2xe-d^3)\sqrt{-x^2e^2+d^2}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*(6*d^2*x^2*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^2 + 3*d^2*x^
2*e^2*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + 2*d^2*x^2*e^2 - (x^3*e^3 - 2*d*x
^2*e^2 + 2*d^2*x*e - d^3)*sqrt(-x^2*e^2 + d^2))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 4.87, size = 461, normalized size = 3.81

$$d^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d),
 True)) - d**2*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x
**2/d**2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)
) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)
) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*
e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) + e**3*Piecewise((-I*d**2*acosh(e*x
/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1
```

```
+ e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x
*sqrt(1 - e**2*x**2/d**2)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(104) = 208.
time = 2.80, size = 211, normalized size = 1.74

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^2\mathrm{sgn}(d)+\frac{3}{2}\,d^2e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^{(-2)}}{8\,x^2}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2}{2\,x}+\frac{\left(d^2e^2-\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2}{x}\right)x^2e^4}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(xe^3-2\,de^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^2*sgn(d) + 3/2*d^2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(
-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 1/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^
2*e^(-2)/x^2 + 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2/x + 1/8*(d^2*e^2 - 4*
(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2/x)*x^2*e^4/(d*e + sqrt(-x^2*e^2 + d^2)*e
)^2 + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^3 - 2*d*e^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

3.111 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=120

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}+de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/6*(-3*e*x+2*d)*(-e^2*x^2+d^2)^(3/2)/x^3+d*e^3*arctan(e*x/(-e^2*x^2+d^2)^
(1/2))-3/2*d*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e^2*(3*e*x+2*d)*(-e^2*
x^2+d^2)^(1/2)/x
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {864, 825, 827, 858, 223, 209, 272, 65, 214}

$$de^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(2d-3ex)\left(d^2-e^2x^2\right)^{3/2}}{6x^3}-\frac{3}{2}de^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (e^2*(2*d + 3*e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - ((2*d - 3*e*x)*(d^2 - e^2*x
^2)^(3/2))/(6*x^3) + d*e^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*d*e^3*Arc
Tanh[Sqrt[d^2 - e^2*x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
```

```
p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)}\,dx = \int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^4}\,dx$$

$$= -\frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3} - \frac{\int \frac{(4d^3e^2-6d^2e^3x)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{4d^2}$$

$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + \frac{\int \frac{12d^4e^3+8d^3e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{8d^2}$$

$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + \frac{1}{2}(3d^2e^3)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,d$$

$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + \frac{1}{4}(3d^2e^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-}}\right.$$

$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) -$$

$$= \frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + de^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) -$$

Mathematica [A]
time = 0.47, size = 139, normalized size = 1.16

$$\frac{\sqrt{d^2-e^2x^2}\,(-2d^3+3d^2ex+8de^2x^2+6e^3x^3)}{6x^3} + 3de^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) - d(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 + 3*d^2*e*x + 8*d*e^2*x^2 + 6*e^3*x^3))/(6*x^3
) + 3*d*e^3*ArcTanh[(Sqrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] - d*(-e^2)^(3
/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(106) = 212$.
time = 0.08, size = 735, normalized size = 6.12

method	result

risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, d\left(-8 e^{2} x^{2}-3 d e x+2 d^{2}\right)}{6 x^{3}}+e^{3} \sqrt{-e^{2} x^{2}+d^{2}}+\frac{e^{4} d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{3 e^{3} d^{2} \ln\left(\frac{2 d^{2}+2}{}\right.}{}$
default	$\frac{e^{3}\left(\frac{\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+d e\left(-\frac{\left(-2 e^{2}\left(x+\frac{d}{e}\right)+2 d e\right)\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8 e^{2}}+\frac{3 d^{2}\left(-\frac{\left(-2 e^{2}\left(x+\frac{d}{e}\right)+2 d e\right) \sqrt{-\left(x+\frac{d}{e}\right)}}{4 e^{2}}\right.}{}\right.\right.}{d^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e^3/d^4*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d
/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-e/d^2*(-1/2/d^2/
x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e
^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^
2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+e^2/d^3*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2
```

```
)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)
+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))))))+1/d*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2
/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5
/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*
d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))-e^3/d^4*(1/
5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1
/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.48, size = 125, normalized size = 1.04

$$d\arcsin\left(\frac{xe}{d}\right)e^3-\frac{3}{2}de^3\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)+\frac{3}{2}\sqrt{-x^2e^2+d^2}\,e^3+\frac{\sqrt{-x^2e^2+d^2}\,de^2}{x}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{2\,x^2}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}d}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] d*arcsin(x*e/d)*e^3 - 3/2*d*e^3*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d
/abs(x)) + 3/2*sqrt(-x^2*e^2 + d^2)*e^3 + sqrt(-x^2*e^2 + d^2)*d*e^2/x + 1/
2*(-x^2*e^2 + d^2)^(3/2)*e/x^2 - 1/3*(-x^2*e^2 + d^2)^(3/2)*d/x^3
```

Fricas [A]
time = 2.09, size = 122, normalized size = 1.02

$$-\frac{12\,dx^3\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^3-9\,dx^3e^3\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-6\,dx^3e^3-(6\,x^3e^3+8\,dx^2e^2+3\,d^2xe-2\,d^3)\sqrt{-x^2e^2+d^2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(12*d*x^3*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^3 - 9*d*x^3*e
^3*log(-(d - sqrt(-x^2*e^2 + d^2))/x) - 6*d*x^3*e^3 - (6*x^3*e^3 + 8*d*x^2*
e^2 + 3*d^2*x*e - 2*d^3)*sqrt(-x^2*e^2 + d^2))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 4.46, size = 457, normalized size = 3.81

$$d^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
```

```
 d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x)
)/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**
2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*
d), True)) - d*e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acos
h(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
, (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**
2*x**2/d**2)), True)) + e**3*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1
)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2
)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I
*e*x/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(102) = 204.
time = 1.47, size = 255, normalized size = 2.12

$$d \arcsin\left(\frac{xe}{d}\right) e^3 \operatorname{sgn}(d) - \frac{3}{2} d e^3 \log\left(\frac{\left|-2\, de - 2\, \sqrt{-x^2 e^2 + d^2}\, e\right| e^{(-2)}}{2\, |x|}\right) + \frac{\left(de^3 - \frac{3\left(de + \sqrt{-x^2e^2 + d^2}\, e\right) de}{x} - \frac{15\left(de + \sqrt{-x^2e^2 + d^2}\, e\right)^2 de^{(-1)}}{x^2}\right) x^3 e^6}{24\left(de + \sqrt{-x^2e^2 + d^2}\, e\right)^3} + \frac{5\left(de + \sqrt{-x^2e^2 + d^2}\, e\right) de}{8\, x} + \frac{\left(de + \sqrt{-x^2e^2 + d^2}\, e\right)^2 de^{(-1)}}{8\, x^2} - \frac{\left(de + \sqrt{-x^2e^2 + d^2}\, e\right)^3 de^{(-3)}}{24\, x^3} + \sqrt{-x^2e^2 + d^2}\, e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] d*arcsin(x*e/d)*e^3*sgn(d) - 3/2*d*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/abs(x)) + 1/24*(d*e^3 - 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d*e/x - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^(-1)/x^2)*x^3*e^6/(d*e + sq
rt(-x^2*e^2 + d^2)*e)^3 + 5/8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e/x + 1/8*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*e^(-1)/x^2 - 1/24*(d*e + sqrt(-x^2*e^2 + d
^2)*e)^3*d*e^(-3)/x^3 + sqrt(-x^2*e^2 + d^2)*e^3
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\, x^2\right)^{5/2}}{x^4\, \left(d + e\, x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

3.112 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=119

$$\frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}-e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/12*(-4*e*x+3*d)*(-e^2*x^2+d^2)^(3/2)/x^4-e^4*arctan(e*x/(-e^2*x^2+d^2)^(
1/2))-3/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/8*e^2*(-8*e*x+3*d)*(-e^2*x^
2+d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {864, 825, 858, 223, 209, 272, 65, 214}

$$e^4\left(-\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(3d-4ex)\left(d^2-e^2x^2\right)^{3/2}}{12x^4}-\frac{3}{8}e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (e^2*(3*d - 8*e*x)*Sqrt[d^2 - e^2*x^2])/(8*x^2) - ((3*d - 4*e*x)*(d^2 - e^2
*x^2)^(3/2))/(12*x^4) - e^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (3*e^4*ArcT
anh[Sqrt[d^2 - e^2*x^2]/d])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)}\,dx = \int \frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{x^5}\,dx$$

$$= -\frac{(3d-4ex)\,(d^2-e^2x^2)^{3/2}}{12x^4} - \frac{\int \frac{(6d^3e^2-8d^2e^3x)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{8d^2}$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\,(d^2-e^2x^2)^{3/2}}{12x^4} + \frac{\int \frac{12d^5e^4-32d^4e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{32d^4}$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\,(d^2-e^2x^2)^{3/2}}{12x^4} + \frac{1}{8}(3de^4)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\,(d^2-e^2x^2)^{3/2}}{12x^4} + \frac{1}{16}(3de^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-}\right.$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\,(d^2-e^2x^2)^{3/2}}{12x^4} - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) -$$

$$= \frac{e^2(3d-8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d-4ex)\,(d^2-e^2x^2)^{3/2}}{12x^4} - e^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) -$$

Mathematica [A]
time = 0.51, size = 137, normalized size = 1.15

$$\frac{\sqrt{d^2-e^2x^2}\,(-6d^3+8d^2ex+15de^2x^2-32e^3x^3)}{24x^4} + \frac{3}{4}e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) + e(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-6*d^3 + 8*d^2*e*x + 15*d*e^2*x^2 - 32*e^3*x^3))/(24*
x^4) + (3*e^4*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/4 + e*(-e^2)
^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(105) = 210$.
time = 0.08, size = 904, normalized size = 7.60

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(32e^3x^3-15d\,e^2x^2-8d^2ex+6d^3)}{24x^4} - \frac{e^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{3e^4d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2-}}{x}\right.}{8\sqrt{d^2}}$

$$3d^2\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\sqrt{-\left(x+\right.}}{4e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^4/d^5*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e
)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+
d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d*(-1/4/d^2/x
^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*
e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*
x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2)
)/x))))))+e^2/d^3*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2
*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2
/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))-e^3/d^4*(
-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2
*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e
^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-e/d^2*(-1/3/d^2/x^
3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2
*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1
/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2)))))))+e^4/d^5*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2
)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(
-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.48, size = 150, normalized size = 1.26

$$-\arcsin\left(\frac{xe}{d}\right)e^4-\frac{3}{8}e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)+\frac{3\sqrt{-x^2e^2+d^2}\,e^4}{8\,d}-\frac{\sqrt{-x^2e^2+d^2}\,e^3}{x}+\frac{3\,(-x^2e^2+d^2)^{\frac{3}{2}}e^2}{8\,dx^2}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{3\,x^3}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}d}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] -arcsin(x*e/d)*e^4 - 3/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/ab
s(x)) + 3/8*sqrt(-x^2*e^2 + d^2)*e^4/d - sqrt(-x^2*e^2 + d^2)*e^3/x + 3/8*(
-x^2*e^2 + d^2)^(3/2)*e^2/(d*x^2) + 1/3*(-x^2*e^2 + d^2)^(3/2)*e/x^3 - 1/4*
(-x^2*e^2 + d^2)^(3/2)*d/x^4
```

Fricas [A]
time = 3.06, size = 112, normalized size = 0.94

$$\frac{48\,x^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^4+9\,x^4e^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-(32\,x^3e^3-15\,dx^2e^2-8\,d^2xe+6\,d^3)\sqrt{-x^2e^2+d^2}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/24*(48*x^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^4 + 9*x^4*e^4*l
og(-(d - sqrt(-x^2*e^2 + d^2))/x) - (32*x^3*e^3 - 15*d*x^2*e^2 - 8*d^2*x*e
+ 6*d^3)*sqrt(-x^2*e^2 + d^2))/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 5.81, size = 541, normalized size = 4.55

$$d^{3}\left(\begin{cases} -\frac{d^{2}}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{3e}{8x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{3}}{8d^{2}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{4}\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{for } \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ \frac{id^{2}}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{3ie}{8x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{3}}{8d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{4}\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{otherwise} \end{cases}\right)-d^{2}e\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3x^{2}}+\frac{e^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{3d^{2}} & \text{for } \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3x^{2}}+\frac{ie^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{3d^{2}} & \text{otherwise} \end{cases}\right)-de^{2}\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{2x}+\frac{e^{2}\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ \frac{id^{2}}{2ex^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie}{2x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{2}\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right)+e^{3}\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^{2}x}{d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for } \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1 \\ -\frac{d}{x\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^{2}x}{d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*s
qrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e
**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*
sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) +
 I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d*
*3), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e*
*3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*s
qrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3
*d**2), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e*
*2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*a
sin(d/(e*x))/(2*d), True)) + e**3*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**
2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x
**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(
d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(101) = 202.
time = 1.27, size = 289, normalized size = 2.43

$$-\arcsin\left(\frac{xe}{d}\right)e^{4}\operatorname{sgn}(d)-\frac{x^{4}\left(\frac{8\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)e^{2}}{x}-\frac{120\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{3}e^{(-2)}}{x^{3}}+\frac{24\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{2}}{x^{2}}-3e^{4}\right)e^{8}}{192\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{4}}-\frac{3}{8}e^{4}\log\left(\frac{\left|-2de-2\sqrt{-x^{2}e^{2}+d^{2}}e\right|e^{(-2)}}{2|x|}\right)-\frac{5\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)e^{2}}{8x}+\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{3}e^{(-2)}}{24x^{3}}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{4}e^{(-4)}}{64x^{4}}+\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}e\right)^{2}}{8x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^4*sgn(d) - 1/192*x^4*(8*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^2
/x - 120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-2)/x^3 + 24*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^2/x^2 - 3*e^4)*e^8/(d*e + sqrt(-x^2*e^2 + d^2)*e)^4 - 3/8*e^4
*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) - 5/8*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)*e^2/x + 1/24*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-
2)/x^3 - 1/64*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-4)/x^4 + 1/8*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^2/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

3.113 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)} dx$

Optimal. Leaf size=108

$$-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}+\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] 1/4*e*(-e^2*x^2+d^2)^(3/2)/x^4-1/5*(-e^2*x^2+d^2)^(5/2)/d/x^5+3/8*e^5*arcta
nh((-e^2*x^2+d^2)^(1/2)/d)/d-3/8*e^3*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {864, 821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5}+\frac{e(d^2-e^2x^2)^{3/2}}{4x^4}+\frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}-\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (-3*e^3*Sqrt[d^2 - e^2*x^2])/(8*x^2) + (e*(d^2 - e^2*x^2)^(3/2))/(4*x^4) -
(d^2 - e^2*x^2)^(5/2)/(5*d*x^5) + (3*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8
*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)}\,dx &= \int \frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{x^6}\,dx\\
&= -\frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - e\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx\\
&= -\frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{1}{2}e\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)\\
&= \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} + \frac{1}{8}(3e^3)\,\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{1}{16}(3e^5)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e}}\right.\\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} + \frac{1}{8}(3e^3)\,\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx\right.\\
&= -\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} + \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5} + \frac{3e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}
\end{aligned}$$

Mathematica [A]

time = 0.49, size = 151, normalized size = 1.40

$$\frac{\sqrt{d^2-e^2x^2}\,(-8d^4+10d^3ex+16d^2e^2x^2-25de^3x^3-8e^4x^4)-15e^5x^5\log\left(d\left(-d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)+15e^5x^5\log\left(d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{40dx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 + 10*d^3*e*x + 16*d^2*e^2*x^2 - 25*d*e^3*x^3 -
 8*e^4*x^4) - 15*e^5*x^5*Log[d*(-d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2])] +
 15*e^5*x^5*Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]])/(40*d*x^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(92) = 184$.
time = 0.07, size = 1100, normalized size = 10.19

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(8e^4x^4+25d\,e^3x^3-16d^2x^2e^2-10d^3ex+8d^4)}{40x^5d}+\frac{3e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$	107
default	Expression too large to display	1100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e^5/d^6*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d
/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d*(-1/5/d^2/x^
5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e
^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)
+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/
2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))))-e/d^2*(-
1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(
7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^
2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d
^2)^(1/2))/x))))))-e^3/d^4*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(
1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^
(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+
e^4/d^5*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2
)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1
/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+e^2/d^3*(
-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2
)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)
+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)
```

```
*x/(-e^2*x^2+d^2)^(1/2))))))-e^5/d^6*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-
e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d
^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.47, size = 144, normalized size = 1.33

$$\frac{3\,e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{8\,d}-\frac{3\,\sqrt{-x^2e^2+d^2}\,e^5}{8\,d^2}-\frac{3\,(-x^2e^2+d^2)^{\frac{3}{2}}e^3}{8\,d^2x^2}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^2}{5\,dx^3}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{4\,x^4}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}d}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="maxima")
```

```
[Out] 3/8*e^5*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d - 3/8*sqrt(-x
^2*e^2 + d^2)*e^5/d^2 - 3/8*(-x^2*e^2 + d^2)^(3/2)*e^3/(d^2*x^2) + 1/5*(-x^
2*e^2 + d^2)^(3/2)*e^2/(d*x^3) + 1/4*(-x^2*e^2 + d^2)^(3/2)*e/x^4 - 1/5*(-x
^2*e^2 + d^2)^(3/2)*d/x^5
```

Fricas [A]
time = 2.58, size = 92, normalized size = 0.85

$$-\frac{15\,x^5e^5\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\left(8\,x^4e^4+25\,dx^3e^3-16\,d^2x^2e^2-10\,d^3xe+8\,d^4\right)\sqrt{-x^2e^2+d^2}}{40\,dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/40*(15*x^5*e^5*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (8*x^4*e^4 + 25*d*x^
3*e^3 - 16*d^2*x^2*e^2 - 10*d^3*x*e + 8*d^4)*sqrt(-x^2*e^2 + d^2))/(d*x^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.62, size = 774, normalized size = 7.17

$$d^3\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
```

```
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) - d**2*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqr
t(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**
2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sq
rt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sqrt(d**2/(e
**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**
2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sq
rt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**3*Piecewise((-e*sqrt(d**2/(
e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) >
1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(87) = 174$.
time = 1.08, size = 366, normalized size = 3.39

$$-\frac{x^5\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^3}{x}+\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e}{x^2}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-1)}}{x^3}-\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-3)}}{x^4}-2\,e^5\right)e^{10}}{320\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d}+\frac{3\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{8\,d}-\frac{\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4e^3}{x}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^4e}{x^2}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^4e^{(-1)}}{x^3}-\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^4e^{(-3)}}{x^4}+\frac{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^4e^{(-5)}}{x^5}}{320\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/320*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^3/x + 10*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*e/x^2 - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-1)/x^3 - 20*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-3)/x^4 - 2*e^5)*e^10/((d*e + sqrt(-x^2*
e^2 + d^2)*e)^5*d) + 3/8*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)
*e^(-2)/abs(x))/d - 1/320*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^3/x + 40
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e/x^2 - 10*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^3*d^4*e^(-1)/x^3 - 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^(-3)/x^4 +
 2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^4*e^(-5)/x^5)/d^5
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

3.114 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=143

$$\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}+\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}$$

```
[Out] -1/24*e^2*(-e^2*x^2+d^2)^(3/2)/d/x^4-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6+1/5*e*(
-e^2*x^2+d^2)^(5/2)/d^2/x^5-1/16*e^6*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/
16*e^4*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {864, 849, 821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6}+\frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5}-\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4}-\frac{e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2}+\frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (e^4*Sqrt[d^2 - e^2*x^2])/(16*d*x^2) - (e^2*(d^2 - e^2*x^2)^(3/2))/(24*d*x^
4) - (d^2 - e^2*x^2)^(5/2)/(6*d*x^6) + (e*(d^2 - e^2*x^2)^(5/2))/(5*d^2*x^5
) - (e^6*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)}\,dx = \int \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{x^7}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{\int \frac{(6d^2e-de^2x)(d^2-e^2x^2)^{3/2}}{x^6}\,dx}{6d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^2\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{6d}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^2\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x, x^2\right)}{12d}$$

$$= -\frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^4\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx,\right.}{16d}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} + \frac{e^6\text{Subst}\left(\int\right.}{}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^4\text{Subst}\left(\int\right.}{}$$

$$= \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{(d^2-e^2x^2)^{5/2}}{6dx^6} + \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^6\tanh^{-1}\left(\right.}{1}$$

Mathematica [**A**]
time = 0.52, size = 123, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\,(-40d^5+48d^4ex+70d^3e^2x^2-96d^2e^3x^3-15de^4x^4+48e^5x^5)+30e^6x^6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{240d^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-40*d^5 + 48*d^4*e*x + 70*d^3*e^2*x^2 - 96*d^2*e^3*x^
3 - 15*d*e^4*x^4 + 48*e^5*x^5) + 30*e^6*x^6*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^
2 - e^2*x^2])/d])/(240*d^2*x^6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(123) = 246.
time = 0.08, size = 1300, normalized size = 9.09

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\,\left(-48 e^{5} x^{5}+15 d\,e^{4} x^{4}+96 d^{2} e^{3} x^{3}-70 x^{2} d^{3} e^{2}-48 d^{4} x e+40 d^{5}\right)}{240 x^{6} d^{2}}-\frac{e^{6} \ln\left(\frac{2 d^{2}+2 \sqrt{d^{2}}\,\sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{16 d \sqrt{d^{2}}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^6/d^7*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e
)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+
d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-e/d^2*(-1/5/d^2
/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/
3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5
/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)
+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))))+e^2/d
^3*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d
^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/
2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x))))))+e^4/d^5*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/
d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+
d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
))))-e^5/d^6*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)
^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1
/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-e^3/
d^4*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)
^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^
(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^
(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))+e^6/d^7*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1
/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2
+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))+1/d*(-1/6/d^2/x^6*(-e^2*x^2+d^2)
^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2
/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-
e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d
^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))))
```

Maxima [A]
time = 0.48, size = 167, normalized size = 1.17

$$-\frac{e^{6} \log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2} e^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{2}}+\frac{\sqrt{-x^{2} e^{2}+d^{2}}\,e^{6}}{16\,d^{3}}+\frac{\left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} e^{4}}{16\,d^{3} x^{2}}-\frac{\left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} e^{3}}{5\,d^{2} x^{3}}+\frac{\left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} e^{2}}{8\,d x^{4}}+\frac{\left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} e}{5\,x^{5}}-\frac{\left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} d}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^2 + 1/16*sq
rt(-x^2*e^2 + d^2)*e^6/d^3 + 1/16*(-x^2*e^2 + d^2)^(3/2)*e^4/(d^3*x^2) - 1/
5*(-x^2*e^2 + d^2)^(3/2)*e^3/(d^2*x^3) + 1/8*(-x^2*e^2 + d^2)^(3/2)*e^2/(d*
x^4) + 1/5*(-x^2*e^2 + d^2)^(3/2)*e/x^5 - 1/6*(-x^2*e^2 + d^2)^(3/2)*d/x^6
```

Fricas [A]
time = 3.84, size = 102, normalized size = 0.71

$$\frac{15\,x^6e^6\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\left(48\,x^5e^5-15\,dx^4e^4-96\,d^2x^3e^3+70\,d^3x^2e^2+48\,d^4xe-40\,d^5\right)\sqrt{-x^2e^2+d^2}}{240\,d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/240*(15*x^6*e^6*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (48*x^5*e^5 - 15*d*x
^4*e^4 - 96*d^2*x^3*e^3 + 70*d^3*x^2*e^2 + 48*d^4*x*e - 40*d^5)*sqrt(-x^2*e
^2 + d^2))/(d^2*x^6)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.83, size = 918, normalized size = 6.42

$$d^3\left(\begin{cases}-\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise}\end{cases}\right)-d^2e\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)-de^2\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)+e^3\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - d**2*e*Piecewise((3*I*d**3*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt
(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2
) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4
*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e*
*6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4
*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - d
*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*
sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) +
e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5
```

```
*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1))
+ I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d
**3), True)) + e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**
3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(116) = 232.
time = 1.11, size = 425, normalized size = 2.97

$$-\frac{x^6\left(\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^4}{x}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^2}{x^2}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-2)}}{x^4}+\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-4)}}{x^5}-\frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3}{x^3}-5\,e^6\right)e^{12}}{1920\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6d^2}-\frac{e^6\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{16\,d^2}+\frac{\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{10}e^4}{x}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^{10}e^2}{x^2}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^{10}e^{(-2)}}{x^4}+\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^{10}e^{(-4)}}{x^5}-\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6d^{10}e^{(-6)}}{x^6}-\frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{10}}{x^3}}{1920\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/1920*x^6*(12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^4/x + 15*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^2*e^2/x^2 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-2)/x^4 +
120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-4)/x^5 - 60*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^3/x^3 - 5*e^6)*e^12/((d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^2) - 1/16*
e^6*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 + 1/1
920*(120*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^10*e^4/x + 15*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^2*d^10*e^2/x^2 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^10*e^(-2
)/x^4 + 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^10*e^(-4)/x^5 - 5*(d*e + sqrt
(-x^2*e^2 + d^2)*e)^6*d^10*e^(-6)/x^6 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3
*d^10/x^3)/d^12
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^7\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

3.115 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx$

Optimal. Leaf size=172

$$-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4}-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}+\frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6}-\frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5}+\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] 1/24*e^3*(-e^2*x^2+d^2)^(3/2)/d^2/x^4-1/7*(-e^2*x^2+d^2)^(5/2)/d/x^7+1/6*e*
(-e^2*x^2+d^2)^(5/2)/d^2/x^6-2/35*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^5+1/16*e^7
*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-1/16*e^5*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {864, 849, 821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{5/2}}{7dx^7}+\frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6}-\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4}-\frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5}+\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] -1/16*(e^5*Sqrt[d^2 - e^2*x^2])/(d^2*x^2) + (e^3*(d^2 - e^2*x^2)^(3/2))/(24
*d^2*x^4) - (d^2 - e^2*x^2)^(5/2)/(7*d*x^7) + (e*(d^2 - e^2*x^2)^(5/2))/(6*
d^2*x^6) - (2*e^2*(d^2 - e^2*x^2)^(5/2))/(35*d^3*x^5) + (e^7*ArcTanh[Sqrt[d
^2 - e^2*x^2]/d])/(16*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)}\,dx = \int \frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{x^8}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{\int \frac{(7d^2e-2de^2x)\,(d^2-e^2x^2)^{3/2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} + \frac{\int \frac{(12d^3e^2-7d^2e^3x)\,(d^2-e^2x^2)^{3/2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^3\int \frac{(d^2-e^2x^2)^{3/2}}{x^5}\,dx}{6d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^3\text{Subst}\left(\int \frac{(d^2-e^2x)^{3/2}}{x^3}\,dx, x\right.}{12d^2}$$

$$= \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} + \frac{e^5\text{Subst}\Big(}{}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e}{35d^3}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e}{35d^3}$$

$$= -\frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} + \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{(d^2-e^2x^2)^{5/2}}{7dx^7} + \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} - \frac{2e^2(d^2-e}{35d^3}$$

Mathematica [A]
time = 0.57, size = 134, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}\,(-240d^6+280d^5ex+384d^4e^2x^2-490d^3e^3x^3-48d^2e^4x^4+105de^5x^5-96e^6x^6)-210e^7x^7\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{1680d^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 + 280*d^5*e*x + 384*d^4*e^2*x^2 - 490*d^3*e^
3*x^3 - 48*d^2*e^4*x^4 + 105*d*e^5*x^5 - 96*e^6*x^6) - 210*e^7*x^7*ArcTanh[
(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(1680*d^3*x^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. 2(148) = 296.
time = 0.08, size = 1325, normalized size = 7.70

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(96e^6x^6-105d\,e^5x^5+48d^2e^4x^4+490d^3e^3x^3-384d^4e^2x^2-280e\,d^5x+240d^6)}{1680x^7d^3}+\frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2}}{x}\right.}{16d^2\sqrt{d^2}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e^7/d^8*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d
/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+e^2/d^3*(-1/5/d^
2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4
/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(
5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2
)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))))-e^3/
d^4*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+
d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3
/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x))))))-e^5/d^6*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2
/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2
+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x
)))))+e^6/d^7*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2
)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(
1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+e^4
/d^5*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2
)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)
^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))-e^7/d^8*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(
1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^
2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))-e/d^2*(-1/6/d^2/x^6*(-e^2*x^2+d
^2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/
d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3
*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2
*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))))-1/7/d^3/x^7*(-e^2*x^2+d^2)^(7/2
)
```

Maxima [A]

time = 0.48, size = 190, normalized size = 1.10

$$\frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{16\,d^3}-\frac{\sqrt{-x^2e^2+d^2}\,e^7}{16\,d^4}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^5}{16\,d^4x^2}+\frac{2\,(-x^2e^2+d^2)^{\frac{3}{2}}e^4}{35\,d^3x^3}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^3}{8\,d^2x^4}+\frac{3\,(-x^2e^2+d^2)^{\frac{3}{2}}e^2}{35\,dx^5}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{6\,x^6}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}d}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/16*e^7*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^3 - 1/16*sqr
t(-x^2*e^2 + d^2)*e^7/d^4 - 1/16*(-x^2*e^2 + d^2)^(3/2)*e^5/(d^4*x^2) + 2/3
5*(-x^2*e^2 + d^2)^(3/2)*e^4/(d^3*x^3) - 1/8*(-x^2*e^2 + d^2)^(3/2)*e^3/(d^
2*x^4) + 3/35*(-x^2*e^2 + d^2)^(3/2)*e^2/(d*x^5) + 1/6*(-x^2*e^2 + d^2)^(3/
2)*e/x^6 - 1/7*(-x^2*e^2 + d^2)^(3/2)*d/x^7
```

Fricas [A]
time = 2.93, size = 112, normalized size = 0.65

$$-\frac{105\,x^{7}e^{7}\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+\left(96\,x^{6}e^{6}-105\,dx^{5}e^{5}+48\,d^{2}x^{4}e^{4}+490\,d^{3}x^{3}e^{3}-384\,d^{4}x^{2}e^{2}-280\,d^{5}xe+240\,d^{6}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{1680\,d^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/1680*(105*x^7*e^7*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (96*x^6*e^6 - 105
*d*x^5*e^5 + 48*d^2*x^4*e^4 + 490*d^3*x^3*e^3 - 384*d^4*x^2*e^2 - 280*d^5*x
*e + 240*d^6)*sqrt(-x^2*e^2 + d^2))/(d^3*x^7)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.64, size = 1037, normalized size = 6.03

$$d^{3}\left(\begin{cases} -\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{7x^{6}}+\frac{e^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{35d^{2}x^{4}}+\frac{4e^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{105d^{4}x^{2}}+\frac{8e^{7}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{105d^{6}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ -\frac{ie\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{7x^{6}}+\frac{ie^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{35d^{2}x^{4}}+\frac{4ie^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{105d^{4}x^{2}}+\frac{8ie^{7}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}{105d^{6}} & \text{otherwise} \end{cases}\right)-d^{2}e\left(\begin{cases} -\frac{d^{2}}{6ex^{7}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{5e}{24x^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{3}}{48d^{2}x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{5}}{16d^{4}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{6}\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^{5}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ \frac{id^{2}}{6ex^{7}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{5ie}{24x^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{3}}{48d^{2}x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{5}}{16d^{4}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{6}\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^{5}} & \text{otherwise} \end{cases}\right)-de^{2}\left(\begin{cases} \frac{3id^{3}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}-\frac{4ide^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}+\frac{2ie^{6}x^{6}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}}-\frac{ie^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1 \\ \frac{3d^{3}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}-\frac{4de^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}+\frac{2e^{6}x^{6}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}}-\frac{e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{otherwise} \end{cases}\right)+e^{3}\left(\begin{cases} -\frac{d^{2}}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{3e}{8x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{3}}{8d^{2}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{4}\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1 \\ \frac{id^{2}}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{3ie}{8x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{3}}{8d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{4}\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - d**2*e*P
iecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d
**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e*
*5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-
d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I
*e**6*asin(d/(e*x))/(16*d**5), True)) - d*e**2*Piecewise((3*I*d**3*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1
+ e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 +
 e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1)
```

```
, (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**
2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**
6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*
sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e**3*Pi
ecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**
2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*aco
sh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d
**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3
/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(139) = 278.
time = 1.09, size = 492, normalized size = 2.86

$$-\frac{x^7\left(\frac{35\left(de+\sqrt{-x^2e^2+d^2}e\right)e^5}{x}+\frac{21\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^3}{x^2}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e}{x^3}+\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^{(-1)}}{x^4}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^5e^{(-3)}}{x^5}-\frac{315\left(de+\sqrt{-x^2e^2+d^2}e\right)^6e^{(-5)}}{x^6}-15e^7\right)e^{14}}{13440\left(de+\sqrt{-x^2e^2+d^2}e\right)^7d^3}+\frac{e^7\log\left(\frac{\left|-2de-2\sqrt{-x^2e^2+d^2}e\right|e^{(-2)}}{2|x|}\right)}{16d^3}-\frac{\frac{315\left(de+\sqrt{-x^2e^2+d^2}e\right)d^{18}e^5}{x}+\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^2d^{18}e^3}{x^2}-\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^3d^{18}e}{x^3}+\frac{105\left(de+\sqrt{-x^2e^2+d^2}e\right)^4d^{18}e^{(-1)}}{x^4}-\frac{21\left(de+\sqrt{-x^2e^2+d^2}e\right)^5d^{18}e^{(-3)}}{x^5}-\frac{35\left(de+\sqrt{-x^2e^2+d^2}e\right)^6d^{18}e^{(-5)}}{x^6}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}e\right)^7d^{18}e^{(-7)}}{x^7}}{13440d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/13440*x^7*(35*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^5/x + 21*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^2*e^3/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e/x^3 + 105
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-1)/x^4 - 105*(d*e + sqrt(-x^2*e^2 + d
^2)*e)^5*e^(-3)/x^5 - 315*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-5)/x^6 - 15*
e^7)*e^14/((d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^3) + 1/16*e^7*log(1/2*abs(-2*
d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 1/13440*(315*(d*e + sq
rt(-x^2*e^2 + d^2)*e)*d^18*e^5/x + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^1
8*e^3/x^2 - 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^18*e/x^3 + 105*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^4*d^18*e^(-1)/x^4 - 21*(d*e + sqrt(-x^2*e^2 + d^2)*e)
^5*d^18*e^(-3)/x^5 - 35*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*d^18*e^(-5)/x^6 +
15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^18*e^(-7)/x^7)/d^21
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^8\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

3.116 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx$

Optimal. Leaf size=201

$$\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4}-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}+\frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7}-\frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6}+\frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5}-$$

[Out] -1/64*e^4*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8+1/7*e*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6+2/35*e^3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5-3/128*e^8*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+3/128*e^6*(-e^2*x^2+d^2)^(1/2)/d^3/x^2

Rubi [A]

time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {864, 849, 821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{5/2}}{8dx^8}+\frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7}-\frac{3e^8\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4}+\frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5}-\frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6}+\frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2}-\frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]

[Out] (3*e^6*Sqrt[d^2 - e^2*x^2])/(128*d^3*x^2) - (e^4*(d^2 - e^2*x^2)^(3/2))/(64*d^3*x^4) - (d^2 - e^2*x^2)^(5/2)/(8*d*x^8) + (e*(d^2 - e^2*x^2)^(5/2))/(7*d^2*x^7) - (e^2*(d^2 - e^2*x^2)^(5/2))/(16*d^3*x^6) + (2*e^3*(d^2 - e^2*x^2)^(5/2))/(35*d^4*x^5) - (3*e^8*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(128*d^4)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^9(d+ex)}\,dx = \int \frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{x^9}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{\int \frac{(8d^2e-3de^2x)\,(d^2-e^2x^2)^{3/2}}{x^8}\,dx}{8d^2}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} + \frac{\int \frac{(21d^3e^2-16d^2e^3x)\,(d^2-e^2x^2)^{3/2}}{x^7}\,dx}{56d^4}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{\int \frac{(96d^4e^3-21d^3e^4x)\,(d^2-e^2x^2)^{3/2}}{x^6}\,d}{336d^6}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} + \frac{e^4\int \frac{(d^2}{}}{1}$$

$$= -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} + \frac{e^4\text{Subst}}{}$$

$$= -\frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{2e^3(d^2-}{35d}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2}{16d^3x}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2}{16d^3x}$$

$$= \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} - \frac{(d^2-e^2x^2)^{5/2}}{8dx^8} + \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{e^2(d^2-e^2}{16d^3x}$$

<u>Mathematica</u> [A]
time = 0.61, size = 145, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\,(-560d^7+640d^6ex+840d^5e^2x^2-1024d^4e^3x^3-70d^3e^4x^4+128d^2e^5x^5-105de^6x^6+256e^7x^7)+210e^8x^8\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{4480d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-560*d^7 + 640*d^6*e*x + 840*d^5*e^2*x^2 - 1024*d^4*e
^3*x^3 - 70*d^3*e^4*x^4 + 128*d^2*e^5*x^5 - 105*d*e^6*x^6 + 256*e^7*x^7) +
210*e^8*x^8*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(4480*d^4*x^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1554 vs. $2(173) = 346$.

time = 0.08, size = 1555, normalized size = 7.74

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-256e^7x^7+105d\,e^6x^6-128d^2e^5x^5+70d^3e^4x^4+1024d^4e^3x^3-840d^5e^2x^2-640d^6ex+560d^7)}{4480x^8d^4} - \frac{3e^8 \ln\left(\frac{2d^2+}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^8/d^9*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e
)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+
d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-e^3/d^4*(-1/5/d
^2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-
4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^
(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/
2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))+e^4
/d^5*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2
+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(
3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^
2*x^2+d^2)^(1/2))/x))))))+e^6/d^7*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^
2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^
2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/
x)))))-e^7/d^8*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^
2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^
(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+1/
d*(-1/8/d^2/x^8*(-e^2*x^2+d^2)^(7/2)+1/8*e^2/d^2*(-1/6/d^2/x^6*(-e^2*x^2+d^
2)^(7/2)-1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d
^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*
(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*
(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))))-e^5/d^6*(-1/3/d^2/x^3*(-e^2*x^2
+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e
^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*
x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2
)))))))+e^8/d^9*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2
*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^
2)^(1/2))/x))))+e^2/d^3*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(7/2)-1/6*e^2/d^2*(-1/
4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/
2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*
```

```
((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2
)^(1/2))/x)))))))+1/7*e/d^4/x^7*(-e^2*x^2+d^2)^(7/2)
```

Maxima [A]
time = 0.48, size = 213, normalized size = 1.06

$$-\frac{3\,e^8\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{128\,d^4}+\frac{3\,\sqrt{-x^2e^2+d^2}\,e^8}{128\,d^5}+\frac{3\,(-x^2e^2+d^2)^{\frac{3}{2}}e^6}{128\,d^5x^2}-\frac{2\,(-x^2e^2+d^2)^{\frac{3}{2}}e^5}{35\,d^4x^3}+\frac{3\,(-x^2e^2+d^2)^{\frac{3}{2}}e^4}{64\,d^3x^4}-\frac{3\,(-x^2e^2+d^2)^{\frac{3}{2}}e^3}{35\,d^2x^5}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^2}{16\,dx^6}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{7\,x^7}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}d}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="maxima")
```

```
[Out] -3/128*e^8*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^4 + 3/128*
sqrt(-x^2*e^2 + d^2)*e^8/d^5 + 3/128*(-x^2*e^2 + d^2)^(3/2)*e^6/(d^5*x^2) -
 2/35*(-x^2*e^2 + d^2)^(3/2)*e^5/(d^4*x^3) + 3/64*(-x^2*e^2 + d^2)^(3/2)*e^
4/(d^3*x^4) - 3/35*(-x^2*e^2 + d^2)^(3/2)*e^3/(d^2*x^5) + 1/16*(-x^2*e^2 +
d^2)^(3/2)*e^2/(d*x^6) + 1/7*(-x^2*e^2 + d^2)^(3/2)*e/x^7 - 1/8*(-x^2*e^2 +
 d^2)^(3/2)*d/x^8
```

Fricas [A]
time = 3.11, size = 122, normalized size = 0.61

$$\frac{105\,x^8e^8\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(256\,x^7e^7-105\,dx^6e^6+128\,d^2x^5e^5-70\,d^3x^4e^4-1024\,d^4x^3e^3+840\,d^5x^2e^2+640\,d^6xe-560\,d^7)\sqrt{-x^2e^2+d^2}}{4480\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4480*(105*x^8*e^8*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (256*x^7*e^7 - 105
*d*x^6*e^6 + 128*d^2*x^5*e^5 - 70*d^3*x^4*e^4 - 1024*d^4*x^3*e^3 + 840*d^5*
x^2*e^2 + 640*d^6*x*e - 560*d^7)*sqrt(-x^2*e^2 + d^2))/(d^4*x^8)
```

Sympy [C] Result contains complex when optimal does not.
time = 28.05, size = 1159, normalized size = 5.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1
)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2/(
e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(4
8*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**
2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*
```

```
e**7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128
*d**7), True)) - d**2*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) +
 e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x*
*2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Ab
s(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e*
*3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x
**2) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6)
, True)) - d*e**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) +
5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
*2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d
/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2
/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(
48*d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) + e**3*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**
6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e*
*4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**
2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**
2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7),
 True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(162) = 324$.

time = 1.79, size = 425, normalized size = 2.11

$$-\frac{x^8\left(\frac{80\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^6}{x}-\frac{112\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^2}{x^3}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-2)}}{x^5}+\frac{1680\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7e^{(-6)}}{x^7}+\frac{280\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4}{x^4}-35\,e^8\right)e^{16}}{71680\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8d^4}-\frac{3\,e^8\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{128\,d^4}+\frac{\frac{1680\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{28}e^6}{x}-\frac{560\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{28}e^2}{x^3}-\frac{112\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^{28}e^{(-2)}}{x^5}+\frac{80\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^7d^{28}e^{(-6)}}{x^7}-\frac{35\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^8d^{28}e^{(-8)}}{x^8}+\frac{280\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^{28}}{x^4}}{71680\,d^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/71680*x^8*(80*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^6/x - 112*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^3*e^2/x^3 - 560*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-2)/x^5
 + 1680*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^(-6)/x^7 + 280*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^4/x^4 - 35*e^8)*e^16/((d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^4) -
 3/128*e^8*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^
4 + 1/71680*(1680*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^28*e^6/x - 560*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^3*d^28*e^2/x^3 - 112*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5
*d^28*e^(-2)/x^5 + 80*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*d^28*e^(-6)/x^7 - 35
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*d^28*e^(-8)/x^8 + 280*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^4*d^28/x^4)/d^32
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^9 (d + e x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

3.117 $\int \frac{x\sqrt{1-x^2}}{1+x}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {799, 794, 222}

$$-\frac{\text{ArcSin}(x)}{2}-\frac{1}{2}\sqrt{1-x^2}\,(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(1 + x),x]
```

```
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) - ArcSin[x]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\int \frac{x\sqrt{1-x^2}}{1+x}\,dx = \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{1}{2}(2-x)\sqrt{1-x^2} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{1}{2}(2-x)\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}(x)$$

Mathematica [A]
time = 0.08, size = 37, normalized size = 1.37

$$\frac{1}{2}(-2+x)\sqrt{1-x^2} + \tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(1 + x),x]
[Out] ((-2 + x)*Sqrt[1 - x^2])/2 + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.08, size = 34, normalized size = 1.26

method	result	size
risch	$-\frac{(x-2)(x^2-1)}{2\sqrt{-x^2+1}} - \frac{\arcsin(x)}{2}$	25
default	$\frac{x\sqrt{-x^2+1}}{2} - \frac{\arcsin(x)}{2} - \sqrt{-(1+x)^2+2+2x}$	34
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+1} + \frac{\operatorname{RootOf}(\_Z^2+1)\ln\left(-\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)}{2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(1+x),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(-x^2+1)^(1/2)-1/2*arcsin(x)-(-(1+x)^2+2+2*x)^(1/2)
```

Maxima [A]
time = 0.48, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^2+1}\,x - \sqrt{-x^2+1} - \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Fricas [A]
time = 2.24, size = 31, normalized size = 1.15

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A]
time = 1.33, size = 29, normalized size = 1.07

$$\begin{cases} \frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2} & \text{for } x>-1\wedge x<1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(1+x),x)
[Out] Piecewise((x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2, (x > -1) & (x <
 1)))
```

Giac [A]
time = 1.33, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(1+x),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^2)^(1/2))/(x + 1),x)
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.118 $\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)} \, dx$

Optimal. Leaf size=51

$$-\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arcsin(a*x)-a*arctanh((-a^2*x^2+1)^(1/2))-(-a*x+1)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {864, 827, 858, 222, 272, 65, 214}

$$-\frac{\sqrt{1-a^2x^2}\,(1-ax)}{x} - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a\text{ArcSin}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] -(((1 - a*x)*Sqrt[1 - a^2*x^2])/x) - a*ArcSin[a*x] - a*ArcTanh[Sqrt[1 - a^2
*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)}\,dx &= \int \frac{(1+ax)\sqrt{1-a^2x^2}}{x^2}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - \frac{1}{2}\int \frac{-2a+2a^2x}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} + a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx - a^2\int \frac{1}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{(1-ax)\sqrt{1-a^2x^2}}{x} - a\sin^{-1}(ax) - a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 95, normalized size = 1.86

$$
\frac{(-1+ax)\sqrt{1-a^2x^2}}{x} + 2a\tanh^{-1}\left(\sqrt{-a^2}\,x - \sqrt{1-a^2x^2}\right) - \sqrt{-a^2}\,\log\left(-\sqrt{-a^2}\,x + \sqrt{1-a^2x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/(x^2*(1 - a*x)),x]
```

```
[Out] ((-1 + a*x)*Sqrt[1 - a^2*x^2])/x + 2*a*ArcTanh[Sqrt[-a^2]*x - Sqrt[1 - a^2*
x^2]] - Sqrt[-a^2]*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(47) = 94$.
time = 0.08, size = 253, normalized size = 4.96

method	result
risch	$\frac{a^2x^2-1}{x\sqrt{-a^2x^2+1}} + a\sqrt{-a^2x^2+1} - \frac{a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}} - a\,\text{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$

default	$-a\left(\frac{\left(-a^2(x-\frac{1}{a})^2-2a(x-\frac{1}{a})\right)^{\frac{3}{2}}}{3}-a\left(-\frac{(-2a^2(x-\frac{1}{a})-2a)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{4a^2}+\frac{\arctan\left(\frac{}{\sqrt{-a^2}}\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(1/3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-a*(-1/4*(-2*a^2*(x-1/a)-2*a)/a^2
*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-
a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))))-1/x*(-a^2*x^2+1)^(5/2)-4*a^2*(1/4*x*(-a
^2*x^2+1)^(3/2)+3/8*x*(-a^2*x^2+1)^(1/2)+3/8/(a^2)^(1/2)*arctan((a^2)^(1/2)
*x/(-a^2*x^2+1)^(1/2)))+a*(1/3*(-a^2*x^2+1)^(3/2)+(-a^2*x^2+1)^(1/2)-arctan
h(1/(-a^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.48, size = 68, normalized size = 1.33

$$-a\arcsin(ax)-a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\sqrt{-a^2x^2+1}\,a-\frac{\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -a*arcsin(a*x) - a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*
x^2 + 1)*a - sqrt(-a^2*x^2 + 1)/x
```

Fricas [A]
time = 1.92, size = 74, normalized size = 1.45

$$\frac{2\,ax\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+ax+\sqrt{-a^2x^2+1}\,(ax-1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + a*x*log((sqrt(-a^2*x^2 + 1)
 - 1)/x) + a*x + sqrt(-a^2*x^2 + 1)*(a*x - 1))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 2.66, size = 170, normalized size = 3.33

$$a\left(\begin{cases}i\sqrt{a^2x^2-1}-\log(ax)+\frac{\log(a^2x^2)}{2}+i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{for } |a^2x^2|>1\\ \sqrt{-a^2x^2+1}+\frac{\log(a^2x^2)}{2}-\log\left(\sqrt{-a^2x^2+1}+1\right) & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{ia^2x}{\sqrt{a^2x^2-1}}+ia\operatorname{acosh}(ax)+\frac{i}{x\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{a^2x}{\sqrt{-a^2x^2+1}}-a\operatorname{asin}(ax)-\frac{1}{x\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/x**2/(-a*x+1),x)
```

```
[Out] a*Piecewise((I*sqrt(a**2*x**2 - 1) - log(a*x) + log(a**2*x**2)/2 + I*asin(1
/(a*x)), Abs(a**2*x**2) > 1), (sqrt(-a**2*x**2 + 1) + log(a**2*x**2)/2 - lo
g(sqrt(-a**2*x**2 + 1) + 1), True)) + Piecewise((-I*a**2*x/sqrt(a**2*x**2 -
 1) + I*a*acosh(a*x) + I/(x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (a**
2*x/sqrt(-a**2*x**2 + 1) - a*asin(a*x) - 1/(x*sqrt(-a**2*x**2 + 1)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(45) =$ 90.
time = 1.06, size = 125, normalized size = 2.45

$$\frac{a^4x}{2\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{a^2\arcsin\left(ax\right)\operatorname{sgn}(a)}{|a|}-\frac{a^2\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\sqrt{-a^2x^2+1}\,a-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{2\,x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - a^2*arcsin(a*x)*sgn(a)
/abs(a) - a^2*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))
/abs(a) + sqrt(-a^2*x^2 + 1)*a - 1/2*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs
(a))
```

Mupad [**B**]
time = 0.05, size = 74, normalized size = 1.45

$$a\sqrt{1-a^2\,x^2}-\frac{\sqrt{1-a^2\,x^2}}{x}-\frac{a^2\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}}+a\,\mathrm{atan}\left(\sqrt{1-a^2\,x^2}\;1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - a^2*x^2)^(3/2)/(x^2*(a*x - 1)),x)
```

```
[Out] a*atan((1 - a^2*x^2)^(1/2)*1i)*1i + a*(1 - a^2*x^2)^(1/2) - (1 - a^2*x^2)^(
1/2)/x - (a^2*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2)
```

3.119 $\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=118

$$\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{3d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

```
[Out] -3/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5+x^3*(-e*x+d)/e^2/(-e^2*x^2+d^
2)^(1/2)-4/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/6*d*(-9*e*x+16*d)*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A]

time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {864, 833, 847, 794, 223, 209}

$$-\frac{3d^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}+\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}}-\frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5}-\frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^3*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) - (4*x^2*Sqrt[d^2 - e^2*x^2])/(3*
e^3) - (d*(16*d - 9*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^5) - (3*d^3*ArcTan[(e*x)
/Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4(d-ex)}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x^2(3d^3-4d^2ex)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} + \frac{\int \frac{x(8d^4e-9d^3e^2x)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^2e^4}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{(3d^3)\int \frac{}{\sqrt{}}}{}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{(3d^3)\,\mathrm{Subs}}{}$$

$$= \frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{3d^3\tan^{-1}}{}$$

Mathematica [A]
time = 0.26, size = 109, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2x^2}\,(-16d^3-7d^2ex+de^2x^2-2e^3x^3)}{6e^5(d+ex)} + \frac{3d^3(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-16*d^3 - 7*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(6*e^5*
(d + e*x)) + (3*d^3*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
)/(2*e^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(104) = 208$.
time = 0.07, size = 215, normalized size = 1.82

method	result
risch	$-\frac{(2e^2x^2-3dex+10d^2)\sqrt{-e^2x^2+d^2}}{6e^5} - \frac{3d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^4\sqrt{e^2}} - \frac{d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^6\left(x+\frac{d}{e}\right)}$

default	$\frac{-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}}{e}-\frac{d\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)}{e^2}-\frac{d^2\sqrt{-e^2x}}{e^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/3*x^2/e^2*(-e^2*x^2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+d^2)^(1/2))-d/
e^2*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2)))-d^2/e^5*(-e^2*x^2+d^2)^(1/2)-d^3/e^4/(e^2)^(1
/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/e^6*d^3/(x+d/e)*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)
```

Maxima [A]
time = 0.48, size = 104, normalized size = 0.88

$$-\frac{3}{2}d^3\arcsin\left(\frac{xe}{d}\right)e^{(-5)}-\frac{1}{3}\sqrt{-x^2e^2+d^2}\,x^2e^{(-3)}+\frac{1}{2}\sqrt{-x^2e^2+d^2}\,dxe^{(-4)}-\frac{5}{3}\sqrt{-x^2e^2+d^2}\,d^2e^{(-5)}-\frac{\sqrt{-x^2e^2+d^2}\,d^3}{xe^6+de^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*d^3*arcsin(x*e/d)*e^(-5) - 1/3*sqrt(-x^2*e^2 + d^2)*x^2*e^(-3) + 1/2*s
qrt(-x^2*e^2 + d^2)*d*x*e^(-4) - 5/3*sqrt(-x^2*e^2 + d^2)*d^2*e^(-5) - sqrt
(-x^2*e^2 + d^2)*d^3/(x*e^6 + d*e^5)
```

Fricas [A]
time = 2.03, size = 108, normalized size = 0.92

$$-\frac{16\,d^3xe+16\,d^4-18\,(d^3xe+d^4)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(2\,x^3e^3-dx^2e^2+7\,d^2xe+16\,d^3)\sqrt{-x^2e^2+d^2}}{6\,(xe^6+de^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^3*x*e + 16*d^4 - 18*(d^3*x*e + d^4)*arctan(-(d - sqrt(-x^2*e^2 +
 d^2))*e^(-1)/x) + (2*x^3*e^3 - d*x^2*e^2 + 7*d^2*x*e + 16*d^3)*sqrt(-x^2*e
^2 + d^2))/(x*e^6 + d*e^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [A]
time = 1.50, size = 92, normalized size = 0.78

$$-\frac{3}{2}\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)+\frac{2\,d^3e^{(-5)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1}-\frac{1}{6}\,\sqrt{-x^2e^2+d^2}\,\left(10\,d^2e^{(-5)}+\left(2\,xe^{(-3)}-3\,de^{(-4)}\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -3/2*d^3*arcsin(x*e/d)*e^(-5)*sgn(d) + 2*d^3*e^(-5)/((d*e + sqrt(-x^2*e^2 +
 d^2)*e)*e^(-2)/x + 1) - 1/6*sqrt(-x^2*e^2 + d^2)*(10*d^2*e^(-5) + (2*x*e^(
-3) - 3*d*e^(-4))*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.120 $\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=91

$$\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}+\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

```
[Out] 3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2
)^(1/2)+1/2*(-3*e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 794, 223, 209}

$$\frac{3d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}+\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}}+\frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (x^2*(d - e*x))/(e^2*Sqrt[d^2 - e^2*x^2]) + ((4*d - 3*e*x)*Sqrt[d^2 - e^2*x
^2])/(2*e^4) + (3*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \int \frac{x^3(d-ex)}{(d^2-e^2x^2)^{3/2}}\,dx \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x(2d^3-3d^2ex)}{\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{(3d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^3} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{(3d^2)\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^3} \\
&= \frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 95, normalized size = 1.04

$$
\frac{\frac{e\sqrt{d^2-e^2x^2}\,(4d^2+dex-e^2x^2)}{d+ex} + 3d^2\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(4*d^2 + d*e*x - e^2*x^2))/(d + e*x) + 3*d^2*Sqrt[-
e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^5)
```

Maple [A]
time = 0.09, size = 159, normalized size = 1.75

method	result
risch	$\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^4}+\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^3\sqrt{e^2}}+\frac{d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)}$
default	$\frac{-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}}{e}+\frac{d\sqrt{-e^2x^2+d^2}}{e^4}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}}+\frac{d^2\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2)))+d/e^4*(-e^2*x^2+d^2)^(1/2)+d^2/e^3/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d^2/e^5/(x+d/e)*(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(1/2)
```

Maxima [A]
time = 0.49, size = 79, normalized size = 0.87

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-4)}-\frac{1}{2}\sqrt{-x^2e^2+d^2}\,xe^{(-3)}+\sqrt{-x^2e^2+d^2}\,de^{(-4)}+\frac{\sqrt{-x^2e^2+d^2}\,d^2}{xe^5+de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-4) - 1/2*sqrt(-x^2*e^2 + d^2)*x*e^(-3) + sqrt(-x^
2*e^2 + d^2)*d*e^(-4) + sqrt(-x^2*e^2 + d^2)*d^2/(x*e^5 + d*e^4)
```

Fricas [A]
time = 2.24, size = 98, normalized size = 1.08

$$\frac{4\,d^2xe+4\,d^3-6\,(d^2xe+d^3)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(x^2e^2-dxe-4\,d^2)\sqrt{-x^2e^2+d^2}}{2\,(xe^5+de^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*d^2*x*e + 4*d^3 - 6*(d^2*x*e + d^3)*arctan(-(d - sqrt(-x^2*e^2 + d^2
))*e^(-1)/x) - (x^2*e^2 - d*x*e - 4*d^2)*sqrt(-x^2*e^2 + d^2))/(x*e^5 + d*e
^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [A]
time = 1.05, size = 81, normalized size = 0.89

$$\frac{3}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{2\,d^2e^{(-4)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1}-\frac{1}{2}\sqrt{-x^2e^2+d^2}\,\left(xe^{(-3)}-2\,de^{(-4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-4)*sgn(d) - 2*d^2*e^(-4)/((d*e + sqrt(-x^2*e^2 +
d^2)*e)*e^(-2)/x + 1) - 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^(-3) - 2*d*e^(-4))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.121 $\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-(-e^2*x^2+d^2)^(1/2)/e^3-d*(-e^2*x^
2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1653, 12, 807, 223, 209}

$$-\frac{d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/e^3) - (d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{\int \frac{de^3x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^2}\\
&= -\frac{\sqrt{d^2-e^2x^2}}{e^3}-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 78, normalized size = 1.01

$$
\frac{(-2d-ex)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{(-e^2)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((-2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*Log[-(Sqrt[-e^2]*x)
 + Sqrt[d^2 - e^2*x^2]])/(-e^2)^(3/2)
```

Maple [A]
time = 0.10, size = 97, normalized size = 1.26

method	result	size
default	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e^{3}}-\frac{d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}-\frac{d \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{e^{4} \left(x+\frac{d}{e}\right)}$	97
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e^{3}}-\frac{d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}-\frac{d \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{e^{4} \left(x+\frac{d}{e}\right)}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^3-d/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2))-1/e^4*d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Maxima [A]
time = 0.48, size = 58, normalized size = 0.75

$$-d \arcsin\left(\frac{xe}{d}\right) e^{(-3)} - \sqrt{-x^{2} e^{2} + d^{2}}\, e^{(-3)} - \frac{\sqrt{-x^{2} e^{2} + d^{2}}\, d}{x e^{4} + d e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -d*arcsin(x*e/d)*e^(-3) - sqrt(-x^2*e^2 + d^2)*e^(-3) - sqrt(-x^2*e^2 + d^2
)*d/(x*e^4 + d*e^3)
```

Fricas [A]
time = 2.40, size = 83, normalized size = 1.08

$$-\frac{2\, d x e + 2\, d^{2} - 2\, (d x e + d^{2}) \arctan\left(-\frac{\left(d - \sqrt{-x^{2} e^{2} + d^{2}}\right) e^{(-1)}}{x}\right) + \sqrt{-x^{2} e^{2} + d^{2}}\, (x e + 2\, d)}{x e^{4} + d e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*d*x*e + 2*d^2 - 2*(d*x*e + d^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-
1)/x) + sqrt(-x^2*e^2 + d^2)*(x*e + 2*d))/(x*e^4 + d*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [A]
time = 1.17, size = 69, normalized size = 0.90

$$-d\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\sqrt{-x^2e^2+d^2}\,e^{(-3)}+\frac{2\,de^{(-3)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] -d*arcsin(x*e/d)*e^(-3)*sgn(d) - sqrt(-x^2*e^2 + d^2)*e^(-3) + 2*d*e^(-3)/(
(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.122 $\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+(-e^2*x^2+d^2)^(1/2)/e^2/(e*x+d)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {807, 223, 209}

$$\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}+\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx, x, \frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e}$$
$$= \frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Mathematica [A]
time = 0.16, size = 71, normalized size = 1.37

$$\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)} + \frac{\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(e^2*(d + e*x)) + (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqr
t[d^2 - e^2*x^2]])/e^3
```

Maple [A]
time = 0.07, size = 74, normalized size = 1.42

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}} + \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)}}{e^3\left(x+\frac{d}{e}\right)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/e^3/(x+d/e)*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Maxima [A]
time = 0.48, size = 37, normalized size = 0.71

$$\arcsin\left(\frac{xe}{d}\right)e^{(-2)} + \frac{\sqrt{-x^2e^2+d^2}}{xe^3+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x*e/d)*e^(-2) + sqrt(-x^2*e^2 + d^2)/(x*e^3 + d*e^2)
```

Fricas [A]
time = 2.77, size = 70, normalized size = 1.35

$$-\frac{2\,(xe+d)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-xe-d-\sqrt{-x^2e^2+d^2}}{xe^3+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*(x*e + d)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - x*e - d - sqrt
(-x^2*e^2 + d^2))/(x*e^3 + d*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [A]
time = 1.34, size = 49, normalized size = 0.94

$$\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{2\,e^{(-2)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e^(-2)*sgn(d) - 2*e^(-2)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/x + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.123 $\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

```
[Out] -(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{665\}$

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Maple [A]
time = 0.06, size = 46, normalized size = 1.48

method	result	size
gosper	$-\frac{-ex+d}{de\sqrt{-e^2x^2+d^2}}$	29
trager	$-\frac{\sqrt{-e^2x^2+d^2}}{de(ex+d)}$	30
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}}{e^2d\left(x+\frac{d}{e}\right)}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/e^2/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Maxima [A]
time = 0.48, size = 29, normalized size = 0.94

$$-\frac{\sqrt{-x^2e^2+d^2}}{dxe^2+d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2*e^2 + d^2)/(d*x*e^2 + d^2*e)
```

Fricas [A]
time = 2.83, size = 35, normalized size = 1.13

$$-\frac{xe+d+\sqrt{-x^2e^2+d^2}}{dxe^2+d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
[Out] -(x*e + d + sqrt(-x^2*e^2 + d^2))/(d*x*e^2 + d^2*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [A]
time = 1.45, size = 38, normalized size = 1.23

$$\frac{2\,e^{(-1)}}{d\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*e^(-1)/(d*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1))
```

Mupad [B]
time = 2.64, size = 29, normalized size = 0.94

$$-\frac{\sqrt{d^2-e^2\,x^2}}{d\,e\,\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] -(d^2 - e^2*x^2)^(1/2)/(d*e*(d + e*x))
```

3.124 $\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] -arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+(-e^2*x^2+d^2)^(1/2)/d^2/(e*x+d)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {871, 12, 272, 65, 214}

$$\frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(d^2*(d + e*x)) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}+\frac{\int \frac{de^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}+\frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d}\\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d}\\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{de^2}\\
&= \frac{\sqrt{d^2-e^2x^2}}{d^2(d+ex)}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 66, normalized size = 1.22

$$\frac{\frac{\sqrt{d^2-e^2x^2}}{d+ex}+2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]/(d + e*x) + 2*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x
^2])/d])/d^2
```

Maple [A]
time = 0.06, size = 88, normalized size = 1.63

method	result	size
default	$\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{e\, d^{2} \left(x+\frac{d}{e}\right)}-\frac{\ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d \sqrt{d^{2}}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/d/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2*e^2 + d^2)*(x*e + d)*x), x)
```

Fricas [A]
time = 2.73, size = 63, normalized size = 1.17

$$\frac{xe+(xe+d)\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+d+\sqrt{-x^{2}e^{2}+d^{2}}}{d^{2}xe+d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*e + (x*e + d)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + d + sqrt(-x^2*e^2 + d
^2))/(d^2*x*e + d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [A]
time = 0.94, size = 75, normalized size = 1.39

$$-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^2}-\frac{2}{d^2\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] -log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 - 2/(d^2
*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.125 $\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-2*(-e^2*x^2+d^2)^(1/2)/d^3/x+(-e^2*x^
2+d^2)^(1/2)/d^2/x/(e*x+d)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {871, 821, 272, 65, 214}

$$\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}-\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(d^3*x) + Sqrt[d^2 - e^2*x^2]/(d^2*x*(d + e*x)) +
(e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}-\frac{\int \frac{-2de^2+e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}-\frac{e\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}-\frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2d^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{d^2e}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{d^3x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x(d+ex)}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 77, normalized size = 0.95

$$
-\frac{\frac{(d+2ex)\sqrt{d^2-e^2x^2}}{x(d+ex)}+2e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((((d + 2*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x)) + 2*e*ArcTanh[(Sqrt[-e^2
]*x - Sqrt[d^2 - e^2*x^2])/d])/d^3)
```

Maple [A]
time = 0.07, size = 108, normalized size = 1.33

method	result	size
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{d^{3} \left(x+\frac{d}{e}\right)}-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{d^{3} x}+\frac{e \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{2} \sqrt{d^{2}}}$	108
risch	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{d^{3} \left(x+\frac{d}{e}\right)}-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{d^{3} x}+\frac{e \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{d^{2} \sqrt{d^{2}}}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^
3/x+e/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2*e^2 + d^2)*(x*e + d)*x^2), x)
```

Fricas [A]
time = 1.43, size = 88, normalized size = 1.09

$$-\frac{x^{2} e^{2}+d x e+\left(x^{2} e^{2}+d x e\right) \log \left(-\frac{d-\sqrt{-x^{2} e^{2}+d^{2}}}{x}\right)+\sqrt{-x^{2} e^{2}+d^{2}}\,(2\,x e+d)}{d^{3} x^{2} e+d^{4} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x^2*e^2 + d*x*e + (x^2*e^2 + d*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) +
sqrt(-x^2*e^2 + d^2)*(2*x*e + d))/(d^3*x^2*e + d^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(74) = 148.
time = 0.79, size = 164, normalized size = 2.02

$$\frac{e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^3}+\frac{x\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{x}+e\right)e^2}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 + 1/2*x
*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/x + e)*e^2/((d*e + sqrt(-x^2*e^2
+ d^2)*e)*d^3*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)) - 1/2*(d*e + s
qrt(-x^2*e^2 + d^2)*e)*e^(-1)/(d^3*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.126 $\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=113

$$-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}+\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}+\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

```
[Out] -3/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-3/2*(-e^2*x^2+d^2)^(1/2)/d^3/x
^2+2*e*(-e^2*x^2+d^2)^(1/2)/d^4/x+(-e^2*x^2+d^2)^(1/2)/d^2/x^2/(e*x+d)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {871, 849, 821, 272, 65, 214}

$$\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)}+\frac{2e\sqrt{d^2-e^2x^2}}{d^4x}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}-\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^4*x) +
Sqrt[d^2 - e^2*x^2]/(d^2*x^2*(d + e*x)) - (3*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2
]/d])/(2*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{\int \frac{-3de^2+2e^3x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{\int \frac{-4d^2e^3+3de^4x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4e^2}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{(3e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}}{2d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{(3e^2)\,\text{Subst}\left(\int \frac{}{x\sqrt{d^2}}\right.}{4d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx,\right.}{2d^3}$$

$$= -\frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} + \frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} - \frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e}}{d}\right.}{2d^4}$$

Mathematica [A]
time = 0.27, size = 93, normalized size = 0.82

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(-d^2+dex+4e^2x^2)}{x^2(d+ex)} + 6e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^2 + d*e*x + 4*e^2*x^2))/(x^2*(d + e*x)) + 6*e^2*A
rcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(2*d^4)
```

Maple [A]
time = 0.08, size = 183, normalized size = 1.62

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^4x^2} + \frac{e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^4\left(x+\frac{d}{e}\right)} - \frac{3e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^3\sqrt{d^2}}$

default	$\frac{e\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{d^{4} \left(x+\frac{d}{e}\right)}+\frac{-\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{2 d^{2} x^{2}}-\frac{e^{2} \ln \left(\frac{2 d^{2}+2 \sqrt{d^{2}}\, \sqrt{-e^{2} x^{2}+d^{2}}}{x}\right)}{2 d^{2} \sqrt{d^{2}}}}{d}+\frac{e\sqrt{-e^{2} x^{2}+}}{d^{4} x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e/d^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/d*(-1/2/d^2/x^2*(-e^2*
x^2+d^2)^(1/2)-1/2*e^2/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^
2)^(1/2))/x))+e*(-e^2*x^2+d^2)^(1/2)/d^4/x-e^2/d^3/(d^2)^(1/2)*ln((2*d^2+2*
(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2*e^2 + d^2)*(x*e + d)*x^3), x)
```

Fricas [A]
time = 1.45, size = 108, normalized size = 0.96

$$\frac{2\,x^3e^3+2\,dx^2e^2+3\,(x^3e^3+dx^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(4\,x^2e^2+dxe-d^2)\sqrt{-x^2e^2+d^2}}{2\,(d^4x^3e+d^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^3*e^3 + 2*d*x^2*e^2 + 3*(x^3*e^3 + d*x^2*e^2)*log(-(d - sqrt(-x^2*
e^2 + d^2))/x) + (4*x^2*e^2 + d*x*e - d^2)*sqrt(-x^2*e^2 + d^2))/(d^4*x^3*e
 + d^5*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(98) = 196.
time = 1.26, size = 233, normalized size = 2.06

$$-\frac{3\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^4}-\frac{x^2\left(\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-2)}}{x^2}+\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}{x}-e^2\right)e^4}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^4\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}-\frac{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^4e^{(-2)}}{x^2}-\frac{4\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4}{x}}{8\,d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] -3/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4 - 1/8*x^2*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-2)/x^2 + 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)/x - e^2)*e^4/((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)) - 1/8*((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^(-2)/x^2 - 4*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4/x)/d^8

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)

[Out] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)

3.127 $\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{x^4(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}}-\frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}-\frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6}-\frac{5d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}$$

```
[Out] 1/3*x^4*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)-5/2*d^2*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))/e^6-1/3*x^2*(-5*e*x+4*d)/e^4/(-e^2*x^2+d^2)^(1/2)-1/6*(-15*e*x+16*d
)*(-e^2*x^2+d^2)^(1/2)/e^6
```

Rubi [A]
time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 833, 794, 223, 209}

$$-\frac{5d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{x^4(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}}-\frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6}-\frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^4*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x^2*(4*d - 5*e*x))/(3*e^4*
Sqrt[d^2 - e^2*x^2]) - ((16*d - 15*e*x)*Sqrt[d^2 - e^2*x^2])/(6*e^6) - (5*d
^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^5(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x^3\left(4d^3-5d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4}$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{(5d^2)}{}$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{(5d^2)}{}$$

$$= \frac{x^4(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2\,\mathrm{ta}}{}$$

Mathematica [A]

time = 0.36, size = 129, normalized size = 1.01

$$\frac{\sqrt{d^2-e^2x^2}\,(16d^4+d^3ex-23d^2e^2x^2-3de^3x^3+3e^4x^4)}{6e^6(-d+ex)(d+ex)^2}+\frac{5d^2(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^4 + d^3*e*x - 23*d^2*e^2*x^2 - 3*d*e^3*x^3 + 3*e
^4*x^4))/(6*e^6*(-d + e*x)*(d + e*x)^2) + (5*d^2*(-e^2)^(3/2)*Log[-(Sqrt[-e
^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(112) = 224.
time = 0.09, size = 350, normalized size = 2.73

method	result
risch	$-\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^6}-\frac{5d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}}-\frac{25d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12e^7\left(x+\frac{d}{e}\right)}-\frac{d^2\sqrt{}}{}$
default	$\frac{-\frac{x^3}{2e^2\sqrt{-e^2x^2+d^2}}+\frac{3d^2\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)}{2e^2}}{e}-\frac{d\left(-\frac{x^2}{e^2\sqrt{-e^2x^2+d^2}}+\frac{2d^2}{e^4\sqrt{-e^2x}}\right.}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/2*x^3/e^2/(-e^2*x^2+d^2)^(1/2)+3/2*d^2/e^2*(x/e^2/(-e^2*x^2+d^2)^(1
/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-d/e^2*(-
x^2/e^2/(-e^2*x^2+d^2)^(1/2)+2*d^2/e^4/(-e^2*x^2+d^2)^(1/2))+d^2/e^3*(x/e^2
/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2)))-d^3/e^6/(-e^2*x^2+d^2)^(1/2)+d^2/e^5*x/(-e^2*x^2+d^2)^(1/2)-d^5/e^
6*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*
(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.48, size = 139, normalized size = 1.09

$$-\frac{5}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-6)}-\frac{x^3e^{(-3)}}{2\sqrt{-x^2e^2+d^2}}+\frac{dx^2e^{(-4)}}{\sqrt{-x^2e^2+d^2}}+\frac{17\,d^2xe^{(-5)}}{6\sqrt{-x^2e^2+d^2}}-\frac{3\,d^3e^{(-6)}}{\sqrt{-x^2e^2+d^2}}+\frac{d^4}{3\left(\sqrt{-x^2e^2+d^2}\,xe^7+\sqrt{-x^2e^2+d^2}\,de^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -5/2*d^2*arcsin(x*e/d)*e^(-6) - 1/2*x^3*e^(-3)/sqrt(-x^2*e^2 + d^2) + d*x^2
*e^(-4)/sqrt(-x^2*e^2 + d^2) + 17/6*d^2*x*e^(-5)/sqrt(-x^2*e^2 + d^2) - 3*d
^3*e^(-6)/sqrt(-x^2*e^2 + d^2) + 1/3*d^4/(sqrt(-x^2*e^2 + d^2)*x*e^7 + sqrt
(-x^2*e^2 + d^2)*d*e^6)
```

Fricas [**A**]
time = 1.99, size = 179, normalized size = 1.40

$$-\frac{16\,d^2x^3e^3+16\,d^3x^2e^2-16\,d^4xe-16\,d^5-30\,(d^2x^3e^3+d^3x^2e^2-d^4xe-d^5)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(3\,x^4e^4-3\,dx^3e^3-23\,d^2x^2e^2+d^3xe+16\,d^4)\sqrt{-x^2e^2+d^2}}{6\,(x^3e^9+dx^2e^8-d^2xe^7-d^3e^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(16*d^2*x^3*e^3 + 16*d^3*x^2*e^2 - 16*d^4*x*e - 16*d^5 - 30*(d^2*x^3*e
^3 + d^3*x^2*e^2 - d^4*x*e - d^5)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)
/x) - (3*x^4*e^4 - 3*d*x^3*e^3 - 23*d^2*x^2*e^2 + d^3*x*e + 16*d^4)*sqrt(-x
^2*e^2 + d^2))/(x^3*e^9 + d*x^2*e^8 - d^2*x*e^7 - d^3*e^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.128 $\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] 1/3*x^3*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)+d*arctan(e*x/(-e^2*x^2+d^2)^(1/2)
)/e^5-1/3*x*(-4*e*x+3*d)/e^4/(-e^2*x^2+d^2)^(1/2)+8/3*(-e^2*x^2+d^2)^(1/2)/
e^5
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 833, 655, 223, 209}

$$\frac{d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\sqrt{d^2-e^2x^2}}{3e^5}-\frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^3*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x*(3*d - 4*e*x))/(3*e^4*Sq
rt[d^2 - e^2*x^2]) + (8*Sqrt[d^2 - e^2*x^2])/(3*e^5) + (d*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/e^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{x^2\left(3d^3-4d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{3d^5-8d^4ex}{\sqrt{d^2-e^2x^2}}\,dx}{3d^4e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\int \frac{1}{\sqrt{d^2-e^2x^2}}\,d}{e^4}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,d\right.}{}$$

$$= \frac{x^3(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e}}\right.}{e^5}$$

Mathematica [A]
time = 0.32, size = 114, normalized size = 1.01

$$\frac{\sqrt{d^2-e^2x^2}\left(-8d^3-5d^2ex+7de^2x^2+3e^3x^3\right)}{3e^5(-d+ex)(d+ex)^2} + \frac{d\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^3 - 5*d^2*e*x + 7*d*e^2*x^2 + 3*e^3*x^3))/(3*e^5
*(-d + e*x)*(d + e*x)^2) + (d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e
^2*x^2]])/e^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(99) = 198$.
time = 0.08, size = 257, normalized size = 2.27

method	result
risch	$\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{e^{5}}+\frac{d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{4}\sqrt{e^{2}}}+\frac{19d\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2de\left(x+\frac{d}{e}\right)}}{12e^{6}\left(x+\frac{d}{e}\right)}-\frac{d\sqrt{-\left(x-\frac{d}{e}\right)^{2} e \cdots}}{4e^{6}\left(\cdots\right.}$
default	$\frac{-\frac{x^{2}}{e^{2}\sqrt{-e^{2}x^{2}+d^{2}}}+\frac{2d^{2}}{e^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{e}-\frac{d\left(\frac{x}{e^{2}\sqrt{-e^{2}x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e^{2}\sqrt{e^{2}}}\right)}{e^{2}}+\frac{d^{2}}{e^{5}\sqrt{-e^{2}x^{2}+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-x^2/e^2/(-e^2*x^2+d^2)^(1/2)+2*d^2/e^4/(-e^2*x^2+d^2)^(1/2))-d/e^2*(x
/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2)))+d^2/e^5/(-e^2*x^2+d^2)^(1/2)-d/e^4*x/(-e^2*x^2+d^2)^(1/2)+1/e^
5*d^4*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*
e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.49, size = 114, normalized size = 1.01

$$d \arcsin\left(\frac{xe}{d}\right) e^{(-5)} - \frac{x^{2}e^{(-3)}}{\sqrt{-x^{2}e^{2}+d^{2}}} - \frac{4\, dxe^{(-4)}}{3\sqrt{-x^{2}e^{2}+d^{2}}} + \frac{3\, d^{2}e^{(-5)}}{\sqrt{-x^{2}e^{2}+d^{2}}} - \frac{d^{3}}{3\left(\sqrt{-x^{2}e^{2}+d^{2}}\, xe^{6} + \sqrt{-x^{2}e^{2}+d^{2}}\, de^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] d*arcsin(x*e/d)*e^(-5) - x^2*e^(-3)/sqrt(-x^2*e^2 + d^2) - 4/3*d*x*e^(-4)/s
qrt(-x^2*e^2 + d^2) + 3*d^2*e^(-5)/sqrt(-x^2*e^2 + d^2) - 1/3*d^3/(sqrt(-x^
2*e^2 + d^2)*x*e^6 + sqrt(-x^2*e^2 + d^2)*d*e^5)
```

Fricas [A]
time = 2.33, size = 165, normalized size = 1.46

$$\frac{8\, dx^{3}e^{3} + 8\, d^{2}x^{2}e^{2} - 8\, d^{3}xe - 8\, d^{4} - 6\left(dx^{3}e^{3} + d^{2}x^{2}e^{2} - d^{3}xe - d^{4}\right) \arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right) + \left(3\, x^{3}e^{3} + 7\, dx^{2}e^{2} - 5\, d^{2}xe - 8\, d^{3}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{3\left(x^{3}e^{8} + dx^{2}e^{7} - d^{2}xe^{6} - d^{3}e^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(8*d*x^3*e^3 + 8*d^2*x^2*e^2 - 8*d^3*x*e - 8*d^4 - 6*(d*x^3*e^3 + d^2*x
^2*e^2 - d^3*x*e - d^4)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (3*x
^3*e^3 + 7*d*x^2*e^2 - 5*d^2*x*e - 8*d^3)*sqrt(-x^2*e^2 + d^2))/(x^3*e^8 +
d*x^2*e^7 - d^2*x*e^6 - d^3*e^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.129 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/3*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/
e^4+1/3*(3*e*x-2*d)/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {864, 833, 792, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}+\frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (x^2*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (2*d - 3*e*x)/(3*e^4*Sqrt[d
^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx\\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d^2e^2}\\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^3}\\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{e^3}\\
&= \frac{x^2(d-ex)}{3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 102, normalized size = 1.15

$$\frac{\left(2d^2-dex-4e^2x^2\right)\sqrt{d^2-e^2x^2}}{3e^4(-d+ex)(d+ex)^2}+\frac{\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^3\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((2*d^2 - d*e*x - 4*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*e^4*(-d + e*x)*(d + e*
x)^2) + Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]/(e^3*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(79) = 158$.
time = 0.07, size = 204, normalized size = 2.29

method	result
default	$\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e}-\frac{d}{e^4\sqrt{-e^2x^2+d^2}}+\frac{x}{e^3\sqrt{-e^2x^2+d^2}}-\frac{d^3\left(-\frac{}{3de\left(x+\frac{d}{e}\right)\sqrt{-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^
2*x^2+d^2)^(1/2)))-d/e^4/(-e^2*x^2+d^2)^(1/2)+1/e^3*x/(-e^2*x^2+d^2)^(1/2)-
d^3/e^4*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-
2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.50, size = 91, normalized size = 1.02

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-4)}+\frac{4\,xe^{(-3)}}{3\sqrt{-x^2e^2+d^2}}-\frac{de^{(-4)}}{\sqrt{-x^2e^2+d^2}}+\frac{d^2}{3\left(\sqrt{-x^2e^2+d^2}\,xe^5+\sqrt{-x^2e^2+d^2}\,de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -arcsin(x*e/d)*e^(-4) + 4/3*x*e^(-3)/sqrt(-x^2*e^2 + d^2) - d*e^(-4)/sqrt(-
x^2*e^2 + d^2) + 1/3*d^2/(sqrt(-x^2*e^2 + d^2)*x*e^5 + sqrt(-x^2*e^2 + d^2)
*d*e^4)
```

Fricas [A]
time = 1.63, size = 148, normalized size = 1.66

$$-\frac{2\,x^3e^3+2\,dx^2e^2-2\,d^2xe-2\,d^3-6\,(x^3e^3+dx^2e^2-d^2xe-d^3)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(4\,x^2e^2+dxe-2\,d^2)\sqrt{-x^2e^2+d^2}}{3\,(x^3e^7+dx^2e^6-d^2xe^5-d^3e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3*e^3 + 2*d*x^2*e^2 - 2*d^2*x*e - 2*d^3 - 6*(x^3*e^3 + d*x^2*e^2
- d^2*x*e - d^3)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (4*x^2*e^2
+ d*x*e - 2*d^2)*sqrt(-x^2*e^2 + d^2))/(x^3*e^7 + d*x^2*e^6 - d^2*x*e^5 - d
^3*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.130 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/3/e^3/(-e^2*x^2+d^2)^(1/2)-1/3*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {869, 12, 267}

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}}-\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] 2/(3*e^3*Sqrt[d^2 - e^2*x^2]) - x^2/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 869

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^n*((a + c*x^2)^(p + 1)/(2*a*e*p*(d + e*x
))), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\int \frac{2dx}{(d^2-e^2x^2)^{3/2}}\,dx}{3de}$$
$$= -\frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}} + \frac{2\int \frac{x}{(d^2-e^2x^2)^{3/2}}\,dx}{3e}$$
$$= \frac{2}{3e^3\sqrt{d^2-e^2x^2}} - \frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.25, size = 60, normalized size = 1.00

$$\frac{\sqrt{d^2-e^2x^2}\,(2d^2+2dex-e^2x^2)}{3de^3(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 + 2*d*e*x - e^2*x^2))/(3*d*e^3*(d - e*x)*(d + e
*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(52) = 104$.
time = 0.07, size = 149, normalized size = 2.48

method	result
gosper	$\frac{(-ex+d)(-e^2x^2+2dex+2d^2)}{3d\,e^3(-e^2x^2+d^2)^{\frac{3}{2}}}$
trager	$\frac{(-e^2x^2+2dex+2d^2)\sqrt{-e^2x^2+d^2}}{3e^3d(ex+d)^2(-ex+d)}$
default	$\frac{1}{e^3\sqrt{-e^2x^2+d^2}} - \frac{x}{d\,e^2\sqrt{-e^2x^2+d^2}} + \frac{d^2\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}} - \frac{}{3e\,d^3\sqrt{-(x}}\right.}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/e^3/(-e^2*x^2+d^2)^(1/2)-1/d/e^2*x/(-e^2*x^2+d^2)^(1/2)+1/e^3*d^2*(-1/3/d
/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2
*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.27, size = 78, normalized size = 1.30

$$-\frac{xe^{(-2)}}{3\sqrt{-x^2e^2+d^2}\,d}+\frac{e^{(-3)}}{\sqrt{-x^2e^2+d^2}}-\frac{d}{3\left(\sqrt{-x^2e^2+d^2}\,xe^4+\sqrt{-x^2e^2+d^2}\,de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d) + e^(-3)/sqrt(-x^2*e^2 + d^2) - 1/3*
d/(sqrt(-x^2*e^2 + d^2)*x*e^4 + sqrt(-x^2*e^2 + d^2)*d*e^3)
```

Fricas [A]
time = 2.36, size = 97, normalized size = 1.62

$$\frac{2\,x^3e^3+2\,dx^2e^2-2\,d^2xe-2\,d^3+(x^2e^2-2\,dxe-2\,d^2)\sqrt{-x^2e^2+d^2}}{3\,(dx^3e^6+d^2x^2e^5-d^3xe^4-d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x^3*e^3 + 2*d*x^2*e^2 - 2*d^2*x*e - 2*d^3 + (x^2*e^2 - 2*d*x*e - 2*d
^2)*sqrt(-x^2*e^2 + d^2))/(d*x^3*e^6 + d^2*x^2*e^5 - d^3*x*e^4 - d^4*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.71, size = 56, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+2\,d\,e\,x-e^2\,x^2\right)}{3\,d\,e^3\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^2 - e^2*x^2 + 2*d*e*x))/(3*d*e^3*(d + e*x)^2*(d
 - e*x))
```

3.131 $\int \frac{x}{(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/3*x/d^2/e/(-e^2*x^2+d^2)^(1/2)+1/3/e^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {807, 197}

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
[Out] x/(3*d^2*e*Sqrt[d^2 - e^2*x^2]) + 1/(3*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3e}\\ &= \frac{x}{3d^2e\sqrt{d^2-e^2x^2}}+\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\,(d^2+dex+e^2x^2)}{3d^2e^2(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(d^2 + d*e*x + e^2*x^2))/(3*d^2*e^2*(d - e*x)*(d + e*x
)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(50) = 100.
time = 0.07, size = 129, normalized size = 2.22

method	result
gosper	$\frac{(-ex+d)(e^2x^2+dex+d^2)}{3d^2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}$
trager	$\frac{(e^2x^2+dex+d^2)\sqrt{-e^2x^2+d^2}}{3e^2d^2(ex+d)^2(-ex+d)}$
default	$\frac{x}{d^2e\sqrt{-e^2x^2+d^2}}-\frac{d\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}\right.}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/d^2/e/(-e^2*x^2+d^2)^(1/2)-d/e^2*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2))
```

Maxima [A]
time = 0.27, size = 61, normalized size = 1.05

$$\frac{xe^{(-1)}}{3\sqrt{-x^2e^2+d^2}\,d^2}+\frac{1}{3\left(\sqrt{-x^2e^2+d^2}\,xe^3+\sqrt{-x^2e^2+d^2}\,de^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*e^(-1)/(sqrt(-x^2*e^2 + d^2)*d^2) + 1/3/(sqrt(-x^2*e^2 + d^2)*x*e^3 +
 sqrt(-x^2*e^2 + d^2)*d*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(47) = 94$.
time = 1.57, size = 95, normalized size = 1.64

$$\frac{x^3e^3 + dx^2e^2 - d^2xe - d^3 - (x^2e^2 + dxe + d^2)\sqrt{-x^2e^2 + d^2}}{3\left(d^2x^3e^5 + d^3x^2e^4 - d^4xe^3 - d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3*e^3 + d*x^2*e^2 - d^2*x*e - d^3 - (x^2*e^2 + d*x*e + d^2)*sqrt(-x^
2*e^2 + d^2))/(d^2*x^3*e^5 + d^3*x^2*e^4 - d^4*x*e^3 - d^5*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{3}{2}}\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.71, size = 52, normalized size = 0.90

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(d^2 + d\,e\,x + e^2\,x^2\right)}{3\,d^2\,e^2\,{\left(d + e\,x\right)}^2\,\left(d - e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + d*e*x))/(3*d^2*e^2*(d + e*x)^2*(d -
 e*x))
```

3.132 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/3*x/d^3/(-e^2*x^2+d^2)^(1/2)-1/3/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {673, 197}

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{3d}\\ &= \frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 60, normalized size = 1.03

$$\frac{\sqrt{d^2-e^2x^2}\,(-d^2+2dex+2e^2x^2)}{3d^3e(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 + 2*d*e*x + 2*e^2*x^2))/(3*d^3*e*(d - e*x)*(d +
e*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(50) = 100.
time = 0.06, size = 104, normalized size = 1.79

method	result	size
gosper	$-\frac{(-ex+d)(-2e^2x^2-2dex+d^2)}{3d^3e(-e^2x^2+d^2)^{\frac{3}{2}}}$	46
trager	$-\frac{(-2e^2x^2-2dex+d^2)\sqrt{-e^2x^2+d^2}}{3d^3(ex+d)^2e(-ex+d)}$	55
default	$\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}}{e}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^
2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.27, size = 62, normalized size = 1.07

$$-\frac{1}{3\left(\sqrt{-x^2e^2+d^2}\,dxe^2+\sqrt{-x^2e^2+d^2}\,d^2e\right)}+\frac{2\,x}{3\,\sqrt{-x^2e^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3/(sqrt(-x^2*e^2 + d^2)*d*x*e^2 + sqrt(-x^2*e^2 + d^2)*d^2*e) + 2/3*x/(s
qrt(-x^2*e^2 + d^2)*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(48) = 96$.
time = 4.10, size = 98, normalized size = 1.69

$$-\frac{x^3e^3 + dx^2e^2 - d^2xe - d^3 + (2\,x^2e^2 + 2\,dxe - d^2)\sqrt{-x^2e^2 + d^2}}{3\,(d^3x^3e^4 + d^4x^2e^3 - d^5xe^2 - d^6e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^3*e^3 + d*x^2*e^2 - d^2*x*e - d^3 + (2*x^2*e^2 + 2*d*x*e - d^2)*sqr
t(-x^2*e^2 + d^2))/(d^3*x^3*e^4 + d^4*x^2*e^3 - d^5*x*e^2 - d^6*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{3}{2}}\left(d + ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.71, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2 - e^2\,x^2}\,\left(-d^2 + 2\,d\,e\,x + 2\,e^2\,x^2\right)}{3\,d^3\,e\,{\left(d + e\,x\right)}^2\,\left(d - e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*e^2*x^2 - d^2 + 2*d*e*x))/(3*d^3*e*(d + e*x)^2*(d
 - e*x))
```

3.133 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] -arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/3*(-2*e*x+3*d)/d^4/(-e^2*x^2+d^2)^(1
/2)+1/3/d^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {871, 837, 12, 272, 65, 214}

$$\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (3*d - 2*e*x)/(3*d^4*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*(d + e*x)*Sqrt[d^2 - e
^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)(d^2-e^2x^2)^{3/2}}\,dx = \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-3de^2+2e^3x}{x(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{3d^3e^4}{x\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^3}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x,\right.}{2d^3}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-}\right.}{d^3e^2}$$

$$= \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Mathematica [A]
time = 0.35, size = 95, normalized size = 1.08

$$\frac{\frac{(4d^2+dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{(d-ex)(d+ex)^2} + 6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (((4*d^2 + d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/((d - e*x)*(d + e*x)^2)
+ 6*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(3*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.
time = 0.06, size = 171, normalized size = 1.94

method	result
default	$-\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}}{d}+\frac{1}{d^2\sqrt{-e^2x^2+d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e
^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/d*(1/d^2/(-e^2*x^
2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2)
)/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)*x), x)
```

Fricas [A]
time = 4.03, size = 150, normalized size = 1.70

$$\frac{4\,x^3e^3+4\,dx^2e^2-4\,d^2xe-4\,d^3+3\,(x^3e^3+dx^2e^2-d^2xe-d^3)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(2\,x^2e^2-dxe-4\,d^2)\sqrt{-x^2e^2+d^2}}{3\,(d^4x^3e^3+d^5x^2e^2-d^6xe-d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*x^3*e^3 + 4*d*x^2*e^2 - 4*d^2*x*e - 4*d^3 + 3*(x^3*e^3 + d*x^2*e^2 -
 d^2*x*e - d^3)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (2*x^2*e^2 - d*x*e - 4
*d^2)*sqrt(-x^2*e^2 + d^2))/(d^4*x^3*e^3 + d^5*x^2*e^2 - d^6*x*e - d^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.134 $\int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{3d^5x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/3*(-3*e*x+4*d)/d^4/x/(-e^2*x^2+d^2)
^(1/2)+1/3/d^2/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2)-8/3*(-e^2*x^2+d^2)^(1/2)/d^5/
x
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {871, 837, 821, 272, 65, 214}

$$\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{3d^5x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*d - 3*e*x)/(3*d^4*x*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x*(d + e*x)*Sqrt[d^2
 - e^2*x^2]) - (8*Sqrt[d^2 - e^2*x^2])/(3*d^5*x) + (e*ArcTanh[Sqrt[d^2 - e^
2*x^2]/d])/d^5
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-4de^2+3e^3x}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-8d^3e^4+3d^2e^5x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\int \frac{}{x\sqrt{}}}{}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} - \frac{e\text{Subs}}{}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{\text{Subst}}{}$$

$$= \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e\tanh}{}$$

Mathematica [A]
time = 0.33, size = 112, normalized size = 0.93

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(3d^3+7d^2ex-5de^2x^2-8e^3x^3)}{x(-d+ex)(d+ex)^2} - 6e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^3 + 7*d^2*e*x - 5*d*e^2*x^2 - 8*e^3*x^3))/(x*(-d
 + e*x)*(d + e*x)^2) - 6*e*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])
/(3*d^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(106) = 212$.
time = 0.08, size = 223, normalized size = 1.86

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x} - \frac{17\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12d^5\left(x+\frac{d}{e}\right)} - \frac{\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)e}}{4d^5\left(x-\frac{d}{e}\right)} + \frac{e\ln\Big(}{}$

default	$\frac{e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^{2}\left(x+\frac{d}{e}\right)+2de}{3e\,d^{3}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}\right)}{d^{2}}+\frac{-\frac{1}{d^{2}x\sqrt{-e^{2}x^{2}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e/d^2*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*
e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/d*(-1/d^2/x/(-e^
2*x^2+d^2)^(1/2)+2*e^2/d^4*x/(-e^2*x^2+d^2)^(1/2))-e/d^2*(1/d^2/(-e^2*x^2+d
^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)*x^2), x)
```

Fricas [A]
time = 2.44, size = 173, normalized size = 1.44

$$-\frac{4\,x^{4}e^{4}+4\,dx^{3}e^{3}-4\,d^{2}x^{2}e^{2}-4\,d^{3}xe+3\,(x^{4}e^{4}+dx^{3}e^{3}-d^{2}x^{2}e^{2}-d^{3}xe)\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+(8\,x^{3}e^{3}+5\,dx^{2}e^{2}-7\,d^{2}xe-3\,d^{3})\sqrt{-x^{2}e^{2}+d^{2}}}{3\,(d^{5}x^{4}e^{3}+d^{6}x^{3}e^{2}-d^{7}x^{2}e-d^{8}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^4*e^4 + 4*d*x^3*e^3 - 4*d^2*x^2*e^2 - 4*d^3*x*e + 3*(x^4*e^4 + d*
x^3*e^3 - d^2*x^2*e^2 - d^3*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (8*x^
3*e^3 + 5*d*x^2*e^2 - 7*d^2*x*e - 3*d^3)*sqrt(-x^2*e^2 + d^2))/(d^5*x^4*e^3
 + d^6*x^3*e^2 - d^7*x^2*e - d^8*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{2}\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.135 $\int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{3/2}} dx$

Optimal. Leaf size=152

$$\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}+\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}$$

```
[Out] -5/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/3*(-4*e*x+5*d)/d^4/x^2/(-e^2
*x^2+d^2)^(1/2)+1/3/d^2/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)-5/2*(-e^2*x^2+d^2)
^(1/2)/d^5/x^2+8/3*e*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {871, 837, 849, 821, 272, 65, 214}

$$\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{5e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (5*d - 4*e*x)/(3*d^4*x^2*Sqrt[d^2 - e^2*x^2]) + 1/(3*d^2*x^2*(d + e*x)*Sqrt
[d^2 - e^2*x^2]) - (5*Sqrt[d^2 - e^2*x^2])/(2*d^5*x^2) + (8*e*Sqrt[d^2 - e^
2*x^2])/(3*d^6*x) - (5*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-5de^2+4e^3x}{x^3(d^2-e^2x^2)^{3/2}}\,dx}{3d^2e^2}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^6e^4}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{\int \frac{-16\ldots}{x^2\sqrt{\ldots}}}{\ldots}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d\ldots}}{3\ldots}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d\ldots}}{3\ldots}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d\ldots}}{3\ldots}$$

$$= \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} + \frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{8e\sqrt{d\ldots}}{3\ldots}$$

Mathematica [A]
time = 0.35, size = 124, normalized size = 0.82

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(3d^4-3d^3ex-23d^2e^2x^2+de^3x^3+16e^4x^4)}{x^2(-d+ex)(d+ex)^2} + 30e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{6d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x - 23*d^2*e^2*x^2 + d*e^3*x^3 + 16*
e^4*x^4))/(x^2*(-d + e*x)*(d + e*x)^2) + 30*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqr
t[d^2 - e^2*x^2])/d])/(6*d^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(132) = 264.
time = 0.09, size = 325, normalized size = 2.14

method	result

risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^6x^2}+\frac{23e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{12d^6\left(x+\frac{d}{e}\right)}-\frac{e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)\cdots}}{4d^6\left(x-\frac{d}{e}\right)}$
default	$-\frac{e^2\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{d^3}+\frac{-\frac{\cdots}{2d^2x^2\sqrt{-\cdots}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/d^3*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(
-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/d*(-1/2/d^2/x
^2/(-e^2*x^2+d^2)^(1/2)+3/2*e^2/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)
^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))-e/d^2*(-1/d^2/x/(
-e^2*x^2+d^2)^(1/2)+2*e^2/d^4*x/(-e^2*x^2+d^2)^(1/2))+e^2/d^3*(1/d^2/(-e^2*
x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/
2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)*x^3), x)
```

Fricas [A]
time = 2.63, size = 188, normalized size = 1.24

$$\frac{14\,x^5e^5+14\,dx^4e^4-14\,d^2x^3e^3-14\,d^3x^2e^2+15\,(x^5e^5+dx^4e^4-d^2x^3e^3-d^3x^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(16\,x^4e^4+dx^3e^3-23\,d^2x^2e^2-3\,d^3xe+3\,d^4)\sqrt{-x^2e^2+d^2}}{6\,(d^6x^5e^3+d^7x^4e^2-d^8x^3e-d^9x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(14*x^5*e^5 + 14*d*x^4*e^4 - 14*d^2*x^3*e^3 - 14*d^3*x^2*e^2 + 15*(x^5*
e^5 + d*x^4*e^4 - d^2*x^3*e^3 - d^3*x^2*e^2)*log(-(d - sqrt(-x^2*e^2 + d^2)
)/x) + (16*x^4*e^4 + d*x^3*e^3 - 23*d^2*x^2*e^2 - 3*d^3*x*e + 3*d^4)*sqrt(-
x^2*e^2 + d^2))/(d^6*x^5*e^3 + d^7*x^4*e^2 - d^8*x^3*e - d^9*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}} \left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(d^2-e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)), x)
```

3.136 $\int \frac{x^7}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$

Optimal. Leaf size=162

$$\frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{7d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-\ldots}}\right)}{2e^8}$$

```
[Out] 1/5*x^6*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^4*(-7*e*x+6*d)/e^4/(-e^2*x
^2+d^2)^(3/2)+7/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^8+1/15*x^2*(-35*e*
x+24*d)/e^6/(-e^2*x^2+d^2)^(1/2)+1/10*(-35*e*x+32*d)*(-e^2*x^2+d^2)^(1/2)/e
^8
```

Rubi [A]
time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 833, 794, 223, 209}

$$\frac{7d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}+\frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}}+\frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8}+\frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^6*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^4*(6*d - 7*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x^2*(24*d - 35*e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2]
) + ((32*d - 35*e*x)*Sqrt[d^2 - e^2*x^2])/(10*e^8) + (7*d^2*ArcTan[(e*x)/Sq
rt[d^2 - e^2*x^2]])/(2*e^8)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^7}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^7(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^5\left(6d^3-7d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^3\left(24d^5-35d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{x(48d^7-105\ldots}{\sqrt{d^2-e\ldots}}}{15d^6e\ldots}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35e\ldots}{1\ldots}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35e\ldots}{1\ldots}$$

$$= \frac{x^6(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^4(6d-7ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{(32d-35e\ldots}{1\ldots}$$

Mathematica [A]
time = 0.52, size = 148, normalized size = 0.91

$$\frac{\frac{e\sqrt{d^2-e^2x^2}\left(96d^6-9d^5ex-249d^4e^2x^2-4d^3e^3x^3+176d^2e^4x^4+15de^5x^5-15e^6x^6\right)}{(d-ex)^2(d+ex)^3}+105d^2\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{30e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(96*d^6 - 9*d^5*e*x - 249*d^4*e^2*x^2 - 4*d^3*e^3*x
^3 + 176*d^2*e^4*x^4 + 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^2*(d + e*x)^3
) + 105*d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(30*e^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(142) = 284$.
time = 0.09, size = 651, normalized size = 4.02

method	result

risch	$\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{2e^8}+\frac{7d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^7\sqrt{e^2}}+\frac{773d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{240e^9\left(x+\frac{d}{e}\right)}+\frac{31d^2\sqrt{}}{}$
default	$\frac{-\frac{x^5}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{5d^2\left(\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}\right)}{2e^2}}{e}-\frac{d\left(-\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4d^2}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/2*x^5/e^2/(-e^2*x^2+d^2)^(3/2)+5/2*d^2/e^2*(1/3*x^3/e^2/(-e^2*x^2+d
^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^
(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))-d/e^2*(-x^4/e^2/(-e^2*x^2+d^2)^(3/2)+4*d^2
/e^2*(x^2/e^2/(-e^2*x^2+d^2)^(3/2)-2/3*d^2/e^4/(-e^2*x^2+d^2)^(3/2)))+d^2/e
^3*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^
2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-d^3/e^4*(x^2/e^2
/(-e^2*x^2+d^2)^(3/2)-2/3*d^2/e^4/(-e^2*x^2+d^2)^(3/2))+d^4/e^5*(1/2*x/e^2/
(-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/
(-e^2*x^2+d^2)^(1/2)))-1/3*d^5/e^8/(-e^2*x^2+d^2)^(3/2)+d^6/e^7*(1/3*x/d^2/
(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))-d^7/e^8*(-1/5/d/e/(x+d
/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*e^2*(x+d/e)+2*d*
e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)
+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]

time = 0.50, size = 265, normalized size = 1.64

$$\frac{d^6}{5\left(\left(-x^2e^2+d^2\right)^{\frac{3}{2}}xe^9+\left(-x^2e^2+d^2\right)^{\frac{3}{2}}de^8\right)}-\frac{x^5e^{(-3)}}{2\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}+\frac{dx^4e^{(-4)}}{\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}+\frac{25\,d^2x^3e^{(-5)}}{2\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}-\frac{65\,d^3x^2e^{(-6)}}{6\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}-\frac{164\,d^4xe^{(-7)}}{15\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}+\frac{53\,d^5e^{(-8)}}{6\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}+\frac{7}{2}d^2\arcsin\left(\frac{xe}{d}\right)e^{(-8)}-\frac{7\,dx^2e^{(-6)}}{6\sqrt{-x^2e^2+d^2}}+\frac{229\,d^2xe^{(-7)}}{30\sqrt{-x^2e^2+d^2}}-\frac{14\,d^3e^{(-8)}}{3\sqrt{-x^2e^2+d^2}}-\frac{7}{6}\sqrt{-x^2e^2+d^2}\,de^{(-8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^6/((-x^2*e^2 + d^2)^(3/2)*x*e^9 + (-x^2*e^2 + d^2)^(3/2)*d*e^8) - 1/2
*x^5*e^(-3)/(-x^2*e^2 + d^2)^(3/2) + d*x^4*e^(-4)/(-x^2*e^2 + d^2)^(3/2) +
25/2*d^2*x^3*e^(-5)/(-x^2*e^2 + d^2)^(3/2) - 65/6*d^3*x^2*e^(-6)/(-x^2*e^2
+ d^2)^(3/2) - 164/15*d^4*x*e^(-7)/(-x^2*e^2 + d^2)^(3/2) + 53/6*d^5*e^(-8)
/(-x^2*e^2 + d^2)^(3/2) + 7/2*d^2*arcsin(x*e/d)*e^(-8) - 7/6*d*x^2*e^(-6)/s
qrt(-x^2*e^2 + d^2) + 229/30*d^2*x*e^(-7)/sqrt(-x^2*e^2 + d^2) - 14/3*d^3*e
^(-8)/sqrt(-x^2*e^2 + d^2) - 7/6*sqrt(-x^2*e^2 + d^2)*d*e^(-8)
```

Fricas [A]
time = 3.34, size = 255, normalized size = 1.57

$$\frac{96\,d^2x^5e^5+96\,d^3x^4e^4-192\,d^4x^3e^3-192\,d^5x^2e^2+96\,d^6xe+96\,d^7-210\,(d^2x^5e^5+d^3x^4e^4-2\,d^4x^3e^3-2\,d^5x^2e^2+d^6xe+d^7)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(15\,x^6e^6-15\,dx^5e^5-176\,d^2x^4e^4+4\,d^3x^3e^3+249\,d^4x^2e^2+9\,d^5xe-96\,d^6)\sqrt{-x^2e^2+d^2}}{30\,(x^5e^{13}+dx^4e^{12}-2\,d^2x^3e^{11}-2\,d^3x^2e^{10}+d^4xe^9+d^5e^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(96*d^2*x^5*e^5 + 96*d^3*x^4*e^4 - 192*d^4*x^3*e^3 - 192*d^5*x^2*e^2 +
 96*d^6*x*e + 96*d^7 - 210*(d^2*x^5*e^5 + d^3*x^4*e^4 - 2*d^4*x^3*e^3 - 2*d
^5*x^2*e^2 + d^6*x*e + d^7)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) -
(15*x^6*e^6 - 15*d*x^5*e^5 - 176*d^2*x^4*e^4 + 4*d^3*x^3*e^3 + 249*d^4*x^2*
e^2 + 9*d^5*x*e - 96*d^6)*sqrt(-x^2*e^2 + d^2))/(x^5*e^13 + d*x^4*e^12 - 2*
d^2*x^3*e^11 - 2*d^3*x^2*e^10 + d^4*x*e^9 + d^5*e^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**7/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{{\left(d^2-e^2\,x^2\right)}^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(x^7/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.137 $\int \frac{x^6}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=148

$$\frac{x^5(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{x^3(5d-6ex)}{15e^4(d^2-e^2x^2)^{3/2}}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}-\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}$$

```
[Out] 1/5*x^5*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^3*(-6*e*x+5*d)/e^4/(-e^2*x
^2+d^2)^(3/2)-d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7+1/5*x*(-8*e*x+5*d)/e^6
/(-e^2*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/e^7
```

Rubi [A]
time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 833, 655, 223, 209}

$$-\frac{d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7}+\frac{x^5(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5e^7}+\frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}}-\frac{x^3(5d-6ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^5*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^3*(5*d - 6*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (x*(5*d - 8*e*x))/(5*e^6*Sqrt[d^2 - e^2*x^2]) - (
16*Sqrt[d^2 - e^2*x^2])/(5*e^7) - (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^7
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^6}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^6(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^4\left(5d^3-6d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x^2\left(15d^5-24d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d^7-48d^6e}{\sqrt{d^2-e^2x}}}{15d^6e^6}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x}}{5e^7}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x}}{5e^7}$$

$$= \frac{x^5(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^3(5d-6ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{16\sqrt{d^2-e^2x}}{5e^7}$$

Mathematica [A]

time = 0.48, size = 136, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2x^2}\,(-48d^5-33d^4ex+87d^3e^2x^2+52d^2e^3x^3-38de^4x^4-15e^5x^5)}{15e^7(-d+ex)^2(d+ex)^3}+\frac{d\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^6\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-48*d^5 - 33*d^4*e*x + 87*d^3*e^2*x^2 + 52*d^2*e^3*x^
3 - 38*d*e^4*x^4 - 15*e^5*x^5))/(15*e^7*(-d + e*x)^2*(d + e*x)^3) + (d*Log[
-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(e^6*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(130) = 260$.
time = 0.08, size = 533, normalized size = 3.60

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{e^7}-\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^6\sqrt{e^2}}-\frac{493d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{240e^8\left(x+\frac{d}{e}\right)}+\frac{25d\sqrt{-\left(x-\cdots\right.}}{\cdots}$
default	$\frac{-\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{4d^2\left(\frac{x^2}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{2d^2}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right)}{e^2}}{e}-\frac{d\left(\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}\cdots\right)}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-x^4/e^2/(-e^2*x^2+d^2)^(3/2)+4*d^2/e^2*(x^2/e^2/(-e^2*x^2+d^2)^(3/2)-
2/3*d^2/e^4/(-e^2*x^2+d^2)^(3/2)))-d/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-
1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2))))+d^2/e^3*(x^2/e^2/(-e^2*x^2+d^2)^(3/2)-2/3*d^2/e^4/(-e
^2*x^2+d^2)^(3/2))-d^3/e^4*(1/2*x/e^2/(-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3
*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+1/3*d^4/e^7/(-
e^2*x^2+d^2)^(3/2)-d^5/e^6*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*
x^2+d^2)^(1/2))+1/e^7*d^6*(-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(3/2)+4/5*e/d*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)))
```

Maxima [A]
time = 0.50, size = 237, normalized size = 1.60

$$-\frac{d^5}{5\left((-x^2e^2+d^2)^{\frac{3}{2}}xe^8+(-x^2e^2+d^2)^{\frac{3}{2}}de^7\right)}-\frac{x^4e^{(-3)}}{(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{5\,dx^3e^{(-4)}}{(-x^2e^2+d^2)^{\frac{3}{2}}}+\frac{20\,d^2x^2e^{(-5)}}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}}+\frac{64\,d^3xe^{(-6)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{14\,d^4e^{(-7)}}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}}-d\arcsin\left(\frac{xe}{d}\right)e^{(-7)}+\frac{x^2e^{(-5)}}{3\sqrt{-x^2e^2+d^2}}-\frac{52\,dxe^{(-6)}}{15\sqrt{-x^2e^2+d^2}}+\frac{4\,d^2e^{(-7)}}{3\sqrt{-x^2e^2+d^2}}+\frac{1}{3}\sqrt{-x^2e^2+d^2}\,e^{(-7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^5/((-x^2*e^2 + d^2)^(3/2)*x*e^8 + (-x^2*e^2 + d^2)^(3/2)*d*e^7) - x^
4*e^(-3)/(-x^2*e^2 + d^2)^(3/2) - 5*d*x^3*e^(-4)/(-x^2*e^2 + d^2)^(3/2) + 2
0/3*d^2*x^2*e^(-5)/(-x^2*e^2 + d^2)^(3/2) + 64/15*d^3*x*e^(-6)/(-x^2*e^2 +
d^2)^(3/2) - 14/3*d^4*e^(-7)/(-x^2*e^2 + d^2)^(3/2) - d*arcsin(x*e/d)*e^(-7
) + 1/3*x^2*e^(-5)/sqrt(-x^2*e^2 + d^2) - 52/15*d*x*e^(-6)/sqrt(-x^2*e^2 +
d^2) + 4/3*d^2*e^(-7)/sqrt(-x^2*e^2 + d^2) + 1/3*sqrt(-x^2*e^2 + d^2)*e^(-7
)
```

Fricas [A]
time = 3.25, size = 240, normalized size = 1.62

$$-\frac{48\,dx^5e^5+48\,d^2x^4e^4-96\,d^3x^3e^3-96\,d^4x^2e^2+48\,d^5xe+48\,d^6-30\,(dx^5e^5+d^2x^4e^4-2\,d^3x^3e^3-2\,d^4x^2e^2+d^5xe+d^6)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(15\,x^5e^5+38\,dx^4e^4-52\,d^2x^3e^3-87\,d^3x^2e^2+33\,d^4xe+48\,d^5)\sqrt{-x^2e^2+d^2}}{15\,(x^5e^{12}+dx^4e^{11}-2\,d^2x^3e^{10}-2\,d^3x^2e^9+d^4xe^8+d^5e^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(48*d*x^5*e^5 + 48*d^2*x^4*e^4 - 96*d^3*x^3*e^3 - 96*d^4*x^2*e^2 + 48
*d^5*x*e + 48*d^6 - 30*(d*x^5*e^5 + d^2*x^4*e^4 - 2*d^3*x^3*e^3 - 2*d^4*x^2
*e^2 + d^5*x*e + d^6)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (15*x^
5*e^5 + 38*d*x^4*e^4 - 52*d^2*x^3*e^3 - 87*d^3*x^2*e^2 + 33*d^4*x*e + 48*d^
5)*sqrt(-x^2*e^2 + d^2))/(x^5*e^12 + d*x^4*e^11 - 2*d^2*x^3*e^10 - 2*d^3*x^
2*e^9 + d^4*x*e^8 + d^5*e^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**6/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(x^6/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.138 $\int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{x^4(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}}-\frac{x^2(4d-5ex)}{15e^4(d^2-e^2x^2)^{3/2}}+\frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

```
[Out] 1/5*x^4*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)-1/15*x^2*(-5*e*x+4*d)/e^4/(-e^2*x
^2+d^2)^(3/2)+arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(-15*e*x+8*d)/e^6/(
-e^2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 833, 792, 223, 209}

$$\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}+\frac{x^4(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}}+\frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}}-\frac{x^2(4d-5ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^4*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d - 5*e*x))/(15*e^4
*(d^2 - e^2*x^2)^(3/2)) + (8*d - 15*e*x)/(15*e^6*Sqrt[d^2 - e^2*x^2]) + Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{x^5(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x^3\left(4d^3-5d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{x\left(8d^5-15d^4ex\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4e^4}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x}}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{1+e}\right.}{}$$

$$= \frac{x^4(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{x^2(4d-5ex)}{15e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.}{e^6}$$

Mathematica [A]
time = 0.43, size = 124, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\left(8d^4-7d^3ex-27d^2e^2x^2+8de^3x^3+23e^4x^4\right)}{15e^6(-d+ex)^2(d+ex)^3} + \frac{\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(8*d^4 - 7*d^3*e*x - 27*d^2*e^2*x^2 + 8*d*e^3*x^3 + 23
*e^4*x^4))/(15*e^6*(-d + e*x)^2*(d + e*x)^3) + (Sqrt[-e^2]*Log[-(Sqrt[-e^2]
*x) + Sqrt[d^2 - e^2*x^2]])/e^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(108) = 216$.
time $= 0.08$, size $= 450$, normalized size $= 3.69$

method	result
default	$\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}}{e}-\frac{d\left(\frac{x^2}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{2d^2}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right)}{e^2}+\frac{d^2\Bigg(-\ldots}{\ldots 2e\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e
^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-d/e^2*(x^2/e^2/
(-e^2*x^2+d^2)^(3/2)-2/3*d^2/e^4/(-e^2*x^2+d^2)^(3/2))+d^2/e^3*(1/2*x/e^2/(
-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(
-e^2*x^2+d^2)^(1/2)))-1/3*d^3/e^6/(-e^2*x^2+d^2)^(3/2)+d^4/e^5*(1/3*x/d^2/(
-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))-d^5/e^6*(-1/5/d/e/(x+d/
e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*e^2*(x+d/e)+2*d*e
)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+
2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(105) = 210$.
time $= 0.49$, size $= 214$, normalized size $= 1.75$

$$\frac{d^4}{5\left(\left(-x^2e^2+d^2\right)^{\frac{3}{2}}xe^7+\left(-x^2e^2+d^2\right)^{\frac{3}{2}}de^6\right)}+\frac{x^3e^{(-3)}}{\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}-\frac{8\,dx^2e^{(-4)}}{3\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}-\frac{4\,d^2xe^{(-5)}}{15\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}+\frac{2\,d^3e^{(-6)}}{\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}+\arcsin\left(\frac{xe}{d}\right)e^{(-6)}-\frac{x^2e^{(-4)}}{3\sqrt{-x^2e^2+d^2}\,d}-\frac{8\,xe^{(-5)}}{15\sqrt{-x^2e^2+d^2}}-\frac{4\,de^{(-6)}}{3\sqrt{-x^2e^2+d^2}}-\frac{\sqrt{-x^2e^2+d^2}\,e^{(-6)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^4/((-x^2*e^2 + d^2)^(3/2)*x*e^7 + (-x^2*e^2 + d^2)^(3/2)*d*e^6) + x^3
*e^(-3)/(-x^2*e^2 + d^2)^(3/2) - 8/3*d*x^2*e^(-4)/(-x^2*e^2 + d^2)^(3/2) -
```

```
4/15*d^2*x*e^(-5)/(-x^2*e^2 + d^2)^(3/2) + 2*d^3*e^(-6)/(-x^2*e^2 + d^2)^(3
/2) + arcsin(x*e/d)*e^(-6) - 1/3*x^2*e^(-4)/(sqrt(-x^2*e^2 + d^2)*d) - 8/15
*x*e^(-5)/sqrt(-x^2*e^2 + d^2) - 4/3*d*e^(-6)/sqrt(-x^2*e^2 + d^2) - 1/3*sq
rt(-x^2*e^2 + d^2)*e^(-6)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(105) = 210$.
time = 2.69, size = 224, normalized size = 1.84

$$\frac{8\,x^5e^5 + 8\,dx^4e^4 - 16\,d^2x^3e^3 - 16\,d^3x^2e^2 + 8\,d^4xe + 8\,d^5 - 30\,(x^5e^5 + dx^4e^4 - 2\,d^2x^3e^3 - 2\,d^3x^2e^2 + d^4xe + d^5)\arctan\left(-\frac{\left(d - \sqrt{-x^2e^2 + d^2}\right)e^{(-1)}}{x}\right) + (23\,x^4e^4 + 8\,dx^3e^3 - 27\,d^2x^2e^2 - 7\,d^3xe + 8\,d^4)\sqrt{-x^2e^2 + d^2}}{15\,(x^5e^{11} + dx^4e^{10} - 2\,d^2x^3e^9 - 2\,d^3x^2e^8 + d^4xe^7 + d^5e^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*x^5*e^5 + 8*d*x^4*e^4 - 16*d^2*x^3*e^3 - 16*d^3*x^2*e^2 + 8*d^4*x*e
 + 8*d^5 - 30*(x^5*e^5 + d*x^4*e^4 - 2*d^2*x^3*e^3 - 2*d^3*x^2*e^2 + d^4*x*
e + d^5)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (23*x^4*e^4 + 8*d*x
^3*e^3 - 27*d^2*x^2*e^2 - 7*d^3*x*e + 8*d^4)*sqrt(-x^2*e^2 + d^2))/(x^5*e^1
1 + d*x^4*e^10 - 2*d^2*x^3*e^9 - 2*d^3*x^2*e^8 + d^4*x*e^7 + d^5*e^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{\frac{5}{2}}\left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**5/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(d^2 - e^2\,x^2\right)}^{5/2}\,\left(d + e\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.139 $\int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}$$

```
[Out] -1/5*x^4*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*d^2/e^5/(-e^2*x^2+d^2)^(3/2
)-4/5/e^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {864, 819, 272, 45}

$$-\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/5*(x^4*(d - e*x))/(d*e*(d^2 - e^2*x^2)^(5/2)) + (4*d^2)/(15*e^5*(d^2 - e
^2*x^2)^(3/2)) - 4/(5*e^5*Sqrt[d^2 - e^2*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= \int \frac{x^4(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx \\ &= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\int \frac{x^3}{(d^2-e^2x^2)^{5/2}}\,dx}{5e} \\ &= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{2\mathrm{Subst}\left(\int \frac{x}{(d^2-e^2x)^{5/2}}\,dx,x,x^2\right)}{5e} \\ &= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{2\mathrm{Subst}\left(\int \left(\frac{d^2}{e^2(d^2-e^2x)^{5/2}}-\frac{1}{e^2(d^2-e^2x)^{3/2}}\right)dx,x,x^2\right)}{5e} \\ &= -\frac{x^4(d-ex)}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4d^2}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{4}{5e^5\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(-8d^4-8d^3ex+12d^2e^2x^2+12de^3x^3-3e^4x^4\right)}{15de^5(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*d^4 - 8*d^3*e*x + 12*d^2*e^2*x^2 + 12*d*e^3*x^3 -
3*e^4*x^4))/(15*d*e^5*(d - e*x)^2*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(73) = 146.
time = 0.06, size = 363, normalized size = 4.27

method	result
gosper	$-\frac{(-ex+d)\left(3e^4x^4-12d\,e^3x^3-12d^2x^2e^2+8d^3ex+8d^4\right)}{15d\,e^5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$

trager	$-\frac{(3e^4x^4-12d\,e^3x^3-12d^2x^2e^2+8d^3ex+8d^4)\sqrt{-e^2x^2+d^2}}{15e^5d(ex+d)^3(-ex+d)^2}$
default	$\frac{\frac{x^2}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{2d^2}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}}{e}-\frac{d\left(\frac{x}{2e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{d^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{2e^2}\right)}{e^2}+\frac{d^2}{3e^5(-e^2x^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(x^2/e^2/(-e^2*x^2+d^2)^(3/2)-2/3*d^2/e^4/(-e^2*x^2+d^2)^(3/2))-d/e^2*(
1/2*x/e^2/(-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+
2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+1/3*d^2/e^5/(-e^2*x^2+d^2)^(3/2)-d^3/e^4*(
1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))+1/e^5*d^4*(-
1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*e^2*
(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2
*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.27, size = 122, normalized size = 1.44

$$-\frac{d^3}{5\left(\left(-x^2e^2+d^2\right)^{\frac{3}{2}}xe^6+\left(-x^2e^2+d^2\right)^{\frac{3}{2}}de^5\right)}+\frac{x^2e^{(-3)}}{\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}-\frac{2\,dxe^{(-4)}}{5\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}-\frac{d^2e^{(-5)}}{3\left(-x^2e^2+d^2\right)^{\frac{3}{2}}}+\frac{xe^{(-4)}}{5\sqrt{-x^2e^2+d^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^3/((-x^2*e^2 + d^2)^(3/2)*x*e^6 + (-x^2*e^2 + d^2)^(3/2)*d*e^5) + x^
2*e^(-3)/(-x^2*e^2 + d^2)^(3/2) - 2/5*d*x*e^(-4)/(-x^2*e^2 + d^2)^(3/2) - 1
/3*d^2*e^(-5)/(-x^2*e^2 + d^2)^(3/2) + 1/5*x*e^(-4)/(sqrt(-x^2*e^2 + d^2)*d
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(69) = 138$.
time = 2.15, size = 156, normalized size = 1.84

$$-\frac{8\,x^5e^5+8\,dx^4e^4-16\,d^2x^3e^3-16\,d^3x^2e^2+8\,d^4xe+8\,d^5+\left(3\,x^4e^4-12\,dx^3e^3-12\,d^2x^2e^2+8\,d^3xe+8\,d^4\right)\sqrt{-x^2e^2+d^2}}{15\left(dx^5e^{10}+d^2x^4e^9-2\,d^3x^3e^8-2\,d^4x^2e^7+d^5xe^6+d^6e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*x^5*e^5 + 8*d*x^4*e^4 - 16*d^2*x^3*e^3 - 16*d^3*x^2*e^2 + 8*d^4*x*
e + 8*d^5 + (3*x^4*e^4 - 12*d*x^3*e^3 - 12*d^2*x^2*e^2 + 8*d^3*x*e + 8*d^4)
*sqrt(-x^2*e^2 + d^2))/(d*x^5*e^10 + d^2*x^4*e^9 - 2*d^3*x^3*e^8 - 2*d^4*x^
2*e^7 + d^5*x*e^6 + d^6*e^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.95, size = 78, normalized size = 0.92

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(8\,d^4+8\,d^3\,e\,x-12\,d^2\,e^2\,x^2-12\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d\,e^5\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(8*d^4 + 3*e^4*x^4 - 12*d*e^3*x^3 - 12*d^2*e^2*x^2
+ 8*d^3*e*x))/(15*d*e^5*(d + e*x)^3*(d - e*x)^2)
```

3.140 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(5/2)+1/15*(3*e*x-2*d)/e^4/(-e^2*x^2+d^
2)^(3/2)-1/5*x/d^2/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {864, 833, 792, 197}

$$\frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (x^2*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d - 3*e*x)/(15*e^4*(d^2
- e^2*x^2)^(3/2)) - x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2} \\
&= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e^3} \\
&= \frac{x^2(d-ex)}{5e^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2d-3ex}{15e^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.31, size = 82, normalized size = 0.90

$$
\frac{\sqrt{d^2-e^2x^2}\left(-2d^4-2d^3ex+3d^2e^2x^2+3de^3x^3+3e^4x^4\right)}{15d^2e^4(d-ex)^2(d+ex)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^4 - 2*d^3*e*x + 3*d^2*e^2*x^2 + 3*d*e^3*x^3 + 3*
e^4*x^4))/(15*d^2*e^4*(d - e*x)^2*(d + e*x)^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(79) = 158$.
time = 0.07, size = 311, normalized size = 3.42

method	result
gosper	$-\frac{(-ex+d)\left(-3e^4x^4-3d\,e^3x^3-3d^2x^2e^2+2d^3ex+2d^4\right)}{15d^2e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$

trager	$-\frac{(-3e^4x^4-3d e^3x^3-3d^2x^2e^2+2d^3ex+2d^4)\sqrt{-e^2x^2+d^2}}{15d^2e^4(ex+d)^3(-ex+d)^2}$
default	$\frac{\frac{x}{2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{d^2\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{2e^2}}{e}-\frac{d}{3e^4(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{d^2\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2 \ldots}}\right.}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/2*x/e^2/(-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(
3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))-1/3*d/e^4/(-e^2*x^2+d^2)^(3/2)+d^2/e^
3*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))-d^3/e^4*(
-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*e^2
*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-
2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.27, size = 100, normalized size = 1.10

$$\frac{d^2}{5\left((-x^2e^2+d^2)^{\frac{3}{2}}xe^5+(-x^2e^2+d^2)^{\frac{3}{2}}de^4\right)}+\frac{2\,xe^{(-3)}}{5\,(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{de^{(-4)}}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{xe^{(-3)}}{5\,\sqrt{-x^2e^2+d^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^2/((-x^2*e^2 + d^2)^(3/2)*x*e^5 + (-x^2*e^2 + d^2)^(3/2)*d*e^4) + 2/5
*x*e^(-3)/(-x^2*e^2 + d^2)^(3/2) - 1/3*d*e^(-4)/(-x^2*e^2 + d^2)^(3/2) - 1/
5*x*e^(-3)/(sqrt(-x^2*e^2 + d^2)*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(76) = 152$.
time = 2.18, size = 159, normalized size = 1.75

$$-\frac{2\,x^5e^5+2\,dx^4e^4-4\,d^2x^3e^3-4\,d^3x^2e^2+2\,d^4xe+2\,d^5-(3\,x^4e^4+3\,dx^3e^3+3\,d^2x^2e^2-2\,d^3xe-2\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(d^2x^5e^9+d^3x^4e^8-2\,d^4x^3e^7-2\,d^5x^2e^6+d^6xe^5+d^7e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*x^5*e^5 + 2*d*x^4*e^4 - 4*d^2*x^3*e^3 - 4*d^3*x^2*e^2 + 2*d^4*x*e
+ 2*d^5 - (3*x^4*e^4 + 3*d*x^3*e^3 + 3*d^2*x^2*e^2 - 2*d^3*x*e - 2*d^4)*sqr
```

```
t(-x^2*e^2 + d^2))/(d^2*x^5*e^9 + d^3*x^4*e^8 - 2*d^4*x^3*e^7 - 2*d^5*x^2*e
^6 + d^6*x*e^5 + d^7*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.84, size = 78, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^4-2\,d^3\,e\,x+3\,d^2\,e^2\,x^2+3\,d\,e^3\,x^3+3\,e^4\,x^4\right)}{15\,d^2\,e^4\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*e^4*x^4 - 2*d^4 + 3*d*e^3*x^3 + 3*d^2*e^2*x^2 - 2
*d^3*e*x))/(15*d^2*e^4*(d + e*x)^3*(d - e*x)^2)
```

3.141 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] -1/5*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/15*(e*x+d)/d/e^3/(-e^2*x^2+d^2)
^(3/2)-2/15*x/d^3/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {869, 792, 197}

$$-\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/5*x^2/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (2*(d + e*x))/(15*d*e^3*(d
^2 - e^2*x^2)^(3/2)) - (2*x)/(15*d^3*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 869

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^n*((a + c*x^2)^(p + 1)/(2*a*e*p*(d + e*x
))), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{x(2d+2ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de}\\
&= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de^2}\\
&= -\frac{x^2}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{15de^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 82, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^4+2d^3ex-3d^2e^2x^2+2de^3x^3+2e^4x^4\right)}{15d^3e^3(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^4 + 2*d^3*e*x - 3*d^2*e^2*x^2 + 2*d*e^3*x^3 + 2*e
^4*x^4))/(15*d^3*e^3*(d - e*x)^2*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(83) = 166.
time = 0.06, size = 234, normalized size = 2.46

method	result
gosper	$\frac{(-ex+d)\left(2e^4x^4+2d\,e^3x^3-3d^2x^2e^2+2d^3ex+2d^4\right)}{15d^3e^3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$\frac{\left(2e^4x^4+2d\,e^3x^3-3d^2x^2e^2+2d^3ex+2d^4\right)\sqrt{-e^2x^2+d^2}}{15d^3e^3(ex+d)^3(-ex+d)^2}$
default	$\frac{1}{3e^3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{d\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{e^2}+\frac{d^2\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{}{6d^2e^2(}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/e^3/(-e^2*x^2+d^2)^(3/2)-d/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^
4/(-e^2*x^2+d^2)^(1/2))+1/e^3*d^2*(-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.27, size = 100, normalized size = 1.05

$$-\frac{d}{5\left((-x^2e^2+d^2)^{\frac{3}{2}}xe^4+(-x^2e^2+d^2)^{\frac{3}{2}}de^3\right)}-\frac{xe^{(-2)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d}+\frac{e^{(-3)}}{3\,(-x^2e^2+d^2)^{\frac{3}{2}}}-\frac{2\,xe^{(-2)}}{15\,\sqrt{-x^2e^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*d/((-x^2*e^2 + d^2)^(3/2)*x*e^4 + (-x^2*e^2 + d^2)^(3/2)*d*e^3) - 1/15
*x*e^(-2)/((-x^2*e^2 + d^2)^(3/2)*d) + 1/3*e^(-3)/(-x^2*e^2 + d^2)^(3/2) -
2/15*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d^3)
```

Fricas [A]
time = 2.01, size = 158, normalized size = 1.66

$$\frac{2\,x^5e^5+2\,dx^4e^4-4\,d^2x^3e^3-4\,d^3x^2e^2+2\,d^4xe+2\,d^5+(2\,x^4e^4+2\,dx^3e^3-3\,d^2x^2e^2+2\,d^3xe+2\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(d^3x^5e^8+d^4x^4e^7-2\,d^5x^3e^6-2\,d^6x^2e^5+d^7xe^4+d^8e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*x^5*e^5 + 2*d*x^4*e^4 - 4*d^2*x^3*e^3 - 4*d^3*x^2*e^2 + 2*d^4*x*e +
 2*d^5 + (2*x^4*e^4 + 2*d*x^3*e^3 - 3*d^2*x^2*e^2 + 2*d^3*x*e + 2*d^4)*sqrt
(-x^2*e^2 + d^2))/(d^3*x^5*e^8 + d^4*x^4*e^7 - 2*d^5*x^3*e^6 - 2*d^6*x^2*e^
5 + d^7*x*e^4 + d^8*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.79, size = 78, normalized size = 0.82

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4+2\,d^3\,e\,x-3\,d^2\,e^2\,x^2+2\,d\,e^3\,x^3+2\,e^4\,x^4\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^4 + 2*e^4*x^4 + 2*d*e^3*x^3 - 3*d^2*e^2*x^2 + 2
*d^3*e*x))/(15*d^3*e^3*(d + e*x)^3*(d - e*x)^2)
```

3.142 $\int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/15*x/d^2/e/(-e^2*x^2+d^2)^(3/2)+1/5/e^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/15
*x/d^4/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {807, 198, 197}

$$\frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] x/(15*d^2*e*(d^2 - e^2*x^2)^(3/2)) + 1/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^(3/
2)) + (2*x)/(15*d^4*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5e}$$
$$= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2e}$$
$$= \frac{x}{15d^2e\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5e^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.30, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\,\left(3d^4+3d^3ex+3d^2e^2x^2-2de^3x^3-2e^4x^4\right)}{15d^4e^2(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*d^4 + 3*d^3*e*x + 3*d^2*e^2*x^2 - 2*d*e^3*x^3 - 2*e
^4*x^4))/(15*d^4*e^2*(d - e*x)^2*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(73) = 146.
time = 0.07, size = 212, normalized size = 2.49

method	result
gosper	$\frac{(-ex+d)\left(-2e^4x^4-2d\,e^3x^3+3d^2x^2e^2+3d^3ex+3d^4\right)}{15d^4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$\frac{\left(-2e^4x^4-2d\,e^3x^3+3d^2x^2e^2+3d^3ex+3d^4\right)\sqrt{-e^2x^2+d^2}}{15d^4(ex+d)^3(-ex+d)^2e^2}$
default	$\frac{\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}}{e} - \frac{d\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\right.}{}\right.}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))-d/e^2*(
-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-2*e^2
*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-
2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.96

$$\frac{1}{5\left((-x^2e^2+d^2)^{\frac{3}{2}}xe^3+(-x^2e^2+d^2)^{\frac{3}{2}}de^2\right)}+\frac{xe^{(-1)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}+\frac{2\,xe^{(-1)}}{15\,\sqrt{-x^2e^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5/((-x^2*e^2 + d^2)^(3/2)*x*e^3 + (-x^2*e^2 + d^2)^(3/2)*d*e^2) + 1/15*x*
e^(-1)/((-x^2*e^2 + d^2)^(3/2)*d^2) + 2/15*x*e^(-1)/(sqrt(-x^2*e^2 + d^2)*d
^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(68) = 136.
time = 2.50, size = 159, normalized size = 1.87

$$\frac{3\,x^5e^5+3\,dx^4e^4-6\,d^2x^3e^3-6\,d^3x^2e^2+3\,d^4xe+3\,d^5-(2\,x^4e^4+2\,dx^3e^3-3\,d^2x^2e^2-3\,d^3xe-3\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(d^4x^5e^7+d^5x^4e^6-2\,d^6x^3e^5-2\,d^7x^2e^4+d^8xe^3+d^9e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^5*e^5 + 3*d*x^4*e^4 - 6*d^2*x^3*e^3 - 6*d^3*x^2*e^2 + 3*d^4*x*e +
 3*d^5 - (2*x^4*e^4 + 2*d*x^3*e^3 - 3*d^2*x^2*e^2 - 3*d^3*x*e - 3*d^4)*sqrt
(-x^2*e^2 + d^2))/(d^4*x^5*e^7 + d^5*x^4*e^6 - 2*d^6*x^3*e^5 - 2*d^7*x^2*e^
4 + d^8*x*e^3 + d^9*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.78, size = 78, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4+3\,d^3\,e\,x+3\,d^2\,e^2\,x^2-2\,d\,e^3\,x^3-2\,e^4\,x^4\right)}{15\,d^4\,e^2\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(3*d^4 - 2*e^4*x^4 - 2*d*e^3*x^3 + 3*d^2*e^2*x^2 + 3
*d^3*e*x))/(15*d^4*e^2*(d + e*x)^3*(d - e*x)^2)
```

3.143 $\int \frac{1}{(d+ex)(d^2-e^2x^2)^{5/2}} \, dx$

Optimal. Leaf size=82

$$\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)-1/5/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+8/15*x
/d^5/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {673, 198, 197}

$$-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (4*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^(
3/2)) + (8*x)/(15*d^5*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx &= -\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d} \\
&= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^3} \\
&= \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$
\frac{\sqrt{d^2-e^2x^2}\left(-3d^4+12d^3ex+12d^2e^2x^2-8de^3x^3-8e^4x^4\right)}{15d^5e(d-ex)^2(d+ex)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3*d^4 + 12*d^3*e*x + 12*d^2*e^2*x^2 - 8*d*e^3*x^3 -
8*e^4*x^4))/(15*d^5*e*(d - e*x)^2*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(70) = 140.
time = 0.07, size = 164, normalized size = 2.00

method	result
gosper	$-\frac{(-ex+d)\left(8e^4x^4+8d\,e^3x^3-12d^2x^2e^2-12d^3ex+3d^4\right)}{15d^5e\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$-\frac{\left(8e^4x^4+8d\,e^3x^3-12d^2x^2e^2-12d^3ex+3d^4\right)\sqrt{-e^2x^2+d^2}}{15d^5(ex+d)^3(-ex+d)^2e}$
default	$\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e^2d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{5d}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(-
2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d
^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.28, size = 81, normalized size = 0.99

$$-\frac{1}{5\left((-x^2e^2+d^2)^{\frac{3}{2}}dxe^2+(-x^2e^2+d^2)^{\frac{3}{2}}d^2e\right)}+\frac{4\,x}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}+\frac{8\,x}{15\,\sqrt{-x^2e^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5/((-x^2*e^2 + d^2)^(3/2)*d*x*e^2 + (-x^2*e^2 + d^2)^(3/2)*d^2*e) + 4/15
*x/((-x^2*e^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-x^2*e^2 + d^2)*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(67) = 134.
time = 2.22, size = 158, normalized size = 1.93

$$-\frac{3\,x^5e^5+3\,dx^4e^4-6\,d^2x^3e^3-6\,d^3x^2e^2+3\,d^4xe+3\,d^5+(8\,x^4e^4+8\,dx^3e^3-12\,d^2x^2e^2-12\,d^3xe+3\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(d^5x^5e^6+d^6x^4e^5-2\,d^7x^3e^4-2\,d^8x^2e^3+d^9xe^2+d^{10}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*x^5*e^5 + 3*d*x^4*e^4 - 6*d^2*x^3*e^3 - 6*d^3*x^2*e^2 + 3*d^4*x*e
+ 3*d^5 + (8*x^4*e^4 + 8*d*x^3*e^3 - 12*d^2*x^2*e^2 - 12*d^3*x*e + 3*d^4)*s
qrt(-x^2*e^2 + d^2))/(d^5*x^5*e^6 + d^6*x^4*e^5 - 2*d^7*x^3*e^4 - 2*d^8*x^2
*e^3 + d^9*x*e^2 + d^10*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.76, size = 78, normalized size = 0.95

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4-12\,d^3\,e\,x-12\,d^2\,e^2\,x^2+8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] -((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 -
 12*d^3*e*x))/(15*d^5*e*(d + e*x)^3*(d - e*x)^2)
```

3.144 $\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] 1/15*(-4*e*x+5*d)/d^4/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/(e*x+d)/(-e^2*x^2+d^2)^(
3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*(-8*e*x+15*d)/d^6/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {871, 837, 12, 272, 65, 214}

$$\frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (5*d - 4*e*x)/(15*d^4*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*(d + e*x)*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 8*e*x)/(15*d^6*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-5de^2+4e^3x}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^3e^4+8d^2e^5x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \int$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \int$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} + \mathrm{S}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \mathrm{S}$$

$$= \frac{5d-4ex}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \mathrm{ta}$$

Mathematica [A]
time = 0.46, size = 118, normalized size = 0.99

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(23d^4+8d^3ex-27d^2e^2x^2-7de^3x^3+8e^4x^4)}{(d-ex)^2(d+ex)^3} + 30\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(23*d^4 + 8*d^3*e*x - 27*d^2*e^2*x^2 - 7*d*e^3*x^3 +
8*e^4*x^4))/((d - e*x)^2*(d + e*x)^3) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2
 - e^2*x^2])/d])/(15*d^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(105) = 210$.
time = 0.06, size = 255, normalized size = 2.14

method	result

default	$-\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\Bigg(-\frac{-2e^{2}\left(x+\frac{d}{e}\right)+2de}{6d^{2}e^{2}\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{-2e^{2}\left(x+\frac{d}{e}\right)+2de}{3e^{2}d^{4}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}}{5d}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*(
-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/
d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/d*(1/3/
d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2
)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(104) = 208.
time = 1.94, size = 224, normalized size = 1.88

$$\frac{23\,x^{5}e^{5}+23\,dx^{4}e^{4}-46\,d^{2}x^{3}e^{3}-46\,d^{3}x^{2}e^{2}+23\,d^{4}xe+23\,d^{5}+15\,(x^{5}e^{5}+dx^{4}e^{4}-2\,d^{2}x^{3}e^{3}-2\,d^{3}x^{2}e^{2}+d^{4}xe+d^{5})\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+(8\,x^{4}e^{4}-7\,dx^{3}e^{3}-27\,d^{2}x^{2}e^{2}+8\,d^{3}xe+23\,d^{4})\sqrt{-x^{2}e^{2}+d^{2}}}{15\,(d^{6}x^{5}e^{5}+d^{7}x^{4}e^{4}-2\,d^{8}x^{3}e^{3}-2\,d^{9}x^{2}e^{2}+d^{10}xe+d^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*x^5*e^5 + 23*d*x^4*e^4 - 46*d^2*x^3*e^3 - 46*d^3*x^2*e^2 + 23*d^4*
x*e + 23*d^5 + 15*(x^5*e^5 + d*x^4*e^4 - 2*d^2*x^3*e^3 - 2*d^3*x^2*e^2 + d^
4*x*e + d^5)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (8*x^4*e^4 - 7*d*x^3*e^3
- 27*d^2*x^2*e^2 + 8*d^3*x*e + 23*d^4)*sqrt(-x^2*e^2 + d^2))/(d^6*x^5*e^5 +
 d^7*x^4*e^4 - 2*d^8*x^3*e^3 - 2*d^9*x^2*e^2 + d^10*x*e + d^11)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.145 $\int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}$$

```
[Out] 1/15*(-5*e*x+6*d)/d^4/x/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x/(e*x+d)/(-e^2*x^2+d^
2)^(3/2)+e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+1/5*(-5*e*x+8*d)/d^6/x/(-e^2
*x^2+d^2)^(1/2)-16/5*(-e^2*x^2+d^2)^(1/2)/d^7/x
```

Rubi [A]
time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {871, 837, 821, 272, 65, 214}

$$\frac{1}{5d^2x(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{16\sqrt{d^2-e^2x^2}}{5d^7x}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7}+\frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}}+\frac{6d-5ex}{15d^4x\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (6*d - 5*e*x)/(15*d^4*x*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x*(d + e*x)*(d^2
- e^2*x^2)^(3/2)) + (8*d - 5*e*x)/(5*d^6*x*Sqrt[d^2 - e^2*x^2]) - (16*Sqrt[
d^2 - e^2*x^2])/(5*d^7*x) + (e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^7
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{5/2}}\,dx = \frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} - \frac{\int \frac{-6de^2+5e^3x}{x^2(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}} + \frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} - \frac{\int \frac{-24d^3e^4+15d^2e^5x}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{15d^6e^4}$$

$$= \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}} + \frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} -$$

$$= \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}} + \frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} -$$

$$= \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}} + \frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} -$$

$$= \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}} + \frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} -$$

$$= \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}} + \frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} -$$

Mathematica [A]
time = 0.46, size = 133, normalized size = 0.86

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\,(15d^5+38d^4ex-52d^3e^2x^2-87d^2e^3x^3+33de^4x^4+48e^5x^5)}{x(d-ex)^2(d+ex)^3} + 30e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(15*d^5 + 38*d^4*e*x - 52*d^3*e^2*x^2 - 87*d^2*
e^3*x^3 + 33*d*e^4*x^4 + 48*e^5*x^5))/(x*(d - e*x)^2*(d + e*x)^3) + 30*e*Ar
cTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/d^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(136) = 272.
time = 0.09, size = 333, normalized size = 2.16

method	result

risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^7 x}-\frac{413\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)}}{240d^7\left(x+\frac{d}{e}\right)}-\frac{23\sqrt{-\left(x-\frac{d}{e}\right)^2 e^2-2d\left(x-\frac{d}{e}\right)e}}{48d^7\left(x-\frac{d}{e}\right)}+\sqrt{}$
default	$\frac{e\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e^2d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right.}}\right.}{5d}\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e/d^2*(-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/6*
(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2
/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/d*(-1/
d^2/x/(-e^2*x^2+d^2)^(3/2)+4*e^2/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/
d^4/(-e^2*x^2+d^2)^(1/2)))-e/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2
/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d
^2)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*x^2), x)
```

Fricas [A]
time = 2.32, size = 249, normalized size = 1.62

$$-\frac{23\,x^6e^6+23\,dx^5e^5-46\,d^2x^4e^4-46\,d^3x^3e^3+23\,d^4x^2e^2+23\,d^5xe+15\,(x^6e^6+dx^5e^5-2\,d^2x^4e^4-2\,d^3x^3e^3+d^4x^2e^2+d^5xe)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(48\,x^5e^5+33\,dx^4e^4-87\,d^2x^3e^3-52\,d^3x^2e^2+38\,d^4xe+15\,d^5)\sqrt{-x^2e^2+d^2}}{15\,(d^7x^6e^5+d^8x^5e^4-2\,d^9x^4e^3-2\,d^{10}x^3e^2+d^{11}x^2e+d^{12}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(23*x^6*e^6 + 23*d*x^5*e^5 - 46*d^2*x^4*e^4 - 46*d^3*x^3*e^3 + 23*d^4
*x^2*e^2 + 23*d^5*x*e + 15*(x^6*e^6 + d*x^5*e^5 - 2*d^2*x^4*e^4 - 2*d^3*x^3
*e^3 + d^4*x^2*e^2 + d^5*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (48*x^5*
```

```
e^5 + 33*d*x^4*e^4 - 87*d^2*x^3*e^3 - 52*d^3*x^2*e^2 + 38*d^4*x*e + 15*d^5)
*sqrt(-x^2*e^2 + d^2))/(d^7*x^6*e^5 + d^8*x^5*e^4 - 2*d^9*x^4*e^3 - 2*d^10*
x^3*e^2 + d^11*x^2*e + d^12*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.146 $\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}$$

```
[Out] 1/15*(-6*e*x+7*d)/d^4/x^2/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x^2/(e*x+d)/(-e^2*x^
2+d^2)^(3/2)-7/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^8+1/15*(-24*e*x+35*d
)/d^6/x^2/(-e^2*x^2+d^2)^(1/2)-7/2*(-e^2*x^2+d^2)^(1/2)/d^7/x^2+16/5*e*(-e^
2*x^2+d^2)^(1/2)/d^8/x
```

Rubi [A]
time = 0.11, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {871, 837, 849, 821, 272, 65, 214}

$$\frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}-\frac{7e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}-\frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2}+\frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}+\frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (7*d - 6*e*x)/(15*d^4*x^2*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^2*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (35*d - 24*e*x)/(15*d^6*x^2*Sqrt[d^2 - e^2*x^2]) -
(7*Sqrt[d^2 - e^2*x^2])/(2*d^7*x^2) + (16*e*Sqrt[d^2 - e^2*x^2])/(5*d^8*x)
- (7*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^8)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-7de^2+6e^3x}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-35d^3e^4+24d^2e^5x}{x^3\left(d^2-e^2x^2\right)^{3/2}}}{15d^6e^4}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2}}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2}}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2}}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2}}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2}}$$

$$= \frac{7d-6ex}{15d^4x^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^2(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2}}$$

Mathematica [A]
time = 0.53, size = 146, normalized size = 0.78

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-15d^6+15d^5ex+176d^4e^2x^2-4d^3e^3x^3-249d^2e^4x^4-9de^5x^5+96e^6x^6\right)}{x^2(d-ex)^2(d+ex)^3}+210e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{30d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^6 + 15*d^5*e*x + 176*d^4*e^2*x^2 - 4*d^3*e^3*x
^3 - 249*d^2*e^4*x^4 - 9*d*e^5*x^5 + 96*e^6*x^6))/(x^2*(d - e*x)^2*(d + e*x
)^3) + 210*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(30*d^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(162) = 324$.
time = 0.11, size = 459, normalized size = 2.47

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2ex+d)}{2d^8x^2}+\frac{673e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{240d^8\left(x+\frac{d}{e}\right)}-\frac{29e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)}}{48d^8\left(x-\frac{d}{e}\right)}$
default	$-\frac{e^2\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e^2d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}\right.}{5d}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/d^3*(-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1
/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/
e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/d*(
-1/2/d^2/x^2/(-e^2*x^2+d^2)^(3/2)+5/2*e^2/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)
+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2
)*(-e^2*x^2+d^2)^(1/2))/x))))-e/d^2*(-1/d^2/x/(-e^2*x^2+d^2)^(3/2)+4*e^2/d^
2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+e^2/d^3*
(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)
^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*x^3), x)
```

Fricas [A]
time = 2.46, size = 265, normalized size = 1.42

$$\frac{116\,x^7e^7+116\,dx^6e^6-232\,d^2x^5e^5-232\,d^3x^4e^4+116\,d^4x^3e^3+116\,d^5x^2e^2+105\,(x^7e^7+dx^6e^6-2\,d^2x^5e^5-2\,d^3x^4e^4+d^4x^3e^3+d^5x^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(96\,x^6e^6-9\,dx^5e^5-249\,d^2x^4e^4-4\,d^3x^3e^3+176\,d^4x^2e^2+15\,d^5xe-15\,d^6)\sqrt{-x^2e^2+d^2}}{30\,(d^8x^7e^5+d^9x^6e^4-2\,d^{10}x^5e^3-2\,d^{11}x^4e^2+d^{12}x^3e+d^{13}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(116*x^7*e^7 + 116*d*x^6*e^6 - 232*d^2*x^5*e^5 - 232*d^3*x^4*e^4 + 116
*d^4*x^3*e^3 + 116*d^5*x^2*e^2 + 105*(x^7*e^7 + d*x^6*e^6 - 2*d^2*x^5*e^5 -
 2*d^3*x^4*e^4 + d^4*x^3*e^3 + d^5*x^2*e^2)*log(-(d - sqrt(-x^2*e^2 + d^2))
/x) + (96*x^6*e^6 - 9*d*x^5*e^5 - 249*d^2*x^4*e^4 - 4*d^3*x^3*e^3 + 176*d^4
*x^2*e^2 + 15*d^5*x*e - 15*d^6)*sqrt(-x^2*e^2 + d^2))/(d^8*x^7*e^5 + d^9*x^
6*e^4 - 2*d^10*x^5*e^3 - 2*d^11*x^4*e^2 + d^12*x^3*e + d^13*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.147 $\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=215

$$\frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-$$

```
[Out] 1/15*(-7*e*x+8*d)/d^4/x^3/(-e^2*x^2+d^2)^(3/2)+1/5/d^2/x^3/(e*x+d)/(-e^2*x^
2+d^2)^(3/2)+7/2*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^9+1/15*(-35*e*x+48*d
)/d^6/x^3/(-e^2*x^2+d^2)^(1/2)-64/15*(-e^2*x^2+d^2)^(1/2)/d^7/x^3+7/2*e*(-e
^2*x^2+d^2)^(1/2)/d^8/x^2-128/15*e^2*(-e^2*x^2+d^2)^(1/2)/d^9/x
```

Rubi [A]

time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {871, 837, 849, 821, 272, 65, 214}

$$\frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}}-\frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}+\frac{7e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}+\frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2}-\frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}+\frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}}+\frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] (8*d - 7*e*x)/(15*d^4*x^3*(d^2 - e^2*x^2)^(3/2)) + 1/(5*d^2*x^3*(d + e*x)*(
d^2 - e^2*x^2)^(3/2)) + (48*d - 35*e*x)/(15*d^6*x^3*Sqrt[d^2 - e^2*x^2]) -
(64*Sqrt[d^2 - e^2*x^2])/(15*d^7*x^3) + (7*e*Sqrt[d^2 - e^2*x^2])/(2*d^8*x^
2) - (128*e^2*Sqrt[d^2 - e^2*x^2])/(15*d^9*x) + (7*e^3*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/(2*d^9)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\int \frac{1}{x^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-8de^2+7e^3x}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-48d^3e^4+35d^2e^5x}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,d}{15d^6e^4}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x}}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x}}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x}}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x}}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x}}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x}}$$

$$= \frac{8d-7ex}{15d^4x^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{1}{5d^2x^3(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x}}$$

Mathematica [A]
time = 0.59, size = 157, normalized size = 0.73

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\,(10d^7-5d^6ex+75d^5e^2x^2+236d^4e^3x^3-244d^3e^4x^4-489d^2e^5x^5+151de^6x^6+256e^7x^7)}{x^3(d-ex)^2(d+ex)^3} + 210e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{30d^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] -1/30*((Sqrt[d^2 - e^2*x^2]*(10*d^7 - 5*d^6*e*x + 75*d^5*e^2*x^2 + 236*d^4*
e^3*x^3 - 244*d^3*e^4*x^4 - 489*d^2*e^5*x^5 + 151*d*e^6*x^6 + 256*e^7*x^7))
/(x^3*(d - e*x)^2*(d + e*x)^3) + 210*e^3*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 -
 e^2*x^2])/d])/d^9
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(187) = 374$.

time = 0.10, size = 569, normalized size = 2.65

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(22e^2x^2-3dex+2d^2)}{6d^9x^3}-\frac{331e^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{80d^9\left(x+\frac{d}{e}\right)}-\frac{35e^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-}}{48d^9\left(x-\frac{d}{e}\right.}$
default	$\frac{e^3\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{6d^2e^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e^2d^4\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}\right.}{5d}\right.}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3/d^4*(-1/5/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+4/5*e/d*(-1/
6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e
^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-e/d^2*
(-1/2/d^2/x^2/(-e^2*x^2+d^2)^(3/2)+5/2*e^2/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2
)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/
2)*(-e^2*x^2+d^2)^(1/2))/x))))+e^2/d^3*(-1/d^2/x/(-e^2*x^2+d^2)^(3/2)+4*e^2
/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+1/d*(
-1/3/d^2/x^3/(-e^2*x^2+d^2)^(3/2)+2*e^2/d^2*(-1/d^2/x/(-e^2*x^2+d^2)^(3/2)+
4*e^2/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))
-e^3/d^4*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/
d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*x^4), x)
```

Fricas [**A**]
time = 2.39, size = 275, normalized size = 1.28

$$-\frac{116\,x^{8}e^{8}+116\,dx^{7}e^{7}-232\,d^{2}x^{6}e^{6}-232\,d^{3}x^{5}e^{5}+116\,d^{4}x^{4}e^{4}+116\,d^{5}x^{3}e^{3}+105\,(x^{8}e^{8}+dx^{7}e^{7}-2\,d^{2}x^{6}e^{6}-2\,d^{3}x^{5}e^{5}+d^{4}x^{4}e^{4}+d^{5}x^{3}e^{3})\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+(256\,x^{7}e^{7}+151\,dx^{6}e^{6}-489\,d^{2}x^{5}e^{5}-244\,d^{3}x^{4}e^{4}+236\,d^{4}x^{3}e^{3}+75\,d^{5}x^{2}e^{2}-5\,d^{6}xe+10\,d^{7})\sqrt{-x^{2}e^{2}+d^{2}}}{30\,(d^{9}x^{8}e^{5}+d^{10}x^{7}e^{4}-2\,d^{11}x^{6}e^{3}-2\,d^{12}x^{5}e^{2}+d^{13}x^{4}e+d^{14}x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(116*x^8*e^8 + 116*d*x^7*e^7 - 232*d^2*x^6*e^6 - 232*d^3*x^5*e^5 + 11
6*d^4*x^4*e^4 + 116*d^5*x^3*e^3 + 105*(x^8*e^8 + d*x^7*e^7 - 2*d^2*x^6*e^6
- 2*d^3*x^5*e^5 + d^4*x^4*e^4 + d^5*x^3*e^3)*log(-(d - sqrt(-x^2*e^2 + d^2)
)/x) + (256*x^7*e^7 + 151*d*x^6*e^6 - 489*d^2*x^5*e^5 - 244*d^3*x^4*e^4 + 2
36*d^4*x^3*e^3 + 75*d^5*x^2*e^2 - 5*d^6*x*e + 10*d^7)*sqrt(-x^2*e^2 + d^2))
/(d^9*x^8*e^5 + d^10*x^7*e^4 - 2*d^11*x^6*e^3 - 2*d^12*x^5*e^2 + d^13*x^4*e
 + d^14*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^4*(d^2 - e^2*x^2)^(5/2)*(d + e*x)), x)
```

3.148 $\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/7*x^2*(-e*x+d)/e^2/(-e^2*x^2+d^2)^(7/2)+1/35*(3*e*x-2*d)/e^4/(-e^2*x^2+d^
2)^(5/2)-1/35*x/d^2/e^3/(-e^2*x^2+d^2)^(3/2)-2/35*x/d^4/e^3/(-e^2*x^2+d^2)^
(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {864, 833, 792, 198, 197}

$$\frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}}-\frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x^2*(d - e*x))/(7*e^2*(d^2 - e^2*x^2)^(7/2)) - (2*d - 3*e*x)/(35*e^4*(d^2
- e^2*x^2)^(5/2)) - x/(35*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) - (2*x)/(35*d^4*e^
3*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{x^3(d-ex)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx$$

$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{\int \frac{x\left(2d^3-3d^2ex\right)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7d^2e^2}$$

$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{3\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35e^3}$$

$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2\int \frac{1}{(d^2-e^2 \cdots}}{35d \cdots}$$

$$= \frac{x^2(d-ex)}{7e^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{2d-3ex}{35e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{x}{35d^2e^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2 \cdots}{35d^4e^3\sqrt{\cdots}}$$

Mathematica [**A**]
time = 0.39, size = 104, normalized size = 0.88

$$\frac{\sqrt{d^2-e^2x^2}\left(-2d^6-2d^5ex+5d^4e^2x^2+5d^3e^3x^3+5d^2e^4x^4-2de^5x^5-2e^6x^6\right)}{35d^4e^4(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^6 - 2*d^5*e*x + 5*d^4*e^2*x^2 + 5*d^3*e^3*x^3 +
5*d^2*e^4*x^4 - 2*d*e^5*x^5 - 2*e^6*x^6))/(35*d^4*e^4*(d - e*x)^3*(d + e*x)
^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(102) = 204$.
time = 0.07, size = 422, normalized size = 3.58

method	result
gosper	$-\frac{(-ex+d)(2e^6x^6+2d\,e^5x^5-5d^2e^4x^4-5d^3e^3x^3-5d^4e^2x^2+2e\,d^5x+2d^6)}{35d^4e^4(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(2e^6x^6+2d\,e^5x^5-5d^2e^4x^4-5d^3e^3x^3-5d^4e^2x^2+2e\,d^5x+2d^6)\sqrt{-e^2x^2+d^2}}{35d^4e^4(ex+d)^4(-ex+d)^3}$
default	$\frac{\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{4e^2}}{e}-\frac{d}{5e^4(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{d^2\left(\frac{x}{5d^2\left(-e^2x\cdots\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(
5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)
)))-1/5*d/e^4/(-e^2*x^2+d^2)^(5/2)+d^2/e^3*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+
4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))-d^
3/e^4*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10
*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^
2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
```

Maxima [A]
time = 0.28, size = 121, normalized size = 1.03

$$\frac{d^2}{7\left((-x^2e^2+d^2)^{\frac{5}{2}}xe^5+(-x^2e^2+d^2)^{\frac{5}{2}}de^4\right)}+\frac{8\,xe^{(-3)}}{35\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{de^{(-4)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{xe^{(-3)}}{35\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}-\frac{2\,xe^{(-3)}}{35\,\sqrt{-x^2e^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/7*d^2/((-x^2*e^2 + d^2)^(5/2)*x*e^5 + (-x^2*e^2 + d^2)^(5/2)*d*e^4) + 8/3
5*x*e^(-3)/(-x^2*e^2 + d^2)^(5/2) - 1/5*d*e^(-4)/(-x^2*e^2 + d^2)^(5/2) - 1
/35*x*e^(-3)/((-x^2*e^2 + d^2)^(3/2)*d^2) - 2/35*x*e^(-3)/(sqrt(-x^2*e^2 +
d^2)*d^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(97) = 194$.
time = 2.32, size = 221, normalized size = 1.87

$$-\frac{2\,x^7e^7+2\,dx^6e^6-6\,d^2x^5e^5-6\,d^3x^4e^4+6\,d^4x^3e^3+6\,d^5x^2e^2-2\,d^6xe-2\,d^7-\left(2\,x^6e^6+2\,dx^5e^5-5\,d^2x^4e^4-5\,d^3x^3e^3-5\,d^4x^2e^2+2\,d^5xe+2\,d^6\right)\sqrt{-x^2e^2+d^2}}{35\,\left(d^4x^7e^{11}+d^5x^6e^{10}-3\,d^6x^5e^9-3\,d^7x^4e^8+3\,d^8x^3e^7+3\,d^9x^2e^6-d^{10}xe^5-d^{11}e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/35*(2*x^7*e^7 + 2*d*x^6*e^6 - 6*d^2*x^5*e^5 - 6*d^3*x^4*e^4 + 6*d^4*x^3*
e^3 + 6*d^5*x^2*e^2 - 2*d^6*x*e - 2*d^7 - (2*x^6*e^6 + 2*d*x^5*e^5 - 5*d^2*
x^4*e^4 - 5*d^3*x^3*e^3 - 5*d^4*x^2*e^2 + 2*d^5*x*e + 2*d^6)*sqrt(-x^2*e^2
+ d^2))/(d^4*x^7*e^11 + d^5*x^6*e^10 - 3*d^6*x^5*e^9 - 3*d^7*x^4*e^8 + 3*d^
8*x^3*e^7 + 3*d^9*x^2*e^6 - d^10*x*e^5 - d^11*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.95, size = 161, normalized size = 1.36

$$\frac{\sqrt{d^2-e^2\,x^2}}{56\,d\,e^4\,{\left(d+e\,x\right)}^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d\,e^4}+\frac{x}{35\,d^2\,e^3}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2\,d}{35\,e^4}-\frac{11\,x}{70\,e^3}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{2\,x\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,e^3\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

```
[Out] (d^2 - e^2*x^2)^(1/2)/(56*d*e^4*(d + e*x)^4) - ((d^2 - e^2*x^2)^(1/2)*(1/(5
6*d*e^4) + x/(35*d^2*e^3)))/((d + e*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1
/2)*((2*d)/(35*e^4) - (11*x)/(70*e^3)))/((d + e*x)^3*(d - e*x)^3) - (2*x*(d
^2 - e^2*x^2)^(1/2))/(35*d^4*e^3*(d + e*x)*(d - e*x))
```

3.149 $\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

```
[Out] -1/7*x^2/d/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+2/35*(2*e*x+d)/d/e^3/(-e^2*x^2+d^
2)^(5/2)-4/105*x/d^3/e^2/(-e^2*x^2+d^2)^(3/2)-8/105*x/d^5/e^2/(-e^2*x^2+d^2
)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {869, 792, 198, 197}

$$-\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}-\frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] -1/7*x^2/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (2*(d + 2*e*x))/(35*d*e^3*
(d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/
(105*d^5*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 869

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^n*((a + c*x^2)^(p + 1)/(2*a*e*p*(d + e*x
))), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{x(2d+4ex)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx}{7de}$$
$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{35de^2}$$
$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^3}$$
$$= -\frac{x^2}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+2ex)}{35de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{4x}{105d^3e^2\left(d^2-e^2x^2\right)^3}$$

Mathematica [A]
time = 0.41, size = 104, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}\left(6d^6+6d^5ex-15d^4e^2x^2+20d^3e^3x^3+20d^2e^4x^4-8de^5x^5-8e^6x^6\right)}{105d^5e^3(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(6*d^6 + 6*d^5*e*x - 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 +
 20*d^2*e^4*x^4 - 8*d*e^5*x^5 - 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^3*(d + e
*x)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(107) = 214$.
time = 0.07, size = 319, normalized size = 2.59

method	result
gosper	$\frac{(-ex+d)\left(-8e^6x^6-8d\,e^5x^5+20d^2e^4x^4+20d^3e^3x^3-15d^4e^2x^2+6e\,d^5x+6d^6\right)}{105d^5e^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$

trager	$\frac{(-8e^6x^6-8d\,e^5x^5+20d^2e^4x^4+20d^3e^3x^3-15d^4e^2x^2+6e\,d^5x+6d^6)\sqrt{-e^2x^2+d^2}}{105d^5(ex+d)^4(-ex+d)^3e^3}$
default	$\frac{1}{5e^3(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)}{e^2}+\frac{d^2\left(-\frac{1}{7de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de(x\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/e^3/(-e^2*x^2+d^2)^(5/2)-d/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*
(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+1/e^3*d^2*
(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e
^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/
6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e
^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
```

Maxima [A]
time = 0.28, size = 121, normalized size = 0.98

$$-\frac{d}{7\left((-x^2e^2+d^2)^{\frac{5}{2}}xe^4+(-x^2e^2+d^2)^{\frac{5}{2}}de^3\right)}-\frac{xe^{(-2)}}{35\,(-x^2e^2+d^2)^{\frac{5}{2}}d}+\frac{e^{(-3)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{4\,xe^{(-2)}}{105\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}-\frac{8\,xe^{(-2)}}{105\,\sqrt{-x^2e^2+d^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/7*d/((-x^2*e^2 + d^2)^(5/2)*x*e^4 + (-x^2*e^2 + d^2)^(5/2)*d*e^3) - 1/35
*x*e^(-2)/((-x^2*e^2 + d^2)^(5/2)*d) + 1/5*e^(-3)/(-x^2*e^2 + d^2)^(5/2) -
4/105*x*e^(-2)/((-x^2*e^2 + d^2)^(3/2)*d^3) - 8/105*x*e^(-2)/(sqrt(-x^2*e^2
 + d^2)*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(101) = 202$.
time = 2.29, size = 220, normalized size = 1.79

$$\frac{6\,x^7e^7+6\,dx^6e^6-18\,d^2x^5e^5-18\,d^3x^4e^4+18\,d^4x^3e^3+18\,d^5x^2e^2-6\,d^6xe-6\,d^7+(8\,x^6e^6+8\,dx^5e^5-20\,d^2x^4e^4-20\,d^3x^3e^3+15\,d^4x^2e^2-6\,d^5xe-6\,d^6)\sqrt{-x^2e^2+d^2}}{105\,(d^5x^7e^{10}+d^6x^6e^9-3\,d^7x^5e^8-3\,d^8x^4e^7+3\,d^9x^3e^6+3\,d^{10}x^2e^5-d^{11}xe^4-d^{12}e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*x^7*e^7 + 6*d*x^6*e^6 - 18*d^2*x^5*e^5 - 18*d^3*x^4*e^4 + 18*d^4*x
^3*e^3 + 18*d^5*x^2*e^2 - 6*d^6*x*e - 6*d^7 + (8*x^6*e^6 + 8*d*x^5*e^5 - 20
*d^2*x^4*e^4 - 20*d^3*x^3*e^3 + 15*d^4*x^2*e^2 - 6*d^5*x*e - 6*d^6)*sqrt(-x
^2*e^2 + d^2))/(d^5*x^7*e^10 + d^6*x^6*e^9 - 3*d^7*x^5*e^8 - 3*d^8*x^4*e^7
+ 3*d^9*x^3*e^6 + 3*d^10*x^2*e^5 - d^11*x*e^4 - d^12*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate(x^2/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.88, size = 161, normalized size = 1.31

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1}{56\,d^2\,e^3}-\frac{4\,x}{105\,d^3\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{2}{35\,e^3}+\frac{3\,x}{70\,d\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}}{56\,d^2\,e^3\,{\left(d+e\,x\right)}^4}-\frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{105\,d^5\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(1/(56*d^2*e^3) - (4*x)/(105*d^3*e^2)))/((d + e*x)^2
*(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*(2/(35*e^3) + (3*x)/(70*d*e^2)))/((d
 + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(56*d^2*e^3*(d + e*x)^4) - (
8*x*(d^2 - e^2*x^2)^(1/2))/(105*d^5*e^2*(d + e*x)*(d - e*x))
```

3.150 $\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=66

$$\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}+\frac{3\sin^{-1}(ax)}{2a^4}$$

```
[Out] 3/2*arcsin(a*x)/a^4+x^2*(-a*x+1)/a^2/(-a^2*x^2+1)^(1/2)+1/2*(-3*a*x+4)*(-a^
2*x^2+1)^(1/2)/a^4
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {864, 833, 794, 222}

$$\frac{3\text{ArcSin}(ax)}{2a^4}+\frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}}+\frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (x^2*(1 - a*x))/(a^2*Sqrt[1 - a^2*x^2]) + ((4 - 3*a*x)*Sqrt[1 - a^2*x^2])/(
2*a^4) + (3*ArcSin[a*x])/(2*a^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}}\,dx &= \int \frac{x^3(1-ax)}{(1-a^2x^2)^{3/2}}\,dx\\ &= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\int \frac{x(2-3ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\ &= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{2a^3}\\ &= \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4} + \frac{3\sin^{-1}(ax)}{2a^4}\end{aligned}$$

Mathematica [**A**]
time = 0.19, size = 83, normalized size = 1.26

$$\frac{\frac{a\sqrt{1-a^2x^2}\,(4+ax-a^2x^2)}{1+ax} + 3\sqrt{-a^2}\,\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((a*Sqrt[1 - a^2*x^2]*(4 + a*x - a^2*x^2))/(1 + a*x) + 3*Sqrt[-a^2]*Log[-(S
qrt[-a^2]*x) + Sqrt[1 - a^2*x^2]])/(2*a^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(58) = 116$.
time = 0.07, size = 134, normalized size = 2.03

method	result

risch	$\frac{(ax-2)(a^2x^2-1)}{2a^4\sqrt{-a^2x^2+1}}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a^3\sqrt{a^2}}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{a^5\left(x+\frac{1}{a}\right)}$
default	$\frac{-\frac{x\sqrt{-a^2x^2+1}}{2a^2}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a^2\sqrt{a^2}}}{a}+\frac{\sqrt{-a^2x^2+1}}{a^4}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^3\sqrt{a^2}}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2 a}}{a^5(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2*x/a^2*(-a^2*x^2+1)^(1/2)+1/2/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x
/(-a^2*x^2+1)^(1/2)))+1/a^4*(-a^2*x^2+1)^(1/2)+1/a^3/(a^2)^(1/2)*arctan((a^
2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^5/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)^(
1/2)
```

Maxima [A]
time = 0.49, size = 68, normalized size = 1.03

$$\frac{\sqrt{-a^2x^2+1}}{a^5x+a^4}-\frac{\sqrt{-a^2x^2+1}\,x}{2\,a^3}+\frac{3\,\arcsin\left(ax\right)}{2\,a^4}+\frac{\sqrt{-a^2x^2+1}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^5*x + a^4) - 1/2*sqrt(-a^2*x^2 + 1)*x/a^3 + 3/2*arcsi
n(a*x)/a^4 + sqrt(-a^2*x^2 + 1)/a^4
```

Fricas [A]
time = 1.99, size = 75, normalized size = 1.14

$$\frac{4\,ax-6\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-(a^2x^2-ax-4)\sqrt{-a^2x^2+1}+4}{2\,(a^5x+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x - 6*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (a^2*x^2
- a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)/(a^5*x + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.07, size = 116, normalized size = 1.76

$$\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2\,a^3\,\sqrt{-a^2}}-\frac{\left(\frac{1}{a^2\,\sqrt{-a^2}}+\frac{x\,\sqrt{-a^2}}{2\,a^3}\right)\,\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3\,\left(x\,\sqrt{-a^2}+\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] (3*asinh(x*(-a^2)^(1/2)))/(2*a^3*(-a^2)^(1/2)) - ((1/(a^2*(-a^2)^(1/2)) + (
x*(-a^2)^(1/2))/(2*a^3))*(1 - a^2*x^2)^(1/2))/(-a^2)^(1/2) - (1 - a^2*x^2)^
(1/2)/(a^3*(x*(-a^2)^(1/2) + (-a^2)^(1/2)/a)*(-a^2)^(1/2))
```

3.151 $\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=55

$$-\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{\sin^{-1}(ax)}{a^3}$$

```
[Out] -arcsin(a*x)/a^3-(-a^2*x^2+1)^(1/2)/a^3-(-a^2*x^2+1)^(1/2)/a^3/(a*x+1)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1653, 12, 807, 222}

$$-\frac{\text{ArcSin}(ax)}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(ax+1)}-\frac{\sqrt{1-a^2x^2}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a^3) - Sqrt[1 - a^2*x^2]/(a^3*(1 + a*x)) - ArcSin[a*x]/
a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\int \frac{a^3x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a^4}\\ &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx}{a}\\ &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^2}\\ &= -\frac{\sqrt{1-a^2x^2}}{a^3}-\frac{\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{\sin^{-1}(ax)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 71, normalized size = 1.29

$$\frac{(-2-ax)\sqrt{1-a^2x^2}}{a^3(1+ax)}-\frac{\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{(-a^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
[Out] ((-2 - a*x)*Sqrt[1 - a^2*x^2])/(a^3*(1 + a*x)) - Log[-(Sqrt[-a^2]*x) + Sqrt
[1 - a^2*x^2]]/(-a^2)^(3/2)
```

Maple [A]
time = 0.06, size = 84, normalized size = 1.53

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}}{a^3}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^2\sqrt{a^2}}-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{a^4\left(x+\frac{1}{a}\right)}$	84

risch	$\frac{a^2x^2-1}{a^3\sqrt{-a^2x^2+1}} - \frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^2\sqrt{a^2}} - \frac{\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2+2\left(x+\frac{1}{a}\right)a}}{a^4\left(x+\frac{1}{a}\right)}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a^2*x^2+1)^(1/2)/a^3-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-1/a^4/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

Maxima [A]
time = 0.48, size = 52, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+1}}{a^4x+a^3} - \frac{\arcsin(ax)}{a^3} - \frac{\sqrt{-a^2x^2+1}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*x^2 + 1)/(a^4*x + a^3) - arcsin(a*x)/a^3 - sqrt(-a^2*x^2 + 1)/a^
3
```

Fricas [A]
time = 2.54, size = 66, normalized size = 1.20

$$-\frac{2\,ax - 2\,(ax+1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + \sqrt{-a^2x^2+1}\,(ax+2) + 2}{a^4x+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^
2 + 1)*(a*x + 2) + 2)/(a^4*x + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [A]
time = 0.82, size = 70, normalized size = 1.27

$$-\frac{\arcsin(ax)\operatorname{sgn}(a)}{a^2|a|}-\frac{\sqrt{-a^2x^2+1}}{a^3}+\frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sgn(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a^2*((sqrt(-
a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

Mupad [B]
time = 0.07, size = 84, normalized size = 1.53

$$\frac{\sqrt{1-a^2\,x^2}}{\left(a\,\sqrt{-a^2}+a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{\operatorname{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] (1 - a^2*x^2)^(1/2)/((a*(-a^2)^(1/2) + a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) -
asinh(x*(-a^2)^(1/2))/(a^2*(-a^2)^(1/2)) - (1 - a^2*x^2)^(1/2)/a^3
```

3.152 $\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=34

$$\frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\sin^{-1}(ax)}{a^2}$$

```
[Out] arcsin(a*x)/a^2+(-a^2*x^2+1)^(1/2)/a^2/(a*x+1)
```

Rubi [A]

time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {807, 222}

$$\frac{\text{ArcSin}(ax)}{a^2}+\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + ArcSin[a*x]/a^2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a}\\ &= \frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\sin^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 67, normalized size = 1.97

$$\frac{\sqrt{1-a^2x^2}}{a^2(1+ax)}+\frac{\sqrt{-a^2}\,\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a^2*(1 + a*x)) + (Sqrt[-a^2]*Log[-(Sqrt[-a^2]*x) + Sqrt[
1 - a^2*x^2]])/a^3
```

Maple [A]
time = 0.06, size = 65, normalized size = 1.91

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a\sqrt{a^2}}+\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2a^2+2\left(x+\frac{1}{a}\right)a}}{a^3\left(x+\frac{1}{a}\right)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/a^3/(x+1/a)*(-(x
+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

Maxima [A]
time = 0.50, size = 33, normalized size = 0.97

$$\frac{\sqrt{-a^2x^2+1}}{a^3x+a^2}+\frac{\arcsin\left(ax\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + arcsin(a*x)/a^2
```

Fricas [A]
time = 2.41, size = 58, normalized size = 1.71

$$\frac{ax-2\left(ax+1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}+1}{a^3x+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*x - 2*(a*x + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 +
 1) + 1)/(a^3*x + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [A]
time = 0.79, size = 52, normalized size = 1.53

$$\frac{\arcsin\left(ax\right)\operatorname{sgn}(a)}{a|a|}-\frac{2}{a\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a*abs(a)) - 2/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*
x) + 1)*abs(a))
```

Mupad [B]
time = 2.60, size = 57, normalized size = 1.68

$$\frac{1}{a^2\,\sqrt{1-a^2\,x^2}}-\frac{x}{a\,\sqrt{1-a^2\,x^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
```

```
[Out] 1/(a^2*(1 - a^2*x^2)^(1/2)) - x/(a*(1 - a^2*x^2)^(1/2)) - (asinh(x*(-a^2)^(
1/2))*(-a^2)^(1/2))/a^3
```

3.153 $\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

```
[Out] -(-a^2*x^2+1)^(1/2)/a/(a*x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{665\}$

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(1+ax)\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

Mathematica [A]
time = 0.12, size = 26, normalized size = 1.00

$$-\frac{\sqrt{1-a^2x^2}}{a(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/(a*(1 + a*x)))
```

Maple [A]
time = 0.06, size = 36, normalized size = 1.38

method	result	size
gosper	$\frac{ax-1}{a\sqrt{-a^2x^2+1}}$	22
trager	$-\frac{\sqrt{-a^2x^2+1}}{a(ax+1)}$	25
default	$-\frac{\sqrt{-\left(x+\frac{1}{a}\right)^2 a^2+2\left(x+\frac{1}{a}\right)a}}{a^2\left(x+\frac{1}{a}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/a^2/(x+1/a)*(-(x+1/a)^2*a^2+2*(x+1/a)*a)^(1/2)
```

Maxima [A]
time = 0.48, size = 23, normalized size = 0.88

$$-\frac{\sqrt{-a^2x^2+1}}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-a^2*x^2 + 1)/(a^2*x + a)
```

Fricas [A]
time = 2.24, size = 28, normalized size = 1.08

$$-\frac{ax+\sqrt{-a^2x^2+1}+1}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(a*x + sqrt(-a^2*x^2 + 1) + 1)/(a^2*x + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [A]
time = 0.93, size = 34, normalized size = 1.31

$$\frac{2}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] 2/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

Mupad [B]
time = 2.59, size = 23, normalized size = 0.88

$$-\frac{\sqrt{1-a^2\,x^2}}{x\,a^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a^2*x^2)^(1/2)*(a*x + 1)),x)
[Out] -(1 - a^2*x^2)^(1/2)/(a + a^2*x)
```

3.154 $\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -arctanh((-a^2*x^2+1)^(1/2))+(-a^2*x^2+1)^(1/2)/(-a*x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {871, 12, 272, 65, 214}

$$\frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{1-ax}+\frac{\int \frac{a^2}{x\sqrt{1-a^2x^2}}\,dx}{a^2}\\ &= \frac{\sqrt{1-a^2x^2}}{1-ax}+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{1-ax}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{1-a^2x^2}}{1-ax}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\ &= \frac{\sqrt{1-a^2x^2}}{1-ax}-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.16, size = 55, normalized size = 1.34

$$\frac{\sqrt{1-a^2x^2}}{1-ax}+2\tanh^{-1}\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(1 - a*x) + 2*ArcTanh[Sqrt[-a^2]*x - Sqrt[1 - a^2*x^2]]
```

Maple [A]
time = 0.06, size = 58, normalized size = 1.41

method	result	size

default	$-\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x)
```

Fricas [A]
time = 2.23, size = 52, normalized size = 1.27

$$\frac{ax+(ax-1)\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-\sqrt{-a^{2}x^{2}+1}-1}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] (a*x + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1) - 1)/
(a*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{ax^{2}\sqrt{-a^{2}x^{2}+1}-x\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
[Out] -Integral(1/(a*x**2*sqrt(-a**2*x**2 + 1) - x*sqrt(-a**2*x**2 + 1)), x)
```

Giac [A]
time = 1.02, size = 74, normalized size = 1.80

$$-\frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{|a|}+\frac{2\,a}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 2
*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))
```

Mupad [B]
time = 2.65, size = 58, normalized size = 1.41

$$\frac{a\sqrt{1-a^2\,x^2}}{\sqrt{-a^2}\,\left(\frac{a}{\sqrt{-a^2}}+x\,\sqrt{-a^2}\right)}-\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a*(1 - a^2*x^2)^(1/2))/((-a^2)^(1/2)*(a/(-a^2)^(1/2) + x*(-a^2)^(1/2))) -
atanh((1 - a^2*x^2)^(1/2))
```

3.155 $\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=64

$$-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -a*arctanh((-a^2*x^2+1)^(1/2))-2*(-a^2*x^2+1)^(1/2)/x+(-a^2*x^2+1)^(1/2)/x/
(-a*x+1)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {871, 821, 272, 65, 214}

$$-\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x^2]/(x*(1 - a*x)) - a*ArcTanh[Sqrt
[1 - a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{x(1-ax)}-\frac{\int \frac{-2a^2-a^3x}{x^2\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}+a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}+\frac{1}{2}a\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a}\\
&= -\frac{2\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x(1-ax)}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 64, normalized size = 1.00

$$
\frac{(1-2ax)\sqrt{1-a^2x^2}}{x(-1+ax)}+2a\tanh^{-1}\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 - 2*a*x)*Sqrt[1 - a^2*x^2])/(x*(-1 + a*x)) + 2*a*ArcTanh[Sqrt[-a^2]*x -
 Sqrt[1 - a^2*x^2]]
```

Maple [A]
time = 0.07, size = 73, normalized size = 1.14

method	result	size
default	$-\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}-\frac{\sqrt{-a^{2}x^{2}+1}}{x}-a\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)$	73
risch	$\frac{a^{2}x^{2}-1}{x\sqrt{-a^{2}x^{2}+1}}-a\left(\frac{\sqrt{-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}+\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^{2}x^{2}+1}}\right)\right)$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-(-a^2*x^2+1)^(1/2)/x-a*arctan
h(1/(-a^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2), x)
```

Fricas [A]
time = 2.47, size = 76, normalized size = 1.19

$$\frac{a^{2}x^{2}-ax+\left(a^{2}x^{2}-ax\right)\log\left(\frac{\sqrt{-a^{2}x^{2}+1}-1}{x}\right)-\sqrt{-a^{2}x^{2}+1}\,(2\,ax-1)}{ax^{2}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + (a^2*x^2 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^
2*x^2 + 1)*(2*a*x - 1))/(a*x^2 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{ax^{3}\sqrt{-a^{2}x^{2}+1}-x^{2}\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 2.59, size = 81, normalized size = 1.27

$$\frac{a^2\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}}-\frac{\sqrt{1-a^2\,x^2}}{x}-a\,\mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a^2*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (1 - a^2*x^2)^(1/2)/x - a*atanh((1 - a^2*x^2)^(1/2))
```

3.156 $\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -3/2*a^2*arctanh((-a^2*x^2+1)^(1/2))-3/2*(-a^2*x^2+1)^(1/2)/x^2-2*a*(-a^2*x
^2+1)^(1/2)/x+(-a^2*x^2+1)^(1/2)/x^2/(-a*x+1)
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {871, 849, 821, 272, 65, 214}

$$-\frac{2a\sqrt{1-a^2x^2}}{x}+\frac{\sqrt{1-a^2x^2}}{x^2(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2x^2}-\frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*x^2) - (2*a*Sqrt[1 - a^2*x^2])/x + Sqrt[1 - a^2*x
^2]/(x^2*(1 - a*x)) - (3*a^2*ArcTanh[Sqrt[1 - a^2*x^2]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}}\,dx &= \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{\int \frac{-3a^2-2a^3x}{x^3\sqrt{1-a^2x^2}}\,dx}{a^2}\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{\int \frac{4a^3+3a^4x}{x^2\sqrt{1-a^2x^2}}\,dx}{2a^2}\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{2}\left(3a^2\right)\int \frac{1}{x\sqrt{1-a^2x^2}}\,d\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-}}\right.\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x\right.\\
&= -\frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3}{2}a^2\tanh^{-1}\left(\sqrt{1-a^2x^2}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 76, normalized size = 0.84

$$\frac{(1+ax-4a^2x^2)\sqrt{1-a^2x^2}}{2x^2(-1+ax)}+3a^2\tanh^{-1}\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - a*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 + a*x - 4*a^2*x^2)*Sqrt[1 - a^2*x^2])/(2*x^2*(-1 + a*x)) + 3*a^2*ArcTan
h[Sqrt[-a^2]*x - Sqrt[1 - a^2*x^2]]
```

Maple [A]
time = 0.07, size = 94, normalized size = 1.04

method	result
default	$-\frac{a\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{3a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}-\frac{a\sqrt{-a^2x^2+1}}{x}$
risch	$\frac{2a^3x^3+a^2x^2-2ax-1}{2x^2\sqrt{-a^2x^2+1}}+\frac{a^2\left(-3\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{2\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{a\left(x-\frac{1}{a}\right)}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/(x-1/a)*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-1/2*(-a^2*x^2+1)^(1/2)/x^2-3/
2*a^2*arctanh(1/(-a^2*x^2+1)^(1/2))-a*(-a^2*x^2+1)^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^3), x)
```

Fricas [A]
time = 1.67, size = 97, normalized size = 1.08

$$\frac{2\,a^3x^3-2\,a^2x^2+3\,(a^3x^3-a^2x^2)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-(4\,a^2x^2-ax-1)\sqrt{-a^2x^2+1}}{2\,(ax^3-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*x^3 - 2*a^2*x^2 + 3*(a^3*x^3 - a^2*x^2)*log((sqrt(-a^2*x^2 + 1)
- 1)/x) - (4*a^2*x^2 - a*x - 1)*sqrt(-a^2*x^2 + 1))/(a*x^3 - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^4\sqrt{-a^2x^2+1} - x^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] -Integral(1/(a*x**4*sqrt(-a**2*x**2 + 1) - x**3*sqrt(-a**2*x**2 + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(78) = 156.
time = 0.76, size = 213, normalized size = 2.37

$$-\frac{\left(a^3+\frac{3\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a}{x}-\frac{20\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}-\frac{3\,a^3\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2\,a\right|}{2\,a^2|x|}\right)}{2\,|a|}-\frac{\frac{4\left(\sqrt{-a^2x^2+1}\,|a|+a\right)a|a|}{x}+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)^2|a|}{ax^2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(a^3 + 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a/x - 20*(sqrt(-a^2*x^2 + 1)*
abs(a) + a)^2/(a*x^2))*a^4*x^2/((sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*((sqrt(-a
^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)) - 3/2*a^3*log(1/2*abs(-2*sqrt(
-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) - 1/8*(4*(sqrt(-a^2*x^2 +
1)*abs(a) + a)*a*abs(a)/x + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*abs(a)/(a*x^2
))/a^2
```

Mupad [B]
time = 2.61, size = 105, normalized size = 1.17

$$\frac{a^3\sqrt{1-a^2x^2}}{\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}}-\frac{a\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}}{2\,x^2}+\frac{a^2\,\mathrm{atan}\left(\sqrt{1-a^2x^2}\;1\mathrm{i}\right)\,3\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(1 - a^2*x^2)^(1/2)*(a*x - 1)),x)
```

```
[Out] (a^2*atan((1 - a^2*x^2)^(1/2)*1i)*3i)/2 - (1 - a^2*x^2)^(1/2)/(2*x^2) - (a*
(1 - a^2*x^2)^(1/2))/x + (a^3*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2
)^(1/2)/a)*(-a^2)^(1/2))
```

3.157 $\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2} dx$

Optimal. Leaf size=229

$$-\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4}+\frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3}-\frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2}+\frac{dx^5(d^2-e^2x^2)^{3/2}}{4e}-\frac{1}{9}$$

```
[Out] -4/21*d^4*x^2*(-e^2*x^2+d^2)^(3/2)/e^4+5/24*d^3*x^3*(-e^2*x^2+d^2)^(3/2)/e^
3-5/21*d^2*x^4*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d*x^5*(-e^2*x^2+d^2)^(3/2)/e-1/
9*x^6*(-e^2*x^2+d^2)^(3/2)-1/2016*d^5*(-315*e*x+256*d)*(-e^2*x^2+d^2)^(3/2)
/e^6-5/64*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6-5/64*d^7*x*(-e^2*x^2+d^2
)^(1/2)/e^5
```

Rubi [A]
time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1823, 847, 794, 201, 223, 209}

$$-\frac{5d^9\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{64e^6}-\frac{1}{9}x^6(d^2-e^2x^2)^{3/2}+\frac{dx^5(d^2-e^2x^2)^{3/2}}{4e}-\frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2}-\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5}-\frac{d^5(256d-315ex)(d^2-e^2x^2)^{3/2}}{2016e^6}-\frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4}+\frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (-5*d^7*x*Sqrt[d^2 - e^2*x^2])/(64*e^5) - (4*d^4*x^2*(d^2 - e^2*x^2)^(3/2))
/(21*e^4) + (5*d^3*x^3*(d^2 - e^2*x^2)^(3/2))/(24*e^3) - (5*d^2*x^4*(d^2 -
e^2*x^2)^(3/2))/(21*e^2) + (d*x^5*(d^2 - e^2*x^2)^(3/2))/(4*e) - (x^6*(d^2
- e^2*x^2)^(3/2))/9 - (d^5*(256*d - 315*e*x)*(d^2 - e^2*x^2)^(3/2))/(2016*e
^6) - (5*d^9*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(64*e^6)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int x^5(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^5(-15d^2e^2+18de^3x)\sqrt{d^2-e^2x^2}\,dx}{9e^2}$$

$$= \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^4(-90d^3e^3+120d^2e^4x)\sqrt{d^2-e^2x^2}\,d}{72e^4}$$

$$= -\frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^3(-480d^4e^4}{}$$

$$= \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2x^2)^{3/2}}{4e} - \frac{1}{9}x^6\left(d^2-e^2x^2\right)^{3}$$

$$= -\frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2}{4e}$$

$$= -\frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2x^2)^{3/2}}{21e^2} + \frac{dx^5(d^2-e^2}{4e}$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2}{21e^2}$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2}{21e^2}$$

$$= -\frac{5d^7x\sqrt{d^2-e^2x^2}}{64e^5} - \frac{4d^4x^2(d^2-e^2x^2)^{3/2}}{21e^4} + \frac{5d^3x^3(d^2-e^2x^2)^{3/2}}{24e^3} - \frac{5d^2x^4(d^2-e^2}{21e^2}$$

Mathematica [A]
time = 0.27, size = 155, normalized size = 0.68

$$\frac{e\sqrt{d^2-e^2x^2}\,(-512d^8+315d^7ex-256d^6e^2x^2+210d^5e^3x^3-192d^4e^4x^4+168d^3e^5x^5+512d^2e^6x^6-1008de^7x^7+448e^8x^8)-315d^9\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{4032e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-512*d^8 + 315*d^7*e*x - 256*d^6*e^2*x^2 + 210*d^5*
e^3*x^3 - 192*d^4*e^4*x^4 + 168*d^3*e^5*x^5 + 512*d^2*e^6*x^6 - 1008*d*e^7*
x^7 + 448*e^8*x^8) - 315*d^9*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^
2*x^2]])/(4032*e^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(197) = 394.
time = 0.09, size = 750, normalized size = 3.28

method	result
risch	$-\frac{\left(-448e^8x^8+1008d\,e^7x^7-512d^2e^6x^6-168d^3e^5x^5+192d^4e^4x^4-210d^5e^3x^3+256d^6e^2x^2-315d^7ex+512d^8\right)\sqrt{-e^2x^2+d^2}}{4032e^6}-$
default	$\frac{-\frac{x^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{9e^2}-\frac{2d^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{63e^4}}{e^2}-\frac{2d\left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+}}{2}\right.}{}\right.}{}\right.}{8e^2}\right.}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(-1/9*x^2*(-e^2*x^2+d^2)^(7/2)/e^2-2/63*d^2/e^4*(-e^2*x^2+d^2)^(7/2))
-2*d/e^3*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)
^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1
/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-3/7*
d^2/e^6*(-e^2*x^2+d^2)^(7/2)-4*d^3/e^5*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*
(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^
```

```
2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+5/e^6*d^4*(1/5*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-d^5/e^7*(1/3/d/e/(x+d/e)^2*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(
5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3
/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 278, normalized size = 1.21

$$-\frac{5}{4} i d^9 \arcsin\left(\frac{xe}{d}+2\right) e^{(-6)} - \frac{85}{64} d^9 \arcsin\left(\frac{xe}{d}\right) e^{(-6)} + \frac{5}{4} \sqrt{x^2e^2+4dxe+3d^2}\, d^7 x e^{(-5)} - \frac{85}{64} \sqrt{-x^2e^2+d^2}\, d^7 x e^{(-5)} + \frac{5}{2} \sqrt{x^2e^2+4dxe+3d^2}\, d^8 e^{(-6)} + \frac{35}{96} \left(-x^2e^2+d^2\right)^{\frac{3}{2}} d^5 x e^{(-5)} - \frac{5}{12} \left(-x^2e^2+d^2\right)^{\frac{3}{2}} d^6 e^{(-6)} - \frac{17}{24} \left(-x^2e^2+d^2\right)^{\frac{5}{2}} d^3 x e^{(-5)} + \left(-x^2e^2+d^2\right)^{\frac{5}{2}} d^4 e^{(-6)} - \frac{\left(-x^2e^2+d^2\right)^{\frac{5}{2}} d^5}{4\left(xe^7+de^6\right)} - \frac{1}{9} \left(-x^2e^2+d^2\right)^{\frac{7}{2}} x^2 e^{(-4)} + \frac{1}{4} \left(-x^2e^2+d^2\right)^{\frac{7}{2}} d x e^{(-5)} - \frac{29}{63} \left(-x^2e^2+d^2\right)^{\frac{7}{2}} d^2 e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -5/4*I*d^9*arcsin(x*e/d + 2)*e^(-6) - 85/64*d^9*arcsin(x*e/d)*e^(-6) + 5/4*
sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^7*x*e^(-5) - 85/64*sqrt(-x^2*e^2 + d^2)*d
^7*x*e^(-5) + 5/2*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^8*e^(-6) + 35/96*(-x^2*
e^2 + d^2)^(3/2)*d^5*x*e^(-5) - 5/12*(-x^2*e^2 + d^2)^(3/2)*d^6*e^(-6) - 17
/24*(-x^2*e^2 + d^2)^(5/2)*d^3*x*e^(-5) + (-x^2*e^2 + d^2)^(5/2)*d^4*e^(-6)
 - 1/4*(-x^2*e^2 + d^2)^(5/2)*d^5/(x*e^7 + d*e^6) - 1/9*(-x^2*e^2 + d^2)^(7
/2)*x^2*e^(-4) + 1/4*(-x^2*e^2 + d^2)^(7/2)*d*x*e^(-5) - 29/63*(-x^2*e^2 +
d^2)^(7/2)*d^2*e^(-6)
```

Fricas [A]
time = 1.96, size = 128, normalized size = 0.56

$$\frac{1}{4032} \left(630\, d^9 \arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right) e^{(-1)}}{x}\right) + \left(448\, x^8 e^8 - 1008\, d x^7 e^7 + 512\, d^2 x^6 e^6 + 168\, d^3 x^5 e^5 - 192\, d^4 x^4 e^4 + 210\, d^5 x^3 e^3 - 256\, d^6 x^2 e^2 + 315\, d^7 x e - 512\, d^8\right) \sqrt{-x^2e^2+d^2}\right) e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(630*d^9*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (448*x^8*e^8
 - 1008*d*x^7*e^7 + 512*d^2*x^6*e^6 + 168*d^3*x^5*e^5 - 192*d^4*x^4*e^4 + 2
10*d^5*x^3*e^3 - 256*d^6*x^2*e^2 + 315*d^7*x*e - 512*d^8)*sqrt(-x^2*e^2 + d
^2))*e^(-6)
```

Sympy [A]
time = 21.93, size = 571, normalized size = 2.49

$$d^2 \left(\begin{cases} -\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6} - \frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4} - \frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2} + \frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\: e \neq 0 \\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise} \end{cases}\right) - 2de \left(\begin{cases} -\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7} + \frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5id^5x^3}{384e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^3x^5}{192e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{7idx^7}{48\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^9}{8d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7} - \frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5d^5x^3}{384e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{d^3x^5}{192e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{7dx^7}{48\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^9}{8d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^2 \left(\begin{cases} -\frac{16d^8\sqrt{d^2-e^2x^2}}{315e^8} - \frac{8d^6x^2\sqrt{d^2-e^2x^2}}{315e^6} - \frac{2d^4x^4\sqrt{d^2-e^2x^2}}{105e^4} - \frac{d^2x^6\sqrt{d^2-e^2x^2}}{63e^2} + \frac{x^8\sqrt{d^2-e^2x^2}}{9} & \text{for}\: e \neq 0 \\ \frac{x^8\sqrt{d^2}}{8} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqr
t(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2)
 + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - 2
*d*e*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sq
rt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2
)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt
(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e
**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sq
rt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) +
 d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2
*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piec
ewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 -
 e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d*
*2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, N
e(e, 0)), (x**8*sqrt(d**2)/8, True))
```

Giac [A]
time = 0.74, size = 332, normalized size = 1.45

$$\frac{\left(161280\,d^{10}\arctan\left(\sqrt{\frac{2\,d}{xe+d}-1}\right)e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+\frac{\left(315\,d^{10}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{17}{2}}e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-18774\,d^{10}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{15}{2}}e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+10458\,d^{10}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{13}{2}}e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-68958\,d^{10}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{11}{2}}e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-8192\,d^{10}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{9}{2}}e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-32418\,d^{10}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{7}{2}}e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-10458\,d^{10}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-2730\,d^{10}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-315\,d^{10}\sqrt{\frac{2\,d}{xe+d}-1}\,e^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)\right)(xe+d)^{9}}{d^{9}}\right)e^{(-16)}}{1032192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/1032192*(161280*d^10*arctan(sqrt(2*d/(x*e + d) - 1))*e^10*sgn(1/(x*e + d)
) + (315*d^10*(2*d/(x*e + d) - 1)^(17/2)*e^10*sgn(1/(x*e + d)) - 18774*d^10
*(2*d/(x*e + d) - 1)^(15/2)*e^10*sgn(1/(x*e + d)) + 10458*d^10*(2*d/(x*e +
d) - 1)^(13/2)*e^10*sgn(1/(x*e + d)) - 68958*d^10*(2*d/(x*e + d) - 1)^(11/2
)*e^10*sgn(1/(x*e + d)) - 8192*d^10*(2*d/(x*e + d) - 1)^(9/2)*e^10*sgn(1/(x
*e + d)) - 32418*d^10*(2*d/(x*e + d) - 1)^(7/2)*e^10*sgn(1/(x*e + d)) - 104
58*d^10*(2*d/(x*e + d) - 1)^(5/2)*e^10*sgn(1/(x*e + d)) - 2730*d^10*(2*d/(x
*e + d) - 1)^(3/2)*e^10*sgn(1/(x*e + d)) - 315*d^10*sqrt(2*d/(x*e + d) - 1)
*e^10*sgn(1/(x*e + d)))*(x*e + d)^9/d^9)*e^(-16)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.158 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=200

$$\frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3}-\frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2}+\frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e}-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}+d^4$$

```
[Out] 8/35*d^3*x^2*(-e^2*x^2+d^2)^(3/2)/e^3-13/48*d^2*x^3*(-e^2*x^2+d^2)^(3/2)/e^
2+2/7*d*x^4*(-e^2*x^2+d^2)^(3/2)/e-1/8*x^5*(-e^2*x^2+d^2)^(3/2)+1/6720*d^4*
(-1365*e*x+1024*d)*(-e^2*x^2+d^2)^(3/2)/e^5+13/128*d^8*arctan(e*x/(-e^2*x^2
+d^2)^(1/2))/e^5+13/128*d^6*x*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A]
time = 0.17, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1823, 847, 794, 201, 223, 209}

$$\frac{13d^8\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5}-\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2}+\frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e}-\frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2}+\frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4}+\frac{d^4(1024d-1365ex)\left(d^2-e^2x^2\right)^{3/2}}{6720e^5}+\frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (13*d^6*x*Sqrt[d^2 - e^2*x^2])/(128*e^4) + (8*d^3*x^2*(d^2 - e^2*x^2)^(3/2)
)/(35*e^3) - (13*d^2*x^3*(d^2 - e^2*x^2)^(3/2))/(48*e^2) + (2*d*x^4*(d^2 -
e^2*x^2)^(3/2))/(7*e) - (x^5*(d^2 - e^2*x^2)^(3/2))/8 + (d^4*(1024*d - 1365
*e*x)*(d^2 - e^2*x^2)^(3/2))/(6720*e^5) + (13*d^8*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/(128*e^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx &= \int x^4(d-ex)^2\sqrt{d^2-e^2x^2}\,dx \\
&= -\frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^4(-13d^2e^2+16de^3x)\sqrt{d^2-e^2x^2}\,dx}{8e^2} \\
&= \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^3(-64d^3e^3+91d^2e^4x)\sqrt{d^2-e^2x^2}\ d}{56e^4} \\
&= -\frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^2(-273d^4e}{} \\
&= \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x\right. \\
&= \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x^2)^{3/2}}{7e} - \frac{1}{8}x^5\left(d^2-e^2x\right. \\
&= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x}{7e} \\
&= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x}{7e} \\
&= \frac{13d^6x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{8d^3x^2(d^2-e^2x^2)^{3/2}}{35e^3} - \frac{13d^2x^3(d^2-e^2x^2)^{3/2}}{48e^2} + \frac{2dx^4(d^2-e^2x}{7e}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 144, normalized size = 0.72

$$
\frac{e\sqrt{d^2-e^2x^2}\left(2048d^7-1365d^6ex+1024d^5e^2x^2-910d^4e^3x^3+768d^3e^4x^4+1960d^2e^5x^5-3840de^6x^6+1680e^7x^7\right)+1365d^8\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{13440e^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(2048*d^7 - 1365*d^6*e*x + 1024*d^5*e^2*x^2 - 910*d^
4*e^3*x^3 + 768*d^3*e^4*x^4 + 1960*d^2*e^5*x^5 - 3840*d*e^6*x^6 + 1680*e^7*
x^7) + 1365*d^8*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(134
40*e^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(172) = 344$.
time = 0.09, size = 696, normalized size = 3.48

method	result

risch	$\frac{(1680e^7x^7-3840d\,e^6x^6+1960d^2e^5x^5+768d^3e^4x^4-910d^4e^3x^3+1024d^5e^2x^2-1365d^6ex+2048d^7)\sqrt{-e^2x^2+d^2}}{13440e^5}+\frac{13d^8\arctan}{}$
default	$\frac{-\frac{x\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{8e^2}+\frac{d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{8e^2}}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] 1/e^2*(-1/8*x*(-e^2*x^2+d^2)^(7/2)/e^2+1/8*d^2/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+2/7*d/e^5*(-e^2*x^2+d^2)^(7/2)+3*d^2/e^4*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))-4/e^5*d^3*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/e^6*d^4*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2

```
)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(1/2))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 256, normalized size = 1.28

$$\frac{7}{8}\,i\,d^8\arcsin\left(\frac{xe}{d}+2\right)e^{(-5)}+\frac{125}{128}\,d^8\arcsin\left(\frac{xe}{d}\right)e^{(-5)}-\frac{7}{8}\,\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^6xe^{(-4)}+\frac{125}{128}\,\sqrt{-x^2e^2+d^2}\,d^6xe^{(-4)}-\frac{7}{4}\,\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^7e^{(-5)}-\frac{67}{192}\,\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^4xe^{(-4)}+\frac{5}{12}\,\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^5e^{(-5)}+\frac{25}{48}\,\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^2xe^{(-4)}-\frac{4}{5}\,\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^3e^{(-5)}+\frac{\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^4}{4\,\left(xe^6+de^5\right)}-\frac{1}{8}\,\left(-x^2e^2+d^2\right)^{\frac{7}{2}}xe^{(-4)}+\frac{2}{7}\,\left(-x^2e^2+d^2\right)^{\frac{7}{2}}de^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 7/8*I*d^8*arcsin(x*e/d + 2)*e^(-5) + 125/128*d^8*arcsin(x*e/d)*e^(-5) - 7/8
*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^6*x*e^(-4) + 125/128*sqrt(-x^2*e^2 + d^2
)*d^6*x*e^(-4) - 7/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^7*e^(-5) - 67/192*(-
x^2*e^2 + d^2)^(3/2)*d^4*x*e^(-4) + 5/12*(-x^2*e^2 + d^2)^(3/2)*d^5*e^(-5)
+ 25/48*(-x^2*e^2 + d^2)^(5/2)*d^2*x*e^(-4) - 4/5*(-x^2*e^2 + d^2)^(5/2)*d^
3*e^(-5) + 1/4*(-x^2*e^2 + d^2)^(5/2)*d^4/(x*e^6 + d*e^5) - 1/8*(-x^2*e^2 +
 d^2)^(7/2)*x*e^(-4) + 2/7*(-x^2*e^2 + d^2)^(7/2)*d*e^(-5)
```

Fricas [A]
time = 2.42, size = 119, normalized size = 0.60

$$-\frac{1}{13440}\left(2730\,d^8\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(1680\,x^7e^7-3840\,dx^6e^6+1960\,d^2x^5e^5+768\,d^3x^4e^4-910\,d^4x^3e^3+1024\,d^5x^2e^2-1365\,d^6xe+2048\,d^7\right)\sqrt{-x^2e^2+d^2}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/13440*(2730*d^8*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (1680*x^7
*e^7 - 3840*d*x^6*e^6 + 1960*d^2*x^5*e^5 + 768*d^3*x^4*e^4 - 910*d^4*x^3*e^
3 + 1024*d^5*x^2*e^2 - 1365*d^6*x*e + 2048*d^7)*sqrt(-x^2*e^2 + d^2))*e^(-5
)
```

Sympy [C] Result contains complex when optimal does not.
time = 26.66, size = 690, normalized size = 3.45

$$d^2\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{5id^8\operatorname{acosh}\left(\frac{ex}{d}\right)}{128e^7}+\frac{5id^7x}{128e^6\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5id^5x^3}{384e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^5}{192e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{7idx^7}{48\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^9}{8d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{5d^8\operatorname{asin}\left(\frac{ex}{d}\right)}{128e^7}-\frac{5d^7x}{128e^6\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5d^5x^3}{384e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^5}{192e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{7dx^7}{48\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^9}{8d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1
+ e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*
d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
```

```
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(1
6*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d*
*2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**
2*x**2/d**2)), True)) - 2*d*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(10
5*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d*
*2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6
*sqrt(d**2)/6, True)) + e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) +
 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*
sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2
)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 +
 e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7)
 - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqr
t(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*
d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d*
*2)), True))
```

Giac [**A**]
time = 0.74, size = 301, normalized size = 1.50

$$-\frac{\left(349440\,d^9\arctan\left(\sqrt{\frac{2\,d}{xe+d}-1}\right)e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)+\frac{\left(1365\,d^9\left(\frac{2\,d}{xe+d}-1\right)^{\frac{15}{2}}e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)-61215\,d^9\left(\frac{2\,d}{xe+d}-1\right)^{\frac{13}{2}}e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)+20517\,d^9\left(\frac{2\,d}{xe+d}-1\right)^{\frac{11}{2}}e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)-141159\,d^9\left(\frac{2\,d}{xe+d}-1\right)^{\frac{9}{2}}e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)-34969\,d^9\left(\frac{2\,d}{xe+d}-1\right)^{\frac{7}{2}}e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)-34853\,d^9\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)-10465\,d^9\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)-1365\,d^9\sqrt{\frac{2\,d}{xe+d}-1}\,e^9\mathrm{sgn}\left(\frac{1}{xe+d}\right)\right)(xe+d)^8}{d^8}\right)e^{(-14)}}{1720320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/1720320*(349440*d^9*arctan(sqrt(2*d/(x*e + d) - 1))*e^9*sgn(1/(x*e + d))
 + (1365*d^9*(2*d/(x*e + d) - 1)^(15/2)*e^9*sgn(1/(x*e + d)) - 61215*d^9*(2
*d/(x*e + d) - 1)^(13/2)*e^9*sgn(1/(x*e + d)) + 20517*d^9*(2*d/(x*e + d) -
1)^(11/2)*e^9*sgn(1/(x*e + d)) - 141159*d^9*(2*d/(x*e + d) - 1)^(9/2)*e^9*s
gn(1/(x*e + d)) - 34969*d^9*(2*d/(x*e + d) - 1)^(7/2)*e^9*sgn(1/(x*e + d))
- 34853*d^9*(2*d/(x*e + d) - 1)^(5/2)*e^9*sgn(1/(x*e + d)) - 10465*d^9*(2*d
/(x*e + d) - 1)^(3/2)*e^9*sgn(1/(x*e + d)) - 1365*d^9*sqrt(2*d/(x*e + d) -
1)*e^9*sgn(1/(x*e + d)))*(x*e + d)^8/d^8)*e^(-14)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.159 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=171

$$-\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2}+\frac{dx^3(d^2-e^2x^2)^{3/2}}{3e}-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}-\frac{d^3(88d-105ex)\left(d^2-e^2\right.}{420e^4}$$

```
[Out] -11/35*d^2*x^2*(-e^2*x^2+d^2)^(3/2)/e^2+1/3*d*x^3*(-e^2*x^2+d^2)^(3/2)/e-1/
7*x^4*(-e^2*x^2+d^2)^(3/2)-1/420*d^3*(-105*e*x+88*d)*(-e^2*x^2+d^2)^(3/2)/e
^4-1/8*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4-1/8*d^5*x*(-e^2*x^2+d^2)^(1
/2)/e^3
```

Rubi [A]
time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1823, 847, 794, 201, 223, 209}

$$-\frac{d^7\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^4}-\frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2}-\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2}+\frac{dx^3(d^2-e^2x^2)^{3/2}}{3e}-\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3}-\frac{d^3(88d-105ex)\left(d^2-e^2x^2\right)^{3/2}}{420e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -1/8*(d^5*x*Sqrt[d^2 - e^2*x^2])/e^3 - (11*d^2*x^2*(d^2 - e^2*x^2)^(3/2))/(
35*e^2) + (d*x^3*(d^2 - e^2*x^2)^(3/2))/(3*e) - (x^4*(d^2 - e^2*x^2)^(3/2))
/7 - (d^3*(88*d - 105*e*x)*(d^2 - e^2*x^2)^(3/2))/(420*e^4) - (d^7*ArcTan[(
e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int x^3(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$
$$= -\frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^3(-11d^2e^2+14de^3x)\sqrt{d^2-e^2x^2}\,dx}{7e^2}$$
$$= \frac{dx^3(d^2-e^2x^2)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x^2(-42d^3e^3+66d^2e^4x)\sqrt{d^2-e^2x^2}\,dx}{42e^4}$$
$$= -\frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2} + \frac{dx^3(d^2-e^2x^2)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x(-132d^4e^4 \cdots}{}$$
$$= -\frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2} + \frac{dx^3(d^2-e^2x^2)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3/2} - \frac{d^3(88d-105e \cdots}{42 \cdots}$$
$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2} + \frac{dx^3(d^2-e^2x^2)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3 \cdots}$$
$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2} + \frac{dx^3(d^2-e^2x^2)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3 \cdots}$$
$$= -\frac{d^5x\sqrt{d^2-e^2x^2}}{8e^3} - \frac{11d^2x^2(d^2-e^2x^2)^{3/2}}{35e^2} + \frac{dx^3(d^2-e^2x^2)^{3/2}}{3e} - \frac{1}{7}x^4\left(d^2-e^2x^2\right)^{3 \cdots}$$

Mathematica [A]
time = 0.26, size = 133, normalized size = 0.78

$$\frac{e\sqrt{d^2-e^2x^2}\,(-176d^6+105d^5ex-88d^4e^2x^2+70d^3e^3x^3+144d^2e^4x^4-280de^5x^5+120e^6x^6)-105d^7\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{840e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(-176*d^6 + 105*d^5*e*x - 88*d^4*e^2*x^2 + 70*d^3*e^
3*x^3 + 144*d^2*e^4*x^4 - 280*d*e^5*x^5 + 120*e^6*x^6) - 105*d^7*Sqrt[-e^2]
*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(840*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(147) = 294$.
time = 0.07, size = 565, normalized size = 3.30

method	result
risch	$-\frac{(-120e^6x^6+280d\,e^5x^5-144d^2e^4x^4-70d^3e^3x^3+88d^4e^2x^2-105e\,d^5x+176d^6)\sqrt{-e^2x^2+d^2}}{840e^4} - \frac{d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8e^3\sqrt{e^2}}$

default	$-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{\frac{7}{2}}}{7 e^{4}}-\frac{2 d \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5 d^{2} \left(\frac{x \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 d^{2} \left(\frac{x \sqrt{-e^{2} x^{2}+d^{2}}}{2}+\frac{d^{2} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{2 \sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{e^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/e^4*(-e^2*x^2+d^2)^(7/2)-2*d/e^3*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(
1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+3/e^4*d^2*(1/5*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-d^3/e^5*(1/3/d/e/(x+d/e)^2*(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5
/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/
2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2))))))
```

<u>Maxima</u> **[C]** Result contains complex when optimal does not.
time = 0.56, size = 234, normalized size = 1.37

$$-\frac{1}{2} i d^{7} \arcsin\left(\frac{xe}{d}+2\right) e^{(-4)}-\frac{5}{8} d^{7} \arcsin\left(\frac{xe}{d}\right) e^{(-4)}+\frac{1}{2} \sqrt{x^{2} e^{2}+4 dxe+3 d^{2}}\, d^{5} x e^{(-3)}-\frac{5}{8} \sqrt{-x^{2} e^{2}+d^{2}}\, d^{5} x e^{(-3)}+\sqrt{x^{2} e^{2}+4 dxe+3 d^{2}}\, d^{6} e^{(-4)}+\frac{1}{3} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} d^{3} x e^{(-3)}-\frac{5}{12} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{3}{2}} d^{4} e^{(-4)}-\frac{1}{3} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{5}{2}} dxe^{(-3)}+\frac{3}{5} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{5}{2}} d^{2} e^{(-4)}-\frac{\left(-x^{2} e^{2}+d^{2}\right)^{\frac{5}{2}} d^{3}}{4 \left(xe^{5}+de^{4}\right)}-\frac{1}{7} \left(-x^{2} e^{2}+d^{2}\right)^{\frac{7}{2}} e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*I*d^7*arcsin(x*e/d + 2)*e^(-4) - 5/8*d^7*arcsin(x*e/d)*e^(-4) + 1/2*sq
rt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^5*x*e^(-3) - 5/8*sqrt(-x^2*e^2 + d^2)*d^5*x
*e^(-3) + sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^6*e^(-4) + 1/3*(-x^2*e^2 + d^2)
^(3/2)*d^3*x*e^(-3) - 5/12*(-x^2*e^2 + d^2)^(3/2)*d^4*e^(-4) - 1/3*(-x^2*e^
2 + d^2)^(5/2)*d*x*e^(-3) + 3/5*(-x^2*e^2 + d^2)^(5/2)*d^2*e^(-4) - 1/4*(-x
^2*e^2 + d^2)^(5/2)*d^3/(x*e^5 + d*e^4) - 1/7*(-x^2*e^2 + d^2)^(7/2)*e^(-4)
```

Fricas [A]
time = 2.86, size = 108, normalized size = 0.63

$$\frac{1}{840}\left(210\,d^7\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(120\,x^6e^6-280\,dx^5e^5+144\,d^2x^4e^4+70\,d^3x^3e^3-88\,d^4x^2e^2+105\,d^5xe-176\,d^6\right)\sqrt{-x^2e^2+d^2}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(210*d^7*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (120*x^6*e^6
- 280*d*x^5*e^5 + 144*d^2*x^4*e^4 + 70*d^3*x^3*e^3 - 88*d^4*x^2*e^2 + 105*d
^5*x*e - 176*d^6)*sqrt(-x^2*e^2 + d^2))*e^(-4)
```

Sympy [A]
time = 7.32, size = 450, normalized size = 2.63

$$d^2\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{id^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{8d^6\sqrt{d^2-e^2x^2}}{105e^6}-\frac{4d^4x^2\sqrt{d^2-e^2x^2}}{105e^4}-\frac{d^2x^4\sqrt{d^2-e^2x^2}}{35e^2}+\frac{x^6\sqrt{d^2-e^2x^2}}{7} & \text{for}\ e\neq 0\\ \frac{x^6\sqrt{d^2}}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**
4*sqrt(d**2)/4, True)) - 2*d*e*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) +
I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1
 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x*
*7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*
x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*
e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e
**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**2*Piecewise((-8*d**6*s
qrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*
e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*
x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

Giac [A]
time = 0.73, size = 270, normalized size = 1.58

$$\frac{\left(13440\,d^{8}\arctan\left(\sqrt{\frac{2\,d}{xe+d}-1}\right)e^{8}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+\frac{\left(105\,d^{8}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{13}{2}}e^{8}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-3780\,d^{8}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{11}{2}}e^{8}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+189\,d^{8}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{9}{2}}e^{8}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-4992\,d^{8}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{7}{2}}e^{8}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-1981\,d^{8}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^{8}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-700\,d^{8}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^{8}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-105\,d^{8}\sqrt{\frac{2\,d}{xe+d}-1}\,e^{8}\mathrm{sgn}\left(\frac{1}{xe+d}\right)\right)(xe+d)^{7}}{d^{7}}\right)e^{(-12)}}{53760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/53760*(13440*d^8*arctan(sqrt(2*d/(x*e + d) - 1))*e^8*sgn(1/(x*e + d)) + (
105*d^8*(2*d/(x*e + d) - 1)^(13/2)*e^8*sgn(1/(x*e + d)) - 3780*d^8*(2*d/(x*
e + d) - 1)^(11/2)*e^8*sgn(1/(x*e + d)) + 189*d^8*(2*d/(x*e + d) - 1)^(9/2)
*e^8*sgn(1/(x*e + d)) - 4992*d^8*(2*d/(x*e + d) - 1)^(7/2)*e^8*sgn(1/(x*e +
 d)) - 1981*d^8*(2*d/(x*e + d) - 1)^(5/2)*e^8*sgn(1/(x*e + d)) - 700*d^8*(2
*d/(x*e + d) - 1)^(3/2)*e^8*sgn(1/(x*e + d)) - 105*d^8*sqrt(2*d/(x*e + d) -
 1)*e^8*sgn(1/(x*e + d)))*(x*e + d)^7/d^7)*e^(-12)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(d^2-e^2\,x^2\right)^{5/2}}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.160 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=142

$$\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}+\frac{2dx^2(d^2-e^2x^2)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{3d^6\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}$$

```
[Out] 2/5*d*x^2*(-e^2*x^2+d^2)^(3/2)/e-1/6*x^3*(-e^2*x^2+d^2)^(3/2)+1/120*d^2*(-4
5*e*x+32*d)*(-e^2*x^2+d^2)^(3/2)/e^3+3/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/
2))/e^3+3/16*d^4*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1823, 847, 794, 201, 223, 209}

$$\frac{3d^6\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}+\frac{2dx^2(d^2-e^2x^2)^{3/2}}{5e}-\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2}+\frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3}+\frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (3*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) + (2*d*x^2*(d^2 - e^2*x^2)^(3/2))/(5
*e) - (x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^2*(32*d - 45*e*x)*(d^2 - e^2*x^2)^
(3/2))/(120*e^3) + (3*d^6*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx = \int x^2(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} - \frac{\int x^2(-9d^2e^2+12de^3x)\sqrt{d^2-e^2x^2}\,dx}{6e^2}$$

$$= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{\int x(-24d^3e^3+45d^2e^4x)\sqrt{d^2-e^2x^2}\,dx}{30e^4}$$

$$= \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + \frac{d^2(32d-45ex)\left(d^2-e^2x^2\right)^{3/2}}{120e^3} + $$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + $$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + $$

$$= \frac{3d^4x\sqrt{d^2-e^2x^2}}{16e^2} + \frac{2dx^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}x^3\left(d^2-e^2x^2\right)^{3/2} + $$

Mathematica [A]
time = 0.22, size = 122, normalized size = 0.86

$$\frac{e\sqrt{d^2-e^2x^2}\,(64d^5-45d^4ex+32d^3e^2x^2+50d^2e^3x^3-96de^4x^4+40e^5x^5)+45d^6\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{240e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(64*d^5 - 45*d^4*e*x + 32*d^3*e^2*x^2 + 50*d^2*e^3*x
^3 - 96*d*e^4*x^4 + 40*e^5*x^5) + 45*d^6*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + S
qrt[d^2 - e^2*x^2]])/(240*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(122) = 244$.
time = 0.06, size = 541, normalized size = 3.81

method	result
risch	$\frac{(40e^5x^5-96d\,e^4x^4+50d^2e^3x^3+32x^2d^3e^2-45d^4xe+64d^5)\sqrt{-e^2x^2+d^2}}{240e^3} + \frac{3d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^2\sqrt{e^2}}$

default	$\frac{\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}}{e^2} - \frac{2d\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de}{5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d
^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e
^2*x^2+d^2)^(1/2)))))-2*d/e^3*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e
*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*
d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1
/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2
)))))+1/e^4*d^2*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3
*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*
d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 215, normalized size = 1.51

$$\frac{1}{8}i\,d^6\arcsin\left(\frac{xe}{d}+2\right)e^{(-3)}+\frac{5}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{8}\,\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^4xe^{(-2)}+\frac{5}{16}\,\sqrt{-x^2e^2+d^2}\,d^4xe^{(-2)}-\frac{1}{4}\,\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^5e^{(-3)}-\frac{7}{24}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^2xe^{(-2)}+\frac{5}{12}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d^3e^{(-3)}+\frac{1}{6}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}xe^{(-2)}-\frac{2}{5}\left(-x^2e^2+d^2\right)^{\frac{5}{2}}de^{(-3)}+\frac{\left(-x^2e^2+d^2\right)^{\frac{5}{2}}d^2}{4\left(xe^4+de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*I*d^6*arcsin(x*e/d + 2)*e^(-3) + 5/16*d^6*arcsin(x*e/d)*e^(-3) - 1/8*sq
rt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^4*x*e^(-2) + 5/16*sqrt(-x^2*e^2 + d^2)*d^4*
x*e^(-2) - 1/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^5*e^(-3) - 7/24*(-x^2*e^2
+ d^2)^(3/2)*d^2*x*e^(-2) + 5/12*(-x^2*e^2 + d^2)^(3/2)*d^3*e^(-3) + 1/6*(-
x^2*e^2 + d^2)^(5/2)*x*e^(-2) - 2/5*(-x^2*e^2 + d^2)^(5/2)*d*e^(-3) + 1/4*(
-x^2*e^2 + d^2)^(5/2)*d^2/(x*e^4 + d*e^3)
```

Fricas [A]
time = 2.23, size = 99, normalized size = 0.70

$$-\frac{1}{240}\left(90\,d^6\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(40\,x^5e^5-96\,dx^4e^4+50\,d^2x^3e^3+32\,d^3x^2e^2-45\,d^4xe+64\,d^5\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(90*d^6*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (40*x^5*e^5 -
 96*d*x^4*e^4 + 50*d^2*x^3*e^3 + 32*d^3*x^2*e^2 - 45*d^4*x*e + 64*d^5)*sqrt
(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.78, size = 541, normalized size = 3.81

$$d^2\left(\begin{cases}-\frac{i d^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{i d^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4}-\frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2}+\frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{i d^6\operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5}+\frac{id^5x}{16e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{id^3x^3}{48e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{5idx^5}{24\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^6\operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5}-\frac{d^5x}{16e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^3x^3}{48e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{5dx^5}{24\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 +
e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(
4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)
/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 -
 e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - 2*d*
e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2
 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*s
qrt(d**2)/4, True)) + e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d*
*5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(
6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)
/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2
*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*
x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(115) = 230$.

time = 0.72, size = 239, normalized size = 1.68

$$-\frac{\left(2880\, d^{7} \arctan\left(\sqrt{\frac{2\, d}{xe+d}-1}\right) e^{7} \mathrm{sgn}\left(\frac{1}{xe+d}\right) + \frac{\left(45\, d^{7} \left(\frac{2\, d}{xe+d}-1\right)^{\frac{11}{2}} e^{7} \mathrm{sgn}\left(\frac{1}{xe+d}\right) - 1025\, d^{7} \left(\frac{2\, d}{xe+d}-1\right)^{\frac{9}{2}} e^{7} \mathrm{sgn}\left(\frac{1}{xe+d}\right) - 174\, d^{7} \left(\frac{2\, d}{xe+d}-1\right)^{\frac{7}{2}} e^{7} \mathrm{sgn}\left(\frac{1}{xe+d}\right) - 594\, d^{7} \left(\frac{2\, d}{xe+d}-1\right)^{\frac{5}{2}} e^{7} \mathrm{sgn}\left(\frac{1}{xe+d}\right) - 255\, d^{7} \left(\frac{2\, d}{xe+d}-1\right)^{\frac{3}{2}} e^{7} \mathrm{sgn}\left(\frac{1}{xe+d}\right) - 45\, d^{7} \sqrt{\frac{2\, d}{xe+d}-1}\, e^{7} \mathrm{sgn}\left(\frac{1}{xe+d}\right)\right) (xe+d)^{6}}{d^{6}}\right) e^{(-10)}}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/7680*(2880*d^7*arctan(sqrt(2*d/(x*e + d) - 1))*e^7*sgn(1/(x*e + d)) + (4
5*d^7*(2*d/(x*e + d) - 1)^(11/2)*e^7*sgn(1/(x*e + d)) - 1025*d^7*(2*d/(x*e
+ d) - 1)^(9/2)*e^7*sgn(1/(x*e + d)) - 174*d^7*(2*d/(x*e + d) - 1)^(7/2)*e^
7*sgn(1/(x*e + d)) - 594*d^7*(2*d/(x*e + d) - 1)^(5/2)*e^7*sgn(1/(x*e + d))
 - 255*d^7*(2*d/(x*e + d) - 1)^(3/2)*e^7*sgn(1/(x*e + d)) - 45*d^7*sqrt(2*d
/(x*e + d) - 1)*e^7*sgn(1/(x*e + d)))*(x*e + d)^6/d^6)*e^(-10)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.161 $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx(d^2-e^2x^2)^{3/2}}{6e}-\frac{2(d^2-e^2x^2)^{5/2}}{15e^2}-\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}$$

```
[Out] -1/6*d*x*(-e^2*x^2+d^2)^(3/2)/e-2/15*(-e^2*x^2+d^2)^(5/2)/e^2-1/3*(-e^2*x^2
+d^2)^(7/2)/e^2/(e*x+d)^2-1/4*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2-1/4*
d^3*x*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {807, 679, 201, 223, 209}

$$-\frac{d^5\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^2}-\frac{dx(d^2-e^2x^2)^{3/2}}{6e}-\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{2(d^2-e^2x^2)^{5/2}}{15e^2}-\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -1/4*(d^3*x*Sqrt[d^2 - e^2*x^2])/e - (d*x*(d^2 - e^2*x^2)^(3/2))/(6*e) - (2
*(d^2 - e^2*x^2)^(5/2))/(15*e^2) - (d^2 - e^2*x^2)^(7/2)/(3*e^2*(d + e*x)^2
) - (d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx &= -\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{2\int \frac{(d^2-e^2x^2)^{5/2}}{d+ex}\,dx}{3e}\\
&= -\frac{2(d^2-e^2x^2)^{5/2}}{15e^2}-\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{(2d)\int (d^2-e^2x^2)^{3/2}\,dx}{3e}\\
&= -\frac{dx(d^2-e^2x^2)^{3/2}}{6e}-\frac{2(d^2-e^2x^2)^{5/2}}{15e^2}-\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{d^3\int \sqrt{d^2-e^2x^2}\,dx}{2e}\\
&= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx(d^2-e^2x^2)^{3/2}}{6e}-\frac{2(d^2-e^2x^2)^{5/2}}{15e^2}-\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\int \cdots}{\cdots}\\
&= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx(d^2-e^2x^2)^{3/2}}{6e}-\frac{2(d^2-e^2x^2)^{5/2}}{15e^2}-\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\text{Sul}\cdots}{\cdots}\\
&= -\frac{d^3x\sqrt{d^2-e^2x^2}}{4e}-\frac{dx(d^2-e^2x^2)^{3/2}}{6e}-\frac{2(d^2-e^2x^2)^{5/2}}{15e^2}-\frac{(d^2-e^2x^2)^{7/2}}{3e^2(d+ex)^2}-\frac{d^5\tan\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 111, normalized size = 0.82

$$\frac{e\sqrt{d^2-e^2x^2}\,(-28d^4+15d^3ex+16d^2e^2x^2-30de^3x^3+12e^4x^4)-15d^5\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{60e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]

[Out] (e*Sqrt[d^2 - e^2*x^2]*(-28*d^4 + 15*d^3*e*x + 16*d^2*e^2*x^2 - 30*d*e^3*x^3 + 12*e^4*x^4) - 15*d^5*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(60*e^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(116) = 232$.

time = 0.07, size = 438, normalized size = 3.22

method	result
risch	$-\frac{\left(-12 e^{4} x^{4}+30 d\, e^{3} x^{3}-16 d^{2} x^{2} e^{2}-15 d^{3} e x +28 d^{4}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{60 e^{2}}-\frac{d^{5} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{4 e \sqrt{e^{2}}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+d e \left(-\frac{\left(-2 e^{2} \left(x+\frac{d}{e}\right)+2 d e \right) \left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8 e^{2}}+\frac{3 d^{2} \left(-\frac{\left(-2 e^{2} \left(x+\frac{d}{e}\right)+2 d e \right) \sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+}}{4 e^{2}}\right.}{}\right.}{e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)

```
[Out] 1/e^2*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2
*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e
)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arcta
n((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-d/e^3*(1/3/d/e/(x+
d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(1/2))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 156, normalized size = 1.15

$$\frac{1}{4} i\, d^5 \arcsin\left(\frac{xe}{d} + 2\right) e^{(-2)} - \frac{1}{4}\sqrt{x^2e^2 + 4\, dxe + 3\, d^2}\, d^3xe^{(-1)} - \frac{1}{2}\sqrt{x^2e^2 + 4\, dxe + 3\, d^2}\, d^4e^{(-2)} + \frac{1}{4}\left(-x^2e^2 + d^2\right)^{\frac{3}{2}} dxe^{(-1)} - \frac{5}{12}\left(-x^2e^2 + d^2\right)^{\frac{3}{2}} d^2e^{(-2)} + \frac{1}{5}\left(-x^2e^2 + d^2\right)^{\frac{5}{2}} e^{(-2)} - \frac{\left(-x^2e^2 + d^2\right)^{\frac{5}{2}} d}{4\left(xe^3 + de^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*d^5*arcsin(x*e/d + 2)*e^(-2) - 1/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^
3*x*e^(-1) - 1/2*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^4*e^(-2) + 1/4*(-x^2*e^2
 + d^2)^(3/2)*d*x*e^(-1) - 5/12*(-x^2*e^2 + d^2)^(3/2)*d^2*e^(-2) + 1/5*(-x
^2*e^2 + d^2)^(5/2)*e^(-2) - 1/4*(-x^2*e^2 + d^2)^(5/2)*d/(x*e^3 + d*e^2)
```

Fricas [A]
time = 2.13, size = 88, normalized size = 0.65

$$\frac{1}{60}\left(30\, d^5 \arctan\left(-\frac{\left(d - \sqrt{-x^2e^2 + d^2}\right)e^{(-1)}}{x}\right) + \left(12\, x^4e^4 - 30\, dx^3e^3 + 16\, d^2x^2e^2 + 15\, d^3xe - 28\, d^4\right)\sqrt{-x^2e^2 + d^2}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(30*d^5*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (12*x^4*e^4 - 3
0*d*x^3*e^3 + 16*d^2*x^2*e^2 + 15*d^3*x*e - 28*d^4)*sqrt(-x^2*e^2 + d^2))*e
^(-2)
```

Sympy [A]
time = 4.01, size = 321, normalized size = 2.36

$$d^2\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2 = 0 \\ -\frac{\left(d^2 - e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1 + \frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1 + \frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1 + \frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1 - \frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1 - \frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1 - \frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{2d^4\sqrt{d^2 - e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2 - e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2 - e^2x^2}}{5} & \text{for}\ e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2
)/(3*e**2), True)) - 2*d*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**
3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/
d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2))
+ 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2
/d**2)), True)) + e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4)
- d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/
5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

Giac [A]
time = 1.32, size = 208, normalized size = 1.53

$$\frac{\left(480\,d^6\arctan\left(\sqrt{\frac{2\,d}{xe+d}-1}\right)e^6\mathrm{sgn}\left(\frac{1}{xe+d}\right)+\frac{\left(15\,d^6\left(\frac{2\,d}{xe+d}-1\right)^{\frac{9}{2}}e^6\mathrm{sgn}\left(\frac{1}{xe+d}\right)-250\,d^6\left(\frac{2\,d}{xe+d}-1\right)^{\frac{7}{2}}e^6\mathrm{sgn}\left(\frac{1}{xe+d}\right)-128\,d^6\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^6\mathrm{sgn}\left(\frac{1}{xe+d}\right)-70\,d^6\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^6\mathrm{sgn}\left(\frac{1}{xe+d}\right)-15\,d^6\sqrt{\frac{2\,d}{xe+d}-1}\,e^6\mathrm{sgn}\left(\frac{1}{xe+d}\right)\right)(xe+d)^5}{d^5}\right)e^{(-8)}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/960*(480*d^6*arctan(sqrt(2*d/(x*e + d) - 1))*e^6*sgn(1/(x*e + d)) + (15*d
^6*(2*d/(x*e + d) - 1)^(9/2)*e^6*sgn(1/(x*e + d)) - 250*d^6*(2*d/(x*e + d)
- 1)^(7/2)*e^6*sgn(1/(x*e + d)) - 128*d^6*(2*d/(x*e + d) - 1)^(5/2)*e^6*sgn
(1/(x*e + d)) - 70*d^6*(2*d/(x*e + d) - 1)^(3/2)*e^6*sgn(1/(x*e + d)) - 15*
d^6*sqrt(2*d/(x*e + d) - 1)*e^6*sgn(1/(x*e + d)))*(x*e + d)^5/d^5)*e^(-8)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2, x)
```

3.162 $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=108

$$\frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d(d^2-e^2x^2)^{3/2}}{12e}+\frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

```
[Out] 5/12*d*(-e^2*x^2+d^2)^(3/2)/e+1/4*(-e*x+d)*(-e^2*x^2+d^2)^(3/2)/e+5/8*d^4*a
rctan(e*x/(-e^2*x^2+d^2)^(1/2))/e+5/8*d^2*x*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {669, 685, 655, 201, 223, 209}

$$\frac{5d^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}+\frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d(d^2-e^2x^2)^{3/2}}{12e}+\frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (5*d^2*x*Sqrt[d^2 - e^2*x^2])/8 + (5*d*(d^2 - e^2*x^2)^(3/2))/(12*e) + ((d
- e*x)*(d^2 - e^2*x^2)^(3/2))/(4*e) + (5*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(8*e)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx &= \int (d-ex)^2\sqrt{d^2-e^2x^2}\,dx\\
&= \frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{4}(5d)\int (d-ex)\sqrt{d^2-e^2x^2}\,dx\\
&= \frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{4}\left(5d^2\right)\int \sqrt{d^2-e^2x^2}\,dx\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{8}\left(5d^4\right)\int \frac{1}{\sqrt{d^2-e\ldots}}\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{1}{8}\left(5d^4\right)\mathrm{Subst}\left(\int \frac{\ldots}{1\ldots}\right.\\
&= \frac{5}{8}d^2x\sqrt{d^2-e^2x^2}+\frac{5d\left(d^2-e^2x^2\right)^{3/2}}{12e}+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}+\frac{5d^4\tan^{-1}\left(\frac{ex}{\sqrt{d^2-\ldots}}\right.}{8e}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 100, normalized size = 0.93

$$\frac{\sqrt{d^2-e^2x^2}\left(16d^3+9d^2ex-16de^2x^2+6e^3x^3\right)}{24e}-\frac{5d^4\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{8\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^3 + 9*d^2*e*x - 16*d*e^2*x^2 + 6*e^3*x^3))/(24*e
) - (5*d^4*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(8*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(92) = 184$.
time = 0.06, size = 244, normalized size = 2.26

method	result
risch	$\frac{(6e^3x^3-16d\,e^2x^2+9d^2ex+16d^3)\sqrt{-e^2x^2+d^2}}{24e}+\frac{5d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}$
default	$\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+\frac{5e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2e^2\left(\right.\right.}{}\right.}{}\right.\right.}{}}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^
2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 115, normalized size = 1.06

$$-\frac{5}{8}i\,d^4\arcsin\left(\frac{xe}{d}+2\right)e^{(-1)}+\frac{5}{4}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^3e^{(-1)}+\frac{5}{8}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^2x+\frac{5}{12}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}de^{(-1)}+\frac{\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}{4\left(xe^2+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -5/8*I*d^4*arcsin(x*e/d + 2)*e^(-1) + 5/4*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d
^3*e^(-1) + 5/8*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^2*x + 5/12*(-x^2*e^2 + d^
2)^(3/2)*d*e^(-1) + 1/4*(-x^2*e^2 + d^2)^(5/2)/(x*e^2 + d*e)
```

Fricas [A]
time = 2.34, size = 79, normalized size = 0.73

$$-\frac{1}{24}\left(30\,d^4\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(6\,x^3e^3-16\,dx^2e^2+9\,d^2xe+16\,d^3\right)\sqrt{-x^2e^2+d^2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(30*d^4*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (6*x^3*e^3 - 1
6*d*x^2*e^2 + 9*d^2*x*e + 16*d^3)*sqrt(-x^2*e^2 + d^2))*e^(-1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.46, size = 350, normalized size = 3.24

$$d^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - 2*d*
e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(
3*e**2), True)) + e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/
(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2
)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*
d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**
2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.
time = 1.63, size = 177, normalized size = 1.64

$$-\frac{\left(240\,d^5\arctan\left(\sqrt{\frac{2\,d}{xe+d}-1}\right)e^5\operatorname{sgn}\left(\frac{1}{xe+d}\right)+\frac{\left(15\,d^5\left(\frac{2\,d}{xe+d}-1\right)^{\frac{7}{2}}e^5\operatorname{sgn}\left(\frac{1}{xe+d}\right)-73\,d^5\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^5\operatorname{sgn}\left(\frac{1}{xe+d}\right)-55\,d^5\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^5\operatorname{sgn}\left(\frac{1}{xe+d}\right)-15\,d^5\sqrt{\frac{2\,d}{xe+d}-1}\,e^5\operatorname{sgn}\left(\frac{1}{xe+d}\right)\right)(xe+d)^4}{d^4}\right)e^{(-6)}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/192*(240*d^5*arctan(sqrt(2*d/(x*e + d) - 1))*e^5*sgn(1/(x*e + d)) + (15*
d^5*(2*d/(x*e + d) - 1)^(7/2)*e^5*sgn(1/(x*e + d)) - 73*d^5*(2*d/(x*e + d)
- 1)^(5/2)*e^5*sgn(1/(x*e + d)) - 55*d^5*(2*d/(x*e + d) - 1)^(3/2)*e^5*sgn(
1/(x*e + d)) - 15*d^5*sqrt(2*d/(x*e + d) - 1)*e^5*sgn(1/(x*e + d)))*(x*e +
d)^4/d^4)*e^(-6)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2\,x^2)^{5/2}}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^2, x)
```

3.163 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx$

Optimal. Leaf size=96

$$d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}-d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)-d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d^3*arctanh(
(-e^2*x^2+d^2)^(1/2)/d)+d*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {866, 1823, 829, 858, 223, 209, 272, 65, 214}

$$d^3\left(-\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+d(d-ex)\sqrt{d^2-e^2x^2}-\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2}-d^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] d*(d - e*x)*Sqrt[d^2 - e^2*x^2] - (d^2 - e^2*x^2)^(3/2)/3 - d^3*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x}\,dx$$

$$= -\frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \frac{(-3d^2e^2+6de^3x)\sqrt{d^2-e^2x^2}}{x}\,dx}{3e^2}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \frac{6d^4e^4-6d^3e^5x}{x\sqrt{d^2-e^2x^2}}\,dx}{6e^4}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + d^4\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \left(d^3e\right)\int \frac{1}{\sqrt{d^2-}}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} + \frac{1}{2}d^4\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - ($$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^4\mathrm{Subst}\left(\int \right.}{}$$

$$= d(d-ex)\sqrt{d^2-e^2x^2} - \frac{1}{3}\left(d^2-e^2x^2\right)^{3/2} - d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3\tanh^{-1}\left(\right.$$

Mathematica [A]
time = 0.31, size = 125, normalized size = 1.30

$$\frac{1}{3}\sqrt{d^2-e^2x^2}\left(2d^2-3dex+e^2x^2\right)+2d^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)+\frac{d^3e\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^2 - 3*d*e*x + e^2*x^2))/3 + 2*d^3*ArcTanh[(Sqrt[-
e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d] + (d^3*e*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2
- e^2*x^2]])/Sqrt[-e^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(86) = 172.
time = 0.06, size = 544, normalized size = 5.67

method	result

default	$-\frac{\frac{\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+d e \left(-\frac{\left(-2 e^{2} \left(x+\frac{d}{e}\right)+2 d e\right) \left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8 e^{2}}+\frac{3 d^{2} \left(-\frac{\left(-2 e^{2} \left(x+\frac{d}{e}\right)+2 d e\right) \sqrt{-\left(x+\frac{d}{e}\right)^{2}}}{4 e^{2}}\right.}{}\right.}{d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+
2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/
e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-1/e/d*(1/3/d/e/(x
+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(1/2))))))+1/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^
2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2
)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.50, size = 101, normalized size = 1.05

$$-d^{3} \arcsin\left(\frac{xe}{d}\right) - d^{3} \log\left(\frac{2\,d^{2}}{|x|} + \frac{2\,\sqrt{-x^{2} e^{2} + d^{2}}\,d}{|x|}\right) - \sqrt{-x^{2} e^{2} + d^{2}}\,dxe + \sqrt{-x^{2} e^{2} + d^{2}}\,d^{2} - \frac{1}{3}\left(-x^{2} e^{2} + d^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -d^3*arcsin(x*e/d) - d^3*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x)
) - sqrt(-x^2*e^2 + d^2)*d*x*e + sqrt(-x^2*e^2 + d^2)*d^2 - 1/3*(-x^2*e^2 +
 d^2)^(3/2)
```

Fricas [A]
time = 2.53, size = 91, normalized size = 0.95

$$2\,d^3\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+d^3\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\frac{1}{3}\left(x^2e^2-3\,dxe+2\,d^2\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 2*d^3*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + d^3*log(-(d - sqrt(-x^
2*e^2 + d^2))/x) + 1/3*(x^2*e^2 - 3*d*x*e + 2*d^2)*sqrt(-x^2*e^2 + d^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.10, size = 267, normalized size = 2.78

$$d^2\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\,\mathrm{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\,\mathrm{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{id^2\,\mathrm{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\,\mathrm{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) -
e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*s
qrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x*
*2) + 1), True)) - 2*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d*
*2)/2, True)) + e**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 -
e**2*x**2)**(3/2)/(3*e**2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^2), x)
```

3.164 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}-\frac{(d^2-e^2x^2)^{3/2}}{x}-\frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -(-e^2*x^2+d^2)^(3/2)/x-1/2*d^2*e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+2*d^2*e*
arctanh((-e^2*x^2+d^2)^(1/2)/d)-1/2*e*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {866, 1821, 829, 858, 223, 209, 272, 65, 214}

$$-\frac{1}{2}d^2e\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{(d^2-e^2x^2)^{3/2}}{x}-\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2}+2d^2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] -1/2*(e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2]) - (d^2 - e^2*x^2)^(3/2)/x - (d^2*e
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/2 + 2*d^2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^2}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{\int \frac{(2d^3e+d^2e^2x)\sqrt{d^2-e^2x^2}}{x}\,dx}{d^2}$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} + \frac{\int \frac{-4d^5e^3-d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{2d^2e^2}$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - (2d^3e)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - \frac{1}{2}(d^2e^2)$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - (d^3e)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right.$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{(2d^3)\,\text{Su}}{}$$

$$= -\frac{1}{2}e(4d+ex)\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{x} - \frac{1}{2}d^2e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 2d^2e\,\tan$$

Mathematica [A]
time = 0.31, size = 128, normalized size = 1.22

$$\frac{1}{2}\left(\frac{\sqrt{d^2-e^2x^2}\,(-2d^2-4dex+e^2x^2)}{x} - 8d^2e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) - d^2\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-2*d^2 - 4*d*e*x + e^2*x^2))/x - 8*d^2*e*ArcTanh[(Sq
rt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] - d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
 Sqrt[d^2 - e^2*x^2]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(93) = 186.
time = 0.08, size = 674, normalized size = 6.42

method	result
risch	$-\frac{d^2\sqrt{-e^2x^2+d^2}}{x} + \frac{e^2x\sqrt{-e^2x^2+d^2}}{2} - \frac{e^2d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}} - 2ed\sqrt{-e^2x^2+d^2} + \frac{2e\,d^3\ln}{}$

default	$\frac{2e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\sqrt{-\left(x+\frac{d}{e}\right)}}{4e^2}\right.}{}\right.\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*e/d^3*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d
/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d^2*(-1/d^2/x*
(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(
-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+1/d^2*(1/3/d/e/(x+d/e)^2*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2))))))-2/d^3*e*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^
2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.49, size = 111, normalized size = 1.06

$$-\frac{1}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e+2\,d^2e\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,xe^2-2\,\sqrt{-x^2e^2+d^2}\,de-\frac{\sqrt{-x^2e^2+d^2}\,d^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*d^2*arcsin(x*e/d)*e + 2*d^2*e*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2
)*d/abs(x)) + 1/2*sqrt(-x^2*e^2 + d^2)*x*e^2 - 2*sqrt(-x^2*e^2 + d^2)*d*e -
 sqrt(-x^2*e^2 + d^2)*d^2/x
```

Fricas [A]
time = 2.65, size = 110, normalized size = 1.05

$$\frac{2\,d^2x\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e-4\,d^2xe\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)-4\,d^2xe+\left(x^2e^2-4\,dxe-2\,d^2\right)\sqrt{-x^2e^2+d^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*x*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e - 4*d^2*x*e*log
(-(d - sqrt(-x^2*e^2 + d^2))/x) - 4*d^2*x*e + (x^2*e^2 - 4*d*x*e - 2*d^2)*s
qrt(-x^2*e^2 + d^2))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 4.46, size = 347, normalized size = 3.30

$$d^2\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e*
*2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1
 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)),
True)) - 2*d*e*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d
/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d
**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**
2/(e**2*x**2) + 1), True)) + e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I
*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d
**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**
2*x**2/d**2)/2, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^2\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^2), x)
```

3.165 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx$

Optimal. Leaf size=110

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d^2-e^2x^2)^{3/2}}{2x^2}+2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(3/2)/x^2+2*d*e^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-1/2*
d*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)+1/2*e*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/
x
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {866, 1821, 827, 858, 223, 209, 272, 65, 214}

$$2de^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x}-\frac{(d^2-e^2x^2)^{3/2}}{2x^2}-\frac{1}{2}de^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] (e*(4*d + e*x)*Sqrt[d^2 - e^2*x^2])/(2*x) - (d^2 - e^2*x^2)^(3/2)/(2*x^2) +
 2*d*e^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - (d*e^2*ArcTanh[Sqrt[d^2 - e^2*
x^2]/d])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^3}\,dx$$
$$= -\frac{(d^2-e^2x^2)^{3/2}}{2x^2} - \frac{\int \frac{(4d^3e-d^2e^2x)\sqrt{d^2-e^2x^2}}{x^2}\,dx}{2d^2}$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{\int \frac{2d^4e^2+8d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{4d^2}$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{1}{2}(d^2e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + (2de^3)\int$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{1}{4}(d^2e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right.$$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}d^2\text{Subst}\Bigg($$
$$= \frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2\tanh^{-}$$

Mathematica [A]
time = 0.33, size = 124, normalized size = 1.13

$$-\frac{(d^2-4dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{2x^2} + de^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) + 2de\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x]
```

```
[Out] -1/2*((d^2 - 4*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/x^2 + d*e^2*ArcTanh[
(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] + 2*d*e*Sqrt[-e^2]*Log[-(Sqrt[-e^2]
*x) + Sqrt[d^2 - e^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(96) = 192.
time = 0.07, size = 816, normalized size = 7.42

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-2e^2x^2-4dex+d^2)}{2x^2} + \frac{2d\,e^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{d^2e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}$

default	$-\frac{3e^2\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\sqrt{-\left(x+\right.}}{4e^2}\right.}{}\right.\right.}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -3*e^2/d^4*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))-2/d^3*e*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-e/d^3*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-

```
1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2
*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*
d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
)))+3/d^4*e^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*(
(-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x))))
```

Maxima [A]
time = 0.48, size = 106, normalized size = 0.96

$$2\,d\arcsin\left(\frac{xe}{d}\right)e^{2}-\frac{1}{2}\,de^{2}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)+\frac{1}{2}\,\sqrt{-x^{2}e^{2}+d^{2}}\,e^{2}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,de}{x}-\frac{\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 2*d*arcsin(x*e/d)*e^2 - 1/2*d*e^2*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)
*d/abs(x)) + 1/2*sqrt(-x^2*e^2 + d^2)*e^2 + 2*sqrt(-x^2*e^2 + d^2)*d*e/x -
1/2*(-x^2*e^2 + d^2)^(3/2)/x^2
```

Fricas [A]
time = 2.76, size = 112, normalized size = 1.02

$$-\frac{8\,dx^{2}\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)e^{2}-dx^{2}e^{2}\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)-2\,dx^{2}e^{2}-\left(2\,x^{2}e^{2}+4\,dxe-d^{2}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*d*x^2*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^2 - d*x^2*e^2*
log(-(d - sqrt(-x^2*e^2 + d^2))/x) - 2*d*x^2*e^2 - (2*x^2*e^2 + 4*d*x*e - d
^2)*sqrt(-x^2*e^2 + d^2))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 4.62, size = 347, normalized size = 3.15

$$d^{2}\left(\begin{cases}-\frac{e\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}{2x}+\frac{e^{2}\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{id^{2}}{2ex^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie}{2x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{2}\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{id}{x\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}+ie\operatorname{acosh}\left(\frac{ex}{d}\right)-\frac{ie^{2}x}{d\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\ -\frac{d}{x\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}-e\operatorname{asin}\left(\frac{ex}{d}\right)+\frac{e^{2}x}{d\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}} & \text{otherwise}\end{cases}\right)+e^{2}\left(\begin{cases}\frac{d^{2}}{ex\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-d\operatorname{acosh}\left(\frac{d}{ex}\right)-\frac{ex}{\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ -\frac{id^{2}}{ex\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+id\operatorname{asin}\left(\frac{d}{ex}\right)+\frac{iex}{\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d),
 True)) - 2*d*e*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*
```

```
x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-
d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x*
*2/d**2)), True)) + e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) -
 d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) >
 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x
/sqrt(-d**2/(e**2*x**2) + 1), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^3\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^2), x)
```

3.166 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx$

Optimal. Leaf size=102

$$\frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{(d^2-e^2x^2)^{3/2}}{3x^3}-e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(3/2)/x^3-e^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-e^3*arct
anh((-e^2*x^2+d^2)^(1/2)/d)+e*(-e*x+d)*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {866, 1821, 825, 858, 223, 209, 272, 65, 214}

$$e^3\left(-\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\right)+\frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2}-\frac{(d^2-e^2x^2)^{3/2}}{3x^3}-e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] (e*(d - e*x)*Sqrt[d^2 - e^2*x^2])/x^2 - (d^2 - e^2*x^2)^(3/2)/(3*x^3) - e^3
*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]] - e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^4}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{3x^3} - \frac{\int \frac{(6d^3e-3d^2e^2x)\sqrt{d^2-e^2x^2}}{x^3}\,dx}{3d^2}$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3x^3} + \frac{\int \frac{12d^5e^3-12d^4e^4x}{x\sqrt{d^2-e^2x^2}}\,dx}{12d^4}$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3x^3} + (de^3)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^4\int \frac{1}{\sqrt{d^2-e}}$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3x^3} + \frac{1}{2}(de^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3x^3} - e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - (de)\text{Subst}\left(\int\right.$$

$$= \frac{e(d-ex)\sqrt{d^2-e^2x^2}}{x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3x^3} - e^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - e^3\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.$$

Mathematica [A]
time = 0.34, size = 121, normalized size = 1.19

$$-\frac{\sqrt{d^2-e^2x^2}\,(d^2-3dex+2e^2x^2)}{3x^3} + 2e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) + (-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x]
```

```
[Out] -1/3*(Sqrt[d^2 - e^2*x^2]*(d^2 - 3*d*e*x + 2*e^2*x^2))/x^3 + 2*e^3*ArcTanh[
(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d] + (-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x)
+ Sqrt[d^2 - e^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 982 vs. 2(92) = 184.
time = 0.07, size = 983, normalized size = 9.64

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(2e^2x^2-3dex+d^2)}{3x^3} - \frac{e^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{e^3d\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{\sqrt{d^2}}$

default	$\frac{4e^3\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+de\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8e^2}+\frac{3d^2\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\sqrt{-\left(x+\frac{d}{e}\right.}}{4e^2}\right.}{}\right.\right.}{d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*e^3/d^5*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/
e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x
+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-2/d^3*e*(-1/2/
d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3
*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2
*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+3/d^4*e^2*(-1/d^2/x*(-e^2*x^2+d^2
)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)
^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+e^2/d^4*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d
```

```
*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/
4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2))))))+1/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e
^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^
2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))-4/d^5*e^3*(1/5*(-e^2*x^2+d^2)
^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1
/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]
time = 0.51, size = 127, normalized size = 1.25

$$-\arcsin\left(\frac{xe}{d}\right)e^3 - e^3\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right) + \frac{\sqrt{-x^2e^2+d^2}\,e^3}{d} - \frac{\sqrt{-x^2e^2+d^2}\,e^2}{x} + \frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{dx^2} - \frac{(-x^2e^2+d^2)^{\frac{3}{2}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -arcsin(x*e/d)*e^3 - e^3*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x)
) + sqrt(-x^2*e^2 + d^2)*e^3/d - sqrt(-x^2*e^2 + d^2)*e^2/x + (-x^2*e^2 + d
^2)^(3/2)*e/(d*x^2) - 1/3*(-x^2*e^2 + d^2)^(3/2)/x^3
```

Fricas [A]
time = 3.37, size = 100, normalized size = 0.98

$$\frac{6\,x^3\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e^3 + 3\,x^3e^3\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right) - (2\,x^2e^2 - 3\,dxe + d^2)\sqrt{-x^2e^2+d^2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*x^3*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e^3 + 3*x^3*e^3*log
(-(d - sqrt(-x^2*e^2 + d^2))/x) - (2*x^2*e^2 - 3*d*x*e + d^2)*sqrt(-x^2*e^2
 + d^2))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 4.18, size = 338, normalized size = 3.31

$$d^2\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases}\right) - 2de\left(\begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*
x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) -
 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))
/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d
), True)) + e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x
**2/d**2)), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: exp(1
)*(1/3*(12*sqrt(2*sageVARd*(sageVARx*exp(1)+sageVARd)^-1/exp(1)*exp(1)-1)*(
2*sageVARd*(sageVARx*exp(1)+sageVARd)^-1/exp(1)*exp(1)-1)^2*exp(1)^2*sign((
sageVARx*exp(1)+
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^4\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^2), x)
```

3.167 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx$

Optimal. Leaf size=108

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(d^2-e^2x^2)^{3/2}}{4x^4}+\frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

```
[Out] -1/4*(-e^2*x^2+d^2)^(3/2)/x^4+2/3*e*(-e^2*x^2+d^2)^(3/2)/d/x^3+5/8*e^4*arct
anh((-e^2*x^2+d^2)^(1/2)/d)/d-5/8*e^2*(-e^2*x^2+d^2)^(1/2)/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {866, 1821, 821, 272, 43, 65, 214}

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2}-\frac{(d^2-e^2x^2)^{3/2}}{4x^4}+\frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3}+\frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] (-5*e^2*Sqrt[d^2 - e^2*x^2])/(8*x^2) - (d^2 - e^2*x^2)^(3/2)/(4*x^4) + (2*e
*(d^2 - e^2*x^2)^(3/2))/(3*d*x^3) + (5*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/
(8*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^5}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{\int \frac{(8d^3e-5d^2e^2x)\sqrt{d^2-e^2x^2}}{x^4}\,dx}{4d^2}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} + \frac{1}{4}(5e^2)\int \frac{\sqrt{d^2-e^2x^2}}{x^3}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} + \frac{1}{8}(5e^2)\,\mathrm{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} - \frac{1}{16}(5e^4)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{d^2-}}\right.$$

$$= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} + \frac{1}{8}(5e^2)\,\mathrm{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,d\right.$$

$$= -\frac{5e^2\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(d^2-e^2x^2)^{3/2}}{4x^4} + \frac{2e(d^2-e^2x^2)^{3/2}}{3dx^3} + \frac{5e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d}$$

Mathematica [A]
time = 0.41, size = 140, normalized size = 1.30

$$-\frac{\sqrt{d^2-e^2x^2}\,(6d^3-16d^2ex+9de^2x^2+16e^3x^3)+15e^4x^4\log\left(d\left(-d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right)-15e^4x^4\log\left(d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{24dx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x]
```

```
[Out] -1/24*(Sqrt[d^2 - e^2*x^2]*(6*d^3 - 16*d^2*e*x + 9*d*e^2*x^2 + 16*e^3*x^3)
+ 15*e^4*x^4*Log[d*(-d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2])] - 15*e^4*x^4*
Log[d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2]])/(d*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. 2(92) = 184.
time = 0.08, size = 1153, normalized size = 10.68

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(16e^3x^3+9d\,e^2x^2-16d^2ex+6d^3)}{24x^4d}+\frac{5e^4\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{8\sqrt{d^2}}$	96
default	Expression too large to display	1153

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -5*e^4/d^6*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))))+3/d^4*e^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))-4/d^5*e^3*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-e^3/d^5*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))-2/d^3*e*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))+5/d^6*e^4*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))

Maxima [A]
time = 0.49, size = 123, normalized size = 1.14

$$\frac{5\,e^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{8\,d}-\frac{5\,\sqrt{-x^2e^2+d^2}\,e^4}{8\,d^2}-\frac{5\,(-x^2e^2+d^2)^{\frac{3}{2}}e^2}{8\,d^2x^2}+\frac{2\,(-x^2e^2+d^2)^{\frac{3}{2}}e}{3\,dx^3}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="maxima")

[Out] 5/8*e^4*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d - 5/8*sqrt(-x^2*e^2 + d^2)*e^4/d^2 - 5/8*(-x^2*e^2 + d^2)^(3/2)*e^2/(d^2*x^2) + 2/3*(-x^2*e^2 + d^2)^(3/2)*e/(d*x^3) - 1/4*(-x^2*e^2 + d^2)^(3/2)/x^4

Fricas [A]

time = 3.42, size = 82, normalized size = 0.76

$$-\frac{15\,x^4e^4\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\left(16\,x^3e^3+9\,dx^2e^2-16\,d^2xe+6\,d^3\right)\sqrt{-x^2e^2+d^2}}{24\,dx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="fricas")

[Out] -1/24*(15*x^4*e^4*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (16*x^3*e^3 + 9*d*x^2*e^2 - 16*d^2*x*e + 6*d^3)*sqrt(-x^2*e^2 + d^2))/(d*x^4)

Sympy [C] Result contains complex when optimal does not.
time = 5.64, size = 422, normalized size = 3.91

$$d^2\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2x}+\frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie}{2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**2,x)

[Out] d**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) - 2*d*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(2*x) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(2*e*x**3*sqrt(-d**2/(e**2*x**2) + 1)) - I*e/(2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**2*asin(d/(e*x))/(2*d), True))

Giac [C] Result contains complex when optimal does not.
time = 1.50, size = 237, normalized size = 2.19

$$\frac{1}{192}\left(\frac{120\,e^3\log\left(\sqrt{\frac{2\,d}{xe+d}-1}+1\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d}-\frac{120\,e^3\log\left(\left|\sqrt{\frac{2\,d}{xe+d}-1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d}+\frac{4\left(15\,e^3\log\left(2\right)-30\,e^3\log\left(i+1\right)+32i\,e^3\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d}-\frac{15\left(\frac{2\,d}{xe+d}-1\right)^{\frac{7}{2}}e^3\operatorname{sgn}\left(\frac{1}{xe+d}\right)+73\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^3\operatorname{sgn}\left(\frac{1}{xe+d}\right)-55\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^3\operatorname{sgn}\left(\frac{1}{xe+d}\right)+15\sqrt{\frac{2\,d}{xe+d}-1}\,e^3\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d\left(\frac{d}{xe+d}-1\right)^4}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="giac")

[Out] 1/192*(120*e^3*log(sqrt(2*d/(x*e + d) - 1) + 1)*sgn(1/(x*e + d))/d - 120*e^3*log(abs(sqrt(2*d/(x*e + d) - 1) - 1))*sgn(1/(x*e + d))/d + 4*(15*e^3*log(2) - 30*e^3*log(I + 1) + 32*I*e^3)*sgn(1/(x*e + d))/d - (15*(2*d/(x*e + d)

```
- 1)^(7/2)*e^3*sgn(1/(x*e + d)) + 73*(2*d/(x*e + d) - 1)^(5/2)*e^3*sgn(1/(x
*e + d)) - 55*(2*d/(x*e + d) - 1)^(3/2)*e^3*sgn(1/(x*e + d)) + 15*sqrt(2*d/
(x*e + d) - 1)*e^3*sgn(1/(x*e + d)))/(d*(d/(x*e + d) - 1)^4))*e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\,x^2\right)^{5/2}}{x^5\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^2), x)
```

3.168 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx$

Optimal. Leaf size=140

$$\frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}+\frac{e(d^2-e^2x^2)^{3/2}}{2dx^4}-\frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3}-\frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(3/2)/x^5+1/2*e*(-e^2*x^2+d^2)^(3/2)/d/x^4-7/15*e^2*(-e
^2*x^2+d^2)^(3/2)/d^2/x^3-1/4*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2+1/4*e
^3*(-e^2*x^2+d^2)^(1/2)/d/x^2
```

Rubi [A]

time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {866, 1821, 849, 821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{3/2}}{5x^5}+\frac{e(d^2-e^2x^2)^{3/2}}{2dx^4}-\frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3}-\frac{e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{4d^2}+\frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (e^3*Sqrt[d^2 - e^2*x^2])/(4*d*x^2) - (d^2 - e^2*x^2)^(3/2)/(5*x^5) + (e*(d
^2 - e^2*x^2)^(3/2))/(2*d*x^4) - (7*e^2*(d^2 - e^2*x^2)^(3/2))/(15*d^2*x^3)
 - (e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(4*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^6}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} - \frac{\int \frac{(10d^3e-7d^2e^2x)\sqrt{d^2-e^2x^2}}{x^5}dx}{5d^2}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} + \frac{\int \frac{(28d^4e^2-10d^3e^3x)\sqrt{d^2-e^2x^2}}{x^4}dx}{20d^4}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\int \frac{\sqrt{d^2-e^2x^2}}{x^3}dx}{2d}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\text{Subst}\left(\int \frac{\sqrt{d^2-e^2x}}{x^2}dx,\right.}{4d}$$

$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} + \frac{e^5\text{Subst}\left(\int\right.}{}$$

$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^3\text{Subst}\left(\int\right.}{}$$

$$= \frac{e^3\sqrt{d^2-e^2x^2}}{4dx^2} - \frac{(d^2-e^2x^2)^{3/2}}{5x^5} + \frac{e(d^2-e^2x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2-e^2x^2)^{3/2}}{15d^2x^3} - \frac{e^5\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.40, size = 112, normalized size = 0.80

$$\frac{\sqrt{d^2-e^2x^2}\,(-12d^4+30d^3ex-16d^2e^2x^2-15de^3x^3+28e^4x^4)+30e^5x^5\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{60d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-12*d^4 + 30*d^3*e*x - 16*d^2*e^2*x^2 - 15*d*e^3*x^3
+ 28*e^4*x^4) + 30*e^5*x^5*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])
/(60*d^2*x^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. 2(120) = 240.
time = 0.07, size = 1349, normalized size = 9.64

method	result	size
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-28e^4x^4+15d\,e^3x^3+16d^2x^2e^2-30d^3ex+12d^4)}{60x^5d^2}-\frac{e^5\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{4d\sqrt{d^2}}$	110
default	Expression too large to display	1349

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 6*e^5/d^7*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/
e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x
+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d^2*(-1/5/d^
2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4
/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(
5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2
)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))))-2/d^
3*e*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+
d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3
/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2
*x^2+d^2)^(1/2))/x))))))-4/d^5*e^3*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e
^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x
^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))
/x)))))+5/d^6*e^4*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2
+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^
2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))
+e^4/d^6*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1
/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^
2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)
/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^
(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))+3/d^4*e^2*(-1/3/d^2/x^3*(
-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1
/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*
x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d
^2)^(1/2)))))))-6/d^7*e^5*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)
^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-
e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [A]

time = 0.48, size = 146, normalized size = 1.04

$$-\frac{e^5\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{4\,d^2}+\frac{\sqrt{-x^2e^2+d^2}\,e^5}{4\,d^3}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^3}{4\,d^3x^2}-\frac{7\,(-x^2e^2+d^2)^{\frac{3}{2}}e^2}{15\,d^2x^3}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{2\,dx^4}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^5*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^2 + 1/4*sqrt
(-x^2*e^2 + d^2)*e^5/d^3 + 1/4*(-x^2*e^2 + d^2)^(3/2)*e^3/(d^3*x^2) - 7/15*
(-x^2*e^2 + d^2)^(3/2)*e^2/(d^2*x^3) + 1/2*(-x^2*e^2 + d^2)^(3/2)*e/(d*x^4)
 - 1/5*(-x^2*e^2 + d^2)^(3/2)/x^5
```

Fricas [A]
time = 4.02, size = 92, normalized size = 0.66

$$\frac{15\,x^5e^5\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\left(28\,x^4e^4-15\,dx^3e^3-16\,d^2x^2e^2+30\,d^3xe-12\,d^4\right)\sqrt{-x^2e^2+d^2}}{60\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*x^5*e^5*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (28*x^4*e^4 - 15*d*x^
3*e^3 - 16*d^2*x^2*e^2 + 30*d^3*x*e - 12*d^4)*sqrt(-x^2*e^2 + d^2))/(d^2*x^
5)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.40, size = 660, normalized size = 4.71

$$d^2\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2
*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e
**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**
2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d*
*2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*
x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5
+ 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 +
15*d*e**2*x**7), True)) - 2*d*e*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*
x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt
(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2
)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqr
t(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) -
 I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**2*Piecewise((-e*sqrt(d**2/(e**2
*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(
```

```
e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(
-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 1.35, size = 265, normalized size = 1.89

$$-\frac{1}{960}\left(\frac{240\,e^4\log\left(\sqrt{\frac{2\,d}{xe+d}-1}+1\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^2}-\frac{240\,e^4\log\left(\left|\sqrt{\frac{2\,d}{xe+d}-1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^2}+\frac{8\,(15\,e^4\log(2)-30\,e^4\log(i+1)+56i\,e^4)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^2}-\frac{15\left(\frac{2d}{xe+d}-1\right)^{\frac{9}{2}}e^4\operatorname{sgn}\left(\frac{1}{xe+d}\right)+250\left(\frac{2d}{xe+d}-1\right)^{\frac{7}{2}}e^4\operatorname{sgn}\left(\frac{1}{xe+d}\right)-128\left(\frac{2d}{xe+d}-1\right)^{\frac{5}{2}}e^4\operatorname{sgn}\left(\frac{1}{xe+d}\right)+70\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}e^4\operatorname{sgn}\left(\frac{1}{xe+d}\right)-15\sqrt{\frac{2\,d}{xe+d}-1}\,e^4\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^2\left(\frac{d}{xe+d}-1\right)^5}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/960*(240*e^4*log(sqrt(2*d/(x*e + d) - 1) + 1)*sgn(1/(x*e + d))/d^2 - 240
*e^4*log(abs(sqrt(2*d/(x*e + d) - 1) - 1))*sgn(1/(x*e + d))/d^2 + 8*(15*e^4
*log(2) - 30*e^4*log(I + 1) + 56*I*e^4)*sgn(1/(x*e + d))/d^2 - (15*(2*d/(x*
e + d) - 1)^(9/2)*e^4*sgn(1/(x*e + d)) + 250*(2*d/(x*e + d) - 1)^(7/2)*e^4*
sgn(1/(x*e + d)) - 128*(2*d/(x*e + d) - 1)^(5/2)*e^4*sgn(1/(x*e + d)) + 70*
(2*d/(x*e + d) - 1)^(3/2)*e^4*sgn(1/(x*e + d)) - 15*sqrt(2*d/(x*e + d) - 1)
*e^4*sgn(1/(x*e + d)))/(d^2*(d/(x*e + d) - 1)^5))*e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^6\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^2), x)
```

3.169 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^7(d+ex)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}-\frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4}+\frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3}+\frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}$$

```
[Out] -1/6*(-e^2*x^2+d^2)^(3/2)/x^6+2/5*e*(-e^2*x^2+d^2)^(3/2)/d/x^5-3/8*e^2*(-e^
2*x^2+d^2)^(3/2)/d^2/x^4+4/15*e^3*(-e^2*x^2+d^2)^(3/2)/d^3/x^3+3/16*e^6*arc
tanh((-e^2*x^2+d^2)^(1/2)/d)/d^3-3/16*e^4*(-e^2*x^2+d^2)^(1/2)/d^2/x^2
```

Rubi [A]
time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {866, 1821, 849, 821, 272, 43, 65, 214}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}}{6x^6}+\frac{2e\left(d^2-e^2x^2\right)^{3/2}}{5dx^5}-\frac{3e^2\left(d^2-e^2x^2\right)^{3/2}}{8d^2x^4}-\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2}+\frac{3e^6\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3}+\frac{4e^3\left(d^2-e^2x^2\right)^{3/2}}{15d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] (-3*e^4*Sqrt[d^2 - e^2*x^2])/(16*d^2*x^2) - (d^2 - e^2*x^2)^(3/2)/(6*x^6) +
 (2*e*(d^2 - e^2*x^2)^(3/2))/(5*d*x^5) - (3*e^2*(d^2 - e^2*x^2)^(3/2))/(8*d
^2*x^4) + (4*e^3*(d^2 - e^2*x^2)^(3/2))/(15*d^3*x^3) + (3*e^6*ArcTanh[Sqrt[
d^2 - e^2*x^2]/d])/(16*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^7(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^7}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} - \frac{\int \frac{(12d^3e-9d^2e^2x)\sqrt{d^2-e^2x^2}}{x^6}dx}{6d^2}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} + \frac{\int \frac{(45d^4e^2-24d^3e^3x)\sqrt{d^2-e^2x^2}}{x^5}dx}{30d^4}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} - \frac{\int \frac{(96d^5e^3-45d^4e^4x)\sqrt{d^2-e^2}}{x^4}}{120d^6}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3} + \frac{(3e^4)}{}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2-e^2x^2)^{3/2}}{15d^3x^3} + \frac{(3e^4)}{}$$

$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2}{15}$$

$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2}{15}$$

$$= -\frac{3e^4\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{6x^6} + \frac{2e(d^2-e^2x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2-e^2x^2)^{3/2}}{8d^2x^4} + \frac{4e^3(d^2}{15}$$

Mathematica [**A**]
time = 0.42, size = 123, normalized size = 0.73

$$-\frac{\sqrt{d^2-e^2x^2}\,(40d^5-96d^4ex+50d^3e^2x^2+32d^2e^3x^3-45de^4x^4+64e^5x^5)+90e^6x^6\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{240d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x]
```

```
[Out] -1/240*(Sqrt[d^2 - e^2*x^2]*(40*d^5 - 96*d^4*e*x + 50*d^3*e^2*x^2 + 32*d^2*
e^3*x^3 - 45*d*e^4*x^4 + 64*e^5*x^5) + 90*e^6*x^6*ArcTanh[(Sqrt[-e^2]*x - S
qrt[d^2 - e^2*x^2])/d])/(d^3*x^6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1549 vs. $2(145) = 290$.
time = 0.07, size = 1550, normalized size = 9.17

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(64e^5x^5-45d\,e^4x^4+32d^2e^3x^3+50x^2d^3e^2-96d^4xe+40d^5)}{240x^6d^3}+\frac{3e^6\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{16d^2\sqrt{d^2}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -7*e^6/d^8*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d
/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(
x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*
arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-2/d^3*e*(-1/5
/d^2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2
)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2
)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(
1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))))+3
/d^4*e^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2
*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^
2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x))))))+5/d^6*e^4*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-
5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-
e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x)))))-6/d^7*e^5*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^
2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x
^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)))))-e^5/d^7*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e
/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*
e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2
*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((
e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))-4/d^5*e^3*(-1/3/d^2/
x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d
^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*
(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*
x^2+d^2)^(1/2)))))))+7/d^8*e^6*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2
+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/
2)*(-e^2*x^2+d^2)^(1/2))/x))))+1/d^2*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(7/2)-1/6
*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2
*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^
2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*
(-e^2*x^2+d^2)^(1/2))/x)))))))
```

Maxima [A]

time = 0.48, size = 169, normalized size = 1.00

$$\frac{3\,e^{6}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-x^{2}e^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{3}}-\frac{3\,\sqrt{-x^{2}e^{2}+d^{2}}\,e^{6}}{16\,d^{4}}-\frac{3\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}e^{4}}{16\,d^{4}x^{2}}+\frac{4\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}e^{3}}{15\,d^{3}x^{3}}-\frac{3\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}e^{2}}{8\,d^{2}x^{4}}+\frac{2\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}e}{5\,dx^{5}}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 3/16*e^6*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^3 - 3/16*sqr
t(-x^2*e^2 + d^2)*e^6/d^4 - 3/16*(-x^2*e^2 + d^2)^(3/2)*e^4/(d^4*x^2) + 4/1
5*(-x^2*e^2 + d^2)^(3/2)*e^3/(d^3*x^3) - 3/8*(-x^2*e^2 + d^2)^(3/2)*e^2/(d^
2*x^4) + 2/5*(-x^2*e^2 + d^2)^(3/2)*e/(d*x^5) - 1/6*(-x^2*e^2 + d^2)^(3/2)/
x^6
```

Fricas [A]
time = 3.37, size = 102, normalized size = 0.60

$$-\frac{45\,x^{6}e^{6}\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+(64\,x^{5}e^{5}-45\,dx^{4}e^{4}+32\,d^{2}x^{3}e^{3}+50\,d^{3}x^{2}e^{2}-96\,d^{4}xe+40\,d^{5})\sqrt{-x^{2}e^{2}+d^{2}}}{240\,d^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/240*(45*x^6*e^6*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (64*x^5*e^5 - 45*d*
x^4*e^4 + 32*d^2*x^3*e^3 + 50*d^3*x^2*e^2 - 96*d^4*x*e + 40*d^5)*sqrt(-x^2*
e^2 + d^2))/(d^3*x^6)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.19, size = 808, normalized size = 4.78

$$d^{2}\left(\begin{cases}-\frac{d^{2}}{6ex^{7}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{5e}{24x^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{3}}{48d^{2}x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{5}}{16d^{4}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{6}\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^{5}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{id^{2}}{6ex^{7}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{5ie}{24x^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{3}}{48d^{2}x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{5}}{16d^{4}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{6}\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^{5}} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}\frac{3id^{3}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}-\frac{4ide^{2}x^{2}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}+\frac{2ie^{6}x^{6}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}}-\frac{ie^{4}x^{4}\sqrt{-1+\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{for}\ \left|\frac{e^{2}x^{2}}{d^{2}}\right|>1\\ \frac{3d^{3}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}-\frac{4de^{2}x^{2}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{2}x^{5}+15e^{2}x^{7}}+\frac{2e^{6}x^{6}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{5}x^{5}+15d^{3}e^{2}x^{7}}-\frac{e^{4}x^{4}\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}}{-15d^{3}x^{5}+15de^{2}x^{7}} & \text{otherwise}\end{cases}\right)+e^{2}\left(\begin{cases}-\frac{d^{2}}{4ex^{5}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{3e}{8x^{3}\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}-\frac{e^{3}}{8d^{2}x\sqrt{\frac{d^{2}}{e^{2}x^{2}}-1}}+\frac{e^{4}\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{for}\ \left|\frac{d^{2}}{e^{2}x^{2}}\right|>1\\ \frac{id^{2}}{4ex^{5}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{3ie}{8x^{3}\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}+\frac{ie^{3}}{8d^{2}x\sqrt{-\frac{d^{2}}{e^{2}x^{2}}+1}}-\frac{ie^{4}\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^{3}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*
sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d
**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*
sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1
)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) - 2*d*e*Piecewise((3*I*d**3*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt
```

```
(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2)
 > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*
d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**
6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*
x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + e*
*2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqr
t(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**
4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sq
rt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I
*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3
), True))
```

Giac [C] Result contains complex when optimal does not.
time = 1.50, size = 293, normalized size = 1.73

$$\frac{1}{7680}\left(\frac{1440\,e^{5}\log\left(\sqrt{\frac{2\,d}{xe+d}-1}+1\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{3}}-\frac{1440\,e^{5}\log\left(\left|\sqrt{\frac{2\,d}{xe+d}-1}-1\right|\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{3}}+\frac{16\left(45\,e^{5}\log\left(2\right)-90\,e^{5}\log\left(i+1\right)+128i\,e^{5}\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{3}}-\frac{45\left(\frac{2d}{xe+d}-1\right)^{\frac{11}{2}}e^{5}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+1025\left(\frac{2d}{xe+d}-1\right)^{\frac{9}{2}}e^{5}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-174\left(\frac{2d}{xe+d}-1\right)^{\frac{7}{2}}e^{5}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+594\left(\frac{2d}{xe+d}-1\right)^{\frac{5}{2}}e^{5}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-255\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}e^{5}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+45\sqrt{\frac{2\,d}{xe+d}-1}\,e^{5}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{3}\left(\frac{d}{xe+d}-1\right)^{6}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/7680*(1440*e^5*log(sqrt(2*d/(x*e + d) - 1) + 1)*sgn(1/(x*e + d))/d^3 - 14
40*e^5*log(abs(sqrt(2*d/(x*e + d) - 1) - 1))*sgn(1/(x*e + d))/d^3 + 16*(45*
e^5*log(2) - 90*e^5*log(I + 1) + 128*I*e^5)*sgn(1/(x*e + d))/d^3 - (45*(2*d
/(x*e + d) - 1)^(11/2)*e^5*sgn(1/(x*e + d)) + 1025*(2*d/(x*e + d) - 1)^(9/2
)*e^5*sgn(1/(x*e + d)) - 174*(2*d/(x*e + d) - 1)^(7/2)*e^5*sgn(1/(x*e + d))
 + 594*(2*d/(x*e + d) - 1)^(5/2)*e^5*sgn(1/(x*e + d)) - 255*(2*d/(x*e + d)
- 1)^(3/2)*e^5*sgn(1/(x*e + d)) + 45*sqrt(2*d/(x*e + d) - 1)*e^5*sgn(1/(x*e
 + d)))/(d^3*(d/(x*e + d) - 1)^6))*e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^7\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^7*(d + e*x)^2), x)
```

3.170 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx$

Optimal. Leaf size=198

$$\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}-\frac{(d^2-e^2x^2)^{3/2}}{7x^7}+\frac{e(d^2-e^2x^2)^{3/2}}{3dx^6}-\frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5}+\frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4}-\frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3}$$

```
[Out] -1/7*(-e^2*x^2+d^2)^(3/2)/x^7+1/3*e*(-e^2*x^2+d^2)^(3/2)/d/x^6-11/35*e^2*(-
e^2*x^2+d^2)^(3/2)/d^2/x^5+1/4*e^3*(-e^2*x^2+d^2)^(3/2)/d^3/x^4-22/105*e^4*
(-e^2*x^2+d^2)^(3/2)/d^4/x^3-1/8*e^7*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/
8*e^5*(-e^2*x^2+d^2)^(1/2)/d^3/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {866, 1821, 849, 821, 272, 43, 65, 214}

$$-\frac{(d^2-e^2x^2)^{3/2}}{7x^7}+\frac{e(d^2-e^2x^2)^{3/2}}{3dx^6}-\frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5}-\frac{e^7\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^4}-\frac{22e^4(d^2-e^2x^2)^{3/2}}{105d^4x^3}+\frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2}+\frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (e^5*Sqrt[d^2 - e^2*x^2])/(8*d^3*x^2) - (d^2 - e^2*x^2)^(3/2)/(7*x^7) + (e*
(d^2 - e^2*x^2)^(3/2))/(3*d*x^6) - (11*e^2*(d^2 - e^2*x^2)^(3/2))/(35*d^2*x
^5) + (e^3*(d^2 - e^2*x^2)^(3/2))/(4*d^3*x^4) - (22*e^4*(d^2 - e^2*x^2)^(3/
2))/(105*d^4*x^3) - (e^7*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(8*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^8(d+ex)^2}\,dx = \int \frac{(d-ex)^2\sqrt{d^2-e^2x^2}}{x^8}\,dx$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} - \frac{\int \frac{(14d^3e-11d^2e^2x)\sqrt{d^2-e^2x^2}}{x^7}\,dx}{7d^2}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} + \frac{\int \frac{(66d^4e^2-42d^3e^3x)\sqrt{d^2-e^2x^2}}{x^6}\,dx}{42d^4}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} - \frac{\int \frac{(210d^5e^3-132d^4e^4x)\sqrt{d^2-e}}{x^5}}{210d^6}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} + \int \frac{(528d}{}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d}{1}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e^2x^2)^{3/2}}{4d^3x^4} - \frac{22e^4(d}{1}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e}{4d^3x}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e}{4d^3x}$$

$$= \frac{e^5\sqrt{d^2-e^2x^2}}{8d^3x^2} - \frac{(d^2-e^2x^2)^{3/2}}{7x^7} + \frac{e(d^2-e^2x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2-e^2x^2)^{3/2}}{35d^2x^5} + \frac{e^3(d^2-e}{4d^3x}$$

Mathematica [A]
time = 0.50, size = 134, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2x^2}\,(-120d^6+280d^5ex-144d^4e^2x^2-70d^3e^3x^3+88d^2e^4x^4-105de^5x^5+176e^6x^6)+210e^7x^7\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{840d^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^6 + 280*d^5*e*x - 144*d^4*e^2*x^2 - 70*d^3*e^3
*x^3 + 88*d^2*e^4*x^4 - 105*d*e^5*x^5 + 176*e^6*x^6) + 210*e^7*x^7*ArcTanh[
(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(840*d^4*x^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. $2(170) = 340$.
time = 0.08, size = 1575, normalized size = 7.95

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-176e^6x^6+105d\,e^5x^5-88d^2e^4x^4+70d^3e^3x^3+144d^4e^2x^2-280e\,d^5x+120d^6)}{840x^7d^4} - \frac{e^7\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e}}{x}\right.}{8d^3\sqrt{d^2}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8*e^7/d^9*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/
e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x
+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+3/d^4*e^2*(-1/
5/d^2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/
2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^
2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^
(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))))-
4/d^5*e^3*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^
2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d
^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)
*(-e^2*x^2+d^2)^(1/2))/x))))))-6/d^7*e^5*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)
-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((
-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^
(1/2))/x)))))+7/d^8*e^6*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e
^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*
x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2
))))))+e^6/d^8*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*
e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d
*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+
2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))+5/d^6*e^4*(-1/3/d^2
/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/
d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2
*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2
*x^2+d^2)^(1/2)))))))-8/d^9*e^7*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^
2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1
/2)*(-e^2*x^2+d^2)^(1/2))/x))))-2/d^3*e*(-1/6/d^2/x^6*(-e^2*x^2+d^2)^(7/2)-
1/6*e^2/d^2*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-
e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2
```

```
+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/
2)*(-e^2*x^2+d^2)^(1/2))/x)))))))-1/7/d^4/x^7*(-e^2*x^2+d^2)^(7/2)
```

Maxima [A]
time = 0.49, size = 192, normalized size = 0.97

$$-\frac{e^7\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-x^2e^2+d^2}\,d}{|x|}\right)}{8\,d^4}+\frac{\sqrt{-x^2e^2+d^2}\,e^7}{8\,d^5}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^5}{8\,d^5x^2}-\frac{22\,(-x^2e^2+d^2)^{\frac{3}{2}}e^4}{105\,d^4x^3}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e^3}{4\,d^3x^4}-\frac{11\,(-x^2e^2+d^2)^{\frac{3}{2}}e^2}{35\,d^2x^5}+\frac{(-x^2e^2+d^2)^{\frac{3}{2}}e}{3\,dx^6}-\frac{(-x^2e^2+d^2)^{\frac{3}{2}}}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*e^7*log(2*d^2/abs(x) + 2*sqrt(-x^2*e^2 + d^2)*d/abs(x))/d^4 + 1/8*sqrt
(-x^2*e^2 + d^2)*e^7/d^5 + 1/8*(-x^2*e^2 + d^2)^(3/2)*e^5/(d^5*x^2) - 22/10
5*(-x^2*e^2 + d^2)^(3/2)*e^4/(d^4*x^3) + 1/4*(-x^2*e^2 + d^2)^(3/2)*e^3/(d^
3*x^4) - 11/35*(-x^2*e^2 + d^2)^(3/2)*e^2/(d^2*x^5) + 1/3*(-x^2*e^2 + d^2)^
(3/2)*e/(d*x^6) - 1/7*(-x^2*e^2 + d^2)^(3/2)/x^7
```

Fricas [A]
time = 4.23, size = 112, normalized size = 0.57

$$\frac{105\,x^7e^7\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(176\,x^6e^6-105\,dx^5e^5+88\,d^2x^4e^4-70\,d^3x^3e^3-144\,d^4x^2e^2+280\,d^5xe-120\,d^6)\sqrt{-x^2e^2+d^2}}{840\,d^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*x^7*e^7*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (176*x^6*e^6 - 105*
d*x^5*e^5 + 88*d^2*x^4*e^4 - 70*d^3*x^3*e^3 - 144*d^4*x^2*e^2 + 280*d^5*x*e
 - 120*d^6)*sqrt(-x^2*e^2 + d^2))/(d^4*x^7)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.39, size = 835, normalized size = 4.22

$$d^2\left(\begin{cases}-\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{7x^6}+\frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{35d^2x^4}+\frac{4e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^4x^2}+\frac{8e^7\sqrt{\frac{d^2}{e^2x^2}-1}}{105d^6} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{7x^6}+\frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{35d^2x^4}+\frac{4ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^4x^2}+\frac{8ie^7\sqrt{-\frac{d^2}{e^2x^2}+1}}{105d^6} & \text{otherwise}\end{cases}\right)-2de\left(\begin{cases}-\frac{d^2}{6ex^7\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{5e}{24x^5\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^3}{48d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}}-\frac{e^5}{16d^4x\sqrt{\frac{d^2}{e^2x^2}-1}}+\frac{e^6\operatorname{acosh}\left(\frac{d}{ex}\right)}{16d^5} & \text{for}\ \left|\frac{d^2}{e^2x^2}\right|>1\\ \frac{id^2}{6ex^7\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{5ie}{24x^5\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^3}{48d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}}+\frac{ie^5}{16d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}}-\frac{ie^6\operatorname{asin}\left(\frac{d}{ex}\right)}{16d^5} & \text{otherwise}\end{cases}\right)+e^2\left(\begin{cases}\frac{3id^3\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4ide^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2ie^6x^6\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{ie^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^3\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}-\frac{4de^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^2x^5+15e^2x^7}+\frac{2e^6x^6\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^5x^5+15d^3e^2x^7}-\frac{e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}}{-15d^3x^5+15de^2x^7} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**8/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e*
*2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*
x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2))
 > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2
*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4
*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 2*d*e*Pi
ecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d*
```

```
*2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**
5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), A
bs(d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) -
 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d
**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*
e**6*asin(d/(e*x))/(16*d**5), True)) + e**2*Piecewise((3*I*d**3*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**
2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (
3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x
**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*s
qrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqr
t(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True))
```

Giac [C] Result contains complex when optimal does not.
time = 1.10, size = 321, normalized size = 1.62

$$-\frac{1}{53760}\left(\frac{6720\,e^{6}\log\left(\sqrt{\frac{2\,d}{xe+d}-1}+1\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{4}}-\frac{6720\,e^{6}\log\left(\left|\sqrt{\frac{2\,d}{xe+d}-1}-1\right|\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{4}}+\frac{32\,\left(105\,e^{6}\log\left(2\right)-210\,e^{6}\log\left(i+1\right)+352i\,e^{6}\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{4}}-\frac{105\,\left(\frac{2\,d}{xe+d}-1\right)^{\frac{13}{2}}e^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+3780\,\left(\frac{2\,d}{xe+d}-1\right)^{\frac{11}{2}}e^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+189\,\left(\frac{2\,d}{xe+d}-1\right)^{\frac{9}{2}}e^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+4992\,\left(\frac{2\,d}{xe+d}-1\right)^{\frac{7}{2}}e^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-1981\,\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)+700\,\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)-105\,\sqrt{\frac{2\,d}{xe+d}-1}\,e^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{4}\left(\frac{d}{xe+d}-1\right)^{7}}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/53760*(6720*e^6*log(sqrt(2*d/(x*e + d) - 1) + 1)*sgn(1/(x*e + d))/d^4 -
6720*e^6*log(abs(sqrt(2*d/(x*e + d) - 1) - 1))*sgn(1/(x*e + d))/d^4 + 32*(1
05*e^6*log(2) - 210*e^6*log(I + 1) + 352*I*e^6)*sgn(1/(x*e + d))/d^4 - (105
*(2*d/(x*e + d) - 1)^(13/2)*e^6*sgn(1/(x*e + d)) + 3780*(2*d/(x*e + d) - 1)
^(11/2)*e^6*sgn(1/(x*e + d)) + 189*(2*d/(x*e + d) - 1)^(9/2)*e^6*sgn(1/(x*e
 + d)) + 4992*(2*d/(x*e + d) - 1)^(7/2)*e^6*sgn(1/(x*e + d)) - 1981*(2*d/(x
*e + d) - 1)^(5/2)*e^6*sgn(1/(x*e + d)) + 700*(2*d/(x*e + d) - 1)^(3/2)*e^6
*sgn(1/(x*e + d)) - 105*sqrt(2*d/(x*e + d) - 1)*e^6*sgn(1/(x*e + d)))/(d^4*
(d/(x*e + d) - 1)^7))*e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^8\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2), x)
```

3.171 $\int \frac{x^4}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] -1/5*d^3*(-e*x+d)^2/e^5/(-e^2*x^2+d^2)^(5/2)+17/15*d^2*(-e*x+d)/e^5/(-e^2*x
^2+d^2)^(3/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-2/15*(-13*e*x+15*d)/e^5/
(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1649, 1828, 12, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{17d^2(d-ex)}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}}-\frac{d^3(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -1/5*(d^3*(d - e*x)^2)/(e^5*(d^2 - e^2*x^2)^(5/2)) + (17*d^2*(d - e*x))/(15
*e^5*(d^2 - e^2*x^2)^(3/2)) - (2*(15*d - 13*e*x))/(15*e^5*Sqrt[d^2 - e^2*x^
2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{x^4(d-ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)\left(\frac{2d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{\frac{11d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}}{(d^2-e^2x^2)^{3/2}}dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{15d}{e^4\sqrt{d^2-}}}{15}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{1}{\sqrt{d^2-}}}{e^4}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{}$$

Mathematica [A]
time = 0.41, size = 113, normalized size = 0.92

$$\frac{\sqrt{d^2-e^2x^2}\,(16d^3+17d^2ex-22de^2x^2-26e^3x^3)}{15e^5(-d+ex)(d+ex)^3} + \frac{\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^4\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(16*d^3 + 17*d^2*e*x - 22*d*e^2*x^2 - 26*e^3*x^3))/(15
*e^5*(-d + e*x)*(d + e*x)^3) + Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]/(
e^4*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(109) = 218.
time = 0.08, size = 363, normalized size = 2.95

method	result

default	$\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}-\frac{2d}{e^5\sqrt{-e^2x^2+d^2}}+\frac{3x}{e^4\sqrt{-e^2x^2+d^2}}-\frac{4d^3\left(-\frac{\ldots}{3de\left(x+\frac{d}{e}\right)\sqrt{\ldots}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2)))-2*d/e^5/(-e^2*x^2+d^2)^(1/2)+3/e^4*x/(-e^2*x^2+d^2)^(1
/2)-4/e^5*d^3*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/
d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/e^6*d^4*
(-1/5/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5*e/d*(-1/3/d/e/
(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2*d*
e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.49, size = 156, normalized size = 1.27

$$-\frac{d^3}{5\left(\sqrt{-x^2e^2+d^2}\,x^2e^7+2\sqrt{-x^2e^2+d^2}\,dxe^6+\sqrt{-x^2e^2+d^2}\,d^2e^5\right)}-\arcsin\left(\frac{xe}{d}\right)e^{(-5)}+\frac{26\,xe^{(-4)}}{15\sqrt{-x^2e^2+d^2}}-\frac{2\,de^{(-5)}}{\sqrt{-x^2e^2+d^2}}+\frac{17\,d^2}{15\left(\sqrt{-x^2e^2+d^2}\,xe^6+\sqrt{-x^2e^2+d^2}\,de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*d^3/(sqrt(-x^2*e^2 + d^2)*x^2*e^7 + 2*sqrt(-x^2*e^2 + d^2)*d*x*e^6 + s
qrt(-x^2*e^2 + d^2)*d^2*e^5) - arcsin(x*e/d)*e^(-5) + 26/15*x*e^(-4)/sqrt(-
x^2*e^2 + d^2) - 2*d*e^(-5)/sqrt(-x^2*e^2 + d^2) + 17/15*d^2/(sqrt(-x^2*e^2
 + d^2)*x*e^6 + sqrt(-x^2*e^2 + d^2)*d*e^5)
```

Fricas [A]
time = 3.63, size = 161, normalized size = 1.31

$$-\frac{16\,x^4e^4+32\,dx^3e^3-32\,d^3xe-16\,d^4-30\left(x^4e^4+2\,dx^3e^3-2\,d^3xe-d^4\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(26\,x^3e^3+22\,dx^2e^2-17\,d^2xe-16\,d^3\right)\sqrt{-x^2e^2+d^2}}{15\left(x^4e^9+2\,dx^3e^8-2\,d^3xe^6-d^4e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(16*x^4*e^4 + 32*d*x^3*e^3 - 32*d^3*x*e - 16*d^4 - 30*(x^4*e^4 + 2*d*
x^3*e^3 - 2*d^3*x*e - d^4)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (
```

```
26*x^3*e^3 + 22*d*x^2*e^2 - 17*d^2*x*e - 16*d^3)*sqrt(-x^2*e^2 + d^2))/(x^4
*e^9 + 2*d*x^3*e^8 - 2*d^3*x*e^6 - d^4*e^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/exp
(1)*(1/32768*(4096/5*sqrt(2*sageVARd*(sageVARx*exp(1)+sageVARd)^-1/exp(1)*e
xp(1)-1)*(2*sageVARd*(sageVARx*exp(1)+sageVARd)^-1/exp(1)*exp(1)-1)^2*exp(1
)^16*(-sign((sag
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(d^2-e^2\,x^2\right)}^{3/2}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(x^4/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.172 $\int \frac{x^3}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{4d(d-ex)}{5e^4(d^2-e^2x^2)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*d^2*(-e*x+d)^2/e^4/(-e^2*x^2+d^2)^(5/2)-4/5*d*(-e*x+d)/e^4/(-e^2*x^2+d^
2)^(3/2)+1/5*(-2*e*x+5*d)/d/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {866, 1649, 651}

$$\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}}-\frac{4d(d-ex)}{5e^4(d^2-e^2x^2)^{3/2}}+\frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (4*d*(d - e*x))/(5*e^4*(d
^2 - e^2*x^2)^(3/2)) + (5*d - 2*e*x)/(5*d*e^4*Sqrt[d^2 - e^2*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

```
&& GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^3(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)\left(-\frac{2d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d} \\
&= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{-\frac{6d^3}{e^3}+\frac{15d^2x}{e^2}}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2} \\
&= \frac{d^2(d-ex)^2}{5e^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4d(d-ex)}{5e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 70, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\,\left(2d^3+4d^2ex+de^2x^2-2e^3x^3\right)}{5de^4(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^3 + 4*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(5*d*e^4*
(d - e*x)*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(87) = 174$.
time = 0.06, size = 309, normalized size = 3.12

method	result
gosper	$\frac{(-ex+d)\left(-2e^3x^3+d\,e^2x^2+4d^2ex+2d^3\right)}{5(ex+d)d\,e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}$
trager	$\frac{\left(-2e^3x^3+d\,e^2x^2+4d^2ex+2d^3\right)\sqrt{-e^2x^2+d^2}}{5d\,e^4(ex+d)^3(-ex+d)}$

default	$\frac{1}{e^4\sqrt{-e^2x^2+d^2}}-\frac{2x}{d\,e^3\sqrt{-e^2x^2+d^2}}+\frac{3d^2\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{}{3e\,d^3\sqrt{-(x}}\right.}{e^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4/(-e^2*x^2+d^2)^(1/2)-2/d/e^3*x/(-e^2*x^2+d^2)^(1/2)+3*d^2/e^4*(-1/3/d
/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2
*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))-d^3/e^5*(-1/5/d/e/(x+d/e)^2/(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5*e/d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.28, size = 143, normalized size = 1.44

$$\frac{d^2}{5\left(\sqrt{-x^2e^2+d^2}\,x^2e^6+2\sqrt{-x^2e^2+d^2}\,dxe^5+\sqrt{-x^2e^2+d^2}\,d^2e^4\right)}-\frac{2\,xe^{(-3)}}{5\sqrt{-x^2e^2+d^2}\,d}+\frac{e^{(-4)}}{\sqrt{-x^2e^2+d^2}}-\frac{4\,d}{5\left(\sqrt{-x^2e^2+d^2}\,xe^5+\sqrt{-x^2e^2+d^2}\,de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*d^2/(sqrt(-x^2*e^2 + d^2)*x^2*e^6 + 2*sqrt(-x^2*e^2 + d^2)*d*x*e^5 + sq
rt(-x^2*e^2 + d^2)*d^2*e^4) - 2/5*x*e^(-3)/(sqrt(-x^2*e^2 + d^2)*d) + e^(-4
)/sqrt(-x^2*e^2 + d^2) - 4/5*d/(sqrt(-x^2*e^2 + d^2)*x*e^5 + sqrt(-x^2*e^2
+ d^2)*d*e^4)
```

Fricas [A]
time = 3.23, size = 109, normalized size = 1.10

$$\frac{2\,x^4e^4+4\,dx^3e^3-4\,d^3xe-2\,d^4+\left(2\,x^3e^3-dx^2e^2-4\,d^2xe-2\,d^3\right)\sqrt{-x^2e^2+d^2}}{5\left(dx^4e^8+2\,d^2x^3e^7-2\,d^4xe^5-d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*x^4*e^4 + 4*d*x^3*e^3 - 4*d^3*x*e - 2*d^4 + (2*x^3*e^3 - d*x^2*e^2 -
 4*d^2*x*e - 2*d^3)*sqrt(-x^2*e^2 + d^2))/(d*x^4*e^8 + 2*d^2*x^3*e^7 - 2*d^
4*x*e^5 - d^5*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.25, size = 171, normalized size = 1.73

$$-\frac{1}{40}\left(\frac{16i\,e^{(-3)}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d}-\frac{5\,e^{(-3)}}{d\sqrt{\frac{2\,d}{xe+d}-1}\;\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{\left(d^4\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^{12}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4-5\,d^4\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^{12}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4+15\,d^4\sqrt{\frac{2\,d}{xe+d}-1}\;e^{12}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4\right)e^{(-15)}}{d^5\mathrm{sgn}\left(\frac{1}{xe+d}\right)^5}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/40*(16*I*e^(-3)*sgn(1/(x*e + d))/d - 5*e^(-3)/(d*sqrt(2*d/(x*e + d) - 1)
*sgn(1/(x*e + d))) - (d^4*(2*d/(x*e + d) - 1)^(5/2)*e^12*sgn(1/(x*e + d))^4
 - 5*d^4*(2*d/(x*e + d) - 1)^(3/2)*e^12*sgn(1/(x*e + d))^4 + 15*d^4*sqrt(2*
d/(x*e + d) - 1)*e^12*sgn(1/(x*e + d))^4)*e^(-15)/(d^5*sgn(1/(x*e + d))^5))
*e^(-1)
```

Mupad [B]
time = 2.97, size = 66, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^3+4\,d^2\,e\,x+d\,e^2\,x^2-2\,e^3\,x^3\right)}{5\,d\,e^4\,\left(d+e\,x\right)^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^3 - 2*e^3*x^3 + d*e^2*x^2 + 4*d^2*e*x))/(5*d*e^
4*(d + e*x)^3*(d - e*x))
```

3.173 $\int \frac{x^2}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx$

Optimal. Leaf size=89

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} - \frac{d}{5e^3(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{7}{15e^3(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/15*x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)-1/5*d/e^3/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2
)+7/15/e^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {866, 1649, 792, 197}

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} - \frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] -1/5*(d*(d - e*x)^2)/(e^3*(d^2 - e^2*x^2)^(5/2)) + (7*(d - e*x))/(15*e^3*(d
^2 - e^2*x^2)^(3/2)) + x/(15*d^2*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{x^2(d-ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx\\ &= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{\int \frac{\left(\frac{2d^2}{e^2}-\frac{5dx}{e}\right)(d-ex)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}\\ &= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15e^2}\\ &= -\frac{d(d-ex)^2}{5e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{7(d-ex)}{15e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 70, normalized size = 0.79

$$\frac{\sqrt{d^2-e^2x^2}\left(4d^3+8d^2ex+2de^2x^2+e^3x^3\right)}{15d^2e^3(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(4*d^3 + 8*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*e
^3*(d - e*x)*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(77) = 154$.
time = 0.08, size = 287, normalized size = 3.22

method	result
gosper	$\frac{(-ex+d)\left(e^3x^3+2d\,e^2x^2+8d^2ex+4d^3\right)}{15(ex+d)d^2e^3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}$

trager	$\frac{(e^3x^3+2d\,e^2x^2+8d^2ex+4d^3)\sqrt{-e^2x^2+d^2}}{15d^2e^3(ex+d)^3(-ex+d)}$
default	$\frac{x}{d^2e^2\sqrt{-e^2x^2+d^2}}-\frac{2d\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.}}\right.}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/d^2/e^2/(-e^2*x^2+d^2)^(1/2)-2*d/e^3*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(1/2))+1/e^4*d^2*(-1/5/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(1/2)+3/5*e/d*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e
/d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.29, size = 124, normalized size = 1.39

$$-\frac{d}{5\left(\sqrt{-x^2e^2+d^2}\,x^2e^5+2\sqrt{-x^2e^2+d^2}\,dxe^4+\sqrt{-x^2e^2+d^2}\,d^2e^3\right)}+\frac{xe^{(-2)}}{15\sqrt{-x^2e^2+d^2}\,d^2}+\frac{7}{15\left(\sqrt{-x^2e^2+d^2}\,xe^4+\sqrt{-x^2e^2+d^2}\,de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*d/(sqrt(-x^2*e^2 + d^2)*x^2*e^5 + 2*sqrt(-x^2*e^2 + d^2)*d*x*e^4 + sqr
t(-x^2*e^2 + d^2)*d^2*e^3) + 1/15*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d^2) + 7/1
5/(sqrt(-x^2*e^2 + d^2)*x*e^4 + sqrt(-x^2*e^2 + d^2)*d*e^3)
```

Fricas [A]
time = 1.80, size = 111, normalized size = 1.25

$$\frac{4\,x^4e^4+8\,dx^3e^3-8\,d^3xe-4\,d^4-(x^3e^3+2\,dx^2e^2+8\,d^2xe+4\,d^3)\sqrt{-x^2e^2+d^2}}{15\,(d^2x^4e^7+2\,d^3x^3e^6-2\,d^5xe^4-d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*x^4*e^4 + 8*d*x^3*e^3 - 8*d^3*x*e - 4*d^4 - (x^3*e^3 + 2*d*x^2*e^2
 + 8*d^2*x*e + 4*d^3)*sqrt(-x^2*e^2 + d^2))/(d^2*x^4*e^7 + 2*d^3*x^3*e^6 - 2
*d^5*x*e^4 - d^6*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.05, size = 171, normalized size = 1.92

$$-\frac{1}{120}\left(-\frac{8i\,e^{(-2)}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^2}-\frac{15\,e^{(-2)}}{d^2\sqrt{\frac{2\,d}{xe+d}-1}\;\mathrm{sgn}\left(\frac{1}{xe+d}\right)}+\frac{\left(3\,d^8\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^8\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4-5\,d^8\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^8\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4-15\,d^8\sqrt{\frac{2\,d}{xe+d}-1}\;e^8\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4\right)e^{(-10)}}{d^{10}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^5}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/120*(-8*I*e^(-2)*sgn(1/(x*e + d))/d^2 - 15*e^(-2)/(d^2*sqrt(2*d/(x*e + d
) - 1)*sgn(1/(x*e + d))) + (3*d^8*(2*d/(x*e + d) - 1)^(5/2)*e^8*sgn(1/(x*e
+ d))^4 - 5*d^8*(2*d/(x*e + d) - 1)^(3/2)*e^8*sgn(1/(x*e + d))^4 - 15*d^8*s
qrt(2*d/(x*e + d) - 1)*e^8*sgn(1/(x*e + d))^4)*e^(-10)/(d^10*sgn(1/(x*e + d
))^5))*e^(-1)
```

Mupad [B]
time = 2.90, size = 66, normalized size = 0.74

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^3+8\,d^2\,e\,x+2\,d\,e^2\,x^2+e^3\,x^3\right)}{15\,d^2\,e^3\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(4*d^3 + e^3*x^3 + 2*d*e^2*x^2 + 8*d^2*e*x))/(15*d^2
*e^3*(d + e*x)^3*(d - e*x))
```

3.174 $\int \frac{x}{(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/15*x/d^3/e/(-e^2*x^2+d^2)^(1/2)+1/5/e^2/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-2/
15/d/e^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {807, 673, 197}

$$-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (4*x)/(15*d^3*e*Sqrt[d^2 - e^2*x^2]) + 1/(5*e^2*(d + e*x)^2*Sqrt[d^2 - e^2*
x^2]) - 2/(15*d*e^2*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{2\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5e}$$

$$= \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}$$

$$= \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}} + \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.30, size = 69, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\left(d^3+2d^2ex+8de^2x^2+4e^3x^3\right)}{15d^3e^2(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
[Out] (Sqrt[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 8*d*e^2*x^2 + 4*e^3*x^3))/(15*d^3*e
^2*(d - e*x)*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(79) = 158.
time = 0.07, size = 262, normalized size = 2.88

method	result
gosper	$\frac{(-ex+d)\left(4e^3x^3+8d\,e^2x^2+2d^2ex+d^3\right)}{15(ex+d)d^3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}$
trager	$\frac{\left(4e^3x^3+8d\,e^2x^2+2d^2ex+d^3\right)\sqrt{-e^2x^2+d^2}}{15d^3(ex+d)^3e^2(-ex+d)}$
default	$\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}}{e^2} - \frac{d\left(-\frac{\ldots}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\ldots}}\ldots\right)}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*
e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))-d/e^3*(-1/5/d/e/(x
+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5*e/d*(-1/3/d/e/(x+d/e)/(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.28, size = 126, normalized size = 1.38

$$\frac{1}{5\left(\sqrt{-x^2e^2+d^2}\,x^2e^4+2\sqrt{-x^2e^2+d^2}\,dxe^3+\sqrt{-x^2e^2+d^2}\,d^2e^2\right)}-\frac{2}{15\left(\sqrt{-x^2e^2+d^2}\,dxe^3+\sqrt{-x^2e^2+d^2}\,d^2e^2\right)}+\frac{4\,xe^{(-1)}}{15\sqrt{-x^2e^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5/(sqrt(-x^2*e^2 + d^2)*x^2*e^4 + 2*sqrt(-x^2*e^2 + d^2)*d*x*e^3 + sqrt(-
x^2*e^2 + d^2)*d^2*e^2) - 2/15/(sqrt(-x^2*e^2 + d^2)*d*x*e^3 + sqrt(-x^2*e^
2 + d^2)*d^2*e^2) + 4/15*x*e^(-1)/(sqrt(-x^2*e^2 + d^2)*d^3)
```

Fricas [A]
time = 2.35, size = 109, normalized size = 1.20

$$\frac{x^4e^4+2\,dx^3e^3-2\,d^3xe-d^4-\left(4\,x^3e^3+8\,dx^2e^2+2\,d^2xe+d^3\right)\sqrt{-x^2e^2+d^2}}{15\left(d^3x^4e^6+2\,d^4x^3e^5-2\,d^6xe^3-d^7e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(x^4*e^4 + 2*d*x^3*e^3 - 2*d^3*x*e - d^4 - (4*x^3*e^3 + 8*d*x^2*e^2 +
2*d^2*x*e + d^3)*sqrt(-x^2*e^2 + d^2))/(d^3*x^4*e^6 + 2*d^4*x^3*e^5 - 2*d^6
*x*e^3 - d^7*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.03, size = 160, normalized size = 1.76

$$-\frac{1}{120}\left(-\frac{32i\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^3}-\frac{15}{d^3\sqrt{\frac{2\,d}{xe+d}-1}\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{3\,d^{12}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4+5\,d^{12}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4-15\,d^{12}\sqrt{\frac{2\,d}{xe+d}-1}\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4}{d^{15}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^5}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/120*(-32*I*sgn(1/(x*e + d))/d^3 - 15/(d^3*sqrt(2*d/(x*e + d) - 1)*sgn(1/
(x*e + d))) - (3*d^12*(2*d/(x*e + d) - 1)^(5/2)*sgn(1/(x*e + d))^4 + 5*d^12
*(2*d/(x*e + d) - 1)^(3/2)*sgn(1/(x*e + d))^4 - 15*d^12*sqrt(2*d/(x*e + d)
- 1)*sgn(1/(x*e + d))^4)/(d^15*sgn(1/(x*e + d))^5))*e^(-2)
```

Mupad [B]
time = 2.88, size = 65, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^3+2\,d^2\,e\,x+8\,d\,e^2\,x^2+4\,e^3\,x^3\right)}{15\,d^3\,e^2\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(d^3 + 4*e^3*x^3 + 8*d*e^2*x^2 + 2*d^2*e*x))/(15*d^3
*e^2*(d + e*x)^3*(d - e*x))
```

3.175 $\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}$$

```
[Out] 2/5*x/d^4/(-e^2*x^2+d^2)^(1/2)-1/5/d/e/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-1/5/d
^2/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {673, 197}

$$-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*x)/(5*d^4*Sqrt[d^2 - e^2*x^2]) - 1/(5*d*e*(d + e*x)^2*Sqrt[d^2 - e^2*x^2
]) - 1/(5*d^2*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{3\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx}{5d}\\
&= -\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}+\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}}{5d^2}\\
&= \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 70, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\,(-2d^3+d^2ex+4de^2x^2+2e^3x^3)}{5d^4e(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-2*d^3 + d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e
*(d - e*x)*(d + e*x)^3)
```

Maple [A]
time = 0.06, size = 156, normalized size = 1.71

method	result
gosper	$-\frac{(-ex+d)(-2e^3x^3-4d\,e^2x^2-d^2ex+2d^3)}{5(ex+d)d^4e(-e^2x^2+d^2)^{\frac{3}{2}}}$
trager	$-\frac{(-2e^3x^3-4d\,e^2x^2-d^2ex+2d^3)\sqrt{-e^2x^2+d^2}}{5d^4(ex+d)^3e(-ex+d)}$
default	$\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{\cdots}{3e\,d^3\sqrt{-(x \cdots}}\right.}{5d}}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(-1/5/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5*e/d*(-1/
3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e
)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.27, size = 129, normalized size = 1.42

$$-\frac{1}{5\left(\sqrt{-x^2e^2+d^2}\,dx^2e^3+2\sqrt{-x^2e^2+d^2}\,d^2xe^2+\sqrt{-x^2e^2+d^2}\,d^3e\right)}-\frac{1}{5\left(\sqrt{-x^2e^2+d^2}\,d^2xe^2+\sqrt{-x^2e^2+d^2}\,d^3e\right)}+\frac{2x}{5\sqrt{-x^2e^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5/(sqrt(-x^2*e^2 + d^2)*d*x^2*e^3 + 2*sqrt(-x^2*e^2 + d^2)*d^2*x*e^2 + s
qrt(-x^2*e^2 + d^2)*d^3*e) - 1/5/(sqrt(-x^2*e^2 + d^2)*d^2*x*e^2 + sqrt(-x^
2*e^2 + d^2)*d^3*e) + 2/5*x/(sqrt(-x^2*e^2 + d^2)*d^4)
```

Fricas [A]
time = 1.68, size = 110, normalized size = 1.21

$$-\frac{2\,x^4e^4+4\,dx^3e^3-4\,d^3xe-2\,d^4+\left(2\,x^3e^3+4\,dx^2e^2+d^2xe-2\,d^3\right)\sqrt{-x^2e^2+d^2}}{5\left(d^4x^4e^5+2\,d^5x^3e^4-2\,d^7xe^2-d^8e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*x^4*e^4 + 4*d*x^3*e^3 - 4*d^3*x*e - 2*d^4 + (2*x^3*e^3 + 4*d*x^2*e^
2 + d^2*x*e - 2*d^3)*sqrt(-x^2*e^2 + d^2))/(d^4*x^4*e^5 + 2*d^5*x^3*e^4 - 2
*d^7*x*e^2 - d^8*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.10, size = 173, normalized size = 1.90

$$\frac{1}{40}\left(\left(\frac{5\,e^{(-3)}}{d^4\sqrt{\frac{2\,d}{xe+d}-1}\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{\left(d^{16}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^{12}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4+5\,d^{16}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^{12}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4+15\,d^{16}\sqrt{\frac{2\,d}{xe+d}-1}\,e^{12}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4\right)e^{(-15)}}{d^{20}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^5}\right)e^3+\frac{16i\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^4}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*((5*e^(-3)/(d^4*sqrt(2*d/(x*e + d) - 1)*sgn(1/(x*e + d))) - (d^16*(2*d
/(x*e + d) - 1)^(5/2)*e^12*sgn(1/(x*e + d))^4 + 5*d^16*(2*d/(x*e + d) - 1)^
(3/2)*e^12*sgn(1/(x*e + d))^4 + 15*d^16*sqrt(2*d/(x*e + d) - 1)*e^12*sgn(1/
(x*e + d))^4)*e^(-15)/(d^20*sgn(1/(x*e + d))^5))*e^3 + 16*I*sgn(1/(x*e + d)
)/d^4)*e^(-1)
```

Mupad [B]
time = 2.85, size = 66, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^3+d^2\,e\,x+4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*e^3*x^3 - 2*d^3 + 4*d*e^2*x^2 + d^2*e*x))/(5*d^4*
e*(d + e*x)^3*(d - e*x))
```

3.176 $\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] 2/5*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+1/15*(-8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(3
/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*(-16*e*x+15*d)/d^5/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 837, 12, 272, 65, 214}

$$\frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}+\frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d - 8*e*x)/(15*d^3*(d^2 - e^
2*x^2)^(3/2)) + (15*d - 16*e*x)/(15*d^5*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt
[d^2 - e^2*x^2]/d]/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx &= \int \frac{(d-ex)^2}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+8dex}{x\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^4e^2+16d^3e^3x}{x\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^6e^2} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^{6}\cdots}{x\sqrt{d^2-\cdots}}}{15d^{10}\cdots} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e\cdots}}}{d^4} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots} \\
&= \frac{2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-8ex}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{\cdots}\right.}{\cdots}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.47, size = 107, normalized size = 0.91

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(26d^3+22d^2ex-17de^2x^2-16e^3x^3\right)}{(d-ex)(d+ex)^3}+30\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(26*d^3 + 22*d^2*e*x - 17*d*e^2*x^2 - 16*e^3*x^3))/((
d - e*x)*(d + e*x)^3) + 30*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])
/(15*d^5)
```

<u>Maple</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(104) = 208$.
time = 0.07, size = 330, normalized size = 2.80

method	result
default	$-\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^{2}\left(x+\frac{d}{e}\right)+2de}{3e\,d^{3}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}}{d^{2}}-\frac{-\frac{\cdots}{5de\left(x+\frac{d}{e}\right)^{2}\sqrt{-\cdots}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2
*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))-1/e/d*(-1/5/d/e/(
x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5*e/d*(-1/3/d/e/(x+d/e)/(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)^2*x), x)
```

Fricas [A]
time = 2.78, size = 162, normalized size = 1.37

$$\frac{26\,x^{4}e^{4}+52\,dx^{3}e^{3}-52\,d^{3}xe-26\,d^{4}+15\,(x^{4}e^{4}+2\,dx^{3}e^{3}-2\,d^{3}xe-d^{4})\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+(16\,x^{3}e^{3}+17\,dx^{2}e^{2}-22\,d^{2}xe-26\,d^{3})\sqrt{-x^{2}e^{2}+d^{2}}}{15\,(d^{5}x^{4}e^{4}+2\,d^{6}x^{3}e^{3}-2\,d^{8}xe-d^{9})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(26*x^4*e^4 + 52*d*x^3*e^3 - 52*d^3*x*e - 26*d^4 + 15*(x^4*e^4 + 2*d*x
^3*e^3 - 2*d^3*x*e - d^4)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (16*x^3*e^3
+ 17*d*x^2*e^2 - 22*d^2*x*e - 26*d^3)*sqrt(-x^2*e^2 + d^2))/(d^5*x^4*e^4 +
2*d^6*x^3*e^3 - 2*d^8*x*e - d^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 1.39, size = 254, normalized size = 2.15

$$-\frac{1}{120}\left(\frac{120\,e^{(-4)}\log\left(\sqrt{\frac{2\,d}{xe+d}-1}+1\right)}{d^5\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{120\,e^{(-4)}\log\left(\left|\sqrt{\frac{2\,d}{xe+d}-1}-1\right|\right)}{d^5\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{15\,e^{(-4)}}{d^5\sqrt{\frac{2\,d}{xe+d}-1}\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{\left(3\,d^{20}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^{16}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4+25\,d^{20}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^{16}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4+165\,d^{20}\sqrt{\frac{2\,d}{xe+d}-1}\,e^{16}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^4\right)e^{(-20)}}{d^{25}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^5}\right)e^4-\frac{(15\,\log(2)-30\,\log(i+1)+32i)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{30\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/120*(120*e^(-4)*log(sqrt(2*d/(x*e + d) - 1) + 1)/(d^5*sgn(1/(x*e + d)))
- 120*e^(-4)*log(abs(sqrt(2*d/(x*e + d) - 1) - 1))/(d^5*sgn(1/(x*e + d))) -
 15*e^(-4)/(d^5*sqrt(2*d/(x*e + d) - 1)*sgn(1/(x*e + d))) - (3*d^20*(2*d/(x
*e + d) - 1)^(5/2)*e^16*sgn(1/(x*e + d))^4 + 25*d^20*(2*d/(x*e + d) - 1)^(3
/2)*e^16*sgn(1/(x*e + d))^4 + 165*d^20*sqrt(2*d/(x*e + d) - 1)*e^16*sgn(1/(
x*e + d))^4)*e^(-20)/(d^25*sgn(1/(x*e + d))^5))*e^4 - 1/30*(15*log(2) - 30*
log(I + 1) + 32*I)*sgn(1/(x*e + d))/d^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.177 $\int \frac{1}{x^2(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

```
[Out] -2/5*e*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)-1/15*e*(-13*e*x+10*d)/d^4/(-e^2*x^
2+d^2)^(3/2)+2*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6-1/15*e*(-41*e*x+30*d)/
d^6/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A]
time = 0.19, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1819, 821, 272, 65, 214}

$$-\frac{2e(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^6x}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}-\frac{e(10d-13ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (e*(10*d - 13*e*x))/(15*d^
4*(d^2 - e^2*x^2)^(3/2)) - (e*(30*d - 41*e*x))/(15*d^6*Sqrt[d^2 - e^2*x^2])
 - Sqrt[d^2 - e^2*x^2]/(d^6*x) + (2*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^6
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x^2 (d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{\int \frac{-5d^2+10dex-8e^2x^2}{x^2(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} + \frac{\int \frac{15d^2-30dex+26e^2x^2}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \cdots$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \cdots$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \cdots$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \cdots$$

$$= -\frac{2e(d-ex)}{5d^2 (d^2-e^2x^2)^{5/2}} - \frac{e(10d-13ex)}{15d^4 (d^2-e^2x^2)^{3/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \cdots$$

Mathematica [A]
time = 0.46, size = 123, normalized size = 0.84

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(15d^4+76d^3ex+32d^2e^2x^2-82de^3x^3-56e^4x^4)}{x(-d+ex)(d+ex)^3} - 60e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(15*d^4 + 76*d^3*e*x + 32*d^2*e^2*x^2 - 82*d*e^3*x^3
- 56*e^4*x^4))/(x*(-d + e*x)*(d + e*x)^3) - 60*e*ArcTanh[(Sqrt[-e^2]*x - Sq
rt[d^2 - e^2*x^2])/d])/(15*d^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(130) = 260.
time = 0.09, size = 379, normalized size = 2.60

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^6x}-\frac{29\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{60d^5e\left(x+\frac{d}{e}\right)^2}-\frac{313\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{120d^6\left(x+\frac{d}{e}\right)}-\frac{\sqrt{-}}{}$
default	$\frac{2e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{d^3}+\frac{-\frac{1}{d^2x\sqrt{-e^2x^2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e/d^3*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-
2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/d^2*(-1/d^2/x/
(-e^2*x^2+d^2)^(1/2)+2*e^2/d^4*x/(-e^2*x^2+d^2)^(1/2))+1/d^2*(-1/5/d/e/(x+d
/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5*e/d*(-1/3/d/e/(x+d/e)/(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2)))-2/d^3*e*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)
^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)^2*x^2), x)
```

Fricas [A]
time = 1.96, size = 185, normalized size = 1.27

$$-\frac{46\,x^5e^5+92\,dx^4e^4-92\,d^3x^2e^2-46\,d^4xe+30\,(x^5e^5+2\,dx^4e^4-2\,d^3x^2e^2-d^4xe)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(56\,x^4e^4+82\,dx^3e^3-32\,d^2x^2e^2-76\,d^3xe-15\,d^4)\sqrt{-x^2e^2+d^2}}{15\,(d^6x^5e^4+2\,d^7x^4e^3-2\,d^9x^2e-d^{10}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(46*x^5*e^5 + 92*d*x^4*e^4 - 92*d^3*x^2*e^2 - 46*d^4*x*e + 30*(x^5*e^
5 + 2*d*x^4*e^4 - 2*d^3*x^2*e^2 - d^4*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/
x) + (56*x^4*e^4 + 82*d*x^3*e^3 - 32*d^2*x^2*e^2 - 76*d^3*x*e - 15*d^4)*sqr
t(-x^2*e^2 + d^2))/(d^6*x^5*e^4 + 2*d^7*x^4*e^3 - 2*d^9*x^2*e - d^10*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(- \left(- d + e x\right) \left(d + e x\right)\right)^{\frac{3}{2}} \left(d + e x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.17, size = 297, normalized size = 2.03

$$\frac{1}{120}\left(\left(\frac{240\,e^{(-5)}\log\left(\sqrt{\frac{2\,d}{xe+d}-1}+1\right)}{d^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{240\,e^{(-5)}\log\left(\left|\sqrt{\frac{2\,d}{xe+d}-1}-1\right|\right)}{d^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}+\frac{30\left(\frac{17\,d}{xe+d}-9\right)e^{(-5)}}{\left(\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}-\sqrt{\frac{2\,d}{xe+d}-1}\right)d^{6}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{\left(3\,d^{24}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^{20}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^{4}+35\,d^{24}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^{20}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^{4}+345\,d^{24}\sqrt{\frac{2\,d}{xe+d}-1}\,e^{20}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^{4}\right)e^{(-25)}}{d^{30}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^{5}}\right)e^{7}+\frac{8\left(15\,e^{2}\log\left(2\right)-30\,e^{2}\log\left(i+1\right)+56 i\,e^{2}\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{6}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*((240*e^(-5)*log(sqrt(2*d/(x*e + d) - 1) + 1)/(d^6*sgn(1/(x*e + d)))
- 240*e^(-5)*log(abs(sqrt(2*d/(x*e + d) - 1) - 1))/(d^6*sgn(1/(x*e + d))) +
 30*(17*d/(x*e + d) - 9)*e^(-5)/(((2*d/(x*e + d) - 1)^(3/2) - sqrt(2*d/(x*e
 + d) - 1))*d^6*sgn(1/(x*e + d))) - (3*d^24*(2*d/(x*e + d) - 1)^(5/2)*e^20*
sgn(1/(x*e + d))^4 + 35*d^24*(2*d/(x*e + d) - 1)^(3/2)*e^20*sgn(1/(x*e + d)
)^4 + 345*d^24*sqrt(2*d/(x*e + d) - 1)*e^20*sgn(1/(x*e + d))^4)*e^(-25)/(d^
30*sgn(1/(x*e + d))^5))*e^7 + 8*(15*e^2*log(2) - 30*e^2*log(I + 1) + 56*I*e
^2)*sgn(1/(x*e + d))/d^6)*e^(-1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.178 $\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=183

$$\frac{2e^2(d-ex)}{5d^3(d^2-e^2x^2)^{5/2}}+\frac{e^2(5d-6ex)}{5d^5(d^2-e^2x^2)^{3/2}}+\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2}}{}\right.}{2d^7}$$

```
[Out] 2/5*e^2*(-e*x+d)/d^3/(-e^2*x^2+d^2)^(5/2)+1/5*e^2*(-6*e*x+5*d)/d^5/(-e^2*x^
2+d^2)^(3/2)-9/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^7+2/5*e^2*(-11*e*x+1
0*d)/d^7/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^6/x^2+2*e*(-e^2*x^
2+d^2)^(1/2)/d^7/x
```

Rubi [A]

time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}}+\frac{2e\sqrt{d^2-e^2x^2}}{d^7x}-\frac{9e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}-\frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}+\frac{e^2(5d-6ex)}{5d^5(d^2-e^2x^2)^{3/2}}+\frac{2e^2(d-ex)}{5d^3(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] (2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^(5/2)) + (e^2*(5*d - 6*e*x))/(5*d^
5*(d^2 - e^2*x^2)^(3/2)) + (2*e^2*(10*d - 11*e*x))/(5*d^7*Sqrt[d^2 - e^2*x^
2]) - Sqrt[d^2 - e^2*x^2]/(2*d^6*x^2) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^7*x) -
 (9*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^7)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{(d-ex)^2}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^2+10dex-10e^2x^2+\frac{8e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^2-30dex+45e^2x^2-\frac{36e^3x^3}{d}}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^2+30d}{x^3\sqrt{d^2-}}}{15}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}$$

$$= \frac{2e^2(d-ex)}{5d^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(5d-6ex)}{5d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2}$$

Mathematica [A]
time = 0.49, size = 136, normalized size = 0.74

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(5d^5-10d^4ex-94d^3e^2x^2-58d^2e^3x^3+83de^4x^4+64e^5x^5)}{x^2(-d+ex)(d+ex)^3}+90e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{10d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 10*d^4*e*x - 94*d^3*e^2*x^2 - 58*d^2*e^3*x^3
 + 83*d*e^4*x^4 + 64*e^5*x^5))/(x^2*(-d + e*x)*(d + e*x)^3) + 90*e^2*ArcTan
h[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(10*d^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(161) = 322$.
time = 0.08, size = 483, normalized size = 2.64

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-4ex+d)}{2d^7x^2}+\frac{181e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{40d^7\left(x+\frac{d}{e}\right)}-\frac{e\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2d\left(x-\frac{d}{e}\right)}}{8d^7\left(x-\frac{d}{e}\right)}$
default	$-\frac{3e^2\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{d^4}+\frac{-}{2d^2x^2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3*e^2/d^4*(-1/3/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3
*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+1/d^2*(-1/2/d
^2/x^2/(-e^2*x^2+d^2)^(1/2)+3/2*e^2/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^2/(
d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))-2/d^3*e*(-1/d
^2/x/(-e^2*x^2+d^2)^(1/2)+2*e^2/d^4*x/(-e^2*x^2+d^2)^(1/2))-e/d^3*(-1/5/d/e
/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/5*e/d*(-1/3/d/e/(x+d/e)/(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/e/d^3*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+3/d^4*e^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^
2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(3/2)*(x*e + d)^2*x^3), x)
```

Fricas [A]
time = 1.82, size = 201, normalized size = 1.10

$$\frac{54\,x^6e^6+108\,dx^5e^5-108\,d^3x^3e^3-54\,d^4x^2e^2+45\,(x^6e^6+2\,dx^5e^5-2\,d^3x^3e^3-d^4x^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(64\,x^5e^5+83\,dx^4e^4-58\,d^2x^3e^3-94\,d^3x^2e^2-10\,d^4xe+5\,d^5)\sqrt{-x^2e^2+d^2}}{10\,(d^7x^6e^4+2\,d^8x^5e^3-2\,d^{10}x^3e-d^{11}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(54*x^6*e^6 + 108*d*x^5*e^5 - 108*d^3*x^3*e^3 - 54*d^4*x^2*e^2 + 45*(x
^6*e^6 + 2*d*x^5*e^5 - 2*d^3*x^3*e^3 - d^4*x^2*e^2)*log(-(d - sqrt(-x^2*e^2
 + d^2))/x) + (64*x^5*e^5 + 83*d*x^4*e^4 - 58*d^2*x^3*e^3 - 94*d^3*x^2*e^2
- 10*d^4*x*e + 5*d^5)*sqrt(-x^2*e^2 + d^2))/(d^7*x^6*e^4 + 2*d^8*x^5*e^3 -
2*d^10*x^3*e - d^11*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 2.41, size = 330, normalized size = 1.80

$$-\frac{1}{40}\left(\left(\left(\frac{180\,e^{(-6)}\log\left(\sqrt{\frac{2\,d}{xe+d}-1}+1\right)}{d^{7}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{180\,e^{(-6)}\log\left(\left|\sqrt{\frac{2\,d}{xe+d}-1}-1\right|\right)}{d^{7}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{5\,e^{(-6)}}{d^{7}\sqrt{\frac{2\,d}{xe+d}-1}\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}+\frac{10\left(5\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{2\,d}{xe+d}-1}\right)e^{(-6)}}{d^{7}\left(\frac{d}{xe+d}-1\right)^{2}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}-\frac{\left(d^{28}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{5}{2}}e^{24}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^{4}+15\,d^{28}\left(\frac{2\,d}{xe+d}-1\right)^{\frac{3}{2}}e^{24}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^{4}+195\,d^{28}\sqrt{\frac{2\,d}{xe+d}-1}\,e^{24}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^{4}\right)e^{(-30)}}{d^{35}\mathrm{sgn}\left(\frac{1}{xe+d}\right)^{5}}\right)e^{9}+\frac{2\left(45\,e^{3}\log\left(2\right)-90\,e^{3}\log\left(i+1\right)+128i\,e^{3}\right)\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^{7}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/40*((180*e^(-6)*log(sqrt(2*d/(x*e + d) - 1) + 1)/(d^7*sgn(1/(x*e + d)))
- 180*e^(-6)*log(abs(sqrt(2*d/(x*e + d) - 1) - 1))/(d^7*sgn(1/(x*e + d))) -
 5*e^(-6)/(d^7*sqrt(2*d/(x*e + d) - 1)*sgn(1/(x*e + d))) + 10*(5*(2*d/(x*e
+ d) - 1)^(3/2) - 3*sqrt(2*d/(x*e + d) - 1))*e^(-6)/(d^7*(d/(x*e + d) - 1)^
2*sgn(1/(x*e + d))) - (d^28*(2*d/(x*e + d) - 1)^(5/2)*e^24*sgn(1/(x*e + d))
^4 + 15*d^28*(2*d/(x*e + d) - 1)^(3/2)*e^24*sgn(1/(x*e + d))^4 + 195*d^28*s
qrt(2*d/(x*e + d) - 1)*e^24*sgn(1/(x*e + d))^4)*e^(-30)/(d^35*sgn(1/(x*e +
d))^5))*e^9 + 2*(45*e^3*log(2) - 90*e^3*log(I + 1) + 128*I*e^3)*sgn(1/(x*e
+ d))/d^7)*e^(-1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(d^2-e^2\,x^2\right)}^{3/2}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(3/2)*(d + e*x)^2), x)
```

3.179 $\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=177

$$\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}-\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{13d^2\tan^{-1}\left(\right.}{2}$$

```
[Out] 1/5*d^4*(-e*x+d)^3/e^6/(-e^2*x^2+d^2)^(5/2)-23/15*d^3*(-e*x+d)^2/e^6/(-e^2*
x^2+d^2)^(3/2)+13/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+127/15*d^2*(-e
*x+d)/e^6/(-e^2*x^2+d^2)^(1/2)+3*d*(-e^2*x^2+d^2)^(1/2)/e^6-1/2*x*(-e^2*x^2
+d^2)^(1/2)/e^5
```

Rubi [A]
time = 0.27, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\frac{13d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6}+\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}}+\frac{3d\sqrt{d^2-e^2x^2}}{e^6}-\frac{x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{d^4(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{23d^3(d-ex)^2}{15e^6\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (23*d^3*(d - e*x)^2)/(15*
e^6*(d^2 - e^2*x^2)^(3/2)) + (127*d^2*(d - e*x))/(15*e^6*Sqrt[d^2 - e^2*x^2
]) + (3*d*Sqrt[d^2 - e^2*x^2])/e^6 - (x*Sqrt[d^2 - e^2*x^2])/(2*e^5) + (13*
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^5(d-ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{(d^2-e^2x^2)^{5/2}}dx}{5d}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{(d-ex)\left(-\frac{37d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right.}{(d^2-e^2x^2)^{3/2}}}{15d^2}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-\frac{90d^5}{e^5}+\frac{45d^4x}{e^4}}{\sqrt{d^2-e}}}{15d}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x}}{e^6}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x}}{e^6}$$

$$= \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}} + \frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x}}{e^6}$$

Mathematica [A]
time = 0.44, size = 118, normalized size = 0.67

$$\frac{\frac{e\sqrt{d^2-e^2x^2}\,(304d^4+717d^3ex+479d^2e^2x^2+45de^3x^3-15e^4x^4)}{(d+ex)^3}+195d^2\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{30e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(304*d^4 + 717*d^3*e*x + 479*d^2*e^2*x^2 + 45*d*e^3
*x^3 - 15*e^4*x^4))/(d + e*x)^3 + 195*d^2*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
Sqrt[d^2 - e^2*x^2]])/(30*e^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(155) = 310$.
time = 0.09, size = 406, normalized size = 2.29

method	result
risch	$\frac{(-ex+6d)\sqrt{-e^2x^2+d^2}}{2e^6}+\frac{13d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5\sqrt{e^2}}+\frac{127d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^7\left(x+\frac{d}{e}\right)}-\frac{23d^3}{}$
default	$\frac{-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}}{e^3}+\frac{3d\sqrt{-e^2x^2+d^2}}{e^6}+\frac{6d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{10d^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^3*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))+3*d*(-e^2*x^2+d^2)^(1/2)/e^6+6*d^2/e^5/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+10*d^2/e^7/(x+d/e)*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-d^5/e^8*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+5/e^7*d^
4*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.48, size = 169, normalized size = 0.95

$$\frac{13}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-6)}+\frac{\sqrt{-x^2e^2+d^2}\,d^4}{5\,(x^3e^9+3\,dx^2e^8+3\,d^2xe^7+d^3e^6)}-\frac{23\,\sqrt{-x^2e^2+d^2}\,d^3}{15\,(x^2e^8+2\,dxe^7+d^2e^6)}-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,xe^{(-5)}+3\,\sqrt{-x^2e^2+d^2}\,de^{(-6)}+\frac{127\,\sqrt{-x^2e^2+d^2}\,d^2}{15\,(xe^7+de^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 13/2*d^2*arcsin(x*e/d)*e^(-6) + 1/5*sqrt(-x^2*e^2 + d^2)*d^4/(x^3*e^9 + 3*d
*x^2*e^8 + 3*d^2*x*e^7 + d^3*e^6) - 23/15*sqrt(-x^2*e^2 + d^2)*d^3/(x^2*e^8
 + 2*d*x*e^7 + d^2*e^6) - 1/2*sqrt(-x^2*e^2 + d^2)*x*e^(-5) + 3*sqrt(-x^2*e
^2 + d^2)*d*e^(-6) + 127/15*sqrt(-x^2*e^2 + d^2)*d^2/(x*e^7 + d*e^6)
```

Fricas [A]
time = 2.51, size = 179, normalized size = 1.01

$$\frac{304\,d^2x^3e^3+912\,d^3x^2e^2+912\,d^4xe+304\,d^5-390\,(d^2x^3e^3+3\,d^3x^2e^2+3\,d^4xe+d^5)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(15\,x^4e^4-45\,dx^3e^3-479\,d^2x^2e^2-717\,d^3xe-304\,d^4)\sqrt{-x^2e^2+d^2}}{30\,(x^3e^9+3\,dx^2e^8+3\,d^2xe^7+d^3e^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(304*d^2*x^3*e^3 + 912*d^3*x^2*e^2 + 912*d^4*x*e + 304*d^5 - 390*(d^2*
x^3*e^3 + 3*d^3*x^2*e^2 + 3*d^4*x*e + d^5)*arctan(-(d - sqrt(-x^2*e^2 + d^2
))*e^(-1)/x) - (15*x^4*e^4 - 45*d*x^3*e^3 - 479*d^2*x^2*e^2 - 717*d^3*x*e -
 304*d^4)*sqrt(-x^2*e^2 + d^2))/(x^3*e^9 + 3*d*x^2*e^8 + 3*d^2*x*e^7 + d^3*
e^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A]
time = 3.13, size = 214, normalized size = 1.21

$$\frac{13}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(xe^{(-5)}-6\,de^{(-6)}\right)-\frac{2\left(\frac{445\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^{(-2)}}{x}+\frac{665\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^{(-4)}}{x^2}+\frac{405\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2e^{(-6)}}{x^3}+\frac{90\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^2e^{(-8)}}{x^4}+107\,d^2\right)e^{(-6)}}{15\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 13/2*d^2*arcsin(x*e/d)*e^(-6)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^(-5) -
 6*d*e^(-6)) - 2/15*(445*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*e^(-2)/x + 665*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^(-4)/x^2 + 405*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^3*d^2*e^(-6)/x^3 + 90*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^2*e^(-8)
/x^4 + 107*d^2)*e^(-6)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^5/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.180 $\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

```
[Out] -1/5*d^3*(-e*x+d)^3/e^5/(-e^2*x^2+d^2)^(5/2)+6/5*d^2*(-e*x+d)^2/e^5/(-e^2*x
^2+d^2)^(3/2)-3*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-24/5*d*(-e*x+d)/e^5/
(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A]
time = 0.22, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {866, 1649, 655, 223, 209}

$$-\frac{3d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}+\frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e^5}-\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*(d^3*(d - e*x)^3)/(e^5*(d^2 - e^2*x^2)^(5/2)) + (6*d^2*(d - e*x)^2)/(5
*e^5*(d^2 - e^2*x^2)^(3/2)) - (24*d*(d - e*x))/(5*e^5*Sqrt[d^2 - e^2*x^2])
- Sqrt[d^2 - e^2*x^2]/e^5 - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^4(d-ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(\frac{3d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)\left(\frac{27d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\frac{45d^4}{e^4}-\frac{15d^3x}{e^3}}{\sqrt{d^2-e^2x}}}{15d^3}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5}$$

$$= -\frac{d^3(d-ex)^3}{5e^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{6d^2(d-ex)^2}{5e^5\left(d^2-e^2x^2\right)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5}$$

Mathematica [A]
time = 0.40, size = 106, normalized size = 0.73

$$\frac{\sqrt{d^2-e^2x^2}\,(-24d^3-57d^2ex-39de^2x^2-5e^3x^3)}{5e^5(d+ex)^3}+\frac{3d(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^3 - 57*d^2*e*x - 39*d*e^2*x^2 - 5*e^3*x^3))/(5*
e^5*(d + e*x)^3) + (3*d*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x
^2]])/e^8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(130) = 260.
time = 0.09, size = 340, normalized size = 2.33

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{e^5}-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}-\frac{24d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^6\left(x+\frac{d}{e}\right)}+\frac{6d^2\sqrt{-\left(x+\right.}}{}$
default	$-\frac{\sqrt{-e^2x^2+d^2}}{e^5}-\frac{3d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^4\sqrt{e^2}}-\frac{6d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^6\left(x+\frac{d}{e}\right)}+\frac{d^4\left(-\frac{\sqrt{-\left(x+\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-e^2*x^2+d^2)^(1/2)/e^5-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+d^2)^(1/2))-6*d/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/e^7*d^
4*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/
e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)))-4/e^6*d^3*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.49, size = 146, normalized size = 1.00

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{(-5)}-\frac{\sqrt{-x^2e^2+d^2}\,d^3}{5\,(x^3e^8+3\,dx^2e^7+3\,d^2xe^6+d^3e^5)}+\frac{6\sqrt{-x^2e^2+d^2}\,d^2}{5\,(x^2e^7+2\,dxe^6+d^2e^5)}-\sqrt{-x^2e^2+d^2}\,e^{(-5)}-\frac{24\sqrt{-x^2e^2+d^2}\,d}{5\,(xe^6+de^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*d*arcsin(x*e/d)*e^(-5) - 1/5*sqrt(-x^2*e^2 + d^2)*d^3/(x^3*e^8 + 3*d*x^2
*e^7 + 3*d^2*x*e^6 + d^3*e^5) + 6/5*sqrt(-x^2*e^2 + d^2)*d^2/(x^2*e^7 + 2*d
*x*e^6 + d^2*e^5) - sqrt(-x^2*e^2 + d^2)*e^(-5) - 24/5*sqrt(-x^2*e^2 + d^2)
*d/(x*e^6 + d*e^5)
```

Fricas [A]
time = 2.74, size = 164, normalized size = 1.12

$$-\frac{24\,dx^3e^3+72\,d^2x^2e^2+72\,d^3xe+24\,d^4-30\,(dx^3e^3+3\,d^2x^2e^2+3\,d^3xe+d^4)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(5\,x^3e^3+39\,dx^2e^2+57\,d^2xe+24\,d^3)\sqrt{-x^2e^2+d^2}}{5\,(x^3e^8+3\,dx^2e^7+3\,d^2xe^6+d^3e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*d*x^3*e^3 + 72*d^2*x^2*e^2 + 72*d^3*x*e + 24*d^4 - 30*(d*x^3*e^3 +
 3*d^2*x^2*e^2 + 3*d^3*x*e + d^4)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)
/x) + (5*x^3*e^3 + 39*d*x^2*e^2 + 57*d^2*x*e + 24*d^3)*sqrt(-x^2*e^2 + d^2)
)/(x^3*e^8 + 3*d*x^2*e^7 + 3*d^2*x*e^6 + d^3*e^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A]
time = 2.12, size = 194, normalized size = 1.33

$$-3\,d\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)-\sqrt{-x^2e^2+d^2}\,e^{(-5)}+\frac{2\left(\frac{80\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^{(-2)}}{x}+\frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{(-4)}}{x^2}+\frac{70\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3de^{(-6)}}{x^3}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4de^{(-8)}}{x^4}+19\,d\right)e^{(-5)}}{5\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -3*d*arcsin(x*e/d)*e^(-5)*sgn(d) - sqrt(-x^2*e^2 + d^2)*e^(-5) + 2/5*(80*(d
*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^(-2)/x + 120*(d*e + sqrt(-x^2*e^2 + d^2)*e
)^2*d*e^(-4)/x^2 + 70*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*e^(-6)/x^3 + 15*(d
```

```
*e + sqrt(-x^2*e^2 + d^2)*e)^4*d*e^(-8)/x^4 + 19*d)*e^(-5)/((d*e + sqrt(-x^
2*e^2 + d^2)*e)*e^(-2)/x + 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{d^2-e^2x^2}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
[Out] int(x^4/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.181 $\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}+\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(-e*x+d)^3/e^4/(-e^2*x^2+d^2)^(5/2)-13/15*d*(-e*x+d)^2/e^4/(-e^2*x^
2+d^2)^(3/2)+arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+32/15*(-e*x+d)/e^4/(-e^2*
x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {866, 1649, 792, 223, 209}

$$\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}+\frac{d^2(d-ex)^3}{5e^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{13d(d-ex)^2}{15e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^(5/2)) - (13*d*(d - e*x)^2)/(15*e^
4*(d^2 - e^2*x^2)^(3/2)) + (32*(d - e*x))/(15*e^4*Sqrt[d^2 - e^2*x^2]) + Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{x^3(d-ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)^2\left(-\frac{3d^3}{e^3}+\frac{5d^2x}{e^2}-\frac{5dx^2}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{\left(-\frac{17d^3}{e^3}+\frac{15d^2x}{e^2}\right)(d-ex)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}}{e^3}$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{1+e^2}\right.}{}$$

$$= \frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{e}{\sqrt{d^2-}}\right.}{e^4}$$

Mathematica [A]

time = 0.35, size = 93, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}\,(22d^2+51dex+32e^2x^2)}{15e^4(d+ex)^3}+\frac{\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(22*d^2 + 51*d*e*x + 32*e^2*x^2))/(15*e^4*(d + e*x)^3)
 + (Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(106) = 212$.
time = 0.06, size = 319, normalized size = 2.66

method	result
default	$\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}}+\frac{3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^5\left(x+\frac{d}{e}\right)}-\frac{d^3\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+3/e^5/(x+d/e)*
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-d^3/e^6*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+3/e
^5*d^2*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+
d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.49, size = 124, normalized size = 1.03

$$\arcsin\left(\frac{xe}{d}\right)e^{(-4)}+\frac{\sqrt{-x^2e^2+d^2}\,d^2}{5\,(x^3e^7+3\,dx^2e^6+3\,d^2xe^5+d^3e^4)}-\frac{13\,\sqrt{-x^2e^2+d^2}\,d}{15\,(x^2e^6+2\,dxe^5+d^2e^4)}+\frac{32\,\sqrt{-x^2e^2+d^2}}{15\,(xe^5+de^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x*e/d)*e^(-4) + 1/5*sqrt(-x^2*e^2 + d^2)*d^2/(x^3*e^7 + 3*d*x^2*e^6
+ 3*d^2*x*e^5 + d^3*e^4) - 13/15*sqrt(-x^2*e^2 + d^2)*d/(x^2*e^6 + 2*d*x*e^
5 + d^2*e^4) + 32/15*sqrt(-x^2*e^2 + d^2)/(x*e^5 + d*e^4)
```

Fricas [A]
time = 2.36, size = 148, normalized size = 1.23

$$\frac{22\,x^3e^3+66\,dx^2e^2+66\,d^2xe+22\,d^3-30\,(x^3e^3+3\,dx^2e^2+3\,d^2xe+d^3)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(32\,x^2e^2+51\,dxe+22\,d^2)\sqrt{-x^2e^2+d^2}}{15\,(x^3e^7+3\,dx^2e^6+3\,d^2xe^5+d^3e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(22*x^3*e^3 + 66*d*x^2*e^2 + 66*d^2*x*e + 22*d^3 - 30*(x^3*e^3 + 3*d*x
^2*e^2 + 3*d^2*x*e + d^3)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (3
2*x^2*e^2 + 51*d*x*e + 22*d^2)*sqrt(-x^2*e^2 + d^2))/(x^3*e^7 + 3*d*x^2*e^6
 + 3*d^2*x*e^5 + d^3*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A]
time = 1.94, size = 169, normalized size = 1.41

$$\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{2\left(\frac{95\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+\frac{145\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{75\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-8)}}{x^4}+22\right)e^{(-4)}}{15\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x*e/d)*e^(-4)*sgn(d) - 2/15*(95*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2
)/x + 145*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-4)/x^2 + 75*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^3*e^(-6)/x^3 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^
4 + 22)*e^(-4)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(x^3/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.182 $\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

```
[Out] -1/5*d*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)^3+8/15*(-e^2*x^2+d^2)^(1/2)/e^3/(e*
x+d)^2-7/15*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {1653, 807, 673, 665}

$$-\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*(d*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^3) + (8*Sqrt[d^2 - e^2*x^2])/(1
5*e^3*(d + e*x)^2) - (7*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
```

```
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx &= \frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{\int \frac{2d^2e^2+de^3x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{(7d)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e^2}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}+\frac{7\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15e^2}\\
&= -\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d+ex)^3}+\frac{8\sqrt{d^2-e^2x^2}}{15e^3(d+ex)^2}-\frac{7\sqrt{d^2-e^2x^2}}{15de^3(d+ex)}
\end{aligned}
$$

Mathematica [A]
time = 0.30, size = 52, normalized size = 0.55

$$\frac{(-2d^2-6dex-7e^2x^2)\sqrt{d^2-e^2x^2}}{15de^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((-2*d^2 - 6*d*e*x - 7*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(83) = 166$.
time = 0.08, size = 288, normalized size = 3.03

method	result
trager	$-\frac{(7e^2x^2+6dex+2d^2)\sqrt{-e^2x^2+d^2}}{15d\,e^3(ex+d)^3}$
gosper	$-\frac{(-ex+d)(7e^2x^2+6dex+2d^2)}{15(ex+d)^2d\,e^3\sqrt{-e^2x^2+d^2}}$
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4d\left(x+\frac{d}{e}\right)}+\frac{d^2\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+}}{3de\left(x+\right.}\right.}{}\right.}{e^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^4/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/e^5*d^2*(-1/5/d/e/(
x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(1/2)))-2*d/e^4*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.50, size = 113, normalized size = 1.19

$$-\frac{\sqrt{-x^2e^2+d^2}\,d}{5\,(x^3e^6+3\,dx^2e^5+3\,d^2xe^4+d^3e^3)}+\frac{8\,\sqrt{-x^2e^2+d^2}}{15\,(x^2e^5+2\,dxe^4+d^2e^3)}-\frac{7\,\sqrt{-x^2e^2+d^2}}{15\,(dxe^4+d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*d/(x^3*e^6 + 3*d*x^2*e^5 + 3*d^2*x*e^4 + d^3*e^3)
 + 8/15*sqrt(-x^2*e^2 + d^2)/(x^2*e^5 + 2*d*x*e^4 + d^2*e^3) - 7/15*sqrt(-x
^2*e^2 + d^2)/(d*x*e^4 + d^2*e^3)
```

Fricas [A]
time = 2.63, size = 98, normalized size = 1.03

$$-\frac{2\,x^3e^3+6\,dx^2e^2+6\,d^2xe+2\,d^3+(7\,x^2e^2+6\,dxe+2\,d^2)\sqrt{-x^2e^2+d^2}}{15\,(dx^3e^6+3\,d^2x^2e^5+3\,d^3xe^4+d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*x^3*e^3 + 6*d*x^2*e^2 + 6*d^2*x*e + 2*d^3 + (7*x^2*e^2 + 6*d*x*e +
 2*d^2)*sqrt(-x^2*e^2 + d^2))/(d*x^3*e^6 + 3*d^2*x^2*e^5 + 3*d^3*x*e^4 + d^
4*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A]
time = 1.89, size = 98, normalized size = 1.03

$$\frac{4\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+1\right)e^{(-3)}}{15\,d\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 10*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*e^(-4)/x^2 + 1)*e^(-3)/(d*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)
/x + 1)^5)
```

Mupad [B]
time = 2.76, size = 48, normalized size = 0.51

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+6\,d\,e\,x+7\,e^2\,x^2\right)}{15\,d\,e^3\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(2*d^2 + 7*e^2*x^2 + 6*d*e*x))/(15*d*e^3*(d + e*x)^
3)
```

3.183 $\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=97

$$\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/e^2/(e*x+d)^3-1/5*(-e^2*x^2+d^2)^(1/2)/d/e^2/(e*x+
d)^2-1/5*(-e^2*x^2+d^2)^(1/2)/d^2/e^2/(e*x+d)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {807, 673, 665}

$$-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*e^2*(d + e*x)^3) - Sqrt[d^2 - e^2*x^2]/(5*d*e^2*(d +
 e*x)^2) - Sqrt[d^2 - e^2*x^2]/(5*d^2*e^2*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
```

```
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx&=\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}+\frac{3\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5e}\\&=\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}+\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{5de}\\&=\frac{\sqrt{d^2-e^2x^2}}{5e^2(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}}{5de^2(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2e^2(d+ex)}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 49, normalized size = 0.51

$$-\frac{\sqrt{d^2-e^2x^2}\,(d^2+3dex+e^2x^2)}{5d^2e^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] -1/5*(Sqrt[d^2 - e^2*x^2]*(d^2 + 3*d*e*x + e^2*x^2))/(d^2*e^2*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(85) = 170.
time = 0.07, size = 240, normalized size = 2.47

method	result
trager	$-\frac{(e^2x^2+3dex+d^2)\sqrt{-e^2x^2+d^2}}{5d^2(ex+d)^3e^2}$
gosper	$-\frac{(-ex+d)(e^2x^2+3dex+d^2)}{5(ex+d)^2d^2e^2\sqrt{-e^2x^2+d^2}}$
default	$-\frac{d\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2d}}{3d^2\left(x+\frac{d}{e}\right)}\right.}{5d}\right.}{e^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -d/e^4*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1
/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/e^3*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.47, size = 117, normalized size = 1.21

$$\frac{\sqrt{-x^2e^2+d^2}}{5\left(x^3e^5+3\,dx^2e^4+3\,d^2xe^3+d^3e^2\right)}-\frac{\sqrt{-x^2e^2+d^2}}{5\left(dx^2e^4+2\,d^2xe^3+d^3e^2\right)}-\frac{\sqrt{-x^2e^2+d^2}}{5\left(d^2xe^3+d^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(-x^2*e^2 + d^2)/(x^3*e^5 + 3*d*x^2*e^4 + 3*d^2*x*e^3 + d^3*e^2) -
1/5*sqrt(-x^2*e^2 + d^2)/(d*x^2*e^4 + 2*d^2*x*e^3 + d^3*e^2) - 1/5*sqrt(-x^
2*e^2 + d^2)/(d^2*x*e^3 + d^3*e^2)
```

Fricas [A]
time = 2.85, size = 94, normalized size = 0.97

$$-\frac{x^3e^3+3\,dx^2e^2+3\,d^2xe+d^3+\left(x^2e^2+3\,dxe+d^2\right)\sqrt{-x^2e^2+d^2}}{5\left(d^2x^3e^5+3\,d^3x^2e^4+3\,d^4xe^3+d^5e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3 + (x^2*e^2 + 3*d*x*e + d^2)*s
qrt(-x^2*e^2 + d^2))/(d^2*x^3*e^5 + 3*d^3*x^2*e^4 + 3*d^4*x*e^3 + d^5*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A]
time = 1.90, size = 128, normalized size = 1.32

$$\frac{2\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}+1\right)e^{(-2)}}{5\,d^2\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 5*(d*e + sqrt(-x^2*e^2 + d
^2)*e)^2*e^(-4)/x^2 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 + 1)*e^
(-2)/(d^2*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5)
```

Mupad [B]
time = 2.59, size = 45, normalized size = 0.46

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x+e^2\,x^2\right)}{5\,d^2\,e^2\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 + e^2*x^2 + 3*d*e*x))/(5*d^2*e^2*(d + e*x)^3)
```

3.184 $\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)^3-2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(e*
x+d)^2-2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(e*x+d)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {673, 665}

$$-\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}-\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^3) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^
2*e*(d + e*x)^2) - (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3} + \frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3} - \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2} + \frac{2\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}$$
$$= -\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3} - \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d+ex)^2} - \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.52

$$\frac{(-7d^2-6dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
[Out] ((-7*d^2 - 6*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x)^3)
```

Maple [A]
time = 0.07, size = 145, normalized size = 1.45

method	result
trager	$-\frac{(2e^2x^2+6dex+7d^2)\sqrt{-e^2x^2+d^2}}{15d^3(ex+d)^3e}$
gosper	$-\frac{(-ex+d)(2e^2x^2+6dex+7d^2)}{15(ex+d)^2d^3e\sqrt{-e^2x^2+d^2}}$
default	$\frac{-\frac{\sqrt{-(x+\frac{d}{e})^2e^2+2de(x+\frac{d}{e})}}{5de(x+\frac{d}{e})^3}+\frac{2e\left(-\frac{\sqrt{-(x+\frac{d}{e})^2e^2+2de(x+\frac{d}{e})}}{3de(x+\frac{d}{e})^2}-\frac{\sqrt{-(x+\frac{d}{e})^2e^2+2de(x}}{3d^2(x+\frac{d}{e})}\right.}{5d}}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/e^3*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/
3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.48, size = 122, normalized size = 1.22

$$-\frac{\sqrt{-x^2e^2+d^2}}{5\,(dx^3e^4+3\,d^2x^2e^3+3\,d^3xe^2+d^4e)}-\frac{2\,\sqrt{-x^2e^2+d^2}}{15\,(d^2x^2e^3+2\,d^3xe^2+d^4e)}-\frac{2\,\sqrt{-x^2e^2+d^2}}{15\,(d^3xe^2+d^4e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)/(d*x^3*e^4 + 3*d^2*x^2*e^3 + 3*d^3*x*e^2 + d^4*e)
 - 2/15*sqrt(-x^2*e^2 + d^2)/(d^2*x^2*e^3 + 2*d^3*x*e^2 + d^4*e) - 2/15*sqr
t(-x^2*e^2 + d^2)/(d^3*x*e^2 + d^4*e)
```

Fricas [A]
time = 2.89, size = 100, normalized size = 1.00

$$-\frac{7\,x^3e^3+21\,dx^2e^2+21\,d^2xe+7\,d^3+(2\,x^2e^2+6\,dxe+7\,d^2)\sqrt{-x^2e^2+d^2}}{15\,(d^3x^3e^4+3\,d^4x^2e^3+3\,d^5xe^2+d^6e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(7*x^3*e^3 + 21*d*x^2*e^2 + 21*d^2*x*e + 7*d^3 + (2*x^2*e^2 + 6*d*x*e
 + 7*d^2)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3*e^4 + 3*d^4*x^2*e^3 + 3*d^5*x*e^2
+ d^6*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A]
time = 2.73, size = 158, normalized size = 1.58

$$\frac{2\left(\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-8)}}{x^4}+7\right)e^{(-1)}}{15\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 40*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^2*e^(-4)/x^2 + 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 + 1
5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^4 + 7)*e^(-1)/(d^3*((d*e + sqrt
(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5)
```

Mupad [B]
time = 2.62, size = 48, normalized size = 0.48

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2+6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 + 6*d*e*x))/(15*d^3*e*(d + e*x)^
3)
```

3.185 $\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=115

$$\frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] 4/5*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)+1/15*(-11*e*x+5*d)/d^2/(-e^2*x^2+d^2)^(3/
2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4+1/15*(-22*e*x+15*d)/d^4/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 837, 12, 272, 65, 214}

$$\frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) + (5*d - 11*e*x)/(15*d^2*(d^2 - e^2
*x^2)^(3/2)) + (15*d - 22*e*x)/(15*d^4*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[
d^2 - e^2*x^2]/d]/d^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+11d^2ex}{x(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int \frac{-15d^5e^2+22d^4e^3x}{x(d^2-e^2x^2)^{3/2}}\,dx}{15d^6e^2}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int -\frac{15d^7e^4}{x\sqrt{d^2-e}}}{15d^{10}e}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2}}}{d^3}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-}\right.}{}$$

$$= \frac{4(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} + \frac{5d-11ex}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d-22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\sqrt{d}\right.}{d^4}$$

Mathematica [A]
time = 0.45, size = 88, normalized size = 0.77

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(32d^2+51dex+22e^2x^2)}{(d+ex)^3} + 30\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(32*d^2 + 51*d*e*x + 22*e^2*x^2))/(d + e*x)^3 + 30*Ar
cTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(15*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(101) = 202.
time = 0.07, size = 332, normalized size = 2.89

method	result
default	$\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{e\,d^{4} \left(x+\frac{d}{e}\right)}-\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}{5 d e \left(x+\frac{d}{e}\right)^{3}}+\frac{2 e \left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e (}}{3 d e \left(x+\frac{d}{e}\right)^{2}}\right.}{e^{2} d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/d^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/e^2/d*(-1/5/d/e/(x+d
/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2)))-1/e/d^2*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/
2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))-1/d^3/(d^2)^(1/2)*
ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2*e^2 + d^2)*(x*e + d)^3*x), x)
```

Fricas [A]
time = 2.67, size = 148, normalized size = 1.29

$$\frac{32\,x^{3}e^{3}+96\,dx^{2}e^{2}+96\,d^{2}xe+32\,d^{3}+15\,(x^{3}e^{3}+3\,dx^{2}e^{2}+3\,d^{2}xe+d^{3})\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+(22\,x^{2}e^{2}+51\,dxe+32\,d^{2})\sqrt{-x^{2}e^{2}+d^{2}}}{15\,(d^{4}x^{3}e^{3}+3\,d^{5}x^{2}e^{2}+3\,d^{6}xe+d^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(32*x^3*e^3 + 96*d*x^2*e^2 + 96*d^2*x*e + 32*d^3 + 15*(x^3*e^3 + 3*d*x
^2*e^2 + 3*d^2*x*e + d^3)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (22*x^2*e^2
+ 51*d*x*e + 32*d^2)*sqrt(-x^2*e^2 + d^2))/(d^4*x^3*e^3 + 3*d^5*x^2*e^2 + 3
*d^6*x*e + d^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [A]
time = 1.83, size = 195, normalized size = 1.70

$$-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^4}-\frac{2\left(\frac{115\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+\frac{185\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{135\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}+\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-8)}}{x^4}+32\right)}{15\,d^4\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4 - 2/15*(
115*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 185*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^2*e^(-4)/x^2 + 135*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 + 45*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^4 + 32)/(d^4*((d*e + sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/x + 1)^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.186 $\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

```
[Out] -4/5*e*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)-1/5*e*(-7*e*x+5*d)/d^3/(-e^2*x^2+d^2
)^(3/2)+3*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5-1/5*e*(-19*e*x+15*d)/d^5/(-
e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A]
time = 0.19, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1819, 821, 272, 65, 214}

$$-\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^5x}+\frac{3e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}-\frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d - 7*e*x))/(5*d^3*(d^
2 - e^2*x^2)^(3/2)) - (e*(15*d - 19*e*x))/(5*d^5*Sqrt[d^2 - e^2*x^2]) - Sqr
t[d^2 - e^2*x^2]/(d^5*x) + (3*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^5
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-16de^2x^2}{x^2\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+42de^2x^2}{x^2\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45d}{x^2\sqrt{d^2-e}}}{15d^6}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x}$$

$$= -\frac{4e(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d-7ex)}{5d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x}$$

Mathematica [A]
time = 0.43, size = 103, normalized size = 0.71

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\,(5d^3+39d^2ex+57de^2x^2+24e^3x^3)}{x(d+ex)^3} + 30e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*((Sqrt[d^2 - e^2*x^2]*(5*d^3 + 39*d^2*e*x + 57*d*e^2*x^2 + 24*e^3*x^3)
)/(x*(d + e*x)^3) + 30*e*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/d
^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(130) = 260.
time = 0.10, size = 349, normalized size = 2.39

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^5x}-\frac{19\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5d^5\left(x+\frac{d}{e}\right)}+\frac{3e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^4\sqrt{d^2}}-\frac{4\sqrt{-}}{}$
default	$-\frac{3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^5\left(x+\frac{d}{e}\right)}+\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2}}{3de\left(x+\frac{d}{e}\right)^2}\right.}{}}{e\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/e/d^2*(-1/5/d/e/(x+d/
e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)))-(-e^2*x^2+d^2)^(1/2)/d^5/x+2/d^3*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(
1/2))+3*e/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2*e^2 + d^2)*(x*e + d)^3*x^2), x)
```

Fricas [A]
time = 2.44, size = 173, normalized size = 1.18

$$-\frac{24\,x^4e^4+72\,dx^3e^3+72\,d^2x^2e^2+24\,d^3xe+15\,(x^4e^4+3\,dx^3e^3+3\,d^2x^2e^2+d^3xe)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(24\,x^3e^3+57\,dx^2e^2+39\,d^2xe+5\,d^3)\sqrt{-x^2e^2+d^2}}{5\,(d^5x^4e^3+3\,d^6x^3e^2+3\,d^7x^2e+d^8x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(24*x^4*e^4 + 72*d*x^3*e^3 + 72*d^2*x^2*e^2 + 24*d^3*x*e + 15*(x^4*e^4
 + 3*d*x^3*e^3 + 3*d^2*x^2*e^2 + d^3*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x
) + (24*x^3*e^3 + 57*d*x^2*e^2 + 39*d^2*x*e + 5*d^3)*sqrt(-x^2*e^2 + d^2))/
(d^5*x^4*e^3 + 3*d^6*x^3*e^2 + 3*d^7*x^2*e + d^8*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(133) = 266$.
time = 2.85, size = 287, normalized size = 1.97

$$\frac{3\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^5}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,d^5x}+\frac{x\left(\frac{121\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{x}+\frac{410\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-3)}}{x^2}+\frac{610\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-5)}}{x^3}+\frac{425\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-7)}}{x^4}+\frac{125\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-9)}}{x^5}+5\,e\right)e^2}{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5 - 1/2
*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/(d^5*x) + 1/10*x*(121*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*e^(-1)/x + 410*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-3)/x^2
 + 610*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-5)/x^3 + 425*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^4*e^(-7)/x^4 + 125*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-9)/x^5
 + 5*e)*e^2/((d*e + sqrt(-x^2*e^2 + d^2)*e)*d^5*((d*e + sqrt(-x^2*e^2 + d^2
)*e)*e^(-2)/x + 1)^5)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.187 $\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=183

$$\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\ldots\right.}{2d\ldots}$$

```
[Out] 4/5*e^2*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)+1/15*e^2*(-31*e*x+25*d)/d^4/(-e^2
*x^2+d^2)^(3/2)-13/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6+1/15*e^2*(-107
*e*x+90*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^5/x^2+3*e*(-
e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A]

time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}}+\frac{3e\sqrt{d^2-e^2x^2}}{d^6x}-\frac{13e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6}-\frac{\sqrt{d^2-e^2x^2}}{2d^5x^2}+\frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (4*e^2*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (e^2*(25*d - 31*e*x))/(15
*d^4*(d^2 - e^2*x^2)^(3/2)) + (e^2*(90*d - 107*e*x))/(15*d^6*Sqrt[d^2 - e^2
*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^5*x^2) + (3*e*Sqrt[d^2 - e^2*x^2])/(d^6*x
) - (13*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^6)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d-ex)^3}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^3+15d^2ex-20de^2x^2+16e^3x^3}{x^3\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^3-45d^2ex+75de^2x^2-62e^3x^3}{x^3\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^3+45}{x^3\sqrt{d}}}{}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2}}{2d^5x^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2}}{2d^5x^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2}}{2d^5x^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2}}{2d^5x^2}$$

$$= \frac{4e^2(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{e^2(25d-31ex)}{15d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2}}{2d^5x^2}$$

Mathematica [A]
time = 0.48, size = 116, normalized size = 0.63

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(-15d^4+45d^3ex+479d^2e^2x^2+717de^3x^3+304e^4x^4)}{x^2(d+ex)^3} + 390e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{30d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 45*d^3*e*x + 479*d^2*e^2*x^2 + 717*d*e^3*x
^3 + 304*e^4*x^4))/(x^2*(d + e*x)^3) + 390*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqrt
[d^2 - e^2*x^2])/d])/(30*d^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(161) = 322$.
time = 0.09, size = 424, normalized size = 2.32

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-6ex+d)}{2d^6x^2}+\frac{107e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^6\left(x+\frac{d}{e}\right)}-\frac{13e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^5\sqrt{d^2}}$
default	$\frac{6e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^6\left(x+\frac{d}{e}\right)}-\frac{-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de}}{3de\left(x+\frac{d}{e}\right)^2}\right.}{}}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6*e/d^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/d^3*(-1/5/d/e/(x+d/e
)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2)))+1/d^3*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(1/2)-1/2*e^2/d^2/(d^2)^(1/2)
*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))+3*e*(-e^2*x^2+d^2)^(1/2)
/d^6/x-3*e/d^4*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3
/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))-6*e^2/d^5/(d^2)^(1/2)*ln
((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2*e^2 + d^2)*(x*e + d)^3*x^3), x)
```

Fricas [A]
time = 2.22, size = 189, normalized size = 1.03

$$\frac{254\,x^5e^5+762\,dx^4e^4+762\,d^2x^3e^3+254\,d^3x^2e^2+195\,(x^5e^5+3\,dx^4e^4+3\,d^2x^3e^3+d^3x^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(304\,x^4e^4+717\,dx^3e^3+479\,d^2x^2e^2+45\,d^3xe-15\,d^4)\sqrt{-x^2e^2+d^2}}{30\,(d^6x^5e^3+3\,d^7x^4e^2+3\,d^8x^3e+d^9x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(254*x^5*e^5 + 762*d*x^4*e^4 + 762*d^2*x^3*e^3 + 254*d^3*x^2*e^2 + 195
*(x^5*e^5 + 3*d*x^4*e^4 + 3*d^2*x^3*e^3 + d^3*x^2*e^2)*log(-(d - sqrt(-x^2*
e^2 + d^2))/x) + (304*x^4*e^4 + 717*d*x^3*e^3 + 479*d^2*x^2*e^2 + 45*d^3*x*
e - 15*d^4)*sqrt(-x^2*e^2 + d^2))/(d^6*x^5*e^3 + 3*d^7*x^4*e^2 + 3*d^8*x^3*
e + d^9*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(156) = 312.
time = 1.51, size = 353, normalized size = 1.93

$$-\frac{13\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^6}-\frac{x^2\left(\frac{2782\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-2)}}{x^2}+\frac{9410\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-4)}}{x^3}+\frac{13645\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-6)}}{x^4}+\frac{9285\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-8)}}{x^5}+\frac{2580\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6e^{(-10)}}{x^6}+\frac{105\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}{x}-15\,e^2\right)e^4}{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}-\frac{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^6e^{(-2)}}{x^2}-\frac{12\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^6}{x}}{8\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -13/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6
 - 1/120*x^2*(2782*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-2)/x^2 + 9410*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^3*e^(-4)/x^3 + 13645*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^4*e^(-6)/x^4 + 9285*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-8)/x^5 + 2580*(
d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-10)/x^6 + 105*(d*e + sqrt(-x^2*e^2 + d^
2)*e)/x - 15*e^2)*e^4/((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^6*((d*e + sqrt(-x
^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5) - 1/8*((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d
^6*e^(-2)/x^2 - 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^6/x)/d^12
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/(x^3*(d^2 - e^2*x^2)^(1/2)*(d + e*x)^3), x)
```

3.188 $\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=204

$$\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}$$

```
[Out] 1/5*d^4*(-e*x+d)^4/e^6/(-e^2*x^2+d^2)^(5/2)-8/5*d^3*(-e*x+d)^3/e^6/(-e^2*x^
2+d^2)^(3/2)+18*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+10*d^2*(-e*x+d)^2/
e^6/(-e^2*x^2+d^2)^(1/2)+59/3*d^2*(-e^2*x^2+d^2)^(1/2)/e^6-2*d*x*(-e^2*x^2+
d^2)^(1/2)/e^5+1/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^4
```

Rubi [A]
time = 0.36, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\frac{18d^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}+\frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}}+\frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6}-\frac{2dx\sqrt{d^2-e^2x^2}}{e^5}+\frac{x^2\sqrt{d^2-e^2x^2}}{3e^4}+\frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(5*e^6*(d^2 - e^2*x^2)^(5/2)) - (8*d^3*(d - e*x)^3)/(5*e^
6*(d^2 - e^2*x^2)^(3/2)) + (10*d^2*(d - e*x)^2)/(e^6*Sqrt[d^2 - e^2*x^2]) +
 (59*d^2*Sqrt[d^2 - e^2*x^2])/(3*e^6) - (2*d*x*Sqrt[d^2 - e^2*x^2])/e^5 + (
x^2*Sqrt[d^2 - e^2*x^2])/(3*e^4) + (18*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^5(d-ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{5d^4x}{e^4}-\frac{5d^3x^2}{e^3}+\frac{5d^2x^3}{e^2}-\frac{5dx^4}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(-\frac{60d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{30d^3x^2}{e^3}+\frac{15d^2x^3}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)\left(-\frac{240d^5}{e^5}+\frac{45d^4x}{e^4}-\frac{15d^3}{e^3}\cdots\right.}{\sqrt{d^2-e^2x^2}}}{15d^3}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{x^2\sqrt{d^2-e^2x^2}}{3e^4} + \frac{\int \frac{720d^6}{e^3}\cdots}{\cdots}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx\sqrt{d^2-e^2x^2}}{e^5} + \frac{x^2\sqrt{\cdots}}{\cdots}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\cdots}{\cdots}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\cdots}{\cdots}$$

$$= \frac{d^4(d-ex)^4}{5e^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{8d^3(d-ex)^3}{5e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{10d^2(d-ex)^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{59d^2\sqrt{d^2-e^2x^2}}{3e^6} - \frac{2dx\cdots}{\cdots}$$

Mathematica [A]
time = 0.41, size = 130, normalized size = 0.64

$$\frac{\sqrt{d^2-e^2x^2}\,(424d^5+1002d^4ex+674d^3e^2x^2+70d^2e^3x^3-15de^4x^4+5e^5x^5)}{15e^6(d+ex)^3} + \frac{18d^3\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(424*d^5 + 1002*d^4*e*x + 674*d^3*e^2*x^2 + 70*d^2*e^3
*x^3 - 15*d*e^4*x^4 + 5*e^5*x^5))/(15*e^6*(d + e*x)^3) + (18*d^3*Sqrt[-e^2]
*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(182) = 364$.
time = 0.08, size = 432, normalized size = 2.12

method	result
risch	$\frac{(e^2x^2-6dex+29d^2)\sqrt{-e^2x^2+d^2}}{3e^6}+\frac{18d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^5\sqrt{e^2}}+\frac{108d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5e^7\left(x+\frac{d}{e}\right)}$
default	$-\frac{(-e^2x^2+d^2)^{\frac{3}{2}}}{3e^6}-\frac{4d\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^5}+\frac{10d^2\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e^6*(-e^2*x^2+d^2)^(3/2)-4*d/e^5*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(
e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))+10/e^6*d^2*((-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)))-5/3/e^9*d^3/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(3/2)-d^5/e^9*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3
/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))-10/e^7*d^3*(-1
/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2))))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.10, size = 188, normalized size = 0.92

$$\frac{424\,d^3x^3e^3+1272\,d^4x^2e^2+1272\,d^5xe+424\,d^6-540\,(d^3x^3e^3+3\,d^4x^2e^2+3\,d^5xe+d^6)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(5\,x^5e^5-15\,dx^4e^4+70\,d^2x^3e^3+674\,d^3x^2e^2+1002\,d^4xe+424\,d^5)\sqrt{-x^2e^2+d^2}}{15\,(x^3e^9+3\,dx^2e^8+3\,d^2xe^7+d^3e^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(424*d^3*x^3*e^3 + 1272*d^4*x^2*e^2 + 1272*d^5*x*e + 424*d^6 - 540*(d^
3*x^3*e^3 + 3*d^4*x^2*e^2 + 3*d^5*x*e + d^6)*arctan(-(d - sqrt(-x^2*e^2 + d
^2))*e^(-1)/x) + (5*x^5*e^5 - 15*d*x^4*e^4 + 70*d^2*x^3*e^3 + 674*d^3*x^2*e
^2 + 1002*d^4*x*e + 424*d^5)*sqrt(-x^2*e^2 + d^2))/(x^3*e^9 + 3*d*x^2*e^8 +
 3*d^2*x*e^7 + d^3*e^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [A]
time = 1.64, size = 224, normalized size = 1.10

$$18\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)+\frac{1}{3}\sqrt{-x^2e^2+d^2}\left(29\,d^2e^{(-6)}+\left(xe^{(-4)}-6\,de^{(-5)}\right)x\right)-\frac{2\left(\frac{385\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3e^{(-2)}}{x}+\frac{575\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^3e^{(-4)}}{x^2}+\frac{355\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^3e^{(-6)}}{x^3}+\frac{80\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3e^{(-8)}}{x^4}+93\,d^3\right)e^{(-6)}}{5\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 18*d^3*arcsin(x*e/d)*e^(-6)*sgn(d) + 1/3*sqrt(-x^2*e^2 + d^2)*(29*d^2*e^(-6
) + (x*e^(-4) - 6*d*e^(-5))*x) - 2/5*(385*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^
3*e^(-2)/x + 575*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*e^(-4)/x^2 + 355*(d*e
 + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*e^(-6)/x^3 + 80*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^4*d^3*e^(-8)/x^4 + 93*d^3)*e^(-6)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-
2)/x + 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
[Out] int((x^5*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.189 $\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=160

$$-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{19d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

```
[Out] -1/5*d^3*(-e*x+d)^4/e^5/(-e^2*x^2+d^2)^(5/2)+19/15*d^2*(-e*x+d)^3/e^5/(-e^2
*x^2+d^2)^(3/2)-19/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-6*d*(-e*x+d)^
2/e^5/(-e^2*x^2+d^2)^(1/2)-1/2*(-e*x+20*d)*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A]
time = 0.26, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {866, 1649, 794, 223, 209}

$$-\frac{19d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}+\frac{19d^2(d-ex)^3}{15e^5\left(d^2-e^2x^2\right)^{3/2}}-\frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}}-\frac{(20d-ex)\sqrt{d^2-e^2x^2}}{2e^5}-\frac{d^3(d-ex)^4}{5e^5\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/5*(d^3*(d - e*x)^4)/(e^5*(d^2 - e^2*x^2)^(5/2)) + (19*d^2*(d - e*x)^3)/(
15*e^5*(d^2 - e^2*x^2)^(3/2)) - (6*d*(d - e*x)^2)/(e^5*Sqrt[d^2 - e^2*x^2])
 - ((20*d - e*x)*Sqrt[d^2 - e^2*x^2])/(2*e^5) - (19*d^2*ArcTan[(e*x)/Sqrt[d
^2 - e^2*x^2]])/(2*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{x^4(d-ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{5d^3x}{e^3}+\frac{5d^2x^2}{e^2}-\frac{5dx^3}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{\int \frac{(d-ex)^2\left(\frac{45d^4}{e^4}-\frac{30d^3x}{e^3}+\frac{15d^2x^2}{e^2}\right)}{(d^2-e^2x^2)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{135d^4}{e^4}-\frac{15d^3x}{e^3}\right)(d-ex)}{\sqrt{d^2-e^2x^2}}\,d}{15d^3}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x}}{2e^5}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x}}{2e^5}$$

$$= -\frac{d^3(d-ex)^4}{5e^5(d^2-e^2x^2)^{5/2}} + \frac{19d^2(d-ex)^3}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{6d(d-ex)^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{(20d-ex)\sqrt{d^2-e^2x}}{2e^5}$$

Mathematica [A]
time = 0.41, size = 121, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\,(-448d^4-1059d^3ex-713d^2e^2x^2-75de^3x^3+15e^4x^4)}{30e^5(d+ex)^3} + \frac{19d^2(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-448*d^4 - 1059*d^3*e*x - 713*d^2*e^2*x^2 - 75*d*e^3*
x^3 + 15*e^4*x^4))/(30*e^5*(d + e*x)^3) + (19*d^2*(-e^2)^(3/2)*Log[-(Sqrt[-
e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(142) = 284.
time = 0.07, size = 408, normalized size = 2.55

method	result

risch	$-\frac{(-ex+8d)\sqrt{-e^2x^2+d^2}}{2e^5}-\frac{19d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^4\sqrt{e^2}}-\frac{199d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^6\left(x+\frac{d}{e}\right)}-$
default	$\frac{\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}}{e^4}-\frac{4d\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{}{\sqrt{-(x}}\right.}{}\right.}{e^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2)))-4/e^5*d*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e
^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+4/3/e
^8*d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+1/e^8*d^4*(-1/5/d/e/(
x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(3/2))+6/e^6*d^2*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 1.81, size = 179, normalized size = 1.12

$$-\frac{448\,d^2x^3e^3+1344\,d^3x^2e^2+1344\,d^4xe+448\,d^5-570\,(d^2x^3e^3+3\,d^3x^2e^2+3\,d^4xe+d^5)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(15\,x^4e^4-75\,dx^3e^3-713\,d^2x^2e^2-1059\,d^3xe-448\,d^4)\sqrt{-x^2e^2+d^2}}{30\,(x^3e^8+3\,dx^2e^7+3\,d^2xe^6+d^3e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/30*(448*d^2*x^3*e^3 + 1344*d^3*x^2*e^2 + 1344*d^4*x*e + 448*d^5 - 570*(d
^2*x^3*e^3 + 3*d^3*x^2*e^2 + 3*d^4*x*e + d^5)*arctan(-(d - sqrt(-x^2*e^2 +
d^2))*e^(-1)/x) - (15*x^4*e^4 - 75*d*x^3*e^3 - 713*d^2*x^2*e^2 - 1059*d^3*x
*e - 448*d^4)*sqrt(-x^2*e^2 + d^2))/(x^3*e^8 + 3*d*x^2*e^7 + 3*d^2*x*e^6 +
d^3*e^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [A]
time = 1.45, size = 214, normalized size = 1.34

$$-\frac{19}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(xe^{(-4)}-8\,de^{(-5)}\right)+\frac{2\left(\frac{685\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2e^{(-2)}}{x}+\frac{1025\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2e^{(-4)}}{x^2}+\frac{615\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2e^{(-6)}}{x^3}+\frac{135\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^2e^{(-8)}}{x^4}+164\,d^2\right)e^{(-5)}}{15\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -19/2*d^2*arcsin(x*e/d)*e^(-5)*sgn(d) + 1/2*sqrt(-x^2*e^2 + d^2)*(x*e^(-4)
- 8*d*e^(-5)) + 2/15*(685*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*e^(-2)/x + 102
5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*e^(-4)/x^2 + 615*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^3*d^2*e^(-6)/x^3 + 135*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^2*e^(
-8)/x^4 + 164*d^2)*e^(-5)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d^2-e^2\,x^2}}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.190 $\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=148

$$\frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}+\frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4}-\frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2}+\frac{4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

```
[Out] 1/5*d^2*(-e^2*x^2+d^2)^(3/2)/e^4/(e*x+d)^4-14/15*d*(-e^2*x^2+d^2)^(3/2)/e^4
/(e*x+d)^3-(-e^2*x^2+d^2)^(3/2)/e^4/(e*x+d)^2+4*d*arctan(e*x/(-e^2*x^2+d^2)
^(1/2))/e^4+8*d*(-e^2*x^2+d^2)^(1/2)/e^4/(e*x+d)
```

Rubi [A]
time = 0.16, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1653, 1651, 673, 665, 677, 223, 209}

$$\frac{4d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}+\frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4}-\frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3}+\frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)}-\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (8*d*Sqrt[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (d^2*(d^2 - e^2*x^2)^(3/2))/(5*
e^4*(d + e*x)^4) - (14*d*(d^2 - e^2*x^2)^(3/2))/(15*e^4*(d + e*x)^3) - (d^2
 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2) + (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

```
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} - \frac{\int \frac{\sqrt{d^2-e^2x^2}\,(2d^3e^2+5d^2e^3x+4de^4x^2)}{(d+ex)^4}\,dx}{e^5}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} - \frac{\int \left(\frac{d^3e^2\sqrt{d^2-e^2x^2}}{(d+ex)^4} - \frac{3d^2e^2\sqrt{d^2-e^2x^2}}{(d+ex)^3} + \frac{4de^2\sqrt{d^2-e^2x^2}}{(d+ex)^2}\right) dx}{e^5}$$

$$= -\frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} - \frac{(4d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^3} + \frac{(3d^2)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^3} - \frac{d^3\int \frac{\sqrt{d^2-}}{(d+}}{e^3}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{d(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} + \frac{(4d)\int \frac{}{\sqrt{}}}{}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} + \frac{(4d)\mathrm{Su}}{}$$

$$= \frac{8d\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d^2(d^2-e^2x^2)^{3/2}}{5e^4(d+ex)^4} - \frac{14d(d^2-e^2x^2)^{3/2}}{15e^4(d+ex)^3} - \frac{(d^2-e^2x^2)^{3/2}}{e^4(d+ex)^2} + \frac{4d\tan^{}}{}$$

Mathematica [A]
time = 0.35, size = 106, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\,(94d^3+222d^2ex+149de^2x^2+15e^3x^3)}{15e^4(d+ex)^3} + \frac{4d\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(94*d^3 + 222*d^2*e*x + 149*d*e^2*x^2 + 15*e^3*x^3))/(
15*e^4*(d + e*x)^3) + (4*d*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*
x^2]])/e^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(134) = 268$.
time = 0.08, size = 349, normalized size = 2.36

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e^4} + \frac{4d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^3\sqrt{e^2}} + \frac{104d\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15e^5\left(x+\frac{d}{e}\right)} - \frac{31d^2\sqrt{-\left(x+\right.}}{}$

default	$\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}+\frac{d e \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^{2}}}}{e^{4}}-\frac{d\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e\left(x+\frac{d}{e}\right.\right.}{e^{7}\left(x+\frac{d}{e}\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-1/e^7*d/(x+d/e)^3*(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(3/2)-d^3/e^7*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))-3*d
/e^5*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(1/2))))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 1.94, size = 164, normalized size = 1.11

$$\frac{94\,dx^{3}e^{3}+282\,d^{2}x^{2}e^{2}+282\,d^{3}xe+94\,d^{4}-120\,(dx^{3}e^{3}+3\,d^{2}x^{2}e^{2}+3\,d^{3}xe+d^{4})\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)+(15\,x^{3}e^{3}+149\,dx^{2}e^{2}+222\,d^{2}xe+94\,d^{3})\sqrt{-x^{2}e^{2}+d^{2}}}{15\,(x^{3}e^{7}+3\,dx^{2}e^{6}+3\,d^{2}xe^{5}+d^{3}e^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(94*d*x^3*e^3 + 282*d^2*x^2*e^2 + 282*d^3*x*e + 94*d^4 - 120*(d*x^3*e^
3 + 3*d^2*x^2*e^2 + 3*d^3*x*e + d^4)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(
-1)/x) + (15*x^3*e^3 + 149*d*x^2*e^2 + 222*d^2*x*e + 94*d^3)*sqrt(-x^2*e^2
+ d^2))/(x^3*e^7 + 3*d*x^2*e^6 + 3*d^2*x*e^5 + d^3*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [A]
time = 1.51, size = 193, normalized size = 1.30

$$4\,d \arcsin\left(\frac{xe}{d}\right) e^{(-4)}\mathrm{sgn}(d) + \sqrt{-x^2e^2+d^2}\, e^{(-4)} - \frac{2\left(\frac{335\left(de+\sqrt{-x^2e^2+d^2}\,e\right)de^{(-2)}}{x} + \frac{505\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2 de^{(-4)}}{x^2} + \frac{285\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3 de^{(-6)}}{x^3} + \frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4 de^{(-8)}}{x^4} + 79\,d\right)e^{(-4)}}{15\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 4*d*arcsin(x*e/d)*e^(-4)*sgn(d) + sqrt(-x^2*e^2 + d^2)*e^(-4) - 2/15*(335*(
d*e + sqrt(-x^2*e^2 + d^2)*e)*d*e^(-2)/x + 505*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^2*d*e^(-4)/x^2 + 285*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*e^(-6)/x^3 + 60*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d*e^(-8)/x^4 + 79*d)*e^(-4)/((d*e + sqrt(-
x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{d^2-e^2\,x^2}}{{(d+e\,x)}^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.191 $\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=115

$$-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4}+\frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -1/5*d*(-e^2*x^2+d^2)^(3/2)/e^3/(e*x+d)^4+3/5*(-e^2*x^2+d^2)^(3/2)/e^3/(e*x
+d)^3-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+
d)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1651, 673, 665, 677, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3}-\frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4}-\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^3
*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]]/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx &= \int \left(\frac{d^2\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4}-\frac{2d\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3}+\frac{\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2}\right)dx\\
&= \frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2}-\frac{(2d)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2}+\frac{d^2\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx}{e^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4}+\frac{2(d^2-e^2x^2)^{3/2}}{3e^3(d+ex)^3}-\frac{\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}+\frac{d\int \sqrt{}}{}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4}+\frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3}-\frac{\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{}{\sqrt{d^2}}\right.}{e^2}\\
&= -\frac{2\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{d(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^4}+\frac{3(d^2-e^2x^2)^{3/2}}{5e^3(d+ex)^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}$$

Mathematica [A]

time = 0.35, size = 91, normalized size = 0.79

$$\frac{(-8d^2 - 19dex - 13e^2x^2)\sqrt{d^2 - e^2x^2}}{5e^3(d + ex)^3} - \frac{\log\left(-\sqrt{-e^2}\,x + \sqrt{d^2 - e^2x^2}\right)}{(-e^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] ((-8*d^2 - 19*d*e*x - 13*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3)
- Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]/(-e^2)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(103) = 206$.
time = 0.08, size = 268, normalized size = 2.33

method	result
default	$\frac{2\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^6\left(x+\frac{d}{e}\right)^3}+\frac{d^2\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^6}+-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right.\right.}{de\left(x+\frac{d}{e}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/e^6/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+1/e^6*d^2*(-1/5/d/e/
(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(3/2))+1/e^4*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*ar
ctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)*x^2/(x*e + d)^4, x)
```

Fricas [A]
time = 1.64, size = 148, normalized size = 1.29

$$-\frac{8\,x^3e^3+24\,dx^2e^2+24\,d^2xe+8\,d^3-10\,(x^3e^3+3\,dx^2e^2+3\,d^2xe+d^3)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(13\,x^2e^2+19\,dxe+8\,d^2)\sqrt{-x^2e^2+d^2}}{5\,(x^3e^6+3\,dx^2e^5+3\,d^2xe^4+d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
[Out] -1/5*(8*x^3*e^3 + 24*d*x^2*e^2 + 24*d^2*x*e + 8*d^3 - 10*(x^3*e^3 + 3*d*x^2
*e^2 + 3*d^2*x*e + d^3)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (13*
x^2*e^2 + 19*d*x*e + 8*d^2)*sqrt(-x^2*e^2 + d^2))/(x^3*e^6 + 3*d*x^2*e^5 +
3*d^2*x*e^4 + d^3*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(-d+ex)\,(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
[Out] Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [A]
time = 1.05, size = 170, normalized size = 1.48

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{2\left(\frac{35\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+\frac{55\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{25\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}+\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-8)}}{x^4}+8\right)e^{(-3)}}{5\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
[Out] -arcsin(x*e/d)*e^(-3)*sgn(d) + 2/5*(35*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2
)/x + 55*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-4)/x^2 + 25*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^3*e^(-6)/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^4
+ 8)*e^(-3)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{d^2-e^2\,x^2}}{{(d+e\,x)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
[Out] int((x^2*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4, x)
```

3.192 $\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=64

$$\frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4}-\frac{4(d^2-e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(3/2)/e^2/(e*x+d)^4-4/15*(-e^2*x^2+d^2)^(3/2)/d/e^2/(e*x
+d)^3
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {807, 665}

$$\frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4}-\frac{4(d^2-e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(3/2)/(5*e^2*(d + e*x)^4) - (4*(d^2 - e^2*x^2)^(3/2))/(15*d
*e^2*(d + e*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4} + \frac{4\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5e}$$
$$= \frac{(d^2-e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2-e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

Mathematica [A]
time = 0.27, size = 52, normalized size = 0.81

$$\frac{\sqrt{d^2-e^2x^2}\,(-d^2-3dex+4e^2x^2)}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
[Out] (Sqrt[d^2 - e^2*x^2]*(-d^2 - 3*d*e*x + 4*e^2*x^2))/(15*d*e^2*(d + e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(56) = 112.
time = 0.06, size = 141, normalized size = 2.20

method	result	size
gosper	$-\frac{(-ex+d)(4ex+d)\sqrt{-e^2x^2+d^2}}{15(ex+d)^3 d\,e^2}$	42
trager	$-\frac{(-4e^2x^2+3dex+d^2)\sqrt{-e^2x^2+d^2}}{15d(ex+d)^3e^2}$	47
default	$-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^5d\left(x+\frac{d}{e}\right)^3} - \frac{d\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4} - \frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^5}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
[Out] -1/3/e^5/d/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-d/e^5*(-1/5/d/e/(
x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(3/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(54) = 108.
time = 0.42, size = 113, normalized size = 1.77

$$\frac{2\sqrt{-x^2e^2+d^2}\,d}{5\,(x^3e^5+3\,dx^2e^4+3\,d^2xe^3+d^3e^2)} - \frac{11\sqrt{-x^2e^2+d^2}}{15\,(x^2e^4+2\,dxe^3+d^2e^2)} + \frac{4\sqrt{-x^2e^2+d^2}}{15\,(dxe^3+d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(-x^2*e^2 + d^2)*d/(x^3*e^5 + 3*d*x^2*e^4 + 3*d^2*x*e^3 + d^3*e^2)
- 11/15*sqrt(-x^2*e^2 + d^2)/(x^2*e^4 + 2*d*x*e^3 + d^2*e^2) + 4/15*sqrt(-x
^2*e^2 + d^2)/(d*x*e^3 + d^2*e^2)
```

Fricas [A]
time = 1.83, size = 96, normalized size = 1.50

$$-\frac{x^3e^3+3\,dx^2e^2+3\,d^2xe+d^3-(4\,x^2e^2-3\,dxe-d^2)\sqrt{-x^2e^2+d^2}}{15\,(dx^3e^5+3\,d^2x^2e^4+3\,d^3xe^3+d^4e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3 - (4*x^2*e^2 - 3*d*x*e - d^2
)*sqrt(-x^2*e^2 + d^2))/(d*x^3*e^5 + 3*d^2*x^2*e^4 + 3*d^3*x*e^3 + d^4*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{-(-d+ex)\,(d+ex)}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(54) = 108.
time = 1.20, size = 128, normalized size = 2.00

$$\frac{2\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-\frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}+1\right)e^{(-2)}}{15\,d\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 2/15*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 5*(d*e + sqrt(-x^2*e^2 +
d^2)*e)^2*e^(-4)/x^2 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 + 1)*
e^(-2)/(d*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5)
```

Mupad [B]
time = 2.90, size = 46, normalized size = 0.72

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+3\,d\,e\,x-4\,e^2\,x^2\right)}{15\,d\,e^2\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(1/2))/(d + e*x)^4,x)
[Out] -((d^2 - e^2*x^2)^(1/2)*(d^2 - 4*e^2*x^2 + 3*d*e*x))/(15*d*e^2*(d + e*x)^3)
```

3.193 $\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=67

$$-\frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4}-\frac{(d^2-e^2x^2)^{3/2}}{15d^2e(d+ex)^3}$$

```
[Out] -1/5*(-e^2*x^2+d^2)^(3/2)/d/e/(e*x+d)^4-1/15*(-e^2*x^2+d^2)^(3/2)/d^2/e/(e*
x+d)^3
```

Rubi [A]

time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {673, 665}

$$-\frac{(d^2-e^2x^2)^{3/2}}{15d^2e(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] -1/5*(d^2 - e^2*x^2)^(3/2)/(d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d^
2*e*(d + e*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx &= -\frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4}+\frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{5d}\\ &= -\frac{(d^2-e^2x^2)^{3/2}}{5de(d+ex)^4}-\frac{(d^2-e^2x^2)^{3/2}}{15d^2e(d+ex)^3}\end{aligned}$$

Mathematica [**A**]
time = 0.37, size = 51, normalized size = 0.76

$$\frac{\sqrt{d^2-e^2x^2}\,(-4d^2+3dex+e^2x^2)}{15d^2e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*d^2 + 3*d*e*x + e^2*x^2))/(15*d^2*e*(d + e*x)^3)
```

Maple [**A**]
time = 0.06, size = 93, normalized size = 1.39

method	result	size
gosper	$-\frac{(-ex+d)(ex+4d)\sqrt{-e^2x^2+d^2}}{15(ex+d)^3d^2e}$	43
trager	$-\frac{(-e^2x^2-3dex+4d^2)\sqrt{-e^2x^2+d^2}}{15d^2(ex+d)^3e}$	49
default	$\frac{-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}}{e^4}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+
d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(57) = 114.
time = 0.36, size = 117, normalized size = 1.75

$$-\frac{2\sqrt{-x^2e^2+d^2}}{5\,(x^3e^4+3\,dx^2e^3+3\,d^2xe^2+d^3e)}+\frac{\sqrt{-x^2e^2+d^2}}{15\,(dx^2e^3+2\,d^2xe^2+d^3e)}+\frac{\sqrt{-x^2e^2+d^2}}{15\,(d^2xe^2+d^3e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-x^2*e^2 + d^2)/(x^3*e^4 + 3*d*x^2*e^3 + 3*d^2*x*e^2 + d^3*e) + 1
/15*sqrt(-x^2*e^2 + d^2)/(d*x^2*e^3 + 2*d^2*x*e^2 + d^3*e) + 1/15*sqrt(-x^2
*e^2 + d^2)/(d^2*x*e^2 + d^3*e)
```

Fricas [**A**]
time = 1.78, size = 100, normalized size = 1.49

$$-\frac{4\,x^3e^3+12\,dx^2e^2+12\,d^2xe+4\,d^3-(x^2e^2+3\,dxe-4\,d^2)\sqrt{-x^2e^2+d^2}}{15\,(d^2x^3e^4+3\,d^3x^2e^3+3\,d^4xe^2+d^5e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(4*x^3*e^3 + 12*d*x^2*e^2 + 12*d^2*x*e + 4*d^3 - (x^2*e^2 + 3*d*x*e -
 4*d^2)*sqrt(-x^2*e^2 + d^2))/(d^2*x^3*e^4 + 3*d^3*x^2*e^3 + 3*d^4*x*e^2 +
d^5*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(57) = 114.
time = 1.55, size = 158, normalized size = 2.36

$$\frac{2\left(\frac{5\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}+\frac{25\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}e^{(-4)}}{x^{2}}+\frac{15\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}e^{(-6)}}{x^{3}}+\frac{15\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}e^{(-8)}}{x^{4}}+4\right)e^{(-1)}}{15\,d^{2}\left(\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 2/15*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 25*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*e^(-4)/x^2 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 + 15
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^4 + 4)*e^(-1)/(d^2*((d*e + sqrt(
-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5)
```

Mupad [B]
time = 2.78, size = 47, normalized size = 0.70

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(-4\,d^2+3\,d\,e\,x+e^2\,x^2\right)}{15\,d^2\,e\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(d + e*x)^4,x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(e^2*x^2 - 4*d^2 + 3*d*e*x))/(15*d^2*e*(d + e*x)^3)
```

3.194 $\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx$

Optimal. Leaf size=110

$$\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

```
[Out] 8/5*d*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)-4/5*e*x/d/(-e^2*x^2+d^2)^(3/2)-arctanh(
(-e^2*x^2+d^2)^(1/2)/d)/d^3+1/5*(-8*e*x+5*d)/d^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 837, 12, 272, 65, 214}

$$-\frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}}+\frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}+\frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*x)/(5*d*(d^2 - e^2*x^2)^(3
/2)) + (5*d - 8*e*x)/(5*d^3*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x
^2]/d]/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+12d^3ex+5d^2e^2x^2}{x(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-24d^3ex}{x(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{15d^6e^2}{x\sqrt{d^2-e^2x^2}}\,dx}{15d^8e^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x\right.}{2d^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2}\right.}{d^2e^2}$$

$$= \frac{8d(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4ex}{5d\left(d^2-e^2x^2\right)^{3/2}} + \frac{5d-8ex}{5d^3\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^3}$$

Mathematica [A]
time = 0.50, size = 88, normalized size = 0.80

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(13d^2+19dex+8e^2x^2)}{(d+ex)^3} + 10\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(13*d^2 + 19*d*e*x + 8*e^2*x^2))/(d + e*x)^3 + 10*Arc
Tanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(5*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(96) = 192.
time = 0.07, size = 415, normalized size = 3.77

method	result
default	$-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}+\frac{d e \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^{2}}}}{d^{4}}+\frac{\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x+\frac{d}{e}\right)\right.}{3 e^{3} d^{3} \left(x+\frac{d}{e}\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^4*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/3/e^3/d^3/(x+d/e)^3*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/e^3/d*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
)-1/e/d^3*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))+1/d^4*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/
2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)/((x*e + d)^4*x), x)
```

Fricas [A]
time = 1.87, size = 148, normalized size = 1.35

$$\frac{13\,x^{3}e^{3}+39\,dx^{2}e^{2}+39\,d^{2}xe+13\,d^{3}+5\left(x^{3}e^{3}+3\,dx^{2}e^{2}+3\,d^{2}xe+d^{3}\right)\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+\left(8\,x^{2}e^{2}+19\,dxe+13\,d^{2}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{5\left(d^{3}x^{3}e^{3}+3\,d^{4}x^{2}e^{2}+3\,d^{5}xe+d^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(13*x^3*e^3 + 39*d*x^2*e^2 + 39*d^2*x*e + 13*d^3 + 5*(x^3*e^3 + 3*d*x^2
*e^2 + 3*d^2*x*e + d^3)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (8*x^2*e^2 + 1
```

```
9*d*x*e + 13*d^2)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3*e^3 + 3*d^4*x^2*e^2 + 3*d^
5*x*e + d^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)**4), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(96) = 192.
time = 1.65, size = 195, normalized size = 1.77

$$-\frac{\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^3}-\frac{2\left(\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+\frac{75\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{55\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}+\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-8)}}{x^4}+13\right)}{5\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3 - 2/5*(4
5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 75*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^2*e^(-4)/x^2 + 55*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 + 20*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^4 + 13)/(d^3*((d*e + sqrt(-x^2*e^2 + d
^2)*e)*e^(-2)/x + 1)^5)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x*(d + e*x)^4), x)
```

3.195 $\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx$

Optimal. Leaf size=143

$$-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

```
[Out] -8/5*e*(-e*x+d)/(-e^2*x^2+d^2)^(5/2)-4/15*e*(-8*e*x+5*d)/d^2/(-e^2*x^2+d^2)
^(3/2)+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-1/15*e*(-79*e*x+60*d)/d^4/(-
e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A]
time = 0.19, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {866, 1819, 821, 272, 65, 214}

$$-\frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^4x}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*(5*d - 8*e*x))/(15*d^2*(d
^2 - e^2*x^2)^(3/2)) - (e*(60*d - 79*e*x))/(15*d^4*Sqrt[d^2 - e^2*x^2]) - S
qrt[d^2 - e^2*x^2]/(d^4*x) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^2(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-27d^2e^2x^2}{x^2(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+64d^2e^2x^2}{x^2(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex}{x^2\sqrt{d^2-e^2x^2}}\,dx}{15d^6}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(4e)\int}{}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} - \frac{(2e)\text{Sul}}{}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4\text{Subst}}{}$$

$$= -\frac{8e(d-ex)}{5\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e(5d-8ex)}{15d^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4e\tanh}{}$$

Mathematica [A]
time = 0.51, size = 103, normalized size = 0.72

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\,(15d^3+149d^2ex+222de^2x^2+94e^3x^3)}{x(d+ex)^3}+120e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4),x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(15*d^3 + 149*d^2*e*x + 222*d*e^2*x^2 + 94*e^3*
x^3))/(x*(d + e*x)^3) + 120*e*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/
d])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(127) = 254.
time = 0.09, size = 499, normalized size = 3.49

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^4x}-\frac{79\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^4\left(x+\frac{d}{e}\right)}+\frac{4e\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{d^3\sqrt{d^2}}-\frac{2\sqrt{-}}{}$
default	$\frac{4e\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d^5}-\frac{2\left(-\left(x+\frac{d}{e}\right)^2e^2+2d\right.}{3e^2d^4\left(x+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4*e/d^5*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^
(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-2/3/e^2/d^4/(x+d/e)^3*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+1/e^2/d^2*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3
/2))+1/d^4*(-1/d^2/x*(-e^2*x^2+d^2)^(3/2)-2*e^2/d^2*(1/2*x*(-e^2*x^2+d^2)^(
1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+3/d^4
*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(1/2))))-4/d^5*e*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((
2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)/((x*e + d)^4*x^2), x)
```

Fricas [A]
time = 2.46, size = 173, normalized size = 1.21

$$-\frac{104\,x^4e^4+312\,dx^3e^3+312\,d^2x^2e^2+104\,d^3xe+60\,(x^4e^4+3\,dx^3e^3+3\,d^2x^2e^2+d^3xe)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(94\,x^3e^3+222\,dx^2e^2+149\,d^2xe+15\,d^3)\sqrt{-x^2e^2+d^2}}{15\,(d^4x^4e^3+3\,d^5x^3e^2+3\,d^6x^2e+d^7x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(104*x^4*e^4 + 312*d*x^3*e^3 + 312*d^2*x^2*e^2 + 104*d^3*x*e + 60*(x^
4*e^4 + 3*d*x^3*e^3 + 3*d^2*x^2*e^2 + d^3*x*e)*log(-(d - sqrt(-x^2*e^2 + d^
2))/x) + (94*x^3*e^3 + 222*d*x^2*e^2 + 149*d^2*x*e + 15*d^3)*sqrt(-x^2*e^2
+ d^2))/(d^4*x^4*e^3 + 3*d^5*x^3*e^2 + 3*d^6*x^2*e + d^7*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^{2}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**2*(d + e*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(130) = 260.
time = 2.09, size = 287, normalized size = 2.01

$$\frac{4\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^{4}}-\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-1)}}{2\,d^{4}x}+\frac{x\left(\frac{491\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-1)}}{x}+\frac{1690\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}e^{(-3)}}{x^{2}}+\frac{2570\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}e^{(-5)}}{x^{3}}+\frac{1815\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}e^{(-7)}}{x^{4}}+\frac{555\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{5}e^{(-9)}}{x^{5}}+15\,e\right)e^{2}}{30\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d^{4}\left(\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 4*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4 - 1/2
*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/(d^4*x) + 1/30*x*(491*(d*e + sqrt(-x
^2*e^2 + d^2)*e)*e^(-1)/x + 1690*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-3)/x^
2 + 2570*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-5)/x^3 + 1815*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^4*e^(-7)/x^4 + 555*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-9)/
x^5 + 15*e)*e^2/((d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*((d*e + sqrt(-x^2*e^2 +
 d^2)*e)*e^(-2)/x + 1)^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^2\,{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^2*(d + e*x)^4), x)
```

3.196 $\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=183

$$\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{19e^2\tanh^{-1}\left(\right.}{2d}$$

```
[Out] 8/5*e^2*(-e*x+d)/d/(-e^2*x^2+d^2)^(5/2)+4/15*e^2*(-13*e*x+10*d)/d^3/(-e^2*x
^2+d^2)^(3/2)-19/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^5+1/15*e^2*(-164*e
*x+135*d)/d^5/(-e^2*x^2+d^2)^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/d^4/x^2+4*e*(-e
^2*x^2+d^2)^(1/2)/d^5/x
```

Rubi [A]

time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}}+\frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}}+\frac{4e\sqrt{d^2-e^2x^2}}{d^5x}-\frac{19e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5}-\frac{\sqrt{d^2-e^2x^2}}{2d^4x^2}+\frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (4*e^2*(10*d - 13*e*x))/(15
*d^3*(d^2 - e^2*x^2)^(3/2)) + (e^2*(135*d - 164*e*x))/(15*d^5*Sqrt[d^2 - e^
2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*d^4*x^2) + (4*e*Sqrt[d^2 - e^2*x^2])/(d^5*
x) - (19*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^5)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^3(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+32de^3x^3}{x^3(d^2-e^2x^2)^{5/2}}\,dx}{5d^2}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-104de^3x^3}{x^3(d^2-e^2x^2)^{3/2}}\,dx}{15d^4}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x}{x^3\sqrt{d^2-e^2x^2}}}{15d^6}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{\int \frac{-120}{x^2\sqrt{}}}{}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d}}{}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d}}{}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d}}{}$$

$$= \frac{8e^2(d-ex)}{5d\left(d^2-e^2x^2\right)^{5/2}} + \frac{4e^2(10d-13ex)}{15d^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e\sqrt{d}}{}$$

Mathematica [A]
time = 0.54, size = 116, normalized size = 0.63

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(-15d^4+75d^3ex+713d^2e^2x^2+1059de^3x^3+448e^4x^4)}{x^2(d+ex)^3} + 570e^2\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{30d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-15*d^4 + 75*d^3*e*x + 713*d^2*e^2*x^2 + 1059*d*e^3*
x^3 + 448*e^4*x^4))/(x^2*(d + e*x)^3) + 570*e^2*ArcTanh[(Sqrt[-e^2]*x - Sqr
t[d^2 - e^2*x^2])/d])/(30*d^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(161) = 322$.

time = 0.08, size = 598, normalized size = 3.27

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-8ex+d)}{2d^5x^2}+\frac{164e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{15d^5\left(x+\frac{d}{e}\right)}-\frac{19e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2d^4\sqrt{d^2}}$
default	$-\frac{10e^2\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d^6}+\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+\right.}{e\,d^5\left(x+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -10*e^2/d^6*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+1/e/d^5/(x+d/e)^3*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+1/d^4*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(3/2)-1/2
*e^2/d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^
2*x^2+d^2)^(1/2))/x)))-1/e/d^3*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))-4/d^5
*e*(-1/d^2/x*(-e^2*x^2+d^2)^(3/2)-2*e^2/d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2
*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-6*e/d^5*(-1/d
/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(1/2))))+10/d^6*e^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d
^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)/((x*e + d)^4*x^3), x)
```

Fricas [A]
time = 2.34, size = 189, normalized size = 1.03

$$\frac{398\,x^5e^5+1194\,dx^4e^4+1194\,d^2x^3e^3+398\,d^3x^2e^2+285\,(x^5e^5+3\,dx^4e^4+3\,d^2x^3e^3+d^3x^2e^2)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(448\,x^4e^4+1059\,dx^3e^3+713\,d^2x^2e^2+75\,d^3xe-15\,d^4)\sqrt{-x^2e^2+d^2}}{30\,(d^5x^5e^3+3\,d^6x^4e^2+3\,d^7x^3e+d^8x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/30*(398*x^5*e^5 + 1194*d*x^4*e^4 + 1194*d^2*x^3*e^3 + 398*d^3*x^2*e^2 + 2
85*(x^5*e^5 + 3*d*x^4*e^4 + 3*d^2*x^3*e^3 + d^3*x^2*e^2)*log(-(d - sqrt(-x^
2*e^2 + d^2))/x) + (448*x^4*e^4 + 1059*d*x^3*e^3 + 713*d^2*x^2*e^2 + 75*d^3
*x*e - 15*d^4)*sqrt(-x^2*e^2 + d^2))/(d^5*x^5*e^3 + 3*d^6*x^4*e^2 + 3*d^7*x
^3*e + d^8*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^3\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(156) = 312.
time = 1.49, size = 353, normalized size = 1.93

$$-\frac{19\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^5}-\frac{x^2\left(\frac{4234\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-2)}}{x^2}+\frac{14330\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-4)}}{x^3}+\frac{20965\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-6)}}{x^4}+\frac{14385\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-8)}}{x^5}+\frac{4080\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6e^{(-10)}}{x^6}+\frac{165\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}{x}-15\,e^2\right)e^4}{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^5\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}-\frac{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^5e^{(-2)}}{x^2}-\frac{16\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^5}{x}}{8\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -19/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5
 - 1/120*x^2*(4234*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-2)/x^2 + 14330*(d*e
 + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-4)/x^3 + 20965*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^4*e^(-6)/x^4 + 14385*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*e^(-8)/x^5 + 4080
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-10)/x^6 + 165*(d*e + sqrt(-x^2*e^2 +
d^2)*e)/x - 15*e^2)*e^4/((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*((d*e + sqrt(
-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^5) - 1/8*((d*e + sqrt(-x^2*e^2 + d^2)*e)^2
*d^5*e^(-2)/x^2 - 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^5/x)/d^10
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2\,x^2}}{x^3\,(d + e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^3*(d + e*x)^4), x)
```

3.197 $\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=210

$$-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{29e^2\sqrt{d^2-e^2x}}{3d^6x}$$

```
[Out] -8/5*e^3*(-e*x+d)/d^2/(-e^2*x^2+d^2)^(5/2)-4/5*e^3*(-6*e*x+5*d)/d^4/(-e^2*x
^2+d^2)^(3/2)+18*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^6-1/5*e^3*(-93*e*x+8
0*d)/d^6/(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/d^4/x^3+2*e*(-e^2*x^
2+d^2)^(1/2)/d^5/x^2-29/3*e^2*(-e^2*x^2+d^2)^(1/2)/d^6/x
```

Rubi [A]
time = 0.31, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$-\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x}-\frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}}+\frac{18e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}+\frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^4x^3}-\frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (4*e^3*(5*d - 6*e*x))/(5
*d^4*(d^2 - e^2*x^2)^(3/2)) - (e^3*(80*d - 93*e*x))/(5*d^6*Sqrt[d^2 - e^2*x
^2]) - Sqrt[d^2 - e^2*x^2]/(3*d^4*x^3) + (2*e*Sqrt[d^2 - e^2*x^2])/(d^5*x^2
) - (29*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^6*x) + (18*e^3*ArcTanh[Sqrt[d^2 - e^2
*x^2]/d])/d^6
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{\sqrt{d^2-e^2x^2}}{x^4(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{-5d^4+20d^3ex-35d^2e^2x^2+40de^3x^3-32e^4x^4}{x^4\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{15d^4-60d^3ex+120d^2e^2x^2-180de^3x^3+144e^4x^4}{x^4\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15d^4}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-15d^4+60d^3ex-135d^2e^2x}{x^4\sqrt{d^2-e^2x}}}{15d^6}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{\int \frac{-180}{}}{}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d}}{d}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d}}{d}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d}}{d}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d}}{d}$$

$$= -\frac{8e^3(d-ex)}{5d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{4e^3(5d-6ex)}{5d^4\left(d^2-e^2x^2\right)^{3/2}} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} + \frac{2e\sqrt{d}}{d}$$

Mathematica [A]
time = 0.62, size = 127, normalized size = 0.60

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(5d^5-15d^4ex+70d^3e^2x^2+674d^2e^3x^3+1002de^4x^4+424e^5x^5\right)}{x^3(d+ex)^3}+540e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d^2 - e^2*x^2]/(x^4*(d + e*x)^4),x]
```

[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(5*d^5 - 15*d^4*e*x + 70*d^3*e^2*x^2 + 674*d^2*e^3*x^3 + 1002*d*e^4*x^4 + 424*e^5*x^5))/(x^3*(d + e*x)^3) + 540*e^3*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/d^6

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(186) = 372$.

time = 0.09, size = 620, normalized size = 2.95

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(29e^2x^2-6dex+d^2)}{3d^6x^3}-\frac{93e^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5d^6\left(x+\frac{d}{e}\right)}+\frac{18e^3\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2}}{x}\right.}{d^5\sqrt{d^2}}$
default	$\frac{20e^3\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{d^7}-\frac{4\left(-\left(x+\frac{d}{e}\right)^2e^2+2d\right.}{3d^6\left(x+\frac{d}{e}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x,method=_RETURNVERBOSE)

[Out] 20*e^3/d^7*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-4/3/d^6/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-4/d^5*e*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(3/2)-1/2*e^2/d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))+1/d^4*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))+10/d^6*e^2*(-1/d^2/x*(-e^2*x^2+d^2)^(3/2)-2*e^2/d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+10*e^2/d^6*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))-1/3/d^6/x^3*(-e^2*x^2+d^2)^(3/2)-20/d^7*e^3*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2*e^2 + d^2)/((x*e + d)^4*x^4), x)
```

Fricas [A]
time = 2.42, size = 199, normalized size = 0.95

$$-\frac{324\,x^{6}e^{6}+972\,dx^{5}e^{5}+972\,d^{2}x^{4}e^{4}+324\,d^{3}x^{3}e^{3}+270\,(x^{6}e^{6}+3\,dx^{5}e^{5}+3\,d^{2}x^{4}e^{4}+d^{3}x^{3}e^{3})\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+(424\,x^{5}e^{5}+1002\,dx^{4}e^{4}+674\,d^{2}x^{3}e^{3}+70\,d^{3}x^{2}e^{2}-15\,d^{4}xe+5\,d^{5})\sqrt{-x^{2}e^{2}+d^{2}}}{15\,(d^{6}x^{6}e^{3}+3\,d^{7}x^{5}e^{2}+3\,d^{8}x^{4}e+d^{9}x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(324*x^6*e^6 + 972*d*x^5*e^5 + 972*d^2*x^4*e^4 + 324*d^3*x^3*e^3 + 27
0*(x^6*e^6 + 3*d*x^5*e^5 + 3*d^2*x^4*e^4 + d^3*x^3*e^3)*log(-(d - sqrt(-x^2
*e^2 + d^2))/x) + (424*x^5*e^5 + 1002*d*x^4*e^4 + 674*d^2*x^3*e^3 + 70*d^3*
x^2*e^2 - 15*d^4*x*e + 5*d^5)*sqrt(-x^2*e^2 + d^2))/(d^6*x^6*e^3 + 3*d^7*x^
5*e^2 + 3*d^8*x^4*e + d^9*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}}{x^{4}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**4/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**4*(d + e*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(179) = 358.
time = 1.46, size = 420, normalized size = 2.00

$$\frac{18\,e^{3}\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^{6}}-\frac{x^{3}\left(\frac{35\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e}{x}-\frac{335\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}e^{(-1)}}{x^{2}}-\frac{7559\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}e^{(-3)}}{x^{3}}-\frac{25195\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{4}e^{(-5)}}{x^{4}}-\frac{36035\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{5}e^{(-7)}}{x^{5}}-\frac{24225\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{6}e^{(-9)}}{x^{6}}-\frac{6585\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{7}e^{(-11)}}{x^{7}}-5\,e^{3}\right)e^{6}}{120\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}d^{6}\left(\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}+1\right)^{5}}-\frac{\frac{117\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)d^{12}e}{x}-\frac{12\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}d^{12}e^{(-1)}}{x^{2}}+\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{3}d^{12}e^{(-3)}}{x^{3}}}{24\,d^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 18*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^6 -
1/120*x^3*(35*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e/x - 335*(d*e + sqrt(-x^2*e^2
```

```
 + d^2)*e)^2*e^(-1)/x^2 - 7559*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-3)/x^3
- 25195*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-5)/x^4 - 36035*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^5*e^(-7)/x^5 - 24225*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*e^(-9
)/x^6 - 6585*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*e^(-11)/x^7 - 5*e^3)*e^6/((d*
e + sqrt(-x^2*e^2 + d^2)*e)^3*d^6*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x
+ 1)^5) - 1/24*(117*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^12*e/x - 12*(d*e + sqr
t(-x^2*e^2 + d^2)*e)^2*d^12*e^(-1)/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d
^12*e^(-3)/x^3)/d^18
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d^2-e^2\,x^2}}{x^4\,{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(1/2)/(x^4*(d + e*x)^4), x)
```

3.198 $\int \frac{x^5 (d^2-e^2x^2)^{5/2}}{(d+ex)^4} \, dx$

Optimal. Leaf size=252

$$\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}+\frac{11d^2\dots}{}$$

```
[Out] 65/4*d^7*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+d^4*(-e*x+d)^4/e^6/(-e^2*x^2+
d^2)^(1/2)+515/21*d^6*(-e^2*x^2+d^2)^(1/2)/e^6-49/4*d^5*x*(-e^2*x^2+d^2)^(1
/2)/e^5+121/21*d^4*x^2*(-e^2*x^2+d^2)^(1/2)/e^4-17/6*d^3*x^3*(-e^2*x^2+d^2)
^(1/2)/e^3+11/7*d^2*x^4*(-e^2*x^2+d^2)^(1/2)/e^2-2/3*d*x^5*(-e^2*x^2+d^2)^(
1/2)/e+1/7*x^6*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\frac{65d^7\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^6}+\frac{1}{7}x^6\sqrt{d^2-e^2x^2}-\frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}+\frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2}+\frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6}-\frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5}+\frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}}+\frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}-\frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^4*(d - e*x)^4)/(e^6*Sqrt[d^2 - e^2*x^2]) + (515*d^6*Sqrt[d^2 - e^2*x^2])
/(21*e^6) - (49*d^5*x*Sqrt[d^2 - e^2*x^2])/(4*e^5) + (121*d^4*x^2*Sqrt[d^2
- e^2*x^2])/(21*e^4) - (17*d^3*x^3*Sqrt[d^2 - e^2*x^2])/(6*e^3) + (11*d^2*x
^4*Sqrt[d^2 - e^2*x^2])/(7*e^2) - (2*d*x^5*Sqrt[d^2 - e^2*x^2])/(3*e) + (x^
6*Sqrt[d^2 - e^2*x^2])/7 + (65*d^7*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(4*e^
6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^5(d-ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^5}{e^5}+\frac{d^4x}{e^4}-\frac{d^3x^2}{e^3}+\frac{d^2x^3}{e^2}-\frac{dx^4}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{28d^8}{e^3}-\frac{91d^7x}{e^2}+\frac{112d^6x^2}{e}-77d^5x^3+56d^4ex^4-55d^3e^2x^5+28}{\sqrt{d^2-e^2x^2}}}{7de^2}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} - \frac{\int \frac{-\frac{168d^8}{e}+546d^7x-672d^6ex^2+}{\sqrt{d^2}}}{}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e} + \frac{1}{7}x^6\sqrt{d^2-e^2x^2} + \int$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x^2}}{7e^2} - \frac{2dx^5\sqrt{d^2-e^2x^2}}{3e}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x^2}}{6e^3} + \frac{11d^2x^4\sqrt{d^2-e^2x}}{7e^2}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2-e^2x}}{6e^3}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}$$

$$= \frac{d^4(d-ex)^4}{e^6\sqrt{d^2-e^2x^2}} + \frac{515d^6\sqrt{d^2-e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2-e^2x^2}}{4e^5} + \frac{121d^4x^2\sqrt{d^2-e^2x^2}}{21e^4}$$

Mathematica [A]
time = 0.37, size = 151, normalized size = 0.60

$$\frac{\frac{e\sqrt{d^2-e^2x^2}\,(2144d^7+779d^6ex-293d^5e^2x^2+162d^4e^3x^3-106d^3e^4x^4+76d^2e^5x^5-44de^6x^6+12e^7x^7)}{d+ex}+1365d^7\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{84e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(2144*d^7 + 779*d^6*e*x - 293*d^5*e^2*x^2 + 162*d^4
*e^3*x^3 - 106*d^3*e^4*x^4 + 76*d^2*e^5*x^5 - 44*d*e^6*x^6 + 12*e^7*x^7))/(
d + e*x) + 1365*d^7*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/
(84*e^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1212 vs. $2(218) = 436$.
time = 0.08, size = 1213, normalized size = 4.81

method	result
risch	$\frac{(12e^6x^6-56d\,e^5x^5+132d^2e^4x^4-238d^3e^3x^3+400d^4e^2x^2-693e\,d^5x+1472d^6)\sqrt{-e^2x^2+d^2}}{84e^6}+\frac{65d^7\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right.}{4e^5\sqrt{e^2}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/e^6*(-e^2*x^2+d^2)^(7/2)-4*d/e^5*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(
1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+10/e^6*d^2*(1/5*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+5/e^8*d^4*(1/d/e/(x+d/e)^3*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/
8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(
-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^
2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
))-d^5/e^9*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)-3*e/d*(1/
d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2
*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(1/2))))))))-10/e^7*d^3*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/
8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(
-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^
2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
)
```

Maxima [C] Result contains complex when optimal does not.

time = 0.54, size = 440, normalized size = 1.75

$$\frac{5}{2}i\,d^{7}\arcsin\left(\frac{xe}{d}+2\right)e^{(-6)}+\frac{75}{4}\,d^{7}\arcsin\left(\frac{xe}{d}\right)e^{(-6)}-\frac{5}{2}\sqrt{x^{2}e^{2}+4\,dxe+3\,d^{2}}\,d^{5}xe^{(-5)}-\frac{5}{4}\sqrt{-x^{2}e^{2}+d^{2}}\,d^{5}xe^{(-5)}-5\sqrt{x^{2}e^{2}+4\,dxe+3\,d^{2}}\,d^{6}e^{(-6)}+\frac{25}{2}\sqrt{-x^{2}e^{2}+d^{2}}\,d^{6}e^{(-6)}-\frac{(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}d^{5}}{2\,(x^{3}e^{9}+3\,dx^{2}e^{8}+3\,d^{2}xe^{7}+d^{3}e^{6})}-\frac{5\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}d^{6}}{2\,(x^{2}e^{8}+2\,dxe^{7}+d^{2}e^{6})}+\frac{15\sqrt{-x^{2}e^{2}+d^{2}}\,d^{7}}{xe^{7}+de^{6}}+\frac{5}{3}\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}d^{3}xe^{(-5)}-\frac{25}{6}\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}d^{4}e^{(-6)}+\frac{5\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}d^{4}}{3\,(x^{2}e^{8}+2\,dxe^{7}+d^{2}e^{6})}+\frac{25\,(-x^{2}e^{2}+d^{2})^{\frac{3}{2}}d^{5}}{6\,(xe^{7}+de^{6})}-\frac{2}{3}\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}dxe^{(-5)}+2\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}d^{2}e^{(-6)}-\frac{5\,(-x^{2}e^{2}+d^{2})^{\frac{5}{2}}d^{3}}{2\,(xe^{7}+de^{6})}-\frac{1}{7}\,(-x^{2}e^{2}+d^{2})^{\frac{7}{2}}e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 5/2*I*d^7*arcsin(x*e/d + 2)*e^(-6) + 75/4*d^7*arcsin(x*e/d)*e^(-6) - 5/2*sq
rt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^5*x*e^(-5) - 5/4*sqrt(-x^2*e^2 + d^2)*d^5*x
*e^(-5) - 5*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^6*e^(-6) + 25/2*sqrt(-x^2*e^2
 + d^2)*d^6*e^(-6) - 1/2*(-x^2*e^2 + d^2)^(5/2)*d^5/(x^3*e^9 + 3*d*x^2*e^8
+ 3*d^2*x*e^7 + d^3*e^6) - 5/2*(-x^2*e^2 + d^2)^(3/2)*d^6/(x^2*e^8 + 2*d*x*
e^7 + d^2*e^6) + 15*sqrt(-x^2*e^2 + d^2)*d^7/(x*e^7 + d*e^6) + 5/3*(-x^2*e^
2 + d^2)^(3/2)*d^3*x*e^(-5) - 25/6*(-x^2*e^2 + d^2)^(3/2)*d^4*e^(-6) + 5/3*
(-x^2*e^2 + d^2)^(5/2)*d^4/(x^2*e^8 + 2*d*x*e^7 + d^2*e^6) + 25/6*(-x^2*e^2
 + d^2)^(3/2)*d^5/(x*e^7 + d*e^6) - 2/3*(-x^2*e^2 + d^2)^(5/2)*d*x*e^(-5) +
 2*(-x^2*e^2 + d^2)^(5/2)*d^2*e^(-6) - 5/2*(-x^2*e^2 + d^2)^(5/2)*d^3/(x*e^
7 + d*e^6) - 1/7*(-x^2*e^2 + d^2)^(7/2)*e^(-6)
```

Fricas [A]
time = 1.71, size = 148, normalized size = 0.59

$$\frac{2144\,d^{7}xe+2144\,d^{8}-2730\,(d^{7}xe+d^{8})\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)+(12\,x^{7}e^{7}-44\,dx^{6}e^{6}+76\,d^{2}x^{5}e^{5}-106\,d^{3}x^{4}e^{4}+162\,d^{4}x^{3}e^{3}-293\,d^{5}x^{2}e^{2}+779\,d^{6}xe+2144\,d^{7})\sqrt{-x^{2}e^{2}+d^{2}}}{84\,(xe^{7}+de^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/84*(2144*d^7*x*e + 2144*d^8 - 2730*(d^7*x*e + d^8)*arctan(-(d - sqrt(-x^2
*e^2 + d^2))*e^(-1)/x) + (12*x^7*e^7 - 44*d*x^6*e^6 + 76*d^2*x^5*e^5 - 106*
d^3*x^4*e^4 + 162*d^4*x^3*e^3 - 293*d^5*x^2*e^2 + 779*d^6*x*e + 2144*d^7)*s
qrt(-x^2*e^2 + d^2))/(x*e^7 + d*e^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [A]
time = 1.38, size = 135, normalized size = 0.54

$$\frac{65}{4}\,d^{7}\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\mathrm{sgn}(d)-\frac{16\,d^{7}e^{(-6)}}{\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}+1}+\frac{1}{84}\left(1472\,d^{6}e^{(-6)}-\left(693\,d^{5}e^{(-5)}-2\left(200\,d^{4}e^{(-4)}-\left(119\,d^{3}e^{(-3)}-2\left(33\,d^{2}e^{(-2)}-\left(14\,de^{(-1)}-3\,x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-x^{2}e^{2}+d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 65/4*d^7*arcsin(x*e/d)*e^(-6)*sgn(d) - 16*d^7*e^(-6)/((d*e + sqrt(-x^2*e^2
+ d^2)*e)*e^(-2)/x + 1) + 1/84*(1472*d^6*e^(-6) - (693*d^5*e^(-5) - 2*(200*
d^4*e^(-4) - (119*d^3*e^(-3) - 2*(33*d^2*e^(-2) - (14*d*e^(-1) - 3*x)*x)*x)
*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.199 $\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=224

$$-\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}}-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4}-\frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{4d\ldots}{}$$

```
[Out] -239/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^5-d^3*(-e*x+d)^4/e^5/(-e^2*x
^2+d^2)^(1/2)-337/15*d^5*(-e^2*x^2+d^2)^(1/2)/e^5+175/16*d^4*x*(-e^2*x^2+d^
2)^(1/2)/e^4-71/15*d^3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3+47/24*d^2*x^3*(-e^2*x^2
+d^2)^(1/2)/e^2-4/5*d*x^4*(-e^2*x^2+d^2)^(1/2)/e+1/6*x^5*(-e^2*x^2+d^2)^(1/
2)
```

Rubi [A]
time = 0.33, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1649, 1829, 655, 223, 209}

$$-\frac{239d^6\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5}+\frac{1}{6}x^5\sqrt{d^2-e^2x^2}-\frac{4dx^4\sqrt{d^2-e^2x^2}}{5e}+\frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2}-\frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5}+\frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4}-\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}}-\frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d^3*(d - e*x)^4)/(e^5*Sqrt[d^2 - e^2*x^2])) - (337*d^5*Sqrt[d^2 - e^2*x^
2])/(15*e^5) + (175*d^4*x*Sqrt[d^2 - e^2*x^2])/(16*e^4) - (71*d^3*x^2*Sqrt[
d^2 - e^2*x^2])/(15*e^3) + (47*d^2*x^3*Sqrt[d^2 - e^2*x^2])/(24*e^2) - (4*d
*x^4*Sqrt[d^2 - e^2*x^2])/(5*e) + (x^5*Sqrt[d^2 - e^2*x^2])/6 - (239*d^6*Ar
cTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^5)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^4(d-ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(\frac{4d^4}{e^4}-\frac{d^3x}{e^3}+\frac{d^2x^2}{e^2}-\frac{dx^3}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} + \frac{\int \frac{-\frac{24d^7}{e^2}+\frac{78d^6x}{e}-96d^5x^2+66d^4ex^3-47d^3e^2x^4+24d^2e^3x}{\sqrt{d^2-e^2x^2}}}{6de^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{\int \frac{120d^7-390d^6ex+480d^5e^2x}{\sqrt{d^2-}}}{30}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e} + \frac{1}{6}x^5\sqrt{d^2-e^2x^2} +$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{4dx^4\sqrt{d^2-e^2x^2}}{5e}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x^2}}{15e^3} + \frac{47d^2x^3\sqrt{d^2-e^2}}{24e^2}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x}}{15e^3}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x}}{15e^3}$$

$$= -\frac{d^3(d-ex)^4}{e^5\sqrt{d^2-e^2x^2}} - \frac{337d^5\sqrt{d^2-e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2-e^2x^2}}{16e^4} - \frac{71d^3x^2\sqrt{d^2-e^2x}}{15e^3}$$

Mathematica [A]
time = 0.40, size = 143, normalized size = 0.64

$$\frac{\sqrt{d^2-e^2x^2}\,(-5632d^6-2047d^5ex+769d^4e^2x^2-426d^3e^3x^3+278d^2e^4x^4-152de^5x^5+40e^6x^6)}{240e^5(d+ex)} + \frac{239d^6(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{16e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5632*d^6 - 2047*d^5*e*x + 769*d^4*e^2*x^2 - 426*d^3*
e^3*x^3 + 278*d^2*e^4*x^4 - 152*d*e^5*x^5 + 40*e^6*x^6))/(240*e^5*(d + e*x)
```

```
) + (239*d^6*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*e
^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(194) = 388$.
time = 0.07, size = 1189, normalized size = 5.31

method	result
risch	$-\frac{(-40e^5x^5+192d\,e^4x^4-470d^2e^3x^3+896x^2d^3e^2-1665d^4xe+3712d^5)\sqrt{-e^2x^2+d^2}}{240e^5}-\frac{239d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16e^4\sqrt{e^2}}-$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d
^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e
^2*x^2+d^2)^(1/2)))))-4/e^5*d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e
*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*
d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1
/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2
)))))-4/e^7*d^3*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d
*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))+1/e^8*d^4*(-1/d/e/(x+d/e)^4*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)-3*e/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d
*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(
7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x
+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2
*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))))+6/e^6*d^
2*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x
/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.52, size = 420, normalized size = 1.88

$$-\frac{9}{4}i\,d^6\arcsin\left(\frac{xe}{d}+2\right)e^{(-5)}-\frac{275}{16}d^6\arcsin\left(\frac{xe}{d}\right)e^{(-5)}+\frac{9}{4}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^4xe^{(-4)}+\frac{5}{16}\sqrt{-x^2e^2+d^2}\,d^4xe^{(-4)}+\frac{9}{2}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^5e^{(-5)}-10\sqrt{-x^2e^2+d^2}\,d^5e^{(-5)}+\frac{(-x^2e^2+d^2)^{\frac{5}{2}}d^4}{2\,(x^3e^8+3\,dx^2e^7+3\,d^2xe^6+d^3e^5)}+\frac{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^5}{2\,(x^2e^7+2\,dxe^6+d^2e^5)}-\frac{15\sqrt{-x^2e^2+d^2}\,d^6}{xe^6+de^5}-\frac{19}{24}(-x^2e^2+d^2)^{\frac{3}{2}}d^2xe^{(-4)}+\frac{5}{2}(-x^2e^2+d^2)^{\frac{3}{2}}d^3e^{(-5)}-\frac{4\,(-x^2e^2+d^2)^{\frac{5}{2}}d^3}{3\,(x^2e^7+2\,dxe^6+d^2e^5)}-\frac{10\,(-x^2e^2+d^2)^{\frac{3}{2}}d^4}{3\,(xe^6+de^5)}+\frac{1}{6}(-x^2e^2+d^2)^{\frac{5}{2}}xe^{(-4)}-\frac{4}{5}(-x^2e^2+d^2)^{\frac{5}{2}}de^{(-5)}+\frac{3\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2}{2\,(xe^6+de^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -9/4*I*d^6*arcsin(x*e/d + 2)*e^(-5) - 275/16*d^6*arcsin(x*e/d)*e^(-5) + 9/4
*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^4*x*e^(-4) + 5/16*sqrt(-x^2*e^2 + d^2)*d
^4*x*e^(-4) + 9/2*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^5*e^(-5) - 10*sqrt(-x^2
*e^2 + d^2)*d^5*e^(-5) + 1/2*(-x^2*e^2 + d^2)^(5/2)*d^4/(x^3*e^8 + 3*d*x^2*
e^7 + 3*d^2*x*e^6 + d^3*e^5) + 5/2*(-x^2*e^2 + d^2)^(3/2)*d^5/(x^2*e^7 + 2*
d*x*e^6 + d^2*e^5) - 15*sqrt(-x^2*e^2 + d^2)*d^6/(x*e^6 + d*e^5) - 19/24*(-
x^2*e^2 + d^2)^(3/2)*d^2*x*e^(-4) + 5/2*(-x^2*e^2 + d^2)^(3/2)*d^3*e^(-5) -
 4/3*(-x^2*e^2 + d^2)^(5/2)*d^3/(x^2*e^7 + 2*d*x*e^6 + d^2*e^5) - 10/3*(-x^
2*e^2 + d^2)^(3/2)*d^4/(x*e^6 + d*e^5) + 1/6*(-x^2*e^2 + d^2)^(5/2)*x*e^(-4
) - 4/5*(-x^2*e^2 + d^2)^(5/2)*d*e^(-5) + 3/2*(-x^2*e^2 + d^2)^(5/2)*d^2/(x
*e^6 + d*e^5)
```

Fricas [**A**]
time = 1.51, size = 139, normalized size = 0.62

$$-\frac{5632\,d^6xe+5632\,d^7-7170\,(d^6xe+d^7)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(40\,x^6e^6-152\,dx^5e^5+278\,d^2x^4e^4-426\,d^3x^3e^3+769\,d^4x^2e^2-2047\,d^5xe-5632\,d^6)\sqrt{-x^2e^2+d^2}}{240\,(xe^6+de^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/240*(5632*d^6*x*e + 5632*d^7 - 7170*(d^6*x*e + d^7)*arctan(-(d - sqrt(-x
^2*e^2 + d^2))*e^(-1)/x) - (40*x^6*e^6 - 152*d*x^5*e^5 + 278*d^2*x^4*e^4 -
426*d^3*x^3*e^3 + 769*d^4*x^2*e^2 - 2047*d^5*x*e - 5632*d^6)*sqrt(-x^2*e^2
+ d^2))/(x*e^6 + d*e^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [**A**]
time = 4.65, size = 124, normalized size = 0.55

$$-\frac{239}{16}\,d^6\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d)+\frac{16\,d^6e^{(-5)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1}-\frac{1}{240}\,(3712\,d^5e^{(-5)}-(1665\,d^4e^{(-4)}-2\,(448\,d^3e^{(-3)}-(235\,d^2e^{(-2)}-4\,(24\,de^{(-1)}-5\,x)x)x)x)x)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -239/16*d^6*arcsin(x*e/d)*e^(-5)*sgn(d) + 16*d^6*e^(-5)/((d*e + sqrt(-x^2*e
^2 + d^2)*e)*e^(-2)/x + 1) - 1/240*(3712*d^5*e^(-5) - (1665*d^4*e^(-4) - 2*
(448*d^3*e^(-3) - (235*d^2*e^(-2) - 4*(24*d*e^(-1) - 5*x)*x)*x)*x)*x)*sqrt(
-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(d^2-e^2 x^2\right)^{5/2}}{(d+e x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.200 $\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx$

Optimal. Leaf size=192

$$\frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}}+\frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4}-\frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3}+\frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{e}+\frac{1}{5}x^4\sqrt{d^2-}$$

```
[Out] 27/2*d^5*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^4+d^2*(-e*x+d)^4/e^4/(-e^2*x^2+
d^2)^(1/2)+101/5*d^4*(-e^2*x^2+d^2)^(1/2)/e^4-19/2*d^3*x*(-e^2*x^2+d^2)^(1/
2)/e^3+18/5*d^2*x^2*(-e^2*x^2+d^2)^(1/2)/e^2-d*x^3*(-e^2*x^2+d^2)^(1/2)/e+1
/5*x^4*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1649, 1829, 655, 223, 209}

$$\frac{27d^5\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}+\frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}+\frac{1}{5}x^4\sqrt{d^2-e^2x^2}-\frac{dx^3\sqrt{d^2-e^2x^2}}{e}+\frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}}+\frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4}-\frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (d^2*(d - e*x)^4)/(e^4*Sqrt[d^2 - e^2*x^2]) + (101*d^4*Sqrt[d^2 - e^2*x^2])
/(5*e^4) - (19*d^3*x*Sqrt[d^2 - e^2*x^2])/(2*e^3) + (18*d^2*x^2*Sqrt[d^2 -
e^2*x^2])/(5*e^2) - (d*x^3*Sqrt[d^2 - e^2*x^2])/e + (x^4*Sqrt[d^2 - e^2*x^2
])/5 + (27*d^5*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^3(d-ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{(d-ex)^3\left(-\frac{4d^3}{e^3}+\frac{d^2x}{e^2}-\frac{dx^2}{e}\right)}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{\frac{20d^6}{e}-65d^5x+80d^4ex^2-54d^3e^2x^3+20d^2e^3x^4}{\sqrt{d^2-e^2x^2}}\,dx}{5de^2}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-80d^6e+260d^5e^2x-380d^4e^3x^2}{\sqrt{d^2-e^2x^2}}}{20de^4}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} + \frac{1}{5}x^4\sqrt{d^2-e^2x^2} + \int$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{e} +$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}$$

$$= \frac{d^2(d-ex)^4}{e^4\sqrt{d^2-e^2x^2}} + \frac{101d^4\sqrt{d^2-e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2-e^2x^2}}{2e^3} + \frac{18d^2x^2\sqrt{d^2-e^2x^2}}{5e^2}$$

Mathematica [A]
time = 0.32, size = 129, normalized size = 0.67

$$\frac{\frac{e\sqrt{d^2-e^2x^2}\,(212d^5+77d^4ex-29d^3e^2x^2+16d^2e^3x^3-8de^4x^4+2e^5x^5)}{d+ex} + 135d^5\sqrt{-e^2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{10e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(212*d^5 + 77*d^4*e*x - 29*d^3*e^2*x^2 + 16*d^2*e^3
*x^3 - 8*d*e^4*x^4 + 2*e^5*x^5))/(d + e*x) + 135*d^5*Sqrt[-e^2]*Log[-(Sqrt[
-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(10*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. 2(168) = 336.
time = 0.08, size = 1086, normalized size = 5.66

method	result
risch	$\frac{(2e^4x^4-10d\,e^3x^3+26d^2x^2e^2-55d^3ex+132d^4)\sqrt{-e^2x^2+d^2}}{10e^4}+\frac{27d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^3\sqrt{e^2}}+\frac{8d^5\sqrt{-\left(x+\frac{d}{e}\right)^2}}{e^5\left(x\right.}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2
*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e
)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arcta
n((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+3/e^6*d^2*(1/d/e/(
x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(1/2)))))))-d^3/e^7*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(7/2)-3*e/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1
/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))))-3*d/e^5*(1/3/d/e/(x+d/e)^2*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(
5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3
/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2))))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 375, normalized size = 1.95

$$\frac{3}{2}i\,d^5\arcsin\left(\frac{xe}{d}+2\right)e^{(-4)}+15\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-4)}-\frac{3}{2}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^3xe^{(-3)}-3\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^4e^{(-4)}+\frac{15}{2}\sqrt{-x^2e^2+d^2}\,d^4e^{(-4)}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}d^3}{2\,(x^3e^7+3\,dx^2e^6+3\,d^2xe^5+d^3e^4)}-\frac{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^4}{2\,(x^2e^6+2\,dxe^5+d^2e^4)}+\frac{15\sqrt{-x^2e^2+d^2}\,d^5}{xe^5+de^4}+\frac{1}{4}(-x^2e^2+d^2)^{\frac{3}{2}}dxe^{(-3)}-\frac{5}{4}(-x^2e^2+d^2)^{\frac{3}{2}}d^2e^{(-4)}+\frac{(-x^2e^2+d^2)^{\frac{5}{2}}d^2}{x^2e^6+2\,dxe^5+d^2e^4}+\frac{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}{2\,(xe^5+de^4)}+\frac{1}{5}(-x^2e^2+d^2)^{\frac{5}{2}}e^{(-4)}-\frac{3\,(-x^2e^2+d^2)^{\frac{5}{2}}d}{4\,(xe^5+de^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 3/2*I*d^5*arcsin(x*e/d + 2)*e^(-4) + 15*d^5*arcsin(x*e/d)*e^(-4) - 3/2*sqrt
(x^2*e^2 + 4*d*x*e + 3*d^2)*d^3*x*e^(-3) - 3*sqrt(x^2*e^2 + 4*d*x*e + 3*d^2
)*d^4*e^(-4) + 15/2*sqrt(-x^2*e^2 + d^2)*d^4*e^(-4) - 1/2*(-x^2*e^2 + d^2)^
(5/2)*d^3/(x^3*e^7 + 3*d*x^2*e^6 + 3*d^2*x*e^5 + d^3*e^4) - 5/2*(-x^2*e^2 +
```

```
 d^2)^(3/2)*d^4/(x^2*e^6 + 2*d*x*e^5 + d^2*e^4) + 15*sqrt(-x^2*e^2 + d^2)*d
^5/(x*e^5 + d*e^4) + 1/4*(-x^2*e^2 + d^2)^(3/2)*d*x*e^(-3) - 5/4*(-x^2*e^2
+ d^2)^(3/2)*d^2*e^(-4) + (-x^2*e^2 + d^2)^(5/2)*d^2/(x^2*e^6 + 2*d*x*e^5 +
 d^2*e^4) + 5/2*(-x^2*e^2 + d^2)^(3/2)*d^3/(x*e^5 + d*e^4) + 1/5*(-x^2*e^2
+ d^2)^(5/2)*e^(-4) - 3/4*(-x^2*e^2 + d^2)^(5/2)*d/(x*e^5 + d*e^4)
```

Fricas [A]
time = 2.09, size = 128, normalized size = 0.67

$$\frac{212\,d^5xe + 212\,d^6 - 270\,(d^5xe + d^6)\arctan\left(-\frac{\left(d - \sqrt{-x^2e^2 + d^2}\right)e^{(-1)}}{x}\right) + (2\,x^5e^5 - 8\,dx^4e^4 + 16\,d^2x^3e^3 - 29\,d^3x^2e^2 + 77\,d^4xe + 212\,d^5)\sqrt{-x^2e^2 + d^2}}{10\,(xe^5 + de^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/10*(212*d^5*x*e + 212*d^6 - 270*(d^5*x*e + d^6)*arctan(-(d - sqrt(-x^2*e^
2 + d^2))*e^(-1)/x) + (2*x^5*e^5 - 8*d*x^4*e^4 + 16*d^2*x^3*e^3 - 29*d^3*x^
2*e^2 + 77*d^4*x*e + 212*d^5)*sqrt(-x^2*e^2 + d^2))/(x*e^5 + d*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(-(-d + ex)\,(d + ex))^{\frac{5}{2}}}{(d + ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [A]
time = 1.69, size = 113, normalized size = 0.59

$$\frac{27}{2}\,d^5\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d) - \frac{16\,d^5e^{(-4)}}{\frac{\left(de + \sqrt{-x^2e^2 + d^2}\,e\right)e^{(-2)}}{x} + 1} + \frac{1}{10}\,(132\,d^4e^{(-4)} - (55\,d^3e^{(-3)} - 2\,(13\,d^2e^{(-2)} - (5\,de^{(-1)} - x)x)x)x)\sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 27/2*d^5*arcsin(x*e/d)*e^(-4)*sgn(d) - 16*d^5*e^(-4)/((d*e + sqrt(-x^2*e^2
+ d^2)*e)*e^(-2)/x + 1) + 1/10*(132*d^4*e^(-4) - (55*d^3*e^(-3) - 2*(13*d^2
*e^(-2) - (5*d*e^(-1) - x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d^2 - e^2\,x^2\right)}^{5/2}}{{\left(d + e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
[Out] int((x^3*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.201 $\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=182

$$-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{(d-ex)^3\sqrt{d^2}}{4e^3}$$

```
[Out] -95/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-d*(-e*x+d)^4/e^3/(-e^2*x^2+d
^2)^(1/2)-95/8*d^3*(-e^2*x^2+d^2)^(1/2)/e^3-95/24*d^2*(-e*x+d)*(-e^2*x^2+d^
2)^(1/2)/e^3-19/12*d*(-e*x+d)^2*(-e^2*x^2+d^2)^(1/2)/e^3-1/4*(-e*x+d)^3*(-e
^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A]
time = 0.13, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1649, 809, 685, 655, 223, 209}

$$-\frac{95d^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3}-\frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3}-\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}-\frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3}-\frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] -((d*(d - e*x)^4)/(e^3*Sqrt[d^2 - e^2*x^2])) - (95*d^3*Sqrt[d^2 - e^2*x^2])
/(8*e^3) - (95*d^2*(d - e*x)*Sqrt[d^2 - e^2*x^2])/(24*e^3) - (19*d*(d - e*x
)^2*Sqrt[d^2 - e^2*x^2])/(12*e^3) - ((d - e*x)^3*Sqrt[d^2 - e^2*x^2])/(4*e^
3) - (95*d^4*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{x^2(d-ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{\left(\frac{4d^2}{e^2}-\frac{dx}{e}\right)(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{d}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(19d)\int \frac{(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{(95d^2)\int}{}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{(d-}{}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2}{12}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2}{12}$$

$$= -\frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2}{12}$$

Mathematica [A]
time = 0.36, size = 121, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\,(-448d^4-163d^3ex+61d^2e^2x^2-26de^3x^3+6e^4x^4)}{24e^3(d+ex)} + \frac{95d^4(-e^2)^{3/2}\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{8e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-448*d^4 - 163*d^3*e*x + 61*d^2*e^2*x^2 - 26*d*e^3*x^
3 + 6*e^4*x^4))/(24*e^3*(d + e*x)) + (95*d^4*(-e^2)^(3/2)*Log[-(Sqrt[-e^2]*
x) + Sqrt[d^2 - e^2*x^2]])/(8*e^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(160) = 320$.
time = 0.07, size = 890, normalized size = 4.89

method	result
risch	$-\frac{\left(-6e^{3}x^{3}+32d\,e^{2}x^{2}-93d^{2}ex+256d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{24e^{3}}-\frac{95d^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8e^{2}\sqrt{e^{2}}}-\frac{8d^{4}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+}}{e^{4}\left(x+\frac{d}{e}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*d/e^5*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d
/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*
e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))+1/e^6*d^2*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(7/2)-3*e/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/
3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2
*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e
)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arcta
n((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))))+1/e^4*(1/3/d/e/
(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.57, size = 335, normalized size = 1.84

$$-\frac{5}{8} i d^4 \arcsin\left(\frac{xe}{d}+2\right) e^{(-3)} - \frac{25}{2} d^4 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} + \frac{5}{8}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^2xe^{(-2)} + \frac{5}{4}\sqrt{x^2e^2+4\,dxe+3\,d^2}\,d^3e^{(-3)} - 5\sqrt{-x^2e^2+d^2}\,d^3e^{(-3)} + \frac{(-x^2e^2+d^2)^{\frac{5}{2}}d^2}{2\,(x^3e^6+3\,dx^2e^5+3\,d^2xe^4+d^3e^3)} + \frac{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^3}{2\,(x^2e^5+2\,dxe^4+d^2e^3)} - \frac{15\sqrt{-x^2e^2+d^2}\,d^4}{xe^4+de^3} + \frac{5}{12}(-x^2e^2+d^2)^{\frac{3}{2}}de^{(-3)} - \frac{2\,(-x^2e^2+d^2)^{\frac{5}{2}}d}{3\,(x^2e^5+2\,dxe^4+d^2e^3)} - \frac{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}{3\,(xe^4+de^3)} + \frac{(-x^2e^2+d^2)^{\frac{5}{2}}}{4\,(xe^4+de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -5/8*I*d^4*arcsin(x*e/d + 2)*e^(-3) - 25/2*d^4*arcsin(x*e/d)*e^(-3) + 5/8*s
qrt(x^2*e^2 + 4*d*x*e + 3*d^2)*d^2*x*e^(-2) + 5/4*sqrt(x^2*e^2 + 4*d*x*e +
3*d^2)*d^3*e^(-3) - 5*sqrt(-x^2*e^2 + d^2)*d^3*e^(-3) + 1/2*(-x^2*e^2 + d^2
)^(5/2)*d^2/(x^3*e^6 + 3*d*x^2*e^5 + 3*d^2*x*e^4 + d^3*e^3) + 5/2*(-x^2*e^2
 + d^2)^(3/2)*d^3/(x^2*e^5 + 2*d*x*e^4 + d^2*e^3) - 15*sqrt(-x^2*e^2 + d^2)
*d^4/(x*e^4 + d*e^3) + 5/12*(-x^2*e^2 + d^2)^(3/2)*d*e^(-3) - 2/3*(-x^2*e^2
 + d^2)^(5/2)*d/(x^2*e^5 + 2*d*x*e^4 + d^2*e^3) - 5/3*(-x^2*e^2 + d^2)^(3/2
)*d^2/(x*e^4 + d*e^3) + 1/4*(-x^2*e^2 + d^2)^(5/2)/(x*e^4 + d*e^3)
```

Fricas [A]
time = 1.86, size = 119, normalized size = 0.65

$$-\frac{448\,d^4xe+448\,d^5-570\,(d^4xe+d^5)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(6\,x^4e^4-26\,dx^3e^3+61\,d^2x^2e^2-163\,d^3xe-448\,d^4)\sqrt{-x^2e^2+d^2}}{24\,(xe^4+de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")

[Out] -1/24*(448*d^4*x*e + 448*d^5 - 570*(d^4*x*e + d^5)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (6*x^4*e^4 - 26*d*x^3*e^3 + 61*d^2*x^2*e^2 - 163*d^3*x*e - 448*d^4)*sqrt(-x^2*e^2 + d^2))/(x*e^4 + d*e^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(-(-d+ex)(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)

[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)

Giac [A]
time = 1.99, size = 102, normalized size = 0.56

$$-\frac{95}{8}d^4\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{16\,d^4e^{(-3)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1}-\frac{1}{24}\left(256\,d^3e^{(-3)}-\left(93\,d^2e^{(-2)}-2\left(16\,de^{(-1)}-3\,x\right)x\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")

[Out] -95/8*d^4*arcsin(x*e/d)*e^(-3)*sgn(d) + 16*d^4*e^(-3)/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1) - 1/24*(256*d^3*e^(-3) - (93*d^2*e^(-2) - 2*(16*d*e^(-1) - 3*x)*x)*x)*sqrt(-x^2*e^2 + d^2)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)

[Out] int((x^2*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)

3.202 $\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx$

Optimal. Leaf size=130

$$\frac{10dx\sqrt{d^2-e^2x^2}}{e}+\frac{20(d^2-e^2x^2)^{3/2}}{3e^2}+\frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2}+\frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4}+\frac{10d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

```
[Out] 20/3*(-e^2*x^2+d^2)^(3/2)/e^2+8*(-e^2*x^2+d^2)^(5/2)/e^2/(e*x+d)^2+(-e^2*x^
2+d^2)^(7/2)/e^2/(e*x+d)^4+10*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^2+10*d
*x*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {807, 677, 679, 201, 223, 209}

$$\frac{10d^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}+\frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4}+\frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2}+\frac{20(d^2-e^2x^2)^{3/2}}{3e^2}+\frac{10dx\sqrt{d^2-e^2x^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (10*d*x*Sqrt[d^2 - e^2*x^2])/e + (20*(d^2 - e^2*x^2)^(3/2))/(3*e^2) + (8*(d
^2 - e^2*x^2)^(5/2))/(e^2*(d + e*x)^2) + (d^2 - e^2*x^2)^(7/2)/(e^2*(d + e*
x)^4) + (10*d^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx = \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{4\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^3}dx}{e}$$

$$= \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{20\int \frac{(d^2-e^2x^2)^{3/2}}{d+ex}dx}{e}$$

$$= \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{(20d)\int \sqrt{d^2-e^2x^2}\,dx}{e}$$

$$= \frac{10dx\sqrt{d^2-e^2x^2}}{e} + \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{(10d^3)\int}{}$$

$$= \frac{10dx\sqrt{d^2-e^2x^2}}{e} + \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{(10d^3)\,\mathrm{S}}{}$$

$$= \frac{10dx\sqrt{d^2-e^2x^2}}{e} + \frac{20(d^2-e^2x^2)^{3/2}}{3e^2} + \frac{8(d^2-e^2x^2)^{5/2}}{e^2(d+ex)^2} + \frac{(d^2-e^2x^2)^{7/2}}{e^2(d+ex)^4} + \frac{10d^3\,\mathrm{tan}}{}$$

Mathematica [A]
time = 0.29, size = 107, normalized size = 0.82

$$\frac{\sqrt{d^2-e^2x^2}\,(47d^3+17d^2ex-5de^2x^2+e^3x^3)}{3e^2(d+ex)} + \frac{10d^3\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(47*d^3 + 17*d^2*e*x - 5*d*e^2*x^2 + e^3*x^3))/(3*e^2*
(d + e*x)) + (10*d^3*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])
/e^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. 2(118) = 236.
time = 0.08, size = 644, normalized size = 4.95

method	result
risch	$\frac{(e^2x^2-6dex+23d^2)\sqrt{-e^2x^2+d^2}}{3e^2} + \frac{10d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}} + \frac{8d^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^3\left(x+\frac{d}{e}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/
(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(1/2)))))))-d/e^5*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(7/2)-3*e/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7
/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(
1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e
^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e
)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))))
```

Maxima [A]
time = 0.51, size = 214, normalized size = 1.65

$$10\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-2)}+\frac{5}{2}\,\sqrt{-x^2e^2+d^2}\,d^2e^{(-2)}-\frac{(-x^2e^2+d^2)^{\frac{5}{2}}d}{2\,(x^3e^5+3\,dx^2e^4+3\,d^2xe^3+d^3e^2)}-\frac{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}{2\,(x^2e^4+2\,dxe^3+d^2e^2)}+\frac{15\,\sqrt{-x^2e^2+d^2}\,d^3}{xe^3+de^2}+\frac{(-x^2e^2+d^2)^{\frac{5}{2}}}{3\,(x^2e^4+2\,dxe^3+d^2e^2)}+\frac{5\,(-x^2e^2+d^2)^{\frac{3}{2}}d}{6\,(xe^3+de^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 10*d^3*arcsin(x*e/d)*e^(-2) + 5/2*sqrt(-x^2*e^2 + d^2)*d^2*e^(-2) - 1/2*(-x
^2*e^2 + d^2)^(5/2)*d/(x^3*e^5 + 3*d*x^2*e^4 + 3*d^2*x*e^3 + d^3*e^2) - 5/2
*(-x^2*e^2 + d^2)^(3/2)*d^2/(x^2*e^4 + 2*d*x*e^3 + d^2*e^2) + 15*sqrt(-x^2*
e^2 + d^2)*d^3/(x*e^3 + d*e^2) + 1/3*(-x^2*e^2 + d^2)^(5/2)/(x^2*e^4 + 2*d*
x*e^3 + d^2*e^2) + 5/6*(-x^2*e^2 + d^2)^(3/2)*d/(x*e^3 + d*e^2)
```

Fricas [A]
time = 1.86, size = 107, normalized size = 0.82

$$\frac{47\,d^3xe+47\,d^4-60\,(d^3xe+d^4)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+(x^3e^3-5\,dx^2e^2+17\,d^2xe+47\,d^3)\sqrt{-x^2e^2+d^2}}{3\,(xe^3+de^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(47*d^3*x*e + 47*d^4 - 60*(d^3*x*e + d^4)*arctan(-(d - sqrt(-x^2*e^2 +
d^2))*e^(-1)/x) + (x^3*e^3 - 5*d*x^2*e^2 + 17*d^2*x*e + 47*d^3)*sqrt(-x^2*e
^2 + d^2))/(x*e^3 + d*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-(-d+ex)(d+ex))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [A]
time = 1.50, size = 91, normalized size = 0.70

$$10\,d^3\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\mathrm{sgn}(d)-\frac{16\,d^3e^{(-2)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1}+\frac{1}{3}\sqrt{-x^2e^2+d^2}\left(23\,d^2e^{(-2)}-\left(6\,de^{(-1)}-x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 10*d^3*arcsin(x*e/d)*e^(-2)*sgn(d) - 16*d^3*e^(-2)/((d*e + sqrt(-x^2*e^2 +
d^2)*e)*e^(-2)/x + 1) + 1/3*sqrt(-x^2*e^2 + d^2)*(23*d^2*e^(-2) - (6*d*e^(-
1) - x)*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x)
```

3.203 $\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4} dx$

Optimal. Leaf size=113

$$-\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)}-\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-\frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

```
[Out] -5/2*(-e^2*x^2+d^2)^(3/2)/e/(e*x+d)-2*(-e^2*x^2+d^2)^(5/2)/e/(e*x+d)^3-15/2
*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e-15/2*d*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A]

time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {677, 679, 223, 209}

$$-\frac{15d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}-\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)}-\frac{15d\sqrt{d^2-e^2x^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (-15*d*Sqrt[d^2 - e^2*x^2])/(2*e) - (5*(d^2 - e^2*x^2)^(3/2))/(2*e*(d + e*x
)) - (2*(d^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^3) - (15*d^2*ArcTan[(e*x)/Sqrt[
d^2 - e^2*x^2]])/(2*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^{5/2}}{(d+ex)^4}\,dx &= -\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-5\int \frac{(d^2-e^2x^2)^{3/2}}{(d+ex)^2}\,dx\\ &= -\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)}-\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-\frac{1}{2}(15d)\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}\,dx\\ &= -\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)}-\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-\frac{1}{2}(15d^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,d\\ &= -\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)}-\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-\frac{1}{2}(15d^2)\,\text{Subst}\left(\int \frac{1}{1+e^2}\right.\\ &= -\frac{15d\sqrt{d^2-e^2x^2}}{2e}-\frac{5(d^2-e^2x^2)^{3/2}}{2e(d+ex)}-\frac{2(d^2-e^2x^2)^{5/2}}{e(d+ex)^3}-\frac{15d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.}{2e}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 95, normalized size = 0.84

$$\frac{\sqrt{d^2-e^2x^2}\,(-24d^2-7dex+e^2x^2)}{2e(d+ex)}+\frac{15d^2\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-24*d^2 - 7*d*e*x + e^2*x^2))/(2*e*(d + e*x)) + (15*d
^2*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(99) = 198.
time = 0.08, size = 347, normalized size = 3.07

method	result

risch	$-\frac{(-ex+8d)\sqrt{-e^2x^2+d^2}}{2e}-\frac{15d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{8d^2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^2\left(x+\frac{d}{e}\right)}$

default

$$-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{de\left(x+\frac{d}{e}\right)^4}-\frac{3e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{de\left(x+\frac{d}{e}\right)^3}+\frac{4e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3de\left(x+\frac{d}{e}\right)^2}+\frac{5e\left(\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}\right.}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)-3*e/d*(1/d/e/(
x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(1/2)))))))
```

Maxima [A]
time = 0.52, size = 128, normalized size = 1.13

$$-\frac{15}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}+\frac{\left(-x^2e^2+d^2\right)^{\frac{5}{2}}}{2\left(x^3e^4+3\,dx^2e^3+3\,d^2xe^2+d^3e\right)}+\frac{5\left(-x^2e^2+d^2\right)^{\frac{3}{2}}d}{2\left(x^2e^3+2\,dxe^2+d^2e\right)}-\frac{15\sqrt{-x^2e^2+d^2}\,d^2}{xe^2+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -15/2*d^2*arcsin(x*e/d)*e^(-1) + 1/2*(-x^2*e^2 + d^2)^(5/2)/(x^3*e^4 + 3*d*
x^2*e^3 + 3*d^2*x*e^2 + d^3*e) + 5/2*(-x^2*e^2 + d^2)^(3/2)*d/(x^2*e^3 + 2*
d*x*e^2 + d^2*e) - 15*sqrt(-x^2*e^2 + d^2)*d^2/(x*e^2 + d*e)
```

Fricas [A]
time = 2.65, size = 98, normalized size = 0.87

$$-\frac{24\,d^2xe+24\,d^3-30\left(d^2xe+d^3\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(x^2e^2-7\,dxe-24\,d^2\right)\sqrt{-x^2e^2+d^2}}{2\left(xe^2+de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^2*x*e + 24*d^3 - 30*(d^2*x*e + d^3)*arctan(-(d - sqrt(-x^2*e^2 +
 d^2))*e^(-1)/x) - (x^2*e^2 - 7*d*x*e - 24*d^2)*sqrt(-x^2*e^2 + d^2))/(x*e^
2 + d*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**4, x)
```

Giac [A]
time = 1.90, size = 80, normalized size = 0.71

$$-\frac{15}{2}\,d^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\mathrm{sgn}(d)+\frac{16\,d^2e^{(-1)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1}-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(8\,de^{(-1)}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -15/2*d^2*arcsin(x*e/d)*e^(-1)*sgn(d) + 16*d^2*e^(-1)/((d*e + sqrt(-x^2*e^2
 + d^2)*e)*e^(-2)/x + 1) - 1/2*sqrt(-x^2*e^2 + d^2)*(8*d*e^(-1) - x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(d + e*x)^4, x)
```

3.204 $\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx$

Optimal. Leaf size=89

$$\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}+4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] 4*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))-d*arctanh((-e^2*x^2+d^2)^(1/2)/d)+8*d*
(-e*x+d)/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {866, 1819, 1823, 858, 223, 209, 272, 65, 214}

$$4d\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+\frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}}+\sqrt{d^2-e^2x^2}-d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] (8*d*(d - e*x))/Sqrt[d^2 - e^2*x^2] + Sqrt[d^2 - e^2*x^2] + 4*d*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]] - d*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x\,(d^2-e^2x^2)^{3/2}}\,dx$$
$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4-4d^3ex+d^2e^2x^2}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$
$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + \frac{\int \frac{d^4e^2+4d^3e^3x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^2e^2}$$
$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + d^2\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx + (4de)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + \frac{1}{2}d^2\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) + (4de)\text{Subst}\Bigg($$
$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{d^2\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{}\right.}{e^2}$$
$$= \frac{8d(d-ex)}{\sqrt{d^2-e^2x^2}} + \sqrt{d^2-e^2x^2} + 4d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Mathematica [A]
time = 0.40, size = 115, normalized size = 1.29

$$\frac{(9d+ex)\sqrt{d^2-e^2x^2}}{d+ex} + 2d\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x}{d} - \frac{\sqrt{d^2-e^2x^2}}{d}\right) - \frac{4de\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

```
[Out] ((9*d + e*x)*Sqrt[d^2 - e^2*x^2])/(d + e*x) + 2*d*ArcTanh[(Sqrt[-e^2]*x)/d
- Sqrt[d^2 - e^2*x^2]/d] - (4*d*e*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]
])/Sqrt[-e^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(81) = 162$.
time = 0.08, size = 1192, normalized size = 13.39

method	result	size
default	Expression too large to display	1192

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^4*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+
2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/
e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-1/e^2/d^2*(1/d/e/
(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(1/2)))))))-1/e^3/d*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(7/2)-3*e/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(
1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))))-1/e/d^3*(1/3/d/e/(x+d/e)^2*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(
3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2))))))+1/d^4*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/
2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*
x^2+d^2)^(1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)/((x*e + d)^4*x), x)
```

Fricas [A]
time = 2.13, size = 112, normalized size = 1.26

$$\frac{9\,dxe + 9\,d^2 - 8\,(dxe + d^2)\arctan\left(-\frac{\left(d - \sqrt{-x^2e^2 + d^2}\right)e^{(-1)}}{x}\right) + (dxe + d^2)\log\left(-\frac{d - \sqrt{-x^2e^2 + d^2}}{x}\right) + \sqrt{-x^2e^2 + d^2}\,(xe + 9\,d)}{xe + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="fricas")
```

[Out] (9*d*x*e + 9*d^2 - 8*(d*x*e + d^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (d*x*e + d^2)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + sqrt(-x^2*e^2 + d^2)*(x*e + 9*d))/(x*e + d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**4,x)

[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x*(d + e*x)**4), x)

Giac [A]
time = 1.78, size = 97, normalized size = 1.09

$$4\,d\arcsin\left(\frac{xe}{d}\right)\operatorname{sgn}(d)-d\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^{2}e^{2}+d^{2}}\,e\right|e^{(-2)}}{2\,|x|}\right)+\sqrt{-x^{2}e^{2}+d^{2}}-\frac{16\,d}{\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="giac")

[Out] 4*d*arcsin(x*e/d)*sgn(d) - d*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + sqrt(-x^2*e^2 + d^2) - 16*d/((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4),x)

[Out] int((d^2 - e^2*x^2)^(5/2)/(x*(d + e*x)^4), x)

3.205 $\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{x^2(d+ex)^4}\,dx$

Optimal. Leaf size=94

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}-e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

```
[Out] -e*arctan(e*x/(-e^2*x^2+d^2)^(1/2))+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)-8*e
*(-e*x+d)/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/x
```

Rubi [A]
time = 0.20, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {866, 1819, 1821, 858, 223, 209, 272, 65, 214}

$$-e\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{x}+4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] (-8*e*(d - e*x))/Sqrt[d^2 - e^2*x^2] - Sqrt[d^2 - e^2*x^2]/x - e*ArcTan[(e*
x)/Sqrt[d^2 - e^2*x^2]] + 4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^2(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^2(d^2-e^2x^2)^{3/2}}\,dx$$
$$= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex+d^2e^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$
$$= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} + \frac{\int \frac{-4d^5e-d^4e^2x}{x\sqrt{d^2-e^2x^2}}\,dx}{d^4}$$
$$= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - (4de)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx - e^2\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - (2de)\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right) - e^2\text{Subst}$$
$$= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{(4d)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx\right.}{e}$$
$$= -\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - e\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right.$$

Mathematica [A]
time = 0.38, size = 114, normalized size = 1.21

$$-\frac{(d+9ex)\sqrt{d^2-e^2x^2}}{x(d+ex)} - 8e\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right) - \sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

```
[Out] -(((d + 9*e*x)*Sqrt[d^2 - e^2*x^2])/(x*(d + e*x))) - 8*e*ArcTanh[(Sqrt[-e^2
]*x - Sqrt[d^2 - e^2*x^2])/d] - Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 -
 e^2*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. 2(86) = 172.
time = 0.09, size = 1322, normalized size = 14.06

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{x} - \frac{e^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{8\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{x+\frac{d}{e}} + \frac{4de\ln\left(\frac{2d^2+2\sqrt{d}\dots}{}\right.}{}$

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4*e/d^5*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)
+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d
/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+2/e/d^3*(1/d/e/(
x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e)
)^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(1/2)))))))+1/e^2/d^2*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(7/2)-3*e/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*
(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))))+1/d^4*(-1/d^2/x*(-e^2*x^2+d^2)^(
7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3
/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-e^2*x^2+d^2)^(1/2))))))+3/d^4*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-
1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2
*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*
d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
)))-4/d^5*e*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-
e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(
1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)/((x*e + d)^4*x^2), x)
```

Fricas [A]
time = 2.49, size = 125, normalized size = 1.33

$$-\frac{8\,x^2e^2+8\,dxe-2\left(x^2e^2+dxe\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+4\left(x^2e^2+dxe\right)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\sqrt{-x^2e^2+d^2}\,(9\,xe+d)}{x^2e+dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -(8*x^2*e^2 + 8*d*x*e - 2*(x^2*e^2 + d*x*e)*arctan(-(d - sqrt(-x^2*e^2 + d^
2))*e^(-1)/x) + 4*(x^2*e^2 + d*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + sq
rt(-x^2*e^2 + d^2)*(9*x*e + d))/(x^2*e + d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^{\frac{5}{2}}}{x^2(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**2*(d + e*x)**4), x)
```

Giac [A]
time = 1.73, size = 170, normalized size = 1.81

$$-\arcsin\left(\frac{xe}{d}\right)e\mathrm{sgn}(d)+4\,e\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)+\frac{x\left(\frac{33\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{x}+e\right)e^2}{2\left(de+\sqrt{-x^2e^2+d^2}\,e\right)\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}-\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-1)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e*sgn(d) + 4*e*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e
)*e^(-2)/abs(x)) + 1/2*x*(33*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/x + e)*e
^2/((d*e + sqrt(-x^2*e^2 + d^2)*e)*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x
 + 1)) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^{5/2}}{x^2\,(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^2*(d + e*x)^4), x)
```

3.206 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4} dx$

Optimal. Leaf size=110

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

```
[Out] -15/2*e^2*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d+8*e^2*(-e*x+d)/d/(-e^2*x^2+d^2)
^(1/2)-1/2*(-e^2*x^2+d^2)^(1/2)/x^2+4*e*(-e^2*x^2+d^2)^(1/2)/d/x
```

Rubi [A]
time = 0.26, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$,
Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} - \frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] (8*e^2*(d - e*x))/(d*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(2*x^2) + (
4*e*Sqrt[d^2 - e^2*x^2])/(d*x) - (15*e^2*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2
*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^3(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^3\,(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{\int \frac{-8d^5e+15d^4e^2x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{2d^4}$$

$$= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{2}(15e^2)\int \frac{1}{x\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} + \frac{1}{4}(15e^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{d^2-e^2x}}\,d\right.$$

$$= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15}{2}\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-}\right.$$

$$= \frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{15e^2\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Mathematica [**A**]
time = 0.42, size = 130, normalized size = 1.18

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(-d^2+7dex+24e^2x^2)}{x^2(d+ex)} + 15e^2\log\left(d\left(-d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)\right) - 15e^2\log\left(d-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-d^2 + 7*d*e*x + 24*e^2*x^2))/(x^2*(d + e*x)) + 15*e
^2*Log[d*(-d - Sqrt[-e^2]*x + Sqrt[d^2 - e^2*x^2])] - 15*e^2*Log[d - Sqrt[-
e^2]*x + Sqrt[d^2 - e^2*x^2]])/(2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(98) = 196$.
time = 0.07, size = 1461, normalized size = 13.28

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-8ex+d)}{2d\,x^2} + \frac{8e\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d\left(x+\frac{d}{e}\right)} - \frac{15e^2\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)}{2\sqrt{d^2}}$

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -10*e^2/d^6*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+
d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*
(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-3/d^4*(1/d/e
/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2)))))))+1/d^4*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(
1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^
(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))-
1/e/d^3*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)-3*e/d*(1/d/e
/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+
d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(1/2))))))))-4/d^5*e*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6
*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*
(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))))))-6*e/d^5*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2
)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/
e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x
+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))+10/d^6*e^2*(1
/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(
1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)/((x*e + d)^4*x^3), x)
```

Fricas [A]
time = 2.52, size = 107, normalized size = 0.97

$$\frac{16\,x^3e^3+16\,dx^2e^2+15\left(x^3e^3+dx^2e^2\right)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+\left(24\,x^2e^2+7\,dxe-d^2\right)\sqrt{-x^2e^2+d^2}}{2\left(dx^3e+d^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(16*x^3*e^3 + 16*d*x^2*e^2 + 15*(x^3*e^3 + d*x^2*e^2)*log(-(d - sqrt(-x
^2*e^2 + d^2))/x) + (24*x^2*e^2 + 7*d*x*e - d^2)*sqrt(-x^2*e^2 + d^2))/(d*x
^3*e + d^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^3\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**3*(d + e*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(95) = 190.
time = 1.43, size = 229, normalized size = 2.08

$$-\frac{15\,e^2\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d}-\frac{x^2\left(\frac{144\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-2)}}{x^2}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)}{x}-e^2\right)e^4}{8\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}-\frac{\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2de^{(-2)}}{x^2}-\frac{16\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d}{x}}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -15/2*e^2*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d -
 1/8*x^2*(144*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*e^(-2)/x^2 + 15*(d*e + sqrt(
-x^2*e^2 + d^2)*e)/x - e^2)*e^4/((d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*((d*e +
 sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)) - 1/8*((d*e + sqrt(-x^2*e^2 + d^2)*
e)^2*d*e^(-2)/x^2 - 16*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d/x)/d^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(d^2-e^2\,x^2\right)}^{5/2}}{x^3\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^3*(d + e*x)^4), x)
```

3.207 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=137

$$-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

```
[Out] 10*e^3*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^2-8*e^3*(-e*x+d)/d^2/(-e^2*x^2+d^2
)^(1/2)-1/3*(-e^2*x^2+d^2)^(1/2)/x^3+2*e*(-e^2*x^2+d^2)^(1/2)/d/x^2-23/3*e^
2*(-e^2*x^2+d^2)^(1/2)/d^2/x
```

Rubi [A]
time = 0.40, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$-\frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x}+\frac{2e\sqrt{d^2-e^2x^2}}{dx^2}-\frac{\sqrt{d^2-e^2x^2}}{3x^3}-\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}}+\frac{10e^3\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] (-8*e^3*(d - e*x))/(d^2*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(3*x^3)
+ (2*e*Sqrt[d^2 - e^2*x^2])/(d*x^2) - (23*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^2*x
) + (10*e^3*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^4(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^4\,(d^2-e^2x^2)^{3/2}}\,dx$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{\int \frac{-12d^5e+23d^4e^2x-24d^3e^3x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{3d^4}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{\int \frac{-46d^6e^2+60d^5e^3x}{x^2\sqrt{d^2-e^2x^2}}\,dx}{6d^6}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{(10e^3)\int -}{x\sqrt{}}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{(5e^3)\,\text{Subst}}{}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{(10e)\text{Subst}}{}$$

$$= -\frac{8e^3(d-ex)}{d^2\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{23e^2\sqrt{d^2-e^2x^2}}{3d^2x} + \frac{10e^3\tanh^{-}}{}$$

Mathematica [**A**]
time = 0.46, size = 103, normalized size = 0.75

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\,(d^3-5d^2ex+17de^2x^2+47e^3x^3)}{x^3(d+ex)} + 60e^3\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

```
[Out] -1/3*((Sqrt[d^2 - e^2*x^2]*(d^3 - 5*d^2*e*x + 17*d*e^2*x^2 + 47*e^3*x^3))/(
x^3*(d + e*x)) + 60*e^3*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/d^
2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1625 vs. 2(123) = 246.
time = 0.10, size = 1626, normalized size = 11.87

method	result
risch	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,(23e^{2}x^{2}-6dex+d^{2})}{3x^{3}d^{2}}-\frac{8e^{2}\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{d^{2}\left(x+\frac{d}{e}\right)}+\frac{10e^{3}\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}}}{x}\right.}{d\sqrt{d^{2}}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 20*e^3/d^7*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d
/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(
x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*
arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+4*e/d^5*(1/d/
e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(1/2)))))))-4/d^5*e*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^
2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^
2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))
))+1/d^4*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)-3*e/d*(1/d/
e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(1/2))))))))+10/d^6*e^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*
(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/
2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))))))+10*e^2/d^6*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^
2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2
*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))+1/d^4*
(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/
2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2
)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-e^2*x^2+d^2)^(1/2)))))))-20/d^7*e^3*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/
3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+
2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)/((x*e + d)^4*x^4), x)
```

Fricas [A]
time = 3.46, size = 117, normalized size = 0.85

$$-\frac{24\,x^4e^4+24\,dx^3e^3+30\,(x^4e^4+dx^3e^3)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(47\,x^3e^3+17\,dx^2e^2-5\,d^2xe+d^3)\sqrt{-x^2e^2+d^2}}{3\,(d^2x^4e+d^3x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(24*x^4*e^4 + 24*d*x^3*e^3 + 30*(x^4*e^4 + d*x^3*e^3)*log(-(d - sqrt(-
x^2*e^2 + d^2))/x) + (47*x^3*e^3 + 17*d*x^2*e^2 - 5*d^2*x*e + d^3)*sqrt(-x^
2*e^2 + d^2))/(d^2*x^4*e + d^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^4\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**4*(d + e*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(118) = 236.
time = 1.34, size = 300, normalized size = 2.19

$$\frac{10\,e^3\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{d^2}-\frac{x^3\left(\frac{11\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e}{x}-\frac{81\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-1)}}{x^2}-\frac{477\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-3)}}{x^3}-e^3\right)e^6}{24\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}-\frac{\frac{93\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4e}{x}-\frac{12\,\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^4e^{(-1)}}{x^2}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^4e^{(-3)}}{x^3}}{24\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 10*e^3*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^2 -
1/24*x^3*(11*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e/x - 81*(d*e + sqrt(-x^2*e^2 +
```

```
 d^2)*e)^2*e^(-1)/x^2 - 477*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-3)/x^3 - e
^3)*e^6/((d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*((d*e + sqrt(-x^2*e^2 + d^2)*
e)*e^(-2)/x + 1)) - 1/24*(93*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e/x - 12*(d
*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*e^(-1)/x^2 + (d*e + sqrt(-x^2*e^2 + d^2)
*e)^3*d^4*e^(-3)/x^3)/d^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^4\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^4*(d + e*x)^4), x)
```

3.208 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx$

Optimal. Leaf size=170

$$\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}-\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-}}{d}\right.}{8d^3}$$

```
[Out] -95/8*e^4*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^3+8*e^4*(-e*x+d)/d^3/(-e^2*x^2+
d^2)^(1/2)-1/4*(-e^2*x^2+d^2)^(1/2)/x^4+4/3*e*(-e^2*x^2+d^2)^(1/2)/d/x^3-31
/8*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^2+32/3*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x
```

Rubi [A]
time = 0.50, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$-\frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2}-\frac{\sqrt{d^2-e^2x^2}}{4x^4}+\frac{4e\sqrt{d^2-e^2x^2}}{3dx^3}+\frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}}-\frac{95e^4\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d^3}+\frac{32e^3\sqrt{d^2-e^2x^2}}{3d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] (8*e^4*(d - e*x))/(d^3*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(4*x^4) +
 (4*e*Sqrt[d^2 - e^2*x^2])/(3*d*x^3) - (31*e^2*Sqrt[d^2 - e^2*x^2])/(8*d^2*
x^2) + (32*e^3*Sqrt[d^2 - e^2*x^2])/(3*d^3*x) - (95*e^4*ArcTanh[Sqrt[d^2 -
e^2*x^2]/d])/(8*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^5(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^5\,(d^2-e^2x^2)^{3/2}}\,dx$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{\int \frac{-16d^5e+31d^4e^2x-32d^3e^3x^2+32d^2e^4x^3}{x^4\sqrt{d^2-e^2x^2}}\,dx}{4d^4}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{\int \frac{-93d^6e^2+128d^5e^3x-96d^4e^4x^2}{x^3\sqrt{d^2-e^2x^2}}\,dx}{12d^6}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{\int \frac{-256d^7e^3+285}{x^2\sqrt{d^2-e}}}{24d^8}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e}}{3d^3x}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e}}{3d^3x}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e}}{3d^3x}$$

$$= \frac{8e^4(d-ex)}{d^3\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2-e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2-e^2x^2}}{8d^2x^2} + \frac{32e^3\sqrt{d^2-e}}{3d^3x}$$

Mathematica [A]
time = 0.47, size = 116, normalized size = 0.68

$$\frac{\frac{\sqrt{d^2-e^2x^2}\,(-6d^4+26d^3ex-61d^2e^2x^2+163de^3x^3+448e^4x^4)}{x^4(d+ex)} + 570e^4\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(-6*d^4 + 26*d^3*e*x - 61*d^2*e^2*x^2 + 163*d*e^3*x^3
 + 448*e^4*x^4))/(x^4*(d + e*x)) + 570*e^4*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2
 - e^2*x^2])/d])/(24*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(148) = 296$.

time = 0.09, size = 1799, normalized size = 10.58

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}\,(-256e^3x^3+93d\,e^2x^2-32d^2ex+6d^3)}{24d^3x^4}+\frac{8e^3\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{d^3\left(x+\frac{d}{e}\right)}-\frac{95e^4\ln\left(\frac{2d^2+2}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -35*e^4/d^8*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+
d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*
(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d^4*(-1/4/
d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)
-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((
-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^
(1/2))/x))))))-5*e^2/d^6*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7
/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(
1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e
^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e
)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)
^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))+10/d^6*e^2*(-1/2/d^2/x^
2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2
*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)
^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))-e/d^5*(-1/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(7/2)-3*e/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/
d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e
)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*
d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e
^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))))-20/d^7*e^3*(-1/d^2/
x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x
*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))-15*e^3/d^7*(1/3/d/e/(x+d/
e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*
e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*
d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(1/2))))))-4/d^5*e*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-4/3*e
^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)
```

```
+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/
2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+35/d^8*e^
4*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^
2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^(5/2)/((x*e + d)^4*x^5), x)
```

Fricas [A]
time = 2.03, size = 129, normalized size = 0.76

$$\frac{192\,x^5e^5+192\,dx^4e^4+285\,(x^5e^5+dx^4e^4)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(448\,x^4e^4+163\,dx^3e^3-61\,d^2x^2e^2+26\,d^3xe-6\,d^4)\sqrt{-x^2e^2+d^2}}{24\,(d^3x^5e+d^4x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="fricas")
[Out] 1/24*(192*x^5*e^5 + 192*d*x^4*e^4 + 285*(x^5*e^5 + d*x^4*e^4)*log(-(d - sqr
t(-x^2*e^2 + d^2))/x) + (448*x^4*e^4 + 163*d*x^3*e^3 - 61*d^2*x^2*e^2 + 26*
d^3*x*e - 6*d^4)*sqrt(-x^2*e^2 + d^2))/(d^3*x^5*e + d^4*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}{x^5\,(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**5*(d + e*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(141) = 282.
time = 1.88, size = 360, normalized size = 2.12

$$-\frac{95\,e^4\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{8\,d^3}-\frac{x^4\left(\frac{29\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^2}{x}+\frac{864\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-2)}}{x^3}+\frac{4128\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-4)}}{x^4}-\frac{160\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2}{x^2}-3\,e^4\right)e^8}{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}+\frac{\frac{1056\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^9e^2}{x}+\frac{32\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^9e^{(-2)}}{x^3}-\frac{3\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^9e^{(-4)}}{x^4}-\frac{192\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^9}{x^2}}{192\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -95/8*e^4*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^3
 - 1/192*x^4*(29*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^2/x + 864*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^3*e^(-2)/x^3 + 4128*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-4)
/x^4 - 160*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2/x^2 - 3*e^4)*e^8/((d*e + sqrt(-
x^2*e^2 + d^2)*e)^4*d^3*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)) + 1/
192*(1056*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^9*e^2/x + 32*(d*e + sqrt(-x^2*e^
2 + d^2)*e)^3*d^9*e^(-2)/x^3 - 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^9*e^(-4
)/x^4 - 192*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^9/x^2)/d^12
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^5\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^5*(d + e*x)^4), x)
```

3.209 $\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx$

Optimal. Leaf size=196

$$-\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{5x^5}+\frac{e\sqrt{d^2-e^2x^2}}{dx^4}-\frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}+\frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2}-\frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\cdots$$

```
[Out] 27/2*e^5*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^4-8*e^5*(-e*x+d)/d^4/(-e^2*x^2+d
^2)^(1/2)-1/5*(-e^2*x^2+d^2)^(1/2)/x^5+e*(-e^2*x^2+d^2)^(1/2)/d/x^4-13/5*e^
2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3+11/2*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-66/5*e
^4*(-e^2*x^2+d^2)^(1/2)/d^4/x
```

Rubi [A]
time = 0.48, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 1821, 821, 272, 65, 214}

$$-\frac{\sqrt{d^2-e^2x^2}}{5x^5}+\frac{e\sqrt{d^2-e^2x^2}}{dx^4}-\frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3}-\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}}+\frac{27e^5\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4}-\frac{66e^4\sqrt{d^2-e^2x^2}}{5d^4x}+\frac{11e^3\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

```
[Out] (-8*e^5*(d - e*x))/(d^4*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(5*x^5)
+ (e*Sqrt[d^2 - e^2*x^2])/(d*x^4) - (13*e^2*Sqrt[d^2 - e^2*x^2])/(5*d^2*x^3
) + (11*e^3*Sqrt[d^2 - e^2*x^2])/(2*d^3*x^2) - (66*e^4*Sqrt[d^2 - e^2*x^2])
/(5*d^4*x) + (27*e^5*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{(d-ex)^4}{x^6\,(d^2-e^2x^2)^{3/2}}\,dx$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\int \frac{-d^4+4d^3ex-7d^2e^2x^2+8de^3x^3-8e^4x^4+\frac{8e^5x^5}{d}}{x^6\sqrt{d^2-e^2x^2}}\,dx}{d^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{\int \frac{-20d^5e+39d^4e^2x-40d^3e^3x^2+40d^2e^4x^3-40de^5x^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{5d^4}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{\int \frac{-156d^6e^2+220d^5e^3x-160d^4e^4x^2+160d^3\cdots}{x^4\sqrt{d^2-e^2x^2}}}{20d^6}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{\int \frac{-660d^7e^3+79\cdots}{x^3\sqrt{d\cdots}}}{\cdots}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-\cdots}}{2d^3x^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-\cdots}}{2d^3x^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-\cdots}}{2d^3x^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-\cdots}}{2d^3x^2}$$

$$= -\frac{8e^5(d-ex)}{d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{5x^5} + \frac{e\sqrt{d^2-e^2x^2}}{dx^4} - \frac{13e^2\sqrt{d^2-e^2x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2-\cdots}}{2d^3x^2}$$

Mathematica [A]
time = 0.58, size = 127, normalized size = 0.65

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\,(2d^5-8d^4ex+16d^3e^2x^2-29d^2e^3x^3+77de^4x^4+212e^5x^5)}{x^5(d+ex)} + 270e^5\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{10d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
[Out] -1/10*((Sqrt[d^2 - e^2*x^2]*(2*d^5 - 8*d^4*e*x + 16*d^3*e^2*x^2 - 29*d^2*e^
3*x^3 + 77*d*e^4*x^4 + 212*e^5*x^5))/(x^5*(d + e*x)) + 270*e^5*ArcTanh[(Sqr
t[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1996 vs. $2(172) = 344$.

time = 0.11, size = 1997, normalized size = 10.19

method	result
risch	$-\frac{\sqrt{-e^{2} x^{2}+d^{2}}\, \left(132 e^{4} x^{4}-55 d\, e^{3} x^{3}+26 d^{2} x^{2} e^{2}-10 d^{3} e x +2 d^{4}\right)}{10 x^{5} d^{4}}-\frac{8 e^{4} \sqrt{-\left(x +\frac{d}{e}\right)^{2} e^{2}+2 d e \left(x +\frac{d}{e}\right)}}{d^{4} \left(x +\frac{d}{e}\right)}+\frac{27 e^{5} \ln}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 56*e^5/d^9*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d
/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(
x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*
arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d^4*(-1/5/d
^2/x^5*(-e^2*x^2+d^2)^(7/2)-2/5*e^2/d^2*(-1/3/d^2/x^3*(-e^2*x^2+d^2)^(7/2)-
4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^
(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/
2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))))-4/d
^5*e*(-1/4/d^2/x^4*(-e^2*x^2+d^2)^(7/2)-3/4*e^2/d^2*(-1/2/d^2/x^2*(-e^2*x^2
+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(
3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^
2*x^2+d^2)^(1/2))/x))))))+6*e^3/d^7*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*
(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2
)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/
e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x
+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))-20/d^7*e^3*(
-1/2/d^2/x^2*(-e^2*x^2+d^2)^(7/2)-5/2*e^2/d^2*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2
*(1/3*(-e^2*x^2+d^2)^(3/2)+d^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*
d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)))))+e^2/d^6*(-1/d/e/(x+d/e)^4*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)-3*e/d*(1/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(7/2)+4*e/d*(1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))
^(7/2)+5/3*e/d*(1/5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*
(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e
^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1
/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))))+35/d^8
*e^4*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^2*x^2+d^2)^(5/2)+5
/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*
d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))))+21*e^4/d^8*(
1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+5/3*e/d*(1/5*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(5/2)+d*e*(-1/8*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+d/e
```

```
)^2*e^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/e)+2*d*e)/e^2*(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))+10/d^6*e^2*(-1/3/d^2/x^3*(-e^2*x^2+
d^2)^(7/2)-4/3*e^2/d^2*(-1/d^2/x*(-e^2*x^2+d^2)^(7/2)-6*e^2/d^2*(1/6*x*(-e^
2*x^2+d^2)^(5/2)+5/6*d^2*(1/4*x*(-e^2*x^2+d^2)^(3/2)+3/4*d^2*(1/2*x*(-e^2*x
^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
))))))-56/d^9*e^5*(1/5*(-e^2*x^2+d^2)^(5/2)+d^2*(1/3*(-e^2*x^2+d^2)^(3/2)+d
^2*((-e^2*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+
d^2)^(1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^(5/2)/((x*e + d)^4*x^6), x)
```

Fricas [A]
time = 2.28, size = 139, normalized size = 0.71

$$-\frac{80\,x^6e^6+80\,dx^5e^5+135\,(x^6e^6+dx^5e^5)\log\left(-\frac{d-\sqrt{-x^2e^2+d^2}}{x}\right)+(212\,x^5e^5+77\,dx^4e^4-29\,d^2x^3e^3+16\,d^3x^2e^2-8\,d^4xe+2\,d^5)\sqrt{-x^2e^2+d^2}}{10\,(d^4x^6e+d^5x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="fricas")
[Out] -1/10*(80*x^6*e^6 + 80*d*x^5*e^5 + 135*(x^6*e^6 + d*x^5*e^5)*log(-(d - sqrt
(-x^2*e^2 + d^2))/x) + (212*x^5*e^5 + 77*d*x^4*e^4 - 29*d^2*x^3*e^3 + 16*d^
3*x^2*e^2 - 8*d^4*x*e + 2*d^5)*sqrt(-x^2*e^2 + d^2))/(d^4*x^6*e + d^5*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}{x^6\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**6*(d + e*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(163) = 326.
time = 1.28, size = 426, normalized size = 2.17

$$\frac{27\,e^5\log\left(\frac{\left|-2\,de-2\,\sqrt{-x^2e^2+d^2}\,e\right|e^{(-2)}}{2\,|x|}\right)}{2\,d^4}-\frac{x^5\left(\frac{9\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^3}{x}-\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e}{x^2}+\frac{185\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-1)}}{x^3}-\frac{870\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-3)}}{x^4}-\frac{3670\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5e^{(-5)}}{x^5}-e^5\right)e^{10}}{160\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^4\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}-\frac{\frac{1110\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^{16}e^3}{x}-\frac{240\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^{16}e}{x^2}+\frac{55\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^{16}e^{(-1)}}{x^3}-\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^{16}e^{(-3)}}{x^4}+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5d^{16}e^{(-5)}}{x^5}}{160\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 27/2*e^5*log(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^4
- 1/160*x^5*(9*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^3/x - 45*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^2*e/x^2 + 185*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-1)/x^3 - 87
0*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-3)/x^4 - 3670*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^5*e^(-5)/x^5 - e^5)*e^10/((d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^4*((d
*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)) - 1/160*(1110*(d*e + sqrt(-x^2*
e^2 + d^2)*e)*d^16*e^3/x - 240*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^16*e/x^2
+ 55*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^16*e^(-1)/x^3 - 10*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^4*d^16*e^(-3)/x^4 + (d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^16*e^
(-5)/x^5)/d^20
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}}{x^6\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)/(x^6*(d + e*x)^4), x)
```

3.210 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx$

Optimal. Leaf size=95

$$\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4}-\frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3}-\frac{\sin^{-1}(ax)}{a^3}$$

```
[Out] 1/5*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^4-3/5*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^3
-arcsin(a*x)/a^3+2*(-a^2*x^2+1)^(1/2)/a^3/(-a*x+1)
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1651, 673, 665, 677, 222}

$$-\frac{\text{ArcSin}(ax)}{a^3}-\frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3}+\frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a^3*(1 - a*x)) + (1 - a^2*x^2)^(3/2)/(5*a^3*(1 - a*x
)^4) - (3*(1 - a^2*x^2)^(3/2))/(5*a^3*(1 - a*x)^3) - ArcSin[a*x]/a^3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx &= \int \left(\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^4}+\frac{2\sqrt{1-a^2x^2}}{a^2(-1+ax)^3}+\frac{\sqrt{1-a^2x^2}}{a^2(-1+ax)^2}\right)dx\\
&= \frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^4}\,dx}{a^2}+\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^2}\,dx}{a^2}+\frac{2\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{a^2}\\
&= \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4}-\frac{2(1-a^2x^2)^{3/2}}{3a^3(1-ax)^3}-\frac{\int \frac{\sqrt{1-a^2x^2}}{(-1+ax)^3}\,dx}{5a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}}}{a^2}\\
&= \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}+\frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4}-\frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3}-\frac{\sin^{-1}(ax)}{a^3}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 82, normalized size = 0.86

$$\frac{(-8+19ax-13a^2x^2)\sqrt{1-a^2x^2}}{5a^3(-1+ax)^3}-\frac{\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{(-a^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^4,x]
```

```
[Out] ((-8 + 19*a*x - 13*a^2*x^2)*Sqrt[1 - a^2*x^2])/(5*a^3*(-1 + a*x)^3) - Log[-
(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]]/(-a^2)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(85) = 170.
time = 0.07, size = 245, normalized size = 2.58

method	result
default	$\frac{\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{a\left(x-\frac{1}{a}\right)^2}+a\left(\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}-\frac{a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{\sqrt{a^2}}\right)}{a^4}$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)+a*((-a^2*(x-1/a)^2-
2*a*(x-1/a))^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(
x-1/a))^(1/2))))+1/a^6*(1/5/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-
1/15/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2))+2/3/a^6/(x-1/a)^3*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4, x)
```

Fricas [A]
time = 1.60, size = 126, normalized size = 1.33

$$\frac{8a^3x^3-24a^2x^2+24ax+10\left(a^3x^3-3a^2x^2+3ax-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\left(13a^2x^2-19ax+8\right)\sqrt{-a^2x^2+1}-8}{5\left(a^6x^3-3a^5x^2+3a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(8*a^3*x^3 - 24*a^2*x^2 + 24*a*x + 10*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - (13*a^2*x^2 - 19*a*x + 8)*sqrt(-a
^2*x^2 + 1) - 8)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}}{\left(ax-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**4,x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x - 1)**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 2.70, size = 220, normalized size = 2.32

$$\frac{4\,a^2\,\sqrt{1-a^2\,x^2}}{15\,\left(a^7\,x^2-2\,a^6\,x+a^5\right)}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{a^2\,\sqrt{-a^2}}-\frac{2\,\sqrt{1-a^2\,x^2}}{5\,\sqrt{-a^2}\,\left(a\,\sqrt{-a^2}-3\,a^2\,x\,\sqrt{-a^2}+3\,a^3\,x^2\,\sqrt{-a^2}-a^4\,x^3\,\sqrt{-a^2}\right)}-\frac{13\,\sqrt{1-a^2\,x^2}}{5\,\left(a\,\sqrt{-a^2}-a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{5\,\sqrt{1-a^2\,x^2}}{3\,\left(a^5\,x^2-2\,a^4\,x+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^4,x)
```

```
[Out] (4*a^2*(1 - a^2*x^2)^(1/2))/(15*(a^5 - 2*a^6*x + a^7*x^2)) - asinh(x*(-a^2)
^(1/2))/(a^2*(-a^2)^(1/2)) - (2*(1 - a^2*x^2)^(1/2))/(5*(-a^2)^(1/2)*(a*(-a
^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2) - a^4*x^3*(-a^2)^
(1/2))) - (13*(1 - a^2*x^2)^(1/2))/(5*(a*(-a^2)^(1/2) - a^2*x*(-a^2)^(1/2))
*(-a^2)^(1/2)) - (5*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2))
```

3.211 $\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx$

Optimal. Leaf size=88

$$\frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}-\frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4}+\frac{23(1-a^2x^2)^{3/2}}{105a^3(1-ax)^3}$$

```
[Out] 1/7*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^5-12/35*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)
^4+23/105*(-a^2*x^2+1)^(3/2)/a^3/(-a*x+1)^3
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {1653, 807, 673, 665}

$$\frac{23(1-a^2x^2)^{3/2}}{105a^3(1-ax)^3}-\frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4}+\frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (1 - a^2*x^2)^(3/2)/(7*a^3*(1 - a*x)^5) - (12*(1 - a^2*x^2)^(3/2))/(35*a^3*
(1 - a*x)^4) + (23*(1 - a^2*x^2)^(3/2))/(105*a^3*(1 - a*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
```

```
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}+\frac{\int \frac{(4a^2-3a^3x)\sqrt{1-a^2x^2}}{(1-ax)^5}\,dx}{a^4}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{a^3(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^4}\,dx}{7a^2}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{23\int \frac{\sqrt{1-a^2x^2}}{(1-ax)^3}\,dx}{35a^2}\\
&= \frac{\left(1-a^2x^2\right)^{3/2}}{7a^3(1-ax)^5}-\frac{12\left(1-a^2x^2\right)^{3/2}}{35a^3(1-ax)^4}+\frac{23\left(1-a^2x^2\right)^{3/2}}{105a^3(1-ax)^3}
\end{aligned}$$

Mathematica [**A**]
time = 0.42, size = 50, normalized size = 0.57

$$\frac{\sqrt{1-a^2x^2}\,\left(2-8ax+13a^2x^2+23a^3x^3\right)}{105a^3(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - a^2*x^2])/(1 - a*x)^5,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2 - 8*a*x + 13*a^2*x^2 + 23*a^3*x^3))/(105*a^3*(-1 + a*
x)^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(76) = 152.
time = 0.07, size = 258, normalized size = 2.93

method	result
gosper	$\frac{\sqrt{-a^{2}x^{2}+1}\,(23a^{2}x^{2}-10ax+2)(ax+1)}{105(ax-1)^{4}a^{3}}$
trager	$\frac{(23a^{3}x^{3}+13a^{2}x^{2}-8ax+2)\sqrt{-a^{2}x^{2}+1}}{105(ax-1)^{4}a^{3}}$
default	$-\frac{2\left(\frac{\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{5a\left(x-\frac{1}{a}\right)^{4}}-\frac{\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{15\left(x-\frac{1}{a}\right)^{3}}\right)}{a^{6}}-\frac{\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3a^{6}\left(x-\frac{1}{a}\right)^{3}}-\frac{\frac{\left(-a^{2}\left(x-\frac{1}{a}\right)^{2}-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{7a\left(x-\frac{1}{a}\right)^{5}}-\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^6*(1/5/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-1/15/(x-1/a)^3*(
-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2))-1/3/a^6/(x-1/a)^3*(-a^2*(x-1/a)^2-2*a*(x
-1/a))^(3/2)-1/a^7*(1/7/a/(x-1/a)^5*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-2/7*
a*(1/5/a/(x-1/a)^4*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-1/15/(x-1/a)^3*(-a^2*
(x-1/a)^2-2*a*(x-1/a))^(3/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(73) = 146$.
time = 0.29, size = 153, normalized size = 1.74

$$\frac{2\sqrt{-a^{2}x^{2}+1}}{7\left(a^{7}x^{4}-4a^{6}x^{3}+6a^{5}x^{2}-4a^{4}x+a^{3}\right)}+\frac{29\sqrt{-a^{2}x^{2}+1}}{35\left(a^{6}x^{3}-3a^{5}x^{2}+3a^{4}x-a^{3}\right)}+\frac{82\sqrt{-a^{2}x^{2}+1}}{105\left(a^{5}x^{2}-2a^{4}x+a^{3}\right)}+\frac{23\sqrt{-a^{2}x^{2}+1}}{105\left(a^{4}x-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-a^2*x^2 + 1)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4*a^4*x + a^3) +
29/35*sqrt(-a^2*x^2 + 1)/(a^6*x^3 - 3*a^5*x^2 + 3*a^4*x - a^3) + 82/105*sqr
t(-a^2*x^2 + 1)/(a^5*x^2 - 2*a^4*x + a^3) + 23/105*sqrt(-a^2*x^2 + 1)/(a^4*
x - a^3)
```

Fricas [A]
time = 1.83, size = 102, normalized size = 1.16

$$\frac{2a^{4}x^{4}-8a^{3}x^{3}+12a^{2}x^{2}-8ax+\left(23a^{3}x^{3}+13a^{2}x^{2}-8ax+2\right)\sqrt{-a^{2}x^{2}+1}+2}{105\left(a^{7}x^{4}-4a^{6}x^{3}+6a^{5}x^{2}-4a^{4}x+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/105*(2*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 8*a*x + (23*a^3*x^3 + 13*a^2*x^
2 - 8*a*x + 2)*sqrt(-a^2*x^2 + 1) + 2)/(a^7*x^4 - 4*a^6*x^3 + 6*a^5*x^2 - 4
*a^4*x + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2\sqrt{-a^2x^2+1}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**5,x)
```

```
[Out] -Integral(x**2*sqrt(-a**2*x**2 + 1)/(a**5*x**5 - 5*a**4*x**4 + 10*a**3*x**3
 - 10*a**2*x**2 + 5*a*x - 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.06, size = 287, normalized size = 3.26

$$\frac{2\,\sqrt{1-a^2\,x^2}}{7\,\left(a^7\,x^4-4\,a^6\,x^3+6\,a^5\,x^2-4\,a^4\,x+a^3\right)}+\frac{4\,\sqrt{1-a^2\,x^2}}{3\,\left(a^5\,x^2-2\,a^4\,x+a^3\right)}+\frac{4\,a\,\sqrt{1-a^2\,x^2}}{35\,\left(a^6\,x^2-2\,a^5\,x+a^4\right)}+\frac{29\,\sqrt{1-a^2\,x^2}}{35\,\sqrt{-a^2}\,\left(a\,\sqrt{-a^2}-3\,a^2\,x\,\sqrt{-a^2}+3\,a^3\,x^2\,\sqrt{-a^2}-a^4\,x^3\,\sqrt{-a^2}\right)}+\frac{23\,\sqrt{1-a^2\,x^2}}{105\,\left(a\,\sqrt{-a^2}-a^2\,x\,\sqrt{-a^2}\right)\,\sqrt{-a^2}}-\frac{2\,a^2\,\sqrt{1-a^2\,x^2}}{3\,\left(a^7\,x^2-2\,a^6\,x+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(1 - a^2*x^2)^(1/2))/(a*x - 1)^5,x)
```

```
[Out] (2*(1 - a^2*x^2)^(1/2))/(7*(a^3 - 4*a^4*x + 6*a^5*x^2 - 4*a^6*x^3 + a^7*x^4
)) + (4*(1 - a^2*x^2)^(1/2))/(3*(a^3 - 2*a^4*x + a^5*x^2)) + (4*a*(1 - a^2*
x^2)^(1/2))/(35*(a^4 - 2*a^5*x + a^6*x^2)) + (29*(1 - a^2*x^2)^(1/2))/(35*(
-a^2)^(1/2)*(a*(-a^2)^(1/2) - 3*a^2*x*(-a^2)^(1/2) + 3*a^3*x^2*(-a^2)^(1/2)
 - a^4*x^3*(-a^2)^(1/2))) + (23*(1 - a^2*x^2)^(1/2))/(105*(a*(-a^2)^(1/2) -
 a^2*x*(-a^2)^(1/2))*(-a^2)^(1/2)) - (2*a^2*(1 - a^2*x^2)^(1/2))/(3*(a^5 -
2*a^6*x + a^7*x^2))
```

3.212 $\int \frac{x^3}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{21}{143e^4(d+ex)^2\,(d^2}$$

```
[Out] -24/5005*x/d^3/e^3/(-e^2*x^2+d^2)^(5/2)+1/13*d^2/e^4/(e*x+d)^4/(-e^2*x^2+d^
2)^(5/2)-30/143*d/e^4/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)+21/143/e^4/(e*x+d)^2/(
-e^2*x^2+d^2)^(5/2)+4/1001/d/e^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2)-32/5005*x/d^5
/e^3/(-e^2*x^2+d^2)^(3/2)-64/5005*x/d^7/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1653, 807, 673, 198, 197}

$$\frac{d^2}{13e^4(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{30d}{143e^4(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{21}{143e^4(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4}{1001de^4(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{64x}{5005d^7e^3\sqrt{d^2-e^2x^2}}-\frac{32x}{5005d^5e^3\left(d^2-e^2x^2\right)^{3/2}}-\frac{24x}{5005d^3e^3\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-24*x)/(5005*d^3*e^3*(d^2 - e^2*x^2)^(5/2)) + d^2/(13*e^4*(d + e*x)^4*(d^2
 - e^2*x^2)^(5/2)) - (30*d)/(143*e^4*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + 2
1/(143*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + 4/(1001*d*e^4*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) - (32*x)/(5005*d^5*e^3*(d^2 - e^2*x^2)^(3/2)) - (64*x)/
(5005*d^7*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
```

```
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{7e^4(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{2d^3e^2-3d^2e^3x-12de^4x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx}{7e^5}$$

$$= -\frac{3d}{14e^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{7e^4(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{-20d^3e^6+3\cdots}{(d+ex)^4(d^2-\cdots}}{56e\cdots}$$

$$= \frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{3d}{14e^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{\cdots}{7e^4(d+ex)^2\cdots}$$

$$= \frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{\cdots}{7e^4(d+ex\cdots}$$

$$= \frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{\cdots}{143e^4(d+e\cdots}$$

$$= \frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{30d}{143e^4(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{\cdots}{143e^4(d+e\cdots}$$

$$= -\frac{24x}{5005d^3e^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{30\cdots}{143e^4(d+ex)^3\cdots}$$

$$= -\frac{24x}{5005d^3e^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{30\cdots}{143e^4(d+ex)^3\cdots}$$

$$= -\frac{24x}{5005d^3e^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{d^2}{13e^4(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{30\cdots}{143e^4(d+ex)^3\cdots}$$

Mathematica [A]
time = 0.62, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\,(90d^9+360d^8ex+315d^7e^2x^2-540d^6e^3x^3+160d^5e^4x^4+776d^4e^5x^5+384d^3e^6x^6-224d^2e^7x^7-256de^8x^8-64e^9x^9)}{5005d^7e^4(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(90*d^9 + 360*d^8*e*x + 315*d^7*e^2*x^2 - 540*d^6*e^3*
x^3 + 160*d^5*e^4*x^4 + 776*d^4*e^5*x^5 + 384*d^3*e^6*x^6 - 224*d^2*e^7*x^7
 - 256*d*e^8*x^8 - 64*e^9*x^9))/(5005*d^7*e^4*(d - e*x)^3*(d + e*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. $2(181) = 362$.
time = 0.07, size = 1212, normalized size = 5.80

method	result
gosper	$\frac{(-ex+d)\left(-64e^{9}x^{9}-256d\,e^{8}x^{8}-224e^{7}x^{7}d^{2}+384e^{6}x^{6}d^{3}+776e^{5}x^{5}d^{4}+160x^{4}d^{5}e^{4}-540d^{6}e^{3}x^{3}+315x^{2}d^{7}e^{2}+360d^{8}xe+90d^{9}\right)}{5005(ex+d)^{3}d^{7}e^{4}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$
trager	$\frac{\left(-64e^{9}x^{9}-256d\,e^{8}x^{8}-224e^{7}x^{7}d^{2}+384e^{6}x^{6}d^{3}+776e^{5}x^{5}d^{4}+160x^{4}d^{5}e^{4}-540d^{6}e^{3}x^{3}+315x^{2}d^{7}e^{2}+360d^{8}xe+90d^{9}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{5005d^{7}(ex+d)^{7}e^{4}(-ex+d)^{3}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10
*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^
2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)
-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
+3*d^2/e^6*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e
/d*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d
/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d
/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e
^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*
(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))-d^3/e^7*(-
1/13/d/e/(x+d/e)^4/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+9/13*e/d*(-1/11/d/e
/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e/d*(-1/9/d/e/(x+d/e)^
2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2
/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e
)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))-3*d/e^5*(-1/9/d/e/(x+d/e)^2/(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d
/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2
/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(171) = 342$.
time = 0.31, size = 365, normalized size = 1.75

$$\frac{d^{2}}{13\left(\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}x^{4}e^{8}+4\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}dx^{3}e^{7}+6\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}x^{2}e^{6}+4\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}xe^{5}+\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{4}e^{4}\right)}-\frac{30\,d}{143\left(\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}x^{3}e^{7}+3\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}dx^{2}e^{6}+3\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}xe^{5}+\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}e^{4}\right)}+\frac{21}{143\left(\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}x^{2}e^{6}+2\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}dxe^{5}+\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}\right)}+\frac{4}{1001\left(\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}dxe^{5}+\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{2}e^{4}\right)}-\frac{24\,xe^{(-3)}}{5005\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}d^{3}}-\frac{32\,xe^{(-3)}}{5005\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{5}}-\frac{64\,xe^{(-3)}}{5005\sqrt{-x^{2}e^{2}+d^{2}}\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13*d^2/((-x^2*e^2 + d^2)^(5/2)*x^4*e^8 + 4*(-x^2*e^2 + d^2)^(5/2)*d*x^3*e
^7 + 6*(-x^2*e^2 + d^2)^(5/2)*d^2*x^2*e^6 + 4*(-x^2*e^2 + d^2)^(5/2)*d^3*x*
```

```
e^5 + (-x^2*e^2 + d^2)^(5/2)*d^4*e^4) - 30/143*d/((-x^2*e^2 + d^2)^(5/2)*x^
3*e^7 + 3*(-x^2*e^2 + d^2)^(5/2)*d*x^2*e^6 + 3*(-x^2*e^2 + d^2)^(5/2)*d^2*x
*e^5 + (-x^2*e^2 + d^2)^(5/2)*d^3*e^4) + 21/143/((-x^2*e^2 + d^2)^(5/2)*x^2
*e^6 + 2*(-x^2*e^2 + d^2)^(5/2)*d*x*e^5 + (-x^2*e^2 + d^2)^(5/2)*d^2*e^4) +
 4/1001/((-x^2*e^2 + d^2)^(5/2)*d*x*e^5 + (-x^2*e^2 + d^2)^(5/2)*d^2*e^4) -
 24/5005*x*e^(-3)/((-x^2*e^2 + d^2)^(5/2)*d^3) - 32/5005*x*e^(-3)/((-x^2*e^
2 + d^2)^(3/2)*d^5) - 64/5005*x*e^(-3)/(sqrt(-x^2*e^2 + d^2)*d^7)
```

Fricas [A]
time = 4.41, size = 291, normalized size = 1.39

$$\frac{90\,x^{10}e^{10}+360\,dx^{9}e^{9}+270\,d^{2}x^{8}e^{8}-720\,d^{3}x^{7}e^{7}-1260\,d^{4}x^{6}e^{6}+1260\,d^{6}x^{4}e^{4}+720\,d^{7}x^{3}e^{3}-270\,d^{8}x^{2}e^{2}-360\,d^{9}xe-90\,d^{10}+\left(64\,x^{9}e^{9}+256\,dx^{8}e^{8}+224\,d^{2}x^{7}e^{7}-384\,d^{3}x^{6}e^{6}-776\,d^{4}x^{5}e^{5}-160\,d^{5}x^{4}e^{4}+540\,d^{6}x^{3}e^{3}-315\,d^{7}x^{2}e^{2}-360\,d^{8}xe-90\,d^{9}\right)\sqrt{-x^{2}e^{2}+d^{2}}}{5005\,\left(d^{7}x^{10}e^{14}+4\,d^{8}x^{9}e^{13}+3\,d^{9}x^{8}e^{12}-8\,d^{10}x^{7}e^{11}-14\,d^{11}x^{6}e^{10}+14\,d^{13}x^{4}e^{8}+8\,d^{14}x^{3}e^{7}-3\,d^{15}x^{2}e^{6}-4\,d^{16}xe^{5}-d^{17}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5005*(90*x^10*e^10 + 360*d*x^9*e^9 + 270*d^2*x^8*e^8 - 720*d^3*x^7*e^7 -
1260*d^4*x^6*e^6 + 1260*d^6*x^4*e^4 + 720*d^7*x^3*e^3 - 270*d^8*x^2*e^2 - 3
60*d^9*x*e - 90*d^10 + (64*x^9*e^9 + 256*d*x^8*e^8 + 224*d^2*x^7*e^7 - 384*
d^3*x^6*e^6 - 776*d^4*x^5*e^5 - 160*d^5*x^4*e^4 + 540*d^6*x^3*e^3 - 315*d^7
*x^2*e^2 - 360*d^8*x*e - 90*d^9)*sqrt(-x^2*e^2 + d^2))/(d^7*x^10*e^14 + 4*d
^8*x^9*e^13 + 3*d^9*x^8*e^12 - 8*d^10*x^7*e^11 - 14*d^11*x^6*e^10 + 14*d^13
*x^4*e^8 + 8*d^14*x^3*e^7 - 3*d^15*x^2*e^6 - 4*d^16*x*e^5 - d^17*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**3/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^4), x)
```

Mupad [B]
time = 3.22, size = 252, normalized size = 1.21

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{107}{4004\,d^2\,e^4}-\frac{1139\,x}{80080\,d^3\,e^3}\right)}{(d+e\,x)^3\,(d-e\,x)^3}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{23}{32032\,d^4\,e^4}+\frac{32\,x}{5005\,d^5\,e^3}\right)}{(d+e\,x)^2\,(d-e\,x)^2}+\frac{\sqrt{d^2-e^2x^2}}{104\,d\,e^4\,(d+e\,x)^7}-\frac{27\,\sqrt{d^2-e^2x^2}}{2288\,d^2\,e^4\,(d+e\,x)^6}-\frac{15\,\sqrt{d^2-e^2x^2}}{2288\,d^3\,e^4\,(d+e\,x)^5}+\frac{23\,\sqrt{d^2-e^2x^2}}{32032\,d^4\,e^4\,(d+e\,x)^4}-\frac{64\,x\,\sqrt{d^2-e^2x^2}}{5005\,d^7\,e^3\,(d+e\,x)\,(d-e\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(107/(4004*d^2*e^4) - (1139*x)/(80080*d^3*e^3)))/((d
 + e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(23/(32032*d^4*e^4) + (32*x
)/(5005*d^5*e^3)))/((d + e*x)^2*(d - e*x)^2) + (d^2 - e^2*x^2)^(1/2)/(104*d
*e^4*(d + e*x)^7) - (27*(d^2 - e^2*x^2)^(1/2))/(2288*d^2*e^4*(d + e*x)^6) -
 (15*(d^2 - e^2*x^2)^(1/2))/(2288*d^3*e^4*(d + e*x)^5) + (23*(d^2 - e^2*x^2
)^(1/2))/(32032*d^4*e^4*(d + e*x)^4) - (64*x*(d^2 - e^2*x^2)^(1/2))/(5005*d
^7*e^3*(d + e*x)*(d - e*x))
```

3.213 $\int \frac{x^2}{(d+ex)^4 (d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=209

$$\frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287de^3(d+ex)^2\left(d^2\right.}$$

```
[Out] 14/2145*x/d^4/e^2/(-e^2*x^2+d^2)^(5/2)-1/13*d/e^3/(e*x+d)^4/(-e^2*x^2+d^2)^
(5/2)+17/143/e^3/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-7/1287/d/e^3/(e*x+d)^2/(-e^
2*x^2+d^2)^(5/2)-7/1287/d^2/e^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+56/6435*x/d^6/
e^2/(-e^2*x^2+d^2)^(3/2)+112/6435*x/d^8/e^2/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1653, 807, 673, 198, 197}

$$-\frac{d}{13e^3(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{17}{143e^3(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287de^3(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{7}{1287d^2e^3(d+ex)\left(d^2-e^2x^2\right)^{5/2}}+\frac{112x}{6435d^8e^2\sqrt{d^2-e^2x^2}}+\frac{56x}{6435d^6e^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{14x}{2145d^4e^2\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (14*x)/(2145*d^4*e^2*(d^2 - e^2*x^2)^(5/2)) - d/(13*e^3*(d + e*x)^4*(d^2 -
e^2*x^2)^(5/2)) + 17/(143*e^3*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*
d*e^3*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 7/(1287*d^2*e^3*(d + e*x)*(d^2 -
 e^2*x^2)^(5/2)) + (56*x)/(6435*d^6*e^2*(d^2 - e^2*x^2)^(3/2)) + (112*x)/(6
435*d^8*e^2*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
```

```
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}} \, dx = \frac{1}{8e^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{3d^2e^2-5de^3x}{(d+ex)^4(d^2-e^2x^2)^{7/2}} \, dx}{8e^4}$$

$$= -\frac{d}{13e^3(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{8e^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{(7d) \int \frac{\cdots}{(d+ex} \cdots}{1\cdots}$$

$$= -\frac{d}{13e^3(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{7 \int \frac{\cdots}{(d+ex)\cdots}}{\cdots}$$

$$= -\frac{d}{13e^3(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{\cdots}{1287de^3(\cdots}$$

$$= -\frac{d}{13e^3(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{\cdots}{1287de^3(\cdots}$$

$$= \frac{14x}{2145d^4e^2 \left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3 (\cdots}$$

$$= \frac{14x}{2145d^4e^2 \left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3 (\cdots}$$

$$= \frac{14x}{2145d^4e^2 \left(d^2-e^2x^2\right)^{5/2}} - \frac{d}{13e^3(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{17}{143e^3(d+ex)^3 (\cdots}$$

Mathematica [A]
time = 0.59, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(200d^9+800d^8ex+700d^7e^2x^2+945d^6e^3x^3-280d^5e^4x^4-1358d^4e^5x^5-672d^3e^6x^6+392d^2e^7x^7+448de^8x^8+112e^9x^9\right)}{6435d^8e^3(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(200*d^9 + 800*d^8*e*x + 700*d^7*e^2*x^2 + 945*d^6*e^3
*x^3 - 280*d^5*e^4*x^4 - 1358*d^4*e^5*x^5 - 672*d^3*e^6*x^6 + 392*d^2*e^7*x
^7 + 448*d*e^8*x^8 + 112*e^9*x^9))/(6435*d^8*e^3*(d - e*x)^3*(d + e*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 984 vs. 2(181) = 362.
time = 0.07, size = 985, normalized size = 4.71

method	result
gosper	$\frac{(-ex+d)\left(112e^9x^9+448d\,e^8x^8+392e^7x^7d^2-672e^6x^6d^3-1358e^5x^5d^4-280x^4d^5e^4+945d^6e^3x^3+700x^2d^7e^2+800d^8xe+200d^9\right)}{6435(ex+d)^3d^8e^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$

trager	$\frac{(112e^9x^9+448d\,e^8x^8+392e^7x^7d^2-672e^6x^6d^3-1358e^5x^5d^4-280x^4d^5e^4+945d^6e^3x^3+700x^2d^7e^2+800d^8xe+200d^9)\sqrt{-e^2x^2+}}{6435d^8(ex+d)^7(-ex+d)^3e^3}$	

$$7e\left(-\frac{1}{7de\left(x+\frac{d}{e}\right)\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\right.\right.}\right.$$

$$8e\left(-\frac{1}{9de\left(x+\frac{d}{e}\right)^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*d/e^5*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e/d
*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e
/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e
)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2
*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-
2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))+1/e^6*d^2*(-
1/13/d/e/(x+d/e)^4/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+9/13*e/d*(-1/11/d/e
/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e/d*(-1/9/d/e/(x+d/e)^
2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2
/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e
)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))+1/e^4*(-1/9/d/e/(x+d/e)^2/(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+
2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(171) = 342$.
time = 0.31, size = 367, normalized size = 1.76

$$-\frac{d}{13\left((-x^2e^2+d^2)^{\frac{5}{2}}x^4e^7+4(-x^2e^2+d^2)^{\frac{5}{2}}dx^3e^6+6(-x^2e^2+d^2)^{\frac{5}{2}}d^2x^2e^5+4(-x^2e^2+d^2)^{\frac{5}{2}}d^3xe^4+(-x^2e^2+d^2)^{\frac{5}{2}}d^4e^3\right)}+\frac{17}{143\left((-x^2e^2+d^2)^{\frac{5}{2}}x^3e^6+3(-x^2e^2+d^2)^{\frac{5}{2}}dx^2e^5+3(-x^2e^2+d^2)^{\frac{5}{2}}d^2xe^4+(-x^2e^2+d^2)^{\frac{5}{2}}d^3e^3\right)}-\frac{7}{1287\left((-x^2e^2+d^2)^{\frac{5}{2}}dx^2e^5+2(-x^2e^2+d^2)^{\frac{5}{2}}d^2xe^4+(-x^2e^2+d^2)^{\frac{5}{2}}d^3e^3\right)}-\frac{7}{1287\left((-x^2e^2+d^2)^{\frac{5}{2}}d^2xe^4+(-x^2e^2+d^2)^{\frac{5}{2}}d^3e^3\right)}+\frac{14xe^{(-2)}}{2145(-x^2e^2+d^2)^{\frac{5}{2}}d^4}+\frac{56xe^{(-2)}}{6435(-x^2e^2+d^2)^{\frac{3}{2}}d^6}+\frac{112xe^{(-2)}}{6435\sqrt{-x^2e^2+d^2}\,d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/13*d/((-x^2*e^2 + d^2)^(5/2)*x^4*e^7 + 4*(-x^2*e^2 + d^2)^(5/2)*d*x^3*e^
6 + 6*(-x^2*e^2 + d^2)^(5/2)*d^2*x^2*e^5 + 4*(-x^2*e^2 + d^2)^(5/2)*d^3*x*e
^4 + (-x^2*e^2 + d^2)^(5/2)*d^4*e^3) + 17/143/((-x^2*e^2 + d^2)^(5/2)*x^3*e
^6 + 3*(-x^2*e^2 + d^2)^(5/2)*d*x^2*e^5 + 3*(-x^2*e^2 + d^2)^(5/2)*d^2*x*e^
4 + (-x^2*e^2 + d^2)^(5/2)*d^3*e^3) - 7/1287/((-x^2*e^2 + d^2)^(5/2)*d*x^2*
e^5 + 2*(-x^2*e^2 + d^2)^(5/2)*d^2*x*e^4 + (-x^2*e^2 + d^2)^(5/2)*d^3*e^3)
- 7/1287/((-x^2*e^2 + d^2)^(5/2)*d^2*x*e^4 + (-x^2*e^2 + d^2)^(5/2)*d^3*e^3
) + 14/2145*x*e^(-2)/((-x^2*e^2 + d^2)^(5/2)*d^4) + 56/6435*x*e^(-2)/((-x^2
*e^2 + d^2)^(3/2)*d^6) + 112/6435*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d^8)
```

Fricas [A]
time = 2.23, size = 292, normalized size = 1.40

$$\frac{200x^{10}e^{10}+800dx^9e^9+600d^2x^8e^8-1600d^3x^7e^7-2800d^4x^6e^6+2800d^6x^4e^4+1600d^7x^3e^3-600d^8x^2e^2-800d^9xe-200d^{10}-(112x^9e^9+448dx^8e^8+392d^2x^7e^7-672d^3x^6e^6-1358d^4x^5e^5-280d^5x^4e^4+945d^6x^3e^3+700d^7x^2e^2+800d^8xe+200d^9)\sqrt{-x^2e^2+d^2}}{6435\left(d^8x^{10}e^{13}+4d^9x^9e^{12}+3d^{10}x^8e^{11}-8d^{11}x^7e^{10}-14d^{12}x^6e^9+14d^{14}x^4e^7+8d^{15}x^3e^6-3d^{16}x^2e^5-4d^{17}xe^4-d^{18}e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/6435*(200*x^10*e^10 + 800*d*x^9*e^9 + 600*d^2*x^8*e^8 - 1600*d^3*x^7*e^7
- 2800*d^4*x^6*e^6 + 2800*d^6*x^4*e^4 + 1600*d^7*x^3*e^3 - 600*d^8*x^2*e^2
- 800*d^9*x*e - 200*d^10 - (112*x^9*e^9 + 448*d*x^8*e^8 + 392*d^2*x^7*e^7 -
 672*d^3*x^6*e^6 - 1358*d^4*x^5*e^5 - 280*d^5*x^4*e^4 + 945*d^6*x^3*e^3 + 7
00*d^7*x^2*e^2 + 800*d^8*x*e + 200*d^9)*sqrt(-x^2*e^2 + d^2))/(d^8*x^10*e^1
3 + 4*d^9*x^9*e^12 + 3*d^10*x^8*e^11 - 8*d^11*x^7*e^10 - 14*d^12*x^6*e^9 +
14*d^14*x^4*e^7 + 8*d^15*x^3*e^6 - 3*d^16*x^2*e^5 - 4*d^17*x*e^4 - d^18*e^3
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**2/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^4), x)
```

Mupad [B]
time = 3.19, size = 252, normalized size = 1.21

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{227}{6864\,d^3\,e^3}-\frac{353\,x}{17160\,d^4\,e^2}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{353}{41184\,d^5\,e^3}-\frac{56\,x}{6435\,d^6\,e^2}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}}{104\,d^2\,e^3\,{\left(d+e\,x\right)}^7}+\frac{\sqrt{d^2-e^2\,x^2}}{2288\,d^3\,e^3\,{\left(d+e\,x\right)}^6}+\frac{37\,\sqrt{d^2-e^2\,x^2}}{5148\,d^4\,e^3\,{\left(d+e\,x\right)}^5}+\frac{353\,\sqrt{d^2-e^2\,x^2}}{41184\,d^5\,e^3\,{\left(d+e\,x\right)}^4}+\frac{112\,x\,\sqrt{d^2-e^2\,x^2}}{6435\,d^8\,e^2\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(227/(6864*d^3*e^3) - (353*x)/(17160*d^4*e^2)))/((d
+ e*x)^3*(d - e*x)^3) - ((d^2 - e^2*x^2)^(1/2)*(353/(41184*d^5*e^3) - (56*x
)/(6435*d^6*e^2)))/((d + e*x)^2*(d - e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(104*d
^2*e^3*(d + e*x)^7) + (d^2 - e^2*x^2)^(1/2)/(2288*d^3*e^3*(d + e*x)^6) + (3
7*(d^2 - e^2*x^2)^(1/2))/(5148*d^4*e^3*(d + e*x)^5) + (353*(d^2 - e^2*x^2)^
(1/2))/(41184*d^5*e^3*(d + e*x)^4) + (112*x*(d^2 - e^2*x^2)^(1/2))/(6435*d^
8*e^2*(d + e*x)*(d - e*x))
```

3.214 $\int \frac{x}{(d+ex)^4(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=211

$$\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^2e^2(d+ex)^2\left(\ldots\right.}$$

```
[Out] 64/2145*x/d^5/e/(-e^2*x^2+d^2)^(5/2)+1/13/e^2/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2
)-4/143/d/e^2/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-32/1287/d^2/e^2/(e*x+d)^2/(-e^
2*x^2+d^2)^(5/2)-32/1287/d^3/e^2/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+256/6435*x/d^
7/e/(-e^2*x^2+d^2)^(3/2)+512/6435*x/d^9/e/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {807, 673, 198, 197}

$$-\frac{32}{1287d^2e^2(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}}+\frac{256x}{6435d^7e\left(d^2-e^2x^2\right)^{3/2}}+\frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{32}{1287d^3e^2(d+ex)\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (64*x)/(2145*d^5*e*(d^2 - e^2*x^2)^(5/2)) + 1/(13*e^2*(d + e*x)^4*(d^2 - e^
2*x^2)^(5/2)) - 4/(143*d*e^2*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*
d^2*e^2*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 32/(1287*d^3*e^2*(d + e*x)*(d^
2 - e^2*x^2)^(5/2)) + (256*x)/(6435*d^7*e*(d^2 - e^2*x^2)^(3/2)) + (512*x)/
(6435*d^9*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
```

```
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{4\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx}{13e}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{32\int \frac{1}{(d+ex)}}{1}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{}{1287d^2e^2(}$$

$$= \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{}{1287d^2e^2(}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\,(d}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\,(d}$$

$$= \frac{64x}{2145d^5e\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{13e^2(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{4}{143de^2(d+ex)^3\,(d}$$

Mathematica [A]
time = 0.58, size = 137, normalized size = 0.65

$$\frac{\sqrt{d^2-e^2x^2}\left(-5d^9-20d^8ex+3200d^7e^2x^2+4320d^6e^3x^3-1280d^5e^4x^4-6208d^4e^5x^5-3072d^3e^6x^6+1792d^2e^7x^7+2048de^8x^8+512e^9x^9\right)}{6435d^9e^2(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-5*d^9 - 20*d^8*e*x + 3200*d^7*e^2*x^2 + 4320*d^6*e^3
*x^3 - 1280*d^5*e^4*x^4 - 6208*d^4*e^5*x^5 - 3072*d^3*e^6*x^6 + 1792*d^2*e^
7*x^7 + 2048*d*e^8*x^8 + 512*e^9*x^9))/(6435*d^9*e^2*(d - e*x)^3*(d + e*x)^
7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(183) = 366$.

time = 0.07, size = 708, normalized size = 3.36

method	result
gosper	$-\frac{(-ex+d)(-512e^9x^9-2048d\,e^8x^8-1792e^7x^7d^2+3072e^6x^6d^3+6208e^5x^5d^4+1280x^4d^5e^4-4320d^6e^3x^3-3200x^2d^7e^2+20d^8xe+5d^9}{6435(ex+d)^3d^9e^2(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(-512e^9x^9-2048d\,e^8x^8-1792e^7x^7d^2+3072e^6x^6d^3+6208e^5x^5d^4+1280x^4d^5e^4-4320d^6e^3x^3-3200x^2d^7e^2+20d^8xe+5d^9)\sqrt{-e^2}}{6435d^9(ex+d)^7(-ex+d)^3e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e/d*(-
1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x
+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2
*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x
+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e
^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))-d/e^5*(-1/13/d/
e/(x+d/e)^4/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+9/13*e/d*(-1/11/d/e/(x+d/e
)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e/d*(-1/9/d/e/(x+d/e)^2/(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(173) = 346$.
time = 0.31, size = 371, normalized size = 1.76

$$\frac{1}{13\left((-x^2e^2+d^2)^{\frac{5}{2}}x^4e^6+4\,(-x^2e^2+d^2)^{\frac{5}{2}}dx^3e^5+6\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2x^2e^4+4\,(-x^2e^2+d^2)^{\frac{5}{2}}d^3xe^3+(-x^2e^2+d^2)^{\frac{5}{2}}d^4e^2\right)}-\frac{4}{143\left((-x^2e^2+d^2)^{\frac{5}{2}}dx^3e^5+3\,(-x^2e^2+d^2)^{\frac{5}{2}}d^2x^2e^4+3\,(-x^2e^2+d^2)^{\frac{5}{2}}d^3xe^3+(-x^2e^2+d^2)^{\frac{5}{2}}d^4e^2\right)}-\frac{32}{1287\left((-x^2e^2+d^2)^{\frac{5}{2}}d^2x^2e^4+2\,(-x^2e^2+d^2)^{\frac{5}{2}}d^3xe^3+(-x^2e^2+d^2)^{\frac{5}{2}}d^4e^2\right)}-\frac{32}{1287\left((-x^2e^2+d^2)^{\frac{5}{2}}d^3xe^3+(-x^2e^2+d^2)^{\frac{5}{2}}d^4e^2\right)}+\frac{64\,xe^{(-1)}}{2145\,(-x^2e^2+d^2)^{\frac{5}{2}}d^5}+\frac{256\,xe^{(-1)}}{6435\,(-x^2e^2+d^2)^{\frac{3}{2}}d^7}+\frac{512\,xe^{(-1)}}{6435\,\sqrt{-x^2e^2+d^2}\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13/((-x^2*e^2 + d^2)^(5/2)*x^4*e^6 + 4*(-x^2*e^2 + d^2)^(5/2)*d*x^3*e^5 +
 6*(-x^2*e^2 + d^2)^(5/2)*d^2*x^2*e^4 + 4*(-x^2*e^2 + d^2)^(5/2)*d^3*x*e^3
+ (-x^2*e^2 + d^2)^(5/2)*d^4*e^2) - 4/143/((-x^2*e^2 + d^2)^(5/2)*d*x^3*e^5
 + 3*(-x^2*e^2 + d^2)^(5/2)*d^2*x^2*e^4 + 3*(-x^2*e^2 + d^2)^(5/2)*d^3*x*e^
3 + (-x^2*e^2 + d^2)^(5/2)*d^4*e^2) - 32/1287/((-x^2*e^2 + d^2)^(5/2)*d^2*x
^2*e^4 + 2*(-x^2*e^2 + d^2)^(5/2)*d^3*x*e^3 + (-x^2*e^2 + d^2)^(5/2)*d^4*e^
2) - 32/1287/((-x^2*e^2 + d^2)^(5/2)*d^3*x*e^3 + (-x^2*e^2 + d^2)^(5/2)*d^4
*e^2) + 64/2145*x*e^(-1)/((-x^2*e^2 + d^2)^(5/2)*d^5) + 256/6435*x*e^(-1)/(
(-x^2*e^2 + d^2)^(3/2)*d^7) + 512/6435*x*e^(-1)/(sqrt(-x^2*e^2 + d^2)*d^9)
```

Fricas [A]
time = 2.86, size = 291, normalized size = 1.38

$$-\frac{5\,x^{10}e^{10}+20\,dx^9e^9+15\,d^2x^8e^8-40\,d^3x^7e^7-70\,d^4x^6e^6+70\,d^6x^4e^4+40\,d^7x^3e^3-15\,d^8x^2e^2-20\,d^9xe-5\,d^{10}+(512\,x^9e^9+2048\,dx^8e^8+1792\,d^2x^7e^7-3072\,d^3x^6e^6-6208\,d^4x^5e^5-1280\,d^5x^4e^4+4320\,d^6x^3e^3+3200\,d^7x^2e^2-20\,d^8xe-5\,d^9)\sqrt{-x^2e^2+d^2}}{6435\,(d^9x^{10}e^{12}+4\,d^{10}x^9e^{11}+3\,d^{11}x^8e^{10}-8\,d^{12}x^7e^9-14\,d^{13}x^6e^8+14\,d^{15}x^4e^6+8\,d^{16}x^3e^5-3\,d^{17}x^2e^4-4\,d^{18}xe^3-d^{19}e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/6435*(5*x^10*e^10 + 20*d*x^9*e^9 + 15*d^2*x^8*e^8 - 40*d^3*x^7*e^7 - 70*
d^4*x^6*e^6 + 70*d^6*x^4*e^4 + 40*d^7*x^3*e^3 - 15*d^8*x^2*e^2 - 20*d^9*x*e
 - 5*d^10 + (512*x^9*e^9 + 2048*d*x^8*e^8 + 1792*d^2*x^7*e^7 - 3072*d^3*x^6
*e^6 - 6208*d^4*x^5*e^5 - 1280*d^5*x^4*e^4 + 4320*d^6*x^3*e^3 + 3200*d^7*x^
2*e^2 - 20*d^8*x*e - 5*d^9)*sqrt(-x^2*e^2 + d^2))/(d^9*x^10*e^12 + 4*d^10*x
^9*e^11 + 3*d^11*x^8*e^10 - 8*d^12*x^7*e^9 - 14*d^13*x^6*e^8 + 14*d^15*x^4*
e^6 + 8*d^16*x^3*e^5 - 3*d^17*x^2*e^4 - 4*d^18*x*e^3 - d^19*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^4), x)
```

Mupad [B]
time = 3.19, size = 252, normalized size = 1.19

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{41}{41184\,d^6\,e^2}+\frac{256\,x}{6435\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{47}{1716\,d^4\,e^2}-\frac{1369\,x}{34320\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}+\frac{\sqrt{d^2-e^2\,x^2}}{104\,d^3\,e^2\,{\left(d+e\,x\right)}^7}+\frac{25\,\sqrt{d^2-e^2\,x^2}}{2288\,d^4\,e^2\,{\left(d+e\,x\right)}^6}+\frac{125\,\sqrt{d^2-e^2\,x^2}}{20592\,d^5\,e^2\,{\left(d+e\,x\right)}^5}-\frac{41\,\sqrt{d^2-e^2\,x^2}}{41184\,d^6\,e^2\,{\left(d+e\,x\right)}^4}+\frac{512\,x\,\sqrt{d^2-e^2\,x^2}}{6435\,d^9\,e\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(41/(41184*d^6*e^2) + (256*x)/(6435*d^7*e)))/((d + e
*x)^2*(d - e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(47/(1716*d^4*e^2) - (1369*x)/(
34320*d^5*e)))/((d + e*x)^3*(d - e*x)^3) + (d^2 - e^2*x^2)^(1/2)/(104*d^3*e
^2*(d + e*x)^7) + (25*(d^2 - e^2*x^2)^(1/2))/(2288*d^4*e^2*(d + e*x)^6) + (
125*(d^2 - e^2*x^2)^(1/2))/(20592*d^5*e^2*(d + e*x)^5) - (41*(d^2 - e^2*x^2
)^(1/2))/(41184*d^6*e^2*(d + e*x)^4) + (512*x*(d^2 - e^2*x^2)^(1/2))/(6435*
d^9*e*(d + e*x)*(d - e*x))
```

3.215 $\int \frac{1}{(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=205

$$\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d^2-\right.}$$

```
[Out] 48/715*x/d^6/(-e^2*x^2+d^2)^(5/2)-1/13/d/e/(e*x+d)^4/(-e^2*x^2+d^2)^(5/2)-9
/143/d^2/e/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2)-8/143/d^3/e/(e*x+d)^2/(-e^2*x^2+d
^2)^(5/2)-8/143/d^4/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+64/715*x/d^8/(-e^2*x^2+d
^2)^(3/2)+128/715*x/d^10/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {673, 198, 197}

$$-\frac{9}{143d^2e(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}-\frac{1}{13de(d+ex)^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}}+\frac{64x}{715d^8\left(d^2-e^2x^2\right)^{3/2}}+\frac{48x}{715d^6\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^4e(d+ex)\left(d^2-e^2x^2\right)^{5/2}}-\frac{8}{143d^3e(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (48*x)/(715*d^6*(d^2 - e^2*x^2)^(5/2)) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x
^2)^(5/2)) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^3*e
*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2
)^(5/2)) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^(3/2)) + (128*x)/(715*d^10*Sqrt[
d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
```

```
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}} \, dx = -\frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} + \frac{9\int \frac{1}{(d+ex)^3 \left(d^2-e^2x^2\right)^{7/2}} \, dx}{13d}$$

$$= -\frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} + \frac{72\int \frac{}{(d+}}{}$$

$$= -\frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{}{143d^3e(}$$

$$= -\frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{}{143d^3e(}$$

$$= \frac{48x}{715d^6 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 (d^2}$$

$$= \frac{48x}{715d^6 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 (d^2}$$

$$= \frac{48x}{715d^6 \left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{13de(d+ex)^4 \left(d^2-e^2x^2\right)^{5/2}} - \frac{9}{143d^2e(d+ex)^3 (d^2}$$

Mathematica [A]
time = 0.02, size = 137, normalized size = 0.67

$$\frac{\sqrt{d^2-e^2x^2} \left(-180d^9-5d^8ex+800d^7e^2x^2+1080d^6e^3x^3-320d^5e^4x^4-1552d^4e^5x^5-768d^3e^6x^6+448d^2e^7x^7+512de^8x^8+128e^9x^9\right)}{715d^{10}e(d-ex)^3(d+ex)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-180*d^9 - 5*d^8*e*x + 800*d^7*e^2*x^2 + 1080*d^6*e^3
*x^3 - 320*d^5*e^4*x^4 - 1552*d^4*e^5*x^5 - 768*d^3*e^6*x^6 + 448*d^2*e^7*x
^7 + 512*d*e^8*x^8 + 128*e^9*x^9))/(715*d^10*e*(d - e*x)^3*(d + e*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(177) = 354$.
time = 0.07, size = 379, normalized size = 1.85

method	result

gosper	$-\frac{(-ex+d)(-128e^9x^9-512d\,e^8x^8-448e^7x^7d^2+768e^6x^6d^3+1552e^5x^5d^4+320x^4d^5e^4-1080d^6e^3x^3-800x^2d^7e^2+5d^8xe+180d^9)}{715(ex+d)^3d^{10}e(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{(-128e^9x^9-512d\,e^8x^8-448e^7x^7d^2+768e^6x^6d^3+1552e^5x^5d^4+320x^4d^5e^4-1080d^6e^3x^3-800x^2d^7e^2+5d^8xe+180d^9)\sqrt{-e^2x^2}}{715d^{10}(ex+d)^7(-ex+d)^3e}$
default	$-\frac{1}{13de\left(x+\frac{d}{e}\right)^4\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{9e\left(-\frac{1}{11de\left(x+\frac{d}{e}\right)^3\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{8e\left(-\frac{1}{9de\left(x+\frac{d}{e}\right)^2\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right.}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(-1/13/d/e/(x+d/e)^4/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+9/13*e/d*(-
1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e/d*(-1/9/d/e/
(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^
2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*
d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/
e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(170) = 340$.
time = 0.32, size = 370, normalized size = 1.80

$$-\frac{1}{13\left((-x^2e^2+d^2)^{\frac{5}{2}}dx^4e^5+4(-x^2e^2+d^2)^{\frac{5}{2}}d^2x^3e^4+6(-x^2e^2+d^2)^{\frac{5}{2}}d^3x^2e^3+4(-x^2e^2+d^2)^{\frac{5}{2}}d^4xe^2+(-x^2e^2+d^2)^{\frac{5}{2}}d^5e\right)}-\frac{9}{143\left((-x^2e^2+d^2)^{\frac{5}{2}}d^2x^3e^4+3(-x^2e^2+d^2)^{\frac{5}{2}}d^3x^2e^3+3(-x^2e^2+d^2)^{\frac{5}{2}}d^4xe^2+(-x^2e^2+d^2)^{\frac{5}{2}}d^5e\right)}-\frac{8}{143\left((-x^2e^2+d^2)^{\frac{5}{2}}d^3x^2e^3+2(-x^2e^2+d^2)^{\frac{5}{2}}d^4xe^2+(-x^2e^2+d^2)^{\frac{5}{2}}d^5e\right)}-\frac{8}{143\left((-x^2e^2+d^2)^{\frac{5}{2}}d^4xe^2+(-x^2e^2+d^2)^{\frac{5}{2}}d^5e\right)}+\frac{48\,x}{715\,(-x^2e^2+d^2)^{\frac{5}{2}}d^6}+\frac{64\,x}{715\,(-x^2e^2+d^2)^{\frac{3}{2}}d^8}+\frac{128\,x}{715\,\sqrt{-x^2e^2+d^2}\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/13/((-x^2*e^2 + d^2)^(5/2)*d*x^4*e^5 + 4*(-x^2*e^2 + d^2)^(5/2)*d^2*x^3*
e^4 + 6*(-x^2*e^2 + d^2)^(5/2)*d^3*x^2*e^3 + 4*(-x^2*e^2 + d^2)^(5/2)*d^4*x
*e^2 + (-x^2*e^2 + d^2)^(5/2)*d^5*e) - 9/143/((-x^2*e^2 + d^2)^(5/2)*d^2*x^
3*e^4 + 3*(-x^2*e^2 + d^2)^(5/2)*d^3*x^2*e^3 + 3*(-x^2*e^2 + d^2)^(5/2)*d^4
*x*e^2 + (-x^2*e^2 + d^2)^(5/2)*d^5*e) - 8/143/((-x^2*e^2 + d^2)^(5/2)*d^3*
x^2*e^3 + 2*(-x^2*e^2 + d^2)^(5/2)*d^4*x*e^2 + (-x^2*e^2 + d^2)^(5/2)*d^5*e
) - 8/143/((-x^2*e^2 + d^2)^(5/2)*d^4*x*e^2 + (-x^2*e^2 + d^2)^(5/2)*d^5*e)
 + 48/715*x/((-x^2*e^2 + d^2)^(5/2)*d^6) + 64/715*x/((-x^2*e^2 + d^2)^(3/2)
*d^8) + 128/715*x/(sqrt(-x^2*e^2 + d^2)*d^10)
```

Fricas [**A**]
time = 5.29, size = 291, normalized size = 1.42

$$-\frac{180\,x^{10}e^{10}+720\,dx^9e^9+540\,d^2x^8e^8-1440\,d^3x^7e^7-2520\,d^4x^6e^6+2520\,d^6x^4e^4+1440\,d^7x^3e^3-540\,d^8x^2e^2-720\,d^9xe-180\,d^{10}+(128\,x^9e^9+512\,dx^8e^8+448\,d^2x^7e^7-768\,d^3x^6e^6-1552\,d^4x^5e^5-320\,d^5x^4e^4+1080\,d^6x^3e^3+800\,d^7x^2e^2-5\,d^8xe-180\,d^9)\sqrt{-x^2e^2+d^2}}{715\,(d^{10}x^{10}e^{11}+4\,d^{11}x^9e^{10}+3\,d^{12}x^8e^9-8\,d^{13}x^7e^8-14\,d^{14}x^6e^7+14\,d^{16}x^4e^5+8\,d^{17}x^3e^4-3\,d^{18}x^2e^3-4\,d^{19}xe^2-d^{20}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/715*(180*x^10*e^10 + 720*d*x^9*e^9 + 540*d^2*x^8*e^8 - 1440*d^3*x^7*e^7
- 2520*d^4*x^6*e^6 + 2520*d^6*x^4*e^4 + 1440*d^7*x^3*e^3 - 540*d^8*x^2*e^2
- 720*d^9*x*e - 180*d^10 + (128*x^9*e^9 + 512*d*x^8*e^8 + 448*d^2*x^7*e^7 -
 768*d^3*x^6*e^6 - 1552*d^4*x^5*e^5 - 320*d^5*x^4*e^4 + 1080*d^6*x^3*e^3 +
800*d^7*x^2*e^2 - 5*d^8*x*e - 180*d^9)*sqrt(-x^2*e^2 + d^2))/(d^10*x^10*e^1
1 + 4*d^11*x^9*e^10 + 3*d^12*x^8*e^9 - 8*d^13*x^7*e^8 - 14*d^14*x^6*e^7 + 1
4*d^16*x^4*e^5 + 8*d^17*x^3*e^4 - 3*d^18*x^2*e^3 - 4*d^19*x*e^2 - d^20*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate(1/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^4), x)
```

Mupad [B]
time = 3.12, size = 242, normalized size = 1.18

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{64\,x}{715\,d^8}+\frac{189}{4576\,d^7\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1139\,x}{5720\,d^6}-\frac{427}{2288\,d^5\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}}{104\,d^4\,e\,{\left(d+e\,x\right)}^7}-\frac{51\,\sqrt{d^2-e^2\,x^2}}{2288\,d^5\,e\,{\left(d+e\,x\right)}^6}-\frac{19\,\sqrt{d^2-e^2\,x^2}}{572\,d^6\,e\,{\left(d+e\,x\right)}^5}-\frac{189\,\sqrt{d^2-e^2\,x^2}}{4576\,d^7\,e\,{\left(d+e\,x\right)}^4}+\frac{128\,x\,\sqrt{d^2-e^2\,x^2}}{715\,d^{10}\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*((64*x)/(715*d^8) + 189/(4576*d^7*e)))/((d + e*x)^2*
(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*((1139*x)/(5720*d^6) - 427/(2288*d^5*
e)))/((d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(104*d^4*e*(d + e*x)
^7) - (51*(d^2 - e^2*x^2)^(1/2))/(2288*d^5*e*(d + e*x)^6) - (19*(d^2 - e^2*
x^2)^(1/2))/(572*d^6*e*(d + e*x)^5) - (189*(d^2 - e^2*x^2)^(1/2))/(4576*d^7
*e*(d + e*x)^4) + (128*x*(d^2 - e^2*x^2)^(1/2))/(715*d^10*(d + e*x)*(d - e*
x))
```

3.216 $\int \frac{1}{x(d+ex)^4 (d^2-e^2x^2)^{7/2}} \, dx$

Optimal. Leaf size=234

$$\frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} +$$

```
[Out] 8/13*d*(-e*x+d)/(-e^2*x^2+d^2)^(13/2)-4/13*e*x/d/(-e^2*x^2+d^2)^(11/2)+1/11
7*(-40*e*x+13*d)/d^3/(-e^2*x^2+d^2)^(9/2)+1/819*(-320*e*x+117*d)/d^5/(-e^2*
x^2+d^2)^(7/2)+1/1365*(-640*e*x+273*d)/d^7/(-e^2*x^2+d^2)^(5/2)+1/819*(-512
*e*x+273*d)/d^9/(-e^2*x^2+d^2)^(3/2)-arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^11+1
/819*(-1024*e*x+819*d)/d^11/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {866, 1819, 837, 12, 272, 65, 214}

$$-\frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}} + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*x)/(13*d*(d^2 - e^2*x^2)
^(11/2)) + (13*d - 40*e*x)/(117*d^3*(d^2 - e^2*x^2)^(9/2)) + (117*d - 320*e
*x)/(819*d^5*(d^2 - e^2*x^2)^(7/2)) + (273*d - 640*e*x)/(1365*d^7*(d^2 - e^
2*x^2)^(5/2)) + (273*d - 512*e*x)/(819*d^9*(d^2 - e^2*x^2)^(3/2)) + (819*d
- 1024*e*x)/(819*d^11*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
/d^11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(d+ex)^4 \left(d^2-e^2x^2\right)^{7/2}}\,dx &= \int \frac{(d-ex)^4}{x\left(d^2-e^2x^2\right)^{15/2}}\,dx \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+44d^3ex+13d^2e^2x^2}{x(d^2-e^2x^2)^{13/2}}\,dx}{13d^2} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-440d^3ex}{x(d^2-e^2x^2)^{11/2}}\,dx}{143d^4} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \cdots \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117\cdots}{819d^5\left(\cdots\right.} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117\cdots}{819d^5\left(\cdots\right.} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117\cdots}{819d^5\left(\cdots\right.} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117\cdots}{819d^5\left(\cdots\right.} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117\cdots}{819d^5\left(\cdots\right.} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117\cdots}{819d^5\left(\cdots\right.} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117\cdots}{819d^5\left(\cdots\right.} \\
&= \frac{8d(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4ex}{13d\left(d^2-e^2x^2\right)^{11/2}} + \frac{13d-40ex}{117d^3\left(d^2-e^2x^2\right)^{9/2}} + \frac{117\cdots}{819d^5\left(\cdots\right.}
\end{aligned}
$$

Mathematica [A]
time = 0.96, size = 173, normalized size = 0.74

$$
\frac{\frac{\sqrt{d^2-e^2x^2}\,(9839d^9+22976d^8ex-4466d^7e^2x^2-56304d^6e^3x^3-34156d^5e^4x^4+40240d^4e^5x^5+45735d^3e^6x^6-1540d^2e^7x^7-16385de^8x^8-5120e^9x^9)}{(d-ex)^3(d+ex)^7}+8190\tanh^{-1}\left(\frac{\sqrt{-e^2}\,x-\sqrt{d^2-e^2x^2}}{d}\right)}{4095d^{11}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(9839*d^9 + 22976*d^8*e*x - 4466*d^7*e^2*x^2 - 56304*
d^6*e^3*x^3 - 34156*d^5*e^4*x^4 + 40240*d^4*e^5*x^5 + 45735*d^3*e^6*x^6 - 1
540*d^2*e^7*x^7 - 16385*d*e^8*x^8 - 5120*e^9*x^9))/((d - e*x)^3*(d + e*x)^7
) + 8190*ArcTanh[(Sqrt[-e^2]*x - Sqrt[d^2 - e^2*x^2])/d])/(4095*d^11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1327 vs. $2(204) = 408$.
time = 0.11, size = 1328, normalized size = 5.68

method	result	size
default	Expression too large to display	1328

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^4*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/1
0*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d
^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2
)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
)-1/e^2/d^2*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*
e/d*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/
d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+
d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*
e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4
*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))-1/e^3/d*(
-1/13/d/e/(x+d/e)^4/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+9/13*e/d*(-1/11/d/
e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e/d*(-1/9/d/e/(x+d/e)
^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^
2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*
e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))-1/e/d^3*(-1/9/d/e/(x+d/e)^2/(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e
^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+
d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^
2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(
-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))+1/d^4*(1/5/d^2/(-e^2*x^2+d^2)^(5/2)
+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-1/d^
2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^4*x), x)
```

Fricas [A]
time = 5.62, size = 402, normalized size = 1.72

$$\frac{9839\,x^{10}e^{10} + 39356\,dx^{9}e^{9} + 29517\,d^{2}x^{8}e^{8} - 78712\,d^{3}x^{7}e^{7} - 137746\,d^{4}x^{6}e^{6} + 137746\,d^{6}x^{4}e^{4} + 78712\,d^{7}x^{3}e^{3} - 29517\,d^{8}x^{2}e^{2} - 39356\,d^{9}xe - 9839\,d^{10} + 4095\,(x^{10}e^{10} + 4\,dx^{9}e^{9} + 3\,d^{2}x^{8}e^{8} - 8\,d^{3}x^{7}e^{7} - 14\,d^{4}x^{6}e^{6} + 14\,d^{6}x^{4}e^{4} + 8\,d^{7}x^{3}e^{3} - 3\,d^{8}x^{2}e^{2} - 4\,d^{9}xe - d^{10})\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right) + (5120\,x^{9}e^{9} + 16385\,dx^{8}e^{8} + 1540\,d^{2}x^{7}e^{7} - 45735\,d^{3}x^{6}e^{6} - 40240\,d^{4}x^{5}e^{5} + 34156\,d^{5}x^{4}e^{4} + 56304\,d^{6}x^{3}e^{3} + 4466\,d^{7}x^{2}e^{2} - 22976\,d^{8}xe - 9839\,d^{9})\sqrt{-x^{2}e^{2}+d^{2}}}{4095\,(d^{11}x^{10}e^{10} + 4\,d^{12}x^{9}e^{9} + 3\,d^{13}x^{8}e^{8} - 8\,d^{14}x^{7}e^{7} - 14\,d^{15}x^{6}e^{6} + 14\,d^{17}x^{4}e^{4} + 8\,d^{18}x^{3}e^{3} - 3\,d^{19}x^{2}e^{2} - 4\,d^{20}xe - d^{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4095*(9839*x^10*e^10 + 39356*d*x^9*e^9 + 29517*d^2*x^8*e^8 - 78712*d^3*x^
7*e^7 - 137746*d^4*x^6*e^6 + 137746*d^6*x^4*e^4 + 78712*d^7*x^3*e^3 - 29517
*d^8*x^2*e^2 - 39356*d^9*x*e - 9839*d^10 + 4095*(x^10*e^10 + 4*d*x^9*e^9 +
3*d^2*x^8*e^8 - 8*d^3*x^7*e^7 - 14*d^4*x^6*e^6 + 14*d^6*x^4*e^4 + 8*d^7*x^3
*e^3 - 3*d^8*x^2*e^2 - 4*d^9*x*e - d^10)*log(-(d - sqrt(-x^2*e^2 + d^2))/x)
 + (5120*x^9*e^9 + 16385*d*x^8*e^8 + 1540*d^2*x^7*e^7 - 45735*d^3*x^6*e^6 -
 40240*d^4*x^5*e^5 + 34156*d^5*x^4*e^4 + 56304*d^6*x^3*e^3 + 4466*d^7*x^2*e
^2 - 22976*d^8*x*e - 9839*d^9)*sqrt(-x^2*e^2 + d^2))/(d^11*x^10*e^10 + 4*d^
12*x^9*e^9 + 3*d^13*x^8*e^8 - 8*d^14*x^7*e^7 - 14*d^15*x^6*e^6 + 14*d^17*x^
4*e^4 + 8*d^18*x^3*e^3 - 3*d^19*x^2*e^2 - 4*d^20*x*e - d^21)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(1/(x*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^4*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,(d^2-e^2\,x^2)^{7/2}\,(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
[Out] int(1/(x*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

3.217 $\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}-\frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}}-\frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}}-\frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}}-\frac{e(12012d-232}{15015d^8\left(d^2-e^2\right.}$$

```
[Out] -8/13*e*(-e*x+d)/(-e^2*x^2+d^2)^(13/2)-4/143*e*(-24*e*x+13*d)/d^2/(-e^2*x^2
+d^2)^(11/2)-1/1287*e*(-1103*e*x+572*d)/d^4/(-e^2*x^2+d^2)^(9/2)-1/9009*e*(
-10111*e*x+5148*d)/d^6/(-e^2*x^2+d^2)^(7/2)-1/15015*e*(-23225*e*x+12012*d)/
d^8/(-e^2*x^2+d^2)^(5/2)-1/9009*e*(-21583*e*x+12012*d)/d^10/(-e^2*x^2+d^2)^
(3/2)+4*e*arctanh((-e^2*x^2+d^2)^(1/2)/d)/d^12-1/9009*e*(-52175*e*x+36036*d
)/d^12/(-e^2*x^2+d^2)^(1/2)-(-e^2*x^2+d^2)^(1/2)/d^12/x
```

Rubi [A]

time = 0.43, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {866, 1819, 821, 272, 65, 214}

$$-\frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}}-\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}}-\frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{d^{12}x}+\frac{4e\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}-\frac{e(12012d-21583ex)}{9009d^{10}\left(d^2-e^2x^2\right)^{3/2}}-\frac{e(12012d-23225ex)}{15015d^8\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5148d-10111ex)}{9009d^6\left(d^2-e^2x^2\right)^{7/2}}-\frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*(13*d - 24*e*x))/(143*d
^2*(d^2 - e^2*x^2)^(11/2)) - (e*(572*d - 1103*e*x))/(1287*d^4*(d^2 - e^2*x^
2)^(9/2)) - (e*(5148*d - 10111*e*x))/(9009*d^6*(d^2 - e^2*x^2)^(7/2)) - (e*
(12012*d - 23225*e*x))/(15015*d^8*(d^2 - e^2*x^2)^(5/2)) - (e*(12012*d - 21
583*e*x))/(9009*d^10*(d^2 - e^2*x^2)^(3/2)) - (e*(36036*d - 52175*e*x))/(90
09*d^12*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(d^12*x) + (4*e*ArcTanh[
Sqrt[d^2 - e^2*x^2]/d])/d^12
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^4\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(d-ex)^4}{x^2\left(d^2-e^2x^2\right)^{15/2}}\,dx$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{\int \frac{-13d^4+52d^3ex-83d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{13/2}}\,dx}{13d^2}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} + \frac{\int \frac{143d^4-572d^3ex+960d^2e^2x^2}{x^2\left(d^2-e^2x^2\right)^{11/2}}\,dx}{143d^4}$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

$$= -\frac{8e(d-ex)}{13\left(d^2-e^2x^2\right)^{13/2}} - \frac{4e(13d-24ex)}{143d^2\left(d^2-e^2x^2\right)^{11/2}} - \frac{e(572d-1103ex)}{1287d^4\left(d^2-e^2x^2\right)^{9/2}} - \cdots$$

Mathematica [A]
time = 0.93, size = 193, normalized size = 0.71

$$\frac{\sqrt{d^2-e^2x^2}\left(45045d^{10}+546316d^9ex+1014094d^8e^2x^2-700504d^7e^3x^3-3157776d^6e^4x^4-1301264d^5e^5x^5+2748320d^4e^6x^6+2496180d^3e^7x^7-350000d^2e^8x^8-1043500de^9x^9-305920e^{10}x^{10}\right)}{45045d^{12}x(-d+ex)^3(d+ex)^7} - \frac{8e\tanh^{-1}\left(\frac{\sqrt{-e^2}x}{d}-\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^4*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(45045*d^10 + 546316*d^9*e*x + 1014094*d^8*e^2*x^2 - 7
00504*d^7*e^3*x^3 - 3157776*d^6*e^4*x^4 - 1301264*d^5*e^5*x^5 + 2748320*d^4
*e^6*x^6 + 2496180*d^3*e^7*x^7 - 350000*d^2*e^8*x^8 - 1043500*d*e^9*x^9 - 3
05920*e^10*x^10))/(45045*d^12*x*(-d + e*x)^3*(d + e*x)^7) - (8*e*ArcTanh[(S
qrt[-e^2]*x)/d - Sqrt[d^2 - e^2*x^2]/d])/d^12
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1428 vs. $2(239) = 478$.
time = 0.16, size = 1429, normalized size = 5.27

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{d^{12}x} - \frac{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)}}{104e^6d^6\left(x+\frac{d}{e}\right)^7} - \frac{103\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)}}{2288e^5d^7\left(x+\frac{d}{e}\right)^6} - \frac{665\sqrt{}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*e/d^5*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/
10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/
d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/
2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
))+2/e/d^3*(-1/11/d/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e
/d*(-1/9/d/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d
/e/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d
/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e
^2*(x+d/e)+2*d*e)/d^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*
(-2*e^2*(x+d/e)+2*d*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))))+1/e^2/d^2*
(-1/13/d/e/(x+d/e)^4/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+9/13*e/d*(-1/11/d
/e/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+8/11*e/d*(-1/9/d/e/(x+d/e
)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d
^2/e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d
*e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))))+1/d^4*(-1/d^2/x/(-e^2*x^2+d^
2)^(5/2)+6*e^2/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2
*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+3/d^4*(-1/9/d/e/(x+d/e)^2
/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+7/9*e/d*(-1/7/d/e/(x+d/e)/(-(x+d/e)^2
*e^2+2*d*e*(x+d/e))^(5/2)+6/7*e/d*(-1/10*(-2*e^2*(x+d/e)+2*d*e)/d^2/e^2/(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+4/5/d^2*(-1/6*(-2*e^2*(x+d/e)+2*d*e)/d^2/
e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/3/e^2/d^4*(-2*e^2*(x+d/e)+2*d*e)
/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))))-4/d^5*e*(1/5/d^2/(-e^2*x^2+d^2)^(
```

```
5/2)+1/d^2*(1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^2*(1/d^2/(-e^2*x^2+d^2)^(1/2)-
1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate(1/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^4*x^2), x)
```

Fricas [A]
time = 3.67, size = 425, normalized size = 1.57

$$-\frac{366136\,x^{11}e^{11}+1464544\,dx^{10}e^{10}+1098408\,d^{2}x^{9}e^{9}-2929088\,d^{3}x^{8}e^{8}-5125904\,d^{4}x^{7}e^{7}+5125904\,d^{6}x^{5}e^{5}+2929088\,d^{7}x^{4}e^{4}-1098408\,d^{8}x^{3}e^{3}-1464544\,d^{9}x^{2}e^{2}-366136\,d^{10}xe+180180\,(x^{11}e^{11}+4\,dx^{10}e^{10}+3\,d^{2}x^{9}e^{9}-8\,d^{3}x^{8}e^{8}-14\,d^{4}x^{7}e^{7}+14\,d^{6}x^{5}e^{5}+8\,d^{7}x^{4}e^{4}-3\,d^{8}x^{3}e^{3}-4\,d^{9}x^{2}e^{2}-d^{10}xe)\log\left(-\frac{d-\sqrt{-x^{2}e^{2}+d^{2}}}{x}\right)+(305920\,x^{10}e^{10}+1043500\,dx^{9}e^{9}+350000\,d^{2}x^{8}e^{8}-2496180\,d^{3}x^{7}e^{7}-2748320\,d^{4}x^{6}e^{6}+1301264\,d^{5}x^{5}e^{5}+3157776\,d^{6}x^{4}e^{4}+700504\,d^{7}x^{3}e^{3}-1014094\,d^{8}x^{2}e^{2}-546316\,d^{9}xe-45045\,d^{10})\sqrt{-x^{2}e^{2}+d^{2}}}{45045\,(d^{12}x^{11}e^{10}+4\,d^{13}x^{10}e^{9}+3\,d^{14}x^{9}e^{8}-8\,d^{15}x^{8}e^{7}-14\,d^{16}x^{7}e^{6}+14\,d^{18}x^{5}e^{4}+8\,d^{19}x^{4}e^{3}-3\,d^{20}x^{3}e^{2}-4\,d^{21}x^{2}e-d^{22}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
[Out] -1/45045*(366136*x^11*e^11 + 1464544*d*x^10*e^10 + 1098408*d^2*x^9*e^9 - 29
29088*d^3*x^8*e^8 - 5125904*d^4*x^7*e^7 + 5125904*d^6*x^5*e^5 + 2929088*d^7
*x^4*e^4 - 1098408*d^8*x^3*e^3 - 1464544*d^9*x^2*e^2 - 366136*d^10*x*e + 18
0180*(x^11*e^11 + 4*d*x^10*e^10 + 3*d^2*x^9*e^9 - 8*d^3*x^8*e^8 - 14*d^4*x^
7*e^7 + 14*d^6*x^5*e^5 + 8*d^7*x^4*e^4 - 3*d^8*x^3*e^3 - 4*d^9*x^2*e^2 - d^
10*x*e)*log(-(d - sqrt(-x^2*e^2 + d^2))/x) + (305920*x^10*e^10 + 1043500*d*
x^9*e^9 + 350000*d^2*x^8*e^8 - 2496180*d^3*x^7*e^7 - 2748320*d^4*x^6*e^6 +
1301264*d^5*x^5*e^5 + 3157776*d^6*x^4*e^4 + 700504*d^7*x^3*e^3 - 1014094*d^
8*x^2*e^2 - 546316*d^9*x*e - 45045*d^10)*sqrt(-x^2*e^2 + d^2))/(d^12*x^11*e
^10 + 4*d^13*x^10*e^9 + 3*d^14*x^9*e^8 - 8*d^15*x^8*e^7 - 14*d^16*x^7*e^6 +
 14*d^18*x^5*e^4 + 8*d^19*x^4*e^3 - 3*d^20*x^3*e^2 - 4*d^21*x^2*e - d^22*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^4*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(d^2-e^2\,x^2\right)}^{7/2}\,{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4),x)
```

```
[Out] int(1/(x^2*(d^2 - e^2*x^2)^(7/2)*(d + e*x)^4), x)
```

3.218 $\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx$

Optimal. Leaf size=102

$$-\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -c^2*(-a^2*x^2+1)^(3/2)/x/(-a*c*x+c)^(3/2)+a*arctanh(c^(1/2)*(-a^2*x^2+1)^(
1/2)/(-a*c*x+c)^(1/2))*c^(1/2)-a*c*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {893, 879, 889, 214}

$$-\frac{c^2\left(1-a^2x^2\right)^{3/2}}{x(c-acx)^{3/2}}-\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] -((a*c*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]) - (c^2*(1 - a^2*x^2)^(3/2))/(x*(
c - a*c*x)^(3/2)) + a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c -
a*c*x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] && !IGtQ[n, 0] && !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2}\,dx &= -\frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}(ac)\int \frac{\sqrt{1-a^2x^2}}{x\sqrt{c-acx}}\,dx \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - \frac{1}{2}a\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - (a^3c^2)\,\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x,\right. \\
&= -\frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} - \frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} + a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.30, size = 81, normalized size = 0.79

$$\frac{\sqrt{c-acx}\left((1+2ax)\sqrt{1-a^2x^2}+ax\sqrt{-1+ax}\,\tan^{-1}\left(\frac{\sqrt{-1+ax}}{\sqrt{1-a^2x^2}}\right)\right)}{x(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - a*c*x]*Sqrt[1 - a^2*x^2])/x^2,x]
```

```
[Out] (Sqrt[c - a*c*x]*((1 + 2*a*x)*Sqrt[1 - a^2*x^2] + a*x*Sqrt[-1 + a*x]*ArcTan
[Sqrt[-1 + a*x]/Sqrt[1 - a^2*x^2]]))/(x*(-1 + a*x))
```

Maple [A]
time = 0.09, size = 95, normalized size = 0.93

method	result

default	$\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)acx+2ax\sqrt{c(ax+1)}\,\sqrt{c}+\sqrt{c(ax+1)}\,\sqrt{c}\right)\sqrt{-c(ax-1)}\,\sqrt{-a^2x^2+1}}{(ax-1)\sqrt{c(ax+1)}\,x\sqrt{c}}$
risch	$\frac{(2a^2x^2+3ax+1)\sqrt{-\frac{(-a^2x^2+1)c}{ax-1}}\,(ax-1)c}{x\sqrt{c(ax+1)}\,\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}}-\frac{a\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{acx+c}}{\sqrt{c}}\right)\sqrt{-\frac{(-a^2x^2+1)c}{ax-1}}\,(ax-1)}{\sqrt{-a^2x^2+1}\,\sqrt{-c(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-arctanh((c*(a*x+1))^(1/2)/c^(1/2))*a*c*x+2*a*x*(c*(a*x+1))^(1/2)*c^(1/2)+
(c*(a*x+1))^(1/2)*c^(1/2))*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c
*(a*x+1))^(1/2)/x/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)/x^2, x)
```

Fricas [A]
time = 2.11, size = 217, normalized size = 2.13

$$\left[\frac{(a^2x^2-ax)\sqrt{c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-2c}{ax^2-x}\right)+2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(2ax+1)}{2(ax^2-x)},\ \frac{(a^2x^2-ax)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,(2ax+1)}{ax^2-x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a^2*x^2 + 1
)*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x), ((a^2*x^2 - a*x)*sqrt(-c)*arctan(sqrt
(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*x^2 +
 1)*sqrt(-a*c*x + c)*(2*a*x + 1))/(a*x^2 - x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(ax-1)}\,\sqrt{-(ax-1)(ax+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))*sqrt(-(a*x - 1)*(a*x + 1))/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-a^2\,x^2}\,\sqrt{c-a\,c\,x}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2,x)
```

```
[Out] int(((1 - a^2*x^2)^(1/2)*(c - a*c*x)^(1/2))/x^2, x)
```

3.219 $\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=39

$$-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

```
[Out] -2*arctanh(c^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*c*x+c)^(1/2))*c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {889, 214}

$$-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - a^2*x^2])/Sqrt[c - a*c*x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c-acx}}{x\sqrt{1-a^2x^2}}\,dx &= \left(2a^2c^2\right)\text{Subst}\left(\int \frac{1}{-a^2c+a^2c^2x^2}\,dx, x, \frac{\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\\ &= -2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)\end{aligned}$$

Mathematica [A]
time = 0.22, size = 47, normalized size = 1.21

$$-\frac{2\sqrt{c-acx}\tan^{-1}\left(\frac{\sqrt{-1+ax}}{\sqrt{1-a^2x^2}}\right)}{\sqrt{-1+ax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(x*Sqrt[1 - a^2*x^2]),x]
[Out] (-2*Sqrt[c - a*c*x]*ArcTan[Sqrt[-1 + a*x]/Sqrt[1 - a^2*x^2]])/Sqrt[-1 + a*x
]
```

Maple [A]
time = 0.07, size = 58, normalized size = 1.49

method	result	size
default	$\frac{2\sqrt{-c(ax-1)}\,\sqrt{-a^2x^2+1}\,\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c(ax+1)}}{\sqrt{c}}\right)}{(ax-1)\sqrt{c(ax+1)}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(-c*(a*x-1))^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x-1)/(c*(a*x+1))^(1/2)*c^(1/2)*a
rctanh((c*(a*x+1))^(1/2)/c^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [A]
time = 2.39, size = 110, normalized size = 2.82

$$\left[\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{c}-2c}{ax^2-x}\right), -2\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\,\sqrt{-acx+c}\,\sqrt{-c}}{a^2cx^2-c}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a^2*x^2 + 1)*sqrt(-a*c*x + c)*sq
rt(c) - 2*c)/(a*x^2 - x)), -2*sqrt(-c)*arctan(sqrt(-a^2*x^2 + 1)*sqrt(-a*c*
x + c)*sqrt(-c)/(a^2*c*x^2 - c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [A]
time = 1.38, size = 57, normalized size = 1.46

$$-\frac{2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{2}\,\sqrt{c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}-\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}\right)}{|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*c^3*(arctan(sqrt(2)*sqrt(c)/sqrt(-c))/(sqrt(-c)*c) - arctan(sqrt(a*c*x +
 c)/sqrt(-c))/(sqrt(-c)*c))/abs(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x*(1 - a^2*x^2)^(1/2)), x)
```

3.220 $\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsin(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(-a*x+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{\text{ArcSin}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}+\sqrt{x}\sqrt{1-ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{1-ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\,\sqrt{1-ax}}\,dx$$
$$= \sqrt{x}\,\sqrt{1-ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\,\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 1.49

$$\sqrt{x}\,\sqrt{1-ax} + \frac{a\log\left(-\sqrt{-a}\,\sqrt{x} + \sqrt{1-ax}\right)}{(-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 - a*x] + (a*Log[-(Sqrt[-a]*Sqrt[x]) + Sqrt[1 - a*x]])/(-a)^(
3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(25) = 50.
time = 0.09, size = 62, normalized size = 1.77

method	result	size
meijerg	$\frac{\sqrt{-a}\left(-2\sqrt{\pi}\,\sqrt{x}\,\sqrt{-a}\,\sqrt{-ax+1}-\frac{2\sqrt{\pi}\,\sqrt{-a}\,\arcsin\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}\right)}{2\sqrt{\pi}\,a}$	57
default	$\sqrt{x}\,\sqrt{-ax+1}+\frac{\sqrt{(-ax+1)\,x}\,\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^2+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{x}\,\sqrt{a}}$	62
risch	$-\frac{\sqrt{x}\,(ax-1)\sqrt{(-ax+1)\,x}}{\sqrt{-x\,(ax-1)}\,\sqrt{-ax+1}}+\frac{\sqrt{(-ax+1)\,x}\,\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^2+x}}\right)}{2\sqrt{-ax+1}\,\sqrt{x}\,\sqrt{a}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(-a*x+1)^(1/2)+1/2*((-a*x+1)*x)^(1/2)/(-a*x+1)^(1/2)/x^(1/2)/a^(1/2
)*arctan(a^(1/2)*(x-1/2/a)/(-a*x^2+x)^(1/2))
```

Maxima [A]
time = 0.51, size = 48, normalized size = 1.37

$$-\frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{a}}+\frac{\sqrt{-ax+1}}{\left(a-\frac{ax-1}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))/sqrt(a) + sqrt(-a*x + 1)/((a - (a
*x - 1)/x)*sqrt(x))
```

Fricas [A]
time = 2.17, size = 92, normalized size = 2.63

$$\left[\frac{2\sqrt{-ax+1}\,a\sqrt{x}-\sqrt{-a}\log\left(-2\,ax+2\sqrt{-ax+1}\sqrt{-a}\sqrt{x}+1\right)}{2\,a},\ \frac{\sqrt{-ax+1}\,a\sqrt{x}-\sqrt{a}\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-a*x + 1)*a*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt(-a*x + 1)*s
qrt(-a)*sqrt(x) + 1))/a, (sqrt(-a*x + 1)*a*sqrt(x) - sqrt(a)*arctan(sqrt(-a
*x + 1)/(sqrt(a)*sqrt(x))))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.79, size = 82, normalized size = 2.34

$$\begin{cases} i\sqrt{x}\sqrt{ax-1}-\frac{i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{for } |ax|>1 \\ -\frac{ax^{\frac{3}{2}}}{\sqrt{-ax+1}}+\frac{\sqrt{x}}{\sqrt{-ax+1}}+\frac{\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(x)*sqrt(a*x - 1) - I*acosh(sqrt(a)*sqrt(x))/sqrt(a), Abs(
a*x) > 1), (-a*x**(3/2)/sqrt(-a*x + 1) + sqrt(x)/sqrt(-a*x + 1) + asin(sqrt
(a)*sqrt(x))/sqrt(a), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16
```

Mupad [B]
time = 2.99, size = 38, normalized size = 1.09

$$\sqrt{x}\,\sqrt{1-a\,x}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{1-a\,x}-1}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(1 - a*x)^(1/2) + (2*atan((a^(1/2)*x^(1/2))/((1 - a*x)^(1/2) - 1)))
/a^(1/2)
```

3.221 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\,\sqrt{1+ax}}\,dx$

Optimal. Leaf size=35

$$\sqrt{x}\,\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}$$

[Out] arcsin(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(-a*x+1)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.138$, Rules used = {862, 52, 56, 222}

$$\frac{\text{ArcSin}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}+\sqrt{x}\,\sqrt{1-ax}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]

[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}}\,dx &= \int \frac{\sqrt{1-ax}}{\sqrt{x}}\,dx\\ &= \sqrt{x}\sqrt{1-ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx\\ &= \sqrt{x}\sqrt{1-ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 1.51, size = 35, normalized size = 1.00

$$\sqrt{x}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 + a*x]),x]
[Out] Sqrt[x]*Sqrt[1 - a*x] + ArcSin[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(25) = 50$.
time = 0.07, size = 76, normalized size = 2.17

method	result	size
default	$\frac{\sqrt{-a^2x^2+1}\,\sqrt{x}\left(2\sqrt{a}\,\sqrt{-x\left(ax-1\right)}+\arctan\left(\frac{2ax-1}{2\sqrt{a}\,\sqrt{-x\left(ax-1\right)}}\right)\right)}{2\sqrt{ax+1}\,\sqrt{-x\left(ax-1\right)}\,\sqrt{a}}$	76
risch	$-\frac{\sqrt{x}\,(ax-1)\sqrt{\frac{x(-a^2x^2+1)}{ax+1}}\,\sqrt{ax+1}}{\sqrt{-x\left(ax-1\right)}\,\sqrt{-a^2x^2+1}}+\frac{\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^2+x}}\right)\sqrt{\frac{x(-a^2x^2+1)}{ax+1}}\,\sqrt{ax+1}}{2\sqrt{a}\,\sqrt{x}\,\sqrt{-a^2x^2+1}}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)/(a*x+1)^(1/2)*(2*a^(1/2)*(-x*(a*x-1))^(1/2)+
arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/(-x*(a*x-1))^(1/2)/a^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(25) = 50.
time = 2.71, size = 199, normalized size = 5.69

$$\left[\frac{4\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{ax+1}\,a\sqrt{x}-(ax+1)\sqrt{-a}\,\log\left(-\frac{8\,a^{3}x^{3}-4\,\sqrt{-a^{2}x^{2}+1}\,(2\,ax-1)\sqrt{ax+1}\,\sqrt{-a}\,\sqrt{x}-7\,ax+1}{ax+1}\right)}{4\,(a^{2}x+a)}, \frac{2\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{ax+1}\,a\sqrt{x}-(ax+1)\sqrt{a}\,\arctan\left(\frac{2\,\sqrt{-a^{2}x^{2}+1}\,\sqrt{ax+1}\,\sqrt{a}\,\sqrt{x}}{2\,a^{2}x^{2}+ax-1}\right)}{2\,(a^{2}x+a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(-a)*log
(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*sqrt(-a)*sqrt
(x) - 7*a*x + 1)/(a*x + 1)))/(a^2*x + a), 1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(a*
x + 1)*a*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(a*x +
 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^2*x + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\,\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(a*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{x}\,\sqrt{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(a*x + 1)^(1/2)), x)
```

3.222 $\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx$

Optimal. Leaf size=34

$$\sqrt{x}\,\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsinh(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(a*x+1)^(1/2)
```

Rubi [**A**]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\sqrt{x}\,\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{1+ax} + \frac{1}{2}\int \frac{1}{\sqrt{x}\,\sqrt{1+ax}}\,dx$$
$$= \sqrt{x}\,\sqrt{1+ax} + \text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\,\sqrt{1+ax} + \frac{\sinh^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.35

$$\sqrt{x}\,\sqrt{1+ax} - \frac{\log\left(-\sqrt{a}\,\sqrt{x} + \sqrt{1+ax}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] - Log[-(Sqrt[a]*Sqrt[x]) + Sqrt[1 + a*x]]/Sqrt[a]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.
time = 0.08, size = 57, normalized size = 1.68

method	result	size
meijerg	$-\frac{-2\sqrt{\pi}\,\sqrt{a}\,\sqrt{x}\,\sqrt{ax+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{a}\,\sqrt{x}\right)}{2\sqrt{a}\,\sqrt{\pi}}$	41
default	$\sqrt{x}\,\sqrt{ax+1}+\frac{\sqrt{(ax+1)\,x}\,\ln\left(\frac{\frac{1}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+x}\right)}{2\sqrt{ax+1}\,\sqrt{x}\,\sqrt{a}}$	57
risch	$\sqrt{x}\,\sqrt{ax+1}+\frac{\sqrt{(ax+1)\,x}\,\ln\left(\frac{\frac{1}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+x}\right)}{2\sqrt{ax+1}\,\sqrt{x}\,\sqrt{a}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(a*x+1)^(1/2)+1/2*((a*x+1)*x)^(1/2)/(a*x+1)^(1/2)/x^(1/2)*ln((1/2+a
*x)/a^(1/2)+(a*x^2+x)^(1/2))/a^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

time = 0.49, size = 68, normalized size = 2.00

$$-\frac{\log\left(-\frac{\sqrt{a}-\frac{\sqrt{ax+1}}{\sqrt{x}}}{\sqrt{a}+\frac{\sqrt{ax+1}}{\sqrt{x}}}\right)}{2\sqrt{a}}-\frac{\sqrt{ax+1}}{\left(a-\frac{ax+1}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(-(sqrt(a) - sqrt(a*x + 1)/sqrt(x))/(sqrt(a) + sqrt(a*x + 1)/sqrt(x
)))/sqrt(a) - sqrt(a*x + 1)/((a - (a*x + 1)/x)*sqrt(x))
```

Fricas [A]
time = 3.08, size = 90, normalized size = 2.65

$$\left[\frac{2\sqrt{ax+1}\,a\sqrt{x}+\sqrt{a}\log\left(2\,ax+2\sqrt{ax+1}\sqrt{a}\sqrt{x}+1\right)}{2\,a},\ \frac{\sqrt{ax+1}\,a\sqrt{x}-\sqrt{-a}\arctan\left(\frac{\sqrt{ax+1}\sqrt{-a}}{a\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(a*x + 1)*a*sqrt(x) + sqrt(a)*log(2*a*x + 2*sqrt(a*x + 1)*sqrt(
a)*sqrt(x) + 1))/a, (sqrt(a*x + 1)*a*sqrt(x) - sqrt(-a)*arctan(sqrt(a*x + 1
)*sqrt(-a)/(a*sqrt(x))))/a]
```

Sympy [A]
time = 0.80, size = 29, normalized size = 0.85

$$\sqrt{x}\sqrt{ax+1}+\frac{\operatorname{asinh}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(a*x + 1) + asinh(sqrt(a)*sqrt(x))/sqrt(a)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{16
```

Mupad [B]
time = 3.00, size = 36, normalized size = 1.06

$$\sqrt{x}\ \sqrt{a\,x+1}+\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a}\ \sqrt{x}}{\sqrt{a\,x+1}-1}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a*x + 1)^(1/2) + (2*atanh((a^(1/2)*x^(1/2))/((a*x + 1)^(1/2) - 1))
)/a^(1/2)
```

3.223 $\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=34

$$\sqrt{x}\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

```
[Out] arcsinh(a^(1/2)*x^(1/2))/a^(1/2)+x^(1/2)*(a*x+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {862, 52, 56, 221}

$$\sqrt{x}\sqrt{ax+1}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}}\,dx &= \int \frac{\sqrt{1+ax}}{\sqrt{x}}\,dx \\
&= \sqrt{x}\sqrt{1+ax}+\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+ax}}\,dx \\
&= \sqrt{x}\sqrt{1+ax}+\text{Subst}\left(\int \frac{1}{\sqrt{1+ax^2}}\,dx,x,\sqrt{x}\right) \\
&= \sqrt{x}\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [**A**]
time = 1.55, size = 34, normalized size = 1.00

$$
\sqrt{x}\sqrt{1+ax}+\frac{\sinh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/(Sqrt[x]*Sqrt[1 - a*x]),x]
```

```
[Out] Sqrt[x]*Sqrt[1 + a*x] + ArcSinh[Sqrt[a]*Sqrt[x]]/Sqrt[a]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(24) = 48.
time = 0.08, size = 86, normalized size = 2.53

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{x}\,\sqrt{-ax+1}\,\left(2\sqrt{(ax+1)\,x}\,\sqrt{a}+\ln\left(\frac{2\sqrt{(ax+1)\,x}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{(ax+1)\,x}\,\sqrt{a}}$	86
risch	$-\frac{(ax+1)\sqrt{x}\,\sqrt{\frac{(-a^2x^2+1)x(-ax+1)}{(ax-1)^2}}\,(ax-1)}{\sqrt{(ax+1)\,x}\,\sqrt{-a^2x^2+1}\,\sqrt{-ax+1}}-\frac{\ln\left(\frac{\frac{1}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+x}\right)\sqrt{\frac{(-a^2x^2+1)x(-ax+1)}{(ax-1)^2}}\,(ax-1)}{2\sqrt{a}\,\sqrt{-a^2x^2+1}\,\sqrt{x}\,\sqrt{-ax+1}}$	153

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/2*(-a^2*x^2+1)^(1/2)*x^(1/2)*(-a*x+1)^(1/2)*(2*((a*x+1)*x)^(1/2)*a^(1/2)
+ln(1/2*(2*((a*x+1)*x)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/((a*x+1)*x)
^(1/2)/a^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(24) = 48.
time = 2.57, size = 208, normalized size = 6.12

$$\left[-\frac{4 \sqrt{-a^2x^2+1}\sqrt{-ax+1}\, a\sqrt{x} - (ax-1)\sqrt{a}\log\left(-\frac{8a^3x^3-4\sqrt{-a^2x^2+1}\,(2ax+1)\sqrt{-ax+1}\sqrt{a}\sqrt{x}-7ax-1}{ax-1}\right)}{4(a^2x-a)}, -\frac{2\sqrt{-a^2x^2+1}\sqrt{-ax+1}\, a\sqrt{x} - (ax-1)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a^2x^2+1}\sqrt{-ax+1}\sqrt{-a}\sqrt{x}}{2a^2x^2-ax-1}\right)}{2(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a^2*x^2 + 1)*sqrt(-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(a)*lo
g(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x + 1)*sqrt(-a*x + 1)*sqrt(a)*sqr
t(x) - 7*a*x - 1)/(a*x - 1)))/(a^2*x - a), -1/2*(2*sqrt(-a^2*x^2 + 1)*sqrt(
-a*x + 1)*a*sqrt(x) - (a*x - 1)*sqrt(-a)*arctan(2*sqrt(-a^2*x^2 + 1)*sqrt(-
a*x + 1)*sqrt(-a)*sqrt(x)/(2*a^2*x^2 - a*x - 1)))/(a^2*x - a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{-ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(-a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{16
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-a^2\,x^2}}{\sqrt{x}\,\sqrt{1-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] int((1 - a^2*x^2)^(1/2)/(x^(1/2)*(1 - a*x)^(1/2)), x)
```

3.224 $\int \sqrt{x}\,\sqrt{1-ax}\,dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{x}\,\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^{3/2}}$$

```
[Out] 1/4*arcsin(a^(1/2)*x^(1/2))/a^(3/2)+1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*
(-a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{\text{ArcSin}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\,\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] -1/4*(Sqrt[x]*Sqrt[1 - a*x])/a + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{x}\sqrt{1-ax}\,dx = \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)}{4a}$$
$$= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 1.02

$$\frac{1}{4}\left(\frac{\sqrt{x}\sqrt{1-ax}\,(-1+2ax)}{a} + \frac{\log\left(-\sqrt{-a}\sqrt{x}+\sqrt{1-ax}\right)}{(-a)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[1 - a*x],x]
```

```
[Out] ((Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x))/a + Log[-(Sqrt[-a]*Sqrt[x]) + Sqrt[1
 - a*x]]/(-a)^(3/2))/4
```

Maple [A]
time = 0.08, size = 84, normalized size = 1.33

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\sqrt{x}\,(-a)^{\frac{3}{2}}(-6ax+3)\sqrt{-ax+1}}{6a} - \frac{\sqrt{\pi}\,(-a)^{\frac{3}{2}}\arcsin\left(\sqrt{a}\sqrt{x}\right)}{2a^{\frac{3}{2}}}}{2\sqrt{-a}\sqrt{\pi}\,a}$	66
default	$-\frac{\sqrt{x}\,(-ax+1)^{\frac{3}{2}}}{2a} + \frac{\sqrt{x}\sqrt{-ax+1} + \frac{\sqrt{(-ax+1)x}\,\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^2+x}}\right)}{2\sqrt{-ax+1}\sqrt{x}\sqrt{a}}}{4a}$	84
risch	$-\frac{(2ax-1)\sqrt{x}\,(ax-1)\sqrt{(-ax+1)x}}{4a\sqrt{-x(ax-1)}\sqrt{-ax+1}} + \frac{\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^2+x}}\right)\sqrt{(-ax+1)x}}{8a^{\frac{3}{2}}\sqrt{x}\sqrt{-ax+1}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*x^(1/2)*(-a*x+1)^(3/2)+1/4/a*(x^(1/2)*(-a*x+1)^(1/2)+1/2*((-a*x+1)*x
)^(1/2)/(-a*x+1)^(1/2)/x^(1/2)/a^(1/2)*arctan(a^(1/2)*(x-1/2/a)/(-a*x^2+x)^
(1/2)))
```

Maxima [A]
time = 0.54, size = 82, normalized size = 1.30

$$\frac{\frac{\sqrt{-ax+1}\,a}{\sqrt{x}}-\frac{(-ax+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{4\left(a^3-\frac{2\,(ax-1)a^2}{x}+\frac{(ax-1)^2a}{x^2}\right)}-\frac{\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\,\sqrt{x}}\right)}{4\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(-a*x + 1)*a/sqrt(x) - (-a*x + 1)^(3/2)/x^(3/2))/(a^3 - 2*(a*x - 1
)*a^2/x + (a*x - 1)^2*a/x^2) - 1/4*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x)))
/a^(3/2)
```

Fricas [A]
time = 1.97, size = 111, normalized size = 1.76

$$\left[\frac{2\,(2\,a^2x-a)\sqrt{-ax+1}\,\sqrt{x}-\sqrt{-a}\,\log\left(-2\,ax+2\,\sqrt{-ax+1}\,\sqrt{-a}\,\sqrt{x}+1\right)}{8\,a^2},\frac{(2\,a^2x-a)\sqrt{-ax+1}\,\sqrt{x}-\sqrt{a}\,\arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\,\sqrt{x}}\right)}{4\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(2*a^2*x - a)*sqrt(-a*x + 1)*sqrt(x) - sqrt(-a)*log(-2*a*x + 2*sqrt
(-a*x + 1)*sqrt(-a)*sqrt(x) + 1))/a^2, 1/4*((2*a^2*x - a)*sqrt(-a*x + 1)*sq
rt(x) - sqrt(a)*arctan(sqrt(-a*x + 1)/(sqrt(a)*sqrt(x))))/a^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.77, size = 148, normalized size = 2.35

$$\begin{cases} \frac{iax^{\frac{5}{2}}}{2\sqrt{ax-1}}-\frac{3ix^{\frac{3}{2}}}{4\sqrt{ax-1}}+\frac{i\sqrt{x}}{4a\sqrt{ax-1}}-\frac{i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{for } |ax|>1 \\ -\frac{ax^{\frac{5}{2}}}{2\sqrt{-ax+1}}+\frac{3x^{\frac{3}{2}}}{4\sqrt{-ax+1}}-\frac{\sqrt{x}}{4a\sqrt{-ax+1}}+\frac{\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((I*a*x**(5/2)/(2*sqrt(a*x - 1)) - 3*I*x**(3/2)/(4*sqrt(a*x - 1))
+ I*sqrt(x)/(4*a*sqrt(a*x - 1)) - I*acosh(sqrt(a)*sqrt(x))/(4*a**(3/2)), Ab
```

```
s(a*x) > 1), (-a*x**(5/2)/(2*sqrt(-a*x + 1)) + 3*x**(3/2)/(4*sqrt(-a*x + 1)
) - sqrt(x)/(4*a*sqrt(-a*x + 1)) + asin(sqrt(a)*sqrt(x))/(4*a**(3/2)), True
))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16
```

Mupad [**B**]
time = 2.60, size = 54, normalized size = 0.86

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{1}{4\,a}\right)\,\sqrt{1-a\,x}-\frac{\ln\left(2\,\sqrt{-a}\,\sqrt{x}\,\sqrt{1-a\,x}-2\,a\,x+1\right)}{8\,{\left(-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1 - a*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - 1/(4*a))*(1 - a*x)^(1/2) - log(2*(-a)^(1/2)*x^(1/2)*(1 - a*x
)^(1/2) - 2*a*x + 1)/(8*(-a)^(3/2))
```

3.225 $\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{x}\sqrt{1-ax}}{4a}+\frac{1}{2}x^{3/2}\sqrt{1-ax}+\frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

```
[Out] 1/4*arcsin(a^(1/2)*x^(1/2))/a^(3/2)+1/2*x^(3/2)*(-a*x+1)^(1/2)-1/4*x^(1/2)*
(-a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {862, 52, 56, 222}

$$\frac{\text{ArcSin}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{1-ax}-\frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] -1/4*(Sqrt[x]*Sqrt[1 - a*x])/a + (x^(3/2)*Sqrt[1 - a*x])/2 + ArcSin[Sqrt[a]
*Sqrt[x]]/(4*a^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}}\,dx &= \int \sqrt{x}\sqrt{1-ax}\,dx\\
&= \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1-ax}}\,dx\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\int \frac{1}{\sqrt{x}\sqrt{1-ax}}\,dx}{8a}\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-ax^2}}\,dx, x, \sqrt{x}\right)}{4a}\\
&= -\frac{\sqrt{x}\sqrt{1-ax}}{4a} + \frac{1}{2}x^{3/2}\sqrt{1-ax} + \frac{\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 1.47, size = 49, normalized size = 0.78

$$
\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}\,(-1+2ax) + \sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 - a^2*x^2])/Sqrt[1 + a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*(-1 + 2*a*x) + ArcSin[Sqrt[a]*Sqrt[x]])/(4*a
^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(43) = 86$.
time = 0.07, size = 92, normalized size = 1.46

method	result	si
default	$\frac{\sqrt{x}\sqrt{-a^2x^2+1}\left(4a^{\frac{3}{2}}x\sqrt{-x(ax-1)}-2\sqrt{a}\sqrt{-x(ax-1)}+\arctan\left(\frac{2ax-1}{2\sqrt{a}\sqrt{-x(ax-1)}}\right)\right)}{8a^{\frac{3}{2}}\sqrt{ax+1}\sqrt{-x(ax-1)}}$	92

risch	$-\frac{(2ax-1)\sqrt{x}\,(ax-1)\sqrt{\frac{x(-a^2x^2+1)}{ax+1}}\,\sqrt{ax+1}}{4a\sqrt{-x\,(ax-1)}\,\sqrt{-a^2x^2+1}}+\frac{\arctan\left(\frac{\sqrt{a}\,\left(x-\frac{1}{2a}\right)}{\sqrt{-a\,x^2+x}}\right)\sqrt{\frac{x(-a^2x^2+1)}{ax+1}}\,\sqrt{ax+1}}{8a^{\frac{3}{2}}\sqrt{x}\,\sqrt{-a^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^(1/2)*(-a^2*x^2+1)^(1/2)/a^(3/2)*(4*a^(3/2)*x*(-x*(a*x-1))^(1/2)-2*a^
(1/2)*(-x*(a*x-1))^(1/2)+arctan(1/2/a^(1/2)*(2*a*x-1)/(-x*(a*x-1))^(1/2)))/
(a*x+1)^(1/2)/(-x*(a*x-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*sqrt(x)/sqrt(a*x + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(43) = 86.
time = 2.48, size = 221, normalized size = 3.51

$$\left[\frac{4\sqrt{-a^2x^2+1}\,(2\,a^2x-a)\sqrt{ax+1}\,\sqrt{x}-(ax+1)\sqrt{-a}\,\log\left(-\frac{8\,a^3x^3-4\sqrt{-a^2x^2+1}\,(2\,ax-1)\sqrt{ax+1}\,\sqrt{-a}\,\sqrt{x}-7\,ax+1}{ax+1}\right)}{16\,(a^3x+a^2)},\ \frac{2\sqrt{-a^2x^2+1}\,(2\,a^2x-a)\sqrt{ax+1}\,\sqrt{x}-(ax+1)\sqrt{a}\,\arctan\left(\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax+1}\,\sqrt{a}\,\sqrt{x}}{2\,a^2x^2+ax-1}\right)}{8\,(a^3x+a^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-a^2*x^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)
*sqrt(-a)*log(-(8*a^3*x^3 - 4*sqrt(-a^2*x^2 + 1)*(2*a*x - 1)*sqrt(a*x + 1)*
sqrt(-a)*sqrt(x) - 7*a*x + 1)/(a*x + 1)))/(a^3*x + a^2), 1/8*(2*sqrt(-a^2*x
^2 + 1)*(2*a^2*x - a)*sqrt(a*x + 1)*sqrt(x) - (a*x + 1)*sqrt(a)*arctan(2*sq
rt(-a^2*x^2 + 1)*sqrt(a*x + 1)*sqrt(a)*sqrt(x)/(2*a^2*x^2 + a*x - 1)))/(a^3
*x + a^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}\,\sqrt{-(ax-1)\,(ax+1)}}{\sqrt{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-a**2*x**2+1)**(1/2)/(a*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-(a*x - 1)*(a*x + 1))/sqrt(a*x + 1), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-a^2*x^2+1)^(1/2)/(a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}\,\sqrt{1-a^2\,x^2}}{\sqrt{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2),x)
```

```
[Out] int((x^(1/2)*(1 - a^2*x^2)^(1/2))/(a*x + 1)^(1/2), x)
```

3.226 $\int (gx)^m(d+ex)^3 \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=250

$$-\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)}+\frac{d^7(11+4m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2}\right.}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(7/2)/g/(8+m)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)^
(7/2)/g^2/(9+m)+d^7*(11+4*m)*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1
/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(8+m)/(1-e^2*x^2/d^2)^(1/2)
+d^6*e*(29+4*m)*(g*x)^(2+m)*hypergeom([-5/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2
)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(9+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1823, 822, 372, 371}

$$-\frac{e\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+2}}{g^2(m+9)}-\frac{3d\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}+\frac{d^7(4m+11)\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^6e(4m+29)\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+9)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m)) - (e*(g*x)^(2 + m)*(
d^2 - e^2*x^2)^(7/2))/(g^2*(9 + m)) + (d^7*(11 + 4*m)*(g*x)^(1 + m)*Sqrt[d^
2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/
(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^6*e*(29 + 4*m)*(g*x)^(2 +
m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x
^2)/d^2])/(g^2*(2 + m)*(9 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)}-\frac{\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}\left(-d^3e^2(9+m)-\cdots\right.}{e^2(9+\cdots}\\
&= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)}+\frac{\int (gx)^m\left(d^3e\cdots\right.}{\cdots}\\
&= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)}+\frac{\left(d^3(11+4m\cdots\right.}{\cdots}\\
&= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)}+\frac{\left(d^7(11+4m\cdots\right.}{\cdots}\\
&= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{7/2}}{g^2(9+m)}+\frac{d^7(11+4m)\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]

time = 0.85, size = 199, normalized size = 0.80

$$
\frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(\frac{d^3\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(\frac{3d^2\,{}_2F_1\left(-\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{2+m}+ex\left(\frac{3d\,{}_2F_1\left(-\frac{5}{2},\frac{3+m}{2};\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)}{3+m}+\frac{ex\,{}_2F_1\left(-\frac{5}{2},\frac{4+m}{2};\frac{6+m}{2};\frac{e^2x^2}{d^2}\right)}{4+m}\right)\right)\right)}{\sqrt{1-\frac{e^2x^2}{d^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*((d^3*Hypergeometric2F1[-5/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[-5/2, (
2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[
-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F
1[-5/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/Sqrt[1 - (e^2*x^2
)/d^2]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (gx)^{m} (ex+d)^{3} \left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(x*e + d)^3*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((d^4*x^3*e^3 + 3*d^5*x^2*e^2 + 3*d^6*x*e + d^7 + (x^7*e^3 + 3*d*x^
6*e^2 + 3*d^2*x^5*e + d^3*x^4)*e^4 - 2*(d^2*x^5*e^3 + 3*d^3*x^4*e^2 + 3*d^4
*x^3*e + d^5*x^2)*e^2)*sqrt(-x^2*e^2 + d^2)*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 21.19, size = 513, normalized size = 2.05

$$\frac{d^{8} g^{m} x x^{m} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}\right)}{2 \Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{3 d^{7} e g^{m} x^{2} x^{m} \Gamma\left(\frac{m}{2}+1\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}\right)}{2 \Gamma\left(\frac{m}{2}+2\right)} + \frac{d^{6} e^{2} g^{m} x^{3} x^{m} \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}\right)}{2 \Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} - \frac{5 d^{5} e^{3} g^{m} x^{4} x^{m} \Gamma\left(\frac{m}{2}+2\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+2 \\ \frac{m}{2}+3 \end{matrix}\middle| \frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}\right)}{2 \Gamma\left(\frac{m}{2}+3\right)} - \frac{5 d^{4} e^{4} g^{m} x^{5} x^{m} \Gamma\left(\frac{m}{2}+\frac{5}{2}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{5}{2} \\ \frac{m}{2}+\frac{7}{2} \end{matrix}\middle| \frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}\right)}{2 \Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{d^{3} e^{5} g^{m} x^{6} x^{m} \Gamma\left(\frac{m}{2}+3\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+3 \\ \frac{m}{2}+4 \end{matrix}\middle| \frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}\right)}{2 \Gamma\left(\frac{m}{2}+4\right)} + \frac{3 d^{2} e^{6} g^{m} x^{7} x^{m} \Gamma\left(\frac{m}{2}+\frac{7}{2}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{7}{2} \\ \frac{m}{2}+\frac{9}{2} \end{matrix}\middle| \frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}\right)}{2 \Gamma\left(\frac{m}{2}+\frac{9}{2}\right)} + \frac{d e^{7} g^{m} x^{8} x^{m} \Gamma\left(\frac{m}{2}+4\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+4 \\ \frac{m}{2}+5 \end{matrix}\middle| \frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}\right)}{2 \Gamma\left(\frac{m}{2}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**8*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**7*e*g**m*x**2*x
**m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2
*I*pi)/d**2)/(2*gamma(m/2 + 2)) + d**6*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)
*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(
2*gamma(m/2 + 5/2)) - 5*d**5*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2
, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3)
) - 5*d**4*e**4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m
/2 + 7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d**3*e
**5*g**m*x**6*x**m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x
**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 4)) + 3*d**2*e**6*g**m*x**7*x**m
*gamma(m/2 + 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/(2*gamma(m/2 + 9/2)) + d*e**7*g**m*x**8*x**m*gamma(m/2 + 4)
*hyper((-1/2, m/2 + 4), (m/2 + 5,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*ga
mma(m/2 + 5))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(x*e + d)^3*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^3,x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^3, x)
```

3.227 $\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=206

$$-\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{d^6(9+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\ {}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{2d^5e(gx)^{2+m}\sqrt{d^2-e}}{g^2(2+}$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(7/2)/g/(8+m)+d^6*(9+2*m)*(g*x)^(1+m)*hypergeom
([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(8
+m)/(1-e^2*x^2/d^2)^(1/2)+2*d^5*e*(g*x)^(2+m)*hypergeom([-5/2, 1+1/2*m],[2+
1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1823, 822, 372, 371}

$$-\frac{\left(d^2-e^2x^2\right)^{7/2}(gx)^{m+1}}{g(m+8)}+\frac{d^6(2m+9)\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+8)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{2d^5e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(7/2))/(g*(8 + m))) + (d^6*(9 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(8 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (2*d^5*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}\,dx &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}-\frac{\int (gx)^m\left(-d^2e^2(9+2m)-2de^3(8+m)x\right)}{e^2(8+m)}\\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}\,dx}{g}+\frac{(d^2(9}{}\\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{\left(2d^5e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right)^{5}}{g\sqrt{1-\frac{e^2x^2}{d^2}}}\\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{7/2}}{g(8+m)}+\frac{d^6(9+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\right.}{g(1+m)(8+m)\sqrt{1-\frac{e^2x^2}{d^2}}}\end{aligned}$$

Mathematica [A]
time = 0.74, size = 174, normalized size = 0.84

$$\frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(d^2(6+5m+m^2)\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)+e(1+m)x\left(2d(3+m)\,{}_2F_1\left(-\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)+e(2+m)x\,{}_2F_1\left(-\frac{5}{2},\frac{3+m}{2};\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)\right)\right)}{(1+m)(2+m)(3+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-
5/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hyperg
```

```
eometric2F1[-5/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hyperg
eometric2F1[-5/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*
(3 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (gx)^m (ex+d)^2 \left(-e^2x^2+d^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(x*e + d)^2*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((d^4*x^2*e^2 + 2*d^5*x*e + d^6 + (x^6*e^2 + 2*d*x^5*e + d^2*x^4)*e
^4 - 2*(d^2*x^4*e^2 + 2*d^3*x^3*e + d^4*x^2)*e^2)*sqrt(-x^2*e^2 + d^2)*(g*x
)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.52, size = 442, normalized size = 2.15

$$\frac{d^7g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{d^6eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)}-\frac{d^5e^2g^mx^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}-\frac{2d^4e^3g^mx^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+3\right)}-\frac{d^3e^4g^mx^5x^m\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{d^2e^5g^mx^6x^m\Gamma\left(\frac{m}{2}+3\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+3\\ \frac{m}{2}+4\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+4\right)}+\frac{de^6g^mx^7x^m\Gamma\left(\frac{m}{2}+\frac{7}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{7}{2}\\ \frac{m}{2}+\frac{9}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**6*e*g**m*x**2*x**
```

```
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/gamma(m/2 + 2) - d**5*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*hyper
((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamm
a(m/2 + 5/2)) - 2*d**4*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2
+ 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 3) - d**3*e
**4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2,),
e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d**2*e**5*g**m*x**
6*x**m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/gamma(m/2 + 4) + d*e**6*g**m*x**7*x**m*gamma(m/2 + 7/2)*hyp
er((-1/2, m/2 + 7/2), (m/2 + 9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*ga
mma(m/2 + 9/2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(x*e + d)^2*(g*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^2,x)
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x)^2, x)
```

3.228 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx$

Optimal. Leaf size=162

$$\frac{d^5(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e(gx)^{2+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^5*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)+d^4*e*(g*x)^(2+m)*hypergeom([-
5/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*
x^2/d^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {822, 372, 371}

$$\frac{d^5\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^5*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) + (d^4*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^{5/2}\,dx = d\int (gx)^m\left(d^2-e^2x^2\right)^{5/2}\,dx+\frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{5/2}\,dx}{g}$$

$$=\frac{\left(d^5\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}\,dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{\left(d^4e\sqrt{d^2-e^2x^2}\right)\int (gx}{g\sqrt{1-}}$$

$$=\frac{d^5(gx)^{1+m}\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^4e(gx)^{2+m}\sqrt{d^2-}}{g^2(2-}$$

Mathematica [A]
time = 0.63, size = 121, normalized size = 0.75

$$\frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\left(d(2+m)\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)+e(1+m)x\,{}_2F_1\left(-\frac{5}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)\right)}{(1+m)(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d*(2 + m)*Hypergeometric2F1[-5/2, (1 +
m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[-5/2, (2 +
m)/2, (4 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (gx)^m\,(ex+d)\left(-e^2x^2+d^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((d^4*x*e + d^5 + (x^5*e + d*x^4)*e^4 - 2*(d^2*x^3*e + d^3*x^2)*e^2
)*sqrt(-x^2*e^2 + d^2)*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.34, size = 374, normalized size = 2.31

$$\frac{d^6 g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\right| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d^5 e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\right| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} - \frac{d^4 e^2 g^m x^3 x^m \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\right| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} - \frac{d^3 e^3 g^m x^4 x^m \Gamma\left(\frac{m}{2}+2\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{2}+2 \\ \frac{m}{2}+3 \end{matrix}\right| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+3\right)} + \frac{d^2 e^4 g^m x^5 x^m \Gamma\left(\frac{m}{2}+\frac{5}{2}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{5}{2} \\ \frac{m}{2}+\frac{7}{2} \end{matrix}\right| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{d e^5 g^m x^6 x^m \Gamma\left(\frac{m}{2}+3\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{2}+3 \\ \frac{m}{2}+4 \end{matrix}\right| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**6*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**5*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/(2*gamma(m/2 + 2)) - d**4*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*h
yper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gam
ma(m/2 + 5/2) - d**3*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 +
2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/gamma(m/2 + 3) + d**2*e**
4*g**m*x**5*x**m*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 7/2)) + d*e**5*g**m*x**6*x**
m*gamma(m/2 + 3)*hyper((-1/2, m/2 + 3), (m/2 + 4,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/(2*gamma(m/2 + 4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(x*e + d)*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d^2-e^2\,x^2\right)^{5/2}\,{\left(g\,x\right)}^m\,\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m*(d + e*x), x)
```

3.229 $\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} \, dx$

Optimal. Leaf size=80

$$\frac{d^4(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^4*(g*x)^(1+m)*hypergeom([-5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {372, 371}

$$\frac{d^4\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
```

```
[Out] (d^4*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (gx)^m \left(d^2-e^2x^2\right)^{5/2} dx = \frac{\left(d^4\sqrt{d^2-e^2x^2}\right)\int (gx)^m \left(1-\frac{e^2x^2}{d^2}\right)^{5/2} dx}{\sqrt{1-\frac{e^2x^2}{d^2}}}$$
$$= \frac{d^4(gx)^{1+m}\sqrt{d^2-e^2x^2}\ {}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A]
time = 0.38, size = 78, normalized size = 0.98

$$\frac{d^4x(gx)^m\sqrt{d^2-e^2x^2}\ {}_2F_1\left(-\frac{5}{2},\frac{1+m}{2};1+\frac{1+m}{2};\frac{e^2x^2}{d^2}\right)}{(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d^2 - e^2*x^2)^(5/2),x]
[Out] (d^4*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-5/2, (1 + m)/2, 1 + (
1 + m)/2, (e^2*x^2)/d^2])/((1 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (gx)^m \left(-e^2x^2+d^2\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
[Out] integral((x^4*e^4 - 2*d^2*x^2*e^2 + d^4)*sqrt(-x^2*e^2 + d^2)*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.18, size = 61, normalized size = 0.76

$$\frac{d^5 g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2),x)
[Out] d**5*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-5/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m,x)
[Out] int((d^2 - e^2*x^2)^(5/2)*(g*x)^m, x)
```

3.230 $\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{d+ex} dx$

Optimal. Leaf size=163

$$\frac{d^3(gx)^{1+m}\sqrt{d^2-e^2x^2}\ {}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e(gx)^{2+m}\sqrt{d^2-e^2x^2}\ {}_2F_1\left(-\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{g^2(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

```
[Out] d^3*(g*x)^(1+m)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*
x^2+d^2)^(1/2)/g/(1+m)/(1-e^2*x^2/d^2)^(1/2)-d^2*e*(g*x)^(2+m)*hypergeom([-
3/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*
x^2/d^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {906, 83, 127, 372, 371}

$$\frac{d^3\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{d^2e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^3*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (
3 + m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (d^2*e*(g*x
)^(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-3/2, (2 + m)/2, (4 + m)/2,
 (e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}\int (gx)^m(d-ex)^{5/2}(d+ex)^{3/2}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= \frac{\left(d\sqrt{d^2-e^2x^2}\right)\int (gx)^m(d-ex)^{3/2}(d+ex)^{3/2}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\left(e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1-}}{g\sqrt{d-e}}$$

$$= d\int (gx)^m \left(d^2-e^2x^2\right)^{3/2}\,dx - \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}\,dx}{g}$$

$$= \frac{\left(d^3\sqrt{d^2-e^2x^2}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{\left(d^2e\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\left(\right.}{g\sqrt{1-\frac{e^2x^2}{d^2}}}$$

$$= \frac{d^3(gx)^{1+m}\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{d^2e(gx)^{2+m}\sqrt{d^2-e^2x^2}\;{}_2}{g^2(2+m)\sqrt{}}$$

Mathematica [A]

time = 0.63, size = 122, normalized size = 0.75

$$\frac{d^2x(gx)^m\sqrt{d^2-e^2x^2}\left(-e(1+m)x\,{}_2F_1\left(-\frac{3}{2},1+\frac{m}{2};2+\frac{m}{2};\frac{e^2x^2}{d^2}\right)+d(2+m)\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)\right)}{(1+m)(2+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (d^2*x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(-(e*(1 + m)*x*Hypergeometric2F1[-3/2, 1
 + m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[-3/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(g*x)^m/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((x^4*e^4 - 2*d^2*x^2*e^2 + d^4)*sqrt(-x^2*e^2 + d^2)*(g*x)^m/(x*e
+ d), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.84, size = 248, normalized size = 1.52

$$\frac{d^4g^mxx^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}-\frac{d^3eg^mx^2x^m\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}-\frac{d^2e^2g^mx^3x^m\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{de^3g^mx^4x^m\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d),x)
```

```
[Out] d**4*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) - d**3*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/(2*gamma(m/2 + 2)) - d**2*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*h
yper((-1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*
gamma(m/2 + 5/2)) + d*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-1/2, m/2 +
 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(g*x)^m/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^{5/2}\,{\left(g\,x\right)}^m}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x),x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x), x)
```

3.231 $\int \frac{(gx)^m(d^2-e^2x^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{(gx)^{1+m}(d^2-e^2x^2)^{3/2}}{g(4+m)}+\frac{d^2(5+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(4+m)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{2de(gx)^{2+m}\sqrt{d^2-e^2x}}{g^2(2+m}$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(3/2)/g/(4+m)+d^2*(5+2*m)*(g*x)^(1+m)*hypergeom
([-1/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g/(1+m)/(4
+m)/(1-e^2*x^2/d^2)^(1/2)-2*d*e*(g*x)^(2+m)*hypergeom([-1/2, 1+1/2*m],[2+1/
2*m],e^2*x^2/d^2)*(-e^2*x^2+d^2)^(1/2)/g^2/(2+m)/(1-e^2*x^2/d^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {866, 1823, 822, 372, 371}

$$-\frac{2de\sqrt{d^2-e^2x^2}\,(gx)^{m+2}\,{}_2F_1\left(-\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{d^2(2m+5)\sqrt{d^2-e^2x^2}\,(gx)^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+4)\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{(d^2-e^2x^2)^{3/2}(gx)^{m+1}}{g(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(3/2))/(g*(4 + m))) + (d^2*(5 + 2*m)*(g*x)
^(1 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2,
(e^2*x^2)/d^2])/(g*(1 + m)*(4 + m)*Sqrt[1 - (e^2*x^2)/d^2]) - (2*d*e*(g*x)^
(2 + m)*Sqrt[d^2 - e^2*x^2]*Hypergeometric2F1[-1/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(g^2*(2 + m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}\,dx = \int (gx)^m(d-ex)^2\sqrt{d^2-e^2x^2}\,dx$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{\int (gx)^m\left(-d^2e^2(5+2m)+2de^3(4+m)x\right)\sqrt{d^2-}}{e^2(4+m)}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{(2de)\int (gx)^{1+m}\sqrt{d^2-e^2x^2}\,dx}{g} + \frac{\left(d^2(5+2m)\right)\int}{}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} - \frac{\left(2de\sqrt{d^2-e^2x^2}\right)\int (gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,dx}{g\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{(}{}$$

$$= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{3/2}}{g(4+m)} + \frac{d^2(5+2m)(gx)^{1+m}\sqrt{d^2-e^2x^2}\;{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+}{2}\right.}{g(1+m)(4+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Mathematica [A]
time = 0.64, size = 173, normalized size = 0.85

$$\frac{x(gx)^m\sqrt{d^2-e^2x^2}\left(d^2(6+5m+m^2)\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)-e(1+m)x\left(2d(3+m)\,{}_2F_1\left(-\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)-e(2+m)x\,{}_2F_1\left(-\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)\right)\right)}{(1+m)(2+m)(3+m)\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[-1/2,
 (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeome
tric2F1[-1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hypergeome
tric2F1[-1/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 +
 m)*Sqrt[1 - (e^2*x^2)/d^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(g*x)^m/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((x^4*e^4 - 2*d^2*x^2*e^2 + d^4)*sqrt(-x^2*e^2 + d^2)*(g*x)^m/(x^2*
e^2 + 2*d*x*e + d^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 33.84, size = 185, normalized size = 0.91

$$\frac{d^3 g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} - \frac{d^2 e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{d e^2 g^m x^3 x^m \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] d**3*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) - d**2*e*g**m*x**2*x**
m*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I
*pi)/d**2)/gamma(m/2 + 2) + d*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*hyper((-
1/2, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m
/2 + 5/2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,{\left(g\,x\right)}^m}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^2, x)
```

3.232 $\int \frac{(gx)^m (d^2-e^2x^2)^{5/2}}{(d+ex)^3} \, dx$

Optimal. Leaf size=250

$$-\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)}+\frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)}+\frac{d^3(5+4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(2+m)\sqrt{d^2-e^2x^2}}-$$

```
[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(1/2)/g/(2+m)+e*(g*x)^(2+m)*(-e^2*x^2+d^2)^
(1/2)/g^2/(3+m)+d^3*(5+4*m)*(g*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2
*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/g/(1+m)/(2+m)/(-e^2*x^2+d^2)^(1/2)-d
^2*e*(11+4*m)*(g*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(
1-e^2*x^2/d^2)^(1/2)/g^2/(2+m)/(3+m)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {866, 1823, 822, 372, 371}

$$-\frac{d^2e(4m+11)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+3)\sqrt{d^2-e^2x^2}}+\frac{e\sqrt{d^2-e^2x^2}\,(gx)^{m+2}}{g^2(m+3)}-\frac{3d\sqrt{d^2-e^2x^2}\,(gx)^{m+1}}{g(m+2)}+\frac{d^3(4m+5)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*Sqrt[d^2 - e^2*x^2])/(g*(2 + m)) + (e*(g*x)^(2 + m)*Sqr
t[d^2 - e^2*x^2])/(g^2*(3 + m)) + (d^3*(5 + 4*m)*(g*x)^(1 + m)*Sqrt[1 - (e^
2*x^2)/d^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/(g
*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2]) - (d^2*e*(11 + 4*m)*(g*x)^(2 + m)*Sqr
t[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)
/d^2])/(g^2*(2 + m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^3}\,dx = \int \frac{(gx)^m(d-ex)^3}{\sqrt{d^2-e^2x^2}}\,dx$$

$$= \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} - \frac{\int \frac{(gx)^m\left(-d^3e^2(3+m)+d^2e^3(11+4m)x-3de^4(3+m)x^2\right)}{\sqrt{d^2-e^2x^2}}\,dx}{e^2(3+m)}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\int \frac{(gx)^m\left(d^3e^4(3+m)(5+4m)-d^2e\right.}{\sqrt{d^2-e^2x}}}{e^4(2+m)(3}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\left(d^3(5+4m)\right)\int \frac{(gx)^m}{\sqrt{d^2-e^2}}}{2+m}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{\left(d^3(5+4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right.}{(2+m)\sqrt{d^2}}$$

$$= -\frac{3d(gx)^{1+m}\sqrt{d^2-e^2x^2}}{g(2+m)} + \frac{e(gx)^{2+m}\sqrt{d^2-e^2x^2}}{g^2(3+m)} + \frac{d^3(5+4m)(gx)^{1+m}\sqrt{1-}}{g(1+m)(2}$$

Mathematica [A]
time = 0.72, size = 245, normalized size = 0.98

$$\frac{x(gx)^m\sqrt{d^2-e^2x^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(d^3(24+26m+9m^2+m^3)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)-e(1+m)x\left(3d^2(12+7m+m^2)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)+e(2+m)x\left(-3d(4+m)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)+e(3+m)x\,{}_2F_1\left(\frac{1}{2},\frac{4+m}{2};\frac{6+m}{2};\frac{e^2x^2}{d^2}\right)\right)\right)\right)}{(1+m)(2+m)(3+m)(4+m)(d-ex)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*Sqrt[d^2 - e^2*x^2]*Sqrt[1 - (e^2*x^2)/d^2]*(d^3*(24 + 26*m + 9*
m^2 + m^3)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*
(1 + m)*x*(3*d^2*(12 + 7*m + m^2)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2] + e*(2 + m)*x*(-3*d*(4 + m)*Hypergeometric2F1[1/2, (3 +
m)/2, (5 + m)/2, (e^2*x^2)/d^2] + e*(3 + m)*x*Hypergeometric2F1[1/2, (4 + m
)/2, (6 + m)/2, (e^2*x^2)/d^2]))))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(d - e*
x)*(d + e*x))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)

[Out] int((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="maxima")

[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(g*x)^m/(x*e + d)^3, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="fricas")

[Out] integral((x^4*e^4 - 2*d^2*x^2*e^2 + d^4)*sqrt(-x^2*e^2 + d^2)*(g*x)^m/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m \left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}}{(d+ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**3,x)

[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**(5/2)/(d + e*x)**3, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x)^m*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")

[Out] integrate((-x^2*e^2 + d^2)^(5/2)*(g*x)^m/(x*e + d)^3, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^{5/2}\,\left(g\,x\right)^m}{\left(d+e\,x\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^3,x)
[Out] int(((d^2 - e^2*x^2)^(5/2)*(g*x)^m)/(d + e*x)^3, x)
```

3.233 $\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=213

$$\frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}}+\frac{(1-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(1+m)\sqrt{d^2-e^2x^2}}+\frac{e(7-4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2}{5d^4g^2(2+m)\sqrt{d^2}}$$

```
[Out] 4/5*(g*x)^(1+m)*(e*x+d)/g/(-e^2*x^2+d^2)^(5/2)+1/5*(1-4*m)*(g*x)^(1+m)*hype
rgeom([5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^3/g
/(1+m)/(-e^2*x^2+d^2)^(1/2)+1/5*e*(7-4*m)*(g*x)^(2+m)*hypergeom([5/2, 1+1/2
*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^4/g^2/(2+m)/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1820, 822, 372, 371}

$$\frac{4(d+ex)(gx)^{m+1}}{5g\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (4*(g*x)^(1 + m)*(d + e*x))/(5*g*(d^2 - e^2*x^2)^(5/2)) + ((1 - 4*m)*(g*x)^
(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2
, (e^2*x^2)/d^2])/(5*d^3*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(7 - 4*m)*(g*x
)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(gx)^m(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(gx)^m\left(-d^3(1-4m)-d^2e(7-4m)x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2}\\ &= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{1}{5}(d(1-4m))\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{5/2}}\,dx + \frac{(e(7-4m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{5/2}}}{5g}\\ &= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{\left((1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^3\sqrt{d^2-e^2x^2}} + \frac{\left(e(7-4m)\sqrt{1-\right.}{5d^4g}\\ &= \frac{4(gx)^{1+m}(d+ex)}{5g\left(d^2-e^2x^2\right)^{5/2}} + \frac{(1-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^3g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(7-}{}\end{aligned}$$

Mathematica [A]
time = 1.08, size = 199, normalized size = 0.93

$$\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(\frac{d^3\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(\frac{3d^2\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{2+m}+ex\left(\frac{3d\,{}_2F_1\left(\frac{7}{2},\frac{3+m}{2};\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)}{3+m}+\frac{ex\,{}_2F_1\left(\frac{7}{2},\frac{4+m}{2};\frac{6+m}{2};\frac{e^2x^2}{d^2}\right)}{4+m}\right)\right)\right)}{d^6\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[7/2, (1 + m)/2,
(3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[7/2, (2
+ m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[7/
2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[7
/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*Sqrt[d^2 - e^2*x
^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (ex+d)^3}{(-e^2x^2+d^2)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(g*x)^m/(-x^2*e^2 + d^2)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*sqrt(-x^2*e^2 + d^2)*(g*
x)^m/(x^8*e^8 - 4*d^2*x^6*e^6 + 6*d^4*x^4*e^4 - 4*d^6*x^2*e^2 + d^8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((g*x)**m*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((x*e + d)^3*(g*x)^m/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.234 $\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=216

$$\frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}}+\frac{(3-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(1+m)\sqrt{d^2-e^2x^2}}+\frac{2e(3-m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}}{5d^5g^2(2+m)\sqrt{d^2}}$$

```
[Out] 2/5*(g*x)^(1+m)*(e*x+d)/d/g/(-e^2*x^2+d^2)^(5/2)+1/5*(3-2*m)*(g*x)^(1+m)*hy
pergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^4
/g/(1+m)/(-e^2*x^2+d^2)^(1/2)+2/5*e*(3-m)*(g*x)^(2+m)*hypergeom([5/2, 1+1/2
*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^5/g^2/(2+m)/(-e^2*x^2+d^
2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1820, 822, 372, 371}

$$\frac{2(d+ex)(gx)^{m+1}}{5dg\left(d^2-e^2x^2\right)^{5/2}}+\frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{5}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{5d^5g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (2*(g*x)^(1 + m)*(d + e*x))/(5*d*g*(d^2 - e^2*x^2)^(5/2)) + ((3 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(5*d^4*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (2*e*(3 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(5*d^5*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(gx)^m(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(gx)^m\left(-d^2(3-2m)-2de(3-m)x\right)}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5d^2} \\
&= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2e(3-m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5dg} - \frac{1}{5}(-3+2m)\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{5/2}} \\
&= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{\left(2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{5/2}}\,dx}{5d^5g\sqrt{d^2-e^2x^2}} - \frac{\left((-3+2m)\sqrt{1\right.}{5d} \\
&= \frac{2(gx)^{1+m}(d+ex)}{5dg\left(d^2-e^2x^2\right)^{5/2}} + \frac{(3-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{5d^4g(1+m)\sqrt{d^2-e^2x^2}} + \frac{2e(3-}{}
\end{aligned}$$

Mathematica [A]
time = 0.88, size = 174, normalized size = 0.81

$$\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(d^2(6+5m+m^2)\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)+e(1+m)x\left(2d(3+m)\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)+e(2+m)x\,{}_2F_1\left(\frac{7}{2},\frac{3+m}{2};\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)\right)\right)}{d^6(1+m)(2+m)(3+m)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[7
/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hyperge
ometric2F1[7/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeo
metric2F1[7/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^6*(1 + m)*(2 + m)
*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^{m} (ex+d)^{2}}{(-e^{2}x^{2}+d^{2})^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(g*x)^m/(-x^2*e^2 + d^2)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*sqrt(-x^2*e^2 + d^2)*(g*x)^m/(x^8*e^8 -
4*d^2*x^6*e^6 + 6*d^4*x^4*e^4 - 4*d^6*x^2*e^2 + d^8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^{m} (d+ex)^{2}}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((g*x)**m*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((x*e + d)^2*(g*x)^m/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m\,(d+e\,x)^2}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x)^2)/(d^2 - e^2*x^2)^(7/2), x)
```

3.235 $\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=124

$$\frac{(gx)^{1+m}\,{}_2F_1\left(1,\frac{1}{2}(-4+m);\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{dg(1+m)\left(d^2-e^2x^2\right)^{5/2}}+\frac{e(gx)^{2+m}\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^2g^2(2+m)\left(d^2-e^2x^2\right)^{5/2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([1, -2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d/g/(1+m)/(-e^
2*x^2+d^2)^(5/2)+e*(g*x)^(2+m)*hypergeom([1, -3/2+1/2*m],[2+1/2*m],e^2*x^2/
d^2)/d^2/g^2/(2+m)/(-e^2*x^2+d^2)^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 162, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {822, 372, 371}

$$\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{7}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^6g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^5g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^5*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) + (e*(g*x)^(2
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^6*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
```

```
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}}\,dx = d\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx + \frac{e\int \frac{(gx)^{1+m}}{(d^2-e^2x^2)^{7/2}}\,dx}{g}$$

$$= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^5\sqrt{d^2-e^2x^2}} + \frac{\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^6g\sqrt{d^2-e^2x^2}}$$

$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^5g(1+m)\sqrt{d^2-e^2x^2}} + \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2}\right.}{d^6g^2(2+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 0.79, size = 121, normalized size = 0.98

$$\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(d(2+m)\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)+e(1+m)x\,{}_2F_1\left(\frac{7}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)\right)}{d^6(1+m)(2+m)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d*(2 + m)*Hypergeometric2F1[7/2, (1 + m
)/2, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[7/2, (2 + m)
/2, (4 + m)/2, (e^2*x^2)/d^2]))/(d^6*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\,(ex+d)}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(g*x)^m/(-x^2*e^2 + d^2)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2*e^2 + d^2)*(x*e + d)*(g*x)^m/(x^8*e^8 - 4*d^2*x^6*e^6 +
6*d^4*x^4*e^4 - 4*d^6*x^2*e^2 + d^8), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 26.87, size = 117, normalized size = 0.94

$$\frac{g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} \frac{7}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2d^6\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} \frac{7}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] g**m*x*x**m*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**2*x**
2*exp_polar(2*I*pi)/d**2)/(2*d**6*gamma(m/2 + 3/2)) + e*g**m*x**2*x**m*gamm
a(m/2 + 1)*hyper((7/2, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I*pi)/d*
*2)/(2*d**7*gamma(m/2 + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(g*x)^m/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m\,(d+e\,x)}{\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((g*x)^m*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x)
```

3.236 $\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=80

$$\frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\,{}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(1+m)\sqrt{d^2-e^2x^2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([7/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/
d^2)^(1/2)/d^6/g/(1+m)/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {372, 371}

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{7}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^6*g*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(gx)^m}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{7/2}}\,dx}{d^6\sqrt{d^2-e^2x^2}}$$
$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{7}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6g(1+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [**A**]
time = 0.55, size = 78, normalized size = 0.98

$$\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{7}{2},\frac{1+m}{2};1+\frac{1+m}{2};\frac{e^2x^2}{d^2}\right)}{d^6(1+m)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/(d^2 - e^2*x^2)^(7/2),x]
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[7/2, (1 + m)/2, 1 + (1
 + m)/2, (e^2*x^2)/d^2])/(d^6*(1 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
[Out] int((g*x)^m/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
[Out] integrate((g*x)^m/(-x^2*e^2 + d^2)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2*e^2 + d^2)*(g*x)^m/(x^8*e^8 - 4*d^2*x^6*e^6 + 6*d^4*x^4*
e^4 - 4*d^6*x^2*e^2 + d^8), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.50, size = 60, normalized size = 0.75

$$\frac{g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} \frac{7}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2d^7\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] g**m*x*x**m*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2,), e**2*x**
2*exp_polar(2*I*pi)/d**2)/(2*d**7*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/(-x^2*e^2 + d^2)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(d^2 - e^2*x^2)^(7/2), x)
```

3.237 $\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=163

$$\frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(1+m)\sqrt{d^2-e^2x^2}}-\frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{9}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{d^8g^2(2+m)\sqrt{d^2-e^2x^2}}$$

```
[Out] (g*x)^(1+m)*hypergeom([9/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/
d^2)^(1/2)/d^7/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-e*(g*x)^(2+m)*hypergeom([9/2, 1
+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^8/g^2/(2+m)/(-e^2*x^
2+d^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {906, 83, 127, 372, 371}

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(m+1)\sqrt{d^2-e^2x^2}}-\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^8g^2(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] ((g*x)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3
 + m)/2, (e^2*x^2)/d^2])/(d^7*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(g*x)^(2
+ m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 + m)/2, (
e^2*x^2)/d^2])/(d^8*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(gx)^m}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\left(\sqrt{d-ex}\,\sqrt{d+ex}\right)\int \frac{(gx)^m}{(d-ex)^{7/2}(d+ex)^{9/2}}\,dx}{\sqrt{d^2-e^2x^2}}$$

$$= \frac{\left(d\sqrt{d-ex}\,\sqrt{d+ex}\right)\int \frac{(gx)^m}{(d-ex)^{9/2}(d+ex)^{9/2}}\,dx}{\sqrt{d^2-e^2x^2}} - \frac{\left(e\sqrt{d-ex}\,\sqrt{d+ex}\right)\int \cdots}{g\sqrt{d^2-e^2x\cdots}}$$

$$= d\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{9/2}}\,dx - \frac{e\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{g}$$

$$= \frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{d^7\sqrt{d^2-e^2x^2}} - \frac{\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{d^8g\sqrt{d^2-e^2x^2}}$$

$$= \frac{(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^7g(1+m)\sqrt{d^2-e^2x^2}} - \frac{e(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\cdots\right.}{d^8g^2(2+m)\sqrt{d^2\cdots}}$$

Mathematica [A]

time = 1.07, size = 122, normalized size = 0.75

$$\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(-e(1+m)x\,{}_2F_1\left(\frac{9}{2},1+\frac{m}{2};2+\frac{m}{2};\frac{e^2x^2}{d^2}\right)+d(2+m)\,{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)\right)}{d^8(1+m)(2+m)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(-(e*(1 + m)*x*Hypergeometric2F1[9/2, 1
+ m/2, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[9/2, (1 + m)/
2, (3 + m)/2, (e^2*x^2)/d^2]))/(d^8*(1 + m)*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(ex+d)\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2*e^2 + d^2)*(g*x)^m/(d^8*x*e + d^9 + (x^9*e + d*x^8)*e^8
- 4*(d^2*x^7*e + d^3*x^6)*e^6 + 6*(d^4*x^5*e + d^5*x^4)*e^4 - 4*(d^6*x^3*e
+ d^7*x^2)*e^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((g*x)**m/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m}{{\left(d^2-e^2\,x^2\right)}^{7/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

```
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)), x)
```

3.238 $\int \frac{(gx)^m}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=217

$$\frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}}+\frac{(7-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(1+m)\sqrt{d^2-e^2x^2}}-\frac{2e(7-m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1}{9d^9g^2(2+m)\sqrt{d^2}}$$

```
[Out] 2/9*(g*x)^(1+m)*(-e*x+d)/d/g/(-e^2*x^2+d^2)^(9/2)+1/9*(7-2*m)*(g*x)^(1+m)*h
ypergeom([9/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^
8/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-2/9*e*(7-m)*(g*x)^(2+m)*hypergeom([9/2, 1+1/
2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^9/g^2/(2+m)/(-e^2*x^2+d
^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {866, 1820, 822, 372, 371}

$$\frac{2(d-ex)(gx)^{m+1}}{9dg\left(d^2-e^2x^2\right)^{9/2}}-\frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{9}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{9d^9g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{9}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (2*(g*x)^(1 + m)*(d - e*x))/(9*d*g*(d^2 - e^2*x^2)^(9/2)) + ((7 - 2*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (1 + m)/2, (3 + m)
/2, (e^2*x^2)/d^2])/(9*d^8*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (2*e*(7 - m)*(g
*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[9/2, (2 + m)/2, (4 +
m)/2, (e^2*x^2)/d^2])/(9*d^9*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(gx)^m}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \int \frac{(gx)^m(d-ex)^2}{\left(d^2-e^2x^2\right)^{11/2}}\,dx \\
&= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{\int \frac{(gx)^m\left(-d^2(7-2m)+2de(7-m)x\right)}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9d^2} \\
&= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{(2e(7-m))\int \frac{(gx)^{1+m}}{\left(d^2-e^2x^2\right)^{9/2}}\,dx}{9dg} - \frac{1}{9}(-7+2m)\int \frac{(}{(d^2-} \\
&= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} - \frac{\left(2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^{1+m}}{\left(1-\frac{e^2x^2}{d^2}\right)^{9/2}}\,dx}{9d^9g\sqrt{d^2-e^2x^2}} - \frac{\left((-7\right.}{} \\
&= \frac{2(gx)^{1+m}(d-ex)}{9dg\left(d^2-e^2x^2\right)^{9/2}} + \frac{(7-2m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{9}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{9d^8g(1+m)\sqrt{d^2-e^2x^2}}
\end{aligned}
$$

Mathematica [A]
time = 1.25, size = 176, normalized size = 0.81

$$\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(d^2(6+5m+m^2)\,{}_2F_1\left(\frac{11}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)-e(1+m)x\left(2d(3+m)\,{}_2F_1\left(\frac{11}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)-e(2+m)x\,{}_2F_1\left(\frac{11}{2},\frac{3+m}{2};\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)\right)\right)}{d^{10}(1+m)(2+m)(3+m)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[1
1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hyperg
eometric2F1[11/2, (2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hyperg
eometric2F1[11/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])))/(d^10*(1 + m)*(2
+ m)*(3 + m)*Sqrt[d^2 - e^2*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(ex+d)^2\,(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2*e^2 + d^2)*(g*x)^m/(d^8*x^2*e^2 + 2*d^9*x*e + d^10 + (x^
10*e^2 + 2*d*x^9*e + d^2*x^8)*e^8 - 4*(d^2*x^8*e^2 + 2*d^3*x^7*e + d^4*x^6)
```

```
*e^6 + 6*(d^4*x^6*e^2 + 2*d^5*x^5*e + d^6*x^4)*e^4 - 4*(d^6*x^4*e^2 + 2*d^7
*x^3*e + d^8*x^2)*e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((g*x)**m/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((g*x)^m/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m}{(d^2-e^2\,x^2)^{7/2}\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2),x)
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2), x)
```

3.239 $\int \frac{(gx)^m}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=214

$$\frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}}+\frac{(7-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\ {}_2F_1\left(\frac{11}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{11d^9g(1+m)\sqrt{d^2-e^2x^2}}-\frac{e(25-4m)(gx)^{2+m}\sqrt{1-\frac{e^2x^2}{d^2}}}{11d^{10}g^2(2+m)\sqrt{}}$$

```
[Out] 4/11*(g*x)^(1+m)*(-e*x+d)/g/(-e^2*x^2+d^2)^(11/2)+1/11*(7-4*m)*(g*x)^(1+m)*
hypergeom([11/2, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/
d^9/g/(1+m)/(-e^2*x^2+d^2)^(1/2)-1/11*e*(25-4*m)*(g*x)^(2+m)*hypergeom([11/
2, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)*(1-e^2*x^2/d^2)^(1/2)/d^10/g^2/(2+m)/(-e
^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {866, 1820, 822, 372, 371}

$$\frac{4(d-ex)(gx)^{m+1}}{11g\left(d^2-e^2x^2\right)^{11/2}}-\frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+2}\,{}_2F_1\left(\frac{11}{2},\frac{m+2}{2};\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(m+2)\sqrt{d^2-e^2x^2}}+\frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\,(gx)^{m+1}\,{}_2F_1\left(\frac{11}{2},\frac{m+1}{2};\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{11d^9g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*(g*x)^(1 + m)*(d - e*x))/(11*g*(d^2 - e^2*x^2)^(11/2)) + ((7 - 4*m)*(g*x
)^(1 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (1 + m)/2, (3 + m
)/2, (e^2*x^2)/d^2])/(11*d^9*g*(1 + m)*Sqrt[d^2 - e^2*x^2]) - (e*(25 - 4*m)
*(g*x)^(2 + m)*Sqrt[1 - (e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (2 + m)/2, (
4 + m)/2, (e^2*x^2)/d^2])/(11*d^10*g^2*(2 + m)*Sqrt[d^2 - e^2*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rubi steps

$$\int \frac{(gx)^m}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \int \frac{(gx)^m(d-ex)^3}{\left(d^2-e^2x^2\right)^{13/2}}\,dx$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} - \frac{\int \frac{(gx)^m\left(-d^3(7-4m)+d^2e(25-4m)x\right)}{\left(d^2-e^2x^2\right)^{11/2}}\,dx}{11d^2}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{1}{11}(d(7-4m))\int \frac{(gx)^m}{\left(d^2-e^2x^2\right)^{11/2}}\,dx - \frac{(e(25-4m)}{}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{\left((7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}\right)\int \frac{(gx)^m}{\left(1-\frac{e^2x^2}{d^2}\right)^{11/2}}\,dx}{11d^9\sqrt{d^2-e^2x^2}} - \frac{\left(e(25\right.}{}$$

$$= \frac{4(gx)^{1+m}(d-ex)}{11g\left(d^2-e^2x^2\right)^{11/2}} + \frac{(7-4m)(gx)^{1+m}\sqrt{1-\frac{e^2x^2}{d^2}}\;{}_2F_1\left(\frac{11}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right.}{11d^9g(1+m)\sqrt{d^2-e^2x^2}}$$

Mathematica [A]
time = 1.51, size = 200, normalized size = 0.93

$$\frac{x(gx)^m\sqrt{1-\frac{e^2x^2}{d^2}}\left(\frac{d^3\,{}_2F_1\left(\frac{13}{2},\frac{1+m}{2};\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(-\frac{3d^2\,{}_2F_1\left(\frac{13}{2},\frac{2+m}{2};\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{2+m}+ex\left(\frac{3d\,{}_2F_1\left(\frac{13}{2},\frac{3+m}{2};\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)}{3+m}-\frac{ex\,{}_2F_1\left(\frac{13}{2},\frac{4+m}{2};\frac{6+m}{2};\frac{e^2x^2}{d^2}\right)}{4+m}\right)\right)\right)}{d^{12}\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (x*(g*x)^m*Sqrt[1 - (e^2*x^2)/d^2]*((d^3*Hypergeometric2F1[13/2, (1 + m)/2,
 (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[13/2,
(2 + m)/2, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1
[13/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric2
F1[13/2, (4 + m)/2, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^12*Sqrt[d^2 -
 e^2*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{(ex+d)^3\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] int((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2*e^2 + d^2)*(g*x)^m/(d^8*x^3*e^3 + 3*d^9*x^2*e^2 + 3*d^10
*x*e + d^11 + (x^11*e^3 + 3*d*x^10*e^2 + 3*d^2*x^9*e + d^3*x^8)*e^8 - 4*(d^
2*x^9*e^3 + 3*d^3*x^8*e^2 + 3*d^4*x^7*e + d^5*x^6)*e^6 + 6*(d^4*x^7*e^3 + 3
*d^5*x^6*e^2 + 3*d^6*x^5*e + d^7*x^4)*e^4 - 4*(d^6*x^5*e^3 + 3*d^7*x^4*e^2
+ 3*d^8*x^3*e + d^9*x^2)*e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
[Out] Integral((g*x)**m/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] integrate((g*x)^m/((-x^2*e^2 + d^2)^(7/2)*(x*e + d)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m}{\left(d^2-e^2\,x^2\right)^{7/2}\,\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3),x)
[Out] int((g*x)^m/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3), x)
```

3.240 $\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=148

$$-\frac{d^5(d^2-e^2x^2)^{1+p}}{2e^6(1+p)}+\frac{d^3(d^2-e^2x^2)^{2+p}}{e^6(2+p)}-\frac{d(d^2-e^2x^2)^{3+p}}{2e^6(3+p)}+\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d^5*(-e^2*x^2+d^2)^(1+p)/e^6/(1+p)+d^3*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-
1/2*d*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+1/7*e*x^7*(-e^2*x^2+d^2)^p*hypergeom([
7/2, -p],[9/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]

time = 0.07, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {778, 272, 45, 372, 371}

$$\frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{d(d^2-e^2x^2)^{p+3}}{2e^6(p+3)}-\frac{d^5(d^2-e^2x^2)^{p+1}}{2e^6(p+1)}+\frac{d^3(d^2-e^2x^2)^{p+2}}{e^6(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^5*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (d^3*(d^2 - e^2*x^2)^(2
+ p))/(e^6*(2 + p)) - (d*(d^2 - e^2*x^2)^(3 + p))/(2*e^6*(3 + p)) + (e*x^7*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (
e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^5(d+ex)\left(d^2-e^2x^2\right)^p\,dx &= d\int x^5\left(d^2-e^2x^2\right)^p\,dx+e\int x^6\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}d\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\\ &= \frac{1}{7}ex^7\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)+\frac{1}{2}d\text{Subst}\left(\int\left(\right.\right.\\ &= -\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{2e^6(1+p)}+\frac{d^3\left(d^2-e^2x^2\right)^{2+p}}{e^6(2+p)}-\frac{d\left(d^2-e^2x^2\right)^{3+p}}{2e^6(3+p)}+\frac{1}{7}ex^7\left(d^2-e^2\right.\end{aligned}$$

Mathematica [A]
time = 0.22, size = 132, normalized size = 0.89

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{7d\left(d^2-e^2x^2\right)\left(2d^4+2d^2e^2(1+p)x^2+e^4\left(2+3p+p^2\right)x^4\right)}{(1+p)(2+p)(3+p)}+2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)\right)}{14e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-7*d*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 +
e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (2*e^7*x^7*Hypergeome
tric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^5(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*x^6*e^6 - (p^2 + p)*d^2*x^4*e^4 - 2*d^4*p*x^2*e^2 - 2*
d^6)*d*e^(p*log(-x^2*e^2 + d^2) - 6)/(p^3 + 6*p^2 + 11*p + 6) + e*integrate
(x^6*e^(p*log(x*e + d) + p*log(-x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^6*e + d*x^5)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(121) = 242$.
time = 2.36, size = 972, normalized size = 6.57

$$d\left(\begin{cases} \frac{x^6 (d^2)^p}{6} & \text{for}\: e = 0 \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2e^4x^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2e^4x^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} & \text{for}\: p = -3 \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{e^4x^4}{-2d^2e^6+2e^8x^2} & \text{for}\: p = -2 \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6} - \frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6} - \frac{d^2x^2}{2e^4} - \frac{x^4}{4e^2} & \text{for}\: p = -1 \\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{e^6p^2x^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{3e^6px^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{2e^6x^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} & \text{otherwise} \end{cases}\right) + \frac{d^{2p}ex^7\,{}_2F_1\left(\begin{matrix}\frac{7}{2}, -p \\ \frac{9}{2}\end{matrix}\middle|\,\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 +
 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*
e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x*
```

```
*2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x*
*2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e*
*8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d*
*2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e
**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e*
*4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e
**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(
p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1
2*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)
**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(
d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e
**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
 + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p*
*2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e*x**7*hyper((7/2,
 -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(d^2-e^2\,x^2\right)}^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x),x)
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.241 $\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=147

$$-\frac{d^4(d^2-e^2x^2)^{1+p}}{2e^5(1+p)}+\frac{d^2(d^2-e^2x^2)^{2+p}}{e^5(2+p)}-\frac{(d^2-e^2x^2)^{3+p}}{2e^5(3+p)}+\frac{1}{5}dx^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d^4*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)+d^2*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-
1/2*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)+1/5*d*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/
2, -p],[7/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {778, 372, 371, 272, 45}

$$\frac{1}{5}dx^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+\frac{d^2(d^2-e^2x^2)^{p+2}}{e^5(p+2)}-\frac{(d^2-e^2x^2)^{p+3}}{2e^5(p+3)}-\frac{d^4(d^2-e^2x^2)^{p+1}}{2e^5(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^4*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) + (d^2*(d^2 - e^2*x^2)^(2
+ p))/(e^5*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^5*(3 + p)) + (d*x^5*(d^2
 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*
x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int x^4\left(d^2-e^2x^2\right)^p\,dx + e\int x^5\left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}e\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)$$
$$= \frac{1}{5}dx^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) + \frac{1}{2}e\text{Subst}\left(\int\left(\right.\right.$$
$$= -\frac{d^4\left(d^2-e^2x^2\right)^{1+p}}{2e^5(1+p)} + \frac{d^2\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)} - \frac{\left(d^2-e^2x^2\right)^{3+p}}{2e^5(3+p)} + \frac{1}{5}dx^5\left(d^2-e^2x\right.$$

Mathematica [A]
time = 0.20, size = 129, normalized size = 0.88

$$\frac{1}{10}\left(d^2-e^2x^2\right)^p\left(-\frac{5\left(d^2-e^2x^2\right)\left(2d^4+2d^2e^2(1+p)x^2+e^4\left(2+3p+p^2\right)x^4\right)}{e^5(1+p)(2+p)(3+p)} + 2dx^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(2*d^4 + 2*d^2*e^2*(1 + p)*x^2 + e^
4*(2 + 3*p + p^2)*x^4))/(e^5*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeome
tric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/10
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)

[Out] int(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")

[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*x^4, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((x^5*e + d*x^4)*(-x^2*e^2 + d^2)^p, x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(119) = 238$.
time = 2.17, size = 972, normalized size = 6.61

$$\frac{dd^{2p}x^5{}_2F_1\left(\begin{matrix}\frac{5}{2}, -p \\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}+e\left(\begin{cases}\frac{x^6(d^2)^p}{6} & \text{for } e=0 \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2e^4x^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2e^4x^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} & \text{for } p=-3 \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{e^4x^4}{-2d^2e^6+2e^8x^2} & \text{for } p=-2 \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6}-\frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6}-\frac{d^2x^2}{2e^4}-\frac{x^4}{4e^2} & \text{for } p=-1 \\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}+\frac{e^6p^2x^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}+\frac{3e^6px^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}+\frac{2e^6x^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**p,x)

[Out] d*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
5 + e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**
4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x*
*2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e
8*x2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d
2*e8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**
8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**

```
8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 +
2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-
2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 +
 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2)
+ e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/
(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2),
 Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 2
2*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3
 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x
**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x*
*4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
 + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(d^2-e^2\,x^2\right)}^p\,\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x),x)
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.242 $\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=120

$$-\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)}+\frac{d\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}+\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d^3*(-e^2*x^2+d^2)^(1+p)/e^4/(1+p)+1/2*d*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p
)+1/5*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/((1-e^2
*x^2/d^2)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {778, 272, 45, 372, 371}

$$\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+\frac{d\left(d^2-e^2x^2\right)^{p+2}}{2e^4(p+2)}-\frac{d^3\left(d^2-e^2x^2\right)^{p+1}}{2e^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^3*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) + (d*(d^2 - e^2*x^2)^(2 +
p))/(2*e^4*(2 + p)) + (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7
/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^3(d+ex)\left(d^2-e^2x^2\right)^p\,dx &= d\int x^3\left(d^2-e^2x^2\right)^p\,dx+e\int x^4\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}d\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\\ &= \frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+\frac{1}{2}d\text{Subst}\left(\int\left(\right.\right.\\ &= -\frac{d^3\left(d^2-e^2x^2\right)^{1+p}}{2e^4(1+p)}+\frac{d\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}+\frac{1}{5}ex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 106, normalized size = 0.88

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{5d\left(d^2-e^2x^2\right)\left(d^2+e^2(1+p)x^2\right)}{(1+p)(2+p)}+2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)\right)}{10e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/((1 + p)
*(2 + p)) + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p))/(10*e^4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)

[Out] int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")

[Out] 1/2*((p + 1)*x^4*e^4 - d^2*p*x^2*e^2 - d^4)*d*e^(p*log(-x^2*e^2 + d^2) - 4)/(p^2 + 3*p + 2) + e*integrate(x^4*e^(p*log(x*e + d) + p*log(-x*e + d)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((x^4*e + d*x^3)*(-x^2*e^2 + d^2)^p, x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(97) = 194.
time = 1.84, size = 382, normalized size = 3.18

$$d\left(\begin{cases} \frac{x^4 (d^2)^p}{4} & \text{for } e = 0 \\ -\frac{d^2 \log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2 \log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2 \log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2 \log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p = -2 \\ -\frac{d^2 \log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2 \log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p = -1 \\ -\frac{d^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4 (d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise} \end{cases}\right) + \frac{d^{2p} e x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**p,x)

[Out] d*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4)

```
- d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4)
+ e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**
4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) + d*
*(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.243 $\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=119

$$-\frac{d^2(d^2-e^2x^2)^{1+p}}{2e^3(1+p)}+\frac{(d^2-e^2x^2)^{2+p}}{2e^3(2+p)}+\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d^2*(-e^2*x^2+d^2)^(1+p)/e^3/(1+p)+1/2*(-e^2*x^2+d^2)^(2+p)/e^3/(2+p)+
1/3*d*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x
^2/d^2)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {778, 372, 371, 272, 45}

$$\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2(d^2-e^2x^2)^{p+1}}{2e^3(p+1)}+\frac{(d^2-e^2x^2)^{p+2}}{2e^3(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^2*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) + (d^2 - e^2*x^2)^(2 + p)/
(2*e^3*(2 + p)) + (d*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2,
(e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^2(d+ex)\left(d^2-e^2x^2\right)^p\,dx &= d\int x^2\left(d^2-e^2x^2\right)^p\,dx+e\int x^3\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}e\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int\\ &= \frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+\frac{1}{2}e\text{Subst}\left(\int\left(\right.\right.\\ &= -\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)}+\frac{1}{3}dx^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2\end{aligned}$$

Mathematica [A]
time = 0.18, size = 103, normalized size = 0.87

$$\frac{1}{6}\left(d^2-e^2x^2\right)^p\left(-\frac{3\left(d^2-e^2x^2\right)\left(d^2+e^2(1+p)x^2\right)}{e^3(1+p)(2+p)}+2dx^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2 + e^2*(1 + p)*x^2))/(e^3*(1 +
p)*(2 + p)) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p))/6
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^3*e + d*x^2)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(95) = 190$.
time = 1.71, size = 382, normalized size = 3.21

$$\frac{dd^{2p}x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}+e\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
3 + e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**
2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*
e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)
), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**
2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e*
```

```
*4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e*
*4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
 e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(d^2-e^2\,x^2\right)}^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x),x)
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.244 $\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=89

$$-\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(1+p)/e^2/(1+p)+1/3*e*x^3*(-e^2*x^2+d^2)^p*hypergeom(
[3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {778, 267, 372, 371}

$$\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d\left(d^2-e^2x^2\right)^{p+1}}{2e^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x(d+ex)\left(d^2-e^2x^2\right)^p\,dx &= d\int x\left(d^2-e^2x^2\right)^p\,dx+e\int x^2\left(d^2-e^2x^2\right)^p\,dx\\ &= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx\\ &= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.13, size = 89, normalized size = 1.00

$$-\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2e^2(1+p)}+\frac{1}{3}ex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int x(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int(x*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2*e^2 + d^2)^(p + 1)*d*e^(-2)/(p + 1) + e*integrate(x^2*e^(p*log(x
*e + d) + p*log(-x*e + d)), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d*x)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [A]

time = 1.45, size = 85, normalized size = 0.96

$$d\left(\begin{cases} \frac{x^2 (d^2)^p}{2} & \text{for}\: e^2 = 0 \\ -\frac{\begin{cases} \frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log\left(d^2 - e^2x^2\right) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right) + \frac{d^{2p} e x^3 {}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| {\frac{e^2 x^2 e^{2i\pi}}{d^2}} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2
)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), Tr
ue)) + d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)
/d**2)/3
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(d^2-e^2\,x^2\right)}^p\,\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x), x)
```

3.245 $\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)}+dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(1+p)/e/(1+p)+d*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],
[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {655, 252, 251}

$$dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/(e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\left(d^2-e^2x^2\right)^p\,dx = -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)} + d\int \left(d^2-e^2x^2\right)^p\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)} + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^p\,dx$$
$$= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)} + dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$

Mathematica [A]
time = 0.15, size = 83, normalized size = 1.00

$$-\frac{\left(d^2-e^2x^2\right)^{1+p}}{2e(1+p)} + dx\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/(e*(1 + p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [A]
time = 1.48, size = 82, normalized size = 0.99

$$dd^{2p}x{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+e\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e
*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)
**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), Tru
e))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p, x)
```

Mupad [B]
time = 4.35, size = 78, normalized size = 0.94

$$\frac{d\,x\,{\left(d^2-e^2\,x^2\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{{\left(1-\frac{e^2\,x^2}{d^2}\right)}^p}-\frac{{\left(d^2-e^2\,x^2\right)}^{p+1}}{2\,e\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x),x)
```

```
[Out] (d*x*(d^2 - e^2*x^2)^p*hypergeom([1/2, -p], 3/2, (e^2*x^2)/d^2))/(1 - (e^2*
x^2)/d^2)^p - (d^2 - e^2*x^2)^(p + 1)/(2*e*(p + 1))
```

3.246 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x}\,dx$

Optimal. Leaf size=104

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d(1+p)}$$

```
[Out] e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2
)^p)-1/2*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)/d/(1+
p)
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {778, 272, 67, 252, 251}

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x}\,dx = d\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx + e\int \left(d^2-e^2x^2\right)^p\,dx$$
$$= \frac{1}{2}d\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \Big(1$$
$$= ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{\left(d^2-e^2x^2\right)^{1+p} {}_2F_1\Big(1}{2d(1}$$

Mathematica [A]
time = 0.15, size = 104, normalized size = 1.00

$$ex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{\left(d^2-e^2x^2\right)^{1+p} {}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 -
 (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(-x^2*e^2 + d^2)^p/x, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.61, size = 78, normalized size = 0.75

$$-\frac{de^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)}+d^{2p}ex\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(
e**2*x**2))/(2*gamma(1 - p)) + d**(2*p)*e*x*hyper((1/2, -p), (3/2,), e**2*x
**2*exp_polar(2*I*pi)/d**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2\,x^2)^p\,(d + e\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x, x)
```

3.247 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

```
[Out] -d*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d
^2)^p)-1/2*e*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)/d
^2/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {778, 372, 371, 272, 67}

$$-\frac{d(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(d^2-e^2x^2\right)^p}{x^2}\,dx = d\int \frac{\left(d^2-e^2x^2\right)^p}{x^2}\,dx + e\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx$$
$$= \frac{1}{2}e\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x, x^2\right) + \left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{(1$$
$$= -\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(1\right.}{2d^2(1}$$

Mathematica [A]
time = 0.18, size = 108, normalized size = 1.00

$$-\frac{d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(-x^2*e^2 + d^2)^p/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.01, size = 82, normalized size = 0.76

$$-\frac{dd^{2p}{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{ee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**2,x)
```

```
[Out] -d*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x -
 e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(
e**2*x**2))/(2*gamma(1 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^2, x)
```

3.248 $\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3} dx$

Optimal. Leaf size=110

$$-\frac{e(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(d^2-e^2x^2)^{1+p} {}_2F_1\left(2,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{2d^3(1+p)}$$

```
[Out] -e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d
^2)^p)-1/2*e^2*(-e^2*x^2+d^2)^(1+p)*hypergeom([2, 1+p],[2+p],1-e^2*x^2/d^2)
/d^3/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {778, 272, 67, 372, 371}

$$-\frac{e(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(d^2-e^2x^2)^{p+1} {}_2F_1\left(2,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -((e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*
(1 - (e^2*x^2)/d^2)^p)) - (e^2*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2,
 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3}\,dx = d\int \frac{(d^2-e^2x^2)^p}{x^3}\,dx + e\int \frac{(d^2-e^2x^2)^p}{x^2}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x^2}\,dx, x, x^2\right) + \left(e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{(1$$

$$= -\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(2\right.}{2d^3(}$$

Mathematica [A]
time = 0.18, size = 106, normalized size = 0.96

$$\frac{1}{2}e\left(d^2-e^2x^2\right)^p\left(-\frac{2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{e(-d^2+e^2x^2)\,{}_2F_1\left(2,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{d^3(1+p)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*((-2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/
(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*Hypergeometric2F1[2, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2])/(d^3*(1 + p))))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)(-e^2x^2+d^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

```
[Out] int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(-x^2*e^2 + d^2)^p/x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.07, size = 85, normalized size = 0.77

$$-\frac{de^{2p}x^{2p}e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2x^2\Gamma(2-p)}-\frac{d^{2p}e_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,),
d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - d**(2*p)*e*hyper((-1/2, -p), (1/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p/x^3, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2\,x^2)^p\,(d + e\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^3,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x))/x^3, x)
```

3.249 $\int x^5(d+ex)^2\,(d^2-e^2x^2)^p\,dx$

Optimal. Leaf size=178

$$-\frac{d^6(d^2-e^2x^2)^{1+p}}{e^6(1+p)}+\frac{5d^4(d^2-e^2x^2)^{2+p}}{2e^6(2+p)}-\frac{2d^2(d^2-e^2x^2)^{3+p}}{e^6(3+p)}+\frac{(d^2-e^2x^2)^{4+p}}{2e^6(4+p)}+\frac{2}{7}dex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)$$

```
[Out] -d^6*(-e^2*x^2+d^2)^(1+p)/e^6/(1+p)+5/2*d^4*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-
2*d^2*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+1/2*(-e^2*x^2+d^2)^(4+p)/e^6/(4+p)+2/7
*d*e*x^7*(-e^2*x^2+d^2)^p*hypergeom([7/2, -p],[9/2],e^2*x^2/d^2)/((1-e^2*x^
2/d^2)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1666, 457, 78, 12, 372, 371}

$$\frac{2}{7}dex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{2d^2(d^2-e^2x^2)^{p+3}}{e^6(p+3)}+\frac{(d^2-e^2x^2)^{p+4}}{2e^6(p+4)}-\frac{d^6(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{5d^4(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^6*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p))) + (5*d^4*(d^2 - e^2*x^2)^(2
+ p))/(2*e^6*(2 + p)) - (2*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (d^
2 - e^2*x^2)^(4 + p)/(2*e^6*(4 + p)) + (2*d*e*x^7*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^5(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= \int 2dex^6\left(d^2-e^2x^2\right)^p\,dx+\int x^5\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^6\left(d^2-e^2x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^6\left(d^2-e^2x\right)^p}{e^4}-\frac{5d^4\left(d^2-e^2x\right)^{1+p}}{e^4}+\frac{4d^2\left(d^2-e^2x\right)^{2+p}}{e^4}-\right.\right.\\ &= -\frac{d^6\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}+\frac{5d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{2d^2\left(d^2-e^2x^2\right)^{3+p}}{e^6(3+p)}+\frac{\left(d^2-e^2x\right.}{2e^6(4+}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 159, normalized size = 0.89

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{14d^6\left(d^2-e^2x^2\right)}{1+p}+\frac{35d^4\left(d^2-e^2x^2\right)^2}{2+p}-\frac{28d^2\left(d^2-e^2x^2\right)^3}{3+p}+\frac{7\left(d^2-e^2x^2\right)^4}{4+p}+4de^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)\right)}{14e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-14*d^6*(d^2 - e^2*x^2))/(1 + p) + (35*d^4*(d^2 - e^2*
x^2)^2)/(2 + p) - (28*d^2*(d^2 - e^2*x^2)^3)/(3 + p) + (7*(d^2 - e^2*x^2)^4
)/(4 + p) + (4*d*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1
 - (e^2*x^2)/d^2)^p))/(14*e^6)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{5}(ex+d)^{2}\left(-e^{2}x^{2}+d^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*x^6*e^6 - (p^2 + p)*d^2*x^4*e^4 - 2*d^4*p*x^2*e^2 - 2*
d^6)*d^2*e^(p*log(-x^2*e^2 + d^2) - 6)/(p^3 + 6*p^2 + 11*p + 6) + integrate
((x^7*e^2 + 2*d*x^6*e)*e^(p*log(x*e + d) + p*log(-x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^7*e^2 + 2*d*x^6*e + d^2*x^5)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(150) = 300.
time = 3.57, size = 2924, normalized size = 16.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e*
*8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2
*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2
*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 +
2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) +
 e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(
2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2),
Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22
*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x*
*2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**
4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
+ e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**
6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6
*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p*
*3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 2*d*d**(2*p)*e*x**7*hype
r((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + e**2*Piecewise((
x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**
4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e + x)/(-
12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 1
1*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*
x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2
 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e + x)/(-1
2*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 27
*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 +
12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e
**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(d/e
 + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x
**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**1
2*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**
4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e +
x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6
), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*
e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**1
2*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4
*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) +
```

```
12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*
x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4)
- 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12
*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4
*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4)
, Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6
*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**1
0*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*
d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e**4*x**
4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*x**2),
Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d*
*4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8
*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**
8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20
*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4*(
d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*
p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e
**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2
 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p +
48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e*
*8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 -
 e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48
*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 +
 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.250 $\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=185

$$-\frac{d^5(d^2-e^2x^2)^{1+p}}{e^5(1+p)}-\frac{x^5(d^2-e^2x^2)^{1+p}}{7+2p}+\frac{2d^3(d^2-e^2x^2)^{2+p}}{e^5(2+p)}-\frac{d(d^2-e^2x^2)^{3+p}}{e^5(3+p)}+\frac{2d^2(6+p)x^5(d^2-e^2x^2)^p\left(1-\right.}{5(7-}$$

```
[Out] -d^5*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)-x^5*(-e^2*x^2+d^2)^(1+p)/(7+2*p)+2*d^3*
(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-d*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)+2/5*d^2*(6+
p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^2)/(7+2*p)/((1-
e^2*x^2/d^2)^p)
```

Rubi [A]

time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1666, 470, 372, 371, 12, 272, 45}

$$\frac{2d^2(p+6)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{x^5(d^2-e^2x^2)^{p+1}}{2p+7}-\frac{d(d^2-e^2x^2)^{p+3}}{e^5(p+3)}-\frac{d^5(d^2-e^2x^2)^{p+1}}{e^5(p+1)}+\frac{2d^3(d^2-e^2x^2)^{p+2}}{e^5(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^5*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p))) - (x^5*(d^2 - e^2*x^2)^(1 +
p))/(7 + 2*p) + (2*d^3*(d^2 - e^2*x^2)^(2 + p))/(e^5*(2 + p)) - (d*(d^2 - e
^2*x^2)^(3 + p))/(e^5*(3 + p)) + (2*d^2*(6 + p)*x^5*(d^2 - e^2*x^2)^p*Hyper
geometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)
^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int 2dex^5\left(d^2-e^2x^2\right)^p\,dx+\int x^4\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx$$

$$=-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+(2de)\int x^5\left(d^2-e^2x^2\right)^p\,dx+\frac{\left(2d^2(6+p)\right)\int x^4(d^2-}{7+2p}$$

$$=-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+(de)\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\frac{\left(2d^2(6+\right.}{}$$

$$=-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{2d^2(6+p)x^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\right.}{5(7+2p)}$$

$$=-\frac{d^5\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{x^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{2d^3\left(d^2-e^2x^2\right)^{2+p}}{e^5(2+p)}-\frac{d(d^2-e^2}{e^5(3+}$$

Mathematica [A]
time = 0.29, size = 186, normalized size = 1.01

$$\frac{1}{35}\left(d^2-e^2x^2\right)^p\left(-\frac{35d^5\left(d^2-e^2x^2\right)}{e^5(1+p)}+\frac{70d^3\left(d^2-e^2x^2\right)^2}{e^5(2+p)}-\frac{35d\left(d^2-e^2x^2\right)^3}{e^5(3+p)}+7d^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+5e^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*d^5*(d^2 - e^2*x^2))/(e^5*(1 + p)) + (70*d^3*(d^2
- e^2*x^2)^2)/(e^5*(2 + p)) - (35*d*(d^2 - e^2*x^2)^3)/(e^5*(3 + p)) + (7*d
^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^
p + (5*e^2*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^
2)/d^2)^p))/35
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4(ex+d)^2\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^6*e^2 + 2*d*x^5*e + d^2*x^4)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(153) = 306$.
time = 2.91, size = 1015, normalized size = 5.49

$$\frac{d^2 d^{2p} x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5} + 2de\left(\begin{cases} \frac{x^6\left(d^2\right)^p}{6} & \text{for}\: e = 0 \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2e^4x^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2e^4x^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} & \text{for}\: p = -3 \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{e^4x^4}{-2d^2e^6+2e^8x^2} & \text{for}\: p = -2 \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6} - \frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6} - \frac{d^2x^2}{2e^4} - \frac{x^4}{4e^2} & \text{for}\: p = -1 \\ -\frac{2d^6\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{2d^4e^2px^2\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4p^2x^4\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4px^4\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{e^6p^2x^6\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{3e^6px^6\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{2e^6x^6\left(d^2-e^2x^2\right)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} & \text{otherwise} \end{cases}\right) + \frac{d^{2p}e^2x^7 {}_2F_1\left(\begin{matrix} \frac{7}{2}, -p \\ \frac{9}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/5 + 2*d*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)
/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*
e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8
*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*
e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*
d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8
*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/
e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4
*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p
**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e*
*6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 -
 e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e*
```

```
*4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 1
2*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 +
 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2
*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)*e**2*x*
*7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.251 $\int x^3(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=149

$$-\frac{d^4(d^2-e^2x^2)^{1+p}}{e^4(1+p)}+\frac{3d^2(d^2-e^2x^2)^{2+p}}{2e^4(2+p)}-\frac{(d^2-e^2x^2)^{3+p}}{2e^4(3+p)}+\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e}{}\right.$$

```
[Out] -d^4*(-e^2*x^2+d^2)^(1+p)/e^4/(1+p)+3/2*d^2*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p)-
1/2*(-e^2*x^2+d^2)^(3+p)/e^4/(3+p)+2/5*d*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([
5/2, -p],[7/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1666, 457, 78, 12, 372, 371}

$$\frac{2}{5}dex^5\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+\frac{3d^2(d^2-e^2x^2)^{p+2}}{2e^4(p+2)}-\frac{(d^2-e^2x^2)^{p+3}}{2e^4(p+3)}-\frac{d^4(d^2-e^2x^2)^{p+1}}{e^4(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^4*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p))) + (3*d^2*(d^2 - e^2*x^2)^(2
+ p))/(2*e^4*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^4*(3 + p)) + (2*d*e*x^
5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 -
 (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^3(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= \int 2dex^4\left(d^2-e^2x^2\right)^p\,dx+\int x^3\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^4\left(d^2-e^2x^2\right)^p\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^4(d^2-e^2x)^p}{e^2}-\frac{3d^2(d^2-e^2x)^{1+p}}{e^2}+\frac{(d^2-e^2x)^{2+p}}{e^2}\right)\,dx,x\right.\\ &= -\frac{d^4(d^2-e^2x^2)^{1+p}}{e^4(1+p)}+\frac{3d^2(d^2-e^2x^2)^{2+p}}{2e^4(2+p)}-\frac{(d^2-e^2x^2)^{3+p}}{2e^4(3+p)}+\frac{2}{5}dex^5\left(d^2-\right.\end{aligned}$$

Mathematica [A]
time = 0.24, size = 138, normalized size = 0.93

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{5\left(d^2-e^2x^2\right)\left(d^4(5+p)+d^2e^2\left(5+6p+p^2\right)x^2+e^4\left(2+3p+p^2\right)x^4\right)}{(1+p)(2+p)(3+p)}+4de^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)\right)}{10e^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]

[Out] ((d^2 - e^2*x^2)^p*((-5*(d^2 - e^2*x^2)*(d^4*(5 + p) + d^2*e^2*(5 + 6*p + p^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (4*d*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(10*e^4)

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3(ex+d)^2\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)

[Out] int(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")

[Out] 1/2*((p + 1)*x^4*e^4 - d^2*p*x^2*e^2 - d^4)*d^2*e^(p*log(-x^2*e^2 + d^2) - 4)/(p^2 + 3*p + 2) + integrate((x^5*e^2 + 2*d*x^4*e)*e^(p*log(x*e + d) + p*log(-x*e + d)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((x^5*e^2 + 2*d*x^4*e + d^2*x^3)*(-x^2*e^2 + d^2)^p, x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(124) = 248.
time = 2.62, size = 1328, normalized size = 8.91

$$d^2\left(\begin{cases} \frac{x^4(d^2)^p}{4} & \text{for}\: e=0 \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for}\: p=-2 \\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for}\: p=-1 \\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise} \end{cases}\right) + \frac{2dd^{2p}ex^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5} + e^2\left(\begin{cases} \frac{x^6(d^2)^p}{6} & \text{for}\: e=0 \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} + \frac{4d^2e^2x^2}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2e^4x^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} - \frac{2e^4x^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} & \text{for}\: p=-3 \\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} - \frac{2d^4}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2} + \frac{e^4x^4}{-2d^2e^6+2e^8x^2} & \text{for}\: p=-2 \\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6} - \frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6} - \frac{d^2x^2}{2e^4} - \frac{x^4}{4e^2} & \text{for}\: p=-1 \\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} - \frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{e^6p^2x^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{3e^6px^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} + \frac{2e^6x^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2
/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2))
, (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2
), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) +
 2*d*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d
**2)/5 + e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x
)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*
d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2
*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2
*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**
2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**
8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d
/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(
4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*
p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e
**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2
- e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e
**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p +
12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(
2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(d^2-e^2\,x^2\right)^p\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.252 $\int x^2(d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=155

$$-\frac{d^3(d^2-e^2x^2)^{1+p}}{e^3(1+p)}-\frac{x^3(d^2-e^2x^2)^{1+p}}{5+2p}+\frac{d(d^2-e^2x^2)^{2+p}}{e^3(2+p)}+\frac{2d^2(4+p)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p\right.}{3(5+2p)}$$

```
[Out] -d^3*(-e^2*x^2+d^2)^(1+p)/e^3/(1+p)-x^3*(-e^2*x^2+d^2)^(1+p)/(5+2*p)+d*(-e^
2*x^2+d^2)^(2+p)/e^3/(2+p)+2/3*d^2*(4+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/
2, -p],[5/2],e^2*x^2/d^2)/(5+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]

time = 0.09, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1666, 470, 372, 371, 12, 272, 45}

$$\frac{2d^2(p+4)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}-\frac{x^3(d^2-e^2x^2)^{p+1}}{2p+5}+\frac{d(d^2-e^2x^2)^{p+2}}{e^3(p+2)}-\frac{d^3(d^2-e^2x^2)^{p+1}}{e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^3*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p))) - (x^3*(d^2 - e^2*x^2)^(1 +
p))/(5 + 2*p) + (d*(d^2 - e^2*x^2)^(2 + p))/(e^3*(2 + p)) + (2*d^2*(4 + p)*
x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5
 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^2(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx=\int 2dex^3\left(d^2-e^2x^2\right)^p\,dx+\int x^2\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx$$

$$=-\frac{x^3(d^2-e^2x^2)^{1+p}}{5+2p}+(2de)\int x^3\left(d^2-e^2x^2\right)^p\,dx+\frac{(2d^2(4+p))\int x^2(d^2-}{5+2p}$$

$$=-\frac{x^3(d^2-e^2x^2)^{1+p}}{5+2p}+(de)\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\,dx,x,x^2\right)+\frac{\Big(2d^2(4+p}{}$$

$$=-\frac{x^3(d^2-e^2x^2)^{1+p}}{5+2p}+\frac{2d^2(4+p)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\right.}{3(5+2p)}$$

$$=-\frac{d^3(d^2-e^2x^2)^{1+p}}{e^3(1+p)}-\frac{x^3(d^2-e^2x^2)^{1+p}}{5+2p}+\frac{d(d^2-e^2x^2)^{2+p}}{e^3(2+p)}+\frac{2d^2(4+p)x^3(}{}$$

Mathematica [A]
time = 0.26, size = 168, normalized size = 1.08

$$\frac{(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(-15d(d^2-e^2x^2)\left(1-\frac{e^2x^2}{d^2}\right)^p (d^2+e^2(1+p)x^2)+5d^2e^3(2+3p+p^2)\,x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+3e^5(2+3p+p^2)\,x^5\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)\right)}{15e^3(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-15*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(d^2 + e^2*
(1 + p)*x^2) + 5*d^2*e^3*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2] + 3*e^5*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2
, (e^2*x^2)/d^2]))/(15*e^3*(1 + p)*(2 + p)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2(ex+d)^2\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4*e^2 + 2*d*x^3*e + d^2*x^2)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(128) = 256$.

time = 2.32, size = 425, normalized size = 2.74

$$\frac{d^2 d^{2p} x^3 {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, -p\\ \frac{5}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3} + 2de\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e = 0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p = -2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4} - \frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p = -1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right) + \frac{d^{2p}e^2x^5 {}_2F_1\left(\left.\begin{matrix}\frac{5}{2}, -p\\ \frac{7}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/3 + 2*d*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(
-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2
) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e*
*4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq
(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2
/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
 + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*
e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4),
True)) + d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*
I*pi)/d**2)/5
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(d^2-e^2x^2\right)^p(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.253 $\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=118

$$-\frac{d^2(d^2-e^2x^2)^{1+p}}{e^2(1+p)}+\frac{(d^2-e^2x^2)^{2+p}}{2e^2(2+p)}+\frac{2}{3}dex^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)$$

```
[Out] -d^2*(-e^2*x^2+d^2)^(1+p)/e^2/(1+p)+1/2*(-e^2*x^2+d^2)^(2+p)/e^2/(2+p)+2/3*
d*e*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/((1-e^2*x^2
/d^2)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1666, 455, 45, 12, 372, 371}

$$\frac{2}{3}dex^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)-\frac{d^2(d^2-e^2x^2)^{p+1}}{e^2(p+1)}+\frac{(d^2-e^2x^2)^{p+2}}{2e^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((d^2*(d^2 - e^2*x^2)^(1 + p))/(e^2*(1 + p))) + (d^2 - e^2*x^2)^(2 + p)/(2
*e^2*(2 + p)) + (2*d*e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2
, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= \int 2dex^2\left(d^2-e^2x^2\right)^p\,dx+\int x\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^2\left(d^2-e^2x^2\right)^p\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int\left(2d^2\left(d^2-e^2x\right)^p-\left(d^2-e^2x\right)^{1+p}\right)\,dx,x,x^2\right)+\left(2de\left(d^2-e^2x\right.\right.\\ &= -\frac{d^2\left(d^2-e^2x^2\right)^{1+p}}{e^2(1+p)}+\frac{\left(d^2-e^2x^2\right)^{2+p}}{2e^2(2+p)}+\frac{2}{3}dex^3\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2\end{aligned}$$

Mathematica [A]
time = 0.18, size = 110, normalized size = 0.93

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{3\left(d^2-e^2x^2\right)\left(d^2(3+p)+e^2(1+p)x^2\right)}{(1+p)(2+p)}+4de^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)\right)}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

[Out] ((d^2 - e^2*x^2)^p*((-3*(d^2 - e^2*x^2)*(d^2*(3 + p) + e^2*(1 + p)*x^2))/((1 + p)*(2 + p)) + (4*d*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(6*e^2)

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(ex+d)^2\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)

[Out] int(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")

[Out] -1/2*(-x^2*e^2 + d^2)^(p + 1)*d^2*e^(-2)/(p + 1) + integrate((x^3*e^2 + 2*d*x^2*e)*e^(p*log(x*e + d) + p*log(-x*e + d)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")

[Out] integral((x^3*e^2 + 2*d*x^2*e + d^2*x)*(-x^2*e^2 + d^2)^p, x)

Sympy [A]
time = 1.97, size = 440, normalized size = 3.73

$$d^2\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(d^2-e^2x^2\right) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{2dd^{2p}ex^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}+e^2\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)

```
[Out] d**2*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x
**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
 True)) + 2*d*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(
2*I*pi)/d**2)/3 + e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-
d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(
-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6
*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e*
*4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e
**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.254 $\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=71

$$-\frac{2^{2+p} d \left(1+\frac{ex}{d}\right)^{-1-p} \left(d^2-e^2x^2\right)^{1+p} {}_2F_1\left(-2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}$$

```
[Out] -2^(2+p)*d*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, -2-p],[2+p
],1/2*(-e*x+d)/d)/e/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {692, 71}

$$-\frac{d 2^{p+2} \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} {}_2F_1\left(-p-2,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((2^(2 + p)*d*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[-2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2 \left(d^2-e^2x^2\right)^p \, dx &= \left(d(d-ex)^{-1-p} \left(1+\frac{ex}{d}\right)^{-1-p} \left(d^2-e^2x^2\right)^{1+p}\right) \int (d-ex)^p \left(1+\frac{ex}{d}\right)^{2+p} \, dx\\ &= -\frac{2^{2+p} d \left(1+\frac{ex}{d}\right)^{-1-p} \left(d^2-e^2x^2\right)^{1+p} {}_2F_1\left(-2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 134, normalized size = 1.89

$$\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-3d(d^2-e^2x^2)\left(1-\frac{e^2x^2}{d^2}\right)^p+3d^2e(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+e^3(1+p)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)\right)}{3e(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
[Out] ((d^2 - e^2*x^2)^p*(-3*d*(d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p + 3*d^2*e*(1
 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + e^3*(1 + p)*x^3*Hy
pergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2]))/(3*e*(1 + p)*(1 - (e^2*x^2)/
d^2)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (ex+d)^2\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [A]
time = 1.80, size = 124, normalized size = 1.75

$$d^2d^{2p}x_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)+2de\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\-\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\\log(d^2-e^2x^2) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right)+\frac{d^{2p}e^2x^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
+ 2*d*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**
2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**
2), True)) + d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_pola
r(2*I*pi)/d**2)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d^2-e^2\,x^2\right)^p\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(d + e*x)^2, x)
```

3.255 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x}\,dx$

Optimal. Leaf size=128

$$-\frac{(d^2-e^2x^2)^{1+p}}{2(1+p)}+2dex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p\right.}{2(1+p)}$$

```
[Out] -1/2*(-e^2*x^2+d^2)^(1+p)/(1+p)+2*d*e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p
],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^(1+p)*hypergeom
([1, 1+p],[2+p],1-e^2*x^2/d^2)/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1666, 457, 81, 67, 12, 252, 251}

$$2dex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/(1 + p) + (2*d*e*x*(d^2 - e^2*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
 p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2\left(d^2-e^2x^2\right)^p}{x}\,dx &= \int 2de\left(d^2-e^2x^2\right)^p\,dx+\int \frac{\left(d^2-e^2x^2\right)^p\left(d^2+e^2x^2\right)}{x}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(d^2+e^2x\right)}{x}\,dx,x,x^2\right)+(2de)\int \left(d^2-e^2x^2\right)^p\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)}+\frac{1}{2}d^2\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx,x,x^2\right)+\Bigg(2de\left(d^2-e^2x^2\right) \\
&= -\frac{\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)}+2dex\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 103, normalized size = 0.80

$$\frac{1}{2}(d^2-e^2x^2)^p\left(4dex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{(d^2-e^2x^2)\left(1+{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)\right)}{1+p}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
)/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)*(1 + Hypergeometric2F1[1, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2]))/(1 + p)))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^2\,(-e^2x^2+d^2)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(-x^2*e^2 + d^2)^p/x, x)
```

Sympy [A]
time = 3.61, size = 136, normalized size = 1.06

$$-\frac{d^{2}e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_{2}F_{1}\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{2\Gamma(1-p)}+2dd^{2p}ex\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)+e^{2}\left(\begin{cases}\frac{x^{2}(d^{2})^{p}}{2} & \text{for } e^{2}=0\\-\frac{\begin{cases}\frac{(d^{2}-e^{2}x^{2})^{p+1}}{p+1} & \text{for } p\neq -1\\\log\left(d^{2}-e^{2}x^{2}\right) & \text{otherwise}\end{cases}}{2e^{2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**
2/(e**2*x**2))/(2*gamma(1 - p)) + 2*d*d**(2*p)*e*x*hyper((1/2, -p), (3/2,),
 e**2*x**2*exp_polar(2*I*pi)/d**2) + e**2*Piecewise((x**2*(d**2)**p/2, Eq(e
**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (lo
g(d**2 - e**2*x**2), True))/(2*e**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x, x)
```

3.256 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2}\,dx$

Optimal. Leaf size=128

$$-\frac{(d^2-e^2x^2)^{1+p}}{x}-2e^2px(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1,1+p;2-\right.}{d(1+p)}$$

```
[Out] -(-e^2*x^2+d^2)^(1+p)/x-2*e^2*p*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2
],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)-e*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p
],[2+p],1-e^2*x^2/d^2)/d/(1+p)
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1821, 778, 272, 67, 252, 251}

$$-2e^2px(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{d(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -((d^2 - e^2*x^2)^(1 + p)/x) - (2*e^2*p*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e*(d^2 - e^2*x^2)
^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2\,(d^2-e^2x^2)^p}{x^2}\,dx &= -\frac{(d^2-e^2x^2)^{1+p}}{x} - \frac{\int \frac{(-2d^3e+2d^2e^2px)\,(d^2-e^2x^2)^p}{x}\,dx}{d^2}\\
&= -\frac{(d^2-e^2x^2)^{1+p}}{x} + (2de)\int \frac{(d^2-e^2x^2)^p}{x}\,dx - \left(2e^2p\right)\int \left(d^2-e^2x^2\right)^p\,dx\\
&= -\frac{(d^2-e^2x^2)^{1+p}}{x} + (de)\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right) - \left(2e^2p\left(d^2-e^2x^2\right.\right.\\
&= -\frac{(d^2-e^2x^2)^{1+p}}{x} - 2e^2px\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 153, normalized size = 1.20

$$\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-d^3(1+p)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)+ex\left(de(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-(d^2-e^2x^2)\left(1-\frac{e^2x^2}{d^2}\right)^p\,{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)\right)\right)}{d(1+p)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-(d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2
)/d^2]) + e*x*(d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
 - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 - (e^2*x^2)/d^2])))/(d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{2} \left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(-x^2*e^2 + d^2)^p/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.51, size = 116, normalized size = 0.91

$$-\frac{d^{2} d^{2p} {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| {\frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}} \right)}{x} - \frac{d e e^{2p} x^{2p} e^{i \pi p} \Gamma\left(-p\right) {}_{2}F_{1}\left(\begin{matrix} -p, -p \\ 1 - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}{\Gamma\left(1 - p\right)} + d^{2p} e^{2} x {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| {\frac{e^{2} x^{2} e^{2 i \pi}}{d^{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**2,x)
```

```
[Out] -d**2*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
x - d*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d
**2/(e**2*x**2))/gamma(1 - p) + d**(2*p)*e**2*x*hyper((1/2, -p), (3/2,), e*
*2*x**2*exp_polar(2*I*pi)/d**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^2, x)
```

3.257 $\int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3}\,dx$

Optimal. Leaf size=139

$$-\frac{(d^2-e^2x^2)^{1+p}}{2x^2}-\frac{2de(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(1-p)\left(d^2-e^2x^2\right)^{1+p}{}_2F_1\left(1,1+p\right.}{2d^2(1+p)}$$

[Out] -1/2*(-e^2*x^2+d^2)^(1+p)/x^2-2*d*e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, -p],[1/2],e^2*x^2/d^2)/x/((1-e^2*x^2/d^2)^p)-1/2*e^2*(1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)/d^2/(1+p)

Rubi [A]
time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1821, 778, 372, 371, 272, 67}

$$-\frac{2de(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{e^2(1-p)\left(d^2-e^2x^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]

[Out] -1/2*(d^2 - e^2*x^2)^(1 + p)/x^2 - (2*d*e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 + p))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^2\,(d^2-e^2x^2)^p}{x^3}\,dx = -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{\int \frac{(-4d^3e-2d^2e^2(1-p)x)\,(d^2-e^2x^2)^p}{x^2}\,dx}{2d^2}$$

$$= -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} + (2de)\int \frac{(d^2-e^2x^2)^p}{x^2}\,dx + \left(e^2(1-p)\right)\int \frac{(d^2-e^2x^2)^p}{x}\,d$$

$$= -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} + \frac{1}{2}\left(e^2(1-p)\right)\mathrm{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x^2\right) + \left(2de(d\right.$$

$$= -\frac{(d^2-e^2x^2)^{1+p}}{2x^2} - \frac{2de(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2}{}$$

Mathematica [A]
time = 0.22, size = 131, normalized size = 0.94

$$\frac{e(d^2-e^2x^2)^p\left(-\frac{4d^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{e(-d^2+e^2x^2)\left({}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)+{}_2F_1\left(2,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)\right)}{1+p}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*((-4*d^3*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (e*(-d^2 + e^2*x^2)*(Hypergeometric2F1[1, 1
 + p, 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e
^2*x^2)/d^2]))/(1 + p)))/(2*d^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{2} \left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

```
[Out] int((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(-x^2*e^2 + d^2)^p/x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.80, size = 139, normalized size = 1.00

$$-\frac{d^{2} e^{2p} x^{2p} e^{i\pi p} \Gamma(1-p)\, {}_{2}F_{1}\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{d^{2}}{e^{2} x^{2}}\right)}{2x^{2}\Gamma\left(2-p\right)} - \frac{2dd^{2p} e_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{e^{2} x^{2} e^{2i\pi}}{d^{2}}\right)}{x} - \frac{e^{2} e^{2p} x^{2p} e^{i\pi p} \Gamma(-p)\, {}_{2}F_{1}\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{d^{2}}{e^{2} x^{2}}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,
), d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - 2*d*d**(2*p)*e*hyper((-1/2, -p
), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - e**2*e**(2*p)*x**(2*p)*exp
(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*gamma(1 -
 p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^2)/x^3, x)
```

3.258 $\int x^5(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=222

$$-\frac{2d^7(d^2-e^2x^2)^{1+p}}{e^6(1+p)}-\frac{ex^7(d^2-e^2x^2)^{1+p}}{9+2p}+\frac{11d^5(d^2-e^2x^2)^{2+p}}{2e^6(2+p)}-\frac{5d^3(d^2-e^2x^2)^{3+p}}{e^6(3+p)}+\frac{3d(d^2-e^2x^2)^{4+p}}{2e^6(4+p)}+\frac{2d^2e(\cdots}{\cdots}$$

```
[Out] -2*d^7*(-e^2*x^2+d^2)^(1+p)/e^6/(1+p)-e*x^7*(-e^2*x^2+d^2)^(1+p)/(9+2*p)+11
/2*d^5*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-5*d^3*(-e^2*x^2+d^2)^(3+p)/e^6/(3+p)+
3/2*d*(-e^2*x^2+d^2)^(4+p)/e^6/(4+p)+2/7*d^2*e*(17+3*p)*x^7*(-e^2*x^2+d^2)^
p*hypergeom([7/2, -p],[9/2],e^2*x^2/d^2)/(9+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1666, 457, 78, 470, 372, 371}

$$\frac{2d^2e(3p+17)x^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7(2p+9)}-\frac{ex^7(d^2-e^2x^2)^{p+1}}{2p+9}+\frac{3d(d^2-e^2x^2)^{p+4}}{2e^6(p+4)}-\frac{2d^7(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{11d^5(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}-\frac{5d^3(d^2-e^2x^2)^{p+3}}{e^6(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^7*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) - (e*x^7*(d^2 - e^2*x^2)^(1
+ p))/(9 + 2*p) + (11*d^5*(d^2 - e^2*x^2)^(2 + p))/(2*e^6*(2 + p)) - (5*d^3
*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (3*d*(d^2 - e^2*x^2)^(4 + p))/(2*
e^6*(4 + p)) + (2*d^2*e*(17 + 3*p)*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[
7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(9 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x^5(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx &= \int x^5\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^6\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\
&= -\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right)\\
&= -\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^7\left(d^2-e^2x\right)^p}{e^4}-\frac{11d^5\left(d^2-e^2x\right)^{1+p}}{e^4}\right.\right.\\
&= -\frac{2d^7\left(d^2-e^2x^2\right)^{1+p}}{e^6(1+p)}-\frac{ex^7\left(d^2-e^2x^2\right)^{1+p}}{9+2p}+\frac{11d^5\left(d^2-e^2x^2\right)^{2+p}}{2e^6(2+p)}-\frac{5d^3\left(d^2\right.}{e^6}
\end{aligned}$$

Mathematica [A]

time = 0.43, size = 205, normalized size = 0.92

$$\frac{(d^2-e^2x^2)^p\left(-\frac{252d^7(d^2-e^2x^2)}{1+p}+\frac{693d^5(d^2-e^2x^2)^2}{2+p}+\frac{189d(d^2-e^2x^2)^4}{4+p}-\frac{630(d^3-de^2x^2)^3}{3+p}+54d^2e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)+14e^9x^9\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{9}{2},-p;\frac{11}{2};\frac{e^2x^2}{d^2}\right)\right)}{126e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-252*d^7*(d^2 - e^2*x^2))/(1 + p) + (693*d^5*(d^2 - e^
2*x^2)^2)/(2 + p) + (189*d*(d^2 - e^2*x^2)^4)/(4 + p) - (630*(d^3 - d*e^2*x
^2)^3)/(3 + p) + (54*d^2*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/
d^2])/(1 - (e^2*x^2)/d^2)^p + (14*e^9*x^9*Hypergeometric2F1[9/2, -p, 11/2,
(e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(126*e^6)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^5(ex+d)^3\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*x^6*e^6 - (p^2 + p)*d^2*x^4*e^4 - 2*d^4*p*x^2*e^2 - 2*
d^6)*d^3*e^(p*log(-x^2*e^2 + d^2) - 6)/(p^3 + 6*p^2 + 11*p + 6) + integrate
((x^8*e^3 + 3*d*x^7*e^2 + 3*d^2*x^6*e)*e^(p*log(x*e + d) + p*log(-x*e + d))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^8*e^3 + 3*d*x^7*e^2 + 3*d^2*x^6*e + d^3*x^5)*(-x^2*e^2 + d^2)^p
, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(190) = 380$.
time = 4.58, size = 2966, normalized size = 13.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**
2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e*
*8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*
x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8
*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2
*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2
*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 +
2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) +
 e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(
2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2),
Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22
*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x*
*2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**
4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
+ e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**
6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6
*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p*
*3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d**2*d**(2*p)*e*x**7*h
yper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + 3*d*e**2*Piec
ewise((x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 +
 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e
+ x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x*
*6) - 11*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12
*e**14*x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**
10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e +
 x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**
6) + 27*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*
x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36
```

```
*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*
log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*
e**14*x**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d*
*2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 +
 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log
(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**
14*x**6), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x*
*2 + 4*e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 +
 4*e**12*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) +
12*d**4*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*
x**4) + 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4
*e**12*x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12
*x**4) - 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 +
4*e**12*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x
**2 + 4*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**1
2*x**4), Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) -
 6*d**6*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 +
 4*e**10*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**
2) + 6*d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e
**4*x**4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4*e**10*
x**2), Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)/(2*e**
8) - d**4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (
-6*d**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 +
100*e**8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p*
*4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2
*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 10
0*e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4
 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**
6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e*
*8*p + 48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4
+ 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*
(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8
*p + 48*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8
*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + ...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*x^5, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(d^2 - e^2 x^2\right)^p (d + e x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^5*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.259 $\int x^4(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=218

$$-\frac{2d^6(d^2-e^2x^2)^{1+p}}{e^5(1+p)}-\frac{3dx^5(d^2-e^2x^2)^{1+p}}{7+2p}+\frac{9d^4(d^2-e^2x^2)^{2+p}}{2e^5(2+p)}-\frac{3d^2(d^2-e^2x^2)^{3+p}}{e^5(3+p)}+\frac{(d^2-e^2x^2)^{4+p}}{2e^5(4+p)}+\frac{2d^3(11+}{}$$

```
[Out] -2*d^6*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)-3*d*x^5*(-e^2*x^2+d^2)^(1+p)/(7+2*p)+
9/2*d^4*(-e^2*x^2+d^2)^(2+p)/e^5/(2+p)-3*d^2*(-e^2*x^2+d^2)^(3+p)/e^5/(3+p)
+1/2*(-e^2*x^2+d^2)^(4+p)/e^5/(4+p)+2/5*d^3*(11+p)*x^5*(-e^2*x^2+d^2)^p*hyp
ergeom([5/2, -p],[7/2],e^2*x^2/d^2)/(7+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1666, 470, 372, 371, 457, 78}

$$-\frac{3dx^5(d^2-e^2x^2)^{p+1}}{2p+7}-\frac{3d^2(d^2-e^2x^2)^{p+3}}{e^5(p+3)}+\frac{(d^2-e^2x^2)^{p+4}}{2e^5(p+4)}-\frac{2d^6(d^2-e^2x^2)^{p+1}}{e^5(p+1)}+\frac{9d^4(d^2-e^2x^2)^{p+2}}{2e^5(p+2)}+\frac{2d^3(p+11)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) - (3*d*x^5*(d^2 - e^2*x^2)^(
1 + p))/(7 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(2 + p))/(2*e^5*(2 + p)) - (3*d^
2*(d^2 - e^2*x^2)^(3 + p))/(e^5*(3 + p)) + (d^2 - e^2*x^2)^(4 + p)/(2*e^5*(
4 + p)) + (2*d^3*(11 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p,
7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^4(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^4\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^5\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx$$

$$=-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^p\left(3d^2e+e^3x\right)\,dx,x,x^2\right)$$

$$=-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^6\left(d^2-e^2x\right)^p}{e^3}-\frac{9d^4\left(d^2-e^2x\right)^{1+p}}{e^3}\right.\right.$$

$$=-\frac{2d^6\left(d^2-e^2x^2\right)^{1+p}}{e^5(1+p)}-\frac{3dx^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{9d^4\left(d^2-e^2x^2\right)^{2+p}}{2e^5(2+p)}-\frac{3d^2(d^2}{e^5}$$

Mathematica [A]

time = 0.41, size = 219, normalized size = 1.00

$$\frac{1}{70}(d^2-e^2x^2)^p\left(-\frac{35(d^2-e^2x^2)\left(6d^6(5+p)+6d^4e^2(5+6p+p^2)\,x^2+3d^2e^4(10+17p+8p^2+p^3)\,x^4+e^6(6+11p+6p^2+p^3)\,x^6\right)}{e^5(1+p)(2+p)(3+p)(4+p)}+14d^3x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+30de^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*(d^2 - e^2*x^2)*(6*d^6*(5 + p) + 6*d^4*e^2*(5 + 6*
p + p^2)*x^2 + 3*d^2*e^4*(10 + 17*p + 8*p^2 + p^3)*x^4 + e^6*(6 + 11*p + 6*
p^2 + p^3)*x^6))/(e^5*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyperg
eometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (30*d*e^2
*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p)
)/70
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4(ex+d)^3\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^7*e^3 + 3*d*x^6*e^2 + 3*d^2*x^5*e + d^3*x^4)*(-x^2*e^2 + d^2)^p
, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1945 vs. $2(185) = 370$.

time = 4.17, size = 2966, normalized size = 13.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/5 + 3*d**2*e*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d*
*2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6
- 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8
*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d*
*2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) -
 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d
**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e
**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(
-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4
/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2
*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**
2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2
*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
+ 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p*
*2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**
3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p
/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d*d**(2*p)*
e**2*x**7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + e*
*3*Piecewise((x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6
*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*l
og(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e
**14*x**6) - 11*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x*
*4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d
**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*lo
g(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e*
*14*x**6) + 27*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2
*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e*
*8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**
4*x**4*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**
```

```
4 + 12*e**14*x**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2
- 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6
*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x
**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 +
 12*e**14*x**6), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e
**10*x**2 + 4*e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10
*x**2 + 4*e**12*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x
**4) + 12*d**4*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4
*e**12*x**4) + 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x
**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 +
4*e**12*x**4) - 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*
x**2 + 4*e**12*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 - 8*d**2*
e**10*x**2 + 4*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 +
 4*e**12*x**4), Eq(p, -3)), (-6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*
x**2) - 6*d**6*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2
*e**8 + 4*e**10*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e*
*10*x**2) + 6*d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3
*d**2*e**4*x**4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8 + 4
*e**10*x**2), Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6*log(d/e + x)
/(2*e**8) - d**4*x**2/(2*e**6) - d**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p,
-1)), (-6*d**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*
p**2 + 100*e**8*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*
e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e*
*4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p*
*2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e*
*8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - d**2*e**6*p
**3*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 +
 100*e**8*p + 48*e**8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**
8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*
p*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 1
00*e**8*p + 48*e**8) + e**8*p**3*x**8*(d**2 - e...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^4*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.260 $\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=193

$$-\frac{2d^5(d^2-e^2x^2)^{1+p}}{e^4(1+p)}-\frac{ex^5(d^2-e^2x^2)^{1+p}}{7+2p}+\frac{7d^3(d^2-e^2x^2)^{2+p}}{2e^4(2+p)}-\frac{3d(d^2-e^2x^2)^{3+p}}{2e^4(3+p)}+\frac{2d^2e(13+3p)x^5(d^2-e^2x^2)}{5}$$

```
[Out] -2*d^5*(-e^2*x^2+d^2)^(1+p)/e^4/(1+p)-e*x^5*(-e^2*x^2+d^2)^(1+p)/(7+2*p)+7/
2*d^3*(-e^2*x^2+d^2)^(2+p)/e^4/(2+p)-3/2*d*(-e^2*x^2+d^2)^(3+p)/e^4/(3+p)+2
/5*d^2*e*(13+3*p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, -p],[7/2],e^2*x^2/d^
2)/(7+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]

time = 0.11, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1666, 457, 78, 470, 372, 371}

$$\frac{2d^2e(3p+13)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5(2p+7)}-\frac{ex^5(d^2-e^2x^2)^{p+1}}{2p+7}-\frac{3d(d^2-e^2x^2)^{p+3}}{2e^4(p+3)}-\frac{2d^5(d^2-e^2x^2)^{p+1}}{e^4(p+1)}+\frac{7d^3(d^2-e^2x^2)^{p+2}}{2e^4(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^5*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) - (e*x^5*(d^2 - e^2*x^2)^(1
+ p))/(7 + 2*p) + (7*d^3*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) - (3*d*(d
^2 - e^2*x^2)^(3 + p))/(2*e^4*(3 + p)) + (2*d^2*e*(13 + 3*p)*x^5*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (
e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^3(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^3\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^4\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx$$

$$=-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right)$$

$$=-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^5\left(d^2-e^2x\right)^p}{e^2}-\frac{7d^3\left(d^2-e^2x\right)^{1+p}}{e^2}\right.\right.$$

$$=-\frac{2d^5\left(d^2-e^2x^2\right)^{1+p}}{e^4(1+p)}-\frac{ex^5\left(d^2-e^2x^2\right)^{1+p}}{7+2p}+\frac{7d^3\left(d^2-e^2x^2\right)^{2+p}}{2e^4(2+p)}-\frac{3d\left(d^2-\right.}{2e^4(}$$

Mathematica [A]

time = 0.34, size = 187, normalized size = 0.97

$$\frac{(d^2-e^2x^2)^p\left(-\frac{35d(d^2-e^2x^2)(d^4(9+p)+d^2e^2(9+10p+p^2)x^2+3e^4(2+3p+p^2)x^4)}{(1+p)(2+p)(3+p)}+42d^2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)+10e^7x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)\right)}{70e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-35*d*(d^2 - e^2*x^2)*(d^4*(9 + p) + d^2*e^2*(9 + 10*p
 + p^2)*x^2 + 3*e^4*(2 + 3*p + p^2)*x^4))/((1 + p)*(2 + p)*(3 + p)) + (42*d
^2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d
^2)^p + (10*e^7*x^7*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(1 - (e
^2*x^2)/d^2)^p))/(70*e^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3(ex+d)^3\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p + 1)*x^4*e^4 - d^2*p*x^2*e^2 - d^4)*d^3*e^(p*log(-x^2*e^2 + d^2) -
4)/(p^2 + 3*p + 2) + integrate((x^6*e^3 + 3*d*x^5*e^2 + 3*d^2*x^4*e)*e^(p*l
og(x*e + d) + p*log(-x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^6*e^3 + 3*d*x^5*e^2 + 3*d^2*x^4*e + d^3*x^3)*(-x^2*e^2 + d^2)^p
, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(165) = 330$.

time = 3.41, size = 1370, normalized size = 7.10

$$d^{3}\left(\begin{cases}\frac{x^{4}(d^{2})^{p}}{4} & \text{for } e=0\\ -\frac{d^{2}\log\left(-\frac{d}{e}+x\right)}{-2d^{2}e^{4}+2e^{6}x^{2}}-\frac{d^{2}\log\left(\frac{d}{e}+x\right)}{-2d^{2}e^{4}+2e^{6}x^{2}}-\frac{d^{2}}{-2d^{2}e^{4}+2e^{6}x^{2}}+\frac{e^{2}x^{2}\log\left(-\frac{d}{e}+x\right)}{-2d^{2}e^{4}+2e^{6}x^{2}}+\frac{e^{2}x^{2}\log\left(\frac{d}{e}+x\right)}{-2d^{2}e^{4}+2e^{6}x^{2}} & \text{for } p=-2\\ -\frac{d^{2}\log\left(-\frac{d}{e}+x\right)}{2e^{4}}-\frac{d^{2}\log\left(\frac{d}{e}+x\right)}{2e^{4}}-\frac{x^{2}}{2e^{2}} & \text{for } p=-1\\ -\frac{d^{4}(d^{2}-e^{2}x^{2})^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}}-\frac{d^{2}e^{2}px^{2}(d^{2}-e^{2}x^{2})^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}}+\frac{e^{4}px^{4}(d^{2}-e^{2}x^{2})^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}}+\frac{e^{4}x^{4}(d^{2}-e^{2}x^{2})^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} & \text{otherwise}\end{cases}\right)+\frac{3d^{2}d^{2p}ex^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{5}+3de^{2}\left(\begin{cases}\frac{x^{6}(d^{2})^{p}}{6} & \text{for } e=0\\ -\frac{2d^{4}\log\left(-\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}-\frac{2d^{4}\log\left(\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}-\frac{3d^{4}}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}+\frac{4d^{2}e^{2}x^{2}\log\left(-\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}+\frac{4d^{2}e^{2}x^{2}\log\left(\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}+\frac{4d^{2}e^{2}x^{2}}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}-\frac{2e^{4}x^{4}\log\left(-\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}}-\frac{2e^{4}x^{4}\log\left(\frac{d}{e}+x\right)}{4d^{4}e^{6}-8d^{2}e^{8}x^{2}+4e^{10}x^{4}} & \text{for } p=-3\\ -\frac{2d^{4}\log\left(-\frac{d}{e}+x\right)}{-2d^{2}e^{6}+2e^{8}x^{2}}-\frac{2d^{4}\log\left(\frac{d}{e}+x\right)}{-2d^{2}e^{6}+2e^{8}x^{2}}-\frac{2d^{4}}{-2d^{2}e^{6}+2e^{8}x^{2}}+\frac{2d^{2}e^{2}x^{2}\log\left(-\frac{d}{e}+x\right)}{-2d^{2}e^{6}+2e^{8}x^{2}}+\frac{2d^{2}e^{2}x^{2}\log\left(\frac{d}{e}+x\right)}{-2d^{2}e^{6}+2e^{8}x^{2}}+\frac{e^{4}x^{4}}{-2d^{2}e^{6}+2e^{8}x^{2}} & \text{for } p=-2\\ -\frac{d^{4}\log\left(-\frac{d}{e}+x\right)}{2e^{6}}-\frac{d^{4}\log\left(\frac{d}{e}+x\right)}{2e^{6}}-\frac{d^{2}x^{2}}{2e^{4}}-\frac{x^{4}}{4e^{2}} & \text{for } p=-1\\ -\frac{2d^{6}(d^{2}-e^{2}x^{2})^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}-\frac{2d^{4}e^{2}px^{2}(d^{2}-e^{2}x^{2})^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}-\frac{d^{2}e^{4}p^{2}x^{4}(d^{2}-e^{2}x^{2})^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}-\frac{d^{2}e^{4}px^{4}(d^{2}-e^{2}x^{2})^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}+\frac{e^{6}p^{2}x^{6}(d^{2}-e^{2}x^{2})^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}+\frac{3e^{6}px^{6}(d^{2}-e^{2}x^{2})^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}}+\frac{2e^{6}x^{6}(d^{2}-e^{2}x^{2})^{p}}{2e^{6}p^{3}+12e^{6}p^{2}+22e^{6}p+12e^{6}} & \text{otherwise}\end{cases}\right)+\frac{d^{2p}e^{3}x^{7}{}_{2}F_{1}\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*
e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2
/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2))
, (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2
), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**
4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) +
e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) +
 3*d**2*d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi
)/d**2)/5 + 3*d*e**2*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-
d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 -
 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*
e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**
6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e*
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(
-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x*
*2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/
(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6
+ 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4
*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) -
 x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 1
2*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)*
*p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x**4
*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) -
 d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e*
*6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e*
*6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**
6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**
2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + d**(2*p)
*e**3*x**7*hyper((7/2, -p), (9/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/7
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^3*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.261 $\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=189

$$-\frac{2d^4(d^2-e^2x^2)^{1+p}}{e^3(1+p)}-\frac{3dx^3(d^2-e^2x^2)^{1+p}}{5+2p}+\frac{5d^2(d^2-e^2x^2)^{2+p}}{2e^3(2+p)}-\frac{(d^2-e^2x^2)^{3+p}}{2e^3(3+p)}+\frac{2d^3(7+p)x^3(d^2-e^2x^2)^p\,(}{3(}$$

```
[Out] -2*d^4*(-e^2*x^2+d^2)^(1+p)/e^3/(1+p)-3*d*x^3*(-e^2*x^2+d^2)^(1+p)/(5+2*p)+
5/2*d^2*(-e^2*x^2+d^2)^(2+p)/e^3/(2+p)-1/2*(-e^2*x^2+d^2)^(3+p)/e^3/(3+p)+2
/3*d^3*(7+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, -p],[5/2],e^2*x^2/d^2)/(5
+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1666, 470, 372, 371, 457, 78}

$$-\frac{3dx^3(d^2-e^2x^2)^{p+1}}{2p+5}+\frac{5d^2(d^2-e^2x^2)^{p+2}}{2e^3(p+2)}-\frac{(d^2-e^2x^2)^{p+3}}{2e^3(p+3)}-\frac{2d^4(d^2-e^2x^2)^{p+1}}{e^3(p+1)}+\frac{2d^3(p+7)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-2*d^4*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) - (3*d*x^3*(d^2 - e^2*x^2)^(
1 + p))/(5 + 2*p) + (5*d^2*(d^2 - e^2*x^2)^(2 + p))/(2*e^3*(2 + p)) - (d^2
- e^2*x^2)^(3 + p)/(2*e^3*(3 + p)) + (2*d^3*(7 + p)*x^3*(d^2 - e^2*x^2)^p*H
ypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5 + 2*p)*(1 - (e^2*x^2)/
d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^2(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx=\int x^2\left(d^2-e^2x^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^3\left(d^2-e^2x^2\right)^p\left(3d^2e+e^3x^2\right)\,dx$$

$$=-\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^p\left(3d^2e+e^3x\right)\,dx,x,x^2\right)$$

$$=-\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{4d^4\left(d^2-e^2x\right)^p}{e}-\frac{5d^2\left(d^2-e^2x\right)^{1+p}}{e}\right.\right.$$

$$=-\frac{2d^4\left(d^2-e^2x^2\right)^{1+p}}{e^3(1+p)}-\frac{3dx^3\left(d^2-e^2x^2\right)^{1+p}}{5+2p}+\frac{5d^2\left(d^2-e^2x^2\right)^{2+p}}{2e^3(2+p)}-\frac{\left(d^2-e^2\right.}{2e^3(3}$$

Mathematica [A]

time = 0.33, size = 187, normalized size = 0.99

$$\frac{1}{30}(d^2-e^2x^2)^p\left(-\frac{15(d^2-e^2x^2)\left(d^4(11+3p)+d^2e^2(11+14p+3p^2)\,x^2+e^4(2+3p+p^2)\,x^4\right)}{e^3(1+p)(2+p)(3+p)}+10d^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+18de^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-15*(d^2 - e^2*x^2)*(d^4*(11 + 3*p) + d^2*e^2*(11 + 14
*p + 3*p^2)*x^2 + e^4*(2 + 3*p + p^2)*x^4))/(e^3*(1 + p)*(2 + p)*(3 + p)) +
 (10*d^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)
/d^2)^p + (18*d*e^2*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(1
- (e^2*x^2)/d^2)^p))/30
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2(ex+d)^3\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^5*e^3 + 3*d*x^4*e^2 + 3*d^2*x^3*e + d^3*x^2)*(-x^2*e^2 + d^2)^p
, x)
```

<u>**Sympy**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(160) = 320$.
time = 3.14, size = 1370, normalized size = 7.25

$$\frac{d^3d^{2p}x^3{}_2F_1\left(\begin{matrix}\frac{3}{2}, -p\\ \frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}+3d^2e\left(\begin{cases}\frac{x^4(d^2)^p}{4} & \text{for } e=0\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}-\frac{d^2}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2}+\frac{e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} & \text{for } p=-2\\ -\frac{d^2\log\left(-\frac{d}{e}+x\right)}{2e^4}-\frac{d^2\log\left(\frac{d}{e}+x\right)}{2e^4}-\frac{x^2}{2e^2} & \text{for } p=-1\\ -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}-\frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4}+\frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} & \text{otherwise}\end{cases}\right)+\frac{3dd^{2p}e^2x^5{}_2F_1\left(\begin{matrix}\frac{5}{2}, -p\\ \frac{7}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{5}+e^3\left(\begin{cases}\frac{x^6(d^2)^p}{6} & \text{for } e=0\\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{3d^4}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}+\frac{4d^2e^2x^2}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2e^4x^4\log\left(-\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4}-\frac{2e^4x^4\log\left(\frac{d}{e}+x\right)}{4d^4e^6-8d^2e^8x^2+4e^{10}x^4} & \text{for } p=-3\\ -\frac{2d^4\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}-\frac{2d^4}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(-\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{2d^2e^2x^2\log\left(\frac{d}{e}+x\right)}{-2d^2e^6+2e^8x^2}+\frac{e^4x^4}{-2d^2e^6+2e^8x^2} & \text{for } p=-2\\ -\frac{d^4\log\left(-\frac{d}{e}+x\right)}{2e^6}-\frac{d^4\log\left(\frac{d}{e}+x\right)}{2e^6}-\frac{d^2x^2}{2e^4}-\frac{x^4}{4e^2} & \text{for } p=-1\\ -\frac{2d^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{2d^4e^2px^2(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4p^2x^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}-\frac{d^2e^4px^4(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}+\frac{e^6p^2x^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}+\frac{3e^6px^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6}+\frac{2e^6x^6(d^2-e^2x^2)^p}{2e^6p^3+12e^6p^2+22e^6p+12e^6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/3 + 3*d**2*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x
)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x
**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2
*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2),
 Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x
**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4
), True)) + 3*d*d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/5 + e**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**
4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*lo
g(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4
*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(
4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e +
 x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e
 + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*
e**8*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/
e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**
2*e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)),
 (-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*
e**4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2
*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*
*2*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*
e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p
/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e
**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x^2*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.262 $\int x(d+ex)^3 \left(d^2-e^2x^2\right)^p \, dx$

Optimal. Leaf size=116

$$-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)}-\frac{3\ 2^{3+p}d^3\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\ {}_2F_1\left(-3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e^2(1+p)(5+2p)}$$

```
[Out] -(e*x+d)^3*(-e^2*x^2+d^2)^(1+p)/e^2/(5+2*p)-3*2^(3+p)*d^3*(1+e*x/d)^(-1-p)*
(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, -3-p],[2+p],1/2*(-e*x+d)/d)/e^2/(2*p^2
+7*p+5)
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {809, 692, 71}

$$-\frac{(d+ex)^3 \left(d^2-e^2x^2\right)^{p+1}}{e^2(2p+5)}-\frac{3d^32^{p+3}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\ {}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e^2(p+1)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(((d + e*x)^3*(d^2 - e^2*x^2)^(1 + p))/(e^2*(5 + 2*p))) - (3*2^(3 + p)*d^3
*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[-3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)])/(e^2*(1 + p)*(5 + 2*p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
```

```
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int x(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx &= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)}+\frac{(3d)\int (d+ex)^3\left(d^2-e^2x^2\right)^p\,dx}{e(5+2p)}\\ &= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)}+\frac{\left(3d^3(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+}\right.}{e(5+2p)}\\ &= -\frac{(d+ex)^3\left(d^2-e^2x^2\right)^{1+p}}{e^2(5+2p)}-\frac{3\ 2^{3+p}d^3\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\ {}_2F_1(-3}{e^2(1+p)(5+2p)}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 159, normalized size = 1.37

$$\frac{\left(d^2-e^2x^2\right)^p\left(-\frac{5d\left(d^2-e^2x^2\right)\left(d^2(5+p)+3e^2(1+p)x^2\right)}{(1+p)(2+p)}+10d^2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+2e^5x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)\right)}{10e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-5*d*(d^2 - e^2*x^2)*(d^2*(5 + p) + 3*e^2*(1 + p)*x^2)
)/((1 + p)*(2 + p)) + (10*d^2*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*
x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2*e^5*x^5*Hypergeometric2F1[5/2, -p, 7/
2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/(10*e^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(ex+d)^3\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2*e^2 + d^2)^(p + 1)*d^3*e^(-2)/(p + 1) + integrate((x^4*e^3 + 3*d
*x^3*e^2 + 3*d^2*x^2*e)*e^(p*log(x*e + d) + p*log(-x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4*e^3 + 3*d*x^3*e^2 + 3*d^2*x^2*e + d^3*x)*(-x^2*e^2 + d^2)^p,
x)
```

Sympy [A]
time = 2.58, size = 479, normalized size = 4.13

$$d^{3}\left(\begin{cases}\frac{x^{2}\left(d^{2}\right)^{p}}{2} & \text{for}\: e^{2} = 0 \\-\frac{\begin{cases}\frac{\left(d^{2}-e^{2}x^{2}\right)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\\log{\left(d^{2}-e^{2}x^{2} \right)} & \text{otherwise}\end{cases}}{2e^{2}} & \text{otherwise}\end{cases}\right) + d^{2}d^{2p}ex^{3}{{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| {\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}} \right)} + 3de^{2}\left(\begin{cases}\frac{x^{4}\left(d^{2}\right)^{p}}{4} & \text{for}\: e = 0 \\-\frac{d^{2}\log{\left(-\frac{d}{e}+x \right)}}{-2d^{2}e^{4}+2e^{6}x^{2}} - \frac{d^{2}\log{\left(\frac{d}{e}+x \right)}}{-2d^{2}e^{4}+2e^{6}x^{2}} - \frac{d^{2}}{-2d^{2}e^{4}+2e^{6}x^{2}} + \frac{e^{2}x^{2}\log{\left(-\frac{d}{e}+x \right)}}{-2d^{2}e^{4}+2e^{6}x^{2}} + \frac{e^{2}x^{2}\log{\left(\frac{d}{e}+x \right)}}{-2d^{2}e^{4}+2e^{6}x^{2}} & \text{for}\: p = -2 \\-\frac{d^{2}\log{\left(-\frac{d}{e}+x \right)}}{2e^{4}} - \frac{d^{2}\log{\left(\frac{d}{e}+x \right)}}{2e^{4}} - \frac{x^{2}}{2e^{2}} & \text{for}\: p = -1 \\-\frac{d^{4}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} - \frac{d^{2}e^{2}px^{2}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} + \frac{e^{4}px^{4}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} + \frac{e^{4}x^{4}\left(d^{2}-e^{2}x^{2}\right)^{p}}{2e^{4}p^{2}+6e^{4}p+4e^{4}} & \text{otherwise}\end{cases}\right) + \frac{d^{2p}e^{3}x^{5}{{}_{2}F_{1}\left(\begin{matrix}\frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| {\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}} \right)}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x
**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2),
 True)) + d**2*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar
(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*lo
g(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4
+ 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x
)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e
**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2
*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p*
*2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 +
6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*
p + 4*e**4), True)) + d**(2*p)*e**3*x**5*hyper((5/2, -p), (7/2,), e**2*x**2
*exp_polar(2*I*pi)/d**2)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^3,x)
[Out] int(x*(d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.263 $\int (d+ex)^3 (d^2-e^2x^2)^p \, dx$

Optimal. Leaf size=73

$$-\frac{2^{3+p}d^2\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(-3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}$$

```
[Out] -2^(3+p)*d^2*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, -3-p],[2
+p],1/2*(-e*x+d)/d)/e/(1+p)
```

Rubi [A]

time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {692, 71}

$$-\frac{d^2 2^{p+3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(-p-3,p+1;p+2;\frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -((2^(3 + p)*d^2*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeomet
ric2F1[-3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3\left(d^2-e^2x^2\right)^p\,dx &= \left(d^2(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{3+p}\,dx\\ &= -\frac{2^{3+p}d^2\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(-3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{e(1+p)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 155 vs. 2(73) = 146.
time = 0.29, size = 155, normalized size = 2.12

$$\frac{1}{2}(d^2-e^2x^2)^p\left(\frac{(-d^2+e^2x^2)\,(d^2(7+3p)+e^2(1+p)x^2)}{e(1+p)(2+p)}+2d^3x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+2de^2x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(((-d^2 + e^2*x^2)*(d^2*(7 + 3*p) + e^2*(1 + p)*x^2))/(e
*(1 + p)*(2 + p)) + (2*d^3*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2]
)/(1 - (e^2*x^2)/d^2)^p + (2*d*e^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2
*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (ex+d)^3\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(-x^2*e^2 + d^2)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(58) = 116$.
time = 2.34, size = 476, normalized size = 6.52

$$d^3 d^{2p} x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right) + 3d^2 e \left(\begin{cases} \frac{x^2 (d^2)^p}{2} & \text{for } e^2 = 0 \\ -\frac{\begin{cases} \frac{(d^2 - e^2 x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2 - e^2 x^2) & \text{otherwise} \end{cases}}{2e^2} & \text{otherwise} \end{cases}\right) + d d^{2p} e^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right) + e^3 \left(\begin{cases} \frac{x^4 (d^2)^p}{4} & \text{for } e = 0 \\ -\frac{d^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} & \text{for } p = -2 \\ -\frac{d^2 \log\left(-\frac{d}{e} + x\right)}{2e^4} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p = -1 \\ -\frac{d^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} - \frac{d^2 e^2 p x^2 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 p x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)
+ 3*d**2*e*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 -
e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*
e**2), True)) + d*d**(2*p)*e**2*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp
_polar(2*I*pi)/d**2) + e**3*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*
log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**
4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e +
 x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/
(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*
p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*
p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2
+ 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**
4*p + 4*e**4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d^2 - e^2\,x^2\right)^p\,\left(d + e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(d + e*x)^3, x)
```

3.264 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x} dx$

Optimal. Leaf size=171

$$-\frac{3d(d^2-e^2x^2)^{1+p}}{2(1+p)}-\frac{ex(d^2-e^2x^2)^{1+p}}{3+2p}+\frac{2d^2e(5+3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{3+2p}-\frac{d(d^2}{}$$

```
[Out] -3/2*d*(-e^2*x^2+d^2)^(1+p)/(1+p)-e*x*(-e^2*x^2+d^2)^(1+p)/(3+2*p)+2*d^2*e*
(5+3*p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/(3+2*p)/(
(1-e^2*x^2/d^2)^p)-1/2*d*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^
2*x^2/d^2)/(1+p)
```

Rubi [A]
time = 0.08, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1666, 457, 81, 67, 396, 252, 251}

$$\frac{2d^2e(3p+5)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{2p+3}-\frac{d(d^2-e^2x^2)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{ex(d^2-e^2x^2)^{p+1}}{2p+3}-\frac{3d(d^2-e^2x^2)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] (-3*d*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (e*x*(d^2 - e^2*x^2)^(1 + p))/
(3 + 2*p) + (2*d^2*e*(5 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -
p, 3/2, (e^2*x^2)/d^2])/((3 + 2*p)*(1 - (e^2*x^2)/d^2)^p) - (d*(d^2 - e^2*x
^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 +
p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^3 (d^2-e^2x^2)^p}{x}\,dx = \int \frac{(d^2-e^2x^2)^p (d^3+3de^2x^2)}{x}\,dx + \int (d^2-e^2x^2)^p (3d^2e+e^3x^2)\,dx$$

$$= -\frac{ex(d^2-e^2x^2)^{1+p}}{3+2p} + \frac{1}{2}\text{Subst}\left(\int \frac{(d^2-e^2x)^p (d^3+3de^2x)}{x}\,dx, x, x^2\right) + \frac{(2d$$

$$= -\frac{3d(d^2-e^2x^2)^{1+p}}{2(1+p)} - \frac{ex(d^2-e^2x^2)^{1+p}}{3+2p} + \frac{1}{2}d^3\text{Subst}\left(\int \frac{(d^2-e^2x)^p}{x}\,dx, x, x$$

$$= -\frac{3d(d^2-e^2x^2)^{1+p}}{2(1+p)} - \frac{ex(d^2-e^2x^2)^{1+p}}{3+2p} + \frac{2d^2e(5+3p)x(d^2-e^2x^2)^p \left(1-\right.}{3+2p}$$

Mathematica [A]
time = 0.29, size = 169, normalized size = 0.99

$$\frac{1}{6}(d^2-e^2x^2)^p \left(-\frac{9d(d^2-e^2x^2)}{1+p} + 18d^2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right) - \frac{3d(d^2-e^2x^2)\,{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)}{1+p} + 2e^3x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-9*d*(d^2 - e^2*x^2))/(1 + p) + (18*d^2*e*x*Hypergeome
tric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*d*(d^2 - e
^2*x^2)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(1 + p) + (2
*e^3*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2
)^p))/6
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3 (-e^2x^2+d^2)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(-x^2*e^2 + d^2)^p/x, x)
```

Sympy [A]
time = 4.67, size = 178, normalized size = 1.04

$$-\frac{d^3 e^{2p} x^{2p} e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + 3d^2 d^{2p} e x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right) + 3de^2\left(\begin{cases}\frac{x^2(d^2)^p}{2} & \text{for } e^2=0\\ -\frac{\begin{cases}\frac{(d^2-e^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(d^2-e^2x^2) & \text{otherwise}\end{cases}}{2e^2} & \text{otherwise}\end{cases}\right) + \frac{d^{2p}e^3x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d**3*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**
2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d**2*d**(2*p)*e*x*hyper((1/2, -p), (3/2
,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**2*(d**2)**p/
2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(d**2 - e**2*x**2), True))/(2*e**2), True)) + d**(2*p)*e**3*x**3*hy
per((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p\,{\left(d+e\,x\right)}^3}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x, x)
```

3.265 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^2}\,dx$

Optimal. Leaf size=159

$$-\frac{e(d^2-e^2x^2)^{1+p}}{2(1+p)}-\frac{d(d^2-e^2x^2)^{1+p}}{x}+2de^2(1-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{3e(d^2$$

```
[Out] -1/2*e*(-e^2*x^2+d^2)^(1+p)/(1+p)-d*(-e^2*x^2+d^2)^(1+p)/x+2*d*e^2*(1-p)*x*
(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
-3/2*e*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)/(1+p)
```

Rubi [A]
time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1821, 1666, 457, 81, 67, 12, 252, 251}

$$2de^2(1-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{3e(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2(p+1)}-\frac{e(d^2-e^2x^2)^{p+1}}{2(p+1)}-\frac{d(d^2-e^2x^2)^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] -1/2*(e*(d^2 - e^2*x^2)^(1 + p))/(1 + p) - (d*(d^2 - e^2*x^2)^(1 + p))/x +
(2*d*e^2*(1 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x
^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometr
ic2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^2}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-3d^4e-2d^3e^2(1-p)x-d^2e^3x^2\right)}{x}\,dx}{d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\int -2d^3e^2(1-p)\left(d^2-e^2x^2\right)^p\,dx}{d^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-3d^4e-d\right.}{x}}{d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} - \frac{\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p\left(-3d^4e-d^2e^3x\right)}{x}\,dx, x, x^2\right)}{2d^2} + \left(2de^2(1-p\right.$$

$$= -\frac{e\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} - \frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} + \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^p}{x}\,dx, x\right.$$

$$= -\frac{e\left(d^2-e^2x^2\right)^{1+p}}{2(1+p)} - \frac{d\left(d^2-e^2x^2\right)^{1+p}}{x} + 2de^2(1-p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right.$$

Mathematica [A]
time = 0.28, size = 158, normalized size = 0.99

$$\frac{\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-2d^3(1+p)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)+ex\left(6de(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(1+3\,{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)\right)\right)\right)}{2(1+p)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^2,x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(-2*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^
2)/d^2] + e*x*(6*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^
2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(1 + 3*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 - (e^2*x^2)/d^2]))))/(2*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\left(-e^2x^2+d^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(-x^2*e^2 + d^2)^p/x^2,
x)
```

Sympy [A]
time = 2.87, size = 177, normalized size = 1.11

$$-\frac{d^{3}d^{2p}{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{x}-\frac{3d^{2}ee^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_{2}F_{1}\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{2\Gamma(1-p)}+3dd^{2p}e^{2}x{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)+e^{3}\left(\begin{cases}\frac{x^{2}(d^{2})^{p}}{2} & \text{for } e^{2}=0\\ -\frac{\begin{cases}\frac{(d^{2}-e^{2}x^{2})^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(d^{2}-e^{2}x^{2}) & \text{otherwise}\end{cases}}{2e^{2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**2,x)
```

```
[Out] -d**3*d**(2*p)*hyper((-1/2, -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/
x - 3*d**2*e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p
,), d**2/(e**2*x**2))/(2*gamma(1 - p)) + 3*d*d**(2*p)*e**2*x*hyper((1/2, -p
), (3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + e**3*Piecewise((x**2*(d**2)*
*p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p,
 -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p\,(d+e\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^2,x)
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^2, x)
```

3.266 $\int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^3}\,dx$

Optimal. Leaf size=166

$$-\frac{d(d^2-e^2x^2)^{1+p}}{2x^2}-\frac{3e(d^2-e^2x^2)^{1+p}}{x}-2e^3(1+3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\frac{e^2(3-$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(1+p)/x^2-3*e*(-e^2*x^2+d^2)^(1+p)/x-2*e^3*(1+3*p)*x*
(-e^2*x^2+d^2)^p*hypergeom([1/2, -p],[3/2],e^2*x^2/d^2)/((1-e^2*x^2/d^2)^p)
-1/2*e^2*(3-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1, 1+p],[2+p],1-e^2*x^2/d^2)
/d/(1+p)
```

Rubi [A]

time = 0.14, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1821, 778, 272, 67, 252, 251}

$$-\frac{e^2(3-p)(d^2-e^2x^2)^{p+1}{}_2F_1\left(1,p+1;p+2;1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)}-\frac{3e(d^2-e^2x^2)^{p+1}}{x}-\frac{d(d^2-e^2x^2)^{p+1}}{2x^2}-2e^3(3p+1)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/x^2 - (3*e*(d^2 - e^2*x^2)^(1 + p))/x - (2
*e^3*(1 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2
)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e^2*(3 - p)*(d^2 - e^2*x^2)^(1 + p)*Hyperg
eometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(d^2-e^2x^2\right)^p}{x^3}\,dx = -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{\int \frac{\left(d^2-e^2x^2\right)^p\left(-6d^4e-2d^3e^2(3-p)x-2d^2e^3x^2\right)}{x^2}\,dx}{2d^2}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\frac{\int \frac{\left(2d^5e^2(3-p)-4d^4e^3(1+3p)x\right)\left(d^2-e^2x^2\right)^p}{x}\,d}{2d^4}$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\left(de^2(3-p)\right)\int \frac{\left(d^2-e^2x^2\right)^p}{x}\,dx-$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}+\frac{1}{2}\left(de^2(3-p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right.}{x}\right.$$

$$= -\frac{d\left(d^2-e^2x^2\right)^{1+p}}{2x^2}-\frac{3e\left(d^2-e^2x^2\right)^{1+p}}{x}-2e^3(1+3p)x\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x}{d^2}\right.$$

Mathematica [A]

time = 0.29, size = 182, normalized size = 1.10

$$\frac{e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-6d^3(1+p)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)+ex\left(2de(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-\left(d^2-e^2x^2\right)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(3\,{}_2F_1\left(1,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)+{}_2F_1\left(2,1+p;2+p;1-\frac{e^2x^2}{d^2}\right)\right)\right)\right)}{2d(1+p)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(d^2 - e^2*x^2)^p)/x^3,x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*(-6*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*
x^2)/d^2] + e*x*(2*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/
d^2] - (d^2 - e^2*x^2)*(1 - (e^2*x^2)/d^2)^p*(3*Hypergeometric2F1[1, 1 + p,
 2 + p, 1 - (e^2*x^2)/d^2] + Hypergeometric2F1[2, 1 + p, 2 + p, 1 - (e^2*x^
2)/d^2]))))/(2*d*(1 + p)*x*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3 (-e^2x^2+d^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)
```

```
[Out] int((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(-x^2*e^2 + d^2)^p/x^3,
x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.33, size = 177, normalized size = 1.07

$$-\frac{d^3e^{2p}x^{2p}e^{i\pi p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2x^2\Gamma(2-p)} - \frac{3d^2d^{2p}e\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\\frac{1}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{x} - \frac{3de^2e^{2p}x^{2p}e^{i\pi p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{2\Gamma(1-p)} + d^{2p}e^3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\\frac{3}{2}\end{matrix}\middle|\frac{e^2x^2e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**p/x**3,x)
```

```
[Out] -d**3*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,
), d**2/(e**2*x**2))/(2*x**2*gamma(2 - p)) - 3*d**2*d**(2*p)*e*hyper((-1/2,
 -p), (1/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - 3*d*e**2*e**(2*p)*x**(2
*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (1 - p,), d**2/(e**2*x**2))/(2*ga
mma(1 - p)) + d**(2*p)*e**3*x*hyper((1/2, -p), (3/2,), e**2*x**2*exp_polar(
2*I*pi)/d**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(-e^2*x^2+d^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p\,\left(d+e\,x\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(d + e*x)^3)/x^3, x)
```

3.267 $\int \frac{x^4(d^2-e^2x^2)^p}{d+ex} dx$

Optimal. Leaf size=148

$$\frac{d^4(d^2-e^2x^2)^p}{2e^5p}-\frac{d^2(d^2-e^2x^2)^{1+p}}{e^5(1+p)}+\frac{(d^2-e^2x^2)^{2+p}}{2e^5(2+p)}+\frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d}$$

```
[Out] 1/2*d^4*(-e^2*x^2+d^2)^p/e^5/p-d^2*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)+1/2*(-e^2
*x^2+d^2)^(2+p)/e^5/(2+p)+1/5*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 1-p],[7/
2],e^2*x^2/d^2)/d/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {864, 778, 372, 371, 272, 45}

$$\frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d}-\frac{d^2(d^2-e^2x^2)^{p+1}}{e^5(p+1)}+\frac{(d^2-e^2x^2)^{p+2}}{2e^5(p+2)}+\frac{d^4(d^2-e^2x^2)^p}{2e^5p}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (d^4*(d^2 - e^2*x^2)^p)/(2*e^5*p) - (d^2*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 +
 p)) + (d^2 - e^2*x^2)^(2 + p)/(2*e^5*(2 + p)) + (x^5*(d^2 - e^2*x^2)^p*Hyp
ergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^2)/d^2])/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(d^2-e^2x^2)^p}{d+ex}\,dx &= \int x^4(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= d\int x^4\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^5\left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= -\left(\frac{1}{2}e\mathrm{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-1+p}\,dx,x,x^2\right)\right) + \frac{\left((d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^{\cdots}}{d}\\ &= \frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d} - \frac{1}{2}e\mathrm{Subst}\left(\int\left(\frac{d^4(d^2-e^2x)^{-\cdots}}{e^4}\cdots\right.\right.\\ &= \frac{d^4(d^2-e^2x^2)^p}{2e^5p} - \frac{d^2(d^2-e^2x^2)^{1+p}}{e^5(1+p)} + \frac{(d^2-e^2x^2)^{2+p}}{2e^5(2+p)} + \frac{x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-\cdots}}{5d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.27, size = 66, normalized size = 0.45

$$\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(5;-p,1-p;6;\frac{ex}{d},-\frac{ex}{d}\right)}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 1 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4(-e^2x^2+d^2)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^4/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^4/(x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*x^4/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^p}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.268 $\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=121

$$-\frac{d^3(d^2-e^2x^2)^p}{2e^4p}+\frac{d(d^2-e^2x^2)^{1+p}}{2e^4(1+p)}-\frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}$$

```
[Out] -1/2*d^3*(-e^2*x^2+d^2)^p/e^4/p+1/2*d*(-e^2*x^2+d^2)^(1+p)/e^4/(1+p)-1/5*e*
x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 1-p],[7/2],e^2*x^2/d^2)/d^2/((1-e^2*x^
2/d^2)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {864, 778, 272, 45, 372, 371}

$$-\frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2}+\frac{d(d^2-e^2x^2)^{p+1}}{2e^4(p+1)}-\frac{d^3(d^2-e^2x^2)^p}{2e^4p}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*(d^3*(d^2 - e^2*x^2)^p)/(e^4*p) + (d*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(
1 + p)) - (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2*
x^2)/d^2])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(d^2-e^2x^2)^p}{d+ex}\,dx &= \int x^3(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx \\
&= d\int x^3\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^4\left(d^2-e^2x^2\right)^{-1+p}\,dx \\
&= \frac{1}{2}d\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-1+p}\,dx,x,x^2\right) - \frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^4\left(1-\right.}{d^2} \\
&= -\frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{1}{2}d\text{Subst}\left(\int\left(\frac{d^2(d^2-e^2x}{e^2}\right.\right. \\
&= -\frac{d^3(d^2-e^2x^2)^p}{2e^4p} + \frac{d(d^2-e^2x^2)^{1+p}}{2e^4(1+p)} - \frac{ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{5}{2},1-p;\frac{7}{2}\right.}{5d^2}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(121) = 242.

time = 0.39, size = 245, normalized size = 2.02

$$\frac{\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(6d^2e(1+p)x\left(1+\frac{ex}{d}\right)^p\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+2e^3(1+p)x^3\left(1+\frac{ex}{d}\right)^p\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)+3d\left(\left(1+\frac{ex}{d}\right)^p\left(-e^2x^2\left(1-\frac{e^2x^2}{d^2}\right)^p+d^2\left(-1+\left(1-\frac{e^2x^2}{d^2}\right)^p\right)\right)+d(d-ex)\left(2-\frac{2e^2x^2}{d^2}\right)^p\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)\right)}{6e^4(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*(6*d^2*e*(1 + p)*x*(1 + (e*x)/d)^p*Hypergeometric2F1[1/2
, -p, 3/2, (e^2*x^2)/d^2] + 2*e^3*(1 + p)*x^3*(1 + (e*x)/d)^p*Hypergeometri
c2F1[3/2, -p, 5/2, (e^2*x^2)/d^2] + 3*d*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e
^2*x^2)/d^2)^p) + d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + d*(d - e*x)*(2 - (2*e
^2*x^2)/d^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(
6*e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^3(-e^2x^2+d^2)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^3/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^3/(x*e + d), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 65.35, size = 5090, normalized size = 42.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d),x)
[Out] Piecewise((3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gam
ma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*g
amma(p + 1)) - 3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p -
1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
- 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*p*acoth(d/(e*x))*gamma(-p - 1/2
)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1
/2)*gamma(p + 1)) + 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p - 1
/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p -
 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(
-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamm
a(-p - 1/2)*gamma(p + 1)) - 6*0**p*d**3*d**(2*p)*acoth(d/(e*x))*gamma(-p -
1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
- 1/2)*gamma(p + 1)) + 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p - 1/2)*gamma(p +
 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p
 + 1)) + 6*0**p*d**2*d**(2*p)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*ga
mma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*
d*d**(2*p)*e**2*p*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*
e**2*x**2*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p +
1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*p*x**3*gam
ma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*g
amma(-p - 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*x**3*gamma(-p - 1/2)*ga
mma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*
gamma(p + 1)) - 3*d**3*d**(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(
p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
 - 1/2)*gamma(p + 1)) - 3*d*d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*e
xp(I*pi*p)*gamma(p)*gamma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1)
+ 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)
*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,),
 d**2/(e**2*x**2))/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p
 - 1/2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p
)*gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(
6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)
), (Abs(e**2*x**2/d**2) > 1) & (Abs(d**2/(e**2*x**2)) > 1)), (3*0**p*d**3*d
**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamm
a(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*
*3*d**(2*p)*p*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)/(6*e*
*4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) -
6*0**p*d**3*d**(2*p)*p*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*
p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 3*0
**p*d**3*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**
4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3
*0**p*d**3*d**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(-p - 1/2)*gamma(p + 1)
```

```
/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p +
1)) - 6*0**p*d**3*d**(2*p)*atanh(d/(e*x))*gamma(-p - 1/2)*gamma(p + 1)/(6*e
**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) +
 6*0**p*d**2*d**(2*p)*e*p*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p
 - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) + 6*0**p*d**2*d
**(2*p)*e*x*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p
+ 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*e**2*p*x**2
*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e*
*4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d*d**(2*p)*e**2*x**2*gamma(-p - 1
/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p -
 1/2)*gamma(p + 1)) + 2*0**p*d**(2*p)*e**3*p*x**3*gamma(-p - 1/2)*gamma(p +
 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p
 + 1)) + 2*0**p*d**(2*p)*e**3*x**3*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*g
amma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*d**3
*d**(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)/(6*
e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1))
- 3*d*d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*ga
mma(-p - 1/2)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/
2)*gamma(p + 1)) - 3*e**3*e**(2*p)*p**2*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*
gamma(-p - 3/2)*hyper((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*
e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1))
- 3*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/2)*hype
r((1 - p, -p - 3/2), (-p - 1/2,), d**2/(e**2*x**2))/(6*e**4*p*gamma(-p - 1/
2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*x^3/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.269 $\int \frac{x^2(d^2-e^2x^2)^p}{d+ex} \, dx$

Optimal. Leaf size=119

$$\frac{d^2(d^2-e^2x^2)^p}{2e^3p} - \frac{(d^2-e^2x^2)^{1+p}}{2e^3(1+p)} + \frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d}$$

```
[Out] 1/2*d^2*(-e^2*x^2+d^2)^p/e^3/p-1/2*(-e^2*x^2+d^2)^(1+p)/e^3/(1+p)+1/3*x^3*(
-e^2*x^2+d^2)^p*hypergeom([3/2, 1-p],[5/2],e^2*x^2/d^2)/d/((1-e^2*x^2/d^2)^
p)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {864, 778, 372, 371, 272, 45}

$$\frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d} + \frac{d^2(d^2-e^2x^2)^p}{2e^3p} - \frac{(d^2-e^2x^2)^{p+1}}{2e^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (d^2*(d^2 - e^2*x^2)^p)/(2*e^3*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^3*(1 + p))
 + (x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2]
)/(3*d*(1 - (e^2*x^2)/d^2)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d^2-e^2x^2)^p}{d+ex}\,dx &= \int x^2(d-ex)\left(d^2-e^2x^2\right)^{-1+p}\,dx \\
&= d\int x^2\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^3\left(d^2-e^2x^2\right)^{-1+p}\,dx \\
&= -\left(\frac{1}{2}e\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-1+p}\,dx,x,x^2\right)\right) + \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(\right.}{d} \\
&= \frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d} - \frac{1}{2}e\text{Subst}\left(\int\left(\frac{d^2(d^2-e^2x)^{-1+}}{e^2}\right.\right. \\
&= \frac{d^2(d^2-e^2x^2)^p}{2e^3p} - \frac{\left(d^2-e^2x^2\right)^{1+p}}{2e^3(1+p)} + \frac{x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right.}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 198, normalized size = 1.66

$$-\frac{\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(\left(1+\frac{ex}{d}\right)^p\left(-e^2x^2\left(1-\frac{e^2x^2}{d^2}\right)^p+d^2\left(-1+\left(1-\frac{e^2x^2}{d^2}\right)^p\right)\right)+2de(1+p)x\left(1+\frac{ex}{d}\right)^p\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+d(d-ex)\left(2-\frac{2e^2x^2}{d^2}\right)^p\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{2e^3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*((d^2 - e^2*x^2)^p*((1 + (e*x)/d)^p*(-(e^2*x^2*(1 - (e^2*x^2)/d^2)^p)
+ d^2*(-1 + (1 - (e^2*x^2)/d^2)^p)) + 2*d*e*(1 + p)*x*(1 + (e*x)/d)^p*Hyper
geometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + d*(d - e*x)*(2 - (2*e^2*x^2)/d^
2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^3*(1 + p)
*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2(-e^2x^2+d^2)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^2/(x*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^2/(x*e + d), x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 12.62, size = 4124, normalized size = 34.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma
(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma
(p + 1)) + 0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*ga
mma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*ga
mma(p + 1)) + 2*0**p*d**2*d**(2*p)*p*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p
+ 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p
+ 1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1
)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1
)) + 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p +
1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p +
1)) + 2*0**p*d**2*d**(2*p)*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*
0**p*d*d**(2*p)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*
gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*x*
gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*
gamma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamma(1/2 - p)*gam
ma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gam
ma(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*
gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d**2*d*
*(2*p)*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)/(2*e**3
*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*
e**(2*p)*p**2*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p,
 -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p
+ 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(
I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/
(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*
gamma(p + 1)) + d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*exp(I*pi*p)*g
amma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma
(1/2 - p)*gamma(p + 1)), (Abs(e**2*x**2/d**2) > 1) & (Abs(d**2/(e**2*x**2))
 > 1)), (-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p
 + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p
 + 1)) + 0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamm
a(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamm
a(p + 1)) + 2*0**p*d**2*d**(2*p)*p*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p +
1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p +
1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2 - p)*gamma(p + 1)/
(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1))
 + 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2) - 1)*gamma(1/2 - p)*gamma(p + 1)
/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)
) + 2*0**p*d**2*d**(2*p)*acoth(d/(e*x))*gamma(1/2 - p)*gamma(p + 1)/(2*e**3
*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0*
*p*d*d**(2*p)*e*p*x*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*ga
mma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*x*ga
```

```
mma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*ga
mma(1/2 - p)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamma(1/2 - p)*gamma
(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma
(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(1/2 - p)*gamma(p + 1)/(2*e**3*p*ga
mma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d**2*d**(
2*p)*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(1/2 - p)/(2*e**3*p*gamma(1/2 -
p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p**2*x
*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/
2 - p,), d**2/(e**2*x**2))/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*g
amma(1/2 - p)*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p
)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2
*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)) +
 d**(2*p)*e**2*p*x**2*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(1/2 - p)/(2*e*
*3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2 - p)*gamma(p + 1)), Abs
(d**2/(e**2*x**2)) > 1), (-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma
(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma
(1/2 - p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*p*log(-d**2/(e**2*x**2) + 1)*g
amma(1/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*g
amma(1/2 - p)*gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*p*atanh(d/(e*x))*gamma(1
/2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1
/2 - p)*gamma(p + 1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(1/2
- p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*gamma(p + 1) + 2*e**3*gamma(1/2
- p)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(1/
2 - p)*gamma(p + 1)/(2*e**3*p*gamma(1/2 - p)*ga...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*x^2/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^p}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.270 $\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=90

$$-\frac{d(d^2-e^2x^2)^p}{2e^2p}-\frac{ex^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^p/e^2/p-1/3*e*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, 1-
p],[5/2],e^2*x^2/d^2)/d^2/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {799, 778, 267, 372, 371}

$$-\frac{ex^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}-\frac{d(d^2-e^2x^2)^p}{2e^2p}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^p)/(e^2*p) - (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometri
c2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(d^2-e^2x^2)^p}{d+ex}\,dx &= \frac{\int x(d^2e-de^2x)\left(d^2-e^2x^2\right)^{-1+p}\,dx}{de}\\ &= d\int x\left(d^2-e^2x^2\right)^{-1+p}\,dx - e\int x^2\left(d^2-e^2x^2\right)^{-1+p}\,dx\\ &= -\frac{d(d^2-e^2x^2)^p}{2e^2p} - \frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int x^2\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d^2}\\ &= -\frac{d(d^2-e^2x^2)^p}{2e^2p} - \frac{ex^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{3}{2},1-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 147, normalized size = 1.63

$$\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{e^2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x),x]
[Out] (2^(-1 + p)*(d^2 - e^2*x^2)^p*(2*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hy
pergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^2*(1 + p)*(1 + (
e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x(-e^2x^2+d^2)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p*x/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p*x/(x*e + d), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.24, size = 427, normalized size = 4.74

$$\begin{cases} \frac{0^{p} d d^{2p} \log\left(\frac{d^{2}}{e^{2}x^{2}}\right)}{2e^{2}} - \frac{0^{p} d d^{2p} \log\left(\frac{d^{2}}{e^{2}x^{2}} - 1\right)}{2e^{2}} - \frac{0^{p} d d^{2p} \operatorname{acoth}\left(\frac{d}{ex}\right)}{e^{2}} + \frac{0^{p} d^{2p} x}{e} - \frac{e^{2p} p x x^{2p} e^{i\pi p} \Gamma(p) \Gamma\left(-p - \frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} 1 - p, -p - \frac{1}{2} \\ \frac{1}{2} - p \end{matrix}\middle| \frac{d^{2}}{e^{2}x^{2}}\right)}{2e\Gamma\left(\frac{1}{2} - p\right)\Gamma(p+1)} - \frac{d^{2p} x^{2} \Gamma(p) \Gamma(1-p) {}_{3}F_{2}\left(\begin{matrix} 2, 1, 1 - p \\ 2, 2 \end{matrix}\middle| \frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{2d\Gamma(-p)\Gamma(p+1)} & \text{for}\: \left|\frac{d^{2}}{e^{2}x^{2}}\right| > 1 \\ \frac{0^{p} d d^{2p} \log\left(\frac{d^{2}}{e^{2}x^{2}}\right)}{2e^{2}} - \frac{0^{p} d d^{2p} \log\left(-\frac{d^{2}}{e^{2}x^{2}} + 1\right)}{2e^{2}} - \frac{0^{p} d d^{2p} \operatorname{atanh}\left(\frac{d}{ex}\right)}{e^{2}} + \frac{0^{p} d^{2p} x}{e} - \frac{e^{2p} p x x^{2p} e^{i\pi p} \Gamma(p) \Gamma\left(-p - \frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} 1 - p, -p - \frac{1}{2} \\ \frac{1}{2} - p \end{matrix}\middle| \frac{d^{2}}{e^{2}x^{2}}\right)}{2e\Gamma\left(\frac{1}{2} - p\right)\Gamma(p+1)} - \frac{d^{2p} x^{2} \Gamma(p) \Gamma(1-p) {}_{3}F_{2}\left(\begin{matrix} 2, 1, 1 - p \\ 2, 2 \end{matrix}\middle| \frac{e^{2}x^{2}e^{2i\pi}}{d^{2}}\right)}{2d\Gamma(-p)\Gamma(p+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d),x)
[Out] Piecewise((0**p*d*d**(2*p)*log(d**2/(e**2*x**2))/(2*e**2) - 0**p*d*d**(2*p)
*log(d**2/(e**2*x**2) - 1)/(2*e**2) - 0**p*d*d**(2*p)*acoth(d/(e*x))/e**2 +
 0**p*d**(2*p)*x/e - e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p -
1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**2))/(2*e*gamma(1/2
- p)*gamma(p + 1)) - d**(2*p)*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p
), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d*gamma(-p)*gamma(p + 1)),
Abs(d**2/(e**2*x**2)) > 1), (0**p*d*d**(2*p)*log(d**2/(e**2*x**2))/(2*e**2)
 - 0**p*d*d**(2*p)*log(-d**2/(e**2*x**2) + 1)/(2*e**2) - 0**p*d*d**(2*p)*at
anh(d/(e*x))/e**2 + 0**p*d**(2*p)*x/e - e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*g
```

```
amma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), d**2/(e**2*x**
2))/(2*e*gamma(1/2 - p)*gamma(p + 1)) - d**(2*p)*x**2*gamma(p)*gamma(1 - p)
*hyper((2, 1, 1 - p), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d*gamma(
-p)*gamma(p + 1)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*x/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d^2-e^2\,x^2\right)}^p}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x),x)
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x), x)
```

3.271 $\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\ {}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

```
[Out] -2^(-1+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 1-p],[2+p],
1/2*(-e*x+d)/d)/d^2/e/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {692, 71}

$$-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\ {}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x),x]
```

```
[Out] -((2^(-1 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx = \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-1+p}dx}{d^2}$$
$$= -\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

Mathematica [A]
time = 0.17, size = 75, normalized size = 1.03

$$-\frac{2^{-1+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{de(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x),x]
[Out] -((2^(-1 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d),x)
[Out] int((-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x*e + d), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.87, size = 321, normalized size = 4.40

$$\begin{cases} \frac{0^p \log\left(-1+\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p \operatorname{acoth}\left(\frac{ex}{d}\right)}{e} + \frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix} 1-p, \frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2e^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^2\Gamma(-p)\Gamma(p+1)} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{0^p \log\left(1-\frac{e^2x^2}{d^2}\right)}{2e} + \frac{0^p \operatorname{atanh}\left(\frac{ex}{d}\right)}{e} + \frac{de^{2p}px^{2p}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix} 1-p, \frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{2e^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{d^{2p}ex^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2d^2\Gamma(-p)\Gamma(p+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] Piecewise((0**p*log(-1 + e**2*x**2/d**2)/(2*e) + 0**p*acoth(e*x/d)/e + d*e*
*(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), d**2/(e**2*x**2))/(2*e**2*x*gamma(3/2 - p)*gamma(p + 1)) + d
**(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2
*exp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), Abs(e**2*x**2/d**
2) > 1), (0**p*log(1 - e**2*x**2/d**2)/(2*e) + 0**p*atanh(e*x/d)/e + d*e**(
2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p),
 (3/2 - p,), d**2/(e**2*x**2))/(2*e**2*x*gamma(3/2 - p)*gamma(p + 1)) + d**
(2*p)*e*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e**2*x**2*e
xp_polar(2*I*pi)/d**2)/(2*d**2*gamma(-p)*gamma(p + 1)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x), x)
```

3.272 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx$

Optimal. Leaf size=104

$$-\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

```
[Out] -e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 1-p],[3/2],e^2*x^2/d^2)/d^2/((1-e^2*x
^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^p*hypergeom([1, p],[1+p],1-e^2*x^2/d^2)/d/p
```

Rubi [A]

time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {864, 778, 272, 67, 252, 251}

$$-\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2}-\frac{(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2dp}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] -((e*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 1 - p, 3/2, (e^2*x^2)/d^2])
/(d^2*(1 - (e^2*x^2)/d^2)^p)) - ((d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p,
1 + p, 1 - (e^2*x^2)/d^2])/(2*d*p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x(d+ex)}\,dx &= \int \frac{(d-ex)\,(d^2-e^2x^2)^{-1+p}}{x}\,dx \\ &= d\int \frac{(d^2-e^2x^2)^{-1+p}}{x}\,dx - e\int \left(d^2-e^2x^2\right)^{-1+p}\,dx \\ &= \frac{1}{2}d\text{Subst}\left(\int \frac{(d^2-e^2x)^{-1+p}}{x}\,dx, x, x^2\right) - \frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \left(1-\frac{e^2x^2}{d^2}\right)^{-}}{d^2} \\ &= -\frac{ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},1-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^2} - \frac{(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;1+p;1-\right.}{2dp}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 151, normalized size = 1.45

$$\frac{2^{-1+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)+d(1+p)\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\,{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)\right)}{d^2p(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] (2^(-1 + p)*(d^2 - e^2*x^2)^p*(p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeome
tric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*(1 + p)*(1/2 + (e*x)/(2*d
))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^2*p*(1 + p)*(1 -
d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x \left(e x +d \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^2*e + d*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.04, size = 355, normalized size = 3.41

$$\begin{cases} - \frac{0^{p} d^{2p} \log{\left(\frac{d^{2}}{e^{2} x^{2}} - 1 \right)}}{2d} - \frac{0^{p} d^{2p} \operatorname{acoth}{\left(\frac{d}{e x} \right)}}{d} + \frac{d e^{2p} p x^{2p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(1 - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, 1 - p \\ 2 - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e^{2} x^{2} \Gamma\left(2 - p\right) \Gamma\left(p + 1\right)} - \frac{e^{2p} p x^{2p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{1}{2} - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{1}{2} - p \\ \frac{3}{2} - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e x \Gamma\left(\frac{3}{2} - p\right) \Gamma\left(p + 1\right)} & \text{for}\: \left|{\frac{d^{2}}{e^{2} x^{2}}}\right| > 1 \\- \frac{0^{p} d^{2p} \log{\left(- \frac{d^{2}}{e^{2} x^{2}} + 1 \right)}}{2d} - \frac{0^{p} d^{2p} \operatorname{atanh}{\left(\frac{d}{e x} \right)}}{d} + \frac{d e^{2p} p x^{2p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(1 - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, 1 - p \\ 2 - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e^{2} x^{2} \Gamma\left(2 - p\right) \Gamma\left(p + 1\right)} - \frac{e^{2p} p x^{2p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{1}{2} - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{1}{2} - p \\ \frac{3}{2} - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e x \Gamma\left(\frac{3}{2} - p\right) \Gamma\left(p + 1\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**(2*p)*log(d**2/(e**2*x**2) - 1)/(2*d) - 0**p*d**(2*p)*a
coth(d/(e*x))/d + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*h
yper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*e**2*x**2*gamma(2 - p)*
gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyp
er((1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2*x**2))/(2*e*x*gamma(3/2 - p)*ga
mma(p + 1)), Abs(d**2/(e**2*x**2)) > 1), (-0**p*d**(2*p)*log(-d**2/(e**2*x*
*2) + 1)/(2*d) - 0**p*d**(2*p)*atanh(d/(e*x))/d + d*e**(2*p)*p*x**(2*p)*exp
(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x
**2))/(2*e**2*x**2*gamma(2 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*p
i*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), d**2/(e**2
*x**2))/(2*e*x*gamma(3/2 - p)*gamma(p + 1)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2\,x^2)^p}{x\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)), x)
```

3.273 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx$

Optimal. Leaf size=106

$$-\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx}+\frac{e(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

```
[Out] -(-e^2*x^2+d^2)^p*hypergeom([-1/2, 1-p],[1/2],e^2*x^2/d^2)/d/x/((1-e^2*x^2/
d^2)^p)+1/2*e*(-e^2*x^2+d^2)^p*hypergeom([1, p],[1+p],1-e^2*x^2/d^2)/d^2/p
```

Rubi [A]
time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {864, 778, 372, 371, 272, 67}

$$\frac{e(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}-\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] -(((d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
*x*(1 - (e^2*x^2)/d^2)^p)) + (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1, p, 1
 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ( !IntegerQ[n] || !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)}\,dx &= \int \frac{(d-ex)\,(d^2-e^2x^2)^{-1+p}}{x^2}\,dx\\ &= d\int \frac{(d^2-e^2x^2)^{-1+p}}{x^2}\,dx - e\int \frac{(d^2-e^2x^2)^{-1+p}}{x}\,dx\\ &= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{(d^2-e^2x)^{-1+p}}{x}\,dx,x,x^2\right)\right)+\frac{\left((d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right.}{x^2}}{d}\\ &= -\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{dx}+\frac{e(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;1+p;1\right.}{2d^2p}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 167, normalized size = 1.58

$$\frac{(d^2-e^2x^2)^p\left(-\frac{2d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{2^pe(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}-\frac{de\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-2*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[1 - p, 1
+ p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (d*e*Hypergeometr
ic2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(2*d^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{2} \left(e x +d \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^3*e + d*x^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.55, size = 450, normalized size = 4.25

$$\begin{cases} - \frac{0^{p} d^{2 p}}{d x} - \frac{0^{p} d^{2 p} e \log{\left(\frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{2}} + \frac{0^{p} d^{2 p} e \log{\left(-1 + \frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{2}} + \frac{0^{p} d^{2 p} e \operatorname{acoth}{\left(\frac{e x}{d} \right)}}{d^{2}} + \frac{d e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{3}{2} - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{3}{2} - p \\ \frac{5}{2} - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e^{2} x^{3} \Gamma\left(\frac{5}{2} - p\right) \Gamma\left(p + 1\right)} - \frac{e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(1 - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, 1 - p \\ 2 - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e x^{2} \Gamma\left(2 - p\right) \Gamma\left(p + 1\right)} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\- \frac{0^{p} d^{2 p}}{d x} - \frac{0^{p} d^{2 p} e \log{\left(\frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{2}} + \frac{0^{p} d^{2 p} e \log{\left(1 - \frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{2}} + \frac{0^{p} d^{2 p} e \operatorname{atanh}{\left(\frac{e x}{d} \right)}}{d^{2}} + \frac{d e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{3}{2} - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{3}{2} - p \\ \frac{5}{2} - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e^{2} x^{3} \Gamma\left(\frac{5}{2} - p\right) \Gamma\left(p + 1\right)} - \frac{e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(1 - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, 1 - p \\ 2 - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e x^{2} \Gamma\left(2 - p\right) \Gamma\left(p + 1\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**(2*p)/(d*x) - 0**p*d**(2*p)*e*log(e**2*x**2/d**2)/(2*d*
*2) + 0**p*d**(2*p)*e*log(-1 + e**2*x**2/d**2)/(2*d**2) + 0**p*d**(2*p)*e*a
coth(e*x/d)/d**2 + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p
)*hyper((1 - p, 3/2 - p), (5/2 - p,), d**2/(e**2*x**2))/(2*e**2*x**3*gamma(
5/2 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1 -
 p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e**2*x**2))/(2*e*x**2*gamma(2 - p
)*gamma(p + 1)), Abs(e**2*x**2/d**2) > 1), (-0**p*d**(2*p)/(d*x) - 0**p*d**
(2*p)*e*log(e**2*x**2/d**2)/(2*d**2) + 0**p*d**(2*p)*e*log(1 - e**2*x**2/d*
*2)/(2*d**2) + 0**p*d**(2*p)*e*atanh(e*x/d)/d**2 + d*e**(2*p)*p*x**(2*p)*ex
p(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,), d**2/
(e**2*x**2))/(2*e**2*x**3*gamma(5/2 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p
)*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), d**2/(e
**2*x**2))/(2*e*x**2*gamma(2 - p)*gamma(p + 1)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2\,x^2)^p}{x^2\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)), x)
```

3.274 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx$

Optimal. Leaf size=108

$$\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x}-\frac{e^2(d^2-e^2x^2)^p\,{}_2F_1\left(2,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

```
[Out] e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 1-p],[1/2],e^2*x^2/d^2)/d^2/x/((1-e^2*x
^2/d^2)^p)-1/2*e^2*(-e^2*x^2+d^2)^p*hypergeom([2, p],[1+p],1-e^2*x^2/d^2)/d
^3/p
```

Rubi [A]

time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {864, 778, 272, 67, 372, 371}

$$\frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x}-\frac{e^2(d^2-e^2x^2)^p\,{}_2F_1\left(2,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^3p}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] (e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2*x^2)/d^2])/(d
^2*x*(1 - (e^2*x^2)/d^2)^p) - (e^2*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2, p
, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^3*p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)}\,dx &= \int \frac{(d-ex)\,(d^2-e^2x^2)^{-1+p}}{x^3}\,dx\\ &= d\int \frac{(d^2-e^2x^2)^{-1+p}}{x^3}\,dx - e\int \frac{(d^2-e^2x^2)^{-1+p}}{x^2}\,dx\\ &= \frac{1}{2}d\text{Subst}\left(\int \frac{(d^2-e^2x)^{-1+p}}{x^2}\,dx, x, x^2\right) - \frac{\left(e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}}{x^2}}{d^2}\\ &= \frac{e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},1-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{e^2(d^2-e^2x^2)^p\,{}_2F_1\left(2,p;1+p;1\right.}{2d^3p}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 219 vs. 2(108) = 216.
time = 0.59, size = 219, normalized size = 2.03

$$\frac{(d^2-e^2x^2)^p\left(\frac{2d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(\frac{d^3\,{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+e^2\left(\frac{\left(2-\frac{2d^2}{e^2x^2}\right)^p(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{d\,{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)\right)\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((2*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + ((d^3*Hypergeometric2F1[1 - p, -p, 2 - p, d^
2/(e^2*x^2)])/((-1 + p)*x^2) + e^2*(((2 - (2*d^2)/(e^2*x^2))^p*(d - e*x)*Hy
pergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (d*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/p))/(1 - d^2/(e
^2*x^2))^p))/(2*d^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{3} \left(e x +d \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^4*e + d*x^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.66, size = 498, normalized size = 4.61

$$\begin{cases} - \frac{0^{p} d^{2 p}}{2 d x^{2}} + \frac{0^{p} d^{2 p} e}{d^{2} x} + \frac{0^{p} d^{2 p} e^{2} \log{\left(\frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{3}} - \frac{0^{p} d^{2 p} e^{2} \log{\left(-1 + \frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{3}} - \frac{0^{p} d^{2 p} e^{2} \operatorname{acoth}{\left(\frac{e x}{d} \right)}}{d^{3}} + \frac{d e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(2 - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, 2 - p \\ 3 - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e^{2} x^{4} \Gamma\left(3 - p\right) \Gamma\left(p + 1\right)} - \frac{e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{3}{2} - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{3}{2} - p \\ \frac{5}{2} - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e x^{3} \Gamma\left(\frac{5}{2} - p\right) \Gamma\left(p + 1\right)} & \text{for}\: \left|{\frac{e^{2} x^{2}}{d^{2}}}\right| > 1 \\ - \frac{0^{p} d^{2 p}}{2 d x^{2}} + \frac{0^{p} d^{2 p} e}{d^{2} x} + \frac{0^{p} d^{2 p} e^{2} \log{\left(\frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{3}} - \frac{0^{p} d^{2 p} e^{2} \log{\left(1 - \frac{e^{2} x^{2}}{d^{2}} \right)}}{2 d^{3}} - \frac{0^{p} d^{2 p} e^{2} \operatorname{atanh}{\left(\frac{e x}{d} \right)}}{d^{3}} + \frac{d e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(2 - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, 2 - p \\ 3 - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e^{2} x^{4} \Gamma\left(3 - p\right) \Gamma\left(p + 1\right)} - \frac{e^{2 p} p x^{2 p} e^{i \pi p} \Gamma\left(p\right) \Gamma\left(\frac{3}{2} - p\right) {{}_{2}F_{1}\left(\begin{matrix} 1 - p, \frac{3}{2} - p \\ \frac{5}{2} - p \end{matrix}\middle| {\frac{d^{2}}{e^{2} x^{2}}} \right)}}{2 e x^{3} \Gamma\left(\frac{5}{2} - p\right) \Gamma\left(p + 1\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d),x)
```

```
[Out] Piecewise((-0**p*d**(2*p)/(2*d*x**2) + 0**p*d**(2*p)*e/(d**2*x) + 0**p*d**(
2*p)*e**2*log(e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*log(-1 + e**2*x
**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*acoth(e*x/d)/d**3 + d*e**(2*p)*p*x*
*(2*p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), d*
*2/(e**2*x**2))/(2*e**2*x**4*gamma(3 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,),
d**2/(e**2*x**2))/(2*e*x**3*gamma(5/2 - p)*gamma(p + 1)), Abs(e**2*x**2/d**
2) > 1), (-0**p*d**(2*p)/(2*d*x**2) + 0**p*d**(2*p)*e/(d**2*x) + 0**p*d**(2
*p)*e**2*log(e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*log(1 - e**2*x**
2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*atanh(e*x/d)/d**3 + d*e**(2*p)*p*x**(
2*p)*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), d**2
/(e**2*x**2))/(2*e**2*x**4*gamma(3 - p)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)
*exp(I*pi*p)*gamma(p)*gamma(3/2 - p)*hyper((1 - p, 3/2 - p), (5/2 - p,), d*
*2/(e**2*x**2))/(2*e*x**3*gamma(5/2 - p)*gamma(p + 1)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)), x)
```

3.275 $\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=179

$$\frac{d^6(d^2-e^2x^2)^{-1+p}}{e^6(1-p)}+\frac{5d^4(d^2-e^2x^2)^p}{2e^6p}-\frac{2d^2(d^2-e^2x^2)^{1+p}}{e^6(1+p)}+\frac{(d^2-e^2x^2)^{2+p}}{2e^6(2+p)}-\frac{2ex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\ {}_2}{7d^3}$$

```
[Out] d^6*(-e^2*x^2+d^2)^(-1+p)/e^6/(1-p)+5/2*d^4*(-e^2*x^2+d^2)^p/e^6/p-2*d^2*(-
e^2*x^2+d^2)^(1+p)/e^6/(1+p)+1/2*(-e^2*x^2+d^2)^(2+p)/e^6/(2+p)-2/7*e*x^7*(
-e^2*x^2+d^2)^p*hypergeom([7/2, 2-p],[9/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2
)^p)
```

Rubi [A]

time = 0.12, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {866, 1666, 457, 78, 12, 372, 371}

$$-\frac{2d^2(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}+\frac{d^6(d^2-e^2x^2)^{p-1}}{e^6(1-p)}+\frac{5d^4(d^2-e^2x^2)^p}{2e^6p}-\frac{2ex^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\ {}_2F_1\left(\frac{7}{2},2-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^6*(d^2 - e^2*x^2)^(-1 + p))/(e^6*(1 - p)) + (5*d^4*(d^2 - e^2*x^2)^p)/(2
*e^6*p) - (2*d^2*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) + (d^2 - e^2*x^2)^(
2 + p)/(2*e^6*(2 + p)) - (2*e*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2,
2 - p, 9/2, (e^2*x^2)/d^2])/(7*d^3*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^5(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int x^5(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= \int -2dex^6\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int x^5\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)\,dx, x, x^2\right) - (2de)\int x^6\left(d^2-e^2x^2\right)^{-2+p}\,d$$
$$= \frac{1}{2}\text{Subst}\left(\int\left(\frac{2d^6(d^2-e^2x)^{-2+p}}{e^4} - \frac{5d^4(d^2-e^2x)^{-1+p}}{e^4} + \frac{4d^2(d^2-e^2x)^p}{e^4} - \frac{(d^2-e^2}{e^4}\right.\right.$$
$$= \frac{d^6(d^2-e^2x^2)^{-1+p}}{e^6(1-p)} + \frac{5d^4(d^2-e^2x^2)^p}{2e^6p} - \frac{2d^2(d^2-e^2x^2)^{1+p}}{e^6(1+p)} + \frac{(d^2-e^2x^2)^{2+p}}{2e^6(2+p)} - \frac{2ex}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.34, size = 66, normalized size = 0.37

$$\frac{x^6(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(6;-p,2-p;7;\frac{ex}{d},-\frac{ex}{d}\right)}{6d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x^6*(d - e*x)^p*(d + e*x)^p*AppellF1[6, -p, 2 - p, 7, (e*x)/d, -((e*x)/d)]
)/(6*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^5(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^5/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^5/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(-(-d+ex)(d+ex))^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x**5*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^5/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int((x^5*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.276 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=184

$$-\frac{d^5(d^2-e^2x^2)^{-1+p}}{e^5(1-p)}-\frac{x^5(d^2-e^2x^2)^{-1+p}}{3+2p}-\frac{2d^3(d^2-e^2x^2)^p}{e^5p}+\frac{d(d^2-e^2x^2)^{1+p}}{e^5(1+p)}+\frac{2(4+p)x^5(d^2-e^2x^2)^p\left(1-\right.}{5d^2(3}$$

```
[Out] -d^5*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)-x^5*(-e^2*x^2+d^2)^(-1+p)/(3+2*p)-2*d^
3*(-e^2*x^2+d^2)^p/e^5/p+d*(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)+2/5*(4+p)*x^5*(-e
^2*x^2+d^2)^p*hypergeom([5/2, 2-p],[7/2],e^2*x^2/d^2)/d^2/(3+2*p)/((1-e^2*x
^2/d^2)^p)
```

Rubi [A]

time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {866, 1666, 470, 372, 371, 12, 272, 45}

$$\frac{2(p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)}-\frac{x^5(d^2-e^2x^2)^{p-1}}{2p+3}+\frac{d(d^2-e^2x^2)^{p+1}}{e^5(p+1)}-\frac{d^5(d^2-e^2x^2)^{p-1}}{e^5(1-p)}-\frac{2d^3(d^2-e^2x^2)^p}{e^5p}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^5*(d^2 - e^2*x^2)^(-1 + p))/(e^5*(1 - p))) - (x^5*(d^2 - e^2*x^2)^(-1
+ p))/(3 + 2*p) - (2*d^3*(d^2 - e^2*x^2)^p)/(e^5*p) + (d*(d^2 - e^2*x^2)^(1
 + p))/(e^5*(1 + p)) + (2*(4 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5
/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^2*(3 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int x^4(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= \int -2dex^5\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int x^4\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx$$
$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - (2de)\int x^5\left(d^2-e^2x^2\right)^{-2+p}\,dx + \frac{(2d^2(4+p))\int x^4(d^2-e^2x}{3+2p}$$
$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - (de)\mathrm{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-2+p}\,dx, x, x^2\right) + \frac{\Big(2(4+p)\,(d^2}{}$$
$$= -\frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} + \frac{2(4+p)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^2(3+2p)}$$
$$= -\frac{d^5\left(d^2-e^2x^2\right)^{-1+p}}{e^5(1-p)} - \frac{x^5\left(d^2-e^2x^2\right)^{-1+p}}{3+2p} - \frac{2d^3(d^2-e^2x^2)^p}{e^5p} + \frac{d(d^2-e^2x^2)^{1+p}}{e^5(1+p)} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.32, size = 66, normalized size = 0.36

$$\frac{x^5(d-ex)^p(d+ex)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}F_1\left(5;-p,2-p;6;\frac{ex}{d},-\frac{ex}{d}\right)}{5d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
[Out] (x^5*(d - e*x)^p*(d + e*x)^p*AppellF1[5, -p, 2 - p, 6, (e*x)/d, -((e*x)/d)]
)/(5*d^2*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p*x^4/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p*x^4/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(-(-d+ex)(d+ex))^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*x^4/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.277 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=150

$$\frac{d^4(d^2-e^2x^2)^{-1+p}}{e^4(1-p)}+\frac{3d^2(d^2-e^2x^2)^p}{2e^4p}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4(1+p)}-\frac{2ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3}$$

```
[Out] d^4*(-e^2*x^2+d^2)^(-1+p)/e^4/(1-p)+3/2*d^2*(-e^2*x^2+d^2)^p/e^4/p-1/2*(-e^
2*x^2+d^2)^(1+p)/e^4/(1+p)-2/5*e*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 2-p],
[7/2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1666, 457, 78, 12, 372, 371}

$$\frac{3d^2(d^2-e^2x^2)^p}{2e^4p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^4(p+1)}+\frac{d^4(d^2-e^2x^2)^{p-1}}{e^4(1-p)}-\frac{2ex^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{5}{2},2-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^4*(d^2 - e^2*x^2)^(-1 + p))/(e^4*(1 - p)) + (3*d^2*(d^2 - e^2*x^2)^p)/(2
*e^4*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^4*(1 + p)) - (2*e*x^5*(d^2 - e^2*x^2
)^p*Hypergeometric2F1[5/2, 2 - p, 7/2, (e^2*x^2)/d^2])/(5*d^3*(1 - (e^2*x^2
)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int x^3(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= \int -2dex^4\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int x^3\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-2+p}\left(d^2+e^2x\right)\,dx, x, x^2\right) - (2de)\int x^4\left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{2d^4(d^2-e^2x)^{-2+p}}{e^2} - \frac{3d^2(d^2-e^2x)^{-1+p}}{e^2} + \frac{(d^2-e^2x)^p}{e^2}\right)\,dx, x, x^2\right) -$$
$$= \frac{d^4(d^2-e^2x^2)^{-1+p}}{e^4(1-p)} + \frac{3d^2(d^2-e^2x^2)^p}{2e^4p} - \frac{(d^2-e^2x^2)^{1+p}}{2e^4(1+p)} - \frac{2ex^5(d^2-e^2x^2)^p\left(1-\frac{e^2}{d}\right.}{5}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(150) = 300.
time = 0.40, size = 332, normalized size = 2.21

$$\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(2d^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p-2d^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^p+2e^2x^2\left(\frac{1}{2}+\frac{ex}{2d}\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^p-8de(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)-6d(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)+d^2\left(1-\frac{e^2x^2}{d^2}\right)^p{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)-dex\left(1-\frac{e^2x^2}{d^2}\right)^p{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{e^4(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*d^2*(1/2 + (e*x)/(2*d))^p - 2*d^2*(1/2 + (
e*x)/(2*d))^p*(1 - (e^2*x^2)/d^2)^p + 2*e^2*x^2*(1/2 + (e*x)/(2*d))^p*(1 -
(e^2*x^2)/d^2)^p - 8*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[
1/2, -p, 3/2, (e^2*x^2)/d^2] - 6*d*(d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d^2*(1 - (e^2*x^2)/d^2)^p
*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - d*e*x*(1 - (e^2*
x^2)/d^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(e^4*
(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^3(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p*x^3/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p*x^3/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(-(-d+ex)(d+ex))^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*x^3/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.278 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2} dx$

Optimal. Leaf size=156

$$-\frac{d^3(d^2-e^2x^2)^{-1+p}}{e^3(1-p)}-\frac{x^3(d^2-e^2x^2)^{-1+p}}{1+2p}-\frac{d(d^2-e^2x^2)^p}{e^3p}+\frac{2(2+p)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},2-\right.}{3d^2(1+2p)}$$

```
[Out] -d^3*(-e^2*x^2+d^2)^(-1+p)/e^3/(1-p)-x^3*(-e^2*x^2+d^2)^(-1+p)/(1+2*p)-d*(-
e^2*x^2+d^2)^p/e^3/p+2/3*(2+p)*x^3*(-e^2*x^2+d^2)^p*hypergeom([3/2, 2-p],[5
/2],e^2*x^2/d^2)/d^2/(1+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {866, 1666, 470, 372, 371, 12, 272, 45}

$$\frac{2(p+2)x^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)}-\frac{x^3(d^2-e^2x^2)^{p-1}}{2p+1}-\frac{d(d^2-e^2x^2)^p}{e^3p}-\frac{d^3(d^2-e^2x^2)^{p-1}}{e^3(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^3*(d^2 - e^2*x^2)^(-1 + p))/(e^3*(1 - p))) - (x^3*(d^2 - e^2*x^2)^(-1
+ p))/(1 + 2*p) - (d*(d^2 - e^2*x^2)^p)/(e^3*p) + (2*(2 + p)*x^3*(d^2 - e^2
*x^2)^p*Hypergeometric2F1[3/2, 2 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 + 2*p)
*(1 - (e^2*x^2)/d^2)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \int x^2(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx$$
$$= \int -2dex^3\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int x^2\left(d^2-e^2x^2\right)^{-2+p}\left(d^2+e^2x^2\right)\,dx$$
$$= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p} - (2de)\int x^3\left(d^2-e^2x^2\right)^{-2+p}\,dx + \frac{(2d^2(2+p))\int x^2(d^2-e^2x}{1+2p}$$
$$= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p} - (de)\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-2+p}\,dx, x, x^2\right) + \frac{\left(2(2+p)\left(d^2-\right.\right.}{}$$
$$= -\frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p} + \frac{2(2+p)x^3(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{3}{2},2-p;\frac{5}{2};\frac{e^2x^2}{d^2}\right)}{3d^2(1+2p)}$$
$$= -\frac{d^3\left(d^2-e^2x^2\right)^{-1+p}}{e^3(1-p)} - \frac{x^3\left(d^2-e^2x^2\right)^{-1+p}}{1+2p} - \frac{d(d^2-e^2x^2)^p}{e^3p} + \frac{2(2+p)x^3(d^2-e^2x^2)}{}$$

Mathematica [A]
time = 0.31, size = 177, normalized size = 1.13

$$\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(4e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(4\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)-{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)\right)}{e^3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(4*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(4
*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometric2F
1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])))/(e^3*(1 + p)*(1 + (e*x)/d)^p*(1
- (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p*x^2/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p*x^2/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}(-(-d+ex)\,(d+ex))^{p}}{(d+ex)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*x^2/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}\,{\left(d^{2}-e^{2}\,x^{2}\right)}^{p}}{{\left(d+e\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.279 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=115

$$\frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2}-\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e^2\,(1-p^2)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(1+p)/e^2/(1-p)/(e*x+d)^2-2^(-1+p)*(1+e*x/d)^(-1-p)*(-e^
2*x^2+d^2)^(1+p)*hypergeom([1+p, 1-p],[2+p],1/2*(-e*x+d)/d)/d^2/e^2/(-p^2+1
)
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {807, 692, 71}

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(1-p)(d+ex)^2}-\frac{2^{p-1}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\,{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^2\,(1-p^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(1 - p)*(d + e*x)^2) - (2^(-1 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^2*e^2*(1 - p^2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^2}\,dx &= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2}+\frac{\int \frac{(d^2-e^2x^2)^p}{d+ex}\,dx}{e(1-p)}\\
&= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2}+\frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right.}{d^2e(1-p)}\\
&= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(1-p)(d+ex)^2}-\frac{2^{-1+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right.}{d^2e^2\left(1-p^2\right)}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 102, normalized size = 0.89

$$\frac{2^{-2+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)+{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{de^2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[1 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-(-d+ex)(d+ex))^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^2, x)
```

3.280 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e(1+p)}$$

```
[Out] -2^(-2+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 2-p],[2+p],
1/2*(-e*x+d)/d)/d^3/e/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {692, 71}

$$-\frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx = \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-2+p}dx}{d^3}$$
$$= -\frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e(1+p)}$$

Mathematica [A]
time = 0.21, size = 75, normalized size = 1.03

$$-\frac{2^{-2+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^2,x]
```

```
[Out] -((2^(-2 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[2 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/(x*e + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^2,x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^2, x)
```

3.281 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx$

Optimal. Leaf size=128

$$\frac{(d^2-e^2x^2)^{-1+p}}{1-p}-\frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;1+p;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}$$

```
[Out] (-e^2*x^2+d^2)^(-1+p)/(1-p)-2*e*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 2-p],[3/
2],e^2*x^2/d^2)/d^3/((1-e^2*x^2/d^2)^p)-1/2*(-e^2*x^2+d^2)^p*hypergeom([1,
p],[1+p],1-e^2*x^2/d^2)/d^2/p
```

Rubi [A]

time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {866, 1666, 457, 80, 67, 12, 252, 251}

$$-\frac{(d^2-e^2x^2)^p\,{}_2F_1\left(1,p;p+1;1-\frac{e^2x^2}{d^2}\right)}{2d^2p}+\frac{(d^2-e^2x^2)^{p-1}}{1-p}-\frac{2ex\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] (d^2 - e^2*x^2)^(-1 + p)/(1 - p) - (2*e*x*(d^2 - e^2*x^2)^p*Hypergeometric2
F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^3*(1 - (e^2*x^2)/d^2)^p) - ((d^2 - e
^2*x^2)^p*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
```

```
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx \\
&= \int -2de\left(d^2-e^2x^2\right)^{-2+p}\,dx + \int \frac{\left(d^2-e^2x^2\right)^{-2+p}(d^2+e^2x^2)}{x}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}(d^2+e^2x)}{x}\,dx, x, x^2\right) - (2de)\int \left(d^2-e^2x^2\right)^{-2+p}\,dx \\
&= \frac{\left(d^2-e^2x^2\right)^{-1+p}}{1-p} + \frac{1}{2}\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-1+p}}{x}\,dx, x, x^2\right) - \frac{\left(2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right.\right.}{} \\
&= \frac{\left(d^2-e^2x^2\right)^{-1+p}}{1-p} - \frac{2ex(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{(d^2-e^2x^2)}{}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 201, normalized size = 1.57

$$
\frac{2^{-2+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(2p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)+p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+2d(1+p)\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)\right)}{d^3p(1+p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] (2^(-2 + p)*(d^2 - e^2*x^2)^p*(2*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + p*(1 - d^2/(e^2*x^2))^p*(
d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d*(1 +
 p)*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))
/(d^3*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{\left(-e^2x^2+d^2\right)^p}{x\left(ex+d\right)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p/(x^3*e^2 + 2*d*x^2*e + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{x\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^2), x)
```

3.282 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx$

Optimal. Leaf size=137

$$-\frac{(d^2-e^2x^2)^{-1+p}}{x}+\frac{2e^2(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{e(d^2-e^2x^2)^{-1+p}\,{}_2F_1\left(1,\cdot\right.}{d(1-p}$$

```
[Out] -(-e^2*x^2+d^2)^(-1+p)/x+2*e^2*(2-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 2-p
],[3/2],e^2*x^2/d^2)/d^4/((1-e^2*x^2/d^2)^p)-e*(-e^2*x^2+d^2)^(-1+p)*hyperg
eom([1, -1+p],[p],1-e^2*x^2/d^2)/d/(1-p)
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1821, 778, 272, 67, 252, 251}

$$-\frac{e(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d(1-p)}-\frac{(d^2-e^2x^2)^{p-1}}{x}+\frac{2e^2(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] -((d^2 - e^2*x^2)^(-1 + p)/x) + (2*e^2*(2 - p)*x*(d^2 - e^2*x^2)^p*Hypergeo
metric2F1[1/2, 2 - p, 3/2, (e^2*x^2)/d^2])/(d^4*(1 - (e^2*x^2)/d^2)^p) - (e
*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2
])/(d*(1 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\,(d^2-e^2x^2)^{-2+p}}{x^2}\,dx \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{x} - \frac{\int \frac{(2d^3e-2d^2e^2(2-p)x)\,(d^2-e^2x^2)^{-2+p}}{x}\,dx}{d^2} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{x} - (2de)\int \frac{(d^2-e^2x^2)^{-2+p}}{x}\,dx + \left(2e^2(2-p)\right)\int \left(d^2-e^2x^2\right)^{-2+p} d \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{x} - (de)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x}\,dx, x, x^2\right) + \frac{\left(2e^2(2-p)\,(d^2-e^2\right.}{} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{x} + \frac{2e^2(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},2-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4} - \frac{e}{}
\end{aligned}
$$

Mathematica [A]
time = 0.46, size = 223, normalized size = 1.63

$$
\frac{(d^2-e^2x^2)^p\left(-\frac{4d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{2^{2+p}e(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{2^pe(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}-\frac{4de\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{4d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-4*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (2^(2 + p)*e*(-d + e*x)*Hypergeometric2F1[1 -
 p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e*(-d
+ e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) - (4*d*e*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p
*(1 - d^2/(e^2*x^2))^p)))/(4*d^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$
\int \frac{(-e^2x^2+d^2)^p}{x^2\,(ex+d)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p/(x^4*e^2 + 2*d*x^3*e + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^2(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{x^2\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^2), x)
```

3.283 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx$

Optimal. Leaf size=143

$$-\frac{(d^2-e^2x^2)^{-1+p}}{2x^2}+\frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x}+\frac{e^2(3-p)\,(d^2-e^2x^2)^{-1+p}\;{}_2F_1\left(1,\right.}{2d^2(1-p)}$$

[Out] -1/2*(-e^2*x^2+d^2)^(-1+p)/x^2+2*e*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 2-p],[1/2],e^2*x^2/d^2)/d^3/x/((1-e^2*x^2/d^2)^p)+1/2*e^2*(3-p)*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e^2*x^2/d^2)/d^2/(1-p)

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1821, 778, 372, 371, 272, 67}

$$\frac{e^2(3-p)\,(d^2-e^2x^2)^{p-1}\;{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}-\frac{(d^2-e^2x^2)^{p-1}}{2x^2}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\;{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]

[Out] -1/2*(d^2 - e^2*x^2)^(-1 + p)/x^2 + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(d^3*x*(1 - (e^2*x^2)/d^2)^p) + (e^2*(3 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 - p))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}}{x^3}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2} - \frac{\int \frac{(4d^3e-2d^2e^2(3-p)x)\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx}{2d^2} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2} - (2de)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x^2}\,dx + \left(e^2(3-p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-2+p}}{x}\,dx \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2} + \frac{1}{2}\left(e^2(3-p)\right)\mathrm{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-2+p}}{x}\,dx, x, x^2\right) - \frac{\left(2e(d^2-e^2\right.}{} \\
&= -\frac{\left(d^2-e^2x^2\right)^{-1+p}}{2x^2} + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^3x} + \frac{e^2(3-p)}{}
\end{aligned}
$$

Mathematica [A]
time = 0.53, size = 283, normalized size = 1.98

$$
\frac{(d^2-e^2x^2)^p\left(\frac{8d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{2d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{3\ 2^{1+p}e^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{2^pe^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{6de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{4d^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2
])/(x*(1 - (e^2*x^2)/d^2)^p) + (2*d^3*Hypergeometric2F1[1 - p, -p, 2 - p, d
^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(1 + p)*e^2*(d -
 e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
+ (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d*e^2*Hypergeometric2F1[-p,
 -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(4*d^5)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$
\int \frac{\left(-e^2x^2+d^2\right)^p}{x^3\left(ex+d\right)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p/(x^5*e^2 + 2*d*x^4*e + d^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d + ex\right)\left(d + ex\right)\right)^{p}}{x^{3}\left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2 - e^2\,x^2\right)^p}{x^3\,\left(d + e\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^2), x)
```

3.284 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{(d^2-e^2x^2)^{-1+p}}{3x^3}-\frac{2e^2(4-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x}-\frac{e^3(d^2-e^2x^2)^{-1+p}\,{}_2F_1\left(2\ldots\right.}{d^3(1-\ldots}$$

```
[Out] -1/3*(-e^2*x^2+d^2)^(-1+p)/x^3-2/3*e^2*(4-p)*(-e^2*x^2+d^2)^p*hypergeom([-1
/2, 2-p],[1/2],e^2*x^2/d^2)/d^4/x/((1-e^2*x^2/d^2)^p)-e^3*(-e^2*x^2+d^2)^(-
1+p)*hypergeom([2, -1+p],[p],1-e^2*x^2/d^2)/d^3/(1-p)
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1821, 778, 272, 67, 372, 371}

$$-\frac{(d^2-e^2x^2)^{p-1}}{3x^3}-\frac{2e^2(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x}-\frac{e^3(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{d^3(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

```
[Out] -1/3*(d^2 - e^2*x^2)^(-1 + p)/x^3 - (2*e^2*(4 - p)*(d^2 - e^2*x^2)^p*Hyperg
eometric2F1[-1/2, 2 - p, 1/2, (e^2*x^2)/d^2])/(3*d^4*x*(1 - (e^2*x^2)/d^2)^
p) - (e^3*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^2
*x^2)/d^2])/(d^3*(1 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\,(d^2-e^2x^2)^{-2+p}}{x^4}\,dx \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} - \frac{\int \frac{(6d^3e-2d^2e^2(4-p)x)\,(d^2-e^2x^2)^{-2+p}}{x^3}\,dx}{3d^2} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} - (2de)\int \frac{(d^2-e^2x^2)^{-2+p}}{x^3}\,dx + \frac{1}{3}\left(2e^2(4-p)\right)\int \frac{(d^2-e^2x^2)^{-2+p}}{x^2} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} - (de)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x^2}\,dx, x, x^2\right) + \frac{\left(2e^2(4-p)\,(d^2-e^2\right.}{} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{3x^3} - \frac{2e^2(4-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},2-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^4x} -
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 334 vs. 2(145) = 290.

time = 0.54, size = 334, normalized size = 2.30

$$
\frac{(d^2-e^2x^2)^p\left(-\frac{4d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3}-\frac{36d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{12d^3e\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{3\,2^{3+p}e^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\,2^pe^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}-\frac{24de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{12d^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-4*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^2
])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (36*d^2*e^2*Hypergeometric2F1[-1/2, -p, 1/
2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (12*d^3*e*Hypergeometric2F1[
1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3
*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/
(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1
[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (24*d*e
^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^
p)))/(12*d^6)
```

Maple [F]

time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{(-e^2x^2+d^2)^p}{x^4\,(ex+d)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p/(x^6*e^2 + 2*d*x^5*e + d^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x^{4}\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^4\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^2), x)
```

3.285 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{(d^2-e^2x^2)^{-1+p}}{4x^4}+\frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3}+\frac{e^4(5-p)\,(d^2-e^2x^2)^{-1+p}\,{}_2F_1\left(\ldots\right.}{4d^4(1-p\ldots}$$

```
[Out] -1/4*(-e^2*x^2+d^2)^(-1+p)/x^4+2/3*e*(-e^2*x^2+d^2)^p*hypergeom([-3/2, 2-p]
,[-1/2],e^2*x^2/d^2)/d^3/x^3/((1-e^2*x^2/d^2)^p)+1/4*e^4*(5-p)*(-e^2*x^2+d^
2)^(-1+p)*hypergeom([2, -1+p],[p],1-e^2*x^2/d^2)/d^4/(1-p)
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1821, 778, 372, 371, 272, 67}

$$-\frac{(d^2-e^2x^2)^{p-1}}{4x^4}+\frac{e^4(5-p)\,(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(2,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{4d^4(1-p)}+\frac{2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

```
[Out] -1/4*(d^2 - e^2*x^2)^(-1 + p)/x^4 + (2*e*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[-3/2, 2 - p, -1/2, (e^2*x^2)/d^2])/(3*d^3*x^3*(1 - (e^2*x^2)/d^2)^p) + (e
^4*(5 - p)*(d^2 - e^2*x^2)^(-1 + p)*Hypergeometric2F1[2, -1 + p, p, 1 - (e^
2*x^2)/d^2])/(4*d^4*(1 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^2}\,dx &= \int \frac{(d-ex)^2\,(d^2-e^2x^2)^{-2+p}}{x^5}\,dx \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{4x^4} - \frac{\int \frac{(8d^3e-2d^2e^2(5-p)x)\,(d^2-e^2x^2)^{-2+p}}{x^4}\,dx}{4d^2} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{4x^4} - (2de)\int \frac{(d^2-e^2x^2)^{-2+p}}{x^4}\,dx + \frac{1}{2}\left(e^2(5-p)\right)\int \frac{(d^2-e^2x^2)^{-2+p}}{x^3}\,d \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{4x^4} + \frac{1}{4}\left(e^2(5-p)\right)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x^2}\,dx, x, x^2\right) - \frac{\Big(2e(d^2-e^{\cdots}}{\cdots} \\
&= -\frac{(d^2-e^2x^2)^{-1+p}}{4x^4} + \frac{2e(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},2-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^3x^3} + \frac{e^4(5-\cdots}{\cdots}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(145) = 290.
time = 0.60, size = 389, normalized size = 2.68

$$
\frac{(d^2-e^2x^2)^p\left(\frac{8d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3}+\frac{48d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{18d^3e^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{15\ 2^{1+p}e^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{6d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right)}{(-2+p)x^4}+\frac{3\ 2^pe^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{30de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{12d^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (48*d^2*e^3*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (18*d^3*e^2*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
 (15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (6*d^5*Hypergeometric2F1[2 - p, -p,
3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (3*2^p*e^4*(d
 - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(
1 + (e*x)/d)^p) + (30*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]
)/(p*(1 - d^2/(e^2*x^2))^p)))/(12*d^7)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{(-e^2x^2+d^2)^p}{x^5\,(ex+d)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x^5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p/(x^7*e^2 + 2*d*x^6*e + d^2*x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^5(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**2,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^2*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{x^5\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2),x)
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^2), x)
```

3.286 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=220

$$-\frac{2d^6(d^2-e^2x^2)^{-2+p}}{e^5(2-p)}-\frac{3dx^5(d^2-e^2x^2)^{-2+p}}{1+2p}+\frac{9d^4(d^2-e^2x^2)^{-1+p}}{2e^5(1-p)}+\frac{3d^2(d^2-e^2x^2)^p}{e^5p}-\frac{(d^2-e^2x^2)^{1+p}}{2e^5(1+p)}+\frac{2(8+\dots}{}$$

```
[Out] -2*d^6*(-e^2*x^2+d^2)^(-2+p)/e^5/(2-p)-3*d*x^5*(-e^2*x^2+d^2)^(-2+p)/(1+2*p
)+9/2*d^4*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)+3*d^2*(-e^2*x^2+d^2)^p/e^5/p-1/2*
(-e^2*x^2+d^2)^(1+p)/e^5/(1+p)+2/5*(8+p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/
2, 3-p],[7/2],e^2*x^2/d^2)/d^3/(1+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1666, 470, 372, 371, 457, 78}

$$-\frac{3dx^5(d^2-e^2x^2)^{p-2}}{2p+1}+\frac{3d^2(d^2-e^2x^2)^p}{e^5p}-\frac{(d^2-e^2x^2)^{p+1}}{2e^5(p+1)}-\frac{2d^6(d^2-e^2x^2)^{p-2}}{e^5(2-p)}+\frac{9d^4(d^2-e^2x^2)^{p-1}}{2e^5(1-p)}+\frac{2(p+8)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (-2*d^6*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (3*d*x^5*(d^2 - e^2*x^2)^
(-2 + p))/(1 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(-1 + p))/(2*e^5*(1 - p)) + (3
*d^2*(d^2 - e^2*x^2)^p)/(e^5*p) - (d^2 - e^2*x^2)^(1 + p)/(2*e^5*(1 + p)) +
 (2*(8 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^3*(1 + 2*p)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^3}\,dx &= \int x^4(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx \\
&= \int x^4\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)\,dx + \int x^5\left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)\,dx \\
&= -\frac{3dx^5(d^2-e^2x^2)^{-2+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-3+p}\left(-3d^2e-e^3x\right)\,dx, x, x^2\right) + \\
&= -\frac{3dx^5(d^2-e^2x^2)^{-2+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int\left(-\frac{4d^6(d^2-e^2x)^{-3+p}}{e^3} + \frac{9d^4(d^2-e^2x)^{-2+p}}{e^3} - \right.\right. \\
&= -\frac{2d^6(d^2-e^2x^2)^{-2+p}}{e^5(2-p)} - \frac{3dx^5(d^2-e^2x^2)^{-2+p}}{1+2p} + \frac{9d^4(d^2-e^2x^2)^{-1+p}}{2e^5(1-p)} + \frac{3d^2(d^2-e^2x^2}{e^5p}
\end{aligned}
$$

Mathematica [A]
time = 0.50, size = 245, normalized size = 1.11

$$
-\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(24de(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(4d\left(\frac{1}{2}+\frac{ex}{2d}\right)^p+4ex\left(\frac{1}{2}+\frac{ex}{2d}\right)^p+24d\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)-8d\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+d\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)\right)}{e^5(1+p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d^2 - e^2*x^2)^p*(24*d*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyp
ergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)
^p*(4*d*(1/2 + (e*x)/(2*d))^p + 4*e*x*(1/2 + (e*x)/(2*d))^p + 24*d*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 8*d*Hypergeometric2F1[2 -
 p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*Hypergeometric2F1[3 - p, 1 + p, 2 +
p, (d - e*x)/(2*d)])))/(e^5*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{x^4\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^4/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^4/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^4/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.287 $\int \frac{x^3\left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=194

$$\frac{2d^5(d^2-e^2x^2)^{-2+p}}{e^4(2-p)}+\frac{ex^5(d^2-e^2x^2)^{-2+p}}{1+2p}-\frac{7d^3(d^2-e^2x^2)^{-1+p}}{2e^4(1-p)}-\frac{3d(d^2-e^2x^2)^p}{2e^4p}-\frac{2e(4+3p)x^5(d^2-e^2x^2)^p\left(\right.}{5d}$$

```
[Out] 2*d^5*(-e^2*x^2+d^2)^(-2+p)/e^4/(2-p)+e*x^5*(-e^2*x^2+d^2)^(-2+p)/(1+2*p)-7
/2*d^3*(-e^2*x^2+d^2)^(-1+p)/e^4/(1-p)-3/2*d*(-e^2*x^2+d^2)^p/e^4/p-2/5*e*(
4+3*p)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 3-p],[7/2],e^2*x^2/d^2)/d^4/(1+
2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.13, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1666, 457, 78, 470, 372, 371}

$$\frac{ex^5(d^2-e^2x^2)^{p-2}}{2p+1}-\frac{3d(d^2-e^2x^2)^p}{2e^4p}+\frac{2d^5(d^2-e^2x^2)^{p-2}}{e^4(2-p)}-\frac{2e(3p+4)x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{5}{2},3-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)}-\frac{7d^3(d^2-e^2x^2)^{p-1}}{2e^4(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^4*(2 - p)) + (e*x^5*(d^2 - e^2*x^2)^(-2
 + p))/(1 + 2*p) - (7*d^3*(d^2 - e^2*x^2)^(-1 + p))/(2*e^4*(1 - p)) - (3*d*
(d^2 - e^2*x^2)^p)/(2*e^4*p) - (2*e*(4 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hyperge
ometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \int x^3(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx$$
$$= \int x^3\left(d^2-e^2x^2\right)^{-3+p}\left(d^3+3de^2x^2\right)\,dx + \int x^4\left(d^2-e^2x^2\right)^{-3+p}\left(-3d^2e-e^3x^2\right)\,dx$$
$$= \frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int x\left(d^2-e^2x\right)^{-3+p}\left(d^3+3de^2x\right)\,dx, x, x^2\right) - \frac{(2d^2e(\cdots}{\cdots}$$
$$= \frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{4d^5\left(d^2-e^2x\right)^{-3+p}}{e^2} - \frac{7d^3\left(d^2-e^2x\right)^{-2+p}}{e^2} + \frac{3d(d^{\cdots}}{\cdots}\right.\right.$$
$$= \frac{2d^5\left(d^2-e^2x^2\right)^{-2+p}}{e^4(2-p)} + \frac{ex^5\left(d^2-e^2x^2\right)^{-2+p}}{1+2p} - \frac{7d^3\left(d^2-e^2x^2\right)^{-1+p}}{2e^4(1-p)} - \frac{3d(d^2-e^2x^2)^p}{2e^4p} - \cdots$$

Mathematica [A]
time = 0.42, size = 202, normalized size = 1.04

$$\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(8e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(12\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)-6\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)\right)}{e^4(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2^(-3 + p)*(d^2 - e^2*x^2)^p*(8*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(1
2*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 6*Hypergeometri
c2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)])))/(e^4*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^
2)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^3/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^3/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(-(-d+ex)\,(d+ex))^p}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^3/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.288 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2}+\frac{2^{-3+p}(4+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\ {}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(1+p)}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(1+p)/e^3/(2-p)/(e*x+d)^3-1/2*(-e^2*x^2+d^2)^(1+p)/e^
3/p/(e*x+d)^2+2^(-3+p)*(4+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeo
m([1+p, 2-p],[2+p],1/2*(-e*x+d)/d)/d^2/e^3/p/(-p^2+p+2)
```

Rubi [A]
time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1653, 807, 692, 71}

$$\frac{2^{p-3}(p+4)(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\ {}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(p+1)}-\frac{(d^2-e^2x^2)^{p+1}}{2e^3p(d+ex)^2}-\frac{d(d^2-e^2x^2)^{p+1}}{2e^3(2-p)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^3*(2 - p)*(d + e*x)^3) - (d^2 - e^2*x^2
)^(1 + p)/(2*e^3*p*(d + e*x)^2) + (2^(-3 + p)*(4 + p)*(1 + (e*x)/d)^(-1 - p
)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/
(2*d)])/(d^2*e^3*(2 - p)*p*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^3}\,dx &= -\frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2}-\frac{\int \frac{(2d^2e^2+2de^3(1+p)x)(d^2-e^2x^2)^p}{(d+ex)^3}\,dx}{2e^4p}\\ &= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2}-\frac{(d(4+p))\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx}{2e^2(2-p)p}\\ &= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2}-\frac{\left((4+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-\right.}{2d^2e^2(2}\\ &= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(2-p)(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^3p(d+ex)^2}+\frac{2^{-3+p}(4+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+}}{d^2e^3(2-p)p(1}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 130, normalized size = 0.83

$$-\frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)-4\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{de^3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[1 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d*e
^3*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^2/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^2/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-\left(-d+ex\right)\left(d+ex\right)\right)^p}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^2/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.289 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=118

$$\frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3} - \frac{3\ 2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\ {}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(1+p)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(1+p)/e^2/(2-p)/(e*x+d)^3-3*2^(-3+p)*(1+e*x/d)^(-1-p)*(-
e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 2-p],[2+p],1/2*(-e*x+d)/d)/d^3/e^2/(-p^2
+p+2)
```

Rubi [A]
time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {807, 692, 71}

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(2-p)(d+ex)^3} - \frac{3\ 2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\ {}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^2(2-p)(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(2 - p)*(d + e*x)^3) - (3*2^(-3 + p)*(1 + (e
*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 +
 p, (d - e*x)/(2*d)])/(d^3*e^2*(2 - p)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^3}\,dx &= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3}+\frac{3\int \frac{(d^2-e^2x^2)^p}{(d+ex)^2}\,dx}{2e(2-p)}\\ &= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3}+\frac{\left(3(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int(d-ex)^p\left(1+\right.}{2d^3e(2-p)}\\ &= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(2-p)(d+ex)^3}-\frac{3\ 2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\ {}_2F_1\left(2-p,1+p;2+p;\frac{d}{}\right.}{d^3e^2(2-p)(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 102, normalized size = 0.86

$$\frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{d^2e^2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d^2*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-(-d+ex)(d+ex))^p}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x)
```

3.290 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=73

$$-\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e(1+p)}$$

```
[Out] -2^(-3+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 3-p],[2+p],
1/2*(-e*x+d)/d)/d^4/e/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {692, 71}

$$-\frac{2^{p-3}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx = \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-3+p}\,dx}{d^4}$$
$$= -\frac{2^{-3+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e(1+p)}$$

Mathematica [A]
time = 0.23, size = 75, normalized size = 1.03

$$-\frac{2^{-3+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^3e(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -((2^(-3 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/(x*e + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^3,x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^3, x)
```

3.291 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx$

Optimal. Leaf size=175

$$\frac{2d(d^2-e^2x^2)^{-2+p}}{2-p}-\frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p}-\frac{2e(4-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}+\frac{(d^2-$$

```
[Out] 2*d*(-e^2*x^2+d^2)^(-2+p)/(2-p)-e*x*(-e^2*x^2+d^2)^(-2+p)/(3-2*p)-2*e*(4-3*
p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^4/(3-2*p)/(
(1-e^2*x^2/d^2)^p)+1/2*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e^2*
x^2/d^2)/d/(1-p)
```

Rubi [A]
time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {866, 1666, 457, 80, 67, 396, 252, 251}

$$\frac{(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d(1-p)}-\frac{ex(d^2-e^2x^2)^{p-2}}{3-2p}+\frac{2d(d^2-e^2x^2)^{p-2}}{2-p}-\frac{2e(4-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^4(3-2p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (2*d*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (e*x*(d^2 - e^2*x^2)^(-2 + p))/(3
- 2*p) - (2*e*(4 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3
/2, (e^2*x^2)/d^2])/(d^4*(3 - 2*p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2
)^(-1 + p)*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d*(1 - p)
)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^3}\,dx &= \int \frac{(d-ex)^3\,(d^2-e^2x^2)^{-3+p}}{x}\,dx \\
&= \int \frac{(d^2-e^2x^2)^{-3+p}\,(d^3+3de^2x^2)}{x}\,dx + \int (d^2-e^2x^2)^{-3+p}\,(-3d^2e-e^3x^2)\,dx \\
&= -\frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p} + \frac{1}{2}\text{Subst}\left(\int \frac{(d^2-e^2x)^{-3+p}\,(d^3+3de^2x)}{x}\,dx, x, x^2\right) - \frac{(2d^2e(4-
\\
&= \frac{2d(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p} + \frac{1}{2}d\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x}\,dx, x, x^2\right) -
\\
&= \frac{2d(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{ex(d^2-e^2x^2)^{-2+p}}{3-2p} - \frac{2e(4-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\Big(}{d^4(3-2p)}
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 328, normalized size = 1.87

$$
\frac{2^{-3+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)+2p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+dp\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)-ep\left(1-\frac{d^2}{e^2x^2}\right)^p x\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)+4d\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)+4dp\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)\right)}{d^4p(1+p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (2^(-3 + p)*(d^2 - e^2*x^2)^p*(4*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*p*(1 - d^2/(e^2*x^2))^p
*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + d*p*(1
 - d^2/(e^2*x^2))^p*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
 - e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)] + 4*d*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p, d^
2/(e^2*x^2)] + 4*d*p*(1/2 + (e*x)/(2*d))^p*Hypergeometric2F1[-p, -p, 1 - p,
 d^2/(e^2*x^2)]))/(d^4*p*(1 + p)*(1 - d^2/(e^2*x^2))^p*(1 + (e*x)/d)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{(-e^2x^2+d^2)^p}{x\,(ex+d)^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p/(x^4*e^3 + 3*d*x^3*e^2 + 3*d^2*x^2*e + d^3*x),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**3,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{x\,(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^3), x)
```

3.292 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p}-\frac{d(d^2-e^2x^2)^{-2+p}}{x}+\frac{2e^2(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5}-\frac{3e(d^2$$

```
[Out] -2*e*(-e^2*x^2+d^2)^(-2+p)/(2-p)-d*(-e^2*x^2+d^2)^(-2+p)/x+2*e^2*(4-p)*x*(-
e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^5/((1-e^2*x^2/d^2)
^p)-3/2*e*(-e^2*x^2+d^2)^(-1+p)*hypergeom([1, -1+p],[p],1-e^2*x^2/d^2)/d^2/
(1-p)
```

Rubi [A]
time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {866, 1821, 1666, 457, 80, 67, 12, 252, 251}

$$-\frac{3e(d^2-e^2x^2)^{p-1}\,{}_2F_1\left(1,p-1;p;1-\frac{e^2x^2}{d^2}\right)}{2d^2(1-p)}-\frac{2e(d^2-e^2x^2)^{p-2}}{2-p}-\frac{d(d^2-e^2x^2)^{p-2}}{x}+\frac{2e^2(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] (-2*e*(d^2 - e^2*x^2)^(-2 + p))/(2 - p) - (d*(d^2 - e^2*x^2)^(-2 + p))/x +
(2*e^2*(4 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2*
x^2)/d^2])/(d^5*(1 - (e^2*x^2)/d^2)^p) - (3*e*(d^2 - e^2*x^2)^(-1 + p)*Hype
rgeometric2F1[1, -1 + p, p, 1 - (e^2*x^2)/d^2])/(2*d^2*(1 - p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^3}\,dx = \int \frac{(d-ex)^3\,(d^2-e^2x^2)^{-3+p}}{x^2}\,dx$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-3+p}(3d^4e-2d^3e^2(4-p)x+d^2e^3x^2)}{x}\,dx}{d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\int -2d^3e^2(4-p)\,(d^2-e^2x^2)^{-3+p}\,dx}{d^2} - \frac{\int \frac{(d^2-e^2x^2)^{-3+p}(3d^4e+d^2e^3x^2)}{x}}{d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{\text{Subst}\left(\int \frac{(d^2-e^2x)^{-3+p}(3d^4e+d^2e^3x)}{x}\,dx, x, x^2\right)}{2d^2} + \left(2de^2(4-p)\right)\int (d^2$$

$$= -\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{d(d^2-e^2x^2)^{-2+p}}{x} - \frac{1}{2}(3e)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-2+p}}{x}\,dx, x, x^2\right) -$$

$$= -\frac{2e(d^2-e^2x^2)^{-2+p}}{2-p} - \frac{d(d^2-e^2x^2)^{-2+p}}{x} + \frac{2e^2(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\big(}{d^5}$$

Mathematica [A]
time = 0.53, size = 280, normalized size = 1.69

$$\frac{(d^2-e^2x^2)^p\left(-\frac{8d^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{3\ 2^{2+p}e(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{2^{2+p}e(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{2^pe(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}-\frac{12de\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{8d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-8*d^2*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^2]
)/(x*(1 - (e^2*x^2)/d^2)^p) + (3*2^(2 + p)*e*(-d + e*x)*Hypergeometric2F1[1
 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^(2 + p
)*e*(-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
 + p)*(1 + (e*x)/d)^p) + (2^p*e*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (12*d*e*Hypergeometric
2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^5)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{x^2\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^5*e^3 + 3*d*x^4*e^2 + 3*d^2*x^3*e + d^3*x^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^2(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2\,x^2)^p}{x^2\,(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^3), x)
```

3.293 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx$

Optimal. Leaf size=173

$$-\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2}+\frac{3e(d^2-e^2x^2)^{-2+p}}{x}-\frac{2e^3(8-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6}+\frac{e^2(\cdots}{\cdots}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(-2+p)/x^2+3*e*(-e^2*x^2+d^2)^(-2+p)/x-2*e^3*(8-3*p)*
x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 3-p],[3/2],e^2*x^2/d^2)/d^6/((1-e^2*x^2/
d^2)^p)+1/2*e^2*(6-p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^
2*x^2/d^2)/d/(2-p)
```

Rubi [A]
time = 0.18, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1821, 778, 272, 67, 252, 251}

$$\frac{e^2(6-p)\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d(2-p)}+\frac{3e(d^2-e^2x^2)^{p-2}}{x}-\frac{d(d^2-e^2x^2)^{p-2}}{2x^2}-\frac{2e^3(8-3p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p{}_2F_1\left(\frac{1}{2},3-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(-2 + p))/x^2 + (3*e*(d^2 - e^2*x^2)^(-2 + p))/x -
(2*e^3*(8 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^
2*x^2)/d^2])/(d^6*(1 - (e^2*x^2)/d^2)^p) + (e^2*(6 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d*(2 - p)
)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^3}\,dx$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}(6d^4e-2d^3e^2(6-p)x+2d^2e^3x^2)}{x^2}\,dx}{2d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2} + \frac{3e(d^2-e^2x^2)^{-2+p}}{x} + \frac{\int \frac{(2d^5e^2(6-p)-4d^4e^3(8-3p)x)\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx}{2d^4}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2} + \frac{3e(d^2-e^2x^2)^{-2+p}}{x} - \left(2e^3(8-3p)\right)\int \left(d^2-e^2x^2\right)^{-3+p}\,dx + \left(d\right.$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2} + \frac{3e(d^2-e^2x^2)^{-2+p}}{x} + \frac{1}{2}\left(de^2(6-p)\right)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-3+p}}{x}\,d\right.$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{2x^2} + \frac{3e(d^2-e^2x^2)^{-2+p}}{x} - \frac{2e^3(8-3p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F}{d^6}$$

Mathematica [A]
time = 0.69, size = 341, normalized size = 1.97

$$\frac{(d^2-e^2x^2)^p\left(\frac{24d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{4d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{3\ 2^{3+p}e^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\ 2^{1+p}e^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{2^pe^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{24de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{8d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((24*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2]])/(x*(1 - (e^2*x^2)/d^2)^p) + (4*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (3*2^(3 + p)*e^2*(d
- e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hy
pergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/
d)^p) + (24*d*e^2*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 -
d^2/(e^2*x^2))^p)))/(8*d^6)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{x^3\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^6*e^3 + 3*d*x^5*e^2 + 3*d^2*x^4*e + d^3*x^3)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x^{3}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{x^3\,{(d+e\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3), x)
```

3.294 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx$

Optimal. Leaf size=179

$$-\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3}+\frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2}-\frac{2e^2(8-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^5x}-\frac{e^3(1}{}$$

[Out] -1/3*d*(-e^2*x^2+d^2)^(-2+p)/x^3+3/2*e*(-e^2*x^2+d^2)^(-2+p)/x^2-2/3*e^2*(8 -p)*(-e^2*x^2+d^2)^p*hypergeom([-1/2, 3-p],[1/2],e^2*x^2/d^2)/d^5/x/((1-e^2 *x^2/d^2)^p)-1/2*e^3*(10-3*p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1 +p],1-e^2*x^2/d^2)/d^2/(2-p)

Rubi [A]
time = 0.18, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {866, 1821, 778, 372, 371, 272, 67}

$$\frac{3e(d^2-e^2x^2)^{p-2}}{2x^2}-\frac{d(d^2-e^2x^2)^{p-2}}{3x^3}-\frac{e^3(10-3p)\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}-\frac{2e^2(8-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2-e^2x^2\right)^p{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^5x}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]

[Out] -1/3*(d*(d^2 - e^2*x^2)^(-2 + p))/x^3 + (3*e*(d^2 - e^2*x^2)^(-2 + p))/(2*x ^2) - (2*e^2*(8 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x^2)/d^2])/(3*d^5*x*(1 - (e^2*x^2)/d^2)^p) - (e^3*(10 - 3*p)*(d^2 - e^ 2*x^2)^(-2 + p)*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2 *d^2*(2 - p))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^3}\,dx = \int \frac{(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}}{x^4}\,dx$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3} - \frac{\int \frac{\left(d^2-e^2x^2\right)^{-3+p}\left(9d^4e-2d^3e^2(8-p)x+3d^2e^3x^2\right)}{x^3}\,dx}{3d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3} + \frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2} + \frac{\int \frac{\left(4d^5e^2(8-p)-6d^4e^3(10-3p)x\right)\left(d^2-e^2x^2\right)^{-3+p}}{x^2}\,dx}{6d^4}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3} + \frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2} - \left(e^3(10-3p)\right)\int \frac{\left(d^2-e^2x^2\right)^{-3+p}}{x}\,dx + \frac{1}{3}(2\ldots$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3} + \frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2} - \frac{1}{2}\left(e^3(10-3p)\right)\text{Subst}\left(\int \frac{\left(d^2-e^2x\right)^{-3+p}}{x}\,d\ldots\right.$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{3x^3} + \frac{3e(d^2-e^2x^2)^{-2+p}}{2x^2} - \frac{2e^2(8-p)\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\ldots\right.}{3d^5x}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 393 vs. 2(179) = 358.

time = 0.64, size = 393, normalized size = 2.20

$$\frac{(d^2-e^2x^2)^p\left(-\frac{8d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3}-\frac{144d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{36d^3e\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{15\ 2^{3+p}e^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\ 2^{3+p}e^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\ 2^pe^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}-\frac{120de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{24d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-8*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^2
])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (144*d^2*e^2*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (36*d^3*e*Hypergeometric2F1
[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (
15*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x
)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeom
etric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) +
 (3*2^p*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*
d)])/((1 + p)*(1 + (e*x)/d)^p) - (120*d*e^3*Hypergeometric2F1[-p, -p, 1 - p
, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(24*d^7)
```

Maple [F]

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^2x^2+d^2\right)^p}{x^4\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^7*e^3 + 3*d*x^6*e^2 + 3*d^2*x^5*e + d^3*x^4)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x^{4}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{x^4\,(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^3), x)
```

3.295 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx$

Optimal. Leaf size=174

$$-\frac{d(d^2-e^2x^2)^{-2+p}}{4x^4}+\frac{e(d^2-e^2x^2)^{-2+p}}{x^3}+\frac{2e^3(4-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^6x}+\frac{e^4(1}{}$$

```
[Out] -1/4*d*(-e^2*x^2+d^2)^(-2+p)/x^4+e*(-e^2*x^2+d^2)^(-2+p)/x^3+2*e^3*(4-p)*(-
e^2*x^2+d^2)^p*hypergeom([-1/2, 3-p],[1/2],e^2*x^2/d^2)/d^6/x/((1-e^2*x^2/d
^2)^p)+1/4*e^4*(10-p)*(-e^2*x^2+d^2)^(-2+p)*hypergeom([2, -2+p],[-1+p],1-e^
2*x^2/d^2)/d^3/(2-p)
```

Rubi [A]
time = 0.19, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1821, 778, 272, 67, 372, 371}

$$-\frac{d(d^2-e^2x^2)^{p-2}}{4x^4}+\frac{e(d^2-e^2x^2)^{p-2}}{x^3}+\frac{2e^3(4-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{1}{2},3-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{d^6x}+\frac{e^4(10-p)\,(d^2-e^2x^2)^{p-2}\,{}_2F_1\left(2,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{4d^3(2-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

```
[Out] -1/4*(d*(d^2 - e^2*x^2)^(-2 + p))/x^4 + (e*(d^2 - e^2*x^2)^(-2 + p))/x^3 +
(2*e^3*(4 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x
^2)/d^2])/(d^6*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(10 - p)*(d^2 - e^2*x^2)^(-2
 + p)*Hypergeometric2F1[2, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(4*d^3*(2 -
p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^3}\,dx = \int \frac{(d-ex)^3\,(d^2-e^2x^2)^{-3+p}}{x^5}\,dx$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{4x^4} - \frac{\int \frac{(d^2-e^2x^2)^{-3+p}(12d^4e-2d^3e^2(10-p)x+4d^2e^3x^2)}{x^4}\,dx}{4d^2}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{4x^4} + \frac{e(d^2-e^2x^2)^{-2+p}}{x^3} + \frac{\int \frac{(6d^5e^2(10-p)-24d^4e^3(4-p)x)(d^2-e^2x^2)^{-3+p}}{x^3}\,dx}{12d^4}$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{4x^4} + \frac{e(d^2-e^2x^2)^{-2+p}}{x^3} - \left(2e^3(4-p)\right)\int \frac{(d^2-e^2x^2)^{-3+p}}{x^2}\,dx + \frac{1}{2}(de$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{4x^4} + \frac{e(d^2-e^2x^2)^{-2+p}}{x^3} + \frac{1}{4}\left(de^2(10-p)\right)\text{Subst}\left(\int \frac{(d^2-e^2x)^{-3+p}}{x^2}\,d\right.$$

$$= -\frac{d(d^2-e^2x^2)^{-2+p}}{4x^4} + \frac{e(d^2-e^2x^2)^{-2+p}}{x^3} + \frac{2e^3(4-p)(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\right.}{d^6x}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 446 vs. 2(174) = 348.

time = 0.73, size = 446, normalized size = 2.56

$$\frac{(d^2-e^2x^2)^p\left(\frac{8d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3}+\frac{80d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{24d^3e^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{15\,2^{2+p}e^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{4d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right)}{(-2+p)x^4}+\frac{5\,2^{1+p}e^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{2^pe^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{60de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{8d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((8*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2]])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (80*d^2*e^3*Hypergeometric2F1[-1/2, -p, 1
/2, (e^2*x^2)/d^2]])/(x*(1 - (e^2*x^2)/d^2)^p) + (24*d^3*e^2*Hypergeometric2
F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)]])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
 (15*2^(2 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (4*d^5*Hypergeometric2F1[2 - p, -p,
3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (5*2^(1 + p)*
e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
+ p)*(1 + (e*x)/d)^p) + (2^p*e^4*(d - e*x)*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (60*d*e^4*Hypergeometr
ic2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(8*d^8)
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{x^5\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x^5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^8*e^3 + 3*d*x^7*e^2 + 3*d^2*x^6*e + d^3*x^5)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{p}}{x^{5}\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**3,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^3*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^5\,{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3),x)
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3), x)
```

3.296 $\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=265

$$-\frac{4d^7(d^2-e^2x^2)^{-3+p}}{e^5(3-p)}+\frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2}-\frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p}+\frac{10d^5(d^2-e^2x^2)^{-2+p}}{e^5(2-p)}-\frac{8d^3(d^2}{e^5}$$

```
[Out] -4*d^7*(-e^2*x^2+d^2)^(-3+p)/e^5/(3-p)+d^2*(13+12*p)*x^5*(-e^2*x^2+d^2)^(-3
+p)/(-4*p^2+1)-e^2*x^7*(-e^2*x^2+d^2)^(-3+p)/(1+2*p)+10*d^5*(-e^2*x^2+d^2)^
(-2+p)/e^5/(2-p)-8*d^3*(-e^2*x^2+d^2)^(-1+p)/e^5/(1-p)-2*d*(-e^2*x^2+d^2)^p
/e^5/p-4/5*(p^2+15*p+16)*x^5*(-e^2*x^2+d^2)^p*hypergeom([5/2, 4-p],[7/2],e^
2*x^2/d^2)/d^4/(-4*p^2+1)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.20, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {866, 1666, 1281, 470, 372, 371, 457, 78}

$$\frac{d^2(12p+13)x^5(d^2-e^2x^2)^{p-3}}{1-4p^2}-\frac{e^2x^7(d^2-e^2x^2)^{p-3}}{2p+1}-\frac{2d(d^2-e^2x^2)^p}{e^5p}-\frac{4d^7(d^2-e^2x^2)^{p-3}}{e^5(3-p)}+\frac{10d^5(d^2-e^2x^2)^{p-2}}{e^5(2-p)}-\frac{4(p^2+15p+16)\,x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{5}{2},4-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right)}{5d^4\,(1-4p^2)}-\frac{8d^3(d^2-e^2x^2)^{p-1}}{e^5(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (-4*d^7*(d^2 - e^2*x^2)^(-3 + p))/(e^5*(3 - p)) + (d^2*(13 + 12*p)*x^5*(d^2
 - e^2*x^2)^(-3 + p))/(1 - 4*p^2) - (e^2*x^7*(d^2 - e^2*x^2)^(-3 + p))/(1 +
 2*p) + (10*d^5*(d^2 - e^2*x^2)^(-2 + p))/(e^5*(2 - p)) - (8*d^3*(d^2 - e^2
*x^2)^(-1 + p))/(e^5*(1 - p)) - (2*d*(d^2 - e^2*x^2)^p)/(e^5*p) - (4*(16 +
15*p + p^2)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 4 - p, 7/2, (e^2*x
^2)/d^2])/(5*d^4*(1 - 4*p^2)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(d^2-e^2x^2)^p}{(d+ex)^4}\,dx &= \int x^4(d-ex)^4\left(d^2-e^2x^2\right)^{-4+p}\,dx \\
&= \int x^5\left(d^2-e^2x^2\right)^{-4+p}\left(-4d^3e-4de^3x^2\right)\,dx+\int x^4\left(d^2-e^2x^2\right)^{-4+p}\left(d^4+6d^2e^2x^2+\right. \\
&= -\frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p}+\frac{1}{2}\text{Subst}\left(\int x^2\left(d^2-e^2x\right)^{-4+p}\left(-4d^3e-4de^3x\right)\,dx,x,x^2\right) \\
&= \frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2}-\frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p}+\frac{1}{2}\text{Subst}\left(\int\left(-\frac{8d^7(d^2-e\right.\right. }{e^3} \\
&= -\frac{4d^7(d^2-e^2x^2)^{-3+p}}{e^5(3-p)}+\frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2}-\frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p}+\frac{10}{} \\
&= -\frac{4d^7(d^2-e^2x^2)^{-3+p}}{e^5(3-p)}+\frac{d^2(13+12p)x^5(d^2-e^2x^2)^{-3+p}}{1-4p^2}-\frac{e^2x^7(d^2-e^2x^2)^{-3+p}}{1+2p}+\frac{10}{}
\end{aligned}
$$

Mathematica [A]
time = 0.54, size = 231, normalized size = 0.87

$$\frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(16e(1+p)x\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)+(d-ex)\left(1-\frac{e^2x^2}{d^2}\right)^p\left(32\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)-24\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+8\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)-{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)\right)}{e^5(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d^2 - e^2*x^2)^p*(16*e*(1 + p)*x*(1/2 + (e*x)/(2*d))^p*Hyperge
ometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*(
32*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] - 24*Hypergeomet
ric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 8*Hypergeometric2F1[3 - p, 1
 + p, 2 + p, (d - e*x)/(2*d)] - Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d -
 e*x)/(2*d)])))/(e^5*(1 + p)*(1 + (e*x)/d)^p*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^4(-e^2x^2+d^2)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^4/(x*e + d)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^4/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*
d^3*x*e + d^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(-(-d+ex)\,(d+ex))^p}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^4/(x*e + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(d^2 - e^2 x^2\right)^p}{\left(d + e\, x\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
[Out] int((x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.297 $\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=211

$$\frac{d^2(d^2-e^2x^2)^{1+p}}{2e^4(3-p)(d+ex)^4}-\frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2}+\frac{3\;2^{-2+p}(2+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+}}{d^2e^4(1-2p)(3-p)}$$

```
[Out] 1/2*d^2*(-e^2*x^2+d^2)^(1+p)/e^4/(3-p)/(e*x+d)^4-d*(1+2*p)*(-e^2*x^2+d^2)^(
1+p)/e^4/(1-2*p)/p/(e*x+d)^3-1/2*(-e^2*x^2+d^2)^(1+p)/e^4/p/(e*x+d)^2+3*2^(
-2+p)*(2+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 3-p],[2+p
],1/2*(-e*x+d)/d)/d^2/e^4/p/(2*p^3-5*p^2-4*p+3)
```

Rubi [A]
time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1653, 807, 692, 71}

$$\frac{3\;2^{p-2}(p+2)\left(d^2-e^2x^2\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^2e^4(1-2p)(3-p)p(p+1)}-\frac{\left(d^2-e^2x^2\right)^{p+1}}{2e^4p(d+ex)^2}-\frac{d(2p+1)\left(d^2-e^2x^2\right)^{p+1}}{e^4(1-2p)p(d+ex)^3}+\frac{d^2\left(d^2-e^2x^2\right)^{p+1}}{2e^4(3-p)(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(3 - p)*(d + e*x)^4) - (d*(1 + 2*p)*(d
^2 - e^2*x^2)^(1 + p))/(e^4*(1 - 2*p)*p*(d + e*x)^3) - (d^2 - e^2*x^2)^(1 +
 p)/(2*e^4*p*(d + e*x)^2) + (3*2^(-2 + p)*(2 + p)*(1 + (e*x)/d)^(-1 - p)*(d
^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d
)])/(d^2*e^4*(1 - 2*p)*(3 - p)*p*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(d^2-e^2x^2)^p}{(d+ex)^4}\,dx &= -\frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2}-\frac{\int \frac{(d^2-e^2x^2)^p(2d^3e^2+2d^2e^3(2+p)x+2de^4(1+2p)x^2)}{(d+ex)^4}\,dx}{2e^5p}\\
&= -\frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2}-\frac{\int \frac{(8d^3e^6(1+p)+2d^2e^7(4+3p+2p^2)x)(d^2-e^2x^2)^p}{(d+ex)^4}}{2e^9(1-2p)p}\\
&= \frac{d^2(d^2-e^2x^2)^{1+p}}{2e^4(3-p)(d+ex)^4}-\frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2}-\frac{(6d^2(2+p))\int}{e^3(1-2p)(}\\
&= \frac{d^2(d^2-e^2x^2)^{1+p}}{2e^4(3-p)(d+ex)^4}-\frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2}-\frac{\left(6(2+p)(d-e\right.}{}\\
&= \frac{d^2(d^2-e^2x^2)^{1+p}}{2e^4(3-p)(d+ex)^4}-\frac{d(1+2p)\left(d^2-e^2x^2\right)^{1+p}}{e^4(1-2p)p(d+ex)^3}-\frac{(d^2-e^2x^2)^{1+p}}{2e^4p(d+ex)^2}+\frac{3\ 2^{-2+p}(2+p)\,(}{}
\end{aligned}$$

Mathematica [A]
time = 0.42, size = 156, normalized size = 0.74

$$\frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-8\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)+12\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)-6\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)+{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{de^4(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-8*Hypergeometric2F1[1 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + 12*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] - 6*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + Hy
pergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d*e^4*(1 + p)*(1 +
 (e*x)/d)^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(-e^{2} x^{2}+d^{2}\right)^{p}}{\left(e x +d \right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^3/(x*e + d)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^3/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*
d^3*x*e + d^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{\left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*x^3/(x*e + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
[Out] int((x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.298 $\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4} dx$

Optimal. Leaf size=163

$$-\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4}+\frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3}-\frac{2^{-3+p}(7+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\ {}_2F_1(3-p,1+p;2+ ...}{d^3e^3(1-2p)(3-p)(1+p)}$$

```
[Out] -1/2*d*(-e^2*x^2+d^2)^(1+p)/e^3/(3-p)/(e*x+d)^4+(-e^2*x^2+d^2)^(1+p)/e^3/(1
-2*p)/(e*x+d)^3-2^(-3+p)*(7+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hyperg
eom([1+p, 3-p],[2+p],1/2*(-e*x+d)/d)/d^3/e^3/(2*p^3-5*p^2-4*p+3)
```

Rubi [A]

time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1653, 807, 692, 71}

$$\frac{(d^2-e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3}-\frac{d(d^2-e^2x^2)^{p+1}}{2e^3(3-p)(d+ex)^4}-\frac{2^{p-3}(p+7)(d^2-e^2x^2)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}\ {}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] -1/2*(d*(d^2 - e^2*x^2)^(1 + p))/(e^3*(3 - p)*(d + e*x)^4) + (d^2 - e^2*x^2
)^(1 + p)/(e^3*(1 - 2*p)*(d + e*x)^3) - (2^(-3 + p)*(7 + p)*(1 + (e*x)/d)^(
-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
 e*x)/(2*d)])/(d^3*e^3*(1 - 2*p)*(3 - p)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d^2-e^2x^2)^p}{(d+ex)^4}\,dx &= \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{\int \frac{(3d^2e^2+2de^3(1+p)x)(d^2-e^2x^2)^p}{(d+ex)^4}\,dx}{e^4(1-2p)} \\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{(d(7+p))\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx}{e^2(1-2p)(3-p)} \\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} + \frac{\left((7+p)(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(\ldots\right.}{d^3e^2(1\ldots} \\
&= -\frac{d(d^2-e^2x^2)^{1+p}}{2e^3(3-p)(d+ex)^4} + \frac{(d^2-e^2x^2)^{1+p}}{e^3(1-2p)(d+ex)^3} - \frac{2^{-3+p}(7+p)\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x\ldots}{d^3e^3(1-2p)(3\ldots}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 130, normalized size = 0.80

$$-\frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(4\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)-4\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)+{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{d^2e^3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(4*Hypergeometric2F1[2 - p, 1 + p
, 2 + p, (d - e*x)/(2*d)] - 4*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e
*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]))/(d^2
*e^3*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2(-e^2x^2+d^2)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^2/(x*e + d)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x^2/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*
d^3*x*e + d^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(-(-d+ex)(d+ex))^p}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x^2/(x*e + d)^4, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
```

```
[Out] int((x^2*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.299 $\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4} dx$

Optimal. Leaf size=118

$$\frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} - \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\ {}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(1+p)}$$

```
[Out] 1/2*(-e^2*x^2+d^2)^(1+p)/e^2/(3-p)/(e*x+d)^4-2^(-2+p)*(1+e*x/d)^(-1-p)*(-e^
2*x^2+d^2)^(1+p)*hypergeom([1+p, 3-p],[2+p],1/2*(-e*x+d)/d)/d^4/e^2/(-p^2+2
*p+3)
```

Rubi [A]
time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {807, 692, 71}

$$\frac{(d^2-e^2x^2)^{p+1}}{2e^2(3-p)(d+ex)^4} - \frac{2^{p-2}\left(\frac{ex}{d}+1\right)^{-p-1}(d^2-e^2x^2)^{p+1}\ {}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^4e^2(3-p)(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (d^2 - e^2*x^2)^(1 + p)/(2*e^2*(3 - p)*(d + e*x)^4) - (2^(-2 + p)*(1 + (e*x
)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p
, (d - e*x)/(2*d)])/(d^4*e^2*(3 - p)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rubi steps

$$\int \frac{x(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} + \frac{2\int \frac{(d^2-e^2x^2)^p}{(d+ex)^3}\,dx}{e(3-p)}$$
$$= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} + \frac{\left(2(d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\right.}{d^4e(3-p)}$$
$$= \frac{(d^2-e^2x^2)^{1+p}}{2e^2(3-p)(d+ex)^4} - \frac{2^{-2+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right.}{d^4e^2(3-p)(1+p)}$$

Mathematica [A]
time = 0.29, size = 102, normalized size = 0.86

$$\frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(-2\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)+{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)\right)}{d^3e^2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]
```

```
[Out] (2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*(-2*Hypergeometric2F1[3 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)
/(2*d)]))/(d^3*e^2*(1 + p)*(1 + (e*x)/d)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x(-e^2x^2+d^2)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x/(x*e + d)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*x/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*d^
3*x*e + d^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-(-d+ex)\,(d+ex))^p}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*x/(x*e + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d^2-e^2\,x^2\right)}^p}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x)
```

```
[Out] int((x*(d^2 - e^2*x^2)^p)/(d + e*x)^4, x)
```

3.300 $\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx$

Optimal. Leaf size=73

$$-\frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-1-p}\left(d^2-e^2x^2\right)^{1+p}\,{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^5e(1+p)}$$

```
[Out] -2^(-4+p)*(1+e*x/d)^(-1-p)*(-e^2*x^2+d^2)^(1+p)*hypergeom([1+p, 4-p],[2+p],
1/2*(-e*x+d)/d)/d^5/e/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {692, 71}

$$-\frac{2^{p-4}\left(\frac{ex}{d}+1\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\,{}_2F_1\left(4-p,p+1;p+2;\frac{d-ex}{2d}\right)}{d^5e(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^5*e*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{(d+ex)^4}\,dx = \frac{\left((d-ex)^{-1-p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\right)\int (d-ex)^p\left(1+\frac{ex}{d}\right)^{-4+p}\,dx}{d^5}$$
$$= -\frac{2^{-4+p}\left(1+\frac{ex}{d}\right)^{-1-p}(d^2-e^2x^2)^{1+p}\;{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^5e(1+p)}$$

Mathematica [A]
time = 0.25, size = 75, normalized size = 1.03

$$-\frac{2^{-4+p}(d-ex)\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\;{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{d^4e(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^4,x]
```

```
[Out] -((2^(-4 + p)*(d - e*x)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[4 - p, 1 + p, 2
 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)*(1 + (e*x)/d)^p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/(x*e + d)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x^2*e^2 + 4*d^3*
x*e + d^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/(x*e + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2-e^2\,x^2)^p}{(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(d + e*x)^4,x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(d + e*x)^4, x)
```

3.301 $\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx$

Optimal. Leaf size=204

$$\frac{4d^2(d^2-e^2x^2)^{-3+p}}{3-p}-\frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p}-\frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)}-\frac{8e(2-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1}{2},\right.}{d^5(5-2p)}$$

```
[Out] 4*d^2*(-e^2*x^2+d^2)^(-3+p)/(3-p)-4*d*e*x*(-e^2*x^2+d^2)^(-3+p)/(5-2*p)-1/2
*(-e^2*x^2+d^2)^(-2+p)/(2-p)-8*e*(2-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1/2, 4
-p],[3/2],e^2*x^2/d^2)/d^5/(5-2*p)/((1-e^2*x^2/d^2)^p)+1/2*(-e^2*x^2+d^2)^(
-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/(2-p)
```

Rubi [A]
time = 0.13, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {866, 1666, 1265, 965, 80, 67, 396, 252, 251}

$$\frac{(d^2-e^2x^2)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2(2-p)}-\frac{4dex(d^2-e^2x^2)^{p-3}}{5-2p}+\frac{4d^2(d^2-e^2x^2)^{p-3}}{3-p}-\frac{(d^2-e^2x^2)^{p-2}}{2(2-p)}-\frac{8e(2-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^5(5-2p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```

```
[Out] (4*d^2*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (4*d*e*x*(d^2 - e^2*x^2)^(-3 + p
))/(5 - 2*p) - (d^2 - e^2*x^2)^(-2 + p)/(2*(2 - p)) - (8*e*(2 - p)*x*(d^2 -
 e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^5*(5 - 2*
p)*(1 - (e^2*x^2)/d^2)^p) + ((d^2 - e^2*x^2)^(-2 + p)*Hypergeometric2F1[1,
-2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*(2 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] || !IntegerQ[m])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x(d+ex)^4}\,dx = \int \frac{(d-ex)^4\,(d^2-e^2x^2)^{-4+p}}{x}\,dx$$

$$= \int (d^2-e^2x^2)^{-4+p}\,(-4d^3e-4de^3x^2)\,dx + \int \frac{(d^2-e^2x^2)^{-4+p}\,(d^4+6d^2e^2x^2+e^4x^4)}{x}$$

$$= -\frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} + \frac{1}{2}\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}\,(d^4+6d^2e^2x+e^4x^2)}{x}\,dx, x, x^2\right)$$

$$= -\frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)} - \frac{\text{Subst}\left(\int \frac{(d^2-e^2x)^{-4+p}(-d^4e^4(2-p)-7d^2e^6(2-p)}{x}\right.}{2e^4(2-p)}$$

$$= \frac{4d^2(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x(d^2-e^2x}{}$$

$$= \frac{4d^2(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{4dex(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{(d^2-e^2x^2)^{-2+p}}{2(2-p)} - \frac{8e(2-p)x(d^2-e^2x}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 417 vs. 2(204) = 408.
time = 0.45, size = 417, normalized size = 2.04

$$\frac{2^{-4+p}\left(1-\frac{d^2}{e^2x^2}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}(d^2-e^2x^2)^p\left(8p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)+4p\left(1-\frac{d^2}{e^2x^2}\right)^p(d-ex)\,{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+2dp\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)-2ep\left(1-\frac{d^2}{e^2x^2}\right)^p x\,{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)+dp\left(1-\frac{d^2}{e^2x^2}\right)^p{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)-ep\left(1-\frac{d^2}{e^2x^2}\right)^p x\,{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)+8d\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)+8dp\left(\frac{1}{2}+\frac{ex}{2d}\right)^p{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)\right)}{d^5p(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x]
```

```
[Out] (2^(-4 + p)*(d^2 - e^2*x^2)^p*(8*p*(1 - d^2/(e^2*x^2))^p*(d - e*x)*Hypergeo
metric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 4*p*(1 - d^2/(e^2*x^2))^p
*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 2*d*p*
(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d
)] - 2*e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (
d - e*x)/(2*d)] + d*p*(1 - d^2/(e^2*x^2))^p*Hypergeometric2F1[4 - p, 1 + p,
```

```
 2 + p, (d - e*x)/(2*d)] - e*p*(1 - d^2/(e^2*x^2))^p*x*Hypergeometric2F1[4
- p, 1 + p, 2 + p, (d - e*x)/(2*d)] + 8*d*(1/2 + (e*x)/(2*d))^p*Hypergeomet
ric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)] + 8*d*p*(1/2 + (e*x)/(2*d))^p*Hypergeo
metric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)]))/(d^5*p*(1 + p)*(1 - d^2/(e^2*x^2)
)^p*(1 + (e*x)/d)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x \left(e x +d \right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^5*e^4 + 4*d*x^4*e^3 + 6*d^2*x^3*e^2 + 4*d^3*
x^2*e + d^4*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{x \left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p}{x\,{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x*(d + e*x)^4), x)
```

3.302 $\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx$

Optimal. Leaf size=207

$$-\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p}-\frac{d^2(d^2-e^2x^2)^{-3+p}}{x}+\frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p}+\frac{4e^2(16-9p+p^2)\,x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)}$$

```
[Out] -4*d*e*(-e^2*x^2+d^2)^(-3+p)/(3-p)-d^2*(-e^2*x^2+d^2)^(-3+p)/x+e^2*x*(-e^2*
x^2+d^2)^(-3+p)/(5-2*p)+4*e^2*(p^2-9*p+16)*x*(-e^2*x^2+d^2)^p*hypergeom([1/
2, 4-p],[3/2],e^2*x^2/d^2)/d^6/(5-2*p)/((1-e^2*x^2/d^2)^p)-2*e*(-e^2*x^2+d^
2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/d/(2-p)
```

Rubi [A]
time = 0.18, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {866, 1821, 1666, 457, 80, 67, 396, 252, 251}

$$-\frac{2e(d^2-e^2x^2)^{p-2}\,{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{d(2-p)}+\frac{e^2x(d^2-e^2x^2)^{p-3}}{5-2p}-\frac{4de(d^2-e^2x^2)^{p-3}}{3-p}-\frac{d^2(d^2-e^2x^2)^{p-3}}{x}+\frac{4e^2(p^2-9p+16)\,x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^6(5-2p)}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

```
[Out] (-4*d*e*(d^2 - e^2*x^2)^(-3 + p))/(3 - p) - (d^2*(d^2 - e^2*x^2)^(-3 + p))/
x + (e^2*x*(d^2 - e^2*x^2)^(-3 + p))/(5 - 2*p) + (4*e^2*(16 - 9*p + p^2)*x*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(
5 - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(d^2 - e^2*x^2)^(-2 + p)*Hypergeomet
ric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d*(2 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x^2(d+ex)^4}\,dx &= \int \frac{(d-ex)^4\,(d^2-e^2x^2)^{-4+p}}{x^2}\,dx \\
&= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(4d^5e-d^4e^2(13-2p)x+4d^3e^3x^2-d^2e^4x^3)}{x}\,dx}{d^2} \\
&= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(4d^5e+4d^3e^3x^2)}{x}\,dx}{d^2} - \frac{\int (d^2-e^2x^2)^{-4+p}\,(-d^4e^2(13-}{d^2} \\
&= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} - \frac{\text{Subst}\Big(\int \frac{(d^2-e^2x)^{-4+p}(4d^5e+4d^3e^3x)}{x}\,dx, x, x^2}{2d^2} \\
&= -\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} - (2de)\text{Subst}\Bigg(\int \frac{(d^2}{} \\
&= -\frac{4de(d^2-e^2x^2)^{-3+p}}{3-p} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{x} + \frac{e^2x(d^2-e^2x^2)^{-3+p}}{5-2p} + \frac{4e^2(16-9p+p^2)}{}
\end{aligned}$$

Mathematica [A]
time = 0.60, size = 337, normalized size = 1.63

$$\frac{(d^2-e^2x^2)^p\left(-16d^2p(1+p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)+2^{5+p}epx(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)+3\ 2^{2+p}epx(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)+2^{2+p}epx(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)+2^pepx(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)-32de(1+p)\left(1-\frac{d^2}{e^2x^2}\right)^{-p}x\,{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)\right)}{16d^6p(1+p)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-16*d^2*p*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, (e^
2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p + (2^(5 + p)*e*p*x*(-d + e*x)*Hypergeome
tric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (3*2^(2 +
p)*e*p*x*(-d + e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
)/(1 + (e*x)/d)^p + (2^(2 + p)*e*p*x*(-d + e*x)*Hypergeometric2F1[3 - p, 1
+ p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p + (2^p*e*p*x*(-d + e*x)*Hyper
geometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + (e*x)/d)^p - (32*d
```

```
*e*(1 + p)*x*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(1 - d^2/(e^2
*x^2))^p))/(16*d^6*p*(1 + p)*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{2} \left(e x +d \right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^6*e^4 + 4*d*x^5*e^3 + 6*d^2*x^4*e^2 + 4*d^3*
x^3*e + d^4*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(- d + e x\right) \left(d + e x\right)\right)^{p}}{x^{2} \left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^2\,\left(d+e\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4), x)
```

3.303 $\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx$

Optimal. Leaf size=211

$$\frac{e^2(11-p)\left(d^2-e^2x^2\right)^{-3+p}}{2(3-p)}-\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2}+\frac{4de(d^2-e^2x^2)^{-3+p}}{x}-\frac{8e^3(4-p)x(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-}}{d^7}$$

```
[Out] 1/2*e^2*(11-p)*(-e^2*x^2+d^2)^(-3+p)/(3-p)-1/2*d^2*(-e^2*x^2+d^2)^(-3+p)/x^
2+4*d*e*(-e^2*x^2+d^2)^(-3+p)/x-8*e^3*(4-p)*x*(-e^2*x^2+d^2)^p*hypergeom([1
/2, 4-p],[3/2],e^2*x^2/d^2)/d^7/((1-e^2*x^2/d^2)^p)+1/2*e^2*(10-p)*(-e^2*x^
2+d^2)^(-2+p)*hypergeom([1, -2+p],[-1+p],1-e^2*x^2/d^2)/d^2/(2-p)
```

Rubi [A]
time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {866, 1821, 1666, 457, 80, 67, 12, 252, 251}

$$\frac{e^2(10-p)\left(d^2-e^2x^2\right)^{p-2}{}_2F_1\left(1,p-2;p-1;1-\frac{e^2x^2}{d^2}\right)}{2d^2(2-p)}+\frac{e^2(11-p)\left(d^2-e^2x^2\right)^{p-3}}{2(3-p)}+\frac{4de(d^2-e^2x^2)^{p-3}}{x}-\frac{d^2(d^2-e^2x^2)^{p-3}}{2x^2}-\frac{8e^3(4-p)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

```
[Out] (e^2*(11 - p)*(d^2 - e^2*x^2)^(-3 + p))/(2*(3 - p)) - (d^2*(d^2 - e^2*x^2)^
(-3 + p))/(2*x^2) + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/x - (8*e^3*(4 - p)*x*(
d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^7*(1
 - (e^2*x^2)/d^2)^p) + (e^2*(10 - p)*(d^2 - e^2*x^2)^(-2 + p)*Hypergeometri
c2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(2 - p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d^2-e^2x^2)^p}{x^3(d+ex)^4}\,dx &= \int \frac{(d-ex)^4\,(d^2-e^2x^2)^{-4+p}}{x^3}\,dx \\
&= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(8d^5e-2d^4e^2(10-p)x+8d^3e^3x^2-2d^2e^4x^3)}{x^2}\,dx}{2d^2} \\
&= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} + \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(2d^6e^2(10-p)-16d^5e^3(4-p)x+2d^4}{x}}{2d^4} \\
&= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} + \frac{\int -16d^5e^3(4-p)\,(d^2-e^2x^2)^{-4+p}\,dx}{2d^4} + \\
&= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} + \frac{\mathrm{Subst}\Big(\int \frac{(d^2-e^2x)^{-4+p}(2d^6e^2(10-p)+2d^4e^4x)}{x}\,d}{4d^4} \\
&= \frac{e^2(11-p)\,(d^2-e^2x^2)^{-3+p}}{2(3-p)} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} + \frac{1}{2}\big(e^2(10-p \\
&= \frac{e^2(11-p)\,(d^2-e^2x^2)^{-3+p}}{2(3-p)} - \frac{d^2(d^2-e^2x^2)^{-3+p}}{2x^2} + \frac{4de(d^2-e^2x^2)^{-3+p}}{x} - \frac{8e^3(4-p)x}{}
\end{aligned}
$$

Mathematica [A]
time = 0.85, size = 399, normalized size = 1.89

$$
\frac{(d^2-e^2x^2)^p\left(\frac{64d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{8d^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{5\ 2^{4+p}e^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\ 2^{3+p}e^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\ 2^{1+p}e^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{2^pe^2(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{80de^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{16d^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((64*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^2*x^2)/d^
2])/(x*(1 - (e^2*x^2)/d^2)^p) + (8*d^3*Hypergeometric2F1[1 - p, -p, 2 - p,
d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) + (5*2^(4 + p)*e^2*(d
- e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
 + (e*x)/d)^p) + (3*2^(3 + p)*e^2*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p,
 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^(1 + p)*e^2*(d -
 e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1
+ (e*x)/d)^p) + (2^p*e^2*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (
```

```
d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (80*d*e^2*Hypergeometric2F1[-p
, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(16*d^7)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{3} \left(e x +d \right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
[Out] int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p/(x^7*e^4 + 4*d*x^6*e^3 + 6*d^2*x^5*e^2 + 4*d^3*
x^4*e + d^4*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d + e x\right) \left(d + e x\right)\right)^{p}}{x^{3} \left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d^2-e^2\,x^2\right)^p}{x^3\,{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x)
[Out] int((d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4), x)
```

3.304 $\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx$

Optimal. Leaf size=210

$$-\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3}+\frac{2de(d^2-e^2x^2)^{-3+p}}{x^2}-\frac{e^2(27-2p)\,(d^2-e^2x^2)^{-3+p}}{3x}+\frac{4e^4(48-17p+p^2)\,x(d^2-e^2x^2)^p\,\Big(}{3d}$$

```
[Out] -1/3*d^2*(-e^2*x^2+d^2)^(-3+p)/x^3+2*d*e*(-e^2*x^2+d^2)^(-3+p)/x^2-1/3*e^2*
(27-2*p)*(-e^2*x^2+d^2)^(-3+p)/x+4/3*e^4*(p^2-17*p+48)*x*(-e^2*x^2+d^2)^p*h
ypergeom([1/2, 4-p],[3/2],e^2*x^2/d^2)/d^8/((1-e^2*x^2/d^2)^p)-2*e^3*(5-p)*
(-e^2*x^2+d^2)^(-3+p)*hypergeom([1, -3+p],[-2+p],1-e^2*x^2/d^2)/d/(3-p)
```

Rubi [A]
time = 0.26, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1821, 778, 272, 67, 252, 251}

$$-\frac{e^2(27-2p)\,(d^2-e^2x^2)^{p-3}}{3x}+\frac{2de(d^2-e^2x^2)^{p-3}}{x^2}-\frac{d^2(d^2-e^2x^2)^{p-3}}{3x^3}-\frac{2e^3(5-p)\,(d^2-e^2x^2)^{p-3}\,{}_2F_1\left(1,p-3;p-2;1-\frac{e^2x^2}{d^2}\right)}{d(3-p)}+\frac{4e^4(p^2-17p+48)\,x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(\frac{1}{2},4-p;\frac{3}{2};\frac{e^2x^2}{d^2}\right)}{3d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

```
[Out] -1/3*(d^2*(d^2 - e^2*x^2)^(-3 + p))/x^3 + (2*d*e*(d^2 - e^2*x^2)^(-3 + p))/
x^2 - (e^2*(27 - 2*p)*(d^2 - e^2*x^2)^(-3 + p))/(3*x) + (4*e^4*(48 - 17*p +
 p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2]
)/(3*d^8*(1 - (e^2*x^2)/d^2)^p) - (2*e^3*(5 - p)*(d^2 - e^2*x^2)^(-3 + p)*H
ypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d*(3 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^4(d+ex)^4}\,dx = \int \frac{(d-ex)^4\,(d^2-e^2x^2)^{-4+p}}{x^4}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(12d^5e-d^4e^2(27-2p)x+12d^3e^3x^2-3d^2e^4x^3)}{x^3}\,dx}{3d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{2de(d^2-e^2x^2)^{-3+p}}{x^2} + \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(2d^6e^2(27-2p)-24d^5e^3(5-p)x+6d^4 \ldots}{x^2}}{6d^4}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{2de(d^2-e^2x^2)^{-3+p}}{x^2} - \frac{e^2(27-2p)\,(d^2-e^2x^2)^{-3+p}}{3x} - \frac{\int \frac{(24d^7e^3(5 \ldots}{}}{}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{2de(d^2-e^2x^2)^{-3+p}}{x^2} - \frac{e^2(27-2p)\,(d^2-e^2x^2)^{-3+p}}{3x} - (4de^3(5- \ldots$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{2de(d^2-e^2x^2)^{-3+p}}{x^2} - \frac{e^2(27-2p)\,(d^2-e^2x^2)^{-3+p}}{3x} - (2de^3(5- \ldots$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{2de(d^2-e^2x^2)^{-3+p}}{x^2} - \frac{e^2(27-2p)\,(d^2-e^2x^2)^{-3+p}}{3x} + \frac{4e^4(48- \ldots}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 452 vs. 2(210) = 420.

time = 0.80, size = 452, normalized size = 2.15

$$\frac{(d^2-e^2x^2)^p\left(-\frac{16d^4\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3}-\frac{480d^2e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}-\frac{96d^3e\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{15\ 2^{5+p}e^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{15\ 2^{3+p}e^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\ 2^{3+p}e^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\ 2^{p}e^3(-d+ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}-\frac{480de^3\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{48d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((-16*d^4*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d^
2])/(x^3*(1 - (e^2*x^2)/d^2)^p) - (480*d^2*e^2*Hypergeometric2F1[-1/2, -p,
1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) - (96*d^3*e*Hypergeometric2F
1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2) +
(15*2^(5 + p)*e^3*(-d + e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*
x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(3 + p)*e^3*(-d + e*x)*Hyperge
ometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p)
 + (3*2^(3 + p)*e^3*(-d + e*x)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d -
e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*e^3*(-d + e*x)*Hypergeometr
ic2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) - (4
80*d*e^3*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2*x^2)])/(p*(1 - d^2/(e^2*
x^2))^p)))/(48*d^8)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-e^2x^2+d^2)^p}{x^4(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)
```

```
[Out] int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p/(x^8*e^4 + 4*d*x^7*e^3 + 6*d^2*x^6*e^2 + 4*d^3*
x^5*e + d^4*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(-d+ex)(d+ex))^p}{x^4(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**4,x)
```

```
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d^2 - e^2\,x^2)^p}{x^4\,(d + e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4), x)
```

3.305 $\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx$

Optimal. Leaf size=216

$$-\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4}+\frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3}-\frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2}+\frac{8e^3(6-p)\,(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

```
[Out] -1/4*d^2*(-e^2*x^2+d^2)^(-3+p)/x^4+4/3*d*e*(-e^2*x^2+d^2)^(-3+p)/x^3-1/4*e^
2*(17-p)*(-e^2*x^2+d^2)^(-3+p)/x^2+8/3*e^3*(6-p)*(-e^2*x^2+d^2)^p*hypergeom
([-1/2, 4-p],[1/2],e^2*x^2/d^2)/d^7/x/((1-e^2*x^2/d^2)^p)+1/4*e^4*(p^2-21*p
+70)*(-e^2*x^2+d^2)^(-3+p)*hypergeom([1, -3+p],[-2+p],1-e^2*x^2/d^2)/d^2/(3
-p)
```

Rubi [A]
time = 0.26, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {866, 1821, 778, 372, 371, 272, 67}

$$-\frac{e^2(17-p)\,(d^2-e^2x^2)^{p-3}}{4x^2}-\frac{d^2(d^2-e^2x^2)^{p-3}}{4x^4}+\frac{4de(d^2-e^2x^2)^{p-3}}{3x^3}+\frac{e^4(p^2-21p+70)\,(d^2-e^2x^2)^{p-3}\,{}_2F_1\left(1,p-3;p-2;1-\frac{e^2x^2}{d^2}\right)}{4d^2(3-p)}+\frac{8e^3(6-p)\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p\,{}_2F_1\left(-\frac{1}{2},4-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{3d^7x}$$

Antiderivative was successfully verified.

```
[In] Int[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

```
[Out] -1/4*(d^2*(d^2 - e^2*x^2)^(-3 + p))/x^4 + (4*d*e*(d^2 - e^2*x^2)^(-3 + p))/
(3*x^3) - (e^2*(17 - p)*(d^2 - e^2*x^2)^(-3 + p))/(4*x^2) + (8*e^3*(6 - p)*
(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 4 - p, 1/2, (e^2*x^2)/d^2])/(3*d^
7*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(70 - 21*p + p^2)*(d^2 - e^2*x^2)^(-3 + p
)*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(4*d^2*(3 - p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d^2-e^2x^2)^p}{x^5(d+ex)^4}\,dx = \int \frac{(d-ex)^4\,(d^2-e^2x^2)^{-4+p}}{x^5}\,dx$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} - \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(16d^5e-2d^4e^2(17-p)x+16d^3e^3x^2-4d^2e^4x^3)}{x^4}\,dx}{4d^2}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} + \frac{\int \frac{(d^2-e^2x^2)^{-4+p}(6d^6e^2(17-p)-32d^5e^3(6-p)x+12d}{x^3}}{12d^4}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2} - \int \frac{(64d^7e^3(6}{}$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2} - \frac{1}{3}\big(8de^3(6$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2} + \frac{1}{4}\big(e^4(70-$$

$$= -\frac{d^2(d^2-e^2x^2)^{-3+p}}{4x^4} + \frac{4de(d^2-e^2x^2)^{-3+p}}{3x^3} - \frac{e^2(17-p)\,(d^2-e^2x^2)^{-3+p}}{4x^2} + \frac{8e^3(6-p)}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 505 vs. 2(216) = 432.
time = 0.91, size = 505, normalized size = 2.34

$$\frac{(d^2-e^2x^2)^p\left(\frac{64d^4e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{3}{2},-p;-\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x^3}+\frac{960d^2e^3\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\frac{e^2x^2}{d^2}\right)}{x}+\frac{240d^3e^2\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(1-p,-p;2-p;\frac{d^2}{e^2x^2}\right)}{(-1+p)x^2}+\frac{105\ 2^{3+p}e^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(1-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{24d^5\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(2-p,-p;3-p;\frac{d^2}{e^2x^2}\right)}{(-2+p)x^4}+\frac{45\ 2^{2+p}e^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(2-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{15\ 2^{1+p}e^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(3-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{3\ 2^pe^4(d-ex)\left(1+\frac{ex}{d}\right)^{-p}{}_2F_1\left(4-p,1+p;2+p;\frac{d-ex}{2d}\right)}{1+p}+\frac{840de^4\left(1-\frac{d^2}{e^2x^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;\frac{d^2}{e^2x^2}\right)}{p}\right)}{48d^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x]
```

```
[Out] ((d^2 - e^2*x^2)^p*((64*d^4*e*Hypergeometric2F1[-3/2, -p, -1/2, (e^2*x^2)/d
^2])/(x^3*(1 - (e^2*x^2)/d^2)^p) + (960*d^2*e^3*Hypergeometric2F1[-1/2, -p,
 1/2, (e^2*x^2)/d^2])/(x*(1 - (e^2*x^2)/d^2)^p) + (240*d^3*e^2*Hypergeometr
ic2F1[1 - p, -p, 2 - p, d^2/(e^2*x^2)])/((-1 + p)*(1 - d^2/(e^2*x^2))^p*x^2
) + (105*2^(3 + p)*e^4*(d - e*x)*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d
- e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (24*d^5*Hypergeometric2F1[2 - p,
 -p, 3 - p, d^2/(e^2*x^2)])/((-2 + p)*(1 - d^2/(e^2*x^2))^p*x^4) + (45*2^(2
 + p)*e^4*(d - e*x)*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e*x)/(2*d)]
)/((1 + p)*(1 + (e*x)/d)^p) + (15*2^(1 + p)*e^4*(d - e*x)*Hypergeometric2F1
[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1 + p)*(1 + (e*x)/d)^p) + (3*2^p*
e^4*(d - e*x)*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/((1
+ p)*(1 + (e*x)/d)^p) + (840*d*e^4*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^
2*x^2)])/(p*(1 - d^2/(e^2*x^2))^p)))/(48*d^9)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}}{x^{5} \left(e x +d \right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
[Out] int((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x^5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p/(x^9*e^4 + 4*d*x^8*e^3 + 6*d^2*x^7*e^2 + 4*d^3*
x^6*e + d^4*x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(-d + e x\right) \left(d + e x\right)\right)^{p}}{x^{5} \left(d + e x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**4,x)
[Out] Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2+d^2)^p/x^5/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p/((x*e + d)^4*x^5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d^2 - e^2\,x^2)^p}{x^5\,(d + e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4),x)
```

```
[Out] int((d^2 - e^2*x^2)^p/(x^5*(d + e*x)^4), x)
```

3.306 $\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=264

$$-\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)}+\frac{2d^3(3+2m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\ 2}{g(1+m)(3+m+2p)}$$

```
[Out] -3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(1+p)/g/(3+m+2*p)-e*(g*x)^(2+m)*(-e^2*x^2+d
^2)^(1+p)/g^2/(4+m+2*p)+2*d^3*(3+2*m+p)*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hyperg
eom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/g/(1+m)/(3+m+2*p)/((1-e^2*x^2/
d^2)^p)+2*d^2*e*(7+2*m+3*p)*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1+1
/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/(4+m+2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.24, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1823, 822, 372, 371}

$$\frac{2d^2e(2m+3p+7)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)(m+2p+4)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p+1}}{g^2(m+2p+4)}-\frac{3d(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}+\frac{2d^3(2m+p+3)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (-3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(1 + p))/(g^2*(4 + m + 2*p)) + (2*d^3*(3 + 2*m + p)*
(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
 (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (2*d^2*e
*(7 + 2*m + 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/
2, -p, (4 + m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(4 + m + 2*p)*(1 - (e^2*x^2)
/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int (gx)^m(d+ex)^3\left(d^2-e^2x^2\right)^p\,dx = -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} - \frac{\int (gx)^m\left(d^2-e^2x^2\right)^p\left(-d^3e^2(4+m+2p)\right.}{e^2}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{\int (gx)^m\left(2d\right.}{}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{(2d^3(3+2m}{}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{\left(2d^3(3+2\right.}{}$$

$$= -\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{1+p}}{g^2(4+m+2p)} + \frac{2d^3(3+2m}{}$$

Mathematica [A]
time = 0.21, size = 194, normalized size = 0.73

$$x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(\frac{d^3\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(\frac{3d^2\,{}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{2+m}+ex\left(\frac{3d\,{}_2F_1\left(\frac{3+m}{2},-p;\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)}{3+m}+\frac{ex\,{}_2F_1\left(\frac{4+m}{2},-p;\frac{6+m}{2};\frac{e^2x^2}{d^2}\right)}{4+m}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]

[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((3*d^2*Hypergeometric2F1[(2 + m)/2, -p,
```

```
(4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3 + m)/2,
 -p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) + (e*x*Hypergeometric2F1[(4 + m)/2,
 -p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(1 - (e^2*x^2)/d^2)^p
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (gx)^m (ex+d)^3 \left(-e^2x^2+d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(-x^2*e^2 + d^2)^p*(g*x)
^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.27, size = 262, normalized size = 0.99

$$\frac{d^3 d^{2p} g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{3d^2 d^{2p} e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{3d d^{2p} e^2 g^m x^3 x^m \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{d^{2p} e^3 g^m x^4 x^m \Gamma\left(\frac{m}{2}+2\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+2 \\ \frac{m}{2}+3 \end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + 3*d**2*d**(2*
```

```
p)*e*g**m*x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x*
*2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2)) + 3*d*d**(2*p)*e**2*g**m*x**3
*x**m*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_p
olar(2*I*pi)/d**2)/(2*gamma(m/2 + 5/2)) + d**(2*p)*e**3*g**m*x**4*x**m*gamm
a(m/2 + 2)*hyper((-p, m/2 + 2), (m/2 + 3,), e**2*x**2*exp_polar(2*I*pi)/d**
2)/(2*gamma(m/2 + 3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^3*(-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d^2-e^2\,x^2\right)}^p\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^3,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^3, x)
```

3.307 $\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=206

$$-\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}+\frac{2d^2(2+m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)(3+m+2p)}+\frac{2de}{\cdots}$$

```
[Out] -(g*x)^(1+m)*(-e^2*x^2+d^2)^(1+p)/g/(3+m+2*p)+2*d^2*(2+m+p)*(g*x)^(1+m)*(-e
^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/g/(1+m)/(3
+m+2*p)/((1-e^2*x^2/d^2)^p)+2*d*e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([-
p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.11, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {1823, 822, 372, 371}

$$\frac{2de(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{2d^2(m+p+2)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)(m+2p+3)}-\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^{p+1}}{g(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] -(((g*x)^(1 + m)*(d^2 - e^2*x^2)^(1 + p))/(g*(3 + m + 2*p))) + (2*d^2*(2 +
m + p)*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3
+ m)/2, (e^2*x^2)/d^2])/(g*(1 + m)*(3 + m + 2*p)*(1 - (e^2*x^2)/d^2)^p) + (
2*d*e*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 +
 m)/2, (e^2*x^2)/d^2])/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
```

```
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int (gx)^m(d+ex)^2\left(d^2-e^2x^2\right)^p\,dx &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}-\frac{\int (gx)^m\left(-2d^2e^2(2+m+p)-2de^3(3+\right.}{e^2(3+m+2p)} \\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}+\frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p\,dx}{g}+\frac{(2d^2(2}{} \\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}+\frac{\left(2de\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+}}{g} \\ &= -\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{1+p}}{g(3+m+2p)}+\frac{2d^2(2+m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\right.}{g(1+m)(3+}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 169, normalized size = 0.82

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2\left(6+5m+m^2\right){}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)+e(1+m)x\left(2d(3+m){}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)+e(2+m)x\,{}_2F_1\left(\frac{3+m}{2},-p;\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)\right)\right)}{(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)
/2, -p, (3 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*(2*d*(3 + m)*Hypergeometric
2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2] + e*(2 + m)*x*Hypergeometric2F
1[(3 + m)/2, -p, (5 + m)/2, (e^2*x^2)/d^2])))/((1 + m)*(2 + m)*(3 + m)*(1 -
 (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(gx\right)^{m}\left(ex+d\right)^{2}\left(-e^{2}x^{2}+d^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.16, size = 192, normalized size = 0.93

$$\frac{d^2 d^{2p} g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\right| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{d d^{2p} e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\right| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{d^{2p} e^2 g^m x^3 x^m \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\right| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d*d**(2*p)*e*
g**m*x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2*ex
p_polar(2*I*pi)/d**2)/gamma(m/2 + 2) + d**(2*p)*e**2*g**m*x**3*x**m*gamma(m
/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), e**2*x**2*exp_polar(2*I*pi)/
d**2)/(2*gamma(m/2 + 5/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^2*(-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d^2 - e^2\,x^2)^p\,(g\,x)^m\,(d + e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^2,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^2, x)
```

3.308 $\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=153

$$\frac{d(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)}+\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{2+m}{2}\right.}{g^2(2+m)}$$

```
[Out] d*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^
2/d^2)/g/(1+m)/((1-e^2*x^2/d^2)^p)+e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom
([-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {822, 372, 371}

$$\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{g^2(m+2)}+\frac{d(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)
/2, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) + (e*(g*x)^(2 + m)*(d
^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, (e^2*x^2)/d^2])
/(g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
```

] && !IGtQ[p, 0]

Rubi steps

$$\int (gx)^m(d+ex)\left(d^2-e^2x^2\right)^p\,dx = d\int (gx)^m\left(d^2-e^2x^2\right)^p\,dx+\frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^p\,dx}{g}$$

$$=\left(d\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p\,dx+\frac{\left(e(d^2-\right.}{}$$

$$=\frac{d(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)}+\frac{e(gx)^2}{}$$

Mathematica [A]
time = 0.10, size = 116, normalized size = 0.76

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d(2+m)\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)+e(1+m)x\,{}_2F_1\left(\frac{2+m}{2},-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)\right)}{(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(d^2 - e^2*x^2)^p,x]
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3
 + m)/2, (e^2*x^2)/d^2] + e*(1 + m)*x*Hypergeometric2F1[(2 + m)/2, -p, (4 +
 m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (gx)^m\,(ex+d)\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
[Out] int((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.29, size = 122, normalized size = 0.80

$$\frac{dd^{2p}g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}-p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{d^{2p}eg^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix}-p, \frac{m}{2}+1 \\ \frac{m}{2}+2\end{matrix}\middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,)
, e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**(2*p)*e*g**m*
x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_pol
ar(2*I*pi)/d**2)/(2*gamma(m/2 + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2-e^2\,x^2\right)}^p\,{\left(g\,x\right)}^m\,\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x),x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x), x)
```

3.309 $\int (gx)^m (d^2-e^2x^2)^p \, dx$

Optimal. Leaf size=75

$$\frac{(gx)^{1+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/
d^2)/g/(1+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]

time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {372, 371}

$$\frac{(gx)^{m+1}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2
, (e^2*x^2)/d^2])/(g*(1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (gx)^m (d^2-e^2x^2)^p \, dx &= \left((d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^p \, dx\\ &= \frac{(gx)^{1+m}(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{g(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 0.97

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;1+\frac{1+m}{2};\frac{e^2x^2}{d^2}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d^2 - e^2*x^2)^p,x]
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2
, (e^2*x^2)/d^2])/((1 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (gx)^m\left(-e^2x^2+d^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p,x)
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.36, size = 61, normalized size = 0.81

$$\frac{d^{2p}g^m x x^m\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\left.\begin{matrix}-p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\right|\frac{e^2x^2e^{2i\pi}}{d^2}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,),
e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2-e^2\,x^2\right)}^p\,{\left(g\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m,x)
```

```
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m, x)
```

3.310 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{d+ex}\,dx$

Optimal. Leaf size=163

$$\frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{dg(1+m)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{2+m}{2}, \dots\right.}{d^2g^2(2+m)}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([1-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2
/d^2)/d/g/(1+m)/((1-e^2*x^2/d^2)^p)-e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeo
m([1-p, 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^2/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]

time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {906, 83, 127, 372, 371}

$$\frac{(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{dg(m+1)}-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^2g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m
)/2, (e^2*x^2)/d^2])/(d*g*(1 + m)*(1 - (e^2*x^2)/d^2)^p) - (e*(g*x)^(2 + m)
*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2, (e^2*x^2)
/d^2])/(d^2*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{d+ex}\,dx = \left((d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^p(d+ex)^{-1+p}\,dx$$

$$= \left(d(d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^{-1+p}(d+ex)^{-1+p}\,dx - \cdots$$

$$= d\int (gx)^m\left(d^2-e^2x^2\right)^{-1+p}\,dx - \frac{e\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}\,dx}{g}$$

$$= \frac{\left(\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-1+p}\,dx}{d} - \frac{\left(e\left(d^2-e^2x^2\right)^p\left(1 \cdots\right.\right.}{\cdots}$$

$$= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{dg(1+m)} - \frac{e(gx)^{2+m}\left(d^2 \cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.13, size = 124, normalized size = 0.76

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(-e(1+m)x\,{}_2F_1\left(1+\frac{m}{2},1-p;2+\frac{m}{2};\frac{e^2x^2}{d^2}\right)+d(2+m)\,{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)\right)}{d^2(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x),x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(-(e*(1 + m)*x*Hypergeometric2F1[1 + m/2, 1 -
p, 2 + m/2, (e^2*x^2)/d^2]) + d*(2 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p,
 (3 + m)/2, (e^2*x^2)/d^2]))/(d^2*(1 + m)*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^{m} \left(-e^{2} x^{2}+d^{2}\right)^{p}}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*(g*x)^m/(x*e + d), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.04, size = 337, normalized size = 2.07

$$-\frac{0^{p} d d^{2p} g^{m} m x^{m} \Phi\left(\frac{d^{2}}{e^{2}x^{2}}, 1, \frac{1}{2}-\frac{m}{2}\right) \Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4e^{2} x \Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^{p} d d^{2p} g^{m} x^{m} \Phi\left(\frac{d^{2}}{e^{2}x^{2}}, 1, \frac{1}{2}-\frac{m}{2}\right) \Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4e^{2} x \Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^{p} d^{2p} g^{m} m x^{m} \Phi\left(\frac{d^{2}}{e^{2}x^{2}}, 1, \frac{m e^{i\pi}}{2}\right) \Gamma\left(-\frac{m}{2}\right)}{4e\Gamma\left(1-\frac{m}{2}\right)} + \frac{d e^{2p} g^{m} p x^{m} x^{2p} e^{i\pi p} \Gamma(p) \Gamma\left(-\frac{m}{2}-p+\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} 1-p, -\frac{m}{2}-p+\frac{1}{2} \\ -\frac{m}{2}-p+\frac{3}{2} \end{matrix}\middle| \frac{d^{2}}{e^{2}x^{2}}\right)}{2e^{2} x \Gamma(p+1) \Gamma\left(-\frac{m}{2}-p+\frac{3}{2}\right)} - \frac{e^{2p} g^{m} p x^{m} x^{2p} e^{i\pi p} \Gamma(p) \Gamma\left(-\frac{m}{2}-p\right) {}_{2}F_{1}\left(\begin{matrix} 1-p, -\frac{m}{2}-p \\ -\frac{m}{2}-p+1 \end{matrix}\middle| \frac{d^{2}}{e^{2}x^{2}}\right)}{2e\Gamma(p+1) \Gamma\left(-\frac{m}{2}-p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

```
[Out] -0**p*d*d**(2*p)*g**m*m*x**m*lerchphi(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma
(1/2 - m/2)/(4*e**2*x*gamma(3/2 - m/2)) + 0**p*d*d**(2*p)*g**m*x**m*lerchph
i(d**2/(e**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*e**2*x*gamma(3/2 - m/
```

```
2)) + 0**p*d**(2*p)*g**m*m*x**m*lerchphi(d**2/(e**2*x**2), 1, m*exp_polar(I
*pi)/2)*gamma(-m/2)/(4*e*gamma(1 - m/2)) + d*e**(2*p)*g**m*p*x**m*x**(2*p)*
exp(I*pi*p)*gamma(p)*gamma(-m/2 - p + 1/2)*hyper((1 - p, -m/2 - p + 1/2), (
-m/2 - p + 3/2,), d**2/(e**2*x**2))/(2*e**2*x*gamma(p + 1)*gamma(-m/2 - p +
 3/2)) - e**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p)
*hyper((1 - p, -m/2 - p), (-m/2 - p + 1,), d**2/(e**2*x**2))/(2*e*gamma(p +
 1)*gamma(-m/2 - p + 1))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^2 - e^2\,x^2)^p\,(g\,x)^m}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x),x)
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x), x)
```

3.311 $\int \frac{(gx)^m (d^2-e^2x^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=214

$$\frac{(gx)^{1+m} (d^2-e^2x^2)^{-1+p}}{g(1-m-2p)}-\frac{2(m+p)(gx)^{1+m} (d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1+m}{2},2-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{d^2g(1+m)(1-m-2p)}-\frac{2e(gx)^{2-}}{}$$

```
[Out] (g*x)^(1+m)*(-e^2*x^2+d^2)^(-1+p)/g/(1-m-2*p)-2*(m+p)*(g*x)^(1+m)*(-e^2*x^2
+d^2)^p*hypergeom([2-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d^2/g/(1+m)/(1-
m-2*p)/((1-e^2*x^2/d^2)^p)-2*e*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([2-p,
 1+1/2*m],[2+1/2*m],e^2*x^2/d^2)/d^3/g^2/(2+m)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]
time = 0.14, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {866, 1823, 822, 372, 371}

$$-\frac{2(m+p)(gx)^{m+1} (d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2},2-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^2g(m+1)(-m-2p+1)}+\frac{(gx)^{m+1} (d^2-e^2x^2)^{p-1}}{g(-m-2p+1)}-\frac{2e(gx)^{m+2} (d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2},2-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^3g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((g*x)^(1 + m)*(d^2 - e^2*x^2)^(-1 + p))/(g*(1 - m - 2*p)) - (2*(m + p)*(g*
x)^(1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 2 - p, (3 + m)/2,
 (e^2*x^2)/d^2])/(d^2*g*(1 + m)*(1 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e
*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 2 - p, (4 + m
)/2, (e^2*x^2)/d^2])/(d^3*g^2*(2 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
```

```
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(gx)^m\left(d^2-e^2x^2\right)^p}{(d+ex)^2}\,dx &= \int (gx)^m(d-ex)^2\left(d^2-e^2x^2\right)^{-2+p}\,dx\\
&= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)}+\frac{\int (gx)^m\left(-2d^2e^2(m+p)-2de^3(1-m-2p)x\right)\left(d^2\right.}{e^2(1-m-2p)}\\
&= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)}-\frac{(2de)\int (gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}\,dx}{g}-\frac{(2d^2(m+p))}{}\\
&= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)}-\frac{\left(2e\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\int (gx)^{1+m}\left(1-\frac{e^2x^2}{d^2}\right.}{d^3g}\\
&= \frac{(gx)^{1+m}\left(d^2-e^2x^2\right)^{-1+p}}{g(1-m-2p)}-\frac{2(m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{1+}{2}\right.}{d^2g(1+m)(1-m-2p)}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 180, normalized size = 0.84

$$\frac{x(gx)^m\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(d^2\left(6+5m+m^2\right){}_2F_1\left(\frac{1+m}{2},2-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)-e(1+m)x\left(2d(3+m){}_2F_1\left(\frac{2+m}{2},2-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)-e(2+m)x\,{}_2F_1\left(\frac{3+m}{2},2-p;\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)\right)\right)}{d^4(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*(d^2*(6 + 5*m + m^2)*Hypergeometric2F1[(1 + m)
/2, 2 - p, (3 + m)/2, (e^2*x^2)/d^2] - e*(1 + m)*x*(2*d*(3 + m)*Hypergeomet
ric2F1[(2 + m)/2, 2 - p, (4 + m)/2, (e^2*x^2)/d^2] - e*(2 + m)*x*Hypergeome
tric2F1[(3 + m)/2, 2 - p, (5 + m)/2, (e^2*x^2)/d^2])))/(d^4*(1 + m)*(2 + m)
*(3 + m)*(1 - (e^2*x^2)/d^2)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (-e^2x^2+d^2)^p}{(ex+d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*(g*x)^m/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m \left(-(-d+ex)(d+ex)\right)^p}{(d+ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)
```

```
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p\,{\left(g\,x\right)}^m}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^2,x)
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^2, x)
```

3.312 $\int \frac{(gx)^m(d^2-e^2x^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=275

$$\frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)}-\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)}-\frac{2(2m+p)(gx)^{1+m}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\Big(\frac{1}{}}{d^3g(1+m)(3-m-2p)}$$

```
[Out] 3*d*(g*x)^(1+m)*(-e^2*x^2+d^2)^(-2+p)/g/(3-m-2*p)-e*(g*x)^(2+m)*(-e^2*x^2+d
^2)^(-2+p)/g^2/(2-m-2*p)-2*(2*m+p)*(g*x)^(1+m)*(-e^2*x^2+d^2)^p*hypergeom([
3-p, 1/2+1/2*m],[3/2+1/2*m],e^2*x^2/d^2)/d^3/g/(1+m)/(3-m-2*p)/((1-e^2*x^2/
d^2)^p)-2*e*(2-2*m-3*p)*(g*x)^(2+m)*(-e^2*x^2+d^2)^p*hypergeom([3-p, 1+1/2*
m],[2+1/2*m],e^2*x^2/d^2)/d^4/g^2/(2+m)/(2-m-2*p)/((1-e^2*x^2/d^2)^p)
```

Rubi [A]

time = 0.28, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {866, 1823, 822, 372, 371}

$$-\frac{e(gx)^{m+2}\left(d^2-e^2x^2\right)^{p-2}}{g^2(-m-2p+2)}+\frac{3d(gx)^{m+1}\left(d^2-e^2x^2\right)^{p-2}}{g(-m-2p+3)}-\frac{2e(-2m-3p+2)(gx)^{m+2}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+2}{2},3-p;\frac{m+4}{2};\frac{e^2x^2}{d^2}\right)}{d^4g^2(m+2)(-m-2p+2)}-\frac{2(2m+p)(gx)^{m+1}\left(d^2-e^2x^2\right)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{m+1}{2},3-p;\frac{m+3}{2};\frac{e^2x^2}{d^2}\right)}{d^3g(m+1)(-m-2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (3*d*(g*x)^(1 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g*(3 - m - 2*p)) - (e*(g*x)^(
2 + m)*(d^2 - e^2*x^2)^(-2 + p))/(g^2*(2 - m - 2*p)) - (2*(2*m + p)*(g*x)^(
1 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 + m)/2, (e^
2*x^2)/d^2])/(d^3*g*(1 + m)*(3 - m - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(2
- 2*m - 3*p)*(g*x)^(2 + m)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2 + m)/2, 3
 - p, (4 + m)/2, (e^2*x^2)/d^2])/(d^4*g^2*(2 + m)*(2 - m - 2*p)*(1 - (e^2*x
^2)/d^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{(gx)^m \left(d^2-e^2x^2\right)^p}{(d+ex)^3}\,dx = \int (gx)^m(d-ex)^3\left(d^2-e^2x^2\right)^{-3+p}\,dx$$

$$= -\frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} + \frac{\int (gx)^m\left(d^2-e^2x^2\right)^{-3+p}\left(d^3e^2(2-m-2p)-2d\right.}{e^2(2-m}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} + \frac{\int (gx)^m\left(-2d^3e^4(2-\right.}{}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{\left(2d^2e(2-2m-3p)\right.}{g(}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{\left(2e(2-2m-3p)\right(}{}$$

$$= \frac{3d(gx)^{1+m}\left(d^2-e^2x^2\right)^{-2+p}}{g(3-m-2p)} - \frac{e(gx)^{2+m}\left(d^2-e^2x^2\right)^{-2+p}}{g^2(2-m-2p)} - \frac{2(2m+p)(gx)^{1+m}\left(\right.}{}$$

Mathematica [A]
time = 0.22, size = 206, normalized size = 0.75

$$\frac{x(gx)^m (d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} \left(\frac{d^3\,{}_2F_1\left(\frac{1+m}{2},3-p;\frac{3+m}{2};\frac{e^2x^2}{d^2}\right)}{1+m}+ex\left(-\frac{3d^2\,{}_2F_1\left(\frac{2+m}{2},3-p;\frac{4+m}{2};\frac{e^2x^2}{d^2}\right)}{2+m}+ex\left(\frac{3d\,{}_2F_1\left(\frac{3+m}{2},3-p;\frac{5+m}{2};\frac{e^2x^2}{d^2}\right)}{3+m}-\frac{ex\,{}_2F_1\left(\frac{4+m}{2},3-p;\frac{6+m}{2};\frac{e^2x^2}{d^2}\right)}{4+m}\right)\right)\right)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*(d^2 - e^2*x^2)^p*((d^3*Hypergeometric2F1[(1 + m)/2, 3 - p, (3 +
 m)/2, (e^2*x^2)/d^2])/(1 + m) + e*x*((-3*d^2*Hypergeometric2F1[(2 + m)/2,
3 - p, (4 + m)/2, (e^2*x^2)/d^2])/(2 + m) + e*x*((3*d*Hypergeometric2F1[(3
+ m)/2, 3 - p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (e*x*Hypergeometric2F1[
(4 + m)/2, 3 - p, (6 + m)/2, (e^2*x^2)/d^2])/(4 + m)))))/(d^6*(1 - (e^2*x^2
)/d^2)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m \left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

```
[Out] int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^2*e^2 + d^2)^p*(g*x)^m/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^
3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m \left(-(-d+ex)\,(d+ex)\right)^p}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)
```

```
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m/(x*e + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d^2-e^2\,x^2\right)}^p\,{\left(g\,x\right)}^m}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^p*(g*x)^m)/(d + e*x)^3, x)
```

3.313 $\int \frac{(gx)^m(1-a^2x^2)^p}{1+ax}\,dx$

Optimal. Leaf size=89

$$\frac{(gx)^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};a^2x^2\right)}{g(1+m)}-\frac{a(gx)^{2+m}\,{}_2F_1\left(\frac{2+m}{2},1-p;\frac{4+m}{2};a^2x^2\right)}{g^2(2+m)}$$

```
[Out] (g*x)^(1+m)*hypergeom([1-p, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/g/(1+m)-a*(g*x)
^(2+m)*hypergeom([1-p, 1+1/2*m],[2+1/2*m],a^2*x^2)/g^2/(2+m)
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {904, 83, 126, 371}

$$\frac{(gx)^{m+1}\,{}_2F_1\left(\frac{m+1}{2},1-p;\frac{m+3}{2};a^2x^2\right)}{g(m+1)}-\frac{a(gx)^{m+2}\,{}_2F_1\left(\frac{m+2}{2},1-p;\frac{m+4}{2};a^2x^2\right)}{g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
```

```
[Out] ((g*x)^(1 + m)*Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2])/(g*
(1 + m)) - (a*(g*x)^(2 + m)*Hypergeometric2F1[(2 + m)/2, 1 - p, (4 + m)/2,
a^2*x^2])/(g^2*(2 + m))
```

Rule 83

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[a, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^p, x], x] + Dist[
b/f, Int[(a + b*x)^n*(c + d*x)^n*(f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n - 1, 0] &&  !RationalQ[
p] &&  !IGtQ[m, 0] && NeQ[m + n + p + 2, 0]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(f*x)^p, x] /; FreeQ[{a, b, c, d, f,
m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && GtQ[a, 0] && GtQ[c, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 904

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p, x
] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m, 0] &&  !
IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(gx)^m \left(1-a^2x^2\right)^p}{1+ax}\,dx &= \int (gx)^m(1-ax)^p(1+ax)^{-1+p}\,dx \\
&= -\frac{a\int (gx)^{1+m}(1-ax)^{-1+p}(1+ax)^{-1+p}\,dx}{g}+\int (gx)^m(1-ax)^{-1+p}(1+ax)^{-1+p} \\
&= -\frac{a\int (gx)^{1+m}\left(1-a^2x^2\right)^{-1+p}\,dx}{g}+\int (gx)^m\left(1-a^2x^2\right)^{-1+p}\,dx \\
&= \frac{(gx)^{1+m}\,{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};a^2x^2\right)}{g(1+m)}-\frac{a(gx)^{2+m}\,{}_2F_1\left(\frac{2+m}{2},1-p;\frac{4+m}{2};a^2x^2\right)}{g^2(2+m)}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 77, normalized size = 0.87

$$x(gx)^m\left(-\frac{ax\,{}_2F_1\left(1+\frac{m}{2},1-p;2+\frac{m}{2};a^2x^2\right)}{2+m}+\frac{{}_2F_1\left(\frac{1+m}{2},1-p;\frac{3+m}{2};a^2x^2\right)}{1+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*x)^m*(1 - a^2*x^2)^p)/(1 + a*x),x]
[Out] x*(g*x)^m*(-((a*x*Hypergeometric2F1[1 + m/2, 1 - p, 2 + m/2, a^2*x^2])/(2 +
 m)) + Hypergeometric2F1[(1 + m)/2, 1 - p, (3 + m)/2, a^2*x^2]/(1 + m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\left(-a^2x^2+1\right)^p}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
[Out] int((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.45, size = 308, normalized size = 3.46

$$\frac{0^p g^m m x^m \Phi\left(\frac{1}{a^2x^2}, 1, \frac{me^{i\pi}}{2}\right)\Gamma\left(-\frac{m}{2}\right)}{4a\Gamma\left(1-\frac{m}{2}\right)} - \frac{0^p g^m m x^m \Phi\left(\frac{1}{a^2x^2}, 1, \frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4a^2x\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} + \frac{0^p g^m x^m \Phi\left(\frac{1}{a^2x^2}, 1, \frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4a^2x\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} - \frac{a^{2p}g^m p x^m x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(-\frac{m}{2}-p\right){}_2F_1\left(\begin{matrix}1-p, -\frac{m}{2}-p\\ -\frac{m}{2}-p+1\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2a\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+1\right)} + \frac{a^{2p}g^m p x^m x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(-\frac{m}{2}-p+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}1-p, -\frac{m}{2}-p+\frac{1}{2}\\ -\frac{m}{2}-p+\frac{3}{2}\end{matrix}\middle|\frac{1}{a^2x^2}\right)}{2a^2x\Gamma(p+1)\Gamma\left(-\frac{m}{2}-p+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(-a**2*x**2+1)**p/(a*x+1),x)
```

```
[Out] 0**p*g**m*m*x**m*lerchphi(1/(a**2*x**2), 1, m*exp_polar(I*pi)/2)*gamma(-m/2
)/(4*a*gamma(1 - m/2)) - 0**p*g**m*m*x**m*lerchphi(1/(a**2*x**2), 1, 1/2 -
m/2)*gamma(1/2 - m/2)/(4*a**2*x*gamma(3/2 - m/2)) + 0**p*g**m*x**m*lerchphi
(1/(a**2*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)/(4*a**2*x*gamma(3/2 - m/2))
- a**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-m/2 - p)*hyper(
(1 - p, -m/2 - p), (-m/2 - p + 1,), 1/(a**2*x**2))/(2*a*gamma(p + 1)*gamma(
-m/2 - p + 1)) + a**(2*p)*g**m*p*x**m*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
m/2 - p + 1/2)*hyper((1 - p, -m/2 - p + 1/2), (-m/2 - p + 3/2,), 1/(a**2*x*
*2))/(2*a**2*x*gamma(p + 1)*gamma(-m/2 - p + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(-a^2*x^2+1)^p/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^p*(g*x)^m/(a*x + 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m\,{\left(1-a^2\,x^2\right)}^p}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(1 - a^2*x^2)^p)/(a*x + 1),x)
[Out] int(((g*x)^m*(1 - a^2*x^2)^p)/(a*x + 1), x)
```

3.314 $\int (gx)^m(d+ex)^n\left(d^2-e^2x^2\right)^p\,dx$

Optimal. Leaf size=96

$$\frac{(gx)^{1+m}(d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^p F_1\left(1+m;-p,-n-p;2+m;\frac{ex}{d},-\frac{ex}{d}\right)}{g(1+m)}$$

```
[Out] (g*x)^(1+m)*(e*x+d)^n*(1+e*x/d)^(-n-p)*(-e^2*x^2+d^2)^p*AppellF1(1+m,-p,-n-
p,2+m,e*x/d,-e*x/d)/g/(1+m)/((1-e*x/d)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {906, 140, 138}

$$\frac{(gx)^{m+1}(d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\left(\frac{ex}{d}+1\right)^{-n-p}F_1\left(m+1;-p,-n-p;m+2;\frac{ex}{d},-\frac{ex}{d}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*(1 + (e*x)/d)^(-n - p)*(d^2 - e^2*x^2)^p*AppellF
1[1 + m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/(g*(1 + m)*(1 - (e*x)/d)^
p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 906

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*
x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int (gx)^m(d+ex)^n\left(d^2-e^2x^2\right)^p\,dx &= \left((d-ex)^{-p}(d+ex)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d-ex)^p(d+ex)^{n+p}\,dx \\
&= \left((d+ex)^{-p}\left(1-\frac{ex}{d}\right)^{-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m(d+ex)^{n+p}\left(1-\frac{ex}{d}\right. \\
&= \left((d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^p\right)\int (gx)^m\left(1-\frac{e}{}\right. \\
&= \frac{(gx)^{1+m}(d+ex)^n\left(1-\frac{ex}{d}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-n-p}\left(d^2-e^2x^2\right)^p F_1\left(1+m;-p\right.}{g(1+m)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.15, size = 90, normalized size = 0.94

$$
\frac{x(gx)^m(d-ex)^p\left(\frac{d-ex}{d}\right)^{-p}(d+ex)^{n+p}\left(\frac{d+ex}{d}\right)^{-n-p}F_1\left(1+m;-p,-n-p;2+m;\frac{ex}{d},-\frac{ex}{d}\right)}{1+m}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(d^2 - e^2*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(d - e*x)^p*(d + e*x)^(n + p)*((d + e*x)/d)^(-n - p)*AppellF1[1
+ m, -p, -n - p, 2 + m, (e*x)/d, -((e*x)/d)])/((1 + m)*((d - e*x)/d)^p)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$
\int (gx)^m(ex+d)^n\left(-e^2x^2+d^2\right)^p\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m*(x*e + d)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
[Out] integral((-x^2*e^2 + d^2)^p*(g*x)^m*(x*e + d)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (gx)^m\,(-(-d+ex)\,(d+ex))^p\,(d+ex)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(-e**2*x**2+d**2)**p,x)
[Out] Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p*(d + e*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(-e^2*x^2+d^2)^p,x, algorithm="giac")
[Out] integrate((-x^2*e^2 + d^2)^p*(g*x)^m*(x*e + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d^2-e^2\,x^2\right)}^p\,{\left(g\,x\right)}^m\,{\left(d+e\,x\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^n,x)
[Out] int((d^2 - e^2*x^2)^p*(g*x)^m*(d + e*x)^n, x)
```

3.315 $\int \frac{x\sqrt{1+x}}{1+x^2}\,dx$

Optimal. Leaf size=214

$$2\sqrt{1+x}+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right.}$$

```
[Out] 2*(1+x)^(1/2)+1/4*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1
/2))^(1/2)-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2)
)^(1/2)+arctan((-2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))/(
2+2*2^(1/2))^(1/2)-arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2)
)^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {839, 841, 1183, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\text{ArcTan}\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}+2\sqrt{x+1}+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 +
 Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] - ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[
1 + x])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(1 + Sqrt[2])] + (Sqrt[(1 + Sqrt[2])
/2]*Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2 - (Sqrt[(1
+ Sqrt[2])/2]*Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 839

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[(d + e*x)^(m -
 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x\sqrt{1+x}}{1+x^2}\,dx = 2\sqrt{1+x} + \int \frac{-1+x}{\sqrt{1+x}\,(1+x^2)}\,dx$$

$$= 2\sqrt{1+x} + 2\text{Subst}\left(\int \frac{-2+x^2}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$

$$= 2\sqrt{1+x} + \frac{\text{Subst}\left(\int \frac{-2\sqrt{2\left(1+\sqrt{2}\right)}-\left(-2-\sqrt{2}\right)x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+x}\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\text{Subst}\left(\int \frac{-2\sqrt{}}{\sqrt{}}\right.}{}$$

$$= 2\sqrt{1+x} - \frac{1}{2}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx, x, \sqrt{1+x}\right) -$$

$$= 2\sqrt{1+x} + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\,\sqrt{1+x}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\left(\right.}$$

$$= 2\sqrt{1+x} + \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right) - \sqrt{\frac{1}{2}\left(-1+\sqrt{\right.}}$$

<u>**Mathematica**</u> [**C**] Result contains complex when optimal does not.
time = 0.08, size = 68, normalized size = 0.32

$$2\sqrt{1+x} - \sqrt{-1+i}\,\tan^{-1}\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\,\sqrt{1+x}\right) - \sqrt{-1-i}\,\tan^{-1}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\,\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 + x])/(1 + x^2),x]
```

```
[Out] 2*Sqrt[1 + x] - Sqrt[-1 + I]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] - Sqrt[-1
 - I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

<u>**Maple**</u> [**A**]
time = 0.51, size = 167, normalized size = 0.78

method	result

derivativedivides	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+x}}{\sqrt{-}}\right.}{\sqrt{-2+2\sqrt{}}}$
default	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+x}}{\sqrt{-}}\right.}{\sqrt{-2+2\sqrt{}}}$
risch	$2\sqrt{1+x}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}}{4}-\frac{\arctan\left(\frac{2\sqrt{1+x}-\sqrt{2+2}}{\sqrt{-2+2\sqrt{2}}}\right.}{\sqrt{-2+2\sqrt{2}}}$
trager	$2\sqrt{1+x}+\frac{\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4-16\_Z^2+1\right)^2-2\right)\ln\left(-\frac{64\,\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(128\_Z^4-\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+x)^(1/2)+1/4*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))*(2+2*2^(1
/2))^(1/2)-(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1
/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/
2))^(1/2))*(2+2*2^(1/2))^(1/2)+(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(
1+x)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 1)*x/(x^2 + 1), x)
```

Fricas [A]
time = 2.72, size = 307, normalized size = 1.43

$$-\frac{1}{8}\cdot 2^{\frac{3}{4}}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}\,\log\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+x+\sqrt{2}+1\right)+\frac{1}{8}\cdot 2^{\frac{3}{4}}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}\,\log\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+x+\sqrt{2}+1\right)+\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{-2\sqrt{2}+4}\,\arctan\left(\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+2x+2\sqrt{2}+2}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}-\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{x+1}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}-\sqrt{2}-1\right)+\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{-2\sqrt{2}+4}\,\arctan\left(\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{-2^{\frac{1}{4}}\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}+2x+2\sqrt{2}+2}\left(\sqrt{2}+2\right)\sqrt{-2\sqrt{2}+4}-\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{x+1}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}+\sqrt{2}+1\right)+2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*2^(1/4)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4)*log(1/2*2^(1/4)*sqrt(x + 1)
*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + x + sqrt(2) + 1) + 1/8*2^(1/4)*(sqrt(
2) + 2)*sqrt(-2*sqrt(2) + 4)*log(-1/2*2^(1/4)*sqrt(x + 1)*(sqrt(2) + 2)*sqr
t(-2*sqrt(2) + 4) + x + sqrt(2) + 1) + 1/2*2^(3/4)*sqrt(-2*sqrt(2) + 4)*arc
tan(1/4*2^(3/4)*sqrt(2^(1/4)*sqrt(x + 1)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4)
 + 2*x + 2*sqrt(2) + 2)*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) - 1/2*2^(3/4)*sq
rt(x + 1)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) - sqrt(2) - 1) + 1/2*2^(3/4)*s
qrt(-2*sqrt(2) + 4)*arctan(1/4*2^(3/4)*sqrt(-2^(1/4)*sqrt(x + 1)*(sqrt(2) +
 2)*sqrt(-2*sqrt(2) + 4) + 2*x + 2*sqrt(2) + 2)*(sqrt(2) + 2)*sqrt(-2*sqrt(
2) + 4) - 1/2*2^(3/4)*sqrt(x + 1)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + sqrt
(2) + 1) + 2*sqrt(x + 1)
```

Sympy [A]
time = 4.07, size = 68, normalized size = 0.32

$$2\sqrt{x+1} - 4\operatorname{RootSum}\left(512t^4 + 32t^2 + 1, \left(t \mapsto t\log\left(-128t^3 + \sqrt{x+1}\right)\right)\right) + 2\operatorname{RootSum}\left(128t^4 + 16t^2 + 1, \left(t \mapsto t\log\left(64t^3 + 4t + \sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*sqrt(x + 1) - 4*RootSum(512*_t**4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*
_t**3 + sqrt(x + 1)))) + 2*RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*
log(64*_t**3 + 4*_t + sqrt(x + 1))))
```

Giac [A]
time = 1.19, size = 167, normalized size = 0.78

$$-\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) + \frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) + 2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*
sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/2*sqrt(2*sqrt(2) - 2)*arctan(-1/2*2^(3
/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) - 1/4*s
qrt(2*sqrt(2) + 2)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2)
+ 1) + 1/4*sqrt(2*sqrt(2) + 2)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) +
 x + sqrt(2) + 1) + 2*sqrt(x + 1)
```

Mupad [B]
time = 0.11, size = 201, normalized size = 0.94

$$2\sqrt{x+1} + \operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} - 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right) - \operatorname{atanh}\left(\frac{\sqrt{x+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{x+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}} - \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}} + \frac{\sqrt{2}\sqrt{x+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}} + 2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(1/2))/(x^2 + 1),x)

[Out] 2*(x + 1)^(1/2) + atanh((x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (x + 1)
^(1/2)/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1
/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/
8 - 2^(1/2)/8)^(1/2) - 2*(2^(1/2)/8 + 1/8)^(1/2)) - atanh((x + 1)^(1/2)/(4*
(1/8 - 2^(1/2)/8)^(1/2)) + (x + 1)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (2^(
1/2)*(x + 1)^(1/2))/(8*(1/8 - 2^(1/2)/8)^(1/2)) + (2^(1/2)*(x + 1)^(1/2))/(
8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/8 - 2^(1/2)/8)^(1/2) + 2*(2^(1/2)/8 + 1/8
)^(1/2))
```

3.316 $\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=255

$$\frac{d(8cd^3-e(4cd^2-ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^5}+\frac{(47cd^2-8ae^2)\,(a+cx^2)^{3/2}}{60c^2e^3}-\frac{13d(d+ex)\,(a+cx^2)^{3/2}}{20ce^3}+\frac{(d+ex)^2\,(a}{5ce}$$

```
[Out] 1/60*(-8*a*e^2+47*c*d^2)*(c*x^2+a)^(3/2)/c^2/e^3-13/20*d*(e*x+d)*(c*x^2+a)^
(3/2)/c/e^3+1/5*(e*x+d)^2*(c*x^2+a)^(3/2)/c/e^3-1/8*d*(-a^2*e^4+4*a*c*d^2*e
^2+8*c^2*d^4)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^6-d^4*arctanh((-
c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^6+1/8
*d*(8*c*d^3-e*(-a*e^2+4*c*d^2)*x)*(c*x^2+a)^(1/2)/c/e^5
```

Rubi [A]
time = 0.39, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1668, 829, 858, 223, 212, 739}

$$-\frac{d(-a^2e^4+4acd^2e^2+8c^2d^4)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6}+\frac{(a+cx^2)^{3/2}\,(47cd^2-8ae^2)}{60c^2e^3}-\frac{d^4\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^6}+\frac{d\sqrt{a+cx^2}\,(8cd^3-ex(4cd^2-ae^2))}{8ce^5}-\frac{13d(a+cx^2)^{3/2}\,(d+ex)}{20ce^3}+\frac{(a+cx^2)^{3/2}\,(d+ex)^2}{5ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(8*c*e^5) + ((47*c*d^
2 - 8*a*e^2)*(a + c*x^2)^(3/2))/(60*c^2*e^3) - (13*d*(d + e*x)*(a + c*x^2)^
(3/2))/(20*c*e^3) + ((d + e*x)^2*(a + c*x^2)^(3/2))/(5*c*e^3) - (d*(8*c^2*d
^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3
/2)*e^6) - (d^4*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e
^2]*Sqrt[a + c*x^2])])/e^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx = \frac{(d+ex)^2\,(a+cx^2)^{3/2}}{5ce^3} + \frac{\int \frac{\sqrt{a+cx^2}\,(-2ad^2e^2-de(3cd^2+4ae^2)x-e^2(11cd^2+2ae^2)x^2-13cde^3x^3)}{d+ex}\,dx}{5ce^4}$$

$$= -\frac{13d(d+ex)\,(a+cx^2)^{3/2}}{20ce^3} + \frac{(d+ex)^2\,(a+cx^2)^{3/2}}{5ce^3} + \frac{\int \frac{\sqrt{a+cx^2}\,(5acd^2e^5+3cde^4(9cd^2-}{d+ex}}{20c^2e^7}$$

$$= \frac{(47cd^2-8ae^2)\,(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\,(a+cx^2)^{3/2}}{20ce^3} + \frac{(d+ex)^2\,(a+cx^2)^{3/2}}{5ce^3} + \int$$

$$= \frac{d(8cd^3-e(4cd^2-ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^5} + \frac{(47cd^2-8ae^2)\,(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\,(a}{20ce^3}$$

$$= \frac{d(8cd^3-e(4cd^2-ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^5} + \frac{(47cd^2-8ae^2)\,(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\,(a}{20ce^3}$$

$$= \frac{d(8cd^3-e(4cd^2-ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^5} + \frac{(47cd^2-8ae^2)\,(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\,(a}{20ce^3}$$

$$= \frac{d(8cd^3-e(4cd^2-ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^5} + \frac{(47cd^2-8ae^2)\,(a+cx^2)^{3/2}}{60c^2e^3} - \frac{13d(d+ex)\,(a}{20ce^3}$$

Mathematica [A]
time = 0.63, size = 230, normalized size = 0.90

$$\frac{e\sqrt{a+cx^2}\,(-16a^2e^4+ace^2(40d^2-15dex+8e^2x^2)+2c^2(60d^4-30d^3ex+20d^2e^2x^2-15de^3x^3+12e^4x^4))+240c^2d^4\sqrt{-cd^2-ae^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)+15\sqrt{c}\,d(8c^2d^4+4acd^2e^2-a^2e^4)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{120c^2e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(-16*a^2*e^4 + a*c*e^2*(40*d^2 - 15*d*e*x + 8*e^2*x^2) +
 2*c^2*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4))
+ 240*c^2*d^4*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a +
 c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + 15*Sqrt[c]*d*(8*c^2*d^4 + 4*a*c*d^2*e^2
- a^2*e^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(120*c^2*e^6)
```

Maple [A]
time = 0.09, size = 428, normalized size = 1.68

method	result

default	$\frac{\frac{x^2(cx^2+a)^{\frac{3}{2}}}{5c}-\frac{2a(cx^2+a)^{\frac{3}{2}}}{15c^2}}{e}-\frac{d\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4c}\right)}{e^2}+\frac{d^2(cx^2+a)^{\frac{3}{2}}}{3e^3c}-\frac{d^3\left(\frac{x}{}\right.}{}$
risch	$-\frac{(-24e^4c^2x^4+30de^3c^2x^3-8ace^4x^2-40c^2d^2e^2x^2+15acde^3x+60c^2d^3ex+16a^2e^4-40acd^2e^2-120c^2d^4)\sqrt{cx^2+a}}{120c^2e^5}+\frac{d\ln\left(x\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/c^2*(c*x^2+a)^(3/2))-d/e^2*(1/4*x*(c*
x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c
*x^2+a)^(1/2))))+1/3*d^2/e^3*(c*x^2+a)^(3/2)/c-d^3/e^4*(1/2*x*(c*x^2+a)^(1/
2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+1/e^5*d^4*((c*(x+d/e)^2-2*c
*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e*ln((-c*d/e+c*(x+d/e))/c^(
1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))-(a*e^2+c*d^2)/e
^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*
e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)
)/(x+d/e)))
```

Maxima [A]
time = 0.34, size = 240, normalized size = 0.94

$$-\sqrt{c}\,d^5\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-6)}+\sqrt{cd^2e^{(-2)}+a}\,d^4\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-5)}-\frac{1}{2}\sqrt{cx^2+a}\,d^3xe^{(-4)}-\frac{ad^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-4)}}{2\sqrt{c}}+\sqrt{cx^2+a}\,d^4e^{(-5)}+\frac{(cx^2+a)^{\frac{3}{2}}x^2e^{(-1)}}{5c}-\frac{(cx^2+a)^{\frac{3}{2}}dxe^{(-2)}}{4c}+\frac{\sqrt{cx^2+a}\,adxe^{(-2)}}{8c}+\frac{a^2d\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-2)}}{8c^{\frac{3}{2}}}+\frac{(cx^2+a)^{\frac{3}{2}}d^2e^{(-3)}}{3c}-\frac{2(cx^2+a)^{\frac{3}{2}}ae^{(-1)}}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -sqrt(c)*d^5*arcsinh(c*x/sqrt(a*c))*e^(-6) + sqrt(c*d^2*e^(-2) + a)*d^4*arc
sinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-5)
- 1/2*sqrt(c*x^2 + a)*d^3*x*e^(-4) - 1/2*a*d^3*arcsinh(c*x/sqrt(a*c))*e^(-4
)/sqrt(c) + sqrt(c*x^2 + a)*d^4*e^(-5) + 1/5*(c*x^2 + a)^(3/2)*x^2*e^(-1)/c
 - 1/4*(c*x^2 + a)^(3/2)*d*x*e^(-2)/c + 1/8*sqrt(c*x^2 + a)*a*d*x*e^(-2)/c
+ 1/8*a^2*d*arcsinh(c*x/sqrt(a*c))*e^(-2)/c^(3/2) + 1/3*(c*x^2 + a)^(3/2)*d
^2*e^(-3)/c - 2/15*(c*x^2 + a)^(3/2)*a*e^(-1)/c^2
```

Fricas [**A**]
time = 13.04, size = 1045, normalized size = 4.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/240*(120*sqrt(c*d^2 + a*e^2)*c^2*d^4*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e +
 a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 +
 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a
^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(60*c
^2*d^3*x*e^2 - 120*c^2*d^4*e - 8*(3*c^2*x^4 + a*c*x^2 - 2*a^2)*e^5 + 15*(2*
c^2*d*x^3 + a*c*d*x)*e^4 - 40*(c^2*d^2*x^2 + a*c*d^2)*e^3)*sqrt(c*x^2 + a))
*e^(-6)/c^2, 1/240*(240*sqrt(-c*d^2 - a*e^2)*c^2*d^4*arctan(-sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^
2)*e^2)) - 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log(-2*c*x^2
- 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(60*c^2*d^3*x*e^2 - 120*c^2*d^4*e -
8*(3*c^2*x^4 + a*c*x^2 - 2*a^2)*e^5 + 15*(2*c^2*d*x^3 + a*c*d*x)*e^4 - 40*(
c^2*d^2*x^2 + a*c*d^2)*e^3)*sqrt(c*x^2 + a))*e^(-6)/c^2, 1/120*(60*sqrt(c*d
^2 + a*e^2)*c^2*d^4*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*
d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^
2 + 2*d*x*e + d^2)) + 15*(8*c^2*d^5 + 4*a*c*d^3*e^2 - a^2*d*e^4)*sqrt(-c)*a
rctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (60*c^2*d^3*x*e^2 - 120*c^2*d^4*e - 8*(
3*c^2*x^4 + a*c*x^2 - 2*a^2)*e^5 + 15*(2*c^2*d*x^3 + a*c*d*x)*e^4 - 40*(c^2
*d^2*x^2 + a*c*d^2)*e^3)*sqrt(c*x^2 + a))*e^(-6)/c^2, 1/120*(120*sqrt(-c*d^
2 - a*e^2)*c^2*d^4*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + 15*(8*c^2*d^5 + 4*a*c*d
^3*e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (60*c^2*d
^3*x*e^2 - 120*c^2*d^4*e - 8*(3*c^2*x^4 + a*c*x^2 - 2*a^2)*e^5 + 15*(2*c^2*
d*x^3 + a*c*d*x)*e^4 - 40*(c^2*d^2*x^2 + a*c*d^2)*e^3)*sqrt(c*x^2 + a))*e^(
-6)/c^2]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**4*sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [**A**]
time = 1.15, size = 252, normalized size = 0.99

$$\frac{2\,(cd^6+ad^4e^2)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-6)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4\,xe^{(-1)}-5\,de^{(-2)}\right)x+\frac{4\,(5\,c^3d^2e^{18}+ac^2e^{20})e^{(-21)}}{c^3}\right)x-\frac{15\,(4\,c^3d^3e^{17}+ac^2de^{19})e^{(-21)}}{c^3}\right)x+\frac{8\,(15\,c^3d^4e^{16}+5\,ac^2d^2e^{18}-2\,a^2ce^{20})e^{(-21)}}{c^3}\right)+\frac{\left(8\,c^{\frac{5}{2}}d^5+4\,ac^{\frac{3}{2}}d^3e^2-a^2\sqrt{c}\,de^4\right)e^{(-6)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^6 + a*d^4*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-6)/sqrt(-c*d^2 - a*e^2) + 1/120*sqrt(c*x^2 + a)*
((2*(3*(4*x*e^(-1) - 5*d*e^(-2))*x + 4*(5*c^3*d^2*e^18 + a*c^2*e^20)*e^(-21
)/c^3)*x - 15*(4*c^3*d^3*e^17 + a*c^2*d*e^19)*e^(-21)/c^3)*x + 8*(15*c^3*d^
4*e^16 + 5*a*c^2*d^2*e^18 - 2*a^2*c*e^20)*e^(-21)/c^3) + 1/8*(8*c^(5/2)*d^5
 + 4*a*c^(3/2)*d^3*e^2 - a^2*sqrt(c)*d*e^4)*e^(-6)*log(abs(-sqrt(c)*x + sqr
t(c*x^2 + a)))/c^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^4*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.317 $\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=211

$$-\frac{(8cd^3-e(4cd^2-ae^2)\,x)\,\sqrt{a+cx^2}}{8ce^4}-\frac{7d(a+cx^2)^{3/2}}{12ce^2}+\frac{(d+ex)\,(a+cx^2)^{3/2}}{4ce^2}+\frac{(8c^2d^4+4acd^2e^2-a^2e^4)\tan$$

```
[Out] -7/12*d*(c*x^2+a)^(3/2)/c/e^2+1/4*(e*x+d)*(c*x^2+a)^(3/2)/c/e^2+1/8*(-a^2*e
^4+4*a*c*d^2*e^2+8*c^2*d^4)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^5+
d^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)
^(1/2)/e^5-1/8*(8*c*d^3-e*(-a*e^2+4*c*d^2)*x)*(c*x^2+a)^(1/2)/c/e^4
```

Rubi [A]
time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1668, 829, 858, 223, 212, 739}

$$\frac{(-a^2e^4+4acd^2e^2+8c^2d^4)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5}+\frac{d^3\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^5}-\frac{\sqrt{a+cx^2}\,(8cd^3-ex(4cd^2-ae^2))}{8ce^4}-\frac{7d(a+cx^2)^{3/2}}{12ce^2}+\frac{(a+cx^2)^{3/2}\,(d+ex)}{4ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -1/8*((8*c*d^3 - e*(4*c*d^2 - a*e^2)*x)*Sqrt[a + c*x^2])/(c*e^4) - (7*d*(a
+ c*x^2)^(3/2))/(12*c*e^2) + ((d + e*x)*(a + c*x^2)^(3/2))/(4*c*e^2) + ((8*
c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8
*c^(3/2)*e^5) + (d^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2])])/e^5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx = \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+cx^2}\,(-ade^2-e(3cd^2+ae^2)x-7cde^2x^2)}{d+ex}\,dx}{4ce^3}$$

$$= -\frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{(-3acde^4+3ce^3(4cd^2-ae^2)x)\sqrt{a+cx^2}}{d+ex}\,dx}{12c^2e^5}$$

$$= -\frac{\left(8cd^3-e(4cd^2-ae^2)\,x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{\left(8cd^3-e(4cd^2-ae^2)\,x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} - \frac{(d^3\cdots}{\cdots}$$

$$= -\frac{\left(8cd^3-e(4cd^2-ae^2)\,x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{(d^3\cdots}{\cdots}$$

$$= -\frac{\left(8cd^3-e(4cd^2-ae^2)\,x\right)\sqrt{a+cx^2}}{8ce^4} - \frac{7d\left(a+cx^2\right)^{3/2}}{12ce^2} + \frac{(d+ex)\left(a+cx^2\right)^{3/2}}{4ce^2} + \frac{(8c\cdots}{\cdots}$$

Mathematica [A]
time = 0.48, size = 199, normalized size = 0.94

$$\frac{\sqrt{c}\,e\sqrt{a+cx^2}\,(ae^2(-8d+3ex)+c(-24d^3+12d^2ex-8de^2x^2+6e^3x^3))-48c^{3/2}d^3\sqrt{-cd^2-ae^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)-3(8c^2d^4+4acd^2e^2-a^2e^4)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{24c^{3/2}e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[c]*e*Sqrt[a + c*x^2]*(a*e^2*(-8*d + 3*e*x) + c*(-24*d^3 + 12*d^2*e*x
- 8*d*e^2*x^2 + 6*e^3*x^3)) - 48*c^(3/2)*d^3*Sqrt[-(c*d^2) - a*e^2]*ArcTan[
(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] - 3*(8*c^2*
d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(24*c^(
3/2)*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(185) = 370$.
time = 0.09, size = 387, normalized size = 1.83

method	result

default	$\frac{\frac{x(cx^2+a)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4c}}{e}-\frac{d(cx^2+a)^{\frac{3}{2}}}{3ce^2}+\frac{d^2\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2-\right.}}{2\sqrt{c}}\right.}{e^3}$
risch	$-\frac{(-6ce^3x^3+8de^2cx^2-3ae^3x-12cd^2ex+8ade^2+24cd^3)\sqrt{cx^2+a}}{24ce^4}-\frac{\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)a^2}{8c^{\frac{3}{2}}e}+\frac{\ln\left(x\sqrt{c}+\sqrt{cx\right.}}{2\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*l
n(x*c^(1/2)+(c*x^2+a)^(1/2))))-1/3*d*(c*x^2+a)^(3/2)/c/e^2+d^2/e^3*(1/2*x*(
c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-d^3/e^4*((c*(x+
d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e*ln((-c*d/e+c*(x
+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))-(a*e^
2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d
/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/
e^2)^(1/2))/(x+d/e)))
```

Maxima [A]

time = 0.32, size = 200, normalized size = 0.95

$$\sqrt{c}\,d^4\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-5)}-\sqrt{cd^2e^{(-2)}+a}\,d^3\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-4)}+\frac{1}{2}\sqrt{cx^2+a}\,d^2xe^{(-3)}+\frac{ad^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-3)}}{2\sqrt{c}}-\sqrt{cx^2+a}\,d^3e^{(-4)}+\frac{(cx^2+a)^{\frac{3}{2}}xe^{(-1)}}{4c}-\frac{\sqrt{cx^2+a}\,axe^{(-1)}}{8c}-\frac{a^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-1)}}{8c^{\frac{3}{2}}}-\frac{(cx^2+a)^{\frac{3}{2}}de^{(-2)}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] sqrt(c)*d^4*arcsinh(c*x/sqrt(a*c))*e^(-5) - sqrt(c*d^2*e^(-2) + a)*d^3*arcs
inh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-4) +
 1/2*sqrt(c*x^2 + a)*d^2*x*e^(-3) + 1/2*a*d^2*arcsinh(c*x/sqrt(a*c))*e^(-3)
/sqrt(c) - sqrt(c*x^2 + a)*d^3*e^(-4) + 1/4*(c*x^2 + a)^(3/2)*x*e^(-1)/c -
1/8*sqrt(c*x^2 + a)*a*x*e^(-1)/c - 1/8*a^2*arcsinh(c*x/sqrt(a*c))*e^(-1)/c^
(3/2) - 1/3*(c*x^2 + a)^(3/2)*d*e^(-2)/c
```

Fricas [A]
time = 10.54, size = 920, normalized size = 4.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*c^2*d^3*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a
*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2
*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*
e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(12*c^2*d^
2*x*e^2 - 24*c^2*d^3*e + 3*(2*c^2*x^3 + a*c*x)*e^4 - 8*(c^2*d*x^2 + a*c*d)*
e^3)*sqrt(c*x^2 + a))*e^(-5)/c^2, -1/48*(48*sqrt(-c*d^2 - a*e^2)*c^2*d^3*ar
ctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c
*d^2 + (a*c*x^2 + a^2)*e^2)) + 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqrt
(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(12*c^2*d^2*x*e^2 -
 24*c^2*d^3*e + 3*(2*c^2*x^3 + a*c*x)*e^4 - 8*(c^2*d*x^2 + a*c*d)*e^3)*sqrt
(c*x^2 + a))*e^(-5)/c^2, 1/24*(12*sqrt(c*d^2 + a*e^2)*c^2*d^3*log(-(2*c^2*d
^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 3*(8*c^2*d^
4 + 4*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) +
(12*c^2*d^2*x*e^2 - 24*c^2*d^3*e + 3*(2*c^2*x^3 + a*c*x)*e^4 - 8*(c^2*d*x^2
 + a*c*d)*e^3)*sqrt(c*x^2 + a))*e^(-5)/c^2, -1/24*(24*sqrt(-c*d^2 - a*e^2)*
c^2*d^3*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2
*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + 3*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2
*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (12*c^2*d^2*x*e^2 - 24*
c^2*d^3*e + 3*(2*c^2*x^3 + a*c*x)*e^4 - 8*(c^2*d*x^2 + a*c*d)*e^3)*sqrt(c*x
^2 + a))*e^(-5)/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A]
time = 0.98, size = 201, normalized size = 0.95

$$-\frac{2\,(cd^5+ad^3e^2)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-5)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{24}\,\sqrt{cx^2+a}\left(\left(2\,(3\,xe^{(-1)}-4\,de^{(-2)})x+\frac{3\,(4\,c^2d^2e^{12}+ace^{14})e^{(-15)}}{c^2}\right)x-\frac{8\,(3\,c^2d^3e^{11}+acde^{13})e^{(-15)}}{c^2}\right)-\frac{(8\,c^2d^4+4\,acd^2e^2-a^2e^4)e^{(-5)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^5 + a*d^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))*e^(-5)/sqrt(-c*d^2 - a*e^2) + 1/24*sqrt(c*x^2 + a)*
((2*(3*x*e^(-1) - 4*d*e^(-2))*x + 3*(4*c^2*d^2*e^12 + a*c*e^14)*e^(-15)/c^2
)*x - 8*(3*c^2*d^3*e^11 + a*c*d*e^13)*e^(-15)/c^2) - 1/8*(8*c^2*d^4 + 4*a*c
*d^2*e^2 - a^2*e^4)*e^(-5)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.318 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=153

$$\frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3}+\frac{(a+cx^2)^{3/2}}{3ce}-\frac{d(2cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^4}-\frac{d^2\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{}{\sqrt{cd^2-}}\right.}{e^4}$$

```
[Out] 1/3*(c*x^2+a)^(3/2)/c/e-1/2*d*(a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(
1/2))/e^4/c^(1/2)-d^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1
/2))*(a*e^2+c*d^2)^(1/2)/e^4+1/2*d*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^3
```

Rubi [A]
time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1668, 12, 829, 858, 223, 212, 739}

$$-\frac{d^2\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4}-\frac{d(ae^2+2cd^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^4}+\frac{d\sqrt{a+cx^2}\,(2d-ex)}{2e^3}+\frac{(a+cx^2)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (d*(2*d - e*x)*Sqrt[a + c*x^2])/(2*e^3) + (a + c*x^2)^(3/2)/(3*c*e) - (d*(2
*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^4) - (d^
2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c
*x^2])])/e^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx = \frac{(a+cx^2)^{3/2}}{3ce} + \frac{\int -\frac{3cdex\sqrt{a+cx^2}}{d+ex}\,dx}{3ce^2}$$

$$= \frac{(a+cx^2)^{3/2}}{3ce} - \frac{d\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx}{e}$$

$$= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce} - \frac{d\int \frac{-acde+c(2cd^2+ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}$$

$$= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce} + \frac{(d^2(cd^2+ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^4} - \frac{(d(2c$$

$$= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce} - \frac{(d^2(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{a}{\sqrt{a}}\right.}{e^4}$$

$$= \frac{d(2d-ex)\sqrt{a+cx^2}}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce} - \frac{d(2cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^4} - \frac{d^2\sqrt{c}}{}$$

Mathematica [A]
time = 0.40, size = 161, normalized size = 1.05

$$\frac{e\sqrt{a+cx^2}\,(6cd^2+2ae^2-3cdex+2ce^2x^2)+12cd^2\sqrt{-cd^2-ae^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)+3\sqrt{c}\,d(2cd^2+ae^2)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{6ce^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2]*(6*c*d^2 + 2*a*e^2 - 3*c*d*e*x + 2*c*e^2*x^2) + 12*c*d^2
*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt
[-(c*d^2) - a*e^2]] + 3*Sqrt[c]*d*(2*c*d^2 + a*e^2)*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2]])/(6*c*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(131) = 262$.
time = 0.08, size = 323, normalized size = 2.11

method	result

default	$\frac{(cx^2+a)^{\frac{3}{2}}}{3ce} - \frac{d\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{e^2} + \frac{d^2\left(\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{ae^2+cd^2}{e^2}} - \frac{\sqrt{c}}{} \right.}{}$
risch	$\frac{(2ce^2x^2-3cdex+2ae^2+6cd^2)\sqrt{cx^2+a}}{6ce^3} - \frac{d\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)a}{2e^2\sqrt{c}} - \frac{d^3\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)\sqrt{c}}{e^4} - \frac{d^2\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^2+a)^(3/2)/c/e-d/e^2*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(
1/2)+(c*x^2+a)^(1/2)))+1/e^3*d^2*((c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2
)/e^2)^(1/2)-c^(1/2)*d/e*ln((-c*d/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e
*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1
/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*
(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]
time = 0.30, size = 140, normalized size = 0.92

$$-\sqrt{c}\,d^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-4)} + \sqrt{cd^2e^{(-2)}+a}\,d^2\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-3)} - \frac{1}{2}\sqrt{cx^2+a}\,dxe^{(-2)} - \frac{ad\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-2)}}{2\sqrt{c}} + \sqrt{cx^2+a}\,d^2e^{(-3)} + \frac{(cx^2+a)^{\frac{3}{2}}e^{(-1)}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -sqrt(c)*d^3*arcsinh(c*x/sqrt(a*c))*e^(-4) + sqrt(c*d^2*e^(-2) + a)*d^2*arc
sinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-3)
- 1/2*sqrt(c*x^2 + a)*d*x*e^(-2) - 1/2*a*d*arcsinh(c*x/sqrt(a*c))*e^(-2)/sq
rt(c) + sqrt(c*x^2 + a)*d^2*e^(-3) + 1/3*(c*x^2 + a)^(3/2)*e^(-1)/c
```

Fricas [A]
time = 3.97, size = 745, normalized size = 4.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c*d^2*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*
d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^
2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c
*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(3*c*d*x*e^2 - 6*c*d^2*e - 2*(c
*x^2 + a)*e^3)*sqrt(c*x^2 + a))*e^(-4)/c, 1/12*(12*sqrt(-c*d^2 - a*e^2)*c*d
^2*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2
+ a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + 3*(2*c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*
x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(3*c*d*x*e^2 - 6*c*d^2*e - 2*(c*
x^2 + a)*e^3)*sqrt(c*x^2 + a))*e^(-4)/c, 1/6*(3*sqrt(c*d^2 + a*e^2)*c*d^2*l
og(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
 a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) +
 3*(2*c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*c*d
*x*e^2 - 6*c*d^2*e - 2*(c*x^2 + a)*e^3)*sqrt(c*x^2 + a))*e^(-4)/c, 1/6*(6*s
qrt(-c*d^2 - a*e^2)*c*d^2*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + 3*(2*c*d^3 + a*d
*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*c*d*x*e^2 - 6*c*d^2*
e - 2*(c*x^2 + a)*e^3)*sqrt(c*x^2 + a))*e^(-4)/c]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A]

time = 0.86, size = 157, normalized size = 1.03

$$\frac{2\,(cd^4+ad^2e^2)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2-ae^2}}+\frac{(2\,cd^3+ade^2)e^{(-4)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,\sqrt{c}}+\frac{1}{6}\,\sqrt{cx^2+a}\left(\left(2\,xe^{(-1)}-3\,de^{(-2)}\right)x+\frac{2\,(3\,cd^2e^7+ae^9)e^{(-10)}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^4 + a*d^2*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/2*(2*c*d^3 + a*d*e^2
)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/6*sqrt(c*x^2 +
a)*((2*x*e^(-1) - 3*d*e^(-2))*x + 2*(3*c*d^2*e^7 + a*e^9)*e^(-10)/c)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x),x)
[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.319 $\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=127

$$-\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2}+\frac{(2cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3}+\frac{d\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{e^3}$$

```
[Out] 1/2*(a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^3/c^(1/2)+d*arctan
h((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^3
-1/2*(-e*x+2*d)*(c*x^2+a)^(1/2)/e^2
```

Rubi [A]
time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {829, 858, 223, 212, 739}

$$\frac{(ae^2+2cd^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3}+\frac{d\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^3}-\frac{\sqrt{a+cx^2}\,(2d-ex)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] -1/2*((2*d - e*x)*Sqrt[a + c*x^2])/e^2 + ((2*c*d^2 + a*e^2)*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*e^3) + (d*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*
e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/e^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx = -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{\int \frac{-acde+c(2cd^2+ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^2}$$
$$= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} - \frac{(d(cd^2+ae^2))\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3} + \frac{(2cd^2+ae^2)\int \frac{1}{\sqrt{a+c$$
$$= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{(d(cd^2+ae^2))\,\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e^3} + \frac{(2cd^2$$
$$= -\frac{(2d-ex)\sqrt{a+cx^2}}{2e^2} + \frac{(2cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,e^3} + \frac{d\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\right.}{e$$

Mathematica [A]
time = 0.32, size = 133, normalized size = 1.05

$$\frac{e(-2d+ex)\sqrt{a+cx^2} - 4d\sqrt{-cd^2-ae^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) - \frac{(2cd^2+ae^2)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x),x]
```

[Out] (e*(-2*d + e*x)*Sqrt[a + c*x^2] - 4*d*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] - ((2*c*d^2 + a*e^ 2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(2*e^3)

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(109) = 218$.
time = 0.08, size = 303, normalized size = 2.39

method	result
default	$\frac{\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}}{e}-\frac{d\left(\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}\right.}{}\right.}{}$
risch	$-\frac{(-ex+2d)\sqrt{c\,x^2+a}}{2e^2}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)a}{2e\sqrt{c}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)\sqrt{c}\,d^2}{e^3}+\frac{d\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd(}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/e*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-d/e ^2*((c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e*ln((- c*d/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1 /2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2* c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e ^2+c*d^2)/e^2)^(1/2))/(x+d/e)))

Maxima [**A**]
time = 0.31, size = 119, normalized size = 0.94

$$\sqrt{c}\,d^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-3)}-\sqrt{cd^2e^{(-2)}+a}\,d\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-2)}+\frac{1}{2}\sqrt{cx^2+a}\,xe^{(-1)}+\frac{a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-1)}}{2\sqrt{c}}-\sqrt{cx^2+a}\,de^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")

[Out] sqrt(c)*d^2*arcsinh(c*x/sqrt(a*c))*e^(-3) - sqrt(c*d^2*e^(-2) + a)*d*arcsin h(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-2) + 1

```
/2*sqrt(c*x^2 + a)*x*e^(-1) + 1/2*a*arcsinh(c*x/sqrt(a*c))*e^(-1)/sqrt(c) -
 sqrt(c*x^2 + a)*d*e^(-2)
```

Fricas [A]
time = 3.95, size = 669, normalized size = 5.27

$$\left[\frac{\left(2\sqrt{cd^2+ae^2}\,cd\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+(2cd^2+ae^2)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\sqrt{cx^2+a}\,(cxe^2-2cde)\right)e^{(-3)}}{4c},\ -\frac{\left(4\sqrt{-cd^2-ae^2}\,cd\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)-(2cd^2+ae^2)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\sqrt{cx^2+a}\,(cxe^2-2cde)\right)e^{(-3)}}{4c},\ \frac{\left(\sqrt{cd^2+ae^2}\,cd\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)-(2cd^2+ae^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+\sqrt{cx^2+a}\,(cxe^2-2cde)\right)e^{(-3)}}{2c},\ -\frac{\left(2\sqrt{-cd^2-ae^2}\,cd\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)+(2cd^2+ae^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-\sqrt{cx^2+a}\,(cxe^2-2cde)\right)e^{(-3)}}{2c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c*d*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2
 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*
e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + (2*c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 -
2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*x^2 + a)*(c*x*e^2 - 2*c*d*e))*e
^(-3)/c, -1/4*(4*sqrt(-c*d^2 - a*e^2)*c*d*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d
*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) -
(2*c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) -
 2*sqrt(c*x^2 + a)*(c*x*e^2 - 2*c*d*e))*e^(-3)/c, 1/2*(sqrt(c*d^2 + a*e^2)*
c*d*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^
2)) - (2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqrt(
c*x^2 + a)*(c*x*e^2 - 2*c*d*e))*e^(-3)/c, -1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d*
arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a
*c*d^2 + (a*c*x^2 + a^2)*e^2)) + (2*c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)
*x/sqrt(c*x^2 + a)) - sqrt(c*x^2 + a)*(c*x*e^2 - 2*c*d*e))*e^(-3)/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A]
time = 1.92, size = 135, normalized size = 1.06

$$-\frac{2\left(cd^3+ade^2\right)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-3)}}{\sqrt{-cd^2-ae^2}}-\frac{\left(2\,c^{\frac{3}{2}}d^2+a\sqrt{c}\,e^2\right)e^{(-3)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c}+\frac{1}{2}\sqrt{cx^2+a}\left(xe^{(-1)}-2\,de^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^3 + a*d*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))*e^(-3)/sqrt(-c*d^2 - a*e^2) - 1/2*(2*c^(3/2)*d^2 + a*
sqrt(c)*e^2)*e^(-3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c + 1/2*sqrt(c*x
^2 + a)*(x*e^(-1) - 2*d*e^(-2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.320 $\int \frac{\sqrt{a+cx^2}}{d+ex} dx$

Optimal. Leaf size=103

$$\frac{\sqrt{a+cx^2}}{e}-\frac{\sqrt{c}\, d\tanh^{-1}\left(\frac{\sqrt{c}\, x}{\sqrt{a+cx^2}}\right)}{e^2}-\frac{\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{e^2}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/e^2-arctanh((-c*d*x+a*e)/(a*e
^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/e^2+(c*x^2+a)^(1/2)/e
```

Rubi [A]

time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {749, 858, 223, 212, 739}

$$-\frac{\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^2}-\frac{\sqrt{c}\, d\tanh^{-1}\left(\frac{\sqrt{c}\, x}{\sqrt{a+cx^2}}\right)}{e^2}+\frac{\sqrt{a+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^2]/e - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e^2 -
(Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*
x^2])])/e^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx &= \frac{\sqrt{a+cx^2}}{e}+\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}\\ &= \frac{\sqrt{a+cx^2}}{e}+\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx-\frac{(cd)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}\\ &= \frac{\sqrt{a+cx^2}}{e}+\left(-a-\frac{cd^2}{e^2}\right)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)-\frac{(cd)\text{Subst}\Big(\int\end{aligned}$$

$$= \frac{\sqrt{a+cx^2}}{e}-\frac{\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e^2}-\frac{\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx}}\right.}{e^2}$$

Mathematica [A]
time = 0.02, size = 110, normalized size = 1.07

$$\frac{e\sqrt{a+cx^2}+2\sqrt{-cd^2-ae^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)+\sqrt{c}\,d\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x),x]
```

```
[Out] (e*Sqrt[a + c*x^2] + 2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e
*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2]])/e^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(89) = 178$.

time = 0.07, size = 261, normalized size = 2.53

method	result
default	$\frac{\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\right)}{e}-\frac{\left(a\,e^{2}\cdots\right.}{\cdots}}{e}$
risch	$\frac{\sqrt{c\,x^{2}+a}}{e}-\frac{\sqrt{c}\,d\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e^{2}}-\frac{\ln\left(\frac{\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x\cdots\right.}{e}\cdots}}{x+\frac{d}{e}}\right)}{e\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*((c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e*ln((
-c*d/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2
*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*
e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]

time = 0.29, size = 83, normalized size = 0.81

$$-\sqrt{c}\,d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-2)}+\sqrt{cd^{2}e^{(-2)}+a}\;\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-1)}+\sqrt{cx^{2}+a}\;e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] -sqrt(c)*d*arcsinh(c*x/sqrt(a*c))*e^(-2) + sqrt(c*d^2*e^(-2) + a)*arcsinh(c
*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-1) + sqrt
(c*x^2 + a)*e^(-1)
```

Fricas [A]

time = 3.44, size = 565, normalized size = 5.49

$$\left[\frac{1}{2}\left(\sqrt{c}\,d\log\left(-2\,cx^{2}+2\,\sqrt{cx^{2}+a}\,\sqrt{c}\,x-a\right)+2\,\sqrt{cx^{2}+a}\,e+\sqrt{cd^{2}+ae^{2}}\,\log\left(-\frac{2\,c^{2}d^{2}x^{2}-2\,acdxe+acd^{2}+2\,\sqrt{cd^{2}+ae^{2}}\,(cdx-ae)\sqrt{cx^{2}+a}+(acx^{2}+2\,a^{2})e^{2}}{x^{2}e^{2}+2\,dxe+d^{2}}\right)\right)e^{(-2)},\ \frac{1}{2}\left(2\,\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^{2}+a}}\right)+2\,\sqrt{cx^{2}+a}\,e+\sqrt{cd^{2}+ae^{2}}\,\log\left(-\frac{2\,c^{2}d^{2}x^{2}-2\,acdxe+acd^{2}+2\,\sqrt{cd^{2}+ae^{2}}\,(cdx-ae)\sqrt{cx^{2}+a}+(acx^{2}+2\,a^{2})e^{2}}{x^{2}e^{2}+2\,dxe+d^{2}}\right)\right)e^{(-2)},\ \frac{1}{2}\left(\sqrt{c}\,d\log\left(-2\,cx^{2}+2\,\sqrt{cx^{2}+a}\,\sqrt{c}\,x-a\right)+2\,\sqrt{-cd^{2}-ae^{2}}\arctan\left(-\frac{\sqrt{-cd^{2}-ae^{2}}\,(cdx-ae)\sqrt{cx^{2}+a}}{c^{2}d^{2}x^{2}+acd^{2}+(acx^{2}+a^{2})e^{2}}\right)+2\,\sqrt{cx^{2}+a}\,e\right)e^{(-2)},\ \left(\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^{2}+a}}\right)+\sqrt{-cd^{2}-ae^{2}}\arctan\left(-\frac{\sqrt{-cd^{2}-ae^{2}}\,(cdx-ae)\sqrt{cx^{2}+a}}{c^{2}d^{2}x^{2}+acd^{2}+(acx^{2}+a^{2})e^{2}}\right)+\sqrt{cx^{2}+a}\,e\right)e^{(-2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(c*
x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^
2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)
*e^2)/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-2), 1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)
*x/sqrt(c*x^2 + a)) + 2*sqrt(c*x^2 + a)*e + sqrt(c*d^2 + a*e^2)*log(-(2*c^2
*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt
(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-2), 1/
2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(-c*d^
2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*
d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + 2*sqrt(c*x^2 + a)*e)*e^(-2), (s
qrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqrt(-c*d^2 - a*e^2)*arctan(
-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2
+ (a*c*x^2 + a^2)*e^2)) + sqrt(c*x^2 + a)*e)*e^(-2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x), x)
```

Giac [A]
time = 1.81, size = 109, normalized size = 1.06

$$\sqrt{c}\, de^{(-2)} \log\left(\left|-\sqrt{c}\, x + \sqrt{cx^2 + a}\right|\right) + \frac{2\,(cd^2 + ae^2) \arctan\left(-\frac{\left(\sqrt{c}\, x - \sqrt{cx^2 + a}\right)e + \sqrt{c}\, d}{\sqrt{-cd^2 - ae^2}}\right) e^{(-2)}}{\sqrt{-cd^2 - ae^2}} + \sqrt{cx^2 + a}\, e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] sqrt(c)*d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a))) + 2*(c*d^2 + a*e^2)
*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2)
)*e^(-2)/sqrt(-c*d^2 - a*e^2) + sqrt(c*x^2 + a)*e^(-1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2 + a}}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(d + e*x), x)
```

3.321 $\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e}+\frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d+arctanh(x*c^(1/2)/(c*x^2+a)^(1/
2))*c^(1/2)/e+arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*
e^2+c*d^2)^(1/2)/d/e
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {910, 272, 65, 214, 858, 223, 212, 739}

$$\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/e + (Sqrt[c*d^2 + a*e^2]*Arc
Tanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*e) - (Sqrt[a]
*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 910

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[Simp[c*d*f + a*e*g - c*(e*
f - d*g)*x, x]*((a + c*x^2)^(p - 1)/(f + g*x)), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex)}\,dx = -\frac{\int \frac{ae-cdx}{(d+ex)\sqrt{a+cx^2}}\,dx}{d} + \frac{a\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e} - \left(\frac{cd}{e}+\frac{ae}{d}\right)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e} - \left(-\frac{cd}{e}-\frac{ae}{d}\right.$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{e} + \frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{de} - \frac{\sqrt{a}\tanh}{}$$

Mathematica [A]
time = 0.23, size = 137, normalized size = 1.18

$$-\frac{2\sqrt{-cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)-2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)+\sqrt{c}\,d\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x)),x]
```

```
[Out] -((2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/
Sqrt[-(c*d^2) - a*e^2]] - 2*Sqrt[a]*e*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])
/Sqrt[a]] + Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(d*e))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(98) = 196$.
time = 0.08, size = 305, normalized size = 2.63

method	result
default	$-\frac{\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}\right)}{e}-}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*((c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e*ln(
(-c*d/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^
(1/2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-
2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a
*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/d*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^
(1/2)*(c*x^2+a)^(1/2))/x))
```

Maxima [**A**]

time = 0.30, size = 103, normalized size = 0.89

$$\frac{\left(\sqrt{c}\,d\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-2)} - \sqrt{cd^2e^{(-2)}+a}\,\operatorname{arsinh}\left(\frac{2\,cdx}{\sqrt{ac}\,|2\,xe+2\,d|} - \frac{2\,ae}{\sqrt{ac}\,|2\,xe+2\,d|}\right)e^{(-1)} - \sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ac}\,|x|}\right)e^{(-1)}\right)e}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] (sqrt(c)*d*arcsinh(c*x/sqrt(a*c))*e^(-2) - sqrt(c*d^2*e^(-2) + a)*arcsinh(2
*c*d*x/(sqrt(a*c)*abs(2*x*e + 2*d)) - 2*a*e/(sqrt(a*c)*abs(2*x*e + 2*d)))*e
^(-1) - sqrt(a)*arcsinh(a/(sqrt(a*c)*abs(x)))*e^(-1))*e/d
```

Fricas [**A**]

time = 3.51, size = 1299, normalized size = 11.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(a)*e
*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(c*d^2 + a*e^2)*
log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x
- a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)))
*e^(-1)/d, -1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*
e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - sqrt(c*d^2 + a*e^2)
*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x
 - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2))
)*e^(-1)/d, 1/2*(sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)
+ sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c
*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c
^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)))*e^(-1)/d, -1/2*(2*sqrt(-c)*d*
arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 +
 a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^
2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e
^2)))*e^(-1)/d, 1/2*(2*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a))*e + sqrt(c
)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + sqrt(c*d^2 + a*e^2)*l
og(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
```

```
a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2))*
e^(-1)/d, -1/2*(2*sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - 2*sqrt(-a
)*arctan(sqrt(-a)/sqrt(c*x^2 + a))*e - sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*
x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^
2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-1)/d, 1/2*(
2*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a))*e + sqrt(c)*d*log(-2*c*x^2 - 2*
sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2
 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 +
 a^2)*e^2)))*e^(-1)/d, -(sqrt(-c)*d*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - sq
rt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a))*e + sqrt(-c*d^2 - a*e^2)*arctan(-sq
rt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (
a*c*x^2 + a^2)*e^2)))*e^(-1)/d]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2 + a}}{x\,\left(d + e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.322 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{a+cx^2}}{dx}-\frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^2-arctanh((-c*d*x+a*e)/(a*e^2+
c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^2-(c*x^2+a)^(1/2)/d/x
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {975, 283, 223, 212, 272, 52, 65, 214, 749, 858, 739}

$$-\frac{\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^2 + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^2} - \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e^2\sqrt{a+cx^2}}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{d^2} - \frac{\sqrt{a+cx^2}}{dx} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} - \frac{e\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{e\int}{}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} + \frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d} - \frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)}{}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2} + \frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a}}\right.}{d^2}$$

Mathematica [A]
time = 0.26, size = 124, normalized size = 1.18

$$-\frac{d\sqrt{a+cx^2} - 2\sqrt{-cd^2-ae^2}\,x\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) + 2\sqrt{a}\,ex\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x)),x]
```

[Out] -((d*Sqrt[a + c*x^2] - 2*Sqrt[-(c*d^2) - a*e^2]*x*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + 2*Sqrt[a]*e*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(d^2*x))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(91) = 182$.
time = 0.10, size = 370, normalized size = 3.52

method	result
risch	$-\frac{\sqrt{c\,x^{2}+a}}{d x}-\frac{e \ln \left(\frac{\frac{2 a\,e^{2}+2 c\,d^{2}}{e^{2}}-\frac{2 c d \left(x +\frac{d}{e}\right)}{e}+2 \sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\, \sqrt{c \left(x +\frac{d}{e}\right)^{2}-\frac{2 c d \left(x +\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x +\frac{d}{e}}\right) a}{d^{2} \sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}-\ln \left(\frac{2 a e \cdots}{\cdots}\right.$
default	$\frac{e \left(\sqrt{c \left(x +\frac{d}{e}\right)^{2}-\frac{2 c d \left(x +\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}-\frac{\sqrt{c}\, d \ln \left(\frac{-\frac{c d}{e}+c \left(x +\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c \left(x +\frac{d}{e}\right)^{2}-\frac{2 c d \left(x +\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\right)}{e}-\cdots\right.}{d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(1/2)/x^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] e/d^2*((c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e*ln((-c*d/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/d*(-1/a/x*(c*x^2+a)^(3/2)+2*c/a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))-e/d^2*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/((x*e + d)*x^2), x)

Fricas [A]
time = 2.62, size = 596, normalized size = 5.68

$$\left[\frac{\sqrt{a}\, xe \log\left(-\frac{cx^2+2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)+\sqrt{cd^2+ae^2}\, x \log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2+2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)-2\sqrt{cx^2+a}\, d}{2d^2x}, \frac{\sqrt{a}\, xe \log\left(-\frac{cx^2+2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)+2\sqrt{-cd^2-ae^2}\, x \arctan\left(-\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)-2\sqrt{cx^2+a}\, d}{2d^2x}, -\frac{2\sqrt{-a}\, x \arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)e-\sqrt{cd^2+ae^2}\, x \log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2+2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+2\sqrt{cx^2+a}\, d}{2d^2x}, -\frac{\sqrt{-a}\, x \arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)e-\sqrt{-cd^2-ae^2}\, x \arctan\left(-\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)+\sqrt{cx^2+a}\, d}{d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*x*e*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqr
t(c*d^2 + a*e^2)*x*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d
^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2
 + 2*d*x*e + d^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), 1/2*(sqrt(a)*x*e*log(-(c
*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(-c*d^2 - a*e^2)*x*arc
tan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*
d^2 + (a*c*x^2 + a^2)*e^2)) - 2*sqrt(c*x^2 + a)*d)/(d^2*x), -1/2*(2*sqrt(-a
)*x*arctan(sqrt(-a)/sqrt(c*x^2 + a))*e - sqrt(c*d^2 + a*e^2)*x*log(-(2*c^2*
d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*sqrt(c*x
^2 + a)*d)/(d^2*x), -(sqrt(-a)*x*arctan(sqrt(-a)/sqrt(c*x^2 + a))*e - sqrt(
-c*d^2 - a*e^2)*x*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a
)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + sqrt(c*x^2 + a)*d)/(d^2*
x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^2\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x)), x)
```

Giac [A]
time = 1.31, size = 145, normalized size = 1.38

$$-\frac{2\,a \arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,d^2}+\frac{2\,a\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2-a\right)d}+\frac{2\,(cd^2+ae^2)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*a*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*d^2) + 2*a
*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*d) + 2*(c*d^2 + a*e^2)*arct
```

```
an(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/(sq
rt(-c*d^2 - a*e^2)*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^2*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.323 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)} \, dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{e\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}-$$

```
[Out] -1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)-e^2*arctanh((c*x^2+a)^(1/
2)/a^(1/2))*a^(1/2)/d^3+e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a
)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^3-1/2*(c*x^2+a)^(1/2)/d/x^2+e*(c*x^2+a)^(1/2
)/d^2/x
```

Rubi [A]
time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {975, 272, 43, 65, 214, 283, 223, 212, 52, 749, 858, 739}

$$-\frac{\sqrt{a}\,e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{e\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3}-\frac{\sqrt{a+cx^2}}{2dx^2}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*Sqrt[a + c*x^2]/(d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (e*Sqrt[c*d^2
+ a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^3
- (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) - (Sqrt[a]*e^2*ArcTanh
[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
```

```
2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x^3(d+ex)}\,dx &= \int \left(\frac{\sqrt{a+cx^2}}{dx^3}-\frac{e\sqrt{a+cx^2}}{d^2x^2}+\frac{e^2\sqrt{a+cx^2}}{d^3x}-\frac{e^3\sqrt{a+cx^2}}{d^3(d+ex)}\right)dx \\
&= \frac{\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d}-\frac{e\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^2}+\frac{e^2\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^3}-\frac{e^3\int \frac{\sqrt{a+cx^2}}{d+ex}\,dx}{d^3} \\
&= -\frac{e^2\sqrt{a+cx^2}}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx,x,x^2\right)}{2d}-\frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{4d}+\frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} \\
&= -\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d^2}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a}\right.}{2d} \\
&= -\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{e\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3}-\frac{c\tan}{}
\end{aligned}
$$

Mathematica [A]

time = 0.44, size = 144, normalized size = 0.90

$$\frac{\frac{d(-d+2ex)\sqrt{a+cx^2}}{x^2}-4e\sqrt{-cd^2-ae^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)+\frac{2(cd^2+2ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] ((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 - 4*e*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(
Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + (2*(c*d^2
+ 2*a*e^2)*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(134) = 268$.
time = 0.09, size = 442, normalized size = 2.76

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-2ex+d)}{2d^2x^2}+\frac{\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)a\,e^2}{d^3\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$
default	$-\frac{e^2\left(\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}\right)}{e}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/d^3*((c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e
*ln((-c*d/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e
^2)^(1/2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/
e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e
)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/d*(-1/2/a/x^2*(c*x^2+a)^(3/2)+1/2*c
/a*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)))-e/d^2*(
-1/a/x*(c*x^2+a)^(3/2)+2*c/a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1
```

```
/2)+(c*x^2+a)^(1/2))))+e^2/d^3*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(
c*x^2+a)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((x*e + d)*x^3), x)
```

Fricas [A]
time = 2.61, size = 737, normalized size = 4.61

$$\left[\frac{2\sqrt{cd^2+ae^2}\,ax^2e\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+(cd^2x^2+2ax^2e^2)\sqrt{a}\log\left(-\frac{cx^2-2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)+2(2adxe-ad^2)\sqrt{cx^2+a}}{4ad^3x^2}, -\frac{4\sqrt{-cd^2-ae^2}\,ax^2\arctan\left(-\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)e-(cd^2x^2+2ax^2e^2)\sqrt{a}\log\left(-\frac{cx^2-2\sqrt{cx^2+a}\sqrt{a}+2a}{x^2}\right)-2(2adxe-ad^2)\sqrt{cx^2+a}}{4ad^3x^2}, \frac{\sqrt{cd^2+ae^2}\,ax^2e\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+(cd^2x^2+2ax^2e^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)+(2adxe-ad^2)\sqrt{cx^2+a}}{2ad^3x^2}, -\frac{2\sqrt{-cd^2-ae^2}\,ax^2\arctan\left(-\frac{\sqrt{-cd^2-ae^2}(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)e-(cd^2x^2+2ax^2e^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)-(2adxe-ad^2)\sqrt{cx^2+a}}{2ad^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="fricas")
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a*x^2*e*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c
*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a
^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + (c*d^2*x^2 + 2*a*x^2*e^2)*sqrt(a)*log
(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*a*d*x*e - a*d^2)*sq
rt(c*x^2 + a))/(a*d^3*x^2), -1/4*(4*sqrt(-c*d^2 - a*e^2)*a*x^2*arctan(-sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*
c*x^2 + a^2)*e^2))*e - (c*d^2*x^2 + 2*a*x^2*e^2)*sqrt(a)*log(-(c*x^2 - 2*sq
rt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*a*d*x*e - a*d^2)*sqrt(c*x^2 + a))/
(a*d^3*x^2), 1/2*(sqrt(c*d^2 + a*e^2)*a*x^2*e*log(-(2*c^2*d^2*x^2 - 2*a*c*d
*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c
*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + (c*d^2*x^2 + 2*a*x^2*e^2)*s
qrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (2*a*d*x*e - a*d^2)*sqrt(c*x^2 +
 a))/(a*d^3*x^2), -1/2*(2*sqrt(-c*d^2 - a*e^2)*a*x^2*arctan(-sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^
2)*e^2))*e - (c*d^2*x^2 + 2*a*x^2*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2
+ a)) - (2*a*d*x*e - a*d^2)*sqrt(c*x^2 + a))/(a*d^3*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^3\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x)), x)
```

Giac [A]
time = 1.01, size = 230, normalized size = 1.44

$$-\frac{2\,(cd^2e+ae^3)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^3}+\frac{(cd^2+2\,ae^2)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d^3}+\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^3cd-2\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2a\sqrt{c}\,e+\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)acd+2\,a^2\sqrt{c}\,e}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2-a\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/
sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^3) + (c*d^2 + 2*a*e^2)*arctan
(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^3) + ((sqrt(c)*x - sq
rt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*sqrt(c)*e + (sqr
t(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^
2 + a))^2 - a)^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.324 $\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=191

$$\frac{e\sqrt{a+cx^2}}{2d^2x^2}-\frac{e^2\sqrt{a+cx^2}}{d^3x}-\frac{(a+cx^2)^{3/2}}{3adx^3}-\frac{e^2\sqrt{cd^2+ae^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^4}+\frac{ce\tanh^{-1}\left(\right.}{2\sqrt{a}}$$

```
[Out] -1/3*(c*x^2+a)^(3/2)/a/d/x^3+1/2*c*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a
^(1/2)+e^3*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^4-e^2*arctanh((-c*d*x
+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^4+1/2*e*(c
*x^2+a)^(1/2)/d^2/x^2-e^2*(c*x^2+a)^(1/2)/d^3/x
```

Rubi [A]
time = 0.15, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {975, 270, 272, 43, 65, 214, 283, 223, 212, 52, 749, 858, 739}

$$\frac{\sqrt{a}\,e^3\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4}-\frac{e^2\sqrt{a+cx^2}}{d^3x}+\frac{e\sqrt{a+cx^2}}{2d^2x^2}+\frac{ce\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^2}-\frac{e^2\sqrt{ae^2+cd^2}\,\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4}-\frac{(a+cx^2)^{3/2}}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (e*Sqrt[a + c*x^2])/(2*d^2*x^2) - (e^2*Sqrt[a + c*x^2])/(d^3*x) - (a + c*x^
2)^(3/2)/(3*a*d*x^3) - (e^2*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt
[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^4 + (c*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[
a]])/(2*Sqrt[a]*d^2) + (Sqrt[a]*e^3*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^4(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^4} - \frac{e\sqrt{a+cx^2}}{d^2x^3} + \frac{e^2\sqrt{a+cx^2}}{d^3x^2} - \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e^4\sqrt{a+cx^2}}{d^4(d+ex)}\right) dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^4} + \frac{e^4\int \text{-}$$

$$= \frac{e^3\sqrt{a+cx^2}}{d^4} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{e\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d^2} + \frac{(ce^2)\int}{}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{(ce)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d^2} - \frac{(ce^2}{}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} + \frac{\sqrt{c}\,e^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d^3} - \frac{e\text{Subst}\left(\int\right.}{}$$

$$= \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{(a+cx^2)^{3/2}}{3adx^3} - \frac{e^2\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+}}\right.}{d^4}$$

Mathematica [A]
time = 0.66, size = 173, normalized size = 0.91

$$-\frac{\frac{d\sqrt{a+cx^2}\,(2ad^2-3adex+2cd^2x^2+6ae^2x^2)}{ax^3} - 12e^2\sqrt{-cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) + \frac{6e(cd^2+2ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -1/6*((d*Sqrt[a + c*x^2]*(2*a*d^2 - 3*a*d*e*x + 2*c*d^2*x^2 + 6*a*e^2*x^2))
/(a*x^3) - 12*e^2*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt
[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]] + (6*e*(c*d^2 + 2*a*e^2)*ArcTanh[(Sqrt
[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. 2(161) = 322.
time = 0.08, size = 465, normalized size = 2.43

method	result

risch	$-\frac{\sqrt{c\,x^{2}+a}\,(6a\,e^{2}x^{2}+2c\,d^{2}x^{2}-3adex+2a\,d^{2})}{6d^{3}x^{3}a}-\frac{e^{3}\ln\left(\frac{\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x\right.}{e}}}{x+\frac{d}{e}}\right.}{d^{4}\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}$
default	$\frac{e^{3}\left(\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\right)}{e}\right.}{d^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e^3/d^4*((c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e*
ln((-c*d/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^
2)^(1/2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e
^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)
+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))-e/d^2*(-1/2/a/x^2*(c*x^2+a)^(3/2)+1/2*
c/a*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)))+e^2/d^
3*(-1/a/x*(c*x^2+a)^(3/2)+2*c/a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c
^(1/2)+(c*x^2+a)^(1/2))))-1/3*(c*x^2+a)^(3/2)/a/d/x^3-e^3/d^4*((c*x^2+a)^(1
/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((x*e + d)*x^4), x)
```

Fricas [A]
time = 2.40, size = 829, normalized size = 4.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*a*x^3*e^2*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e +
a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 +
2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 3*(c*d^2*x^3*e + 2*a*x^3*e^3)*sqrt
(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*c*d^3*x^2 +
6*a*d*x^2*e^2 - 3*a*d^2*x*e + 2*a*d^3)*sqrt(c*x^2 + a))/(a*d^4*x^3), 1/12*(
12*sqrt(-c*d^2 - a*e^2)*a*x^3*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sq
rt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^2 + 3*(c*d^2
*x^3*e + 2*a*x^3*e^3)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(2*c*d^3*x^2 + 6*a*d*x^2*e^2 - 3*a*d^2*x*e + 2*a*d^3)*sqrt(c*x^2
 + a))/(a*d^4*x^3), 1/6*(3*sqrt(c*d^2 + a*e^2)*a*x^3*e^2*log(-(2*c^2*d^2*x^
2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 3*(c*d^2*x^3*e +
 2*a*x^3*e^3)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (2*c*d^3*x^2 + 6*
a*d*x^2*e^2 - 3*a*d^2*x*e + 2*a*d^3)*sqrt(c*x^2 + a))/(a*d^4*x^3), 1/6*(6*s
qrt(-c*d^2 - a*e^2)*a*x^3*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^2 - 3*(c*d^2*x^3
*e + 2*a*x^3*e^3)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (2*c*d^3*x^2
+ 6*a*d*x^2*e^2 - 3*a*d^2*x*e + 2*a*d^3)*sqrt(c*x^2 + a))/(a*d^4*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x^4 (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**4*(d + e*x)), x)
```

Giac [A]
time = 1.39, size = 309, normalized size = 1.62

$$\frac{2\left(cd^2e^2 + ae^4\right)\arctan\left(-\frac{\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)e + \sqrt{c}\,d}{\sqrt{-cd^2 - ae^2}}\right)}{\sqrt{-cd^2 - ae^2}\,d^4} - \frac{\left(cd^2e + 2\,ae^3\right)\arctan\left(-\frac{\sqrt{c}\,x - \sqrt{cx^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d^4} - \frac{3\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)^5 cde - 6\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)^4 c^{\frac{3}{2}}d^2 - 6\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)^4 a\sqrt{c}\,e^2 - 3\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)a^2cde - 2\,a^2c^{\frac{3}{2}}d^2 + 12\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)^2 a^2\sqrt{c}\,e^2 - 6\,a^3\sqrt{c}\,e^2}{3\left(\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)^2 - a\right)^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] 2*(c*d^2*e^2 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)
/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^4) - (c*d^2*e + 2*a*e^3)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*d^4) - 1/3*(3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*c*d*e - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(3/2)
*d^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*sqrt(c)*e^2 - 3*(sqrt(c)*x - sqr
t(c*x^2 + a))*a^2*c*d*e - 2*a^2*c^(3/2)*d^2 + 12*(sqrt(c)*x - sqrt(c*x^2 +
```

```
a))^2*a^2*sqrt(c)*e^2 - 6*a^3*sqrt(c)*e^2)/(((sqrt(c)*x - sqrt(c*x^2 + a))^
2 - a)^3*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+a}}{x^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^4*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.325 $\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=274

$$-\frac{\sqrt{a+cx^2}}{4dx^4}-\frac{c\sqrt{a+cx^2}}{8adx^2}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}+\frac{e^3\sqrt{a+cx^2}}{d^4x}+\frac{e(a+cx^2)^{3/2}}{3ad^2x^3}+\frac{e^3\sqrt{cd^2+ae^2}\tanh^{-1}\left(\frac{ae-}{\sqrt{cd^2+ae}}\right.}{d^5}$$

```
[Out] 1/3*e*(c*x^2+a)^(3/2)/a/d^2/x^3+1/8*c^2*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^
(3/2)/d-1/2*c*e^2*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-e^4*arctanh(
(c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^5+e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)
^(1/2)/(c*x^2+a)^(1/2))*(a*e^2+c*d^2)^(1/2)/d^5-1/4*(c*x^2+a)^(1/2)/d/x^4-1
/8*c*(c*x^2+a)^(1/2)/a/d/x^2-1/2*e^2*(c*x^2+a)^(1/2)/d^3/x^2+e^3*(c*x^2+a)^
(1/2)/d^4/x
```

Rubi [A]
time = 0.20, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {975, 272, 43, 44, 65, 214, 270, 283, 223, 212, 52, 749, 858, 739}

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,e^4\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5}+\frac{e^3\sqrt{a+cx^2}}{d^4x}-\frac{e^2\sqrt{a+cx^2}}{2d^3x^2}-\frac{ce^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d^3}+\frac{e(a+cx^2)^{3/2}}{3ad^2x^3}+\frac{e^3\sqrt{ae^2+cd^2}\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5}-\frac{c\sqrt{a+cx^2}}{8adx^2}-\frac{\sqrt{a+cx^2}}{4dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -1/4*Sqrt[a + c*x^2]/(d*x^4) - (c*Sqrt[a + c*x^2])/(8*a*d*x^2) - (e^2*Sqrt[
a + c*x^2])/(2*d^3*x^2) + (e^3*Sqrt[a + c*x^2])/(d^4*x) + (e*(a + c*x^2)^(3
/2))/(3*a*d^2*x^3) + (e^3*Sqrt[c*d^2 + a*e^2]*ArcTanh[(a*e - c*d*x)/(Sqrt[c
*d^2 + a*e^2]*Sqrt[a + c*x^2])])/d^5 + (c^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]
])/(8*a^(3/2)*d) - (c*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d^3)
 - (Sqrt[a]*e^4*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^5} - \frac{e\sqrt{a+cx^2}}{d^2x^4} + \frac{e^2\sqrt{a+cx^2}}{d^3x^3} - \frac{e^3\sqrt{a+cx^2}}{d^4x^2} + \frac{e^4\sqrt{a+cx^2}}{d^5x} - \frac{e^5\sqrt{a}}{d^5(d}\right.$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x^5}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^4}\,dx}{d^2} + \frac{e^2\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d^3} - \frac{e^3\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^4} + \frac{e^4\int}{}$$

$$= -\frac{e^4\sqrt{a+cx^2}}{d^5} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^3}\,dx, x, x^2\right)}{2d} + \frac{e^2\text{S}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{c\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\right.}{8d}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} - \frac{\sqrt{c}\,e^3\,\text{ta}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{4dx^4} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x} + \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{cd^2}}{}$$

Mathematica [A]
time = 0.80, size = 215, normalized size = 0.78

$$\frac{\sqrt{a}\left(d\sqrt{a+cx^2}\,(cd^2x^2(-3d+8ex)+a(-6d^3+8d^2ex-12de^2x^2+24e^3x^3))-48ae^3\sqrt{-cd^2-ae^2}\,x^4\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)\right)-6(c^2d^4-4acd^2e^2-8a^2e^4)\,x^4\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{24a^{3/2}d^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[a]*(d*Sqrt[a + c*x^2]*(c*d^2*x^2*(-3*d + 8*e*x) + a*(-6*d^3 + 8*d^2*e
*x - 12*d*e^2*x^2 + 24*e^3*x^3)) - 48*a*e^3*Sqrt[-(c*d^2) - a*e^2]*x^4*ArcT
an[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]]) - 6*(c^
2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*x^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2]
)/Sqrt[a]])/(24*a^(3/2)*d^5*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(228) = 456$.
time = 0.09, size = 558, normalized size = 2.04

method	result

risch	$-\frac{\sqrt{c x^2+a}\,(-24a e^3x^3-8c d^2e x^3+12ad e^2x^2+3c d^3x^2-8a d^2ex+6a d^3)}{24d^4x^4a}+\frac{a e^4\ln\left(\frac{\frac{2a e^2+2c d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\ \cdots}{\cdots}\right)}{d^5\sqrt{\frac{a e \cdots}{}}}$
default	$-\frac{e^4\left(\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2+c d^2}{e^2}}-\frac{\sqrt{c}\,d\ln\left(\frac{-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2+c d^2}{e^2}}\right)}{e}\ \cdots\right)}{d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^4/d^5*((c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c^(1/2)*d/e
*ln((-c*d/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e
^2)^(1/2))-(a*e^2+c*d^2)/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/
e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e
)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/d*(-1/4/a/x^4*(c*x^2+a)^(3/2)-1/4*c
/a*(-1/2/a/x^2*(c*x^2+a)^(3/2)+1/2*c/a*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a
^(1/2)*(c*x^2+a)^(1/2))/x))))+e^2/d^3*(-1/2/a/x^2*(c*x^2+a)^(3/2)+1/2*c/a*(
(c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)))-e^3/d^4*(-1
/a/x*(c*x^2+a)^(3/2)+2*c/a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2
)+(c*x^2+a)^(1/2))))+1/3*e*(c*x^2+a)^(3/2)/a/d^2/x^3+e^4/d^5*((c*x^2+a)^(1/
2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((x*e + d)*x^5), x)
```

Fricas [A]
time = 2.37, size = 1016, normalized size = 3.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/48*(24*sqrt(c*d^2 + a*e^2)*a^2*x^4*e^3*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e
 + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2
 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 3*(c^2*d^4*x^4 - 4*a*c*d^2*x^4*
e^2 - 8*a^2*x^4*e^4)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)
/x^2) - 2*(3*a*c*d^4*x^2 - 24*a^2*d*x^3*e^3 + 12*a^2*d^2*x^2*e^2 + 6*a^2*d^
4 - 8*(a*c*d^3*x^3 + a^2*d^3*x)*e)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), -1/48*(4
8*sqrt(-c*d^2 - a*e^2)*a^2*x^4*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^3 + 3*(c^2*
d^4*x^4 - 4*a*c*d^2*x^4*e^2 - 8*a^2*x^4*e^4)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c
*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*a*c*d^4*x^2 - 24*a^2*d*x^3*e^3 + 12*a^
2*d^2*x^2*e^2 + 6*a^2*d^4 - 8*(a*c*d^3*x^3 + a^2*d^3*x)*e)*sqrt(c*x^2 + a))
/(a^2*d^5*x^4), 1/24*(12*sqrt(c*d^2 + a*e^2)*a^2*x^4*e^3*log(-(2*c^2*d^2*x^
2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 3*(c^2*d^4*x^4 -
 4*a*c*d^2*x^4*e^2 - 8*a^2*x^4*e^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a
)) - (3*a*c*d^4*x^2 - 24*a^2*d*x^3*e^3 + 12*a^2*d^2*x^2*e^2 + 6*a^2*d^4 - 8
*(a*c*d^3*x^3 + a^2*d^3*x)*e)*sqrt(c*x^2 + a))/(a^2*d^5*x^4), -1/24*(24*sqr
t(-c*d^2 - a*e^2)*a^2*x^4*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c
*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^3 + 3*(c^2*d^4*x
^4 - 4*a*c*d^2*x^4*e^2 - 8*a^2*x^4*e^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2
 + a)) + (3*a*c*d^4*x^2 - 24*a^2*d*x^3*e^3 + 12*a^2*d^2*x^2*e^2 + 6*a^2*d^4
 - 8*(a*c*d^3*x^3 + a^2*d^3*x)*e)*sqrt(c*x^2 + a))/(a^2*d^5*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^5\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**5*(d + e*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(224) = 448.
time = 1.53, size = 596, normalized size = 2.18

$$-\frac{2\,(cd^2e^3+ae^5)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}\,d^5}-\frac{(c^2d^4-4\,acd^2e^2-8\,a^2e^4)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,ad^5}+\frac{3\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^7c^2d^3-24\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^6ac^{\frac{3}{2}}d^2e+21\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^5ac^2d^3+12\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^7acde^2+24\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^4a^2c^{\frac{3}{2}}d^2e+21\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^3a^2c^2d^3-12\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^5a^2cde^2-24\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^6a^2\sqrt{c}\,e^3-8\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2a^3c^{\frac{3}{2}}d^2e+3\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)a^3c^2d^3-12\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^3a^3cde^2+72\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^4a^3\sqrt{c}\,e^3+8\,a^4c^{\frac{3}{2}}d^2e+12\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)a^4cde^2-72\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2a^4\sqrt{c}\,e^3+24\,a^5\sqrt{c}\,e^3}{12\left(\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2-a\right)^4ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*e^3 + a*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d
)/sqrt(-c*d^2 - a*e^2))/(sqrt(-c*d^2 - a*e^2)*d^5) - 1/4*(c^2*d^4 - 4*a*c*d
^2*e^2 - 8*a^2*e^4)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-
a)*a*d^5) + 1/12*(3*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^2*d^3 - 24*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*a*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a*c^2*d^3 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c*d*e^2 + 24*(sqrt(c)*x -
sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a
^2*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*d*e^2 - 24*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*a^2*sqrt(c)*e^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a
^3*c^(3/2)*d^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d^3 - 12*(sqrt(c
)*x - sqrt(c*x^2 + a))^3*a^3*c*d*e^2 + 72*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a
^3*sqrt(c)*e^3 + 8*a^4*c^(3/2)*d^2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4
*c*d*e^2 - 72*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*e^3 + 24*a^5*sqrt
(c)*e^3)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^4*a*d^4)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.326 $\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=195

$$\frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3}-\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3}+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3}-\frac{d(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right.}{2c^{3/2}e^4}$$

```
[Out] -1/2*d*(-a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^4-d^4*
arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^4/(a*e^2+c*d^2)
^(1/2)+1/6*(-4*a*e^2+11*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^3-7/6*d*(e*x+d)*(c*x^2
+a)^(1/2)/c/e^3+1/3*(e*x+d)^2*(c*x^2+a)^(1/2)/c/e^3
```

Rubi [A]
time = 0.31, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1668, 858, 223, 212, 739}

$$-\frac{d(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}+\frac{\sqrt{a+cx^2}\,(11cd^2-4ae^2)}{6c^2e^3}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}}-\frac{7d\sqrt{a+cx^2}\,(d+ex)}{6ce^3}+\frac{\sqrt{a+cx^2}\,(d+ex)^2}{3ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((11*c*d^2 - 4*a*e^2)*Sqrt[a + c*x^2])/(6*c^2*e^3) - (7*d*(d + e*x)*Sqrt[a
+ c*x^2])/(6*c*e^3) + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^3) - (d*(2*c*d^2
 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*Sqrt[c*d^2 +
 a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{-2ad^2e^2-de(cd^2+4ae^2)x-e^2(5cd^2+2ae^2)x^2-7cde^3x^3}{(d+ex)\sqrt{a+cx^2}}\,dx}{3ce^4}$$

$$= -\frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{3acd^2e^5+cde^4(5cd^2-ae^2)x+ce^5(11cd^2}{(d+ex)\sqrt{a+cx^2}}}{6c^2e^7}$$

$$= \frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{\int \frac{3c}{}}{}$$

$$= \frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} + \frac{d^4\int}{}$$

$$= \frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d^4S}{}$$

$$= \frac{(11cd^2-4ae^2)\sqrt{a+cx^2}}{6c^2e^3} - \frac{7d(d+ex)\sqrt{a+cx^2}}{6ce^3} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^3} - \frac{d(2}{}$$

Mathematica [A]
time = 0.55, size = 161, normalized size = 0.83

$$\frac{\frac{e\sqrt{a+cx^2}\,(-4ae^2+c(6d^2-3dex+2e^2x^2))}{c^2}-\frac{12d^4\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{3d(2cd^2-ae^2)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{c^{3/2}}}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(-4*a*e^2 + c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c^2 - (12
*d^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]]
)/Sqrt[-(c*d^2) - a*e^2] + (3*d*(2*c*d^2 - a*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a
 + c*x^2]])/c^(3/2))/(6*e^4)
```

Maple [A]
time = 0.08, size = 258, normalized size = 1.32

method	result
risch	$-\frac{\left(-2c\,e^2x^2+3cdex+4a\,e^2-6c\,d^2\right)\sqrt{c\,x^2+a}}{6c^2e^3}+\frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)a}{2c^{\frac{3}{2}}\,e^2}-\frac{d^3\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^4\sqrt{c}}-\frac{d^4\ln}{}$
default	$\frac{\frac{x^2\sqrt{c\,x^2+a}}{3c}-\frac{2a\sqrt{c\,x^2+a}}{3c^2}}{e}-\frac{d\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)}{e^2}+\frac{d^2\sqrt{c\,x^2+a}}{e^3c}-\frac{d^3\ln\left(x\sqrt{\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/3*x^2/c*(c*x^2+a)^(1/2)-2/3*a/c^2*(c*x^2+a)^(1/2))-d/e^2*(1/2*x/c*(c
*x^2+a)^(1/2)-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+d^2/e^3/c*(c*x^2
+a)^(1/2)-d^3/e^4*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/e^5*d^4/((a*e^2+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e
^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Maxima [A]
time = 0.31, size = 166, normalized size = 0.85

$$\frac{d^4\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-5)}}{\sqrt{cd^2e^{(-2)}+a}}-\frac{d^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-4)}}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,x^2e^{(-1)}}{3\,c}-\frac{\sqrt{cx^2+a}\,dxe^{(-2)}}{2\,c}+\frac{ad\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-2)}}{2\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+a}\,d^2e^{(-3)}}{c}-\frac{2\,\sqrt{cx^2+a}\,ae^{(-1)}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*
e^(-5)/sqrt(c*d^2*e^(-2) + a) - d^3*arcsinh(c*x/sqrt(a*c))*e^(-4)/sqrt(c) +
 1/3*sqrt(c*x^2 + a)*x^2*e^(-1)/c - 1/2*sqrt(c*x^2 + a)*d*x*e^(-2)/c + 1/2*
a*d*arcsinh(c*x/sqrt(a*c))*e^(-2)/c^(3/2) + sqrt(c*x^2 + a)*d^2*e^(-3)/c -
2/3*sqrt(c*x^2 + a)*a*e^(-1)/c^2
```

Fricas [A]
time = 6.81, size = 1021, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*c^2*d^4*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*
c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*
a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 3*(2*c^2*d^5 + a*c*d^3*e^2 - a^2*d*e
^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(3*c^2*d^3*
x*e^2 - 6*c^2*d^4*e + 3*a*c*d*x*e^4 - 2*(a*c*x^2 - 2*a^2)*e^5 - 2*(c^2*d^2*
x^2 + a*c*d^2)*e^3)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), 1/12*(12*sq
rt(-c*d^2 - a*e^2)*c^2*d^4*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) - 3*(2*c^2*d^5 +
a*c*d^3*e^2 - a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
 - a) - 2*(3*c^2*d^3*x*e^2 - 6*c^2*d^4*e + 3*a*c*d*x*e^4 - 2*(a*c*x^2 - 2*a
^2)*e^5 - 2*(c^2*d^2*x^2 + a*c*d^2)*e^3)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*
c^2*e^6), 1/6*(3*sqrt(c*d^2 + a*e^2)*c^2*d^4*log(-(2*c^2*d^2*x^2 - 2*a*c*d*
x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*
x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 3*(2*c^2*d^5 + a*c*d^3*e^2 -
 a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*c^2*d^3*x*e^2
- 6*c^2*d^4*e + 3*a*c*d*x*e^4 - 2*(a*c*x^2 - 2*a^2)*e^5 - 2*(c^2*d^2*x^2 +
a*c*d^2)*e^3)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6), 1/6*(6*sqrt(-c*d^
2 - a*e^2)*c^2*d^4*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 +
a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + 3*(2*c^2*d^5 + a*c*d^3*
e^2 - a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*c^2*d^3*x
*e^2 - 6*c^2*d^4*e + 3*a*c*d*x*e^4 - 2*(a*c*x^2 - 2*a^2)*e^5 - 2*(c^2*d^2*x
^2 + a*c*d^2)*e^3)*sqrt(c*x^2 + a))/(c^3*d^2*e^4 + a*c^2*e^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A]
time = 1.42, size = 163, normalized size = 0.84

$$\frac{2\,d^4\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{6}\,\sqrt{cx^2+a}\left(x\left(\frac{2\,xe^{(-1)}}{c}-\frac{3\,de^{(-2)}}{c}\right)+\frac{2\,(3\,c^2d^2e^7-2\,ace^9)e^{(-10)}}{c^3}\right)+\frac{\left(2\,c^{\frac{3}{2}}d^3-a\sqrt{c}\,de^2\right)e^{(-4)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/6*sqrt(c*x^2 + a)*(x*(2*x*e^(-1)/c -
 3*d*e^(-2)/c) + 2*(3*c^2*d^2*e^7 - 2*a*c*e^9)*e^(-10)/c^3) + 1/2*(2*c^(3/2
)*d^3 - a*sqrt(c)*d*e^2)*e^(-4)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.327 $\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=152

$$-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^2}+\frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3\sqrt{cd^2+ae^2}}$$

```
[Out] 1/2*(-a*e^2+2*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^3+d^3*arc
tanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^3/(a*e^2+c*d^2)^(1
/2)-3/2*d*(c*x^2+a)^(1/2)/c/e^2+1/2*(e*x+d)*(c*x^2+a)^(1/2)/c/e^2
```

Rubi [A]
time = 0.18, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1668, 858, 223, 212, 739}

$$\frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}}-\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{\sqrt{a+cx^2}\,(d+ex)}{2ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-3*d*Sqrt[a + c*x^2])/(2*c*e^2) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^2) +
((2*c*d^2 - a*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^3) +
(d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^3*Sqr
t[c*d^2 + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(d+ex)\sqrt{a+cx^2}}\,dx &= \frac{(d+ex)\sqrt{a+cx^2}}{2ce^2}+\frac{\int \frac{-ade^2-e(cd^2+ae^2)x-3cde^2x^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{2ce^3}\\ &= -\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^2}+\frac{\int \frac{-acde^4+ce^3(2cd^2-ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{2c^2e^5}\\ &= -\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^2}-\frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^3}+\frac{(2cd^2-ae^2}{}\\ &= -\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^2}+\frac{d^3\mathrm{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right.}{e^3}\\ &= -\frac{3d\sqrt{a+cx^2}}{2ce^2}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^2}+\frac{(2cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3}+\end{aligned}$$

Mathematica [A]
time = 0.40, size = 137, normalized size = 0.90

$$\frac{\frac{e(-2d+ex)\sqrt{a+cx^2}}{c}+\frac{4d^3\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{(-2cd^2+ae^2)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{c^{3/2}}}{2e^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e*x)*Sqrt[a + c*x^2]),x]

[Out] ((e*(-2*d + e*x)*Sqrt[a + c*x^2])/c + (4*d^3*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/Sqrt[-(c*d^2) - a*e^2] + ((-2*c*d^2 + a*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/c^(3/2))/(2*e^3)

Maple [A]
time = 0.10, size = 216, normalized size = 1.42

method	result
risch	$-\frac{(-ex+2d)\sqrt{c\,x^{2}+a}}{2c\,e^{2}}-\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)a}{2c^{\frac{3}{2}}e}+\frac{d^{2}\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e^{3}\sqrt{c}}+\frac{d^{3}\ln\left(\frac{\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}}{}\right.}{}$
default	$\frac{\frac{x\sqrt{c\,x^{2}+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2c^{\frac{3}{2}}}}{e}-\frac{d\sqrt{c\,x^{2}+a}}{c\,e^{2}}+\frac{d^{2}\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e^{3}\sqrt{c}}+\frac{d^{3}\ln\left(\frac{\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}-\frac{2cd(}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/e*(1/2*x/c*(c*x^2+a)^(1/2)-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-d*(c*x^2+a)^(1/2)/c/e^2+d^2/e^3*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d^3/e^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))

Maxima [A]
time = 0.30, size = 127, normalized size = 0.84

$$-\frac{d^{3}\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-4)}}{\sqrt{cd^{2}e^{(-2)}+a}}+\frac{d^{2}\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-3)}}{\sqrt{c}}+\frac{\sqrt{cx^{2}+a}\,xe^{(-1)}}{2\,c}-\frac{a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-1)}}{2\,c^{\frac{3}{2}}}-\frac{\sqrt{cx^{2}+a}\,de^{(-2)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] -d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-4)/sqrt(c*d^2*e^(-2) + a) + d^2*arcsinh(c*x/sqrt(a*c))*e^(-3)/sqrt(c)

```
+ 1/2*sqrt(c*x^2 + a)*x*e^(-1)/c - 1/2*a*arcsinh(c*x/sqrt(a*c))*e^(-1)/c^(3
/2) - sqrt(c*x^2 + a)*d*e^(-2)/c
```

Fricas [A]
time = 6.43, size = 881, normalized size = 5.80

$$\left[\frac{2\sqrt{cd^2+ae^2}\,c^2d^3\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)-(2c^2d^4+acd^2e^2-a^2e^4)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2(c^2d^2xe^2-2c^2d^3e+acxe^4-2acde^3)\sqrt{cx^2+a}}{4(c^3d^2e^3+ac^2e^5)},\ -\frac{4\sqrt{-cd^2-ae^2}\,c^2d^3\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)+(2c^2d^4+acd^2e^2-a^2e^4)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2(c^2d^2xe^2-2c^2d^3e+acxe^4-2acde^3)\sqrt{cx^2+a}}{4(c^3d^2e^3+ac^2e^5)},\ \frac{\sqrt{cd^2+ae^2}\,c^2d^3\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)-(2c^2d^4+acd^2e^2-a^2e^4)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+(c^2d^2xe^2-2c^2d^3e+acxe^4-2acde^3)\sqrt{cx^2+a}}{2(c^3d^2e^3+ac^2e^5)},\ -\frac{2\sqrt{-cd^2-ae^2}\,c^2d^3\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)+(2c^2d^4+acd^2e^2-a^2e^4)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-(c^2d^2xe^2-2c^2d^3e+acxe^4-2acde^3)\sqrt{cx^2+a}}{2(c^3d^2e^3+ac^2e^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*c^2*d^3*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c
*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a
^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*s
qrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c^2*d^2*x*e^2 -
 2*c^2*d^3*e + a*c*x*e^4 - 2*a*c*d*e^3)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c
^2*e^5), -1/4*(4*sqrt(-c*d^2 - a*e^2)*c^2*d^3*arctan(-sqrt(-c*d^2 - a*e^2)*
(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)
) + (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2
 + a)*sqrt(c)*x - a) - 2*(c^2*d^2*x*e^2 - 2*c^2*d^3*e + a*c*x*e^4 - 2*a*c*d
*e^3)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), 1/2*(sqrt(c*d^2 + a*e^2)*
c^2*d^3*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e
+ d^2)) - (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sq
rt(c*x^2 + a)) + (c^2*d^2*x*e^2 - 2*c^2*d^3*e + a*c*x*e^4 - 2*a*c*d*e^3)*sq
rt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5), -1/2*(2*sqrt(-c*d^2 - a*e^2)*c^2*
d^3*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2
 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + (2*c^2*d^4 + a*c*d^2*e^2 - a^2*e^4)*sq
rt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^2*x*e^2 - 2*c^2*d^3*e +
a*c*x*e^4 - 2*a*c*d*e^3)*sqrt(c*x^2 + a))/(c^3*d^2*e^3 + a*c^2*e^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A]
time = 1.20, size = 129, normalized size = 0.85

$$-\frac{2\,d^3\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-3)}}{\sqrt{-cd^2-ae^2}}+\frac{1}{2}\sqrt{cx^2+a}\left(\frac{xe^{(-1)}}{c}-\frac{2\,de^{(-2)}}{c}\right)-\frac{(2\,cd^2-ae^2)e^{(-3)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*d^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 -
a*e^2))*e^(-3)/sqrt(-c*d^2 - a*e^2) + 1/2*sqrt(c*x^2 + a)*(x*e^(-1)/c - 2*d
*e^(-2)/c) - 1/2*(2*c*d^2 - a*e^2)*e^(-3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.328 $\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{a+cx^2}}{ce}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^2/c^(1/2)-d^2*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^2/(a*e^2+c*d^2)^(1/2)+(c*x^2+a)^(1/2
)/c/e
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1668, 12, 858, 223, 212, 739}

$$-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}+\frac{\sqrt{a+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e
^2) - (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e
^2*Sqrt[c*d^2 + a*e^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(d+ex)\sqrt{a+cx^2}}\,dx &= \frac{\sqrt{a+cx^2}}{ce}-\frac{\int \frac{cdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2}+\frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{e^2}-\frac{d^2\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\right.}{e^2}\\
&= \frac{\sqrt{a+cx^2}}{ce}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\sqrt{cd^2+ae^2}}
\end{aligned}
$$

Mathematica [A]

time = 0.34, size = 116, normalized size = 1.06

$$\frac{\frac{e\sqrt{a+cx^2}}{c}-\frac{2d^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{d\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2])/c - (2*d^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^
2])/Sqrt[-(c*d^2) - a*e^2]])/Sqrt[-(c*d^2) - a*e^2] + (d*Log[-(Sqrt[c]*x) +
 Sqrt[a + c*x^2]])/Sqrt[c])/e^2
```

Maple [A]
time = 0.07, size = 172, normalized size = 1.58

method	result
default	$\frac{\sqrt{c\,x^2+a}}{ce}-\frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^2\sqrt{c}}-\frac{d^2\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\right.}{e}}}{x+\frac{d}{e}}\right.}{e^3\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$
risch	$\frac{\sqrt{c\,x^2+a}}{ce}-\frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^2\sqrt{c}}-\frac{d^2\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\right.}{e}}}{x+\frac{d}{e}}\right.}{e^3\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+a)^(1/2)/c/e-d/e^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/e^3*d^2/(
(a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+
c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x
+d/e))
```

Maxima [A]
time = 0.29, size = 89, normalized size = 0.82

$$\frac{d^2\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-3)}}{\sqrt{cd^2e^{(-2)}+a}}-\frac{d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-2)}}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,e^{(-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] d^2*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*
e^(-3)/sqrt(c*d^2*e^(-2) + a) - d*arcsinh(c*x/sqrt(a*c))*e^(-2)/sqrt(c) + s
qrt(c*x^2 + a)*e^(-1)/c
```

Fricas [**A**]
time = 2.62, size = 721, normalized size = 6.61

$$\left[\frac{\sqrt{cd^2+ae^2}\,cd^2\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2+2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+(cd^3+ade^2)\sqrt{c}\,\log\left(-2cx^2+2\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2(cd^2e+ae^3)\sqrt{cx^2+a}}{2(c^2d^2e^2+ace^4)}, \frac{2\sqrt{-cd^2-ae^2}\,cd^2\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)+(cd^3+ade^2)\sqrt{c}\,\log\left(-2cx^2+2\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2(cd^2e+ae^3)\sqrt{cx^2+a}}{2(c^2d^2e^2+ace^4)}, \frac{\sqrt{cd^2+ae^2}\,cd^2\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2+2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+2(cd^3+ade^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+2(cd^2e+ae^3)\sqrt{cx^2+a}}{2(c^2d^2e^2+ace^4)}, \frac{\sqrt{-cd^2-ae^2}\,cd^2\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)+(cd^3+ade^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+(cd^2e+ae^3)\sqrt{cx^2+a}}{c^2d^2e^2+ace^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2
 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*
e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 +
2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^
2*d^2*e^2 + a*c*e^4), 1/2*(2*sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(-sqrt(-c*d^2
 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 +
 a^2)*e^2)) + (c*d^3 + a*d*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4)
, 1/2*(sqrt(c*d^2 + a*e^2)*c*d^2*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^
2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)
*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt
(-c)*x/sqrt(c*x^2 + a)) + 2*(c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2
 + a*c*e^4), (sqrt(-c*d^2 - a*e^2)*c*d^2*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + (
c*d^3 + a*d*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c*d^2*e + a
*e^3)*sqrt(c*x^2 + a))/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [**A**]
time = 1.54, size = 105, normalized size = 0.96

$$\frac{2\,d^2\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}}+\frac{de^{(-2)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,e^{(-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))*e^(-2)/sqrt(-c*d^2 - a*e^2) + d*e^(-2)*log(abs(-sqrt(c)*x + sqrt(c*x
^2 + a)))/sqrt(c) + sqrt(c*x^2 + a)*e^(-1)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{c\,x^2+a}\,\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.329 $\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e}+\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{e\sqrt{cd^2+ae^2}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e/c^(1/2)+d*arctanh((-c*d*x+a*e)/(a*e^2+
c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e/(a*e^2+c*d^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {858, 223, 212, 739}

$$\frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*e) + (d*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e*Sqrt[c*d^2 + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e} - \frac{d\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{e}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e} + \frac{d\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\sqrt{cd^2+ae^2}}$$

Mathematica [A]
time = 0.25, size = 98, normalized size = 1.14

$$\frac{\frac{2d\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}} - \frac{\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*d*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]
])/Sqrt[-(c*d^2) - a*e^2] - Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/Sqrt[c])/e
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(74) = 148.
time = 0.06, size = 151, normalized size = 1.76

method	result
default	$\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e\sqrt{c}} + \frac{d\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/
e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Maxima [A]
time = 0.31, size = 71, normalized size = 0.83

$$-\frac{d\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-2)}}{\sqrt{cd^2e^{(-2)}+a}}+\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-1)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -d*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e
^(-2)/sqrt(c*d^2*e^(-2) + a) + arcsinh(c*x/sqrt(a*c))*e^(-1)/sqrt(c)
```

Fricas [A]
time = 2.71, size = 620, normalized size = 7.21

$$\left[\frac{\sqrt{cd^2+ae^2}\,cd\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+(cd^2+ae^2)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}\,x-a\right)}{2(c^2d^2e+ace^3)}, -\frac{2\sqrt{-cd^2-ae^2}\,cd\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)-(cd^2+ae^2)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}\,x-a\right)}{2(c^2d^2e+ace^3)}, \frac{\sqrt{cd^2+ae^2}\,cd\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)-2(cd^2+ae^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)}{2(c^2d^2e+ace^3)}, -\frac{\sqrt{-cd^2-ae^2}\,cd\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)+(cd^2+ae^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)}{c^2d^2e+ace^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*c*d*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 -
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^
2)/(x^2*e^2 + 2*d*x*e + d^2)) + (c*d^2 + a*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sq
rt(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), -1/2*(2*sqrt(-c*d^2 -
a*e^2)*c*d*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*
d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) - (c*d^2 + a*e^2)*sqrt(c)*log(-2*
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a))/(c^2*d^2*e + a*c*e^3), 1/2*(sqrt(
c*d^2 + a*e^2)*c*d*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d
^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2
 + 2*d*x*e + d^2)) - 2*(c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^
2 + a)))/(c^2*d^2*e + a*c*e^3), -(sqrt(-c*d^2 - a*e^2)*c*d*arctan(-sqrt(-c*
d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^
2 + a^2)*e^2)) + (c*d^2 + a*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
))/(c^2*d^2*e + a*c*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A]
time = 1.11, size = 88, normalized size = 1.02

$$-\frac{2\,d\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^{(-1)}}{\sqrt{-cd^2-ae^2}}-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*d*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))*e^(-1)/sqrt(-c*d^2 - a*e^2) - e^(-1)*log(abs(-sqrt(c)*x + sqrt(c*x^2
+ a)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.330 $\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}$$

```
[Out] -arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)^(1
/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {739, 212}

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])]/Sqrt[c*d^2 +
 a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx &= -\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\sqrt{cd^2+ae^2}}\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 64, normalized size = 1.19

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])
/Sqrt[-(c*d^2) - a*e^2]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(48) = 96$.
time = 0.07, size = 127, normalized size = 2.35

method	result	size
default	$-\frac{\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((
a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/
2))/(x+d/e))
```

Maxima [**A**]
time = 0.28, size = 53, normalized size = 0.98

$$\frac{\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-1)}}{\sqrt{cd^2e^{(-2)}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-
1)/sqrt(c*d^2*e^(-2) + a)
```

Fricas [A]
time = 3.18, size = 205, normalized size = 3.80

$$\left[\frac{\log\left(-\frac{2\,c^2d^2x^2-2\,acdxe+acd^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2\,a^2)e^2}{x^2e^2+2\,dxe+d^2}\right)}{2\,\sqrt{cd^2+ae^2}},\ \frac{\sqrt{-cd^2-ae^2}\,\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)}{cd^2+ae^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c
*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d
^2))/sqrt(c*d^2 + a*e^2), sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2
))/(c*d^2 + a*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A]
time = 1.48, size = 59, normalized size = 1.09

$$\frac{2\,\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2
))/sqrt(-c*d^2 - a*e^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.331 $\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)+e*arctanh((-c*d*x+a*e)/(a*e^2+c
*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c*d^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {975, 272, 65, 214, 739, 212}

$$\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d*Sqrt[c*
d^2 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(d+ex)\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{dx\sqrt{a+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+cx^2}}\right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{d} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} \\
&= \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\sqrt{cd^2+ae^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d}
\end{aligned}$$

Mathematica [A]

time = 0.25, size = 104, normalized size = 1.21

$$\frac{2\left(\frac{e\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*((e*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2
]])/Sqrt[-(c*d^2) - a*e^2] + ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]]
/Sqrt[a]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(74) = 148$.
time = 0.08, size = 158, normalized size = 1.84

method	result
default	$\frac{\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right.}{d\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a
*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2
))/(x+d/e))-1/d/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(x*e + d)*x), x)
```

Fricas [A]
time = 3.75, size = 625, normalized size = 7.27

$$\left[\frac{\sqrt{cd^2+ae^2}\,ae\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+(cd^2+ae^2)\sqrt{a}\,\log\left(-\frac{cx^2-2\sqrt{cx^2+a}\,\sqrt{a}+2a}{x^2}\right)}{2(acd^3+a^2de^2)},\ -\frac{2\sqrt{-cd^2-ae^2}\,a\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)e-(cd^2+ae^2)\sqrt{a}\,\log\left(-\frac{cx^2-2\sqrt{cx^2+a}\,\sqrt{a}+2a}{x^2}\right)}{2(acd^3+a^2de^2)},\ \frac{\sqrt{cd^2+ae^2}\,ae\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2-2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+2(cd^2+ae^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)}{2(acd^3+a^2de^2)},\ -\frac{\sqrt{-cd^2-ae^2}\,a\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)e-(cd^2+ae^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)}{acd^3+a^2de^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*e*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 -
 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^
2)/(x^2*e^2 + 2*d*x*e + d^2)) + (c*d^2 + a*e^2)*sqrt(a)*log(-(c*x^2 - 2*sqr
t(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), -1/2*(2*sqrt(-c*d^
2 - a*e^2)*a*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^
2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e - (c*d^2 + a*e^2)*sqrt(a)*log
(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*c*d^3 + a^2*d*e^2), 1/
2*(sqrt(c*d^2 + a*e^2)*a*e*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*
sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/
(x^2*e^2 + 2*d*x*e + d^2)) + 2*(c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqr
t(c*x^2 + a)))/(a*c*d^3 + a^2*d*e^2), -(sqrt(-c*d^2 - a*e^2)*a*arctan(-sqrt
(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*
c*x^2 + a^2)*e^2))*e - (c*d^2 + a*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2
+ a)))/(a*c*d^3 + a^2*d*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.332 $\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{a+cx^2}}{adx}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)-e^2*arctanh((-c*d*x+a*e)/(a*
e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c*d^2)^(1/2)-(c*x^2+a)^(1/2)/a
/d/x
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.318$, Rules used = {975, 270, 272, 65, 214, 739, 212}

$$-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2
]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2 + a*e^2]) + (e*ArcTanh[Sqrt[a + c*x^2]
/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}}\,dx &= \int \left(\frac{1}{dx^2\sqrt{a+cx^2}}-\frac{e}{d^2x\sqrt{a+cx^2}}+\frac{e^2}{d^2(d+ex)\sqrt{a+cx^2}}\right)dx\\
&= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d}-\frac{e\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2}+\frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2}\\
&= -\frac{\sqrt{a+cx^2}}{adx}-\frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d^2}-\frac{e^2\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sqrt{a+cx^2}}{adx}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}-\frac{e\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x\right.}{cd^2}\\
&= -\frac{\sqrt{a+cx^2}}{adx}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.34, size = 128, normalized size = 1.15

$$-\frac{\frac{d\sqrt{a+cx^2}}{ax}+\frac{2e^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}+\frac{2e\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((d*Sqrt[a + c*x^2])/(a*x) + (2*e^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
+ c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/Sqrt[-(c*d^2) - a*e^2] + (2*e*ArcTanh[(S
qrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^2)
```

Maple [A]
time = 0.07, size = 180, normalized size = 1.62

method	result
default	$-\frac{e\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}-\frac{\sqrt{c\,x^2+a}}{adx}+\frac{e\ln\left(\frac{2a\ldots\right.}{\ldots}$
risch	$-\frac{e\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}-\frac{\sqrt{c\,x^2+a}}{adx}+\frac{e\ln\left(\frac{2a\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*
((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2))/(x+d/e))-(c*x^2+a)^(1/2)/a/d/x+e/d^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2
+a)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(x*e + d)*x^2), x)
```

Fricas [A]
time = 3.02, size = 747, normalized size = 6.73

$$\left[\frac{\sqrt{cd^2+ae^2}\,axe^2\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2+2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)+(cd^2xe+axe^3)\sqrt{a}\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\,\sqrt{a}+2a}{x^2}\right)-2\,(cd^3+ade^2)\sqrt{cx^2+a}}{2\,(acd^4x+a^2d^2xe^2)}, \frac{2\sqrt{-cd^2-ae^2}\,ax\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)e^2+(cd^2xe+axe^3)\sqrt{a}\log\left(-\frac{cx^2+2\sqrt{cx^2+a}\,\sqrt{a}+2a}{x^2}\right)-2\,(cd^3+ade^2)\sqrt{cx^2+a}}{2\,(acd^4x+a^2d^2xe^2)}, \frac{\sqrt{cd^2+ae^2}\,axe^2\log\left(-\frac{2c^2d^2x^2-2acdxe+acd^2+2\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2a^2)e^2}{x^2e^2+2dxe+d^2}\right)-2\,(cd^2xe+axe^3)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)-2\,(cd^3+ade^2)\sqrt{cx^2+a}}{2\,(acd^4x+a^2d^2xe^2)}, \frac{\sqrt{-cd^2-ae^2}\,ax\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)e^2-(cd^2xe+axe^3)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2+a}}\right)-(cd^3+ade^2)\sqrt{cx^2+a}}{acd^4x+a^2d^2xe^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*a*x*e^2*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d
^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2
)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + (c*d^2*x*e + a*x*e^3)*sqrt(a)*log(-(c*x
^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2
 + a))/(a*c*d^4*x + a^2*d^2*x*e^2), 1/2*(2*sqrt(-c*d^2 - a*e^2)*a*x*arctan(
-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2
+ (a*c*x^2 + a^2)*e^2))*e^2 + (c*d^2*x*e + a*x*e^3)*sqrt(a)*log(-(c*x^2 + 2
*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))
/(a*c*d^4*x + a^2*d^2*x*e^2), 1/2*(sqrt(c*d^2 + a*e^2)*a*x*e^2*log(-(2*c^2*
d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(
c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 2*(c*d^2*x
*e + a*x*e^3)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - 2*(c*d^3 + a*d*e^
2)*sqrt(c*x^2 + a))/(a*c*d^4*x + a^2*d^2*x*e^2), (sqrt(-c*d^2 - a*e^2)*a*x*
arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a
*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^2 - (c*d^2*x*e + a*x*e^3)*sqrt(-a)*arctan(
sqrt(-a)/sqrt(c*x^2 + a)) - (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/(a*c*d^4*x +
 a^2*d^2*x*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [A]
time = 1.45, size = 142, normalized size = 1.28

$$2\,c\left(\frac{\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^2}{\sqrt{-cd^2-ae^2}\,cd^2}-\frac{\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,cd^2}+\frac{1}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2-a\right)\sqrt{c}\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*(arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*
e^2))*e^2/(sqrt(-c*d^2 - a*e^2)*c*d^2) - arctan(-(sqrt(c)*x - sqrt(c*x^2 +
a))/sqrt(-a))*e/(sqrt(-a)*c*d^2) + 1/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)
*sqrt(c)*d))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.333 $\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=168

$$-\frac{\sqrt{a+cx^2}}{2adx^2}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a}\cdots}{\cdots}\right)}{\sqrt{a}\,d^3}$$

```
[Out] 1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d-e^2*arctanh((c*x^2+a)^(1/2
)/a^(1/2))/d^3/a^(1/2)+e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+
a)^(1/2))/d^3/(a*e^2+c*d^2)^(1/2)-1/2*(c*x^2+a)^(1/2)/a/d/x^2+e*(c*x^2+a)^(
1/2)/a/d^2/x
```

Rubi [A]

time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {975, 272, 44, 65, 214, 270, 739, 212}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}+\frac{e\sqrt{a+cx^2}}{ad^2x}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*Sqrt[a + c*x^2]/(a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (e^3*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 +
a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - (e^2*ArcTanh
[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{dx^3\sqrt{a+cx^2}} - \frac{e}{d^2x^2\sqrt{a+cx^2}} + \frac{e^2}{d^3x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)\sqrt{a+cx^2}}\right.$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)\sqrt{}}}{d}$$

$$= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{e^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x\right.}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{c\text{Subst}(}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} - \frac{e^2\tanh}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}} + \frac{c\tanh^{-}}{}$$

Mathematica [A]
time = 0.51, size = 198, normalized size = 1.18

$$\frac{\sqrt{a}\left(d(cd^2+ae^2)(-d+2ex)\sqrt{a+cx^2} - 4ae^3\sqrt{-cd^2-ae^2}\,x^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)\right) - 2(c^2d^4-acd^2e^2-2a^2e^4)\,x^2\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^3(cd^2+ae^2)\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[a]*(d*(c*d^2 + a*e^2)*(-d + 2*e*x)*Sqrt[a + c*x^2] - 4*a*e^3*Sqrt[-(c
*d^2) - a*e^2]*x^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*
d^2) - a*e^2]]) - 2*(c^2*d^4 - a*c*d^2*e^2 - 2*a^2*e^4)*x^2*ArcTanh[(Sqrt[c
]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(2*a^(3/2)*d^3*(c*d^2 + a*e^2)*x^2)
```

Maple [A]
time = 0.08, size = 235, normalized size = 1.40

method	result

risch	$-\frac{\sqrt{c x^2+a}\,(-2ex+d)}{2a\,d^2 x^2}+\frac{e^2\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^3\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$
default	$\frac{e^2\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^3\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}+\frac{-\frac{\sqrt{c\,x^2+a}}{2a\,x^2}+\frac{c\ln\left(2a+\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/d^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2
*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^
(1/2))/(x+d/e))+1/d*(-1/2/a/x^2*(c*x^2+a)^(1/2)+1/2*c/a^(3/2)*ln((2*a+2*a^(
1/2)*(c*x^2+a)^(1/2))/x))+e*(c*x^2+a)^(1/2)/a/d^2/x-e^2/d^3/a^(1/2)*ln((2*a
+2*a^(1/2)*(c*x^2+a)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(x*e + d)*x^3), x)
```

Fricas [A]
time = 3.57, size = 969, normalized size = 5.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*d^2 + a*e^2)*a^2*x^2*e^3*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e +
 a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 +
 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - (c^2*d^4*x^2 - a*c*d^2*x^2*e^2 -
2*a^2*x^2*e^4)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2)
```

```
+ 2*(2*a*c*d^3*x*e - a*c*d^4 + 2*a^2*d*x*e^3 - a^2*d^2*e^2)*sqrt(c*x^2 + a)
)/(a^2*c*d^5*x^2 + a^3*d^3*x^2*e^2), -1/4*(4*sqrt(-c*d^2 - a*e^2)*a^2*x^2*a
rctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*
c*d^2 + (a*c*x^2 + a^2)*e^2))*e^3 + (c^2*d^4*x^2 - a*c*d^2*x^2*e^2 - 2*a^2*
x^2*e^4)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2
*a*c*d^3*x*e - a*c*d^4 + 2*a^2*d*x*e^3 - a^2*d^2*e^2)*sqrt(c*x^2 + a))/(a^2
*c*d^5*x^2 + a^3*d^3*x^2*e^2), 1/2*(sqrt(c*d^2 + a*e^2)*a^2*x^2*e^3*log(-(2
*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*
sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - (c^2*
d^4*x^2 - a*c*d^2*x^2*e^2 - 2*a^2*x^2*e^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*
x^2 + a)) + (2*a*c*d^3*x*e - a*c*d^4 + 2*a^2*d*x*e^3 - a^2*d^2*e^2)*sqrt(c*
x^2 + a))/(a^2*c*d^5*x^2 + a^3*d^3*x^2*e^2), -1/2*(2*sqrt(-c*d^2 - a*e^2)*a
^2*x^2*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*
x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^3 + (c^2*d^4*x^2 - a*c*d^2*x^2*e^2
- 2*a^2*x^2*e^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (2*a*c*d^3*x*e
 - a*c*d^4 + 2*a^2*d*x*e^3 - a^2*d^2*e^2)*sqrt(c*x^2 + a))/(a^2*c*d^5*x^2 +
 a^3*d^3*x^2*e^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [**A**]
time = 1.98, size = 239, normalized size = 1.42

$$-c^{\frac{3}{2}}\left(\frac{2\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^3}{\sqrt{-cd^2-ae^2}\,c^{\frac{3}{2}}d^3}+\frac{(cd^2-2\,ae^2)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ac^{\frac{3}{2}}d^3}-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^3\sqrt{c}\,d-2\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2ae+\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)a\sqrt{c}\,d+2\,a^2e}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2-a\right)^2acd^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -c^(3/2)*(2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d
^2 - a*e^2))*e^3/(sqrt(-c*d^2 - a*e^2)*c^(3/2)*d^3) + (c*d^2 - 2*a*e^2)*arc
tan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^(3/2)*d^3) - ((s
qrt(c)*x - sqrt(c*x^2 + a))^3*sqrt(c)*d - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*sqrt(c)*d + 2*a^2*e)/(((sqrt(c)*x -
sqrt(c*x^2 + a))^2 - a)^2*a*c*d^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.334 $\int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{a(ae+cdx)}{c^2\,(cd^2+ae^2)\,\sqrt{a+cx^2}}+\frac{\sqrt{a+cx^2}}{c^2e}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{e^2\,(cd^2+ae^2)^{3/2}}$$

```
[Out] -d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^2-d^4*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^2/(a*e^2+c*d^2)^(3/2)+a*(c*d*x+a*e)/
c^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)+(c*x^2+a)^(1/2)/c^2/e
```

Rubi [A]
time = 0.20, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1661, 1668, 858, 223, 212, 739}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2}+\frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}\,(ae^2+cd^2)}+\frac{\sqrt{a+cx^2}}{c^2e}-\frac{d^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^2\,(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a*e + c*d*x))/(c^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + Sqrt[a + c*x^2]/(
c^2*e) - (d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^2) - (d^4*ArcT
anh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e
^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{\frac{a^2d^2}{cd^2+ae^2}-ax^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{\int \frac{\frac{a^2cd^2e^2}{cd^2+ae^2}+acdex}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac^2e^2}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce^2} + \frac{d^4\int \frac{1}{(d+ex)\sqrt{a+\cdots}}}{e^2\left(cd^2+a\cdots\right)}$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce^2} - \cdots$$

$$= \frac{a(ae+cdx)}{c^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} - \frac{d^4\tanh^{-1}\cdots}{\cdots}$$

Mathematica [A]
time = 0.72, size = 198, normalized size = 1.36

$$\frac{2d^4\sqrt{-cd^2-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right) + \frac{(cd^2+ae^2)\left(2a^2e^3+c^2d^2ex^2+ace(d^2+dex+e^2x^2)+\sqrt{c}\,d(cd^2+ae^2)\sqrt{a+cx^2}\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)\right)}{c^2\sqrt{a+cx^2}}}{\left(cd^2e+ae^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (2*d^4*Sqrt[-(c*d^2) - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2]
)/Sqrt[-(c*d^2) - a*e^2]] + ((c*d^2 + a*e^2)*(2*a^2*e^3 + c^2*d^2*e*x^2 + a
*c*e*(d^2 + d*e*x + e^2*x^2) + Sqrt[c]*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]))/(c^2*Sqrt[a + c*x^2]))/(c*d^2*e + a*e^3
)^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(130) = 260.
time = 0.10, size = 438, normalized size = 3.00

method	result

risch	$\frac{\sqrt{c\,x^{2}+a}}{c^{2} e}-\frac{d \ln\left(x \sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{c^{\frac{3}{2}} e^{2}}+\frac{c\,d^{4} \ln\left(\frac{\frac{2 a\,e^{2}+2 c\,d^{2}}{e^{2}}-\frac{2 c d \left(x+\frac{d}{e}\right)}{e}+2 \sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\, \sqrt{c \left(x+\frac{d}{e}\right)^{2}-\frac{2 c d \left(x \right.}{e}}}{x+\frac{d}{e}}\right)}{e^{3} \left(\sqrt{-a c}\, e+c d\right) \left(\sqrt{-a c}\, e-c d\right) \sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}$
default	$\frac{\frac{x^{2}}{c \sqrt{c\,x^{2}+a}}+\frac{2 a}{c^{2} \sqrt{c\,x^{2}+a}}}{e}-\frac{d \left(-\frac{x}{c \sqrt{c\,x^{2}+a}}+\frac{\ln\left(x \sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{c^{\frac{3}{2}}}\right)}{e^{2}}-\frac{d^{2}}{e^{3} c \sqrt{c\,x^{2}+a}}-\frac{d^{3} x}{e^{4} a \sqrt{c\,x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(x^2/c/(c*x^2+a)^(1/2)+2*a/c^2/(c*x^2+a)^(1/2))-d/e^2*(-x/c/(c*x^2+a)^(
1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-d^2/e^3/c/(c*x^2+a)^(1/2)-d^3
/e^4*x/a/(c*x^2+a)^(1/2)+1/e^5*d^4*(1/(a*e^2+c*d^2)*e^2/(c*(x+d/e)^2-2*c*d/
e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*c*d*e/(a*e^2+c*d^2)*(2*c*(x+d/e)-2*c*d
/e)/(4*c*(a*e^2+c*d^2)/e^2-4*c^2*d^2/e^2)/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e
^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a
*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*
c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]

time = 0.34, size = 242, normalized size = 1.66

$$\frac{c d^{5} x}{\sqrt{c x^{2}+a}\, a c d^{2} e^{4}+\sqrt{c x^{2}+a}\, a^{2} e^{6}}+\frac{d^{4} \operatorname{arsinh}\left(\frac{c d x}{\sqrt{a c}\,|x e+d|}-\frac{a e}{\sqrt{a c}\,|x e+d|}\right) e^{(-5)}}{\left(c d^{2} e^{(-2)}+a\right)^{\frac{3}{2}}}-\frac{d^{3} x e^{(-4)}}{\sqrt{c x^{2}+a}\, a}+\frac{d^{4}}{\sqrt{c x^{2}+a}\, c d^{2} e^{3}+\sqrt{c x^{2}+a}\, a e^{5}}+\frac{x^{2} e^{(-1)}}{\sqrt{c x^{2}+a}\, c}+\frac{d x e^{(-2)}}{\sqrt{c x^{2}+a}\, c}-\frac{d \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right) e^{(-2)}}{c^{\frac{3}{2}}}-\frac{d^{2} e^{(-3)}}{\sqrt{c x^{2}+a}\, c}+\frac{2 a e^{(-1)}}{\sqrt{c x^{2}+a}\, c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d^5*x/(sqrt(c*x^2 + a)*a*c*d^2*e^4 + sqrt(c*x^2 + a)*a^2*e^6) + d^4*arcsi
nh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-5)/(c
*d^2*e^(-2) + a)^(3/2) - d^3*x*e^(-4)/(sqrt(c*x^2 + a)*a) + d^4/(sqrt(c*x^2
 + a)*c*d^2*e^3 + sqrt(c*x^2 + a)*a*e^5) + x^2*e^(-1)/(sqrt(c*x^2 + a)*c) +
 d*x*e^(-2)/(sqrt(c*x^2 + a)*c) - d*arcsinh(c*x/sqrt(a*c))*e^(-2)/c^(3/2) -
 d^2*e^(-3)/(sqrt(c*x^2 + a)*c) + 2*a*e^(-1)/(sqrt(c*x^2 + a)*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(127) = 254$.
time = 9.20, size = 1493, normalized size = 10.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^3*d^5*x^2 + a*c^2*d^5 + (a^2*c*d*x^2 + a^3*d)*e^4 + 2*(a*c^2*d^3*x
^2 + a^2*c*d^3)*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a
*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) +
(a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*(a*c^2*d^3*x*e^2 + a^
2*c*d*x*e^4 + (a^2*c*x^2 + 2*a^3)*e^5 + (2*a*c^2*d^2*x^2 + 3*a^2*c*d^2)*e^3
 + (c^3*d^4*x^2 + a*c^2*d^4)*e)*sqrt(c*x^2 + a))/((a^2*c^3*x^2 + a^3*c^2)*e
^6 + 2*(a*c^4*d^2*x^2 + a^2*c^3*d^2)*e^4 + (c^5*d^4*x^2 + a*c^4*d^4)*e^2),
1/2*(2*(c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 -
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a
^2)*e^2)) + (c^3*d^5*x^2 + a*c^2*d^5 + (a^2*c*d*x^2 + a^3*d)*e^4 + 2*(a*c^2
*d^3*x^2 + a^2*c*d^3)*e^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)
*x - a) + 2*(a*c^2*d^3*x*e^2 + a^2*c*d*x*e^4 + (a^2*c*x^2 + 2*a^3)*e^5 + (2
*a*c^2*d^2*x^2 + 3*a^2*c*d^2)*e^3 + (c^3*d^4*x^2 + a*c^2*d^4)*e)*sqrt(c*x^2
 + a))/((a^2*c^3*x^2 + a^3*c^2)*e^6 + 2*(a*c^4*d^2*x^2 + a^2*c^3*d^2)*e^4 +
 (c^5*d^4*x^2 + a*c^4*d^4)*e^2), 1/2*(2*(c^3*d^5*x^2 + a*c^2*d^5 + (a^2*c*d
*x^2 + a^3*d)*e^4 + 2*(a*c^2*d^3*x^2 + a^2*c*d^3)*e^2)*sqrt(-c)*arctan(sqrt
(-c)*x/sqrt(c*x^2 + a)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c*d^2 + a*e^2)*log
(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2
*(a*c^2*d^3*x*e^2 + a^2*c*d*x*e^4 + (a^2*c*x^2 + 2*a^3)*e^5 + (2*a*c^2*d^2*
x^2 + 3*a^2*c*d^2)*e^3 + (c^3*d^4*x^2 + a*c^2*d^4)*e)*sqrt(c*x^2 + a))/((a^
2*c^3*x^2 + a^3*c^2)*e^6 + 2*(a*c^4*d^2*x^2 + a^2*c^3*d^2)*e^4 + (c^5*d^4*x
^2 + a*c^4*d^4)*e^2), ((c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c*d^2 - a*e^2)*arcta
n(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^
2 + (a*c*x^2 + a^2)*e^2)) + (c^3*d^5*x^2 + a*c^2*d^5 + (a^2*c*d*x^2 + a^3*d
)*e^4 + 2*(a*c^2*d^3*x^2 + a^2*c*d^3)*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(
c*x^2 + a)) + (a*c^2*d^3*x*e^2 + a^2*c*d*x*e^4 + (a^2*c*x^2 + 2*a^3)*e^5 +
(2*a*c^2*d^2*x^2 + 3*a^2*c*d^2)*e^3 + (c^3*d^4*x^2 + a*c^2*d^4)*e)*sqrt(c*x
^2 + a))/((a^2*c^3*x^2 + a^3*c^2)*e^6 + 2*(a*c^4*d^2*x^2 + a^2*c^3*d^2)*e^4
 + (c^5*d^4*x^2 + a*c^4*d^4)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(127) = 254.
time = 1.05, size = 299, normalized size = 2.05

$$\frac{2\,d^4\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{\left(cd^2e^2+ae^4\right)\sqrt{-cd^2-ae^2}}+\frac{de^{(-2)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}+\frac{\left(\frac{\left(c^4d^4e^5+2\,ac^3d^2e^7+a^2c^2e^9\right)x}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}}+\frac{ac^3d^3e^6+a^2c^2de^8}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}}\right)x+\frac{ac^3d^4e^5+3\,a^2c^2d^2e^7+2\,a^3ce^9}{c^5d^4e^6+2\,ac^4d^2e^8+a^2c^3e^{10}}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2*e^2 + a*e^4)*sqrt(-c*d^2 - a*e^2)) + d*e^(-2)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2) + (((c^4*d^4*e^5 + 2*a*c^3*d^2*e^7 + a^2*c
^2*e^9)*x/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^10) + (a*c^3*d^3*e^6 +
 a^2*c^2*d*e^8)/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^10))*x + (a*c^3*
d^4*e^5 + 3*a^2*c^2*d^2*e^7 + 2*a^3*c*e^9)/(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 +
 a^2*c^3*e^10))/sqrt(c*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^4/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.335 $\int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e\left(cd^2+ae^2\right)^{3/2}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e+d^3*arctanh((-c*d*x+a*e)/(a*e^
2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e/(a*e^2+c*d^2)^(3/2)+a*(-e*x+d)/c/(a*e^2+c
*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 858, 223, 212, 739}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e}+\frac{a(d-ex)}{c\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{d^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]]/(c^(3/2)*e) + (d^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]
*Sqrt[a + c*x^2])])/(e*(c*d^2 + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{-\frac{a^2de}{cd^2+ae^2}-ax}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac} \\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{ce} - \frac{d^3\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{e\left(cd^2+ae^2\right)} \\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{ce} + \frac{d^3\mathrm{Subst}\left(\int \frac{\ldots}{c\ldots}\right.}{\ldots} \\
&= \frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e} + \frac{d^3\tanh^{-1}\left(\frac{ae-cd\ldots}{\sqrt{cd^2+ae^2}\ldots}\right)}{e\left(cd^2+ae^2\right)^{3/\ldots}}
\end{aligned}$$

Mathematica [A]
time = 0.57, size = 137, normalized size = 1.11

$$\frac{a(d-ex)}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{2d^3\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{e\left(-cd^2-ae^2\right)^{3/2}} - \frac{\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{c^{3/2}e}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e*x)*(a + c*x^2)^(3/2)),x]

[Out] (a*(d - e*x))/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*d^3*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(e*(-(c*d^2) - a*e^2)^(3/2)) - Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/(c^(3/2)*e)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(109) = 218$.
time = 0.07, size = 397, normalized size = 3.23

method	result
default	$\frac{-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}}{e}+\frac{d}{e^2 c\sqrt{c\,x^2+a}}+\frac{d^2 x}{e^3 a\sqrt{c\,x^2+a}}-\frac{d^3\left(\frac{e^2 \cdots}{\left(a\,e^2+c\,d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2 - \cdots}} \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/e*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+d/e^2/c/(c*x^2+a)^(1/2)+d^2/e^3*x/a/(c*x^2+a)^(1/2)-d^3/e^4*(1/(a*e^2+c*d^2)*e^2/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*c*d*e/(a*e^2+c*d^2)*(2*c*(x+d/e)-2*c*d/e)/(4*c*(a*e^2+c*d^2)/e^2-4*c^2*d^2/e^2)/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))

Maxima [A]
time = 0.33, size = 204, normalized size = 1.66

$$-\frac{cd^4x}{\sqrt{cx^2+a}\,acd^2e^3+\sqrt{cx^2+a}\,a^2e^5}-\frac{d^3\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-4)}}{\left(cd^2e^{(-2)}+a\right)^{\frac{3}{2}}}+\frac{d^2xe^{(-3)}}{\sqrt{cx^2+a}\,a}-\frac{d^3}{\sqrt{cx^2+a}\,cd^2e^2+\sqrt{cx^2+a}\,ae^4}-\frac{xe^{(-1)}}{\sqrt{cx^2+a}\,c}+\frac{\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-1)}}{c^{\frac{3}{2}}}+\frac{de^{(-2)}}{\sqrt{cx^2+a}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")

[Out] -c*d^4*x/(sqrt(c*x^2 + a)*a*c*d^2*e^3 + sqrt(c*x^2 + a)*a^2*e^5) - d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-4)/(c*d^2*e^(-2) + a)^(3/2) + d^2*x*e^(-3)/(sqrt(c*x^2 + a)*a) - d^3/(sqrt(c*x^2 + a)*c*d^2*e^2 + sqrt(c*x^2 + a)*a*e^4) - x*e^(-1)/(sqrt(c*x^2 + a)*c) + arcsinh(c*x/sqrt(a*c))*e^(-1)/c^(3/2) + d*e^(-2)/(sqrt(c*x^2 + a)*c)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(109) = 218$.

time = 9.96, size = 1304, normalized size = 10.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^3*d^4*x^2 + a*c^2*d^4 + (a^2*c*x^2 + a^3)*e^4 + 2*(a*c^2*d^2*x^2 +
 a^2*c*d^2)*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d
*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c
*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 2*(a*c^2*d^2*x*e^2 - a*c^2*
d^3*e + a^2*c*x*e^4 - a^2*c*d*e^3)*sqrt(c*x^2 + a))/((a^2*c^3*x^2 + a^3*c^2
)*e^5 + 2*(a*c^4*d^2*x^2 + a^2*c^3*d^2)*e^3 + (c^5*d^4*x^2 + a*c^4*d^4)*e),
 -1/2*(2*(c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2
 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 +
 a^2)*e^2)) - (c^3*d^4*x^2 + a*c^2*d^4 + (a^2*c*x^2 + a^3)*e^4 + 2*(a*c^2*d
^2*x^2 + a^2*c*d^2)*e^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
 - a) + 2*(a*c^2*d^2*x*e^2 - a*c^2*d^3*e + a^2*c*x*e^4 - a^2*c*d*e^3)*sqrt(
c*x^2 + a))/((a^2*c^3*x^2 + a^3*c^2)*e^5 + 2*(a*c^4*d^2*x^2 + a^2*c^3*d^2)*
e^3 + (c^5*d^4*x^2 + a*c^4*d^4)*e), -1/2*(2*(c^3*d^4*x^2 + a*c^2*d^4 + (a^2
*c*x^2 + a^3)*e^4 + 2*(a*c^2*d^2*x^2 + a^2*c*d^2)*e^2)*sqrt(-c)*arctan(sqrt
(-c)*x/sqrt(c*x^2 + a)) - (c^3*d^3*x^2 + a*c^2*d^3)*sqrt(c*d^2 + a*e^2)*log
(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2
*(a*c^2*d^2*x*e^2 - a*c^2*d^3*e + a^2*c*x*e^4 - a^2*c*d*e^3)*sqrt(c*x^2 + a
))/((a^2*c^3*x^2 + a^3*c^2)*e^5 + 2*(a*c^4*d^2*x^2 + a^2*c^3*d^2)*e^3 + (c^
5*d^4*x^2 + a*c^4*d^4)*e), -((c^3*d^3*x^2 + a*c^2*d^3)*sqrt(-c*d^2 - a*e^2)
*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 +
a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + (c^3*d^4*x^2 + a*c^2*d^4 + (a^2*c*x^2 + a
^3)*e^4 + 2*(a*c^2*d^2*x^2 + a^2*c*d^2)*e^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqr
t(c*x^2 + a)) + (a*c^2*d^2*x*e^2 - a*c^2*d^3*e + a^2*c*x*e^4 - a^2*c*d*e^3)
*sqrt(c*x^2 + a))/((a^2*c^3*x^2 + a^3*c^2)*e^5 + 2*(a*c^4*d^2*x^2 + a^2*c^3
*d^2)*e^3 + (c^5*d^4*x^2 + a*c^4*d^4)*e)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+cx^2)^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(109) = 218.
time = 0.86, size = 219, normalized size = 1.78

$$-\frac{2\,d^3\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2e+ae^3)\sqrt{-cd^2-ae^2}}-\frac{\frac{(ac^2d^2e^3+a^2ce^5)x}{c^4d^4e^2+2\,ac^3d^2e^4+a^2c^2e^6}-\frac{ac^2d^3e^2+a^2cde^4}{c^4d^4e^2+2\,ac^3d^2e^4+a^2c^2e^6}}{\sqrt{cx^2+a}}-\frac{e^{(-1)}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")

[Out] -2*d^3*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/((c*d^2*e + a*e^3)*sqrt(-c*d^2 - a*e^2)) - ((a*c^2*d^2*e^3 + a^2*c*e^5)*x/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6) - (a*c^2*d^3*e^2 + a^2*c*d*e^4)/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6))/sqrt(c*x^2 + a) - e^(-1)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(c\,x^2+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x)),x)

[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x)), x)

3.336 $\int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{ae+cdx}{c\,(cd^2+ae^2)\,\sqrt{a+cx^2}}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

[Out] -d^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)^(3/2)+(-c*d*x-a*e)/c/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1661, 12, 739, 212}

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}\,(ae^2+cd^2)}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]

[Out] -((a*e + c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])) - (d^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{acd^2}{(cd^2+ae^2)(d+ex)\sqrt{a+cx^2}}\,dx}{ac}\\
&= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{d^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{d^2\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= -\frac{ae+cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{d^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.44, size = 106, normalized size = 1.12

$$\frac{-ae-cdx}{c\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{2d^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*e) - c*d*x)/(c*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (2*d^2*ArcTan[(Sqrt[
c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^
2)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(88) = 176.
time = 0.07, size = 355, normalized size = 3.74

method	result
default	$-\frac{1}{ec\sqrt{c\,x^{2}+a}}-\frac{dx}{e^{2}a\sqrt{c\,x^{2}+a}}+\frac{d^{2}\left(\frac{e^{2}}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}+\frac{}{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{4c\left(a\right.}{}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/c/(c*x^2+a)^(1/2)-d/e^2*x/a/(c*x^2+a)^(1/2)+1/e^3*d^2*(1/(a*e^2+c*d^2)
*e^2/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*c*d*e/(a*e^2+c
*d^2)*(2*c*(x+d/e)-2*c*d/e)/(4*c*(a*e^2+c*d^2)/e^2-4*c^2*d^2/e^2)/(c*(x+d/e
)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^
2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

<u>Maxima</u> [**A**]
time = 0.31, size = 168, normalized size = 1.77

$$\frac{cd^{3}x}{\sqrt{cx^{2}+a}\,acd^{2}e^{2}+\sqrt{cx^{2}+a}\,a^{2}e^{4}}+\frac{d^{2}\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-3)}}{\left(cd^{2}e^{(-2)}+a\right)^{\frac{3}{2}}}-\frac{dxe^{(-2)}}{\sqrt{cx^{2}+a}\,a}+\frac{d^{2}}{\sqrt{cx^{2}+a}\,cd^{2}e+\sqrt{cx^{2}+a}\,ae^{3}}-\frac{e^{(-1)}}{\sqrt{cx^{2}+a}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d^3*x/(sqrt(c*x^2 + a)*a*c*d^2*e^2 + sqrt(c*x^2 + a)*a^2*e^4) + d^2*arcsi
nh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-3)/(c
*d^2*e^(-2) + a)^(3/2) - d*x*e^(-2)/(sqrt(c*x^2 + a)*a) + d^2/(sqrt(c*x^2 +
 a)*c*d^2*e + sqrt(c*x^2 + a)*a*e^3) - e^(-1)/(sqrt(c*x^2 + a)*c)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(87) = 174.
time = 2.39, size = 445, normalized size = 4.68

$$\left[\frac{\left(c^{2}d^{2}x^{2}+acd^{2}\right)\sqrt{cd^{2}+ae^{2}}\log\left(-\frac{2\,c^{2}d^{2}x^{2}-2\,acdxe+acd^{2}+2\,\sqrt{cd^{2}+ae^{2}}\,(cdx-ae)\sqrt{cx^{2}+a}+(acx^{2}+2\,a^{2})e^{2}}{x^{2}e^{2}+2\,dxe+d^{2}}\right)-2\left(c^{2}d^{3}x+acdxe^{2}+acd^{2}e+a^{2}e^{3}\right)\sqrt{cx^{2}+a}}{2\left(c^{4}d^{4}x^{2}+ac^{3}d^{4}+(a^{2}c^{2}x^{2}+a^{3}c)e^{4}+2\,(ac^{3}d^{2}x^{2}+a^{2}c^{2}d^{2})e^{2}\right)},\ \frac{\left(c^{2}d^{2}x^{2}+acd^{2}\right)\sqrt{-cd^{2}-ae^{2}}\arctan\left(-\frac{\sqrt{-cd^{2}-ae^{2}}\,(cdx-ae)\sqrt{cx^{2}+a}}{c^{2}d^{2}x^{2}+acd^{2}+(acx^{2}+a^{2})e^{2}}\right)-\left(c^{2}d^{3}x+acdxe^{2}+acd^{2}e+a^{2}e^{3}\right)\sqrt{cx^{2}+a}}{c^{4}d^{4}x^{2}+ac^{3}d^{4}+(a^{2}c^{2}x^{2}+a^{3}c)e^{4}+2\,(ac^{3}d^{2}x^{2}+a^{2}c^{2}d^{2})e^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^2*x^2 + a*c*d^2)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a
*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) +
(a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 2*(c^2*d^3*x + a*c*d*x*
e^2 + a*c*d^2*e + a^2*e^3)*sqrt(c*x^2 + a))/(c^4*d^4*x^2 + a*c^3*d^4 + (a^2
*c^2*x^2 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^2 + a^2*c^2*d^2)*e^2), ((c^2*d^2*x^2
 + a*c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)
*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) - (c^2*d^3*
x + a*c*d*x*e^2 + a*c*d^2*e + a^2*e^3)*sqrt(c*x^2 + a))/(c^4*d^4*x^2 + a*c^
3*d^4 + (a^2*c^2*x^2 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^2 + a^2*c^2*d^2)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+cx^2)^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A]
time = 1.26, size = 174, normalized size = 1.83

$$-\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}}-\frac{\frac{(c^2d^3+acde^2)x}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}+\frac{acd^2e+a^2e^3}{c^3d^4+2\,ac^2d^2e^2+a^2ce^4}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*d^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a
*e^2))/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) - ((c^2*d^3 + a*c*d*e^2)*x/(c
^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4) + (a*c*d^2*e + a^2*e^3)/(c^3*d^4 + 2*
a*c^2*d^2*e^2 + a^2*c*e^4))/sqrt(c*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.337 $\int \frac{x}{(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{d-ex}{(cd^2+ae^2)\sqrt{a+cx^2}}+\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

```
[Out] d*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)
^(3/2)+(e*x-d)/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {837, 12, 739, 212}

$$\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}-\frac{d-ex}{\sqrt{a+cx^2}\,(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((d - e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2])) + (d*e*ArcTanh[(a*e - c*d*x)
/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{\int \frac{acde}{(d+ex)\sqrt{a+cx^2}}\,dx}{ac\left(cd^2+ae^2\right)}\\
&= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{(de)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{(de)\operatorname{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= -\frac{d-ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{de\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.40, size = 99, normalized size = 1.12

$$
\frac{-d+ex}{\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{2de\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-d + e*x)/((c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*d*e*ArcTan[(Sqrt[c]*(d +
e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(80) = 160.
time = 0.08, size = 335, normalized size = 3.81

method	result
default	$\frac{x}{ea\sqrt{c\,x^{2}+a}}-\frac{d\left(\frac{e^{2}}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}+\frac{2cde\left(2c\left(x+\frac{d}{e}\right.\right.}{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{4c\left(a\,e^{2}+c\,d^{2}\right)}{e^{2}}-\frac{4c^{2}d^{2}}{e^{2}}\right)\sqrt{c\left(x\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*x/a/(c*x^2+a)^(1/2)-d/e^2*(1/(a*e^2+c*d^2)*e^2/(c*(x+d/e)^2-2*c*d/e*(x+
d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+2*c*d*e/(a*e^2+c*d^2)*(2*c*(x+d/e)-2*c*d/e)/(
4*c*(a*e^2+c*d^2)/e^2-4*c^2*d^2/e^2)/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*
d^2)/e^2)^(1/2)-1/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+
c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e
*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]
time = 0.30, size = 147, normalized size = 1.67

$$-\frac{cd^{2}x}{\sqrt{cx^{2}+a}\,acd^{2}e+\sqrt{cx^{2}+a}\,a^{2}e^{3}}-\frac{d\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-2)}}{\left(cd^{2}e^{(-2)}+a\right)^{\frac{3}{2}}}+\frac{xe^{(-1)}}{\sqrt{cx^{2}+a}\,a}-\frac{d}{\sqrt{cx^{2}+a}\,cd^{2}+\sqrt{cx^{2}+a}\,ae^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -c*d^2*x/(sqrt(c*x^2 + a)*a*c*d^2*e + sqrt(c*x^2 + a)*a^2*e^3) - d*arcsinh(
c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-2)/(c*d^
2*e^(-2) + a)^(3/2) + x*e^(-1)/(sqrt(c*x^2 + a)*a) - d/(sqrt(c*x^2 + a)*c*d
^2 + sqrt(c*x^2 + a)*a*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(81) = 162.
time = 2.74, size = 416, normalized size = 4.73

$$\left[\frac{(cdx^{2}+ad)\sqrt{cd^{2}+ae^{2}}\,e\log\left(-\frac{2c^{2}d^{2}x^{2}-2acdxe+acd^{2}-2\sqrt{cd^{2}+ae^{2}}\,(cdx-ae)\sqrt{cx^{2}+a}+(acx^{2}+2a^{2})e^{2}}{x^{2}e^{2}+2dxe+d^{2}}\right)+2(cd^{2}xe-cd^{3}+axe^{3}-ade^{2})\sqrt{cx^{2}+a}}{2(c^{3}d^{4}x^{2}+ac^{2}d^{4}+(a^{2}cx^{2}+a^{3})e^{4}+2(ac^{2}d^{2}x^{2}+a^{2}cd^{2})e^{2})},-\frac{(cdx^{2}+ad)\sqrt{-cd^{2}-ae^{2}}\arctan\left(-\frac{\sqrt{-cd^{2}-ae^{2}}\,(cdx-ae)\sqrt{cx^{2}+a}}{c^{2}d^{2}x^{2}+acd^{2}+(acx^{2}+a^{2})e^{2}}\right)e-(cd^{2}xe-cd^{3}+axe^{3}-ade^{2})\sqrt{cx^{2}+a}}{c^{3}d^{4}x^{2}+ac^{2}d^{4}+(a^{2}cx^{2}+a^{3})e^{4}+2(ac^{2}d^{2}x^{2}+a^{2}cd^{2})e^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*x^2 + a*d)*sqrt(c*d^2 + a*e^2)*e*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x
*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x
^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*(c*d^2*x*e - c*d^3 + a*x*e^
3 - a*d*e^2)*sqrt(c*x^2 + a))/(c^3*d^4*x^2 + a*c^2*d^4 + (a^2*c*x^2 + a^3)*
e^4 + 2*(a*c^2*d^2*x^2 + a^2*c*d^2)*e^2), -((c*d*x^2 + a*d)*sqrt(-c*d^2 - a
*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x
^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e - (c*d^2*x*e - c*d^3 + a*x*e^3 - a*d
*e^2)*sqrt(c*x^2 + a))/(c^3*d^4*x^2 + a*c^2*d^4 + (a^2*c*x^2 + a^3)*e^4 + 2
*(a*c^2*d^2*x^2 + a^2*c*d^2)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+cx^2)^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A]
time = 0.79, size = 162, normalized size = 1.84

$$\frac{2\,d\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}}+\frac{\frac{(cd^2e+ae^3)x}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{cd^3+ade^2}{c^2d^4+2\,acd^2e^2+a^2e^4}}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*d*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^
2))*e/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2)) + ((c*d^2*e + a*e^3)*x/(c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4) - (c*d^3 + a*d*e^2)/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4))/sqrt(c*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(x/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.338 $\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}$$

```
[Out] -e^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+(c*d*x+a*e)/a/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {755, 12, 739, 212}

$$\frac{ae+cdx}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (e^2*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)\left(a+cx^2\right)^{3/2}}\,dx &= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{ae^2}{(d+ex)\sqrt{a+cx^2}}\,dx}{a\left(cd^2+ae^2\right)}\\
&= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{e^2\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}\\
&= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^2\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx,x,\frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}\\
&= \frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 104, normalized size = 1.11

$$\frac{ae+cdx}{a\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{2e^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*e + c*d*x)/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (2*e^2*ArcTan[(Sqrt[c]*
(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^
(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(86) = 172.
time = 0.07, size = 315, normalized size = 3.35

method	result

default	$\frac{\frac{e^2}{\left(a\,e^2+c\,d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}+\frac{2cde\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(a\,e^2+c\,d^2\right)\left(\frac{4c\left(a\,e^2+c\,d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}}}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/(a*e^2+c*d^2)*e^2/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1
/2)+2*c*d*e/(a*e^2+c*d^2)*(2*c*(x+d/e)-2*c*d/e)/(4*c*(a*e^2+c*d^2)/e^2-4*c^
2*d^2/e^2)/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+c
*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)
+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2
)^(1/2))/(x+d/e)))
```

Maxima [A]
time = 0.29, size = 123, normalized size = 1.31

$$\frac{cdx}{\sqrt{cx^2+a}\,acd^2+\sqrt{cx^2+a}\,a^2e^2}+\frac{\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-1)}}{\left(cd^2e^{(-2)}+a\right)^{\frac{3}{2}}}+\frac{1}{\sqrt{cx^2+a}\,cd^2e^{(-1)}+\sqrt{cx^2+a}\,ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*d*x/(sqrt(c*x^2 + a)*a*c*d^2 + sqrt(c*x^2 + a)*a^2*e^2) + arcsinh(c*d*x/(
sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-1)/(c*d^2*e^(-2
) + a)^(3/2) + 1/(sqrt(c*x^2 + a)*c*d^2*e^(-1) + sqrt(c*x^2 + a)*a*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(85) = 170.
time = 3.02, size = 432, normalized size = 4.60

$$\left[\frac{(acx^2+a^2)\sqrt{cd^2+ae^2}\,e^2\log\left(-\frac{2\,c^2d^2x^2-2\,acdxe+acd^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2\,a^2)e^2}{x^2e^2+2\,dxe+d^2}\right)+2\,(c^2d^3x+acdxe^2+acd^2e+a^2e^3)\sqrt{cx^2+a}}{2\,(ac^3d^4x^2+a^2c^2d^4+(a^3cx^2+a^4)e^4+2\,(a^2c^2d^2x^2+a^3cd^2)e^2)},\ \frac{(acx^2+a^2)\sqrt{-cd^2-ae^2}\,\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)e^2+(c^2d^3x+acdxe^2+acd^2e+a^2e^3)\sqrt{cx^2+a}}{ac^3d^4x^2+a^2c^2d^4+(a^3cx^2+a^4)e^4+2\,(a^2c^2d^2x^2+a^3cd^2)e^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*x^2 + a^2)*sqrt(c*d^2 + a*e^2)*e^2*log(-(2*c^2*d^2*x^2 - 2*a*c*d
*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c
*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*(c^2*d^3*x + a*c*d*x*e^2
```

```
+ a*c*d^2*e + a^2*e^3)*sqrt(c*x^2 + a))/(a*c^3*d^4*x^2 + a^2*c^2*d^4 + (a^3
*c*x^2 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^2 + a^3*c*d^2)*e^2), ((a*c*x^2 + a^2)*
sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
+ a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^2 + (c^2*d^3*x + a*c*
d*x*e^2 + a*c*d^2*e + a^2*e^3)*sqrt(c*x^2 + a))/(a*c^3*d^4*x^2 + a^2*c^2*d^
4 + (a^3*c*x^2 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^2 + a^3*c*d^2)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(85) = 170.
time = 1.18, size = 172, normalized size = 1.83

$$\frac{\frac{(c^2d^3+acde^2)x}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}+\frac{acd^2e+a^2e^3}{ac^2d^4+2\,a^2cd^2e^2+a^3e^4}}{\sqrt{cx^2+a}}-\frac{2\,\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^2}{\left(cd^2+ae^2\right)\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^2*d^3 + a*c*d*e^2)*x/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4) + (a*c*d^2
*e + a^2*e^3)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))/sqrt(c*x^2 + a) - 2*
arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*
e^2/((c*d^2 + a*e^2)*sqrt(-c*d^2 - a*e^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.339 $\int \frac{1}{x(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{ad\sqrt{a+cx^2}}-\frac{e(ae+cdx)}{ad\left(cd^2+ae^2\right)\sqrt{a+cx^2}}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c*d^
2)^(3/2)-arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d+1/a/d/(c*x^2+a)^(1/2)-e
*(c*d*x+a*e)/a/d/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {975, 272, 53, 65, 214, 755, 12, 739, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{e(ae+cdx)}{ad\sqrt{a+cx^2}\left(ae^2+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}+\frac{1}{ad\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (e*(a*e + c*d*x))/(a*d*(c*d^2 + a*e^2)*Sqrt[a + c
*x^2]) + (e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])
/(d*(c*d^2 + a*e^2)^(3/2)) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)(a+cx^2)^{3/2}}\,dx = \int \left(\frac{1}{dx(a+cx^2)^{3/2}} - \frac{e}{d(d+ex)(a+cx^2)^{3/2}}\right)dx$$

$$= \frac{\int \frac{1}{x(a+cx^2)^{3/2}}\,dx}{d} - \frac{e\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx}{d}$$

$$= -\frac{e(ae+cdx)}{ad(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx, x, x^2\right)}{2d} - \frac{e\int \frac{ae^2}{(d+ex)\sqrt{a-}}}{ad(cd^2+}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{e(ae+cdx)}{ad(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2ad}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{e(ae+cdx)}{ad(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx}\right.}{acd}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{e(ae+cdx)}{ad(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+c}}\right.}{d(cd^2+ae^2)^{3/2}}$$

Mathematica [A]
time = 0.75, size = 144, normalized size = 0.98

$$\frac{c(d-ex)}{a(cd^2+ae^2)\sqrt{a+cx^2}} - \frac{2e^3\tan^{-1}\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{d(-cd^2-ae^2)^{3/2}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (c*(d - e*x))/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) - (2*e^3*ArcTan[(Sqrt[c]*
(d + e*x) - e*Sqrt[a + c*x^2]])/Sqrt[-(c*d^2) - a*e^2]])/(d*(-(c*d^2) - a*e^
2)^(3/2)) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(a^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(131) = 262.
time = 0.06, size = 363, normalized size = 2.47

method	result

default	$-\frac{\frac{e^2}{\left(a e^2+c d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2+c d^2}{e^2}}}+\frac{2cde\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(a e^2+c d^2\right)\left(\frac{4c\left(a e^2+c d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/(a*e^2+c*d^2)*e^2/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2)+2*c*d*e/(a*e^2+c*d^2)*(2*c*(x+d/e)-2*c*d/e)/(4*c*(a*e^2+c*d^2)/e^2-4*c
^2*d^2/e^2)/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2+
c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e
)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^
2)^(1/2))/(x+d/e)))+1/d*(1/a/(c*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(c
*x^2+a)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(x*e + d)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(131) = 262.
time = 2.93, size = 1284, normalized size = 8.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*x^2 + a^3)*sqrt(c*d^2 + a*e^2)*e^3*log(-(2*c^2*d^2*x^2 - 2*a*c
*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a
*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + (c^3*d^4*x^2 + a*c^2*d^4
+ (a^2*c*x^2 + a^3)*e^4 + 2*(a*c^2*d^2*x^2 + a^2*c*d^2)*e^2)*sqrt(a)*log(-(
c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c^2*d^3*x*e - a*c^2*d^
4 + a^2*c*d*x*e^3 - a^2*c*d^2*e^2)*sqrt(c*x^2 + a))/(a^2*c^3*d^5*x^2 + a^3*
```

```
c^2*d^5 + (a^4*c*d*x^2 + a^5*d)*e^4 + 2*(a^3*c^2*d^3*x^2 + a^4*c*d^3)*e^2),
 -1/2*(2*(a^2*c*x^2 + a^3)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2
)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^
2))*e^3 - (c^3*d^4*x^2 + a*c^2*d^4 + (a^2*c*x^2 + a^3)*e^4 + 2*(a*c^2*d^2*x
^2 + a^2*c*d^2)*e^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 2*(a*c^2*d^3*x*e - a*c^2*d^4 + a^2*c*d*x*e^3 - a^2*c*d^2*e^2)*sqrt(
c*x^2 + a))/(a^2*c^3*d^5*x^2 + a^3*c^2*d^5 + (a^4*c*d*x^2 + a^5*d)*e^4 + 2*
(a^3*c^2*d^3*x^2 + a^4*c*d^3)*e^2), 1/2*((a^2*c*x^2 + a^3)*sqrt(c*d^2 + a*e
^2)*e^3*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e
+ d^2)) + 2*(c^3*d^4*x^2 + a*c^2*d^4 + (a^2*c*x^2 + a^3)*e^4 + 2*(a*c^2*d^2
*x^2 + a^2*c*d^2)*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - 2*(a*c^2
*d^3*x*e - a*c^2*d^4 + a^2*c*d*x*e^3 - a^2*c*d^2*e^2)*sqrt(c*x^2 + a))/(a^2
*c^3*d^5*x^2 + a^3*c^2*d^5 + (a^4*c*d*x^2 + a^5*d)*e^4 + 2*(a^3*c^2*d^3*x^2
 + a^4*c*d^3)*e^2), -((a^2*c*x^2 + a^3)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-
c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*
x^2 + a^2)*e^2))*e^3 - (c^3*d^4*x^2 + a*c^2*d^4 + (a^2*c*x^2 + a^3)*e^4 + 2
*(a*c^2*d^2*x^2 + a^2*c*d^2)*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a))
 + (a*c^2*d^3*x*e - a*c^2*d^4 + a^2*c*d*x*e^3 - a^2*c*d^2*e^2)*sqrt(c*x^2 +
 a))/(a^2*c^3*d^5*x^2 + a^3*c^2*d^5 + (a^4*c*d*x^2 + a^5*d)*e^4 + 2*(a^3*c^
2*d^3*x^2 + a^4*c*d^3)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(3/2),x)
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)),x)
[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.340 $\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}}$$

```
[Out] -e^4*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c
*d^2)^(3/2)+e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-e/a/d^2/(c*x^2+a
)^(1/2)-1/a/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)+e^2*(c*d*x+a*e)
/a/d^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {975, 277, 197, 272, 53, 65, 214, 755, 12, 739, 212}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{e^4\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*Sqrt[a + c*x^2
]) - (e^4*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^
2*(c*d^2 + a*e^2)^(3/2)) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
```

```
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^{3/2}}\,dx = \int \left(\frac{1}{dx^2\left(a+cx^2\right)^{3/2}} - \frac{e}{d^2x\left(a+cx^2\right)^{3/2}} + \frac{e^2}{d^2(d+ex)\left(a+cx^2\right)^{3/2}}\right)dx$$

$$= \frac{\int \frac{1}{x^2(a+cx^2)^{3/2}}\,dx}{d} - \frac{e\int \frac{1}{x(a+cx^2)^{3/2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{(d+ex)(a+cx^2)^{3/2}}\,dx}{d^2}$$

$$= -\frac{1}{adx\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+cx^2}} - \frac{(2c)\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{ad} - \frac{e\mathrm{Sul}}{}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+}}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+}}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\left(cd^2+ae^2\right)\sqrt{a+}}$$

Mathematica [A]
time = 0.71, size = 178, normalized size = 0.92

$$\frac{-\frac{d\left(a^2e^2+2c^2d^2x^2+ac\left(d^2+dex+e^2x^2\right)\right)}{a^2\left(cd^2+ae^2\right)x\sqrt{a+cx^2}} + \frac{2e^4\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}} - \frac{2e\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((d*(a^2*e^2 + 2*c^2*d^2*x^2 + a*c*(d^2 + d*e*x + e^2*x^2)))/(a^2*(c*d^2
+ a*e^2)*x*Sqrt[a + c*x^2])) + (2*e^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
+ c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) - (2*e*ArcTanh[
(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/a^(3/2))/d^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(174) = 348$.
time = 0.10, size = 403, normalized size = 2.08

method	result
risch	$-\frac{\sqrt{c x^2+a}}{a^2 d x}+\frac{c e^3 \ln\left(\frac{\frac{2a e^2+2c d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^2\left(\sqrt{-ac}\,e+cd\right)\left(\sqrt{-ac}\,e-cd\right)\sqrt{\frac{a e^2+c d^2}{e^2}}}+\frac{e\ln\left(2 \cdots\right.}{\cdots}$
default	$\frac{e\left(\frac{e^2}{\left(a e^2+c d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2+c d^2}{e^2}}}+\frac{2cde\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(a e^2+c d^2\right)\left(\frac{4c\left(a e^2+c d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}\cdots}}\cdots\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e/d^2*(1/(a*e^2+c*d^2)*e^2/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^
(1/2)+2*c*d*e/(a*e^2+c*d^2)*(2*c*(x+d/e)-2*c*d/e)/(4*c*(a*e^2+c*d^2)/e^2-4*
c^2*d^2/e^2)/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*e^2
+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/
e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e
^2)^(1/2))/(x+d/e)))+1/d*(-1/a/x/(c*x^2+a)^(1/2)-2*c/a^2*x/(c*x^2+a)^(1/2))
-e/d^2*(1/a/(c*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(x*e + d)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(174) = 348$.
time = 3.82, size = 1527, normalized size = 7.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*x^3 + a^3*x)*sqrt(c*d^2 + a*e^2)*e^4*log(-(2*c^2*d^2*x^2 - 2*a
*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) +
(a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + ((a^2*c*x^3 + a^3*x)*e^
5 + 2*(a*c^2*d^2*x^3 + a^2*c*d^2*x)*e^3 + (c^3*d^4*x^3 + a*c^2*d^4*x)*e)*sq
rt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*c^3*d^5*x^
2 + a*c^2*d^4*x*e + a*c^2*d^5 + a^2*c*d^2*x*e^3 + (a^2*c*d*x^2 + a^3*d)*e^4
 + (3*a*c^2*d^3*x^2 + 2*a^2*c*d^3)*e^2)*sqrt(c*x^2 + a))/(a^2*c^3*d^6*x^3 +
 a^3*c^2*d^6*x + (a^4*c*d^2*x^3 + a^5*d^2*x)*e^4 + 2*(a^3*c^2*d^4*x^3 + a^4
*c*d^4*x)*e^2), 1/2*(2*(a^2*c*x^3 + a^3*x)*sqrt(-c*d^2 - a*e^2)*arctan(-sqr
t(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a
*c*x^2 + a^2)*e^2))*e^4 + ((a^2*c*x^3 + a^3*x)*e^5 + 2*(a*c^2*d^2*x^3 + a^2
*c*d^2*x)*e^3 + (c^3*d^4*x^3 + a*c^2*d^4*x)*e)*sqrt(a)*log(-(c*x^2 + 2*sqrt
(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*c^3*d^5*x^2 + a*c^2*d^4*x*e + a*c^2*
d^5 + a^2*c*d^2*x*e^3 + (a^2*c*d*x^2 + a^3*d)*e^4 + (3*a*c^2*d^3*x^2 + 2*a^
2*c*d^3)*e^2)*sqrt(c*x^2 + a))/(a^2*c^3*d^6*x^3 + a^3*c^2*d^6*x + (a^4*c*d^
2*x^3 + a^5*d^2*x)*e^4 + 2*(a^3*c^2*d^4*x^3 + a^4*c*d^4*x)*e^2), 1/2*((a^2*
c*x^3 + a^3*x)*sqrt(c*d^2 + a*e^2)*e^4*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e +
a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 +
2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 2*((a^2*c*x^3 + a^3*x)*e^5 + 2*(a*
c^2*d^2*x^3 + a^2*c*d^2*x)*e^3 + (c^3*d^4*x^3 + a*c^2*d^4*x)*e)*sqrt(-a)*ar
ctan(sqrt(-a)/sqrt(c*x^2 + a)) - 2*(2*c^3*d^5*x^2 + a*c^2*d^4*x*e + a*c^2*d
^5 + a^2*c*d^2*x*e^3 + (a^2*c*d*x^2 + a^3*d)*e^4 + (3*a*c^2*d^3*x^2 + 2*a^2
*c*d^3)*e^2)*sqrt(c*x^2 + a))/(a^2*c^3*d^6*x^3 + a^3*c^2*d^6*x + (a^4*c*d^2
*x^3 + a^5*d^2*x)*e^4 + 2*(a^3*c^2*d^4*x^3 + a^4*c*d^4*x)*e^2), ((a^2*c*x^3
 + a^3*x)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*s
qrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^4 - ((a^2*c
*x^3 + a^3*x)*e^5 + 2*(a*c^2*d^2*x^3 + a^2*c*d^2*x)*e^3 + (c^3*d^4*x^3 + a*
c^2*d^4*x)*e)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (2*c^3*d^5*x^2 +
a*c^2*d^4*x*e + a*c^2*d^5 + a^2*c*d^2*x*e^3 + (a^2*c*d*x^2 + a^3*d)*e^4 + (
3*a*c^2*d^3*x^2 + 2*a^2*c*d^3)*e^2)*sqrt(c*x^2 + a))/(a^2*c^3*d^6*x^3 + a^3
*c^2*d^6*x + (a^4*c*d^2*x^3 + a^5*d^2*x)*e^4 + 2*(a^3*c^2*d^4*x^3 + a^4*c*d
^4*x)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A]
time = 1.69, size = 266, normalized size = 1.37

$$-\frac{\frac{\left(ac^3d^3+a^2c^2de^2\right)x}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}+\frac{a^2c^2d^2e+a^3ce^3}{a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4}}{\sqrt{cx^2+a}}-\frac{2\,\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2-ae^2}}\right)e^4}{\left(cd^4+ad^2e^2\right)\sqrt{-cd^2-ae^2}}-\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,ad^2}+\frac{2\,\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)^2-a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a*c^3*d^3 + a^2*c^2*d*e^2)*x/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4) +
 (a^2*c^2*d^2*e + a^3*c*e^3)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))/sqr
t(c*x^2 + a) - 2*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(
-c*d^2 - a*e^2))*e^4/((c*d^4 + a*d^2*e^2)*sqrt(-c*d^2 - a*e^2)) - 2*arctan(
-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*a*d^2) + 2*sqrt(c)/(((
sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)*a*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(c\,x^2+a\right)}^{3/2}\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.341 $\int \frac{1}{x^3(d+ex)(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=276

$$-\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}}-\frac{1}{2adx^2\sqrt{a+cx^2}}+\frac{e}{ad^2x\sqrt{a+cx^2}}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{e^3(ae+cdx)}{ad^3\left(cd^2+ae^2\right)\sqrt{a\cdots}}$$

```
[Out] e^5*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^3/(a*e^2+c*
d^2)^(3/2)+3/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(5/2)/d-e^2*arctanh((c*
x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^3-3/2*c/a^2/d/(c*x^2+a)^(1/2)+e^2/a/d^3/(c*
x^2+a)^(1/2)-1/2/a/d/x^2/(c*x^2+a)^(1/2)+e/a/d^2/x/(c*x^2+a)^(1/2)+2*c*e*x/
a^2/d^2/(c*x^2+a)^(1/2)-e^3*(c*d*x+a*e)/a/d^3/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {975, 272, 44, 53, 65, 214, 277, 197, 755, 12, 739, 212}

$$-\frac{e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{2cex}{a^2d^2\sqrt{a+cx^2}}-\frac{3c}{2a^2d\sqrt{a+cx^2}}+\frac{e^2}{ad^3\sqrt{a+cx^2}}+\frac{e}{ad^2x\sqrt{a+cx^2}}+\frac{e^5\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\left(ae^2+cd^2\right)^{3/2}}-\frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{1}{2adx^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-3*c)/(2*a^2*d*Sqrt[a + c*x^2]) + e^2/(a*d^3*Sqrt[a + c*x^2]) - 1/(2*a*d*x
^2*Sqrt[a + c*x^2]) + e/(a*d^2*x*Sqrt[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*Sqrt
[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a*d^3*(c*d^2 + a*e^2)*Sqrt[a + c*x^2])
+ (e^5*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^3*(
c*d^2 + a*e^2)^(3/2)) + (3*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(5/2)*d
) - (e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)(a+cx^2)^{3/2}}\,dx = \int \left(\frac{1}{dx^3(a+cx^2)^{3/2}} - \frac{e}{d^2x^2(a+cx^2)^{3/2}} + \frac{e^2}{d^3x(a+cx^2)^{3/2}} - \frac{e^3}{d^3(d+ex)(a}\right.$$

$$= \frac{\int \frac{1}{x^3(a+cx^2)^{3/2}}\,dx}{d} - \frac{e\int \frac{1}{x^2(a+cx^2)^{3/2}}\,dx}{d^2} + \frac{e^2\int \frac{1}{x(a+cx^2)^{3/2}}\,dx}{d^3} - \frac{e^3\int \frac{1}{(d+ex)(a+cx^2)}}{d^3}$$

$$= \frac{e}{ad^2x\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3(cd^2+ae^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x^2(a+cx)^{3/2}}\,dx, x, x^2\right.}{2d}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{}{ad^3}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{}{ad^3}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{}{ad^3}$$

$$= \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{}{ad^3}$$

Mathematica [A]
time = 0.81, size = 217, normalized size = 0.79

$$-\frac{\frac{d(c^2d^2x^2(3d-4ex)+a^2e^2(d-2ex)+ac(d^3-2d^2ex+de^2x^2-2e^3x^3))}{a^2(cd^2+ae^2)x^2\sqrt{a+cx^2}} + \frac{4e^5\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} + \frac{2(3cd^2-2ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{5/2}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -1/2*((d*(c^2*d^2*x^2*(3*d - 4*e*x) + a^2*e^2*(d - 2*e*x) + a*c*(d^3 - 2*d^
2*e*x + d*e^2*x^2 - 2*e^3*x^3)))/(a^2*(c*d^2 + a*e^2)*x^2*Sqrt[a + c*x^2])
+ (4*e^5*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e
^2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*(3*c*d^2 - 2*a*e^2)*ArcTanh[(Sqrt[c]*x
- Sqrt[a + c*x^2])/Sqrt[a]])/a^(5/2))/d^3
```

Maple [A]
time = 0.10, size = 479, normalized size = 1.74

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-2ex+d)}{2a^2d^2x^2}-\frac{c\,e^4\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^3\left(\sqrt{-ac}\,e+cd\right)\left(\sqrt{-ac}\,e-cd\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$
default	$-\frac{e^2\left(\frac{e^2}{\left(a\,e^2+c\,d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}+\frac{2cde\left(2c\left(x+\frac{d}{e}\right)-\frac{2cd}{e}\right)}{\left(a\,e^2+c\,d^2\right)\left(\frac{4c\left(a\,e^2+c\,d^2\right)}{e^2}-\frac{4c^2d^2}{e^2}\right)\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd(}{}}}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/d^3*(1/(a*e^2+c*d^2)*e^2/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^
2)^(1/2)+2*c*d*e/(a*e^2+c*d^2)*(2*c*(x+d/e)-2*c*d/e)/(4*c*(a*e^2+c*d^2)/e^2
-4*c^2*d^2/e^2)/(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-1/(a*
e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x
+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2
)/e^2)^(1/2))/(x+d/e)))+1/d*(-1/2/a/x^2/(c*x^2+a)^(1/2)-3/2*c/a*(1/a/(c*x^2
+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)))-e/d^2*(-1/a/x/(
c*x^2+a)^(1/2)-2*c/a^2*x/(c*x^2+a)^(1/2))+e^2/d^3*(1/a/(c*x^2+a)^(1/2)-1/a^
(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(x*e + d)*x^3), x)
```

Fricas [A]
time = 3.32, size = 1961, normalized size = 7.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/4*(2*(a^3*c*x^4 + a^4*x^2)*sqrt(c*d^2 + a*e^2)*e^5*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - (3*c^4*d^6*x^4 + 3*a*c^3*d^6*x^2 - 2*(a^3*c*x^4 + a^4*x^2)*e^6 - (a^2*c^2*d^2*x^4 + a^3*c*d^2*x^2)*e^4 + 4*(a*c^3*d^4*x^4 + a^2*c^2*d^4*x^2)*e^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(3*a*c^3*d^6*x^2 + a^2*c^2*d^6 - 2*(a^3*c*d*x^3 + a^4*d*x)*e^5 + (a^3*c*d^2*x^2 + a^4*d^2)*e^4 - 2*(3*a^2*c^2*d^3*x^3 + 2*a^3*c*d^3*x)*e^3 + 2*(2*a^2*c^2*d^4*x^2 + a^3*c*d^4)*e^2 - 2*(2*a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e)*sqrt(c*x^2 + a))/(a^3*c^3*d^7*x^4 + a^4*c^2*d^7*x^2 + (a^5*c*d^3*x^4 + a^6*d^3*x^2)*e^4 + 2*(a^4*c^2*d^5*x^4 + a^5*c*d^5*x^2)*e^2), -1/4*(4*(a^3*c*x^4 + a^4*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^5 + (3*c^4*d^6*x^4 + 3*a*c^3*d^6*x^2 - 2*(a^3*c*x^4 + a^4*x^2)*e^6 - (a^2*c^2*d^2*x^4 + a^3*c*d^2*x^2)*e^4 + 4*(a*c^3*d^4*x^4 + a^2*c^2*d^4*x^2)*e^2)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*a*c^3*d^6*x^2 + a^2*c^2*d^6 - 2*(a^3*c*d*x^3 + a^4*d*x)*e^5 + (a^3*c*d^2*x^2 + a^4*d^2)*e^4 - 2*(3*a^2*c^2*d^3*x^3 + 2*a^3*c*d^3*x)*e^3 + 2*(2*a^2*c^2*d^4*x^2 + a^3*c*d^4)*e^2 - 2*(2*a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e)*sqrt(c*x^2 + a))/(a^3*c^3*d^7*x^4 + a^4*c^2*d^7*x^2 + (a^5*c*d^3*x^4 + a^6*d^3*x^2)*e^4 + 2*(a^4*c^2*d^5*x^4 + a^5*c*d^5*x^2)*e^2), 1/2*((a^3*c*x^4 + a^4*x^2)*sqrt(c*d^2 + a*e^2)*e^5*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - (3*c^4*d^6*x^4 + 3*a*c^3*d^6*x^2 - 2*(a^3*c*x^4 + a^4*x^2)*e^6 - (a^2*c^2*d^2*x^4 + a^3*c*d^2*x^2)*e^4 + 4*(a*c^3*d^4*x^4 + a^2*c^2*d^4*x^2)*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (3*a*c^3*d^6*x^2 + a^2*c^2*d^6 - 2*(a^3*c*d*x^3 + a^4*d*x)*e^5 + (a^3*c*d^2*x^2 + a^4*d^2)*e^4 - 2*(3*a^2*c^2*d^3*x^3 + 2*a^3*c*d^3*x)*e^3 + 2*(2*a^2*c^2*d^4*x^2 + a^3*c*d^4)*e^2 - 2*(2*a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e)*sqrt(c*x^2 + a))/(a^3*c^3*d^7*x^4 + a^4*c^2*d^7*x^2 + (a^5*c*d^3*x^4 + a^6*d^3*x^2)*e^4 + 2*(a^4*c^2*d^5*x^4 + a^5*c*d^5*x^2)*e^2), -1/2*(2*(a^3*c*x^4 + a^4*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2))*e^5 + (3*c^4*d^6*x^4 + 3*a*c^3*d^6*x^2 - 2*(a^3*c*x^4 + a^4*x^2)*e^6 - (a^2*c^2*d^2*x^4 + a^3*c*d^2*x^2)*e^4 + 4*(a*c^3*d^4*x^4 + a^2*c^2*d^4*x^2)*e^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (3*a*c^3*d^6*x^2 + a^2*c^2*d^6 - 2*(a^3*c*d*x^3 + a^4*d*x)*e^5 + (a^3*c*d^2*x^2 + a^4*d^2)*e^4 - 2*(3*a^2*c^2*d^3*x^3 + 2*a^3*c*d^3*x)*e^3 + 2*(2*a^2*c^2*d^4*x^2 + a^3*c*d^4)*e^2 - 2*(2*a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e)*sqrt(c*x^2 + a))/(a^3*c^3*d^7*x^4 + a^4*c^2*d^7*x^2 + (a^5*c*d^3*x^4 + a^6*d^3*x^2)*e^4 + 2*(a^4*c^2*d^5*x^4 + a^5*c*d^5*x^2)*e^2)]

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + cx^2)^{\frac{3}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

Giac [A]
time = 1.78, size = 358, normalized size = 1.30

$$\frac{\frac{(a^2c^3d^2e+a^3c^2e^3)x}{a^4c^2d^4+2a^5cd^2e^2+a^6e^4} - \frac{a^2c^3d^3+a^3c^2de^2}{a^4c^2d^4+2a^5cd^2e^2+a^6e^4}}{\sqrt{cx^2+a}} - \frac{2\arctan\left(-\frac{(\sqrt{c}x-\sqrt{cx^2+a})e+\sqrt{c}d}{\sqrt{-cd^2-ae^2}}\right)e^5}{(cd^5+ad^3e^2)\sqrt{-cd^2-ae^2}} - \frac{(3cd^2-2ae^2)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2d^3} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3cd - 2\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2a\sqrt{c}e + \left(\sqrt{c}x-\sqrt{cx^2+a}\right)acd + 2a^2\sqrt{c}e}{\left(\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^2-a\right)^2a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^2*c^3*d^2*e + a^3*c^2*e^3)*x/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4)
- (a^2*c^3*d^3 + a^3*c^2*d*e^2)/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 + a^6*e^4))/
sqrt(c*x^2 + a) - 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/s
qrt(-c*d^2 - a*e^2))*e^5/((c*d^5 + a*d^3*e^2)*sqrt(-c*d^2 - a*e^2)) - (3*c*
d^2 - 2*a*e^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^
2*d^3) + ((sqrt(c)*x - sqrt(c*x^2 + a))^3*c*d - 2*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a*sqrt(c)*e + (sqrt(c)*x - sqrt(c*x^2 + a))*a*c*d + 2*a^2*sqrt(c)*e)
/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^2*a^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \left(c\,x^2 + a\right)^{3/2} \left(d + e\,x\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(3/2)*(d + e*x)), x)
```

3.342 $\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=244

$$\frac{(13cd^2-2ae^2)\sqrt{a+cx^2}}{3c^2e^4}+\frac{d^5\sqrt{a+cx^2}}{e^4(cd^2+ae^2)(d+ex)}-\frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4}+\frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4}-\frac{d(4cd^2-a$$

```
[Out] -d*(-a*e^2+4*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^5-d^4*(5*a
*e^2+4*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^5
/(a*e^2+c*d^2)^(3/2)+1/3*(-2*a*e^2+13*c*d^2)*(c*x^2+a)^(1/2)/c^2/e^4+d^5*(c
*x^2+a)^(1/2)/e^4/(a*e^2+c*d^2)/(e*x+d)-5/3*d*(e*x+d)*(c*x^2+a)^(1/2)/c/e^4
+1/3*(e*x+d)^2*(c*x^2+a)^(1/2)/c/e^4
```

Rubi [A]
time = 0.56, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1665, 1668, 858, 223, 212, 739}

$$-\frac{d(4cd^2-ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5}+\frac{\sqrt{a+cx^2}\,(13cd^2-2ae^2)}{3c^2e^4}+\frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)(ae^2+cd^2)}-\frac{d^4(5ae^2+4cd^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5(ae^2+cd^2)^{3/2}}-\frac{5d\sqrt{a+cx^2}\,(d+ex)}{3ce^4}+\frac{\sqrt{a+cx^2}\,(d+ex)^2}{3ce^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((13*c*d^2 - 2*a*e^2)*Sqrt[a + c*x^2])/(3*c^2*e^4) + (d^5*Sqrt[a + c*x^2])/
(e^4*(c*d^2 + a*e^2)*(d + e*x)) - (5*d*(d + e*x)*Sqrt[a + c*x^2])/(3*c*e^4)
 + ((d + e*x)^2*Sqrt[a + c*x^2])/(3*c*e^4) - (d*(4*c*d^2 - a*e^2)*ArcTanh[(
Sqrt[c]*x)/Sqrt[a + c*x^2]])/(c^(3/2)*e^5) - (d^4*(4*c*d^2 + 5*a*e^2)*ArcTa
nh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^5*(c*d^2 + a*e^
2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{-\frac{ad^4}{e^3}+\frac{d^3\left(cd^2+ae^2\right)x}{e^4}-\frac{d^2\left(cd^2+ae^2\right)x^2}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^3-\frac{\left(cd^2+ae^2\right)x^4}{e}}{(d+ex)\sqrt{a+cx^2}} \cdots}{cd^2+ae^2} \\
&= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{\int \frac{-ad^2e\left(cd^2-2ae^2\right)+4d\left(cd^2+ae^2\right)^2x+\cdots}{(d+ex)\sqrt{\cdots}}}{3ce^4\left(cd\cdots\right.} \\
&= \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \frac{(d+ex)^2\sqrt{a+cx^2}}{3ce^4} - \frac{\int \frac{-6a\cdots}{\cdots}}{\cdots} \\
&= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \cdots \\
&= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \cdots \\
&= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \cdots \\
&= \frac{\left(13cd^2-2ae^2\right)\sqrt{a+cx^2}}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4\left(cd^2+ae^2\right)(d+ex)} - \frac{5d(d+ex)\sqrt{a+cx^2}}{3ce^4} + \cdots
\end{aligned}
$$

Mathematica [**A**]
time = 1.24, size = 251, normalized size = 1.03

$$
\frac{\frac{e\sqrt{a+cx^2}\left(-2a^2e^4(d+ex)+ace^2\left(7d^3+4d^2ex-2de^2x^2+e^3x^3\right)+c^2d^2\left(12d^3+6d^2ex-2de^2x^2+e^3x^3\right)\right)}{c^2\left(cd^2+ae^2\right)(d+ex)}+\frac{6d^4\left(4cd^2+5ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}+\frac{3\left(4cd^3-ade^2\right)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{c^{3/2}}}{3e^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(-2*a^2*e^4*(d + e*x) + a*c*e^2*(7*d^3 + 4*d^2*e*x - 2*
d*e^2*x^2 + e^3*x^3) + c^2*d^2*(12*d^3 + 6*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)
))/(c^2*(c*d^2 + a*e^2)*(d + e*x)) + (6*d^4*(4*c*d^2 + 5*a*e^2)*ArcTan[(Sqr
t[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*
e^2)^(3/2) + (3*(4*c*d^3 - a*d*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/c^
(3/2))/(3*e^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(218) = 436.
time = 0.10, size = 477, normalized size = 1.95

method	result
risch	$-\frac{\left(-c\,e^{2} x^{2}+3 c d e x+2 a\,e^{2}-9 c\,d^{2}\right) \sqrt{c\,x^{2}+a}}{3 c^{2} e^{4}}+\frac{d \ln\left(x \sqrt{c}+\sqrt{c\,x^{2}+a}\right) a}{c^{\frac{3}{2}} e^{3}}-\frac{4 d^{3} \ln\left(x \sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{e^{5} \sqrt{c}}-\frac{5 d^{4} \mathrm{l}\ldots}{\ldots}$
default	$\frac{\frac{x^{2} \sqrt{c\,x^{2}+a}}{3 c}-\frac{2 a \sqrt{c\,x^{2}+a}}{3 c^{2}}}{e^{2}}-\frac{2 d \left(\frac{x \sqrt{c\,x^{2}+a}}{2 c}-\frac{a \ln\left(x \sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2 c^{\frac{3}{2}}}\right)}{e^{3}}+\frac{3 d^{2} \sqrt{c\,x^{2}+a}}{e^{4} c}-\frac{4 d^{3} \ln\left(\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/3*x^2/c*(c*x^2+a)^(1/2)-2/3*a/c^2*(c*x^2+a)^(1/2))-2*d/e^3*(1/2*x/
c*(c*x^2+a)^(1/2)-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+3*d^2/e^4/c*
(c*x^2+a)^(1/2)-4*d^3/e^5*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-5/e^6*d^4/(
(a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+
c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x
+d/e))-d^5/e^7*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(
a*e^2+c*d^2)/e^2)^(1/2)-c*d*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2
*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2
-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]

time = 0.33, size = 266, normalized size = 1.09

$$-\frac{c d^{6} \operatorname{arsinh}\left(\frac{c d x}{\sqrt{a c} |x e+d|}-\frac{a e}{\sqrt{a c} |x e+d|}\right) e^{(-8)}}{\left(c d^{2} e^{(-2)}+a\right)^{\frac{3}{2}}}+\frac{5 d^{4} \operatorname{arsinh}\left(\frac{c d x}{\sqrt{a c} |x e+d|}-\frac{a e}{\sqrt{a c} |x e+d|}\right) e^{(-6)}}{\sqrt{c d^{2} e^{(-2)}+a}}+\frac{\sqrt{c x^{2}+a} d^{5}}{c d^{2} x e^{5}+c d^{3} e^{4}+a x e^{7}+a d e^{6}}-\frac{4 d^{3} \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right) e^{(-5)}}{\sqrt{c}}+\frac{\sqrt{c x^{2}+a} x^{2} e^{(-2)}}{3 c}-\frac{\sqrt{c x^{2}+a} d x e^{(-3)}}{c}+\frac{a d \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right) e^{(-3)}}{c^{\frac{3}{2}}}+\frac{3 \sqrt{c x^{2}+a} d^{2} e^{(-4)}}{c}-\frac{2 \sqrt{c x^{2}+a} a e^{(-2)}}{3 c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -c*d^6*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)
))*e^(-8)/(c*d^2*e^(-2) + a)^(3/2) + 5*d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e
 + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-6)/sqrt(c*d^2*e^(-2) + a) + sqrt
(c*x^2 + a)*d^5/(c*d^2*x*e^5 + c*d^3*e^4 + a*x*e^7 + a*d*e^6) - 4*d^3*arcsi
nh(c*x/sqrt(a*c))*e^(-5)/sqrt(c) + 1/3*sqrt(c*x^2 + a)*x^2*e^(-2)/c - sqrt(
```

```
c*x^2 + a)*d*x*e^(-3)/c + a*d*arcsinh(c*x/sqrt(a*c))*e^(-3)/c^(3/2) + 3*sqr
t(c*x^2 + a)*d^2*e^(-4)/c - 2/3*sqrt(c*x^2 + a)*a*e^(-2)/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(211) = 422$.
time = 69.12, size = 1960, normalized size = 8.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(4*c^3*d^7*x*e + 4*c^3*d^8 + 7*a*c^2*d^5*x*e^3 + 7*a*c^2*d^6*e^2 +
2*a^2*c*d^3*x*e^5 + 2*a^2*c*d^4*e^4 - a^3*d*x*e^7 - a^3*d^2*e^6)*sqrt(c)*lo
g(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 3*(4*c^3*d^6*x*e + 4*c^3*d^
7 + 5*a*c^2*d^4*x*e^3 + 5*a*c^2*d^5*e^2)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^
2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*(6*c^3*d^6
*x*e^2 + 12*c^3*d^7*e + (a^2*c*x^3 - 2*a^3*x)*e^8 - 2*(a^2*c*d*x^2 + a^3*d)
*e^7 + 2*(a*c^2*d^2*x^3 + a^2*c*d^2*x)*e^6 - (4*a*c^2*d^3*x^2 - 5*a^2*c*d^3
)*e^5 + (c^3*d^4*x^3 + 10*a*c^2*d^4*x)*e^4 - (2*c^3*d^5*x^2 - 19*a*c^2*d^5)
*e^3)*sqrt(c*x^2 + a))/(c^4*d^4*x*e^6 + c^4*d^5*e^5 + 2*a*c^3*d^2*x*e^8 + 2
*a*c^3*d^3*e^7 + a^2*c^2*x*e^10 + a^2*c^2*d*e^9), 1/6*(6*(4*c^3*d^6*x*e + 4
*c^3*d^7 + 5*a*c^2*d^4*x*e^3 + 5*a*c^2*d^5*e^2)*sqrt(-c*d^2 - a*e^2)*arctan
(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2
 + (a*c*x^2 + a^2)*e^2)) + 3*(4*c^3*d^7*x*e + 4*c^3*d^8 + 7*a*c^2*d^5*x*e^3
 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^3*x*e^5 + 2*a^2*c*d^4*e^4 - a^3*d*x*e^7 - a^
3*d^2*e^6)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c
^3*d^6*x*e^2 + 12*c^3*d^7*e + (a^2*c*x^3 - 2*a^3*x)*e^8 - 2*(a^2*c*d*x^2 +
a^3*d)*e^7 + 2*(a*c^2*d^2*x^3 + a^2*c*d^2*x)*e^6 - (4*a*c^2*d^3*x^2 - 5*a^2
*c*d^3)*e^5 + (c^3*d^4*x^3 + 10*a*c^2*d^4*x)*e^4 - (2*c^3*d^5*x^2 - 19*a*c^
2*d^5)*e^3)*sqrt(c*x^2 + a))/(c^4*d^4*x*e^6 + c^4*d^5*e^5 + 2*a*c^3*d^2*x*e
^8 + 2*a*c^3*d^3*e^7 + a^2*c^2*x*e^10 + a^2*c^2*d*e^9), 1/6*(6*(4*c^3*d^7*x
*e + 4*c^3*d^8 + 7*a*c^2*d^5*x*e^3 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^3*x*e^5 +
2*a^2*c*d^4*e^4 - a^3*d*x*e^7 - a^3*d^2*e^6)*sqrt(-c)*arctan(sqrt(-c)*x/sqr
t(c*x^2 + a)) + 3*(4*c^3*d^6*x*e + 4*c^3*d^7 + 5*a*c^2*d^4*x*e^3 + 5*a*c^2*
d^5*e^2)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 +
2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2
)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*(6*c^3*d^6*x*e^2 + 12*c^3*d^7*e + (a^2*c*x
^3 - 2*a^3*x)*e^8 - 2*(a^2*c*d*x^2 + a^3*d)*e^7 + 2*(a*c^2*d^2*x^3 + a^2*c*
d^2*x)*e^6 - (4*a*c^2*d^3*x^2 - 5*a^2*c*d^3)*e^5 + (c^3*d^4*x^3 + 10*a*c^2*
d^4*x)*e^4 - (2*c^3*d^5*x^2 - 19*a*c^2*d^5)*e^3)*sqrt(c*x^2 + a))/(c^4*d^4*
x*e^6 + c^4*d^5*e^5 + 2*a*c^3*d^2*x*e^8 + 2*a*c^3*d^3*e^7 + a^2*c^2*x*e^10
+ a^2*c^2*d*e^9), 1/3*(3*(4*c^3*d^6*x*e + 4*c^3*d^7 + 5*a*c^2*d^4*x*e^3 + 5
*a*c^2*d^5*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x -
```

```
a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + 3*(4*
c^3*d^7*x*e + 4*c^3*d^8 + 7*a*c^2*d^5*x*e^3 + 7*a*c^2*d^6*e^2 + 2*a^2*c*d^3
*x*e^5 + 2*a^2*c*d^4*e^4 - a^3*d*x*e^7 - a^3*d^2*e^6)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (6*c^3*d^6*x*e^2 + 12*c^3*d^7*e + (a^2*c*x^3 - 2*a
^3*x)*e^8 - 2*(a^2*c*d*x^2 + a^3*d)*e^7 + 2*(a*c^2*d^2*x^3 + a^2*c*d^2*x)*e
^6 - (4*a*c^2*d^3*x^2 - 5*a^2*c*d^3)*e^5 + (c^3*d^4*x^3 + 10*a*c^2*d^4*x)*e
^4 - (2*c^3*d^5*x^2 - 19*a*c^2*d^5)*e^3)*sqrt(c*x^2 + a))/(c^4*d^4*x*e^6 +
c^4*d^5*e^5 + 2*a*c^3*d^2*x*e^8 + 2*a*c^3*d^3*e^7 + a^2*c^2*x*e^10 + a^2*c^
2*d*e^9)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)**2/(c*x**2+a)**(1/2),x)
[Out] Integral(x**5/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(x^5/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.343 $\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=204

$$-\frac{5d\sqrt{a+cx^2}}{2ce^3}-\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)}+\frac{(d+ex)\sqrt{a+cx^2}}{2ce^3}+\frac{\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}+\frac{d^3(3cd^2 \cdots}{}$$

```
[Out] 1/2*(-a*e^2+6*c*d^2)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)/e^4+d^3*(4*
a*e^2+3*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/e^
4/(a*e^2+c*d^2)^(3/2)-5/2*d*(c*x^2+a)^(1/2)/c/e^3-d^4*(c*x^2+a)^(1/2)/e^3/(
a*e^2+c*d^2)/(e*x+d)+1/2*(e*x+d)*(c*x^2+a)^(1/2)/c/e^3
```

Rubi [A]

time = 0.34, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1665, 1668, 858, 223, 212, 739}

$$\frac{\left(6cd^2-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4}-\frac{d^4\sqrt{a+cx^2}}{e^3(d+ex)\left(ae^2+cd^2\right)}+\frac{d^3\left(4ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{e^4\left(ae^2+cd^2\right)^{3/2}}-\frac{5d\sqrt{a+cx^2}}{2ce^3}+\frac{\sqrt{a+cx^2}\,(d+ex)}{2ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-5*d*Sqrt[a + c*x^2])/(2*c*e^3) - (d^4*Sqrt[a + c*x^2])/(e^3*(c*d^2 + a*e^
2)*(d + e*x)) + ((d + e*x)*Sqrt[a + c*x^2])/(2*c*e^3) + ((6*c*d^2 - a*e^2)*
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)*e^4) + (d^3*(3*c*d^2 + 4*a
*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^4*(c
*d^2 + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} - \frac{\int \frac{\frac{ad^3}{e^2}-\frac{d^2\left(cd^2+ae^2\right)x}{e^3}+d\left(a+\frac{cd^2}{e^2}\right)x^2-\frac{\left(cd^2+ae^2\right)x^3}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} - \frac{\int \frac{ade\left(3cd^2+ae^2\right)-\left(c^2d^4-a^2e^4\right)x+5cd\ldots}{(d+ex)\sqrt{a+cx^2}}}{2ce^3\left(cd^2+ae^2\right)\ldots}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} - \frac{\int \frac{acde^3\left(3cd^2+a\ldots\right.}{(d\ldots}}{2c\ldots}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\ldots}{2\ldots}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\ldots}{\ldots}$$

$$= -\frac{5d\sqrt{a+cx^2}}{2ce^3} - \frac{d^4\sqrt{a+cx^2}}{e^3\left(cd^2+ae^2\right)(d+ex)} + \frac{(d+ex)\sqrt{a+cx^2}}{2ce^3} + \frac{\left(6cd^2-ae^2\right)\ldots}{\ldots}$$

Mathematica [A]
time = 1.03, size = 210, normalized size = 1.03

$$\frac{\frac{e\sqrt{a+cx^2}\left(cd^2\left(-6d^2-3dex+e^2x^2\right)+ae^2\left(-4d^2-3dex+e^2x^2\right)\right)}{c\left(cd^2+ae^2\right)(d+ex)} - \frac{4d^3\left(3cd^2+4ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}} + \frac{\left(-6cd^2+ae^2\right)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{c^{3/2}}}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(c*d^2*(-6*d^2 - 3*d*e*x + e^2*x^2) + a*e^2*(-4*d^2 - 3
*d*e*x + e^2*x^2)))/(c*(c*d^2 + a*e^2)*(d + e*x)) - (4*d^3*(3*c*d^2 + 4*a*e
^2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])
/(-(c*d^2) - a*e^2)^(3/2) + ((-6*c*d^2 + a*e^2)*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2]])/c^(3/2))/(2*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(180) = 360.
time = 0.09, size = 435, normalized size = 2.13

method	result

risch	$-\frac{(-ex+4d)\sqrt{c\,x^2+a}}{2c\,e^3}-\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)a}{2e^2c^{\frac{3}{2}}}+\frac{3d^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^4\sqrt{c}}+\frac{4d^3\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd(x\cdots}{e}}{\cdots}\right)}{\cdots}$
default	$\frac{\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}}{e^2}-\frac{2d\sqrt{c\,x^2+a}}{c\,e^3}+\frac{3d^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^4\sqrt{c}}+\frac{4d^3\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}\cdots}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/2*x/c*(c*x^2+a)^(1/2)-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))
-2*d*(c*x^2+a)^(1/2)/c/e^3+3*d^2/e^4*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+
4/e^5*d^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)
+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2
)^(1/2))/(x+d/e))+1/e^6*d^4*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2-2*c*
d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c*d*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)
*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]

time = 0.32, size = 227, normalized size = 1.11

$$\frac{cd^5\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-7)}}{(cd^2e^{(-2)}+a)^{\frac{3}{2}}}-\frac{4\,d^3\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-5)}}{\sqrt{cd^2e^{(-2)}+a}}-\frac{\sqrt{cx^2+a}\,d^4}{cd^2xe^4+cd^3e^3+axe^6+ade^5}+\frac{3\,d^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-4)}}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,xe^{(-2)}}{2\,c}-\frac{a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-2)}}{2\,c^{\frac{3}{2}}}-\frac{2\,\sqrt{cx^2+a}\,de^{(-3)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] c*d^5*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d))
)*e^(-7)/(c*d^2*e^(-2) + a)^(3/2) - 4*d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e
+ d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-5)/sqrt(c*d^2*e^(-2) + a) - sqrt(
c*x^2 + a)*d^4/(c*d^2*x*e^4 + c*d^3*e^3 + a*x*e^6 + a*d*e^5) + 3*d^2*arcsin
h(c*x/sqrt(a*c))*e^(-4)/sqrt(c) + 1/2*sqrt(c*x^2 + a)*x*e^(-2)/c - 1/2*a*ar
csinh(c*x/sqrt(a*c))*e^(-2)/c^(3/2) - 2*sqrt(c*x^2 + a)*d*e^(-3)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(174) = 348$.

time = 78.12, size = 1723, normalized size = 8.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((6*c^3*d^6*x*e + 6*c^3*d^7 + 11*a*c^2*d^4*x*e^3 + 11*a*c^2*d^5*e^2 +
 4*a^2*c*d^2*x*e^5 + 4*a^2*c*d^3*e^4 - a^3*x*e^7 - a^3*d*e^6)*sqrt(c)*log(-
2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(3*c^3*d^5*x*e + 3*c^3*d^6 +
 4*a*c^2*d^3*x*e^3 + 4*a*c^2*d^4*e^2)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x
^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2
 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*(3*c^3*d^5*x*
e^2 + 6*c^3*d^6*e + 6*a*c^2*d^3*x*e^4 - a^2*c*x^2*e^7 + 3*a^2*c*d*x*e^6 - 2
*(a*c^2*d^2*x^2 - 2*a^2*c*d^2)*e^5 - (c^3*d^4*x^2 - 10*a*c^2*d^4)*e^3)*sqrt
(c*x^2 + a))/(c^4*d^4*x*e^5 + c^4*d^5*e^4 + 2*a*c^3*d^2*x*e^7 + 2*a*c^3*d^3
*e^6 + a^2*c^2*x*e^9 + a^2*c^2*d*e^8), -1/4*(4*(3*c^3*d^5*x*e + 3*c^3*d^6 +
 4*a*c^2*d^3*x*e^3 + 4*a*c^2*d^4*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*
d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^
2 + a^2)*e^2)) + (6*c^3*d^6*x*e + 6*c^3*d^7 + 11*a*c^2*d^4*x*e^3 + 11*a*c^2
*d^5*e^2 + 4*a^2*c*d^2*x*e^5 + 4*a^2*c*d^3*e^4 - a^3*x*e^7 - a^3*d*e^6)*sqr
t(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(3*c^3*d^5*x*e^2 +
 6*c^3*d^6*e + 6*a*c^2*d^3*x*e^4 - a^2*c*x^2*e^7 + 3*a^2*c*d*x*e^6 - 2*(a*c
^2*d^2*x^2 - 2*a^2*c*d^2)*e^5 - (c^3*d^4*x^2 - 10*a*c^2*d^4)*e^3)*sqrt(c*x^
2 + a))/(c^4*d^4*x*e^5 + c^4*d^5*e^4 + 2*a*c^3*d^2*x*e^7 + 2*a*c^3*d^3*e^6
 + a^2*c^2*x*e^9 + a^2*c^2*d*e^8), -1/2*((6*c^3*d^6*x*e + 6*c^3*d^7 + 11*a*c
^2*d^4*x*e^3 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^2*x*e^5 + 4*a^2*c*d^3*e^4 - a^3
*x*e^7 - a^3*d*e^6)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (3*c^3*d^
5*x*e + 3*c^3*d^6 + 4*a*c^2*d^3*x*e^3 + 4*a*c^2*d^4*e^2)*sqrt(c*d^2 + a*e^2
)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*
x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)
) + (3*c^3*d^5*x*e^2 + 6*c^3*d^6*e + 6*a*c^2*d^3*x*e^4 - a^2*c*x^2*e^7 + 3*
a^2*c*d*x*e^6 - 2*(a*c^2*d^2*x^2 - 2*a^2*c*d^2)*e^5 - (c^3*d^4*x^2 - 10*a*c
^2*d^4)*e^3)*sqrt(c*x^2 + a))/(c^4*d^4*x*e^5 + c^4*d^5*e^4 + 2*a*c^3*d^2*x*
e^7 + 2*a*c^3*d^3*e^6 + a^2*c^2*x*e^9 + a^2*c^2*d*e^8), -1/2*(2*(3*c^3*d^5*
x*e + 3*c^3*d^6 + 4*a*c^2*d^3*x*e^3 + 4*a*c^2*d^4*e^2)*sqrt(-c*d^2 - a*e^2)
*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 +
a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + (6*c^3*d^6*x*e + 6*c^3*d^7 + 11*a*c^2*d^4
*x*e^3 + 11*a*c^2*d^5*e^2 + 4*a^2*c*d^2*x*e^5 + 4*a^2*c*d^3*e^4 - a^3*x*e^7
 - a^3*d*e^6)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (3*c^3*d^5*x*e^
2 + 6*c^3*d^6*e + 6*a*c^2*d^3*x*e^4 - a^2*c*x^2*e^7 + 3*a^2*c*d*x*e^6 - 2*(
a*c^2*d^2*x^2 - 2*a^2*c*d^2)*e^5 - (c^3*d^4*x^2 - 10*a*c^2*d^4)*e^3)*sqrt(c
```

```
*x^2 + a))/(c^4*d^4*x*e^5 + c^4*d^5*e^4 + 2*a*c^3*d^2*x*e^7 + 2*a*c^3*d^3*e
^6 + a^2*c^2*x*e^9 + a^2*c^2*d*e^8)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^4/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.344 $\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a+cx^2}}{ce^2}+\frac{d^3\sqrt{a+cx^2}}{e^2\left(cd^2+ae^2\right)(d+ex)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^3}-\frac{d^2\left(2cd^2+3ae^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^3\left(cd^2+ae^2\right)^{3/2}}$$

```
[Out] -d^2*(3*a*e^2+2*c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(
1/2))/e^3/(a*e^2+c*d^2)^(3/2)-2*d*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^3/c^
(1/2)+(c*x^2+a)^(1/2)/c/e^2+d^3*(c*x^2+a)^(1/2)/e^2/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]
time = 0.21, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1665, 1668, 858, 223, 212, 739}

$$-\frac{d^2\left(3ae^2+2cd^2\right)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\left(ae^2+cd^2\right)^{3/2}}+\frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)\left(ae^2+cd^2\right)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^3}+\frac{\sqrt{a+cx^2}}{ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*e^2) + (d^3*Sqrt[a + c*x^2])/(e^2*(c*d^2 + a*e^2)*(d + e
*x)) - (2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*e^3) - (d^2*(2*c
*d^2 + 3*a*e^2)*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])
])/(e^3*(c*d^2 + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{\int \frac{-\frac{ad^2}{e}+d\left(a+\frac{cd^2}{e^2}\right)x-\frac{(cd^2+ae^2)x^2}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{\int \frac{-acd^2e+2cd(cd^2+ae^2)x}{(d+ex)\sqrt{a+cx^2}}\,dx}{ce^2(cd^2+ae^2)}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{(2d)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^3} + \frac{(d^2(2cd^2+3ae^2}{e^3(}$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{(2d)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^3} -$$

$$= \frac{\sqrt{a+cx^2}}{ce^2} + \frac{d^3\sqrt{a+cx^2}}{e^2(cd^2+ae^2)(d+ex)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^3} - \frac{d^2(2cd^2+3}{}$$

Mathematica [A]
time = 0.86, size = 173, normalized size = 1.08

$$\frac{\frac{e\sqrt{a+cx^2}\,(ae^2(d+ex)+cd^2(2d+ex))}{c(cd^2+ae^2)(d+ex)} + \frac{2d^2(2cd^2+3ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} + \frac{2d\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((e*Sqrt[a + c*x^2]*(a*e^2*(d + e*x) + c*d^2*(2*d + e*x)))/(c*(c*d^2 + a*e^
2)*(d + e*x)) + (2*d^2*(2*c*d^2 + 3*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sq
rt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*d*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/e^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(144) = 288$.
time = 0.08, size = 390, normalized size = 2.44

method	result

risch	$\frac{\sqrt{c\,x^2+a}}{c\,e^2}-\frac{2d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^3\sqrt{c}}-\frac{3d^2\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd(}{}}}{x+\frac{d}{e}}\right)}{e^4\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$
default	$\frac{\sqrt{c\,x^2+a}}{c\,e^2}-\frac{2d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^3\sqrt{c}}-\frac{3d^2\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd(}{}}}{x+\frac{d}{e}}\right)}{e^4\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+a)^(1/2)/c/e^2-2*d/e^3*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-3/e^4*d
^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*
e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)
)/(x+d/e))-d^3/e^5*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2-2*c*d/e*(x+d/
e)+(a*e^2+c*d^2)/e^2)^(1/2)-c*d*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*l
n((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/
e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]
time = 0.31, size = 189, normalized size = 1.18

$$-\frac{cd^4\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-6)}}{(cd^2e^{(-2)}+a)^{\frac{3}{2}}}+\frac{3\,d^2\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-4)}}{\sqrt{cd^2e^{(-2)}+a}}+\frac{\sqrt{cx^2+a}\,d^3}{cd^2xe^3+cd^3e^2+axe^5+ade^4}-\frac{2\,d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-3)}}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,e^{(-2)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -c*d^4*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)
))*e^(-6)/(c*d^2*e^(-2) + a)^(3/2) + 3*d^2*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e
 + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-4)/sqrt(c*d^2*e^(-2) + a) + sqrt
(c*x^2 + a)*d^3/(c*d^2*x*e^3 + c*d^3*e^2 + a*x*e^5 + a*d*e^4) - 2*d*arcsinh
(c*x/sqrt(a*c))*e^(-3)/sqrt(c) + sqrt(c*x^2 + a)*e^(-2)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(139) = 278$.

time = 11.06, size = 1376, normalized size = 8.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(c^2*d^5*x*e + c^2*d^6 + 2*a*c*d^3*x*e^3 + 2*a*c*d^4*e^2 + a^2*d*x*
e^5 + a^2*d^2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)
+ (2*c^2*d^4*x*e + 2*c^2*d^5 + 3*a*c*d^2*x*e^3 + 3*a*c*d^3*e^2)*sqrt(c*d^2
+ a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2
)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e
 + d^2)) + 2*(c^2*d^4*x*e^2 + 2*c^2*d^5*e + 2*a*c*d^2*x*e^4 + 3*a*c*d^3*e^3
 + a^2*x*e^6 + a^2*d*e^5)*sqrt(c*x^2 + a))/(c^3*d^4*x*e^4 + c^3*d^5*e^3 + 2
*a*c^2*d^2*x*e^6 + 2*a*c^2*d^3*e^5 + a^2*c*x*e^8 + a^2*c*d*e^7), ((2*c^2*d^
4*x*e + 2*c^2*d^5 + 3*a*c*d^2*x*e^3 + 3*a*c*d^3*e^2)*sqrt(-c*d^2 - a*e^2)*a
rctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*
c*d^2 + (a*c*x^2 + a^2)*e^2)) + (c^2*d^5*x*e + c^2*d^6 + 2*a*c*d^3*x*e^3 +
2*a*c*d^4*e^2 + a^2*d*x*e^5 + a^2*d^2*e^4)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*
x^2 + a)*sqrt(c)*x - a) + (c^2*d^4*x*e^2 + 2*c^2*d^5*e + 2*a*c*d^2*x*e^4 +
3*a*c*d^3*e^3 + a^2*x*e^6 + a^2*d*e^5)*sqrt(c*x^2 + a))/(c^3*d^4*x*e^4 + c^
3*d^5*e^3 + 2*a*c^2*d^2*x*e^6 + 2*a*c^2*d^3*e^5 + a^2*c*x*e^8 + a^2*c*d*e^7
), 1/2*(4*(c^2*d^5*x*e + c^2*d^6 + 2*a*c*d^3*x*e^3 + 2*a*c*d^4*e^2 + a^2*d*
x*e^5 + a^2*d^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c^2*d
^4*x*e + 2*c^2*d^5 + 3*a*c*d^2*x*e^3 + 3*a*c*d^3*e^2)*sqrt(c*d^2 + a*e^2)*l
og(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x -
 a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) +
 2*(c^2*d^4*x*e^2 + 2*c^2*d^5*e + 2*a*c*d^2*x*e^4 + 3*a*c*d^3*e^3 + a^2*x*e
^6 + a^2*d*e^5)*sqrt(c*x^2 + a))/(c^3*d^4*x*e^4 + c^3*d^5*e^3 + 2*a*c^2*d^2
*x*e^6 + 2*a*c^2*d^3*e^5 + a^2*c*x*e^8 + a^2*c*d*e^7), ((2*c^2*d^4*x*e + 2*
c^2*d^5 + 3*a*c*d^2*x*e^3 + 3*a*c*d^3*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(-sqr
t(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a
*c*x^2 + a^2)*e^2)) + 2*(c^2*d^5*x*e + c^2*d^6 + 2*a*c*d^3*x*e^3 + 2*a*c*d^
4*e^2 + a^2*d*x*e^5 + a^2*d^2*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 +
a)) + (c^2*d^4*x*e^2 + 2*c^2*d^5*e + 2*a*c*d^2*x*e^4 + 3*a*c*d^3*e^3 + a^2*
x*e^6 + a^2*d*e^5)*sqrt(c*x^2 + a))/(c^3*d^4*x*e^4 + c^3*d^5*e^3 + 2*a*c^2*
d^2*x*e^6 + 2*a*c^2*d^3*e^5 + a^2*c*x*e^8 + a^2*c*d*e^7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + cx^2}\,(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.345 $\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{d^2\sqrt{a+cx^2}}{e\left(cd^2+ae^2\right)(d+ex)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}+\frac{d(cd^2+2ae^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{e^2\left(cd^2+ae^2\right)^{3/2}}$$

```
[Out] d*(2*a*e^2+c*d^2)*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))
/e^2/(a*e^2+c*d^2)^(3/2)+arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/e^2/c^(1/2)-d^2
*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1665, 858, 223, 212, 739}

$$-\frac{d^2\sqrt{a+cx^2}}{e(d+ex)\left(ae^2+cd^2\right)}+\frac{d(2ae^2+cd^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\left(ae^2+cd^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((d^2*Sqrt[a + c*x^2])/(e*(c*d^2 + a*e^2)*(d + e*x))) + ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]]/(Sqrt[c]*e^2) + (d*(c*d^2 + 2*a*e^2)*ArcTanh[(a*e - c*d*
x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(e^2*(c*d^2 + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{d^2\sqrt{a+cx^2}}{e\,(cd^2+ae^2)\,(d+ex)} - \frac{\int \frac{ad-\frac{(cd^2+ae^2)x}{e}}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\,(cd^2+ae^2)\,(d+ex)} + \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{e^2} - \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right)\int \frac{1}{(d+ex)\sqrt{a+cx}}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\,(cd^2+ae^2)\,(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\left(d\left(2a+\frac{cd^2}{e^2}\right)\right.}{}$$

$$= -\frac{d^2\sqrt{a+cx^2}}{e\,(cd^2+ae^2)\,(d+ex)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,e^2} + \frac{d\left(2a+\frac{cd^2}{e^2}\right)\tanh^{-1}\left(\frac{}{\sqrt{cd}}\right.}{(cd^2+ae}$$

Mathematica [A]
time = 0.69, size = 146, normalized size = 1.07

$$-\frac{\frac{d^2e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{2d(cd^2+2ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} + \frac{\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e*x)^2*Sqrt[a + c*x^2]),x]

[Out] -(((d^2*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) + (2*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) + Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]/Sqrt[c])/e^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(123) = 246$.
time = 0.07, size = 369, normalized size = 2.69

method	result
default	$\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{e^2\sqrt{c}}+\frac{2d\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^3\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/e^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+2*d/e^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))+1/e^4*d^2*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c*d*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))

Maxima [A]
time = 0.31, size = 170, normalized size = 1.24

$$\frac{cd^3\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-5)}}{\left(cd^2e^{(-2)}+a\right)^{\frac{3}{2}}}-\frac{2\,d\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-3)}}{\sqrt{cd^2e^{(-2)}+a}}-\frac{\sqrt{cx^2+a}\,d^2}{cd^2xe^2+cd^3e+axe^4+ade^3}+\frac{\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e^{(-2)}}{\sqrt{c}}$$

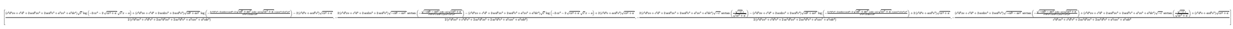
Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] c*d^3*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-5)/(c*d^2*e^(-2) + a)^(3/2) - 2*d*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-3)/sqrt(c*d^2*e^(-2) + a) - sqrt(c*

```
x^2 + a)*d^2/(c*d^2*x*e^2 + c*d^3*e + a*x*e^4 + a*d*e^3) + arcsinh(c*x/sqrt
(a*c))*e^(-2)/sqrt(c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(119) = 238$.
time = 10.63, size = 1208, normalized size = 8.82



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*e^3 + 2*a*c*d^3*e^2 + a^2*x*e^5
+ a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + (c^2
*d^3*x*e + c^2*d^4 + 2*a*c*d*x*e^3 + 2*a*c*d^2*e^2)*sqrt(c*d^2 + a*e^2)*log
(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a
*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 2
*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^4*x*e^3 + c^3*d^5*e^2 +
2*a*c^2*d^2*x*e^5 + 2*a*c^2*d^3*e^4 + a^2*c*x*e^7 + a^2*c*d*e^6), -1/2*(2*(
c^2*d^3*x*e + c^2*d^4 + 2*a*c*d*x*e^3 + 2*a*c*d^2*e^2)*sqrt(-c*d^2 - a*e^2)
*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 +
a*c*d^2 + (a*c*x^2 + a^2)*e^2)) - (c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*e^3
+ 2*a*c*d^3*e^2 + a^2*x*e^5 + a^2*d*e^4)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^
2 + a)*sqrt(c)*x - a) + 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d
^4*x*e^3 + c^3*d^5*e^2 + 2*a*c^2*d^2*x*e^5 + 2*a*c^2*d^3*e^4 + a^2*c*x*e^7
+ a^2*c*d*e^6), -1/2*(2*(c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*e^3 + 2*a*c*d^
3*e^2 + a^2*x*e^5 + a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a))
- (c^2*d^3*x*e + c^2*d^4 + 2*a*c*d*x*e^3 + 2*a*c*d^2*e^2)*sqrt(c*d^2 + a*e^
2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d
*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2
)) + 2*(c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^4*x*e^3 + c^3*d^5*
e^2 + 2*a*c^2*d^2*x*e^5 + 2*a*c^2*d^3*e^4 + a^2*c*x*e^7 + a^2*c*d*e^6), -((
c^2*d^3*x*e + c^2*d^4 + 2*a*c*d*x*e^3 + 2*a*c*d^2*e^2)*sqrt(-c*d^2 - a*e^2)
*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 +
a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + (c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*e^3
+ 2*a*c*d^3*e^2 + a^2*x*e^5 + a^2*d*e^4)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) + (c^2*d^4*e + a*c*d^2*e^3)*sqrt(c*x^2 + a))/(c^3*d^4*x*e^3 + c^3
*d^5*e^2 + 2*a*c^2*d^2*x*e^5 + 2*a*c^2*d^3*e^4 + a^2*c*x*e^7 + a^2*c*d*e^6)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + cx^2}\,(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.346 $\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)\,(d+ex)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\,\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

```
[Out] -a*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)+d*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]

time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {821, 739, 212}

$$\frac{d\sqrt{a+cx^2}}{(d+ex)\,(ae^2+cd^2)}-\frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\,\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (a*e*ArcTanh[(a*e - c*d*x
)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{(ae)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$

$$= \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{(ae)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$

$$= \frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{ae\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

Mathematica [A]
time = 0.44, size = 100, normalized size = 1.11

$$\frac{d\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{2ae\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)^2*Sqrt[a + c*x^2]),x]
[Out] (d*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) + (2*a*e*ArcTan[(Sqrt[c]*(d
 + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3
/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(82) = 164$.
time = 0.07, size = 344, normalized size = 3.82

method	result
default	$-\frac{\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}} - \frac{d\left(-\frac{e^2\sqrt{c\left(x+\frac{d}{e}\right)^2-\ldots}}{\left(a\,e^2+c\,\ldots\right.}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*
((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(
1/2))/(x+d/e))-d/e^3*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2-2*c*d/e*(x+
d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c*d*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)
*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+
d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]
time = 0.33, size = 149, normalized size = 1.66

$$-\frac{cd^2 \operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|} - \frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-4)}}{\left(cd^2e^{(-2)} + a\right)^{\frac{3}{2}}} + \frac{\operatorname{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|} - \frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-2)}}{\sqrt{cd^2e^{(-2)} + a}} + \frac{\sqrt{cx^2+a}\,d}{cd^2xe + cd^3 + axe^3 + ade^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -c*d^2*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)
))*e^(-4)/(c*d^2*e^(-2) + a)^(3/2) + arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d))
 - a*e/(sqrt(a*c)*abs(x*e + d)))*e^(-2)/sqrt(c*d^2*e^(-2) + a) + sqrt(c*x^2
 + a)*d/(c*d^2*x*e + c*d^3 + a*x*e^3 + a*d*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.
time = 1.85, size = 368, normalized size = 4.09

$$\left[\frac{\sqrt{cd^2+ae^2}\,(axe^2+ade)\log\left(-\frac{2\,c^2d^2x^2-2\,acdxe+acd^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2\,a^2)e^2}{x^2e^2+2\,dxe+d^2}\right)+2\,(cd^3+ade^2)\sqrt{cx^2+a}}{2\,(c^2d^4xe+c^2d^5+2\,acd^2xe^3+2\,acd^3e^2+a^2xe^5+a^2de^4)}, \frac{\sqrt{-cd^2-ae^2}\,(axe^2+ade)\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)+(cd^3+ade^2)\sqrt{cx^2+a}}{c^2d^4xe+c^2d^5+2\,acd^2xe^3+2\,acd^3e^2+a^2xe^5+a^2de^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 + a*e^2)*(a*x*e^2 + a*d*e)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x
*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x
^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + 2*(c*d^3 + a*d*e^2)*sqrt(c*x^
2 + a))/(c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*e^3 + 2*a*c*d^3*e^2 + a^2*x*e^
5 + a^2*d*e^4), (sqrt(-c*d^2 - a*e^2)*(a*x*e^2 + a*d*e)*arctan(-sqrt(-c*d^2
 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 +
 a^2)*e^2)) + (c*d^3 + a*d*e^2)*sqrt(c*x^2 + a))/(c^2*d^4*x*e + c^2*d^5 + 2
*a*c*d^2*x*e^3 + 2*a*c*d^3*e^2 + a^2*x*e^5 + a^2*d*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.347 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=91

$$-\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

```
[Out] -c*d*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2
)^(3/2)-e*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {745, 739, 212}

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)}-\frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (c*d*ArcTanh[(a*e - c*
d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{(cd)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{cd^2+ae^2}$$
$$= -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{(cd)\text{Subst}\left(\int \frac{1}{cd^2+ae^2-x^2}\,dx, x, \frac{ae-cdx}{\sqrt{a+cx^2}}\right)}{cd^2+ae^2}$$
$$= -\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{cd\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{(cd^2+ae^2)^{3/2}}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 1.11

$$-\frac{e\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{2cd\tan^{-1}\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + (2*c*d*ArcTan[(Sqrt[c]
*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)
^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(83) = 166.
time = 0.09, size = 215, normalized size = 2.36

method	result
default	$\frac{-\frac{e^2\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{\left(a\,e^2+c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{cde\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\right.}{e}}}{x+\frac{d}{e}}\right.}{\left(a\,e^2+c\,d^2\right)\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d
^2)/e^2)^(1/2)-c*d*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c
*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*
(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [A]
time = 0.31, size = 93, normalized size = 1.02

$$\frac{cd\,\mathrm{arsinh}\left(\frac{cdx}{\sqrt{ac}\,|xe+d|}-\frac{ae}{\sqrt{ac}\,|xe+d|}\right)e^{(-3)}}{\left(cd^2e^{(-2)}+a\right)^{\frac{3}{2}}}-\frac{\sqrt{cx^2+a}}{cd^3e^{(-1)}+cd^2x+axe^2+ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] c*d*arcsinh(c*d*x/(sqrt(a*c)*abs(x*e + d)) - a*e/(sqrt(a*c)*abs(x*e + d)))*
e^(-3)/(c*d^2*e^(-2) + a)^(3/2) - sqrt(c*x^2 + a)/(c*d^3*e^(-1) + c*d^2*x +
 a*x*e^2 + a*d*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(84) = 168.
time = 1.69, size = 373, normalized size = 4.10

$$\left[\frac{(cdxe+cd^2)\sqrt{cd^2+ae^2}\,\log\left(-\frac{2\,c^2d^2x^2-2\,acdxe+acd^2+2\,\sqrt{cd^2+ae^2}\,(cdx-ae)\sqrt{cx^2+a}+(acx^2+2\,a^2)e^2}{x^2e^2+2\,dxe+d^2}\right)-2\,(cd^2e+ae^3)\sqrt{cx^2+a}}{2\,(c^2d^4xe+c^2d^5+2\,acd^2xe^3+2\,acd^3e^2+a^2xe^5+a^2de^4)},\ \frac{(cdxe+cd^2)\sqrt{-cd^2-ae^2}\,\arctan\left(-\frac{\sqrt{-cd^2-ae^2}\,(cdx-ae)\sqrt{cx^2+a}}{c^2d^2x^2+acd^2+(acx^2+a^2)e^2}\right)-(cd^2e+ae^3)\sqrt{cx^2+a}}{c^2d^4xe+c^2d^5+2\,acd^2xe^3+2\,acd^3e^2+a^2xe^5+a^2de^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d*x*e + c*d^2)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x
*e + a*c*d^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x
^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) - 2*(c*d^2*e + a*e^3)*sqrt(c*x^
2 + a))/(c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*e^3 + 2*a*c*d^3*e^2 + a^2*x*e^
5 + a^2*d*e^4), ((c*d*x*e + c*d^2)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2
 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 +
 a^2)*e^2)) - (c*d^2*e + a*e^3)*sqrt(c*x^2 + a))/(c^2*d^4*x*e + c^2*d^5 + 2
*a*c*d^2*x*e^3 + 2*a*c*d^3*e^2 + a^2*x*e^5 + a^2*d*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2), x)

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^2)^(1/2)*(d + e*x)^2),x)

[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x)^2), x)

3.348 $\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=179

$$\frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}}-\frac{\tanh^{-1}}{}$$

```
[Out] c*e*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/(a*e^2+c*d^2)
^(3/2)-arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)+e*arctanh((-c*d*x+a*e)/
(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+c*d^2)^(1/2)+e^2*(c*x^2+a)^
(1/2)/d/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]
time = 0.10, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {975, 272, 65, 214, 745, 739, 212}

$$\frac{e^2\sqrt{a+cx^2}}{d(d+ex)\left(ae^2+cd^2\right)}+\frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}}+\frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\left(ae^2+cd^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e^2*Sqrt[a + c*x^2])/(d*(c*d^2 + a*e^2)*(d + e*x)) + (c*e*ArcTanh[(a*e - c
*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(c*d^2 + a*e^2)^(3/2) + (e*Ar
cTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*Sqrt[c*d^2
 + a*e^2]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{d^2x\sqrt{a+cx^2}} - \frac{e}{d(d+ex)^2\sqrt{a+cx^2}} - \frac{e}{d^2(d+ex)\sqrt{a+cx^2}}\right) dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2} - \frac{e\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^2} - \frac{e\int \frac{1}{(d+ex)^2\sqrt{a+cx^2}}\,dx}{d}$$

$$= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} + \frac{e\text{Subst}\left(\int \frac{1}{cd^2+ae^2}\right.}{}$$

$$= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{e\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\sqrt{cd^2+ae^2}} + \frac{\text{Subst}\left(\int \frac{}{-\frac{a}{c}}\right.}{}$$

$$= \frac{e^2\sqrt{a+cx^2}}{d\left(cd^2+ae^2\right)(d+ex)} + \frac{ce\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{\left(cd^2+ae^2\right)^{3/2}} + \frac{e\tanh^{-1}\left(\right.}{d}$$

Mathematica [A]
time = 0.70, size = 153, normalized size = 0.85

$$\frac{e\left(\frac{de\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{2\left(2cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{\left(-cd^2-ae^2\right)^{3/2}}\right) + \frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (e*((d*e*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)) - (2*(2*c*d^2 + a*e^2
)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(
-(c*d^2) - a*e^2)^(3/2)) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]
])/Sqrt[a])/d^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(159) = 318.
time = 0.07, size = 376, normalized size = 2.10

method	result

default	$\frac{\ln\left(\frac{\frac{2a\,e^2+2c\,d^2}{e^2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^2\sqrt{\frac{a\,e^2+c\,d^2}{e^2}}}-\frac{-\frac{e^2\sqrt{c\left(x+\frac{d}{e}\right)^2-\frac{2cd\left(x\cdots\right.}{e}}}{\left(a\,e^2+c\,d^2\right)\left(x+\cdots\right.}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d^2/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))-1/e/d*(-1/(a*e^2+c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c*d*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)))-1/d^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + a)*(x*e + d)^2*x), x)

Fricas [A]
time = 3.19, size = 1213, normalized size = 6.78

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*((2*a*c*d^2*x*e^2 + 2*a*c*d^3*e + a^2*x*e^4 + a^2*d*e^3)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)) + (c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*e^3 + 2*a*c*d^3*e^2 + a^2*x*e^5 + a^2*d*e^4)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^6*x*e + a*c^2*d^7

```
+ 2*a^2*c*d^4*x*e^3 + 2*a^2*c*d^5*e^2 + a^3*d^2*x*e^5 + a^3*d^3*e^4), -1/2*
(2*(2*a*c*d^2*x*e^2 + 2*a*c*d^3*e + a^2*x*e^4 + a^2*d*e^3)*sqrt(-c*d^2 - a*
e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^
2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) - (c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*
e^3 + 2*a*c*d^3*e^2 + a^2*x*e^5 + a^2*d*e^4)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c
*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a)
)/(a*c^2*d^6*x*e + a*c^2*d^7 + 2*a^2*c*d^4*x*e^3 + 2*a^2*c*d^5*e^2 + a^3*d^
2*x*e^5 + a^3*d^3*e^4), 1/2*(2*(c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x*e^3 + 2
*a*c*d^3*e^2 + a^2*x*e^5 + a^2*d*e^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 +
 a)) + (2*a*c*d^2*x*e^2 + 2*a*c*d^3*e + a^2*x*e^4 + a^2*d*e^3)*sqrt(c*d^2 +
 a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e
+ d^2)) + 2*(a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^6*x*e + a*c
^2*d^7 + 2*a^2*c*d^4*x*e^3 + 2*a^2*c*d^5*e^2 + a^3*d^2*x*e^5 + a^3*d^3*e^4)
, -((2*a*c*d^2*x*e^2 + 2*a*c*d^3*e + a^2*x*e^4 + a^2*d*e^3)*sqrt(-c*d^2 - a
*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x
^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) - (c^2*d^4*x*e + c^2*d^5 + 2*a*c*d^2*x
*e^3 + 2*a*c*d^3*e^2 + a^2*x*e^5 + a^2*d*e^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt
(c*x^2 + a)) - (a*c*d^3*e^2 + a^2*d*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^6*x*e +
a*c^2*d^7 + 2*a^2*c*d^4*x*e^3 + 2*a^2*c*d^5*e^2 + a^3*d^2*x*e^5 + a^3*d^3*e
^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(160) = 320.
time = 1.23, size = 600, normalized size = 3.35

$$\left(\frac{\sqrt{c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}}\,d^2e^2\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{cd^5\operatorname{sgn}\left(\frac{1}{xe+d}\right)^2+ad^3e^2\operatorname{sgn}\left(\frac{1}{xe+d}\right)^2}-\frac{\left(2\sqrt{cd^2+ae^2}\,cd^2\arctan\left(\frac{\left(\sqrt{c}\,d-\sqrt{cd^2+ae^2}\right)e^{(-1)}}{\sqrt{-a}}\right)e+2\sqrt{-a}\,cd^2e^2\log\left(\left|-cd+\sqrt{cd^2+ae^2}\,\sqrt{c}\right|\right)+2\sqrt{cd^2+ae^2}\,a\arctan\left(\frac{\left(\sqrt{c}\,d-\sqrt{cd^2+ae^2}\right)e^{(-1)}}{\sqrt{-a}}\right)e^3+\sqrt{cd^2+ae^2}\,\sqrt{-a}\,\sqrt{c}\,de^2+\sqrt{-a}\,ae^4\log\left(\left|-cd+\sqrt{cd^2+ae^2}\,\sqrt{c}\right|\right)\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{\sqrt{cd^2+ae^2}\,\sqrt{-a}\,cd^4+\sqrt{cd^2+ae^2}\,\sqrt{-a}\,ad^2e^2}+\frac{(2\,cd^2e^2+ae^4)\log\left(\left|-cd+\sqrt{cd^2+ae^2}\left(\sqrt{c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}}+\frac{\sqrt{cd^2e^2+ae^4}\,e^{(-1)}}{xe+d}\right)\right|\right)}{(cd^4+ad^2e^2)\sqrt{cd^2+ae^2}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}+\frac{2\arctan\left(\frac{\left(d\left(\sqrt{c-\frac{2\,cd}{xe+d}+\frac{cd^2}{(xe+d)^2}+\frac{ae^2}{(xe+d)^2}}+\frac{\sqrt{cd^2e^2+ae^4}\,e^{(-1)}}{xe+d}\right)-\sqrt{cd^2+ae^2}\right)e^{(-1)}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,d^2\operatorname{sgn}\left(\frac{1}{xe+d}\right)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)*d^2*e^2*
sgn(1/(x*e + d))/(c*d^5*sgn(1/(x*e + d))^2 + a*d^3*e^2*sgn(1/(x*e + d))^2)
- (2*sqrt(c*d^2 + a*e^2)*c*d^2*arctan((sqrt(c)*d - sqrt(c*d^2 + a*e^2))*e^(
```

```
-1)/sqrt(-a))*e + 2*sqrt(-a)*c*d^2*e^2*log(abs(-c*d + sqrt(c*d^2 + a*e^2)*s
qrt(c))) + 2*sqrt(c*d^2 + a*e^2)*a*arctan((sqrt(c)*d - sqrt(c*d^2 + a*e^2))
*e^(-1)/sqrt(-a))*e^3 + sqrt(c*d^2 + a*e^2)*sqrt(-a)*sqrt(c)*d*e^2 + sqrt(-
a)*a*e^4*log(abs(-c*d + sqrt(c*d^2 + a*e^2)*sqrt(c))))*sgn(1/(x*e + d))/(sq
rt(c*d^2 + a*e^2)*sqrt(-a)*c*d^4 + sqrt(c*d^2 + a*e^2)*sqrt(-a)*a*d^2*e^2)
+ (2*c*d^2*e^2 + a*e^4)*log(abs(-c*d + sqrt(c*d^2 + a*e^2)*(sqrt(c - 2*c*d/
(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2) + sqrt(c*d^2*e^2 + a*e^4
)*e^(-1)/(x*e + d))))/((c*d^4 + a*d^2*e^2)*sqrt(c*d^2 + a*e^2)*sgn(1/(x*e +
 d))) + 2*arctan((d*(sqrt(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(
x*e + d)^2) + sqrt(c*d^2*e^2 + a*e^4)*e^(-1)/(x*e + d)) - sqrt(c*d^2 + a*e^
2))*e^(-1)/sqrt(-a))*e/(sqrt(-a)*d^2*sgn(1/(x*e + d))))*e^(-1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.349 $\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=212

$$-\frac{\sqrt{a+cx^2}}{ad^2x}-\frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)}-\frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d\left(cd^2+ae^2\right)^{3/2}}-\frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^3\sqrt{cd^2+ae^2}}$$

```
[Out] -c*e^2*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d/(a*e^2+c
*d^2)^(3/2)+2*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-2*e^2*arctanh(
(-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^3/(a*e^2+c*d^2)^(1/2)-(
c*x^2+a)^(1/2)/a/d^2/x-e^3*(c*x^2+a)^(1/2)/d^2/(a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]
time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {975, 270, 272, 65, 214, 745, 739, 212}

$$\frac{2e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}-\frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\left(ae^2+cd^2\right)^{3/2}}-\frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)\left(ae^2+cd^2\right)}-\frac{\sqrt{a+cx^2}}{ad^2x}-\frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d^2*x)) - (e^3*Sqrt[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*(
d + e*x)) - (c*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^
2])])/(d*(c*d^2 + a*e^2)^(3/2)) - (2*e^2*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2
+ a*e^2]*Sqrt[a + c*x^2])])/(d^3*Sqrt[c*d^2 + a*e^2]) + (2*e*ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \left(\frac{1}{d^2x^2\sqrt{a+cx^2}} - \frac{2e}{d^3x\sqrt{a+cx^2}} + \frac{e^2}{d^2(d+ex)^2\sqrt{a+cx^2}} + \frac{2}{d^3(d+ex}\right.$$

$$= \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3} + \frac{(2e^2)\int \frac{1}{(d+ex)\sqrt{a+cx^2}}\,dx}{d^3} +$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{d^3} -$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{2e^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx}}\right.}{d^3\sqrt{cd^2+ae^2}}$$

$$= -\frac{\sqrt{a+cx^2}}{ad^2x} - \frac{e^3\sqrt{a+cx^2}}{d^2\left(cd^2+ae^2\right)(d+ex)} - \frac{ce^2\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx}}\right.}{d\left(cd^2+ae^2\right)^{3/2}}$$

Mathematica [A]
time = 1.01, size = 183, normalized size = 0.86

$$\frac{-\frac{d\sqrt{a+cx^2}\left(cd^2(d+ex)+ae^2(d+2ex)\right)}{a(cd^2+ae^2)x(d+ex)} + \frac{2e^2\left(3cd^2+2ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} - \frac{4e\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-((d*Sqrt[a + c*x^2]*(c*d^2*(d + e*x) + a*e^2*(d + 2*e*x)))/(a*(c*d^2 + a*
e^2)*x*(d + e*x))) + (2*e^2*(3*c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) -
 e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^2]])/(-(c*d^2) - a*e^2)^(3/2) - (4*
e*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a])/d^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(190) = 380.
time = 0.08, size = 395, normalized size = 1.86

method	result

default	$-\frac{2e\ln\left(\frac{\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{d^{3}\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}-\frac{\sqrt{c\,x^{2}+a}}{a\,d^{2}x}+\frac{-\frac{e^{2}\sqrt{\cdots}}{\cdots}}{\cdots}\cdots$
risch	$-\frac{\sqrt{c\,x^{2}+a}}{a\,d^{2}x}-\frac{2e\ln\left(\frac{\frac{2a\,e^{2}+2c\,d^{2}}{e^{2}}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x+\frac{d}{e}\right)^{2}-\frac{2cd\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}{x+\frac{d}{e}}\right)}{d^{3}\sqrt{\frac{a\,e^{2}+c\,d^{2}}{e^{2}}}}-\frac{e^{2}\sqrt{c\cdots}}{\cdots}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*e/d^3/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+
2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)
^(1/2))/(x+d/e))-(c*x^2+a)^(1/2)/a/d^2/x+1/d^2*(-1/(a*e^2+c*d^2)*e^2/(x+d/e
)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c*d*e/(a*e^2+c*d^2)
/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^
2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/
(x+d/e)))+2/d^3*e/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(x*e + d)^2*x^2), x)
```

Fricas [A]
time = 4.18, size = 1479, normalized size = 6.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((3*a*c*d^2*x^2*e^3 + 3*a*c*d^3*x*e^2 + 2*a^2*x^2*e^5 + 2*a^2*d*x*e^4)
*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c
*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e
^2 + 2*d*x*e + d^2)) + 2*(c^2*d^4*x^2*e^2 + c^2*d^5*x*e + 2*a*c*d^2*x^2*e^4
 + 2*a*c*d^3*x*e^3 + a^2*x^2*e^6 + a^2*d*x*e^5)*sqrt(a)*log(-(c*x^2 + 2*sqr
t(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(c^2*d^5*x*e + c^2*d^6 + 3*a*c*d^3*x*e
^3 + 2*a*c*d^4*e^2 + 2*a^2*d*x*e^5 + a^2*d^2*e^4)*sqrt(c*x^2 + a))/(a*c^2*d
^7*x^2*e + a*c^2*d^8*x + 2*a^2*c*d^5*x^2*e^3 + 2*a^2*c*d^6*x*e^2 + a^3*d^3*
x^2*e^5 + a^3*d^4*x*e^4), ((3*a*c*d^2*x^2*e^3 + 3*a*c*d^3*x*e^2 + 2*a^2*x^2
*e^5 + 2*a^2*d*x*e^4)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*
d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) +
 (c^2*d^4*x^2*e^2 + c^2*d^5*x*e + 2*a*c*d^2*x^2*e^4 + 2*a*c*d^3*x*e^3 + a^2
*x^2*e^6 + a^2*d*x*e^5)*sqrt(a)*log(-(c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(a) + 2
*a)/x^2) - (c^2*d^5*x*e + c^2*d^6 + 3*a*c*d^3*x*e^3 + 2*a*c*d^4*e^2 + 2*a^2
*d*x*e^5 + a^2*d^2*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7*x^2*e + a*c^2*d^8*x + 2
*a^2*c*d^5*x^2*e^3 + 2*a^2*c*d^6*x*e^2 + a^3*d^3*x^2*e^5 + a^3*d^4*x*e^4),
-1/2*(4*(c^2*d^4*x^2*e^2 + c^2*d^5*x*e + 2*a*c*d^2*x^2*e^4 + 2*a*c*d^3*x*e^
3 + a^2*x^2*e^6 + a^2*d*x*e^5)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) -
(3*a*c*d^2*x^2*e^3 + 3*a*c*d^3*x*e^2 + 2*a^2*x^2*e^5 + 2*a^2*d*x*e^4)*sqrt(
c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 + 2*sqrt(c*d^2 +
 a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2
*d*x*e + d^2)) + 2*(c^2*d^5*x*e + c^2*d^6 + 3*a*c*d^3*x*e^3 + 2*a*c*d^4*e^2
 + 2*a^2*d*x*e^5 + a^2*d^2*e^4)*sqrt(c*x^2 + a))/(a*c^2*d^7*x^2*e + a*c^2*d
^8*x + 2*a^2*c*d^5*x^2*e^3 + 2*a^2*c*d^6*x*e^2 + a^3*d^3*x^2*e^5 + a^3*d^4*
x*e^4), ((3*a*c*d^2*x^2*e^3 + 3*a*c*d^3*x*e^2 + 2*a^2*x^2*e^5 + 2*a^2*d*x*e
^4)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sqrt(c*
x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) - 2*(c^2*d^4*x^2*e^
2 + c^2*d^5*x*e + 2*a*c*d^2*x^2*e^4 + 2*a*c*d^3*x*e^3 + a^2*x^2*e^6 + a^2*d
*x*e^5)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (c^2*d^5*x*e + c^2*d^6
+ 3*a*c*d^3*x*e^3 + 2*a*c*d^4*e^2 + 2*a^2*d*x*e^5 + a^2*d^2*e^4)*sqrt(c*x^2
 + a))/(a*c^2*d^7*x^2*e + a*c^2*d^8*x + 2*a^2*c*d^5*x^2*e^3 + 2*a^2*c*d^6*x
*e^2 + a^3*d^3*x^2*e^5 + a^3*d^4*x*e^4)]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.350 $\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=268

$$-\frac{\sqrt{a+cx^2}}{2ad^2x^2}+\frac{2e\sqrt{a+cx^2}}{ad^3x}+\frac{e^4\sqrt{a+cx^2}}{d^3\left(cd^2+ae^2\right)(d+ex)}+\frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{cd^2+ae^2}\sqrt{a+cx^2}}\right)}{d^2\left(cd^2+ae^2\right)^{3/2}}+\frac{3e^3\tanh^{-1}\left(\right.}{d}$$

```
[Out] c*e^3*arctanh((-c*d*x+a*e)/(a*e^2+c*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^2/(a*e^2+
c*d^2)^(3/2)+1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-3*e^2*arcta
nh((c*x^2+a)^(1/2)/a^(1/2))/d^4/a^(1/2)+3*e^3*arctanh((-c*d*x+a*e)/(a*e^2+c
*d^2)^(1/2)/(c*x^2+a)^(1/2))/d^4/(a*e^2+c*d^2)^(1/2)-1/2*(c*x^2+a)^(1/2)/a/
d^2/x^2+2*e*(c*x^2+a)^(1/2)/a/d^3/x+e^4*(c*x^2+a)^(1/2)/d^3/(a*e^2+c*d^2)/(
e*x+d)
```

Rubi [A]

time = 0.16, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {975, 272, 44, 65, 214, 270, 745, 739, 212}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^4}+\frac{2e\sqrt{a+cx^2}}{ad^3x}+\frac{ce^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\left(ae^2+cd^2\right)^{3/2}}-\frac{\sqrt{a+cx^2}}{2ad^2x^2}+\frac{3e^3\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}}+\frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*Sqrt[a + c*x^2]/(a*d^2*x^2) + (2*e*Sqrt[a + c*x^2])/(a*d^3*x) + (e^4*S
qrt[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*ArcTanh[(a*e - c*d
*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2)) + (
3*e^3*ArcTanh[(a*e - c*d*x)/(Sqrt[c*d^2 + a*e^2]*Sqrt[a + c*x^2])])/(d^4*Sq
rt[c*d^2 + a*e^2]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d^2) -
 (3*e^2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^4)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}}\,dx = \int\left(\frac{1}{d^2x^3\sqrt{a+cx^2}} - \frac{2e}{d^3x^2\sqrt{a+cx^2}} + \frac{3e^2}{d^4x\sqrt{a+cx^2}} - \frac{e^3}{d^3(d+ex)^2\sqrt{a+\right.$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(2e)\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^3} + \frac{(3e^2)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^4} - \frac{(3e}{}$$

$$= \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} +$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{3e^3\tanh^{-1}\left(\frac{}{\sqrt{cd^2}}\right.}{d^4\sqrt{cd}}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{}{\sqrt{cd^2}}\right.}{d^2(cd^2}$$

$$= -\frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{e^4\sqrt{a+cx^2}}{d^3(cd^2+ae^2)(d+ex)} + \frac{ce^3\tanh^{-1}\left(\frac{}{\sqrt{cd^2}}\right.}{d^2(cd^2}$$

Mathematica [A]
time = 1.23, size = 224, normalized size = 0.84

$$\frac{\frac{d\sqrt{a+cx^2}\,(cd^2(-d^2+3dex+4e^2x^2)+ae^2(-d^2+3dex+6e^2x^2))}{a(cd^2+ae^2)x^2(d+ex)} - \frac{4e^3(4cd^2+3ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+cx^2}}{\sqrt{-cd^2-ae^2}}\right)}{(-cd^2-ae^2)^{3/2}} + \frac{2(-cd^2+6ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] ((d*Sqrt[a + c*x^2]*(c*d^2*(-d^2 + 3*d*e*x + 4*e^2*x^2) + a*e^2*(-d^2 + 3*d
*e*x + 6*e^2*x^2)))/(a*(c*d^2 + a*e^2)*x^2*(d + e*x)) - (4*e^3*(4*c*d^2 + 3
*a*e^2)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + c*x^2])/Sqrt[-(c*d^2) - a*e^
2]])/(-(c*d^2) - a*e^2)^(3/2) + (2*(-(c*d^2) + 6*a*e^2)*ArcTanh[(Sqrt[c]*x
- Sqrt[a + c*x^2])/Sqrt[a]])/a^(3/2))/(2*d^4)
```

Maple [A]
time = 0.11, size = 453, normalized size = 1.69

method	result

risch	$-\frac{\sqrt{c x^2+a}\,(-4ex+d)}{2 a d^3 x^2} + \frac{3e^2 \ln\left(\frac{\frac{2a e^2+2c d^2}{e^2} - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + 2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2 - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + \frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^4\sqrt{\frac{a e^2+c d^2}{e^2}}}$
default	$\frac{3e^2 \ln\left(\frac{\frac{2a e^2+2c d^2}{e^2} - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + 2\sqrt{\frac{a e^2+c d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2 - \frac{2cd\left(x+\frac{d}{e}\right)}{e} + \frac{a e^2+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{d^4\sqrt{\frac{a e^2+c d^2}{e^2}}} + \frac{-\frac{\sqrt{c x^2+a}}{2a x^2} + \frac{c \ln\left(2a+2\right.}{}}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*e^2/d^4/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)
+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2
)^(1/2))/(x+d/e))+1/d^2*(-1/2/a/x^2*(c*x^2+a)^(1/2)+1/2*c/a^(3/2)*ln((2*a+2
*a^(1/2)*(c*x^2+a)^(1/2))/x))+2*e*(c*x^2+a)^(1/2)/a/d^3/x-e/d^3*(-1/(a*e^2+
c*d^2)*e^2/(x+d/e)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-c*
d*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e
*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2-2*c*d/e*(x+d/e)+(a*e^2+c*
d^2)/e^2)^(1/2))/(x+d/e)))-3/d^4*e^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1
/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(x*e + d)^2*x^3), x)
```

Fricas [A]
time = 5.03, size = 1910, normalized size = 7.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(4*a^2*c*d^2*x^3*e^4 + 4*a^2*c*d^3*x^2*e^3 + 3*a^3*x^3*e^6 + 3*a^3*
d*x^2*e^5)*sqrt(c*d^2 + a*e^2)*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2
- 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e
^2)/(x^2*e^2 + 2*d*x*e + d^2)) - (c^3*d^6*x^3*e + c^3*d^7*x^2 - 4*a*c^2*d^4
*x^3*e^3 - 4*a*c^2*d^5*x^2*e^2 - 11*a^2*c*d^2*x^3*e^5 - 11*a^2*c*d^3*x^2*e^
4 - 6*a^3*x^3*e^7 - 6*a^3*d*x^2*e^6)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(3*a*c^2*d^6*x*e - a*c^2*d^7 + 6*a^2*c*d^4*x*e^3
+ 6*a^3*d*x^2*e^6 + 3*a^3*d^2*x*e^5 + (10*a^2*c*d^3*x^2 - a^3*d^3)*e^4 + 2*
(2*a*c^2*d^5*x^2 - a^2*c*d^5)*e^2)*sqrt(c*x^2 + a))/(a^2*c^2*d^8*x^3*e + a^
2*c^2*d^9*x^2 + 2*a^3*c*d^6*x^3*e^3 + 2*a^3*c*d^7*x^2*e^2 + a^4*d^4*x^3*e^5
 + a^4*d^5*x^2*e^4), -1/4*(4*(4*a^2*c*d^2*x^3*e^4 + 4*a^2*c*d^3*x^2*e^3 + 3
*a^3*x^3*e^6 + 3*a^3*d*x^2*e^5)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 -
a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^
2)*e^2)) + (c^3*d^6*x^3*e + c^3*d^7*x^2 - 4*a*c^2*d^4*x^3*e^3 - 4*a*c^2*d^5
*x^2*e^2 - 11*a^2*c*d^2*x^3*e^5 - 11*a^2*c*d^3*x^2*e^4 - 6*a^3*x^3*e^7 - 6*
a^3*d*x^2*e^6)*sqrt(a)*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2)
- 2*(3*a*c^2*d^6*x*e - a*c^2*d^7 + 6*a^2*c*d^4*x*e^3 + 6*a^3*d*x^2*e^6 + 3*
a^3*d^2*x*e^5 + (10*a^2*c*d^3*x^2 - a^3*d^3)*e^4 + 2*(2*a*c^2*d^5*x^2 - a^2
*c*d^5)*e^2)*sqrt(c*x^2 + a))/(a^2*c^2*d^8*x^3*e + a^2*c^2*d^9*x^2 + 2*a^3*
c*d^6*x^3*e^3 + 2*a^3*c*d^7*x^2*e^2 + a^4*d^4*x^3*e^5 + a^4*d^5*x^2*e^4), -
1/2*((c^3*d^6*x^3*e + c^3*d^7*x^2 - 4*a*c^2*d^4*x^3*e^3 - 4*a*c^2*d^5*x^2*e
^2 - 11*a^2*c*d^2*x^3*e^5 - 11*a^2*c*d^3*x^2*e^4 - 6*a^3*x^3*e^7 - 6*a^3*d*
x^2*e^6)*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (4*a^2*c*d^2*x^3*e^4 +
 4*a^2*c*d^3*x^2*e^3 + 3*a^3*x^3*e^6 + 3*a^3*d*x^2*e^5)*sqrt(c*d^2 + a*e^2)
*log(-(2*c^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x
 - a*e)*sqrt(c*x^2 + a) + (a*c*x^2 + 2*a^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2))
 - (3*a*c^2*d^6*x*e - a*c^2*d^7 + 6*a^2*c*d^4*x*e^3 + 6*a^3*d*x^2*e^6 + 3*a
^3*d^2*x*e^5 + (10*a^2*c*d^3*x^2 - a^3*d^3)*e^4 + 2*(2*a*c^2*d^5*x^2 - a^2*
c*d^5)*e^2)*sqrt(c*x^2 + a))/(a^2*c^2*d^8*x^3*e + a^2*c^2*d^9*x^2 + 2*a^3*c
*d^6*x^3*e^3 + 2*a^3*c*d^7*x^2*e^2 + a^4*d^4*x^3*e^5 + a^4*d^5*x^2*e^4), -1
/2*(2*(4*a^2*c*d^2*x^3*e^4 + 4*a^2*c*d^3*x^2*e^3 + 3*a^3*x^3*e^6 + 3*a^3*d*
x^2*e^5)*sqrt(-c*d^2 - a*e^2)*arctan(-sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)*sq
rt(c*x^2 + a)/(c^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + a^2)*e^2)) + (c^3*d^6*x^3
*e + c^3*d^7*x^2 - 4*a*c^2*d^4*x^3*e^3 - 4*a*c^2*d^5*x^2*e^2 - 11*a^2*c*d^2
*x^3*e^5 - 11*a^2*c*d^3*x^2*e^4 - 6*a^3*x^3*e^7 - 6*a^3*d*x^2*e^6)*sqrt(-a)
*arctan(sqrt(-a)/sqrt(c*x^2 + a)) - (3*a*c^2*d^6*x*e - a*c^2*d^7 + 6*a^2*c*
d^4*x*e^3 + 6*a^3*d*x^2*e^6 + 3*a^3*d^2*x*e^5 + (10*a^2*c*d^3*x^2 - a^3*d^3
)*e^4 + 2*(2*a*c^2*d^5*x^2 - a^2*c*d^5)*e^2)*sqrt(c*x^2 + a))/(a^2*c^2*d^8*
x^3*e + a^2*c^2*d^9*x^2 + 2*a^3*c*d^6*x^3*e^3 + 2*a^3*c*d^7*x^2*e^2 + a^4*d
^4*x^3*e^5 + a^4*d^5*x^2*e^4)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.351 $\int x^2(a+bx)^n\left(c+dx^2\right)dx$

Optimal. Leaf size=135

$$\frac{a^2(b^2c+a^2d)\,(a+bx)^{1+n}}{b^5(1+n)}-\frac{2a(b^2c+2a^2d)\,(a+bx)^{2+n}}{b^5(2+n)}+\frac{(b^2c+6a^2d)\,(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad(a+bx)^{4+n}}{b^5(4+n)}+\frac{d(a+}{b^5(5}$$

```
[Out] a^2*(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^5/(1+n)-2*a*(2*a^2*d+b^2*c)*(b*x+a)^(2+n)
/b^5/(2+n)+(6*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^5/
(4+n)+d*(b*x+a)^(5+n)/b^5/(5+n)
```

Rubi [A]
time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {962}

$$\frac{a^2(a^2d+b^2c)\,(a+bx)^{n+1}}{b^5(n+1)}-\frac{2a(2a^2d+b^2c)\,(a+bx)^{n+2}}{b^5(n+2)}+\frac{(6a^2d+b^2c)\,(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] (a^2*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (2*a*(b^2*c + 2*a^2
*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + ((b^2*c + 6*a^2*d)*(a + b*x)^(3 + n)
)/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5
 + n))/(b^5*(5 + n))
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)dx &= \int\left(\frac{(a^2b^2c+a^4d)\,(a+bx)^n}{b^4}-\frac{2(ab^2c+2a^3d)\,(a+bx)^{1+n}}{b^4}+\frac{(b^2c+6a^2d)\,(a}{b^4}\right.\\ &= \frac{a^2(b^2c+a^2d)\,(a+bx)^{1+n}}{b^5(1+n)}-\frac{2a(b^2c+2a^2d)\,(a+bx)^{2+n}}{b^5(2+n)}+\frac{(b^2c+6a^2d)\,(a+}{b^5(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 114, normalized size = 0.84

$$\frac{(a+bx)^{1+n}\left(\frac{a^2b^2c+a^4d}{1+n}-\frac{2a(b^2c+2a^2d)(a+bx)}{2+n}+\frac{(b^2c+6a^2d)(a+bx)^2}{3+n}-\frac{4ad(a+bx)^3}{4+n}+\frac{d(a+bx)^4}{5+n}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^2*c + a^4*d)/(1 + n) - (2*a*(b^2*c + 2*a^2*d)*(a
 + b*x))/(2 + n) + ((b^2*c + 6*a^2*d)*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*
x)^3)/(4 + n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(135) = 270$.
time = 0.08, size = 320, normalized size = 2.37

method	result
norman	$\frac{d\,x^5 e^{n\ln(bx+a)}}{5+n}+\frac{nda\,x^4 e^{n\ln(bx+a)}}{b(n^2+9n+20)}+\frac{(b^2c\,n^2+9b^2cn+12a^2d+20b^2c)an\,x^2 e^{n\ln(bx+a)}}{b^3(n^4+14n^3+71n^2+154n+120)}+\frac{2a^3(b^2c\,n^2+9b^2cn+12a^2d+20b^2c)e^{n\ln(}}{b^5(n^5+15n^4+85n^3+225n^2+274n+1}$
gosper	$(bx+a)^{1+n}(b^4d\,n^4x^4+10b^4d\,n^3x^4-4a\,b^3d\,n^3x^3+b^4c\,n^4x^2+35b^4d\,n^2x^4-24a\,b^3d\,n^2x^3+12b^4c\,n^3x^2+50b^4dn\,x^4+12a^2b^2d\,n^2x^2-2a\,b^3c$
risch	$(b^5d\,n^4x^5+a\,b^4d\,n^4x^4+10b^5d\,n^3x^5+6a\,b^4d\,n^3x^4+b^5c\,n^4x^3+35b^5d\,n^2x^5-4a^2b^3d\,n^3x^3+a\,b^4c\,n^4x^2+11a\,b^4d\,n^2x^4+12b^5c\,n^3x^3+50b^5d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] d/(5+n)*x^5*exp(n*ln(b*x+a))+n*d*a/b/(n^2+9*n+20)*x^4*exp(n*ln(b*x+a))+(b^2
*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)*a/b^3*n/(n^4+14*n^3+71*n^2+154*n+120)*x
^2*exp(n*ln(b*x+a))+2*a^3*(b^2*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)/b^5/(n^5+
15*n^4+85*n^3+225*n^2+274*n+120)*exp(n*ln(b*x+a))-(-b^2*c*n^2+4*a^2*d*n-9*b
^2*c*n-20*b^2*c)/b^2/(n^3+12*n^2+47*n+60)*x^3*exp(n*ln(b*x+a))-2/b^4*n*a^2*
(b^2*c*n^2+9*b^2*c*n+12*a^2*d+20*b^2*c)/(n^5+15*n^4+85*n^3+225*n^2+274*n+12
0)*x*exp(n*ln(b*x+a))
```

Maxima [A]
time = 0.28, size = 210, normalized size = 1.56

$$\frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^n c}{(n^3+6n^2+11n+6)b^3}+\frac{((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+12(n^2+n)a^3b^2x^2-24a^4bnx+24a^5)(bx+a)^n d}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n +
24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n
)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)
^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(135) = 270$.

time = 2.01, size = 368, normalized size = 2.73

$$\frac{(2\,a^3b^2cn^2+18\,a^3b^2cn+40\,a^3b^2c+24\,a^5d+(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24\,b^5d)x^5+(ab^4dn^4+6\,ab^4dn^3+11\,ab^4dn^2+6\,ab^4dn)x^4+(b^5cn^4+40\,b^5c+4\,(3\,b^5c-a^2b^3d)n^3+(49\,b^5c-12\,a^2b^3d)n^2+2\,(39\,b^5c-4\,a^2b^3d)n)x^3+(ab^4cn^4+10\,ab^4cn^3+(29\,ab^4c+12\,a^3b^2d)n^2+4\,(5\,ab^4c+3\,a^3b^2d)n)x^2-2\,(a^2b^3cn^3+9\,a^2b^3cn^2+4\,(5\,a^2b^3c+3\,a^4bd)n)x)(bx+a)^n}{b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^2*c*n^2 + 18*a^3*b^2*c*n + 40*a^3*b^2*c + 24*a^5*d + (b^5*d*n^4 +
10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5 + (a*b^4*d*n^4 + 6
*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + (b^5*c*n^4 + 40*b^5*c +
4*(3*b^5*c - a^2*b^3*d)*n^3 + (49*b^5*c - 12*a^2*b^3*d)*n^2 + 2*(39*b^5*c -
 4*a^2*b^3*d)*n)*x^3 + (a*b^4*c*n^4 + 10*a*b^4*c*n^3 + (29*a*b^4*c + 12*a^3
*b^2*d)*n^2 + 4*(5*a*b^4*c + 3*a^3*b^2*d)*n)*x^2 - 2*(a^2*b^3*c*n^3 + 9*a^2
*b^3*c*n^2 + 4*(5*a^2*b^3*c + 3*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^
5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4134 vs. $2(122) = 244$.
time = 1.33, size = 4134, normalized size = 30.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(
12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**
9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48
*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4
8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a**
3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) - a**2*b**2*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**
7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b*
*9*x**4) + 108*a**2*b**2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b*
*7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*a*b**3*c*x/(12*a**4*b**5 + 48*
a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**
3*d*x**3*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b
**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 6*b**4*c*x**2/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b*
*9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-12*a**4*d*lo
g(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 22
*a**4*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 36*a*
```

*3*b*d*x*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3) - 54*a**3*b*d*x/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**
8*x**3) - a**2*b**2*c/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3) - 36*a**2*b**2*d*x**2*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*
a*b**7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d*x**2/(3*a**3*b**5 + 9*a**2*b**6
*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 3*a*b**3*c*x/(3*a**3*b**5 + 9*a**2*b**6
*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 12*a*b**3*d*x**3*log(a/b + x)/(3*a**3*b
5 + 9*a2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 3*b**4*c*x**2/(3*a**3*
b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + 3*b**4*d*x**4/(3*a**3
*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3), Eq(n, -4)), (12*a**4*
d*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2
*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5
 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*
b**7*x**2) + 2*a**2*b**2*c*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*
x**2) + 3*a**2*b**2*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b*
*2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*
c*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a*b**3*c*x/(2
*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*b**5 + 4*a
*b**6*x + 2*b**7*x**2) + 2*b**4*c*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6
*x + 2*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), E
q(n, -3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*
b**5 + 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**
2*b**2*c*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 6*a**2*b**2*c/(3*a*b**5 + 3*b
6*x) + 6*a2*b**2*d*x**2/(3*a*b**5 + 3*b**6*x) - 6*a*b**3*c*x*log(a/b +
x)/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x) + 3*b**4*c
*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5 + 3*b**6*x), Eq(n, -2))
, (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*c*log(a/b + x)/b**3 + a*
*2*d*x**2/(2*b**3) - a*c*x/b**2 - a*d*x**3/(3*b**2) + c*x**2/(2*b) + d*x**4
/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 2*a**3*b**2*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 18*a**3*b**2*c*n*(
a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*
b**5*n + 120*b**5) + 40*a**3*b**2*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*
*2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
*2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*x**2*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 2
*a**2*b**3*c*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 18*a**2*b**3*c*n**2*x*(a + b*x)**
n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 1
20*b**5) - 40*a**2*b**3*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b
5*n3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3
*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27

```
4*b**5*n + 120*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 27...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(135) = 270.
time = 1.85, size = 624, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + (b*x + a)^n*b^5*c*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 +
35*(b*x + a)^n*b^5*d*n^2*x^5 + (b*x + a)^n*a*b^4*c*n^4*x^2 + 12*(b*x + a)^n
*b^5*c*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x + a)^n*a*b^4*d*n
^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + 10*(b*x + a)^n*a*b^4*c*n^3*x^2 + 49*(
b*x + a)^n*b^5*c*n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b*x + a)^n
*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 - 2*(b*x + a)^n*a^2*b^3*c*n^3*x +
 29*(b*x + a)^n*a*b^4*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 + 78*(b*
x + a)^n*b^5*c*n*x^3 - 8*(b*x + a)^n*a^2*b^3*d*n*x^3 - 18*(b*x + a)^n*a^2*b
^3*c*n^2*x + 20*(b*x + a)^n*a*b^4*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^2
+ 40*(b*x + a)^n*b^5*c*x^3 + 2*(b*x + a)^n*a^3*b^2*c*n^2 - 40*(b*x + a)^n*a
^2*b^3*c*n*x - 24*(b*x + a)^n*a^4*b*d*n*x + 18*(b*x + a)^n*a^3*b^2*c*n + 40
*(b*x + a)^n*a^3*b^2*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*b
^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Mupad [B]
time = 2.82, size = 363, normalized size = 2.69

$$(a+b\,x)^n\,\left(\frac{2\,a^3\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{d\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{x^3\,\left(n^2+3\,n+2\right)\,\left(-4\,d\,a^2\,n+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{2\,a^2\,n\,x\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,n\,x^2\,\left(n+1\right)\,\left(12\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,d\,n\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((2*a^3*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^5*(27
4*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d*x^5*(50*n + 35*n^2 + 10*
n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^3*(3*
n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 4*a^2*d*n + 9*b^2*c*n))/(b^2*(274*n +
225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (2*a^2*n*x*(12*a^2*d + 20*b^2*c +
 b^2*c*n^2 + 9*b^2*c*n))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (a*n*x^2*(n + 1)*(12*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^3*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2
 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.352 $\int x(a+bx)^n(c+dx^2)\,dx$

Optimal. Leaf size=102

$$-\frac{a(b^2c+a^2d)(a+bx)^{1+n}}{b^4(1+n)}+\frac{(b^2c+3a^2d)(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

```
[Out] -a*(a^2*d+b^2*c)*(b*x+a)^(1+n)/b^4/(1+n)+(3*a^2*d+b^2*c)*(b*x+a)^(2+n)/b^4/
(2+n)-3*a*d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.062$, Rules used = {786}

$$-\frac{a(a^2d+b^2c)(a+bx)^{n+1}}{b^4(n+1)}+\frac{(3a^2d+b^2c)(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] -((a*(b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + ((b^2*c + 3*a^2*d)
*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n))
 + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx)^n\left(c+dx^2\right)\,dx=\int\left(\frac{a(-b^2c-a^2d)(a+bx)^n}{b^3}+\frac{(b^2c+3a^2d)(a+bx)^{1+n}}{b^3}-\frac{3ad(a+bx)^{2+n}}{b^3}+\right.$$

$$=-\frac{a(b^2c+a^2d)(a+bx)^{1+n}}{b^4(1+n)}+\frac{(b^2c+3a^2d)(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(}{b}$$

Mathematica [A]
time = 0.10, size = 109, normalized size = 1.07

$$\frac{(a+bx)^{1+n}\left(-6a^3d+6a^2bd(1+n)x+b^3(3+4n+n^2)\,x(c(4+n)+d(2+n)x^2)-ab^2(c(12+7n+n^2)+3d(2+3n+n^2)\,x^2)\right)}{b^4(1+n)(2+n)(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(-6*a^3*d + 6*a^2*b*d*(1 + n)*x + b^3*(3 + 4*n + n^2)*x*
(c*(4 + n) + d*(2 + n)*x^2) - a*b^2*(c*(12 + 7*n + n^2) + 3*d*(2 + 3*n + n^
2)*x^2)))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [A]
time = 0.06, size = 195, normalized size = 1.91

method	result
gosper	$-\frac{(bx+a)^{1+n}(-b^3dn^3x^3-6b^3dn^2x^3+3ab^2dn^2x^2-b^3cn^3x-11b^3dnx^3+9ab^2dnx^2-8b^3cn^2x-6dx^3b^3-6a^2bdnx+ab^2cn^2+6adx^2}{b^4(n^4+10n^3+35n^2+50n+24)}$
norman	$\frac{dx^4 e^{n\ln(bx+a)}}{4+n}+\frac{na(b^2cn^2+7b^2cn+6a^2d+12b^2c)x\, e^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}+\frac{ndax^3 e^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{a^2(b^2cn^2+7b^2cn+6a^2d+12b^2c)e^{n\ln(bx}}{b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{(-b^4dn^3x^4-ab^3dn^3x^3-6b^4dn^2x^4-3ab^3dn^2x^3-b^4cn^3x^2-11b^4dnx^4+3a^2b^2dn^2x^2-ab^3cn^3x-2ab^3dnx^3-8b^4cn^2x^2-6dx^4b}{(3+n)(4+n)(2+n)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-b^3*c*n^3*
x-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-8*b^3*c*n^2*x-6*b^3*d*x^3-6*a^2*b*d*n*x+a*
b^2*c*n^2+6*a*b^2*d*x^2-19*b^3*c*n*x-6*a^2*b*d*x+7*a*b^2*c*n-12*b^3*c*x+6*a
^3*d+12*a*b^2*c)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.29, size = 146, normalized size = 1.43

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n c}{(n^2+3n+2)b^2}+\frac{((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4)(bx+a)^n d}{(n^4+10n^3+35n^2+50n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n
)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2
+ 50*n + 24)*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(102) = 204$.
time = 2.33, size = 250, normalized size = 2.45

$$-\frac{(a^2b^2cn^2+7a^2b^2cn+12a^2b^2c+6a^4d-(b^4dn^3+6b^4dn^2+11b^4dn+6b^4d)x^4-(ab^3dn^3+3ab^3dn^2+2ab^3dn)x^3-(b^4cn^3+12b^4c+(8b^4c-3a^2b^2d)n^2+(19b^4c-3a^2b^2d)n)x^2-(ab^3cn^3+7ab^3cn^2+6(2ab^3c+a^3bd)n)x)(bx+a)^n}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -(a^2*b^2*c*n^2 + 7*a^2*b^2*c*n + 12*a^2*b^2*c + 6*a^4*d - (b^4*d*n^3 + 6*b
^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 - (a*b^3*d*n^3 + 3*a*b^3*d*n^2 + 2*a*b
^3*d*n)*x^3 - (b^4*c*n^3 + 12*b^4*c + (8*b^4*c - 3*a^2*b^2*d)*n^2 + (19*b^4
*c - 3*a^2*b^2*d)*n)*x^2 - (a*b^3*c*n^3 + 7*a*b^3*c*n^2 + 6*(2*a*b^3*c + a^
3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24
*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. $2(90) = 180$.
time = 0.83, size = 2181, normalized size = 21.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6
*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*
log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
+ 27*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**
3) - a*b**2*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
 + 18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*
x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a
*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b +
 x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*
b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) -
 a*b**2*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*x**2*log(a/
b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*c*x/(2*a**2*b**4 +
 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b*
*6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3
*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 2*a*b**2*c*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c/(2*a*b**4 + 2*
b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*x*log(a/b + x)/(
2*a*b**4 + 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**
3*d*log(a/b + x)/b**4 + a**2*d*x/b**3 - a*c*log(a/b + x)/b**2 - a*d*x**2/(2
*b**2) + c*x/b + d*x**3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n*
*4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a
 + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
- a**2*b**2*c*n**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 7*a**2*b**2*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
```

```
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 12*a**2*b**2*c*(a + b*x)**n/(b**
4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d
*n**2*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*
n + 24*b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
+ 35*b**4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*x*(a + b*x)**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 7*a*b**3*c*n**2
*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b
**4) + 12*a*b**3*c*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**
2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*c*n**3*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 8*b**4*c*n**2*x**2*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 19*b**4*c
*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + 12*b**4*c*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4
*n**2 + 50*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 1
0*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a +
 b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
11*b**4*d*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(102) = 204$.
time = 2.30, size = 410, normalized size = 4.02

$$\frac{(bx+a)^{n}b^{4}dn^{3}x^{4}+(bx+a)^{n}ab^{3}dn^{3}x^{3}+6\,(bx+a)^{n}b^{4}dn^{2}x^{4}+(bx+a)^{n}b^{4}cn^{3}x^{2}+3\,(bx+a)^{n}ab^{3}dn^{2}x^{3}+11\,(bx+a)^{n}b^{4}dnx^{4}+(bx+a)^{n}ab^{3}cn^{3}x+8\,(bx+a)^{n}b^{4}cn^{2}x^{2}-3\,(bx+a)^{n}a^{2}b^{2}dn^{2}x^{2}+2\,(bx+a)^{n}ab^{3}dnx^{3}+6\,(bx+a)^{n}b^{4}dx^{4}+7\,(bx+a)^{n}ab^{3}cn^{2}x+19\,(bx+a)^{n}b^{4}cnx^{2}-3\,(bx+a)^{n}a^{2}b^{2}dnx^{2}-(bx+a)^{n}a^{2}b^{2}cn^{2}+12\,(bx+a)^{n}ab^{3}cnx+6\,(bx+a)^{n}a^{3}bdnx+12\,(bx+a)^{n}b^{4}cx^{2}-7\,(bx+a)^{n}a^{2}b^{2}cn-12\,(bx+a)^{n}a^{2}b^{2}c-6\,(bx+a)^{n}a^{4}d}{b^{4}n^{4}+10\,b^{4}n^{3}+35\,b^{4}n^{2}+50\,b^{4}n+24\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + (b*x + a)^n*b^4*c*n^3*x^2 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 1
1*(b*x + a)^n*b^4*d*n*x^4 + (b*x + a)^n*a*b^3*c*n^3*x + 8*(b*x + a)^n*b^4*c
*n^2*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a*b^3*d*n*x^3 +
6*(b*x + a)^n*b^4*d*x^4 + 7*(b*x + a)^n*a*b^3*c*n^2*x + 19*(b*x + a)^n*b^4*
c*n*x^2 - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 - (b*x + a)^n*a^2*b^2*c*n^2 + 12*(b
*x + a)^n*a*b^3*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 12*(b*x + a)^n*b^4*c*x^
2 - 7*(b*x + a)^n*a^2*b^2*c*n - 12*(b*x + a)^n*a^2*b^2*c - 6*(b*x + a)^n*a^
4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Mupad [**B**]
time = 2.70, size = 255, normalized size = 2.50

$$(a+b\,x)^n\,\left(\frac{d\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{a^2\,\left(6\,d\,a^2+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{x^2\,\left(n+1\right)\,\left(-3\,d\,a^2\,n+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x\,\left(6\,d\,a^2+c\,b^2\,n^2+7\,c\,b^2\,n+12\,c\,b^2\right)}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,d\,n\,x^3\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
 + 24) - (a^2*(6*a^2*d + 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^4*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24)) + (x^2*(n + 1)*(12*b^2*c + b^2*c*n^2 - 3*a^2*d*n
 + 7*b^2*c*n))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x*(6*a^2*d
+ 12*b^2*c + b^2*c*n^2 + 7*b^2*c*n))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (a*d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.353 $\int (a+bx)^n (c+dx^2)\, dx$

Optimal. Leaf size=70

$$\frac{(b^2c+a^2d)(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}$$

```
[Out] (a^2*d+b^2*c)*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x+
a)^(3+n)/b^3/(3+n)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {711}

$$\frac{(a^2d+b^2c)(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n
))/(b^3*(2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n (c+dx^2)\, dx &= \int \left(\frac{(b^2c+a^2d)(a+bx)^n}{b^2}-\frac{2ad(a+bx)^{1+n}}{b^2}+\frac{d(a+bx)^{2+n}}{b^2}\right) dx\\ &= \frac{(b^2c+a^2d)(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 0.93

$$\frac{(a+bx)^{1+n}(2a^2d-2abd(1+n)x+b^2(2+n)(c(3+n)+d(1+n)x^2))}{b^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2),x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2*d - 2*a*b*d*(1 + n)*x + b^2*(2 + n)*(c*(3 + n) +
d*(1 + n)*x^2)))/(b^3*(1 + n)*(2 + n)*(3 + n))
```

Maple [A]
time = 0.07, size = 100, normalized size = 1.43

method	result
gosper	$\frac{(bx+a)^{1+n}\left(b^{2} d\,n^{2} x^{2}+3 b^{2} dn\,x^{2}-2 abdnx+b^{2} c\,n^{2}+2 d\,x^{2} b^{2}-2 adxb+5 b^{2} cn+2 a^{2} d+6 b^{2} c\right)}{b^{3}\left(n^{3}+6 n^{2}+11 n+6\right)}$
risch	$\frac{\left(b^{3} d\,n^{2} x^{3}+a\,b^{2} d\,n^{2} x^{2}+3 b^{3} dn\,x^{3}+a\,b^{2} dn\,x^{2}+b^{3} c\,n^{2} x+2 d\,x^{3} b^{3}-2 a^{2} bdnx+a\,b^{2} c\,n^{2}+5 b^{3} cnx+5 a\,b^{2} cn+6 b^{3} cx+2 a^{3} d+6 a\,b^{2} c\right)(bx+a)^{n}}{(2+n)(3+n)(1+n)b^{3}}$
norman	$\frac{d\,x^{3} {\mathrm e}^{n \ln(bx+a)}}{3+n}+\frac{a\left(b^{2} c\,n^{2}+5 b^{2} cn+2 a^{2} d+6 b^{2} c\right){\mathrm e}^{n \ln(bx+a)}}{b^{3}\left(n^{3}+6 n^{2}+11 n+6\right)}+\frac{adn\,x^{2} {\mathrm e}^{n \ln(bx+a)}}{b\left(n^{2}+5 n+6\right)}-\frac{\left(-b^{2} c\,n^{2}+2 a^{2} dn-5 b^{2} cn-6 b^{2} c\right)x\,{\mathrm e}^{n \ln(bx+a)}}{b^{2}\left(n^{3}+6 n^{2}+11 n+6\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*d*n^2*x^2+3*b^2*d*n*x^2-2*a*b*d*n*x+b^2*c*n^2+2*b^2*d*x^
2-2*a*b*d*x+5*b^2*c*n+2*a^2*d+6*b^2*c)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.29, size = 89, normalized size = 1.27

$$\frac{(bx+a)^{n+1}c}{b(n+1)}+\frac{\left((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^nd}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^
2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*d/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(70) = 140.
time = 2.36, size = 148, normalized size = 2.11

$$\frac{(ab^2cn^2+5\,ab^2cn+6\,ab^2c+2\,a^3d+(b^3dn^2+3\,b^3dn+2\,b^3d)x^3+(ab^2dn^2+ab^2dn)x^2+(b^3cn^2+6\,b^3c+(5\,b^3c-2\,a^2bd)n)x)(bx+a)^n}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] (a*b^2*c*n^2 + 5*a*b^2*c*n + 6*a*b^2*c + 2*a^3*d + (b^3*d*n^2 + 3*b^3*d*n +
 2*b^3*d)*x^3 + (a*b^2*d*n^2 + a*b^2*d*n)*x^2 + (b^3*c*n^2 + 6*b^3*c + (5*b
^3*c - 2*a^2*b*d)*n)*x)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(61) = 122$.

time = 0.48, size = 952, normalized size = 13.60

$$\begin{cases} a^n\left(cx + \frac{dx^3}{3}\right) & \text{for}\ b = 0 \\ \frac{2a^2d\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2d}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abdx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abdx}{2a^2b^3+4ab^4x+2b^5x^2} - \frac{b^2c}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2dx^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} & \text{for}\ n = -3 \\ -\frac{2a^2d\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2d}{ab^3+b^4x} - \frac{2abdx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{b^2c}{ab^3+b^4x} + \frac{b^2dx^2}{ab^3+b^4x} & \text{for}\ n = -2 \\ \frac{a^2d\log\left(\frac{a}{b}+x\right)}{b^3} - \frac{adx}{b^2} + \frac{c\log\left(\frac{a}{b}+x\right)}{b} + \frac{dx^2}{2b} & \text{for}\ n = -1 \\ \frac{2a^3d(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} - \frac{2a^2bdnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2cn^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{5ab^2cn(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{6ab^2c(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2dn^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2dnx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{b^3cn^2x(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{5b^3cnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{6b^3cx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{b^3dn^2x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{3b^3dnx^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{2b^3dx^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**3/3), Eq(b, 0)), (2*a**2*d*log(a/b + x)/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d/(2*a**2*b**3 + 4*a*b**4*x + 2*
b**5*x**2) + 4*a*b*d*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2
) + 4*a*b*d*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**3 + 4
*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*d*log(a/b + x)/(a*b**3 + b**
4*x) - 2*a**2*d/(a*b**3 + b**4*x) - 2*a*b*d*x*log(a/b + x)/(a*b**3 + b**4*x
) - b**2*c/(a*b**3 + b**4*x) + b**2*d*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (
a**2*d*log(a/b + x)/b**3 - a*d*x/b**2 + c*log(a/b + x)/b + d*x**2/(2*b), Eq
(n, -1)), (2*a**3*d*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b
**3) - 2*a**2*b*d*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6
*b**3) + a*b**2*c*n**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 5*a*b**2*c*n*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 6*a*b**2*c*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*
b**3) + a*b**2*d*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*
n + 6*b**3) + a*b**2*d*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b*
*3*n + 6*b**3) + b**3*c*n**2*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3) + 5*b**3*c*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b
**3*n + 6*b**3) + 6*b**3*c*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**
3*n + 6*b**3) + b**3*d*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11
*b**3*n + 6*b**3) + 3*b**3*d*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 +
 11*b**3*n + 6*b**3) + 2*b**3*d*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
+ 11*b**3*n + 6*b**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(70) = 140$.

time = 1.39, size = 237, normalized size = 3.39

$$\frac{(bx+a)^n b^3dn^2x^3 + (bx+a)^n ab^2dn^2x^2 + 3\,(bx+a)^n b^3dnx^3 + (bx+a)^n b^3cn^2x + (bx+a)^n ab^2dnx^2 + 2\,(bx+a)^n b^3dx^3 + (bx+a)^n ab^2cn^2 + 5\,(bx+a)^n b^3cnx - 2\,(bx+a)^n a^2bdnx + 5\,(bx+a)^n ab^2cn + 6\,(bx+a)^n b^3cx + 6\,(bx+a)^n ab^2c + 2\,(bx+a)^n a^3d}{b^3n^3 + 6\,b^3n^2 + 11\,b^3n + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*d*n^2*x^3 + (b*x + a)^n*a*b^2*d*n^2*x^2 + 3*(b*x + a)^n*b^
3*d*n*x^3 + (b*x + a)^n*b^3*c*n^2*x + (b*x + a)^n*a*b^2*d*n*x^2 + 2*(b*x +
```

```
a)^n*b^3*d*x^3 + (b*x + a)^n*a*b^2*c*n^2 + 5*(b*x + a)^n*b^3*c*n*x - 2*(b*x
 + a)^n*a^2*b*d*n*x + 5*(b*x + a)^n*a*b^2*c*n + 6*(b*x + a)^n*b^3*c*x + 6*(
b*x + a)^n*a*b^2*c + 2*(b*x + a)^n*a^3*d)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n +
 6*b^3)
```

Mupad [B]
time = 2.63, size = 163, normalized size = 2.33

$$(a+b\,x)^n\,\left(\frac{d\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{x\,\left(-2\,d\,a^2\,b\,n+c\,b^3\,n^2+5\,c\,b^3\,n+6\,c\,b^3\right)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,\left(2\,d\,a^2+c\,b^2\,n^2+5\,c\,b^2\,n+6\,c\,b^2\right)}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,d\,n\,x^2\,\left(n+1\right)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*b^3*c
 + b^3*c*n^2 + 5*b^3*c*n - 2*a^2*b*d*n))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (
a*(2*a^2*d + 6*b^2*c + b^2*c*n^2 + 5*b^2*c*n))/(b^3*(11*n + 6*n^2 + n^3 + 6
)) + (a*d*n*x^2*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6)))
```

3.354 $\int \frac{(a+bx)^n(c+dx^2)}{x} dx$

Optimal. Leaf size=77

$$-\frac{ad(a+bx)^{1+n}}{b^2(1+n)}+\frac{d(a+bx)^{2+n}}{b^2(2+n)}-\frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] -a*d*(b*x+a)^(1+n)/b^2/(1+n)+d*(b*x+a)^(2+n)/b^2/(2+n)-c*(b*x+a)^(1+n)*hype
rgeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {966, 81, 67}

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)}+\frac{d(a+bx)^{n+2}}{b^2(n+2)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2))/x,x]
```

```
[Out] -((a*d*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (d*(a + b*x)^(2 + n))/(b^2*(2 +
n)) - (c*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])
/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
```

```
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n\left(c+dx^2\right)}{x}\,dx &= \frac{d(a+bx)^{2+n}}{b^2(2+n)}+\frac{\int \frac{(a+bx)^n\left(b^2c(2+n)-abd(2+n)x\right)}{x}\,dx}{b^2(2+n)}\\ &= -\frac{ad(a+bx)^{1+n}}{b^2(1+n)}+\frac{d(a+bx)^{2+n}}{b^2(2+n)}+c\int \frac{(a+bx)^n}{x}\,dx\\ &= -\frac{ad(a+bx)^{1+n}}{b^2(1+n)}+\frac{d(a+bx)^{2+n}}{b^2(2+n)}-\frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.83

$$-\frac{(a+bx)^{1+n}\left(ad(a-b(1+n)x)+b^2c(2+n)\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)\right)}{ab^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2))/x,x]
[Out] -(((a + b*x)^(1 + n)*(a*d*(a - b*(1 + n)*x) + b^2*c*(2 + n)*Hypergeometric2
F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/(a*b^2*(1 + n)*(2 + n)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\left(d\,x^2+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)/x,x)
[Out] int((b*x+a)^n*(d*x^2+c)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="fricas")
[Out] integral((d*x^2 + c)*(b*x + a)^n/x, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(61) = 122$.
time = 2.58, size = 345, normalized size = 4.48

$$-\frac{b^{n} c n \left(\frac{a}{b}+x\right)^{n} \Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma(n+2)}-\frac{b^{n} c \left(\frac{a}{b}+x\right)^{n} \Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma(n+2)}+d\left(\begin{cases}\frac{a^{n} x^{2}}{2} & \text{for}\ b=0\\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^{2}+b^{3}x}+\frac{a}{ab^{2}+b^{3}x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^{2}+b^{3}x} & \text{for}\ n=-2\\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^{2}}+\frac{x}{b} & \text{for}\ n=-1\\ -\frac{a^{2}(a+bx)^{n}}{b^{2}n^{2}+3b^{2}n+2b^{2}}+\frac{abnx(a+bx)^{n}}{b^{2}n^{2}+3b^{2}n+2b^{2}}+\frac{b^{2}nx^{2}(a+bx)^{n}}{b^{2}n^{2}+3b^{2}n+2b^{2}}+\frac{b^{2}x^{2}(a+bx)^{n}}{b^{2}n^{2}+3b^{2}n+2b^{2}} & \text{otherwise}\end{cases}\right)-\frac{bb^{n} c n x \left(\frac{a}{b}+x\right)^{n} \Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}-\frac{bb^{n} c x \left(\frac{a}{b}+x\right)^{n} \Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)/x,x)
[Out] -b**n*c*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(n +
 2) - b**n*c*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) + d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**
3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2))
, (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 +
 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2)
+ b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a +
 b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) - b*b**n*c*n*x*(a/b + x)**
n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c*x*
(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)/x,x, algorithm="giac")
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)\,{\left(a+b\,x\right)}^n}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^2)*(a + b*x)^n)/x, x)
```

3.355 $\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=232

$$\frac{a^2(b^2c+a^2d)^2\,(a+bx)^{1+n}}{b^7(1+n)}-\frac{2a(b^2c+a^2d)\,(b^2c+3a^2d)\,(a+bx)^{2+n}}{b^7(2+n)}+\frac{(b^4c^2+12a^2b^2cd+15a^4d^2)\,(a+bx)^{3+}}{b^7(3+n)}$$

```
[Out] a^2*(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^7/(1+n)-2*a*(a^2*d+b^2*c)*(3*a^2*d+b^2*
c)*(b*x+a)^(2+n)/b^7/(2+n)+(15*a^4*d^2+12*a^2*b^2*c*d+b^4*c^2)*(b*x+a)^(3+n
)/b^7/(3+n)-4*a*d*(5*a^2*d+2*b^2*c)*(b*x+a)^(4+n)/b^7/(4+n)+d*(15*a^2*d+2*b
^2*c)*(b*x+a)^(5+n)/b^7/(5+n)-6*a*d^2*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+a)^(
7+n)/b^7/(7+n)
```

Rubi [A]

time = 0.10, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {962}

$$\frac{a^2(a^2d+b^2c)^2\,(a+bx)^{n+1}}{b^7(n+1)}-\frac{2a(a^2d+b^2c)\,(3a^2d+b^2c)\,(a+bx)^{n+2}}{b^7(n+2)}-\frac{4ad(5a^2d+2b^2c)\,(a+bx)^{n+4}}{b^7(n+4)}+\frac{d(15a^2d+2b^2c)\,(a+bx)^{n+5}}{b^7(n+5)}+\frac{(15a^4d^2+12a^2b^2cd+b^4c^2)\,(a+bx)^{n+3}}{b^7(n+3)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (2*a*(b^2*c + a^2
*d)*(b^2*c + 3*a^2*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) + ((b^4*c^2 + 12*a^2
*b^2*c*d + 15*a^4*d^2)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d*(2*b^2*c +
 5*a^2*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (d*(2*b^2*c + 15*a^2*d)*(a + b
*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d
^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^2\right)^2\,dx &= \int \left(\frac{(ab^2c+a^3d)^2\,(a+bx)^n}{b^6}+\frac{2a(-b^2c-3a^2d)\,(b^2c+a^2d)\,(a+bx)^{1+n}}{b^6}+\right.\\ &= \frac{a^2(b^2c+a^2d)^2\,(a+bx)^{1+n}}{b^7(1+n)}-\frac{2a(b^2c+a^2d)\,(b^2c+3a^2d)\,(a+bx)^{2+n}}{b^7(2+n)}+\frac{(b^4c}{}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 199, normalized size = 0.86

$$\frac{(a+bx)^{1+n}\left(\frac{(ab^2c+a^3d)^2}{1+n}-\frac{2a(b^2c+a^2d)(b^2c+3a^2d)(a+bx)}{2+n}+\frac{(b^4c^2+12a^2b^2cd+15a^4d^2)(a+bx)^2}{3+n}-\frac{4ad(2b^2c+5a^2d)(a+bx)^3}{4+n}+\frac{d(2b^2c+15a^2d)(a+bx)^4}{5+n}-\frac{6ad^2(a+bx)^5}{6+n}+\frac{d^2(a+bx)^6}{7+n}\right)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^2*c + a^3*d)^2/(1 + n) - (2*a*(b^2*c + a^2*d)*(b^2
*c + 3*a^2*d)*(a + b*x))/(2 + n) + ((b^4*c^2 + 12*a^2*b^2*c*d + 15*a^4*d^2)
*(a + b*x)^2)/(3 + n) - (4*a*d*(2*b^2*c + 5*a^2*d)*(a + b*x)^3)/(4 + n) + (
d*(2*b^2*c + 15*a^2*d)*(a + b*x)^4)/(5 + n) - (6*a*d^2*(a + b*x)^5)/(6 + n)
 + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(232) = 464$.
time = 0.08, size = 754, normalized size = 3.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(7+n)*x^7*exp(n*ln(b*x+a))+n*a/b*d^2/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))
+(b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+312*a^2*b^2
*c*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840*b^4*c^2)*a/b^5*n/(n^6
+27*n^5+295*n^4+1665*n^3+5104*n^2+8028*n+5040)*x^2*exp(n*ln(b*x+a))+2*a^3*(
b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+312*a^2*b^2*c
*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840*b^4*c^2)/b^7/(n^7+28*n^
6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*exp(n*ln(b*x+a))-(-b^4*
c^2*n^4+8*a^2*b^2*c*d*n^3-22*b^4*c^2*n^3+104*a^2*b^2*c*d*n^2-179*b^4*c^2*n^
2+120*a^4*d^2*n+336*a^2*b^2*c*d*n-638*b^4*c^2*n-840*b^4*c^2)/b^4/(n^5+25*n^
4+245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(b*x+a))-2*d*(-b^2*c*n^2+3*a^2*
d*n-13*b^2*c*n-42*b^2*c)/b^2/(n^3+18*n^2+107*n+210)*x^5*exp(n*ln(b*x+a))-2/
b^6*n*a^2*(b^4*c^2*n^4+22*b^4*c^2*n^3+24*a^2*b^2*c*d*n^2+179*b^4*c^2*n^2+31
2*a^2*b^2*c*d*n+638*b^4*c^2*n+360*a^4*d^2+1008*a^2*b^2*c*d+840*b^4*c^2)/(n^
7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*x*exp(n*ln(b*x+a
))+2*(b^2*c*n^2+13*b^2*c*n+15*a^2*d+42*b^2*c)*a/b^3*d*n/(n^4+22*n^3+179*n^2
+638*n+840)*x^4*exp(n*ln(b*x+a))
```

Maxima [A]
time = 0.29, size = 447, normalized size = 1.93

$$\frac{\left((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^n c^2}{(n^3+6n^2+11n+6)b^3}+\frac{2\left((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+12(n^2+n)a^3b^2x^2-24a^4bnx+24a^5\right)(bx+a)^n cd}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}+\frac{\left((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n^2+120n)ab^6x^6-6(n^5+10n^4+35n^3+50n^2+24n)a^2b^5x^5+30(n^4+6n^3+11n^2+6n)a^3b^4x^4-120(n^3+3n^2+2n)a^4b^3x^3+360(n^2+n)a^5b^2x^2-720a^6bnx+720a^7\right)(bx+a)^n d^2}{(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 +
6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(232) = 464$.

time = 2.20, size = 1027, normalized size = 4.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^4*c^2*n^4 + 44*a^3*b^4*c^2*n^3 + 1680*a^3*b^4*c^2 + 2016*a^5*b^2*c
*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*n^4 + 735*b^
7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^7 + (a*b^6*d
^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^3 + 274*a*b^
6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 + 2*(b^7*c*d*n^6 + 1008*b^7*c*d + (23*b^7*
c*d - 3*a^2*b^5*d^2)*n^5 + 3*(69*b^7*c*d - 10*a^2*b^5*d^2)*n^4 + 5*(185*b^7
*c*d - 21*a^2*b^5*d^2)*n^3 + 2*(1072*b^7*c*d - 75*a^2*b^5*d^2)*n^2 + 36*(67
*b^7*c*d - 2*a^2*b^5*d^2)*n)*x^5 + 2*(a*b^6*c*d*n^6 + 19*a*b^6*c*d*n^5 + (1
31*a*b^6*c*d + 15*a^3*b^4*d^2)*n^4 + (401*a*b^6*c*d + 90*a^3*b^4*d^2)*n^3 +
 15*(36*a*b^6*c*d + 11*a^3*b^4*d^2)*n^2 + 18*(14*a*b^6*c*d + 5*a^3*b^4*d^2)
*n)*x^4 + (b^7*c^2*n^6 + 1680*b^7*c^2 + (25*b^7*c^2 - 8*a^2*b^5*c*d)*n^5 +
(247*b^7*c^2 - 128*a^2*b^5*c*d)*n^4 + (1219*b^7*c^2 - 664*a^2*b^5*c*d - 120
*a^4*b^3*d^2)*n^3 + 8*(389*b^7*c^2 - 152*a^2*b^5*c*d - 45*a^4*b^3*d^2)*n^2
+ 4*(949*b^7*c^2 - 168*a^2*b^5*c*d - 60*a^4*b^3*d^2)*n)*x^3 + 2*(179*a^3*b^
4*c^2 + 24*a^5*b^2*c*d)*n^2 + (a*b^6*c^2*n^6 + 23*a*b^6*c^2*n^5 + 3*(67*a*b
^6*c^2 + 8*a^3*b^4*c*d)*n^4 + (817*a*b^6*c^2 + 336*a^3*b^4*c*d)*n^3 + 2*(73
9*a*b^6*c^2 + 660*a^3*b^4*c*d + 180*a^5*b^2*d^2)*n^2 + 24*(35*a*b^6*c^2 + 4
2*a^3*b^4*c*d + 15*a^5*b^2*d^2)*n)*x^2 + 4*(319*a^3*b^4*c^2 + 156*a^5*b^2*c
*d)*n - 2*(a^2*b^5*c^2*n^5 + 22*a^2*b^5*c^2*n^4 + (179*a^2*b^5*c^2 + 24*a^4
*b^3*c*d)*n^3 + 2*(319*a^2*b^5*c^2 + 156*a^4*b^3*c*d)*n^2 + 24*(35*a^2*b^5*
c^2 + 42*a^4*b^3*c*d + 15*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*n^7 + 28*b^7*n^
6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n
 + 5040*b^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14317 vs. $2(218) = 436$.

time = 4.61, size = 14317, normalized size = 61.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**3/3 + 2*c*d*x**5/5 + d**2*x**7/7), Eq(b, 0)), (60*
a**6*d**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13
*x**6) + 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 9
00*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x
 + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*
b**12*x**5 + 60*b**13*x**6) - 4*a**4*b**2*c*d/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a*
*6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900
*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*
x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10
*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 24*a**3*b
**3*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*
b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200
*a**3*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**
4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
 + 60*b**13*x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*
x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a
*b**12*x**5 + 60*b**13*x**6) - a**2*b**4*c**2/(60*a**6*b**7 + 360*a**5*b**8
*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) - 60*a**2*b**4*c*d*x**2/(60*a**6*b**7 + 360*a
**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**
4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x
)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x*
*3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b*
*4*d**2*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a*
*3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 6
*a*b**5*c**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*
a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) -
 80*a*b**5*c*d*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x*
*6) + 360*a*b**5*d**2*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 9
00*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 360*a*b**5*d**2*x**5/(60*a**6*b**7 + 360*a**5*b**
```

```
8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360
*a*b**12*x**5 + 60*b**13*x**6) - 15*b**6*c**2*x**2/(60*a**6*b**7 + 360*a**5
*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 +
 360*a*b**12*x**5 + 60*b**13*x**6) - 60*b**6*c*d*x**4/(60*a**6*b**7 + 360*a
**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**
4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 60*b**6*d**2*x**6*log(a/b + x)/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-180*a
**6*d**2*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2
+ 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 411*a**6*d**2/(
30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 +
 150*a*b**11*x**4 + 30*b**12*x**5) - 900*a**5*b*d**2*x*log(a/b + x)/(30*a**
5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a
*b**11*x**4 + 30*b**12*x**5) - 1875*a**5*b*d**2*x/(30*a**5*b**7 + 150*a**4*
b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b
**12*x**5) - 12*a**4*b**2*c*d/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b*
*9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 1800*a*
*4*b**2*d**2*x**2*log(a/b + x)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b
**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 3300*a
**4*b**2*d**2*x**2/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 3
00*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 60*a**3*b**3*c*d*x
/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3
 + 150*a*b**11*x**4 + 30*b**12*x**5) - 1800*a**3*b**3*d**2*x**3*log(a/b + x
)/(30*a**5*b**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**
3 + 150*a*b**11*x**4 + 30*b**12*x**5) - 2700*a**3*b**3*d**2*x**3/(30*a**5*b
**7 + 150*a**4*b**8*x + 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b*
*11*x**4 + 30*b**12*x**5) - a**2*b**4*c**2/(30*a**5*b**7 + 150*a**4*b**8*x
+ 300*a**3*b**9*x**2 + 300*a**2*b**10*x**3 + 150*a*b**11*x**4 + 30*b**12*x*
*5) - 120*a**2*b**4*c*d*x**2/(30*a**5*b**7 + 15...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. $2(232) = 464$.
time = 3.52, size = 1750, normalized size = 7.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^5 + 15*(b*x + a)^n*a*b^6*d
^2*n^5*x^6 + 175*(b*x + a)^n*b^7*d^2*n^4*x^7 + 2*(b*x + a)^n*a*b^6*c*d*n^6*
x^4 + 46*(b*x + a)^n*b^7*c*d*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^2*n^5*x^5 +
85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7 + (b*x +
 a)^n*b^7*c^2*n^6*x^3 + 38*(b*x + a)^n*a*b^6*c*d*n^5*x^4 + 414*(b*x + a)^n*
```

```
b^7*c*d*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^
6*d^2*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^2*n^2*x^7 + (b*x + a)^n*a*b^6*c^2*n^
6*x^2 + 25*(b*x + a)^n*b^7*c^2*n^5*x^3 - 8*(b*x + a)^n*a^2*b^5*c*d*n^5*x^3
+ 262*(b*x + a)^n*a*b^6*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 +
1850*(b*x + a)^n*b^7*c*d*n^3*x^5 - 210*(b*x + a)^n*a^2*b^5*d^2*n^3*x^5 + 27
4*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b^7*d^2*n*x^7 + 23*(b*x
+ a)^n*a*b^6*c^2*n^5*x^2 + 247*(b*x + a)^n*b^7*c^2*n^4*x^3 - 128*(b*x + a)^
n*a^2*b^5*c*d*n^4*x^3 + 802*(b*x + a)^n*a*b^6*c*d*n^3*x^4 + 180*(b*x + a)^n
*a^3*b^4*d^2*n^3*x^4 + 4288*(b*x + a)^n*b^7*c*d*n^2*x^5 - 300*(b*x + a)^n*a
^2*b^5*d^2*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*
d^2*x^7 - 2*(b*x + a)^n*a^2*b^5*c^2*n^5*x + 201*(b*x + a)^n*a*b^6*c^2*n^4*x
^2 + 24*(b*x + a)^n*a^3*b^4*c*d*n^4*x^2 + 1219*(b*x + a)^n*b^7*c^2*n^3*x^3
- 664*(b*x + a)^n*a^2*b^5*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^2*n^3*x^3
 + 1080*(b*x + a)^n*a*b^6*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*n^2*x^4
 + 4824*(b*x + a)^n*b^7*c*d*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 - 44*
(b*x + a)^n*a^2*b^5*c^2*n^4*x + 817*(b*x + a)^n*a*b^6*c^2*n^3*x^2 + 336*(b*
x + a)^n*a^3*b^4*c*d*n^3*x^2 + 3112*(b*x + a)^n*b^7*c^2*n^2*x^3 - 1216*(b*x
 + a)^n*a^2*b^5*c*d*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 504*(b*
x + a)^n*a*b^6*c*d*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^2*n*x^4 + 2016*(b*x +
a)^n*b^7*c*d*x^5 + 2*(b*x + a)^n*a^3*b^4*c^2*n^4 - 358*(b*x + a)^n*a^2*b^5*
c^2*n^3*x - 48*(b*x + a)^n*a^4*b^3*c*d*n^3*x + 1478*(b*x + a)^n*a*b^6*c^2*n
^2*x^2 + 1320*(b*x + a)^n*a^3*b^4*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2
*n^2*x^2 + 3796*(b*x + a)^n*b^7*c^2*n*x^3 - 672*(b*x + a)^n*a^2*b^5*c*d*n*x
^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 44*(b*x + a)^n*a^3*b^4*c^2*n^3 - 1
276*(b*x + a)^n*a^2*b^5*c^2*n^2*x - 624*(b*x + a)^n*a^4*b^3*c*d*n^2*x + 840
*(b*x + a)^n*a*b^6*c^2*n*x^2 + 1008*(b*x + a)^n*a^3*b^4*c*d*n*x^2 + 360*(b*
x + a)^n*a^5*b^2*d^2*n*x^2 + 1680*(b*x + a)^n*b^7*c^2*x^3 + 358*(b*x + a)^n
*a^3*b^4*c^2*n^2 + 48*(b*x + a)^n*a^5*b^2*c*d*n^2 - 1680*(b*x + a)^n*a^2*b^
5*c^2*n*x - 2016*(b*x + a)^n*a^4*b^3*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 1276*(b*x + a)^n*a^3*b^4*c^2*n + 624*(b*x + a)^n*a^5*b^2*c*d*n + 1680*(
b*x + a)^n*a^3*b^4*c^2 + 2016*(b*x + a)^n*a^5*b^2*c*d + 720*(b*x + a)^n*a^7
*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 1
3132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

Mupad [B]
time = 3.12, size = 932, normalized size = 4.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (2*a^3*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2
*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*d + 312*a^2*b^2*c*d*n
+ 24*a^2*b^2*c*d*n^2))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 32
```

```
2*n^5 + 28*n^6 + n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 7
35*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1
960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)
*(840*b^4*c^2 - 120*a^4*d^2*n + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^
2*n^3 + b^4*c^2*n^4 - 336*a^2*b^2*c*d*n - 104*a^2*b^2*c*d*n^2 - 8*a^2*b^2*c
*d*n^3))/(b^4*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6
 + n^7 + 5040)) - (2*a^2*n*x*(a + b*x)^n*(360*a^4*d^2 + 840*b^4*c^2 + 638*b
^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 1008*a^2*b^2*c*
d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^6*(13068*n + 13132*n^2 + 67
69*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^5*(a + b*x)^n*
(42*b^2*c + b^2*c*n^2 - 3*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^
6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(360*a^4*d^2 + 840*b^
4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 10
08*a^2*b^2*c*d + 312*a^2*b^2*c*d*n + 24*a^2*b^2*c*d*n^2))/(b^5*(13068*n + 1
3132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n
*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(15*a^2*d + 42*b^2*c + b^2*c*n^2
+ 13*b^2*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
28*n^6 + n^7 + 5040))
```

3.356 $\int x(a+bx)^n (c+dx^2)^2 \, dx$

Optimal. Leaf size=185

$$-\frac{a(b^2c+a^2d)^2 (a+bx)^{1+n}}{b^6(1+n)}+\frac{(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{2+n}}{b^6(2+n)}-\frac{2ad(3b^2c+5a^2d)(a+bx)^{3+n}}{b^6(3+n)}+\frac{2d(b^2c+}{b}$$

```
[Out] -a*(a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^6/(1+n)+(a^2*d+b^2*c)*(5*a^2*d+b^2*c)*(b
*x+a)^(2+n)/b^6/(2+n)-2*a*d*(5*a^2*d+3*b^2*c)*(b*x+a)^(3+n)/b^6/(3+n)+2*d*(
5*a^2*d+b^2*c)*(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d^2*
(b*x+a)^(6+n)/b^6/(6+n)
```

Rubi [A]
time = 0.07, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {786}

$$-\frac{a(a^2d+b^2c)^2 (a+bx)^{n+1}}{b^6(n+1)}+\frac{(a^2d+b^2c)(5a^2d+b^2c)(a+bx)^{n+2}}{b^6(n+2)}-\frac{2ad(5a^2d+3b^2c)(a+bx)^{n+3}}{b^6(n+3)}+\frac{2d(5a^2d+b^2c)(a+bx)^{n+4}}{b^6(n+4)}-\frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^2(a+bx)^{n+6}}{b^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^6*(1 + n))) + ((b^2*c + a^2*d)
*(b^2*c + 5*a^2*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) - (2*a*d*(3*b^2*c + 5*a
^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (2*d*(b^2*c + 5*a^2*d)*(a + b*x)^(
4 + n))/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a
 + b*x)^(6 + n))/(b^6*(6 + n))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n (c+dx^2)^2 \, dx &= \int \left(-\frac{a(b^2c+a^2d)^2 (a+bx)^n}{b^5}+\frac{(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{1+n}}{b^5}-\frac{2ad(3}{}\right.\\ &= -\frac{a(b^2c+a^2d)^2 (a+bx)^{1+n}}{b^6(1+n)}+\frac{(b^2c+a^2d)(b^2c+5a^2d)(a+bx)^{2+n}}{b^6(2+n)}-\frac{2ad(3b^2}{}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 279, normalized size = 1.51

$$\frac{(a+bx)^{1+n}\left(-120a^5d^2+120a^4bd^2(1+n)x-12a^3b^2d(c(30+11n+n^2)+5d(2+3n+n^2)x^2)+4a^2b^3d(1+n)x(3c(30+11n+n^2)+5d(6+5n+n^2)x^2)+b^5(15+23n+9n^2+n^3)x(c^2(24+10n+n^2)+2cd(12+8n+n^2)x^2+d^2(8+6n+n^2)x^4)-ab^4(c^2(360+342n+119n^2+18n^3+n^4)+6cd(60+112n+65n^2+14n^3+n^4)x^2+5d^2(24+50n+35n^2+10n^3+n^4)x^4)\right)}{b^6(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-120*a^5*d^2 + 120*a^4*b*d^2*(1 + n)*x - 12*a^3*b^2*d*(
c*(30 + 11*n + n^2) + 5*d*(2 + 3*n + n^2)*x^2) + 4*a^2*b^3*d*(1 + n)*x*(3*c
*(30 + 11*n + n^2) + 5*d*(6 + 5*n + n^2)*x^2) + b^5*(15 + 23*n + 9*n^2 + n^
3)*x*(c^2*(24 + 10*n + n^2) + 2*c*d*(12 + 8*n + n^2)*x^2 + d^2*(8 + 6*n + n
^2)*x^4) - a*b^4*(c^2*(360 + 342*n + 119*n^2 + 18*n^3 + n^4) + 6*c*d*(60 +
112*n + 65*n^2 + 14*n^3 + n^4)*x^2 + 5*d^2*(24 + 50*n + 35*n^2 + 10*n^3 + n
^4)*x^4)))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(185) = 370$.
time = 0.08, size = 601, normalized size = 3.25

method	result
norman	$\frac{d^{2} x^{6} {\mathrm e}^{n \ln \left(b x +a \right)}}{6+n}+\frac{n a \left(b^{4} c^{2} n^{4}+18 b^{4} c^{2} n^{3}+12 a^{2} b^{2} c d \,n^{2}+119 b^{4} c^{2} n^{2}+132 a^{2} b^{2} c d n +342 b^{4} c^{2} n +120 a^{4} d^{2}+360 a^{2} b^{2} c d +360 b^{4} c^{2}\right) x \,{\mathrm e}^{n \ln}}{b^{5} \left(n^{6}+21 n^{5}+175 n^{4}+735 n^{3}+1624 n^{2}+1764 n +720\right)}$
gosper	$-\frac{\left(b x +a \right)^{1+n} \left(-b^{5} d^{2} n^{5} x^{5}-15 b^{5} d^{2} n^{4} x^{5}+5 a \,b^{4} d^{2} n^{4} x^{4}-2 b^{5} c d \,n^{5} x^{3}-85 b^{5} d^{2} n^{3} x^{5}+50 a \,b^{4} d^{2} n^{3} x^{4}-34 b^{5} c d \,n^{4} x^{3}-225 b^{5} d^{2} n^{2} x^{5}-20 a^{2}\right.}{}$
risch	$-\frac{\left(-b^{6} d^{2} n^{5} x^{6}-a \,b^{5} d^{2} n^{5} x^{5}-15 b^{6} d^{2} n^{4} x^{6}-10 a \,b^{5} d^{2} n^{4} x^{5}-2 b^{6} c d \,n^{5} x^{4}-85 b^{6} d^{2} n^{3} x^{6}+5 a^{2} b^{4} d^{2} n^{4} x^{4}-2 a \,b^{5} c d \,n^{5} x^{3}-35 a \,b^{5} d^{2} n^{3} x^{5}-3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(6+n)*x^6*exp(n*ln(b*x+a))+1/b^5*n*a*(b^4*c^2*n^4+18*b^4*c^2*n^3+12*a^2
*b^2*c*d*n^2+119*b^4*c^2*n^2+132*a^2*b^2*c*d*n+342*b^4*c^2*n+120*a^4*d^2+36
0*a^2*b^2*c*d+360*b^4*c^2)/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)
*x*exp(n*ln(b*x+a))+n*a/b*d^2/(n^2+11*n+30)*x^5*exp(n*ln(b*x+a))-a^2*(b^4*c
^2*n^4+18*b^4*c^2*n^3+12*a^2*b^2*c*d*n^2+119*b^4*c^2*n^2+132*a^2*b^2*c*d*n+
342*b^4*c^2*n+120*a^4*d^2+360*a^2*b^2*c*d+360*b^4*c^2)/b^6/(n^6+21*n^5+175*
n^4+735*n^3+1624*n^2+1764*n+720)*exp(n*ln(b*x+a))-(-b^4*c^2*n^4+6*a^2*b^2*c
*d*n^3-18*b^4*c^2*n^3+66*a^2*b^2*c*d*n^2-119*b^4*c^2*n^2+60*a^4*d^2*n+180*a
^2*b^2*c*d*n-342*b^4*c^2*n-360*b^4*c^2)/b^4/(n^5+20*n^4+155*n^3+580*n^2+104
4*n+720)*x^2*exp(n*ln(b*x+a))-d*(-2*b^2*c*n^2+5*a^2*d*n-22*b^2*c*n-60*b^2*c
)/b^2/(n^3+15*n^2+74*n+120)*x^4*exp(n*ln(b*x+a))+2*(b^2*c*n^2+11*b^2*c*n+10
*a^2*d+30*b^2*c)*a/b^3*d*n/(n^4+18*n^3+119*n^2+342*n+360)*x^3*exp(n*ln(b*x+
a))
```

Maxima [A]
time = 0.29, size = 335, normalized size = 1.81

$$\frac{{\left(b^{2} {\left(n + 1\right)} x^{2} + a b n x - a^{2}\right)} {\left(b x + a\right)}^{n} c^{2}}{{\left(n^{2} + 3 \, n + 2\right)} b^{2}} + \frac{2 \, {\left({\left(n^{3} + 6 \, n^{2} + 11 \, n + 6\right)} b^{4} x^{4} + {\left(n^{3} + 3 \, n^{2} + 2 \, n\right)} a b^{3} x^{3} - 3 \, {\left(n^{2} + n\right)} a^{2} b^{2} x^{2} + 6 \, a^{3} b n x - 6 \, a^{4}\right)} {\left(b x + a\right)}^{n} c d}{{\left(n^{4} + 10 \, n^{3} + 35 \, n^{2} + 50 \, n + 24\right)} b^{4}} + \frac{{\left({\left(n^{5} + 15 \, n^{4} + 85 \, n^{3} + 225 \, n^{2} + 274 \, n + 120\right)} b^{6} x^{6} + {\left(n^{5} + 10 \, n^{4} + 35 \, n^{3} + 50 \, n^{2} + 24 \, n\right)} a b^{5} x^{5} - 5 \, {\left(n^{4} + 6 \, n^{3} + 11 \, n^{2} + 6 \, n\right)} a^{2} b^{4} x^{4} + 20 \, {\left(n^{3} + 3 \, n^{2} + 2 \, n\right)} a^{3} b^{3} x^{3} - 60 \, {\left(n^{2} + n\right)} a^{4} b^{2} x^{2} + 120 \, a^{5} b n x - 120 \, a^{6}\right)} {\left(b x + a\right)}^{n} d^{2}}{{\left(n^{6} + 21 \, n^{5} + 175 \, n^{4} + 735 \, n^{3} + 1624 \, n^{2} + 1764 \, n + 720\right)} b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 3
5*n^2 + 50*n + 24)*b^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*
b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^
3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^
2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d^2/((n^6 + 21*n^
5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(185) = 370$.
time = 2.03, size = 757, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^4*c^2*n^4 + 18*a^2*b^4*c^2*n^3 + 360*a^2*b^4*c^2 + 360*a^4*b^2*c*d
+ 120*a^6*d^2 - (b^6*d^2*n^5 + 15*b^6*d^2*n^4 + 85*b^6*d^2*n^3 + 225*b^6*d^
2*n^2 + 274*b^6*d^2*n + 120*b^6*d^2)*x^6 - (a*b^5*d^2*n^5 + 10*a*b^5*d^2*n^
4 + 35*a*b^5*d^2*n^3 + 50*a*b^5*d^2*n^2 + 24*a*b^5*d^2*n)*x^5 - (2*b^6*c*d*
n^5 + 360*b^6*c*d + (34*b^6*c*d - 5*a^2*b^4*d^2)*n^4 + 2*(107*b^6*c*d - 15*
a^2*b^4*d^2)*n^3 + (614*b^6*c*d - 55*a^2*b^4*d^2)*n^2 + 6*(132*b^6*c*d - 5*
a^2*b^4*d^2)*n)*x^4 - 2*(a*b^5*c*d*n^5 + 14*a*b^5*c*d*n^4 + 5*(13*a*b^5*c*d
 + 2*a^3*b^3*d^2)*n^3 + 2*(56*a*b^5*c*d + 15*a^3*b^3*d^2)*n^2 + 20*(3*a*b^5
*c*d + a^3*b^3*d^2)*n)*x^3 + (119*a^2*b^4*c^2 + 12*a^4*b^2*c*d)*n^2 - (b^6*
c^2*n^5 + 360*b^6*c^2 + (19*b^6*c^2 - 6*a^2*b^4*c*d)*n^4 + (137*b^6*c^2 - 7
2*a^2*b^4*c*d)*n^3 + (461*b^6*c^2 - 246*a^2*b^4*c*d - 60*a^4*b^2*d^2)*n^2 +
 6*(117*b^6*c^2 - 30*a^2*b^4*c*d - 10*a^4*b^2*d^2)*n)*x^2 + 6*(57*a^2*b^4*c
^2 + 22*a^4*b^2*c*d)*n - (a*b^5*c^2*n^5 + 18*a*b^5*c^2*n^4 + (119*a*b^5*c^2
 + 12*a^3*b^3*c*d)*n^3 + 6*(57*a*b^5*c^2 + 22*a^3*b^3*c*d)*n^2 + 120*(3*a*b
^5*c^2 + 3*a^3*b^3*c*d + a^5*b*d^2)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5
 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8940 vs. $2(170) = 340$.
time = 2.67, size = 8940, normalized size = 48.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**2/2 + c*d*x**4/2 + d**2*x**6/6), Eq(b, 0)), (60*a*
*5*d**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 +
 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d**2/(60
*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 30
0*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*d**2*x*log(a/b + x)/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 625*a**4*b*d**2*x/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
**5) - 6*a**3*b**2*c*d/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d
**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d
**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b*
*9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 30*a**2*b**3*c*d*x/(60*a**5*b
**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**
10*x**4 + 60*b**11*x**5) + 600*a**2*b**3*d**2*x**3*log(a/b + x)/(60*a**5*b*
*6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**1
0*x**4 + 60*b**11*x**5) + 900*a**2*b**3*d**2*x**3/(60*a**5*b**6 + 300*a**4*
b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b*
*11*x**5) - 3*a*b**4*c**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x
**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 60*a*b**4*c*
d*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4*log(a/b +
x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**
3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d**2*x**4/(60*a**5*b**6
+ 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x
**4 + 60*b**11*x**5) - 15*b**5*c**2*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600
*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) -
60*b**5*c*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600
*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d**2*x**5*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d**2*lo
g(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x
**3 + 12*b**10*x**4) - 125*a**5*d**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d**2*x*log(a/b
+ x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 +
12*b**10*x**4) - 440*a**4*b*d**2*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2
*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*a**3*b**2*c*d/(12*a**4*b**
6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
360*a**3*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a*
*2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*d**2*x**2/(1
2*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**1
0*x**4) - 24*a**2*b**3*c*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*
x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d**2*x**3*log(a/b +
x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12
```

```
*b**10*x**4) - 240*a**2*b**3*d**2*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*
a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a*b**4*c**2/(12*a**4*b**
6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
36*a*b**4*c*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*
a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b*
*6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
 4*b**5*c**2*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b*
*9*x**3 + 12*b**10*x**4) - 24*b**5*c*d*x**3/(12*a**4*b**6 + 48*a**3*b**7*x
+ 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*d**2*x**5/(
12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**
10*x**4), Eq(n, -5)), (60*a**5*d**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**
7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*d**2/(6*a**3*b**6 + 18*a**2*
b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*d**2*x*log(a/b + x)/(6*
a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a**4*b*d**
2*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 12*a**3
*b**2*c*d*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b
**9*x**3) + 22*a**3*b**2*c*d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2
 + 6*b**9*x**3) + 180*a**3*b**2*d**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a*
*2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d**2*x**2/(6*a**3
*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 36*a**2*b**3*c*d*x
*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3)
 + 54*a**2*b**3*c*d*x/(6*a**3*b**6 + 18*a**2*b*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(185) = 370$.
time = 2.76, size = 1266, normalized size = 6.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d^2*n^5*x^6 + (b*x + a)^n*a*b^5*d^2*n^5*x^5 + 15*(b*x + a)
^n*b^6*d^2*n^4*x^6 + 2*(b*x + a)^n*b^6*c*d*n^5*x^4 + 10*(b*x + a)^n*a*b^5*d
^2*n^4*x^5 + 85*(b*x + a)^n*b^6*d^2*n^3*x^6 + 2*(b*x + a)^n*a*b^5*c*d*n^5*x
^3 + 34*(b*x + a)^n*b^6*c*d*n^4*x^4 - 5*(b*x + a)^n*a^2*b^4*d^2*n^4*x^4 + 3
5*(b*x + a)^n*a*b^5*d^2*n^3*x^5 + 225*(b*x + a)^n*b^6*d^2*n^2*x^6 + (b*x +
a)^n*b^6*c^2*n^5*x^2 + 28*(b*x + a)^n*a*b^5*c*d*n^4*x^3 + 214*(b*x + a)^n*b
^6*c*d*n^3*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n^3*x^4 + 50*(b*x + a)^n*a*b^5*
d^2*n^2*x^5 + 274*(b*x + a)^n*b^6*d^2*n*x^6 + (b*x + a)^n*a*b^5*c^2*n^5*x +
 19*(b*x + a)^n*b^6*c^2*n^4*x^2 - 6*(b*x + a)^n*a^2*b^4*c*d*n^4*x^2 + 130*(
b*x + a)^n*a*b^5*c*d*n^3*x^3 + 20*(b*x + a)^n*a^3*b^3*d^2*n^3*x^3 + 614*(b*
x + a)^n*b^6*c*d*n^2*x^4 - 55*(b*x + a)^n*a^2*b^4*d^2*n^2*x^4 + 24*(b*x + a
)^n*a*b^5*d^2*n*x^5 + 120*(b*x + a)^n*b^6*d^2*x^6 + 18*(b*x + a)^n*a*b^5*c^
2*n^4*x + 137*(b*x + a)^n*b^6*c^2*n^3*x^2 - 72*(b*x + a)^n*a^2*b^4*c*d*n^3*
```

x^2 + 224*(b*x + a)^n*a*b^5*c*d*n^2*x^3 + 60*(b*x + a)^n*a^3*b^3*d^2*n^2*x^
3 + 792*(b*x + a)^n*b^6*c*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*d^2*n*x^4 - (b*x
 + a)^n*a^2*b^4*c^2*n^4 + 119*(b*x + a)^n*a*b^5*c^2*n^3*x + 12*(b*x + a)^n*
a^3*b^3*c*d*n^3*x + 461*(b*x + a)^n*b^6*c^2*n^2*x^2 - 246*(b*x + a)^n*a^2*b
^4*c*d*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n^2*x^2 + 120*(b*x + a)^n*a*b^5
*c*d*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d^2*n*x^3 + 360*(b*x + a)^n*b^6*c*d*x^4
 - 18*(b*x + a)^n*a^2*b^4*c^2*n^3 + 342*(b*x + a)^n*a*b^5*c^2*n^2*x + 132*(
b*x + a)^n*a^3*b^3*c*d*n^2*x + 702*(b*x + a)^n*b^6*c^2*n*x^2 - 180*(b*x + a
)^n*a^2*b^4*c*d*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d^2*n*x^2 - 119*(b*x + a)^n*
a^2*b^4*c^2*n^2 - 12*(b*x + a)^n*a^4*b^2*c*d*n^2 + 360*(b*x + a)^n*a*b^5*c^
2*n*x + 360*(b*x + a)^n*a^3*b^3*c*d*n*x + 120*(b*x + a)^n*a^5*b*d^2*n*x + 3
60*(b*x + a)^n*b^6*c^2*x^2 - 342*(b*x + a)^n*a^2*b^4*c^2*n - 132*(b*x + a)^
n*a^4*b^2*c*d*n - 360*(b*x + a)^n*a^2*b^4*c^2 - 360*(b*x + a)^n*a^4*b^2*c*d
 - 120*(b*x + a)^n*a^6*d^2)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n
^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)

Mupad [B]
time = 3.05, size = 723, normalized size = 3.91

$$\frac{d^2x^6(a+bx)^n(n^5+15n^4+85n^3+225n^2+274n+120)}{n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720}-\frac{a^2(a+bx)^n(120a^4d^2+12a^2b^2cdn^2+132a^2b^2cdn+360a^2b^2cd+b^4c^2n^4+18b^4c^2n^3+119b^4c^2n^2+342b^4c^2n+360b^4c^2)}{b^6(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}+\frac{x^2(n+1)(a+bx)^n(-60a^4d^2n-6a^2b^2cdn^3-66a^2b^2cdn^2-180a^2b^2cdn+b^4c^2n^4+18b^4c^2n^3+119b^4c^2n^2+342b^4c^2n+360b^4c^2)}{b^4(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}+\frac{dx^4(a+bx)^n(-5da^2n+2cb^2n^2+22cb^2n+60cb^2)(n^3+6n^2+11n+6)}{b^2(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}+\frac{anx(a+bx)^n(120a^4d^2+12a^2b^2cdn^2+132a^2b^2cdn+360a^2b^2cd+b^4c^2n^4+18b^4c^2n^3+119b^4c^2n^2+342b^4c^2n+360b^4c^2)}{b^5(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}+\frac{ad^2nx^5(a+bx)^n(n^4+10n^3+35n^2+50n+24)}{b(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}+\frac{2adnx^3(a+bx)^n(n^2+3n+2)(10da^2+cb^2n^2+11cb^2n+30cb^2)}{b^3(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c + d*x^2)^2*(a + b*x)^n,x)

[Out] (d^2*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (a^2*(a + b*x)^n*
(120*a^4*d^2 + 360*b^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n
^3 + b^4*c^2*n^4 + 360*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2
))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^
2*(n + 1)*(a + b*x)^n*(360*b^4*c^2 - 60*a^4*d^2*n + 342*b^4*c^2*n + 119*b^4
*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 - 180*a^2*b^2*c*d*n - 66*a^2*b^2*c*
d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 2
1*n^5 + n^6 + 720)) + (d*x^4*(a + b*x)^n*(60*b^2*c + 2*b^2*c*n^2 - 5*a^2*d*
n + 22*b^2*c*n)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3
 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*n*x*(a + b*x)^n*(120*a^4*d^2 + 360*b
^4*c^2 + 342*b^4*c^2*n + 119*b^4*c^2*n^2 + 18*b^4*c^2*n^3 + b^4*c^2*n^4 + 3
60*a^2*b^2*c*d + 132*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5*(1764*n + 16
24*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d^2*n*x^5*(a + b*x)^
n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(10
*a^2*d + 30*b^2*c + b^2*c*n^2 + 11*b^2*c*n))/(b^3*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720))

3.357 $\int (a+bx)^n \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=140

$$\frac{\left(b^2c+a^2d\right)^2 (a+bx)^{1+n}}{b^5(1+n)}-\frac{4ad(b^2c+a^2d)\,(a+bx)^{2+n}}{b^5(2+n)}+\frac{2d(b^2c+3a^2d)\,(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad^2(a+bx)^{4+n}}{b^5(4+n)}+\frac{d^2(a+\cdots}{b^5(\cdots}$$

```
[Out] (a^2*d+b^2*c)^2*(b*x+a)^(1+n)/b^5/(1+n)-4*a*d*(a^2*d+b^2*c)*(b*x+a)^(2+n)/b
^5/(2+n)+2*d*(3*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d^2*(b*x+a)^(4+n)/
b^5/(4+n)+d^2*(b*x+a)^(5+n)/b^5/(5+n)
```

Rubi [A]
time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {711}

$$\frac{\left(a^2d+b^2c\right)^2 (a+bx)^{n+1}}{b^5(n+1)}-\frac{4ad(a^2d+b^2c)\,(a+bx)^{n+2}}{b^5(n+2)}+\frac{2d(3a^2d+b^2c)\,(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad^2(a+bx)^{n+4}}{b^5(n+4)}+\frac{d^2(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((b^2*c + a^2*d)^2*(a + b*x)^(1 + n))/(b^5*(1 + n)) - (4*a*d*(b^2*c + a^2*d
)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (2*d*(b^2*c + 3*a^2*d)*(a + b*x)^(3 +
n))/(b^5*(3 + n)) - (4*a*d^2*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d^2*(a + b
*x)^(5 + n))/(b^5*(5 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n \left(c+dx^2\right)^2 \, dx &= \int \left(\frac{\left(b^2c+a^2d\right)^2 (a+bx)^n}{b^4}-\frac{4ad(b^2c+a^2d)\,(a+bx)^{1+n}}{b^4}+\frac{2d(b^2c+3a^2d)\,(a+\cdots}{b^4}\right.\\ &= \frac{\left(b^2c+a^2d\right)^2 (a+bx)^{1+n}}{b^5(1+n)}-\frac{4ad(b^2c+a^2d)\,(a+bx)^{2+n}}{b^5(2+n)}+\frac{2d(b^2c+3a^2d)\,(a+b\cdots}{b^5(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 184, normalized size = 1.31

$$\frac{(a+bx)^{1+n}\left(24a^4d^2-24a^3bd^2(1+n)x+4a^2b^2d(c(20+9n+n^2)+3d(2+3n+n^2)\,x^2)-4ab^3d(1+n)x(c(20+9n+n^2)+d(6+5n+n^2)\,x^2)+b^4(8+6n+n^2)\,(c^2(15+8n+n^2)+2cd(5+6n+n^2)\,x^2+d^2(3+4n+n^2)\,x^4)\right)}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(24*a^4*d^2 - 24*a^3*b*d^2*(1 + n)*x + 4*a^2*b^2*d*(c*(2
0 + 9*n + n^2) + 3*d*(2 + 3*n + n^2)*x^2) - 4*a*b^3*d*(1 + n)*x*(c*(20 + 9*
n + n^2) + d*(6 + 5*n + n^2)*x^2) + b^4*(8 + 6*n + n^2)*(c^2*(15 + 8*n + n^
2) + 2*c*d*(5 + 6*n + n^2)*x^2 + d^2*(3 + 4*n + n^2)*x^4)))/(b^5*(1 + n)*(2
 + n)*(3 + n)*(4 + n)*(5 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(140) = 280$.
time = 0.07, size = 420, normalized size = 3.00

method	result
gosper	$\frac{(bx+a)^{1+n}\left(b^4d^2n^4x^4+10b^4d^2n^3x^4-4a\,b^3d^2n^3x^3+2b^4cd\,n^4x^2+35b^4d^2n^2x^4-24a\,b^3d^2n^2x^3+24b^4cd\,n^3x^2+50b^4d^2n\,x^4+12a^2b^2d^2n^2\right.}{}$
norman	$\frac{d^2x^5\mathrm{e}^{n\ln(bx+a)}}{5+n}+\frac{a\left(b^4c^2n^4+14b^4c^2n^3+4a^2b^2cd\,n^2+71b^4c^2n^2+36a^2b^2cdn+154b^4c^2n+24a^4d^2+80a^2b^2cd+120b^4c^2\right)\mathrm{e}^{n\ln(bx+a)}}{b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}$
risch	$\frac{\left(b^5d^2n^4x^5+a\,b^4d^2n^4x^4+10b^5d^2n^3x^5+6a\,b^4d^2n^3x^4+2b^5cd\,n^4x^3+35b^5d^2n^2x^5-4a^2b^3d^2n^3x^3+2a\,b^4cd\,n^4x^2+11a\,b^4d^2n^2x^4+24b^5cd\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^4*d^2*n^4*x^4+10*b^4*d^2*n^3*x^4-4*a*b^3*d^2*n^3*x^3+2*b^4
*c*d*n^4*x^2+35*b^4*d^2*n^2*x^4-24*a*b^3*d^2*n^2*x^3+24*b^4*c*d*n^3*x^2+50*
b^4*d^2*n*x^4+12*a^2*b^2*d^2*n^2*x^2-4*a*b^3*c*d*n^3*x-44*a*b^3*d^2*n*x^3+b
^4*c^2*n^4+98*b^4*c*d*n^2*x^2+24*b^4*d^2*x^4+36*a^2*b^2*d^2*n*x^2-40*a*b^3*
c*d*n^2*x-24*a*b^3*d^2*x^3+14*b^4*c^2*n^3+156*b^4*c*d*n*x^2-24*a^3*b*d^2*n*
x+4*a^2*b^2*c*d*n^2+24*a^2*b^2*d^2*x^2-116*a*b^3*c*d*n*x+71*b^4*c^2*n^2+80*
b^4*c*d*x^2-24*a^3*b*d^2*x+36*a^2*b^2*c*d*n-80*a*b^3*c*d*x+154*b^4*c^2*n+24
*a^4*d^2+80*a^2*b^2*c*d+120*b^4*c^2)/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+1
20)
```

Maxima [A]
time = 0.28, size = 235, normalized size = 1.68

$$\frac{(bx+a)^{n+1}c^2}{b(n+1)}+\frac{2\left((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^ncd}{(n^3+6\,n^2+11\,n+6)b^3}+\frac{\left((n^4+10\,n^3+35\,n^2+50\,n+24)b^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3x^3+12\,(n^2+n)a^3b^2x^2-24\,a^4bnx+24\,a^5\right)(bx+a)^nd^2}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*
a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c*d/((n^3 + 6*n^2 + 11*n + 6)*
b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2
 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^
```

```
2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d^2/((n^5 + 15*n^4 + 85*n^3 + 22
5*n^2 + 274*n + 120)*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(140) = 280.
time = 2.13, size = 519, normalized size = 3.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^4*c^2*n^4 + 14*a*b^4*c^2*n^3 + 120*a*b^4*c^2 + 80*a^3*b^2*c*d + 24*a^5
*d^2 + (b^5*d^2*n^4 + 10*b^5*d^2*n^3 + 35*b^5*d^2*n^2 + 50*b^5*d^2*n + 24*b
^5*d^2)*x^5 + (a*b^4*d^2*n^4 + 6*a*b^4*d^2*n^3 + 11*a*b^4*d^2*n^2 + 6*a*b^4
*d^2*n)*x^4 + 2*(b^5*c*d*n^4 + 40*b^5*c*d + 2*(6*b^5*c*d - a^2*b^3*d^2)*n^3
 + (49*b^5*c*d - 6*a^2*b^3*d^2)*n^2 + 2*(39*b^5*c*d - 2*a^2*b^3*d^2)*n)*x^3
 + (71*a*b^4*c^2 + 4*a^3*b^2*c*d)*n^2 + 2*(a*b^4*c*d*n^4 + 10*a*b^4*c*d*n^3
 + (29*a*b^4*c*d + 6*a^3*b^2*d^2)*n^2 + 2*(10*a*b^4*c*d + 3*a^3*b^2*d^2)*n)
*x^2 + 2*(77*a*b^4*c^2 + 18*a^3*b^2*c*d)*n + (b^5*c^2*n^4 + 120*b^5*c^2 + 2
*(7*b^5*c^2 - 2*a^2*b^3*c*d)*n^3 + (71*b^5*c^2 - 36*a^2*b^3*c*d)*n^2 + 2*(7
7*b^5*c^2 - 40*a^2*b^3*c*d - 12*a^4*b*d^2)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*
b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5097 vs. 2(128) = 256.
time = 1.52, size = 5097, normalized size = 36.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5), Eq(b, 0)), (12*a**4*
d**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a
*b**8*x**3 + 12*b**9*x**4) + 25*a**4*d**2/(12*a**4*b**5 + 48*a**3*b**6*x +
72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d**2*x*log(a
/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
 + 12*b**9*x**4) + 88*a**3*b*d**2*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**
2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 2*a**2*b**2*c*d/(12*a**4*b**
5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 7
2*a**2*b**2*d**2*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2
*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**2*d**2*x**2/(12*a
**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x*
*4) - 8*a*b**3*c*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d**2*x**3*log(a/b + x)/(12*a**4*b
```

5 + 48*a3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) +
48*a*b**3*d**2*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 4
8*a*b**8*x**3 + 12*b**9*x**4) - 3*b**4*c**2/(12*a**4*b**5 + 48*a**3*b**6*x
+ 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 12*b**4*c*d*x**2/(12
*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*
x**4) + 12*b**4*d**2*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-12*a**4*d**2
*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) -
22*a**4*d**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) -
36*a**3*b*d**2*x*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2
+ 3*b**8*x**3) - 54*a**3*b*d**2*x/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*
x**2 + 3*b**8*x**3) - 2*a**2*b**2*c*d/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b*
*7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d**2*x**2*log(a/b + x)/(3*a**3*b**5 +
9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 36*a**2*b**2*d**2*x**2/(3*a
3*b5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 6*a*b**3*c*d*x/(3
*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) - 12*a*b**3*d**2*
x**3*log(a/b + x)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**
3) - b**4*c**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3)
- 6*b**4*c*d*x**2/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**
3) + 3*b**4*d**2*x**4/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8
*x**3), Eq(n, -4)), (12*a**4*d**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x +
2*b**7*x**2) + 18*a**4*d**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a
3*b*d2*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**
3*b*d**2*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*a**2*b**2*c*d*log(a
/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 6*a**2*b**2*c*d/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b**2*d**2*x**2*log(a/b + x)/(2*a
2*b5 + 4*a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x*log(a/b + x)/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) + 8*a*b**3*c*d*x/(2*a**2*b**5 + 4*a*b**6*x
+ 2*b**7*x**2) - 4*a*b**3*d**2*x**3/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**
2) - b**4*c**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 4*b**4*c*d*x**2*l
og(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + b**4*d**2*x**4/(2*a*
*2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -3)), (-12*a**4*d**2*log(a/b + x
)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d**2/(3*a*b**5 + 3*b**6*x) - 12*a**3*b*d*
*2*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d*log(a/b + x)/(3*
a*b**5 + 3*b**6*x) - 12*a**2*b**2*c*d/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d
2*x2/(3*a*b**5 + 3*b**6*x) - 12*a*b**3*c*d*x*log(a/b + x)/(3*a*b**5 + 3
*b**6*x) - 2*a*b**3*d**2*x**3/(3*a*b**5 + 3*b**6*x) - 3*b**4*c**2/(3*a*b**5
+ 3*b**6*x) + 6*b**4*c*d*x**2/(3*a*b**5 + 3*b**6*x) + b**4*d**2*x**4/(3*a*
b**5 + 3*b**6*x), Eq(n, -2)), (a**4*d**2*log(a/b + x)/b**5 - a**3*d**2*x/b*
*4 + 2*a**2*c*d*log(a/b + x)/b**3 + a**2*d**2*x**2/(2*b**3) - 2*a*c*d*x/b**
2 - a*d**2*x**3/(3*b**2) + c**2*log(a/b + x)/b + c*d*x**2/b + d**2*x**4/(4*
b), Eq(n, -1)), (24*a**5*d**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b
5*n3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d**2*n*x*(a +
b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**
5*n + 120*b**5) + 4*a**3*b**2*c*d*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*

```
*4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 36*a**3*b**2*c
*d*n*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 80*a**3*b**2*c*d*(a + b*x)**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b
**2*d**2*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d**2*n*x**2*(a + b*x)
**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n +
 120*b**5) - 4*a**2*b**3*c*d*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 1...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 851 vs. 2(140) = 280.
time = 1.79, size = 851, normalized size = 6.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d^2*n^4*x^5 + (b*x + a)^n*a*b^4*d^2*n^4*x^4 + 10*(b*x + a)
^n*b^5*d^2*n^3*x^5 + 2*(b*x + a)^n*b^5*c*d*n^4*x^3 + 6*(b*x + a)^n*a*b^4*d^
2*n^3*x^4 + 35*(b*x + a)^n*b^5*d^2*n^2*x^5 + 2*(b*x + a)^n*a*b^4*c*d*n^4*x^
2 + 24*(b*x + a)^n*b^5*c*d*n^3*x^3 - 4*(b*x + a)^n*a^2*b^3*d^2*n^3*x^3 + 11
*(b*x + a)^n*a*b^4*d^2*n^2*x^4 + 50*(b*x + a)^n*b^5*d^2*n*x^5 + (b*x + a)^n
*b^5*c^2*n^4*x + 20*(b*x + a)^n*a*b^4*c*d*n^3*x^2 + 98*(b*x + a)^n*b^5*c*d*
n^2*x^3 - 12*(b*x + a)^n*a^2*b^3*d^2*n^2*x^3 + 6*(b*x + a)^n*a*b^4*d^2*n*x^
4 + 24*(b*x + a)^n*b^5*d^2*x^5 + (b*x + a)^n*a*b^4*c^2*n^4 + 14*(b*x + a)^n
*b^5*c^2*n^3*x - 4*(b*x + a)^n*a^2*b^3*c*d*n^3*x + 58*(b*x + a)^n*a*b^4*c*d
*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n^2*x^2 + 156*(b*x + a)^n*b^5*c*d*n*x
^3 - 8*(b*x + a)^n*a^2*b^3*d^2*n*x^3 + 14*(b*x + a)^n*a*b^4*c^2*n^3 + 71*(b
*x + a)^n*b^5*c^2*n^2*x - 36*(b*x + a)^n*a^2*b^3*c*d*n^2*x + 40*(b*x + a)^n
*a*b^4*c*d*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d^2*n*x^2 + 80*(b*x + a)^n*b^5*c*
d*x^3 + 71*(b*x + a)^n*a*b^4*c^2*n^2 + 4*(b*x + a)^n*a^3*b^2*c*d*n^2 + 154*
(b*x + a)^n*b^5*c^2*n*x - 80*(b*x + a)^n*a^2*b^3*c*d*n*x - 24*(b*x + a)^n*a
^4*b*d^2*n*x + 154*(b*x + a)^n*a*b^4*c^2*n + 36*(b*x + a)^n*a^3*b^2*c*d*n +
 120*(b*x + a)^n*b^5*c^2*x + 120*(b*x + a)^n*a*b^4*c^2 + 80*(b*x + a)^n*a^3
*b^2*c*d + 24*(b*x + a)^n*a^5*d^2)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225
*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Mupad [B]
time = 2.84, size = 496, normalized size = 3.54

$$(a+b\,x)^n\,\left(\frac{a\,\left(24\,a^4\,d^2+4\,a^2\,b^2\,c\,d\,n^2+36\,a^2\,b^2\,c\,d\,n+80\,a^2\,b^2\,c\,d+b^4\,c^2\,n^4+14\,b^4\,c^2\,n^3+71\,b^4\,c^2\,n^2+154\,b^4\,c^2\,n+120\,b^4\,c^2\right)}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{d^2\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{x\,\left(-24\,a^4\,b\,d^2\,n-4\,a^2\,b^3\,c\,d\,n^3-36\,a^2\,b^3\,c\,d\,n^2-80\,a^2\,b^3\,c\,d\,n+b^5\,c^2\,n^4+14\,b^5\,c^2\,n^3+71\,b^5\,c^2\,n^2+154\,b^5\,c^2\,n+120\,b^5\,c^2\right)}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{2\,d\,x^3\,\left(n^2+3\,n+2\right)\,\left(-2\,d\,a^2\,n+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,d^2\,n\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{2\,a\,d\,n\,x^2\,\left(n+1\right)\,\left(6\,d\,a^2+c\,b^2\,n^2+9\,c\,b^2\,n+20\,c\,b^2\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((a*(24*a^4*d^2 + 120*b^4*c^2 + 154*b^4*c^2*n + 71*b^4*c^2*n^2
+ 14*b^4*c^2*n^3 + b^4*c^2*n^4 + 80*a^2*b^2*c*d + 36*a^2*b^2*c*d*n + 4*a^2*
b^2*c*d*n^2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (d^2*
x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4
 + n^5 + 120) + (x*(120*b^5*c^2 + 154*b^5*c^2*n + 71*b^5*c^2*n^2 + 14*b^5*c
^2*n^3 + b^5*c^2*n^4 - 24*a^4*b*d^2*n - 80*a^2*b^3*c*d*n - 36*a^2*b^3*c*d*n
^2 - 4*a^2*b^3*c*d*n^3))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (2*d*x^3*(3*n + n^2 + 2)*(20*b^2*c + b^2*c*n^2 - 2*a^2*d*n + 9*b^2*c*
n))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d^2*n*x^4*(1
1*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (2*a*d*n*x^2*(n + 1)*(6*a^2*d + 20*b^2*c + b^2*c*n^2 + 9*b^2*c*n))/(b^3*
(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.358 $\int \frac{(a+bx)^n(c+dx^2)^2}{x}\,dx$

Optimal. Leaf size=148

$$-\frac{ad(2b^2c+a^2d)\,(a+bx)^{1+n}}{b^4(1+n)}+\frac{d(2b^2c+3a^2d)\,(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)}+\frac{d^2(a+bx)^{4+n}}{b^4(4+n)}-\frac{c^2(a+bx)^{1+n}}{}$$

```
[Out] -a*d*(a^2*d+2*b^2*c)*(b*x+a)^(1+n)/b^4/(1+n)+d*(3*a^2*d+2*b^2*c)*(b*x+a)^(2
+n)/b^4/(2+n)-3*a*d^2*(b*x+a)^(3+n)/b^4/(3+n)+d^2*(b*x+a)^(4+n)/b^4/(4+n)-c
^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A]
time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {966, 1634, 67}

$$-\frac{ad(a^2d+2b^2c)\,(a+bx)^{n+1}}{b^4(n+1)}+\frac{d(3a^2d+2b^2c)\,(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)}+\frac{d^2(a+bx)^{n+4}}{b^4(n+4)}-\frac{c^2(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2)^2)/x,x]
```

```
[Out] -((a*d*(2*b^2*c + a^2*d)*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (d*(2*b^2*c +
3*a^2*d)*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^(3 + n))/(b^
4*(3 + n)) + (d^2*(a + b*x)^(4 + n))/(b^4*(4 + n)) - (c^2*(a + b*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(a+bx)^n\left(c+dx^2\right)^2}{x}\,dx = \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + \frac{\int \frac{(a+bx)^n\left(b^4c^2(4+n)-a^3bd^2(4+n)x+b^2d\left(2b^2c-3a^2d\right)(4+n)x^2-3ab^3d^2(4+n)x^3\right.}{x}}{b^4(4+n)}$$

$$= \frac{d^2(a+bx)^{4+n}}{b^4(4+n)} + \frac{\int \left(-abd\left(2b^2c+a^2d\right)(4+n)(a+bx)^n + \frac{\left(4b^4c^2+b^4c^2n\right)(a+bx)^n}{x}\right.}{b}$$

$$= -\frac{ad\left(2b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)} + \frac{d\left(2b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)}$$

$$= -\frac{ad\left(2b^2c+a^2d\right)(a+bx)^{1+n}}{b^4(1+n)} + \frac{d\left(2b^2c+3a^2d\right)(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad^2(a+bx)^{3+n}}{b^4(3+n)}$$

Mathematica [A]
time = 0.14, size = 132, normalized size = 0.89

$$(a+bx)^{1+n}\left(-\frac{ad\left(2b^2c+a^2d\right)}{b^4(1+n)}+\frac{d\left(2b^2c+3a^2d\right)(a+bx)}{b^4(2+n)}-\frac{3ad^2(a+bx)^2}{b^4(3+n)}+\frac{d^2(a+bx)^3}{b^4(4+n)}-\frac{c^2\,{}_2F_1\left(1,1+n;2+n;\frac{a+bx}{a}\right)}{a+an}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2)^2)/x,x]
[Out] (a + b*x)^(1 + n)*(-((a*d*(2*b^2*c + a^2*d))/(b^4*(1 + n))) + (d*(2*b^2*c +
 3*a^2*d)*(a + b*x))/(b^4*(2 + n)) - (3*a*d^2*(a + b*x)^2)/(b^4*(3 + n)) +
(d^2*(a + b*x)^3)/(b^4*(4 + n)) - (c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*x)/a])/(a + a*n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\left(d\,x^2+c\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^2/x,x)
[Out] int((b*x+a)^n*(d*x^2+c)^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x + a)^n/x, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. 2(129) = 258.
time = 3.71, size = 1678, normalized size = 11.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**2/x,x)
```

```
[Out] -b**n*c**2*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**2*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 2*c*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*
b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x),
Eq(n, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b
**2*n**2 + 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n
+ 2*b**2) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2
*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + d**2*Piecewise
((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*
x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2*b**5*x +
 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b**4 + 18
*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a/b + x)
/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*
x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3
*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*
```

```
x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6
*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x*log(
a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a**2*b**
4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4
*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x
**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3/(2*a*
b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2
*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)),
(-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b), Eq(n
, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 +
35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b*x)**n/(b
**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2
*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(b**4*n**4
 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n*x**3*(a
+ b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) +
 b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*
b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*x**4*(a
+ b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4),
True)) - b*b**n*c**2*n*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n
 + 1)/(a*gamma(n + 2)) - b*b**n*c**2*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1,
n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^2/x,x, algorithm="giac")
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^2\,{\left(a+b\,x\right)}^n}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^2*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^2)^2*(a + b*x)^n)/x, x)
```

3.359 $\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=343

$$\frac{a^2(b^2c+a^2d)^3\,(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a(b^2c+a^2d)^2\,(b^2c+4a^2d)\,(a+bx)^{2+n}}{b^9(2+n)}+\frac{(b^2c+a^2d)\,(b^4c^2+17a^2b^2cd+28a^4d^2)}{b^9(3+n)}$$

```
[Out] a^2*(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(a^2*d+b^2*c)^2*(4*a^2*d+b^
2*c)*(b*x+a)^(2+n)/b^9/(2+n)+(a^2*d+b^2*c)*(28*a^4*d^2+17*a^2*b^2*c*d+b^4*c
^2)*(b*x+a)^(3+n)/b^9/(3+n)-4*a*d*(14*a^4*d^2+15*a^2*b^2*c*d+3*b^4*c^2)*(b*
x+a)^(4+n)/b^9/(4+n)+d*(70*a^4*d^2+45*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(5+n)/
b^9/(5+n)-2*a*d^2*(28*a^2*d+9*b^2*c)*(b*x+a)^(6+n)/b^9/(6+n)+d^2*(28*a^2*d+
3*b^2*c)*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*(b*x+a
)^(9+n)/b^9/(9+n)
```

Rubi [A]

time = 0.14, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {962}

$$-\frac{2ad^2(28a^2d+9b^2c)\,(a+bx)^{n+6}}{b^9(n+6)}+\frac{d^2(28a^2d+3b^2c)\,(a+bx)^{n+7}}{b^9(n+7)}+\frac{a^2(a^2d+b^2c)^3\,(a+bx)^{n+1}}{b^9(n+1)}-\frac{2a(a^2d+b^2c)^2\,(4a^2d+b^2c)\,(a+bx)^{n+2}}{b^9(n+2)}+\frac{(a^2d+b^2c)\,(28a^4d^2+17a^2b^2cd+b^4c^2)\,(a+bx)^{n+3}}{b^9(n+3)}-\frac{4ad(14a^4d^2+15a^2b^2cd+3b^4c^2)\,(a+bx)^{n+4}}{b^9(n+4)}+\frac{d(70a^4d^2+45a^2b^2cd+3b^4c^2)\,(a+bx)^{n+5}}{b^9(n+5)}-\frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^3(a+bx)^{n+9}}{b^9(n+9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^2*c + a^2
*d)^2*(b^2*c + 4*a^2*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^2*c + a^2*d)
*(b^4*c^2 + 17*a^2*b^2*c*d + 28*a^4*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) -
 (4*a*d*(3*b^4*c^2 + 15*a^2*b^2*c*d + 14*a^4*d^2)*(a + b*x)^(4 + n))/(b^9*(
4 + n)) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d + 70*a^4*d^2)*(a + b*x)^(5 + n))/(
b^9*(5 + n)) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)^(6 + n))/(b^9*(6 + n
)) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*
(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^2\right)^3\,dx=\int\left(\frac{a^2(b^2c+a^2d)^3\,(a+bx)^n}{b^8}-\frac{2a(b^2c+a^2d)^2\,(b^2c+4a^2d)\,(a+bx)^{1+n}}{b^8}+\right.$$
$$=\frac{a^2(b^2c+a^2d)^3\,(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a(b^2c+a^2d)^2\,(b^2c+4a^2d)\,(a+bx)^{2+n}}{b^9(2+n)}+\frac{(b^2}{}$$

Mathematica [A]
time = 0.23, size = 302, normalized size = 0.88

$$\frac{(a+bx)^{1+n}\left(\frac{a^2(b^2c+a^2d)^3}{1+n}-\frac{2a(b^2c+a^2d)^2(b^2c+4a^2d)(a+bx)}{2+n}+\frac{(b^2c+a^2d)(b^4c^2+17a^2b^2cd+28a^4d^2)(a+bx)^2}{3+n}-\frac{4ad(3b^4c^2+15a^2b^2cd+14a^4d^2)(a+bx)^3}{4+n}+\frac{d(3b^4c^2+45a^2b^2cd+70a^4d^2)(a+bx)^4}{5+n}-\frac{2ad^2(9b^2c+28a^2d)(a+bx)^5}{6+n}+\frac{d^2(3b^2c+28a^2d)(a+bx)^6}{7+n}-\frac{8ad^3(a+bx)^7}{8+n}+\frac{d^3(a+bx)^8}{9+n}\right)}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^2*c + a^2*d)^3)/(1 + n) - (2*a*(b^2*c + a^2*d)^
2*(b^2*c + 4*a^2*d)*(a + b*x))/(2 + n) + ((b^2*c + a^2*d)*(b^4*c^2 + 17*a^2
*b^2*c*d + 28*a^4*d^2)*(a + b*x)^2)/(3 + n) - (4*a*d*(3*b^4*c^2 + 15*a^2*b^
2*c*d + 14*a^4*d^2)*(a + b*x)^3)/(4 + n) + (d*(3*b^4*c^2 + 45*a^2*b^2*c*d +
 70*a^4*d^2)*(a + b*x)^4)/(5 + n) - (2*a*d^2*(9*b^2*c + 28*a^2*d)*(a + b*x)
^5)/(6 + n) + (d^2*(3*b^2*c + 28*a^2*d)*(a + b*x)^6)/(7 + n) - (8*a*d^3*(a
+ b*x)^7)/(8 + n) + (d^3*(a + b*x)^8)/(9 + n)))/b^9
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. 2(343) = 686.
time = 0.10, size = 2232, normalized size = 6.51

method	result	size
gosper	Expression too large to display	2232
risch	Expression too large to display	2558

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^8*d^3*n^8*x^8+36*b^8*d^3*n^7*x^8-8*a*b^7*d^3*n^7*x^7+3*b^8
*c*d^2*n^8*x^6+546*b^8*d^3*n^6*x^8-224*a*b^7*d^3*n^6*x^7+114*b^8*c*d^2*n^7*
x^6+4536*b^8*d^3*n^5*x^8+56*a^2*b^6*d^3*n^6*x^6-18*a*b^7*c*d^2*n^7*x^5-2576
*a*b^7*d^3*n^5*x^7+3*b^8*c^2*d*n^8*x^4+1812*b^8*c*d^2*n^6*x^6+22449*b^8*d^3
*n^4*x^8+1176*a^2*b^6*d^3*n^5*x^6-576*a*b^7*c*d^2*n^6*x^5-15680*a*b^7*d^3*n
^4*x^7+120*b^8*c^2*d*n^7*x^4+15666*b^8*c*d^2*n^5*x^6+67284*b^8*d^3*n^3*x^8-
336*a^3*b^5*d^3*n^5*x^5+90*a^2*b^6*c*d^2*n^6*x^4+9800*a^2*b^6*d^3*n^4*x^6-1
2*a*b^7*c^2*d*n^7*x^3-7416*a*b^7*c*d^2*n^5*x^5-54152*a*b^7*d^3*n^3*x^7+b^8*
c^3*n^8*x^2+2010*b^8*c^2*d*n^6*x^4+80157*b^8*c*d^2*n^4*x^6+118124*b^8*d^3*n
```

```
^2*x^8-5040*a^3*b^5*d^3*n^4*x^5+2430*a^2*b^6*c*d^2*n^5*x^4+41160*a^2*b^6*d^
3*n^3*x^6-432*a*b^7*c^2*d*n^6*x^3-49500*a*b^7*c*d^2*n^4*x^5-105056*a*b^7*d^
3*n^2*x^7+42*b^8*c^3*n^7*x^2+18300*b^8*c^2*d*n^5*x^4+246876*b^8*c*d^2*n^3*x
^6+109584*b^8*d^3*n*x^8+1680*a^4*b^4*d^3*n^4*x^4-360*a^3*b^5*c*d^2*n^5*x^3-
28560*a^3*b^5*d^3*n^3*x^5+36*a^2*b^6*c^2*d*n^6*x^2+24930*a^2*b^6*c*d^2*n^4*
x^4+90944*a^2*b^6*d^3*n^2*x^6-2*a*b^7*c^3*n^7*x-6312*a*b^7*c^2*d*n^5*x^3-18
3942*a*b^7*c*d^2*n^3*x^5-104544*a*b^7*d^3*n*x^7+744*b^8*c^3*n^6*x^2+98319*b
^8*c^2*d*n^4*x^4+442908*b^8*c*d^2*n^2*x^6+40320*b^8*d^3*x^8+16800*a^4*b^4*d
^3*n^3*x^4-8280*a^3*b^5*c*d^2*n^4*x^3-75600*a^3*b^5*d^3*n^2*x^5+1188*a^2*b^
6*c^2*d*n^5*x^2+122850*a^2*b^6*c*d^2*n^3*x^4+98784*a^2*b^6*d^3*n*x^6-80*a*b
^7*c^3*n^6*x-47952*a*b^7*c^2*d*n^4*x^3-377604*a*b^7*c*d^2*n^2*x^5-40320*a*b
^7*d^3*x^7+7218*b^8*c^3*n^5*x^2+316380*b^8*c^2*d*n^3*x^4+417744*b^8*c*d^2*n
*x^6-6720*a^5*b^3*d^3*n^3*x^3+1080*a^4*b^4*c*d^2*n^4*x^2+58800*a^4*b^4*d^3*
n^2*x^4-72*a^3*b^5*c^2*d*n^5*x-66600*a^3*b^5*c*d^2*n^3*x^3-92064*a^3*b^5*d^
3*n*x^5+2*a^2*b^6*c^3*n^6+15372*a^2*b^6*c^2*d*n^4*x^2+305460*a^2*b^6*c*d^2*
n^2*x^4+40320*a^2*b^6*d^3*x^6-1328*a*b^7*c^3*n^5*x-201468*a*b^7*c^2*d*n^3*x
^3-391824*a*b^7*c*d^2*n*x^5+41619*b^8*c^3*n^4*x^2+589140*b^8*c^2*d*n^2*x^4+
155520*b^8*c*d^2*x^6-40320*a^5*b^3*d^3*n^2*x^3+21600*a^4*b^4*c*d^2*n^3*x^2+
84000*a^4*b^4*d^3*n*x^4-2232*a^3*b^5*c^2*d*n^4*x-225000*a^3*b^5*c*d^2*n^2*x
^3-40320*a^3*b^5*d^3*x^5+78*a^2*b^6*c^3*n^5+97740*a^2*b^6*c^2*d*n^3*x^2+360
720*a^2*b^6*c*d^2*n*x^4-11780*a*b^7*c^3*n^4*x-459648*a*b^7*c^2*d*n^2*x^3-15
5520*a*b^7*c*d^2*x^5+144468*b^8*c^3*n^3*x^2+572400*b^8*c^2*d*n*x^4+20160*a^
6*b^2*d^3*n^2*x^2-2160*a^5*b^3*c*d^2*n^3*x-73920*a^5*b^3*d^3*n*x^3+72*a^4*b
^4*c^2*d*n^4+135000*a^4*b^4*c*d^2*n^2*x^2+40320*a^4*b^4*d^3*x^4-26280*a^3*b
^5*c^2*d*n^3*x-321840*a^3*b^5*c*d^2*n*x^3+1250*a^2*b^6*c^3*n^4+311184*a^2*b
^6*c^2*d*n^2*x^2+155520*a^2*b^6*c*d^2*x^4-59678*a*b^7*c^3*n^3*x-517968*a*b^
7*c^2*d*n*x^3+290276*b^8*c^3*n^2*x^2+217728*b^8*c^2*d*x^4+60480*a^6*b^2*d^3
*n*x^2-38880*a^5*b^3*c*d^2*n^2*x-40320*a^5*b^3*d^3*x^3+2160*a^4*b^4*c^2*d*n
^3+270000*a^4*b^4*c*d^2*n*x^2-142920*a^3*b^5*c^2*d*n^2*x-155520*a^3*b^5*c*d
^2*x^3+10530*a^2*b^6*c^3*n^3+445392*a^2*b^6*c^2*d*n*x^2-169580*a*b^7*c^3*n^
2*x-217728*a*b^7*c^2*d*x^3+301872*b^8*c^3*n*x^2-40320*a^7*b*d^3*n*x+2160*a^
6*b^2*c*d^2*n^2+40320*a^6*b^2*d^3*x^2-192240*a^5*b^3*c*d^2*n*x+24120*a^4*b^
4*c^2*d*n^2+155520*a^4*b^4*c*d^2*x^2-336528*a^3*b^5*c^2*d*n*x+49148*a^2*b^6
*c^3*n^2+217728*a^2*b^6*c^2*d*x^2-241392*a*b^7*c^3*n*x+120960*b^8*c^3*x^2-4
0320*a^7*b*d^3*x+36720*a^6*b^2*c*d^2*n-155520*a^5*b^3*c*d^2*x+118800*a^4*b^
4*c^2*d*n-217728*a^3*b^5*c^2*d*x+120432*a^2*b^6*c^3*n-120960*a*b^7*c^3*x+40
320*a^8*d^3+155520*a^6*b^2*c*d^2+217728*a^4*b^4*c^2*d+120960*a^2*b^6*c^3)/b
^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+723680*n^3+1172700*n^2
+1026576*n+362880)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(343) = 686$.
time = 0.32, size = 795, normalized size = 2.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*
n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x
+ a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + 3*((n^
6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 +
15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 +
35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b
^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 -
720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*
n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^8 + 36*n^7 + 546*n^
6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^9*x
^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2
+ 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764
*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x
^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*
n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)
*(b*x + a)^n*d^3/((n^9 + 45*n^8 + 870*n^7 + 9450*n^6 + 63273*n^5 + 269325*n
^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)*b^9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2165 vs. $2(343) = 686$.
time = 1.94, size = 2165, normalized size = 6.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^3*n^6 + 78*a^3*b^6*c^3*n^5 + 120960*a^3*b^6*c^3 + 217728*a^5*b
^4*c^2*d + 155520*a^7*b^2*c*d^2 + 40320*a^9*d^3 + (b^9*d^3*n^8 + 36*b^9*d^3
*n^7 + 546*b^9*d^3*n^6 + 4536*b^9*d^3*n^5 + 22449*b^9*d^3*n^4 + 67284*b^9*d
^3*n^3 + 118124*b^9*d^3*n^2 + 109584*b^9*d^3*n + 40320*b^9*d^3)*x^9 + (a*b^
8*d^3*n^8 + 28*a*b^8*d^3*n^7 + 322*a*b^8*d^3*n^6 + 1960*a*b^8*d^3*n^5 + 676
9*a*b^8*d^3*n^4 + 13132*a*b^8*d^3*n^3 + 13068*a*b^8*d^3*n^2 + 5040*a*b^8*d^
3*n)*x^8 + (3*b^9*c*d^2*n^8 + 155520*b^9*c*d^2 + 2*(57*b^9*c*d^2 - 4*a^2*b^
7*d^3)*n^7 + 12*(151*b^9*c*d^2 - 14*a^2*b^7*d^3)*n^6 + 14*(1119*b^9*c*d^2 -
 100*a^2*b^7*d^3)*n^5 + 21*(3817*b^9*c*d^2 - 280*a^2*b^7*d^3)*n^4 + 28*(881
7*b^9*c*d^2 - 464*a^2*b^7*d^3)*n^3 + 36*(12303*b^9*c*d^2 - 392*a^2*b^7*d^3)
*n^2 + 144*(2901*b^9*c*d^2 - 40*a^2*b^7*d^3)*n)*x^7 + (3*a*b^8*c*d^2*n^8 +
96*a*b^8*c*d^2*n^7 + 4*(309*a*b^8*c*d^2 + 14*a^3*b^6*d^3)*n^6 + 30*(275*a*b
```

```
^8*c*d^2 + 28*a^3*b^6*d^3)*n^5 + (30657*a*b^8*c*d^2 + 4760*a^3*b^6*d^3)*n^4
 + 6*(10489*a*b^8*c*d^2 + 2100*a^3*b^6*d^3)*n^3 + 8*(8163*a*b^8*c*d^2 + 191
8*a^3*b^6*d^3)*n^2 + 960*(27*a*b^8*c*d^2 + 7*a^3*b^6*d^3)*n)*x^6 + 3*(b^9*c
^2*d*n^8 + 72576*b^9*c^2*d + 2*(20*b^9*c^2*d - 3*a^2*b^7*c*d^2)*n^7 + 2*(33
5*b^9*c^2*d - 81*a^2*b^7*c*d^2)*n^6 + 2*(3050*b^9*c^2*d - 831*a^2*b^7*c*d^2
 - 56*a^4*b^5*d^3)*n^5 + (32773*b^9*c^2*d - 8190*a^2*b^7*c*d^2 - 1120*a^4*b
^5*d^3)*n^4 + 4*(26365*b^9*c^2*d - 5091*a^2*b^7*c*d^2 - 980*a^4*b^5*d^3)*n^
3 + 4*(49095*b^9*c^2*d - 6012*a^2*b^7*c*d^2 - 1400*a^4*b^5*d^3)*n^2 + 48*(3
975*b^9*c^2*d - 216*a^2*b^7*c*d^2 - 56*a^4*b^5*d^3)*n)*x^5 + 2*(625*a^3*b^6
*c^3 + 36*a^5*b^4*c^2*d)*n^4 + 3*(a*b^8*c^2*d*n^8 + 36*a*b^8*c^2*d*n^7 + 2*
(263*a*b^8*c^2*d + 15*a^3*b^6*c*d^2)*n^6 + 6*(666*a*b^8*c^2*d + 115*a^3*b^6
*c*d^2)*n^5 + (16789*a*b^8*c^2*d + 5550*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^
4 + 6*(6384*a*b^8*c^2*d + 3125*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^3 + 4*(10
791*a*b^8*c^2*d + 6705*a^3*b^6*c*d^2 + 1540*a^5*b^4*d^3)*n^2 + 96*(189*a*b^
8*c^2*d + 135*a^3*b^6*c*d^2 + 35*a^5*b^4*d^3)*n)*x^4 + 270*(39*a^3*b^6*c^3
 + 8*a^5*b^4*c^2*d)*n^3 + (b^9*c^3*n^8 + 120960*b^9*c^3 + 6*(7*b^9*c^3 - 2*a
^2*b^7*c^2*d)*n^7 + 12*(62*b^9*c^3 - 33*a^2*b^7*c^2*d)*n^6 + 6*(1203*b^9*c^
3 - 854*a^2*b^7*c^2*d - 60*a^4*b^5*c*d^2)*n^5 + 3*(13873*b^9*c^3 - 10860*a^
2*b^7*c^2*d - 2400*a^4*b^5*c*d^2)*n^4 + 12*(12039*b^9*c^3 - 8644*a^2*b^7*c^
2*d - 3750*a^4*b^5*c*d^2 - 560*a^6*b^3*d^3)*n^3 + 4*(72569*b^9*c^3 - 37116*
a^2*b^7*c^2*d - 22500*a^4*b^5*c*d^2 - 5040*a^6*b^3*d^3)*n^2 + 48*(6289*b^9*
c^3 - 1512*a^2*b^7*c^2*d - 1080*a^4*b^5*c*d^2 - 280*a^6*b^3*d^3)*n)*x^3 + 4
*(12287*a^3*b^6*c^3 + 6030*a^5*b^4*c^2*d + 540*a^7*b^2*c*d^2)*n^2 + (a*b^8*
c^3*n^8 + 40*a*b^8*c^3*n^7 + 4*(166*a*b^8*c^3 + 9*a^3*b^6*c^2*d)*n^6 + 62*(
95*a*b^8*c^3 + 18*a^3*b^6*c^2*d)*n^5 + (29839*a*b^8*c^3 + 13140*a^3*b^6*c^2
*d + 1080*a^5*b^4*c*d^2)*n^4 + 10*(8479*a*b^8*c^3 + 7146*a^3*b^6*c^2*d + 19
44*a^5*b^4*c*d^2)*n^3 + 24*(5029*a*b^8*c^3 + 7011*a^3*b^6*c^2*d + 4005*a^5*
b^4*c*d^2 + 840*a^7*b^2*d^3)*n^2 + 576*(105*a*b^8*c^3 + 189*a^3*b^6*c^2*d +
 135*a^5*b^4*c*d^2 + 35*a^7*b^2*d^3)*n)*x^2 + 48*(2509*a^3*b^6*c^3 + 2475*a
^5*b^4*c^2*d + 765*a^7*b^2*c*d^2)*n - 2*(a^2*b^7*c^3*n^7 + 39*a^2*b^7*c^3*n
^6 + (625*a^2*b^7*c^3 + 36*a^4*b^5*c^2*d)*n^5 + 135*(39*a^2*b^7*c^3 + 8*a^4
*b^5*c^2*d)*n^4 + 2*(12287*a^2*b^7*c^3 + 6030*a^4*b^5*c^2*d + 540*a^6*b^3*c
*d^2)*n^3 + 24*(2509*a^2*b^7*c^3 + 2475*a^4*b^5*c^2*d + 765*a^6*b^3*c*d^2)*
n^2 + 576*(105*a^2*b^7*c^3 + 189*a^4*b^5*c^2*d + 135*a^6*b^3*c*d^2 + 35*a^8
*b*d^3)*n)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^
6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 102
6576*b^9*n + 362880*b^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 35984 vs. $2(328) = 656$.
time = 12.14, size = 35984, normalized size = 104.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x**3/3 + 3*c**2*d*x**5/5 + 3*c*d**2*x**7/7 + d**3*x**
9/9), Eq(b, 0)), (840*a**8*d**3*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**
10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**
4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840
*b**17*x**8) + 2283*a**8*d**3/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a*
*6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*
b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) +
6720*a**7*b*d**3*x*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*
a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**
3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8)
+ 17424*a**7*b*d**3*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11
*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x*
*5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 45*a**6*
b**2*c*d**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47
040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520
*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 23520*a**6*b**2*d*
*3*x**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*
x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**
5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 57624*a**
6*b**2*d**3*x**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2
 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 +
23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 360*a**5*b**3
*c*d**2*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 4704
0*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a
**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 47040*a**5*b**3*d**3
*x**3*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x*
*2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5
+ 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 107408*a**5
*b**3*d**3*x**3/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2
+ 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 2
3520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 9*a**4*b**4*c*
*2*d/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**
5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b
**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 1260*a**4*b**4*c*d**2*x**
2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b
**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**1
5*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 58800*a**4*b**4*d**3*x**4*lo
g(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 470
40*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*
a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 122500*a**4*b**4*d*
*3*x**4/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*
a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**
2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 72*a**3*b**5*c**2*d*x/
```

```
(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**
12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*
x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 2520*a**3*b**5*c*d**2*x**3/(84
0*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*
x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**
6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 47040*a**3*b**5*d**3*x**5*log(a/b
 + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a*
*5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*
b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 86240*a**3*b**5*d**3*x**
5/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b
**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**1
5*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 5*a**2*b**6*c**3/(840*a**8*b
**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 5
8800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720
*a*b**16*x**7 + 840*b**17*x**8) - 252*a**2*b**6*c**2*d*x**2/(840*a**8*b**9
+ 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800
*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b
**16*x**7 + 840*b**17*x**8) - 3150*a**2*b**6*c*d**2*x**4/(840*a**8*b**9 + 6
720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a*
*4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**1
6*x**7 + 840*b**17*x**8) + 23520*a**2*b**6*d**3*x**6*log(a/b + x)/(840*a**8
*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 +
 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 67
20*a*b**16*x**7 + 840*b**17*x**8) + 35280*a**2*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3713 vs. $2(343) = 686$.
time = 1.24, size = 3713, normalized size = 10.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^3*n^8*x^9 + (b*x + a)^n*a*b^8*d^3*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^3*n^7*x^9 + 3*(b*x + a)^n*b^9*c*d^2*n^8*x^7 + 28*(b*x + a)^n*a*b^8
*d^3*n^7*x^8 + 546*(b*x + a)^n*b^9*d^3*n^6*x^9 + 3*(b*x + a)^n*a*b^8*c*d^2*
n^8*x^6 + 114*(b*x + a)^n*b^9*c*d^2*n^7*x^7 - 8*(b*x + a)^n*a^2*b^7*d^3*n^7
*x^7 + 322*(b*x + a)^n*a*b^8*d^3*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^3*n^5*x^9
 + 3*(b*x + a)^n*b^9*c^2*d*n^8*x^5 + 96*(b*x + a)^n*a*b^8*c*d^2*n^7*x^6 + 1
812*(b*x + a)^n*b^9*c*d^2*n^6*x^7 - 168*(b*x + a)^n*a^2*b^7*d^3*n^6*x^7 + 1
960*(b*x + a)^n*a*b^8*d^3*n^5*x^8 + 22449*(b*x + a)^n*b^9*d^3*n^4*x^9 + 3*(
b*x + a)^n*a*b^8*c^2*d*n^8*x^4 + 120*(b*x + a)^n*b^9*c^2*d*n^7*x^5 - 18*(b*
x + a)^n*a^2*b^7*c*d^2*n^7*x^5 + 1236*(b*x + a)^n*a*b^8*c*d^2*n^6*x^6 + 56*
(b*x + a)^n*a^3*b^6*d^3*n^6*x^6 + 15666*(b*x + a)^n*b^9*c*d^2*n^5*x^7 - 140
```

0*(b*x + a)^n*a^2*b^7*d^3*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^3*n^4*x^8 + 67284*(b*x + a)^n*b^9*d^3*n^3*x^9 + (b*x + a)^n*b^9*c^3*n^8*x^3 + 108*(b*x + a)^n*a*b^8*c^2*d*n^7*x^4 + 2010*(b*x + a)^n*b^9*c^2*d*n^6*x^5 - 486*(b*x + a)^n*a^2*b^7*c*d^2*n^6*x^5 + 8250*(b*x + a)^n*a*b^8*c*d^2*n^5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^3*n^5*x^6 + 80157*(b*x + a)^n*b^9*c*d^2*n^4*x^7 - 5880*(b*x + a)^n*a^2*b^7*d^3*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^3*n^3*x^8 + 118124*(b*x + a)^n*b^9*d^3*n^2*x^9 + (b*x + a)^n*a*b^8*c^3*n^8*x^2 + 42*(b*x + a)^n*b^9*c^3*n^7*x^3 - 12*(b*x + a)^n*a^2*b^7*c^2*d*n^7*x^3 + 1578*(b*x + a)^n*a*b^8*c^2*d*n^6*x^4 + 90*(b*x + a)^n*a^3*b^6*c*d^2*n^6*x^4 + 18300*(b*x + a)^n*b^9*c^2*d*n^5*x^5 - 4986*(b*x + a)^n*a^2*b^7*c*d^2*n^5*x^5 - 336*(b*x + a)^n*a^4*b^5*d^3*n^5*x^5 + 30657*(b*x + a)^n*a*b^8*c*d^2*n^4*x^6 + 4760*(b*x + a)^n*a^3*b^6*d^3*n^4*x^6 + 246876*(b*x + a)^n*b^9*c*d^2*n^3*x^7 - 12992*(b*x + a)^n*a^2*b^7*d^3*n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^3*n^2*x^8 + 109584*(b*x + a)^n*b^9*d^3*n*x^9 + 40*(b*x + a)^n*a*b^8*c^3*n^7*x^2 + 744*(b*x + a)^n*b^9*c^3*n^6*x^3 - 396*(b*x + a)^n*a^2*b^7*c^2*d*n^6*x^3 + 11988*(b*x + a)^n*a*b^8*c^2*d*n^5*x^4 + 2070*(b*x + a)^n*a^3*b^6*c*d^2*n^5*x^4 + 98319*(b*x + a)^n*b^9*c^2*d*n^4*x^5 - 24570*(b*x + a)^n*a^2*b^7*c*d^2*n^4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^3*n^4*x^5 + 62934*(b*x + a)^n*a*b^8*c*d^2*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^3*n^3*x^6 + 442908*(b*x + a)^n*b^9*c*d^2*n^2*x^7 - 14112*(b*x + a)^n*a^2*b^7*d^3*n^2*x^7 + 5040*(b*x + a)^n*a*b^8*d^3*n*x^8 + 40320*(b*x + a)^n*b^9*d^3*x^9 - 2*(b*x + a)^n*a^2*b^7*c^3*n^7*x + 664*(b*x + a)^n*a*b^8*c^3*n^6*x^2 + 36*(b*x + a)^n*a^3*b^6*c^2*d*n^6*x^2 + 7218*(b*x + a)^n*b^9*c^3*n^5*x^3 - 5124*(b*x + a)^n*a^2*b^7*c^2*d*n^5*x^3 - 360*(b*x + a)^n*a^4*b^5*c*d^2*n^5*x^3 + 50367*(b*x + a)^n*a*b^8*c^2*d*n^4*x^4 + 16650*(b*x + a)^n*a^3*b^6*c*d^2*n^4*x^4 + 1680*(b*x + a)^n*a^5*b^4*d^3*n^4*x^4 + 316380*(b*x + a)^n*b^9*c^2*d*n^3*x^5 - 61092*(b*x + a)^n*a^2*b^7*c*d^2*n^3*x^5 - 11760*(b*x + a)^n*a^4*b^5*d^3*n^3*x^5 + 65304*(b*x + a)^n*a*b^8*c*d^2*n^2*x^6 + 15344*(b*x + a)^n*a^3*b^6*d^3*n^2*x^6 + 417744*(b*x + a)^n*b^9*c*d^2*n*x^7 - 5760*(b*x + a)^n*a^2*b^7*d^3*n*x^7 - 78*(b*x + a)^n*a^2*b^7*c^3*n^6*x + 5890*(b*x + a)^n*a*b^8*c^3*n^5*x^2 + 1116*(b*x + a)^n*a^3*b^6*c^2*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^3*n^4*x^3 - 32580*(b*x + a)^n*a^2*b^7*c^2*d*n^4*x^3 - 7200*(b*x + a)^n*a^4*b^5*c*d^2*n^4*x^3 + 114912*(b*x + a)^n*a*b^8*c^2*d*n^3*x^4 + 56250*(b*x + a)^n*a^3*b^6*c*d^2*n^3*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n^3*x^4 + 589140*(b*x + a)^n*b^9*c^2*d*n^2*x^5 - 72144*(b*x + a)^n*a^2*b^7*c*d^2*n^2*x^5 - 16800*(b*x + a)^n*a^4*b^5*d^3*n^2*x^5 + 25920*(b*x + a)^n*a*b^8*c*d^2*n*x^6 + 6720*(b*x + a)^n*a^3*b^6*d^3*n*x^6 + 155520*(b*x + a)^n*b^9*c*d^2*x^7 + 2*(b*x + a)^n*a^3*b^6*c^3*n^6 - 1250*(b*x + a)^n*a^2*b^7*c^3*n^5*x - 72*(b*x + a)^n*a^4*b^5*c^2*d*n^5*x + 29839*(b*x + a)^n*a*b^8*c^3*n^4*x^2 + 13140*(b*x + a)^n*a^3*b^6*c^2*d*n^4*x^2 + 1080*(b*x + a)^n*a^5*b^4*c*d^2*n^4*x^2 + 144468*(b*x + a)^n*b^9*c^3*n^3*x^3 - 103728*(b*x + a)^n*a^2*b^7*c^2*d*n^3*x^3 - 45000*(b*x + a)^n*a^4*b^5*c*d^2*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^3*n^3*x^3 + 129492*(b*x + a)^n*a*b^8*c^2*d*n^2*x^4 + 80460*(b*x + a)^n*a^3*b^6*c*d^2*n^2*x^4 + 18480*(b*x + a)^n*a^5*b^4*d^3*n^2*x^4 + 572400*(b*x + a)^n*b^9*c^2*d*n*x^5 - 31104*(b*x + a)^n*a^2*b^7*c*d^2*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^3*n*x

```
^5 + 78*(b*x + a)^n*a^3*b^6*c^3*n^5 - 10530*(b*x + a)^n*a^2*b^7*c^3*n^4*x -
 2160*(b*x + a)^n*a^4*b^5*c^2*d*n^4*x + 84790*(b*x + a)^n*a*b^8*c^3*n^3*x^2
 + 71460*(b*x + a)^n*a^3*b^6*c^2*d*n^3*x^2 + 19440*(b*x + a)^n*a^5*b^4*c*d^
2*n^3*x^2 + 290276*(b*x + a)^n*b^9*c^3*n^2*x^3 - 148464*(b*x + a)^n*a^2*b^7
*c^2*d*n^2*x^3 - 90000*(b*x + a)^n*a^4*b^5*c*d^2*n^2*x^3 - 20160*(b*x + a)^
n*a^6*b^3*d^3*n^2*x^3 + 54432*(b*x + a)^n*a*b^8*c^2*d*n*x^4 + 38880*(b*x +
a)^n*a^3*b^6*c*d^2*n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^3*n*x^4 + 217728*(b*
x + a)^n*b^9*c^2*d*x^5 + 1250*(b*x + a)^n*a^3*b^6*c^3*n^4 + 72*(b*x + a)^n*
a^5*b^4*c^2*d*n^4 - 49148*(b*x + a)^n*a^2*b^7*c^3*n^3*x - 24120*(b*x + a)^n
*a^4*b^5*c^2*d*n^3*x - 2160*(b*x + a)^n*a^6*b^3*c*d^2*n^3*x + 120696*(b*x +
 a)^n*a*b^8*c^3*n^2*x^2 + 168264*(b*x + a)^n*a^...
```

Mupad [B]
time = 3.81, size = 1796, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*n^
3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880) +
(2*a^3*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 24574
*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b^6*c^
3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^2*d*n
+ 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d^2*n^2
+ 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^9*(1026576*n + 1172700
*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
n^9 + 362880)) - (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(6720*a^6*d^3*n - 60480*b
^6*c^3 - 60216*b^6*c^3*n - 24574*b^6*c^3*n^2 - 5265*b^6*c^3*n^3 - 625*b^6*c
^3*n^4 - 39*b^6*c^3*n^5 - b^6*c^3*n^6 + 36288*a^2*b^4*c^2*d*n + 25920*a^4*b
^2*c*d^2*n + 19800*a^2*b^4*c^2*d*n^2 + 6120*a^4*b^2*c*d^2*n^2 + 4020*a^2*b^
4*c^2*d*n^3 + 360*a^4*b^2*c*d^2*n^3 + 360*a^2*b^4*c^2*d*n^4 + 12*a^2*b^4*c^
2*d*n^5))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n
^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (3*d*x^5*(a + b*x)^n*(5
0*n + 35*n^2 + 10*n^3 + n^4 + 24)*(3024*b^4*c^2 - 112*a^4*d^2*n + 1650*b^4*
c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 + b^4*c^2*n^4 - 432*a^2*b^2*c*d*n
- 102*a^2*b^2*c*d*n^2 - 6*a^2*b^2*c*d*n^3))/(b^4*(1026576*n + 1172700*n^2 +
 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 +
362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^
6*c^3*n + 24574*b^6*c^3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c
^3*n^5 + b^6*c^3*n^6 + 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a
^2*b^4*c^2*d*n + 18360*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4
*b^2*c*d^2*n^2 + 1080*a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^8*(1026
```

576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*
n^7 + 45*n^8 + n^9 + 362880)) + (d^2*x^7*(a + b*x)^n*(216*b^2*c + 3*b^2*c*n
^2 - 8*a^2*d*n + 51*b^2*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^
5 + n^6 + 720))/(b^2*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 6
3273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (a*n*x^2*(n + 1)*
(a + b*x)^n*(20160*a^6*d^3 + 60480*b^6*c^3 + 60216*b^6*c^3*n + 24574*b^6*c^
3*n^2 + 5265*b^6*c^3*n^3 + 625*b^6*c^3*n^4 + 39*b^6*c^3*n^5 + b^6*c^3*n^6 +
 108864*a^2*b^4*c^2*d + 77760*a^4*b^2*c*d^2 + 59400*a^2*b^4*c^2*d*n + 18360
*a^4*b^2*c*d^2*n + 12060*a^2*b^4*c^2*d*n^2 + 1080*a^4*b^2*c*d^2*n^2 + 1080*
a^2*b^4*c^2*d*n^3 + 36*a^2*b^4*c^2*d*n^4))/(b^7*(1026576*n + 1172700*n^2 +
723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 3
62880)) + (a*d^3*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n
^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b*(1026576*n + 1172700*n^2 + 723680*n
^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))
 + (a*d^2*n*x^6*(a + b*x)^n*(56*a^2*d + 216*b^2*c + 3*b^2*c*n^2 + 51*b^2*c*n
)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(1026576*n + 117270
0*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
 n^9 + 362880)) + (3*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(560*a^
4*d^2 + 3024*b^4*c^2 + 1650*b^4*c^2*n + 335*b^4*c^2*n^2 + 30*b^4*c^2*n^3 +
b^4*c^2*n^4 + 2160*a^2*b^2*c*d + 510*a^2*b^2*c*d*n + 30*a^2*b^2*c*d*n^2))/(
b^5*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n
^6 + 870*n^7 + 45*n^8 + n^9 + 362880))

3.360 $\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=282

$$-\frac{a\left(b^2c+a^2d\right)^3 (a+bx)^{1+n}}{b^8(1+n)}+\frac{\left(b^2c+a^2d\right)^2 \left(b^2c+7a^2d\right) (a+bx)^{2+n}}{b^8(2+n)}-\frac{3ad\left(b^2c+a^2d\right) \left(3b^2c+7a^2d\right) (a+bx)^{3+n}}{b^8(3+n)}$$

```
[Out] -a*(a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^8/(1+n)+(a^2*d+b^2*c)^2*(7*a^2*d+b^2*c)*
(b*x+a)^(2+n)/b^8/(2+n)-3*a*d*(a^2*d+b^2*c)*(7*a^2*d+3*b^2*c)*(b*x+a)^(3+n)
/b^8/(3+n)+d*(35*a^4*d^2+30*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(4+n)/b^8/(4+n)-
5*a*d^2*(7*a^2*d+3*b^2*c)*(b*x+a)^(5+n)/b^8/(5+n)+3*d^2*(7*a^2*d+b^2*c)*(b*
x+a)^(6+n)/b^8/(6+n)-7*a*d^3*(b*x+a)^(7+n)/b^8/(7+n)+d^3*(b*x+a)^(8+n)/b^8/
(8+n)
```

Rubi [A]

time = 0.11, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{786\}$

$$-\frac{5ad^2\left(7a^2d+3b^2c\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{3d^2\left(7a^2d+b^2c\right)(a+bx)^{n+6}}{b^8(n+6)}-\frac{a\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(a^2d+b^2c\right)^2\left(7a^2d+b^2c\right)(a+bx)^{n+2}}{b^8(n+2)}-\frac{3ad\left(a^2d+b^2c\right)\left(7a^2d+3b^2c\right)(a+bx)^{n+3}}{b^8(n+3)}+\frac{d\left(35a^4d^2+30a^2b^2cd+3b^4c^2\right)(a+bx)^{n+4}}{b^8(n+4)}-\frac{7ad^3(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^3(a+bx)^{n+8}}{b^8(n+8)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] -((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d)
^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a^2
*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2 +
30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^2*(3
*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7*a^2*
d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^8*(7 +
 n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx)^n \left(c+dx^2\right)^3 \, dx = \int \left(-\frac{a\left(b^2c+a^2d\right)^3 (a+bx)^n}{b^7}+\frac{\left(b^2c+a^2d\right)^2 \left(b^2c+7a^2d\right) (a+bx)^{1+n}}{b^7}+\frac{3ad(\ldots\right.$$

$$= -\frac{a\left(b^2c+a^2d\right)^3 (a+bx)^{1+n}}{b^8(1+n)}+\frac{\left(b^2c+a^2d\right)^2 \left(b^2c+7a^2d\right) (a+bx)^{2+n}}{b^8(2+n)}-\frac{3ad(b^2\ldots}{\ldots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 578 vs. 2(282) = 564.
time = 0.38, size = 578, normalized size = 2.05

$$\frac{(a+bx)^{1+n}\left(-5040a^7d^3+5040a^6bd^3(1+n)x-360a^5b^2d^2(c(56+15n+n^2)+7d(2+3n+n^2)x^2)+120a^4b^3d^2(1+n)x(3c(56+15n+n^2)+7d(6+5n+n^2)x^2)-6a^3b^4d(3c^2(1680+1066n+251n^2+26n^3+n^4)+30cd(112+198n+103n^2+18n^3+n^4)x^2+35d^2(24+50n+35n^2+10n^3+n^4)x^4)+6a^2b^5d(1+n)x(3c^2(1680+1066n+251n^2+26n^3+n^4)+10cd(336+370n+137n^2+20n^3+n^4)x^2+7d^2(120+154n+71n^2+14n^3+n^4)x^4)+b^7(105+176n+86n^2+16n^3+n^4)x(c^3(192+104n+18n^2+n^3)+3c^2d(96+76n+16n^2+n^3)x^2+3cd^2(64+56n+14n^2+n^3)x^4+d^3(48+44n+12n^2+n^3)x^6)-ab^6(c^3(20160+24552n+12154n^2+3135n^3+445n^4+33n^5+n^6)+9c^2d(3360+7172n+5380n^2+1871n^3+331n^4+29n^5+n^6)x^2+15cd^2(1344+3160n+2734n^2+1135n^3+241n^4+25n^5+n^6)x^4+7d^3(720+1764n+1624n^2+735n^3+175n^4+21n^5+n^6)x^6)\right)}{b^8(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)(7+n)(8+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-5040*a^7*d^3 + 5040*a^6*b*d^3*(1 + n)*x - 360*a^5*b^2*
d^2*(c*(56 + 15*n + n^2) + 7*d*(2 + 3*n + n^2)*x^2) + 120*a^4*b^3*d^2*(1 +
n)*x*(3*c*(56 + 15*n + n^2) + 7*d*(6 + 5*n + n^2)*x^2) - 6*a^3*b^4*d*(3*c^2
*(1680 + 1066*n + 251*n^2 + 26*n^3 + n^4) + 30*c*d*(112 + 198*n + 103*n^2 +
 18*n^3 + n^4)*x^2 + 35*d^2*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^4) + 6*a^
2*b^5*d*(1 + n)*x*(3*c^2*(1680 + 1066*n + 251*n^2 + 26*n^3 + n^4) + 10*c*d*
(336 + 370*n + 137*n^2 + 20*n^3 + n^4)*x^2 + 7*d^2*(120 + 154*n + 71*n^2 +
14*n^3 + n^4)*x^4) + b^7*(105 + 176*n + 86*n^2 + 16*n^3 + n^4)*x*(c^3*(192
+ 104*n + 18*n^2 + n^3) + 3*c^2*d*(96 + 76*n + 16*n^2 + n^3)*x^2 + 3*c*d^2*
(64 + 56*n + 14*n^2 + n^3)*x^4 + d^3*(48 + 44*n + 12*n^2 + n^3)*x^6) - a*b^
6*(c^3*(20160 + 24552*n + 12154*n^2 + 3135*n^3 + 445*n^4 + 33*n^5 + n^6) +
9*c^2*d*(3360 + 7172*n + 5380*n^2 + 1871*n^3 + 331*n^4 + 29*n^5 + n^6)*x^2
+ 15*c*d^2*(1344 + 3160*n + 2734*n^2 + 1135*n^3 + 241*n^4 + 25*n^5 + n^6)*x
^4 + 7*d^3*(720 + 1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6)*x^6
)))/(b^8*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)*(8 + n))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. 2(282) = 564.
time = 0.09, size = 1214, normalized size = 4.30

method	result
norman	$\frac{d^3x^8e^{n\ln(bx+a)}}{8+n}+\frac{na(b^6c^3n^6+33b^6c^3n^5+18a^2b^4c^2d\,n^4+445b^6c^3n^4+468a^2b^4c^2d\,n^3+3135b^6c^3n^3+360a^4b^2c\,d^2n^2+4518a^2b^4c^2d\,n}{b^7(n^8+36n^7+546n^6+4536n^5}$
gosper	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3/(8+n)*x^8*exp(n*ln(b*x+a))+1/b^7*n*a*(b^6*c^3*n^6+33*b^6*c^3*n^5+18*a^2
*b^4*c^2*d*n^4+445*b^6*c^3*n^4+468*a^2*b^4*c^2*d*n^3+3135*b^6*c^3*n^3+360*a
^4*b^2*c*d^2*n^2+4518*a^2*b^4*c^2*d*n^2+12154*b^6*c^3*n^2+5400*a^4*b^2*c*d^
2*n+19188*a^2*b^4*c^2*d*n+24552*b^6*c^3*n+5040*a^6*d^3+20160*a^4*b^2*c*d^2+
30240*a^2*b^4*c^2*d+20160*b^6*c^3)/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+6
7284*n^3+118124*n^2+109584*n+40320)*x*exp(n*ln(b*x+a))+n*a/b*d^3/(n^2+15*n+
56)*x^7*exp(n*ln(b*x+a))-a^2*(b^6*c^3*n^6+33*b^6*c^3*n^5+18*a^2*b^4*c^2*d*n
```

```
^4+445*b^6*c^3*n^4+468*a^2*b^4*c^2*d*n^3+3135*b^6*c^3*n^3+360*a^4*b^2*c*d^2
*n^2+4518*a^2*b^4*c^2*d*n^2+12154*b^6*c^3*n^2+5400*a^4*b^2*c*d^2*n+19188*a^
2*b^4*c^2*d*n+24552*b^6*c^3*n+5040*a^6*d^3+20160*a^4*b^2*c*d^2+30240*a^2*b^
4*c^2*d+20160*b^6*c^3)/b^8/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3
+118124*n^2+109584*n+40320)*exp(n*ln(b*x+a))-(-b^6*c^3*n^6+9*a^2*b^4*c^2*d*
n^5-33*b^6*c^3*n^5+234*a^2*b^4*c^2*d*n^4-445*b^6*c^3*n^4+180*a^4*b^2*c*d^2*
n^3+2259*a^2*b^4*c^2*d*n^3-3135*b^6*c^3*n^3+2700*a^4*b^2*c*d^2*n^2+9594*a^2
*b^4*c^2*d*n^2-12154*b^6*c^3*n^2+2520*a^6*d^3*n+10080*a^4*b^2*c*d^2*n+15120
*a^2*b^4*c^2*d*n-24552*b^6*c^3*n-20160*b^6*c^3)/b^6/(n^7+35*n^6+511*n^5+402
5*n^4+18424*n^3+48860*n^2+69264*n+40320)*x^2*exp(n*ln(b*x+a))-(-3*b^2*c*n^2
+7*a^2*d*n-45*b^2*c*n-168*b^2*c)/b^2*d^2/(n^3+21*n^2+146*n+336)*x^6*exp(n*l
n(b*x+a))-3*(-b^4*c^2*n^4+5*a^2*b^2*c*d*n^3-26*b^4*c^2*n^3+75*a^2*b^2*c*d*n
^2-251*b^4*c^2*n^2+70*a^4*d^2*n+280*a^2*b^2*c*d*n-1066*b^4*c^2*n-1680*b^4*c
^2)*d/b^4/(n^5+30*n^4+355*n^3+2070*n^2+5944*n+6720)*x^4*exp(n*ln(b*x+a))+3*
(b^2*c*n^2+15*b^2*c*n+14*a^2*d+56*b^2*c)*n*a/b^3*d^2/(n^4+26*n^3+251*n^2+10
66*n+1680)*x^5*exp(n*ln(b*x+a))+3*(b^4*c^2*n^4+26*b^4*c^2*n^3+20*a^2*b^2*c*
d*n^2+251*b^4*c^2*n^2+300*a^2*b^2*c*d*n+1066*b^4*c^2*n+280*a^4*d^2+1120*a^2
*b^2*c*d+1680*b^4*c^2)*a/b^5*d*n/(n^6+33*n^5+445*n^4+3135*n^3+12154*n^2+245
52*n+20160)*x^3*exp(n*ln(b*x+a))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(282) = 564$.
time = 0.30, size = 625, normalized size = 2.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 +
 35*n^2 + 50*n + 24)*b^4) + 3*((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 1
20)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 +
6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60
*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c*d^2/((n^6 +
 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + ((n^7 + 28*n^
6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (
n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 -
 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^
5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^
2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)
*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*d^3/((n^8 + 36*n^7 +
546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)
*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. $2(282) = 564$.
time = 3.31, size = 1675, normalized size = 5.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^3*n^6 + 33*a^2*b^6*c^3*n^5 + 20160*a^2*b^6*c^3 + 30240*a^4*b^4*
c^2*d + 20160*a^6*b^2*c*d^2 + 5040*a^8*d^3 - (b^8*d^3*n^7 + 28*b^8*d^3*n^6
+ 322*b^8*d^3*n^5 + 1960*b^8*d^3*n^4 + 6769*b^8*d^3*n^3 + 13132*b^8*d^3*n^2
 + 13068*b^8*d^3*n + 5040*b^8*d^3)*x^8 - (a*b^7*d^3*n^7 + 21*a*b^7*d^3*n^6
+ 175*a*b^7*d^3*n^5 + 735*a*b^7*d^3*n^4 + 1624*a*b^7*d^3*n^3 + 1764*a*b^7*d
^3*n^2 + 720*a*b^7*d^3*n)*x^7 - (3*b^8*c*d^2*n^7 + 20160*b^8*c*d^2 + (90*b^
8*c*d^2 - 7*a^2*b^6*d^3)*n^6 + 3*(366*b^8*c*d^2 - 35*a^2*b^6*d^3)*n^5 + 5*(
1404*b^8*c*d^2 - 119*a^2*b^6*d^3)*n^4 + 9*(2803*b^8*c*d^2 - 175*a^2*b^6*d^3
)*n^3 + 2*(25245*b^8*c*d^2 - 959*a^2*b^6*d^3)*n^2 + 24*(2143*b^8*c*d^2 - 35
*a^2*b^6*d^3)*n)*x^6 - 3*(a*b^7*c*d^2*n^7 + 25*a*b^7*c*d^2*n^6 + (241*a*b^7
*c*d^2 + 14*a^3*b^5*d^3)*n^5 + 5*(227*a*b^7*c*d^2 + 28*a^3*b^5*d^3)*n^4 + 2
*(1367*a*b^7*c*d^2 + 245*a^3*b^5*d^3)*n^3 + 20*(158*a*b^7*c*d^2 + 35*a^3*b^
5*d^3)*n^2 + 336*(4*a*b^7*c*d^2 + a^3*b^5*d^3)*n)*x^5 + (445*a^2*b^6*c^3 +
18*a^4*b^4*c^2*d)*n^4 - 3*(b^8*c^2*d*n^7 + 10080*b^8*c^2*d + (32*b^8*c^2*d
- 5*a^2*b^6*c*d^2)*n^6 + (418*b^8*c^2*d - 105*a^2*b^6*c*d^2)*n^5 + (2864*b^
8*c^2*d - 785*a^2*b^6*c*d^2 - 70*a^4*b^4*d^3)*n^4 + (10993*b^8*c^2*d - 2535
*a^2*b^6*c*d^2 - 420*a^4*b^4*d^3)*n^3 + 2*(11656*b^8*c^2*d - 1765*a^2*b^6*c
*d^2 - 385*a^4*b^4*d^3)*n^2 + 12*(2073*b^8*c^2*d - 140*a^2*b^6*c*d^2 - 35*a
^4*b^4*d^3)*n)*x^4 + 3*(1045*a^2*b^6*c^3 + 156*a^4*b^4*c^2*d)*n^3 - 3*(a*b^
7*c^2*d*n^7 + 29*a*b^7*c^2*d*n^6 + (331*a*b^7*c^2*d + 20*a^3*b^5*c*d^2)*n^5
 + (1871*a*b^7*c^2*d + 360*a^3*b^5*c*d^2)*n^4 + 20*(269*a*b^7*c^2*d + 103*a
^3*b^5*c*d^2 + 14*a^5*b^3*d^3)*n^3 + 4*(1793*a*b^7*c^2*d + 990*a^3*b^5*c*d^
2 + 210*a^5*b^3*d^3)*n^2 + 560*(6*a*b^7*c^2*d + 4*a^3*b^5*c*d^2 + a^5*b^3*d
^3)*n)*x^3 + 2*(6077*a^2*b^6*c^3 + 2259*a^4*b^4*c^2*d + 180*a^6*b^2*c*d^2)*
n^2 - (b^8*c^3*n^7 + 20160*b^8*c^3 + (34*b^8*c^3 - 9*a^2*b^6*c^2*d)*n^6 + (
478*b^8*c^3 - 243*a^2*b^6*c^2*d)*n^5 + (3580*b^8*c^3 - 2493*a^2*b^6*c^2*d -
 180*a^4*b^4*c*d^2)*n^4 + (15289*b^8*c^3 - 11853*a^2*b^6*c^2*d - 2880*a^4*b
^4*c*d^2)*n^3 + 2*(18353*b^8*c^3 - 12357*a^2*b^6*c^2*d - 6390*a^4*b^4*c*d^2
 - 1260*a^6*b^2*d^3)*n^2 + 72*(621*b^8*c^3 - 210*a^2*b^6*c^2*d - 140*a^4*b^
4*c*d^2 - 35*a^6*b^2*d^3)*n)*x^2 + 36*(682*a^2*b^6*c^3 + 533*a^4*b^4*c^2*d
+ 150*a^6*b^2*c*d^2)*n - (a*b^7*c^3*n^7 + 33*a*b^7*c^3*n^6 + (445*a*b^7*c^3
 + 18*a^3*b^5*c^2*d)*n^5 + 3*(1045*a*b^7*c^3 + 156*a^3*b^5*c^2*d)*n^4 + 2*(
6077*a*b^7*c^3 + 2259*a^3*b^5*c^2*d + 180*a^5*b^3*c*d^2)*n^3 + 36*(682*a*b^
7*c^3 + 533*a^3*b^5*c^2*d + 150*a^5*b^3*c*d^2)*n^2 + 5040*(4*a*b^7*c^3 + 6*
a^3*b^5*c^2*d + 4*a^5*b^3*c*d^2 + a^7*b*d^3)*n)*x)*(b*x + a)^n/(b^8*n^8 + 3
```

```
6*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 11
8124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 24687 vs. $2(265) = 530$.
time = 7.11, size = 24687, normalized size = 87.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x**2/2 + 3*c**2*d*x**4/4 + c*d**2*x**6/2 + d**3*x**8/
8), Eq(b, 0)), (420*a**7*d**3*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*
x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 +
8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1089*a**7*d**3
/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**1
1*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 +
 420*b**15*x**7) + 2940*a**6*b*d**3*x*log(a/b + x)/(420*a**7*b**8 + 2940*a*
*6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12
*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 7203*a
**6*b*d**3*x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 147
00*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*
b**14*x**6 + 420*b**15*x**7) - 30*a**5*b**2*c*d**2/(420*a**7*b**8 + 2940*a*
*6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12
*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 8820*a
**5*b**2*d**3*x**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a*
*5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b
**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 20139*a**5*b**2*d**3*x**2
/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**1
1*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 +
 420*b**15*x**7) - 210*a**4*b**3*c*d**2*x/(420*a**7*b**8 + 2940*a**6*b**9*x
 + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8
820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 14700*a**4*b**3
*d**3*x**3*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10
*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**
5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 30625*a**4*b**3*d**3*x**3/(420*a*
*7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 +
 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**
15*x**7) - 9*a**3*b**4*c**2*d/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5
*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**
13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 630*a**3*b**4*c*d**2*x**2/(
420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*
x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 4
20*b**15*x**7) + 14700*a**3*b**4*d**3*x**4*log(a/b + x)/(420*a**7*b**8 + 29
```

```
40*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*
b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 2
6950*a**3*b**4*d**3*x**4/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**1
0*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x*
*5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 63*a**2*b**5*c**2*d*x/(420*a**7*
b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14
700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*
x**7) - 1050*a**2*b**5*c*d**2*x**3/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820
*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**
2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 8820*a**2*b**5*d**3*x*
*5*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 +
14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940
*a*b**14*x**6 + 420*b**15*x**7) + 13230*a**2*b**5*d**3*x**5/(420*a**7*b**8
+ 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a
**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7)
 - 10*a*b**6*c**3/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2
+ 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 29
40*a*b**14*x**6 + 420*b**15*x**7) - 189*a*b**6*c**2*d*x**2/(420*a**7*b**8 +
 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a*
*3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7)
- 1050*a*b**6*c*d**2*x**4/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**
10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x
**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 2940*a*b**6*d**3*x**6*log(a/b +
 x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b
**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**
6 + 420*b**15*x**7) + 2940*a*b**6*d**3*x**6/(420*a**7*b**8 + 2940*a**6*b**9
*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 +
 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 70*b**7*c**3*
x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**
11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6
+ 420*b**15*x**7) - 315*b**7*c**2*d*x**3/(420*a**7*b**8 + 2940*a**6*b**9*x
+ 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 88
20*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 630*b**7*c*d**2*
x**5/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2851 vs. $2(282) = 564$.
time = 1.60, size = 2851, normalized size = 10.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^3*n^7*x^8 + (b*x + a)^n*a*b^7*d^3*n^7*x^7 + 28*(b*x + a)
^n*b^8*d^3*n^6*x^8 + 3*(b*x + a)^n*b^8*c*d^2*n^7*x^6 + 21*(b*x + a)^n*a*b^7
```

*d^3*n^6*x^7 + 322*(b*x + a)^n*b^8*d^3*n^5*x^8 + 3*(b*x + a)^n*a*b^7*c*d^2*n^7*x^5 + 90*(b*x + a)^n*b^8*c*d^2*n^6*x^6 - 7*(b*x + a)^n*a^2*b^6*d^3*n^6*x^6 + 175*(b*x + a)^n*a*b^7*d^3*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^3*n^4*x^8 + 3*(b*x + a)^n*b^8*c^2*d*n^7*x^4 + 75*(b*x + a)^n*a*b^7*c*d^2*n^6*x^5 + 1098*(b*x + a)^n*b^8*c*d^2*n^5*x^6 - 105*(b*x + a)^n*a^2*b^6*d^3*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^3*n^4*x^7 + 6769*(b*x + a)^n*b^8*d^3*n^3*x^8 + 3*(b*x + a)^n*a*b^7*c^2*d*n^7*x^3 + 96*(b*x + a)^n*b^8*c^2*d*n^6*x^4 - 15*(b*x + a)^n*a^2*b^6*c*d^2*n^6*x^4 + 723*(b*x + a)^n*a*b^7*c*d^2*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^3*n^5*x^5 + 7020*(b*x + a)^n*b^8*c*d^2*n^4*x^6 - 595*(b*x + a)^n*a^2*b^6*d^3*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^3*n^3*x^7 + 13132*(b*x + a)^n*b^8*d^3*n^2*x^8 + (b*x + a)^n*b^8*c^3*n^7*x^2 + 87*(b*x + a)^n*a*b^7*c^2*d*n^6*x^3 + 1254*(b*x + a)^n*b^8*c^2*d*n^5*x^4 - 315*(b*x + a)^n*a^2*b^6*c*d^2*n^5*x^4 + 3405*(b*x + a)^n*a*b^7*c*d^2*n^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^3*n^4*x^5 + 25227*(b*x + a)^n*b^8*c*d^2*n^3*x^6 - 1575*(b*x + a)^n*a^2*b^6*d^3*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^3*n^2*x^7 + 13068*(b*x + a)^n*b^8*d^3*n*x^8 + (b*x + a)^n*a*b^7*c^3*n^7*x + 34*(b*x + a)^n*b^8*c^3*n^6*x^2 - 9*(b*x + a)^n*a^2*b^6*c^2*d*n^6*x^2 + 993*(b*x + a)^n*a*b^7*c^2*d*n^5*x^3 + 60*(b*x + a)^n*a^3*b^5*c*d^2*n^5*x^3 + 8592*(b*x + a)^n*b^8*c^2*d*n^4*x^4 - 2355*(b*x + a)^n*a^2*b^6*c*d^2*n^4*x^4 - 210*(b*x + a)^n*a^4*b^4*d^3*n^4*x^4 + 8202*(b*x + a)^n*a*b^7*c*d^2*n^3*x^5 + 1470*(b*x + a)^n*a^3*b^5*d^3*n^3*x^5 + 50490*(b*x + a)^n*b^8*c*d^2*n^2*x^6 - 1918*(b*x + a)^n*a^2*b^6*d^3*n^2*x^6 + 720*(b*x + a)^n*a*b^7*d^3*n*x^7 + 5040*(b*x + a)^n*b^8*d^3*x^8 + 33*(b*x + a)^n*a*b^7*c^3*n^6*x + 478*(b*x + a)^n*b^8*c^3*n^5*x^2 - 243*(b*x + a)^n*a^2*b^6*c^2*d*n^5*x^2 + 5613*(b*x + a)^n*a*b^7*c^2*d*n^4*x^3 + 1080*(b*x + a)^n*a^3*b^5*c*d^2*n^4*x^3 + 32979*(b*x + a)^n*b^8*c^2*d*n^3*x^4 - 7605*(b*x + a)^n*a^2*b^6*c*d^2*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^3*n^3*x^4 + 9480*(b*x + a)^n*a*b^7*c*d^2*n^2*x^5 + 2100*(b*x + a)^n*a^3*b^5*d^3*n^2*x^5 + 51432*(b*x + a)^n*b^8*c*d^2*n*x^6 - 840*(b*x + a)^n*a^2*b^6*d^3*n*x^6 - (b*x + a)^n*a^2*b^6*c^3*n^6 + 445*(b*x + a)^n*a*b^7*c^3*n^5*x + 18*(b*x + a)^n*a^3*b^5*c^2*d*n^5*x + 3580*(b*x + a)^n*b^8*c^3*n^4*x^2 - 2493*(b*x + a)^n*a^2*b^6*c^2*d*n^4*x^2 - 180*(b*x + a)^n*a^4*b^4*c*d^2*n^4*x^2 + 16140*(b*x + a)^n*a*b^7*c^2*d*n^3*x^3 + 6180*(b*x + a)^n*a^3*b^5*c*d^2*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^3*n^3*x^3 + 69936*(b*x + a)^n*b^8*c^2*d*n^2*x^4 - 10590*(b*x + a)^n*a^2*b^6*c*d^2*n^2*x^4 - 2310*(b*x + a)^n*a^4*b^4*d^3*n^2*x^4 + 4032*(b*x + a)^n*a*b^7*c*d^2*n*x^5 + 1008*(b*x + a)^n*a^3*b^5*d^3*n*x^5 + 20160*(b*x + a)^n*b^8*c*d^2*x^6 - 33*(b*x + a)^n*a^2*b^6*c^3*n^5 + 3135*(b*x + a)^n*a*b^7*c^3*n^4*x + 468*(b*x + a)^n*a^3*b^5*c^2*d*n^4*x + 15289*(b*x + a)^n*b^8*c^3*n^3*x^2 - 11853*(b*x + a)^n*a^2*b^6*c^2*d*n^3*x^2 - 2880*(b*x + a)^n*a^4*b^4*c*d^2*n^3*x^2 + 21516*(b*x + a)^n*a*b^7*c^2*d*n^2*x^3 + 11880*(b*x + a)^n*a^3*b^5*c*d^2*n^2*x^3 + 2520*(b*x + a)^n*a^5*b^3*d^3*n^2*x^3 + 74628*(b*x + a)^n*b^8*c^2*d*n*x^4 - 5040*(b*x + a)^n*a^2*b^6*c*d^2*n*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^3*n*x^4 - 445*(b*x + a)^n*a^2*b^6*c^3*n^4 - 18*(b*x + a)^n*a^4*b^4*c^2*d*n^4 + 12154*(b*x + a)^n*a*b^7*c^3*n^3*x + 4518*(b*x + a)^n*a^3*b^5*c^2*d*n^3*x + 360*(b*x + a)^n*a^5*b^3*c*d^2*n^3*x + 36706*(b*x + a)^n*b^8*c^3*n^2*x^2 - 24714*(b*x + a)^n*a^2

```
*b^6*c^2*d*n^2*x^2 - 12780*(b*x + a)^n*a^4*b^4*c*d^2*n^2*x^2 - 2520*(b*x +
a)^n*a^6*b^2*d^3*n^2*x^2 + 10080*(b*x + a)^n*a*b^7*c^2*d*n*x^3 + 6720*(b*x
+ a)^n*a^3*b^5*c*d^2*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^3*n*x^3 + 30240*(b*
x + a)^n*b^8*c^2*d*x^4 - 3135*(b*x + a)^n*a^2*b^6*c^3*n^3 - 468*(b*x + a)^n
*a^4*b^4*c^2*d*n^3 + 24552*(b*x + a)^n*a*b^7*c^3*n^2*x + 19188*(b*x + a)^n*
a^3*b^5*c^2*d*n^2*x + 5400*(b*x + a)^n*a^5*b^3*c*d^2*n^2*x + 44712*(b*x + a
)^n*b^8*c^3*n*x^2 - 15120*(b*x + a)^n*a^2*b^6*c^2*d*n*x^2 - 10080*(b*x + a)
^n*a^4*b^4*c*d^2*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^3*n*x^2 - 12154*(b*x +
a)^n*a^2*b^6*c^3*n^2 - 4518*(b*x + a)^n*a^4*b^4*c^2*d*n^2 - 360*(b*x + a)^n
*a^6*b^2*c*d^2*n^2 + 20160*(b*x + a)^n*a*b^7*c^3*n*x + 30240*(b*x + a)^n*a^
3*b^5*c^2*d*n*x + 20160*(b*x + a)^n*a^5*b^3*c*d^2*n*x + 5040*(b*x + a)^n*a^
7*b*d^3*n*x + 20160*(b*x + a)^n*b^8*c^3*x^2 - 24552*(b*x + a)^n*a^2*b^6*c^3
*n - 19188*(b*x + a)^n*a^4*b^4*c^2*d*n - 5400*(b*x + a)^n*a^6*b^2*c*d^2*n -
 20160*(b*x + a)^n*a^2*b^6*c^3 - 30240*(b*x + a)^n*a^4*b^4*c^2*d - 20160*(b
*x + a)^n*a^6*b^2*c*d^2 - 5040*(b*x + a)^n*a^8*d^3)/(b^8*n^8 + 36*b^8*n^7 +
 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 118124*b^8*n^
2 + 109584*b^8*n + 40320*b^8)
```

Mupad [B]
time = 3.48, size = 1459, normalized size = 5.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^3 +
20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3 + 44
5*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 20160*
a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a^2*b^4
*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*b^4*c^2
*d*n^4))/(b^8*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 5
46*n^6 + 36*n^7 + n^8 + 40320)) - (x^2*(n + 1)*(a + b*x)^n*(2520*a^6*d^3*n
- 20160*b^6*c^3 - 24552*b^6*c^3*n - 12154*b^6*c^3*n^2 - 3135*b^6*c^3*n^3 -
445*b^6*c^3*n^4 - 33*b^6*c^3*n^5 - b^6*c^3*n^6 + 15120*a^2*b^4*c^2*d*n + 10
080*a^4*b^2*c*d^2*n + 9594*a^2*b^4*c^2*d*n^2 + 2700*a^4*b^2*c*d^2*n^2 + 225
9*a^2*b^4*c^2*d*n^3 + 180*a^4*b^2*c*d^2*n^3 + 234*a^2*b^4*c^2*d*n^4 + 9*a^2
*b^4*c^2*d*n^5))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (d^2*x^6*(a + b*x)^n*(168*b^2*c +
 3*b^2*c*n^2 - 7*a^2*d*n + 45*b^2*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 +
 n^5 + 120))/(b^2*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5
 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*d*x^4*(a + b*x)^n*(11*n + 6*n^2 +
n^3 + 6)*(1680*b^4*c^2 - 70*a^4*d^2*n + 1066*b^4*c^2*n + 251*b^4*c^2*n^2 +
```

```
26*b^4*c^2*n^3 + b^4*c^2*n^4 - 280*a^2*b^2*c*d*n - 75*a^2*b^2*c*d*n^2 - 5*a
^2*b^2*c*d*n^3))/(b^4*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)^n*(5040*a^6*d^3
+ 20160*b^6*c^3 + 24552*b^6*c^3*n + 12154*b^6*c^3*n^2 + 3135*b^6*c^3*n^3 +
445*b^6*c^3*n^4 + 33*b^6*c^3*n^5 + b^6*c^3*n^6 + 30240*a^2*b^4*c^2*d + 2016
0*a^4*b^2*c*d^2 + 19188*a^2*b^4*c^2*d*n + 5400*a^4*b^2*c*d^2*n + 4518*a^2*b
^4*c^2*d*n^2 + 360*a^4*b^2*c*d^2*n^2 + 468*a^2*b^4*c^2*d*n^3 + 18*a^2*b^4*c
^2*d*n^4))/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 +
 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*d^3*n*x^7*(a + b*x)^n*(1764*n + 1624
*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 +
 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*a
*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(280*a^4*d^2 + 1680*b^4*c^2 + 1066*b^4
*c^2*n + 251*b^4*c^2*n^2 + 26*b^4*c^2*n^3 + b^4*c^2*n^4 + 1120*a^2*b^2*c*d
+ 300*a^2*b^2*c*d*n + 20*a^2*b^2*c*d*n^2))/(b^5*(109584*n + 118124*n^2 + 67
284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (3*a*d^
2*n*x^5*(a + b*x)^n*(14*a^2*d + 56*b^2*c + b^2*c*n^2 + 15*b^2*c*n)*(50*n +
35*n^2 + 10*n^3 + n^4 + 24))/(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 2244
9*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

3.361 $\int (a+bx)^n \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=223

$$\frac{\left(b^2c+a^2d\right)^3 (a+bx)^{1+n}}{b^7(1+n)}-\frac{6ad\left(b^2c+a^2d\right)^2 (a+bx)^{2+n}}{b^7(2+n)}+\frac{3d\left(b^2c+a^2d\right)\left(b^2c+5a^2d\right)(a+bx)^{3+n}}{b^7(3+n)}-\frac{4ad^2(3b^2c}{}$$

```
[Out] (a^2*d+b^2*c)^3*(b*x+a)^(1+n)/b^7/(1+n)-6*a*d*(a^2*d+b^2*c)^2*(b*x+a)^(2+n)
/b^7/(2+n)+3*d*(a^2*d+b^2*c)*(5*a^2*d+b^2*c)*(b*x+a)^(3+n)/b^7/(3+n)-4*a*d^
2*(5*a^2*d+3*b^2*c)*(b*x+a)^(4+n)/b^7/(4+n)+3*d^2*(5*a^2*d+b^2*c)*(b*x+a)^(
5+n)/b^7/(5+n)-6*a*d^3*(b*x+a)^(6+n)/b^7/(6+n)+d^3*(b*x+a)^(7+n)/b^7/(7+n)
```

Rubi [A]
time = 0.08, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {711}

$$-\frac{4ad^2\left(5a^2d+3b^2c\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{3d^2\left(5a^2d+b^2c\right)(a+bx)^{n+5}}{b^7(n+5)}+\frac{\left(a^2d+b^2c\right)^3(a+bx)^{n+1}}{b^7(n+1)}-\frac{6ad\left(a^2d+b^2c\right)^2(a+bx)^{n+2}}{b^7(n+2)}+\frac{3d\left(a^2d+b^2c\right)\left(5a^2d+b^2c\right)(a+bx)^{n+3}}{b^7(n+3)}-\frac{6ad^3(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^3(a+bx)^{n+7}}{b^7(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^7*(1 + n)) - (6*a*d*(b^2*c + a^2*d
)^2*(a + b*x)^(2 + n))/(b^7*(2 + n)) + (3*d*(b^2*c + a^2*d)*(b^2*c + 5*a^2*
d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) - (4*a*d^2*(3*b^2*c + 5*a^2*d)*(a + b*x
)^(4 + n))/(b^7*(4 + n)) + (3*d^2*(b^2*c + 5*a^2*d)*(a + b*x)^(5 + n))/(b^7
*(5 + n)) - (6*a*d^3*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d^3*(a + b*x)^(7 +
 n))/(b^7*(7 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^2\right)^3 \, dx = \int \left(\frac{\left(b^2c+a^2d\right)^3 (a+bx)^n}{b^6}-\frac{6ad\left(b^2c+a^2d\right)^2 (a+bx)^{1+n}}{b^6}+\frac{3d\left(b^2c+a^2d\right)(b^2}{}\right.$$

$$= \frac{\left(b^2c+a^2d\right)^3 (a+bx)^{1+n}}{b^7(1+n)}-\frac{6ad\left(b^2c+a^2d\right)^2 (a+bx)^{2+n}}{b^7(2+n)}+\frac{3d\left(b^2c+a^2d\right)(b^2c}{b^7(3}$$

Mathematica [A]
time = 0.24, size = 413, normalized size = 1.85

$$\frac{(a+bx)^{1+n}\left(720a^6d^3-720a^5bd^3(1+n)x+72a^4b^2d^2\left(c\left(42+13n+n^2\right)+5d\left(2+3n+n^2\right)x^2\right)-24a^3b^3d^2(1+n)x\left(3c\left(42+13n+n^2\right)+5d\left(6+5n+n^2\right)x^2\right)+6a^2b^4d\left(c^2\left(840+638n+179n^2+22n^3+n^4\right)+6cd\left(84+152n+83n^2+16n^3+n^4\right)x^2+5d^2\left(24+50n+35n^2+10n^3+n^4\right)x^4\right)-6ab^5d(1+n)x\left(c^2\left(840+638n+179n^2+22n^3+n^4\right)+2cd\left(252+288n+113n^2+18n^3+n^4\right)x^2+d^2\left(120+154n+71n^2+14n^3+n^4\right)x^4\right)+b^6\left(48+44n+12n^2+n^3\right)\left(c^3\left(105+71n+15n^2+n^3\right)+3c^2d\left(35+47n+13n^2+n^3\right)x^2+3cd^2\left(21+31n+11n^2+n^3\right)x^4+d^3\left(15+23n+9n^2+n^3\right)x^6\right)\right)}{b^7(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)(7+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(720*a^6*d^3 - 720*a^5*b*d^3*(1 + n)*x + 72*a^4*b^2*d^2*
(c*(42 + 13*n + n^2) + 5*d*(2 + 3*n + n^2)*x^2) - 24*a^3*b^3*d^2*(1 + n)*x*
(3*c*(42 + 13*n + n^2) + 5*d*(6 + 5*n + n^2)*x^2) + 6*a^2*b^4*d*(c^2*(840 +
 638*n + 179*n^2 + 22*n^3 + n^4) + 6*c*d*(84 + 152*n + 83*n^2 + 16*n^3 + n^
4)*x^2 + 5*d^2*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^4) - 6*a*b^5*d*(1 + n)
*x*(c^2*(840 + 638*n + 179*n^2 + 22*n^3 + n^4) + 2*c*d*(252 + 288*n + 113*n
^2 + 18*n^3 + n^4)*x^2 + d^2*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)*x^4) + b
^6*(48 + 44*n + 12*n^2 + n^3)*(c^3*(105 + 71*n + 15*n^2 + n^3) + 3*c^2*d*(3
5 + 47*n + 13*n^2 + n^3)*x^2 + 3*c*d^2*(21 + 31*n + 11*n^2 + n^3)*x^4 + d^3
*(15 + 23*n + 9*n^2 + n^3)*x^6)))/(b^7*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 +
 n)*(6 + n)*(7 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(223) = 446.
time = 0.09, size = 958, normalized size = 4.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3/(7+n)*x^7*exp(n*ln(b*x+a))+a*(b^6*c^3*n^6+27*b^6*c^3*n^5+6*a^2*b^4*c^2*
d*n^4+295*b^6*c^3*n^4+132*a^2*b^4*c^2*d*n^3+1665*b^6*c^3*n^3+72*a^4*b^2*c*d
^2*n^2+1074*a^2*b^4*c^2*d*n^2+5104*b^6*c^3*n^2+936*a^4*b^2*c*d^2*n+3828*a^2
*b^4*c^2*d*n+8028*b^6*c^3*n+720*a^6*d^3+3024*a^4*b^2*c*d^2+5040*a^2*b^4*c^2
*d+5040*b^6*c^3)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*
n+5040)*exp(n*ln(b*x+a))+a*d^3*n/b/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))-(-b^6
*c^3*n^6+6*a^2*b^4*c^2*d*n^5-27*b^6*c^3*n^5+132*a^2*b^4*c^2*d*n^4-295*b^6*c
^3*n^4+72*a^4*b^2*c*d^2*n^3+1074*a^2*b^4*c^2*d*n^3-1665*b^6*c^3*n^3+936*a^4
*b^2*c*d^2*n^2+3828*a^2*b^4*c^2*d*n^2-5104*b^6*c^3*n^2+720*a^6*d^3*n+3024*a
^4*b^2*c*d^2*n+5040*a^2*b^4*c^2*d*n-8028*b^6*c^3*n-5040*b^6*c^3)/b^6/(n^7+2
8*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*x*exp(n*ln(b*x+a))-
3*(-b^2*c*n^2+2*a^2*d*n-13*b^2*c*n-42*b^2*c)/b^2*d^2/(n^3+18*n^2+107*n+210)
*x^5*exp(n*ln(b*x+a))-3*(-b^4*c^2*n^4+4*a^2*b^2*c*d*n^3-22*b^4*c^2*n^3+52*a
^2*b^2*c*d*n^2-179*b^4*c^2*n^2+40*a^4*d^2*n+168*a^2*b^2*c*d*n-638*b^4*c^2*n
-840*b^4*c^2)/b^4*d/(n^5+25*n^4+245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(
b*x+a))+3*(b^2*c*n^2+13*b^2*c*n+10*a^2*d+42*b^2*c)*d^2*a/b^3*n/(n^4+22*n^3+
179*n^2+638*n+840)*x^4*exp(n*ln(b*x+a))+3*(b^4*c^2*n^4+22*b^4*c^2*n^3+12*a^
2*b^2*c*d*n^2+179*b^4*c^2*n^2+156*a^2*b^2*c*d*n+638*b^4*c^2*n+120*a^4*d^2+5
```

```
04*a^2*b^2*c*d+840*b^4*c^2)*d*a/b^5*n/(n^6+27*n^5+295*n^4+1665*n^3+5104*n^2
+8028*n+5040)*x^2*exp(n*ln(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(223) = 446$.
time = 0.30, size = 472, normalized size = 2.12

$$\frac{(bx+a)^{n+1}c^3}{b(n+1)} + \frac{3\left((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3\right)(bx+a)^nc^2d}{(n^3+6n^2+11n+6)b^3} + \frac{3\left((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+12(n^2+n)a^3b^2x^2-24a^4bnx+24a^5\right)(bx+a)^ncd^2}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5} + \frac{\left((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n^2+120n)ab^6x^6-6(n^5+10n^4+35n^3+50n^2+24n)a^2b^5x^5+30(n^4+6n^3+11n^2+6n)a^3b^4x^4-120(n^3+3n^2+2n)a^4b^3x^3+360(n^2+n)a^5b^2x^2-720a^6bnx+720a^7\right)(bx+a)^nd^3}{(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*
a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c^2*d/((n^3 + 6*n^2 + 11*n + 6
)*b^3) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11
*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^
3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d^2/((n^5 + 15*n^4 + 85*n^
3 + 225*n^2 + 274*n + 120)*b^5) + ((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624
*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 +
 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 +
 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*
b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*
d^3/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n +
5040)*b^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(223) = 446$.
time = 2.32, size = 1244, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^3*n^6 + 27*a*b^6*c^3*n^5 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3
024*a^5*b^2*c*d^2 + 720*a^7*d^3 + (b^7*d^3*n^6 + 21*b^7*d^3*n^5 + 175*b^7*d
^3*n^4 + 735*b^7*d^3*n^3 + 1624*b^7*d^3*n^2 + 1764*b^7*d^3*n + 720*b^7*d^3)
*x^7 + (a*b^6*d^3*n^6 + 15*a*b^6*d^3*n^5 + 85*a*b^6*d^3*n^4 + 225*a*b^6*d^3
*n^3 + 274*a*b^6*d^3*n^2 + 120*a*b^6*d^3*n)*x^6 + 3*(b^7*c*d^2*n^6 + 1008*b
^7*c*d^2 + (23*b^7*c*d^2 - 2*a^2*b^5*d^3)*n^5 + (207*b^7*c*d^2 - 20*a^2*b^5
*d^3)*n^4 + 5*(185*b^7*c*d^2 - 14*a^2*b^5*d^3)*n^3 + 4*(536*b^7*c*d^2 - 25*
a^2*b^5*d^3)*n^2 + 12*(201*b^7*c*d^2 - 4*a^2*b^5*d^3)*n)*x^5 + (295*a*b^6*c
^3 + 6*a^3*b^4*c^2*d)*n^4 + 3*(a*b^6*c*d^2*n^6 + 19*a*b^6*c*d^2*n^5 + (131*
a*b^6*c*d^2 + 10*a^3*b^4*d^3)*n^4 + (401*a*b^6*c*d^2 + 60*a^3*b^4*d^3)*n^3
+ 10*(54*a*b^6*c*d^2 + 11*a^3*b^4*d^3)*n^2 + 12*(21*a*b^6*c*d^2 + 5*a^3*b^4
*d^3)*n)*x^4 + 3*(555*a*b^6*c^3 + 44*a^3*b^4*c^2*d)*n^3 + 3*(b^7*c^2*d*n^6
```

```
+ 1680*b^7*c^2*d + (25*b^7*c^2*d - 4*a^2*b^5*c*d^2)*n^5 + (247*b^7*c^2*d -
64*a^2*b^5*c*d^2)*n^4 + (1219*b^7*c^2*d - 332*a^2*b^5*c*d^2 - 40*a^4*b^3*d^
3)*n^3 + 8*(389*b^7*c^2*d - 76*a^2*b^5*c*d^2 - 15*a^4*b^3*d^3)*n^2 + 4*(949
*b^7*c^2*d - 84*a^2*b^5*c*d^2 - 20*a^4*b^3*d^3)*n)*x^3 + 2*(2552*a*b^6*c^3
+ 537*a^3*b^4*c^2*d + 36*a^5*b^2*c*d^2)*n^2 + 3*(a*b^6*c^2*d*n^6 + 23*a*b^6
*c^2*d*n^5 + 3*(67*a*b^6*c^2*d + 4*a^3*b^4*c*d^2)*n^4 + (817*a*b^6*c^2*d +
168*a^3*b^4*c*d^2)*n^3 + 2*(739*a*b^6*c^2*d + 330*a^3*b^4*c*d^2 + 60*a^5*b^
2*d^3)*n^2 + 24*(35*a*b^6*c^2*d + 21*a^3*b^4*c*d^2 + 5*a^5*b^2*d^3)*n)*x^2
+ 12*(669*a*b^6*c^3 + 319*a^3*b^4*c^2*d + 78*a^5*b^2*c*d^2)*n + (b^7*c^3*n^
6 + 5040*b^7*c^3 + 3*(9*b^7*c^3 - 2*a^2*b^5*c^2*d)*n^5 + (295*b^7*c^3 - 132
*a^2*b^5*c^2*d)*n^4 + 3*(555*b^7*c^3 - 358*a^2*b^5*c^2*d - 24*a^4*b^3*c*d^2
)*n^3 + 4*(1276*b^7*c^3 - 957*a^2*b^5*c^2*d - 234*a^4*b^3*c*d^2)*n^2 + 36*(
223*b^7*c^3 - 140*a^2*b^5*c^2*d - 84*a^4*b^3*c*d^2 - 20*a^6*b*d^3)*n)*x)*(b
*x + a)^n/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3
 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15990 vs. $2(207) = 414$.
time = 4.45, size = 15990, normalized size = 71.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7), Eq(
b, 0)), (60*a**6*d**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + 147*a**6*d**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + 360*a**5*b*d**3*x*log(a/b + x)/(60*a**6*b**7 + 360*a**
5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4
+ 360*a*b**12*x**5 + 60*b**13*x**6) + 822*a**5*b*d**3*x/(60*a**6*b**7 + 360
*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x
**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 6*a**4*b**2*c*d**2/(60*a**6*b**7
+ 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b*
*11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**3*x**2*log(
a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b
**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*
a**4*b**2*d**3*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x*
*6) - 36*a**3*b**3*c*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9
*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*
b**13*x**6) + 1200*a**3*b**3*d**3*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**
5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4
```

+ 360*a*b**12*x**5 + 60*b**13*x**6) + 2200*a**3*b**3*d**3*x**3/(60*a**6*b**
7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*
b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 3*a**2*b**4*c**2*d/(60*a**
6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*
a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 90*a**2*b**4*c*d**2*x
2/(60*a6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*
x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**2*b
4*d3*x**4*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*
x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b
13*x6) + 1350*a**2*b**4*d**3*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900
*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*
x**5 + 60*b**13*x**6) - 18*a*b**5*c**2*d*x/(60*a**6*b**7 + 360*a**5*b**8*x
+ 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b
12*x5 + 60*b**13*x**6) - 120*a*b**5*c*d**2*x**3/(60*a**6*b**7 + 360*a**
5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4
+ 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d**3*x**5*log(a/b + x)/(60
*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d**3*x
5/(60*a6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*
x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 10*b**6*c*
*3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x
3 + 900*a2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 45*b**6*c**
2*d*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b
10*x3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 90*b*
*6*c*d**2*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*
a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) +
60*b**6*d**3*x**6*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*
b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 +
60*b**13*x**6), Eq(n, -7)), (-60*a**6*d**3*log(a/b + x)/(10*a**5*b**7 + 50
*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 +
10*b**12*x**5) - 137*a**6*d**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b
9*x2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**
5*b*d**3*x*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2
+ 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 625*a**5*b*d**3
*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**
3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 6*a**4*b**2*c*d**2/(10*a**5*b**7 + 5
0*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4
+ 10*b**12*x**5) - 600*a**4*b**2*d**3*x**2*log(a/b + x)/(10*a**5*b**7 + 50*
a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 +
10*b**12*x**5) - 1100*a**4*b**2*d**3*x**2/(10*a**5*b**7 + 50*a**4*b**8*x +
100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5)
- 30*a**3*b**3*c*d**2*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**
2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 600*a**3*b**3*
d**3*x**3*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2
+ 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 900*a**3*b**3*d*

```
*3*x**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**1
0*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - a**...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2085 vs. $2(223) = 446$.
time = 1.25, size = 2085, normalized size = 9.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^3*n^6*x^7 + (b*x + a)^n*a*b^6*d^3*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^3*n^5*x^7 + 3*(b*x + a)^n*b^7*c*d^2*n^6*x^5 + 15*(b*x + a)^n*a*b^6
*d^3*n^5*x^6 + 175*(b*x + a)^n*b^7*d^3*n^4*x^7 + 3*(b*x + a)^n*a*b^6*c*d^2*
n^6*x^4 + 69*(b*x + a)^n*b^7*c*d^2*n^5*x^5 - 6*(b*x + a)^n*a^2*b^5*d^3*n^5*
x^5 + 85*(b*x + a)^n*a*b^6*d^3*n^4*x^6 + 735*(b*x + a)^n*b^7*d^3*n^3*x^7 +
3*(b*x + a)^n*b^7*c^2*d*n^6*x^3 + 57*(b*x + a)^n*a*b^6*c*d^2*n^5*x^4 + 621*
(b*x + a)^n*b^7*c*d^2*n^4*x^5 - 60*(b*x + a)^n*a^2*b^5*d^3*n^4*x^5 + 225*(b
*x + a)^n*a*b^6*d^3*n^3*x^6 + 1624*(b*x + a)^n*b^7*d^3*n^2*x^7 + 3*(b*x + a
)^n*a*b^6*c^2*d*n^6*x^2 + 75*(b*x + a)^n*b^7*c^2*d*n^5*x^3 - 12*(b*x + a)^n
*a^2*b^5*c*d^2*n^5*x^3 + 393*(b*x + a)^n*a*b^6*c*d^2*n^4*x^4 + 30*(b*x + a)
^n*a^3*b^4*d^3*n^4*x^4 + 2775*(b*x + a)^n*b^7*c*d^2*n^3*x^5 - 210*(b*x + a)
^n*a^2*b^5*d^3*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^3*n^2*x^6 + 1764*(b*x + a)
^n*b^7*d^3*n*x^7 + (b*x + a)^n*b^7*c^3*n^6*x + 69*(b*x + a)^n*a*b^6*c^2*d*n
^5*x^2 + 741*(b*x + a)^n*b^7*c^2*d*n^4*x^3 - 192*(b*x + a)^n*a^2*b^5*c*d^2*
n^4*x^3 + 1203*(b*x + a)^n*a*b^6*c*d^2*n^3*x^4 + 180*(b*x + a)^n*a^3*b^4*d^
3*n^3*x^4 + 6432*(b*x + a)^n*b^7*c*d^2*n^2*x^5 - 300*(b*x + a)^n*a^2*b^5*d^
3*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^3*n*x^6 + 720*(b*x + a)^n*b^7*d^3*x^7 +
 (b*x + a)^n*a*b^6*c^3*n^6 + 27*(b*x + a)^n*b^7*c^3*n^5*x - 6*(b*x + a)^n*a
^2*b^5*c^2*d*n^5*x + 603*(b*x + a)^n*a*b^6*c^2*d*n^4*x^2 + 36*(b*x + a)^n*a
^3*b^4*c*d^2*n^4*x^2 + 3657*(b*x + a)^n*b^7*c^2*d*n^3*x^3 - 996*(b*x + a)^n
*a^2*b^5*c*d^2*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^3*n^3*x^3 + 1620*(b*x +
a)^n*a*b^6*c*d^2*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^3*n^2*x^4 + 7236*(b*x
+ a)^n*b^7*c*d^2*n*x^5 - 144*(b*x + a)^n*a^2*b^5*d^3*n*x^5 + 27*(b*x + a)^n
*a*b^6*c^3*n^5 + 295*(b*x + a)^n*b^7*c^3*n^4*x - 132*(b*x + a)^n*a^2*b^5*c^
2*d*n^4*x + 2451*(b*x + a)^n*a*b^6*c^2*d*n^3*x^2 + 504*(b*x + a)^n*a^3*b^4*
c*d^2*n^3*x^2 + 9336*(b*x + a)^n*b^7*c^2*d*n^2*x^3 - 1824*(b*x + a)^n*a^2*b
^5*c*d^2*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^3*n^2*x^3 + 756*(b*x + a)^n*a*
b^6*c*d^2*n*x^4 + 180*(b*x + a)^n*a^3*b^4*d^3*n*x^4 + 3024*(b*x + a)^n*b^7*
c*d^2*x^5 + 295*(b*x + a)^n*a*b^6*c^3*n^4 + 6*(b*x + a)^n*a^3*b^4*c^2*d*n^4
 + 1665*(b*x + a)^n*b^7*c^3*n^3*x - 1074*(b*x + a)^n*a^2*b^5*c^2*d*n^3*x -
72*(b*x + a)^n*a^4*b^3*c*d^2*n^3*x + 4434*(b*x + a)^n*a*b^6*c^2*d*n^2*x^2 +
 1980*(b*x + a)^n*a^3*b^4*c*d^2*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^3*n^2*x
^2 + 11388*(b*x + a)^n*b^7*c^2*d*n*x^3 - 1008*(b*x + a)^n*a^2*b^5*c*d^2*n*x
```

```
^3 - 240*(b*x + a)^n*a^4*b^3*d^3*n*x^3 + 1665*(b*x + a)^n*a*b^6*c^3*n^3 + 1
32*(b*x + a)^n*a^3*b^4*c^2*d*n^3 + 5104*(b*x + a)^n*b^7*c^3*n^2*x - 3828*(b
*x + a)^n*a^2*b^5*c^2*d*n^2*x - 936*(b*x + a)^n*a^4*b^3*c*d^2*n^2*x + 2520*
(b*x + a)^n*a*b^6*c^2*d*n*x^2 + 1512*(b*x + a)^n*a^3*b^4*c*d^2*n*x^2 + 360*
(b*x + a)^n*a^5*b^2*d^3*n*x^2 + 5040*(b*x + a)^n*b^7*c^2*d*x^3 + 5104*(b*x
+ a)^n*a*b^6*c^3*n^2 + 1074*(b*x + a)^n*a^3*b^4*c^2*d*n^2 + 72*(b*x + a)^n*
a^5*b^2*c*d^2*n^2 + 8028*(b*x + a)^n*b^7*c^3*n*x - 5040*(b*x + a)^n*a^2*b^5
*c^2*d*n*x - 3024*(b*x + a)^n*a^4*b^3*c*d^2*n*x - 720*(b*x + a)^n*a^6*b*d^3
*n*x + 8028*(b*x + a)^n*a*b^6*c^3*n + 3828*(b*x + a)^n*a^3*b^4*c^2*d*n + 93
6*(b*x + a)^n*a^5*b^2*c*d^2*n + 5040*(b*x + a)^n*b^7*c^3*x + 5040*(b*x + a)
^n*a*b^6*c^3 + 5040*(b*x + a)^n*a^3*b^4*c^2*d + 3024*(b*x + a)^n*a^5*b^2*c*
d^2 + 720*(b*x + a)^n*a^7*d^3)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b
^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

Mupad [B]
time = 3.16, size = 1144, normalized size = 5.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*(720*a^7*d^3 + 5040*a*b^6*c^3 + 5040*a^3*b^4*c^2*d + 3024*a^5*
b^2*c*d^2 + 5104*a*b^6*c^3*n^2 + 1665*a*b^6*c^3*n^3 + 295*a*b^6*c^3*n^4 + 2
7*a*b^6*c^3*n^5 + a*b^6*c^3*n^6 + 8028*a*b^6*c^3*n + 3828*a^3*b^4*c^2*d*n +
 936*a^5*b^2*c*d^2*n + 1074*a^3*b^4*c^2*d*n^2 + 72*a^5*b^2*c*d^2*n^2 + 132*
a^3*b^4*c^2*d*n^3 + 6*a^3*b^4*c^2*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*
n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (x*(a + b*x)^n*(720*a^6*
b*d^3*n - 8028*b^7*c^3*n - 5104*b^7*c^3*n^2 - 1665*b^7*c^3*n^3 - 295*b^7*c^
3*n^4 - 27*b^7*c^3*n^5 - b^7*c^3*n^6 - 5040*b^7*c^3 + 5040*a^2*b^5*c^2*d*n
+ 3024*a^4*b^3*c*d^2*n + 3828*a^2*b^5*c^2*d*n^2 + 936*a^4*b^3*c*d^2*n^2 + 1
074*a^2*b^5*c^2*d*n^3 + 72*a^4*b^3*c*d^2*n^3 + 132*a^2*b^5*c^2*d*n^4 + 6*a^
2*b^5*c^2*d*n^5))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5
 + 28*n^6 + n^7 + 5040)) + (d^3*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^
3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n
^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (3*d^2*x^5*(a + b*x)^n*(42*b^2*c + b^
2*c*n^2 - 2*a^2*d*n + 13*b^2*c*n)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b^2
*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040
)) + (3*d*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(840*b^4*c^2 - 40*a^4*d^2*n + 638
*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 - 168*a^2*b^2*c
*d*n - 52*a^2*b^2*c*d*n^2 - 4*a^2*b^2*c*d*n^3))/(b^4*(13068*n + 13132*n^2 +
 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (a*d^3*n*x^6*(a +
b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b*(13068*n + 13132
*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (3*a*d^2*n*x
^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(10*a^2*d + 42*b^2*c + b^2*c*n^2 +
```

```
13*b^2*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28
*n^6 + n^7 + 5040)) + (3*a*d*n*x^2*(n + 1)*(a + b*x)^n*(120*a^4*d^2 + 840*b
^4*c^2 + 638*b^4*c^2*n + 179*b^4*c^2*n^2 + 22*b^4*c^2*n^3 + b^4*c^2*n^4 + 5
04*a^2*b^2*c*d + 156*a^2*b^2*c*d*n + 12*a^2*b^2*c*d*n^2))/(b^5*(13068*n + 1
3132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))
```

3.362 $\int \frac{(a+bx)^n (c+dx^2)^3}{x} \, dx$

Optimal. Leaf size=246

$$-\frac{ad(3b^4c^2+3a^2b^2cd+a^4d^2)\,(a+bx)^{1+n}}{b^6(1+n)}+\frac{d(3b^4c^2+9a^2b^2cd+5a^4d^2)\,(a+bx)^{2+n}}{b^6(2+n)}-\frac{ad^2(9b^2c+10a^2d)\,(a+$$

```
[Out] -a*d*(a^4*d^2+3*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(1+n)/b^6/(1+n)+d*(5*a^4*d^2
+9*a^2*b^2*c*d+3*b^4*c^2)*(b*x+a)^(2+n)/b^6/(2+n)-a*d^2*(10*a^2*d+9*b^2*c)*
(b*x+a)^(3+n)/b^6/(3+n)+d^2*(10*a^2*d+3*b^2*c)*(b*x+a)^(4+n)/b^6/(4+n)-5*a*
d^3*(b*x+a)^(5+n)/b^6/(5+n)+d^3*(b*x+a)^(6+n)/b^6/(6+n)-c^3*(b*x+a)^(1+n)*h
ypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A]
time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {966, 1634, 67}

$$-\frac{ad^2(10a^2d+9b^2c)\,(a+bx)^{n+3}}{b^6(n+3)}+\frac{d^2(10a^2d+3b^2c)\,(a+bx)^{n+4}}{b^6(n+4)}-\frac{ad(a^4d^2+3a^2b^2cd+3b^4c^2)\,(a+bx)^{n+1}}{b^6(n+1)}+\frac{d(5a^4d^2+9a^2b^2cd+3b^4c^2)\,(a+bx)^{n+2}}{b^6(n+2)}-\frac{5ad^3(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^3(a+bx)^{n+6}}{b^6(n+6)}-\frac{c^3(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

```
[Out] -((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2)*(a + b*x)^(1 + n))/(b^6*(1 + n
))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x)^(2 + n))/(b^6*(2
 + n)) - (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (d^
2*(3*b^2*c + 10*a^2*d)*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d^3*(a + b*x
)^(5 + n))/(b^6*(5 + n)) + (d^3*(a + b*x)^(6 + n))/(b^6*(6 + n)) - (c^3*(a
+ b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
```

```
!IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(a+bx)^n\left(c+dx^2\right)^3}{x}\,dx = \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + \frac{\int \frac{(a+bx)^n\left(b^6c^3(6+n)-a^5bd^3(6+n)x+b^2d\left(3b^4c^2-5a^4d^2\right)(6+n)x^2-10a^3b^3d^3(6+n)\cdots}{x}}{b^6(6+n)}$$

$$= \frac{d^3(a+bx)^{6+n}}{b^6(6+n)} + \frac{\int\left(-abd\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(6+n)(a+bx)^n+\frac{(6b^6c^3+b^6c\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx\cdots}{b^6(2+n)}$$

$$= -\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)(a+bx)^{1+n}}{b^6(1+n)} + \frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx\cdots}{b^6(2+n)}$$

Mathematica [A]
time = 0.19, size = 226, normalized size = 0.92

$$(a+bx)^{1+n}\left(-\frac{ad\left(3b^4c^2+3a^2b^2cd+a^4d^2\right)}{b^6(1+n)}+\frac{d\left(3b^4c^2+9a^2b^2cd+5a^4d^2\right)(a+bx)}{b^6(2+n)}-\frac{ad^2\left(9b^2c+10a^2d\right)(a+bx)^2}{b^6(3+n)}+\frac{d^2\left(3b^2c+10a^2d\right)(a+bx)^3}{b^6(4+n)}-\frac{5ad^3(a+bx)^4}{b^6(5+n)}+\frac{d^3(a+bx)^5}{b^6(6+n)}-\frac{c^3\,{}_2F_1\left(1,1+n;2+n;\frac{a+bx}{a}\right)}{a+an}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^2)^3)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*(-((a*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2))/(b^6*(1 +
n))) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x))/(b^6*(2 + n))
- (a*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^2)/(b^6*(3 + n)) + (d^2*(3*b^2*c +
10*a^2*d)*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^3*(a + b*x)^4)/(b^6*(5 + n))
+ (d^3*(a + b*x)^5)/(b^6*(6 + n)) - (c^3*Hypergeometric2F1[1, 1 + n, 2 + n,
 (a + b*x)/a])/(a + a*n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\left(d\,x^2+c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^2+c)^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x + a)^n/x, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4007 vs. $2(226) = 452$.
time = 5.67, size = 5692, normalized size = 23.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**2+c)**3/x,x)
```

```
[Out] -b**n*c**3*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**3*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 3*c**2*d*Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/
(a*b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x
), Eq(n, -2)), (-a*log(a/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n
/(b**2*n**2 + 3*b**2*n + 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2
*n + 2*b**2) + b**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b
**2*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) + 3*c*d**2*Pi
ecewise((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a**
2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2*b
**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*
```

```
a*b**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 6*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
6*b**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b
*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2*a
**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2*b
**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**
3/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3
*a*b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(n,
 -2)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b
), Eq(n, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n*
*2 + 50*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*
n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b*x
)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a*
*2*b**2*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b
**4*n + 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3
+ 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(b*
*4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n*x
**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) + b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**
2 + 50*b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b*
*4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)**n
/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*x
**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4), True)) + d**3*Piecewise((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b +
x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**
3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b
**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**
11*x**5) + 300*a**4*b*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*
a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 6
25*a**4*b*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2
*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*x**2*log(a/b
 + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*
x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*x**2/(60*a**5*b**
6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10
*x**4 + 60*b**11*x**5) + 600*a**2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 30
0*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
+ 60*b**11*x**5) + 900*a**2*b**3*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600
*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) +
300*a*b**4*x**4*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**
8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**
4*x**4/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*a
**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*
```

```
a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*log(a/b + x)/(12*a**4*
b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4)
 - 125*a**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*
x**3 + 12*b**10*x**4) - 240*a**4*b*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b
**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 440*a**4*b*x/
(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b*
*10*x**4) - 360*a**3*b**2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x
+ 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^2+c)^3/x,x, algorithm="giac")
[Out] integrate((d*x^2 + c)^3*(b*x + a)^n/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^2+c\right)^3\,\left(a+b\,x\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^3*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^2)^3*(a + b*x)^n)/x, x)
```

3.363 $\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=250

$$\frac{(cd^2-ae^2)\,(d+ex)^{1+n}}{c^2e^3(1+n)}-\frac{2d(d+ex)^{2+n}}{ce^3(2+n)}+\frac{(d+ex)^{3+n}}{ce^3(3+n)}+\frac{(-a)^{3/2}(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}}\right.}{2c^2\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}$$

```
[Out] (-a*e^2+c*d^2)*(e*x+d)^(1+n)/c^2/e^3/(1+n)-2*d*(e*x+d)^(2+n)/c/e^3/(2+n)+(e
*x+d)^(3+n)/c/e^3/(3+n)+1/2*(-a)^(3/2)*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+
n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/c^2/(1+n)/(-e*(-a)^(1/2)+d*c^
(1/2))-1/2*(-a)^(3/2)*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2
)/(e*(-a)^(1/2)+d*c^(1/2)))/c^2/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.28, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1643, 726, 70}

$$\frac{(cd^2-ae^2)\,(d+ex)^{n+1}}{c^2e^3(n+1)}+\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c^2(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{(-a)^{3/2}(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2c^2(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}-\frac{2d(d+ex)^{n+2}}{ce^3(n+2)}+\frac{(d+ex)^{n+3}}{ce^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((c*d^2 - a*e^2)*(d + e*x)^(1 + n))/(c^2*e^3*(1 + n)) - (2*d*(d + e*x)^(2 +
 n))/(c*e^3*(2 + n)) + (d + e*x)^(3 + n)/(c*e^3*(3 + n)) + ((-a)^(3/2)*(d +
 e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[
c]*d - Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - ((-a)^(3/2)
*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{(cd^2-ae^2)(d+ex)^n}{c^2e^2} - \frac{2d(d+ex)^{1+n}}{ce^2} + \frac{(d+ex)^{2+n}}{ce^2} + \frac{a^2(d+ex)^n}{c^2(a+cx^2)}\right) dx$$

$$= \frac{(cd^2-ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{a^2\int \frac{(d+ex)^n}{a+cx^2}\,dx}{c^2}$$

$$= \frac{(cd^2-ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{a^2\int \left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \right.}{c^2}$$

$$= \frac{(cd^2-ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} - \frac{(-a)^{3/2}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2c^2}$$

$$= \frac{(cd^2-ae^2)(d+ex)^{1+n}}{c^2e^3(1+n)} - \frac{2d(d+ex)^{2+n}}{ce^3(2+n)} + \frac{(d+ex)^{3+n}}{ce^3(3+n)} + \frac{(-a)^{3/2}(d+ex)^{1+n}\,{}_2F_1\left(1,\right.}{2c^2\left(\sqrt{c}\,d-\right.}$$

Mathematica [A]
time = 0.41, size = 217, normalized size = 0.87

$$\frac{(d+ex)^{1+n}\left(\frac{2(cd^2-ae^2)}{e^3(1+n)} - \frac{4cd(d+ex)}{e^3(2+n)} + \frac{2c(d+ex)^2}{e^3(3+n)} + \frac{(-a)^{3/2}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{\sqrt{-a}\,a\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*(c*d^2 - a*e^2))/(e^3*(1 + n)) - (4*c*d*(d + e*x))/(
e^3*(2 + n)) + (2*c*(d + e*x)^2)/(e^3*(3 + n)) + ((-a)^(3/2)*Hypergeometric
2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[
c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt[-a]*a*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*
(1 + n))))/(2*c^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^4*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^n*x^4/(c*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^n*x^4/(c*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x**4*(d + e*x)**n/(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n*x^4/(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^n)/(a + c*x^2),x)
```

```
[Out] int((x^4*(d + e*x)^n)/(a + c*x^2), x)
```

3.364 $\int \frac{x^3(d+ex)^n}{a+cx^2} dx$

Optimal. Leaf size=209

$$-\frac{d(d+ex)^{1+n}}{ce^2(1+n)}+\frac{(d+ex)^{2+n}}{ce^2(2+n)}+\frac{a(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}+\frac{a(d+ex)^{1+n}\,{}_2F_1\left(1,1+n\right.}{2c^{3/2}\left(\sqrt{c}\,d+\sqrt{}\right.}$$

```
[Out] -d*(e*x+d)^(1+n)/c/e^2/(1+n)+(e*x+d)^(2+n)/c/e^2/(2+n)+1/2*a*(e*x+d)^(1+n)*
hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/c^(3/2)
/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*a*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+
n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/c^(3/2)/(1+n)/(e*(-a)^(1/2)+d*
c^(1/2))
```

Rubi [A]
time = 0.15, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1643, 845, 70}

$$\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c^{3/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}+\frac{a(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2c^{3/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}-\frac{d(d+ex)^{n+1}}{ce^2(n+1)}+\frac{(d+ex)^{n+2}}{ce^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -((d*(d + e*x)^(1 + n))/(c*e^2*(1 + n))) + (d + e*x)^(2 + n)/(c*e^2*(2 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)
) + (a*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3(d+ex)^n}{a+cx^2}\,dx = \int \left(-\frac{d(d+ex)^n}{ce}+\frac{(d+ex)^{1+n}}{ce}-\frac{ax(d+ex)^n}{c\,(a+cx^2)}\right)dx$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)}+\frac{(d+ex)^{2+n}}{ce^2(2+n)}-\frac{a\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{c}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)}+\frac{(d+ex)^{2+n}}{ce^2(2+n)}-\frac{a\int \left(-\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{(d+ex)^n}{2\sqrt{c}\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)}{c}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)}+\frac{(d+ex)^{2+n}}{ce^2(2+n)}+\frac{a\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2c^{3/2}}-\frac{a\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2c^{3/2}}$$

$$= -\frac{d(d+ex)^{1+n}}{ce^2(1+n)}+\frac{(d+ex)^{2+n}}{ce^2(2+n)}+\frac{a(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}+$$

Mathematica [A]
time = 0.20, size = 168, normalized size = 0.80

$$\frac{(d+ex)^{1+n}\left(-\frac{2\sqrt{c}\,(d-e(1+n)x)}{e^2(2+n)}+\frac{a\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{c}\,d-\sqrt{-a}\,e}+\frac{a\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2c^{3/2}(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*Sqrt[c]*(d - e*(1 + n)*x))/(e^2*(2 + n)) + (a*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
)/(Sqrt[c]*d - Sqrt[-a]*e) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]
*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(2*c^(3/2
)*(1 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(e x +d \right)^{n}}{c \,x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x^3*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^n*x^3/(c*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^n*x^3/(c*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(d + e x\right)^{n}}{a + c x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x**3*(d + e*x)**n/(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n*x^3/(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(d+e\,x\right)}^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^n)/(a + c*x^2),x)
```

```
[Out] int((x^3*(d + e*x)^n)/(a + c*x^2), x)
```

3.365 $\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=194

$$\frac{(d+ex)^{1+n}}{ce(1+n)}+\frac{\sqrt{-a}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{\sqrt{-a}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{2c\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+}$$

```
[Out] (e*x+d)^(1+n)/c/e/(1+n)+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*
c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(-a)^(1/2)/c/(1+n)/(-e*(-a)^(1/2)+d*c^(1
/2))-1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/
2)+d*c^(1/2)))*(-a)^(1/2)/c/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.16, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1643, 726, 70}

$$\frac{\sqrt{-a}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{\sqrt{-a}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2c(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}+\frac{(d+ex)^{n+1}}{ce(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] (d + e*x)^(1 + n)/(c*e*(1 + n)) + (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*c*(
Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (Sqrt[-a]*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*c*
(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx &= \int \left(\frac{(d+ex)^n}{c}-\frac{a(d+ex)^n}{c\,(a+cx^2)}\right)dx\\
&= \frac{(d+ex)^{1+n}}{ce(1+n)}-\frac{a\int \frac{(d+ex)^n}{a+cx^2}\,dx}{c}\\
&= \frac{(d+ex)^{1+n}}{ce(1+n)}-\frac{a\int \left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx}{c}\\
&= \frac{(d+ex)^{1+n}}{ce(1+n)}-\frac{\sqrt{-a}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2c}-\frac{\sqrt{-a}\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2c}\\
&= \frac{(d+ex)^{1+n}}{ce(1+n)}+\frac{\sqrt{-a}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{\sqrt{-a}\,(d+ex}{2}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 170, normalized size = 0.88

$$\frac{(d+ex)^{1+n}\left(2(cd^2+ae^2)+e\left(\sqrt{-a}\,\sqrt{c}\,d-ae\right)\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)-e\left(\sqrt{-a}\,\sqrt{c}\,d+ae\right)\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)\right)}{2ce\,(cd^2+ae^2)\,(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*(2*(c*d^2 + a*e^2) + e*(Sqrt[-a]*Sqrt[c]*d - a*e)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
- e*(Sqrt[-a]*Sqrt[c]*d + a*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*
(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]))/(2*c*e*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^n/(c*x^2+a),x)
[Out] int(x^2*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((x*e + d)^n*x^2/(c*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
[Out] integral((x*e + d)^n*x^2/(c*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**n/(c*x**2+a),x)
[Out] Integral(x**2*(d + e*x)**n/(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((x*e + d)^n*x^2/(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(d+e\,x\right)}^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^n)/(a + c*x^2),x)
[Out] int((x^2*(d + e*x)^n)/(a + c*x^2), x)
```

3.366 $\int \frac{x(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=163

$$-\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2\sqrt{c}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2\sqrt{c}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}$$

```
[Out] -1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+
d*c^(1/2)))/(1+n)/c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*(e*x+d)^(1+n)*hyper
geom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(1+n)/c^(1/2)
/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.07, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {845, 70}

$$-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2\sqrt{c}\,(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2\sqrt{c}\,(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -1/2*((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*
x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) -
 ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx = \int \left(-\frac{(d+ex)^n}{2\sqrt{c}\,\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{(d+ex)^n}{2\sqrt{c}\,\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2\sqrt{c}}+\frac{\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2\sqrt{c}}$$

$$= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2\sqrt{c}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{2\sqrt{c}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1}$$

Mathematica [A]
time = 0.08, size = 151, normalized size = 0.93

$$-\frac{(d+ex)^{1+n}\left(\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)+\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)\right)}{2\sqrt{c}\,(cd^2+ae^2)\,(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] -1/2*((d + e*x)^(1 + n)*((Sqrt[c]*d + Sqrt[-a]*e)*Hypergeometric2F1[1, 1 +
n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (Sqrt[c]*d - Sqrt
[-a]*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d +
 Sqrt[-a]*e)]))/(Sqrt[c]*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int(x*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^n*x/(c*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^n*x/(c*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Integral(x*(d + e*x)**n/(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n*x/(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(d+e\,x\right)}^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^n)/(a + c*x^2),x)
```

```
[Out] int((x*(d + e*x)^n)/(a + c*x^2), x)
```

3.367 $\int \frac{(d+ex)^n}{a+cx^2} dx$

Optimal. Leaf size=167

$$\frac{(d+ex)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+n)}-\frac{(d+ex)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+n)}$$

```
[Out] 1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d
*c^(1/2)))/(1+n)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*(e*x+d)^(1+n)*hyp
ergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(1+n)/(-a)^
(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.06, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {726, 70}

$$\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(n+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (
(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2\sqrt{-a}} - \frac{\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2\sqrt{-a}}$$

$$= \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}$$

Mathematica [A]
time = 0.09, size = 145, normalized size = 0.87

$$\frac{(d+ex)^{1+n}\left(\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{c}\,d-\sqrt{-a}\,e}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2\sqrt{-a}\,(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(a + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)]/(Sqrt[c]*d - Sqrt[-a]*e) - Hypergeometric2F1[1, 1
 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(Sqrt[c]*d + Sqr
t[-a]*e)))/(2*Sqrt[-a]*(1 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/(c*x^2+a),x)
```

```
[Out] int((e*x+d)^n/(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((x*e + d)^n/(c*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
[Out] integral((x*e + d)^n/(c*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^n}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/(c*x**2+a),x)
[Out] Integral((d + e*x)**n/(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a),x, algorithm="giac")
[Out] integrate((x*e + d)^n/(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(a + c*x^2),x)
[Out] int((d + e*x)^n/(a + c*x^2), x)
```

3.368 $\int \frac{(d+ex)^n}{x(a+cx^2)} dx$

Optimal. Leaf size=207

$$\frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}+\frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2a\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}-$$

```
[Out] -(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],1+e*x/d)/a/d/(1+n)+1/2*(e*x+d)^(1+n
)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*c^(1/
2)/a/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[
2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a/(1+n)/(e*(-a)^(1/2
)+d*c^(1/2))
```

Rubi [A]

time = 0.13, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {975, 67, 845, 70}

$$\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}+\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2a(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{ex}{d}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x*(a + c*x^2)),x]
```

```
[Out] (Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) +
(Sqrt[c]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d +
 e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*a*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) -
((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e*x)/d])/(a*d*(1
 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx = \int \left(\frac{(d+ex)^n}{ax} - \frac{cx(d+ex)^n}{a\,(a+cx^2)}\right)dx$$

$$= \frac{\int \frac{(d+ex)^n}{x}\,dx}{a} - \frac{c\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{a}$$

$$= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)} - \frac{c\int \left(-\frac{(d+ex)^n}{2\sqrt{c}\,\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{(d+ex)}{2\sqrt{c}\,\left(\sqrt{-a}\right.}\right.}{a}$$

$$= -\frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{ad(1+n)} + \frac{\sqrt{c}\,\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2a} - \frac{\sqrt{c}\,\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}}}{2a}$$

$$= \frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2\right.}{2a\left(\sqrt{c}\,d+\sqrt{-a}\,e\right.}$$

Mathematica [A]
time = 0.19, size = 189, normalized size = 0.91

$$\frac{(d+ex)^{1+n}\left(\left(cd^2+\sqrt{-a}\,\sqrt{c}\,de\right)\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)+\left(cd^2-\sqrt{-a}\,\sqrt{c}\,de\right)\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)-2(cd^2+ae^2)\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)\right)}{2ad\,(cd^2+ae^2)\,(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x*(a + c*x^2)),x]
```

```
[Out] ((d + e*x)^(1 + n)*((c*d^2 + Sqrt[-a]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)] + (c*d^2 - Sqrt[-a
```

```
]*Sqrt[c]*d*e)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)] - 2*(c*d^2 + a*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n,
 1 + (e*x)/d]))/(2*a*d*(c*d^2 + a*e^2)*(1 + n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{x\,(c\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x/(c*x^2+a),x)
[Out] int((e*x+d)^n/x/(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="maxima")
[Out] integrate((x*e + d)^n/((c*x^2 + a)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="fricas")
[Out] integral((x*e + d)^n/(c*x^3 + a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x/(c*x**2+a),x)
[Out] Integral((d + e*x)**n/(x*(a + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a),x, algorithm="giac")
[Out] integrate((x*e + d)^n/((c*x^2 + a)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{x\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(x*(a + c*x^2)),x)
[Out] int((d + e*x)^n/(x*(a + c*x^2)), x)
```

3.369 $\int \frac{(d+ex)^n}{x^2(a+cx^2)}\,dx$

Optimal. Leaf size=207

$$\frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2(-a)^{3/2}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}+\frac{e(d+$$

```
[Out] e*(e*x+d)^(1+n)*hypergeom([2, 1+n],[2+n],1+e*x/d)/a/d^2/(1+n)+1/2*c*(e*x+d)
^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/
(-a)^(3/2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))-1/2*c*(e*x+d)^(1+n)*hypergeom([1
, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(3/2)/(1+n)/(e*
(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.15, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {975, 67, 726, 70}

$$\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}-\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2(-a)^{3/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}+\frac{e(d+ex)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{ad^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

```
[Out] (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))
/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n))
 - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*
x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(3/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 +
n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])
/(a*d^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex)^n}{x^2\,(a+cx^2)}\,dx = \int \left(\frac{(d+ex)^n}{ax^2}-\frac{c(d+ex)^n}{a\,(a+cx^2)}\right)dx$$

$$=\frac{\int \frac{(d+ex)^n}{x^2}\,dx}{a}-\frac{c\int \frac{(d+ex)^n}{a+cx^2}\,dx}{a}$$

$$=\frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)}-\frac{c\int \left(\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{\sqrt{-a}\,(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right.}{a}$$

$$=\frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{ad^2(1+n)}-\frac{c\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2(-a)^{3/2}}-\frac{c\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2(-a)^{3/2}}$$

$$=\frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{2(-a)^{3/2}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)}$$

Mathematica [A]
time = 0.23, size = 167, normalized size = 0.81

$$\frac{(d+ex)^{1+n}\left(-\frac{c\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{-a}\,\sqrt{c}\,d+ae}+\frac{c\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{-a}\,\sqrt{c}\,d-ae}+\frac{2e\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{d^2}\right)}{2a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x^2*(a + c*x^2)),x]
```

```
[Out] ((d + e*x)^(1 + n)*(-((c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e
*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[c]*d + a*e)) + (c*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sq
rt[-a]*Sqrt[c]*d - a*e) + (2*e*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)
/d])/d^2))/(2*a*(1 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{x^2\,(c\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x^2/(c*x^2+a),x)
```

```
[Out] int((e*x+d)^n/x^2/(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^n/((c*x^2 + a)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^n/(c*x^4 + a*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x**2/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n/((c*x^2 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{x^2\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(x^2*(a + c*x^2)),x)
```

```
[Out] int((d + e*x)^n/(x^2*(a + c*x^2)), x)
```

3.370 $\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=332

$$\frac{(d+ex)^{1+n}}{c^2e(1+n)}+\frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2\left(cd^2+ae^2\right)\left(a+cx^2\right)}+\frac{\left(3\sqrt{-a}\,cd^2+a\sqrt{c}\,den+\sqrt{-a}\,ae^2(3+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+\right.}{4c^2\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(1+n)}$$

```
[Out] (e*x+d)^(1+n)/c^2/e/(1+n)+1/2*a*(c*d*x+a*e)*(e*x+d)^(1+n)/c^2/(a*e^2+c*d^2)
/(c*x^2+a)-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-
a)^(1/2)+d*c^(1/2)))*(3*c*d^2*(-a)^(1/2)+a*e^2*(3+n)*(-a)^(1/2)-a*d*e*n*c^(
1/2))/c^2/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))+1/4*(e*x+d)^(1+n)*hy
pergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(3*c*d^2*
(-a)^(1/2)+a*e^2*(3+n)*(-a)^(1/2)+a*d*e*n*c^(1/2))/c^2/(a*e^2+c*d^2)/(1+n)/
(-e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.31, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1663, 1643, 70}

$$\frac{(d+ex)^{n+1}\left(3\sqrt{-a}\,cd^2+a\sqrt{c}\,den+\sqrt{-a}\,ae^2(n+3)\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4c^2(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(3\sqrt{-a}\,cd^2-a\sqrt{c}\,den+\sqrt{-a}\,ae^2(n+3)\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4c^2(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}+\frac{a(d+ex)^{n+1}(ae+cdx)}{2c^2\left(a+cx^2\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}}{c^2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] (d + e*x)^(1 + n)/(c^2*e*(1 + n)) + (a*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*
c^2*(c*d^2 + a*e^2)*(a + c*x^2)) + ((3*Sqrt[-a]*c*d^2 + a*Sqrt[c]*d*e*n + S
qrt[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d - Sqrt[-a
]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((3*Sqrt[-a]*c*d^2 - a*Sqrt[c]*d*e*n + Sqrt
[-a]*a*e^2*(3 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^2*(Sqrt[c]*d + Sqrt[-a]*e
)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} - \frac{\int \frac{(d+ex)^n\left(\frac{a^2\left(cd^2+ae^2(1+n)\right)}{c^2}+\frac{a^2denx}{c}-2a\left(d^2+\frac{ae^2}{c}\right)x^2\right)}{a+cx^2}\,dx}{2a(cd^2+ae^2)}$$

$$= \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} - \frac{\int \left(-\frac{2a(cd^2+ae^2)(d+ex)^n}{c^2} + \frac{\left(-\frac{a^3den}{c^{3/2}}+\sqrt{-a}\left(\frac{3a^2d^2}{c}+\frac{3a^3e^2}{c^2}+\frac{a^3e^2n}{c^2}\right)\right.}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)}\right.}{2a\,(cd^2}$$

$$= \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} - \frac{\left(3\sqrt{-a}\,cd^2-a\sqrt{c}\,den+\sqrt{-a}\,ae^2(3+n)\right)}{4c^2(cd^2+ae^2)}$$

$$= \frac{(d+ex)^{1+n}}{c^2e(1+n)} + \frac{a(ae+cdx)(d+ex)^{1+n}}{2c^2(cd^2+ae^2)(a+cx^2)} + \frac{\left(3\sqrt{-a}\,cd^2+a\sqrt{c}\,den+\sqrt{-a}\,ae^2(3+n)\right)}{4c^2\left(\sqrt{c}\,d-\sqrt{-}\right.}$$

Mathematica [A]
time = 0.67, size = 413, normalized size = 1.24

$$\frac{(d+ex)^{1+n}\left(\frac{4}{e+en}+\frac{2a(ae+cdx)}{(cd^2+ae^2)(a+cx^2)}+\frac{4\sqrt{-a}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{4\sqrt{-a}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}+\frac{a\left(\frac{\left(cd^2-ae^2(-1+n)+\sqrt{-a}\,\sqrt{c}\,den\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{c}\,d-\sqrt{-a}\,e}-\frac{\left(cd^2-ae^2(-1+n)-\sqrt{-a}\,\sqrt{c}\,den\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{-a}\,(cd^2+ae^2)(1+n)}\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*(4/(e + e*n) + (2*a*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a +
 c*x^2)) + (4*Sqrt[-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (4*Sqrt[
-a]*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqr
t[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n
) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e
^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/(Sq
rt[-a]*(c*d^2 + a*e^2)*(1 + n))))/(4*c^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(e x +d \right)^{n}}{\left(c \,x^{2}+a \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^4*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^n*x^4/(c*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^n*x^4/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n*x^4/(c*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d+e\,x\right)}^n}{{\left(c\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^n)/(a + c*x^2)^2,x)
```

```
[Out] int((x^4*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.371 $\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=297

$$\frac{a(d-ex)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)}+\frac{\left(\sqrt{-a}\,den-\frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(1+n)}-\frac{(2c\cdots}{\cdots}$$

```
[Out] 1/2*a*(-e*x+d)*(e*x+d)^(1+n)/c/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*hy
pergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(d*e*n*(-
a)^(1/2)+(-2*c*d^2-a*e^2*(2+n))/c^(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/
2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e
*(-a)^(1/2)+d*c^(1/2)))*(2*c*d^2+a*e^2*(2+n)+d*e*n*(-a)^(1/2)*c^(1/2))/c^(3
/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.27, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1663, 845, 70}

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2(n+2)+2cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4c^{3/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(\sqrt{-a}\,den-\frac{ae^2(n+2)+2cd^2}{\sqrt{c}}\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4c(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}+\frac{a(d-ex)(d+ex)^{n+1}}{2c\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] (a*(d - e*x)*(d + e*x)^(1 + n))/(2*c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((Sqrt[
-a]*d*e*n - (2*c*d^2 + a*e^2*(2 + n))/Sqrt[c])*(d + e*x)^(1 + n)*Hypergeome
tric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*
c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((2*c*d^2 + Sqrt[-a]*
Sqrt[c]*d*e*n + a*e^2*(2 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*c^(3/2)*(Sqrt[c]
*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
```

```
]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^3(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{a(d-ex)(d+ex)^{1+n}}{2c\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\int \frac{(d+ex)^n\left(\frac{a^2den}{c}-\frac{a\left(2cd^2+ae^2(2+n)\right)x}{c}\right)}{a+cx^2}\,dx}{2a\,(cd^2+ae^2)}$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\int \left(\frac{\left(\frac{\sqrt{-a}\,a^2den}{c}+\frac{a^2\left(2cd^2+ae^2(2+n)\right)}{c^{3/2}}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(\frac{\sqrt{-a}\,a^2den}{c}-\frac{a^2(\cdots}{\cdots}\right.}{2a\left(\sqrt{-\cdots}\right.}\right.}{2a\,(cd^2+ae^2)}$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\left(2cd^2+\sqrt{-a}\,\sqrt{c}\,den+ae^2(2+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{4c^{3/2}\,(cd^2+ae^2)} - \cdots$$

$$= \frac{a(d-ex)(d+ex)^{1+n}}{2c\,(cd^2+ae^2)\,(a+cx^2)} + \frac{\left(\sqrt{-a}\,den-\frac{2cd^2+ae^2(2+n)}{\sqrt{c}}\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+\cdots\right.}{4c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(cd^2+ae^2)\,(1+n)}$$

Mathematica [A]
time = 0.71, size = 247, normalized size = 0.83

$$-\frac{(d+ex)^{1+n}\left(\frac{2a\sqrt{c}\,(-d+ex)}{a+cx^2}+\frac{\left(2cd^2-\sqrt{-a}\,\sqrt{c}\,den+ae^2(2+n)\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}+\frac{\left(2cd^2+\sqrt{-a}\,\sqrt{c}\,den+ae^2(2+n)\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}\right)}{4c^{3/2}\,(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -1/4*((d + e*x)^(1 + n)*((2*a*Sqrt[c]*(-d + e*x))/(a + c*x^2) + ((2*c*d^2 -
 Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometric2F1[1, 1 + n, 2 + n,
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(
1 + n)) + ((2*c*d^2 + Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(2 + n))*Hypergeometri
c2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt
[c]*d + Sqrt[-a]*e)*(1 + n))))/(c^(3/2)*(c*d^2 + a*e^2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{3}(ex+d)^{n}}{(c\,x^{2}+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^n*x^3/(c*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^n*x^3/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((x*e + d)^n*x^3/(c*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(d+e\,x\right)}^n}{{\left(c\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int((x^3*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.372 $\int \frac{x^2(d+ex)^n}{\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=306

$$-\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)}+\frac{\left(cd^2-\sqrt{-a}\,\sqrt{c}\,den+ae^2(1+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}}\right.}{4\sqrt{-a}\,c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(1+n)}$$

```
[Out] -1/2*(c*d*x+a*e)*(e*x+d)^(1+n)/c/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*
hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+
a*e^2*(1+n)-d*e*n*(-a)^(1/2)*c^(1/2))/c/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/(-e*
(-a)^(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^
(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1+n)+d*e*n*(-a)^(1/2)*c^(1/2)
)/c/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.35, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1663, 845, 70}

$$\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\,\sqrt{c}\,den+ae^2(n+1)+cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,c(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\,\sqrt{c}\,den+ae^2(n+1)+cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,c(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}-\frac{(d+ex)^{n+1}(ae+cdx)}{2c\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -1/2*((a*e + c*d*x)*(d + e*x)^(1 + n))/(c*(c*d^2 + a*e^2)*(a + c*x^2)) + ((
c*d^2 - Sqrt[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4
*Sqrt[-a]*c*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((c*d^2 + S
qrt[-a]*Sqrt[c]*d*e*n + a*e^2*(1 + n))*(d + e*x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*Sqrt[-a]
*c*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
```

```
]
```

Rule 1663

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + c*x^2)^(p + 1)*((a*(e*f - d*g)
+ (c*d*f + a*e*g)*x)/(2*a*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*a*(p +
1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(c*d^2 + a*e^2)*Q + c*d^2*f*(2*p + 3) - a*e*(d*g*m - e*f*(m + 2*p + 3
)) + e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, c, d, e, m
}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[m,
0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2}\,dx = -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^n\left(-\frac{a\left(cd^2+ae^2(1+n)\right)}{c}-adenx\right)}{a+cx^2}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(\frac{\left(\frac{a^2den}{\sqrt{c}}-\frac{\sqrt{-a}\,a\left(cd^2+ae^2(1+n)\right)}{c}\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(-\frac{a^2den}{\sqrt{c}}-\frac{\sqrt{-a}\,a\cdots}{\cdots}\right.}{2a\left(\sqrt{-\cdots}\right.}\right.}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(cd^2-\sqrt{-a}\,\sqrt{c}\,den+ae^2(1+n)\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{4\sqrt{-a}\,c\left(cd^2+ae^2\right)} - \cdots$$

$$= -\frac{(ae+cdx)(d+ex)^{1+n}}{2c\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(cd^2-\sqrt{-a}\,\sqrt{c}\,den+ae^2(1+n)\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1\cdots\right.}{4\sqrt{-a}\,c\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)\cdots}$$

Mathematica [A]
time = 0.59, size = 403, normalized size = 1.32

$$\frac{(d+ex)^{1+n}\left(-\frac{2(ae+cdx)}{(cd^2+ae^2)(a+cx^2)}+\frac{2\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{2\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{-a}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}+\frac{a\left(\frac{\left(cd^2-ae^2(-1+n)+\sqrt{-a}\,\sqrt{c}\,den\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{c}\,d-\sqrt{-a}\,e}-\frac{\left(cd^2-ae^2(-1+n)-\sqrt{-a}\,\sqrt{c}\,den\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{(-a)^{3/2}(cd^2+ae^2)(1+n)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*(a*e + c*d*x))/((c*d^2 + a*e^2)*(a + c*x^2)) + (2*H
ypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]
*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (2*Hypergeometric2F1[1,
 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a]*(Sq
rt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*(((c*d^2 - a*e^2*(-1 + n) + Sqrt[-a]*Sq
rt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c
]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 - a*e^2*(-1 + n) - S
qrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))/((-a)^(3/2)*(c*d^2
 + a*e^2)*(1 + n))))/(4*c)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2(ex+d)^n}{(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^n*x^2/(c*x^2 + a)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^n*x^2/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n*x^2/(c*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(d+e\,x\right)}^n}{{\left(c\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^n)/(a + c*x^2)^2,x)
```

```
[Out] int((x^2*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.373 $\int \frac{x(d+ex)^n}{(a+cx^2)^2} \, dx$

Optimal. Leaf size=279

$$-\frac{(d-ex)(d+ex)^{1+n}}{2\,(cd^2+ae^2)\,(a+cx^2)}+\frac{e\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)n(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{c}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(cd^2+ae^2)\,(1+n)}+\frac{e\left(\sqrt{-a}\,\sqrt{c}\right.}{}$$

```
[Out] -1/2*(-e*x+d)*(e*x+d)^(1+n)/(a*e^2+c*d^2)/(c*x^2+a)+1/4*e*n*(e*x+d)^(1+n)*h
ypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(e*(-a)^
(1/2)+d*c^(1/2))/(a*e^2+c*d^2)/(1+n)/(-a)^(1/2)/c^(1/2)/(-e*(-a)^(1/2)+d*c^
(1/2))+1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-
a)^(1/2)+d*c^(1/2)))*(a*e+d*(-a)^(1/2)*c^(1/2))/a/(a*e^2+c*d^2)/(1+n)/c^(1/
2)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.21, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {837, 845, 70}

$$\frac{en\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{c}\,(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ae^2+cd^2)}+\frac{en\left(\sqrt{-a}\,\sqrt{c}\,d+ae\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4a\sqrt{c}\,(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ae^2+cd^2)}-\frac{(d-ex)(d+ex)^{n+1}}{2\,(a+cx^2)\,(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] -1/2*((d - e*x)*(d + e*x)^(1 + n))/((c*d^2 + a*e^2)*(a + c*x^2)) + (e*(Sqrt
[c]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
(Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*Sqrt[-a]*Sqrt[c]*(Sqrt[c]
*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(Sqrt[-a]*Sqrt[c]*d + a*e)*n
*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[-a]*e)])/(4*a*Sqrt[c]*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*
e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
```

```
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx = -\frac{(d-ex)(d+ex)^{1+n}}{2(cd^2+ae^2)(a+cx^2)} - \frac{\int \frac{(d+ex)^n(-acden+ace^2nx)}{a+cx^2}\,dx}{2ac(cd^2+ae^2)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2(cd^2+ae^2)(a+cx^2)} - \frac{\int \left(\frac{\left(-\sqrt{-a}\,acden-a^2\sqrt{c}\,e^2n\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(-\sqrt{-a}\,acden+a^2\sqrt{c}\,e^2\right.}{2a\left(\sqrt{-a}+\sqrt{c}\right.}\right.}{2ac(cd^2+ae^2)}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2(cd^2+ae^2)(a+cx^2)} + \frac{\left(e\left(\sqrt{-a}\,\sqrt{c}\,d-ae\right)n\right)\int \frac{(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{4a\sqrt{c}\,(cd^2+ae^2)} + \frac{\left(e\left(\sqrt{-a}\,d+\right.\right.}{}$$

$$= -\frac{(d-ex)(d+ex)^{1+n}}{2(cd^2+ae^2)(a+cx^2)} + \frac{e\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)n(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}}{\sqrt{c}\,d-}\right.}{4\sqrt{-a}\,\sqrt{c}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(cd^2+ae^2)(1+n)}$$

Mathematica [A]
time = 0.29, size = 230, normalized size = 0.82

$$\frac{(d+ex)^{1+n}\left(-\frac{2ac(d-ex)}{a+cx^2} - \frac{\left(\sqrt{-a}\,cden-a\sqrt{c}\,e^2n\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{\left(\sqrt{-a}\,cden+a\sqrt{c}\,e^2n\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}\right)}{4ac(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*a*c*(d - e*x))/(a + c*x^2) - ((Sqrt[-a]*c*d*e*n - a
*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqr
```

```
t[c]*d - Sqrt[-a]*e)])/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + ((Sqrt[-a]*c*d*
e*n + a*Sqrt[c]*e^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*c*(
c*d^2 + a*e^2))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x(ex+d)^n}{(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int(x*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^n*x/(c*x^2 + a)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^n*x/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
[Out] integrate((x*e + d)^n*x/(c*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(d+e\,x\right)}^n}{{\left(c\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^n)/(a + c*x^2)^2,x)
[Out] int((x*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.374 $\int \frac{(d+ex)^n}{(a+cx^2)^2} dx$

Optimal. Leaf size=304

$$\frac{(ae+cdx)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)}-\frac{\left(cd^2+ae^2(1-n)+\sqrt{-a}\,\sqrt{c}\,den\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right.}{4(-a)^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(1+n)}$$

```
[Out] 1/2*(c*d*x+a*e)*(e*x+d)^(1+n)/a/(a*e^2+c*d^2)/(c*x^2+a)+1/4*(e*x+d)^(1+n)*h
ypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*
e^2*(1-n)-d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(3/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^
(1/2)+d*c^(1/2))-1/4*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)+d*e*n*(-a)^(1/2)*c^(1/2))/(-
a)^(3/2)/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]
time = 0.28, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {755, 845, 70}

$$-\frac{(d+ex)^{n+1}\left(\sqrt{-a}\,\sqrt{c}\,den+ae^2(1-n)+cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}\left(-\sqrt{-a}\,\sqrt{c}\,den+ae^2(1-n)+cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}+\frac{(d+ex)^{n+1}(ae+cdx)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(a + c*x^2)^2,x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) - ((c*d
^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-
a)^(3/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + ((c*d^2 + a*e^
2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1,
1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*(-a)^(3/2)*
(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
```

```
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rubi steps

$$\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx = \frac{(ae+cdx)(d+ex)^{1+n}}{2a\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\int \frac{(d+ex)^n(-cd^2-ae^2(1-n)+cdenx)}{a+cx^2}\,dx}{2a\,(cd^2+ae^2)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\int \left(\frac{\left(\sqrt{-a}\,(-cd^2-ae^2(1-n))-a\sqrt{c}\,den\right)(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)} + \frac{\left(\sqrt{-a}\,(-cd^2-ae^2\right.}{2a\left(\sqrt{\right.}}\right.}{2a\,(cd^2+ae^2)}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\,(cd^2+ae^2)\,(a+cx^2)} + \frac{\left(cd^2+ae^2(1-n)-\sqrt{-a}\,\sqrt{c}\,den\right)\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{4(-a)^{3/2}\,(cd^2+ae^2)} + \frac{(cd}{}$$

$$= \frac{(ae+cdx)(d+ex)^{1+n}}{2a\,(cd^2+ae^2)\,(a+cx^2)} - \frac{\left(cd^2+ae^2(1-n)+\sqrt{-a}\,\sqrt{c}\,den\right)(d+ex)^{1+n}\,{}_2F_1\left(1,1+n\right.}{4(-a)^{3/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(cd^2+ae^2)\,(1}$$

Mathematica [A]
time = 0.30, size = 253, normalized size = 0.83

$$\frac{(d+ex)^{1+n}\left(\frac{2(ae+cdx)}{a+cx^2}+\frac{\left(cd^2-ae^2(-1+n)+\sqrt{-a}\,\sqrt{c}\,den\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}+\frac{\left(-cd^2+ae^2(-1+n)+\sqrt{-a}\,\sqrt{c}\,den\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)}\right)}{4a\,(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(a + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*(a*e + c*d*x))/(a + c*x^2) + ((c*d^2 - a*e^2*(-1 + n
) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d
+ e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(1 +
n)) + ((-(c*d^2) + a*e^2*(-1 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2
```

```
F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[-a
]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n))))/(4*a*(c*d^2 + a*e^2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/(c*x^2+a)^2,x)
[Out] int((e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((x*e + d)^n/(c*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((x*e + d)^n/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/(c*x**2+a)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n/(c*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(a + c*x^2)^2,x)
```

```
[Out] int((d + e*x)^n/(a + c*x^2)^2, x)
```

3.375 $\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=489

$$\frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)}+\frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a^2\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}+\frac{\sqrt{c}\,e\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)n(d+}{4(-a)^{3/2}\left(\sqrt{c}\,d\right.}$$

```
[Out] 1/2*c*(-e*x+d)*(e*x+d)^(1+n)/a/(a*e^2+c*d^2)/(c*x^2+a)-(e*x+d)^(1+n)*hyperg
eom([1, 1+n],[2+n],1+e*x/d)/a^2/d/(1+n)+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n
],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a^2/(1+n)/(-e*(-
a)^(1/2)+d*c^(1/2))+1/2*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1
/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)/a^2/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))+1/
4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)
+d*c^(1/2)))*c^(1/2)*(e*(-a)^(1/2)+d*c^(1/2))/(-a)^(3/2)/(a*e^2+c*d^2)/(1+n
)/(-e*(-a)^(1/2)+d*c^(1/2))-1/4*e*n*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],
(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*c^(1/2)*(a*e+d*(-a)^(1/2)*c^(1/2)
)/a^2/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))
```

Rubi [A]

time = 0.41, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {975, 67, 837, 845, 70}

$$-\frac{\sqrt{c}\,en\left(\sqrt{-a}\,\sqrt{c}\,d+ae\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4a^2(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a^2(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}+\frac{\sqrt{c}\,(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2a^2(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}-\frac{(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{ex}{d}+1\right)}{a^2d(n+1)}+\frac{\sqrt{c}\,en\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4(-a)^{3/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)}+\frac{c(d-ex)(d+ex)^{n+1}}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```

```
[Out] (c*(d - e*x)*(d + e*x)^(1 + n))/(2*a*(c*d^2 + a*e^2)*(a + c*x^2)) + (Sqrt[c
]*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/
(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a^2*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (Sqrt
[c]*e*(Sqrt[c]*d + Sqrt[-a]*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-a)^(3/2)*(Sq
rt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (Sqrt[c]*(d + e*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-
a]*e)])/(2*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) - (Sqrt[c]*e*(Sqrt[-a]*Sqr
t[c]*d + a*e)*n*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*a^2*(Sqrt[c]*d + Sqrt[-a]*e)*(c
*d^2 + a*e^2)*(1 + n)) - ((d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
 n, 1 + (e*x)/d])/(a^2*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex)^n}{x\left(a+cx^2\right)^2}\,dx = \int \left(\frac{(d+ex)^n}{a^2x} - \frac{cx(d+ex)^n}{a\left(a+cx^2\right)^2} - \frac{cx(d+ex)^n}{a^2\left(a+cx^2\right)}\right)\,dx$$

$$= \frac{\int \frac{(d+ex)^n}{x}\,dx}{a^2} - \frac{c\int \frac{x(d+ex)^n}{a+cx^2}\,dx}{a^2} - \frac{c\int \frac{x(d+ex)^n}{(a+cx^2)^2}\,dx}{a}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} - \frac{c\int\left(-\frac{(d+\cdots}{2\sqrt{c}\left(\sqrt{-\cdots}\right.}\right.}{}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d(1+n)} + \frac{\sqrt{c}\int \frac{(d+ex)^n}{\sqrt{-a}-\sqrt{c}\cdots}}{2a^2}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a^2\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{\sqrt{c}\,(\cdots}{}$$

$$= \frac{c(d-ex)(d+ex)^{1+n}}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\sqrt{c}\,(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2a^2\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{\sqrt{c}\,e\cdots}{}$$

Mathematica [A]
time = 0.68, size = 391, normalized size = 0.80

$$\frac{(d+ex)^{1+n}\left(\frac{2ac(d-ex)}{(cd^2+ae^2)(a+cx^2)} - \frac{4\,{}_2F_1\left(1,1+n;2+n;\frac{d+ex}{d}\right)}{d+dn} + \frac{2\sqrt{c}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{2\sqrt{c}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)} + \frac{\sqrt{c}\,en\left(\left(\sqrt{-a}\,cd^2-2a\sqrt{c}\,de+(-a)^{3/2}e^2\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)+\left(-\sqrt{-a}\,cd^2-2a\sqrt{c}\,de+\sqrt{-a}\,ae^2\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)\right)}{(cd^2+ae^2)^2(1+n)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x*(a + c*x^2)^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((2*a*c*(d - e*x))/((c*d^2 + a*e^2)*(a + c*x^2)) - (4*Hy
pergeometric2F1[1, 1 + n, 2 + n, (d + e*x)/d])/(d + d*n) + (2*Sqrt[c]*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]
)/((Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) + (2*Sqrt[c]*Hypergeometric2F1[1, 1 +
n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/((Sqrt[c]*d + Sqrt
[-a]*e)*(1 + n)) + (Sqrt[c]*e*n*((Sqrt[-a]*c*d^2 - 2*a*Sqrt[c]*d*e + (-a)^(
3/2)*e^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d
 - Sqrt[-a]*e)] + (-(Sqrt[-a]*c*d^2) - 2*a*Sqrt[c]*d*e + Sqrt[-a]*a*e^2)*Hy
pergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*
e)]))/((c*d^2 + a*e^2)^2*(1 + n))))/(4*a^2)
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{x\,(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x/(c*x^2+a)^2,x)
[Out] int((e*x+d)^n/x/(c*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((x*e + d)^n/((c*x^2 + a)^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((x*e + d)^n/(c^2*x^5 + 2*a*c*x^3 + a^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^n}{x\,(a+cx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x/(c*x**2+a)**2,x)
[Out] Integral((d + e*x)**n/(x*(a + c*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n/((c*x^2 + a)^2*x), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{x\,\left(c\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(x*(a + c*x^2)^2),x)
```

```
[Out] int((d + e*x)^n/(x*(a + c*x^2)^2), x)
```

3.376 $\int \frac{(d+ex)^n}{x^2(a+cx^2)^2} \, dx$

Optimal. Leaf size=513

$$-\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2(cd^2+ae^2)(a+cx^2)}-\frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{5/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)}-\frac{c\left(cd^2+ae^2(1-n)+\sqrt{-a}\right.}{4(-a)^{5/}}$$

[Out] -1/2*c*(c*d*x+a*e)*(e*x+d)^(1+n)/a^2/(a*e^2+c*d^2)/(c*x^2+a)+e*(e*x+d)^(1+n)*hypergeom([2, 1+n],[2+n],1+e*x/d)/a^2/d^2/(1+n)-1/2*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(5/2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))+1/2*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))/(-a)^(5/2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))+1/4*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)-d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(5/2)/(a*e^2+c*d^2)/(1+n)/(e*(-a)^(1/2)+d*c^(1/2))-1/4*c*(e*x+d)^(1+n)*hypergeom([1, 1+n],[2+n],(e*x+d)*c^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)))*(c*d^2+a*e^2*(1-n)+d*e*n*(-a)^(1/2)*c^(1/2))/(-a)^(5/2)/(a*e^2+c*d^2)/(1+n)/(-e*(-a)^(1/2)+d*c^(1/2))

Rubi [A]
time = 0.49, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {975, 67, 755, 845, 70, 726}

$$-\frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2(a+cx^2)(ae^2+cd^2)}+\frac{e(d+ex)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{ex}{d}+1\right)}{a^2d^2(n+1)}-\frac{c(d+ex)^{n+1}\left(\sqrt{-a}\sqrt{c}\,den+ae^2(1-n)+cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{4(-a)^{5/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ae^2+cd^2)}+\frac{c(d+ex)^{n+1}\left(-\sqrt{-a}\sqrt{c}\,den+ae^2(1-n)+cd^2\right)\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{4(-a)^{5/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ae^2+cd^2)}-\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{5/2}(n+1)\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)}+\frac{c(d+ex)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{2(-a)^{5/2}(n+1)\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]

[Out] -1/2*(c*(a*e + c*d*x)*(d + e*x)^(1 + n))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2)) - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + n)) - (c*(c*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(4*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (c*(c*d^2 + a*e^2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(4*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d^2*(1 + n))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[m]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex)^n}{x^2\left(a+cx^2\right)^2}\,dx = \int \left(\frac{(d+ex)^n}{a^2x^2} - \frac{c(d+ex)^n}{a\left(a+cx^2\right)^2} - \frac{c(d+ex)^n}{a^2\left(a+cx^2\right)}\right)\,dx$$

$$= \frac{\int \frac{(d+ex)^n}{x^2}\,dx}{a^2} - \frac{c\int \frac{(d+ex)^n}{a+cx^2}\,dx}{a^2} - \frac{c\int \frac{(d+ex)^n}{(a+cx^2)^2}\,dx}{a}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} - \frac{c\int \Bigg(\frac{\sqrt{-}}{2a\Big(\sqrt{}}}{}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e(d+ex)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)} + \frac{c\int \frac{(d+ex}{\sqrt{-a}-}}{2(-a)}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{5/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} + \frac{c(}{}$$

$$= -\frac{c(ae+cdx)(d+ex)^{1+n}}{2a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{c(d+ex)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{2(-a)^{5/2}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(1+n)} - \frac{c(}{}$$

Mathematica [A]

time = 0.53, size = 437, normalized size = 0.85

$$\frac{1}{4}(d+ex)^{1+n}\left(-\frac{2c(ae+cdx)}{a^2\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{2c\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{(-a)^{5/2}\left(-\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)} + \frac{2c\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{(-a)^{5/2}\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(1+n)} + \frac{ac\left(\frac{\left(cd^2-ae^2(-1+n)+\sqrt{-a}\,\sqrt{c}\,den\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d-\sqrt{-a}\,e}\right)}{\sqrt{c}\,d-\sqrt{-a}\,e} - \frac{\left(cd^2-ae^2(-1+n)-\sqrt{-a}\,\sqrt{c}\,den\right){}_2F_1\left(1,1+n;2+n;\frac{\sqrt{c}\,(d+ex)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{\sqrt{c}\,d+\sqrt{-a}\,e}\right)}{(-a)^{7/2}\left(cd^2+ae^2\right)(1+n)} + \frac{4e\,{}_2F_1\left(2,1+n;2+n;1+\frac{ex}{d}\right)}{a^2d^2(1+n)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^n/(x^2*(a + c*x^2)^2),x]
```

```
[Out] ((d + e*x)^(1 + n)*((-2*c*(a*e + c*d*x))/(a^2*(c*d^2 + a*e^2)*(a + c*x^2))
+ (2*c*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d -
Sqrt[-a]*e)])/((-a)^(5/2)*(-(Sqrt[c]*d) + Sqrt[-a]*e)*(1 + n)) + (2*c*Hyper
geometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]
)/((-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + n)) + (a*c*(((c*d^2 - a*e^2*(-1
 + n) + Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]
*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(Sqrt[c]*d - Sqrt[-a]*e) - ((c*d^2 -
 a*e^2*(-1 + n) - Sqrt[-a]*Sqrt[c]*d*e*n)*Hypergeometric2F1[1, 1 + n, 2 + n
, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e)))
/((-a)^(7/2)*(c*d^2 + a*e^2)*(1 + n)) + (4*e*Hypergeometric2F1[2, 1 + n, 2
+ n, 1 + (e*x)/d])/(a^2*d^2*(1 + n))))/4
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^n}{x^2\,(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
[Out] int((e*x+d)^n/x^2/(c*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((x*e + d)^n/((c*x^2 + a)^2*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="fricas")
[Out] integral((x*e + d)^n/(c^2*x^6 + 2*a*c*x^4 + a^2*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**n/x**2/(c*x**2+a)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^n/x^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^n/((c*x^2 + a)^2*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^n}{x^2\,{\left(c\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^n/(x^2*(a + c*x^2)^2),x)
```

```
[Out] int((d + e*x)^n/(x^2*(a + c*x^2)^2), x)
```

3.377 $\int (gx)^m(d+ex)^n \left(a+cx^2\right)^2 \, dx$

Optimal. Leaf size=399

$$-\frac{cd(2+m)\left(cd^2\left(12+7m+m^2\right)+2ae^2\left(20+m^2+9n+n^2+m(9+2n)\right)\right)(gx)^{1+m}(d+ex)^{1+n}}{e^4g(2+m+n)(3+m+n)(4+m+n)(5+m+n)}+\frac{c(cd^2(12+$$

```
[Out] -c*d*(2+m)*(c*d^2*(m^2+7*m+12)+2*a*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))*(g*x)^(1
+m)*(e*x+d)^(1+n)/e^4/g/(2+m+n)/(3+m+n)/(4+m+n)/(5+m+n)+c*(c*d^2*(m^2+7*m+1
2)+2*a*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))*(g*x)^(2+m)*(e*x+d)^(1+n)/e^3/g^2/(3
+m+n)/(4+m+n)/(5+m+n)-c^2*d*(4+m)*(g*x)^(3+m)*(e*x+d)^(1+n)/e^2/g^3/(4+m+n)
/(5+m+n)+c^2*(g*x)^(4+m)*(e*x+d)^(1+n)/e/g^4/(5+m+n)+(a^2*e^4*(2+m+n)*(3+m+
n)*(4+m+n)*(5+m+n)+c*d^2*(1+m)*(2+m)*(c*d^2*(m^2+7*m+12)+2*a*e^2*(20+m^2+9*
n+n^2+m*(9+2*n))))*(g*x)^(1+m)*(e*x+d)^n*hypergeom([-n, 1+m],[2+m],-e*x/d)/
e^4/g/(1+m)/(2+m+n)/(3+m+n)/(4+m+n)/(5+m+n)/((1+e*x/d)^n)
```

Rubi [A]

time = 0.50, antiderivative size = 377, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {966, 1637, 81, 68, 66}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(\frac{a^2}{m+1}+\frac{cd^2(m+2)\left(2ae^2\left(m^2+m(2n+9)+n^2+9n+20\right)+cd^2\left(m^2+7m+12\right)\right)}{e^4(m+n+2)(m+n+3)(m+n+4)(m+n+5)}\right){}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)}{g}-\frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}\left(2ae^2(m^2+m(2n+9)+n^2+9n+20)+cd^2(m^2+7m+12)\right)}{e^4g(m+n+2)(m+n+3)(m+n+4)(m+n+5)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}\left(2ae^2(m^2+m(2n+9)+n^2+9n+20)+cd^2(m^2+7m+12)\right)}{e^3g^2(m+n+3)(m+n+4)(m+n+5)}-\frac{c^2d(m+4)(gx)^{m+3}(d+ex)^{n+1}}{e^2g^3(m+n+4)(m+n+5)}+\frac{c^2(gx)^{m+4}(d+ex)^{n+1}}{eg^4(m+n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

```
[Out] -((c*d*(2 + m)*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*
(9 + 2*n)))*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^4*g*(2 + m + n)*(3 + m + n)
*(4 + m + n)*(5 + m + n))) + (c*(c*d^2*(12 + 7*m + m^2) + 2*a*e^2*(20 + m^2
 + 9*n + n^2 + m*(9 + 2*n)))*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e^3*g^2*(3 +
 m + n)*(4 + m + n)*(5 + m + n)) - (c^2*d*(4 + m)*(g*x)^(3 + m)*(d + e*x)^(
1 + n))/(e^2*g^3*(4 + m + n)*(5 + m + n)) + (c^2*(g*x)^(4 + m)*(d + e*x)^(1
 + n))/(e*g^4*(5 + m + n)) + ((a^2/(1 + m) + (c*d^2*(2 + m)*(c*d^2*(12 + 7*
m + m^2) + 2*a*e^2*(20 + m^2 + 9*n + n^2 + m*(9 + 2*n))))/(e^4*(2 + m + n)*
(3 + m + n)*(4 + m + n)*(5 + m + n)))*(g*x)^(1 + m)*(d + e*x)^n*Hypergeomet
ric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + (e*x)/d)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rule 1637

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Dist[1/(d*b^q*(
m + n + q + 1)), Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q +
 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)^
(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
x] && PolyQ[Px, x] && GtQ[Expon[Px, x], 2]
```

Rubi steps

$$
\begin{aligned}
\int (gx)^m (d+ex)^n \left(a+cx^2\right)^2 \, dx &= \frac{c^2 (gx)^{4+m} (d+ex)^{1+n}}{eg^4(5+m+n)} + \frac{\int (gx)^m (d+ex)^n \left(a^2 eg^4(5+m+n) + 2aceg^4(5\right.}{eg^4(5+m+}\\
&= -\frac{c^2 d(4+m)(gx)^{3+m}(d+ex)^{1+n}}{e^2 g^3 (4+m+n)(5+m+n)} + \frac{c^2 (gx)^{4+m}(d+ex)^{1+n}}{eg^4(5+m+n)} + \int (gx)^m (d+\\
&= \frac{c\left(cd^2(12+7m+m^2) + 2ae^2(20+m^2+9n+n^2+m(9+2n))\right)(gx)^{2+m}}{e^3 g^2 (3+m+n)(4+m+n)(5+m+n)}\\
&= -\frac{cd(2+m)\left(cd^2(12+7m+m^2) + 2ae^2(20+m^2+9n+n^2+m(9+2n)\right.}{e^4 g(2+m+n)(3+m+n)(4+m+n)(5+m+}\\
&= -\frac{cd(2+m)\left(cd^2(12+7m+m^2) + 2ae^2(20+m^2+9n+n^2+m(9+2n)\right.}{e^4 g(2+m+n)(3+m+n)(4+m+n)(5+m+}\\
&= -\frac{cd(2+m)\left(cd^2(12+7m+m^2) + 2ae^2(20+m^2+9n+n^2+m(9+2n)\right.}{e^4 g(2+m+n)(3+m+n)(4+m+n)(5+m+}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 275, normalized size = 0.69

$$
\frac{x(gx)^m (d+ex)^n \left(1+\frac{ex}{d}\right)^{-n} \left(c^2 d^4 \,{}_2F_1\left(1+m,-4-n;2+m;-\frac{ex}{d}\right) - 4c^2 d^4 \,{}_2F_1\left(1+m,-3-n;2+m;-\frac{ex}{d}\right) + 6c^2 d^4 \,{}_2F_1\left(1+m,-2-n;2+m;-\frac{ex}{d}\right) + 2acd^2 e^2 \,{}_2F_1\left(1+m,-2-n;2+m;-\frac{ex}{d}\right) - 4c^2 d^4 \,{}_2F_1\left(1+m,-1-n;2+m;-\frac{ex}{d}\right) - 4acd^2 e^2 \,{}_2F_1\left(1+m,-1-n;2+m;-\frac{ex}{d}\right) + c^2 d^4 \,{}_2F_1\left(1+m,-n;2+m;-\frac{ex}{d}\right) + 2acd^2 e^2 \,{}_2F_1\left(1+m,-n;2+m;-\frac{ex}{d}\right) + a^2 e^4 \,{}_2F_1\left(1+m,-n;2+m;-\frac{ex}{d}\right)\right)}{e^4(1+m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2)^2,x]
```

```
[Out] (x*(g*x)^m*(d + e*x)^n*(c^2*d^4*Hypergeometric2F1[1 + m, -4 - n, 2 + m, -((
e*x)/d)] - 4*c^2*d^4*Hypergeometric2F1[1 + m, -3 - n, 2 + m, -((e*x)/d)] +
6*c^2*d^4*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] + 2*a*c*d^2*e
^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*x)/d)] - 4*c^2*d^4*Hypergeo
metric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] - 4*a*c*d^2*e^2*Hypergeometric2
F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + c^2*d^4*Hypergeometric2F1[1 + m, -n,
 2 + m, -((e*x)/d)] + 2*a*c*d^2*e^2*Hypergeometric2F1[1 + m, -n, 2 + m, -((
e*x)/d)] + a^2*e^4*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^4*(
1 + m)*(1 + (e*x)/d)^n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$
\int \left(gx\right)^{m} \left(ex+d\right)^{n} \left(c\,x^{2}+a\right)^{2}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^2*(g*x)^m*(x*e + d)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(g*x)^m*(x*e + d)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.96, size = 131, normalized size = 0.33

$$\frac{a^2 d^n g^m x x^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)} + \frac{2acd^n g^m x^3 x^m \Gamma(m+3)\, {}_2F_1\left(\begin{matrix} -n, m+3 \\ m+4 \end{matrix}\middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)} + \frac{c^2 d^n g^m x^5 x^m \Gamma(m+5)\, {}_2F_1\left(\begin{matrix} -n, m+5 \\ m+6 \end{matrix}\middle| \frac{exe^{i\pi}}{d}\right)}{\Gamma(m+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(c*x**2+a)**2,x)
```

```
[Out] a**2*d**n*g**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp_pol
ar(I*pi)/d)/gamma(m + 2) + 2*a*c*d**n*g**m*x**3*x**m*gamma(m + 3)*hyper((-n
, m + 3), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4) + c**2*d**n*g**m*x*
*5*x**m*gamma(m + 5)*hyper((-n, m + 5), (m + 6,), e*x*exp_polar(I*pi)/d)/ga
mma(m + 6)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^2*(g*x)^m*(x*e + d)^n, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,x)^m\,{\left(c\,x^2+a\right)}^2\,{\left(d+e\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^2*(d + e*x)^n,x)
[Out] int((g*x)^m*(a + c*x^2)^2*(d + e*x)^n, x)
```

3.378 $\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx$

Optimal. Leaf size=164

$$-\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)}+\frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)}+\frac{(cd^2(1+m)(2+m)+ae^2(2+m+n)(3+m+n}{e^2g(1+m)(}$$

```
[Out] -c*d*(2+m)*(g*x)^(1+m)*(e*x+d)^(1+n)/e^2/g/(2+m+n)/(3+m+n)+c*(g*x)^(2+m)*(e
*x+d)^(1+n)/e/g^2/(3+m+n)+(c*d^2*(1+m)*(2+m)+a*e^2*(2+m+n)*(3+m+n))*(g*x)^(
1+m)*(e*x+d)^n*hypergeom([-n, 1+m],[2+m],-e*x/d)/e^2/g/(1+m)/(2+m+n)/(3+m+n
)/((1+e*x/d)^n)
```

Rubi [A]
time = 0.09, antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {966, 81, 68, 66}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(\frac{a}{m+1}+\frac{cd^2(m+2)}{e^2(m+n+2)(m+n+3)}\right){}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)}{g}-\frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}}{e^2g(m+n+2)(m+n+3)}+\frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

```
[Out] -((c*d*(2 + m)*(g*x)^(1 + m)*(d + e*x)^(1 + n))/(e^2*g*(2 + m + n)*(3 + m +
 n))) + (c*(g*x)^(2 + m)*(d + e*x)^(1 + n))/(e*g^2*(3 + m + n)) + ((a/(1 +
m) + (c*d^2*(2 + m))/(e^2*(2 + m + n)*(3 + m + n)))*(g*x)^(1 + m)*(d + e*x)
^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)])/(g*(1 + (e*x)/d)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\int (gx)^m(d+ex)^n\left(a+cx^2\right)\,dx = \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \frac{\int (gx)^m(d+ex)^n\left(aeg^2(3+m+n)-cdg^2(2+m\right.}{eg^2(3+m+n)}$$
$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \left(a+\frac{cd^2(1}{e^2(2+m}\right.$$
$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \left(\left(a+\frac{cd^2}{e^2(2+}\right.\right.$$
$$= -\frac{cd(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^2g(2+m+n)(3+m+n)} + \frac{c(gx)^{2+m}(d+ex)^{1+n}}{eg^2(3+m+n)} + \frac{\left(a+\frac{cd^2(1+m}{e^2(2+m+n)}\right.}{}$$

Mathematica [A]
time = 0.07, size = 113, normalized size = 0.69

$$\frac{x(gx)^m(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\left(cd^2\,{}_2F_1\left(1+m,-2-n;2+m;-\frac{ex}{d}\right)-2cd^2\,{}_2F_1\left(1+m,-1-n;2+m;-\frac{ex}{d}\right)+\left(cd^2+ae^2\right)\,{}_2F_1\left(1+m,-n;2+m;-\frac{ex}{d}\right)\right)}{e^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^n*(a + c*x^2),x]
```

```
[Out] (x*(g*x)^m*(d + e*x)^n*(c*d^2*Hypergeometric2F1[1 + m, -2 - n, 2 + m, -((e*
x)/d)] - 2*c*d^2*Hypergeometric2F1[1 + m, -1 - n, 2 + m, -((e*x)/d)] + (c*d
^2 + a*e^2)*Hypergeometric2F1[1 + m, -n, 2 + m, -((e*x)/d)]))/(e^2*(1 + m)*
(1 + (e*x)/d)^n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (gx)^m\,(ex+d)^n\,\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

```
[Out] int((g*x)^m*(e*x+d)^n*(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)*(g*x)^m*(x*e + d)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)*(g*x)^m*(x*e + d)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.10, size = 82, normalized size = 0.50

$$\frac{ad^n g^m x x^m \Gamma(m+1)\,{}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+2)}+\frac{cd^n g^m x^3 x^m \Gamma(m+3)\,{}_2F_1\left(\begin{matrix} -n, m+3 \\ m+4 \end{matrix}\middle|\frac{exe^{i\pi}}{d}\right)}{\Gamma(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n*(c*x**2+a),x)
```

```
[Out] a*d**n*g**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), e*x*exp_polar(
I*pi)/d)/gamma(m + 2) + c*d**n*g**m*x**3*x**m*gamma(m + 3)*hyper((-n, m + 3
), (m + 4,), e*x*exp_polar(I*pi)/d)/gamma(m + 4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n*(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)*(g*x)^m*(x*e + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,x)^m\,\left(c\,x^2+a\right)\,\left(d+e\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)*(d + e*x)^n,x)
[Out] int((g*x)^m*(a + c*x^2)*(d + e*x)^n, x)
```

3.379 $\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$

Optimal. Leaf size=148

$$\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{2ag(1+m)}+\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1\right.}{2ag(1}$$

```
[Out] 1/2*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,-n,1,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2
))/a/g/(1+m)/((1+e*x/d)^n)+1/2*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,1,-n,2+m,
x*c^(1/2)/(-a)^(1/2),-e*x/d)/a/g/(1+m)/((1+e*x/d)^n)
```

Rubi [A]

time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {926, 140, 138}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{2ag(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{2ag(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(2*a*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e
*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(2*a
*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
```

```
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx = \int \left(\frac{\sqrt{-a}\,(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)}+\frac{\sqrt{-a}\,(gx)^m(d+ex)^n}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2\sqrt{-a}}-\frac{\int \frac{(gx)^m(d+ex)^n}{\sqrt{-a}+\sqrt{c}\,x}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\left((d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right)^n}{\sqrt{-a}-\sqrt{c}\,x}\,dx}{2\sqrt{-a}}-\frac{\left((d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int \frac{(gx)^m\left(1+\frac{ex}{d}\right.}{\sqrt{-a}+\sqrt{c}}}{2\sqrt{-a}}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{2ag(1+m)}+\frac{(gx)^{1+m}(d}{}$$

Mathematica [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m(d+ex)^n}{a+cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2),x]
[Out] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2), x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\,(ex+d)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
[Out] int((g*x)^m*(e*x+d)^n/(c*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m*(x*e + d)^n/(c*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((g*x)^m*(x*e + d)^n/(c*x^2 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m*(x*e + d)^n/(c*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m\,(d+e\,x)^n}{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2),x)
```

```
[Out] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2), x)
```

3.380 $\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=295

$$\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(1+m)}+\frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1-\right.}{4a^2g(1+}$$

```
[Out] 1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,-n,1,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2
))/a^2/g/(1+m)/((1+e*x/d)^n)+1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,1,-n,2+
m,x*c^(1/2)/(-a)^(1/2),-e*x/d)/a^2/g/(1+m)/((1+e*x/d)^n)+1/4*(g*x)^(1+m)*(e
*x+d)^n*AppellF1(1+m,-n,2,2+m,-e*x/d,-x*c^(1/2)/(-a)^(1/2))/a^2/g/(1+m)/((1
+e*x/d)^n)+1/4*(g*x)^(1+m)*(e*x+d)^n*AppellF1(1+m,2,-n,2+m,x*c^(1/2)/(-a)^(
1/2),-e*x/d)/a^2/g/(1+m)/((1+e*x/d)^n)
```

Rubi [A]
time = 0.29, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {975, 140, 138, 926}

$$\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{ex}{d},\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,2;m+2;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(m+1)}+\frac{(gx)^{m+1}(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}F_1\left(m+1;-n,2;m+2;-\frac{ex}{d},\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), -((Sqr
t[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d +
 e*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4
*a^2*g*(1 + m)*(1 + (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m
, -n, 2, 2 + m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1 + m)*(1 +
 (e*x)/d)^n) + ((g*x)^(1 + m)*(d + e*x)^n*AppellF1[1 + m, -n, 2, 2 + m, -((
e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1 + m)*(1 + (e*x)/d)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
```

```
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx = \int \left(-\frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\,\sqrt{c}\,-cx\right)^2} - \frac{c(gx)^m(d+ex)^n}{4a\left(\sqrt{-a}\,\sqrt{c}\,+cx\right)^2} - \frac{c(gx)^m(d+ex)^n}{2a\left(-ac-c^2x^2\right)}\right)dx$$

$$= -\frac{c\int \frac{(gx)^m(d+ex)^n}{\left(\sqrt{-a}\,\sqrt{c}\,-cx\right)^2}\,dx}{4a} - \frac{c\int \frac{(gx)^m(d+ex)^n}{\left(\sqrt{-a}\,\sqrt{c}\,+cx\right)^2}\,dx}{4a} - \frac{c\int \frac{(gx)^m(d+ex)^n}{-ac-c^2x^2}\,dx}{2a}$$

$$= -\frac{c\int \left(-\frac{\sqrt{-a}\,(gx)^m(d+ex)^n}{2ac\left(\sqrt{-a}\,-\sqrt{c}\,x\right)} - \frac{\sqrt{-a}\,(gx)^m(d+ex)^n}{2ac\left(\sqrt{-a}\,+\sqrt{c}\,x\right)}\right)dx}{2a} - \frac{\left(c(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}\right)\int}{4a}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(}{}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,2;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(}{}$$

$$= \frac{(gx)^{1+m}(d+ex)^n\left(1+\frac{ex}{d}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{ex}{d},-\frac{\sqrt{c}\,x}{\sqrt{-a}}\right)}{4a^2g(1+m)} + \frac{(gx)^{1+m}(}{}$$

Mathematica [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m(d+ex)^n}{(a+cx^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]
```

```
[Out] Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (ex+d)^n}{(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
```

```
[Out] int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((g*x)^m*(x*e + d)^n/(c*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x)^m*(x*e + d)^n/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x)^m*(x*e + d)^n/(c*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m\,(d+e\,x)^n}{(c\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x)
```

```
[Out] int(((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x)
```

3.381 $\int x^5(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{a^2d(a+bx^2)^{1+p}}{2b^3(1+p)}-\frac{ad(a+bx^2)^{2+p}}{b^3(2+p)}+\frac{d(a+bx^2)^{3+p}}{2b^3(3+p)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*a^2*d*(b*x^2+a)^(1+p)/b^3/(1+p)-a*d*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*d*(b*
x^2+a)^(3+p)/b^3/(3+p)+1/7*e*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/2],-b*x
^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {778, 272, 45, 372, 371}

$$\frac{a^2d(a+bx^2)^{p+1}}{2b^3(p+1)}-\frac{ad(a+bx^2)^{p+2}}{b^3(p+2)}+\frac{d(a+bx^2)^{p+3}}{2b^3(p+3)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*d*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (d*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*x^7*(a + b*x^2)^p
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5(d+ex)\left(a+bx^2\right)^p\,dx = d\int x^5\left(a+bx^2\right)^p\,dx + e\int x^6\left(a+bx^2\right)^p\,dx$$
$$= \frac{1}{2}d\text{Subst}\left(\int x^2(a+bx)^p\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^6\Bigg(1$$
$$= \frac{1}{7}ex^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right) + \frac{1}{2}d\text{Subst}\Bigg(\int\Bigg(\frac{a^2(a-}{b}$$
$$= \frac{a^2d\left(a+bx^2\right)^{1+p}}{2b^3(1+p)} - \frac{ad\left(a+bx^2\right)^{2+p}}{b^3(2+p)} + \frac{d\left(a+bx^2\right)^{3+p}}{2b^3(3+p)} + \frac{1}{7}ex^7\left(a+bx^2\right)^p\Bigg(1+$$

Mathematica [A]
time = 0.14, size = 112, normalized size = 0.90

$$\frac{1}{14}\left(a+bx^2\right)^p\left(\frac{7d\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+2ex^7\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((7*d*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
+ p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*e*x^7*Hypergeometric2F1[7/2
, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^5(ex+d)\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^6, x) + 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p
)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)*(b*x^2 + a)^p*d/((p^3 + 6*p^2 + 11*p +
 6)*b^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^6*e + d*x^5)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(104) = 208$.
time = 13.62, size = 950, normalized size = 7.60

$$\frac{a^p e x^7 {}_2F_1\left(\begin{matrix} \frac{7}{2}, -p \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7} + d\left(\begin{cases} \frac{a^p x^6}{6} & \text{for}\ b = 0 \\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2b^2x^4\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2b^2x^4\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} & \text{for}\ p = -3 \\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} + \frac{b^2x^4}{2ab^3+2b^4x^2} & \text{for}\ p = -2 \\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3} + \frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b} & \text{for}\ p = -1 \\ \frac{2a^3(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} - \frac{2a^2bpx^2(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2p^2x^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2px^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{b^3p^2x^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{3b^3px^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{2b^3x^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d*Piecew
ise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b
**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
```

```
x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**
4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*
x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p,
 -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x
 + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x
 + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2
), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b)
)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)
**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p
**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
 + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p +
12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*
b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p*
*2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x*e + d)*(b*x^2 + a)^p*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(b\,x^2+a\right)}^p\,\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p*(d + e*x),x)
[Out] int(x^5*(a + b*x^2)^p*(d + e*x), x)
```

3.382 $\int x^4(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{a^2e(a+bx^2)^{1+p}}{2b^3(1+p)}-\frac{ae(a+bx^2)^{2+p}}{b^3(2+p)}+\frac{e(a+bx^2)^{3+p}}{2b^3(3+p)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*a^2*e*(b*x^2+a)^(1+p)/b^3/(1+p)-a*e*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*e*(b*
x^2+a)^(3+p)/b^3/(3+p)+1/5*d*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x
^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.278$, Rules used = {778, 372, 371, 272, 45}

$$\frac{a^2e(a+bx^2)^{p+1}}{2b^3(p+1)}-\frac{ae(a+bx^2)^{p+2}}{b^3(p+2)}+\frac{e(a+bx^2)^{p+3}}{2b^3(p+3)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (a^2*e*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*e*(a + b*x^2)^(2 + p))/(b^
3*(2 + p)) + (e*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*x^5*(a + b*x^2)^p
*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^4(d+ex)\left(a+bx^2\right)^p\,dx &= d\int x^4\left(a+bx^2\right)^p\,dx+e\int x^5\left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}e\text{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^2\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\Bigg(1\\ &= \frac{1}{5}dx^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+\frac{1}{2}e\text{Subst}\Bigg(\int\Bigg(\frac{a^2(a-}{b}\\ &= \frac{a^2e\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}-\frac{ae\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{e\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\Bigg(1+\end{aligned}$$

Mathematica [A]
time = 0.12, size = 112, normalized size = 0.90

$$\frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5e\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+2dx^5\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p
+ p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (2*d*x^5*Hypergeometric2F1[5/2
, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^4(ex+d)\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^5*e + d*x^4)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(104) = 208$.
time = 9.27, size = 950, normalized size = 7.60

$$\frac{a^p d x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{b x^2 e^{i\pi}}{a}\right)}{5} + e\left(\begin{cases} \frac{a^p x^6}{6} & \text{for}\: b = 0 \\ \frac{2a^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2a^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2b^2x^4 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2b^2x^4 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} & \text{for}\: p = -3 \\ -\frac{2a^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2abx^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} + \frac{b^2x^4}{2ab^3+2b^4x^2} & \text{for}\: p = -2 \\ \frac{a^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3} + \frac{a^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b} & \text{for}\: p = -1 \\ \frac{2a^3\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} - \frac{2a^2bpx^2\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2p^2x^4\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2px^4\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{b^3p^2x^6\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{3b^3px^6\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{2b^3x^6\left(a+bx^2\right)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + e*Piecew
ise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b
**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4
) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**
4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*
```

```
x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p,
 -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x
 + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x
 + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2
), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b)
)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)
**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a
 + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p
**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
 + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p +
12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*
b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p*
*2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x*e + d)*(b*x^2 + a)^p*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(b\,x^2+a\right)}^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x),x)
[Out] int(x^4*(a + b*x^2)^p*(d + e*x), x)
```

3.383 $\int x^3(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=100

$$-\frac{ad(a+bx^2)^{1+p}}{2b^2(1+p)}+\frac{d(a+bx^2)^{2+p}}{2b^2(2+p)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] -1/2*a*d*(b*x^2+a)^(1+p)/b^2/(1+p)+1/2*d*(b*x^2+a)^(2+p)/b^2/(2+p)+1/5*e*x^
5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {778, 272, 45, 372, 371}

$$-\frac{ad(a+bx^2)^{p+1}}{2b^2(p+1)}+\frac{d(a+bx^2)^{p+2}}{2b^2(p+2)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*d*(a + b*x^2)^(1 + p))/(b^2*(1 + p)) + (d*(a + b*x^2)^(2 + p))/(2*b
^2*(2 + p)) + (e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2
)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^3(d+ex)\left(a+bx^2\right)^p\,dx &= d\int x^3\left(a+bx^2\right)^p\,dx+e\int x^4\left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}d\text{Subst}\left(\int x(a+bx)^p\,dx,x,x^2\right)+\left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^4\left(1\right.\\ &= \frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+\frac{1}{2}d\text{Subst}\left(\int\left(-\frac{a(a}{}\right.\right.\\ &= -\frac{ad\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{d\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},\right.\end{aligned}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.87

$$\frac{1}{10}\left(a+bx^2\right)^p\left(-\frac{5d\left(a+bx^2\right)\left(a-b(1+p)x^2\right)}{b^2(1+p)(2+p)}+2ex^5\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p)
) + (2*e*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)
^p))/10
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3(ex+d)\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] e*integrate((b*x^2 + a)^p*x^4, x) + 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)
*(b*x^2 + a)^p*d/((p^2 + 3*p + 2)*b^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4*e + d*x^3)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(82) = 164$.
time = 7.51, size = 364, normalized size = 3.64

$$\frac{a^p e x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + d\left(\begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d*Piecew
ise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2
) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x
**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x +
 sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/
(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*
(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)
**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2
```

```
*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**
2*p + 4*b**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x*e + d)*(b*x^2 + a)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(b\,x^2+a\right)}^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p*(d + e*x),x)
[Out] int(x^3*(a + b*x^2)^p*(d + e*x), x)
```

3.384 $\int x^2(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=100

$$-\frac{ae(a+bx^2)^{1+p}}{2b^2(1+p)}+\frac{e(a+bx^2)^{2+p}}{2b^2(2+p)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] -1/2*a*e*(b*x^2+a)^(1+p)/b^2/(1+p)+1/2*e*(b*x^2+a)^(2+p)/b^2/(2+p)+1/3*d*x^
3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]

time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {778, 372, 371, 272, 45}

$$-\frac{ae(a+bx^2)^{p+1}}{2b^2(p+1)}+\frac{e(a+bx^2)^{p+2}}{2b^2(p+2)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*e*(a + b*x^2)^(1 + p))/(b^2*(1 + p)) + (e*(a + b*x^2)^(2 + p))/(2*b
^2*(2 + p)) + (d*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^2(d+ex)\left(a+bx^2\right)^p\,dx &= d\int x^2\left(a+bx^2\right)^p\,dx+e\int x^3\left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}e\text{Subst}\left(\int x(a+bx)^p\,dx,x,x^2\right)+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1\right.\\ &= \frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+\frac{1}{2}e\text{Subst}\left(\int\left(-\frac{a(a}{}\right.\right.\\ &= -\frac{ae\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{1}{3}dx^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},\right.\end{aligned}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.87

$$\frac{1}{6}\left(a+bx^2\right)^p\left(-\frac{3e\left(a+bx^2\right)\left(a-b(1+p)x^2\right)}{b^2(1+p)(2+p)}+2dx^3\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)*(a + b*x^2)^p,x]
[Out] ((a + b*x^2)^p*((-3*e*(a + b*x^2)*(a - b*(1 + p)*x^2))/(b^2*(1 + p)*(2 + p)
) + (2*d*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)
^p))/6
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2(ex+d)\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x^2*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^3*e + d*x^2)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(82) = 164.
time = 5.00, size = 364, normalized size = 3.64

$$\frac{a^p d x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + e\left(\begin{cases} \frac{a^p x^4}{4} & \text{for}\ b = 0 \\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for}\ p = -2 \\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for}\ p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + e*Piecew
ise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2
) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x
**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x +
 sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/
(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*
(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)
**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2
*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**
2*p + 4*b**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x*e + d)*(b*x^2 + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x),x)
[Out] int(x^2*(a + b*x^2)^p*(d + e*x), x)
```

3.385 $\int x(d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{d(a+bx^2)^{1+p}}{2b(1+p)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*d*(b*x^2+a)^(1+p)/b/(1+p)+1/3*e*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/
2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {778, 267, 372, 371}

$$\frac{d(a+bx^2)^{p+1}}{2b(p+1)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e*x^3*(a + b*x^2)^p*Hypergeometric
2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x(d+ex)\left(a+bx^2\right)^p\,dx &= d\int x\left(a+bx^2\right)^p\,dx+e\int x^2\left(a+bx^2\right)^p\,dx\\ &= \frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= \frac{d\left(a+bx^2\right)^{1+p}}{2b(1+p)}+\frac{1}{3}ex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 71, normalized size = 0.95

$$\frac{1}{6}\left(a+bx^2\right)^p\left(\frac{3d\left(a+bx^2\right)}{b(1+p)}+2ex^3\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((3*d*(a + b*x^2))/(b*(1 + p)) + (2*e*x^3*Hypergeometric2F1[
3/2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/6
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x(ex+d)\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int(x*(e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
[Out] e*integrate((b*x^2 + a)^p*x^2, x) + 1/2*(b*x^2 + a)^(p + 1)*d/(b*(p + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((x^2*e + d*x)*(b*x^2 + a)^p, x)
```

Sympy [A]
time = 4.16, size = 65, normalized size = 0.87

$$\frac{a^{p}ex^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3}+d\left(\begin{cases}\frac{a^{p}x^{2}}{2} & \text{for}\ b=0\\\frac{\begin{cases}\frac{(a+bx^{2})^{p+1}}{p+1} & \text{for}\ p\neq -1\\\log\left(a+bx^{2}\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x**2+a)**p,x)
[Out] a**p*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d*Piecew
ise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x*e + d)*(b*x^2 + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(b\,x^2+a\right)}^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p*(d + e*x),x)
[Out] int(x*(a + b*x^2)^p*(d + e*x), x)
```

3.386 $\int (d+ex)\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=70

$$\frac{e\left(a+bx^2\right)^{1+p}}{2b(1+p)}+dx\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*e*(b*x^2+a)^(1+p)/b/(1+p)+d*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*
x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]

time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {655, 252, 251}

$$dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] (e*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (d*x*(a + b*x^2)^p*Hypergeometric2F
1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\left(a+bx^2\right)^p\,dx=\frac{e(a+bx^2)^{1+p}}{2b(1+p)}+d\int\left(a+bx^2\right)^p\,dx$$
$$=\frac{e(a+bx^2)^{1+p}}{2b(1+p)}+\left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p\,dx$$
$$=\frac{e(a+bx^2)^{1+p}}{2b(1+p)}+dx\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Mathematica [A]
time = 0.11, size = 98, normalized size = 1.40

$$\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(bex^2\left(1+\frac{bx^2}{a}\right)^p+ae\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+2bd(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2b(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(b*e*x^2*(1 + (b*x^2)/a)^p + a*e*(-1 + (1 + (b*x^2)/a)^p) +
2*b*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(2*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (ex+d)\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(b*x^2 + a)^p, x)
```

Sympy [A]
time = 2.78, size = 61, normalized size = 0.87

$$a^{p}dx{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)+e\left(\begin{cases}\frac{a^{p}x^{2}}{2} & \text{for}\: b=0\\ \frac{\begin{cases}\frac{(a+bx^{2})^{p+1}}{p+1} & \text{for}\: p\neq -1\\ \log\left(a+bx^{2}\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p,x)
```

```
[Out] a**p*d*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + e*Piecewise((
a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p, -1
)), (log(a + b*x**2), True))/(2*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p, x)
```

Mupad [B]
time = 3.36, size = 65, normalized size = 0.93

$$\frac{e\,{\left(b\,x^2+a\right)}^{p+1}}{2\,b\,\left(p+1\right)}+\frac{d\,x\,{\left(b\,x^2+a\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(d + e*x),x)
```

```
[Out] (e*(a + b*x^2)^(p + 1))/(2*b*(p + 1)) + (d*x*(a + b*x^2)^p*hypergeom([1/2,
-p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

3.387 $\int \frac{(d+ex)(a+bx^2)^p}{x} dx$

Optimal. Leaf size=88

$$ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] e*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)-1/2*d*(
b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {778, 272, 67, 252, 251}

$$ex\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d\left(a+bx^2\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x,x]
```

```
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+bx^2\right)^p}{x}\,dx &= d\int \frac{\left(a+bx^2\right)^p}{x}\,dx + e\int \left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right.\\ &= ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right) - \frac{d\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+\right.}{2a(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 88, normalized size = 1.00

$$ex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right) - \frac{d\left(a+bx^2\right)^{1+p}{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x,x]
```

```
[Out] (e*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p - (d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*x^2)/a])/(2*a*(1 + p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(b\,x^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(b*x^2 + a)^p/x, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.03, size = 65, normalized size = 0.74

$$a^p e x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{b x^2 e^{i\pi}}{a}\right) - \frac{b^p d x^{2p} \Gamma(-p)\, {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{a e^{i\pi}}{b x^2}\right)}{2\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x,x)
```

```
[Out] a**p*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*x**(2*
p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma
(1 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d+e\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x))/x,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x))/x, x)
```

3.388 $\int \frac{(d+ex)(a+bx^2)^p}{x^2} dx$

Optimal. Leaf size=91

$$-\frac{d(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{e(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

```
[Out] -d*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)-1/2*e
*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {778, 372, 371, 272, 67}

$$-\frac{d(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{e(a+bx^2)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^2}\,dx = d\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx + e\int \frac{\left(a+bx^2\right)^p}{x}\,dx$$

$$= \frac{1}{2}e\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)}{x^2}$$

$$= -\frac{d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p\right.}{2a(1+p)}$$

Mathematica [**A**]
time = 0.10, size = 91, normalized size = 1.00

$$-\frac{d\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (b*x^2)/a])/(2*a*(1 + p))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(b\,x^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(b*x^2 + a)^p/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.15, size = 68, normalized size = 0.75

$$-\frac{a^p d\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, -p\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}-\frac{b^p e x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p, -p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*e*x**(
2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gam
ma(1 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d+e\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x))/x^2,x)
[Out] int(((a + b*x^2)^p*(d + e*x))/x^2, x)
```

3.389 $\int \frac{(d+ex)(a+bx^2)^p}{x^3}\,dx$

Optimal. Leaf size=92

$$-\frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}+\frac{bd(a+bx^2)^{1+p}{}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

```
[Out] -e*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)+1/2*b
*d*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],1+b*x^2/a)/a^2/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {778, 272, 67, 372, 371}

$$\frac{bd(a+bx^2)^{p+1}{}_2F_1\left(2,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -((e*(a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 +
(b*x^2)/a)^p)) + (b*d*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, 1 + (b*x^2)/a])/(2*a^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx^2\right)^p}{x^3}\,dx = d\int \frac{\left(a+bx^2\right)^p}{x^3}\,dx + e\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{x^2}\,dx, x, x^2\right) + \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right.}{x^2}$$

$$= -\frac{e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} + \frac{bd\left(a+bx^2\right)^{1+p}\,{}_2F_1\left(2,1+\right.}{2a^2(1+p)}$$

Mathematica [A]
time = 0.10, size = 89, normalized size = 0.97

$$\frac{1}{2}\left(a+bx^2\right)^p\left(-\frac{2e\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} + \frac{bd\left(a+bx^2\right)\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)}{a^2(1+p)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x^2)^p)/x^3,x]
```

```
[Out] ((a + b*x^2)^p*((-2*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1
 + (b*x^2)/a)^p) + (b*d*(a + b*x^2)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x^2)/a])/(a^2*(1 + p))))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)\left(b\,x^2+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(b*x^2+a)^p/x^3,x)
```

```
[Out] int((e*x+d)*(b*x^2+a)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(b*x^2 + a)^p/x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.81, size = 71, normalized size = 0.77

$$-\frac{a^p e\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p d x^{2p} \Gamma(1-p)\, {}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{a e^{i\pi}}{bx^2}\right)}{2x^2 \Gamma(2-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x**2+a)**p/x**3,x)
```

```
[Out] -a**p*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d*x**(
2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**2))/
(2*x**2*gamma(2 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(b*x^2 + a)^p/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d+e\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x))/x^3,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x))/x^3, x)
```

3.390 $\int x^5(d+ex)^2 (a+bx^2)^p \, dx$

Optimal. Leaf size=188

$$\frac{a^2(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^4(1+p)}-\frac{a(2bd^2-3ae^2)(a+bx^2)^{2+p}}{2b^4(2+p)}+\frac{(bd^2-3ae^2)(a+bx^2)^{3+p}}{2b^4(3+p)}+\frac{e^2(a+bx^2)^{4+p}}{2b^4(4+p)}+\frac{2}{7}dex^7$$

```
[Out] 1/2*a^2*(-a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^4/(1+p)-1/2*a*(-3*a*e^2+2*b*d^2)*(
b*x^2+a)^(2+p)/b^4/(2+p)+1/2*(-3*a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+1/2
*e^2*(b*x^2+a)^(4+p)/b^4/(4+p)+2/7*d*e*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],
[9/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1666, 457, 78, 12, 372, 371}

$$\frac{a^2(bd^2-ae^2)(a+bx^2)^{p+1}}{2b^4(p+1)}-\frac{a(2bd^2-3ae^2)(a+bx^2)^{p+2}}{2b^4(p+2)}+\frac{(bd^2-3ae^2)(a+bx^2)^{p+3}}{2b^4(p+3)}+\frac{e^2(a+bx^2)^{p+4}}{2b^4(p+4)}+\frac{2}{7}dex^7(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (a^2*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) - (a*(2*b*d^2 - 3
*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^4*(2 + p)) + ((b*d^2 - 3*a*e^2)*(a + b*x^
2)^(3 + p))/(2*b^4*(3 + p)) + (e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (
2*d*e*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(
1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int x^5(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^6\left(a+bx^2\right)^p\,dx+\int x^5\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^6\left(a+bx^2\right)^p\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2\left(-bd^2+ae^2\right)(a+bx)^p}{b^3}+\frac{a\left(-2bd^2+3ae^2\right)(a+bx)^{1+p}}{b^3}+\right.\right.\\
&= \frac{a^2\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}-\frac{a\left(2bd^2-3ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{\left(bd^2-3ae^2\right)(a}{2b^4(3+}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 205, normalized size = 1.09

$$
\frac{1}{14}\left(a+bx^2\right)^p\left(\frac{7d^2\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+\frac{7e^2\left(a+bx^2\right)\left(-6a^3+6a^2b(1+p)x^2-3ab^2\left(2+3p+p^2\right)x^4+b^3\left(6+11p+6p^2+p^3\right)x^6\right)}{b^4(1+p)(2+p)(3+p)(4+p)}+4dex^7\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((7*d^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*
p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*e^2*(a + b*x^2)*(-6*a^3 +
 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6*p^2
+ p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (4*d*e*x^7*Hypergeomet
ric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/14
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{5}(ex+d)^{2}\left(b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p*d^2/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((x^7*e^2 + 2*
d*x^6*e)*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^7*e^2 + 2*d*x^6*e + d^2*x^5)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(163) = 326.
time = 19.15, size = 2883, normalized size = 15.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(e*x+d)**2*(b*x**2+a)**p,x)

[Out] 2*a**p*d*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True)) + e**2*Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**

```
4) - 6*a*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**
6*x**4) - 6*a*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 +
4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4), Eq(
p, -3)), (6*a**3*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*log(
x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2)
+ 6*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**
2*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 +
4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log(x
 - sqrt(-a/b))/(2*b**4) - a**3*log(x + sqrt(-a/b))/(2*b**4) + a**2*x**2/(2*
b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/
(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3
*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*
b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20
*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a +
 b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b*
*4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**
4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4
*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x*
*6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p
+ 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 7
0*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2*b
**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*
x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*
p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(b\,x^2+a\right)}^p\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^5*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.391 $\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=177

$$\frac{a^2de\left(a+bx^2\right)^{1+p}}{b^3(1+p)}+\frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}-\frac{2ade\left(a+bx^2\right)^{2+p}}{b^3(2+p)}+\frac{de\left(a+bx^2\right)^{3+p}}{b^3(3+p)}-\frac{\left(5ae^2-bd^2(7+2p)\right)x^5\left(a+bx^2\right.}{5b(7}$$

```
[Out] a^2*d*e*(b*x^2+a)^(1+p)/b^3/(1+p)+e^2*x^5*(b*x^2+a)^(1+p)/b/(7+2*p)-2*a*d*e
*(b*x^2+a)^(2+p)/b^3/(2+p)+d*e*(b*x^2+a)^(3+p)/b^3/(3+p)-1/5*(5*a*e^2-b*d^2
*(7+2*p))*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/b/(7+2*p)/((1
+b*x^2/a)^p)
```

Rubi [A]
time = 0.11, antiderivative size = 169, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1666, 470, 372, 371, 12, 272, 45}

$$\frac{a^2de\left(a+bx^2\right)^{p+1}}{b^3(p+1)}-\frac{2ade\left(a+bx^2\right)^{p+2}}{b^3(p+2)}+\frac{de\left(a+bx^2\right)^{p+3}}{b^3(p+3)}+\frac{1}{5}x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{5ae^2}{2bp+7b}\right)\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+\frac{e^2x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*e*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (e^2*x^5*(a + b*x^2)^(1 + p))
/(b*(7 + 2*p)) - (2*a*d*e*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (d*e*(a + b*
x^2)^(3 + p))/(b^3*(3 + p)) + ((d^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x
^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x^4(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^5\left(a+bx^2\right)^p\,dx + \int x^4\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx \\
&= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} + (2de)\int x^5\left(a+bx^2\right)^p\,dx - \left(-d^2+\frac{5ae^2}{7b+2bp}\right)\int x^4\left(a+\right. \\
&= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} + (de)\text{Subst}\left(\int x^2(a+bx)^p\,dx, x, x^2\right) - \left(\left(-d^2+\frac{5ae^2}{7b+2}\right.\right. \\
&= \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} + \frac{1}{5}\left(d^2-\frac{5ae^2}{7b+2bp}\right)x^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-\right. \\
&= \frac{a^2de\left(a+bx^2\right)^{1+p}}{b^3(1+p)} + \frac{e^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)} - \frac{2ade\left(a+bx^2\right)^{2+p}}{b^3(2+p)} + \frac{de\left(a+bx^2\right)^{3+p}}{b^3(3+p)}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 156, normalized size = 0.88

$$\frac{1}{35}(a+bx^2)^p\left(\frac{35de(a+bx^2)\,(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4)}{b^3(1+p)(2+p)(3+p)}+7d^2x^5\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+5e^2x^7\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((35*d*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (7*d^2*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (5*e^2*x^7*Hypergeometri
c2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/35
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4(ex+d)^2\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^6*e^2 + 2*d*x^5*e + d^2*x^4)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(151) = 302$.

time = 18.00, size = 986, normalized size = 5.57

$$\frac{a^p d^2 x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2}, -p\\ \frac{7}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + \frac{a^p e^2 x^7 {}_2F_1\left(\begin{matrix}\frac{7}{2}, -p\\ \frac{9}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7} + 2de\left(\begin{cases} \frac{a^p x^6}{6} & \text{for } b = 0\\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{4abx^2}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2b^2x^4\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} + \frac{2b^2x^4\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} & \text{for } p = -3\\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} + \frac{b^2x^4}{2ab^3+2b^4x^2} & \text{for } p = -2\\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3} + \frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b} & \text{for } p = -1\\ \frac{2a^3(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} - \frac{2a^2bpx^2(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2p^2x^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{ab^2px^4(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{b^3p^2x^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{3b^3px^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} + \frac{2b^3x^6(a+bx^2)^p}{2b^3p^3+12b^3p^2+22b^3p+12b^3} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*
e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 2*d*e*Piec
ewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8
*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a
*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*
x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**
5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x
**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**
2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(
p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log
(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2)
 - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log
(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x*
*2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/
b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**
2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*
(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2
*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**
3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p
+ 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 2
2*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*
p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12
*b**3*p**2 + 22*b**3*p + 12*b**3), True))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p*x^4, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^4*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.392 $\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=149

$$-\frac{a(bd^2-ae^2)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{(bd^2-2ae^2)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{e^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1$$

```
[Out] -1/2*a*(-a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^3/(1+p)+1/2*(-2*a*e^2+b*d^2)*(b*x^2
+a)^(2+p)/b^3/(2+p)+1/2*e^2*(b*x^2+a)^(3+p)/b^3/(3+p)+2/5*d*e*x^5*(b*x^2+a)
^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1666, 457, 78, 12, 372, 371}

$$-\frac{a(bd^2-ae^2)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{(bd^2-2ae^2)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{e^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{2}{5}dex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + ((b*d^2 - 2*a*
e^2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(2*b^
3*(3 + p)) + (2*d*e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*
x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x^3(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^4\left(a+bx^2\right)^p\,dx+\int x^3\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^4\left(a+bx^2\right)^p\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a\left(-bd^2+ae^2\right)(a+bx)^p}{b^2}+\frac{\left(bd^2-2ae^2\right)(a+bx)^{1+p}}{b^2}+\frac{e^2(a+\right.\right.\\
&= -\frac{a\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{\left(bd^2-2ae^2\right)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{e^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 152, normalized size = 1.02

$$\frac{1}{10}\left(a+bx^2\right)^p\left(\frac{5d^2\left(a+bx^2\right)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)}+\frac{5e^2\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+4dex^5\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*d^2*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
p)) + (5*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x
^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (4*d*e*x^5*Hypergeometric2F1[5/2, -p,
7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/10
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{3}(ex+d)^{2}\left(b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^2/((p^2 + 3*p + 2)*
b^2) + integrate((x^5*e^2 + 2*d*x^4*e)*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^5*e^2 + 2*d*x^4*e + d^2*x^3)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(124) = 248.
time = 10.34, size = 1294, normalized size = 8.68

$$\frac{2a^{p}dex^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{5}+d^{2}\left(\begin{cases}\frac{a^{p}x^{4}}{4} & \text{for}\: b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}} & \text{for}\: p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^{2}}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^{2}}+\frac{x^{2}}{2b} & \text{for}\: p=-1\\ -\frac{a^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{abpx^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}px^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}x^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}} & \text{otherwise}\end{cases}\right)+e^{2}\left(\begin{cases}\frac{a^{p}x^{6}}{6} & \text{for}\: b=0\\ \frac{2a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{3a^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2b^{2}x^{4}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2b^{2}x^{4}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}} & \text{for}\: p=-3\\ -\frac{2a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2a^{2}}{2ab^{3}+2b^{4}x^{2}}-\frac{2abx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2abx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}+\frac{b^{2}x^{4}}{2ab^{3}+2b^{4}x^{2}} & \text{for}\: p=-2\\ \frac{a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^{3}}+\frac{a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^{3}}-\frac{ax^{2}}{2b^{2}}+\frac{x^{4}}{4b} & \text{for}\: p=-1\\ \frac{2a^{3}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}-\frac{2a^{2}bpx^{2}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{ab^{2}p^{2}x^{4}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{ab^{2}px^{4}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{b^{3}p^{2}x^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{3b^{3}px^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{2b^{3}x^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**2
*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b*
*3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2
*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*
log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(
-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)),
(-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a +
b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/
(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2
+ 6*b**2*p + 4*b**2), True)) + e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a
**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**
2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/
(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b)
)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/
b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 +
 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*
b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b
))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4
*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(
2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4
*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt
(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x*
*4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*
p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/
(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*
x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6
*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**
3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3),
True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(b\,x^2+a\right)}^p\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p*(d + e*x)^2,x)
[Out] int(x^3*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.393 $\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=152

$$-\frac{ade\left(a+bx^2\right)^{1+p}}{b^2(1+p)}+\frac{e^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{de\left(a+bx^2\right)^{2+p}}{b^2(2+p)}-\frac{\left(3ae^2-bd^2(5+2p)\right)x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1}{3b(5+2p)}$$

```
[Out] -a*d*e*(b*x^2+a)^(1+p)/b^2/(1+p)+e^2*x^3*(b*x^2+a)^(1+p)/b/(5+2*p)+d*e*(b*x
^2+a)^(2+p)/b^2/(2+p)-1/3*(3*a*e^2-b*d^2*(5+2*p))*x^3*(b*x^2+a)^p*hypergeom
([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.09, antiderivative size = 144, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.350$, Rules used = {1666, 470, 372, 371, 12, 272, 45}

$$-\frac{ade\left(a+bx^2\right)^{p+1}}{b^2(p+1)}+\frac{de\left(a+bx^2\right)^{p+2}}{b^2(p+2)}+\frac{1}{3}x^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{3ae^2}{2bp+5b}\right){}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+\frac{e^2x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] -((a*d*e*(a + b*x^2)^(1 + p))/(b^2*(1 + p))) + (e^2*x^3*(a + b*x^2)^(1 + p)
)/(b*(5 + 2*p)) + (d*e*(a + b*x^2)^(2 + p))/(b^2*(2 + p)) + ((d^2 - (3*a*e^
2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^
2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\int x^2(d+ex)^2\left(a+bx^2\right)^p\,dx=\int 2dex^3\left(a+bx^2\right)^p\,dx+\int x^2\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx$$

$$=\frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)}+(2de)\int x^3\left(a+bx^2\right)^p\,dx-\left(-d^2+\frac{3ae^2}{5b+2bp}\right)\int x^2(a+$$

$$=\frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)}+(de)\text{Subst}\left(\int x(a+bx)^p\,dx,x,x^2\right)-\left(\left(-d^2+\frac{3ae^2}{5b+2b}\right.\right.$$

$$=\frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)}+\frac{1}{3}\left(d^2-\frac{3ae^2}{5b+2bp}\right)x^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-\right.$$

$$=-\frac{ade(a+bx^2)^{1+p}}{b^2(1+p)}+\frac{e^2x^3(a+bx^2)^{1+p}}{b(5+2p)}+\frac{de(a+bx^2)^{2+p}}{b^2(2+p)}+\frac{1}{3}\left(d^2-\frac{3ae^2}{5b+2bp}\right.$$

Mathematica [A]
time = 0.23, size = 139, normalized size = 0.91

$$\frac{1}{15}(a+bx^2)^p\left(5d^2x^3\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+\frac{3e\left(-\frac{5d(a+bx^2)(a-b(1+p)x^2)}{b^2}+e(2+3p+p^2)x^5\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)}{(1+p)(2+p)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((5*d^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^p + (3*e*((-5*d*(a + b*x^2)*(a - b*(1 + p)*x^2))/b^2 + (e*(2
 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p))/((1 + p)*(2 + p))))/15
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2(ex+d)^2\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x^2*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4*e^2 + 2*d*x^3*e + d^2*x^2)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(126) = 252$.
time = 9.65, size = 400, normalized size = 2.63

$$\frac{a^p d^2 x^3 {}_2F_1\left(\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + \frac{a^p e^2 x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2}, -p \\ \frac{7}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + 2de \left(\begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} & \text{for } p = -2 \\ -\frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{abpx^2 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2px^4 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2x^4 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p*
e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 2*d*e*Piec
ewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x*
*2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3
*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x
 + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b)
)/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**
2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**
2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b
**2*p + 4*b**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(b\,x^2+a\right)^p \left(d+e\,x\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x^2*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.394 $\int x(d+ex)^2 (a+bx^2)^p \, dx$

Optimal. Leaf size=113

$$\frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2(1+p)}+\frac{e^2(a+bx^2)^{2+p}}{2b^2(2+p)}+\frac{2}{3}dex^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/2*(-a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^2/(1+p)+1/2*e^2*(b*x^2+a)^(2+p)/b^2/(2
+p)+2/3*d*e*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2/a
)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1666, 455, 45, 12, 372, 371}

$$\frac{(bd^2-ae^2)(a+bx^2)^{p+1}}{2b^2(p+1)}+\frac{e^2(a+bx^2)^{p+2}}{2b^2(p+2)}+\frac{2}{3}dex^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^2*(a + b*x^2)^(2
 + p))/(2*b^2*(2 + p)) + (2*d*e*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p
, 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x(d+ex)^2\left(a+bx^2\right)^p\,dx &= \int 2dex^2\left(a+bx^2\right)^p\,dx+\int x\left(a+bx^2\right)^p\left(d^2+e^2x^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int (a+bx)^p\left(d^2+e^2x\right)\,dx,x,x^2\right)+(2de)\int x^2\left(a+bx^2\right)^p\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\left(bd^2-ae^2\right)(a+bx)^p}{b}+\frac{e^2(a+bx)^{1+p}}{b}\right)\,dx,x,x^2\right)+\left(2de\left(a+\right.\right.\\ &= \frac{\left(bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+\frac{2}{3}dex^3\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2\end{aligned}$$

Mathematica [A]
time = 0.20, size = 184, normalized size = 1.63

$$\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(3b^2x^2\left(1+\frac{bx^2}{a}\right)^p\left(d^2(2+p)+e^2(1+p)x^2\right)-3a^2e^2\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+3ab\left(e^2px^2\left(1+\frac{bx^2}{a}\right)^p+d^2(2+p)\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)\right)+4b^2de(2+3p+p^2)\,x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)}{6b^2(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(3*b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + e^2*(1 + p)*x^2)
 - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + 3*a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p
```

```
+ d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 4*b^2*d*e*(2 + 3*p + p^2)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)]))/(6*b^2*(1 + p)*(2 + p)*(1 + (b
*x^2)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(ex+d)^2 \left(b\,x^2+a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int(x*(e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)*d^2/(b*(p + 1)) + integrate((x^3*e^2 + 2*d*x^2*e)*(
b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^2 + 2*d*x^2*e + d^2*x)*(b*x^2 + a)^p, x)
```

Sympy [A]
time = 5.47, size = 408, normalized size = 3.61

$$\frac{2a^p dex^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + d^2\left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\: b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log\left(a+bx^2\right) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} \frac{a^p x^4}{4} & \text{for}\: b = 0 \\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for}\: p = -2 \\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for}\: p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + d**2
*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p +
1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**2*Piecewise((a*
*p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*l
og(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) +
b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-
a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**2
) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*
x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*
b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 +
 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4
*b**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(b\,x^2+a\right)}^p\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p*(d + e*x)^2,x)
```

```
[Out] int(x*(a + b*x^2)^p*(d + e*x)^2, x)
```

3.395 $\int (d+ex)^2 (a+bx^2)^p \, dx$

Optimal. Leaf size=133

$$\frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)}+\frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)}-\frac{\left(ae^2-bd^2(3+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\right.}{b(3+2p)}$$

```
[Out] d*e*(2+p)*(b*x^2+a)^(1+p)/b/(2*p^2+5*p+3)+e*(e*x+d)*(b*x^2+a)^(1+p)/b/(3+2*
p)-(a*e^2-b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/
b/(3+2*p)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {757, 655, 252, 251}

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+3b}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e(d+ex)\left(a+bx^2\right)^{p+1}}{b(2p+3)}+\frac{de(p+2)\left(a+bx^2\right)^{p+1}}{b(p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] (d*e*(2 + p)*(a + b*x^2)^(1 + p))/(b*(1 + p)*(3 + 2*p)) + (e*(d + e*x)*(a +
 b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\int (d+ex)^2 \left(a+bx^2\right)^p \, dx = \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \frac{\int \left(-ae^2+bd^2(3+2p)+2bde(2+p)x\right)\left(a+bx^2\right)^p \, dx}{b(3+2p)}$$
$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(d^2-\frac{ae^2}{3b+2bp}\right)\int \left(a+bx\right.$$
$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(\left(d^2-\frac{ae^2}{3b+2bp}\right)\left(a+bx^2\right.\right.$$
$$= \frac{de(2+p)\left(a+bx^2\right)^{1+p}}{b(1+p)(3+2p)} + \frac{e(d+ex)\left(a+bx^2\right)^{1+p}}{b(3+2p)} + \left(d^2-\frac{ae^2}{3b+2bp}\right)x\left(a+bx^2\right)$$

Mathematica [A]
time = 0.18, size = 133, normalized size = 1.00

$$\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(3bd^2(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+e\left(3d\left(bx^2\left(1+\frac{bx^2}{a}\right)^p+a\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)\right)+be(1+p)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)\right)}{3b(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(3*b*d^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)] + e*(3*d*(b*x^2*(1 + (b*x^2)/a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + b*e*
(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(3*b*(1 + p)*(
1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex+d\right)^{2}\left(b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(b*x^2 + a)^p, x)
```

Sympy [A]
time = 5.28, size = 97, normalized size = 0.73

$$a^{p}d^{2}x\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)+\frac{a^{p}e^{2}x^{3}\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3}+2de\left(\begin{cases}\frac{a^{p}x^{2}}{2} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^{2})^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(a+bx^{2}\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p,x)
[Out] a**p*d**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e**2*
x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 2*d*e*Piecewise
((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^p\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(d + e*x)^2,x)
[Out] int((a + b*x^2)^p*(d + e*x)^2, x)
```

3.396 $\int \frac{(d+ex)^2(a+bx^2)^p}{x}\,dx$

Optimal. Leaf size=118

$$\frac{e^2(a+bx^2)^{1+p}}{2b(1+p)}+2dex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d^2(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1\right.}{2a(1+p)}$$

```
[Out] 1/2*e^2*(b*x^2+a)^(1+p)/b/(1+p)+2*d*e*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/
2],-b*x^2/a)/((1+b*x^2/a)^p)-1/2*d^2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+
p],1+b*x^2/a)/a/(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1666, 457, 81, 67, 12, 252, 251}

$$-\frac{d^2(a+bx^2)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+2dex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e^2(a+bx^2)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x,x]
```

```
[Out] (e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (2*d*e*x*(a + b*x^2)^p*Hypergeome
tric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\,(a+bx^2)^p}{x}\,dx &= \int 2de\left(a+bx^2\right)^p\,dx+\int \frac{(a+bx^2)^p\,(d^2+e^2x^2)}{x}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p\,(d^2+e^2x)}{x}\,dx,x,x^2\right)+(2de)\int \left(a+bx^2\right)^p\,dx\\ &= \frac{e^2(a+bx^2)^{1+p}}{2b(1+p)}+\frac{1}{2}d^2\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\left(2de\left(a+bx^2\right)^p\left(1+\frac{bx}{a}\right.\right.\\ &= \frac{e^2(a+bx^2)^{1+p}}{2b(1+p)}+2dex\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-\frac{d^2(a+}{}\end{aligned}$$

Mathematica [A]

time = 0.16, size = 101, normalized size = 0.86

$$\frac{1}{2}(a+bx^2)^p\left(4dex\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{(a+bx^2)\left(ae^2-bd^2\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)\right)}{ab(1+p)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x,x]
[Out] ((a + b*x^2)^p*((4*d*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1
+ (b*x^2)/a)^p + ((a + b*x^2)*(a*e^2 - b*d^2*Hypergeometric2F1[1, 1 + p, 2
+ p, 1 + (b*x^2)/a]))/(a*b*(1 + p))))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^2\,(b\,x^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x,x)
[Out] int((e*x+d)^2*(b*x^2+a)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="maxima")
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="fricas")
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(b*x^2 + a)^p/x, x)
```

Sympy [A]
time = 5.08, size = 109, normalized size = 0.92

$$2a^pdex{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)-\frac{b^pd^2x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}+e^2\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p/x,x)
```

```
[Out] 2*a**p*d*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**2
*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(
2*gamma(1 - p)) + e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a +
b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^p\,{\left(d+e\,x\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x, x)
```

3.397 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^2}\,dx$

Optimal. Leaf size=127

$$-\frac{d^2(a+bx^2)^{1+p}}{ax}+\frac{(ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{de(a+bx^2)^{1+p}\,{}_2F_1\big(1}{a(1}$$

```
[Out] -d^2*(b*x^2+a)^(1+p)/a/x+(a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p*hypergeom([1/2
, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^p)-d*e*(b*x^2+a)^(1+p)*hypergeom([1, 1
+p],[2+p],1+b*x^2/a)/a/(1+p)
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1821, 778, 272, 67, 252, 251}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2(2p+1))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{d^2(a+bx^2)^{p+1}}{ax}-\frac{de(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(a*x)) + ((a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^
2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) -
 (d*e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a]
)/(a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2\left(a+bx^2\right)^p}{x^2}\,dx &= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax}-\frac{\int \frac{\left(-2ade-\left(ae^2+bd^2(1+2p)\right)x\right)\left(a+bx^2\right)^p}{x}\,dx}{a}\\
&= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax}+(2de)\int \frac{\left(a+bx^2\right)^p}{x}\,dx+\frac{\left(ae^2+bd^2(1+2p)\right)\int \left(a+bx^2\right)^p\,d}{a}\\
&= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax}+(de)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right)+\frac{\Big(\left(ae^2+bd^2(1+2p)\right)}{}\\
&= -\frac{d^2\left(a+bx^2\right)^{1+p}}{ax}+\frac{\left(ae^2+bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\right.}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 134, normalized size = 1.06

$$
-\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(ad^2(1+p)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)+ex\left(-ae(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+d\left(a+bx^2\right)\left(1+\frac{bx^2}{a}\right)^p{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)\right)\right)}{a(1+p)x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^2,x]
```

```
[Out] -(((a + b*x^2)^p*(a*d^2*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/
a)] + e*x*(-(a*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]) +
 d*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*
x^2)/a])))/(a*(1 + p)*x*(1 + (b*x^2)/a)^p))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2\,(b\,x^2+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p/x^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(b*x^2 + a)^p/x^2, x)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 5.39, size = 95, normalized size = 0.75

$$-\frac{a^p d^2 {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} + a^p e^2 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{b^p d e x^{2p} \Gamma(-p)\, {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{a e^{i\pi}}{bx^2}\right)}{\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d**2*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p*e**
2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d*e*x**(2*p)*
gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/gamma(1 - p
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d+e\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x^2,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x^2, x)
```

3.398 $\int \frac{(d+ex)^2(a+bx^2)^p}{x^3}\,dx$

Optimal. Leaf size=127

$$-\frac{d^2(a+bx^2)^{1+p}}{2ax^2}-\frac{2de(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{(ae^2+bd^2p)\,(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+\right.}{2a^2(1+p)}$$

```
[Out] -1/2*d^2*(b*x^2+a)^(1+p)/a/x^2-2*d*e*(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2]
,-b*x^2/a)/x/((1+b*x^2/a)^p)-1/2*(b*d^2*p+a*e^2)*(b*x^2+a)^(1+p)*hypergeom(
[1, 1+p],[2+p],1+b*x^2/a)/a^2/(1+p)
```

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1821, 778, 372, 371, 272, 67}

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{d^2(a+bx^2)^{p+1}}{2ax^2}-\frac{2de(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -1/2*(d^2*(a + b*x^2)^(1 + p))/(a*x^2) - (2*d*e*(a + b*x^2)^p*Hypergeometri
c2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - ((a*e^2 + b*d^2*
p)*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(
2*a^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2\,(a+bx^2)^p}{x^3}\,dx &= -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} - \frac{\int \frac{(-4ade-2(ae^2+bd^2p)x)(a+bx^2)^p}{x^2}\,dx}{2a}\\ &= -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} + (2de)\int \frac{(a+bx^2)^p}{x^2}\,dx + \frac{(ae^2+bd^2p)\int \frac{(a+bx^2)^p}{x}\,dx}{a}\\ &= -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} + \frac{(ae^2+bd^2p)\,\mathrm{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2a} + \Bigg(2de(a+bx^2)^p \Bigg(\\ &= -\frac{d^2(a+bx^2)^{1+p}}{2ax^2} - \frac{2de(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{(ae^2+\cdots}{}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 119, normalized size = 0.94

$$\frac{1}{2}(a+bx^2)^p\left(-\frac{4de\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{(a+bx^2)\left(ae^2\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)-bd^2\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)\right)}{a^2(1+p)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x^2)^p)/x^3,x]
```

```
[Out] ((a + b*x^2)^p*((-4*d*e*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^p) - ((a + b*x^2)*(a*e^2*Hypergeometric2F1[1, 1 + p, 2 + p,
 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a]))
/(a^2*(1 + p))))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{2} \left(b\,x^{2}+a\right)^{p}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(b*x^2+a)^p/x^3,x)
```

```
[Out] int((e*x+d)^2*(b*x^2+a)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(b*x^2 + a)^p/x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.19, size = 119, normalized size = 0.94

$$-\frac{2a^{p}de\,{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{x} - \frac{b^{p}d^{2}x^{2p}\Gamma(1-p)\,{}_{2}F_{1}\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^{2}}\right)}{2x^{2}\Gamma\left(2-p\right)} - \frac{b^{p}e^{2}x^{2p}\Gamma(-p)\,{}_{2}F_{1}\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^{2}}\right)}{2\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(b*x**2+a)**p/x**3,x)
```

```
[Out] -2*a**p*d*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x - b**p*d*
*2*x**(2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*
x**2))/(2*x**2*gamma(2 - p)) - b**p*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p),
 (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2*(b*x^2 + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d+e\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^2)/x^3,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^2)/x^3, x)
```

3.399 $\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=247

$$\frac{a^2d(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}-\frac{ad(2bd^2-9ae^2)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{d(bd^2-9ae^2)\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}$$

```
[Out] 1/2*a^2*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^4/(1+p)+e^3*x^7*(b*x^2+a)^(1+p
)/b/(9+2*p)-1/2*a*d*(-9*a*e^2+2*b*d^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+1/2*d*(-9*
a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+3/2*d*e^2*(b*x^2+a)^(4+p)/b^4/(4+p)-
1/7*e*(7*a*e^2-3*b*d^2*(9+2*p))*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/2],-
b*x^2/a)/b/(9+2*p)/((1+b*x^2/a)^p)
```

Rubi [A]

time = 0.16, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {1666, 457, 78, 470, 372, 371}

$$\frac{a^2d(bd^2-3ae^2)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{ad(2bd^2-9ae^2)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{d(bd^2-9ae^2)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{3de^2\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{1}{7}ex^7\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{7ae^2}{2bp+9b}\right){}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)+\frac{e^3x^7\left(a+bx^2\right)^{p+1}}{b(2p+9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (a^2*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (e^3*x^7*(a
 + b*x^2)^(1 + p))/(b*(9 + 2*p)) - (a*d*(2*b*d^2 - 9*a*e^2)*(a + b*x^2)^(2
+ p))/(2*b^4*(2 + p)) + (d*(b*d^2 - 9*a*e^2)*(a + b*x^2)^(3 + p))/(2*b^4*(3
 + p)) + (3*d*e^2*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (e*(3*d^2 - (7*a*e
^2)/(9*b + 2*b*p))*x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x
^2)/a)])/(7*(1 + (b*x^2)/a)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^5(d+ex)^3\left(a+bx^2\right)^p\,dx&=\int x^5\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^6\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\&=\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}+\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right)+\left(e\left(3\right.\right.\\&=\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2d\left(-bd^2+3ae^2\right)(a+bx)^p}{b^3}+\frac{ad\left(-2bd^2\right.}{}\right.\right.\\&=\frac{a^2d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{e^3x^7\left(a+bx^2\right)^{1+p}}{b(9+2p)}-\frac{ad\left(2bd^2-9ae^2\right)\left(a+bx^2\right.}{2b^4(2+p)}\end{aligned}$$

Mathematica [A]

time = 0.38, size = 249, normalized size = 1.01

$$\frac{1}{126}(a+bx^2)^p\left(\frac{63d^3(a+bx^2)\,(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4)}{b^3(1+p)(2+p)(3+p)}+\frac{189de^2(a+bx^2)\,(-6a^3+6a^2b(1+p)x^2-3ab^2(2+3p+p^2)\,x^4+b^3(6+11p+6p^2+p^3)\,x^6)}{b^4(1+p)(2+p)(3+p)(4+p)}+54d^2ex^7\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)+14e^3x^9\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{9}{2},-p;\frac{11}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((63*d^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3
*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (189*d*e^2*(a + b*x^2)*(-6*
a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6
*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (54*d^2*e*x^7*Hyp
ergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (14*e^3*x^9
*Hypergeometric2F1[9/2, -p, 11/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/126
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^5(ex+d)^3\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)
*(b*x^2 + a)^p*d^3/((p^3 + 6*p^2 + 11*p + 6)*b^3) + integrate((x^8*e^3 + 3*
d*x^7*e^2 + 3*d^2*x^6*e)*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^8*e^3 + 3*d*x^7*e^2 + 3*d^2*x^6*e + d^3*x^5)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(218) = 436.
time = 34.49, size = 2919, normalized size = 11.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] 3*a**p*d**2*e*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + a
**p*e**3*x**9*hyper((9/2, -p), (11/2,), b*x**2*exp_polar(I*pi)/a)/9 + d**3*
Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 +
 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**
5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b
**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4
*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b*
*2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2
*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4),
 Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2
*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x
**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2
*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**
4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt
(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b
*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x
**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*
b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**
3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2
 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b
**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3
+ 12*b**3*p**2 + 22*b**3*p + 12*b**3), True)) + 3*d*e**2*Piecewise((a**p*x*
*8/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x
**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x + sqrt(-a/b))/(12*a**3*
b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**
3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**
2*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 +
12*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*
b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 +
 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x
- sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*
x**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**
2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2
*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x - sqrt(-a/b
```

```
))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b
**3*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*
x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(x - sqrt(-a/b))/(4*a**2*b**4
 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(x + sqrt(-a/b))/(4*a**2*b**4 +
 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**
6*x**4) - 12*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 +
 4*b**6*x**4) - 12*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*
x**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*
x**4) - 6*a*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*
b**6*x**4) - 6*a*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2
 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4),
Eq(p, -3)), (6*a**3*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*l
og(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**
2) + 6*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*
x**2*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4
 + 4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*lo
g(x - sqrt(-a/b))/(2*b**4) - a**3*log(x + sqrt(-a/b))/(2*b**4) + a**2*x**2/
(2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)*
*p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a
**3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 1
00*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 +
 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(
a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48
*b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*
b**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b
**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p
*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4
*p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3
+ 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(
2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4
*p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(b\,x^2+a\right)}^p\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^p*(d + e*x)^3,x)
[Out] int(x^5*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.400 $\int x^4(d+ex)^3 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=249

$$\frac{a^2e(3bd^2-ae^2)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{3de^2x^5(a+bx^2)^{1+p}}{b(7+2p)}-\frac{3ae(2bd^2-ae^2)\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}+\frac{3e(bd^2-ae^2)\left(a+bx^2\right)^{3}}{2b^4(3+p)}$$

```
[Out] 1/2*a^2*e*(-a*e^2+3*b*d^2)*(b*x^2+a)^(1+p)/b^4/(1+p)+3*d*e^2*x^5*(b*x^2+a)^
(1+p)/b/(7+2*p)-3/2*a*e*(-a*e^2+2*b*d^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+3/2*e*(-
a*e^2+b*d^2)*(b*x^2+a)^(3+p)/b^4/(3+p)+1/2*e^3*(b*x^2+a)^(4+p)/b^4/(4+p)-1/
5*d*(15*a*e^2-b*d^2*(7+2*p))*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x
^2/a)/b/(7+2*p)/((1+b*x^2/a)^p)
```

Rubi [A]

time = 0.16, antiderivative size = 241, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1666, 470, 372, 371, 457, 78}

$$\frac{a^2e(3bd^2-ae^2)\left(a+bx^2\right)^{p+1}}{2b^4(p+1)}-\frac{3ae(2bd^2-ae^2)\left(a+bx^2\right)^{p+2}}{2b^4(p+2)}+\frac{3e(bd^2-ae^2)\left(a+bx^2\right)^{p+3}}{2b^4(p+3)}+\frac{e^3\left(a+bx^2\right)^{p+4}}{2b^4(p+4)}+\frac{1}{5}dx^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{15ae^2}{2bp+7b}\right){}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+\frac{3de^2x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (a^2*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*d*e^2*x^
5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) - (3*a*e*(2*b*d^2 - a*e^2)*(a + b*x^2)
^(2 + p))/(2*b^4*(2 + p)) + (3*e*(b*d^2 - a*e^2)*(a + b*x^2)^(3 + p))/(2*b^
4*(3 + p)) + (e^3*(a + b*x^2)^(4 + p))/(2*b^4*(4 + p)) + (d*(d^2 - (15*a*e^
2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^
2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^4(d+ex)^3\left(a+bx^2\right)^p\,dx &= \int x^4\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)dx+\int x^5\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)dx\\ &= \frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^p\left(3d^2e+e^3x\right)dx,x,x^2\right)+\Big(d\\ &= \frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2e\left(-3bd^2+ae^2\right)(a+bx)^p}{b^3}+\frac{3ae(-2\right.\right.\\ &= \frac{a^2e\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{3de^2x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}-\frac{3ae\left(2bd^2-ae^2\right)(a+b}{2b^4(2+p)}\end{aligned}$$

Mathematica [A]

time = 0.35, size = 249, normalized size = 1.00

$$\frac{1}{70}(a+bx^2)^p\left(\frac{105d^2e(a+bx^2)\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{b^3(1+p)(2+p)(3+p)}+\frac{35e^3(a+bx^2)\left(-6a^3+6a^2b(1+p)x^2-3ab^2(2+3p+p^2)\,x^4+b^3(6+11p+6p^2+p^3)\,x^6\right)}{b^4(1+p)(2+p)(3+p)(4+p)}+14d^3x^5\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+30de^2x^7\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((105*d^2*e*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2
+ 3*p + p^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (35*e^3*(a + b*x^2)*(-6*
a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6
*p^2 + p^3)*x^6))/(b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)) + (14*d^3*x^5*Hyper
geometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (30*d*e^2*x^7
*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4(ex+d)^3\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^7*e^3 + 3*d*x^6*e^2 + 3*d^2*x^5*e + d^3*x^4)*(b*x^2 + a)^p, x)
```

<u>**Sympy**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. $2(216) = 432$.

time = 23.60, size = 2919, normalized size = 11.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*a**
p*d*e**2*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + 3*d**2
*e*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*
b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 +
4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2
+ 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2
*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
+ 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**
4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a
**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**
4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x
**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*
b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + s
qrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a
+ b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*
p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) +
 a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p +
 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*
b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p
**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 1
2*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p*
*3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True)) + e**3*Piecewise((a**p*x**
8/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x*
*2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x + sqrt(-a/b))/(12*a**3*b
**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3
*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2
*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 1
2*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b
**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 +
36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x -
 sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x
**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2
 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/(12*a**3*b**4 + 36*a**2*
b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x - sqrt(-a/b)
```

```
)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b*
*3*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x
**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(x - sqrt(-a/b))/(4*a**2*b**4
+ 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(x + sqrt(-a/b))/(4*a**2*b**4 +
8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6
*x**4) - 12*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 +
4*b**6*x**4) - 12*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x
**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x
**4) - 6*a*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b
**6*x**4) - 6*a*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2
+ 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4), E
q(p, -3)), (6*a**3*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*lo
g(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2
) + 6*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x
**2*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4
+ 4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)), (-a**3*log
(x - sqrt(-a/b))/(2*b**4) - a**3*log(x + sqrt(-a/b))/(2*b**4) + a**2*x**2/(
2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**
p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a*
*3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 10
0*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 +
20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a
 + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*
b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b
**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b*
*4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*
x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*
p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 +
 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2
*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*
p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p...
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(b\,x^2+a\right)}^p\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x^4*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.401 $\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx$

Optimal. Leaf size=207

$$-\frac{ad(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{d(bd^2-6ae^2)\left(a+bx^2\right)^{2+p}}{2b^3(2+p)}+\frac{3de^2\left(a+bx^2\right)^{3+p}}{2b^3(3+p)}-\frac{e(5ae^2-$$

```
[Out] -1/2*a*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^3/(1+p)+e^3*x^5*(b*x^2+a)^(1+p)
/b/(7+2*p)+1/2*d*(-6*a*e^2+b*d^2)*(b*x^2+a)^(2+p)/b^3/(2+p)+3/2*d*e^2*(b*x^
2+a)^(3+p)/b^3/(3+p)-1/5*e*(5*a*e^2-3*b*d^2*(7+2*p))*x^5*(b*x^2+a)^p*hyperg
eom([5/2, -p],[7/2],-b*x^2/a)/b/(7+2*p)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.13, antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1666, 457, 78, 470, 372, 371}

$$-\frac{ad(bd^2-3ae^2)\left(a+bx^2\right)^{p+1}}{2b^3(p+1)}+\frac{d(bd^2-6ae^2)\left(a+bx^2\right)^{p+2}}{2b^3(p+2)}+\frac{3de^2\left(a+bx^2\right)^{p+3}}{2b^3(p+3)}+\frac{1}{5}ex^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{5ae^2}{2bp+7b}\right){}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+\frac{e^3x^5\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (e^3*x^5*(
a + b*x^2)^(1 + p))/(b*(7 + 2*p)) + (d*(b*d^2 - 6*a*e^2)*(a + b*x^2)^(2 + p
))/(2*b^3*(2 + p)) + (3*d*e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*(3*
d^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p,
 7/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\int x^3(d+ex)^3\left(a+bx^2\right)^p\,dx=\int x^3\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^4\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx$$

$$=\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right)+\left(e\left(3d\right.\right.$$

$$=\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{ad\left(-bd^2+3ae^2\right)(a+bx)^p}{b^2}+\frac{\left(bd^3-6ade^2\right.}{b}\right.\right.$$

$$=-\frac{ad\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{e^3x^5\left(a+bx^2\right)^{1+p}}{b(7+2p)}+\frac{d\left(bd^2-6ae^2\right)\left(a+bx^2\right)}{2b^3(2+p)}$$

Mathematica [A]
time = 0.29, size = 196, normalized size = 0.95

$$\frac{1}{70}\left(a+bx^2\right)^p\left(\frac{35d^3\left(a+bx^2\right)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)}+\frac{105de^2\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+42d^2ex^5\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+10e^3x^7\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((35*d^3*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2 +
 p)) + (105*d*e^2*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p
^2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (42*d^2*e*x^5*Hypergeometric2F1[5
/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (10*e^3*x^7*Hypergeometric2
F1[7/2, -p, 9/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/70
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{3}(ex+d)^{3}\left(b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^3*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*d^3/((p^2 + 3*p + 2)*
b^2) + integrate((x^6*e^3 + 3*d*x^5*e^2 + 3*d^2*x^4*e)*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^6*e^3 + 3*d*x^5*e^2 + 3*d^2*x^4*e + d^3*x^3)*(b*x^2 + a)^p, x)
```

Sympy [A]
time = 19.07, size = 1329, normalized size = 6.42

$$\frac{3a^{p}d^{2}ex^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{5}{2},-p\\\frac{7}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{5}+\frac{a^{p}e^{3}x^{7}{}_{2}F_{1}\left(\begin{matrix}\frac{7}{2},-p\\\frac{9}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{7}+d^{3}\left(\begin{cases}\frac{a^{p}x^{4}}{4} & \text{for}\: b=0\\\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}} & \text{for}\: p=-2\\-\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^{2}}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^{2}}+\frac{x^{2}}{2b} & \text{for}\: p=-1\\-\frac{a^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{abpx^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}px^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}x^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}} & \text{otherwise}\end{cases}\right)+3de^{2}\left(\begin{cases}\frac{a^{p}x^{6}}{6} & \text{for}\: b=0\\\frac{2a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{3a^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2b^{2}x^{4}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2b^{2}x^{4}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}} & \text{for}\: p=-3\\-\frac{2a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2a^{2}}{2ab^{3}+2b^{4}x^{2}}-\frac{2abx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2abx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}+\frac{b^{2}x^{4}}{2ab^{3}+2b^{4}x^{2}} & \text{for}\: p=-2\\\frac{a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^{3}}+\frac{a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^{3}}-\frac{ax^{2}}{2b^{2}}+\frac{x^{4}}{4b} & \text{for}\: p=-1\\\frac{2a^{3}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}-\frac{2a^{2}bpx^{2}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{ab^{2}p^{2}x^{4}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{ab^{2}px^{4}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{b^{3}p^{2}x^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{3b^{3}px^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{2b^{3}x^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x+d)**3*(b*x**2+a)**p,x)

[Out] 3*a**p*d**2*e*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*e**3*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7 + d**3*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True)) + 3*d*e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p*x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(b\,x^2+a\right)^p\,(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^p*(d + e*x)^3,x)
[Out] int(x^3*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.402 $\int x^2(d+ex)^3\,(a+bx^2)^p\;dx$

Optimal. Leaf size=210

$$-\frac{ae(3bd^2-ae^2)\,(a+bx^2)^{1+p}}{2b^3(1+p)}+\frac{3de^2x^3(a+bx^2)^{1+p}}{b(5+2p)}+\frac{e(3bd^2-2ae^2)\,(a+bx^2)^{2+p}}{2b^3(2+p)}+\frac{e^3(a+bx^2)^{3+p}}{2b^3(3+p)}-\frac{d(9ae^2-$$

```
[Out] -1/2*a*e*(-a*e^2+3*b*d^2)*(b*x^2+a)^(1+p)/b^3/(1+p)+3*d*e^2*x^3*(b*x^2+a)^(
1+p)/b/(5+2*p)+1/2*e*(-2*a*e^2+3*b*d^2)*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*e^3*(
b*x^2+a)^(3+p)/b^3/(3+p)-1/3*d*(9*a*e^2-b*d^2*(5+2*p))*x^3*(b*x^2+a)^p*hype
rgeom([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.13, antiderivative size = 202, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {1666, 470, 372, 371, 457, 78}

$$-\frac{ae(3bd^2-ae^2)\,(a+bx^2)^{p+1}}{2b^3(p+1)}+\frac{e(3bd^2-2ae^2)\,(a+bx^2)^{p+2}}{2b^3(p+2)}+\frac{e^3(a+bx^2)^{p+3}}{2b^3(p+3)}+\frac{1}{3}dx^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{9ae^2}{2bp+5b}\right){}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+\frac{3de^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] -1/2*(a*e*(3*b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) + (3*d*e^2*x
^3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (e*(3*b*d^2 - 2*a*e^2)*(a + b*x^2)^
(2 + p))/(2*b^3*(2 + p)) + (e^3*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*(
d^2 - (9*a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p,
 5/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^p)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^2(d+ex)^3\left(a+bx^2\right)^p\,dx&=\int x^2\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^3\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\&=\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int x(a+bx)^p\left(3d^2e+e^3x\right)\,dx,x,x^2\right)+\Bigg(d\\&=\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{ae\left(-3bd^2+ae^2\right)(a+bx)^p}{b^2}+\frac{(3bd^2e-\right.\right.\\&=-\frac{ae\left(3bd^2-ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^3(1+p)}+\frac{3de^2x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{e\left(3bd^2-2ae^2\right)(a+}{2b^3(2+p)}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 196, normalized size = 0.93

$$\frac{1}{30}\left(a+bx^2\right)^p\left(\frac{45d^2e\left(a+bx^2\right)\left(-a+b(1+p)x^2\right)}{b^2(1+p)(2+p)}+\frac{15e^3\left(a+bx^2\right)\left(2a^2-2ab(1+p)x^2+b^2\left(2+3p+p^2\right)x^4\right)}{b^3(1+p)(2+p)(3+p)}+10d^3x^3\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+18de^2x^5\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*((45*d^2*e*(a + b*x^2)*(-a + b*(1 + p)*x^2))/(b^2*(1 + p)*(2
 + p)) + (15*e^3*(a + b*x^2)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^
2)*x^4))/(b^3*(1 + p)*(2 + p)*(3 + p)) + (10*d^3*x^3*Hypergeometric2F1[3/2,
 -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p + (18*d*e^2*x^5*Hypergeometric2F
1[5/2, -p, 7/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/30
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{2}(ex+d)^{3}\left(b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x^2*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^5*e^3 + 3*d*x^4*e^2 + 3*d^2*x^3*e + d^3*x^2)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(180) = 360.
time = 12.57, size = 1329, normalized size = 6.33

$$\frac{a^{p}d^{3}x^{3}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},-p\\\frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3}+\frac{3a^{p}de^{2}x^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{5}{2},-p\\\frac{7}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{5}+3d^{2}e\left(\begin{cases}\frac{a^{p}x^{4}}{4} & \text{for } b=0\\\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{a}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}}+\frac{bx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{2}+2b^{3}x^{2}} & \text{for } p=-2\\-\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^{2}}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^{2}}+\frac{x^{2}}{2b} & \text{for } p=-1\\-\frac{a^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{abpx^{2}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}px^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}}+\frac{b^{2}x^{4}\left(a+bx^{2}\right)^{p}}{2b^{2}p^{2}+6b^{2}p+4b^{2}} & \text{otherwise}\end{cases}\right)+e^{3}\left(\begin{cases}\frac{a^{p}x^{6}}{6} & \text{for } b=0\\\frac{2a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{3a^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{4abx^{2}}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2b^{2}x^{4}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}}+\frac{2b^{2}x^{4}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^{2}b^{3}+8ab^{4}x^{2}+4b^{5}x^{4}} & \text{for } p=-3\\-\frac{2a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2a^{2}}{2ab^{3}+2b^{4}x^{2}}-\frac{2abx^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}-\frac{2abx^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^{3}+2b^{4}x^{2}}+\frac{b^{2}x^{4}}{2ab^{3}+2b^{4}x^{2}} & \text{for } p=-2\\\frac{a^{2}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^{3}}+\frac{a^{2}\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^{3}}-\frac{ax^{2}}{2b^{2}}+\frac{x^{4}}{4b} & \text{for } p=-1\\\frac{2a^{3}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}-\frac{2a^{2}bpx^{2}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{ab^{2}p^{2}x^{4}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{ab^{2}px^{4}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{b^{3}p^{2}x^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{3b^{3}px^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}}+\frac{2b^{3}x^{6}\left(a+bx^{2}\right)^{p}}{2b^{3}p^{3}+12b^{3}p^{2}+22b^{3}p+12b^{3}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + 3*a**
p*d*e**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + 3*d**2
*e*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*
b**3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 +
 2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**
2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqr
t(-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1))
, (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a
+ b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**
p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**
2 + 6*b**2*p + 4*b**2), True)) + e**3*Piecewise((a**p*x**6/6, Eq(b, 0)), (2
*a**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a
**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**
2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/
b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-
a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b
**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**
2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a
/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b*
*4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))
/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b*
*4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sq
rt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) +
x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2
 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12
*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**
3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**
p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a +
b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x*
*6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b
**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(b x^2+a\right)^p (d+e x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^p*(d + e*x)^3,x)
[Out] int(x^2*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.403 $\int x(d+ex)^3 \left(a+bx^2\right)^p \, dx$

Optimal. Leaf size=167

$$\frac{d(bd^2-3ae^2)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{3de^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}-\frac{e(ae^2-bd^2(5+2p))\,x^3\left(a+bx^2\right)^p\left(1+\right.}{b(5+2p)}$$

```
[Out] 1/2*d*(-3*a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^2/(1+p)+e^3*x^3*(b*x^2+a)^(1+p)/b/
(5+2*p)+3/2*d*e^2*(b*x^2+a)^(2+p)/b^2/(2+p)-e*(a*e^2-b*d^2*(5+2*p))*x^3*(b*
x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/b/(5+2*p)/((1+b*x^2/a)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 159, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1666, 455, 45, 470, 372, 371}

$$\frac{d(bd^2-3ae^2)\left(a+bx^2\right)^{p+1}}{2b^2(p+1)}+\frac{3de^2\left(a+bx^2\right)^{p+2}}{2b^2(p+2)}+ex^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{ae^2}{2bp+5b}\right){}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+\frac{e^3x^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^3*x^3*(a + b
*x^2)^(1 + p))/(b*(5 + 2*p)) + (3*d*e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)
) + (e*(d^2 - (a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/
2, -p, 5/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int x(d+ex)^3\left(a+bx^2\right)^p\,dx &= \int x\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)\,dx+\int x^2\left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx\\
&= \frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int (a+bx)^p\left(d^3+3de^2x\right)\,dx,x,x^2\right)+\left(3e\left(d^2 -\right.\right.\\
&= \frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{1}{2}\text{Subst}\left(\int\left(\frac{\left(bd^3-3ade^2\right)(a+bx)^p}{b}+\frac{3de^2(a+bx)^{1+p}}{b}\right.\right.\\
&= \frac{d\left(bd^2-3ae^2\right)\left(a+bx^2\right)^{1+p}}{2b^2(1+p)}+\frac{e^3x^3\left(a+bx^2\right)^{1+p}}{b(5+2p)}+\frac{3de^2\left(a+bx^2\right)^{2+p}}{2b^2(2+p)}+e\left(d^2 -\right.
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 228, normalized size = 1.37

$$\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(5d\left(b^2x^2\left(1+\frac{bx^2}{a}\right)^p\left(d^2(2+p)+3e^2(1+p)x^2\right)-3a^2e^2\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+ab\left(3e^2px^2\left(1+\frac{bx^2}{a}\right)^p+d^2(2+p)\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)\right)\right)+10b^2d^2e\left(2+3p+p^2\right)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+2b^2e^3\left(2+3p+p^2\right)x^5\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)}{10b^2(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(5*d*(b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + 3*e^2*(1 + p)
*x^2) - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(3*e^2*p*x^2*(1 + (b*x^2)/
a)^p + d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p))) + 10*b^2*d^2*e*(2 + 3*p + p^2
)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 2*b^2*e^3*(2 + 3*p +
p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)]))/(10*b^2*(1 + p)*(2
 + p)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x(ex+d)^3\left(b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int(x*(e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)^(p + 1)*d^3/(b*(p + 1)) + integrate((x^4*e^3 + 3*d*x^3*e^2
+ 3*d^2*x^2*e)*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4*e^3 + 3*d*x^3*e^2 + 3*d^2*x^2*e + d^3*x)*(b*x^2 + a)^p, x)
```

Sympy [A]
time = 10.25, size = 440, normalized size = 2.63

$$a^p d^2 e x^3 {}_2F_1\left(\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^3 x^5 {}_2F_1\left(\begin{matrix}\frac{5}{2}, -p \\ \frac{7}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5} + d^3\left(\begin{cases}\frac{a^p x^2}{2} & \text{for}\: b = 0 \\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log\left(a + bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right) + 3de^2\left(\begin{cases}\frac{a^p x^4}{4} & \text{for}\: b = 0 \\ \frac{a\log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for}\: p = -2 \\ -\frac{a\log\left(x - \sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a\log\left(x + \sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for}\: p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*e*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + a**p*
e**3*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + d**3*Piece
wise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne
(p, -1)), (log(a + b*x**2), True))/(2*b), True)) + 3*d*e**2*Piecewise((a**p
*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*log
(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*
x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-a/
b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**2)
- a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x*
*2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6
*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b
**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(b\,x^2+a\right)}^p\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int(x*(a + b*x^2)^p*(d + e*x)^3, x)
```

3.404 $\int (d+ex)^3 (a+bx^2)^p \, dx$

Optimal. Leaf size=176

$$-\frac{e(ae^2-3bd^2(2+p))(a+bx^2)^{1+p}}{2b^2(1+p)(2+p)}+\frac{3de^2x(a+bx^2)^{1+p}}{b(3+2p)}+\frac{e^3x^2(a+bx^2)^{1+p}}{2b(2+p)}-\frac{d(3ae^2-bd^2(3+2p))\,x(a+b}{b}$$

```
[Out] -1/2*e*(a*e^2-3*b*d^2*(2+p))*(b*x^2+a)^(1+p)/b^2/(1+p)/(2+p)+3*d*e^2*x*(b*x
^2+a)^(1+p)/b/(3+2*p)+1/2*e^3*x^2*(b*x^2+a)^(1+p)/b/(2+p)-d*(3*a*e^2-b*d^2*
(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/(3+2*p)/((1+b*
x^2/a)^p)
```

Rubi [A]

time = 0.10, antiderivative size = 169, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {757, 794, 252, 251}

$$-\frac{e(a+bx^2)^{p+1}\left((2p+3)\left(ae^2-bd^2(2p+5)\right)-2bde(p+1)(p+3)x\right)}{2b^2(p+2)\left(2p^2+5p+3\right)}+dx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(d^2-\frac{3ae^2}{2bp+3b}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{e(d+ex)^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] (e*(d + e*x)^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) - (e*((3 + 2*p)*(a*e^2 -
b*d^2*(5 + 2*p)) - 2*b*d*e*(1 + p)*(3 + p)*x)*(a + b*x^2)^(1 + p))/(2*b^2*(
2 + p)*(3 + 5*p + 2*p^2)) + (d*(d^2 - (3*a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2
)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
```

```
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx^2\right)^p \, dx = \frac{e(d+ex)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} + \frac{\int (d+ex) \left(-2(ae^2-bd^2(2+p))+2bde(3+p)x\right) (a}{2b(2+p)}$$
$$= \frac{e(d+ex)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e((3+2p) \left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x}{2b^2(2+p) \left(3+5p+2p^2\right)}$$
$$= \frac{e(d+ex)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e((3+2p) \left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x}{2b^2(2+p) \left(3+5p+2p^2\right)}$$
$$= \frac{e(d+ex)^2 \left(a+bx^2\right)^{1+p}}{2b(2+p)} - \frac{e((3+2p) \left(ae^2-bd^2(5+2p)\right)-2bde(1+p)(3+p)x}{2b^2(2+p) \left(3+5p+2p^2\right)}$$

Mathematica [A]
time = 0.28, size = 223, normalized size = 1.27

$$\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(2b^2d^3(2+3p+p^2)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+e\left(b^2x^2\left(1+\frac{bx^2}{a}\right)^p(3d^2(2+p)+e^2(1+p)x^2)-a^2e^2\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+ab\left(e^2px^2\left(1+\frac{bx^2}{a}\right)^p+3d^2(2+p)\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)\right)+2b^2de(2+3p+p^2)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)\right)}{2b^2(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x^2)^p,x]
```

```
[Out] ((a + b*x^2)^p*(2*b^2*d^3*(2 + 3*p + p^2)*x*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*x^2)/a)] + e*(b^2*x^2*(1 + (b*x^2)/a)^p*(3*d^2*(2 + p) + e^2*(1 + p)*
x^2) - a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + a*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p
+ 3*d^2*(2 + p)*(-1 + (1 + (b*x^2)/a)^p)) + 2*b^2*d*e*(2 + 3*p + p^2)*x^3*H
ypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(2*b^2*(1 + p)*(2 + p)*(1 +
 (b*x^2)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex+d\right)^{3} \left(b\,x^{2}+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(b*x^2 + a)^p, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(148) = 296$.
time = 6.71, size = 437, normalized size = 2.48

$$a^p d^3 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{b x^2 e^{i\pi}}{a}\right) + a^p d e^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{b x^2 e^{i\pi}}{a}\right) + 3d^2 e \left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\: b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log(a+bx^2) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right) + e^3 \left(\begin{cases} \frac{a^p x^4}{4} & \text{for}\: b = 0 \\ \frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} & \text{for}\: p = -2 \\ -\frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for}\: p = -1 \\ -\frac{a^2 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{abpx^2 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2px^4 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} + \frac{b^2x^4 (a+bx^2)^p}{2b^2p^2 + 6b^2p + 4b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*e**
2*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*d**2*e*Piecew
ise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(
p, -1)), (log(a + b*x**2), True))/(2*b), True)) + e**3*Piecewise((a**p*x**4
/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*log(x +
sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*
log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-a/b))/(
2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**2) - a*l
og(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**
```

```
p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p*
*2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2
*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2),
 True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^2+a\right)^p\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p*(d + e*x)^3,x)
[Out] int((a + b*x^2)^p*(d + e*x)^3, x)
```

3.405 $\int \frac{(d+ex)^3(a+bx^2)^p}{x}\,dx$

Optimal. Leaf size=171

$$\frac{3de^2(a+bx^2)^{1+p}}{2b(1+p)}+\frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)}-\frac{e(ae^2-3bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(3+2p)}-$$

```
[Out] 3/2*d*e^2*(b*x^2+a)^(1+p)/b/(1+p)+e^3*x*(b*x^2+a)^(1+p)/b/(3+2*p)-e*(a*e^2-
3*b*d^2*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/(3+2*p
)/((1+b*x^2/a)^p)-1/2*d^3*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/
a)/a/(1+p)
```

Rubi [A]

time = 0.09, antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1666, 457, 81, 67, 396, 252, 251}

$$-\frac{d^3(a+bx^2)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3d^2-\frac{ae^2}{2bp+3b}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{3de^2(a+bx^2)^{p+1}}{2b(p+1)}+\frac{e^3x(a+bx^2)^{p+1}}{b(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

```
[Out] (3*d*e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e^3*x*(a + b*x^2)^(1 + p))/(
b*(3 + 2*p)) + (e*(3*d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p - (d^3*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2] && IGtQ[m, -2] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x}\,dx = \int \frac{\left(a+bx^2\right)^p\left(d^3+3de^2x^2\right)}{x}\,dx + \int \left(a+bx^2\right)^p\left(3d^2e+e^3x^2\right)\,dx$$
$$= \frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)} + \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p\left(d^3+3de^2x\right)}{x}\,dx, x, x^2\right) + \left(e\left(3d^2 - \right.\right.$$
$$= \frac{3de^2(a+bx^2)^{1+p}}{2b(1+p)} + \frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)} + \frac{1}{2}d^3\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right) + \left(e\right.$$
$$= \frac{3de^2(a+bx^2)^{1+p}}{2b(1+p)} + \frac{e^3x(a+bx^2)^{1+p}}{b(3+2p)} + e\left(3d^2 - \frac{ae^2}{3b+2bp}\right)x\left(a+bx^2\right)^p\left(1+\right.$$

Mathematica [A]
time = 0.26, size = 170, normalized size = 0.99

$$\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(18abd^2e(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)-3bd^3(a+bx^2)\left(1+\frac{bx^2}{a}\right)^p{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)+ae^2\left(9d(a+bx^2)\left(1+\frac{bx^2}{a}\right)^p+2be(1+p)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)\right)}{6ab(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x,x]
```

```
[Out] ((a + b*x^2)^p*(18*a*b*d^2*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b
*x^2)/a)] - 3*b*d^3*(a + b*x^2)*(1 + (b*x^2)/a)^p*Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*x^2)/a] + a*e^2*(9*d*(a + b*x^2)*(1 + (b*x^2)/a)^p + 2*b*e
*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])))/(6*a*b*(1 + p
)*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3\,(b\,x^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(b*x^2 + a)^p/x, x)
```

Sympy [A]
time = 7.37, size = 144, normalized size = 0.84

$$3a^{p}d^{2}ex\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)+\frac{a^{p}e^{3}x^{3}\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3}-\frac{b^{p}d^{3}x^{2p}\Gamma(-p)\,{}_{2}F_{1}\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{2\Gamma(1-p)}+3de^{2}\left(\begin{cases}\frac{a^{p}x^{2}}{2} & \text{for}\ b=0\\ \frac{\begin{cases}\frac{(a+bx^{2})^{p+1}}{p+1} & \text{for}\ p\neq-1\\ \log(a+bx^{2}) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x,x)
```

```
[Out] 3*a**p*d**2*e*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*e
**3*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 - b**p*d**3*x
**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*
gamma(1 - p)) + 3*d*e**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a
+ b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), Tru
e))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d+e\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x, x)
```

3.406 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^2}\,dx$

Optimal. Leaf size=159

$$\frac{e^3(a+bx^2)^{1+p}}{2b(1+p)}-\frac{d^3(a+bx^2)^{1+p}}{ax}+\frac{d(3ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{3d^2\cdots}{}$$

```
[Out] 1/2*e^3*(b*x^2+a)^(1+p)/b/(1+p)-d^3*(b*x^2+a)^(1+p)/a/x+d*(3*a*e^2+b*d^2*(1
+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^p)-
3/2*d^2*e*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/(1+p)
```

Rubi [A]
time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1821, 1666, 457, 81, 67, 12, 252, 251}

$$-\frac{d^3(a+bx^2)^{p+1}}{ax}+\frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2+bd^2(2p+1))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{3d^2e(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a(p+1)}+\frac{e^3(a+bx^2)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

```
[Out] (e^3*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) - (d^3*(a + b*x^2)^(1 + p))/(a*x) +
 (d*(3*a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (3*d^2*e*(a + b*x^2)^(1 + p)*Hy
pergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*(1 + p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1666

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Module[{q = E
xpon[Pq, x], k}, Int[x^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2)^p, x] + Int[x^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2)^p, x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2] && IGtQ[m, -2] &&  !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3\,(a+bx^2)^p}{x^2}\,dx &= -\frac{d^3(a+bx^2)^{1+p}}{ax} - \frac{\int \frac{(a+bx^2)^p(-3ad^2e-d(3ae^2+bd^2(1+2p))x-ae^3x^2)}{x}\,dx}{a} \\
&= -\frac{d^3(a+bx^2)^{1+p}}{ax} + \frac{\int d(3ae^2+bd^2(1+2p))\,(a+bx^2)^p\,dx}{a} - \frac{\int \frac{(a+bx^2)^p(-3ad^2e-}{x}}{a} \\
&= -\frac{d^3(a+bx^2)^{1+p}}{ax} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(-3ad^2e-ae^3x)}{x}\,dx,x,x^2\right)}{2a} + \frac{(d(3ae^2+bd^2(1}{} \\
&= \frac{e^3(a+bx^2)^{1+p}}{2b(1+p)} - \frac{d^3(a+bx^2)^{1+p}}{ax} + \frac{1}{2}(3d^2e)\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,x^2\right) + \\
&= \frac{e^3(a+bx^2)^{1+p}}{2b(1+p)} - \frac{d^3(a+bx^2)^{1+p}}{ax} + \frac{d(3ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx}{a}\right.}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 154, normalized size = 0.97

$$
\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(-2abd^3(1+p)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)+ex\left(6abde(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+(a+bx^2)\left(1+\frac{bx^2}{a}\right)^p\left(ae^2-3bd^2\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)\right)\right)\right)}{2ab(1+p)x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^2,x]
```

```
[Out] ((a + b*x^2)^p*(-2*a*b*d^3*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^
2)/a)] + e*x*(6*a*b*d*e*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)
/a)] + (a + b*x^2)*(1 + (b*x^2)/a)^p*(a*e^2 - 3*b*d^2*Hypergeometric2F1[1,
1 + p, 2 + p, 1 + (b*x^2)/a]))))/(2*a*b*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{(ex+d)^3\,(b\,x^2+a)^p}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(b*x^2 + a)^p/x^2, x)
```

Sympy [A]
time = 5.99, size = 143, normalized size = 0.90

$$-\frac{a^p d^3 {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} + 3a^p de^2 x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{3b^p d^2 e x^{2p}\Gamma(-p)\, {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)} + e^3 \left(\begin{cases} \frac{a^p x^2}{2} & \text{for}\: b = 0 \\ \frac{\begin{cases} \frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log(a+bx^2) & \text{otherwise} \end{cases}}{2b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x**2,x)
```

```
[Out] -a**p*d**3*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + 3*a**p*d
*e**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - 3*b**p*d**2*e*
x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**2))/(2
*gamma(1 - p)) + e**3*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b
*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^p\,\left(d+e\,x\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x^2,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x^2, x)
```

3.407 $\int \frac{(d+ex)^3(a+bx^2)^p}{x^3}\,dx$

Optimal. Leaf size=168

$$-\frac{d^3(a+bx^2)^{1+p}}{2ax^2}-\frac{3d^2e(a+bx^2)^{1+p}}{ax}+\frac{e(ae^2+3bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-$$

```
[Out] -1/2*d^3*(b*x^2+a)^(1+p)/a/x^2-3*d^2*e*(b*x^2+a)^(1+p)/a/x+e*(a*e^2+3*b*d^2
*(1+2*p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/a/((1+b*x^2/a)^
p)-1/2*d*(b*d^2*p+3*a*e^2)*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2
/a)/a^2/(1+p)
```

Rubi [A]
time = 0.15, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1821, 778, 272, 67, 252, 251}

$$-\frac{d(a+bx^2)^{p+1}(3ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2(p+1)}-\frac{d^3(a+bx^2)^{p+1}}{2ax^2}+\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+3bd^2(2p+1))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{a}-\frac{3d^2e(a+bx^2)^{p+1}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -1/2*(d^3*(a + b*x^2)^(1 + p))/(a*x^2) - (3*d^2*e*(a + b*x^2)^(1 + p))/(a*x
) + (e*(a*e^2 + 3*b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -
p, 3/2, -((b*x^2)/a)])/(a*(1 + (b*x^2)/a)^p) - (d*(3*a*e^2 + b*d^2*p)*(a +
b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*(1
 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 778

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] && !IntegerQ[2*p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx^2\right)^p}{x^3}\,dx = -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{\int \frac{\left(a+bx^2\right)^p\left(-6ad^2e-2d\left(3ae^2+bd^2p\right)x-2ae^3x^2\right)}{x^2}\,dx}{2a}$$

$$= -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax} + \frac{\int \frac{\left(2ad\left(3ae^2+bd^2p\right)+2ae\left(ae^2+3bd^2(1+2p)\right)x\right)\left(a+bx\right.}{x}}{2a^2}$$

$$= -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax} + \frac{\left(d\left(3ae^2+bd^2p\right)\right)\int \frac{\left(a+bx^2\right)^p}{x}\,dx}{a} + \frac{(e(ae^2}{}$$

$$= -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax} + \frac{\left(d\left(3ae^2+bd^2p\right)\right)\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x,\right.}{2a}$$

$$= -\frac{d^3\left(a+bx^2\right)^{1+p}}{2ax^2} - \frac{3d^2e\left(a+bx^2\right)^{1+p}}{ax} + \frac{e\left(ae^2+3bd^2(1+2p)\right)x\left(a+bx^2\right)^p\left(1+\right.}{a}$$

Mathematica [A]
time = 0.28, size = 174, normalized size = 1.04

$$-\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(6a^2d^2e(1+p)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)+x\left(-2a^2e^3(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+d\left(a+bx^2\right)\left(1+\frac{bx^2}{a}\right)^p\left(3ae^2\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)-bd^2\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^2}{a}\right)\right)\right)\right)}{2a^2(1+p)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x^2)^p)/x^3,x]
```

```
[Out] -1/2*((a + b*x^2)^p*(6*a^2*d^2*e*(1 + p)*Hypergeometric2F1[-1/2, -p, 1/2, -
((b*x^2)/a)] + x*(-2*a^2*e^3*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b
*x^2)/a)] + d*(a + b*x^2)*(1 + (b*x^2)/a)^p*(3*a*e^2*Hypergeometric2F1[1, 1
 + p, 2 + p, 1 + (b*x^2)/a] - b*d^2*Hypergeometric2F1[2, 1 + p, 2 + p, 1 +
(b*x^2)/a]))))/(a^2*(1 + p)*x*(1 + (b*x^2)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3 \left(b\,x^2+a\right)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(b*x^2+a)^p/x^3,x)
```

```
[Out] int((e*x+d)^3*(b*x^2+a)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(b*x^2 + a)^p/x^3, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.80, size = 150, normalized size = 0.89

$$-\frac{3a^p d^2 e\, {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{x} + a^p e^3 x\, {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{b^p d^3 x^{2p} \Gamma(1-p)\, {}_2F_1\left(\left.\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\right| \frac{a e^{i\pi}}{bx^2}\right)}{2x^2 \Gamma(2-p)} - \frac{3b^p d e^2 x^{2p} \Gamma(-p)\, {}_2F_1\left(\left.\begin{matrix} -p, -p \\ 1-p \end{matrix}\right| \frac{a e^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(b*x**2+a)**p/x**3,x)
```

```
[Out] -3*a**p*d**2*e*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x + a**p
*e**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) - b**p*d**3*x**(
2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I*pi)/(b*x**2))/
(2*x**2*gamma(2 - p)) - 3*b**p*d*e**2*x**(2*p)*gamma(-p)*hyper((-p, -p), (1
 - p,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1 - p))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(b*x^2+a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(b*x^2 + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p\,(d+e\,x)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^p*(d + e*x)^3)/x^3,x)
```

```
[Out] int(((a + b*x^2)^p*(d + e*x)^3)/x^3, x)
```

3.408 $\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=199

$$\frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2e^3(1+p)}+\frac{(a+bx^2)^{2+p}}{2b^2e(2+p)}+\frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}-\frac{d^4(a+bx^2)}{2}$$

[Out] 1/2*(-a*e^2+b*d^2)*(b*x^2+a)^(1+p)/b^2/e^3/(1+p)+1/2*(b*x^2+a)^(2+p)/b^2/e/(2+p)+1/5*x^5*(b*x^2+a)^p*AppellF1(5/2,1,-p,7/2,e^2*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p)-1/2*d^4*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2)/(1+p)

Rubi [A]

time = 0.15, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {973, 525, 524, 457, 90, 70}

$$\frac{x^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d}+\frac{(bd^2-ae^2)(a+bx^2)^{p+1}}{2b^2e^3(p+1)}+\frac{(a+bx^2)^{p+2}}{2b^2e(p+2)}-\frac{d^4(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^3(p+1)(ae^2+bd^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x),x]

[Out] ((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*e^3*(1 + p)) + (a + b*x^2)^(2 + p)/(2*b^2*e*(2 + p)) + (x^5*(a + b*x^2)^p*AppellF1[5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(5*d*(1 + (b*x^2)/a)^p) - (d^4*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^3*(b*d^2 + a*e^2)*(1 + p))

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx = d\int \frac{x^4(a+bx^2)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^5(a+bx^2)^p}{d^2-e^2x^2}\,dx$$

$$= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{x^2(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^4\left(1+\right.}{d^2-e}$$

$$= \frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d} - \frac{1}{2}e\text{Subst}\left(\int\left(\frac{(-bd^2+ae^2)(}{be^4}\right.\right.$$

$$= \frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2e^3(1+p)} + \frac{(a+bx^2)^{2+p}}{2b^2e(2+p)} + \frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\right.}{5d}$$

$$= \frac{(bd^2-ae^2)(a+bx^2)^{1+p}}{2b^2e^3(1+p)} + \frac{(a+bx^2)^{2+p}}{2b^2e(2+p)} + \frac{x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\right.}{5d}$$

Mathematica [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx^2)^p}{d+ex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x),x]
[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x), x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4(b\,x^2+a)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d),x)
[Out] int(x^4*(b*x^2+a)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4/(x*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4/(x*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^p}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x^4*(a + b*x^2)^p)/(d + e*x), x)
```

3.409 $\int \frac{x^3(a+bx^2)^p}{d+ex} dx$

Optimal. Leaf size=163

$$-\frac{d(a+bx^2)^{1+p}}{2be^2(1+p)}-\frac{ex^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2}+\frac{d^3(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p\right.}{2e^2\left(bd^2+ae^2\right)(1+p)}$$

```
[Out] -1/2*d*(b*x^2+a)^(1+p)/b/e^2/(1+p)-1/5*e*x^5*(b*x^2+a)^p*AppellF1(5/2,1,-p,
7/2,e^2*x^2/d^2,-b*x^2/a)/d^2/((1+b*x^2/a)^p)+1/2*d^3*(b*x^2+a)^(1+p)*hyper
geom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)/(1+p)
```

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {973, 457, 81, 70, 525, 524}

$$-\frac{ex^5(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2}+\frac{d^3(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)\left(ae^2+bd^2\right)}-\frac{d(a+bx^2)^{p+1}}{2be^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] -1/2*(d*(a + b*x^2)^(1 + p))/(b*e^2*(1 + p)) - (e*x^5*(a + b*x^2)^p*AppellF
1[5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(5*d^2*(1 + (b*x^2)/a)^p)
+ (d^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x
^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^2)*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx^2)^p}{d+ex}\,dx &= d\int \frac{x^3(a+bx^2)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^4(a+bx^2)^p}{d^2-e^2x^2}\,dx\\ &= \frac{1}{2}d\text{Subst}\left(\int \frac{x(a+bx)^p}{d^2-e^2x}\,dx,x,x^2\right) - \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^4\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\ &= -\frac{d(a+bx^2)^{1+p}}{2be^2(1+p)} - \frac{ex^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3\text{Subst}\left(\int\right.}{}\\ &= -\frac{d(a+bx^2)^{1+p}}{2be^2(1+p)} - \frac{ex^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{5}{2};-p,1;\frac{7}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3(a+bx^2)}{}\end{aligned}$$

Mathematica [A]

time = 0.34, size = 260, normalized size = 1.60

$$\frac{(a+bx^2)^p\left(-\frac{3d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{p}+\frac{e\left(1+\frac{bx^2}{a}\right)^{-p}\left(6bd^2(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+e\left(-3d\left(bx^2\left(1+\frac{bx^2}{a}\right)^p+a\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)\right)+2be(1+p)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)\right)}{b(1+p)}\right)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*((-3*d^3*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(
d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) + (e*(6*b*d^2*(1 + p)*x*H
ypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + e*(-3*d*(b*x^2*(1 + (b*x^2)/
a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + 2*b*e*(1 + p)*x^3*Hypergeometric2F1[3/
2, -p, 5/2, -((b*x^2)/a)])))/(b*(1 + p)*(1 + (b*x^2)/a)^p)))/(6*e^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^3 (b\,x^2+a)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x^3*(b*x^2+a)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^3/(x*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^3/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(b\,x^2+a\right)}^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x), x)
```

3.410 $\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=161

$$\frac{(a+bx^2)^{1+p}}{2be(1+p)}+\frac{x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2(\cdots}{bd^2\cdots}\right.}{2e\left(bd^2+ae^2\right)(1+p)}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/e/(1+p)+1/3*x^3*(b*x^2+a)^p*AppellF1(3/2,1,-p,5/2,e^2
*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p)-1/2*d^2*(b*x^2+a)^(1+p)*hypergeom([1,
1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2)/(1+p)
```

Rubi [A]
time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {973, 525, 524, 457, 81, 70}

$$\frac{x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d}-\frac{d^2(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)(ae^2+bd^2)}+\frac{(a+bx^2)^{p+1}}{2be(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e*(1 + p)) + (x^3*(a + b*x^2)^p*AppellF1[3/2, -p,
1, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d*(1 + (b*x^2)/a)^p) - (d^2*(a + b
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
 a*e^2)])/(2*e*(b*d^2 + a*e^2)*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx^2)^p}{d+ex}\,dx &= d\int \frac{x^2(a+bx^2)^p}{d^2-e^2x^2}\,dx - e\int \frac{x^3(a+bx^2)^p}{d^2-e^2x^2}\,dx \\
&= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{x(a+bx)^p}{d^2-e^2x}\,dx, x, x^2\right)\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{x^2\left(1+\frac{bx^2}{a}\right.}{d^2-e^2x} \\
&= \frac{(a+bx^2)^{1+p}}{2be(1+p)} + \frac{x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d} - \frac{d^2\text{Subst}\left(\int \frac{(a+b}{d^2-e}\right.}{2e} \\
&= \frac{(a+bx^2)^{1+p}}{2be(1+p)} + \frac{x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{3}{2};-p,1;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d} - \frac{d^2(a+bx^2)^{1+p}}{2e(}
\end{aligned}$$

Mathematica [A]

time = 0.27, size = 227, normalized size = 1.41

$$\frac{(a+bx^2)^p\left(ae^2p+be^2px^2-ae^2p\left(1+\frac{bx^2}{a}\right)^{-p}+bd^2(1+p)\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)-2bdep(1+p)x\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2be^3p(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*(a*e^2*p + b*e^2*p*x^2 - (a*e^2*p)/(1 + (b*x^2)/a)^p + (b*d^
2*(1 + p)*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (
d + Sqrt[-(a/b)]*e)/(d + e*x)])/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*
(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*b*d*e*p*(1 + p)*x*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*b*e^3*p*(1 + p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2(b\,x^2+a)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x^2*(b*x^2+a)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2/(x*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(b\,x^2+a\right)}^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x),x)
```

```
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x), x)
```

3.411 $\int \frac{x(a+bx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=173

$$-\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e}+\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e}+\frac{d(a-}{}$$

```
[Out] -x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e/((1+b*x^2/a)^p
)+x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/((1+b*x^2/a)^p)+1/2*d
*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e
^2+b*d^2)/(1+p)
```

Rubi [A]

time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {858, 252, 251, 771, 441, 440, 455, 70}

$$-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e}+\frac{d(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] -((x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(e*(1 + (b*x^2)/a)^p)) + (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -
((b*x^2)/a)])/(e*(1 + (b*x^2)/a)^p) + (d*(a + b*x^2)^(1 + p)*Hypergeometric
2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)
*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(a+bx^2)^p}{d+ex}\,dx &= \frac{\int (a+bx^2)^p\,dx}{e} - \frac{d\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e} \\
&= -\frac{d\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e} + \frac{\left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx}{e} \\
&= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} - d\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx - \frac{d^2\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{e} \\
&= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e} - \frac{1}{2}d\,\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx, x, x^2\right) - \\
&= -\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e} + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\right.}{e}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 172, normalized size = 0.99

$$
\frac{(a+bx^2)^p\left(-\frac{d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} + 2ex\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2e^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*(-((d*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(
d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d +
 e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*e*x*Hypergeometric2F1[
1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$
\int \frac{x(b\,x^2+a)^p}{ex+d}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d),x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx^2)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*x**2)**p/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(b\,x^2+a\right)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^p)/(d + e*x),x)
[Out] int((x*(a + b*x^2)^p)/(d + e*x), x)
```

3.412 $\int \frac{(a+bx^2)^p}{d+ex} \, dx$

Optimal. Leaf size=125

$$\frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{1+p} {}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2(bd^2+ae^2)(1+p)}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d/((1+b*x^2/a)^p)
-1/2*e*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2)
)/(a*e^2+b*d^2)/(1+p)
```

Rubi [A]

time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {771, 441, 440, 455, 70}

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(d + e*x),x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d
*(1 + (b*x^2)/a)^p) - (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^p}{d+ex}\,dx &= \int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx\\ &= d\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx+e\int \frac{x(a+bx^2)^p}{-d^2+e^2x^2}\,dx\\ &= \frac{1}{2}e\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx,x,x^2\right)+\left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx\\ &= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d}-\frac{e(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p\right.}{2\left(bd^2+ae^2\right)(1+p)}\end{aligned}$$

Mathematica [A]

time = 0.15, size = 131, normalized size = 1.05

$$\frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{2ep}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x),x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*
x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*p*((e*(-Sqrt[-(a/b)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d),x)
[Out] int((b*x^2+a)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^p}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d),x)
[Out] Integral((a + b*x**2)**p/(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(d + e*x),x)
[Out] int((a + b*x^2)^p/(d + e*x), x)
```

3.413 $\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx$

Optimal. Leaf size=176

$$-\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d\left(bd^2+ae^2\right)(1+p)}-\frac{(a+}{}$$

```
[Out] -e*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^2/((1+b*x^2/
a)^p)+1/2*e^2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2
+b*d^2))/d/(a*e^2+b*d^2)/(1+p)-1/2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p]
,1+b*x^2/a)/a/d/(1+p)
```

Rubi [A]

time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {973, 457, 88, 67, 70, 441, 440}

$$-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2(a+bx^2)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d(p+1)\left(ae^2+bd^2\right)}-\frac{(a+bx^2)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^2*(1 + (b*x^2)/a)^p)) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1,
1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d*(b*d^2 + a*e^2)*(1 +
 p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/
a])/(2*a*d*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !I
ntegerQ[p] && !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x(d+ex)}\,dx = d\int \frac{(a+bx^2)^p}{x\,(d^2-e^2x^2)}\,dx - e\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{x\,(d^2-e^2x)}\,dx, x, x^2\right) - \left(e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{d^2-e^2x^2}\,dx$$

$$= -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d} + \frac{e^2}{}$$

$$= -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+\right.}{2d\,(bd^2+ae^2)\,(1+p}$$

Mathematica [A]
time = 0.25, size = 170, normalized size = 0.97

$$\frac{(a+bx^2)^p\left(-\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)+\left(1+\frac{a}{bx^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)\right)}{2dp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*(-(AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))
^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p
, -(a/(b*x^2))]/(1 + a/(b*x^2))^p))/(2*d*p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/x/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((x*e + d)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(x^2*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^p}{x (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d),x)
```

```
[Out] Integral((a + b*x**2)**p/(x*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((x*e + d)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2 + a)^p}{x\,\left(d + e\,x\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x*(d + e*x)),x)
```

```
[Out] int((a + b*x^2)^p/(x*(d + e*x)), x)
```

3.414 $\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx$

Optimal. Leaf size=178

$$-\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx}-\frac{e^3(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;\frac{e^2(a+bx^2)}{bd^2+ae^2}\right)}{2d^2\left(bd^2+ae^2\right)(1+p)}+\frac{e(a+$$

```
[Out] -(b*x^2+a)^p*AppellF1(-1/2,1,-p,1/2,e^2*x^2/d^2,-b*x^2/a)/d/x/((1+b*x^2/a)^
p)-1/2*e^3*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*
d^2))/d^2/(a*e^2+b*d^2)/(1+p)+1/2*e*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p
],1+b*x^2/a)/a/d^2/(1+p)
```

Rubi [A]
time = 0.11, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {973, 525, 524, 457, 88, 67, 70}

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx}-\frac{e^3(a+bx^2)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)\left(ae^2+bd^2\right)}+\frac{e(a+bx^2)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2ad^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] -(((a + b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(
d*x*(1 + (b*x^2)/a)^p)) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 +
 p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 +
p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)
/a])/(2*a*d^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] && !IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !I
ntegerQ[p] && !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)}\,dx = d\int \frac{(a+bx^2)^p}{x^2\left(d^2-e^2x^2\right)}\,dx - e\int \frac{(a+bx^2)^p}{x\left(d^2-e^2x^2\right)}\,dx$$

$$= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{(a+bx)^p}{x\left(d^2-e^2x\right)}\,dx, x, x^2\right)\right) + \left(d(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)}{x^2\left(d^2-e^2\right.}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^2} - \frac{e}{}$$

$$= -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{dx} - \frac{e^3(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+\right.}{2d^2\left(bd^2+ae^2\right)(1+}$$

Mathematica [**A**]
time = 0.39, size = 214, normalized size = 1.20

$$\frac{(a+bx^2)^p\left(\frac{e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} - \frac{2d\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e\left(1+\frac{a}{bx^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)}{p}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*((e*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d
+ e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e
*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (2*d*Hypergeometric2F1[-1/2,
 -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-p, -
p, 1 - p, -(a/(b*x^2))])/(p*(1 + a/(b*x^2))^p)))/(2*d^2)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^2\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d),x)
```

```
[Out] int((b*x^2+a)^p/x^2/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((x*e + d)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(x^3*e + d*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((x*e + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2+a)^p}{x^2\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)),x)
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)), x)
```

3.415 $\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx$

Optimal. Leaf size=213

$$-\frac{(a+bx^2)^{1+p}}{2adx^2}+\frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x}+\frac{e^4(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{e^2}{b\ldots}\right.}{2d^3(bd^2+ae^2)(1+p)}$$

```
[Out] -1/2*(b*x^2+a)^(1+p)/a/d/x^2+e*(b*x^2+a)^p*AppellF1(-1/2,1,-p,1/2,e^2*x^2/d
^2,-b*x^2/a)/d^2/x/((1+b*x^2/a)^p)+1/2*e^4*(b*x^2+a)^(1+p)*hypergeom([1, 1+
p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(1+p)-1/2*(b*d^2*p+
a*e^2)*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a^2/d^3/(1+p)
```

Rubi [A]

time = 0.16, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {973, 457, 105, 162, 67, 70, 525, 524}

$$-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2a^2d^3(p+1)}+\frac{e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x}+\frac{e^4(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^3(p+1)(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(a + b*x^2)^(1 + p)/(a*d*x^2) + (e*(a + b*x^2)^p*AppellF1[-1/2, -p, 1,
 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^2*x*(1 + (b*x^2)/a)^p) + (e^4*(a + b
*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
 a*e^2)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a*e^2 + b*d^2*p)*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a^2*d^3*(1 +
p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^3(d+ex)}\,dx = d\int \frac{(a+bx^2)^p}{x^3\left(d^2-e^2x^2\right)}\,dx - e\int \frac{(a+bx^2)^p}{x^2\left(d^2-e^2x^2\right)}\,dx$$

$$= \frac{1}{2}d\text{Subst}\left(\int \frac{(a+bx)^p}{x^2\left(d^2-e^2x\right)}\,dx, x, x^2\right) - \left(e\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2\left(d^2-e^2x^2\right)}\,d$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} - \frac{\text{Subst}\left(\int \frac{(a+bx)}{}\right.}{}$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4\text{Subst}\left(\int \frac{(a+b}{d^2-}\right.}{2d}$$

$$= -\frac{(a+bx^2)^{1+p}}{2adx^2} + \frac{e(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(-\frac{1}{2};-p,1;\frac{1}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2x} + \frac{e^4(a+bx^2)^{1+p}}{2d^3}$$

Mathematica [A]
time = 0.34, size = 256, normalized size = 1.20

$$\frac{(a+bx^2)^p\left(-\frac{e^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{p}+\frac{2de\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}+\left(1+\frac{a}{bx^2}\right)^{-p}\left(\frac{d^2\,{}_2F_1\left(1-p,-p;2-p;-\frac{a}{bx^2}\right)}{(-1+p)x^2}+\frac{e^2\,{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)}{p}\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(x^3*(d + e*x)),x]
```

```
[Out] ((a + b*x^2)^p*(-((e^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)
/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d
 + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p)) + (2*d*e*Hypergeometric2F
1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^p) + ((d^2*Hypergeometri
c2F1[1 - p, -p, 2 - p, -(a/(b*x^2))])/((-1 + p)*x^2) + (e^2*Hypergeometric2
F1[-p, -p, 1 - p, -(a/(b*x^2))])/p)/(1 + a/(b*x^2))^p))/(2*d^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^3\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^3/(e*x+d),x)
[Out] int((b*x^2+a)^p/x^3/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((x*e + d)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(x^4*e + d*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**3/(e*x+d),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((x*e + d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^p}{x^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x^3*(d + e*x)),x)
[Out] int((a + b*x^2)^p/(x^3*(d + e*x)), x)
```

3.416 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=392

$$-\frac{d(4+3p)\left(a+bx^2\right)^{1+p}}{be^3(1+p)(3+2p)}-\frac{d^4\left(a+bx^2\right)^{1+p}}{e^3\left(bd^2+ae^2\right)(d+ex)}+\frac{(d+ex)\left(a+bx^2\right)^{1+p}}{be^3(3+2p)}-\frac{2d^2\left(2ae^2+bd^2(2+p)\right)x\left(a+bx^2\right)}{e^4\,(}$$

```
[Out] -d*(4+3*p)*(b*x^2+a)^(1+p)/b/e^3/(1+p)/(3+2*p)-d^4*(b*x^2+a)^(1+p)/e^3/(a*e
^2+b*d^2)/(e*x+d)+(e*x+d)*(b*x^2+a)^(1+p)/b/e^3/(3+2*p)-2*d^2*(2*a*e^2+b*d^
2*(2+p))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^4/(a*e
^2+b*d^2)/((1+b*x^2/a)^p)-(a^2*e^4-2*a*b*d^2*e^2*(4+3*p)-2*b^2*d^4*(2*p^2+7
*p+6))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b/e^4/(a*e^2+b*d^2
)/(3+2*p)/((1+b*x^2/a)^p)+d^3*(2*a*e^2+b*d^2*(2+p))*(b*x^2+a)^(1+p)*hyperge
om([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A]

time = 0.58, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1665, 1668, 858, 252, 251, 771, 441, 440, 455, 70}

$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2e^4-2abd^2e^2(3p+4)-2b^2d^4\left(2p^2+7p+6\right)\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{be^4(2p+3)\left(ae^2+bd^2\right)}-\frac{2d^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(p+2)\right)F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4\left(ae^2+bd^2\right)}-\frac{d^4\left(a+bx^2\right)^{p+1}}{e^3(d+ex)\left(ae^2+bd^2\right)}+\frac{d^3\left(a+bx^2\right)^{p+1}\left(2ae^2+bd^2(p+2)\right){}_2F_1\left(1,p+1;p+2;\frac{e^2\left(bx^2+a\right)}{bd^2+ae^2}\right)}{e^3(p+1)\left(ae^2+bd^2\right)^2}-\frac{d(3p+4)\left(a+bx^2\right)^{p+1}}{be^3(p+1)(2p+3)}+\frac{(d+ex)\left(a+bx^2\right)^{p+1}}{be^3(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d*(4 + 3*p)*(a + b*x^2)^(1 + p))/(b*e^3*(1 + p)*(3 + 2*p))) - (d^4*(a +
b*x^2)^(1 + p))/(e^3*(b*d^2 + a*e^2)*(d + e*x)) + ((d + e*x)*(a + b*x^2)^(1
 + p))/(b*e^3*(3 + 2*p)) - (2*d^2*(2*a*e^2 + b*d^2*(2 + p))*x*(a + b*x^2)^p
*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^
2)*(1 + (b*x^2)/a)^p) - ((a^2*e^4 - 2*a*b*d^2*e^2*(4 + 3*p) - 2*b^2*d^4*(6
+ 7*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b*e^4*(b*d^2 + a*e^2)*(3 + 2*p)*(1 + (b*x^2)/a)^p) + (d^3*(2*a*e^2 + b
*d^2*(2 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(
a + b*x^2))/(b*d^2 + a*e^2)])/(e^3*(b*d^2 + a*e^2)^2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx = -\frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} - \frac{\int \frac{(a+bx^2)^p\left(\frac{ad^3}{e^2}-\frac{d^2\left(ae^2+2bd^2(1+p)\right)x}{e^3}+d\left(a+\frac{bd^2}{e^2}\right)x^2-\frac{\left(bd^2+ae^2\right)x^3}{e}\right)}{d+ex}\,dx}{bd^2+ae^2}$$

$$= -\frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{\int \frac{(a+bx^2)^p\left(ade\left(ae^2+2bd^2(2+p)\right)+\left(a^2e^4-\cdots\right.\right.}{d+\cdots}}{be^3\left(bd^2+\cdots\right.}$$

$$= -\frac{d(4+3p)(a+bx^2)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{\int \frac{(2abde\cdots}{\cdots}}{\cdots}$$

$$= -\frac{d(4+3p)(a+bx^2)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{(2d^3(2a\cdots}{\cdots}$$

$$= -\frac{d(4+3p)(a+bx^2)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{(2d^3(2a\cdots}{\cdots}$$

$$= -\frac{d(4+3p)(a+bx^2)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{(a^2e^4-\cdots}{\cdots}$$

$$= -\frac{d(4+3p)(a+bx^2)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{(a^2e^4-\cdots}{\cdots}$$

$$= -\frac{d(4+3p)(a+bx^2)^{1+p}}{be^3(1+p)(3+2p)} - \frac{d^4(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)(d+ex)} + \frac{(d+ex)(a+bx^2)^{1+p}}{be^3(3+2p)} - \frac{2d^2(2a\cdots}{\cdots}$$

Mathematica [F]
time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2,x]
[Out] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^2, x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4(b\,x^2+a)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
[Out] int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^4/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^4/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^p}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^4*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.417 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=321

$$\frac{(a+bx^2)^{1+p}}{2be^2(1+p)}+\frac{d^3(a+bx^2)^{1+p}}{e^2\left(bd^2+ae^2\right)(d+ex)}+\frac{d(3ae^2+bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\right.}{e^3\left(bd^2+ae^2\right)}$$

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/e^2/(1+p)+d^3*(b*x^2+a)^(1+p)/e^2/(a*e^2+b*d^2)/(e*x+
d)+d*(3*a*e^2+b*d^2*(3+2*p))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^
2,-b*x^2/a)/e^3/(a*e^2+b*d^2)/((1+b*x^2/a)^p)-d*(2*a*e^2+b*d^2*(3+2*p))*x*(
b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^3/(a*e^2+b*d^2)/((1+b*x^2/
a)^p)-1/2*d^2*(3*a*e^2+b*d^2*(3+2*p))*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2
+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^2/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A]
time = 0.35, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1665, 1668, 858, 252, 251, 771, 441, 440, 455, 70}

$$\frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2+bd^2(2p+3))\,F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3\left(ae^2+bd^2\right)}-\frac{d^2(a+bx^2)^{p+1}\left(3ae^2+bd^2(2p+3)\right)\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)\left(ae^2+bd^2\right)^2}-\frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(2ae^2+bd^2(2p+3)\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^3\left(ae^2+bd^2\right)}+\frac{d^3(a+bx^2)^{p+1}}{e^2(d+ex)\left(ae^2+bd^2\right)}+\frac{(a+bx^2)^{p+1}}{2be^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^2*(1 + p)) + (d^3*(a + b*x^2)^(1 + p))/(e^2*(b*d
^2 + a*e^2)*(d + e*x)) + (d*(3*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*App
ellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2 + a*e^2)*(
1 + (b*x^2)/a)^p) - (d*(2*a*e^2 + b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^3*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)
^p) - (d^2*(3*a*e^2 + b*d^2*(3 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2 + a*e^
2)^2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^2}\,dx = \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{\int \frac{(a+bx^2)^p\left(-\frac{ad^2}{e}+d\left(a+\frac{2bd^2(1+p)}{e^2}\right)x-\frac{(bd^2+ae^2)x^2}{e}\right)}{d+ex}dx}{bd^2+ae^2}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{\int \frac{(-2abd^2e(1+p)+2bd(1+p)(2ae^2+bd^2(3+2p))x)(a+bx^2)^p}{d+ex}}{2be^2(bd^2+ae^2)(1+p)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{(d(2ae^2+bd^2(3+2p)))\int (a+bx^2)^p\,dx}{e^3(bd^2+ae^2)} + \frac{(d}{}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} + \frac{(d^2(3ae^2+bd^2(3+2p)))\int\left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+}{-d^2+}\right.}{e^3(bd^2+ae^2)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{d(2ae^2+bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-}}{e^3(bd^2+ae^2)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} - \frac{d(2ae^2+bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-}}{e^3(bd^2+ae^2)}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^2(1+p)} + \frac{d^3(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)(d+ex)} + \frac{d(3ae^2+bd^2(3+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-}}{e^3(bd^2+ae^2)}$$

Mathematica [A]
time = 0.62, size = 343, normalized size = 1.07

$$\frac{(a+bx^2)^p\left(-\frac{2d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{3d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{p}+e\left(\frac{e\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{-p}\right)}{b+bp}-4dx\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*((-2*d^3*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)
]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a
/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (
3*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d +
Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(S
qrt[-(a/b)] + x))/(d + e*x))^p) + e*((e*(a + b*x^2 - a/(1 + (b*x^2)/a)^p))/
(b + b*p) - (4*d*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x
^2)/a)^p)))/(2*e^4)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(b \,x^{2}+a \right)^{p}}{\left(e x +d \right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
[Out] int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^3/(x*e + d)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^3/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^2+a\right)}^p}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.418 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=281

$$-\frac{d^2(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)\,(d+ex)}-\frac{2(ae^2+bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2\,(bd^2+ae^2)}+\frac{(ae^2+2}{}$$

```
[Out] -d^2*(b*x^2+a)^(1+p)/e/(a*e^2+b*d^2)/(e*x+d)-2*(a*e^2+b*d^2*(1+p))*x*(b*x^2
+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^2/(a*e^2+b*d^2)/((1+b*x
^2/a)^p)+(a*e^2+2*b*d^2*(1+p))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x
^2/a)/e^2/(a*e^2+b*d^2)/((1+b*x^2/a)^p)+d*(a*e^2+b*d^2*(1+p))*(b*x^2+a)^(1+
p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2)^2/
(1+p)
```

Rubi [A]

time = 0.21, antiderivative size = 277, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1665, 858, 252, 251, 771, 441, 440, 455, 70}

$$-\frac{2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2(p+1))\,F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^2\,(ae^2+bd^2)}+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a+\frac{2bd^2(p+1)}{e^2}\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{ae^2+bd^2}+\frac{d(a+bx^2)^{p+1}\,(ae^2+bd^2(p+1))\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e(p+1)\,(ae^2+bd^2)^2}-\frac{d^2(a+bx^2)^{p+1}}{e(d+ex)\,(ae^2+bd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] -((d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x))) - (2*(a*e^2 + b*
d^2*(1 + p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e^2*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) + ((a + (2*b*d^2*(1 + p))
/e^2)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/((b*d^
2 + a*e^2)*(1 + (b*x^2)/a)^p) + (d*(a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 +
 p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/
(e*(b*d^2 + a*e^2)^2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^2}\,dx = -\frac{d^2(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)\,(d+ex)} - \frac{\int \frac{\left(ad-\frac{(ae^2+2bd^2(1+p))x}{e}\right)(a+bx^2)^p}{d+ex}\,dx}{bd^2+ae^2}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)\,(d+ex)} - \frac{(2d(ae^2+bd^2(1+p)))\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e^2\,(bd^2+ae^2)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)\int (a}{bd^2+ae}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)\,(d+ex)} - \frac{(2d(ae^2+bd^2(1+p)))\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2}+\frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e^2\,(bd^2+ae^2)} + \frac{((}{}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)\,(d+ex)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right.}{bd^2+ae^2}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)\,(d+ex)} + \frac{\left(a+\frac{2bd^2(1+p)}{e^2}\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right.}{bd^2+ae^2}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)\,(d+ex)} - \frac{2(ae^2+bd^2(1+p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2}\right.}{e^2\,(bd^2+ae^2)}$$

Mathematica [A]
time = 0.42, size = 300, normalized size = 1.07

$$\frac{(a+bx^2)^p\left(\frac{d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{(-1+2p)(d+ex)}-\frac{d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{p}+ex\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*((d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)
] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d*A
ppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(
a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a
/b)] + x))/(d + e*x))^p) + (e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(1 + (b*x^2)/a)^p))/e^3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(b\,x^{2}+a \right)^{p}}{\left(e x +d \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^2/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^2/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(b\,x^2+a\right)}^p}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x)^2,x)
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.419 $\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=273

$$\frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)}+\frac{(ae^2+bd^2(1+2p))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\,(bd^2+ae^2)}-\frac{bd(1+2p)x(\cdots}{\cdots}$$

```
[Out] d*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(e*x+d)+(a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p
*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d/e/(a*e^2+b*d^2)/((1+b*x^2/a)
^p)-b*d*(1+2*p)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/(a*e^2+
b*d^2)/((1+b*x^2/a)^p)-1/2*(a*e^2+b*d^2*(1+2*p))*(b*x^2+a)^(1+p)*hypergeom(
[1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A]
time = 0.15, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {849, 858, 252, 251, 771, 441, 440, 455, 70}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2(2p+1))\,F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de\,(ae^2+bd^2)}-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e\,(ae^2+bd^2)}-\frac{(a+bx^2)^{p+1}(ae^2+bd^2(2p+1))\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^2}+\frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2+bd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d + e*x)) + ((a*e^2 + b*d^2*(1 +
2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2
])/(d*e*(b*d^2 + a*e^2)*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*x*(a + b*x^2)^p
*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(b*d^2 + a*e^2)*(1 + (b*
x^2)/a)^p) - ((a*e^2 + b*d^2*(1 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2
F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^
2*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx &= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} - \frac{\int \frac{(-ae+bd(1+2p)x)(a+bx^2)^p}{d+ex}\,dx}{bd^2+ae^2} \\
&= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} - \frac{(bd(1+2p))\int (a+bx^2)^p\,dx}{e(bd^2+ae^2)} + \frac{(ae^2+bd^2(1+2p))\int \frac{(a+bx^2)^p}{d+ex}\,dx}{e(bd^2+ae^2)} \\
&= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} + \frac{(ae^2+bd^2(1+2p))\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{e(bd^2+ae^2)} - \frac{\left(bd(1+2p\right.}{} \\
&= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} - \frac{bd(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)} + \frac{(ae^2}{} \\
&= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} - \frac{bd(1+2p)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(bd^2+ae^2)} + \frac{(ae^2}{} \\
&= \frac{d(a+bx^2)^{1+p}}{(bd^2+ae^2)(d+ex)} + \frac{(ae^2+bd^2(1+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a}\right.}{de(bd^2+ae^2)}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 223, normalized size = 0.82

$$\frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(-2dpF_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)+(-1+2p)(d+ex)F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)\right)}{2e^2p(-1+2p)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*(-2*d*p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + (-1 + 2*p)*(d + e*x)*Appel
lF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)
]*e)/(d + e*x)]))/(2*e^2*p*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p
*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x(b\,x^2+a)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d)^2,x)
[Out] int(x*(b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx^2)^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d)**2,x)
[Out] Integral(x*(a + b*x**2)**p/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(b\,x^2+a\right)}^p}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] int((x*(a + b*x^2)^p)/(d + e*x)^2, x)
```

3.420 $\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx$

Optimal. Leaf size=244

$$\frac{e^2x(a+bx^2)^{1+p}}{(bd^2+ae^2)(d^2-e^2x^2)}-\frac{2bpx(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{bd^2+ae^2}+\frac{b(1+2p)x(a+bx^2)^p\left(1-\right.}{bd^2}$$

[Out] e^2*x*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)-2*b*p*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/(a*e^2+b*d^2)/((1+b*x^2/a)^p)+b*(1+2*p)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/(a*e^2+b*d^2)/((1+b*x^2/a)^p)-b*d*e*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(1+p)

Rubi [A]
time = 0.12, antiderivative size = 191, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {771, 441, 440, 455, 70, 525, 524}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2}+\frac{e^2x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^4}-\frac{bde(a+bx^2)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)(ae^2+bd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^p/(d + e*x)^2,x]

[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^2*(1 + (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^4*(1 + (b*x^2)/a)^p) - (b*d*e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/((b*d^2 + a*e^2)^2*(1 + p))

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx = \int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+bx^2)^p}{(d^2-e^2x^2)^2} + \frac{e^2x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\right) dx$$

$$= d^2 \int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx - (2de)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx + e^2 \int \frac{x^2(a+bx^2)^p}{(-d^2+e^2x^2)^2}\,dx$$

$$= -\left((de)\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^2}\,dx, x, x^2\right)\right) + \left(d^2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^2}{a}\right.}{(d^2-e^2}$$

$$= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{3}{2}\right.}{3d^4}$$

Mathematica [A]
time = 0.22, size = 141, normalized size = 0.58

$$\frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{e(-1+2p)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x)^2,x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(e*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int((b*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*x**2)**p/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^p}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(d + e*x)^2,x)
```

```
[Out] int((a + b*x^2)^p/(d + e*x)^2, x)
```

3.421 $\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}$$

```
[Out] -e*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b*x^2/
a)^p)-e*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b
*x^2/a)^p)-1/3*e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2
/a)/d^5/((1+b*x^2/a)^p)+1/2*e^2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^
2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)/(1+p)-1/2*(b*x^2+a)^(1+p)*hype
rgeom([1, 1+p],[2+p],1+b*x^2/a)/a/d^2/(1+p)+b*e^2*(b*x^2+a)^(1+p)*hypergeom
([2, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A]

time = 0.29, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {975, 272, 67, 771, 441, 440, 455, 70, 525, 524}

$$-\frac{e^3x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^5}-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}+\frac{e^2(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)\left(ae^2+bd^2\right)}+\frac{be^2(a+bx^2)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)\left(ae^2+bd^2\right)^2}-\frac{(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2ad^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] -((e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2]
)/(d^3*(1 + (b*x^2)/a)^p)) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -
((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)
^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^5*(1 + (b*x
^2)/a)^p) + (e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^
2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^2*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*
x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^2*(1
 + p)) + (b*e^2*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2
*(a + b*x^2))/(b*d^2 + a*e^2)])/((b*d^2 + a*e^2)^2*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x(d+ex)^2}\,dx = \int \left(\frac{(a+bx^2)^p}{d^2x} - \frac{e(a+bx^2)^p}{d(d+ex)^2} - \frac{e(a+bx^2)^p}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x}\,dx}{d^2} - \frac{e\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^2} - \frac{e\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^2} - \frac{e\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{d^2} - \frac{e\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+\cdots}{(d^2-e^2\cdots}\right.}{\cdots}$$

$$= -\frac{(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} - \frac{e\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{d} - (de)\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx\cdots$$

$$= -\frac{(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^2(1+p)} + e^2\text{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^2}\,dx, x, x^2\right) - \frac{e^2\text{S}\cdots}{\cdots}$$

$$= -\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} - \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\cdots\right.}{d^3}$$

Mathematica [A]

time = 0.39, size = 303, normalized size = 0.82

$$\frac{(a+bx^2)^p\left(-\frac{2d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{(-1+2p)(d+ex)} + \frac{-\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)+\left(1+\frac{a}{bx^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)}{p}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)^2),x]
```

```
[Out] ((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b
)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (-(
AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-
(a/b)]*e)/(d + e*x)]/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b
)] + x))/(d + e*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))]/(1
 + a/(b*x^2))^p)/p))/(2*d^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{x\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d)^2,x)
```

```
[Out] int((b*x^2+a)^p/x/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((x*e + d)^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(x^3*e^2 + 2*d*x^2*e + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{p}}{x\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*x**2)**p/(x*(d + e*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((x*e + d)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^p}{x\,(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x*(d + e*x)^2),x)
```

```
[Out] int((a + b*x^2)^p/(x*(d + e*x)^2), x)
```

3.422 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx$

Optimal. Leaf size=421

$$\frac{2e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right.}{d^4}$$

```
[Out] 2*e^2*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x
^2/a)^p)+e^2*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/
((1+b*x^2/a)^p)+1/3*e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-
b*x^2/a)/d^6/((1+b*x^2/a)^p)-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/
a)/d^2/x/((1+b*x^2/a)^p)-e^3*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(
b*x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(1+p)+e*(b*x^2+a)^(1+p)*hypergeom
([1, 1+p],[2+p],1+b*x^2/a)/a/d^3/(1+p)-b*e^3*(b*x^2+a)^(1+p)*hypergeom([2,
1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^2/(1+p)
```

Rubi [A]
time = 0.30, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {975, 372, 371, 272, 67, 771, 441, 440, 455, 70, 525, 524}

$$\frac{e^4x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}+\frac{2e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{e(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{ad^3(p+1)}-\frac{be^3(a+bx^2)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d(p+1)\left(ae^2+bd^2\right)^2}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x}-\frac{e^3(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^3(p+1)\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^
2])/(d^4*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2
, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x
^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^6*(1 + (
b*x^2)/a)^p) - ((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)
])/(d^2*x*(1 + (b*x^2)/a)^p) - (e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1
, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^3*(b*d^2 + a*e^2)*(1
 + p)) + (e*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x
^2)/a])/(a*d^3*(1 + p)) - (b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1
 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d*(b*d^2 + a*e^2)^2*(1 +
p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2}\,dx = \int \left(\frac{(a+bx^2)^p}{d^2x^2} - \frac{2e(a+bx^2)^p}{d^3x} + \frac{e^2(a+bx^2)^p}{d^2(d+ex)^2} + \frac{2e^2(a+bx^2)^p}{d^3(d+ex)} \right) dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x^2}\,dx}{d^2} - \frac{(2e)\int \frac{(a+bx^2)^p}{x}\,dx}{d^3} + \frac{(2e^2)\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^3} + \frac{e^2\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d^2}$$

$$= -\frac{e\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{d^3} + \frac{(2e^2)\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right) dx}{d^3} + \frac{e^2\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2}\right.}{}$$

$$= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x} + \frac{e(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1\right.}{ad^3(1+p)}$$

$$= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^2x} + \frac{e(a+bx^2)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1\right.}{ad^3(1+p)}$$

$$= \frac{2e^2x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} + \frac{e^2x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} F}{d^4}$$

Mathematica [A]
time = 0.53, size = 342, normalized size = 0.81

$$\frac{(a+bx^2)^p \left(\frac{de\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{(-1+2p)(d+ex)} + \frac{e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{p} - \frac{d\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x} - \frac{e\left(1+\frac{a}{bx^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)}{p} \right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^2),x]
```

```
[Out] ((a + b*x^2)^p*((d*e*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e
)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)
] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (e*A
ppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(
a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a
/b)] + x))/(d + e*x))^p) - (d*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)
])/(x*(1 + (b*x^2)/a)^p) - (e*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))
])/(p*(1 + a/(b*x^2))^p)))/d^3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^2\,(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
```

```
[Out] int((b*x^2+a)^p/x^2/(e*x+d)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((x*e + d)^2*x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(x^4*e^2 + 2*d*x^3*e + d^2*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((x*e + d)^2*x^2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^p}{x^2\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)^2),x)
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)^2), x)
```

3.423 $\int \frac{x^4(a+bx^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=449

$$\frac{(a+bx^2)^{1+p}}{2be^3(1+p)}-\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2}+\frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)}+\frac{d(6a^2e^4+3abd^2e^2(4+3p)+b$$

```
[Out] 1/2*(b*x^2+a)^(1+p)/b/e^3/(1+p)-1/2*d^4*(b*x^2+a)^(1+p)/e^3/(a*e^2+b*d^2)/(
e*x+d)^2+d^3*(4*a*e^2+b*d^2*(3+p))*(b*x^2+a)^(1+p)/e^3/(a*e^2+b*d^2)^2/(e*x
+d)+d*(6*a^2*e^4+3*a*b*d^2*e^2*(4+3*p)+b^2*d^4*(2*p^2+7*p+6))*x*(b*x^2+a)^p
*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e^4/(a*e^2+b*d^2)^2/((1+b*x^2/
a)^p)-d*(3*a^2*e^4+2*a*b*d^2*e^2*(5+4*p)+b^2*d^4*(2*p^2+7*p+6))*x*(b*x^2+a)
^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^4/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)-
1/2*d^2*(6*a^2*e^4+3*a*b*d^2*e^2*(4+3*p)+b^2*d^4*(2*p^2+7*p+6))*(b*x^2+a)^(
1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e^3/(a*e^2+b*d^2
)^3/(1+p)
```

Rubi [A]
time = 0.61, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1665, 1668, 858, 252, 251, 771, 441, 440, 455, 70}

$$\frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4(2p^2+7p+6))F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^4(ae^2+bd^2)^2}-\frac{dx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3a^2e^4+2abd^2e^2(4p+5)+b^2d^4(2p^2+7p+6))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^4(ae^2+bd^2)^2}-\frac{d^2(a+bx^2)^{p+1}(6a^2e^4+3abd^2e^2(3p+4)+b^2d^4(2p^2+7p+6))\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^3(p+1)(ae^2+bd^2)^3}-\frac{d^4(a+bx^2)^{p+1}}{2e^3(d+ex)^2(ae^2+bd^2)}+\frac{d^3(a+bx^2)^{p+1}(4ae^2+bd^2(p+3))}{e^3(d+ex)(ae^2+bd^2)^2}+\frac{(a+bx^2)^{p+1}}{2be^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (a + b*x^2)^(1 + p)/(2*b*e^3*(1 + p)) - (d^4*(a + b*x^2)^(1 + p))/(2*e^3*(b
*d^2 + a*e^2)*(d + e*x)^2) + (d^3*(4*a*e^2 + b*d^2*(3 + p))*(a + b*x^2)^(1
+ p))/(e^3*(b*d^2 + a*e^2)^2*(d + e*x)) + (d*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4
+ 3*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/
2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p)
- (d*(3*a^2*e^4 + 2*a*b*d^2*e^2*(5 + 4*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a
 + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^4*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - (d^2*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2
*d^4*(6 + 7*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 +
 p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^3*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{x^4(a+bx^2)^p}{(d+ex)^3}\,dx = -\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} - \frac{\int \frac{(a+bx^2)^p\left(\frac{2ad^3}{e^2}-\frac{2d^2(ae^2+bd^2(1+p))x}{e^3}+2d\left(a+\frac{bd^2}{e^2}\right)x^2-2\left(\frac{bd^2}{e}+ae\right)x^3\cdots}{(d+ex)^2}}{2(bd^2+ae^2)}$$

$$= -\frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} + \frac{\int \frac{(a+bx^2)^p\left(2ad^2\left(3a\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} + \frac{\int \cdots}{\cdots}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} + \frac{(d^2(\cdots}{\cdots}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} + \frac{(d^2(\cdots}{\cdots}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} - \frac{d(3\cdots}{\cdots}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} - \frac{d(3\cdots}{\cdots}$$

$$= \frac{(a+bx^2)^{1+p}}{2be^3(1+p)} - \frac{d^4(a+bx^2)^{1+p}}{2e^3(bd^2+ae^2)(d+ex)^2} + \frac{d^3(4ae^2+bd^2(3+p))(a+bx^2)^{1+p}}{e^3(bd^2+ae^2)^2(d+ex)} + \frac{d(6\cdots}{\cdots}$$

Mathematica [A]
time = 0.93, size = 462, normalized size = 1.03

$$\frac{(a+bx^2)^p\left(\frac{ae^2}{b+bp}+\frac{e^2x^2}{1+p}-\frac{8d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{d^4\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+p)(d+ex)^2}+\frac{6d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}-6dex\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((a*e^2)/(b + b*p) + (e^2*x^2)/(1 + p) - (8*d^3*AppellF1[1 -
 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)
/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(
a/b)] + x))/(d + e*x))^p*(d + e*x)) + (d^4*AppellF1[2 - 2*p, -p, -p, 3 - 2*
```

```
p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 +
p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))
^p*(d + e*x)^2) + (6*d^2*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/
(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) - (6*d*e*x*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p))/(2*e^5)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^4(b\,x^2+a)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^4/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^4/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^4/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^p}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^4*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.424 $\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=416

$$\frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2}-\frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)}-\frac{(3a^2e^4+abd^2e^2(6+7p)+b^2d^4(3+5p+2p^2)}{e^3(}$$

```
[Out] 1/2*d^3*(b*x^2+a)^(1+p)/e^2/(a*e^2+b*d^2)/(e*x+d)^2-d^2*(3*a*e^2+b*d^2*(2+p
))*(b*x^2+a)^(1+p)/e^2/(a*e^2+b*d^2)^2/(e*x+d)-(3*a^2*e^4+a*b*d^2*e^2*(6+7*
p)+b^2*d^4*(2*p^2+5*p+3))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-
b*x^2/a)/e^3/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+(a^2*e^4+a*b*d^2*e^2*(5+6*p)+b
^2*d^4*(2*p^2+5*p+3))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^3
/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+1/2*d*(3*a^2*e^4+a*b*d^2*e^2*(6+7*p)+b^2*d
^4*(2*p^2+5*p+3))*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a
*e^2+b*d^2))/e^2/(a*e^2+b*d^2)^3/(1+p)
```

Rubi [A]
time = 0.39, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1665, 858, 252, 251, 771, 441, 440, 455, 70}

$$-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3a^2e^4+abd^2e^2(7p+6)+b^2d^4(2p^2+5p+3))\,F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e^3(ae^2+bd^2)^2}+\frac{d(a+bx^2)^{p+1}(3a^2e^4+abd^2e^2(7p+6)+b^2d^4(2p^2+5p+3))\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)(ae^2+bd^2)^3}+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(a^2e^4+abd^2e^2(6p+5)+b^2d^4(2p^2+5p+3))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^3(ae^2+bd^2)^2}-\frac{d^2(a+bx^2)^{p+1}(3ae^2+bd^2(p+2))}{e^2(d+ex)(ae^2+bd^2)^2}+\frac{d^3(a+bx^2)^{p+1}}{2e^2(d+ex)^2(ae^2+bd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d^3*(a + b*x^2)^(1 + p))/(2*e^2*(b*d^2 + a*e^2)*(d + e*x)^2) - (d^2*(3*a*e
^2 + b*d^2*(2 + p))*(a + b*x^2)^(1 + p))/(e^2*(b*d^2 + a*e^2)^2*(d + e*x))
- ((3*a^2*e^4 + a*b*d^2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*x*(a + b
*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2
 + a*e^2)^2*(1 + (b*x^2)/a)^p) + ((a^2*e^4 + a*b*d^2*e^2*(5 + 6*p) + b^2*d^
4*(3 + 5*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x
^2)/a)])/(e^3*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (d*(3*a^2*e^4 + a*b*d^
2*e^2*(6 + 7*p) + b^2*d^4*(3 + 5*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeome
tric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^2*(b*d^2
+ a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^3(a+bx^2)^p}{(d+ex)^3}\,dx = \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{\int \frac{(a+bx^2)^p\left(-\frac{2ad^2}{e}+2d\left(a+\frac{bd^2(1+p)}{e^2}\right)x-2\left(\frac{bd^2}{e}+ae\right)x^2\right)}{(d+ex)^2}\,dx}{2(bd^2+ae^2)}$$

$$= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} + \frac{\int \left(-\frac{2ad\left(2ae^2+bd^2(1+p)\right)}{e}+\cdots\right.}{\cdots}$$

$$= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} + \frac{(a^2e^4+abd^2e^2(5+6\cdots}{\cdots}$$

$$= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} - \frac{(d(3a^2e^4+abd^2e^2(6\cdots}{\cdots}$$

$$= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} + \frac{(a^2e^4+abd^2e^2(5+6\cdots}{\cdots}$$

$$= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} + \frac{(a^2e^4+abd^2e^2(5+6\cdots}{\cdots}$$

$$= \frac{d^3(a+bx^2)^{1+p}}{2e^2(bd^2+ae^2)(d+ex)^2} - \frac{d^2(3ae^2+bd^2(2+p))(a+bx^2)^{1+p}}{e^2(bd^2+ae^2)^2(d+ex)} - \frac{(3a^2e^4+abd^2e^2(6+\cdots}{\cdots}$$

Mathematica [A]

time = 0.74, size = 436, normalized size = 1.05

$$\frac{(a+bx^2)^p\left(\frac{6d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}-\frac{d^3\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+p)(d+ex)^2}-\frac{3d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}+2ex\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((6*d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/
b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d
^3*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d +
Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p
*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) - (3*d*AppellF1[-2*p, -p
, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*
x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d +
e*x))^p) + (2*e*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p))/(2*e^4)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(b\,x^{2}+a \right)^{p}}{\left(e x +d \right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x^3/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x^3/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x^3/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^2+a\right)}^p}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x^3*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.425 $\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=396

$$-\frac{d^2(a+bx^2)^{1+p}}{2e(bd^2+ae^2)(d+ex)^2}+\frac{d(2ae^2+bd^2(1+p))(a+bx^2)^{1+p}}{e(bd^2+ae^2)^2(d+ex)}+\frac{(a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2}{de^2}$$

```
[Out] -1/2*d^2*(b*x^2+a)^(1+p)/e/(a*e^2+b*d^2)/(e*x+d)^2+d*(2*a*e^2+b*d^2*(1+p))*
(b*x^2+a)^(1+p)/e/(a*e^2+b*d^2)^2/(e*x+d)+(a^2*e^4+a*b*d^2*e^2*(2+5*p)+b^2*
d^4*(2*p^2+3*p+1))*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a
)/d/e^2/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)-b*d*(1+2*p)*(2*a*e^2+b*d^2*(1+p))*x
*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e^2/(a*e^2+b*d^2)^2/((1+b*
x^2/a)^p)-1/2*(a^2*e^4+a*b*d^2*e^2*(2+5*p)+b^2*d^4*(2*p^2+3*p+1))*(b*x^2+a)
^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/e/(a*e^2+b*d^2
)^3/(1+p)
```

Rubi [A]
time = 0.37, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1665, 849, 858, 252, 251, 771, 441, 440, 455, 70}

$$\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(a^2e^4+abd^2e^2(5p+2)+b^2d^4(2p^2+3p+1))\,F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{de^2(ae^2+bd^2)^2}-\frac{(a+bx^2)^{p+1}(a^2e^4+abd^2e^2(5p+2)+b^2d^4(2p^2+3p+1))\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)(ae^2+bd^2)^3}-\frac{bd(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(2ae^2+bd^2(p+1))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e^2(ae^2+bd^2)^2}+\frac{d(a+bx^2)^{p+1}(2ae^2+bd^2(p+1))}{e(d+ex)(ae^2+bd^2)^2}-\frac{d^2(a+bx^2)^{p+1}}{2e(d+ex)^2(ae^2+bd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] -1/2*(d^2*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)*(d + e*x)^2) + (d*(2*a*e^
2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p))/(e*(b*d^2 + a*e^2)^2*(d + e*x)) + (
(a^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1 + 3*p + 2*p^2))*x*(a + b*x^2)
^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d*e^2*(b*d^2 +
a*e^2)^2*(1 + (b*x^2)/a)^p) - (b*d*(1 + 2*p)*(2*a*e^2 + b*d^2*(1 + p))*x*(a
 + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^2*(b*d^2 + a*
e^2)^2*(1 + (b*x^2)/a)^p) - ((a^2*e^4 + a*b*d^2*e^2*(2 + 5*p) + b^2*d^4*(1
+ 3*p + 2*p^2))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2
*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
```

```
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)^p}{(d+ex)^3}\,dx = -\frac{d^2(a+bx^2)^{1+p}}{2e\,(bd^2+ae^2)\,(d+ex)^2} - \frac{\int \frac{\left(2ad-\frac{2\left(ae^2+bd^2(1+p)\right)x}{e}\right)(a+bx^2)^p}{(d+ex)^2}\,dx}{2\,(bd^2+ae^2)}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e\,(bd^2+ae^2)\,(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)^2\,(d+ex)} + \frac{\int \frac{\left(2a\left(ae^2+bd^2p\right)-\frac{2bd(1+2}{}\right.}{}}{2\,(bd}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e\,(bd^2+ae^2)\,(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)^2\,(d+ex)} - \frac{(bd(1+2p)\,(2ae^2+b}{e^2\,(b}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e\,(bd^2+ae^2)\,(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)^2\,(d+ex)} + \frac{(a^2e^4+abd^2e^2(2+5}{}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e\,(bd^2+ae^2)\,(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)^2\,(d+ex)} - \frac{bd(1+2p)\,(2ae^2+bd}{}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e\,(bd^2+ae^2)\,(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)^2\,(d+ex)} - \frac{bd(1+2p)\,(2ae^2+bd}{}$$

$$= -\frac{d^2(a+bx^2)^{1+p}}{2e\,(bd^2+ae^2)\,(d+ex)^2} + \frac{d(2ae^2+bd^2(1+p))\,(a+bx^2)^{1+p}}{e\,(bd^2+ae^2)^2\,(d+ex)} + \frac{(a^2e^4+abd^2e^2(2+5}{}$$

Mathematica [A]
time = 0.42, size = 290, normalized size = 0.73

$$\frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(-\frac{4dF_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{d^2F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{(-1+p)(d+ex)^2}+\frac{F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*((-4*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*(d + e*x)) + (d^
2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + S
qrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*(d + e*x)^2) + AppellF1[-2*p, -p, -p,
1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]/p)
)/(2*e^3*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d +
 e*x))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(b\,x^{2}+a \right)^{p}}{\left(e x +d \right)^{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)
[Out] int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*x^2/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*x^2/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b x^{2}\right)^{p}}{\left(d + e x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**p/(e*x+d)**3,x)
[Out] Integral(x**2*(a + b*x**2)**p/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*x^2/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(b\,x^2+a\right)^p}{\left(d+e\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int((x^2*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.426 $\int \frac{x(a+bx^2)^p}{(d+ex)^3} dx$

Optimal. Leaf size=336

$$\frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} - \frac{bp(3ae^2+bd^2(1+2p))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};\cdot\right.}{e(bd^2+ae^2)^2}$$

```
[Out] 1/2*d*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(e*x+d)^2-(b*d^2*p+a*e^2)*(b*x^2+a)^(1+
p)/(a*e^2+b*d^2)^2/(e*x+d)-b*p*(3*a*e^2+b*d^2*(1+2*p))*x*(b*x^2+a)^p*Appell
F1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/e/(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+b*(
1+2*p)*(b*d^2*p+a*e^2)*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/e/
(a*e^2+b*d^2)^2/((1+b*x^2/a)^p)+1/2*b*d*p*(3*a*e^2+b*d^2*(1+2*p))*(b*x^2+a)
^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^
3/(1+p)
```

Rubi [A]
time = 0.27, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {849, 858, 252, 251, 771, 441, 440, 455, 70}

$$-\frac{bpx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3ae^2+bd^2(2p+1))F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{e(ae^2+bd^2)^2}+\frac{b(2p+1)x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ae^2+bd^2p)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{e(ae^2+bd^2)^2}+\frac{bdp(a+bx^2)^{p+1}(3ae^2+bd^2(2p+1))\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^3}-\frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)(ae^2+bd^2)^2}+\frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (d*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d + e*x)^2) - ((a*e^2 + b*d^2*p
)*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)^2*(d + e*x)) - (b*p*(3*a*e^2 + b*d^
2*(1 + 2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x
^2)/d^2])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*(1 + 2*p)*(a*e^2 + b
*d^2*p)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(
b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*d*p*(3*a*e^2 + b*d^2*(1 + 2*p))*(a
 + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d
^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx = \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{\int \frac{(-2ae+2bdpx)(a+bx^2)^p}{(d+ex)^2}\,dx}{2(bd^2+ae^2)}$$

$$= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} + \frac{\int \frac{(2abde(1-p)+2b(1+2p)(ae^2+bd^2p)x)(a \cdots}{d+ex}}{2(bd^2+ae^2)^2}$$

$$= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} + \frac{(b(1+2p)(ae^2+bd^2p))\int (a+ \cdots}{e(bd^2+ae^2)^2}$$

$$= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} - \frac{(bdp(3ae^2+bd^2(1+2p)))\int \left(\frac{d(\cdots}{d \cdots}\right.}{e(bd^2+ae \cdots}$$

$$= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} + \frac{b(1+2p)(ae^2+bd^2p)x(a+bx \cdots}{e(bd \cdots}$$

$$= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} + \frac{b(1+2p)(ae^2+bd^2p)x(a+bx \cdots}{e(bd \cdots}$$

$$= \frac{d(a+bx^2)^{1+p}}{2(bd^2+ae^2)(d+ex)^2} - \frac{(ae^2+bd^2p)(a+bx^2)^{1+p}}{(bd^2+ae^2)^2(d+ex)} - \frac{bp(3ae^2+bd^2(1+2p))x(a+bx \cdots}{e \cdots}$$

Mathematica [A]
time = 0.31, size = 229, normalized size = 0.68

$$\frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^p\left(2(-1+p)(d+ex)F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)+d(1-2p)F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)\right)}{2e^2(-1+p)(-1+2p)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*(2*(-1 + p)*(d + e*x)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d
 - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)] + d*(1 - 2*p)*
AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqr
t[-(a/b)]*e)/(d + e*x)]))/(2*e^2*(-1 + p)*(-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x
))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x(b\,x^2+a)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int(x*(b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*x/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*x/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx^2)^p}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Integral(x*(a + b*x**2)**p/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p*x/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(b\,x^2+a\right)}^p}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^p)/(d + e*x)^3,x)
[Out] int((x*(a + b*x^2)^p)/(d + e*x)^3, x)
```

3.427 $\int \frac{(a+bx^2)^p}{(d+ex)^3} \, dx$

Optimal. Leaf size=322

$$-\frac{d^2e(a+bx^2)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2}+\frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}+\frac{e^2x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)}{}$$

```
[Out] -1/4*d^2*e*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2+x*(b*x^2+a)^p*App
ellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^3/((1+b*x^2/a)^p)+e^2*x^3*(b*x^2
+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+1/4*b
*e*(2*a*e^2+b*d^2*(1+p))*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^2*(b*x^
2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(1+p)-3/2*b^2*d^2*e*(b*x^2+a)^(1+p)*hyp
ergeom([3, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(1+p)
```

Rubi [A]
time = 0.23, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {771, 441, 440, 455, 70, 525, 524, 457, 79}

$$\frac{e^2x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3}-\frac{3b^2d^2e(a+bx^2)^{p+1}\,{}_2F_1\left(3,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)^3}+\frac{be(a+bx^2)^{p+1}\left(2ae^2+bd^2(p+1)\right)\,{}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4(p+1)\left(ae^2+bd^2\right)^3}-\frac{d^2e(a+bx^2)^{p+1}}{4\left(d^2-e^2x^2\right)^2\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(d + e*x)^3,x]
```

```
[Out] -1/4*(d^2*e*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (x*(
a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^3*(
1 + (b*x^2)/a)^p) + (e^2*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((b*x
^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (b*e*(2*a*e^2 + b*d^2*(1
+ p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^
2))/(b*d^2 + a*e^2)])/(4*(b*d^2 + a*e^2)^3*(1 + p)) - (3*b^2*d^2*e*(a + b*x
^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a
*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx &= \int \left(\frac{d^3(a+bx^2)^p}{(d^2-e^2x^2)^3} - \frac{3d^2ex(a+bx^2)^p}{(d^2-e^2x^2)^3} + \frac{3de^2x^2(a+bx^2)^p}{(d^2-e^2x^2)^3} + \frac{e^3x^3(a+bx^2)^p}{(-d^2+e^2x^2)^3}\right)dx \\
&= d^3\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx - \left(3d^2e\right)\int \frac{x(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx + \left(3de^2\right)\int \frac{x^2(a+bx^2)^p}{(d^2-e^2x^2)^3}\,dx + e^3\int \frac{x^3}{(-\cdots} \\
&= -\left(\frac{1}{2}\left(3d^2e\right)\operatorname{Subst}\left(\int \frac{(a+bx)^p}{(d^2-e^2x)^3}\,dx, x, x^2\right)\right) + \frac{1}{2}e^3\operatorname{Subst}\left(\int \frac{x(a+bx)^p}{(-d^2+e^2x)^3}\,dx, x, x^2\right) \cdots \\
&= -\frac{d^2e(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2x^3\cdots}{\cdots} \\
&= -\frac{d^2e(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3} + \frac{e^2x^3\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 142, normalized size = 0.44

$$\frac{\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}(a+bx^2)^pF_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{2e(-1+p)(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(d + e*x)^3,x]
```

```
[Out] ((a + b*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d +
 e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(2*e*(-1 + p)*((e*(-Sqrt[-(a/b)] +
x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/(e*x+d)^3,x)
```

```
[Out] int((b*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*x**2)**p/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^p}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(d + e*x)^3,x)
[Out] int((a + b*x^2)^p/(d + e*x)^3, x)
```

3.428 $\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx$

Optimal. Leaf size=700

$$\frac{de^2(a+bx^2)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2}-\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}}{}$$

```
[Out] 1/4*d*e^2*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2-e*x*(b*x^2+a)^p*Ap
pellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-e*x*(b*x^2+a)
^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-e*x*(b*x
^2+a)^p*AppellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^4/((1+b*x^2/a)^p)-1/3
*e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^6/((1+b*
x^2/a)^p)-e^3*x^3*(b*x^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d
^6/((1+b*x^2/a)^p)+1/2*e^2*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*
x^2+a)/(a*e^2+b*d^2))/d^3/(a*e^2+b*d^2)/(1+p)-1/2*(b*x^2+a)^(1+p)*hypergeom
([1, 1+p],[2+p],1+b*x^2/a)/a/d^3/(1+p)+b*e^2*(b*x^2+a)^(1+p)*hypergeom([2,
1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^2/(1+p)-1/4*b*e^2*(
2*a*e^2+b*d^2*(1+p))*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^2*(b*x^2+a)
/(a*e^2+b*d^2))/d/(a*e^2+b*d^2)^3/(1+p)+3/2*b^2*d*e^2*(b*x^2+a)^(1+p)*hyper
geom([3, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(1+p)
```

Rubi [A]
time = 0.54, antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {975, 272, 67, 771, 441, 440, 455, 70, 525, 524, 457, 79}

$$-\frac{e^3x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^6}-\frac{e^3x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6}-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}-\frac{ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4}+\frac{3b^2de^2(a+bx^2)^{p+1}\,{}_2F_1\left(3,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)\left(ae^2+bd^2\right)^3}-\frac{(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2ad^3(p+1)}-\frac{be^2(a+bx^2)^{p+1}\left(2ae^2+bd^2(p+1)\right)\,{}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d(p+1)\left(ae^2+bd^2\right)^3}+\frac{be^2(a+bx^2)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d(p+1)\left(ae^2+bd^2\right)^2}+\frac{de^2(a+bx^2)^{p+1}}{4\left(d^2-e^2x^2\right)^2\left(ae^2+bd^2\right)}+\frac{e^2(a+bx^2)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^3(p+1)\left(ae^2+bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] (d*e^2*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) - (e*x*(a
 + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1
 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a
), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) - (e*x*(a + b*x^2)^p*AppellF1[1/
2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1 + (b*x^2)/a)^p) - (e^3
*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/
(3*d^6*(1 + (b*x^2)/a)^p) - (e^3*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2
, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^6*(1 + (b*x^2)/a)^p) + (e^2*(a + b*x^2)^
(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2
)])/(2*d^3*(b*d^2 + a*e^2)*(1 + p)) - ((a + b*x^2)^(1 + p)*Hypergeometric2F
1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^3*(1 + p)) + (b*e^2*(a + b*x^2)^(
1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)
```

```
])/(d*(b*d^2 + a*e^2)^2*(1 + p)) - (b*e^2*(2*a*e^2 + b*d^2*(1 + p))*(a + b*
x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
a*e^2)])/(4*d*(b*d^2 + a*e^2)^3*(1 + p)) + (3*b^2*d*e^2*(a + b*x^2)^(1 + p)
*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*
(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
```

```
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
```

```
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x(d+ex)^3}\,dx = \int \left(\frac{(a+bx^2)^p}{d^3x} - \frac{e(a+bx^2)^p}{d(d+ex)^3} - \frac{e(a+bx^2)^p}{d^2(d+ex)^2} - \frac{e(a+bx^2)^p}{d^3(d+ex)}\right)dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x}\,dx}{d^3} - \frac{e\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^3} - \frac{e\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d^2} - \frac{e\int \frac{(a+bx^2)^p}{(d+ex)^3}\,dx}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^3} - \frac{e\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right)dx}{d^3} - \frac{e\int \left(\frac{d^2(a+bx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex(a+b}{(d^2-e^2x}\right.}{d^2}$$

$$= -\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)} - e\int \frac{(a+bx^2)^p}{(d^2-e^2x^2)^2}\,dx - \frac{e\int \frac{(a+bx^2)^p}{d^2-e^2x^2}\,dx}{d^2} - ($$

$$= -\frac{(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^2}{a}\right)}{2ad^3(1+p)} - \frac{e^2\text{Subst}\left(\int \frac{(a+bx)^p}{-d^2+e^2x}\,dx, x, x^2\right)}{2d^3} + \frac{e^2\text{Subst}\left(\right.}{}$$

$$= \frac{de^2(a+bx^2)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2} - \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} - \frac{ex(a}{}$$

$$= \frac{de^2(a+bx^2)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2} - \frac{ex(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4} - \frac{ex(a}{}$$

Mathematica [A]
time = 0.72, size = 434, normalized size = 0.62

$$\frac{(a+bx^2)^p\left(-\frac{2d\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{(-1+2p)(d+ex)} - \frac{d^2\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{(-1+p)(d+ex)^2} + \frac{-\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)+\left(1+\frac{a}{bx^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)}{p}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(x*(d + e*x)^3),x]
```

```
[Out] ((a + b*x^2)^p*((-2*d*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*
e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b
)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) - (d^
2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + S
qrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*
((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) + (-(AppellF1[-2*p, -p, -
```

```
p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)]
/(((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^
p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))]/(1 + a/(b*x^2))^p)/p))
/(2*d^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{x \left(e x +d \right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x/(e*x+d)^3,x)
[Out] int((b*x^2+a)^p/x/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/((x*e + d)^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(x^4*e^3 + 3*d*x^3*e^2 + 3*d^2*x^2*e + d^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b x^{2}\right)^{p}}{x \left(d + e x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x/(e*x+d)**3,x)
[Out] Integral((a + b*x**2)**p/(x*(d + e*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/((x*e + d)^3*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^2+a)^p}{x\,(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x*(d + e*x)^3),x)
[Out] int((a + b*x^2)^p/(x*(d + e*x)^3), x)
```

3.429 $\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx$

Optimal. Leaf size=754

$$-\frac{e^3(a+bx^2)^{1+p}}{4\left(bd^2+ae^2\right)\left(d^2-e^2x^2\right)^2}+\frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5}+\frac{2e^2x(a+bx^2)^p\left(1+\right.}{}$$

[Out] -1/4*e^3*(b*x^2+a)^(1+p)/(a*e^2+b*d^2)/(-e^2*x^2+d^2)^2+3*e^2*x*(b*x^2+a)^p*AppellF1(1/2,1,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+2*e^2*x*(b*x^2+a)^p*AppellF1(1/2,2,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+e^2*x*(b*x^2+a)^p*AppellF1(1/2,3,-p,3/2,e^2*x^2/d^2,-b*x^2/a)/d^5/((1+b*x^2/a)^p)+2/3*e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,2,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^7/((1+b*x^2/a)^p)+e^4*x^3*(b*x^2+a)^p*AppellF1(3/2,3,-p,5/2,e^2*x^2/d^2,-b*x^2/a)/d^7/((1+b*x^2/a)^p)-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/d^3/x/((1+b*x^2/a)^p)-3/2*e^3*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^4/(a*e^2+b*d^2)/(1+p)+3/2*e*(b*x^2+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^2/a)/a/d^4/(1+p)-2*b*e^3*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)^2/(1+p)+1/4*b*e^3*(2*a*e^2+b*d^2*(1+p))*(b*x^2+a)^(1+p)*hypergeom([2, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/d^2/(a*e^2+b*d^2)^3/(1+p)-3/2*b^2*e^3*(b*x^2+a)^(1+p)*hypergeom([3, 1+p],[2+p],e^2*(b*x^2+a)/(a*e^2+b*d^2))/(a*e^2+b*d^2)^3/(1+p)

Rubi [A]
time = 0.58, antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {975, 372, 371, 272, 67, 771, 441, 440, 455, 70, 525, 524, 457, 79}

[illegible]

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]

[Out] -1/4*(e^3*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2) + (3*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (2*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (2*e^4*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^7*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^7*(1 + (b*x^2)/a)^p) - ((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(d^3*x*(1 + (b*x^2)/a)^p) - (3*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (

```
e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*d^4*(b*d^2 + a*e^2)*(1 + p)) + (3*e*(
a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*
d^4*(1 + p)) - (2*b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 +
 p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^2)^2*(1 + p)) + (
b*e^3*(2*a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1
+ p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(4*d^2*(b*d^2 + a*e^2)^3*(1
 + p)) - (3*b^2*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p,
(e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^3}\,dx = \int \left(\frac{(a+bx^2)^p}{d^3x^2} - \frac{3e(a+bx^2)^p}{d^4x} + \frac{e^2(a+bx^2)^p}{d^2(d+ex)^3} + \frac{2e^2(a+bx^2)^p}{d^3(d+ex)^2} + \frac{3e^2(a+bx^2)^p}{d^4(d+ex)}\right) dx$$

$$= \frac{\int \frac{(a+bx^2)^p}{x^2}\,dx}{d^3} - \frac{(3e)\int \frac{(a+bx^2)^p}{x}\,dx}{d^4} + \frac{(3e^2)\int \frac{(a+bx^2)^p}{d+ex}\,dx}{d^4} + \frac{(2e^2)\int \frac{(a+bx^2)^p}{(d+ex)^2}\,dx}{d^3} + \frac{e^2\int \frac{(a+\cdots}{(d+\cdots}}{d\cdots}$$

$$= -\frac{(3e)\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^2\right)}{2d^4} + \frac{(3e^2)\int \left(\frac{d(a+bx^2)^p}{d^2-e^2x^2} + \frac{ex(a+bx^2)^p}{-d^2+e^2x^2}\right) dx}{d^4} + \frac{(2e^2)\int \left(\frac{d^2(a+\cdots}{(d^2-e\cdots}\right.}{\cdots}$$

$$= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} + \frac{3e(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1\cdots\right.}{2ad^4(1+p)}$$

$$= -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} + \frac{3e(a+bx^2)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1\cdots\right.}{2ad^4(1+p)}$$

$$= -\frac{e^3(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \cdots$$

$$= -\frac{e^3(a+bx^2)^{1+p}}{4(bd^2+ae^2)(d^2-e^2x^2)^2} + \frac{3e^2x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} + \cdots$$

Mathematica [A]

time = 0.78, size = 478, normalized size = 0.63

$$\frac{(a+bx^2)^p\left(\frac{4de\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(1-2p;-p,-p;2-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{(-1+2p)(d+ex)}+\frac{d^2e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(2-2p;-p,-p;3-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{(-1+p)(d+ex)^2}+\frac{3e\left(\frac{e\left(-\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{d-\sqrt{-\frac{a}{b}}\,e}{d+ex},\frac{d+\sqrt{-\frac{a}{b}}\,e}{d+ex}\right)}{p}-\frac{2d\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}-\frac{3e\left(1+\frac{a}{bx^2}\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{a}{bx^2}\right)}{p}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]
```

```
[Out] ((a + b*x^2)^p*((4*d*e*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]
*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/
b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)) + (d
^2*e*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d
+ Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))
^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x)^2) + (3*e*AppellF1[-2*p,
-p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d +
e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d
+ e*x))^p) - (2*d*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (
b*x^2)/a)^p) - (3*e*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*x^2))])/(p*(1 +
 a/(b*x^2))^p)))/(2*d^4)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^2+a)^p}{x^2\,(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

```
[Out] int((b*x^2+a)^p/x^2/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/((x*e + d)^3*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(x^5*e^3 + 3*d*x^4*e^2 + 3*d^2*x^3*e + d^3*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**p/x**2/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^p/x^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/((x*e + d)^3*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^2+a\right)}^p}{x^2\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^p/(x^2*(d + e*x)^3),x)
```

```
[Out] int((a + b*x^2)^p/(x^2*(d + e*x)^3), x)
```

3.430 $\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=276

$$\frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)}-\frac{d(3ae^2(1+m)-cd^2(3+m+2p))\,(gx)^{1+m}\left(a+cx^2\right)^p}{cg(1+m)(3+m+$$

```
[Out] 3*d*e^2*(g*x)^(1+m)*(c*x^2+a)^(1+p)/c/g/(3+m+2*p)+e^3*(g*x)^(2+m)*(c*x^2+a)
^(1+p)/c/g^2/(4+m+2*p)-d*(3*a*e^2*(1+m)-c*d^2*(3+m+2*p))*(g*x)^(1+m)*(c*x^2
+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/c/g/(1+m)/(3+m+2*p)/(
(c*x^2/a+1)^p)-e*(a*e^2*(2+m)-3*c*d^2*(4+m+2*p))*(g*x)^(2+m)*(c*x^2+a)^p*hy
pergeom([-p, 1+1/2*m],[2+1/2*m],-c*x^2/a)/c/g^2/(2+m)/(4+m+2*p)/((c*x^2/a+1
)^p)
```

Rubi [A]
time = 0.32, antiderivative size = 254, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1823, 822, 372, 371}

$$\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{3d^2}{m+2}-\frac{ae^2}{c(m+2p+4)}\right){}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{g^2}+\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{d^2}{m+1}-\frac{3ae^2}{c(m+2p+3)}\right){}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g}+\frac{3de^2(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{cg(m+2p+3)}+\frac{e^3(gx)^{m+2}\left(a+cx^2\right)^{p+1}}{cg^2(m+2p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] (3*d*e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + (e^3*(g*x
)^(2 + m)*(a + c*x^2)^(1 + p))/(c*g^2*(4 + m + 2*p)) + (d*(d^2/(1 + m) - (3
*a*e^2)/(c*(3 + m + 2*p)))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1
 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + (c*x^2)/a)^p) + (e*((3*d^2)/
(2 + m) - (a*e^2)/(c*(4 + m + 2*p)))*(g*x)^(2 + m)*(a + c*x^2)^p*Hypergeome
tric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(1 + (c*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int (gx)^m(d+ex)^3\left(a+cx^2\right)^p\,dx = \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{\int (gx)^m\left(a+cx^2\right)^p\left(cd^3(4+m+2p)-e(ae^2(2\right.}{c(}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{\int (gx)^m\left(-cd(4+\right.}{}$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \left(d\left(d^2-\frac{3ae^2(1}{c(3+m}\right.\right.$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \left(d\left(d^2-\frac{3ae^2(1}{c(3+m}\right.\right.$$

$$= \frac{3de^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)} + \frac{e^3(gx)^{2+m}\left(a+cx^2\right)^{1+p}}{cg^2(4+m+2p)} + \frac{d\left(d^2-\frac{3ae^2(1+m)}{c(3+m+2p)}\right)}{}$$

Mathematica [A]
time = 0.18, size = 186, normalized size = 0.67

$$x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(\frac{3d^2ex\,{}_2F_1\left(1+\frac{m}{2},-p;2+\frac{m}{2};-\frac{cx^2}{a}\right)}{2+m}+\frac{e^3x^3\,{}_2F_1\left(2+\frac{m}{2},-p;3+\frac{m}{2};-\frac{cx^2}{a}\right)}{4+m}+\frac{d^3\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{1+m}+\frac{3de^2x^2\,{}_2F_1\left(\frac{3+m}{2},-p;\frac{5+m}{2};-\frac{cx^2}{a}\right)}{3+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^3*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*((3*d^2*e*x*Hypergeometric2F1[1 + m/2, -p, 2 + m/2
, -((c*x^2)/a)])/(2 + m) + (e^3*x^3*Hypergeometric2F1[2 + m/2, -p, 3 + m/2,
 -((c*x^2)/a)])/(4 + m) + (d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(1 + m) + (3*d*e^2*x^2*Hypergeometric2F1[(3 + m)/2, -p, (5 +
 m)/2, -((c*x^2)/a)])/(3 + m)))/(1 + (c*x^2)/a)^p
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(g x \right)^{m} \left(e x +d \right)^{3} \left(c \,x^{2}+a \right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)*(c*x^2 + a)^p*(g*x)^m, x
)
```

Sympy [C] Result contains complex when optimal does not.
time = 121.87, size = 235, normalized size = 0.85

$$\frac{a^{p} d^{3} g^{m} x x^{m} \Gamma\left(\frac{m}{2} + \frac{1}{2}\right) {{}_{2}F_{1}\left(\begin{matrix} - p, \frac{m}{2} + \frac{1}{2} \\ \frac{m}{2} + \frac{3}{2} \end{matrix}\middle| {\frac{c x^{2} e^{i \pi}}{a}} \right)}}{2 \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{3 a^{p} d^{2} e g^{m} x^{2} x^{m} \Gamma\left(\frac{m}{2} + 1\right) {{}_{2}F_{1}\left(\begin{matrix} - p, \frac{m}{2} + 1 \\ \frac{m}{2} + 2 \end{matrix}\middle| {\frac{c x^{2} e^{i \pi}}{a}} \right)}}{2 \Gamma\left(\frac{m}{2} + 2\right)} + \frac{3 a^{p} d e^{2} g^{m} x^{3} x^{m} \Gamma\left(\frac{m}{2} + \frac{3}{2}\right) {{}_{2}F_{1}\left(\begin{matrix} - p, \frac{m}{2} + \frac{3}{2} \\ \frac{m}{2} + \frac{5}{2} \end{matrix}\middle| {\frac{c x^{2} e^{i \pi}}{a}} \right)}}{2 \Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{a^{p} e^{3} g^{m} x^{4} x^{m} \Gamma\left(\frac{m}{2} + 2\right) {{}_{2}F_{1}\left(\begin{matrix} - p, \frac{m}{2} + 2 \\ \frac{m}{2} + 3 \end{matrix}\middle| {\frac{c x^{2} e^{i \pi}}{a}} \right)}}{2 \Gamma\left(\frac{m}{2} + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**3*(c*x**2+a)**p,x)
```

```
[Out] a**p*d**3*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,),
 c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + 3*a**p*d**2*e*g**m*x**2*x
**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*pi)/
a)/(2*gamma(m/2 + 2)) + 3*a**p*d*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*hyper
((-p, m/2 + 3/2), (m/2 + 5/2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 5/
2)) + a**p*e**3*g**m*x**4*x**m*gamma(m/2 + 2)*hyper((-p, m/2 + 2), (m/2 + 3
,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3*(c*x^2 + a)^p*(g*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,x)^m\,\left(c\,x^2+a\right)^p\,\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^3,x)
```

```
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^3, x)
```

3.431 $\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=205

$$\frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}-\frac{\left(ae^2(1+m)-cd^2(3+m+2p)\right)(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+}{2}\right.}{cg(1+m)(3+m+2p)}$$

```
[Out] e^2*(g*x)^(1+m)*(c*x^2+a)^(1+p)/c/g/(3+m+2*p)-(a*e^2*(1+m)-c*d^2*(3+m+2*p))
*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/c/
g/(1+m)/(3+m+2*p)/((c*x^2/a+1)^p)+2*d*e*(g*x)^(2+m)*(c*x^2+a)^p*hypergeom([
-p, 1+1/2*m],[2+1/2*m],-c*x^2/a)/g^2/(2+m)/((c*x^2/a+1)^p)
```

Rubi [A]
time = 0.13, antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1823, 822, 372, 371}

$$\frac{(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(\frac{d^2}{m+1}-\frac{ae^2}{c(m+2p+3)}\right){}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g}+\frac{2de(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{g^2(m+2)}+\frac{e^2(gx)^{m+1}\left(a+cx^2\right)^{p+1}}{cg(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (e^2*(g*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*g*(3 + m + 2*p)) + ((d^2/(1 + m)
 - (a*e^2)/(c*(3 + m + 2*p)))*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1
[(1 + m)/2, -p, (3 + m)/2, -((c*x^2)/a)])/(g*(1 + (c*x^2)/a)^p) + (2*d*e*(g
*x)^(2 + m)*(a + c*x^2)^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*
x^2)/a)])/(g^2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
```

```
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int (gx)^m(d+ex)^2\left(a+cx^2\right)^p\,dx &= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\int (gx)^m\left(-ae^2(1+m)+cd^2(3+m+2p)+2c\cdots\right.}{c(3+m+2p)}\\ &= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{(2de)\int (gx)^{1+m}\left(a+cx^2\right)^p\,dx}{g}+\left(d^2-\frac{ae^2(1\cdots}{c(3+\cdots}\right.\\ &= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\left(2de\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^{1+m}\left(1+\cdots\right.}{g}\\ &= \frac{e^2(gx)^{1+m}\left(a+cx^2\right)^{1+p}}{cg(3+m+2p)}+\frac{\left(d^2-\frac{ae^2(1+m)}{c(3+m+2p)}\right)(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)\cdots}{g(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 162, normalized size = 0.79

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(2de\left(3+4m+m^2\right)x\,{}_2F_1\left(1+\frac{m}{2},-p;2+\frac{m}{2};-\frac{cx^2}{a}\right)+(2+m)\left(d^2(3+m)\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)+e^2(1+m)x^2\,{}_2F_1\left(\frac{3+m}{2},-p;\frac{5+m}{2};-\frac{cx^2}{a}\right)\right)\right)}{(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*(2*d*e*(3 + 4*m + m^2)*x*Hypergeometric2F1[1 + m/2
, -p, 2 + m/2, -((c*x^2)/a)] + (2 + m)*(d^2*(3 + m)*Hypergeometric2F1[(1 +
m)/2, -p, (3 + m)/2, -((c*x^2)/a)] + e^2*(1 + m)*x^2*Hypergeometric2F1[(3 +
 m)/2, -p, (5 + m)/2, -((c*x^2)/a)])))/((1 + m)*(2 + m)*(3 + m)*(1 + (c*x^2
)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (gx)^m (ex+d)^2 \left(c\,x^2+a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)*(c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 78.48, size = 172, normalized size = 0.84

$$\frac{a^p d^2 g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{a^p d e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{a^p e^2 g^m x^3 x^m \Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)**2*(c*x**2+a)**p,x)
```

```
[Out] a**p*d**2*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,),
 c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + a**p*d*e*g**m*x**2*x**m*g
amma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*pi)/a)/ga
mma(m/2 + 2) + a**p*e**2*g**m*x**3*x**m*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3
/2), (m/2 + 5/2,), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 5/2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)^2*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x*e + d)^2*(c*x^2 + a)^p*(g*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,x)^m\,\left(c\,x^2+a\right)^p\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^2,x)
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x)^2, x)
```

3.432 $\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx$

Optimal. Leaf size=135

$$\frac{d(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)}+\frac{e(gx)^{2+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{2+m}{2},-\right.}{g^2(2+m)}$$

```
[Out] d*(g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/g
/(1+m)/((c*x^2/a+1)^p)+e*(g*x)^(2+m)*(c*x^2+a)^p*hypergeom([-p, 1+1/2*m],[2
+1/2*m],-c*x^2/a)/g^2/(2+m)/((c*x^2/a+1)^p)
```

Rubi [A]
time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {822, 372, 371}

$$\frac{d(gx)^{m+1}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}+\frac{e(gx)^{m+2}\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2}{2},-p;\frac{m+4}{2};-\frac{cx^2}{a}\right)}{g^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (d*(g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p) + (e*(g*x)^(2 + m)*(a + c*x^2)
^p*Hypergeometric2F1[(2 + m)/2, -p, (4 + m)/2, -((c*x^2)/a)])/(g^2*(2 + m)*
(1 + (c*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
```

```
] && !IGtQ[p, 0]
```

Rubi steps

$$\int (gx)^m(d+ex)\left(a+cx^2\right)^p\,dx = d\int (gx)^m\left(a+cx^2\right)^p\,dx + \frac{e\int (gx)^{1+m}\left(a+cx^2\right)^p\,dx}{g}$$

$$= \left(d\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p\,dx + \frac{\left(e\left(a+cx^2\right)^p\left(1\right.\right.}{}$$

$$= \frac{d(gx)^{1+m}\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)} + \frac{e(gx)^{2+m}\left(a\right.}{}$$

Mathematica [A]
time = 0.05, size = 106, normalized size = 0.79

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(e(1+m)x\,{}_2F_1\left(1+\frac{m}{2},-p;2+\frac{m}{2};-\frac{cx^2}{a}\right)+d(2+m)\,{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)\right)}{(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(d + e*x)*(a + c*x^2)^p,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*(e*(1 + m)*x*Hypergeometric2F1[1 + m/2, -p, 2 + m/
2, -((c*x^2)/a)] + d*(2 + m)*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(c*x^2)/a)]))/((1 + m)*(2 + m)*(1 + (c*x^2)/a)^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (gx)^m\,(ex+d)\left(c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

```
[Out] int((g*x)^m*(e*x+d)*(c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 40.47, size = 109, normalized size = 0.81

$$\frac{a^p d g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{a^p e g^m x^2 x^m \Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+1 \\ \frac{m}{2}+2 \end{matrix}\middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(c*x**2+a)**p,x)
```

```
[Out] a**p*d*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), c*
x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + a**p*e*g**m*x**2*x**m*gamma(
m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), c*x**2*exp_polar(I*pi)/a)/(2*gamm
a(m/2 + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(e*x+d)*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x*e + d)*(c*x^2 + a)^p*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,x)^m\,\left(c\,x^2+a\right)^p\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^p*(d + e*x),x)
```

```
[Out] int((g*x)^m*(a + c*x^2)^p*(d + e*x), x)
```

3.433 $\int (gx)^m (a+cx^2)^p \, dx$

Optimal. Leaf size=66

$$\frac{(gx)^{1+m}(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)}$$

```
[Out] (g*x)^(1+m)*(c*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-c*x^2/a)/g/(
1+m)/((c*x^2/a+1)^p)
```

Rubi [A]

time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {372, 371}

$$\frac{(gx)^{m+1}(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2},-p;\frac{m+3}{2};-\frac{cx^2}{a}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*x)^m*(a + c*x^2)^p,x]
```

```
[Out] ((g*x)^(1 + m)*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -(
(c*x^2)/a)])/(g*(1 + m)*(1 + (c*x^2)/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (gx)^m (a+cx^2)^p \, dx &= \left((a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int (gx)^m\left(1+\frac{cx^2}{a}\right)^p dx\\ &= \frac{(gx)^{1+m}(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2},-p;\frac{3+m}{2};-\frac{cx^2}{a}\right)}{g(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 0.97

$$\frac{x(gx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2},-p;1+\frac{1+m}{2};-\frac{cx^2}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*x)^m*(a + c*x^2)^p,x]
[Out] (x*(g*x)^m*(a + c*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -(
(c*x^2)/a)])/((1 + m)*(1 + (c*x^2)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (gx)^m \left(c\,x^2+a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p,x)
[Out] int((g*x)^m*(c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(g*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.69, size = 54, normalized size = 0.82

$$\frac{a^p g^m x x^m \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\right| \frac{cx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p,x)
```

```
[Out] a**p*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), c*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (g\,x)^m\,\left(c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(a + c*x^2)^p,x)
```

```
[Out] int((g*x)^m*(a + c*x^2)^p, x)
```

3.434 $\int \frac{(gx)^m(a+cx^2)^p}{d+ex}\,dx$

Optimal. Leaf size=157

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,1;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(1+m)}-\frac{ex^2(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{2+m}{2};\right.}{d^2(2+m)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d/(1+m)/((c*x^2/a+1)^p)-e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,1,-p,
2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^2/(2+m)/((c*x^2/a+1)^p)
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {973, 525, 524}

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(m+1)}-\frac{ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,1;\frac{m+4}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d*(1 + m)*(1 + (c*x^2)/a)^p) - (e*x^2*(g*x)^m*(a + c*x^2
)^p*AppellF1[(2 + m)/2, -p, 1, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^
2*(2 + m)*(1 + (c*x^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 973

```
Int[(((g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_
Symbol] :> Dist[d*((g*x)^n/x^n), Int[(x^n*(a + c*x^2)^p)/(d^2 - e^2*x^2), x
], x] - Dist[e*((g*x)^n/x^n), Int[(x^(n + 1)*(a + c*x^2)^p)/(d^2 - e^2*x^2)
, x], x] /; FreeQ[{a, c, d, e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !I
ntegerQ[p] &&  !IntegersQ[n, 2*p]
```

Rubi steps

$$\int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx = \left(dx^{-m}(gx)^m\right)\int \frac{x^m(a+cx^2)^p}{d^2-e^2x^2}\,dx - \left(ex^{-m}(gx)^m\right)\int \frac{x^{1+m}(a+cx^2)^p}{d^2-e^2x^2}\,dx$$

$$= \left(dx^{-m}(gx)^m\,(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{d^2-e^2x^2}\,dx - \left(ex^{-m}(gx)^m\,(a+\right.$$

$$= \frac{x(gx)^m\,(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,1;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d(1+m)} - \frac{ex^2(gx)^m\,(a+c}{}$$

Mathematica [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\,(a+cx^2)^p}{d+ex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x),x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x), x]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m\,(c\,x^2+a)^p}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(x*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(g*x)^m/(x*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,x)^m\,(c\,x^2+a)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x),x)
```

```
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x), x)
```

3.435 $\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} \, dx$

Optimal. Leaf size=238

$$\frac{x(gx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{1+m}{2};-p,2;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(1+m)}-\frac{2ex^2(gx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{2+m}{2};\right.}{d^3(2+m)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,2,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d^2/(1+m)/((c*x^2/a+1)^p)-2*e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,2
,-p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^3/(2+m)/((c*x^2/a+1)^p)+e^2*x^3*(g*x)^m
*(c*x^2+a)^p*AppellF1(3/2+1/2*m,2,-p,5/2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^4/(3
+m)/((c*x^2/a+1)^p)
```

Rubi [A]

time = 0.18, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {976, 525, 524}

$$\frac{x(gx)^m \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} F_1\left(\frac{m+1}{2};-p,2;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(m+1)}+\frac{e^2x^3(gx)^m \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} F_1\left(\frac{m+3}{2};-p,2;\frac{m+5}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(m+3)}-\frac{2ex^2(gx)^m \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} F_1\left(\frac{m+2}{2};-p,2;\frac{m+4}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 2, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^2*(1 + m)*(1 + (c*x^2)/a)^p) - (2*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 2, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^3*(2 + m)*(1 + (c*x^2)/a)^p) + (e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF1[
(3 + m)/2, -p, 2, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^4*(3 + m)*(1
+ (c*x^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 976

```
Int[((g_.)*(x_))^(n_.)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_),
 x_Symbol] :> Dist[(g*x)^n/x^n, Int[ExpandIntegrand[x^n*(a + c*x^2)^p, (d/(
d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(-m), x], x], x] /; FreeQ[{a, c, d,
 e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[m, 0] &&  !IntegerQ[p] &&
  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2}\,dx = \left(x^{-m}(gx)^m\right)\int \left(\frac{d^2x^m(a+cx^2)^p}{(d^2-e^2x^2)^2} - \frac{2dex^{1+m}(a+cx^2)^p}{(d^2-e^2x^2)^2} + \frac{e^2x^{2+m}(a+cx^2)^p}{(-d^2+e^2x^2)^2}\right) dx$$

$$= \left(d^2x^{-m}(gx)^m\right)\int \frac{x^m(a+cx^2)^p}{(d^2-e^2x^2)^2}\,dx - \left(2dex^{-m}(gx)^m\right)\int \frac{x^{1+m}(a+cx^2)^p}{(d^2-e^2x^2)^2}\,dx + (e^2$$

$$= \left(d^2x^{-m}(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{(d^2-e^2x^2)^2}\,dx - \left(2dex^{-m}(gx)^m\right.$$

$$= \frac{x(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{1+m}{2};-p,2;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^2(1+m)} - \frac{2ex^2(gx)^m (a+}{}$$

Mathematica [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (c\,x^2+a)^p}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
```

```
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,x)^m\,{(c\,x^2+a)}^p}{{(d+e\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2,x)
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^2, x)
```

3.436 $\int \frac{(gx)^m(a+cx^2)^p}{(d+ex)^3}\,dx$

Optimal. Leaf size=321

$$\frac{x(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1+m}{2};-p,3;\frac{3+m}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(1+m)}-\frac{3ex^2(gx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{2+m}{2}\right.}{d^4(2+m)}$$

```
[Out] x*(g*x)^m*(c*x^2+a)^p*AppellF1(1/2+1/2*m,3,-p,3/2+1/2*m,e^2*x^2/d^2,-c*x^2/
a)/d^3/(1+m)/((c*x^2/a+1)^p)-3*e*x^2*(g*x)^m*(c*x^2+a)^p*AppellF1(1+1/2*m,3
,-p,2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^4/(2+m)/((c*x^2/a+1)^p)+3*e^2*x^3*(g*x)
^m*(c*x^2+a)^p*AppellF1(3/2+1/2*m,3,-p,5/2+1/2*m,e^2*x^2/d^2,-c*x^2/a)/d^5/
(3+m)/((c*x^2/a+1)^p)-e^3*x^4*(g*x)^m*(c*x^2+a)^p*AppellF1(2+1/2*m,3,-p,3+1
/2*m,e^2*x^2/d^2,-c*x^2/a)/d^6/(4+m)/((c*x^2/a+1)^p)
```

Rubi [A]
time = 0.25, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {976, 525, 524}

$$-\frac{e^3x^4(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+4}{2};-p,3;\frac{m+6}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^6(m+4)}+\frac{3e^2x^3(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+3}{2};-p,3;\frac{m+5}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5(m+3)}-\frac{3ex^2(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+2}{2};-p,3;\frac{m+4}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^4(m+2)}+\frac{x(gx)^m\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};-p,3;\frac{m+3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
```

```
[Out] (x*(g*x)^m*(a + c*x^2)^p*AppellF1[(1 + m)/2, -p, 3, (3 + m)/2, -((c*x^2)/a)
, (e^2*x^2)/d^2])/(d^3*(1 + m)*(1 + (c*x^2)/a)^p) - (3*e*x^2*(g*x)^m*(a + c
*x^2)^p*AppellF1[(2 + m)/2, -p, 3, (4 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])
/(d^4*(2 + m)*(1 + (c*x^2)/a)^p) + (3*e^2*x^3*(g*x)^m*(a + c*x^2)^p*AppellF
1[(3 + m)/2, -p, 3, (5 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^5*(3 + m)*(
1 + (c*x^2)/a)^p) - (e^3*x^4*(g*x)^m*(a + c*x^2)^p*AppellF1[(4 + m)/2, -p,
3, (6 + m)/2, -((c*x^2)/a), (e^2*x^2)/d^2])/(d^6*(4 + m)*(1 + (c*x^2)/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 976

```
Int[((g_.)*(x_))^(n_.)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_),
 x_Symbol] :> Dist[(g*x)^n/x^n, Int[ExpandIntegrand[x^n*(a + c*x^2)^p, (d/(
d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(-m), x], x], x] /; FreeQ[{a, c, d,
 e, g, n, p}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[m, 0] &&  !IntegerQ[p] &&
  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx = (x^{-m}(gx)^m) \int \left(\frac{d^3x^m(a+cx^2)^p}{(d^2-e^2x^2)^3} - \frac{3d^2ex^{1+m}(a+cx^2)^p}{(d^2-e^2x^2)^3} + \frac{3de^2x^{2+m}(a+cx^2)^p}{(d^2-e^2x^2)^3} + \right.$$

$$= (d^3x^{-m}(gx)^m) \int \frac{x^m(a+cx^2)^p}{(d^2-e^2x^2)^3} dx - (3d^2ex^{-m}(gx)^m) \int \frac{x^{1+m}(a+cx^2)^p}{(d^2-e^2x^2)^3} dx + (3d$$

$$= \left(d^3x^{-m}(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right) \int \frac{x^m\left(1+\frac{cx^2}{a}\right)^p}{(d^2-e^2x^2)^3} dx - \left(3d^2ex^{-m}(gx)^m \right($$

$$= \frac{x(gx)^m (a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{1+m}{2}; -p, 3; \frac{3+m}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^3(1+m)} - \frac{3ex^2(gx)^m (a+}{}$$

Mathematica [**F**]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]
[Out] Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(gx)^m (c\,x^2+a)^p}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
[Out] int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(g*x)^m/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,x\right)}^m\,{\left(c\,x^2+a\right)}^p}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] int(((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x)
```

3.437 $\int \frac{x^3 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{d+ex}\, dx$

Optimal. Leaf size=345

$$\frac{1}{24}\left(\frac{a}{cd} - \frac{7d}{e^2}\right) x^2 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2} + \frac{x^3 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{4e} - \frac{(105c^3d^6 - 25ac^2d^4e$$

```
[Out] 1/128*(-a*e^2+c*d^2)*(5*a^3*e^6+9*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+35*c^3*d^6
)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)+1/24*(a/c/d-7*d/e^2)*x^
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/4*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/e-1/192*(105*c^3*d^6-25*a*c^2*d^4*e^2-17*a^2*c*d^2*e^4-15*a^
3*e^6-2*c*d*e*(-5*a^2*e^4-6*a*c*d^2*e^2+35*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A]
time = 0.31, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 846, 793, 635, 212}

$$-\frac{(-15a^3e^6 - 2cdex(-5a^2e^4 - 6acd^2e^2 + 35c^2d^4) - 17a^2cd^2e^4 - 25ac^2d^4e^2 + 105c^3d^6)\sqrt{x(ae^2+cd^2) + ade + cdex^2}}{192c^3d^3e^4} + \frac{(cd^2 - ae^2)(5a^3e^6 + 9a^2cd^2e^4 + 15ac^2d^4e^2 + 35c^3d^6)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}} + \frac{1}{24}x^2\left(\frac{a}{cd} - \frac{7d}{e^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2} + \frac{x^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] ((a/(c*d) - (7*d)/e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/24
+ (x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((105*c^3*d^6 -
 25*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6 - 2*c*d*e*(35*c^2*d^4 - 6
*a*c*d^2*e^2 - 5*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
192*c^3*d^3*e^4) + ((c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*
c*d^2*e^4 + 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(7/2)*d^(7
/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{d + ex}\, dx = \int \frac{x^3(ae + cdx)}{\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}\, dx$$

$$= \frac{x^3 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{4e} + \frac{\int \frac{x^2\left(-3acd^2e - \frac{1}{2}cd\left(7cd^2 - ae^2\right.\right.}{\sqrt{ade + (cd^2 + ae^2)\, x +}}}{4cde}$$

$$= \frac{1}{24}\left(\frac{a}{cd} - \frac{7d}{e^2}\right) x^2 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2} + \frac{x^3 \sqrt{ade + (}}{}$$

$$= \frac{1}{24}\left(\frac{a}{cd} - \frac{7d}{e^2}\right) x^2 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2} + \frac{x^3 \sqrt{ade + (}}{}$$

$$= \frac{1}{24}\left(\frac{a}{cd} - \frac{7d}{e^2}\right) x^2 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2} + \frac{x^3 \sqrt{ade + (}}{}$$

$$= \frac{1}{24}\left(\frac{a}{cd} - \frac{7d}{e^2}\right) x^2 \sqrt{ade + (cd^2 + ae^2)\, x + cdex^2} + \frac{x^3 \sqrt{ade + (}}{}$$

Mathematica [**A**]
time = 1.11, size = 275, normalized size = 0.80

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(e\sqrt{ae+cdx}\,\sqrt{d+ex}\,(15a^3e^6+a^2cde^4(17d-10ex)+ac^2d^2e^2(25d^2-12dex+8e^2x^2)+c^3d^3(-105d^3+70d^2ex-56de^2x^2+48e^3x^3))+3\sqrt{\frac{e}{cd}}\,(-35c^4d^8+20ac^3d^6e^2+6a^2c^2d^4e^4+4a^3cd^2e^6+5a^4e^8)\log\left(-\sqrt{\frac{e}{cd}}\,\sqrt{ae+cdx}+\sqrt{d+ex}\right)\right)}{192c^3d^3e^5\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(e*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(15*a^3
*e^6 + a^2*c*d*e^4*(17*d - 10*e*x) + a*c^2*d^2*e^2*(25*d^2 - 12*d*e*x + 8*e
^2*x^2) + c^3*d^3*(-105*d^3 + 70*d^2*e*x - 56*d*e^2*x^2 + 48*e^3*x^3)) + 3*
Sqrt[e/(c*d)]*(-35*c^4*d^8 + 20*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c
*d^2*e^6 + 5*a^4*e^8)*Log[-(Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]) + Sqrt[d + e*x
]]))/(192*c^3*d^3*e^5*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(315) = 630$.
time = 0.10, size = 955, normalized size = 2.77

method	result

default	$\frac{x\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{4cde}-\frac{5\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{3cde}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\left(2cdex+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right.}}{4cde}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-5/8*(a*e^2+c*d^2)/
c/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-1/2*(a*e^2+c*d^2)/
c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))
)-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2
+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1
/2)))-d/e^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-1/2*(a*e^2+c
*d^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d
^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^
(1/2)))+d^2/e^3*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d
*e)^(1/2))-d^3/e^4*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^
2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e
)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see 'assume?' for
 more d
```

Fricas [A]
time = 3.75, size = 653, normalized size = 1.89

$$\left[-\frac{3\,(35\,c^4d^8 - 20\,ac^3d^6e^2 - 6\,a^2c^2d^4e^4 - 4\,a^3cd^2e^6 - 5\,a^4e^8)\sqrt{cd}\,e^{\frac{1}{2}}\log\left(8\,c^2d^3xe + c^2d^4 + 8\,acdxe^3 + a^2e^4 - 4\,\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,(2\,cdxe + cd^2 + ae^2)\sqrt{cd}\,e^{\frac{1}{2}} + 2\,(4\,c^2d^2x^2 + 3\,acd^2)e^2\right) - 4\,(70\,c^4d^6xe^2 - 105\,c^4d^7e - 10\,a^2c^2d^2xe^6 + 15\,a^3cde^7 + (8\,ac^3d^3x^2 + 17\,a^2c^2d^3)e^5 + 12\,(4\,c^4d^4x^3 - ac^3d^4x)e^4 - (56\,c^4d^5x^2 - 25\,ac^3d^5)e^3)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,e^{(-5)}}{768\,c^4d^4}, -\frac{3\,(35\,c^4d^8 - 20\,ac^3d^6e^2 - 6\,a^2c^2d^4e^4 - 4\,a^3cd^2e^6 - 5\,a^4e^8)\sqrt{-cde}\arctan\left(\frac{\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,(2\,cdxe + cd^2 + ae^2)\sqrt{-cde}}{2\,(c^2d^3xe + acdxe^3 + (c^2d^2x^2 + acd^2)e^2)}\right) - 2\,(70\,c^4d^6xe^2 - 105\,c^4d^7e - 10\,a^2c^2d^2xe^6 + 15\,a^3cde^7 + (8\,ac^3d^3x^2 + 17\,a^2c^2d^3)e^5 + 12\,(4\,c^4d^4x^3 - ac^3d^4x)e^4 - (56\,c^4d^5x^2 - 25\,ac^3d^5)e^3)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,e^{(-5)}}{384\,c^4d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x
*e^3 + a^2*e^4 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e +
 c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) - 4*
(70*c^4*d^6*x*e^2 - 105*c^4*d^7*e - 10*a^2*c^2*d^2*x*e^6 + 15*a^3*c*d*e^7 +
 (8*a*c^3*d^3*x^2 + 17*a^2*c^2*d^3)*e^5 + 12*(4*c^4*d^4*x^3 - a*c^3*d^4*x)*
e^4 - (56*c^4*d^5*x^2 - 25*a*c^3*d^5)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^
2 + a*d)*e))*e^(-5)/(c^4*d^4), -1/384*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6
*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqr
t(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-
c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2*(70*c
^4*d^6*x*e^2 - 105*c^4*d^7*e - 10*a^2*c^2*d^2*x*e^6 + 15*a^3*c*d*e^7 + (8*a
*c^3*d^3*x^2 + 17*a^2*c^2*d^3)*e^5 + 12*(4*c^4*d^4*x^3 - a*c^3*d^4*x)*e^4 -
 (56*c^4*d^5*x^2 - 25*a*c^3*d^5)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e))*e^(-5)/(c^4*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**3*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Giac [A]
time = 2.48, size = 292, normalized size = 0.85

$$\frac{1}{192}\sqrt{cdx^2e + cd^2x + axe^2 + ade}\left(2\left(4\left(6\,xe^{(-1)} - \frac{(7\,c^3d^4e^2 - ac^2d^2e^4)e^{(-4)}}{c^3d^3}\right)x + \frac{(35\,c^3d^5e - 6\,ac^2d^3e^3 - 5\,a^2cde^5)e^{(-4)}}{c^3d^3}\right)x - \frac{(105\,c^3d^6 - 25\,ac^2d^4e^2 - 17\,a^2cd^2e^4 - 15\,a^3e^6)e^{(-4)}}{c^3d^3}\right) - \frac{(35\,c^4d^8 - 20\,ac^3d^6e^2 - 6\,a^2c^2d^4e^4 - 4\,a^3cd^2e^6 - 5\,a^4e^8)e^{\left(-\frac{9}{2}\right)}\log\left(\left|-cd^2 - 2\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)\sqrt{cd}\,e^{\frac{1}{2}} - ae^2\right|\right)}{128\,\sqrt{cd}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/192*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(6*x*e^(-1) - (7*c^
3*d^4*e^2 - a*c^2*d^2*e^4)*e^(-4)/(c^3*d^3))*x + (35*c^3*d^5*e - 6*a*c^2*d^
3*e^3 - 5*a^2*c*d*e^5)*e^(-4)/(c^3*d^3))*x - (105*c^3*d^6 - 25*a*c^2*d^4*e^
2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6)*e^(-4)/(c^3*d^3)) - 1/128*(35*c^4*d^8 -
20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*e^(-9/2
)*log(abs(-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))*sqrt(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d)*c^3*d^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.438 $\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=251

$$\frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3e}+\frac{((5cd^2-3ae^2)(3cd^2+ae^2)-2cde(5cd^2-ae^2)x)\sqrt{ade+(cd^2+ae^2)x}}{24c^2d^2e^3}$$

```
[Out] -1/16*(-a*e^2+c*d^2)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*c*d*e
*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/c^(5/2)/d^(5/2)/e^(7/2)+1/3*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/e+1/24*((-3*a*e^2+5*c*d^2)*(a*e^2+3*c*d^2)-2*c*d*e*(-a*e^2+5*c*d^2)*x)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A]
time = 0.17, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {865, 846, 793, 635, 212}

$$-\frac{(cd^2-ae^2)(a^2e^4+2acd^2e^2+5c^2d^4)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}+\frac{((5cd^2-3ae^2)(ae^2+3cd^2)-2cdex(5cd^2-ae^2))\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24c^2d^2e^3}+\frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) + (((5*c*d^2 - 3*a*
e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(24*c^2*d^2*e^3) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 2*
a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[
d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(5/2)*d^(5/
2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx=\int \frac{x^2(ae+cdx)}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$$

$$=\frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e}+\frac{\int \frac{x\left(-2acd^2e-\frac{1}{2}cd\left(5cd^2-ae^2\right)x\right.}{}}{\sqrt{ade+\left(cd^2+ae^2\right)x+c}\ldots}{3cde}$$

$$=\frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e}+\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)\ldots\right.}{}$$

$$=\frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e}+\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)\ldots\right.}{}$$

$$=\frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3e}+\frac{\left(\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)\ldots\right.}{}$$

Mathematica [A]
time = 0.78, size = 218, normalized size = 0.87

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(-e\sqrt{ae+cdx}\,\sqrt{d+ex}\,(3a^2e^4+2acde^2(2d-ex)+c^2d^2(-15d^2+10dex-8e^2x^2))-3\sqrt{\frac{e}{cd}}\,(-5c^3d^6+3ac^2d^4e^2+a^2cd^2e^4+a^3e^6)\log\left(-\sqrt{\frac{e}{cd}}\,\sqrt{ae+cdx}+\sqrt{d+ex}\right)\right)}{24c^2d^2e^4\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(e*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(3*a^
2*e^4 + 2*a*c*d*e^2*(2*d - e*x) + c^2*d^2*(-15*d^2 + 10*d*e*x - 8*e^2*x^2))
) - 3*Sqrt[e/(c*d)]*(-5*c^3*d^6 + 3*a*c^2*d^4*e^2 + a^2*c*d^2*e^4 + a^3*e^6
)*Log[-(Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]) + Sqrt[d + e*x]]))/(24*c^2*d^2*e^4
*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(225) = 450$.
time = 0.08, size = 512, normalized size = 2.04

method	result
default	$\frac{\frac{\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{3cde}-\frac{\left(a\,e^2+c\,d^2\right)\left(\frac{\left(2cdex+a\,e^2+c\,d^2\right)\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2}}{4cde}+\frac{\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right.}{}\right.}{2cde}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/
d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e
*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-
d/e^2*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))
+1/e^3*d^2*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)
*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 3.25, size = 517, normalized size = 2.06

$$\left[-\frac{\left(3\,{\left(5\,c^{3}d^{6} - 3\,ac^{2}d^{4}e^{2} - a^{2}cd^{2}e^{4} - a^{3}e^{6}\right)}\sqrt{cd}\,e^{\frac{1}{2}}\log\left(8\,c^{2}d^{3}xe + c^{2}d^{4} + 8\,acdxe^{3} + a^{2}e^{4} + 4\,\sqrt{cd^{2}x + axe^{2} + {\left(cdx^{2} + ad\right)}e}\,{\left(2\,cdxe + cd^{2} + ae^{2}\right)}\sqrt{cd}\,e^{\frac{1}{2}} + 2\,{\left(4\,c^{2}d^{2}x^{2} + 3\,acd^{2}\right)}e^{2}\right) + 4\,{\left(10\,c^{3}d^{4}xe^{2} - 15\,c^{3}d^{5}e - 2\,ac^{2}d^{2}xe^{4} + 3\,a^{2}cde^{5} - 4\,{\left(2\,c^{3}d^{3}x^{2} - ac^{2}d^{3}\right)}e^{3}\right)}\sqrt{cd^{2}x + axe^{2} + {\left(cdx^{2} + ad\right)}e}\right)e^{\left(-4\right)}}{96\,c^{3}d^{3}}, \frac{\left(3\,{\left(5\,c^{3}d^{6} - 3\,ac^{2}d^{4}e^{2} - a^{2}cd^{2}e^{4} - a^{3}e^{6}\right)}\sqrt{-cde}\arctan\left(\frac{\sqrt{cd^{2}x + axe^{2} + {\left(cdx^{2} + ad\right)}e}\,{\left(2\,cdxe + cd^{2} + ae^{2}\right)}\sqrt{-cde}}{2\,{\left(c^{2}d^{3}xe + acdxe^{3} + {\left(c^{2}d^{2}x^{2} + acd^{2}\right)}e^{2}\right)}}\right) - 2\,{\left(10\,c^{3}d^{4}xe^{2} - 15\,c^{3}d^{5}e - 2\,ac^{2}d^{2}xe^{4} + 3\,a^{2}cde^{5} - 4\,{\left(2\,c^{3}d^{3}x^{2} - ac^{2}d^{3}\right)}e^{3}\right)}\sqrt{cd^{2}x + axe^{2} + {\left(cdx^{2} + ad\right)}e}\right)e^{\left(-4\right)}}{48\,c^{3}d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d)
*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e
^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 4*(10*c^3*d^4*x*e^2 - 15*c^3*
d^5*e - 2*a*c^2*d^2*x*e^4 + 3*a^2*c*d*e^5 - 4*(2*c^3*d^3*x^2 - a*c^2*d^3)*e
^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-4)/(c^3*d^3), 1/48*(3*
(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan
(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2
)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) -
 2*(10*c^3*d^4*x*e^2 - 15*c^3*d^5*e - 2*a*c^2*d^2*x*e^4 + 3*a^2*c*d*e^5 - 4
*(2*c^3*d^3*x^2 - a*c^2*d^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*
e))*e^(-4)/(c^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{\left(d + ex\right)\left(ae + cdx\right)}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x**2*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Giac [A]
time = 1.40, size = 217, normalized size = 0.86

$$\frac{1}{24}\,\sqrt{cdx^{2}e + cd^{2}x + axe^{2} + ade}\left(2\,\left(4\,xe^{\left(-1\right)} - \frac{{\left(5\,c^{2}d^{3}e - acde^{3}\right)}e^{\left(-3\right)}}{c^{2}d^{2}}\right)x + \frac{{\left(15\,c^{2}d^{4} - 4\,acd^{2}e^{2} - 3\,a^{2}e^{4}\right)}e^{\left(-3\right)}}{c^{2}d^{2}}\right) + \frac{{\left(5\,c^{3}d^{6} - 3\,ac^{2}d^{4}e^{2} - a^{2}cd^{2}e^{4} - a^{3}e^{6}\right)}e^{\left(-\frac{7}{2}\right)}\log\left({\left| -cd^{2} - 2\,{\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^{2}e + cd^{2}x + axe^{2} + ade}\right)}\sqrt{cd}\,e^{\frac{1}{2}} - ae^{2} \right|}\right)}{16\,\sqrt{cd}\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/24*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*x*e^(-1) - (5*c^2*d^
3*e - a*c*d*e^3)*e^(-3)/(c^2*d^2))*x + (15*c^2*d^4 - 4*a*c*d^2*e^2 - 3*a^2*
e^4)*e^(-3)/(c^2*d^2)) + 1/16*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4
- a^3*e^6)*e^(-7/2)*log(abs(-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))*sqrt(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d)*c^2*
d^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.439 $\int \frac{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=207

$$-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2cde(d+ex)}+\frac{\left(cd^2-ae^2\right)\left(3cd^2+ae^2\right)\cdots}{\cdots}$$

```
[Out] 1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e/(e*x+d)+1/8*(-a*e^2+c*d^2
)*(a*e^2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-1/4*(a/
c/d+3*d/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {808, 678, 635, 212}

$$\frac{\left(cd^2-ae^2\right)\left(ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2cde(d+ex)}-\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] -1/4*((a/(c*d) + (3*d)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*c*d*e*(d + e*x)) + ((c*d^2
 - a*e^2)*(3*c*d^2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*
Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d
^(3/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
```

```
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{2cde(d+ex)} + \frac{1}{4}\left(-\frac{3d}{e}-\frac{ae}{cd}\right)\int \frac{\sqrt{ade+(\cdots}}{\cdots}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2} + \frac{(ade+(cd^2+\cdots}{2cde(\cdots}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2} + \frac{(ade+(cd^2+\cdots}{2cde(\cdots}$$

$$= -\frac{1}{4}\left(\frac{a}{cd}+\frac{3d}{e^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2} + \frac{(ade+(cd^2+\cdots}{2cde(\cdots}$$

Mathematica [A]

time = 0.49, size = 161, normalized size = 0.78

$$\frac{e(ae+cdx)(d+ex)\left(ae^2+cd(-3d+2ex)\right)+\sqrt{\frac{e}{cd}}\left(-3c^2d^4+2acd^2e^2+a^2e^4\right)\sqrt{ae+cdx}\,\sqrt{d+ex}\,\log\left(-\sqrt{\frac{e}{cd}}\,\sqrt{ae+cdx}+\sqrt{d+ex}\right)}{4cde^3\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

```
[Out] (e*(a*e + c*d*x)*(d + e*x)*(a*e^2 + c*d*(-3*d + 2*e*x)) + Sqrt[e/(c*d)]*(-3
*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Log[-(S
qrt[e/(c*d)]*Sqrt[a*e + c*d*x]) + Sqrt[d + e*x]])/(4*c*d*e^3*Sqrt[(a*e + c*
d*x)*(d + e*x)])
```

Maple [A]
time = 0.08, size = 291, normalized size = 1.41

method	result
default	$\frac{\frac{\left(2 c d e x+a\,e^{2}+c\,d^{2}\right)\sqrt{a d e+\left(a\,e^{2}+c\,d^{2}\right)x+c d e\,x^{2}}}{4 c d e}+\frac{\left(4 a c\,d^{2} e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\ln\left(\frac{\frac{1}{2} a\,e^{2}+\frac{1}{2} c\,d^{2}+c d e x}{\sqrt{c d e}}+\sqrt{a d e+\left(a\,e^{2}\right.}\right.}{8 c d e\sqrt{c d e}}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e
*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-d
/e^2*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1
/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 3.03, size = 409, normalized size = 1.98

$$\left[-\frac{\left(\left(3\,c^{2}d^{4}-2\,acd^{2}e^{2}-a^{2}e^{4}\right)\sqrt{cd}\,e^{\frac{1}{2}}\log\left(8\,c^{2}d^{3}xe+c^{2}d^{4}+8\,acdxe^{3}+a^{2}e^{4}-4\,\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,(2\,cdxe+cd^{2}+ae^{2})\sqrt{cd}\,e^{\frac{1}{2}}+2\,(4\,c^{2}d^{2}x^{2}+3\,acd^{2})e^{2}\right)-4\,(2\,c^{2}d^{2}xe^{2}-3\,c^{2}d^{3}e+acde^{3})\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\right)e^{(-3)}}{16\,c^{2}d^{2}},\ -\frac{\left(\left(3\,c^{2}d^{4}-2\,acd^{2}e^{2}-a^{2}e^{4}\right)\sqrt{-cde}\,\arctan\left(\frac{\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,(2\,cdxe+cd^{2}+ae^{2})\sqrt{-cde}}{2\,(c^{2}d^{3}xe+acdxe^{3}+(c^{2}d^{2}x^{2}+acd^{2})e^{2})}\right)-2\,(2\,c^{2}d^{2}xe^{2}-3\,c^{2}d^{3}e+acde^{3})\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\right)e^{(-3)}}{8\,c^{2}d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/16*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d)*e^(1/2)*log(8*c^2*d
^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*
```

```
d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^
2*x^2 + 3*a*c*d^2)*e^2) - 4*(2*c^2*d^2*x*e^2 - 3*c^2*d^3*e + a*c*d*e^3)*sqr
t(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-3)/(c^2*d^2), -1/8*((3*c^2*d^
4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2
 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e
 + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2*(2*c^2*d^2*x*e^2 - 3*c^2
*d^3*e + a*c*d*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-3)/(c^
2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Giac [A]
time = 1.26, size = 160, normalized size = 0.77

$$\frac{1}{4}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\,xe^{(-1)}-\frac{(3\,cd^2-ae^2)e^{(-2)}}{cd}\right)-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-cd^2-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)\sqrt{cd}\,e^{\frac{1}{2}}-ae^2\right|\right)}{8\sqrt{cd}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] 1/4*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*x*e^(-1) - (3*c*d^2 - a*
e^2)*e^(-2)/(c*d)) - 1/8*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*e^(-5/2)*log
(abs(-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e))*sqrt(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d)*c*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

3.440 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e}-\frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

```
[Out] -1/2*(-a*e^2+c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c^(1/2)/d^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {678, 635, 212}

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\,e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(c
*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
```

```
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(2cd^2e-e(cd^2+ae^2))\int \frac{}{\sqrt{ade}}}{2e^2}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(2cd^2e-e(cd^2+ae^2))\,\mathrm{Subst}\Big(\int}{}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e} - \frac{(cd^2-ae^2)\tanh^{-1}\Big(\frac{}{2\sqrt{c}\sqrt{d}\sqrt{}}}{2\sqrt{c}}$$

Mathematica [A]
time = 0.13, size = 107, normalized size = 0.82

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(e+\frac{\sqrt{\frac{e}{cd}}\,(cd^2-ae^2)\log\left(-\sqrt{\frac{e}{cd}}\sqrt{ae+cdx}+\sqrt{d+ex}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(e + (Sqrt[e/(c*d)]*(c*d^2 - a*e^2)*Log[-(Sq
rt[e/(c*d)]*Sqrt[a*e + c*d*x]) + Sqrt[d + e*x]])/(Sqrt[a*e + c*d*x]*Sqrt[d
+ e*x])))/e^2
```

Maple [A]
time = 0.10, size = 131, normalized size = 1.00

method	result
default	$\frac{\sqrt{cde\left(x+\frac{d}{e}\right)^2+(a\,e^2-c\,d^2)\left(x+\frac{d}{e}\right)}+\frac{(a\,e^2-c\,d^2)\ln\left(\frac{\frac{a\,e^2}{2}-\frac{c\,d^2}{2}+cde\left(x+\frac{d}{e}\right)}{\sqrt{cde}}+\sqrt{cde\left(x+\frac{d}{e}\right)^2+(a\,e^2-c}\right.}{2\sqrt{cde}}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/e*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/
2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^
2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 1.39, size = 334, normalized size = 2.55

$$\left[\frac{\left(4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,cde-(cd^2-ae^2)\sqrt{cd}\,e^{\frac{1}{2}}\log\left(8c^2d^3xe+c^2d^4+8acdxe^3+a^2e^4+4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2cdxe+cd^2+ae^2)\sqrt{cd}\,e^{\frac{1}{2}}+2(4c^2d^2x^2+3acd^2)e^2\right)\right)e^{(-2)}}{4cd}, \frac{\left(2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,cde+(cd^2-ae^2)\sqrt{-cde}\arctan\left(\frac{\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2cdxe+cd^2+ae^2)\sqrt{-cde}}{2(c^2d^3xe+acdxe^3+(c^2d^2x^2+acd^2)e^2)}\right)\right)e^{(-2)}}{2cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/4*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*c*d*e - (c*d^2 - a*e^2)
*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 +
4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*s
qrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2))*e^(-2)/(c*d), 1/2*(2
*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*c*d*e + (c*d^2 - a*e^2)*sqrt(-
c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e +
c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c
*d^2)*e^2)))*e^(-2)/(c*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

<u>Giac</u> [**A**]
time = 2.28, size = 128, normalized size = 0.98

$$\sqrt{cdx^2e + cd^2x + axe^2 + ade}\, e^{(-1)} + \frac{(cd^2 - ae^2)\sqrt{cd}\, e^{\left(-\frac{3}{2}\right)} \log\left(\left|-\sqrt{cd}\, cd^2 e^{\frac{1}{2}} - 2\left(\sqrt{cd}\, xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)cde - \sqrt{cd}\, ae^{\frac{5}{2}}\right|\right)}{2\, cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*e^(-1) + 1/2*(c*d^2 - a*e^2)*sq
rt(c*d)*e^(-3/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2))
)/(c*d)
```

<u>Mupad</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x), x)
```

3.441 $\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x(d+ex)}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{c}\,\sqrt{d}\,\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\,\sqrt{e}\,\tanh^{-1}\left(\frac{2ade+(cd^2\cdots}{2\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ade+(c\cdots}}\right.}{\sqrt{d}}$$

```
[Out] arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)-arctanh(1/2*(2*a*d*e+(a*e
^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))*a^(1/2)*e^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {863, 857, 635, 212, 738}

$$\frac{\sqrt{c}\,\sqrt{d}\,\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\,\sqrt{e}\,\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (Sqrt[a]*Sqrt
[e]*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[d]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx &= \int \frac{ae+cdx}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx\\ &= (cd)\int \frac{1}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx+(ae)\int \frac{}{x\sqrt{ade+(cd^2}}\\ &= (2cd)\mathrm{Subst}\left(\int \frac{1}{4cde-x^2}\,dx, x, \frac{cd^2+ae^2+2cdex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)\\ &= \frac{\sqrt{c}\,\sqrt{d}\,\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}} -\end{aligned}$$

Mathematica [A]
time = 0.37, size = 167, normalized size = 0.99

$$-\frac{2\sqrt{c}\sqrt{\frac{e}{cd}}\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{c}\left(-ex+\sqrt{\frac{e}{cd}}\sqrt{ae+cdx}\sqrt{d+ex}\right)}{\sqrt{a}\,e}\right)+\sqrt{c}\,d\log\left(-\sqrt{\frac{e}{cd}}\sqrt{ae+cdx}+\sqrt{d+ex}\right)\right)}{e\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

[Out] (-2*Sqrt[c]*Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*e*ArcTan
h[(Sqrt[c]*(-(e*x) + Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]))/(Sqrt[
a]*e)] + Sqrt[c]*d*Log[-(Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]) + Sqrt[d + e*x]])
)/(e*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(136) = 272$.
time = 0.07, size = 308, normalized size = 1.83

method	result
default	$-\frac{\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(\frac{\frac{a\,e^{2}}{2}-\frac{c\,d^{2}}{2}+cde\left(x+\frac{d}{e}\right)}{\sqrt{cde}}+\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\right.}\right.}{2\sqrt{cde}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x,method=_RETURNVERBO
SE)

[Out] -1/d*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1
/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))+1/d*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2
*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/x))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="m
axima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d

Fricas [A]
time = 2.54, size = 935, normalized size = 5.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/2*sqrt(c*d)*e^(-1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e
^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e^2 + c*d^2*e +
 a*e^3)*sqrt(c*d)*e^(-1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 1/2*sqrt(
a/d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3
 - 4*(c*d^3*x + a*d*x*e^2 + 2*a*d^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
a*d)*e)*sqrt(a/d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2), 1/2*sq
rt(a/d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*
e^3 - 4*(c*d^3*x + a*d*x*e^2 + 2*a*d^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*sqrt(a/d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - sq
rt(-c*d*e^(-1))*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c
*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e^(-1))/(c^2*d^3*x + a*c*d*x*e^2 + (c^2*d
^2*x^2 + a*c*d^2)*e)), 1/2*sqrt(c*d)*e^(-1/2)*log(8*c^2*d^3*x*e + c^2*d^4 +
 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2
*c*d*x*e^2 + c*d^2*e + a*e^3)*sqrt(c*d)*e^(-1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c
*d^2)*e^2) + sqrt(-a*e/d)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*e/d)/(a*c*d^2*x*e + a^2*x*e^3 +
 (a*c*d*x^2 + a^2*d)*e^2)), -sqrt(-c*d*e^(-1))*arctan(1/2*sqrt(c*d^2*x + a*
x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e^(-1))/(c
^2*d^3*x + a*c*d*x*e^2 + (c^2*d^2*x^2 + a*c*d^2)*e)) + sqrt(-a*e/d)*arctan(
1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*
e)*sqrt(-a*e/d)/(a*c*d^2*x*e + a^2*x*e^3 + (a*c*d*x^2 + a^2*d)*e^2))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x*(d + e*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)), x)
```

3.442 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{dx}-\frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{2\sqrt{a}\,d^{3/2}\sqrt{e}}$$

```
[Out] -1/2*(-a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)-(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {863, 820, 738, 212}

$$-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}-\frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*Arc
Tanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx = \int \frac{ae+cdx}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{dx} - \frac{(-2acd^2e+ae(cd^2+ae^2))\int \frac{}{x}}{2}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{dx} + \frac{(-2acd^2e+ae(cd^2+ae^2))\,\mathrm{Sub}}{}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{dx} - \frac{(cd^2-ae^2)\tanh^{-1}\left(\frac{}{2\sqrt{a}\sqrt{d}}\right.}{2\sqrt{a}}$$

Mathematica [A]
time = 0.66, size = 141, normalized size = 1.03

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(-d+\frac{(-cd^2x+ae^2x)\tanh^{-1}\left(\frac{\sqrt{c}\left(-ex+\sqrt{\frac{e}{cd}}\sqrt{ae+cdx}\sqrt{d+ex}\right)}{\sqrt{a}\,e}\right)}{\sqrt{a}\sqrt{c}\sqrt{\frac{e}{cd}}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-d + ((-(c*d^2*x) + a*e^2*x)*ArcTanh[(Sqrt[
c]*(-(e*x) + Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]))/(Sqrt[a]*e)])/
(Sqrt[a]*Sqrt[c]*Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(d^2*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(117) = 234$.
time = 0.10, size = 708, normalized size = 5.17

method	result
default	$\frac{e\left(\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}+\frac{\left(a e^2-c d^2\right)\ln\left(\frac{\frac{a e^2}{2}-\frac{c d^2}{2}+cde\left(x+\frac{d}{e}\right)}{\sqrt{cde}}+\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a e^2-\right.}\right.}{2\sqrt{cde}}\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] e/d^2*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((
1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))+1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln(
(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/
2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*
d*e)^(1/2)))-e/d^2*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^
2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)
*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((x*e + d)*x^2), x)
```

Fricas [A]
time = 2.62, size = 356, normalized size = 2.60

$$\left[-\frac{\left(4 \sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\, ade + (cd^2x - axe^2)\sqrt{ad}\, e^{\frac{1}{2}} \log\left(\frac{c^2d^4x^2+8\,acd^3xe+a^2x^2e^4+8\,a^2dxe^3+4\,(cd^2x+axe^2+2\,ade)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,\sqrt{ad}\, e^{\frac{1}{2}}+2\,(3\,acd^2x^2+4\,a^2d^2)e^2}{x^2}\right)\right)e^{(-1)}}{4\,ad^2x}, -\frac{\left(2\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\, ade - (cd^2x - axe^2)\sqrt{-ade}\, \arctan\left(\frac{(cd^2x+axe^2+2\,ade)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,\sqrt{-ade}}{2\,(acd^3xe+a^2dxe^3+(acd^2x^2+a^2d^2)e^2)}\right)\right)e^{(-1)}}{2\,ad^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*a*d*e + (c*d^2*x - a*x
*e^2)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*
a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d
*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2))
*e^(-1)/(a*d^2*x), -1/2*(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*a*d*
e - (c*d^2*x - a*x*e^2)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e +
a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)))*e^(-1)/(a*d^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^2\,(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**2*(d + e*x)), x)
```

Giac [A]
time = 1.15, size = 229, normalized size = 1.67

$$\frac{(cd^2 - ae^2)\arctan\left(-\frac{\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}\,d} - \frac{\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)cd^2 + 2\sqrt{cd}\,ade^{\frac{3}{2}} + \left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)ae^2}{\left(ade - \left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] (c*d^2 - a*e^2)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) - ((sqrt(c*d)*x*e^(1/2) - s
qrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d^2 + 2*sqrt(c*d)*a*d*e^(3/2)
 + (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a*e^
```

```
2)/((a*d*e - (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*
d*e))^2)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

3.443 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2dx^2}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4x}+\frac{(cd^2-ae^2)(cd^2+3ae^2)\tanh$$

```
[Out] 1/8*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^
(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5
/2)/e^(3/2)-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2-1/4*(c/a/e-3*
e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A]
time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 848, 820, 738, 212}

$$\frac{(cd^2-ae^2)(3ae^2+cd^2)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4x}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^2) - ((c/(a*e) - (3*e
)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*x) + ((c*d^2 - a*e^2
)*(c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d
]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(5/2)
*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)}\,dx = \int \frac{ae+cdx}{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2dx^2} - \frac{\int \frac{-\frac{1}{2}ae(cd^2-3ae^2)+acde^2x}{x^2\sqrt{ade+(cd^2+ae^2)x+c}}}{2ade}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{4x}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{4x}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{4x}$$

Mathematica [A]
time = 1.43, size = 198, normalized size = 0.98

$$\frac{-\sqrt{a}\,e(ae+cdx)(d+ex)\left(cd^2x+ae(2d-3ex)\right)+\sqrt{c}\,\sqrt{\frac{e}{cd}}\left(c^2d^4+2acd^2e^2-3a^2e^4\right)x^2\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{c}\left(-ex+\sqrt{\frac{e}{cd}}\sqrt{ae+cdx}\,\sqrt{d+ex}\right)}{\sqrt{a}\,e}\right)}{4a^{3/2}d^2e^2x^2\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

```
[Out] (-(Sqrt[a]*e*(a*e + c*d*x)*(d + e*x)*(c*d^2*x + a*e*(2*d - 3*e*x))) + Sqrt[
c]*Sqrt[e/(c*d)]*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*x^2*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*(-(e*x) + Sqrt[e/(c*d)]*Sqrt[a*e + c*d*x]*
Sqrt[d + e*x]))/(Sqrt[a]*e)])/(4*a^(3/2)*d^2*e^2*x^2*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(176) = 352$.
time = 0.09, size = 1353, normalized size = 6.70

method	result	size
default	Expression too large to display	1353

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] -e^2/d^3*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*l
n((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2
-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))+1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(
1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*
a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/(c*d*e)^(1/2))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(
a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*
e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-
e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)
/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a
```

```
*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a
*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^
2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+e^2/d^3*((a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e
*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*
d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((x*e + d)*x^3), x)
```

Fricas [A]
time = 2.37, size = 445, normalized size = 2.20

$$\left[-\frac{\left(\left(c^2d^4x^2+2\,acd^2x^2e^2-3\,a^2x^2e^4\right)\sqrt{ad}\,e^{\frac{1}{2}}\log\left(\frac{c^2d^4x^2+8\,acd^3xe+a^2x^2e^4+8\,a^2dxe^3-4\,(cd^2x+axe^2+2\,ade)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{ad}\,e^{\frac{1}{2}}+2\,(3\,acd^2x^2+4\,a^2d^2)e^2}{x^2}\right)+4\,\left(acd^3xe-3\,a^2dxe^3+2\,a^2d^2e^2\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\right)e^{(-2)}}{16\,a^2d^3x^2},\ -\frac{\left(\left(c^2d^4x^2+2\,acd^2x^2e^2-3\,a^2x^2e^4\right)\sqrt{-ade}\,\arctan\left(\frac{(cd^2x+axe^2+2\,ade)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{-ade}}{2\,(acd^3xe+a^2dxe^3+(acd^2x^2+a^2d^2)e^2)}\right)+2\,\left(acd^3xe-3\,a^2dxe^3+2\,a^2d^2e^2\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\right)e^{(-2)}}{8\,a^2d^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/16*((c^2*d^4*x^2 + 2*a*c*d^2*x^2*e^2 - 3*a^2*x^2*e^4)*sqrt(a*d)*e^(1/2)
*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*
x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d
)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) + 4*(a*c*d^3*x*e - 3*a^
2*d*x*e^3 + 2*a^2*d^2*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(
-2)/(a^2*d^3*x^2), -1/8*((c^2*d^4*x^2 + 2*a*c*d^2*x^2*e^2 - 3*a^2*x^2*e^4)*
sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^
2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x
^2 + a^2*d^2)*e^2)) + 2*(a*c*d^3*x*e - 3*a^2*d*x*e^3 + 2*a^2*d^2*e^2)*sqrt(
c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)/(a^2*d^3*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{x^3\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**3*(d + e*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(175) = 350$.
time = 1.27, size = 517, normalized size = 2.56

$$-\frac{\left(c^2d^4+2\,acd^2e^2-3\,a^2e^4\right)\arctan\left(-\frac{\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}}{\sqrt{-ade}}\right)e^{(-1)}}{4\,\sqrt{-ade}\,ad^2}+\frac{\left(\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)ac^2d^5e+\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^3c^2d^4+8\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^2\sqrt{cd}\,acd^3e^{\frac{3}{2}}+10\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)a^2cd^3e^3+2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^3acd^2e^2+8\sqrt{cd}\,a^3d^2e^{\frac{9}{2}}+5\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)a^3de^5-3\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^3a^2e^4\right)e^{(-1)}}{4\left(ade-\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^2\right)^2ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -1/4*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*arctan(-(sqrt(c*d)*x*e^(1/2) - s
qrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))*e^(-1)/(sqrt(-a*d
*e)*a*d^2) + 1/4*((sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
 + a*d*e))*a*c^2*d^5*e + (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
a*x*e^2 + a*d*e))^3*c^2*d^4 + 8*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d
^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a*c*d^3*e^(3/2) + 10*(sqrt(c*d)*x*e^(1
/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^2*c*d^3*e^3 + 2*(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a*c*d^2*e^2
 + 8*sqrt(c*d)*a^3*d^2*e^(9/2) + 5*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))*a^3*d*e^5 - 3*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x
^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^2*e^4)*e^(-1)/((a*d*e - (sqrt(c*d)*x
*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2)^2*a*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

3.444 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=286

$$-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3dx^3}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12x^2}+\frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)\sqrt{}}{24a^2\cdots}$$

```
[Out] -1/16*(-a*e^2+c*d^2)*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*a*d*e
+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/d/x^3-1/12*(c/a/e-5*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2+1/2
4*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/a^2/d^3/e^2/x
```

Rubi [A]
time = 0.25, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 848, 820, 738, 212}

$$\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{24a^2d^3e^2x}-\frac{\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12x^2}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] -1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^3) - ((c/(a*e) - (5*e
)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*x^2) + ((3*c*d^2 -
5*a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24
*a^2*d^3*e^2*x) - ((c*d^2 - a*e^2)*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*Ar
cTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)}\,dx = \int \frac{ae+cdx}{x^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\int \frac{-\frac{1}{2}ae(cd^2-5ae^2)+2acde^2x}{x^3\sqrt{ade+(cd^2+ae^2)\,x+c}}}{3ade}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{12x^2}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{12x^2}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{12x^2}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{12x^2}$$

Mathematica [A]
time = 10.20, size = 210, normalized size = 0.73

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\,(3c^2d^4x^2-2acd^2ex(d-2ex)+a^2e^2(-8d^2+10dex-15e^2x^2))}{x^3}-\frac{3(c^3d^6+ac^2d^4e^2+3a^2cd^2e^4-5a^3e^6)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{24a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 - 2
*a*c*d^2*e*x*(d - 2*e*x) + a^2*e^2*(-8*d^2 + 10*d*e*x - 15*e^2*x^2)))/x^3 -
 (3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(Sqrt[d
]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*S
qrt[d + e*x])))/(24*a^(5/2)*d^(7/2)*e^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2058 vs. 2(256) = 512.
time = 0.07, size = 2059, normalized size = 7.20

method	result	size
default	Expression too large to display	2059

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] e^3/d^4*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln
((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-
c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))-e/d^2*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^
(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2
*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*
(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a
*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))
+e^2/d^3*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d
^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/
2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d
*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*
x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/d*(-1/3/a/d/e/x^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(a*e^2+c*d^2)/a/d/e*(-1/2/a/d
/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/
a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e
)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2
)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e
)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/x))))-e^3/d^4*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/
2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+
```

```
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((x*e + d)*x^4), x)
```

Fricas [A]
time = 6.44, size = 565, normalized size = 1.98

$$\left[-\frac{\left(3\left(c^3d^6x^3+ac^2d^4x^3e^2+3a^2cd^2x^3e^4-5a^3x^3e^6\right)\sqrt{ad}\,e^{\frac{1}{2}}\log\left(\frac{c^2d^4x^2+8acd^3xe+a^2x^2e^4+8a^2dxe^3+4\left(cd^2x+axe^2+2ade\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{ad}\,e^{\frac{1}{2}}+2\left(3acd^2x^2+4a^2d^2\right)e^2}{x^2}\right)-4\left(3ac^2d^5x^2e-2a^2cd^4xe^2-15a^3dx^2e^5+10a^3d^2xe^4+4\left(a^2cd^3x^2-2a^3d^3\right)e^3\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\right)e^{(-3)}}{96a^3d^4x^3},\ \frac{\left(3\left(c^3d^6x^3+ac^2d^4x^3e^2+3a^2cd^2x^3e^4-5a^3x^3e^6\right)\sqrt{-ade}\arctan\left(\frac{\left(cd^2x+axe^2+2ade\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{-ade}}{2\left(acd^3xe+a^2dxe^3+\left(acd^2x^2+a^2d^2\right)e^2\right)}\right)+2\left(3ac^2d^5x^2e-2a^2cd^4xe^2-15a^3dx^2e^5+10a^3d^2xe^4+4\left(a^2cd^3x^2-2a^3d^3\right)e^3\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\right)e^{(-3)}}{48a^3d^4x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6*x^3 + a*c^2*d^4*x^3*e^2 + 3*a^2*c*d^2*x^3*e^4 - 5*a^3*x^
3*e^6)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8
*a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*
d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2)
 - 4*(3*a*c^2*d^5*x^2*e - 2*a^2*c*d^4*x*e^2 - 15*a^3*d*x^2*e^5 + 10*a^3*d^2
*x*e^4 + 4*(a^2*c*d^3*x^2 - 2*a^3*d^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x
^2 + a*d)*e))*e^(-3)/(a^3*d^4*x^3), 1/48*(3*(c^3*d^6*x^3 + a*c^2*d^4*x^3*e^
2 + 3*a^2*c*d^2*x^3*e^4 - 5*a^3*x^3*e^6)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x +
 a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e
)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(3*a*c^2*d
^5*x^2*e - 2*a^2*c*d^4*x*e^2 - 15*a^3*d*x^2*e^5 + 10*a^3*d^2*x*e^4 + 4*(a^2
*c*d^3*x^2 - 2*a^3*d^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e
^(-3)/(a^3*d^4*x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(252) = 504.
time = 1.49, size = 925, normalized size = 3.23

$$\frac{\left(c^{3}d^{6}+ac^{2}d^{4}e^{2}+3a^{2}cd^{2}e^{4}-5a^{3}e^{6}\right)\arctan\left(-\frac{\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}}{\sqrt{-ade}}\right)e^{(-2)}}{8\sqrt{-ade}\,a^{2}d^{3}}-\frac{\left(3\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)a^{2}c^{3}d^{8}e^{2}+8\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{3}ac^{3}d^{7}e-3\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{5}c^{3}d^{6}+48\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{2}\sqrt{cd}\,a^{2}c^{2}d^{6}e^{\frac{5}{2}}+51\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)a^{3}c^{2}d^{6}e^{4}+72\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{3}a^{2}c^{2}d^{5}e^{3}-3\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{5}ac^{2}d^{4}e^{2}+16\sqrt{cd}\,a^{4}cd^{5}e^{\frac{11}{2}}+144\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{2}\sqrt{cd}\,a^{3}cd^{4}e^{\frac{9}{2}}+105\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)a^{4}cd^{4}e^{6}+24\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{3}a^{3}cd^{3}e^{5}-9\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{5}a^{2}cd^{2}e^{4}+48\sqrt{cd}\,a^{5}d^{3}e^{\frac{15}{2}}+33\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)a^{5}d^{2}e^{8}-40\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{3}a^{4}de^{7}+15\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{5}a^{3}e^{6}\right)e^{(-2)}}{24\left(ade-\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^{2}e+cd^{2}x+axe^{2}+ade}\right)^{2}\right)^{3}a^{2}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/8*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*arctan(-(sqrt(c
*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))*
e^(-2)/(sqrt(-a*d*e)*a^2*d^3) - 1/24*(3*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2
*e + c*d^2*x + a*x*e^2 + a*d*e))*a^2*c^3*d^8*e^2 + 8*(sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a*c^3*d^7*e - 3*(sqrt(c*d)*
x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*c^3*d^6 + 48*(sq
rt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d
)*a^2*c^2*d^6*e^(5/2) + 51*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))*a^3*c^2*d^6*e^4 + 72*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^
2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^2*c^2*d^5*e^3 - 3*(sqrt(c*d)*x*e^(1/2
) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a*c^2*d^4*e^2 + 16*sqrt(
c*d)*a^4*c*d^5*e^(11/2) + 144*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2
*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^3*c*d^4*e^(9/2) + 105*(sqrt(c*d)*x*e^(
1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^4*c*d^4*e^6 + 24*(sqr
t(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^3*c*d^3
*e^5 - 9*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)
)^5*a^2*c*d^2*e^4 + 48*sqrt(c*d)*a^5*d^3*e^(15/2) + 33*(sqrt(c*d)*x*e^(1/2)
 - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^5*d^2*e^8 - 40*(sqrt(c*d)
*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^4*d*e^7 + 15*
(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^3*e
^6)*e^(-2)/((a*d*e - (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))^2)^3*a^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^4\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

3.445 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=389

$$-\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4dx^4}-\frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24x^3}+\frac{(5c^2d^4+6acd^2e^2-35a^2e^4)}{96a^2}$$

```
[Out] 1/128*(-a*e^2+c*d^2)*(35*a^3*e^6+15*a^2*c*d^2*e^4+9*a*c^2*d^4*e^2+5*c^3*d^6
)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/4*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/d/x^4-1/24*(c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/x^3+1/96*(-35*a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2-1/192*(-105*a^3*e^6+25*a^2*c*d^2
*e^4+17*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a
^3/d^4/e^3/x
```

Rubi [A]
time = 0.38, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 848, 820, 738, 212}

$$\frac{(-35a^2e^4+6acd^2e^2+5c^2d^4)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{96a^2d^3e^2x^2}-\frac{(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2+15c^3d^6)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{192a^3d^4e^3x}+\frac{(cd^2-ae^2)(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{128a^{7/2}d^{9/2}e^{7/2}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4dx^4}-\frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] -1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^4) - ((c/(a*e) - (7*e
)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*x^3) + ((5*c^2*d^4
+ 6*a*c*d^2*e^2 - 35*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(96*a^2*d^3*e^2*x^2) - ((15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*e^4 -
 105*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(192*a^3*d^4*e^3
*x) + ((c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 35
*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(7/2)*d^(9/2)*e^(7/2)
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5(d+ex)}\,dx = \int \frac{ae+cdx}{x^5\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4} - \frac{\int \frac{-\frac{1}{2}ae(cd^2-7ae^2)+3acde^2x}{x^4\sqrt{ade+(cd^2+ae^2)\,x+c}}}{4ade}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{24x^3}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{24x^3}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{24x^3}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{24x^3}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)}}{24x^3}$$

Mathematica [A]
time = 10.27, size = 273, normalized size = 0.70

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\,(-15c^3d^6x^3+ac^2d^4ex^2(10d-17ex)+a^2cd^2e^2x(-8d^2+12dex-25e^2x^2)+a^3e^3(-48d^3+56d^2ex-70de^2x^2+105e^3x^3))}{x^4}+\frac{3(5c^4d^8+4ac^3d^6e^2+6a^2c^2d^4e^4+20a^3cd^2e^6-35a^4e^8)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{192a^{7/2}d^{9/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-15*c^3*d^6*x^3 +
 a*c^2*d^4*e*x^2*(10*d - 17*e*x) + a^2*c*d^2*e^2*x*(-8*d^2 + 12*d*e*x - 25*
e^2*x^2) + a^3*e^3*(-48*d^3 + 56*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))/x^
4 + (3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6
- 35*a^4*e^8)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d +
 e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*a^(7/2)*d^(9/2)*e^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3470 vs. $2(355) = 710$.
time = 0.10, size = 3471, normalized size = 8.92

method	result	size
default	Expression too large to display	3471

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] -e^4/d^5*((c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*l
n((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2
-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))+1/d*(-1/4/a/d/e/x^4*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)-5/8*(a*e^2+c*d^2)/a/d/e*(-1/3/a/d/e/x^3*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(a*e^2+c*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)
/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e
/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/
e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2
*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/x))))-1/4*c/a*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-
1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^
2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+
c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a
*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)
/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2
*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e
^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+e^2/d^3*(-1/2/a/d/e/x^2
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/
x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2
)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/
```

```
e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/x)))-e^3/d^4*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e
^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2
+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/
a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x
)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-e/
d^2*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(a*e^2+c*d^
2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/4*(a*e^2
+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^
2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*l
n((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+2*c/a*(1/4*(2*c
*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+1/2*c/a*((a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d
^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^
(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+e^4/d^5*((a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^
(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((x*e + d)*x^5), x)
```

Fricas [A]
time = 13.76, size = 719, normalized size = 1.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8*x^4 + 4*a*c^3*d^6*x^4*e^2 + 6*a^2*c^2*d^4*x^4*e^4 + 2
0*a^3*c*d^2*x^4*e^6 - 35*a^4*x^4*e^8)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 +
8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c
*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) + 4*(15*a*c^3*d^7*x^3*e - 10*a^2*c^2*d^6*x^
2*e^2 - 105*a^4*d*x^3*e^7 + 70*a^4*d^2*x^2*e^6 + (25*a^3*c*d^3*x^3 - 56*a^4
*d^3*x)*e^5 - 12*(a^3*c*d^4*x^2 - 4*a^4*d^4)*e^4 + (17*a^2*c^2*d^5*x^3 + 8*
a^3*c*d^5*x)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-4)/(a^4*
d^5*x^4), -1/384*(3*(5*c^4*d^8*x^4 + 4*a*c^3*d^6*x^4*e^2 + 6*a^2*c^2*d^4*x^
4*e^4 + 20*a^3*c*d^2*x^4*e^6 - 35*a^4*x^4*e^8)*sqrt(-a*d*e)*arctan(1/2*(c*d
^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(
-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(15*
a*c^3*d^7*x^3*e - 10*a^2*c^2*d^6*x^2*e^2 - 105*a^4*d*x^3*e^7 + 70*a^4*d^2*x
^2*e^6 + (25*a^3*c*d^3*x^3 - 56*a^4*d^3*x)*e^5 - 12*(a^3*c*d^4*x^2 - 4*a^4*
d^4)*e^4 + (17*a^2*c^2*d^5*x^3 + 8*a^3*c*d^5*x)*e^3)*sqrt(c*d^2*x + a*x*e^2
 + (c*d*x^2 + a*d)*e))*e^(-4)/(a^4*d^5*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. 2(347) = 694.
time = 1.77, size = 1501, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -1/64*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 -
 35*a^4*e^8)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))/sqrt(-a*d*e))*e^(-3)/(sqrt(-a*d*e)*a^3*d^4) + 1/192*(15*(sqrt
(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^3*c^4*d^11
```

e^3 + 73(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e
))^3*a^2*c^4*d^10*e^2 - 55*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))^5*a*c^4*d^9*e + 15*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))^7*c^4*d^8 + 384*(sqrt(c*d)*x*e^(1/2) - sqrt
(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^3*c^3*d^9*e^(7/2) +
396*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^4
*c^3*d^9*e^5 + 980*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^
2 + a*d*e))^3*a^3*c^3*d^8*e^4 - 44*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))^5*a^2*c^3*d^7*e^3 + 12*(sqrt(c*d)*x*e^(1/2) - s
qrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a*c^3*d^6*e^2 + 128*sqrt(c*d)
*a^5*c^2*d^8*e^(13/2) + 1792*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*
x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^4*c^2*d^7*e^(11/2) + 768*(sqrt(c*d)*x*e
^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^3*c^2*d
^6*e^(9/2) + 1170*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
 + a*d*e))*a^5*c^2*d^7*e^7 + 2238*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c
*d^2*x + a*x*e^2 + a*d*e))^3*a^4*c^2*d^6*e^6 - 66*(sqrt(c*d)*x*e^(1/2) - sq
rt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^3*c^2*d^5*e^5 + 18*(sqrt(c*d
)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^2*c^2*d^4*e^
4 + 256*sqrt(c*d)*a^6*c*d^6*e^(17/2) + 2432*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^5*c*d^5*e^(15/2) + 1212*
(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^6*c*d
^5*e^9 + 292*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*
d*e))^3*a^5*c*d^4*e^8 - 220*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
 + a*x*e^2 + a*d*e))^5*a^4*c*d^3*e^7 + 60*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x
^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^3*c*d^2*e^6 + 384*sqrt(c*d)*a^7*d^4*
e^(21/2) + 279*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
a*d*e))*a^7*d^3*e^11 - 511*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))^3*a^6*d^2*e^10 + 385*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^
2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^5*d*e^9 - 105*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^4*e^8)*e^(-3)/((a*d*e - (s
qrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2)^4*a^3*
d^4)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^5\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

3.446 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=449

$$\frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^4d^4e^5}+\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right.$$

```
[Out] 1/20*(a/c/d-3*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/6*x^3*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e-1/960*(105*c^3*d^6-21*a*c^2*d^4*e^2
-33*a^2*c*d^2*e^4-35*a^3*e^6-6*c*d*e*(-7*a^2*e^4-6*a*c*d^2*e^2+21*c^2*d^4)*
x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4-1/1024*(-a*e^2+c*d^2
)^3*(7*a^3*e^6+15*a^2*c*d^2*e^4+21*a*c^2*d^4*e^2+21*c^3*d^6)*arctanh(1/2*(2
*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)+1/512*(-7*a^4*e^8-8*a^3*c*d^2*e^6-6*a^
2*c^2*d^4*e^4+21*c^4*d^8)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^4/d^4/e^5
```

Rubi [A]
time = 0.35, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 846, 793, 626, 635, 212}

$$-\frac{\left(-35a^3e^6-6cdex\left(-7a^2e^4-6acd^2e^2+21c^2d^4\right)-33a^2cd^2e^4-21ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960c^3d^3e^4}-\frac{\left(7a^3e^6+15a^2cd^2e^4+21ac^2d^4e^2+21c^3d^6\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}}+\frac{\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^4d^4e^5}+\frac{1}{20}x^2\left(\frac{a}{cd}-\frac{3d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}+\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a^3*c*d^2*e^6 - 7*a^4*e^8)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*c^4*d^4*
e^5) + ((a/(c*d) - (3*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/20 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - ((10
5*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e*(21*
c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(960*c^3*d^3*e^4) - ((c*d^2 - a*e^2)^3*(21*c^3*d^6 + 21*a*c^2*d
^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
1024*c^(9/2)*d^(9/2)*e^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \int x^3(ae+cdx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$

$$= \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{6e} + \frac{\int x^2\left(-3acd^2e-\frac{3}{2}cd(3cd^2\right.}{}$$

$$= \frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{x^3(ade+(}{}$$

$$= \frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{x^3(ade+(}{}$$

$$= \frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{}}{512c^4d^4e^5}$$

$$= \frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{}}{512c^4d^4e^5}$$

$$= \frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{}}{512c^4d^4e^5}$$

Mathematica [A]
time = 1.33, size = 385, normalized size = 0.86

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(-105a^5e^{10}+5a^4cde^8(11d+14ex)+2a^3c^2d^2e^6(27d^2-16dex-28e^2x^2)+6a^2c^3d^3e^4(13d^3-6d^2ex+4de^2x^2+8e^3x^3)+ac^4d^4e^2(-525d^4+336d^3ex-264d^2e^2x^2+224de^3x^3+1664e^4x^4)+c^5d^5(315d^5-210d^4ex+168d^3e^2x^2-144d^2e^3x^3+128de^4x^4+1280e^5x^5)\right)-\frac{15(cd^2-ae^2)^3(21c^3d^6+21ac^2d^4e^2+15a^2cd^2e^4+7a^3e^6)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{7680c^{9/2}d^{9/2}e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]

[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10 + 5*
a^4*c*d*e^8*(11*d + 14*e*x) + 2*a^3*c^2*d^2*e^6*(27*d^2 - 16*d*e*x - 28*e^2
*x^2) + 6*a^2*c^3*d^3*e^4*(13*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 + 8*e^3*x^3) +
a*c^4*d^4*e^2*(-525*d^4 + 336*d^3*e*x - 264*d^2*e^2*x^2 + 224*d*e^3*x^3 + 1
664*e^4*x^4) + c^5*d^5*(315*d^5 - 210*d^4*e*x + 168*d^3*e^2*x^2 - 144*d^2*e
^3*x^3 + 128*d*e^4*x^4 + 1280*e^5*x^5)) - (15*(c*d^2 - a*e^2)^3*(21*c^3*d^6
 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(Sqrt[c]*Sqrt[d
]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d +
e*x])))/(7680*c^(9/2)*d^(9/2)*e^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(415) = 830$.
time = 0.09, size = 1426, normalized size = 3.18

method	result	size
default	Expression too large to display	1426

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/6*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-7/12*(a*e^2+c*d^2)
/c/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-1/2*(a*e^2+c*d^2)
/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d
^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-1/6*a/c*(1/8*(2*c*d*e*x+a*e^
2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*
e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(c*d*e)^(1/2))))-d/e^2*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/
e-1/2*(a*e^2+c*d^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(
2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))+d^2/e^3*(
1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3
/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/
e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-d^3/e^4*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-
c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/
c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d
/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 3.36, size = 1013, normalized size = 2.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 4*a^3
*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(c*d)
*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e
^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 4*(210*c^6*d^10*x*e^2 - 315*c
^6*d^11*e - 70*a^4*c^2*d^2*x*e^10 + 105*a^5*c*d*e^11 + (56*a^3*c^3*d^3*x^2
- 55*a^4*c^2*d^3)*e^9 - 16*(3*a^2*c^4*d^4*x^3 - 2*a^3*c^3*d^4*x)*e^8 - 2*(8
32*a*c^5*d^5*x^4 + 12*a^2*c^4*d^5*x^2 + 27*a^3*c^3*d^5)*e^7 - 4*(320*c^6*d^
6*x^5 + 56*a*c^5*d^6*x^3 - 9*a^2*c^4*d^6*x)*e^6 - 2*(64*c^6*d^7*x^4 - 132*a
*c^5*d^7*x^2 + 39*a^2*c^4*d^7)*e^5 + 48*(3*c^6*d^8*x^3 - 7*a*c^5*d^8*x)*e^4
 - 21*(8*c^6*d^9*x^2 - 25*a*c^5*d^9)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e))*e^(-6)/(c^5*d^5), 1/15360*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2
+ 15*a^2*c^4*d^8*e^4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*
e^10 - 7*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^
2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x
*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2*(210*c^6*d^10*x*e^2 - 315*c^6*d^11
*e - 70*a^4*c^2*d^2*x*e^10 + 105*a^5*c*d*e^11 + (56*a^3*c^3*d^3*x^2 - 55*a^
4*c^2*d^3)*e^9 - 16*(3*a^2*c^4*d^4*x^3 - 2*a^3*c^3*d^4*x)*e^8 - 2*(832*a*c^
5*d^5*x^4 + 12*a^2*c^4*d^5*x^2 + 27*a^3*c^3*d^5)*e^7 - 4*(320*c^6*d^6*x^5 +
 56*a*c^5*d^6*x^3 - 9*a^2*c^4*d^6*x)*e^6 - 2*(64*c^6*d^7*x^4 - 132*a*c^5*d^
7*x^2 + 39*a^2*c^4*d^7)*e^5 + 48*(3*c^6*d^8*x^3 - 7*a*c^5*d^8*x)*e^4 - 21*(
8*c^6*d^9*x^2 - 25*a*c^5*d^9)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e))*e^(-6)/(c^5*d^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.75, size = 486, normalized size = 1.08

$$\frac{1}{7680}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(2\left(8\left(10\,cdx+\frac{(c^6d^7e^4+13\,ac^5d^5e^6)e^{(-5)}}{c^5d^5}\right)x-\frac{(9\,c^6d^8e^3-14\,ac^5d^6e^5-3\,a^2c^4d^4e^7)e^{(-5)}}{c^5d^5}\right)x+\frac{(21\,c^6d^9e^2-33\,ac^5d^7e^4+3\,a^2c^4d^5e^6-7\,a^3c^3d^3e^8)e^{(-5)}}{c^5d^5}\right)x-\frac{(105\,c^6d^{10}e-168\,ac^5d^8e^3+18\,a^2c^4d^6e^5+16\,a^3c^3d^4e^7-35\,a^4c^2d^2e^9)e^{(-5)}}{c^5d^5}\right)x+\frac{(315\,c^6d^{11}-525\,ac^5d^9e^2+78\,a^2c^4d^7e^4+54\,a^3c^3d^5e^6+55\,a^4c^2d^3e^8-105\,a^5cde^{10})e^{(-5)}}{c^5d^5}\right)+\frac{(21\,c^6d^{12}-42\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4+4\,a^3c^3d^6e^6+3\,a^4c^2d^4e^8+6\,a^5cd^2e^{10}-7\,a^6e^{12})e^{\left(-\frac{11}{2}\right)}\log\left(\left|-cd^2-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)\sqrt{cd}\,e^{\frac{1}{2}}-ae^2\right|\right)}{1024\sqrt{cd}\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/7680*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(10*c*d*x +
(c^6*d^7*e^4 + 13*a*c^5*d^5*e^6)*e^(-5)/(c^5*d^5))*x - (9*c^6*d^8*e^3 - 14*
a*c^5*d^6*e^5 - 3*a^2*c^4*d^4*e^7)*e^(-5)/(c^5*d^5))*x + (21*c^6*d^9*e^2 -
33*a*c^5*d^7*e^4 + 3*a^2*c^4*d^5*e^6 - 7*a^3*c^3*d^3*e^8)*e^(-5)/(c^5*d^5))
*x - (105*c^6*d^10*e - 168*a*c^5*d^8*e^3 + 18*a^2*c^4*d^6*e^5 + 16*a^3*c^3*
d^4*e^7 - 35*a^4*c^2*d^2*e^9)*e^(-5)/(c^5*d^5))*x + (315*c^6*d^11 - 525*a*c
^5*d^9*e^2 + 78*a^2*c^4*d^7*e^4 + 54*a^3*c^3*d^5*e^6 + 55*a^4*c^2*d^3*e^8 -
 105*a^5*c*d*e^10)*e^(-5)/(c^5*d^5)) + 1/1024*(21*c^6*d^12 - 42*a*c^5*d^10*
e^2 + 15*a^2*c^4*d^8*e^4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*
d^2*e^10 - 7*a^6*e^12)*e^(-11/2)*log(abs(-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*sqrt(c*d)*e^(1/2) - a*e^2))/(s
qrt(c*d)*c^4*d^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.447 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=352

$$-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^3d^3e^4}+\frac{x^2(ade+(cd^2$$

```
[Out] 1/5*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/240*(35*c^2*d^4-12*a*c*
d^2*e^2-15*a^2*e^4-6*c*d*e*(-3*a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)/c^2/d^2/e^3+1/256*(-a*e^2+c*d^2)^3*(3*a^2*e^4+6*a*c*d^2*e^2+7
*c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-1/128*(-a*e^2+c
*d^2)*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A]

time = 0.21, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {865, 846, 793, 626, 635, 212}

$$\frac{\left(-15a^2e^4-6cdex(7cd^2-3ae^2)-12acd^2e^2+35c^2d^4\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{240c^2d^2e^3}+\frac{\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{7/2}d^{7/2}e^{9/2}}-\frac{\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^3d^3e^4}+\frac{x^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -1/128*((c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(c*d^2 + a*
e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*e^4)
 + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) + ((35*c^2*d^4
 - 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(240*c^2*d^2*e^3) + ((c*d^2 - a*e^2)^3*
(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
(256*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\int x^2(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}+\frac{\int x\left(-2acd^2e-\frac{1}{2}cd(7cd^2\ \cdots\right.}{\cdots}$$

$$=\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5e}+\frac{\left(35c^2d^4-12acd^2e^2-15a^2\cdots\right.}{\cdots}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{a\cdots}}{128c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{a\cdots}}{128c^3d^3e^4}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{a\cdots}}{128c^3d^3e^4}$$

Mathematica [A]
time = 0.77, size = 303, normalized size = 0.86

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\left(45a^4e^8-30a^3cde^6(d+ex)-6a^2c^2d^2e^4\left(6d^2-3dex-4e^2x^2\right)+2ac^3d^3e^2\left(95d^3-61d^2ex+48de^2x^2+264e^3x^3\right)+c^4d^4\left(-105d^4+70d^3ex-56d^2e^2x^2+48de^3x^3+384e^4x^4\right)\right)+\frac{15\left(cd^2-ae^2\right)^3\left(7c^2d^4+6acd^2e^2+3a^2e^4\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{1920c^{7/2}d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(45*a^4*e^8 - 30*a^
3*c*d*e^6*(d + e*x) - 6*a^2*c^2*d^2*e^4*(6*d^2 - 3*d*e*x - 4*e^2*x^2) + 2*a
*c^3*d^3*e^2*(95*d^3 - 61*d^2*e*x + 48*d*e^2*x^2 + 264*e^3*x^3) + c^4*d^4*(
-105*d^4 + 70*d^3*e*x - 56*d^2*e^2*x^2 + 48*d*e^3*x^3 + 384*e^4*x^4)) + (15
*(c*d^2 - a*e^2)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(Sqrt[c]
*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sq
rt[d + e*x])))/(1920*c^(7/2)*d^(7/2)*e^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. 2(322) = 644.
time = 0.08, size = 797, normalized size = 2.26

method	result

default	$\frac{\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{5cde}-\frac{\left(a\,e^2+c\,d^2\right)\left(\frac{\left(2cdex+a\,e^2+c\,d^2\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4acd^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\left(\frac{\left(2cdex+a\,e^2+\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/
d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)
/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-d/e^2*(1/8*(2*c*d*e*x+a*e^2+c*d
^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1
/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c
*d*e)^(1/2)))+1/e^3*d^2*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+
1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c
*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
1/2))/(c*d*e)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 2.90, size = 821, normalized size = 2.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2
*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*
e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2
 + 3*a*c*d^2)*e^2) - 4*(70*c^5*d^8*x*e^2 - 105*c^5*d^9*e - 30*a^3*c^2*d^2*x
*e^8 + 45*a^4*c*d*e^9 + 6*(4*a^2*c^3*d^3*x^2 - 5*a^3*c^2*d^3)*e^7 + 6*(88*a
*c^4*d^4*x^3 + 3*a^2*c^3*d^4*x)*e^6 + 12*(32*c^5*d^5*x^4 + 8*a*c^4*d^5*x^2
- 3*a^2*c^3*d^5)*e^5 + 2*(24*c^5*d^6*x^3 - 61*a*c^4*d^6*x)*e^4 - 2*(28*c^5*
d^7*x^2 - 95*a*c^4*d^7)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e
^(-5)/(c^4*d^4), -1/3840*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6
*e^4 + 2*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^
2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2))
- 2*(70*c^5*d^8*x*e^2 - 105*c^5*d^9*e - 30*a^3*c^2*d^2*x*e^8 + 45*a^4*c*d*e
^9 + 6*(4*a^2*c^3*d^3*x^2 - 5*a^3*c^2*d^3)*e^7 + 6*(88*a*c^4*d^4*x^3 + 3*a^
2*c^3*d^4*x)*e^6 + 12*(32*c^5*d^5*x^4 + 8*a*c^4*d^5*x^2 - 3*a^2*c^3*d^5)*e^
5 + 2*(24*c^5*d^6*x^3 - 61*a*c^4*d^6*x)*e^4 - 2*(28*c^5*d^7*x^2 - 95*a*c^4*
d^7)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-5)/(c^4*d^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.96, size = 386, normalized size = 1.10

$$\frac{1}{1920}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(6\left(8\,cdx+\frac{(c^5d^6e^3+11\,ac^4d^4e^5)e^{(-4)}}{c^4d^4}\right)x-\frac{(7\,c^5d^7e^2-12\,ac^4d^5e^4-3\,a^2c^3d^3e^6)e^{(-4)}}{c^4d^4}\right)x+\frac{(35\,c^5d^8e-61\,ac^4d^6e^3+9\,a^2c^3d^4e^5-15\,a^3c^2d^2e^7)e^{(-4)}}{c^4d^4}\right)x-\frac{(105\,c^5d^9-190\,ac^4d^7e^2+36\,a^2c^3d^5e^4+30\,a^3c^2d^3e^6-45\,a^4cde^8)e^{(-4)}}{c^4d^4}\right)-\frac{(7\,c^5d^{10}-15\,ac^4d^8e^2+6\,a^2c^3d^6e^4+2\,a^3c^2d^4e^6+3\,a^4cd^2e^8-3\,a^5e^{10})e^{(-\frac{9}{2})}\log\left(\left|-cd^2-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)\sqrt{cd}\,e^{\frac{1}{2}}-ae^2\right|\right)}{256\sqrt{cd}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/1920*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(6*(8*c*d*x + (c^5
*d^6*e^3 + 11*a*c^4*d^4*e^5)*e^(-4)/(c^4*d^4))*x - (7*c^5*d^7*e^2 - 12*a*c^
4*d^5*e^4 - 3*a^2*c^3*d^3*e^6)*e^(-4)/(c^4*d^4))*x + (35*c^5*d^8*e - 61*a*c
^4*d^6*e^3 + 9*a^2*c^3*d^4*e^5 - 15*a^3*c^2*d^2*e^7)*e^(-4)/(c^4*d^4))*x -
(105*c^5*d^9 - 190*a*c^4*d^7*e^2 + 36*a^2*c^3*d^5*e^4 + 30*a^3*c^2*d^3*e^6
- 45*a^4*c*d*e^8)*e^(-4)/(c^4*d^4)) - 1/256*(7*c^5*d^10 - 15*a*c^4*d^8*e^2
+ 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*e^(
-9/2)*log(abs(-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
a*x*e^2 + a*d*e))*sqrt(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d)*c^3*d^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.448 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=295

$$\frac{\left(cd^2-ae^2\right)\left(5cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2e^3}-\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(ade+\left(cd^2+\right.\right.$$

```
[Out] -1/24*(3*a/c/d+5*d/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/4*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e/(e*x+d)-1/128*(-a*e^2+c*d^2)^3*(3*a*
e^2+5*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+1/64*(-a*e^2
+c*d^2)*(3*a*e^2+5*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A]
time = 0.17, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {808, 678, 626, 635, 212}

$$-\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}}+\frac{\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64c^2d^2e^3}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4cde(d+ex)}-\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)*(5*c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c^2*d^2*e^3) - (((3*a)/(c*d) + (5*d)
/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/24 + (a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2)/(4*c*d*e*(d + e*x)) - ((c*d^2 - a*e^2)^3*(5*c*
d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*c^(5/2)*d^(5/2)*e^(7
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{4cde(d+ex)} + \frac{1}{8}\left(-\frac{5d}{e}-\frac{3ae}{cd}\right)\int \frac{(ade+(\ldots}{\ldots}$$

$$= -\frac{1}{24}\left(\frac{3a}{cd}+\frac{5d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} + \frac{(ade+(cd\ldots}{4\ldots}$$

$$= \frac{(cd^2-ae^2)(5cd^2+3ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae\ldots}}{64c^2d^2e^3}$$

$$= \frac{(cd^2-ae^2)(5cd^2+3ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae\ldots}}{64c^2d^2e^3}$$

$$= \frac{(cd^2-ae^2)(5cd^2+3ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae\ldots}}{64c^2d^2e^3}$$

Mathematica [A]

time = 0.55, size = 236, normalized size = 0.80

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}\,(-9a^3e^6+3a^2cde^4(3d+2ex)+ac^2d^2e^2(-31d^2+20dex+72e^2x^2)+c^3d^3(15d^3-10d^2ex+8de^2x^2+48e^3x^3))-\frac{3(cd^2-ae^2)^3(5cd^2+3ae^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{192c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-9*a^3*e^6 + 3*a^2
*c*d*e^4*(3*d + 2*e*x) + a*c^2*d^2*e^2*(-31*d^2 + 20*d*e*x + 72*e^2*x^2) +
c^3*d^3*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - (3*(c*d^2 - a*e
^2)^3*(5*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*
Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*c^(5/2)*d^(5/
2)*e^(7/2))
```

Maple [A]
time = 0.08, size = 483, normalized size = 1.64

method	result
default	$\frac{\left(2cdex+a\,e^2+c\,d^2\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\left(\frac{\left(2cdex+a\,e^2+c\,d^2\right)\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cde}}{4cde}\right.}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3
/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)
/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-d/e^2*(1/3*(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^
2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/
c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 2.58, size = 653, normalized size = 2.21

$$\left[-\frac{3\left(5c^4d^8-12ac^3d^6e^2+6a^2c^2d^4e^4+4a^3cd^2e^6-3a^4e^8\right)\sqrt{cd}\,e^{\frac{1}{2}}\log\left(8c^2d^3xe+c^2d^4+8acdxe^3+a^2e^4+4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2cdxe+cd^2+ae^2)\sqrt{cd}\,e^{\frac{1}{2}}+2(4c^2d^2x^2+3acd^2)e^2\right)+4\left(10c^4d^6xe^2-15c^4d^7e-6a^2c^2d^2xe^6+9a^3cde^7-9(8ac^3d^3x^2+a^2c^2d^3)e^5-4(12c^4d^4x^3+5ac^3d^4x)e^4-(8c^4d^5x^2-31ac^3d^5)e^3\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}{768c^3d^3}e^{(-4)}, \frac{3\left(5c^4d^8-12ac^3d^6e^2+6a^2c^2d^4e^4+4a^3cd^2e^6-3a^4e^8\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2cdxe+cd^2+ae^2)\sqrt{-cde}}{2(c^2d^3xe+acdxe^3+(c^2d^2x^2+acd^2)e^2)}\right)-2\left(10c^4d^6xe^2-15c^4d^7e-6a^2c^2d^2xe^6+9a^3cde^7-9(8ac^3d^3x^2+a^2c^2d^3)e^5-4(12c^4d^4x^3+5ac^3d^4x)e^4-(8c^4d^5x^2-31ac^3d^5)e^3\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}{384c^3d^3}e^{(-4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/768*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*
e^6 - 3*a^4*e^8)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*
e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e +
c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 4*(
10*c^4*d^6*x*e^2 - 15*c^4*d^7*e - 6*a^2*c^2*d^2*x*e^6 + 9*a^3*c*d*e^7 - 9*(
8*a*c^3*d^3*x^2 + a^2*c^2*d^3)*e^5 - 4*(12*c^4*d^4*x^3 + 5*a*c^3*d^4*x)*e^4
 - (8*c^4*d^5*x^2 - 31*a*c^3*d^5)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
a*d)*e))*e^(-4)/(c^3*d^3), 1/384*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c
^2*d^4*e^4 + 4*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^
2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)
/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2*(10*c^4*d^6
*x*e^2 - 15*c^4*d^7*e - 6*a^2*c^2*d^2*x*e^6 + 9*a^3*c*d*e^7 - 9*(8*a*c^3*d^
3*x^2 + a^2*c^2*d^3)*e^5 - 4*(12*c^4*d^4*x^3 + 5*a*c^3*d^4*x)*e^4 - (8*c^4*
d^5*x^2 - 31*a*c^3*d^5)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e
^(-4)/(c^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(x*((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

Giac [A]
time = 2.50, size = 298, normalized size = 1.01

$$\frac{1}{192}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(6\,cdx+\frac{(c^4d^5e^2+9\,ac^3d^3e^4)e^{(-3)}}{c^3d^3}\right)x-\frac{(5\,c^4d^6e-10\,ac^3d^4e^3-3\,a^2c^2d^2e^5)e^{(-3)}}{c^3d^3}\right)x+\frac{(15\,c^4d^7-31\,ac^3d^5e^2+9\,a^2c^2d^3e^4-9\,a^3cde^6)e^{(-3)}}{c^3d^3}\right)+\frac{(5\,c^4d^8-12\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6-3\,a^4e^8)e^{\left(-\frac{7}{2}\right)}\log\left(\left|-cd^2-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)\sqrt{cd}\,e^{\frac{1}{2}}-ae^2\right|\right)}{128\,\sqrt{cd}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] 1/192*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(6*c*d*x + (c^4*d^5
*e^2 + 9*a*c^3*d^3*e^4)*e^(-3)/(c^3*d^3))*x - (5*c^4*d^6*e - 10*a*c^3*d^4*e
^3 - 3*a^2*c^2*d^2*e^5)*e^(-3)/(c^3*d^3))*x + (15*c^4*d^7 - 31*a*c^3*d^5*e^
2 + 9*a^2*c^2*d^3*e^4 - 9*a^3*c*d*e^6)*e^(-3)/(c^3*d^3)) + 1/128*(5*c^4*d^8
 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 - 3*a^4*e^8)*e^(-
7/2)*log(abs(-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))*sqrt(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d)*c^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

3.449 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=201

$$\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e}+\frac{(cd^2}{\quad}$$

```
[Out] 1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/16*(-a*e^2+c*d^2)^3*arctanh
(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)+1/8*(a/c/d-d/e^2)*(2*c*d*e*x+a*e
^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {678, 626, 635, 212}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}+\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/8 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) +
((c*d^2 - a*e^2)^3*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*c^(3/2)*d^(3/2)*e
^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3e}-\frac{\left(2cd^2e-e\left(cd^2+ae^2\right)\right)\int\sqrt{ade}}{2e^2}$$

$$=\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}$$

$$=\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}$$

$$=\frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}$$

Mathematica [A]
time = 0.14, size = 180, normalized size = 0.90

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\left(3a^2e^4+2acde^2(4d+7ex)+c^2d^2\left(-3d^2+2dex+8e^2x^2\right)\right)+\frac{3\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\,\sqrt{d+ex}}\right)}{24c^{3/2}d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^2*e^4 + 2*a*c*
d*e^2*(4*d + 7*e*x) + c^2*d^2*(-3*d^2 + 2*d*e*x + 8*e^2*x^2)) + (3*(c*d^2 -
 a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x
])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^(3/2)*d^(3/2)*e^(5/2))
```

Maple [A]
time = 0.09, size = 230, normalized size = 1.14

method	result
default	$\frac{\left(cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{\left(2cde\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{4cde}-\cdots\right)}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/e*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1
/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/
(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(173) = 346$.
time = 0.30, size = 451, normalized size = 2.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] 1/16*c^3*d^6*e^(-5/2)*log(c*d^2*e^(-1) + 2*c*d*x + a*e + 2*sqrt(c*d*x^2*e +
 c*d^2*x + a*x*e^2 + a*d*e)*sqrt(c*d)*e^(-1/2))/(c*d)^(3/2) - 3/16*a*c^2*d^
4*e^(-1/2)*log(c*d^2*e^(-1) + 2*c*d*x + a*e + 2*sqrt(c*d*x^2*e + c*d^2*x +
a*x*e^2 + a*d*e)*sqrt(c*d)*e^(-1/2))/(c*d)^(3/2) - 1/4*sqrt(c*d*x^2*e + c*d
^2*x + a*x*e^2 + a*d*e)*c*d^2*x*e^(-1) - 1/8*sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e)*c*d^3*e^(-2) + 3/16*a^2*c*d^2*e^(3/2)*log(c*d^2*e^(-1) + 2*c*
d*x + a*e + 2*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*sqrt(c*d)*e^(-1/2
))/(c*d)^(3/2) + 1/4*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*a*x*e - 1/
16*a^3*e^(7/2)*log(c*d^2*e^(-1) + 2*c*d*x + a*e + 2*sqrt(c*d*x^2*e + c*d^2*
x + a*x*e^2 + a*d*e)*sqrt(c*d)*e^(-1/2))/(c*d)^(3/2) + 1/3*(c*d*x^2*e + c*d
^2*x + a*x*e^2 + a*d*e)^(3/2)*e^(-1) + 1/8*sqrt(c*d*x^2*e + c*d^2*x + a*x*e
^2 + a*d*e)*a^2*e^2/(c*d)
```

Fricas [A]
time = 2.71, size = 511, normalized size = 2.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d)
*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 - 4*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e
^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) - 4*(2*c^3*d^4*x*e^2 - 3*c^3*d^
5*e + 14*a*c^2*d^2*x*e^4 + 3*a^2*c*d*e^5 + 8*(c^3*d^3*x^2 + a*c^2*d^3)*e^3)
*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-3)/(c^2*d^2), -1/48*(3*(c
^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1
/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*
sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2
*(2*c^3*d^4*x*e^2 - 3*c^3*d^5*e + 14*a*c^2*d^2*x*e^4 + 3*a^2*c*d*e^5 + 8*(c
^3*d^3*x^2 + a*c^2*d^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e
^(-3)/(c^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x), x)
```

Giac [A]
time = 2.46, size = 223, normalized size = 1.11

$$\frac{1}{24}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\,cdx+\frac{(c^3d^4e+7\,ac^2d^2e^3)e^{(-2)}}{c^2d^2}\right)x-\frac{(3\,c^3d^5-8\,ac^2d^3e^2-3\,a^2cde^4)e^{(-2)}}{c^2d^2}\right)-\frac{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-cd^2-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)\sqrt{cd}\,e^{\frac{1}{2}}-ae^2\right|\right)}{16\sqrt{cd}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] 1/24*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*c*d*x + (c^3*d^4*e +
 7*a*c^2*d^2*e^3)*e^(-2)/(c^2*d^2))*x - (3*c^3*d^5 - 8*a*c^2*d^3*e^2 - 3*a^
2*c*d*e^4)*e^(-2)/(c^2*d^2)) - 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^
2*e^4 - a^3*e^6)*e^(-5/2)*log(abs(-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*
d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*sqrt(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d
)*c*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x), x)
```

3.450 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=251

$$\frac{\left(cd^2+5ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e}-\frac{\left(c^2d^4-6acd^2e^2-3a^2e^4\right)\tanh^{-1}\left(\frac{cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade}}\right.}{8\sqrt{c}\sqrt{d}\,e^{3/2}}$$

```
[Out] -1/8*(-3*a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)
/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c
^(1/2)/d^(1/2)-a^(3/2)*e^(3/2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2
)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*d^(1/2)+1/4*(2*c
*d*e*x+5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A]
time = 0.17, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 828, 857, 635, 212, 738}

$$-a^{3/2}\sqrt{d}\,e^{3/2}\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}\,e^{3/2}}+\frac{\left(5ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] ((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(4*e) - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(8*Sqrt[c]*Sqrt[d]*e^(3/2)) - a^(3/2)*Sqrt[d]*e^(3/2)*ArcTanh[(2*a*
d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d + ex)}\,dx = \int \frac{(ae + cdx)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x}\,dx$$

$$= \frac{(cd^2 + 5ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{4e} - \frac{\int \frac{-4a^2cd^2e}{x\sqrt{ade}}}{}$$

$$= \frac{(cd^2 + 5ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{4e} + (a^2de^2)\int$$

$$= \frac{(cd^2 + 5ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{4e} - (2a^2de^2)\,\mathrm{S}$$

$$= \frac{(cd^2 + 5ae^2 + 2cdex)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{4e} - \frac{(c^2d^4 - 6a}{}$$

Mathematica [A]
time = 0.55, size = 232, normalized size = 0.92

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,(5ae^2+cd(d+2ex))+(-c^2d^4+6acd^2e^2+3a^2e^4)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{ae+cdx}}\right)-8a^{3/2}\sqrt{c}\,de^3\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}{\sqrt{d}\,\sqrt{ae+cdx}}\right)\right)}{4\sqrt{c}\,\sqrt{d}\,e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*(5*a*e^2 + c*d*(d + 2*e*x)) + (-(c^2*d^4) + 6*a*c*d^2*e^2 +
3*a^2*e^4)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*
x])] - 8*a^(3/2)*Sqrt[c]*d*e^3*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqr
t[d]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[c]*Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(213) = 426$.
time = 0.08, size = 607, normalized size = 2.42

method	result

default	$-\frac{\frac{\left(cde\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(a e^2-c d^2\right)\left(\frac{\left(2cde\left(x+\frac{d}{e}\right)+a e^2-c d^2\right)\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}}{4cde}-\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/d*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(
1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d
/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))
/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
))+1/d*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*
(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*
e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*
d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)
^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 8.89, size = 1303, normalized size = 5.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/16*(8*sqrt(a*d)*a*c*d*e^(7/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2
*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e
^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e
^2)/x^2) - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d)*e^(1/2)*log(8*c^
2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 +
(c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2
*d^2*x^2 + 3*a*c*d^2)*e^2) + 4*(2*c^2*d^2*x*e^2 + c^2*d^3*e + 5*a*c*d*e^3)*
sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)/(c*d), 1/8*(4*sqrt(a*d)
*a*c*d*e^(7/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e
^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*
d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) + (c^2*d^
4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e
^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x
*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(2*c^2*d^2*x*e^2 + c^2
*d^3*e + 5*a*c*d*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)/(
c*d), 1/16*(16*sqrt(-a*d*e)*a*c*d*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*
sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2
*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2))*e^3 - (c^2*d^4 - 6*a*c*d^2*e^2 - 3
*a^2*e^4)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a
^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 +
 a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 4*(2*c^2*d
^2*x*e^2 + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d
)*e))*e^(-2)/(c*d), 1/8*(8*sqrt(-a*d*e)*a*c*d*arctan(1/2*(c*d^2*x + a*x*e^2
 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d
^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2))*e^3 + (c^2*d^4 - 6*a*c
*d^2*e^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d
*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*
d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(2*c^2*d^2*x*e^2 + c^2*d^3*e +
5*a*c*d*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)/(c*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x*(d + e*x)), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)), x)
```

3.451 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=240

$$-\frac{(ae-cdx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x}+\frac{\sqrt{c}\,\sqrt{d}\,(cd^2+3ae^2)\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ade+(cd^2+ae^2}}\right)}{2\sqrt{e}}$$

```
[Out] 1/2*(3*a*e^2+c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)-1/2*(
a*e^2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*a^(1/2)*e^(1/2)/d^(1/2)-(-c*d*x+
a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A]
time = 0.18, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 826, 857, 635, 212, 738}

$$-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(ae-cdx)}{x}+\frac{\sqrt{c}\,\sqrt{d}\,(3ae^2+cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{e}}-\frac{\sqrt{a}\,\sqrt{e}\,(ae^2+3cd^2)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] -(((a*e - c*d*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (Sqrt[c]
*Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*S
qrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[e]) -
 (Sqrt[a]*Sqrt[e]*(3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(
2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
*Sqrt[d])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{3/2}}{x^2(d + ex)} dx = \int \frac{(ae + cdx)\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{x^2} dx$$

$$= -\frac{(ae - cdx)\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{x} - \frac{1}{2}\int \frac{-ae(3cd^2 + ae}{x\sqrt{ade + (cd}}$$

$$= -\frac{(ae - cdx)\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{x} + \frac{1}{2}\left(ae\left(3cd^2 + ae^2\right)\right)$$

$$= -\frac{(ae - cdx)\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{x} - \left(ae\left(3cd^2 + ae^2\right)\right) \text{Su}$$

$$= -\frac{(ae - cdx)\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{x} + \frac{\sqrt{c}\,\sqrt{d}\,(cd^2 + 3ae^2}{}$$

Mathematica [A]
time = 0.43, size = 213, normalized size = 0.89

$$-\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{d}\,\sqrt{e}\,(ae-cdx)\sqrt{ae+cdx}\,\sqrt{d+ex}-\sqrt{c}\,d(cd^2+3ae^2)\,x\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{ae+cdx}}\right)+\sqrt{a}\,e(3cd^2+ae^2)\,x\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}{\sqrt{d}\,\sqrt{ae+cdx}}\right)\right)}{\sqrt{d}\,\sqrt{e}\,x\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[d]*Sqrt[e]*(a*e - c*d*x)*Sqrt[a*e
+ c*d*x]*Sqrt[d + e*x] - Sqrt[c]*d*(c*d^2 + 3*a*e^2)*x*ArcTanh[(Sqrt[c]*Sqr
t[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])] + Sqrt[a]*e*(3*c*d^2 + a*e
^2)*x*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])
)/(Sqrt[d]*Sqrt[e]*x*Sqrt[(a*e + c*d*x)*(d + e*x)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(202) = 404$.
time = 0.08, size = 1300, normalized size = 5.42

method	result	size
default	Expression too large to display	1300

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] e/d^2*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*
(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+
```

```
d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e)
)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2
)))+1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^
2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/
4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8
*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*
d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*
c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*
e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^2+c*d
^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1
/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c
*d*e)^(1/2))))-e/d^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^
2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+
c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/
2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/
2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d
*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((x*e + d)*x^2), x)
```

Fricas [A]
time = 3.68, size = 1212, normalized size = 5.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/4*((c*d^2*x + 3*a*x*e^2)*sqrt(c*d)*e^(-1/2)*log(8*c^2*d^3*x*e + c^2*d^4
+ 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(
```

```
2*c*d*x*e^2 + c*d^2*e + a*e^3)*sqrt(c*d)*e^(-1/2) + 2*(4*c^2*d^2*x^2 + 3*a*
c*d^2)*e^2) + (3*c*d^2*x + a*x*e^2)*sqrt(a/d)*e^(1/2)*log((c^2*d^4*x^2 + 8*
a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^3*x + a*d*x*e^2 + 2*a*d^
2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a/d)*e^(1/2) + 2*(3*a
*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*(c*d*x - a*e))/x, 1/4*((3*c*d^2*x + a*x*e^2)*sqrt(a/d)*e^(1/2)*log((
c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^3*x + a*
d*x*e^2 + 2*a*d^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a/d)*
e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 2*(c*d^2*x + 3*a*x*e^2)
*sqrt(-c*d*e^(-1))*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(
2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e^(-1))/(c^2*d^3*x + a*c*d*x*e^2 + (c^
2*d^2*x^2 + a*c*d^2)*e)) + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c
*d*x - a*e))/x, 1/4*((c*d^2*x + 3*a*x*e^2)*sqrt(c*d)*e^(-1/2)*log(8*c^2*d^3
*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e)*(2*c*d*x*e^2 + c*d^2*e + a*e^3)*sqrt(c*d)*e^(-1/2) + 2*(4*c^2
*d^2*x^2 + 3*a*c*d^2)*e^2) + 2*(3*c*d^2*x + a*x*e^2)*sqrt(-a*e/d)*arctan(1/
2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
*sqrt(-a*e/d)/(a*c*d^2*x*e + a^2*x*e^3 + (a*c*d*x^2 + a^2*d)*e^2)) + 4*sqrt
(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*x - a*e))/x, -1/2*((c*d^2*x +
3*a*x*e^2)*sqrt(-c*d*e^(-1))*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
 a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e^(-1))/(c^2*d^3*x + a*c*d*x
*e^2 + (c^2*d^2*x^2 + a*c*d^2)*e)) - (3*c*d^2*x + a*x*e^2)*sqrt(-a*e/d)*arc
tan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*sqrt(-a*e/d)/(a*c*d^2*x*e + a^2*x*e^3 + (a*c*d*x^2 + a^2*d)*e^2)) -
2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*x - a*e))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}}{x^2(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**2*(d + e*x)), x)
```

Giac [A]
time = 1.37, size = 373, normalized size = 1.55

$$\sqrt{cdx^2e+cd^2x+axe^2+ade}\,cd+\frac{(3\,acd^2e+a^2e^3)\arctan\left(-\frac{\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}}-\frac{\left(\sqrt{cd}\,c^2d^3e^{\frac{1}{2}}+3\,\sqrt{cd}\,acde^{\frac{5}{2}}\right)e^{(-1)}\log\left(\left|-\sqrt{cd}\,cd^2e^{\frac{1}{2}}-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)cde-\sqrt{cd}\,ae^{\frac{5}{2}}\right|\right)}{2\,cd}-\frac{\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)acd^2e+2\,\sqrt{cd}\,a^2de^{\frac{5}{2}}+\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)a^2e^3}{ade-\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*c*d + (3*a*c*d^2*e + a^2*e^3)*a
rctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/
sqrt(-a*d*e))/sqrt(-a*d*e) - 1/2*(sqrt(c*d)*c^2*d^3*e^(1/2) + 3*sqrt(c*d)*a
*c*d*e^(5/2))*e^(-1)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1
/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5
/2)))/(c*d) - ((sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
a*d*e))*a*c*d^2*e + 2*sqrt(c*d)*a^2*d*e^(5/2) + (sqrt(c*d)*x*e^(1/2) - sqrt
(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^2*e^3)/(a*d*e - (sqrt(c*d)*x*e^(
1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)), x)
```

3.452 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=256

$$-\frac{(2ade+(5cd^2+ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^2}+c^{3/2}d^{3/2}\sqrt{e}\,\tanh^{-1}\left(\frac{cd^2+ae^2+2c}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ade+(cd^2}\right.$$

```
[Out] -1/8*(-a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*
x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)
/a^(1/2)/e^(1/2)+c^(3/2)*d^(3/2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2
)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*e^(1/2)-1/4*(2*a
*d*e+(a*e^2+5*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2
```

Rubi [A]
time = 0.18, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 824, 857, 635, 212, 738}

$$-\frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{a}\,d^{3/2}\sqrt{e}}+c^{3/2}d^{3/2}\sqrt{e}\,\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\right)-\frac{(x(ae^2+5cd^2)+2ade)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*((2*a*d*e + (5*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(d*x^2) + c^(3/2)*d^(3/2)*Sqrt[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*
x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
 - ((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2
)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)])/(8*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx = \int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^3}\,dx$$

$$= -\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2} - \frac{\int \frac{-\frac{1}{2}c}{x\sqrt{}}}{}$$

$$= -\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2} + \left(c^2d^2e\right.$$

$$= -\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2} + \left(2c^2d^2\right.$$

$$= -\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2} + c^{3/2}d^{3/}$$

Mathematica [A]
time = 0.57, size = 243, normalized size = 0.95

$$-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}\left(5cd^2x+ae(2d+ex)\right)-8\sqrt{a}c^{3/2}d^3ex^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)+\left(3c^2d^4+6acd^2e^2-a^2e^4\right)x^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)\right)}{4\sqrt{a}d^{3/2}\sqrt{e}x^2\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*(5*c*d^2*x + a*e*(2*d + e*x)) - 8*Sqrt[a]*c^(3/2)*d^3*e
*x^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])] +
 (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*x^2*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d
+ e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])]))/(Sqrt[a]*d^(3/2)*Sqrt[e]*x^2*Sqrt[(a
*e + c*d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2437 vs. $2(218) = 436$.
time = 0.09, size = 2438, normalized size = 9.52

method	result	size
default	Expression too large to display	2438

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] -e^2/d^3*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^
2)*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*
(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d
/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(
1/2)))+1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*
e^2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*
d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*
e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+
a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*
e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a
*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e
^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2
*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(c*d*e)^(1/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+
1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(
c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d
^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2
)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))-e/d^2*(-
1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)/a/d/e*(
1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e
*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^
2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*
e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a
*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2
))))+e^2/d^3*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)
*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)
/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*
e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+
1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(
```

```
c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((x*e + d)*x^3), x)
```

Fricas [A]
time = 5.26, size = 1379, normalized size = 5.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(8*sqrt(c*d)*a*c*d^3*x^2*e^(3/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*
d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*
e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) -
 (3*c^2*d^4*x^2 + 6*a*c*d^2*x^2*e^2 - a^2*x^2*e^4)*sqrt(a*d)*e^(1/2)*log((c
^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(c*d^2*x + a*x
*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/
2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 4*(5*a*c*d^3*x*e + a^2*d*x*e
^3 + 2*a^2*d^2*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-1)/(a*
d^2*x^2), -1/16*(16*sqrt(-c*d*e)*a*c*d^3*x^2*arctan(1/2*sqrt(c*d^2*x + a*x*
e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*
x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2))*e + (3*c^2*d^4*x^2 + 6*a*
c*d^2*x^2*e^2 - a^2*x^2*e^4)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3
*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c
*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2
+ 4*a^2*d^2)*e^2)/x^2) + 4*(5*a*c*d^3*x*e + a^2*d*x*e^3 + 2*a^2*d^2*e^2)*sq
rt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-1)/(a*d^2*x^2), 1/8*(4*sqrt(
c*d)*a*c*d^3*x^2*e^(3/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*
e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*
e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + (3*c^2*d^4*x^
2 + 6*a*c*d^2*x^2*e^2 - a^2*x^2*e^4)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x
*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a
*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) - 2*(5*a*c*d^3*x*e
 + a^2*d*x*e^3 + 2*a^2*d^2*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
```

```
)*e^(-1)/(a*d^2*x^2), -1/8*(8*sqrt(-c*d*e)*a*c*d^3*x^2*arctan(1/2*sqrt(c*d^
2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)
/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2))*e - (3*c^2*d^4*
x^2 + 6*a*c*d^2*x^2*e^2 - a^2*x^2*e^4)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a
*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/
(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(5*a*c*d^3*x
*e + a^2*d*x*e^3 + 2*a^2*d^2*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*
e))*e^(-1)/(a*d^2*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(215) = 430.
time = 1.81, size = 647, normalized size = 2.53

$$-\sqrt{cd}\,cde^{\frac{1}{2}}\log\left(\left|-\sqrt{cd}\,cd^2e^{\frac{1}{2}}-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)cde-\sqrt{cd}\,ae^{\frac{5}{2}}\right|\right)+\frac{\left(3c^2d^4+6acd^2e^2-a^2e^4\right)\arctan\left(-\frac{\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}\,d}-\frac{3\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)ac^2d^5e-5\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^3c^2d^4+8\sqrt{cd}\,a^2cd^4e^{\frac{5}{2}}-16\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^2\sqrt{cd}\,acd^3e^{\frac{3}{2}}-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)a^2cd^3e^3-10\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^3acd^2e^2-8\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^2\sqrt{cd}\,a^2de^{\frac{7}{2}}-\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)a^3de^5-\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^3a^2e^4}{4\left(ade-\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)^2\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -sqrt(c*d)*c*d*e^(1/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^
(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^
(5/2))) + 1/4*(3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(-(sqrt(c*d)*x*e^
(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))/(sqrt(-a
*d*e)*d) - 1/4*(3*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
 + a*d*e))*a*c^2*d^5*e - 5*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))^3*c^2*d^4 + 8*sqrt(c*d)*a^2*c*d^4*e^(5/2) - 16*(sqrt(c*
d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a*c
*d^3*e^(3/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
+ a*d*e))*a^2*c*d^3*e^3 - 10*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*
x + a*x*e^2 + a*d*e))^3*a*c*d^2*e^2 - 8*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2
*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^2*d*e^(7/2) - (sqrt(c*d)*x*e
^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^3*d*e^5 - (sqrt(c*d
)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^2*e^4)/((a*d
*e - (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2)
^2*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^3\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)), x)
```

3.453 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=211

$$-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3dx^3}+\frac{(cd^2}{}$$

```
[Out] -1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^3+1/16*(-a*e^2+c*d^2)^3*ar
ctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)-1/8*(c/a/e-e/d^2)*(2*a*d*
e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 820, 734, 738, 212}

$$\frac{\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8x^2}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] -1/8*((c/(a*e) - e/d^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/x^2 - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
/(3*d*x^3) + ((c*d^2 - a*e^2)^3*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sq
rt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*a^
(3/2)*d^(5/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx=\int \frac{(ae+cdx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^4}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3dx^3}-\frac{\left(-2acd^2e+ae(cd^2+ae^2)\right)\int}{}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}$$

$$=-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2}$$

Mathematica [A]

time = 0.37, size = 201, normalized size = 0.95

$$\frac{(-cd^2+ae^2)^3\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\,(3c^2d^4x^2+2acd^2ex(7d+4ex)+a^2e^2(8d^2+2dex-3e^2x^2))}{(cd^2-ae^2)^3x^3}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{24a^{3/2}d^{5/2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

```
[Out] ((-(c*d^2) + a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[
e]*(3*c^2*d^4*x^2 + 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*x
- 3*e^2*x^2)))/((c*d^2 - a*e^2)^3*x^3) - (3*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d
 + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/
(24*a^(3/2)*d^(5/2)*e^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4329 vs. $2(185) = 370$.

time = 0.08, size = 4330, normalized size = 20.52

method	result	size
default	Expression too large to display	4330

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] e^3/d^4*(1/3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2
)*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(
x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/
e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1
/2)))-e/d^2*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/4*(a*
e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a
*e^2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c
*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d
*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))
+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a
*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+
a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*
e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/
2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
```

(1/2))/(c*d*e)^(1/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2
+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*
d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+e^2/d^3
(-1/a/d/e/x(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)/a/d/
e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*
d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c
*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c
*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2
)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d
/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2
)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^
2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(
1/2))))+1/d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-1/6*(a*
e^2+c*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/
4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3
/2*(a*e^2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*
e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^
2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(
1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((
1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a
*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+4*c/a*(1/8*(2*c*d
*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c
*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*l
n((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/(c*d*e)^(1/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*
a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e
^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2
+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+2/
3*c/a*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+3/2*(a*e^2+c*d^2)

```
/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*
(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*
e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((x*e + d)*x^4), x)
```

Fricas [A]
time = 2.85, size = 565, normalized size = 2.68

$$\left[-\frac{\left(3\left(c^{3}d^{6}x^{3}-3ac^{2}d^{4}x^{3}e^{2}+3a^{2}cd^{2}x^{3}e^{4}-a^{3}x^{3}e^{6}\right)\sqrt{ad}\,e^{\frac{1}{2}}\log\left(\frac{c^{2}d^{4}x^{2}+8acd^{3}xe+a^{2}x^{2}e^{4}+8a^{2}dxe^{3}-4\left(cd^{2}x+axe^{2}+2ade\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{ad}\,e^{\frac{1}{2}}+2\left(3acd^{2}x^{2}+4a^{2}d^{2}\right)e^{2}}{x^{2}}\right)+4\left(3ac^{2}d^{5}x^{2}e+14a^{2}cd^{4}xe^{2}-3a^{3}dx^{2}e^{5}+2a^{3}d^{2}xe^{4}+8\left(a^{2}cd^{3}x^{2}+a^{3}d^{3}\right)e^{3}\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\right)e^{(-2)}}{96\,a^{2}d^{3}x^{3}},\ -\frac{\left(3\left(c^{3}d^{6}x^{3}-3ac^{2}d^{4}x^{3}e^{2}+3a^{2}cd^{2}x^{3}e^{4}-a^{3}x^{3}e^{6}\right)\sqrt{-ade}\arctan\left(\frac{\left(cd^{2}x+axe^{2}+2ade\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{-ade}}{2\left(acd^{3}xe+a^{2}dxe^{3}+\left(acd^{2}x^{2}+a^{2}d^{2}\right)e^{2}\right)}\right)+2\left(3ac^{2}d^{5}x^{2}e+14a^{2}cd^{4}xe^{2}-3a^{3}dx^{2}e^{5}+2a^{3}d^{2}xe^{4}+8\left(a^{2}cd^{3}x^{2}+a^{3}d^{3}\right)e^{3}\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\right)e^{(-2)}}{48\,a^{2}d^{3}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6*x^3 - 3*a*c^2*d^4*x^3*e^2 + 3*a^2*c*d^2*x^3*e^4 - a^3*x^
3*e^6)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8
*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*
d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2)
 + 4*(3*a*c^2*d^5*x^2*e + 14*a^2*c*d^4*x*e^2 - 3*a^3*d*x^2*e^5 + 2*a^3*d^2*
x*e^4 + 8*(a^2*c*d^3*x^2 + a^3*d^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
+ a*d)*e))*e^(-2)/(a^2*d^3*x^3), -1/48*(3*(c^3*d^6*x^3 - 3*a*c^2*d^4*x^3*e^
2 + 3*a^2*c*d^2*x^3*e^4 - a^3*x^3*e^6)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a
*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/
(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(3*a*c^2*d^5
*x^2*e + 14*a^2*c*d^4*x*e^2 - 3*a^3*d*x^2*e^5 + 2*a^3*d^2*x*e^4 + 8*(a^2*c*
d^3*x^2 + a^3*d^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)
/(a^2*d^3*x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**4/(e*x+d),x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(185) = 370$.
time = 1.40, size = 1022, normalized size = 4.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -1/8*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))
*e^(-1)/(sqrt(-a*d*e)*a*d^2) + 1/24*(3*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))*a^2*c^3*d^8*e^2 - 8*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a*c^3*d^7*e - 3*(sqrt(c*d)*x
*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*c^3*d^6 - 48*(sqr
t(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)
*a*c^2*d^5*e^(3/2) - 9*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e))*a^3*c^2*d^6*e^4 - 72*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e
+ c*d^2*x + a*x*e^2 + a*d*e))^3*a^2*c^2*d^5*e^3 - 39*(sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a*c^2*d^4*e^2 - 16*sqrt(c*d
)*a^4*c*d^5*e^(11/2) - 48*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
 a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^3*c*d^4*e^(9/2) - 96*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^2*c*d^3*e^(7/2
) - 39*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*
a^4*c*d^4*e^6 - 72*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^
2 + a*d*e))^3*a^3*c*d^3*e^5 - 9*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d
^2*x + a*x*e^2 + a*d*e))^5*a^2*c*d^2*e^4 - 48*(sqrt(c*d)*x*e^(1/2) - sqrt(c
*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^4*d^2*e^(13/2) - 3*(sq
rt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^5*d^2*e^
8 - 8*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3
*a^4*d*e^7 + 3*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
a*d*e))^5*a^3*e^6)*e^(-1)/((a*d*e - (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
 c*d^2*x + a*x*e^2 + a*d*e))^2)^3*a*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)), x)
```

3.454 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=295

$$\frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64a^2d^3e^2x^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{4dx^4}$$

```
[Out] -1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^4-1/24*(3*c/a/e-5*e/d^2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3-1/128*(-a*e^2+c*d^2)^3*(5*a*e^2+
3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)+1/64*(-a*e^2+c
*d^2)*(5*a*e^2+3*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2
```

Rubi [A]
time = 0.24, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

$$-\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}}+\frac{\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64a^2d^3e^2x^2}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}-\frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] ((c*d^2 - a*e^2)*(3*c*d^2 + 5*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*a^2*d^3*e^2*x^2) - (a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*d*x^4) - (((3*c)/(a*e) - (5*e)/d^2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*x^3) - ((c*d^2 - a*e^2)^3*(3*
c*d^2 + 5*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(128*a^(5/2)*d^(7/2)*
e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
```

```
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)}\,dx = \int \frac{(ae + cdx)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^5}\,dx$$

$$= -\frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{4dx^4} - \frac{\int \frac{\left(-\frac{1}{2}ae(3cd^2-5ae^2)+acde^2x\right)\sqrt{a}}{}}{4}$$

$$= -\frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae} - \frac{5e}{d^2}\right)(ade + (cd^2 + ae^2}{24x^3}$$

$$= \frac{(cd^2 - ae^2)(3cd^2 + 5ae^2)(2ade + (cd^2 + ae^2)x)\sqrt{ade + (cd^2 + c}}{64a^2d^3e^2x^2}$$

$$= \frac{(cd^2 - ae^2)(3cd^2 + 5ae^2)(2ade + (cd^2 + ae^2)x)\sqrt{ade + (cd^2 + c}}{64a^2d^3e^2x^2}$$

$$= \frac{(cd^2 - ae^2)(3cd^2 + 5ae^2)(2ade + (cd^2 + ae^2)x)\sqrt{ade + (cd^2 + c}}{64a^2d^3e^2x^2}$$

Mathematica [A]
time = 0.57, size = 247, normalized size = 0.84

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\,(-9c^3d^6x^3+3ac^2d^4ex^2(2d+3ex)+a^2cd^2e^2x(72d^2+20dex-31e^2x^2)+a^3e^3(48d^3+8d^2ex-10de^2x^2+15e^3x^3))}{x^4} - \frac{3(cd^2-ae^2)^3(3cd^2+5ae^2)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{192a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-9*c^3*d^6*x^3
+ 3*a*c^2*d^4*e*x^2*(2*d + 3*e*x) + a^2*c*d^2*e^2*x*(72*d^2 + 20*d*e*x - 31
*e^2*x^2) + a^3*e^3*(48*d^3 + 8*d^2*e*x - 10*d*e^2*x^2 + 15*e^3*x^3)))/x^4)
 - (3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d
 + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/
(192*a^(5/2)*d^(7/2)*e^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7420 vs. 2(265) = 530.
time = 0.08, size = 7421, normalized size = 25.16

method	result	size
default	Expression too large to display	7421

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((x*e + d)*x^5), x)
```

Fricas [A]
time = 12.11, size = 721, normalized size = 2.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/768*(3*(3*c^4*d^8*x^4 - 4*a*c^3*d^6*x^4*e^2 - 6*a^2*c^2*d^4*x^4*e^4 + 1
2*a^3*c*d^2*x^4*e^6 - 5*a^4*x^4*e^8)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8
*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^2 + 2*a*d*e
)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*
d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 4*(9*a*c^3*d^7*x^3*e - 6*a^2*c^2*d^6*x^2*e
^2 - 15*a^4*d*x^3*e^7 + 10*a^4*d^2*x^2*e^6 + (31*a^3*c*d^3*x^3 - 8*a^4*d^3*
x)*e^5 - 4*(5*a^3*c*d^4*x^2 + 12*a^4*d^4)*e^4 - 9*(a^2*c^2*d^5*x^3 + 8*a^3*
c*d^5*x)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-3)/(a^3*d^4*
x^4), 1/384*(3*(3*c^4*d^8*x^4 - 4*a*c^3*d^6*x^4*e^2 - 6*a^2*c^2*d^4*x^4*e^4
 + 12*a^3*c*d^2*x^4*e^6 - 5*a^4*x^4*e^8)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x +
 a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e
)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(9*a*c^3*d
^7*x^3*e - 6*a^2*c^2*d^6*x^2*e^2 - 15*a^4*d*x^3*e^7 + 10*a^4*d^2*x^2*e^6 +
(31*a^3*c*d^3*x^3 - 8*a^4*d^3*x)*e^5 - 4*(5*a^3*c*d^4*x^2 + 12*a^4*d^4)*e^4
 - 9*(a^2*c^2*d^5*x^3 + 8*a^3*c*d^5*x)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x
^2 + a*d)*e))*e^(-3)/(a^3*d^4*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. 2(262) = 524.
time = 1.75, size = 1640, normalized size = 5.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/64*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 -
5*a^4*e^8)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^
2 + a*d*e))/sqrt(-a*d*e))*e^(-2)/(sqrt(-a*d*e)*a^2*d^3) - 1/192*(9*(sqrt(c*
d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^3*c^4*d^11*e^
3 - 33*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^
3*a^2*c^4*d^10*e^2 - 33*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))^5*a*c^4*d^9*e + 9*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))^7*c^4*d^8 - 384*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^2*c^3*d^8*e^(5/2) - 12*(
sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^4*c^3*
d^9*e^5 - 724*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a
*d*e))^3*a^3*c^3*d^8*e^4 - 596*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^
2*x + a*x*e^2 + a*d*e))^5*a^2*c^3*d^7*e^3 - 12*(sqrt(c*d)*x*e^(1/2) - sqrt(
c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a*c^3*d^6*e^2 - 768*(sqrt(c*d)*x*
e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^4*c^2*
d^7*e^(11/2) - 1536*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e
^2 + a*d*e))^4*sqrt(c*d)*a^3*c^2*d^6*e^(9/2) - 384*(sqrt(c*d)*x*e^(1/2) - s
qrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^2*c^2*d^5*e^(7/2)
 - 402*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*
a^5*c^2*d^7*e^7 - 1854*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e))^3*a^4*c^2*d^6*e^6 - 1086*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^
2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^3*c^2*d^5*e^5 - 18*(sqrt(c*d)*x*e^(1/
2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^2*c^2*d^4*e^4 - 128*s
qrt(c*d)*a^6*c*d^6*e^(17/2) - 1024*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^5*c*d^5*e^(15/2) - 1536*(sqrt(c*d
```

```
)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^4*
c*d^4*e^(13/2) - 348*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))*a^6*c*d^5*e^9 - 900*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c
*d^2*x + a*x*e^2 + a*d*e))^3*a^5*c*d^4*e^8 - 132*(sqrt(c*d)*x*e^(1/2) - sqr
t(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^4*c*d^3*e^7 + 36*(sqrt(c*d)*x
*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^3*c*d^2*e^6 - 3
84*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sq
rt(c*d)*a^6*d^3*e^(19/2) - 15*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2
*x + a*x*e^2 + a*d*e))*a^7*d^3*e^11 - 73*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^
2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^6*d^2*e^10 + 55*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^5*d*e^9 - 15*(sqrt(c*d)*
x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^4*e^8)*e^(-2)/
((a*d*e - (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e
))^2)^4*a^2*d^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)), x)
```

3.455 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=395

$$-\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^3d^4e^3x^2}-\frac{(ade+(cd$$

```
[Out] -1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^5-1/40*(3*c/a/e-7*e/d^2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4+1/240*(-35*a^2*e^4+12*a*c*d^2*e^
2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^3+1/256
*(-a*e^2+c*d^2)^3*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+
(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/128*(-a*e^2+c*d^2)*(7*a^2*e^4+6*a*c*d^2*e^
2+3*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^3/d^4/e^3/x^2
```

Rubi [A]
time = 0.32, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

$$\frac{\left(-35a^2e^4+12acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{240a^2d^3e^2x^3}+\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256a^{7/2}d^{9/2}e^{7/2}}-\frac{\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^3d^4e^3x^2}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}-\frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{40x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

```
[Out] -1/128*((c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a^3*d^4*e^
3*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(5*d*x^5) - (((3*c)/
(a*e) - (7*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(40*x^4)
+ ((15*c^2*d^4 + 12*a*c*d^2*e^2 - 35*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2))/(240*a^2*d^3*e^2*x^3) + ((c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6
*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*
Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(256*a^(7/2)
*d^(9/2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^6(d + ex)}\,dx = \int \frac{(ae + cdx)\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^6}\,dx$$

$$= -\frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{5dx^5} - \frac{\int \frac{\left(-\frac{1}{2}ae(3cd^2 - 7ae^2) + 2acde^2x\right)\sqrt{}}{}}{}$$

$$= -\frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae} - \frac{7e}{d^2}\right)(ade + (cd^2 + ae^2}{40x^4}$$

$$= -\frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae} - \frac{7e}{d^2}\right)(ade + (cd^2 + ae^2}{40x^4}$$

$$= -\frac{(cd^2 - ae^2)\,(3c^2d^4 + 6acd^2e^2 + 7a^2e^4)\,(2ade + (cd^2 + ae^2)\,x)\,\sqrt{}}{128a^3d^4e^3x^2}$$

$$= -\frac{(cd^2 - ae^2)\,(3c^2d^4 + 6acd^2e^2 + 7a^2e^4)\,(2ade + (cd^2 + ae^2)\,x)\,\sqrt{}}{128a^3d^4e^3x^2}$$

$$= -\frac{(cd^2 - ae^2)\,(3c^2d^4 + 6acd^2e^2 + 7a^2e^4)\,(2ade + (cd^2 + ae^2)\,x)\,\sqrt{}}{128a^3d^4e^3x^2}$$

Mathematica [**A**]
time = 0.84, size = 317, normalized size = 0.80

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\,(45c^4d^8x^4-30ac^3d^6ex^3(d+ex)+6a^2c^2d^4e^2x^2(4d^2+3dex-6e^2x^2)+2a^3cd^2e^3x(264d^3+48d^2ex-61de^2x^2+95e^3x^3)+a^4e^4(384d^4+48d^3ex-56d^2e^2x^2+70de^3x^3-105e^4x^4))}{x^5}+\frac{15(cd^2-ae^2)^3(3c^2d^4+6acd^2e^2+7a^2e^4)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{1920a^{7/2}d^{9/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(45*c^4*d^8*x^4
- 30*a*c^3*d^6*e*x^3*(d + e*x) + 6*a^2*c^2*d^4*e^2*x^2*(4*d^2 + 3*d*e*x - 6
*e^2*x^2) + 2*a^3*c*d^2*e^3*x*(264*d^3 + 48*d^2*e*x - 61*d*e^2*x^2 + 95*e^3
*x^3) + a^4*e^4*(384*d^4 + 48*d^3*e*x - 56*d^2*e^2*x^2 + 70*d*e^3*x^3 - 105
*e^4*x^4)))/x^5) + (15*(c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2
*e^4)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])
/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(1920*a^(7/2)*d^(9/2)*e^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10574 vs. $2(361) = 722$.
time = 0.08, size = 10575, normalized size = 26.77

method	result	size
default	Expression too large to display	10575

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((x*e + d)*x^6), x)
```

Fricas [A]
time = 27.84, size = 907, normalized size = 2.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(3*c^5*d^10*x^5 - 3*a*c^4*d^8*x^5*e^2 - 2*a^2*c^3*d^6*x^5*e^4
- 6*a^3*c^2*d^4*x^5*e^6 + 15*a^4*c*d^2*x^5*e^8 - 7*a^5*x^5*e^10)*sqrt(a*d)*
e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*
(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*s
qrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) + 4*(45*a*c^4*d^
9*x^4*e - 30*a^2*c^3*d^8*x^3*e^2 - 105*a^5*d*x^4*e^9 + 70*a^5*d^2*x^3*e^8 +
 2*(95*a^4*c*d^3*x^4 - 28*a^5*d^3*x^2)*e^7 - 2*(61*a^4*c*d^4*x^3 - 24*a^5*d
^4*x)*e^6 - 12*(3*a^3*c^2*d^5*x^4 - 8*a^4*c*d^5*x^2 - 32*a^5*d^5)*e^5 + 6*(
3*a^3*c^2*d^6*x^3 + 88*a^4*c*d^6*x)*e^4 - 6*(5*a^2*c^3*d^7*x^4 - 4*a^3*c^2*
d^7*x^2)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-4)/(a^4*d^5*
x^5), -1/3840*(15*(3*c^5*d^10*x^5 - 3*a*c^4*d^8*x^5*e^2 - 2*a^2*c^3*d^6*x^5
*e^4 - 6*a^3*c^2*d^4*x^5*e^6 + 15*a^4*c*d^2*x^5*e^8 - 7*a^5*x^5*e^10)*sqrt(
-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (
c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 +
a^2*d^2)*e^2)) + 2*(45*a*c^4*d^9*x^4*e - 30*a^2*c^3*d^8*x^3*e^2 - 105*a^5*d
```

```
*x^4*e^9 + 70*a^5*d^2*x^3*e^8 + 2*(95*a^4*c*d^3*x^4 - 28*a^5*d^3*x^2)*e^7 -
 2*(61*a^4*c*d^4*x^3 - 24*a^5*d^4*x)*e^6 - 12*(3*a^3*c^2*d^5*x^4 - 8*a^4*c*
d^5*x^2 - 32*a^5*d^5)*e^5 + 6*(3*a^3*c^2*d^6*x^3 + 88*a^4*c*d^6*x)*e^4 - 6*
(5*a^2*c^3*d^7*x^4 - 4*a^3*c^2*d^7*x^2)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e))*e^(-4)/(a^4*d^5*x^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. 2(354) = 708.
time = 1.52, size = 2381, normalized size = 6.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -1/128*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2*d^4*e^
6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*
x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))*e^(-3)/(sqrt(-a*d*e)*a^3*
d^4) + 1/1920*(45*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
 + a*d*e))*a^4*c^5*d^14*e^4 - 210*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c
*d^2*x + a*x*e^2 + a*d*e))^3*a^3*c^5*d^13*e^3 - 384*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^2*c^5*d^12*e^2 + 210*(sqrt
(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a*c^5*d^11
*e - 45*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))
^9*c^5*d^10 - 3840*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^
2 + a*d*e))^4*sqrt(c*d)*a^3*c^4*d^11*e^(7/2) - 45*(sqrt(c*d)*x*e^(1/2) - sq
rt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^5*c^4*d^12*e^6 - 7470*(sqrt(c*
d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^4*c^4*d^11*
e^5 - 9600*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*
e))^5*a^3*c^4*d^10*e^4 - 210*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*
x + a*x*e^2 + a*d*e))^7*a^2*c^4*d^9*e^3 + 45*(sqrt(c*d)*x*e^(1/2) - sqrt(c*
d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a*c^4*d^8*e^2 - 7680*(sqrt(c*d)*x*e
^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^5*c^3*d
^10*e^(13/2) - 26880*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))^4*sqrt(c*d)*a^4*c^3*d^9*e^(11/2) - 11520*(sqrt(c*d)*x*e^(1/2)
```

```
 - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^3*c^3*d^8*e^(
9/2) - 3870*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d
*e))*a^6*c^3*d^10*e^8 - 34420*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2
*x + a*x*e^2 + a*d*e))^3*a^5*c^3*d^9*e^7 - 37120*(sqrt(c*d)*x*e^(1/2) - sqr
t(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^4*c^3*d^8*e^6 - 5260*(sqrt(c*
d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^3*c^3*d^7*e
^5 + 30*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))
^9*a^2*c^3*d^6*e^4 - 768*sqrt(c*d)*a^7*c^2*d^9*e^(19/2) - 23040*(sqrt(c*d)*
x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^6*c^
2*d^8*e^(17/2) - 53760*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e))^4*sqrt(c*d)*a^5*c^2*d^7*e^(15/2) - 19200*(sqrt(c*d)*x*e^(1/
2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^4*c^2*d^6*e
^(13/2) - 7770*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
a*d*e))*a^7*c^2*d^8*e^10 - 41820*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*
d^2*x + a*x*e^2 + a*d*e))^3*a^6*c^2*d^7*e^9 - 30720*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^5*c^2*d^6*e^8 - 420*(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^4*c^2*d^5
*e^7 + 90*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e
))^9*a^3*c^2*d^4*e^6 - 1280*sqrt(c*d)*a^8*c*d^7*e^(23/2) - 16640*(sqrt(c*d)
*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^7*c
*d^6*e^(21/2) - 28160*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e))^4*sqrt(c*d)*a^6*c*d^5*e^(19/2) - 3615*(sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^8*c*d^6*e^12 - 12570*(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^7*c*d^5*e
^11 - 1920*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*
e))^5*a^6*c*d^4*e^10 + 1050*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
 + a*x*e^2 + a*d*e))^7*a^5*c*d^3*e^9 - 225*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*
x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^4*c*d^2*e^8 - 3840*(sqrt(c*d)*x*e^(
1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^8*d^4*e^(
25/2) - 105*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d
*e))*a^9*d^4*e^14 - 790*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))^3*a^8*d^3*e^13 + 896*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e
 + c*d^2*x + a*x*e^2 + a*d*e))^5*a^7*d^2*e^12 - 490*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^6*d*e^11 + 105*(sqrt(c*d)*
x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^5*e^10)*e^(-3)
/((a*d*e - (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*
e))^2)^5*a^3*d^4)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^6\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)), x)
```

3.456 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=498

$$\frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512a^4d^5e^4x^2}-\frac{\left(ade+\left(cd^2\right.\right.}{}$$

```
[Out] -1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^6-1/20*(c/a/e-3*e/d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5+1/160*(-21*a^2*e^4+6*a*c*d^2*e^2+7
*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/960*(-1
05*a^3*e^6+21*a^2*c*d^2*e^4+33*a*c^2*d^4*e^2+35*c^3*d^6)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^3-1/1024*(-a*e^2+c*d^2)^3*(21*a^3*e^6+2
1*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d
^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(
9/2)/d^(11/2)/e^(9/2)+1/512*(-21*a^4*e^8+6*a^2*c^2*d^4*e^4+8*a*c^3*d^6*e^2+
7*c^4*d^8)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/a^4/d^5/e^4/x^2
```

Rubi [A]
time = 0.45, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$,
Rules used = {863, 848, 820, 734, 738, 212}

$$\frac{\left(-21a^2e^4+6acd^2e^2+7c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4}+\frac{\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512a^4d^5e^4x^2}-\frac{\left(-105a^3e^6+21a^2cd^2e^4+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3}-\frac{\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\left(cd^2-ae^2\right)^3\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024a^{9/2}d^{11/2}e^{9/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] ((7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^4*e^8)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*a^4*d^
5*e^4*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(6*d*x^6) - ((c/
(a*e) - (3*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(20*x^5)
+ ((7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(160*a^2*d^3*e^2*x^4) - ((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 2
1*a^2*c*d^2*e^4 - 105*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
))/(960*a^3*d^4*e^3*x^3) - ((c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2
 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*
Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(102
4*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
```

```
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx = \int \frac{(ae+cdx)\sqrt{ade + \left(cd^2 + ae^2\right)x + cdex^2}}{x^7}\,dx$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{6dx^6} - \frac{\int \frac{\left(-\frac{3}{2}ae\left(cd^2-3ae^2\right)+3acde^2x\right)\sqrt{ad}}{x}}{6a}$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right)\left(ade + \left(cd^2 + ae^2\right)\right.}{20x^5}$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right)\left(ade + \left(cd^2 + ae^2\right)\right.}{20x^5}$$

$$= -\frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right)\left(ade + \left(cd^2 + ae^2\right)\right.}{20x^5}$$

$$= \frac{\left(7c^4d^8 + 8ac^3d^6e^2 + 6a^2c^2d^4e^4 - 21a^4e^8\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{}}{512a^4d^5e^4x^2}$$

$$= \frac{\left(7c^4d^8 + 8ac^3d^6e^2 + 6a^2c^2d^4e^4 - 21a^4e^8\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{}}{512a^4d^5e^4x^2}$$

$$= \frac{\left(7c^4d^8 + 8ac^3d^6e^2 + 6a^2c^2d^4e^4 - 21a^4e^8\right)\left(2ade + \left(cd^2 + ae^2\right)x\right)\sqrt{}}{512a^4d^5e^4x^2}$$

Mathematica [A]
time = 1.17, size = 402, normalized size = 0.81

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-105c^5d^{10}x^5+5ac^4d^8ex^4(14d+11ex)-2a^2c^3d^6e^2x^3\left(28d^2+16dex-27e^2x^2\right)+6a^3c^2d^4e^3x^2\left(8d^3+4d^2ex-6de^2x^2+13e^3x^3\right)+a^4cd^2e^4x\left(1664d^4+224d^3ex-264d^2e^2x^2+336de^3x^3-525e^4x^4\right)+a^5e^5\left(1280d^5+128d^4ex-144d^3e^2x^2+168d^2e^3x^3-210de^4x^4+315e^5x^5\right)\right)}{x^6}-\frac{15\left(cd^2-ae^2\right)^3\left(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{7680a^{9/2}d^{11/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^5*d^10*x
^5 + 5*a*c^4*d^8*e*x^4*(14*d + 11*e*x) - 2*a^2*c^3*d^6*e^2*x^3*(28*d^2 + 16
*d*e*x - 27*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(8*d^3 + 4*d^2*e*x - 6*d*e^2*x
^2 + 13*e^3*x^3) + a^4*c*d^2*e^4*x*(1664*d^4 + 224*d^3*e*x - 264*d^2*e^2*x^
2 + 336*d*e^3*x^3 - 525*e^4*x^4) + a^5*e^5*(1280*d^5 + 128*d^4*e*x - 144*d^
3*e^2*x^2 + 168*d^2*e^3*x^3 - 210*d*e^4*x^4 + 315*e^5*x^5)))/x^6) - (15*(c*
```

```
d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^
6)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(S
qrt[a*e + c*d*x]*Sqrt[d + e*x])))/(7680*a^(9/2)*d^(11/2)*e^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 16882 vs. $2(460) = 920$.
time = 0.09, size = 16883, normalized size = 33.90

method	result	size
default	Expression too large to display	16883

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((x*e + d)*x^7), x)
```

Fricas [A]
time = 90.62, size = 1117, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(7*c^6*d^12*x^6 - 6*a*c^5*d^10*x^6*e^2 - 3*a^2*c^4*d^8*x^6*e^
4 - 4*a^3*c^3*d^6*x^6*e^6 - 15*a^4*c^2*d^4*x^6*e^8 + 42*a^5*c*d^2*x^6*e^10
- 21*a^6*x^6*e^12)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2
*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a
*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^
2)*e^2)/x^2) - 4*(105*a*c^5*d^11*x^5*e - 70*a^2*c^4*d^10*x^4*e^2 - 315*a^6*
d*x^5*e^11 + 210*a^6*d^2*x^4*e^10 + 21*(25*a^5*c*d^3*x^5 - 8*a^6*d^3*x^3)*e
^9 - 48*(7*a^5*c*d^4*x^4 - 3*a^6*d^4*x^2)*e^8 - 2*(39*a^4*c^2*d^5*x^5 - 132
*a^5*c*d^5*x^3 + 64*a^6*d^5*x)*e^7 + 4*(9*a^4*c^2*d^6*x^4 - 56*a^5*c*d^6*x^
```

```
2 - 320*a^6*d^6)*e^6 - 2*(27*a^3*c^3*d^7*x^5 + 12*a^4*c^2*d^7*x^3 + 832*a^5
*c*d^7*x)*e^5 + 16*(2*a^3*c^3*d^8*x^4 - 3*a^4*c^2*d^8*x^2)*e^4 - (55*a^2*c^
4*d^9*x^5 - 56*a^3*c^3*d^9*x^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*
d)*e))*e^(-5)/(a^5*d^6*x^6), 1/15360*(15*(7*c^6*d^12*x^6 - 6*a*c^5*d^10*x^6
*e^2 - 3*a^2*c^4*d^8*x^6*e^4 - 4*a^3*c^3*d^6*x^6*e^6 - 15*a^4*c^2*d^4*x^6*e
^8 + 42*a^5*c*d^2*x^6*e^10 - 21*a^6*x^6*e^12)*sqrt(-a*d*e)*arctan(1/2*(c*d^
2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-
a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(105*
a*c^5*d^11*x^5*e - 70*a^2*c^4*d^10*x^4*e^2 - 315*a^6*d*x^5*e^11 + 210*a^6*d
^2*x^4*e^10 + 21*(25*a^5*c*d^3*x^5 - 8*a^6*d^3*x^3)*e^9 - 48*(7*a^5*c*d^4*x
^4 - 3*a^6*d^4*x^2)*e^8 - 2*(39*a^4*c^2*d^5*x^5 - 132*a^5*c*d^5*x^3 + 64*a^
6*d^5*x)*e^7 + 4*(9*a^4*c^2*d^6*x^4 - 56*a^5*c*d^6*x^2 - 320*a^6*d^6)*e^6 -
 2*(27*a^3*c^3*d^7*x^5 + 12*a^4*c^2*d^7*x^3 + 832*a^5*c*d^7*x)*e^5 + 16*(2*
a^3*c^3*d^8*x^4 - 3*a^4*c^2*d^8*x^2)*e^4 - (55*a^2*c^4*d^9*x^5 - 56*a^3*c^3
*d^9*x^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-5)/(a^5*d^6
*x^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**7/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3289 vs. $2(449) = 898$.
time = 3.62, size = 3289, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/512*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^
6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*arctan(-(sqrt(c*d
)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))*e^
(-4)/(sqrt(-a*d*e)*a^4*d^5) - 1/7680*(105*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x
^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^5*c^6*d^17*e^5 - 595*(sqrt(c*d)*x*e^(1
/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^4*c^6*d^16*e^4 - 168
6*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^3
*c^6*d^15*e^3 + 1386*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))^7*a^2*c^6*d^14*e^2 - 595*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
```

e + c*d^2*x + a*x*e^2 + a*d*e))^9*a*c^6*d^13*e + 105*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*c^6*d^12 - 15360*(sqrt(c*d
)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^4*
c^5*d^14*e^(9/2) - 90*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e))*a^6*c^5*d^15*e^7 - 30210*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2
*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^5*c^5*d^14*e^6 - 53412*(sqrt(c*d)*x*e^
(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^4*c^5*d^13*e^5 - 1
188*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a
^3*c^5*d^12*e^4 + 510*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e))^9*a^2*c^5*d^11*e^3 - 90*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))^11*a*c^5*d^10*e^2 - 30720*(sqrt(c*d)*x*e^(1
/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^6*c^4*d^13
e^(15/2) - 153600(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^
2 + a*d*e))^4*sqrt(c*d)*a^5*c^4*d^12*e^(13/2) - 97280*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^4*c^4*d^11*e^(
11/2) - 15405*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a
*d*e))*a^7*c^4*d^13*e^9 - 199425*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*
d^2*x + a*x*e^2 + a*d*e))^3*a^6*c^4*d^12*e^8 - 332370*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^5*c^4*d^11*e^7 - 86610*(
sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^4*c^
4*d^10*e^6 + 255*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
+ a*d*e))^9*a^3*c^4*d^9*e^5 - 45*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*
d^2*x + a*x*e^2 + a*d*e))^11*a^2*c^4*d^8*e^4 - 3072*sqrt(c*d)*a^8*c^3*d^12*
e^(21/2) - 135168*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
+ a*d*e))^2*sqrt(c*d)*a^7*c^3*d^11*e^(19/2) - 506880*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^6*c^3*d^10*e^(
17/2) - 337920*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
a*d*e))^6*sqrt(c*d)*a^5*c^3*d^9*e^(15/2) - 30720*(sqrt(c*d)*x*e^(1/2) - sqr
t(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(c*d)*a^4*c^3*d^8*e^(13/2)
- 46140*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))
*a^8*c^3*d^11*e^11 - 419500*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))^3*a^7*c^3*d^10*e^10 - 581400*(sqrt(c*d)*x*e^(1/2) - sq
rt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^6*c^3*d^9*e^9 - 135960*(sqrt
(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^5*c^3*d^
8*e^8 + 340*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d
*e))^9*a^4*c^3*d^7*e^7 - 60*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))^11*a^3*c^3*d^6*e^6 - 6144*sqrt(c*d)*a^9*c^2*d^10*e^(25
/2) - 193536*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*
d*e))^2*sqrt(c*d)*a^8*c^2*d^9*e^(23/2) - 552960*(sqrt(c*d)*x*e^(1/2) - sqrt
(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^7*c^2*d^8*e^(21/2) -
261120*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))
^6*sqrt(c*d)*a^6*c^2*d^7*e^(19/2) - 46305*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x
^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^9*c^2*d^9*e^13 - 305925*(sqrt(c*d)*x*e
^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^8*c^2*d^8*e^12 -
279450*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^

```
5*a^7*c^2*d^7*e^11 - 2970*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
 a*x*e^2 + a*d*e))^7*a^6*c^2*d^6*e^10 + 1275*(sqrt(c*d)*x*e^(1/2) - sqrt(c*
d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^5*c^2*d^5*e^9 - 225*(sqrt(c*d)*x*
e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*a^4*c^2*d^4*e^8 -
 5120*sqrt(c*d)*a^10*c*d^8*e^(29/2) - 92160*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^9*c*d^7*e^(27/2) - 18432
0*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqr
t(c*d)*a^8*c*d^6*e^(25/2) - 14730*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c
*d^2*x + a*x*e^2 + a*d*e))*a^10*c*d^7*e^15 - 65010*(sqrt(c*d)*x*e^(1/2) - s
qrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^9*c*d^6*e^14 - 10116*(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^8*c*d^5*e
^13 + 8316*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*
e))^7*a^7*c*d^4*e^12 - 3570*(sqrt(c*d)*x*e^(1/2...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^7\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)), x)
```

3.457 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=574

$$-\frac{3(cd^2-ae^2)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{16384c^5d^5e^6}$$

```
[Out] 1/2048*(-a*e^2+c*d^2)*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*
d^6)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^
4/e^5+1/112*(5*a/c/d-11*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+
1/8*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-1/4480*(231*c^3*d^6-15*a*
c^2*d^4*e^2-95*a^2*c*d^2*e^4-105*a^3*e^6-10*c*d*e*(-15*a^2*e^4-10*a*c*d^2*e
^2+33*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/e^4+3/327
68*(-a*e^2+c*d^2)^5*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*d^
6)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(11/2)/d^(11/2)/e^(13/2)-3/16384*(-a*e^2+c*d
^2)^3*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33*c^3*d^6)*(2*c*d*e*x+
a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e^6
```

Rubi [A]

time = 0.43, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 846, 793, 626, 635, 212}

$$-\frac{\left(-105a^3e^6-10cdex\left(-15a^2e^4-10acd^2e^2+33c^2d^4\right)-95a^2cd^2e^4-15ac^2d^4e^2+231c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{4480c^3d^3e^4}+\frac{3\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{32768c^{11/2}d^{11/2}e^{13/2}}-\frac{3\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16384c^5d^5e^6}+\frac{\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2048c^4d^4e^5}+\frac{1}{112}x^2\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}+\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 1
5*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(16384*c^5*d^5*e^6) + ((c*d^2 - a*e^2)*(33*c^3*d^6 + 45*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*c^4*d^4*e^5) + (((5*a)/(c*d) - (
11*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/112 + (x^3*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*e) - ((231*c^3*d^6 - 15*a*c
^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*c*d*e*(33*c^2*d^4 - 10*a*c
*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4
480*c^3*d^3*e^4) + (3*(c*d^2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35
*a^2*c*d^2*e^4 + 15*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*c^(1
1/2)*d^(11/2)*e^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \int x^3(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}\,dx$$

$$= \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{8e} + \frac{\int x^2\left(-3acd^2e-\frac{1}{2}cd(11c\right.}{}$$

$$= \frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} + \frac{x^3(ade}{}$$

$$= \frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} + \frac{x^3(ade}{}$$

$$= \frac{(cd^2-ae^2)\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)(cd^2+a}{2048c^4d^4e^5}$$

$$= -\frac{3(cd^2-ae^2)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)(cd^2}{16384c^5d^5e^6}$$

$$= -\frac{3(cd^2-ae^2)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)(cd^2}{16384c^5d^5e^6}$$

$$= -\frac{3(cd^2-ae^2)^3\left(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6\right)(cd^2}{16384c^5d^5e^6}$$

Mathematica [A]
time = 1.49, size = 549, normalized size = 0.96

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(1575*a^7*e^14 - 52
5*a^6*c*d*e^12*(7*d + 2*e*x) + 35*a^5*c^2*d^2*e^10*(29*d^2 + 68*d*e*x + 24*
e^2*x^2) + 5*a^4*c^3*d^3*e^8*(185*d^3 - 110*d^2*e*x - 376*d*e^2*x^2 - 144*e
^3*x^3) + 5*a^3*c^4*d^4*e^6*(265*d^4 - 120*d^3*e*x + 80*d^2*e^2*x^2 + 320*d
*e^3*x^3 + 128*e^4*x^4) + a^2*c^5*d^5*e^4*(-11193*d^5 + 7034*d^4*e*x - 5488
*d^3*e^2*x^2 + 4640*d^2*e^3*x^3 + 137600*d*e^4*x^4 + 103680*e^5*x^5) + a*c^
6*d^6*e^2*(11445*d^6 - 7476*d^5*e*x + 5928*d^4*e^2*x^2 - 5056*d^3*e^3*x^3 +
 4480*d^2*e^4*x^4 + 212480*d*e^5*x^5 + 168960*e^6*x^6) + c^7*d^7*(-3465*d^7
 + 2310*d^6*e*x - 1848*d^5*e^2*x^2 + 1584*d^4*e^3*x^3 - 1408*d^3*e^4*x^4 +
1280*d^2*e^5*x^5 + 87040*d*e^6*x^6 + 71680*e^7*x^7)) + (105*(c*d^2 - a*e^2)
```

```
^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*ArcTanh[
(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[a*e + c
*d*x]*Sqrt[d + e*x])))/(573440*c^(11/2)*d^(11/2)*e^(13/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1894 vs. 2(536) = 1072.
time = 0.07, size = 1895, normalized size = 3.30

method	result	size
default	Expression too large to display	1895

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/8*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-9/16*(a*e^2+c*d^2)
/c/d/e*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-1/2*(a*e^2+c*d^2)
/c/d/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*
d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+
1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(
c*d*e)^(1/2)))))-1/8*a/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(
2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/
d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))))-d/e^2*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(7/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/d/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/
e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+
c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*
e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))))+d^2/e^3*(1/12*(2*c*d*e*x
+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4
*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d
*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))))-d^3/e^4
*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*
(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
```

```
^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*
(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln(
(1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c
*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 2.23, size = 1485, normalized size = 2.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2293760*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4 -
40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c^2*
d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3
*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*
x^2 + 3*a*c*d^2)*e^2) + 4*(2310*c^8*d^14*x*e^2 - 3465*c^8*d^15*e - 1050*a^6
*c^2*d^2*x*e^14 + 1575*a^7*c*d*e^15 + 105*(8*a^5*c^3*d^3*x^2 - 35*a^6*c^2*d
^3)*e^13 - 20*(36*a^4*c^4*d^4*x^3 - 119*a^5*c^3*d^4*x)*e^12 + 5*(128*a^3*c^
5*d^5*x^4 - 376*a^4*c^4*d^5*x^2 + 203*a^5*c^3*d^5)*e^11 + 10*(10368*a^2*c^6
*d^6*x^5 + 160*a^3*c^5*d^6*x^3 - 55*a^4*c^4*d^6*x)*e^10 + 5*(33792*a*c^7*d^
7*x^6 + 27520*a^2*c^6*d^7*x^4 + 80*a^3*c^5*d^7*x^2 + 185*a^4*c^4*d^7)*e^9 +
 40*(1792*c^8*d^8*x^7 + 5312*a*c^7*d^8*x^5 + 116*a^2*c^6*d^8*x^3 - 15*a^3*c
^5*d^8*x)*e^8 + (87040*c^8*d^9*x^6 + 4480*a*c^7*d^9*x^4 - 5488*a^2*c^6*d^9*
x^2 + 1325*a^3*c^5*d^9)*e^7 + 2*(640*c^8*d^10*x^5 - 2528*a*c^7*d^10*x^3 + 3
517*a^2*c^6*d^10*x)*e^6 - (1408*c^8*d^11*x^4 - 5928*a*c^7*d^11*x^2 + 11193*
a^2*c^6*d^11)*e^5 + 12*(132*c^8*d^12*x^3 - 623*a*c^7*d^12*x)*e^4 - 21*(88*c
^8*d^13*x^2 - 545*a*c^7*d^13)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e))*e^(-7)/(c^6*d^6), -1/1146880*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 +
```

```
140*a^2*c^6*d^12*e^4 - 40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3
*d^6*e^10 - 20*a^6*c^2*d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(-c*
d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*
d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d
^2)*e^2)) - 2*(2310*c^8*d^14*x*e^2 - 3465*c^8*d^15*e - 1050*a^6*c^2*d^2*x*e
^14 + 1575*a^7*c*d*e^15 + 105*(8*a^5*c^3*d^3*x^2 - 35*a^6*c^2*d^3)*e^13 - 2
0*(36*a^4*c^4*d^4*x^3 - 119*a^5*c^3*d^4*x)*e^12 + 5*(128*a^3*c^5*d^5*x^4 -
376*a^4*c^4*d^5*x^2 + 203*a^5*c^3*d^5)*e^11 + 10*(10368*a^2*c^6*d^6*x^5 + 1
60*a^3*c^5*d^6*x^3 - 55*a^4*c^4*d^6*x)*e^10 + 5*(33792*a*c^7*d^7*x^6 + 2752
0*a^2*c^6*d^7*x^4 + 80*a^3*c^5*d^7*x^2 + 185*a^4*c^4*d^7)*e^9 + 40*(1792*c^
8*d^8*x^7 + 5312*a*c^7*d^8*x^5 + 116*a^2*c^6*d^8*x^3 - 15*a^3*c^5*d^8*x)*e^
8 + (87040*c^8*d^9*x^6 + 4480*a*c^7*d^9*x^4 - 5488*a^2*c^6*d^9*x^2 + 1325*a
^3*c^5*d^9)*e^7 + 2*(640*c^8*d^10*x^5 - 2528*a*c^7*d^10*x^3 + 3517*a^2*c^6*
d^10*x)*e^6 - (1408*c^8*d^11*x^4 - 5928*a*c^7*d^11*x^2 + 11193*a^2*c^6*d^11
)*e^5 + 12*(132*c^8*d^12*x^3 - 623*a*c^7*d^12*x)*e^4 - 21*(88*c^8*d^13*x^2
- 545*a*c^7*d^13)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-7)/
(c^6*d^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.21, size = 738, normalized size = 1.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/573440*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(10*(4*(14
*c^2*d^2*x*e + (17*c^9*d^10*e^7 + 33*a*c^8*d^8*e^9)*e^(-7)/(c^7*d^7))*x + (
c^9*d^11*e^6 + 166*a*c^8*d^9*e^8 + 81*a^2*c^7*d^7*e^10)*e^(-7)/(c^7*d^7))*x
 - (11*c^9*d^12*e^5 - 35*a*c^8*d^10*e^7 - 1075*a^2*c^7*d^8*e^9 - 5*a^3*c^6*
d^6*e^11)*e^(-7)/(c^7*d^7))*x + (99*c^9*d^13*e^4 - 316*a*c^8*d^11*e^6 + 290
*a^2*c^7*d^9*e^8 + 100*a^3*c^6*d^7*e^10 - 45*a^4*c^5*d^5*e^12)*e^(-7)/(c^7*
d^7))*x - (231*c^9*d^14*e^3 - 741*a*c^8*d^12*e^5 + 686*a^2*c^7*d^10*e^7 - 5
0*a^3*c^6*d^8*e^9 + 235*a^4*c^5*d^6*e^11 - 105*a^5*c^4*d^4*e^13)*e^(-7)/(c^
7*d^7))*x + (1155*c^9*d^15*e^2 - 3738*a*c^8*d^13*e^4 + 3517*a^2*c^7*d^11*e^
```

```
6 - 300*a^3*c^6*d^9*e^8 - 275*a^4*c^5*d^7*e^10 + 1190*a^5*c^4*d^5*e^12 - 52
5*a^6*c^3*d^3*e^14)*e^(-7)/(c^7*d^7))*x - (3465*c^9*d^16*e - 11445*a*c^8*d^
14*e^3 + 11193*a^2*c^7*d^12*e^5 - 1325*a^3*c^6*d^10*e^7 - 925*a^4*c^5*d^8*e
^9 - 1015*a^5*c^4*d^6*e^11 + 3675*a^6*c^3*d^4*e^13 - 1575*a^7*c^2*d^2*e^15)
*e^(-7)/(c^7*d^7)) - 3/32768*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^
6*d^12*e^4 - 40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10
- 20*a^6*c^2*d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*e^(-13/2)*log(abs(
-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*
e))*sqrt(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d)*c^5*d^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
[Out] int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.458 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=452

$$\frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024c^4d^4e^5}-\frac{\left(cd^2-ae^2\right)\left(9c\right.}{}$$

```
[Out] -1/384*(-a*e^2+c*d^2)*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^2
+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4+1/7*x^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e+1/840*(63*c^2*d^4-20*a*c*d^2*e^2-35*a^2*
e^4-10*c*d*e*(-5*a*e^2+9*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/
c^2/d^2/e^3-1/2048*(-a*e^2+c*d^2)^5*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*ar
ctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2)/e^(11/2)+1/1024*(-a*e^2+c*d^2)^3*(5
*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^5
```

Rubi [A]
time = 0.27, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {865, 846, 793, 626, 635, 212}

$$\frac{\left(-35a^2e^4-10cdex(9cd^2-5ae^2)-20acd^2e^2+63c^2d^4\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{840c^2d^2e^3}-\frac{\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2048c^{9/2}d^{9/2}e^{11/2}}+\frac{\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{1024c^4d^4e^5}-\frac{\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{384c^3d^3e^4}+\frac{x^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^3*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2
+ 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*c^4*d^4*e^5
) - ((c*d^2 - a*e^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^
2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*c^3*d^3*
e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*e) + ((63*c^2
*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(840*c^2*d^2*e^3) - ((c*d^2 - a*e^
2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*
d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(2048*c^(9/2)*d^(9/2)*e^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \int x^2(ae + cdx)\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}\,dx$$

$$= \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{7e} + \frac{\int x\left(-2acd^2e - \frac{1}{2}cd(9cd^2 -\right.}{}$$

$$= \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{7e} + \frac{(63c^2d^4 - 20acd^2e^2 - 35a^2}{}$$

$$= -\frac{\left(cd^2 - ae^2\right)\left(9c^2d^4 + 10acd^2e^2 + 5a^2e^4\right)\left(cd^2 + ae^2 + 2cdex\right)(ac}{384c^3d^3e^4}$$

$$= \frac{\left(cd^2 - ae^2\right)^3\left(9c^2d^4 + 10acd^2e^2 + 5a^2e^4\right)\left(cd^2 + ae^2 + 2cdex\right)\sqrt{a}}{1024c^4d^4e^5}$$

$$= \frac{\left(cd^2 - ae^2\right)^3\left(9c^2d^4 + 10acd^2e^2 + 5a^2e^4\right)\left(cd^2 + ae^2 + 2cdex\right)\sqrt{a}}{1024c^4d^4e^5}$$

$$= \frac{\left(cd^2 - ae^2\right)^3\left(9c^2d^4 + 10acd^2e^2 + 5a^2e^4\right)\left(cd^2 + ae^2 + 2cdex\right)\sqrt{a}}{1024c^4d^4e^5}$$

Mathematica [**A**]
time = 1.24, size = 479, normalized size = 1.06

$$\frac{(cd^2-ae^2)^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(-525a^6e^{12}+350a^5cde^{10}(4d+ex)-35a^4c^2d^2e^8(15d^2+26dex+8e^2x^2)-60a^3c^3d^3e^6(10d^3-5d^2ex-12de^2x^2-4e^3x^3)+a^2c^4d^4e^4(3689d^4-2332d^3ex+1824d^2e^2x^2+33520de^3x^3+23680e^4x^4)+2ac^5d^5e^2(-1680d^5+1099d^4ex-872d^3e^2x^2+744d^2e^3x^3+24320de^4x^4+18560e^5x^5)+3c^6d^6(315d^6-210d^5ex+168d^4e^2x^2-144d^3e^3x^3+128d^2e^4x^4+6400de^5x^5+5120e^6x^6)\right)}{(cd^2-ae^2)^5(ae+cdx)(d+ex)}-\frac{105(9c^2d^4+10acd^2e^2+5a^2e^4)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{(ae+cdx)^{3/2}(d+ex)^{3/2}}\right)}{107520c^{9/2}d^{9/2}e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e
]*(-525*a^6*e^12 + 350*a^5*c*d*e^10*(4*d + e*x) - 35*a^4*c^2*d^2*e^8*(15*d^
2 + 26*d*e*x + 8*e^2*x^2) - 60*a^3*c^3*d^3*e^6*(10*d^3 - 5*d^2*e*x - 12*d*e
^2*x^2 - 4*e^3*x^3) + a^2*c^4*d^4*e^4*(3689*d^4 - 2332*d^3*e*x + 1824*d^2*e
^2*x^2 + 33520*d*e^3*x^3 + 23680*e^4*x^4) + 2*a*c^5*d^5*e^2*(-1680*d^5 + 10
99*d^4*e*x - 872*d^3*e^2*x^2 + 744*d^2*e^3*x^3 + 24320*d*e^4*x^4 + 18560*e^
5*x^5) + 3*c^6*d^6*(315*d^6 - 210*d^5*e*x + 168*d^4*e^2*x^2 - 144*d^3*e^3*x
^3 + 128*d^2*e^4*x^4 + 6400*d*e^5*x^5 + 5120*e^6*x^6)))/((c*d^2 - a*e^2)^5*
(a*e + c*d*x)*(d + e*x)) - (105*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*Ar
cTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/((a*e +
 c*d*x)^(3/2)*(d + e*x)^(3/2))))/(107520*c^(9/2)*d^(9/2)*e^(11/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(418) = 836.
time = 0.08, size = 1080, normalized size = 2.39

method	result
default	$\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{7}{2}}}{7cde}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\left(2cdex+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{5}{2}}}{12cde}+\frac{5\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdex+a\,e^{2}+\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e-1/2*(a*e^2+c*d^2)/c/
d/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2
)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d
*e)^(1/2)))))-d/e^2*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d
*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c
*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*l
n((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/(c*d*e)^(1/2))))+1/e^3*d^2*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*
(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(
c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/
4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(
c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)))
)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 2.36, size = 1235, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/430080*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15*
a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e^1
2 - 5*a^7*e^14)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e
^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c
*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 4*(6
30*c^7*d^12*x*e^2 - 945*c^7*d^13*e - 350*a^5*c^2*d^2*x*e^12 + 525*a^6*c*d*e
^13 + 280*(a^4*c^3*d^3*x^2 - 5*a^5*c^2*d^3)*e^11 - 10*(24*a^3*c^4*d^4*x^3 -
 91*a^4*c^3*d^4*x)*e^10 - 5*(4736*a^2*c^5*d^5*x^4 + 144*a^3*c^4*d^5*x^2 - 1
05*a^4*c^3*d^5)*e^9 - 20*(1856*a*c^6*d^6*x^5 + 1676*a^2*c^5*d^6*x^3 + 15*a^
3*c^4*d^6*x)*e^8 - 8*(1920*c^7*d^7*x^6 + 6080*a*c^6*d^7*x^4 + 228*a^2*c^5*d
^7*x^2 - 75*a^3*c^4*d^7)*e^7 - 4*(4800*c^7*d^8*x^5 + 372*a*c^6*d^8*x^3 - 58
3*a^2*c^5*d^8*x)*e^6 - (384*c^7*d^9*x^4 - 1744*a*c^6*d^9*x^2 + 3689*a^2*c^5
*d^9)*e^5 + 2*(216*c^7*d^10*x^3 - 1099*a*c^6*d^10*x)*e^4 - 168*(3*c^7*d^11*
x^2 - 20*a*c^6*d^11)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-
6)/(c^5*d^5), 1/215040*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^
10*e^4 - 15*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a
^6*c*d^2*e^12 - 5*a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2
+ (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e
+ a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2*(630*c^7*d^12*x*e^2 - 945
*c^7*d^13*e - 350*a^5*c^2*d^2*x*e^12 + 525*a^6*c*d*e^13 + 280*(a^4*c^3*d^3*
x^2 - 5*a^5*c^2*d^3)*e^11 - 10*(24*a^3*c^4*d^4*x^3 - 91*a^4*c^3*d^4*x)*e^10
 - 5*(4736*a^2*c^5*d^5*x^4 + 144*a^3*c^4*d^5*x^2 - 105*a^4*c^3*d^5)*e^9 - 2
0*(1856*a*c^6*d^6*x^5 + 1676*a^2*c^5*d^6*x^3 + 15*a^3*c^4*d^6*x)*e^8 - 8*(1
```

```
920*c^7*d^7*x^6 + 6080*a*c^6*d^7*x^4 + 228*a^2*c^5*d^7*x^2 - 75*a^3*c^4*d^7
)*e^7 - 4*(4800*c^7*d^8*x^5 + 372*a*c^6*d^8*x^3 - 583*a^2*c^5*d^8*x)*e^6 -
(384*c^7*d^9*x^4 - 1744*a*c^6*d^9*x^2 + 3689*a^2*c^5*d^9)*e^5 + 2*(216*c^7*
d^10*x^3 - 1099*a*c^6*d^10*x)*e^4 - 168*(3*c^7*d^11*x^2 - 20*a*c^6*d^11)*e^
3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-6)/(c^5*d^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.14, size = 612, normalized size = 1.35

$$\frac{1}{107520}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(2\left(8\left(10\left(12c^2d^2xe+\frac{(15c^8d^9e^6+29ac^7d^7e^8)e^{(-6)}}{c^6d^6}\right)x+\frac{(3c^8d^{10}e^5+380ac^7d^8e^7+185a^2c^6d^6e^9)e^{(-6)}}{c^6d^6}\right)x-\frac{(27c^8d^{11}e^4-93ac^7d^9e^6-2095a^2c^6d^7e^8-15a^3c^5d^5e^{10})e^{(-6)}}{c^6d^6}\right)x+\frac{(63c^8d^{12}e^3-218ac^7d^{10}e^5+228a^2c^6d^8e^7+90a^3c^5d^6e^9-35a^4c^4d^4e^{11})e^{(-6)}}{c^6d^6}\right)x-\frac{(315c^8d^{13}e^2-1099ac^7d^{11}e^4+1166a^2c^6d^9e^6-150a^3c^5d^7e^8+455a^4c^4d^5e^{10}-175a^5c^3d^3e^{12})e^{(-6)}}{c^6d^6}\right)x+\frac{(945c^8d^{14}e-3360ac^7d^{12}e^3+3689a^2c^6d^{10}e^5-600a^3c^5d^8e^7-525a^4c^4d^6e^9+1400a^5c^3d^4e^{11}-525a^6c^2d^2e^{13})e^{(-6)}}{c^6d^6}\right)+\frac{(9c^7d^{14}-35ac^6d^{12}e^2+45a^2c^5d^{10}e^4-15a^3c^4d^8e^6-5a^4c^3d^6e^8-9a^5c^2d^4e^{10}+15a^6cd^2e^{12}-5a^7e^{14})e^{\left(-\frac{11}{2}\right)}\log\left(\left|-cd^2-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)\sqrt{cd}\,e^{\frac{1}{2}}-ae^2\right|\right)}{2048\sqrt{cd}\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/107520*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(10*(12*c^
2*d^2*x*e + (15*c^8*d^9*e^6 + 29*a*c^7*d^7*e^8)*e^(-6)/(c^6*d^6))*x + (3*c^
8*d^10*e^5 + 380*a*c^7*d^8*e^7 + 185*a^2*c^6*d^6*e^9)*e^(-6)/(c^6*d^6))*x -
 (27*c^8*d^11*e^4 - 93*a*c^7*d^9*e^6 - 2095*a^2*c^6*d^7*e^8 - 15*a^3*c^5*d^
5*e^10)*e^(-6)/(c^6*d^6))*x + (63*c^8*d^12*e^3 - 218*a*c^7*d^10*e^5 + 228*a
^2*c^6*d^8*e^7 + 90*a^3*c^5*d^6*e^9 - 35*a^4*c^4*d^4*e^11)*e^(-6)/(c^6*d^6)
)*x - (315*c^8*d^13*e^2 - 1099*a*c^7*d^11*e^4 + 1166*a^2*c^6*d^9*e^6 - 150*
a^3*c^5*d^7*e^8 + 455*a^4*c^4*d^5*e^10 - 175*a^5*c^3*d^3*e^12)*e^(-6)/(c^6*
d^6))*x + (945*c^8*d^14*e - 3360*a*c^7*d^12*e^3 + 3689*a^2*c^6*d^10*e^5 - 6
00*a^3*c^5*d^8*e^7 - 525*a^4*c^4*d^6*e^9 + 1400*a^5*c^3*d^4*e^11 - 525*a^6*
c^2*d^2*e^13)*e^(-6)/(c^6*d^6)) + 1/2048*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 +
45*a^2*c^5*d^10*e^4 - 15*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^
4*e^10 + 15*a^6*c*d^2*e^12 - 5*a^7*e^14)*e^(-11/2)*log(abs(-c*d^2 - 2*(sqrt
(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*sqrt(c*d)*e^
(1/2) - a*e^2))/(sqrt(c*d)*c^4*d^4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
[Out] int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.459 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal. Leaf size=381

$$-\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^4}+\frac{\left(cd^2-ae^2\right)\left(7cd^2+5ae^2\right)}{}$$

```
[Out] 1/192*(-a*e^2+c*d^2)*(5*a*e^2+7*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e^3-1/60*(5*a/c/d+7*d/e^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d
/e/(e*x+d)+1/1024*(-a*e^2+c*d^2)^5*(5*a*e^2+7*c*d^2)*arctanh(1/2*(2*c*d*e*x
+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/c^(7/2)/d^(7/2)/e^(9/2)-1/512*(-a*e^2+c*d^2)^3*(5*a*e^2+7*c*d^2)*(2*c*d
*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4
```

Rubi [A]
time = 0.23, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {808, 678, 626, 635, 212}

$$\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024c^{7/2}d^{7/2}e^{9/2}}-\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512c^3d^3e^4}+\frac{\left(5ae^2+7cd^2\right)\left(cd^2-ae^2\right)\left(ae^2+cd^2+2cdex\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{192c^2d^2e^3}+\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{7/2}}{6cde(d+ex)}-\frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] -1/512*((c*d^2 - a*e^2)^3*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*e^4) + ((c*d^2 - a*e^2
)*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(192*c^2*d^2*e^3) - (((5*a)/(c*d) + (7*d)/e^2)*(a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/60 + (a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(7/2)/(6*c*d*e*(d + e*x)) + ((c*d^2 - a*e^2)^5*(7*c*d^2 + 5*a*e^
2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(1024*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
```

```
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}dx = \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{6cde(d+ex)} + \frac{1}{12}\left(-\frac{7d}{e}-\frac{5ae}{cd}\right)\int \frac{(ade+$$

$$= -\frac{1}{60}\left(\frac{5a}{cd}+\frac{7d}{e^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2} + \frac{(ade+(cd}{6}$$

$$= \frac{(cd^2-ae^2)(7cd^2+5ae^2)(cd^2+ae^2+2cdex)(ade+(cd^2+ae^2)}{192c^2d^2e^3}$$

$$= -\frac{(cd^2-ae^2)^3(7cd^2+5ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+}}{512c^3d^3e^4}$$

$$= -\frac{(cd^2-ae^2)^3(7cd^2+5ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+}}{512c^3d^3e^4}$$

$$= -\frac{(cd^2-ae^2)^3(7cd^2+5ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+}}{512c^3d^3e^4}$$

Mathematica [A]

time = 0.95, size = 388, normalized size = 1.02

$$\frac{(cd^2-ae^2)^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\,(75a^5e^{10}-5a^4cde^8(49d+10ex)+10a^3c^2d^2e^6(15d^2+16dex+4e^2x^2)-6a^2c^3d^3e^4(91d^3-58d^2ex-564de^2x^2-360e^3x^3)+ac^4d^4e^2(415d^4-272d^3ex+216d^2e^2x^2+4448de^3x^3+3200e^4x^4)+c^5d^5(-105d^5+70d^4ex-56d^3e^2x^2+48d^2e^3x^3+1664de^4x^4+1280e^5x^5))}{(cd^2-ae^2)^5(ae+cdx)(d+ex)}+\frac{15(7cd^2+5ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{(ae+cdx)^{3/2}(d+ex)^{3/2}}\right)}{7680c^{7/2}d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e
]*(75*a^5*e^10 - 5*a^4*c*d*e^8*(49*d + 10*e*x) + 10*a^3*c^2*d^2*e^6*(15*d^2
 + 16*d*e*x + 4*e^2*x^2) - 6*a^2*c^3*d^3*e^4*(91*d^3 - 58*d^2*e*x - 564*d*e
^2*x^2 - 360*e^3*x^3) + a*c^4*d^4*e^2*(415*d^4 - 272*d^3*e*x + 216*d^2*e^2*
x^2 + 4448*d*e^3*x^3 + 3200*e^4*x^4) + c^5*d^5*(-105*d^5 + 70*d^4*e*x - 56*
d^3*e^2*x^2 + 48*d^2*e^3*x^3 + 1664*d*e^4*x^4 + 1280*e^5*x^5)))/((c*d^2 - a
*e^2)^5*(a*e + c*d*x)*(d + e*x)) + (15*(7*c*d^2 + 5*a*e^2)*ArcTanh[(Sqrt[e]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/((a*e + c*d*x)^(3/2)*
(d + e*x)^(3/2))))/(7680*c^(7/2)*d^(7/2)*e^(9/2))
```

Maple [A]

time = 0.08, size = 673, normalized size = 1.77

method	result

default	$\frac{\frac{\left(2cdex+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{5}{2}}}{12cde}+\frac{5\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdex+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2
)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d
*e)^(1/2))))-d/e^2*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(
a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+a
*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-
c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*
(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 2.68, size = 1015, normalized size = 2.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [-1/30720*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3
*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(c*
d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 - 4*sqrt(c
*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)
*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) - 4*(70*c^6*d^10*x*e^2 - 105*
c^6*d^11*e - 50*a^4*c^2*d^2*x*e^10 + 75*a^5*c*d*e^11 + 5*(8*a^3*c^3*d^3*x^2
 - 49*a^4*c^2*d^3)*e^9 + 80*(27*a^2*c^4*d^4*x^3 + 2*a^3*c^3*d^4*x)*e^8 + 2*
(1600*a*c^5*d^5*x^4 + 1692*a^2*c^4*d^5*x^2 + 75*a^3*c^3*d^5)*e^7 + 4*(320*c
^6*d^6*x^5 + 1112*a*c^5*d^6*x^3 + 87*a^2*c^4*d^6*x)*e^6 + 2*(832*c^6*d^7*x^
4 + 108*a*c^5*d^7*x^2 - 273*a^2*c^4*d^7)*e^5 + 16*(3*c^6*d^8*x^3 - 17*a*c^5
*d^8*x)*e^4 - (56*c^6*d^9*x^2 - 415*a*c^5*d^9)*e^3)*sqrt(c*d^2*x + a*x*e^2
+ (c*d*x^2 + a*d)*e))*e^(-5)/(c^4*d^4), -1/15360*(15*(7*c^6*d^12 - 30*a*c^5
*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 +
18*a^5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*
e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*
x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2*(70*c^6*d^10*x*e^2 -
105*c^6*d^11*e - 50*a^4*c^2*d^2*x*e^10 + 75*a^5*c*d*e^11 + 5*(8*a^3*c^3*d^3
*x^2 - 49*a^4*c^2*d^3)*e^9 + 80*(27*a^2*c^4*d^4*x^3 + 2*a^3*c^3*d^4*x)*e^8
+ 2*(1600*a*c^5*d^5*x^4 + 1692*a^2*c^4*d^5*x^2 + 75*a^3*c^3*d^5)*e^7 + 4*(3
20*c^6*d^6*x^5 + 1112*a*c^5*d^6*x^3 + 87*a^2*c^4*d^6*x)*e^6 + 2*(832*c^6*d^
7*x^4 + 108*a*c^5*d^7*x^2 - 273*a^2*c^4*d^7)*e^5 + 16*(3*c^6*d^8*x^3 - 17*a
*c^5*d^8*x)*e^4 - (56*c^6*d^9*x^2 - 415*a*c^5*d^9)*e^3)*sqrt(c*d^2*x + a*x*
e^2 + (c*d*x^2 + a*d)*e))*e^(-5)/(c^4*d^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.12, size = 499, normalized size = 1.31

$$\frac{1}{7680}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(2\left(4\left(2\left(8\left(10\,c^2d^2xe+\frac{(13\,c^7d^8e^5+25\,ac^6d^6e^7)e^{(-5)}}{c^5d^5}\right)x+\frac{(3\,c^7d^9e^4+278\,ac^6d^7e^6+135\,a^2c^5d^5e^8)e^{(-5)}}{c^5d^5}\right)x-\frac{(7\,c^7d^{10}e^3-27\,ac^6d^8e^5-423\,a^2c^5d^6e^7-5\,a^3c^4d^4e^9)e^{(-5)}}{c^5d^5}\right)x+\frac{(35\,c^7d^{11}e^2-136\,ac^6d^9e^4+174\,a^2c^5d^7e^6+80\,a^3c^4d^5e^8-25\,a^4c^3d^3e^{10})e^{(-5)}}{c^5d^5}\right)x-\frac{(105\,c^7d^{12}e-415\,ac^6d^{10}e^3+546\,a^2c^5d^8e^5-150\,a^3c^4d^6e^7+245\,a^4c^3d^4e^9-75\,a^5c^2d^2e^{11})e^{(-5)}}{c^5d^5}\right)-\frac{(7\,c^6d^{12}-30\,ac^5d^{10}e^2+45\,a^2c^4d^8e^4-20\,a^3c^3d^6e^6-15\,a^4c^2d^4e^8+18\,a^5cd^2e^{10}-5\,a^6e^{12})e^{\left(-\frac{9}{2}\right)}\log\left(\left|-cd^2-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)\sqrt{cd}\,e^{\frac{1}{2}}-ae^2\right|\right)}{1024\,\sqrt{cd}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="g
iac")
```

```
[Out] 1/7680*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*(10*c^2*d^2*
x*e + (13*c^7*d^8*e^5 + 25*a*c^6*d^6*e^7)*e^(-5)/(c^5*d^5))*x + (3*c^7*d^9*
e^4 + 278*a*c^6*d^7*e^6 + 135*a^2*c^5*d^5*e^8)*e^(-5)/(c^5*d^5))*x - (7*c^7
*d^10*e^3 - 27*a*c^6*d^8*e^5 - 423*a^2*c^5*d^6*e^7 - 5*a^3*c^4*d^4*e^9)*e^(
-5)/(c^5*d^5))*x + (35*c^7*d^11*e^2 - 136*a*c^6*d^9*e^4 + 174*a^2*c^5*d^7*e
^6 + 80*a^3*c^4*d^5*e^8 - 25*a^4*c^3*d^3*e^10)*e^(-5)/(c^5*d^5))*x - (105*c
^7*d^12*e - 415*a*c^6*d^10*e^3 + 546*a^2*c^5*d^8*e^5 - 150*a^3*c^4*d^6*e^7
+ 245*a^4*c^3*d^4*e^9 - 75*a^5*c^2*d^2*e^11)*e^(-5)/(c^5*d^5)) - 1/1024*(7*
c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15
*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*e^(-9/2)*log(abs(-c*d^2
- 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*sqr
t(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d)*c^3*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

```
[Out] int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

3.460 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex} \, dx$

Optimal. Leaf size=274

$$\frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3}+\frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\left(ad\right.$$

```
[Out] 1/16*(a/c/d-d/e^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-3/256*(-a*e^2+c*d^2)^
5*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)+3/128*(-a*e^2+c*d^2)^3*
(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3
```

Rubi [A]
time = 0.11, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {678, 626, 635, 212}

$$-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}}+\frac{3\left(cd^2-ae^2\right)^3\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128c^2d^2e^3}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}+\frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 +
2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/16 + (a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - (3*(c*d^2 - a*e^2)^5*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(256*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5e} - \frac{(2cd^2e-e(cd^2+ae^2))\int (ade-}{2e}$$

$$= \frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)(cd^2+ae^2+2cdex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^3$$

$$= \frac{3(cd^2-ae^2)^3(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2e^3}$$

$$= \frac{3(cd^2-ae^2)^3(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2e^3}$$

$$= \frac{3(cd^2-ae^2)^3(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^2d^2e^3}$$

Mathematica [A]
time = 0.33, size = 267, normalized size = 0.97

$$\frac{(cd^2-ae^2)^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)^3\left(15c^4d^4-\frac{15e^4(ae+cdx)^4}{(d+ex)^4}+\frac{70cde^3(ae+cdx)^3}{(d+ex)^3}+\frac{128c^2d^2e^2(ae+cdx)^2}{(d+ex)^2}-\frac{70c^3d^3e(ae+cdx)}{d+ex}\right)}{(cd^2-ae^2)^5(ae+cdx)}-\frac{15\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{(ae+cdx)^{3/2}(d+ex)^{3/2}}\right)}{640c^{5/2}d^{5/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

```
[Out] ((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e
]*(d + e*x)^3*(15*c^4*d^4 - (15*e^4*(a*e + c*d*x)^4)/(d + e*x)^4 + (70*c*d*
```

```
e^3*(a*e + c*d*x)^3)/(d + e*x)^3 + (128*c^2*d^2*e^2*(a*e + c*d*x)^2)/(d + e
*x)^2 - (70*c^3*d^3*e*(a*e + c*d*x))/(d + e*x)))/((c*d^2 - a*e^2)^5*(a*e +
c*d*x)) - (15*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d +
 e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(640*c^(5/2)*d^(5/2)*e^(7/
2))
```

Maple [A]
time = 0.09, size = 327, normalized size = 1.19

method	result
default	$\frac{\frac{\left(cde\left(x+\frac{d}{e}\right)^{2}+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\left(a e^{2}-c d^{2}\right)\left(\frac{\left(2cde\left(x+\frac{d}{e}\right)+a e^{2}-c d^{2}\right)\left(cde\left(x+\frac{d}{e}\right)^{2}+\left(a e^{2}-c d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8cde}-\frac{3\left(a e^{2}-c d^{2}\right)^{2}\left(\frac{(2cde(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/e*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1
/8*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d
/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*
ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(241) = 482$.
time = 0.35, size = 896, normalized size = 3.27

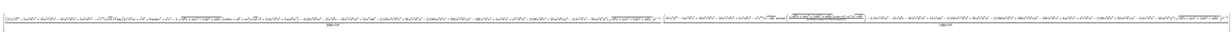
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] -3/256*c^4*d^9*e^(-7/2)*log(c*d^2*e^(-1) + 2*c*d*x + a*e + 2*sqrt(c*d*x^2*e
 + c*d^2*x + a*x*e^2 + a*d*e)*sqrt(c*d)*e^(-1/2))/(c*d)^(3/2) + 15/256*a*c^
3*d^7*e^(-3/2)*log(c*d^2*e^(-1) + 2*c*d*x + a*e + 2*sqrt(c*d*x^2*e + c*d^2*
x + a*x*e^2 + a*d*e)*sqrt(c*d)*e^(-1/2))/(c*d)^(3/2) + 3/64*sqrt(c*d*x^2*e
```

```
+ c*d^2*x + a*x*e^2 + a*d*e)*c^2*d^5*x*e^(-2) + 3/128*sqrt(c*d*x^2*e + c*d^
2*x + a*x*e^2 + a*d*e)*c^2*d^6*e^(-3) - 15/128*a^2*c^2*d^5*e^(1/2)*log(c*d^
2*e^(-1) + 2*c*d*x + a*e + 2*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*sq
rt(c*d)*e^(-1/2))/(c*d)^(3/2) - 3/64*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a
*d*e)*a*c*d^4*e^(-1) + 15/128*a^3*c*d^3*e^(5/2)*log(c*d^2*e^(-1) + 2*c*d*x
+ a*e + 2*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*sqrt(c*d)*e^(-1/2))/(
c*d)^(3/2) - 9/64*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*a*c*d^3*x - 1
/8*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^(3/2)*c*d^2*x*e^(-1) - 1/16*(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^(3/2)*c*d^3*e^(-2) + 9/64*sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e)*a^2*d*x*e^2 - 15/256*a^4*d*e^(9/2)*log(c*d^2
*e^(-1) + 2*c*d*x + a*e + 2*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*sqr
t(c*d)*e^(-1/2))/(c*d)^(3/2) + 1/8*(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^
(3/2)*a*x*e - 3/64*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*a^3*x*e^4/(c
*d) + 3/64*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*a^3*e^3/c + 1/5*(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)^(5/2)*e^(-1) + 3/256*a^5*e^(13/2)*log(c
*d^2*e^(-1) + 2*c*d*x + a*e + 2*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)
*sqrt(c*d)*e^(-1/2))/((c*d)^(3/2)*c*d) + 1/16*(c*d*x^2*e + c*d^2*x + a*x*e^
2 + a*d*e)^(3/2)*a^2*e^2/(c*d) - 3/128*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e)*a^4*e^5/(c^2*d^2)
```

Fricas [A]
time = 2.76, size = 815, normalized size = 2.97



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] [1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d
^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e +
c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3
*a*c*d^2)*e^2) - 4*(10*c^5*d^8*x*e^2 - 15*c^5*d^9*e - 10*a^3*c^2*d^2*x*e^8
+ 15*a^4*c*d*e^9 - 2*(124*a^2*c^3*d^3*x^2 + 35*a^3*c^2*d^3)*e^7 - 2*(168*a*
c^4*d^4*x^3 + 233*a^2*c^3*d^4*x)*e^6 - 128*(c^5*d^5*x^4 + 4*a*c^4*d^5*x^2 +
 a^2*c^3*d^5)*e^5 - 2*(88*c^5*d^6*x^3 + 23*a*c^4*d^6*x)*e^4 - 2*(4*c^5*d^7*
x^2 - 35*a*c^4*d^7)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-4
)/(c^3*d^3), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 -
10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sq
rt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(
-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2*(10*
c^5*d^8*x*e^2 - 15*c^5*d^9*e - 10*a^3*c^2*d^2*x*e^8 + 15*a^4*c*d*e^9 - 2*(1
24*a^2*c^3*d^3*x^2 + 35*a^3*c^2*d^3)*e^7 - 2*(168*a*c^4*d^4*x^3 + 233*a^2*c
^3*d^4*x)*e^6 - 128*(c^5*d^5*x^4 + 4*a*c^4*d^5*x^2 + a^2*c^3*d^5)*e^5 - 2*(
```

```
88*c^5*d^6*x^3 + 23*a*c^4*d^6*x)*e^4 - 2*(4*c^5*d^7*x^2 - 35*a*c^4*d^7)*e^3
)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-4)/(c^3*d^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.75, size = 396, normalized size = 1.45

$$\frac{1}{640}\,\sqrt{cdx^2e+cd^2x+axe^2+ade}\,\left(2\,\left(4\,\left(2\,\left(8\,c^2d^2xe+\frac{(11\,c^6d^7e^4+21\,ac^5d^5e^6)e^{(-4)}}{c^4d^4}\right)x+\frac{(c^6d^8e^3+64\,ac^5d^6e^5+31\,a^2c^4d^4e^7)e^{(-4)}}{c^4d^4}\right)x-\frac{(5\,c^6d^9e^2-23\,ac^5d^7e^4-233\,a^2c^4d^5e^6-5\,a^3c^3d^3e^8)e^{(-4)}}{c^4d^4}\right)x+\frac{(15\,c^6d^{10}e-70\,ac^5d^8e^3+128\,a^2c^4d^6e^5+70\,a^3c^3d^4e^7-15\,a^4c^2d^2e^9)e^{(-4)}}{c^4d^4}\right)+\frac{3\,(c^5d^{10}-5\,ac^4d^8e^2+10\,a^2c^3d^6e^4-10\,a^3c^2d^4e^6+5\,a^4cd^2e^8-a^5e^{10})e^{\left(-\frac{7}{2}\right)}\log\left(\left|-cd^2-2\,\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)\sqrt{cd}\,e^{\frac{1}{2}}-ae^2\right|\right)}{256\,\sqrt{cd}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] 1/640*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*(2*(8*c^2*d^2*x*e +
 (11*c^6*d^7*e^4 + 21*a*c^5*d^5*e^6)*e^(-4)/(c^4*d^4))*x + (c^6*d^8*e^3 + 6
4*a*c^5*d^6*e^5 + 31*a^2*c^4*d^4*e^7)*e^(-4)/(c^4*d^4))*x - (5*c^6*d^9*e^2
- 23*a*c^5*d^7*e^4 - 233*a^2*c^4*d^5*e^6 - 5*a^3*c^3*d^3*e^8)*e^(-4)/(c^4*d
^4))*x + (15*c^6*d^10*e - 70*a*c^5*d^8*e^3 + 128*a^2*c^4*d^6*e^5 + 70*a^3*c
^3*d^4*e^7 - 15*a^4*c^2*d^2*e^9)*e^(-4)/(c^4*d^4)) + 3/256*(c^5*d^10 - 5*a*
c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a
^5*e^10)*e^(-7/2)*log(abs(-c*d^2 - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e
+ c*d^2*x + a*x*e^2 + a*d*e))*sqrt(c*d)*e^(1/2) - a*e^2))/(sqrt(c*d)*c^2*d^
2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x), x)
```

3.461 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal. Leaf size=394

$$-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cde^2}$$

```
[Out] 1/24*(6*c*d*e*x+11*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e
-a^(5/2)*d^(3/2)*e^(5/2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1
/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/128*(-5*a^4*e^8+60*a
^3*c*d^2*e^6+90*a^2*c^2*d^4*e^4-20*a*c^3*d^6*e^2+3*c^4*d^8)*arctanh(1/2*(2*
c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-1/64*(3*c^3*d^6-11*a*c^2*d^4*e^2-83*a^2*
c*d^2*e^4-5*a^3*e^6+2*c*d*e*(-5*a*e^2+c*d^2)*(a*e^2+3*c*d^2)*x)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2
```

Rubi [A]
time = 0.28, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {863, 828, 857, 635, 212, 738}

$$-a^{5/2}d^{3/2}e^{5/2}\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(-5a^3e^6-83a^2cd^2e^4-11ac^2d^4e^2+2cdex(cd^2-5ae^2)\left(ae^2+3cd^2\right)+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64cde^2}+\frac{\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128c^{3/2}d^{3/2}e^{5/2}}+\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] -1/64*((3*c^3*d^6 - 11*a*c^2*d^4*e^2 - 83*a^2*c*d^2*e^4 - 5*a^3*e^6 + 2*c*d
*e*(c*d^2 - 5*a*e^2)*(3*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(c*d*e^2) + ((3*c*d^2 + 11*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(24*e) + ((3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*
a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(c*d^2 + a*e^2 + 2*
c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])])/(128*c^(3/2)*d^(3/2)*e^(5/2)) - a^(5/2)*d^(3/2)*e^(5/2)*ArcTanh[(2*
a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)} dx = \int \frac{(ae + cdx)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x} dx$$

$$= \frac{(3cd^2 + 11ae^2 + 6cdex)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{24e} - \int \frac{(-8a^2$$

$$= -\frac{(3c^3d^6 - 11ac^2d^4e^2 - 83a^2cd^2e^4 - 5a^3e^6 + 2cde(cd^2 - 5ae^2)(3cd}{64cde^2}$$

$$= -\frac{(3c^3d^6 - 11ac^2d^4e^2 - 83a^2cd^2e^4 - 5a^3e^6 + 2cde(cd^2 - 5ae^2)(3cd}{64cde^2}$$

$$= -\frac{(3c^3d^6 - 11ac^2d^4e^2 - 83a^2cd^2e^4 - 5a^3e^6 + 2cde(cd^2 - 5ae^2)(3cd}{64cde^2}$$

$$= -\frac{(3c^3d^6 - 11ac^2d^4e^2 - 83a^2cd^2e^4 - 5a^3e^6 + 2cde(cd^2 - 5ae^2)(3cd}{64cde^2}$$

Mathematica [A]
time = 1.02, size = 342, normalized size = 0.87

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,(15a^3e^6+a^2cde^4(337d+118ex)+ac^2d^2e^2(57d^2+244dex+136e^2x^2)+c^3(-9d^6+6d^5ex+72d^4e^2x^2+48d^3e^3x^3))-384a^{5/2}c^{3/2}d^3e^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ae+cdx}}{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}\right)+3(3c^4d^8-20ac^3d^6e^2+90a^2c^2d^4e^4+60a^3cd^2e^6-5a^4e^8)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}\right)\right)}{192c^{3/2}d^{3/2}e^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*(15*a^3*e^6 + a^2*c*d*e^4*(337*d + 118*e*x) + a*c^2*d^2*e^2*
(57*d^2 + 244*d*e*x + 136*e^2*x^2) + c^3*(-9*d^6 + 6*d^5*e*x + 72*d^4*e^2*x
^2 + 48*d^3*e^3*x^3)) - 384*a^(5/2)*c^(3/2)*d^3*e^5*ArcTanh[(Sqrt[d]*Sqrt[a
*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 3*(3*c^4*d^8 - 20*a*c^3*d^6
*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(Sqrt[e]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(192*c^(3/2)*d^(3/2)*
e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(352) = 704$.
time = 0.08, size = 997, normalized size = 2.53

method	result

default	$-\frac{\left(cde\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\frac{\left(a\,e^2-c\,d^2\right)\left(\frac{\left(2cde\left(x+\frac{d}{e}\right)+a\,e^2-c\,d^2\right)\left(cde\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8cde}-\frac{3\left(a\,e^2-c\,d^2\right)^2\left(\frac{(2c}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/d*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(
1/8*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d
/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/
d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e
*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))+1/d*(1/5*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d
/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*
x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a
*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2
*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*
a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)
^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^
2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(
1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 54.81, size = 1817, normalized size = 4.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="f
ricas")
```

```
[Out] [1/768*(384*sqrt(a*d)*a^2*c^2*d^3*e^(11/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e
 + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2
*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*
a^2*d^2)*e^2)/x^2) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 +
 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^
4 + 8*a*c*d*x*e^3 + a^2*e^4 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d
^2)*e^2) + 4*(6*c^4*d^6*x*e^2 - 9*c^4*d^7*e + 118*a^2*c^2*d^2*x*e^6 + 15*a^
3*c*d*e^7 + (136*a*c^3*d^3*x^2 + 337*a^2*c^2*d^3)*e^5 + 4*(12*c^4*d^4*x^3 +
 61*a*c^3*d^4*x)*e^4 + 3*(24*c^4*d^5*x^2 + 19*a*c^3*d^5)*e^3)*sqrt(c*d^2*x
+ a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-3)/(c^2*d^2), 1/384*(192*sqrt(a*d)*a^2*
c^2*d^3*e^(11/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x
*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 3*(3*
c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*
e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2
*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^
2*x^2 + a*c*d^2)*e^2)) + 2*(6*c^4*d^6*x*e^2 - 9*c^4*d^7*e + 118*a^2*c^2*d^2
*x*e^6 + 15*a^3*c*d*e^7 + (136*a*c^3*d^3*x^2 + 337*a^2*c^2*d^3)*e^5 + 4*(12
*c^4*d^4*x^3 + 61*a*c^3*d^4*x)*e^4 + 3*(24*c^4*d^5*x^2 + 19*a*c^3*d^5)*e^3)
*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-3)/(c^2*d^2), 1/768*(768*
sqrt(-a*d*e)*a^2*c^2*d^3*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^
2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3
+ (a*c*d^2*x^2 + a^2*d^2)*e^2))*e^5 - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*
a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(c*d)*e^(1/2)*log(8*c^2
*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 - 4*sqrt(c*d^2*x + a*x*e^2 + (
c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*
d^2*x^2 + 3*a*c*d^2)*e^2) + 4*(6*c^4*d^6*x*e^2 - 9*c^4*d^7*e + 118*a^2*c^2*
d^2*x*e^6 + 15*a^3*c*d*e^7 + (136*a*c^3*d^3*x^2 + 337*a^2*c^2*d^3)*e^5 + 4*
(12*c^4*d^4*x^3 + 61*a*c^3*d^4*x)*e^4 + 3*(24*c^4*d^5*x^2 + 19*a*c^3*d^5)*e
^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-3)/(c^2*d^2), 1/384*(3
```

```
84*sqrt(-a*d*e)*a^2*c^2*d^3*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c
*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e
^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2))*e^5 - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 +
90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*
sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqr
t(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(6
*c^4*d^6*x*e^2 - 9*c^4*d^7*e + 118*a^2*c^2*d^2*x*e^6 + 15*a^3*c*d*e^7 + (13
6*a*c^3*d^3*x^2 + 337*a^2*c^2*d^3)*e^5 + 4*(12*c^4*d^4*x^3 + 61*a*c^3*d^4*x
)*e^4 + 3*(24*c^4*d^5*x^2 + 19*a*c^3*d^5)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*
d*x^2 + a*d)*e))*e^(-3)/(c^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{5}{2}}}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x/(e*x+d),x)
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x*(d + e*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="g
iac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)), x)
```

3.462 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$

Optimal. Leaf size=352

$$\frac{\left(c^2d^4+28acd^2e^2+19a^2e^4+2cde\left(cd^2+7ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8e}-\frac{(3ae-cdx)\left(ade+\left(cd^2+\right.\right.}{3x}$$

```
[Out] -1/3*(-c*d*x+3*a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x-1/16*(-5*a^3*
e^6-45*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+c^3*d^6)*arctanh(1/2*(2*c*d*e*x+a*e^2
+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^
(3/2)/c^(1/2)/d^(1/2)-1/2*a^(3/2)*e^(3/2)*(3*a*e^2+5*c*d^2)*arctanh(1/2*(2*
a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))*d^(1/2)+1/8*(c^2*d^4+28*a*c*d^2*e^2+19*a^2*e^4+2*c*d*e*(7*a*e^
2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e
```

Rubi [A]
time = 0.27, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {863, 826, 828, 857, 635, 212, 738}

$$-\frac{1}{2}a^{3/2}\sqrt{d}\,e^{3/2}\left(3ae^2+5cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)+\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8e}-\frac{\left(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16\sqrt{c}\sqrt{d}\,e^{3/2}}-\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] ((c^2*d^4 + 28*a*c*d^2*e^2 + 19*a^2*e^4 + 2*c*d*e*(c*d^2 + 7*a*e^2)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*e) - ((3*a*e - c*d*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*x) - ((c^3*d^6 - 15*a*c^2*d^4*e^2
- 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt
[c]*Sqrt[d]*e^(3/2)) - (a^(3/2)*Sqrt[d]*e^(3/2)*(5*c*d^2 + 3*a*e^2)*ArcTanh
[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx = \int \frac{(ae+cdx)\,(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2}\,dx$$

$$= -\frac{(3ae-cdx)\,(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3x} - \frac{1}{2}\int \frac{(-ae(5cd^2 -$$

$$= \frac{(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x)\sqrt{ade+(cd^2+a}}{8e}$$

$$= \frac{(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x)\sqrt{ade+(cd^2+a}}{8e}$$

$$= \frac{(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x)\sqrt{ade+(cd^2+a}}{8e}$$

$$= \frac{(c^2d^4+28acd^2e^2+19a^2e^4+2cde(cd^2+7ae^2)x)\sqrt{ade+(cd^2+a}}{8e}$$

Mathematica [A]
time = 1.03, size = 309, normalized size = 0.88

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,(3a^2e^3(-8d+11ex)+2acde^2x(34d+13ex)+c^2d^2x(3d^2+14dex+8e^2x^2))-24a^{3/2}\sqrt{c}\,de^3(5cd^2+3ae^2)\,x\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ae+cdx}}{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}\right)-3(c^3d^6-15ac^2d^4e^2-45a^2cd^2e^4-5a^3e^6)\,x\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}\right)\right)}{24\sqrt{c}\,\sqrt{d}\,e^{3/2}x\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*(3*a^2*e^3*(-8*d + 11*e*x) + 2*a*c*d*e^2*x*(34*d + 13*e*x) +
 c^2*d^2*x*(3*d^2 + 14*d*e*x + 8*e^2*x^2)) - 24*a^(3/2)*Sqrt[c]*d*e^3*(5*c*
d^2 + 3*a*e^2)*x*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[
d + e*x])] - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*
x*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(2
4*Sqrt[c]*Sqrt[d]*e^(3/2)*x*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2075 vs. $2(308) = 616$.

time = 0.09, size = 2076, normalized size = 5.90

method	result	size
default	Expression too large to display	2076

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] e/d^2*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*
(1/8*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c
/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/
e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))+1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(7/2)+5/2*(a*e^2+c*d^2)/a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c
/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*
e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))
+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*
(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*
e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*
d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)
^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d
^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*
x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))))-e/d^2*(1/5*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+
c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a
*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^
2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*
e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
```

```
x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^
2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(
1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((
1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a
*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((x*e + d)*x^2), x)
```

Fricas [A]
time = 19.07, size = 1697, normalized size = 4.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/96*(3*(c^3*d^6*x - 15*a*c^2*d^4*x*e^2 - 45*a^2*c*d^2*x*e^4 - 5*a^3*x*e^
6)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4
+ 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)
*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) - 24*(5*a*c^2*d^3*x
*e^3 + 3*a^2*c*d*x*e^5)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e
+ a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*
x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a
^2*d^2)*e^2)/x^2) - 4*(14*c^3*d^4*x^2*e^2 + 3*c^3*d^5*x*e + 33*a^2*c*d*x*e^
5 + 2*(13*a*c^2*d^2*x^2 - 12*a^2*c*d^2)*e^4 + 4*(2*c^3*d^3*x^3 + 17*a*c^2*d
^3*x)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)/(c*d*x), 1/4
8*(12*(5*a*c^2*d^3*x*e^3 + 3*a^2*c*d*x*e^5)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*
x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 +
2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*
(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) + 3*(c^3*d^6*x - 15*a*c^2*d^4*x*e^2 -
 45*a^2*c*d^2*x*e^4 - 5*a^3*x*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a
*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d
^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(14*c^3*d^4*x^2*e^
2 + 3*c^3*d^5*x*e + 33*a^2*c*d*x*e^5 + 2*(13*a*c^2*d^2*x^2 - 12*a^2*c*d^2)*
e^4 + 4*(2*c^3*d^3*x^3 + 17*a*c^2*d^3*x)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d
```

```
*x^2 + a*d)*e))*e^(-2)/(c*d*x), -1/96*(3*(c^3*d^6*x - 15*a*c^2*d^4*x*e^2 -
45*a^2*c*d^2*x*e^4 - 5*a^3*x*e^6)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2
*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*
c*d^2)*e^2) - 48*(5*a*c^2*d^3*x*e^3 + 3*a^2*c*d*x*e^5)*sqrt(-a*d*e)*arctan(
1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*
e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2))
- 4*(14*c^3*d^4*x^2*e^2 + 3*c^3*d^5*x*e + 33*a^2*c*d*x*e^5 + 2*(13*a*c^2*d^
2*x^2 - 12*a^2*c*d^2)*e^4 + 4*(2*c^3*d^3*x^3 + 17*a*c^2*d^3*x)*e^3)*sqrt(c*
d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)/(c*d*x), 1/48*(24*(5*a*c^2*d^3
*x*e^3 + 3*a^2*c*d*x*e^5)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*
d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e
+ a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 3*(c^3*d^6*x - 15*a*c^2*d^4
*x*e^2 - 45*a^2*c*d^2*x*e^4 - 5*a^3*x*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e
)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(14*c^3*d^
4*x^2*e^2 + 3*c^3*d^5*x*e + 33*a^2*c*d*x*e^5 + 2*(13*a*c^2*d^2*x^2 - 12*a^2
*c*d^2)*e^4 + 4*(2*c^3*d^3*x^3 + 17*a*c^2*d^3*x)*e^3)*sqrt(c*d^2*x + a*x*e^
2 + (c*d*x^2 + a*d)*e))*e^(-2)/(c*d*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.11, size = 504, normalized size = 1.43

$$\frac{1}{24}\sqrt{cdx^2e + cd^2x + axe^2 + ade}\left(2\left(4c^2d^2xe + \frac{(7c^4d^5e^2 + 13ac^3d^3e^4)e^{(-2)}}{c^2d^2}\right)x + \frac{(3c^4d^6e + 68ac^3d^4e^3 + 33a^2c^2d^2e^5)e^{(-2)}}{c^2d^2}\right) + \frac{(5a^2cd^3e^2 + 3a^3de^4)\arctan\left(-\frac{\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}} + \frac{\left(\sqrt{cd}c^3d^6e^{\frac{1}{2}} - 15\sqrt{cd}ac^2d^4e^{\frac{5}{2}} - 45\sqrt{cd}a^2cd^2e^{\frac{9}{2}} - 5\sqrt{cd}a^3e^{\frac{13}{2}}\right)e^{(-2)}\log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}} - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)cde - \sqrt{cd}ae^{\frac{5}{2}}\right|\right)}{16cd} - \frac{\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)a^2cd^3e^2 + 2\sqrt{cd}a^3d^2e^{\frac{7}{2}} + \left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)a^3de^4}{ade - \left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/24*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*(2*(4*c^2*d^2*x*e + (7*c^4
*d^5*e^2 + 13*a*c^3*d^3*e^4)*e^(-2)/(c^2*d^2))*x + (3*c^4*d^6*e + 68*a*c^3*
d^4*e^3 + 33*a^2*c^2*d^2*e^5)*e^(-2)/(c^2*d^2)) + (5*a^2*c*d^3*e^2 + 3*a^3*
d*e^4)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
a*d*e))/sqrt(-a*d*e))/sqrt(-a*d*e) + 1/16*(sqrt(c*d)*c^3*d^6*e^(1/2) - 15*s
qrt(c*d)*a*c^2*d^4*e^(5/2) - 45*sqrt(c*d)*a^2*c*d^2*e^(9/2) - 5*sqrt(c*d)*a
^3*e^(13/2))*e^(-2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/
```

```
2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/
2)))/(c*d) - ((sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a
*d*e))*a^2*c*d^3*e^2 + 2*sqrt(c*d)*a^3*d^2*e^(7/2) + (sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^3*d*e^4)/(a*d*e - (sqrt(c*d
)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

3.463 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx$

Optimal. Leaf size=339

$$-\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x}-\frac{(ae-cdx)\,(ade+(cd^2+ae^2)\,x+$$

```
[Out] -1/2*(-c*d*x+a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2+3/8*(5*a^2*e^
4+10*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/
2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*c^(1/2)*d^(1/2)/e^(1/2)
-3/8*(a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*
x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*a^(1/2)
*e^(1/2)/d^(1/2)-3/4*(a*e*(a*e^2+3*c*d^2)-c*d*(3*a*e^2+c*d^2)*x)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
```

Rubi [A]
time = 0.25, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 826, 857, 635, 212, 738}

$$\frac{3\sqrt{c}\,\sqrt{d}\,(5a^2e^4+10acd^2e^2+c^2d^4)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{e}}-\frac{3\sqrt{a}\,\sqrt{e}\,(a^2e^4+10acd^2e^2+5c^2d^4)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{d}}-\frac{(ae-cdx)\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2x^2}-\frac{3(ae(ae^2+3cd^2)-cdx(3ae^2+cd^2))\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (-3*(a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(4*x) - ((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(2*x^2) + (3*Sqrt[c]*Sqrt[d]*(c^2*d^4 + 10*a*c*d^2*e^2 +
 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*Sqrt[e]) - (3*Sqrt[a]*Sqr
t[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e
^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*Sqrt[d])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3}\,dx$$

$$=-\frac{(ae-cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2x^2}-\frac{3}{8}\int \frac{(-2ae(3cd^2$$

$$=-\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)\,x)\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{4x}$$

$$=-\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)\,x)\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{4x}$$

$$=-\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)\,x)\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{4x}$$

$$=-\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)\,x)\sqrt{ade+\left(cd^2+ae^2\right)x+c}}{4x}$$

Mathematica [A]
time = 0.93, size = 287, normalized size = 0.85

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{d}\,\sqrt{e}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,(-9acdex(d-ex)+c^2d^2x^2(5d+2ex)-a^2e^2(2d+5ex))-3\sqrt{a}\,e(5c^2d^4+10acd^2e^2+a^2e^4)\,x^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ae+cdx}}{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}\right)+3\sqrt{c}\,d(c^2d^4+10acd^2e^2+5a^2e^4)\,x^2\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}\right)\right)}{4\sqrt{d}\,\sqrt{e}\,x^2\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d
+ e*x]*(-9*a*c*d*e*x*(d - e*x) + c^2*d^2*x^2*(5*d + 2*e*x) - a^2*e^2*(2*d +
 5*e*x)) - 3*Sqrt[a]*e*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*x^2*ArcTanh[(
Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 3*Sqrt[c]*d*(
c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*x^2*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x
])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(4*Sqrt[d]*Sqrt[e]*x^2*Sqrt[(a*e + c*
d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3892 vs. $2(295) = 590$.
time = 0.09, size = 3893, normalized size = 11.48

method	result	size
default	Expression too large to display	3893

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -e^2/d^3*(1/5*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/c/d/e*(1/4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/c/d/e*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/c/d/e*ln((1/2*a*e^2-1/2*c*d^2+c*d*e*(x+d/e))/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2))))+1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+3/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+5/2*(a*e^2+c*d^2)/a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))))+5/2*c/a*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))))-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+5/2*(a*e^2+c*d^2)/a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2*c*

```
d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*
ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*
e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/(c*d*e)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2
+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c
*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))))+6*c/
a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2)+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/
c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c
*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e
)^(1/2)))))+e^2/d^3*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+1/2*(a*e^2
+c*d^2)*(1/8*(2*c*d*e*x+a*e^2+c*d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/c/d/e*(1/4*(2*c*d*e*x+a*e^2+c*
d^2)/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/8*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((x*e + d)*x^3), x)
```

Fricas [A]
time = 8.84, size = 1581, normalized size = 4.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(c^2*d^4*x^2 + 10*a*c*d^2*x^2*e^2 + 5*a^2*x^2*e^4)*sqrt(c*d)*e^(-1
/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x
+ a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e^2 + c*d^2*e + a*e^3)*sqrt(c*d)*e^
(-1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 3*(5*c^2*d^4*x^2 + 10*a*c*d^2
*x^2*e^2 + a^2*x^2*e^4)*sqrt(a/d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e
+ a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^3*x + a*d*x*e^2 + 2*a*d^2*e)*sqrt(c*
d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a/d)*e^(1/2) + 2*(3*a*c*d^2*x^2 +
 4*a^2*d^2)*e^2)/x^2) + 4*(5*c^2*d^3*x^2 - 5*a^2*x*e^3 + (9*a*c*d*x^2 - 2*a
^2*d)*e^2 + (2*c^2*d^2*x^3 - 9*a*c*d^2*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e))/x^2, 1/16*(3*(5*c^2*d^4*x^2 + 10*a*c*d^2*x^2*e^2 + a^2*x^2*e
^4)*sqrt(a/d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^
2*d*x*e^3 - 4*(c*d^3*x + a*d*x*e^2 + 2*a*d^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*sqrt(a/d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2
) - 6*(c^2*d^4*x^2 + 10*a*c*d^2*x^2*e^2 + 5*a^2*x^2*e^4)*sqrt(-c*d*e^(-1))*
arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 +
 a*e^2)*sqrt(-c*d*e^(-1))/(c^2*d^3*x + a*c*d*x*e^2 + (c^2*d^2*x^2 + a*c*d^2
)*e)) + 4*(5*c^2*d^3*x^2 - 5*a^2*x*e^3 + (9*a*c*d*x^2 - 2*a^2*d)*e^2 + (2*c
^2*d^2*x^3 - 9*a*c*d^2*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/x
^2, 1/16*(3*(c^2*d^4*x^2 + 10*a*c*d^2*x^2*e^2 + 5*a^2*x^2*e^4)*sqrt(c*d)*e^
(-1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2
*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e^2 + c*d^2*e + a*e^3)*sqrt(c*d)
*e^(-1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 6*(5*c^2*d^4*x^2 + 10*a*c*
d^2*x^2*e^2 + a^2*x^2*e^4)*sqrt(-a*e/d)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a
*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*e/d)/(a*c*d^2*x*e
 + a^2*x*e^3 + (a*c*d*x^2 + a^2*d)*e^2)) + 4*(5*c^2*d^3*x^2 - 5*a^2*x*e^3 +
 (9*a*c*d*x^2 - 2*a^2*d)*e^2 + (2*c^2*d^2*x^3 - 9*a*c*d^2*x)*e)*sqrt(c*d^2*
x + a*x*e^2 + (c*d*x^2 + a*d)*e))/x^2, -1/8*(3*(c^2*d^4*x^2 + 10*a*c*d^2*x^
2*e^2 + 5*a^2*x^2*e^4)*sqrt(-c*d*e^(-1))*arctan(1/2*sqrt(c*d^2*x + a*x*e^2
+ (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e^(-1))/(c^2*d^3
*x + a*c*d*x*e^2 + (c^2*d^2*x^2 + a*c*d^2)*e)) - 3*(5*c^2*d^4*x^2 + 10*a*c*
d^2*x^2*e^2 + a^2*x^2*e^4)*sqrt(-a*e/d)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a
*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*e/d)/(a*c*d^2*x*e
 + a^2*x*e^3 + (a*c*d*x^2 + a^2*d)*e^2)) - 2*(5*c^2*d^3*x^2 - 5*a^2*x*e^3 +
 (9*a*c*d*x^2 - 2*a^2*d)*e^2 + (2*c^2*d^2*x^3 - 9*a*c*d^2*x)*e)*sqrt(c*d^2*
x + a*x*e^2 + (c*d*x^2 + a*d)*e))/x^2]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(291) = 582.
time = 1.78, size = 785, normalized size = 2.32

$$\frac{1}{4}\left(2c^2d^2xe + \frac{(5c^3d^4e + 9ac^2d^2e^3)e^{(-1)}}{cd}\right)\sqrt{cdx^2e + cd^2x + axe^2 + ade} + \frac{3(5ac^2d^4e + 10a^2cd^2e^3 + a^3e^5)\arctan\left(-\frac{\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}} - \frac{3\left(\sqrt{cd}\,c^3d^5e^{\frac{1}{2}} + 10\sqrt{cd}\,ac^2d^3e^{\frac{5}{2}} + 5\sqrt{cd}\,a^2cde^{\frac{9}{2}}\right)e^{(-1)}\log\left(\left|-\sqrt{cd}\,cd^2e^{\frac{1}{2}} - 2\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)cde - \sqrt{cd}\,ae^{\frac{5}{2}}\right|\right)}{8cd} - \frac{7\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)a^2c^2d^5e^2 - 9\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^3ac^2d^4e + 16\sqrt{cd}\,a^3cd^4e^{\frac{7}{2}} - 24\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2\sqrt{cd}\,a^2cd^3e^{\frac{5}{2}} + 6\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)a^3cd^3e^4 - 18\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^3a^2cd^2e^3 + 8\sqrt{cd}\,a^4d^2e^{\frac{11}{2}} - 16\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2\sqrt{cd}\,a^3de^{\frac{9}{2}} + 3\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)a^4de^6 - 5\left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^3a^3e^5}{4\left(ade - \left(\sqrt{cd}\,xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 1/4*(2*c^2*d^2*x*e + (5*c^3*d^4*e + 9*a*c^2*d^2*e^3)*e^(-1)/(c*d))*sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e) + 3/4*(5*a*c^2*d^4*e + 10*a^2*c*d^2*e^3
 + a^3*e^5)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e
^2 + a*d*e))/sqrt(-a*d*e))/sqrt(-a*d*e) - 3/8*(sqrt(c*d)*c^3*d^5*e^(1/2) +
10*sqrt(c*d)*a*c^2*d^3*e^(5/2) + 5*sqrt(c*d)*a^2*c*d*e^(9/2))*e^(-1)*log(ab
s(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^
2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c*d) - 1/4*(7*(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^2*c^2*d^5*e
^2 - 9*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^
3*a*c^2*d^4*e + 16*sqrt(c*d)*a^3*c*d^4*e^(7/2) - 24*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^2*c*d^3*e^(5/2)
+ 6*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^3
*c*d^3*e^4 - 18*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e))^3*a^2*c*d^2*e^3 + 8*sqrt(c*d)*a^4*d^2*e^(11/2) - 16*(sqrt(c*d)*x*e
^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^3*d*e^(
9/2) + 3*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)
)*a^4*d*e^6 - 5*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e))^3*a^3*e^5)/(a*d*e - (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*
x + a*x*e^2 + a*d*e))^2)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

3.464 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx$

Optimal. Leaf size=371

$$-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8dx}-\frac{\left(4ade+3\left(3cd^2+ae^2\right)x\right)}{}$$

```
[Out] -1/12*(4*a*d*e+3*(a*e^2+3*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
/d/x^3-1/16*(-a^3*e^6+15*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+5*c^3*d^6)*arctanh(
1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)+1/2*c^(3/2)*d^(3/2)*(5*a*e^2+3*
c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))*e^(1/2)-1/8*(5*c^2*d^4+12*a*c*d^2*e^2-a^2*
e^4-2*c*d*e*(a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x
```

Rubi [A]
time = 0.29, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {863, 824, 826, 857, 635, 212, 738}

$$-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8dx}-\frac{\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16\sqrt{a}d^{3/2}\sqrt{e}}+\frac{1}{2}c^{3/2}d^{3/2}\sqrt{e}\left(5ae^2+3cd^2\right)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] -1/8*((5*c^2*d^4 + 12*a*c*d^2*e^2 - a^2*e^4 - 2*c*d*e*(7*c*d^2 + a*e^2)*x)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d*x) - ((4*a*d*e + 3*(3*c*d^2
 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*d*x^3) + (c
^(3/2)*d^(3/2)*Sqrt[e]*(3*c*d^2 + 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e
*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
])/2 - ((5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh
[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])])/(16*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4}\,dx$$

$$=-\frac{\left(4ade+3\left(3cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12dx^3}-\int \frac{(-$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+c\right.}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+c\right.}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+c\right.}}{8dx}$$

$$=-\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+c\right.}}{8dx}$$

Mathematica [A]
time = 1.06, size = 316, normalized size = 0.85

$$-\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,\left(3c^2d^3x^2(11d-8ex)+2acd^2ex(13d+34ex)+a^2e^2\left(8d^2+14dex+3e^2x^2\right)\right)+3\left(5c^3d^6+45ac^2d^4e^2+15a^2cd^2e^4-a^3e^6\right)x^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ae+cdx}}{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}\right)-24\sqrt{a}\,c^{3/2}d^3e\left(3cd^2+5ae^2\right)x^3\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}\right)\right)}{24\sqrt{a}\,d^{3/2}\sqrt{e}\,x^3\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

```
[Out] -1/24*(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e +
c*d*x]*Sqrt[d + e*x]*(3*c^2*d^3*x^2*(11*d - 8*e*x) + 2*a*c*d^2*e*x*(13*d +
34*e*x) + a^2*e^2*(8*d^2 + 14*d*e*x + 3*e^2*x^2)) + 3*(5*c^3*d^6 + 45*a*c^2
*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*
x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] - 24*Sqrt[a]*c^(3/2)*d^3*e*(3*c*d^2 +
5*a*e^2)*x^3*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d +
e*x])]))/(Sqrt[a]*d^(3/2)*Sqrt[e]*x^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6849 vs. $2(327) = 654$.
time = 0.07, size = 6850, normalized size = 18.46

method	result	size
default	Expression too large to display	6850

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((x*e + d)*x^4), x)
```

Fricas [A]
time = 10.65, size = 1765, normalized size = 4.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/96*(24*(3*a*c^2*d^5*x^3*e + 5*a^2*c*d^3*x^3*e^3)*sqrt(c*d)*e^(1/2)*log(8
*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2
 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*
c^2*d^2*x^2 + 3*a*c*d^2)*e^2) - 3*(5*c^3*d^6*x^3 + 45*a*c^2*d^4*x^3*e^2 + 1
5*a^2*c*d^2*x^3*e^4 - a^3*x^3*e^6)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a
*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*
sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^
2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 4*(33*a*c^2*d^5*x^2*e + 3*a^3*d*x^2*e^5 + 14
*a^3*d^2*x*e^4 + 4*(17*a^2*c*d^3*x^2 + 2*a^3*d^3)*e^3 - 2*(12*a*c^2*d^4*x^3
 - 13*a^2*c*d^4*x)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-1)
/(a*d^2*x^3), -1/96*(3*(5*c^3*d^6*x^3 + 45*a*c^2*d^4*x^3*e^2 + 15*a^2*c*d^2
*x^3*e^4 - a^3*x^3*e^6)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e
```

```
+ a^2*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*
x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a
^2*d^2)*e^2)/x^2) + 48*(3*a*c^2*d^5*x^3*e + 5*a^2*c*d^3*x^3*e^3)*sqrt(-c*d*
e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^
2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2
)*e^2)) + 4*(33*a*c^2*d^5*x^2*e + 3*a^3*d*x^2*e^5 + 14*a^3*d^2*x*e^4 + 4*(1
7*a^2*c*d^3*x^2 + 2*a^3*d^3)*e^3 - 2*(12*a*c^2*d^4*x^3 - 13*a^2*c*d^4*x)*e^
2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-1)/(a*d^2*x^3), 1/48*(1
2*(3*a*c^2*d^5*x^3*e + 5*a^2*c*d^3*x^3*e^3)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3
*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*
x^2 + 3*a*c*d^2)*e^2) + 3*(5*c^3*d^6*x^3 + 45*a*c^2*d^4*x^3*e^2 + 15*a^2*c*
d^2*x^3*e^4 - a^3*x^3*e^6)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a
*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e
 + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) - 2*(33*a*c^2*d^5*x^2*e + 3*
a^3*d*x^2*e^5 + 14*a^3*d^2*x*e^4 + 4*(17*a^2*c*d^3*x^2 + 2*a^3*d^3)*e^3 - 2
*(12*a*c^2*d^4*x^3 - 13*a^2*c*d^4*x)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e))*e^(-1)/(a*d^2*x^3), 1/48*(3*(5*c^3*d^6*x^3 + 45*a*c^2*d^4*x^3*e
^2 + 15*a^2*c*d^2*x^3*e^4 - a^3*x^3*e^6)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x +
 a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e
)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) - 24*(3*a*c^2*
d^5*x^3*e + 5*a^2*c*d^3*x^3*e^3)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3
*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) - 2*(33*a*c^2*d^5*x^2*e
+ 3*a^3*d*x^2*e^5 + 14*a^3*d^2*x*e^4 + 4*(17*a^2*c*d^3*x^2 + 2*a^3*d^3)*e^3
 - 2*(12*a*c^2*d^4*x^3 - 13*a^2*c*d^4*x)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d
*x^2 + a*d)*e))*e^(-1)/(a*d^2*x^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. 2(321) = 642.
time = 1.56, size = 1237, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm=
"giac")
```

```
[Out] sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)*c^2*d^2*e - 1/2*(3*sqrt(c*d)*c^
3*d^4*e^(3/2) + 5*sqrt(c*d)*a*c^2*d^2*e^(7/2))*e^(-1)*log(abs(-sqrt(c*d)*c*
d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c*d) + 1/8*(5*c^3*d^6 + 45*a*c^2*d^
4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) - 1/24*
(15*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^2
*c^3*d^8*e^2 - 40*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
 + a*d*e))^3*a*c^3*d^7*e + 33*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2
*x + a*x*e^2 + a*d*e))^5*c^3*d^6 + 48*sqrt(c*d)*a^3*c^2*d^7*e^(7/2) - 144*(
sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c
*d)*a^2*c^2*d^6*e^(5/2) + 144*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2
*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a*c^2*d^5*e^(3/2) + 39*(sqrt(c*d)*x*e^(1
/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^3*c^2*d^6*e^4 - 72*(sq
rt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^2*c^2*
d^5*e^3 + 153*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a
*d*e))^5*a*c^2*d^4*e^2 + 112*sqrt(c*d)*a^4*c*d^5*e^(11/2) - 240*(sqrt(c*d)*
x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^3*c*
d^4*e^(9/2) + 288*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
 + a*d*e))^4*sqrt(c*d)*a^2*c*d^3*e^(7/2) + 45*(sqrt(c*d)*x*e^(1/2) - sqrt(c
*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^4*c*d^4*e^6 - 24*(sqrt(c*d)*x*e^(1
/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^3*c*d^3*e^5 + 99*(sq
rt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^2*c*d^
2*e^4 + 48*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*
e))^4*sqrt(c*d)*a^3*d*e^(11/2) - 3*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))*a^5*d^2*e^8 + 8*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^4*d*e^7 + 3*(sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^3*e^6)/((a*d*e - (sqrt(c*
d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2)^3*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

3.465 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx$

Optimal. Leaf size=404

$$-\frac{\left(2ade\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64ad^2ex^2}$$

```
[Out] -1/24*(6*a*d*e+(3*a*e^2+11*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/d/x^4+1/128*(-3*a^4*e^8+20*a^3*c*d^2*e^6-90*a^2*c^2*d^4*e^4-60*a*c^3*d^6*
e^2+5*c^4*d^8)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)+c^(5/2)*
d^(5/2)*e^(3/2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-1/64*(2*a*d*e*(-a*e^2+5*c*d^2)*(3
*a*e^2+c*d^2)+(-3*a^3*e^6+11*a^2*c*d^2*e^4+83*a*c^2*d^4*e^2+5*c^3*d^6)*x)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/e/x^2
```

Rubi [A]

time = 0.29, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 824, 857, 635, 212, 738}

$$-\frac{\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64ad^2ex^2}+\frac{\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{128a^{3/2}d^{5/2}e^{3/2}}+c^{5/2}d^{5/2}e^{3/2}\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] -1/64*((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (5*c^3*d^6 + 83*a*c^2
*d^4*e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(a*d^2*e*x^2) - ((6*a*d*e + (11*c*d^2 + 3*a*e^2)*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*d*x^4) + c^(5/2)*d^(5/2)*e^(3/2)*
ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])] + ((5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2
*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(2*a*d*e + (c*d^2 + a*
e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])])/(128*a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5}\,dx$$

$$=-\frac{\left(6ade+\left(11cd^2+3ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24dx^4}-\int$$

$$=-\frac{\left(2ade(5cd^2-ae^2)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade(5cd^2-ae^2)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade(5cd^2-ae^2)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e\right.\right.}{64ad^2ex^2}$$

$$=-\frac{\left(2ade(5cd^2-ae^2)\left(cd^2+3ae^2\right)+\left(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e\right.\right.}{64ad^2ex^2}$$

Mathematica [**A**]
time = 1.11, size = 358, normalized size = 0.89

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(-\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,(15c^3d^6x^3+ac^2d^4ex^2(118d+337ex)+a^2cd^2e^2x(136d^2+244dex+57e^2x^2)+3a^3e^3(16d^3+24d^2ex+2de^2x^2-3e^3x^3))+3(5c^4d^8-60ac^3d^6e^2-90a^2c^2d^4e^4+20a^3cd^2e^6-3a^4e^8)\,x^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ae+cdx}}{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}\right)+384a^{3/2}c^{5/2}d^5e^3x^4\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}\right)\right)}{192a^{3/2}d^{5/2}e^{3/2}x^4\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*(15*c^3*d^6*x^3 + a*c^2*d^4*e*x^2*(118*d + 337*e*x) + a^2*
c*d^2*e^2*x*(136*d^2 + 244*d*e*x + 57*e^2*x^2) + 3*a^3*e^3*(16*d^3 + 24*d^2
*e*x + 2*d*e^2*x^2 - 3*e^3*x^3))) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^
2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*x^4*ArcTanh[(Sqrt[d]*Sqrt[a*e
 + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 384*a^(3/2)*c^(5/2)*d^5*e^3*x
^4*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(
192*a^(3/2)*d^(5/2)*e^(3/2)*x^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11684 vs. $2(362) = 724$.
time = 0.08, size = 11685, normalized size = 28.92

method	result	size

default	Expression too large to display	11685

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((x*e + d)*x^5), x)
```

Fricas [A]
time = 26.42, size = 1945, normalized size = 4.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/768*(384*sqrt(c*d)*a^2*c^2*d^5*x^4*e^(7/2)*log(8*c^2*d^3*x*e + c^2*d^4 +
 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2
*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)
*e^2) - 3*(5*c^4*d^8*x^4 - 60*a*c^3*d^6*x^4*e^2 - 90*a^2*c^2*d^4*x^4*e^4 +
20*a^3*c*d^2*x^4*e^6 - 3*a^4*x^4*e^8)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 +
8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c
*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 4*(15*a*c^3*d^7*x^3*e + 118*a^2*c^2*d^6*x
^2*e^2 - 9*a^4*d*x^3*e^7 + 6*a^4*d^2*x^2*e^6 + 3*(19*a^3*c*d^3*x^3 + 24*a^4
*d^3*x)*e^5 + 4*(61*a^3*c*d^4*x^2 + 12*a^4*d^4)*e^4 + (337*a^2*c^2*d^5*x^3
+ 136*a^3*c*d^5*x)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)
/(a^2*d^3*x^4), -1/768*(768*sqrt(-c*d*e)*a^2*c^2*d^5*x^4*arctan(1/2*sqrt(c*
d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*
e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2))*e^3 + 3*(5*c^
4*d^8*x^4 - 60*a*c^3*d^6*x^4*e^2 - 90*a^2*c^2*d^4*x^4*e^4 + 20*a^3*c*d^2*x^
4*e^6 - 3*a^4*x^4*e^8)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e +
```

```
a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x
 + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^
2*d^2)*e^2)/x^2) + 4*(15*a*c^3*d^7*x^3*e + 118*a^2*c^2*d^6*x^2*e^2 - 9*a^4*
d*x^3*e^7 + 6*a^4*d^2*x^2*e^6 + 3*(19*a^3*c*d^3*x^3 + 24*a^4*d^3*x)*e^5 + 4
*(61*a^3*c*d^4*x^2 + 12*a^4*d^4)*e^4 + (337*a^2*c^2*d^5*x^3 + 136*a^3*c*d^5
*x)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)/(a^2*d^3*x^4),
 1/384*(192*sqrt(c*d)*a^2*c^2*d^5*x^4*e^(7/2)*log(8*c^2*d^3*x*e + c^2*d^4 +
 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2
*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)
*e^2) - 3*(5*c^4*d^8*x^4 - 60*a*c^3*d^6*x^4*e^2 - 90*a^2*c^2*d^4*x^4*e^4 +
20*a^3*c*d^2*x^4*e^6 - 3*a^4*x^4*e^8)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*
x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(
a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) - 2*(15*a*c^3*d^7
*x^3*e + 118*a^2*c^2*d^6*x^2*e^2 - 9*a^4*d*x^3*e^7 + 6*a^4*d^2*x^2*e^6 + 3*
(19*a^3*c*d^3*x^3 + 24*a^4*d^3*x)*e^5 + 4*(61*a^3*c*d^4*x^2 + 12*a^4*d^4)*e
^4 + (337*a^2*c^2*d^5*x^3 + 136*a^3*c*d^5*x)*e^3)*sqrt(c*d^2*x + a*x*e^2 +
(c*d*x^2 + a*d)*e))*e^(-2)/(a^2*d^3*x^4), -1/384*(384*sqrt(-c*d*e)*a^2*c^2*
d^5*x^4*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e +
 c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*
c*d^2)*e^2))*e^3 + 3*(5*c^4*d^8*x^4 - 60*a*c^3*d^6*x^4*e^2 - 90*a^2*c^2*d^4
*x^4*e^4 + 20*a^3*c*d^2*x^4*e^6 - 3*a^4*x^4*e^8)*sqrt(-a*d*e)*arctan(1/2*(c
*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqr
t(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(1
5*a*c^3*d^7*x^3*e + 118*a^2*c^2*d^6*x^2*e^2 - 9*a^4*d*x^3*e^7 + 6*a^4*d^2*x
^2*e^6 + 3*(19*a^3*c*d^3*x^3 + 24*a^4*d^3*x)*e^5 + 4*(61*a^3*c*d^4*x^2 + 12
*a^4*d^4)*e^4 + (337*a^2*c^2*d^5*x^3 + 136*a^3*c*d^5*x)*e^3)*sqrt(c*d^2*x +
 a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-2)/(a^2*d^3*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1783 vs. 2(353) = 706.
time = 1.60, size = 1783, normalized size = 4.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm=
"giac")

[Out] -sqrt(c*d)*c^2*d^2*e^(3/2)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*
x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*
a*e^(5/2))) - 1/64*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*
a^3*c*d^2*e^6 - 3*a^4*e^8)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))*e^(-1)/(sqrt(-a*d*e)*a*d^2) + 1/1
92*(15*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*
a^3*c^4*d^11*e^3 - 55*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e))^3*a^2*c^4*d^10*e^2 + 73*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))^5*a*c^4*d^9*e + 15*(sqrt(c*d)*x*e^(1/2) - s
qrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*c^4*d^8 + 384*(sqrt(c*d)*x*e^
(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a*c^3*d^7*
e^(3/2) - 180*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a
*d*e))*a^4*c^3*d^9*e^5 + 660*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*
x + a*x*e^2 + a*d*e))^3*a^3*c^3*d^8*e^4 + 276*(sqrt(c*d)*x*e^(1/2) - sqrt(c
*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^2*c^3*d^7*e^3 + 588*(sqrt(c*d)*x
*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a*c^3*d^6*e^2 - 5
12*sqrt(c*d)*a^5*c^2*d^8*e^(13/2) + 2048*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^
2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^4*c^2*d^7*e^(11/2) - 1152*(
sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c
*d)*a^3*c^2*d^6*e^(9/2) + 2304*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^
2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^2*c^2*d^5*e^(7/2) + 114*(sqrt(c*d)*x*
e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^5*c^2*d^7*e^7 + 13
74*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^
4*c^2*d^6*e^6 + 990*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e
^2 + a*d*e))^5*a^3*c^2*d^5*e^5 + 882*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e
+ c*d^2*x + a*x*e^2 + a*d*e))^7*a^2*c^2*d^4*e^4 + 768*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^5*c*d^5*e^(15/
2) + 768*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)
)^4*sqrt(c*d)*a^4*c*d^4*e^(13/2) + 1152*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2
*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^3*c*d^3*e^(11/2) + 60*(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^6*c*d^5*e^9
 + 548*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^
3*a^5*c*d^4*e^8 + 676*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e))^5*a^4*c*d^3*e^7 + 60*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
 c*d^2*x + a*x*e^2 + a*d*e))^7*a^3*c*d^2*e^6 + 384*(sqrt(c*d)*x*e^(1/2) - s
qrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^5*d^2*e^(17/2) -
9*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^7*d
^3*e^11 + 33*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*
d*e))^3*a^6*d^2*e^10 + 33*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
 a*x*e^2 + a*d*e))^5*a^5*d*e^9 - 9*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))^7*a^4*e^8)*e^(-1)/((a*d*e - (sqrt(c*d)*x*e^(1/2
) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2)^4*a*d^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^5\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

3.466 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx$

Optimal. Leaf size=289

$$\frac{3\left(cd^2-ae^2\right)^3\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128a^2d^3e^2x^2}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade\right.}{16x^4}$$

```
[Out] -1/16*(c/a/e-e/d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/x^4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^5-3/256*(-a*
e^2+c*d^2)^5*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)+3/128*(-a*
e^2+c*d^2)^3*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^2/d^3/e^2/x^2
```

Rubi [A]
time = 0.21, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {863, 820, 734, 738, 212}

$$-\frac{3\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}}+\frac{3\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128a^2d^3e^2x^2}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] (3*(c*d^2 - a*e^2)^3*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(128*a^2*d^3*e^2*x^2) - ((c/(a*e) - e/d^2)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(16*x^4)
- (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*d*x^5) - (3*(c*d^2 - a*e
^2)^5*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(256*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
```

```
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx = \int \frac{(ae+cdx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6}\,dx$$

$$= -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5dx^5} - \frac{(-2acd^2e+ae(cd^2+ae^2))}{2a}$$

$$= -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3}}{16x^4}$$

$$= \frac{3(cd^2-ae^2)^3(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cde}}{128a^2d^3e^2x^2}$$

$$= \frac{3(cd^2-ae^2)^3(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cde}}{128a^2d^3e^2x^2}$$

$$= \frac{3(cd^2-ae^2)^3(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cde}}{128a^2d^3e^2x^2}$$

Mathematica [A]
time = 0.72, size = 271, normalized size = 0.94

$$\frac{(-cd^2+ae^2)^5((ae+cdx)(d+ex))^{3/2}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)^3\left(15a^4e^4-\frac{15d^4(ae+cdx)^4}{(d+ex)^4}+\frac{70ad^3e(ae+cdx)^3}{(d+ex)^3}+\frac{128a^2d^2e^2(ae+cdx)^2}{(d+ex)^2}-\frac{70a^3de^3(ae+cdx)}{d+ex}\right)}{(-cd^2+ae^2)^5x^5(ae+cdx)}+\frac{15\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{(ae+cdx)^{3/2}(d+ex)^{3/2}}\right)}{640a^{5/2}d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

```
[Out] ((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*(-((Sqrt[a]*Sqrt[d]*S
qrt[e]*(d + e*x)^3*(15*a^4*e^4 - (15*d^4*(a*e + c*d*x)^4)/(d + e*x)^4 + (70
*a*d^3*e*(a*e + c*d*x)^3)/(d + e*x)^3 + (128*a^2*d^2*e^2*(a*e + c*d*x)^2)/(
d + e*x)^2 - (70*a^3*d*e^3*(a*e + c*d*x))/(d + e*x)))/((-(c*d^2) + a*e^2)^5
*x^5*(a*e + c*d*x))) + (15*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqr
t[e]*Sqrt[d + e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(640*a^(5/2)*
d^(7/2)*e^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19538 vs. $2(259) = 518$.
time = 0.08, size = 19539, normalized size = 67.61

method	result	size

default	Expression too large to display	19539

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((x*e + d)*x^6), x)
```

Fricas [A]
time = 21.73, size = 901, normalized size = 3.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [1/2560*(15*(c^5*d^10*x^5 - 5*a*c^4*d^8*x^5*e^2 + 10*a^2*c^3*d^6*x^5*e^4 -
10*a^3*c^2*d^4*x^5*e^6 + 5*a^4*c*d^2*x^5*e^8 - a^5*x^5*e^10)*sqrt(a*d)*e^(1
/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d
^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(
a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) + 4*(15*a*c^4*d^9*x^
4*e - 10*a^2*c^3*d^8*x^3*e^2 - 15*a^5*d*x^4*e^9 + 10*a^5*d^2*x^3*e^8 + 2*(3
5*a^4*c*d^3*x^4 - 4*a^5*d^3*x^2)*e^7 - 2*(23*a^4*c*d^4*x^3 + 88*a^5*d^4*x)*
e^6 - 128*(a^3*c^2*d^5*x^4 + 4*a^4*c*d^5*x^2 + a^5*d^5)*e^5 - 2*(233*a^3*c^
2*d^6*x^3 + 168*a^4*c*d^6*x)*e^4 - 2*(35*a^2*c^3*d^7*x^4 + 124*a^3*c^2*d^7*
x^2)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-3)/(a^3*d^4*x^5)
, 1/1280*(15*(c^5*d^10*x^5 - 5*a*c^4*d^8*x^5*e^2 + 10*a^2*c^3*d^6*x^5*e^4 -
 10*a^3*c^2*d^4*x^5*e^6 + 5*a^4*c*d^2*x^5*e^8 - a^5*x^5*e^10)*sqrt(-a*d*e)*
arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
+ a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)
*e^2)) + 2*(15*a*c^4*d^9*x^4*e - 10*a^2*c^3*d^8*x^3*e^2 - 15*a^5*d*x^4*e^9
+ 10*a^5*d^2*x^3*e^8 + 2*(35*a^4*c*d^3*x^4 - 4*a^5*d^3*x^2)*e^7 - 2*(23*a^4
```

```
*c*d^4*x^3 + 88*a^5*d^4*x)*e^6 - 128*(a^3*c^2*d^5*x^4 + 4*a^4*c*d^5*x^2 + a
^5*d^5)*e^5 - 2*(233*a^3*c^2*d^6*x^3 + 168*a^4*c*d^6*x)*e^4 - 2*(35*a^2*c^3
*d^7*x^4 + 124*a^3*c^2*d^7*x^2)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*
d)*e))*e^(-3)/(a^3*d^4*x^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2480 vs. 2(258) = 516.
time = 1.55, size = 2480, normalized size = 8.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm=
"giac")
```

```
[Out] 3/128*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6
 + 5*a^4*c*d^2*e^8 - a^5*e^10)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))*e^(-2)/(sqrt(-a*d*e)*a^2*d^3)
 - 1/640*(15*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*
d*e))*a^4*c^5*d^14*e^4 - 70*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
 + a*x*e^2 + a*d*e))^3*a^3*c^5*d^13*e^3 + 128*(sqrt(c*d)*x*e^(1/2) - sqrt(c
*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^2*c^5*d^12*e^2 + 70*(sqrt(c*d)*x
*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a*c^5*d^11*e - 15
*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*c^5*
d^10 + 1280*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d
*e))^6*sqrt(c*d)*a^2*c^4*d^10*e^(5/2) - 75*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*
x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^5*c^4*d^12*e^6 + 350*(sqrt(c*d)*x*e^(
1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^4*c^4*d^11*e^5 + 32
00*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^
3*c^4*d^10*e^4 + 2210*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e))^7*a^2*c^4*d^9*e^3 + 75*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e
 + c*d^2*x + a*x*e^2 + a*d*e))^9*a*c^4*d^8*e^2 + 6400*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^4*c^3*d^9*e^(1
1/2) + 6400*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d
*e))^6*sqrt(c*d)*a^3*c^3*d^8*e^(9/2) + 2560*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(c*d)*a^2*c^3*d^7*e^(7/2) + 150*
```

```
(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^6*c^3
*d^10*e^8 + 5700*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
+ a*d*e))^3*a^5*c^3*d^9*e^7 + 12800*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
 c*d^2*x + a*x*e^2 + a*d*e))^5*a^4*c^3*d^8*e^6 + 7100*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^3*c^3*d^7*e^5 + 1130*(sq
rt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^2*c^3*
d^6*e^4 + 256*sqrt(c*d)*a^7*c^2*d^9*e^(19/2) + 2560*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^6*c^2*d^8*e^(17/
2) + 14080*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*
e))^4*sqrt(c*d)*a^5*c^2*d^7*e^(15/2) + 11520*(sqrt(c*d)*x*e^(1/2) - sqrt(c*
d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^4*c^2*d^6*e^(13/2) + 38
40*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sq
rt(c*d)*a^3*c^2*d^5*e^(11/2) + 1130*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
 c*d^2*x + a*x*e^2 + a*d*e))*a^7*c^2*d^8*e^10 + 7100*(sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^6*c^2*d^7*e^9 + 12800*(sq
rt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^5*c^2*
d^6*e^8 + 5700*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
a*d*e))^7*a^4*c^2*d^5*e^7 + 150*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d
^2*x + a*x*e^2 + a*d*e))^9*a^3*c^2*d^4*e^6 + 2560*(sqrt(c*d)*x*e^(1/2) - sq
rt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^7*c*d^6*e^(21/2) +
 6400*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4
*sqrt(c*d)*a^6*c*d^5*e^(19/2) + 6400*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e
+ c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^5*c*d^4*e^(17/2) + 75*(sqrt(c*d
)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^8*c*d^6*e^12 +
 2210*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3
*a^7*c*d^5*e^11 + 3200*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e))^5*a^6*c*d^4*e^10 + 350*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^5*c*d^3*e^9 - 75*(sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^4*c*d^2*e^8 + 1280*(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a
^7*d^3*e^(23/2) - 15*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))*a^9*d^4*e^14 + 70*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d
^2*x + a*x*e^2 + a*d*e))^3*a^8*d^3*e^13 + 128*(sqrt(c*d)*x*e^(1/2) - sqrt(c
*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^7*d^2*e^12 - 70*(sqrt(c*d)*x*e^(
1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^6*d*e^11 + 15*(sqrt
(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^5*e^10)*
e^(-2)/((a*d*e - (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
+ a*d*e))^2)^5*a^2*d^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^6\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

3.467 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx$

Optimal. Leaf size=386

$$-\frac{\left(cd^2-ae^2\right)^3\left(5cd^2+7ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512a^3d^4e^3x^2}+\frac{\left(cd^2-ae^2\right)\left(5cd^2+7ae^2\right)}{}$$

```
[Out] 1/192*(-a*e^2+c*d^2)*(7*a*e^2+5*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(5/2)/d/x^6-1/60*(5*c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(5/2)/x^5+1/1024*(-a*e^2+c*d^2)^5*(7*a*e^2+5*c*d^2)*arctanh(1/2*(2*a*d*e
+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)-1/512*(-a*e^2+c*d^2)^3*(7*a*e^2+5*c*d^2)*(2
*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3
/x^2
```

Rubi [A]
time = 0.32, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

$$\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}}-\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512a^3d^4e^3x^2}+\frac{\left(7ae^2+5cd^2\right)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{192a^2d^3e^2x^4}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}-\frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] -1/512*((c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a^3*d^4*e^3*x^2) + ((c*d^2 -
 a*e^2)*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(192*a^2*d^3*e^2*x^4) - (a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/2)/(6*d*x^6) - (((5*c)/(a*e) - (7*e)/d^2)*(a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(60*x^5) + ((c*d^2 - a*e^2)^5*(5*c*d^2
+ 7*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(1024*a^(7/2)*d^(9/2)*e^(7/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)}\,dx = \int \frac{(ae+cdx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^7}\,dx$$

$$= -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6dx^6} - \frac{\int \frac{\left(-\frac{1}{2}ae(5cd^2-7ae^2)+acde^2x\right)(ade+}{x^6}}{6ade}$$

$$= -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)}{60x^5}$$

$$= \frac{(cd^2-ae^2)(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)}{192a^2d^3e^2x^4}$$

$$= -\frac{(cd^2-ae^2)^3(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+}}{512a^3d^4e^3x^2}$$

$$= -\frac{(cd^2-ae^2)^3(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+}}{512a^3d^4e^3x^2}$$

$$= -\frac{(cd^2-ae^2)^3(5cd^2+7ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+}}{512a^3d^4e^3x^2}$$

Mathematica [A]
time = 0.97, size = 404, normalized size = 1.05

$$\frac{(-cd^2+ae^2)^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\,(75c^5d^{10}x^5-5ac^4d^8ex^4(10d+49ex)+10a^2c^3d^6e^2x^3(4d^2+16dex+15e^2x^2)+6a^3c^2d^4e^3x^2(360d^3+564d^2ex+58de^2x^2-91e^3x^3)+a^4cd^2e^4x(3200d^4+4448d^3ex+216d^2e^2x^2-272de^3x^3+415e^4x^4)+a^5e^5(1280d^5+1664d^4ex+48d^3e^2x^2-56d^2e^3x^3+70de^4x^4-105e^5x^5))}{(cd^2-ae^2)^5x^6(ae+cdx)(d+ex)}-\frac{15(5cd^2+7ae^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{(ae+cdx)^{3/2}(d+ex)^{3/2}}\right)}{7680a^{7/2}d^{9/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

```
[Out] ((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[a]*Sqrt[d]*Sqr
t[e]*(75*c^5*d^10*x^5 - 5*a*c^4*d^8*e*x^4*(10*d + 49*e*x) + 10*a^2*c^3*d^6*
e^2*x^3*(4*d^2 + 16*d*e*x + 15*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(360*d^3 +
564*d^2*e*x + 58*d*e^2*x^2 - 91*e^3*x^3) + a^4*c*d^2*e^4*x*(3200*d^4 + 4448
*d^3*e*x + 216*d^2*e^2*x^2 - 272*d*e^3*x^3 + 415*e^4*x^4) + a^5*e^5*(1280*d
^5 + 1664*d^4*e*x + 48*d^3*e^2*x^2 - 56*d^2*e^3*x^3 + 70*d*e^4*x^4 - 105*e^
5*x^5)))/((c*d^2 - a*e^2)^5*x^6*(a*e + c*d*x)*(d + e*x)) - (15*(5*c*d^2 + 7
*a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])
])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(7680*a^(7/2)*d^(9/2)*e^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32290 vs. $2(352) = 704$.
time = 0.07, size = 32291, normalized size = 83.66

method	result	size
default	Expression too large to display	32291

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((x*e + d)*x^7), x)
```

Fricas [A]
time = 85.94, size = 1115, normalized size = 2.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/30720*(15*(5*c^6*d^12*x^6 - 18*a*c^5*d^10*x^6*e^2 + 15*a^2*c^4*d^8*x^6*
e^4 + 20*a^3*c^3*d^6*x^6*e^6 - 45*a^4*c^2*d^4*x^6*e^8 + 30*a^5*c*d^2*x^6*e^
10 - 7*a^6*x^6*e^12)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a
^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x +
 a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*
d^2)*e^2)/x^2) + 4*(75*a*c^5*d^11*x^5*e - 50*a^2*c^4*d^10*x^4*e^2 - 105*a^6
*d*x^5*e^11 + 70*a^6*d^2*x^4*e^10 + (415*a^5*c*d^3*x^5 - 56*a^6*d^3*x^3)*e^
9 - 16*(17*a^5*c*d^4*x^4 - 3*a^6*d^4*x^2)*e^8 - 2*(273*a^4*c^2*d^5*x^5 - 10
8*a^5*c*d^5*x^3 - 832*a^6*d^5*x)*e^7 + 4*(87*a^4*c^2*d^6*x^4 + 1112*a^5*c*d
^6*x^2 + 320*a^6*d^6)*e^6 + 2*(75*a^3*c^3*d^7*x^5 + 1692*a^4*c^2*d^7*x^3 +
1600*a^5*c*d^7*x)*e^5 + 80*(2*a^3*c^3*d^8*x^4 + 27*a^4*c^2*d^8*x^2)*e^4 - 5
*(49*a^2*c^4*d^9*x^5 - 8*a^3*c^3*d^9*x^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*
d*x^2 + a*d)*e))*e^(-4)/(a^4*d^5*x^6), -1/15360*(15*(5*c^6*d^12*x^6 - 18*a*
c^5*d^10*x^6*e^2 + 15*a^2*c^4*d^8*x^6*e^4 + 20*a^3*c^3*d^6*x^6*e^6 - 45*a^4
*c^2*d^4*x^6*e^8 + 30*a^5*c*d^2*x^6*e^10 - 7*a^6*x^6*e^12)*sqrt(-a*d*e)*arc
```

```
tan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^
2)) + 2*(75*a*c^5*d^11*x^5*e - 50*a^2*c^4*d^10*x^4*e^2 - 105*a^6*d*x^5*e^11
 + 70*a^6*d^2*x^4*e^10 + (415*a^5*c*d^3*x^5 - 56*a^6*d^3*x^3)*e^9 - 16*(17*
a^5*c*d^4*x^4 - 3*a^6*d^4*x^2)*e^8 - 2*(273*a^4*c^2*d^5*x^5 - 108*a^5*c*d^5
*x^3 - 832*a^6*d^5*x)*e^7 + 4*(87*a^4*c^2*d^6*x^4 + 1112*a^5*c*d^6*x^2 + 32
0*a^6*d^6)*e^6 + 2*(75*a^3*c^3*d^7*x^5 + 1692*a^4*c^2*d^7*x^3 + 1600*a^5*c*
d^7*x)*e^5 + 80*(2*a^3*c^3*d^8*x^4 + 27*a^4*c^2*d^8*x^2)*e^4 - 5*(49*a^2*c^
4*d^9*x^5 - 8*a^3*c^3*d^9*x^3)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d
)*e))*e^(-4)/(a^4*d^5*x^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3430 vs. 2(347) = 694.
time = 1.91, size = 3430, normalized size = 8.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -1/512*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d^
6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*arctan(-(sqrt(
c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))
*e^(-3)/(sqrt(-a*d*e)*a^3*d^4) + 1/7680*(75*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^5*c^6*d^17*e^5 - 425*(sqrt(c*d)*x*e^
(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^4*c^6*d^16*e^4 + 9
90*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^
3*c^6*d^15*e^3 + 990*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))^7*a^2*c^6*d^14*e^2 - 425*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))^9*a*c^6*d^13*e + 75*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*c^6*d^12 + 15360*(sqrt(c*d)
*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^3*c
^5*d^13*e^(7/2) - 270*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e))*a^6*c^5*d^15*e^7 + 1530*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*
e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^5*c^5*d^14*e^6 + 42516*(sqrt(c*d)*x*e^(
```

1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^4*c^5*d^13*e^5 + 39
444*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a
^3*c^5*d^12*e^4 + 1530*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e))^9*a^2*c^5*d^11*e^3 - 270*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^
2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*a*c^5*d^10*e^2 + 76800*(sqrt(c*d)*x*e^
(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^5*c^4*d^
12*e^(13/2) + 143360*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))^6*sqrt(c*d)*a^4*c^4*d^11*e^(11/2) + 61440*(sqrt(c*d)*x*e^(1/2
) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(c*d)*a^3*c^4*d^10*e
^(9/2) + 225*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*
d*e))*a^7*c^4*d^13*e^9 + 75525*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^
2*x + a*x*e^2 + a*d*e))^3*a^6*c^4*d^12*e^8 + 279450*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^5*c^4*d^11*e^7 + 233370*(s
qrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^4*c^4
*d^10*e^6 + 44805*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
 + a*d*e))^9*a^3*c^4*d^9*e^5 + 225*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))^11*a^2*c^4*d^8*e^4 + 46080*(sqrt(c*d)*x*e^(1/2)
 - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^7*c^3*d^11*e^
(19/2) + 307200*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e))^4*sqrt(c*d)*a^6*c^3*d^10*e^(17/2) + 460800*(sqrt(c*d)*x*e^(1/2) -
sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^5*c^3*d^9*e^(15/
2) + 184320*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d
*e))^8*sqrt(c*d)*a^4*c^3*d^8*e^(13/2) + 15360*(sqrt(c*d)*x*e^(1/2) - sqrt(c
*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^10*sqrt(c*d)*a^3*c^3*d^7*e^(11/2) +
15660*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a
^8*c^3*d^11*e^11 + 203100*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
 a*x*e^2 + a*d*e))^3*a^7*c^3*d^10*e^10 + 526200*(sqrt(c*d)*x*e^(1/2) - sqrt
(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^6*c^3*d^9*e^9 + 372600*(sqrt(c
*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^5*c^3*d^8*
e^8 + 64860*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d
*e))^9*a^4*c^3*d^7*e^7 + 300*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*
x + a*x*e^2 + a*d*e))^11*a^3*c^3*d^6*e^6 + 3072*sqrt(c*d)*a^9*c^2*d^10*e^(2
5/2) + 73728*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*
d*e))^2*sqrt(c*d)*a^8*c^2*d^9*e^(23/2) + 368640*(sqrt(c*d)*x*e^(1/2) - sqrt
(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^7*c^2*d^8*e^(21/2) +
 430080*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))
^6*sqrt(c*d)*a^6*c^2*d^7*e^(19/2) + 138240*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*
x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(c*d)*a^5*c^2*d^6*e^(17/2) + 1468
5*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^9*c
^2*d^9*e^13 + 142065*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*
e^2 + a*d*e))^3*a^8*c^2*d^8*e^12 + 313650*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x
^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^7*c^2*d^7*e^11 + 160050*(sqrt(c*d)*x
*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^6*c^2*d^6*e^10
+ 3825*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^
9*a^5*c^2*d^5*e^9 - 675*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a

```
*x*e^2 + a*d*e))^11*a^4*c^2*d^4*e^8 + 30720*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d
*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^9*c*d^7*e^(27/2) + 12288
0*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqr
t(c*d)*a^8*c*d^6*e^(25/2) + 128000*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^7*c*d^5*e^(23/2) + 450*(sqrt(c*d)
*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^10*c*d^7*e^15 +
 28170*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + ...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^7\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

3.468 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx$

Optimal. Leaf size=500

$$\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{1024a^4d^5e^4x^2}-\frac{\left(cd^2-ae^2\right.}{}$$

[Out] -1/384*(-a*e^2+c*d^2)*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^4-1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^7-1/84*(5*c/a/e-9*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6+1/840*(-63*a^2*e^4+20*a*c*d^2*e^2+35*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2/d^3/e^2/x^5-1/2048*(-a*e^2+c*d^2)^5*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(9/2)/d^(11/2)/e^(9/2)+1/1024*(-a*e^2+c*d^2)^3*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^5/e^4/x^2

Rubi [A]
time = 0.41, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

$$\frac{\left(-63a^2e^4+20acd^2e^2+35c^2d^4\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}-\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^5\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2048a^{9/2}d^{11/2}e^{9/2}}+\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)^3\left(x(ae^2+cd^2)+2ade\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{1024a^4d^5e^4x^2}-\frac{\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(cd^2-ae^2\right)\left(x(ae^2+cd^2)+2ade\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{384a^3d^4e^3x^4}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{84x^6}$$

Antiderivative was successfully verified.

[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]

[Out] ((c*d^2 - a*e^2)^3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*a^4*d^5*e^4*x^2) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(384*a^3*d^4*e^3*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(7*d*x^7) - (((5*c)/(a*e) - (9*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(84*x^6) + ((35*c^2*d^4 + 20*a*c*d^2*e^2 - 63*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(840*a^2*d^3*e^2*x^5) - ((c*d^2 - a*e^2)^5*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2048*a^(9/2)*d^(11/2)*e^(9/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
```

```
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx=\int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^8}\,dx$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-9ae^2\right)+2acde^2x\right)(ad}{x^7}}{7ade}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right.\right.}{84x^6}$$

$$=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right.\right.}{84x^6}$$

$$=-\frac{\left(cd^2-ae^2\right)\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)(}{384a^3d^4e^3x^4}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{}}{1024a^4d^5e^4x^2}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{}}{1024a^4d^5e^4x^2}$$

$$=\frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{}}{1024a^4d^5e^4x^2}$$

Mathematica [A]

time = 1.30, size = 497, normalized size = 0.99

$$\frac{\left(-cd^2+ae^2\right)^5\left((ae+cdx)(d+ex)\right)^{3/2}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-525c^6d^{12}x^6+350ac^5d^{10}ex^5(d+4ex)-35a^2c^4d^8e^2x^4\left(8d^2+26dex+15e^2x^2\right)+60a^3c^3d^6e^3x^3\left(4d^3+12d^2ex+5de^2x^2-10e^3x^3\right)+a^4c^2d^4e^4x^2\left(23680d^4+33520d^3ex+1824d^2e^2x^2-2332de^3x^3+3689e^4x^4\right)+2a^5cd^2e^5x\left(18560d^5+24320d^4ex+744d^3e^2x^2-872d^2e^3x^3+1099de^4x^4-1680e^5x^5\right)+3a^6e^6\left(5120d^6+6400d^5ex+128d^4e^2x^2-144d^3e^3x^3+168d^2e^4x^4-210de^5x^5+315e^6x^6\right)\right)}{\left(cd^2-ae^2\right)^5x^7(ae+cdx)(d+ex)}+\frac{105\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{(ae+cdx)^{3/2}(d+ex)^{3/2}}\right)}{107520a^{9/2}d^{11/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] ((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[a]*Sqrt[d]*Sqr
t[e]*(-525*c^6*d^12*x^6 + 350*a*c^5*d^10*e*x^5*(d + 4*e*x) - 35*a^2*c^4*d^8
*e^2*x^4*(8*d^2 + 26*d*e*x + 15*e^2*x^2) + 60*a^3*c^3*d^6*e^3*x^3*(4*d^3 +
12*d^2*e*x + 5*d*e^2*x^2 - 10*e^3*x^3) + a^4*c^2*d^4*e^4*x^2*(23680*d^4 + 3
3520*d^3*e*x + 1824*d^2*e^2*x^2 - 2332*d*e^3*x^3 + 3689*e^4*x^4) + 2*a^5*c*
d^2*e^5*x*(18560*d^5 + 24320*d^4*e*x + 744*d^3*e^2*x^2 - 872*d^2*e^3*x^3 +
```

```
1099*d*e^4*x^4 - 1680*e^5*x^5) + 3*a^6*e^6*(5120*d^6 + 6400*d^5*e*x + 128*d
^4*e^2*x^2 - 144*d^3*e^3*x^3 + 168*d^2*e^4*x^4 - 210*d*e^5*x^5 + 315*e^6*x^
6)))/((c*d^2 - a*e^2)^5*x^7*(a*e + c*d*x)*(d + e*x)) + (105*(5*c^2*d^4 + 10
*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt
[e]*Sqrt[d + e*x])]])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(107520*a^(9/2
)*d^(11/2)*e^(9/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 45105 vs. $2(462) = 924$.
time = 0.08, size = 45106, normalized size = 90.21

method	result	size
default	Expression too large to display	45106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((x*e + d)*x^8), x)
```

Fricas [**A**]
time = 176.66, size = 1355, normalized size = 2.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] [-1/430080*(105*(5*c^7*d^14*x^7 - 15*a*c^6*d^12*x^7*e^2 + 9*a^2*c^5*d^10*x^
7*e^4 + 5*a^3*c^4*d^8*x^7*e^6 + 15*a^4*c^3*d^6*x^7*e^8 - 45*a^5*c^2*d^4*x^7
*e^10 + 35*a^6*c*d^2*x^7*e^12 - 9*a^7*x^7*e^14)*sqrt(a*d)*e^(1/2)*log((c^2*
d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^
2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2)
+ 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 4*(525*a*c^6*d^13*x^6*e - 350*a
```

```
^2*c^5*d^12*x^5*e^2 - 945*a^7*d*x^6*e^13 + 630*a^7*d^2*x^5*e^12 + 168*(20*a
^6*c*d^3*x^6 - 3*a^7*d^3*x^4)*e^11 - 2*(1099*a^6*c*d^4*x^5 - 216*a^7*d^4*x^
3)*e^10 - (3689*a^5*c^2*d^5*x^6 - 1744*a^6*c*d^5*x^4 + 384*a^7*d^5*x^2)*e^9
 + 4*(583*a^5*c^2*d^6*x^5 - 372*a^6*c*d^6*x^3 - 4800*a^7*d^6*x)*e^8 + 8*(75
*a^4*c^3*d^7*x^6 - 228*a^5*c^2*d^7*x^4 - 6080*a^6*c*d^7*x^2 - 1920*a^7*d^7)
*e^7 - 20*(15*a^4*c^3*d^8*x^5 + 1676*a^5*c^2*d^8*x^3 + 1856*a^6*c*d^8*x)*e^
6 + 5*(105*a^3*c^4*d^9*x^6 - 144*a^4*c^3*d^9*x^4 - 4736*a^5*c^2*d^9*x^2)*e^
5 + 10*(91*a^3*c^4*d^10*x^5 - 24*a^4*c^3*d^10*x^3)*e^4 - 280*(5*a^2*c^5*d^1
1*x^6 - a^3*c^4*d^11*x^4)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))
*e^(-5)/(a^5*d^6*x^7), 1/215040*(105*(5*c^7*d^14*x^7 - 15*a*c^6*d^12*x^7*e^
2 + 9*a^2*c^5*d^10*x^7*e^4 + 5*a^3*c^4*d^8*x^7*e^6 + 15*a^4*c^3*d^6*x^7*e^8
 - 45*a^5*c^2*d^4*x^7*e^10 + 35*a^6*c*d^2*x^7*e^12 - 9*a^7*x^7*e^14)*sqrt(-
a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a
^2*d^2)*e^2)) + 2*(525*a*c^6*d^13*x^6*e - 350*a^2*c^5*d^12*x^5*e^2 - 945*a^
7*d*x^6*e^13 + 630*a^7*d^2*x^5*e^12 + 168*(20*a^6*c*d^3*x^6 - 3*a^7*d^3*x^4
)*e^11 - 2*(1099*a^6*c*d^4*x^5 - 216*a^7*d^4*x^3)*e^10 - (3689*a^5*c^2*d^5*
x^6 - 1744*a^6*c*d^5*x^4 + 384*a^7*d^5*x^2)*e^9 + 4*(583*a^5*c^2*d^6*x^5 -
372*a^6*c*d^6*x^3 - 4800*a^7*d^6*x)*e^8 + 8*(75*a^4*c^3*d^7*x^6 - 228*a^5*c
^2*d^7*x^4 - 6080*a^6*c*d^7*x^2 - 1920*a^7*d^7)*e^7 - 20*(15*a^4*c^3*d^8*x^
5 + 1676*a^5*c^2*d^8*x^3 + 1856*a^6*c*d^8*x)*e^6 + 5*(105*a^3*c^4*d^9*x^6 -
 144*a^4*c^3*d^9*x^4 - 4736*a^5*c^2*d^9*x^2)*e^5 + 10*(91*a^3*c^4*d^10*x^5
- 24*a^4*c^3*d^10*x^3)*e^4 - 280*(5*a^2*c^5*d^11*x^6 - a^3*c^4*d^11*x^4)*e^
3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))*e^(-5)/(a^5*d^6*x^7)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4505 vs. 2(452) = 904.
time = 1.81, size = 4505, normalized size = 9.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm=
"giac")
```

[Out] 1/1024*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*d^10*e^4 + 5*a^3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 35*a^6*c*d^2*e^12 - 9*a^7*e^14)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))*e^(-4)/(sqrt(-a*d*e)*a^4*d^5) - 1/107520*(525*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^6*c^7*d^20*e^6 - 3500*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^5*c^7*d^19*e^5 + 9905*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^4*c^7*d^18*e^4 + 15360*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^3*c^7*d^17*e^3 - 9905*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^2*c^7*d^16*e^2 + 3500*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*a*c^7*d^15*e - 525*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^13*c^7*d^14 + 215040*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^4*c^6*d^16*e^(9/2) - 1575*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^7*c^6*d^18*e^8 + 10500*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^6*c^6*d^17*e^7 + 615405*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^5*c^6*d^16*e^6 + 752640*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^4*c^6*d^15*e^5 + 29715*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^3*c^6*d^14*e^4 - 10500*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*a^2*c^6*d^13*e^3 + 1575*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^13*a*c^6*d^12*e^2 + 1075200*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^6*c^5*d^15*e^(15/2) + 2938880*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^5*c^5*d^14*e^(13/2) + 1576960*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(c*d)*a^4*c^5*d^13*e^(11/2) + 945*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^8*c^5*d^16*e^10 + 1068900*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^7*c^5*d^15*e^9 + 5823909*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^6*c^5*d^14*e^8 + 6730752*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^5*c^5*d^13*e^7 + 1745499*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^4*c^5*d^12*e^6 + 6300*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*a^3*c^5*d^11*e^5 - 945*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^13*a^2*c^5*d^10*e^4 + 645120*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^8*c^4*d^14*e^(21/2) + 6666240*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^7*c^4*d^13*e^(19/2) + 14192640*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^6*c^4*d^12*e^(17/2) + 8171520*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(c*d)*a^5*c^4*d^11*e^(15/2) + 1075200*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^10*sqrt(c*d)*a^4*c^4*d^10*e^(13/2) + 215565*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^9*c^4*d^14*e^12 + 4655700

```
*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^8*
c^4*d^13*e^11 + 17499825*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
a*x*e^2 + a*d*e))^5*a^7*c^4*d^12*e^10 + 18170880*(sqrt(c*d)*x*e^(1/2) - sqr
t(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^6*c^4*d^11*e^9 + 5294415*(sqr
t(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^5*c^4*d
^10*e^8 + 290220*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
+ a*d*e))^11*a^4*c^4*d^9*e^7 - 525*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
c*d^2*x + a*x*e^2 + a*d*e))^13*a^3*c^4*d^8*e^6 + 30720*sqrt(c*d)*a^10*c^3*d
^13*e^(27/2) + 1935360*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e))^2*sqrt(c*d)*a^9*c^3*d^12*e^(25/2) + 13332480*(sqrt(c*d)*x*e
^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^8*c^3*d
^11*e^(23/2) + 23654400*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))^6*sqrt(c*d)*a^7*c^3*d^10*e^(21/2) + 12257280*(sqrt(c*d)*x*
e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(c*d)*a^6*c^3*
d^9*e^(19/2) + 1505280*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e))^10*sqrt(c*d)*a^5*c^3*d^8*e^(17/2) + 431655*(sqrt(c*d)*x*e^(
1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^10*c^3*d^12*e^14 + 62
25660*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3
*a^9*c^3*d^11*e^13 + 19168275*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2
*x + a*x*e^2 + a*d*e))^5*a^8*c^3*d^10*e^12 + 16665600*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^7*c^3*d^9*e^11 + 3625965
*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^8\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

3.469 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

Optimal. Leaf size=628

$$-\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{16384a^5d^6e^5x^2}$$

```
[Out] 1/2048*(-a*e^2+c*d^2)*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*
d^6)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^4/
d^5/e^4/x^4-1/8*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^8-1/112*(5*c/a/
e-11*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7+1/448*(-33*a^2*e^4+
10*a*c*d^2*e^2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2/d^3/
e^2/x^6-1/4480*(-231*a^3*e^6+15*a^2*c*d^2*e^4+95*a*c^2*d^4*e^2+105*c^3*d^6)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^3/d^4/e^3/x^5+3/32768*(-a*e^2+c*
d^2)^5*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*d^6)*arctanh(1/
2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/a^(11/2)/d^(13/2)/e^(11/2)-3/16384*(-a*e^2+c*d^2)^3*(33*a
^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15*c^3*d^6)*(2*a*d*e+(a*e^2+c*d^2)
*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^5/d^6/e^5/x^2
```

Rubi [A]
time = 0.57, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {863, 848, 820, 734, 738, 212}

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (-3*(c*d^2 - a*e^2)^3*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 3
3*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(16384*a^5*d^6*e^5*x^2) + ((c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^
2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*a^4*d^5*e^4*x^4) - (a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(8*d*x^8) - (((5*c)/(a*e) - (11*e)
/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*x^7) + ((15*c^2*d
^4 + 10*a*c*d^2*e^2 - 33*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2))/(448*a^2*d^3*e^2*x^6) - ((105*c^3*d^6 + 95*a*c^2*d^4*e^2 + 15*a^2*c*d
^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4480*
a^3*d^4*e^3*x^5) + (3*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45
*a^2*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(32768*a^
(11/2)*d^(13/2)*e^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx = \int \frac{(ae+cdx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^9}\,dx$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\int \frac{\left(-\frac{1}{2}ae\left(5cd^2-11ae^2\right)+3acde^2x\right)(ade}{x^8}}{8ade}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)\right.}{112x^7}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)\right.}{112x^7}$$

$$= -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)\right.}{112x^7}$$

$$= \frac{\left(cd^2-ae^2\right)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)(2ade+(cd}{2048a^4d^5e^4x^4}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)(2ade+}{16384a^5d^6e^5x^2}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)(2ade+}{16384a^5d^6e^5x^2}$$

$$= -\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)(2ade+}{16384a^5d^6e^5x^2}$$

Mathematica [A]
time = 1.67, size = 572, normalized size = 0.91

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(1575c^7d^{14}x^7-525ac^6d^{12}ex^6(2d+7ex)+35a^2c^5d^{10}e^2x^5\left(24d^2+68dex+29e^2x^2\right)-5a^3c^4d^8e^3x^4\left(144d^3+376d^2ex+110de^2x^2-185e^3x^3\right)+5a^4c^3d^6e^4x^3\left(128d^4+320d^3ex+80d^2e^2x^2-120de^3x^3+265e^4x^4\right)+a^5c^2d^4e^5x^2\left(103680d^5+137600d^4ex+4640d^3e^2x^2-5488d^2e^3x^3+7034de^4x^4-11193e^5x^5\right)+a^6cd^2e^6x\left(168960d^6+212480d^5ex+4480d^4e^2x^2-5056d^3e^3x^3+5928d^2e^4x^4-7476de^5x^5+11445e^6x^6\right)+a^7e^7\left(71680d^7+87040d^6ex+1280d^5e^2x^2-1408d^4e^3x^3+1584d^3e^4x^4-1848d^2e^5x^5+2310de^6x^6-3465e^7x^7\right)\right)}{x^8}+\frac{105\left(cd^2-ae^2\right)^5\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{573440a^{11/2}d^{13/2}e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(1575*c^7*d^14*x
^7 - 525*a*c^6*d^12*e*x^6*(2*d + 7*e*x) + 35*a^2*c^5*d^10*e^2*x^5*(24*d^2 +
 68*d*e*x + 29*e^2*x^2) - 5*a^3*c^4*d^8*e^3*x^4*(144*d^3 + 376*d^2*e*x + 11
0*d*e^2*x^2 - 185*e^3*x^3) + 5*a^4*c^3*d^6*e^4*x^3*(128*d^4 + 320*d^3*e*x +
 80*d^2*e^2*x^2 - 120*d*e^3*x^3 + 265*e^4*x^4) + a^5*c^2*d^4*e^5*x^2*(10368
0*d^5 + 137600*d^4*e*x + 4640*d^3*e^2*x^2 - 5488*d^2*e^3*x^3 + 7034*d*e^4*x
^4 - 11193*e^5*x^5) + a^6*c*d^2*e^6*x*(168960*d^6 + 212480*d^5*e*x + 4480*d
^4*e^2*x^2 - 5056*d^3*e^3*x^3 + 5928*d^2*e^4*x^4 - 7476*d*e^5*x^5 + 11445*e
^6*x^6) + a^7*e^7*(71680*d^7 + 87040*d^6*e*x + 1280*d^5*e^2*x^2 - 1408*d^4*
e^3*x^3 + 1584*d^3*e^4*x^4 - 1848*d^2*e^5*x^5 + 2310*d*e^6*x^6 - 3465*e^7*x
^7)))/x^8) + (105*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2
*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[
e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(573440*a^(11/2)*d^
(13/2)*e^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70735 vs. 2(586) = 1172.
time = 0.08, size = 70736, normalized size = 112.64

method	result	size
default	Expression too large to display	70736

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((x*e + d)*x^9), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5747 vs. 2(570) = 1140.
time = 2.25, size = 5747, normalized size = 9.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm=
"giac")
```

```
[Out] -3/16384*(15*c^8*d^16 - 40*a*c^7*d^14*e^2 + 20*a^2*c^6*d^12*e^4 + 8*a^3*c^5
*d^10*e^6 + 10*a^4*c^4*d^8*e^8 + 40*a^5*c^3*d^6*e^10 - 140*a^6*c^2*d^4*e^12
 + 120*a^7*c*d^2*e^14 - 33*a^8*e^16)*arctan(-(sqrt(c*d)*x*e^(1/2) - sqrt(c*
d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))/sqrt(-a*d*e))*e^(-5)/(sqrt(-a*d*e)*a^
5*d^6) + 1/573440*(1575*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))*a^7*c^8*d^23*e^7 - 12075*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x
^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^6*c^8*d^22*e^6 + 40215*(sqrt(c*d)*x*
e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^5*c^8*d^21*e^5 +
 88045*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^
7*a^4*c^8*d^20*e^4 - 75795*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))^9*a^3*c^8*d^19*e^3 + 40215*(sqrt(c*d)*x*e^(1/2) - sqrt(
c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*a^2*c^8*d^18*e^2 - 12075*(sqrt(c
*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^13*a*c^8*d^17*
e + 1575*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e)
)^15*c^8*d^16 + 1146880*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))^6*sqrt(c*d)*a^5*c^7*d^19*e^(11/2) - 4200*(sqrt(c*d)*x*e^(1
/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*a^8*c^7*d^21*e^9 + 32200
*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^7*
c^7*d^20*e^8 + 3333400*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*
x*e^2 + a*d*e))^5*a^6*c^7*d^19*e^7 + 5117320*(sqrt(c*d)*x*e^(1/2) - sqrt(c*
d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^7*a^5*c^7*d^18*e^6 + 202120*(sqrt(c*d
```

)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^4*c^7*d^17*e
^5 - 107240*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d
*e))^11*a^3*c^7*d^16*e^4 + 32200*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*
d^2*x + a*x*e^2 + a*d*e))^13*a^2*c^7*d^15*e^3 - 4200*(sqrt(c*d)*x*e^(1/2) -
 sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^15*a*c^7*d^14*e^2 + 5734400*(
sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c
*d)*a^7*c^6*d^18*e^(17/2) + 20643840*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e
+ c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^6*c^6*d^17*e^(15/2) + 13762560*
(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(
c*d)*a^5*c^6*d^16*e^(13/2) + 2100*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c
*d^2*x + a*x*e^2 + a*d*e))*a^9*c^6*d^19*e^11 + 5718300*(sqrt(c*d)*x*e^(1/2)
 - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^8*c^6*d^18*e^10 + 41341
300*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a
^7*c^6*d^17*e^9 + 62321980*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))^7*a^6*c^6*d^16*e^8 + 21034300*(sqrt(c*d)*x*e^(1/2) - sq
rt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^5*c^6*d^15*e^7 + 53620*(sqrt
(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*a^4*c^6*d
^14*e^6 - 16100*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e))^13*a^3*c^6*d^13*e^5 + 2100*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
 c*d^2*x + a*x*e^2 + a*d*e))^15*a^2*c^6*d^12*e^4 + 3440640*(sqrt(c*d)*x*e^(
1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^9*c^5*d^1
7*e^(23/2) + 48168960*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x
*e^2 + a*d*e))^4*sqrt(c*d)*a^8*c^5*d^16*e^(21/2) + 136478720*(sqrt(c*d)*x*e
^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^7*c^5*d
^15*e^(19/2) + 105512960*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
a*x*e^2 + a*d*e))^8*sqrt(c*d)*a^6*c^5*d^14*e^(17/2) + 19496960*(sqrt(c*d)*x
*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^10*sqrt(c*d)*a^5*c^
5*d^13*e^(15/2) + 1147720*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x +
 a*x*e^2 + a*d*e))*a^10*c^5*d^17*e^13 + 34399960*(sqrt(c*d)*x*e^(1/2) - sqr
t(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^3*a^9*c^5*d^16*e^12 + 175494088*(
sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^5*a^8*c^
5*d^15*e^11 + 248341016*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a
*x*e^2 + a*d*e))^7*a^7*c^5*d^14*e^10 + 104522264*(sqrt(c*d)*x*e^(1/2) - sqr
t(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^9*a^6*c^5*d^13*e^9 + 10113992*(sq
rt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^11*a^5*c^5
*d^12*e^8 - 6440*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2
+ a*d*e))^13*a^4*c^5*d^11*e^7 + 840*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
 c*d^2*x + a*x*e^2 + a*d*e))^15*a^3*c^5*d^10*e^6 + 163840*sqrt(c*d)*a^11*c^
4*d^16*e^(29/2) + 14745600*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x
+ a*x*e^2 + a*d*e))^2*sqrt(c*d)*a^10*c^4*d^15*e^(27/2) + 141066240*(sqrt(c*
d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^4*sqrt(c*d)*a^9
*c^4*d^14*e^(25/2) + 344064000*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^
2*x + a*x*e^2 + a*d*e))^6*sqrt(c*d)*a^8*c^4*d^13*e^(23/2) + 260341760*(sqrt
(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))^8*sqrt(c*d)*
a^7*c^4*d^12*e^(21/2) + 57344000*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e + c*

```
d^2*x + a*x*e^2 + a*d*e))^10*sqrt(c*d)*a^6*c^4*...
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^9\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

3.470 $\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=271

$$-\frac{3(3cd^2+ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4c^2d^2e^3}-\frac{2d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e^3(cd^2-ae^2)(d+ex)}+\frac{(d+ex)\sqrt{ade+(cd}{2cd}$$

```
[Out] 3/8*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c
^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(
5/2)/e^(7/2)-3/4*(a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^
2/d^2/e^3-2*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(-a*e^2+c*d^2)/
(e*x+d)+1/2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^3
```

Rubi [A]

time = 0.22, antiderivative size = 298, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 832, 793, 635, 212}

$$\frac{3(a^2e^4+2acd^2e^2+5c^2d^4)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}-\frac{((5cd^2-3ae^2)(ae^2+3cd^2)-2cdex(5cd^2-ae^2))\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e^3(cd^2-ae^2)}-\frac{2dx^2(cdx(cd^2-ae^2)+ae(cd^2-ae^2))}{e(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(e*(c*d^2 - a*e^2)
^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (((5*c*d^2 - 3*a*e^2)*(3*
c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(4*c^2*d^2*e^3*(c*d^2 - a*e^2)) + (3*(5*c^2*d^4 + 2*a*c*d^2
*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(7/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{x^3(ae+cdx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{e\,(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{2\int \frac{x(2a}{}}{}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{e\,(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{((5cd^2}{}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{e\,(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{((5cd^2}{}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{e\,(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{((5cd^2}{}$$

Mathematica [A]
time = 0.48, size = 277, normalized size = 1.02

$$\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,(3a^3e^5(d+ex)+a^2cde^3(4d^2+5dex+e^2x^2)+c^3d^4x(-15d^2-5dex+2e^2x^2)-ac^2d^2e(15d^3+d^2ex-4de^2x^2+2e^3x^3))+3(5c^3d^6-3ac^2d^4e^2-a^2cd^2e^4-a^3e^6)\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}\right)}{4c^{5/2}d^{5/2}e^{7/2}\,(cd^2-ae^2)\,\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^3*e^5*(d + e*x) + a^2*c*d*e^3*(4*d^2 + 5*d*e*
x + e^2*x^2) + c^3*d^4*x*(-15*d^2 - 5*d*e*x + 2*e^2*x^2) - a*c^2*d^2*e*(15*
d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)) + 3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2
- a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(4*c^(5/2)*d^(5/2)*e^
(7/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(243) = 486.
time = 0.09, size = 514, normalized size = 1.90

method	result

default	$\frac{\frac{x\sqrt{ade+(a\,e^2+c\,d^2)\,x+cde\,x^2}}{2cde}-\frac{3\left(a\,e^2+c\,d^2\right)\left(\frac{\sqrt{ade+(a\,e^2+c\,d^2)\,x+cde\,x^2}}{cde}-\frac{\left(a\,e^2+c\,d^2\right)\ln\left(\frac{\frac{1}{2}a\,e^2+\frac{1}{2}c}{\sqrt{c}}\right.}{}\right.}{4cde}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/2*x/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(a*e^2+c*d^2)/
c/d/e*(1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/
d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-1/2*a/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c
*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-d/e^2*(
1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/e*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/(c*d*e)^(1/2))+d^2/e^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+2*d^3/e^4/(a*e^
2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 3.70, size = 729, normalized size = 2.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/16*(3*(5*c^3*d^6*x*e + 5*c^3*d^7 - 3*a*c^2*d^4*x*e^3 - 3*a*c^2*d^5*e^2 -
 a^2*c*d^2*x*e^5 - a^2*c*d^3*e^4 - a^3*x*e^7 - a^3*d*e^6)*sqrt(c*d)*e^(1/2)
*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a
*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) +
 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) - 4*(5*c^3*d^5*x*e^2 + 15*c^3*d^6*e - 2
*a*c^2*d^3*x*e^4 - 3*a^2*c*d*x*e^6 + (2*a*c^2*d^2*x^2 - 3*a^2*c*d^2)*e^5 -
2*(c^3*d^4*x^2 + 2*a*c^2*d^4)*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e))/(c^4*d^5*x*e^5 + c^4*d^6*e^4 - a*c^3*d^3*x*e^7 - a*c^3*d^4*e^6), -1/8*
(3*(5*c^3*d^6*x*e + 5*c^3*d^7 - 3*a*c^2*d^4*x*e^3 - 3*a*c^2*d^5*e^2 - a^2*c
*d^2*x*e^5 - a^2*c*d^3*e^4 - a^3*x*e^7 - a^3*d*e^6)*sqrt(-c*d*e)*arctan(1/2
*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sq
rt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(
5*c^3*d^5*x*e^2 + 15*c^3*d^6*e - 2*a*c^2*d^3*x*e^4 - 3*a^2*c*d*x*e^6 + (2*a
*c^2*d^2*x^2 - 3*a^2*c*d^2)*e^5 - 2*(c^3*d^4*x^2 + 2*a*c^2*d^4)*e^3)*sqrt(c
*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(c^4*d^5*x*e^5 + c^4*d^6*e^4 - a*c^3
*d^3*x*e^7 - a*c^3*d^4*e^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [A]
time = 1.17, size = 249, normalized size = 0.92

$$-\frac{2\,d^3e^{(-3)}}{\sqrt{cd}\,de^{\frac{1}{2}}+\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)e}+\frac{1}{4}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(\frac{2\,xe^{(-2)}}{cd}-\frac{(7\,cd^2e^5+3\,ae^7)e^{(-8)}}{c^2d^2}\right)-\frac{3\left(5\sqrt{cd}\,c^2d^4e^{\frac{1}{2}}+2\sqrt{cd}\,acd^2e^{\frac{5}{2}}+\sqrt{cd}\,a^2e^{\frac{9}{2}}\right)e^{(-4)}\log\left(\left|-\sqrt{cd}\,cd^2e^{\frac{1}{2}}-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)cde-\sqrt{cd}\,ae^{\frac{5}{2}}\right|\right)}{8\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] -2*d^3*e^(-3)/(sqrt(c*d)*d*e^(1/2) + (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e
+ c*d^2*x + a*x*e^2 + a*d*e))*e) + 1/4*sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 +
 a*d*e)*(2*x*e^(-2)/(c*d) - (7*c*d^2*e^5 + 3*a*e^7)*e^(-8)/(c^2*d^2)) - 3/8
*(5*sqrt(c*d)*c^2*d^4*e^(1/2) + 2*sqrt(c*d)*a*c*d^2*e^(5/2) + sqrt(c*d)*a^2
*e^(9/2))*e^(-4)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2)
- sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2))
)/(c^3*d^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.471 $\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cde^2}+\frac{2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e^2(cd^2-ae^2)(d+ex)}-\frac{(3cd^2+ae^2)\tanh^{-1}\left(\frac{\cdots}{2\sqrt{c}\sqrt{d}\sqrt{e}\ \cdots}\right)}{2c^{3/2}d^{3/2}\cdots}$$

```
[Out] -1/2*(a*e^2+3*c*d^2)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^
(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2+2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]
time = 0.22, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1652, 806, 635, 212}

$$-\frac{(ae^2+3cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}}+\frac{2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)(cd^2-ae^2)}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cde^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d*e^2) + (2*d^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(c*d^2 - a*e^2)*(d + e*x)) - ((3*c*d^
2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(2*c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
```

```
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 1652

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{\int \frac{-\frac{1}{2}de\left(cd^2+ae^2\right)-\frac{1}{2}e^2(}{(d+ex)\sqrt{ade+\left(cd^2+a\right.}}}{cde^3}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{e^2\left(cd^2-ae^2\right)(d+}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{e^2\left(cd^2-ae^2\right)(d+}$$

$$= \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cde^2} + \frac{2d^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{e^2\left(cd^2-ae^2\right)(d+}$$

Mathematica [A]
time = 0.29, size = 201, normalized size = 1.03

$$\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(-a^2e^3(d+ex)+c^2d^3x(3d+ex)+acde\left(3d^2-e^2x^2\right)\right)-\left(3c^2d^4-2acd^2e^2-a^2e^4\right)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{c^{3/2}d^{3/2}e^{5/2}\left(cd^2-ae^2\right)\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[e]*(-(a^2*e^3*(d + e*x)) + c^2*d^3*x*(3*d + e*x) + a*
c*d*e*(3*d^2 - e^2*x^2)) - (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*Sqrt[a*e +
 c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*
Sqrt[d + e*x])])/(c^(3/2)*d^(3/2)*e^(5/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x
)*(d + e*x)])
```

Maple [A]
time = 0.09, size = 259, normalized size = 1.33

method	result
default	$\frac{\frac{\sqrt{ade+(a e^2+c d^2) x+cde x^2}}{cde}-\frac{(a e^2+c d^2) \ln\left(\frac{\frac{1}{2} a e^2+\frac{1}{2} c d^2+cdex}{\sqrt{cde}}+\sqrt{ade+(a e^2+c d^2) x+cde x^2}\right)}{2cde\sqrt{cde}}}{e} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/c/d/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/
e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))/(c*d*e)^(1/2))-d/e^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-2/e^3*d^2/(a
*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 4.03, size = 563, normalized size = 2.89

$$\left[\frac{(3c^2d^4xe+3c^2d^5-2acd^2xe^3-2acd^3e^2-a^2xe^5-a^2de^4)\sqrt{cd}\,e^{\frac{1}{2}}\log\left(8c^2d^3xe+c^2d^4+8acdxe^3+a^2e^4-4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2cdxe+cd^2+ae^2)\sqrt{cd}\,e^{\frac{1}{2}}+2(4c^2d^2x^2+3acd^2)e^2\right)+4(c^2d^3xe^2+3c^2d^4e-acdxe^4-acd^2e^3)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}{4(c^3d^4xe^4+c^3d^5e^3-ac^2d^2xe^6-ac^2d^3e^5)}, \frac{(3c^2d^4xe+3c^2d^5-2acd^2xe^3-2acd^3e^2-a^2xe^5-a^2de^4)\sqrt{-cde}\arctan\left(\frac{\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2cdxe+cd^2+ae^2)\sqrt{-cde}}{2(c^2d^3xe+acdxe^3+(c^2d^2x^2+acd^2)e^2)}\right)+2(c^2d^3xe^2+3c^2d^4e-acdxe^4-acd^2e^3)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}{2(c^3d^4xe^4+c^3d^5e^3-ac^2d^2xe^6-ac^2d^3e^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/4*((3*c^2*d^4*x*e + 3*c^2*d^5 - 2*a*c*d^2*x*e^3 - 2*a*c*d^3*e^2 - a^2*x*
e^5 - a^2*d*e^4)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*
e^3 + a^2*e^4 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e +
c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) + 4*(
c^2*d^3*x*e^2 + 3*c^2*d^4*e - a*c*d*x*e^4 - a*c*d^2*e^3)*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e))/(c^3*d^4*x*e^4 + c^3*d^5*e^3 - a*c^2*d^2*x*e^6 -
 a*c^2*d^3*e^5), 1/2*((3*c^2*d^4*x*e + 3*c^2*d^5 - 2*a*c*d^2*x*e^3 - 2*a*c*
d^3*e^2 - a^2*x*e^5 - a^2*d*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3
*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(c^2*d^3*x*e^2 + 3*c
^2*d^4*e - a*c*d*x*e^4 - a*c*d^2*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e))/(c^3*d^4*x*e^4 + c^3*d^5*e^3 - a*c^2*d^2*x*e^6 - a*c^2*d^3*e^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [A]
time = 1.73, size = 202, normalized size = 1.04

$$\frac{2\,d^2e^{(-2)}}{\sqrt{cd}\,de^{\frac{1}{2}}+\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)e}+\frac{\sqrt{cdx^2e+cd^2x+axe^2+ade}\,e^{(-2)}}{cd}+\frac{\left(3\,\sqrt{cd}\,cd^2e^{\frac{1}{2}}+\sqrt{cd}\,ae^{\frac{5}{2}}\right)e^{(-3)}\log\left(\left|-\sqrt{cd}\,cd^2e^{\frac{1}{2}}-2\left(\sqrt{cd}\,xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2+ade}\right)cde-\sqrt{cd}\,ae^{\frac{5}{2}}\right|\right)}{2\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] 2*d^2*e^(-2)/(sqrt(c*d)*d*e^(1/2) + (sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^2*e +
 c*d^2*x + a*x*e^2 + a*d*e))*e) + sqrt(c*d*x^2*e + c*d^2*x + a*x*e^2 + a*d*
e)*e^(-2)/(c*d) + 1/2*(3*sqrt(c*d)*c*d^2*e^(1/2) + sqrt(c*d)*a*e^(5/2))*e^(
-3)*log(abs(-sqrt(c*d)*c*d^2*e^(1/2) - 2*(sqrt(c*d)*x*e^(1/2) - sqrt(c*d*x^
2*e + c*d^2*x + a*x*e^2 + a*d*e))*c*d*e - sqrt(c*d)*a*e^(5/2)))/(c^2*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.472 $\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$-\frac{2d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(cd^2-ae^2)(d+ex)}+\frac{\tanh^{-1}\left(\frac{cd^2+ae^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}$$

```
[Out] arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/e^(3/2)/c^(1/2)/d^(1/2)-2*d*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]
time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {806, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}-\frac{2d\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)(cd^2-ae^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d +
e*x)) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
```

```
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{2d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(cd^2-ae^2)(d+ex)} + \frac{\int \frac{1}{\sqrt{ade+(cd^2+c}}}{e}$$

$$= -\frac{2d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(cd^2-ae^2)(d+ex)} + \frac{2\text{Subst}\left(\int \frac{1}{4cde-x^2}\,d\right.}{}$$

$$= -\frac{2d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(cd^2-ae^2)(d+ex)} + \frac{\tanh^{-1}\left(\frac{}{2\sqrt{c}\sqrt{d}}\right.}{}$$

Mathematica [A]
time = 0.20, size = 130, normalized size = 0.94

$$\frac{2\left(-\frac{d^{3/2}\sqrt{e}\,(ae+cdx)}{cd^2-ae^2}+\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}\right)}{\sqrt{c}}\right)}{\sqrt{d}\,e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*(-((d^(3/2)*Sqrt[e]*(a*e + c*d*x))/(c*d^2 - a*e^2)) + (Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d
+ e*x])])/Sqrt[c]))/(Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.09, size = 131, normalized size = 0.94

method	result

default	$\frac{\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdex}{\sqrt{cde}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}}\right)}{e\sqrt{cde}}+\frac{2d\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{e^{2}\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+2*d/e^2/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [A]
time = 4.10, size = 430, normalized size = 3.09

$$\left[-\frac{4\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\,cd^{2}e-\left(cd^{2}xe+cd^{3}-axe^{3}-ade^{2}\right)\sqrt{cd}\,e^{\frac{1}{2}}\log\left(8\,c^{2}d^{3}xe+c^{2}d^{4}+8\,acdxe^{3}+a^{2}e^{4}+4\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\left(2\,cdxe+cd^{2}+ae^{2}\right)\sqrt{cd}\,e^{\frac{1}{2}}+2\left(4\,c^{2}d^{2}x^{2}+3\,acd^{2}\right)e^{2}\right)}{2\left(c^{2}d^{3}xe^{3}+c^{2}d^{4}e^{2}-acdxe^{5}-acd^{2}e^{4}\right)},\ -\frac{2\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\,cd^{2}e+\left(cd^{2}xe+cd^{3}-axe^{3}-ade^{2}\right)\sqrt{-cde}\,\arctan\left(\frac{\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\left(2\,cdxe+cd^{2}+ae^{2}\right)\sqrt{-cde}}{2\left(c^{2}d^{3}xe+acdxe^{3}+\left(c^{2}d^{2}x^{2}+acd^{2}\right)e^{2}\right)}\right)}{c^{2}d^{3}xe^{3}+c^{2}d^{4}e^{2}-acdxe^{5}-acd^{2}e^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/2*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*c*d^2*e - (c*d^2*x*e +
 c*d^3 - a*x*e^3 - a*d*e^2)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 +
 8*a*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2
*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)
*e^2))/(c^2*d^3*x*e^3 + c^2*d^4*e^2 - a*c*d*x*e^5 - a*c*d^2*e^4), -(2*sqrt(
c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*c*d^2*e + (c*d^2*x*e + c*d^3 - a*x*e
^3 - a*d*e^2)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*c*d*x*e^3
```

```
+ (c^2*d^2*x^2 + a*c*d^2)*e^2)))/(c^2*d^3*x*e^3 + c^2*d^4*e^2 - a*c*d*x*e^5
 - a*c*d^2*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,0,5]%%%},[2,2]%%%}+%%%{%%%{-2,[1,2,3]%%%},[2,1]%%%}+
%%%{%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.473 $\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {664}

$$\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)\,(cd^2-ae^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x))
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.81

$$\frac{2(ae+cdx)}{(cd^2-ae^2)\,\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

[Out] (2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [A]
time = 0.08, size = 65, normalized size = 1.25

method	result	size
trager	$-\frac{2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{(a\,e^2-c\,d^2)(ex+d)}$	50
gosper	$-\frac{2(cdx+ae)}{(a\,e^2-c\,d^2)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	51
default	$-\frac{2\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{e\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}$	65

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/e/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for more d

Fricas [A]
time = 3.91, size = 57, normalized size = 1.10

$$\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}{cd^2xe+cd^3-axe^3-ade^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)/(c*d^2*x*e + c*d^3 - a*x*e^3 - a*d*e^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,0,1]%%%},[2]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1,0,
%%%{-1
```

Mupad [B]
time = 2.64, size = 50, normalized size = 0.96

$$-\frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(a\,e^2-c\,d^2\right)\,\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] -(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e^2 - c*d^2)*(d + e*
x))
```

3.474 $\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\tanh^{-1}\left(\frac{2ade+\left(cd^2+ae^2\right)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{\sqrt{a}\,d^{3/2}\sqrt{e}}$$

```
[Out] -arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(3/2)/a^(1/2)/e^(1/2)-2*e*(c*d*x+a*e)/d/(-a*
e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {865, 836, 12, 738, 212}

$$-\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}\,d^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{2\int -\frac{\cdots}{2x\cdots}}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\int \frac{\cdots}{x\sqrt{ad\cdots}}}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{2\mathrm{Subst}\Big(\cdots}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{\tanh^{-1}\Big(\cdots}{\cdots}$$

Mathematica [A]

time = 0.19, size = 131, normalized size = 0.92

$$\frac{2\left(-\frac{\sqrt{d}\,e^{3/2}(ae+cdx)}{cd^2-ae^2}-\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ae+cdx}}{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}\right)}{\sqrt{a}}\right)}{d^{3/2}\sqrt{e}\,\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]

[Out] (2*(-((Sqrt[d]*e^(3/2)*(a*e + c*d*x))/(c*d^2 - a*e^2)) - (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/Sqrt[a]))/(d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [A]
time = 0.09, size = 136, normalized size = 0.95

method	result
default	$\frac{2\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{d\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}-\frac{\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd}}{x}\right.}{d\sqrt{ade}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(x*e + d)*x), x)

Fricas [A]
time = 4.98, size = 443, normalized size = 3.10

$$\left[-\frac{4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,ade^2-(cd^2xe+cd^3-axe^3-ade^2)\sqrt{ad}\,e^{\frac{1}{2}}\log\left(\frac{c^2d^4x^2+8\,acd^3xe+a^2x^2e^4+8\,a^2dxe^3-4\,(cd^2x+axe^2+2\,ade)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{ad}\,e^{\frac{1}{2}}+2\,(3\,acd^2x^2+4\,a^2d^2)e^2}{x^2}\right)}{2\,(acd^4xe^2+acd^5e-a^2d^2xe^4-a^2d^3e^3)},\ -\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,ade^2-(cd^2xe+cd^3-axe^3-ade^2)\sqrt{-ade}\,\arctan\left(\frac{(cd^2x+axe^2+2\,ade)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{-ade}}{2\,(acd^3xe+a^2dxe^3+(acd^2x^2+a^2d^2)e^2)}\right)}{acd^4xe^2+acd^5e-a^2d^2xe^4-a^2d^3e^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/2*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*a*d*e^2 - (c*d^2*x*e +
 c*d^3 - a*x*e^3 - a*d*e^2)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*
x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*
d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 +
 4*a^2*d^2)*e^2)/x^2))/(a*c*d^4*x*e^2 + a*c*d^5*e - a^2*d^2*x*e^4 - a^2*d^3
*e^3), -(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*a*d*e^2 - (c*d^2*x*e
 + c*d^3 - a*x*e^3 - a*d*e^2)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 +
2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*
x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)))/(a*c*d^4*x*e^2 + a*c*d^5
*e - a^2*d^2*x*e^4 - a^2*d^3*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{(d+ex)\,(ae+cdx)}\,\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,1,5]%%%},[2,2]%%%}+%%%{%%%{-2,[1,3,3]%%%},[2,1]%%%}+
%%%{%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.475 $\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=229

$$-\frac{2e(ae+cdx)}{d(cd^2-ae^2)\,x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}-\frac{(cd^2-3ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{ad^2e\,(cd^2-ae^2)\,x}+\frac{(cd^2+3ae}{}$$

```
[Out] 1/2*(3*a*e^2+c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)-2*e
*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(-3
*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/e/(-a*e^2+c*d^2
)/x
```

Rubi [A]

time = 0.19, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {865, 836, 820, 738, 212}

$$\frac{(3ae^2+cd^2)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}-\frac{(cd^2-3ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{ad^2ex(cd^2-ae^2)}-\frac{2e(ae+cdx)}{dx(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]) - ((c*d^2 - 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(a*d^2*e*(c*d^2 - a*e^2)*x) + ((c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d
^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(2*a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\,(cd^2-ae^2)\,x\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{2\int \cdots}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\,(cd^2-ae^2)\,x\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2 \cdots}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\,(cd^2-ae^2)\,x\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2 \cdots}{\cdots}$$

$$= -\frac{2e(ae+cdx)}{d\,(cd^2-ae^2)\,x\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2 \cdots}{\cdots}$$

Mathematica [A]
time = 0.27, size = 201, normalized size = 0.88

$$\frac{\sqrt{a}\,\sqrt{d}\,\sqrt{e}\,\left(-c^2d^3x(d+ex)+a^2e^3(d+3ex)-acde(d^2-3e^2x^2)\right)+\left(c^2d^4+2acd^2e^2-3a^2e^4\right)x\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{ae+cdx}}{\sqrt{a}\,\sqrt{e}\,\sqrt{d+ex}}\right)}{a^{3/2}d^{5/2}e^{3/2}\left(cd^2-ae^2\right)x\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c^2*d^3*x*(d + e*x)) + a^2*e^3*(d + 3*e*x) - a*
c*d*e*(d^2 - 3*e^2*x^2)) + (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*x*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e
]*Sqrt[d + e*x])])/(a^(3/2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)*x*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Maple [A]
time = 0.09, size = 270, normalized size = 1.18

method	result
default	$-\frac{2e\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{d^2\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}+\frac{-\frac{\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2}}{adex}+\frac{\left(a\,e^2+c\,d^2\right)\ln\left(\frac{2ade \cdots}{\cdots}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -2*e/d^2/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2
)+1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^2)
/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+e/d^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c
*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(x*e + d)*x^2), x)
```

Fricas [A]
time = 9.24, size = 599, normalized size = 2.62

$$\left[\frac{(c^2d^4x^2e + c^2d^5x + 2\,acd^2x^2e^3 + 2\,acd^3xe^2 - 3\,a^2x^2e^5 - 3\,a^2dxe^4)\sqrt{ad}\,e^{\frac{1}{2}}\log\left(\frac{c^2d^4x^2 + 8\,acd^3xe + a^2x^2e^4 + 8\,a^2dxe^3 + 4\,(cd^2x + axe^2 + 2\,ade)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,\sqrt{ad}\,e^{\frac{1}{2}} + 2\,(3\,acd^2x^2 + 4\,a^2d^2)e^2}{x^2}\right) - 4\,(acd^3xe^2 + acd^4e - 3\,a^2dxe^4 - a^2d^2e^3)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}}{4\,(a^2cd^5x^2e^3 + a^2cd^6xe^2 - a^3d^3x^2e^5 - a^3d^4xe^4)}, -\frac{(c^2d^4x^2e + c^2d^5x + 2\,acd^2x^2e^3 + 2\,acd^3xe^2 - 3\,a^2x^2e^5 - 3\,a^2dxe^4)\sqrt{-ade}\,\arctan\left(\frac{(cd^2x + axe^2 + 2\,ade)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,\sqrt{-ade}}{2\,(acd^3xe + a^2dxe^3 + (acd^2x^2 + a^2d^2)e^2)}\right) + 2\,(acd^3xe^2 + acd^4e - 3\,a^2dxe^4 - a^2d^2e^3)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}}{2\,(a^2cd^5x^2e^3 + a^2cd^6xe^2 - a^3d^3x^2e^5 - a^3d^4xe^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/4*((c^2*d^4*x^2*e + c^2*d^5*x + 2*a*c*d^2*x^2*e^3 + 2*a*c*d^3*x*e^2 - 3*
a^2*x^2*e^5 - 3*a^2*d*x*e^4)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3
*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c
*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2
+ 4*a^2*d^2)*e^2)/x^2) - 4*(a*c*d^3*x*e^2 + a*c*d^4*e - 3*a^2*d*x*e^4 - a^2
*d^2*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(a^2*c*d^5*x^2*e^3 +
 a^2*c*d^6*x*e^2 - a^3*d^3*x^2*e^5 - a^3*d^4*x*e^4), -1/2*((c^2*d^4*x^2*e +
 c^2*d^5*x + 2*a*c*d^2*x^2*e^3 + 2*a*c*d^3*x*e^2 - 3*a^2*x^2*e^5 - 3*a^2*d*
x*e^4)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x +
 a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*
c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(a*c*d^3*x*e^2 + a*c*d^4*e - 3*a^2*d*x*e^4 -
 a^2*d^2*e^3)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(a^2*c*d^5*x^2*e
^3 + a^2*c*d^6*x*e^2 - a^3*d^3*x^2*e^5 - a^3*d^4*x*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{(d+ex)(ae+cdx)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,0,1]%%%},[6,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1,
0,%%%{
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.476 $\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2e(ae+cdx)}{d\,(cd^2-ae^2)\,x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}-\frac{(cd^2-5ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2ad^2e\,(cd^2-ae^2)\,x^2}+\frac{(3cd^2-5a$$

```
[Out] -3/8*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x
)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/
d^(7/2)/e^(5/2)-2*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^2/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-1/2*(-5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a/d^2/e/(-a*e^2+c*d^2)/x^2+1/4*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)/x
```

Rubi [A]
time = 0.33, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {865, 836, 848, 820, 738, 212}

$$\frac{(3cd^2-5ae^2)\,(3ae^2+cd^2)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4a^2d^3e^2x\,(cd^2-ae^2)}-\frac{3(5a^2e^4+2acd^2e^2+c^2d^4)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}-\frac{(cd^2-5ae^2)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2\,(cd^2-ae^2)}-\frac{2e(ae+cdx)}{dx^2\,(cd^2-ae^2)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) - ((c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(2*a*d^2*e*(c*d^2 - a*e^2)*x^2) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^
2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*a^2*d^3*e^2*(c*d^2 - a*e
^2)*x) - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(8*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{ae+cdx}{x^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{2\int -$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2$$

$$= -\frac{2e(ae+cdx)}{d\left(cd^2-ae^2\right)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(cd^2$$

Mathematica [A]
time = 0.45, size = 283, normalized size = 0.86

$$\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(3c^3d^5x^2(d+ex)+a^3e^4(2d^2-5dex-15e^2x^2)+ac^2d^3ex(d^2+5dex+4e^2x^2)-a^2cde^2(2d^3-4d^2ex+de^2x^2+15e^3x^3)\right)-3(c^3d^6+ac^2d^4e^2+3a^2cd^2e^4-5a^3e^6)x^2\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{4a^{5/2}d^{7/2}e^{5/2}\left(cd^2-ae^2\right)x^2\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^3*d^5*x^2*(d + e*x) + a^3*e^4*(2*d^2 - 5*d*e*
x - 15*e^2*x^2) + a*c^2*d^3*e*x*(d^2 + 5*d*e*x + 4*e^2*x^2) - a^2*c*d*e^2*(
2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 15*e^3*x^3)) - 3*(c^3*d^6 + a*c^2*d^4*e^2 +
 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*x^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(
Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(4*a^(5/2)*d^(
7/2)*e^(5/2)*(c*d^2 - a*e^2)*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.08, size = 545, normalized size = 1.66

method	result

default	$\frac{2e^2\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}{d^3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}+\frac{-\frac{\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2}}{2ade\,x^2}-\frac{3\left(a\,e^2+c\,d^2\right)\left(-\frac{\sqrt{a\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2*e^2/d^3/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2)+1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(a*e^2+c
*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+
c*d^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))+1/2*c/a/(a*d*e)^(1/2)*ln((2*a*d*e+
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)
)-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(a*e^2+c*d^
2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-e^2/d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e
^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(x*e + d)*x^3), x)
```

Fricas [**A**]
time = 8.39, size = 799, normalized size = 2.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] [1/16*(3*(c^3*d^6*x^3*e + c^3*d^7*x^2 + a*c^2*d^4*x^3*e^3 + a*c^2*d^5*x^2*e
^2 + 3*a^2*c*d^2*x^3*e^5 + 3*a^2*c*d^3*x^2*e^4 - 5*a^3*x^3*e^7 - 5*a^3*d*x^
```

```
2*e^6)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8
*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*
d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2)
 + 4*(3*a*c^2*d^6*x*e + 2*a^2*c*d^4*x*e^3 - 15*a^3*d*x^2*e^6 - 5*a^3*d^2*x*
e^5 + 2*(2*a^2*c*d^3*x^2 + a^3*d^3)*e^4 + (3*a*c^2*d^5*x^2 - 2*a^2*c*d^5)*e
^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(a^3*c*d^6*x^3*e^4 + a^3*c
*d^7*x^2*e^3 - a^4*d^4*x^3*e^6 - a^4*d^5*x^2*e^5), 1/8*(3*(c^3*d^6*x^3*e +
c^3*d^7*x^2 + a*c^2*d^4*x^3*e^3 + a*c^2*d^5*x^2*e^2 + 3*a^2*c*d^2*x^3*e^5 +
 3*a^2*c*d^3*x^2*e^4 - 5*a^3*x^3*e^7 - 5*a^3*d*x^2*e^6)*sqrt(-a*d*e)*arctan
(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2))
 + 2*(3*a*c^2*d^6*x*e + 2*a^2*c*d^4*x*e^3 - 15*a^3*d*x^2*e^6 - 5*a^3*d^2*x*
e^5 + 2*(2*a^2*c*d^3*x^2 + a^3*d^3)*e^4 + (3*a*c^2*d^5*x^2 - 2*a^2*c*d^5)*e
^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(a^3*c*d^6*x^3*e^4 + a^3*c
*d^7*x^2*e^3 - a^4*d^4*x^3*e^6 - a^4*d^5*x^2*e^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{(d+ex)\,(ae+cdx)}\,\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%%{1,[0,3,9]%%%},[2,4]%%%}+%%%{%%%{-4,[1,5,7]%%%},[2,3]%%%}+
%%%{%%
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
[Out] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

3.477 $\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=515

$$-\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}-\frac{2x^2(ade(cd^2-ae^2)\,(7c^2d^4-12acd^2e^2-3a^2e^4)+(cd^2-}{3cde^2\,(cd^2-ae^2)^4\,\sqrt{ade+(cd}}$$

```
[Out] -2/3*d*x^4*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+5/8*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4
)*arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2)-2/3*x^2*(a*d*e*(-a*e^2+
c*d^2)*(-3*a^2*e^4-12*a*c*d^2*e^2+7*c^2*d^4)+(-a*e^2+c*d^2)*(-3*a^3*e^6-a^2
*c*d^2*e^4-11*a*c^2*d^4*e^2+7*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^4/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/12*(105*c^4*d^8-190*a*c^3*d^6*e^2+36*a^2*
c^2*d^4*e^4+30*a^3*c*d^2*e^6-45*a^4*e^8-2*c*d*e*(-15*a^3*e^6+9*a^2*c*d^2*e^
4-61*a*c^2*d^4*e^2+35*c^3*d^6)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^3/d^3/e^4/(-a*e^2+c*d^2)^3
```

Rubi [A]
time = 0.41, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 832, 793, 635, 212}

$$\frac{5(3a^2e^4+6acd^2e^2+7c^2d^4)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}}-\frac{2x^2(ade(cd^2-ae^2)(-3a^2e^4-12acd^2e^2+7c^2d^4)+x(cd^2-ae^2)(-3a^3e^6-a^2cd^2e^4-11ac^2d^4e^2+7c^3d^6))}{3cde^2(cd^2-ae^2)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{(-45a^4e^8+30a^3cd^2e^6+36a^2c^2d^4e^4-2cdex(-15a^3e^6+9a^2cd^2e^4-61ac^2d^4e^2+35c^3d^6)-190ac^3d^6e^2+105c^4d^8)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12c^3d^3e^4(cd^2-ae^2)^3}-\frac{2dx^4(cdx(cd^2-ae^2)+ae(cd^2-ae^2))}{3e(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^4*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x^2*(a*d*e*(c*d^2
- a*e^2)*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(7*c^3*
d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 -
 a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((105*c^4*d^8 - 19
0*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^4*e^8 - 2*c*
d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*e^6)*x)*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^3*d^3*e^4*(c*d^2 - a*e^2)^3)
+ (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(8*c^(7/2)*d^(7/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^5(ae+cdx)}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx$$

$$=-\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)x)}{3e(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}+\frac{2\int \frac{x}{}}{}$$

$$=-\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)x)}{3e(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}-\frac{2x^2(c}{}$$

$$=-\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)x)}{3e(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}-\frac{2x^2(c}{}$$

$$=-\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)x)}{3e(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}-\frac{2x^2(c}{}$$

$$=-\frac{2dx^4(ae(cd^2-ae^2)+cd(cd^2-ae^2)x)}{3e(cd^2-ae^2)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}-\frac{2x^2(c}{}$$

Mathematica [A]
time = 0.83, size = 388, normalized size = 0.75

$$\frac{-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(-45a^5e^9(d+ex)^2+15a^4cde^7(2d-ex)(d+ex)^2+6a^3c^2d^2e^5(d+ex)^2(6d^2+2dex+e^2x^2)+c^5d^8x(105d^3+140d^2ex+21de^2x^2-6e^3x^3)-2a^2c^3d^4e^3(95d^4+111d^3ex-6d^2e^2x^2-9de^3x^3+9e^4x^4)+ac^4d^6e(105d^4-50d^3ex-237d^2e^2x^2-48de^3x^3+18e^4x^4)\right)}{(cd^2-ae^2)^3}+15(7c^2d^4+6acd^2e^2+3a^2e^4)(ae+cdx)^{3/2}(d+ex)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{12c^{7/2}d^{7/2}e^{9/2}((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-45*a^5*e^9*(d + e*x)^2 + 15*a^4
*c*d*e^7*(2*d - e*x)*(d + e*x)^2 + 6*a^3*c^2*d^2*e^5*(d + e*x)^2*(6*d^2 + 2
*d*e*x + e^2*x^2) + c^5*d^8*x*(105*d^3 + 140*d^2*e*x + 21*d*e^2*x^2 - 6*e^3
*x^3) - 2*a^2*c^3*d^4*e^3*(95*d^4 + 111*d^3*e*x - 6*d^2*e^2*x^2 - 9*d*e^3*x
^3 + 9*e^4*x^4) + a*c^4*d^6*e*(105*d^4 - 50*d^3*e*x - 237*d^2*e^2*x^2 - 48*
d*e^3*x^3 + 18*e^4*x^4)))/(c*d^2 - a*e^2)^3) + 15*(7*c^2*d^4 + 6*a*c*d^2*e^
2 + 3*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(12*c^(7/2)*d^(7/2)*e^(9/2)*(
(a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1922 vs. $2(485) = 970$.
time = 0.11, size = 1923, normalized size = 3.73

method	result	size
default	Expression too large to display	1923

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(1/2*x^3/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2+c*d^2
)/c/d/e*(x^2/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2
)/c/d/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)
/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/
e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-2*a/c*(-1/c/d/e
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e
^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)))-3/2*a/c*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e
^2+c*d^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*
d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(
c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)))-d/e^2
*(x^2/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/c/d/e
*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d/e*
(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*
d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-2*a/c*(-1/c/d/e/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c*d^
2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
)+d^2/e^3*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^
2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/
d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-d^3/e^4*(-1/c
/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x
+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))+2*d^4/e^5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-d^5/e^6*(-2/3/(a*e^2-c*d^2)/(x+d
/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3
*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2
))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. 2(466) = 932.
time = 16.74, size = 2161, normalized size = 4.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/48*(15*(7*c^6*d^13*x - 3*a^6*x^2*e^13 - 3*(a^5*c*d*x^3 + 2*a^6*d*x)*e^12
 - 3*(a^5*c*d^2*x^2 + a^6*d^2)*e^11 + 3*(a^4*c^2*d^3*x^3 + a^5*c*d^3*x)*e^1
0 + (8*a^4*c^2*d^4*x^2 + 3*a^5*c*d^4)*e^9 + (2*a^3*c^3*d^5*x^3 + 7*a^4*c^2*
d^5*x)*e^8 + 2*(5*a^3*c^3*d^6*x^2 + a^4*c^2*d^6)*e^7 + 2*(3*a^2*c^4*d^7*x^3
 + 7*a^3*c^3*d^7*x)*e^6 - 3*(a^2*c^4*d^8*x^2 - 2*a^3*c^3*d^8)*e^5 - 3*(5*a*
c^5*d^9*x^3 + 8*a^2*c^4*d^9*x)*e^4 - (23*a*c^5*d^10*x^2 + 15*a^2*c^4*d^10)*
e^3 + (7*c^6*d^11*x^3 - a*c^5*d^11*x)*e^2 + 7*(2*c^6*d^12*x^2 + a*c^5*d^12)
*e)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4
 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2
)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2) - 4*(105*c^6*d^12*
x*e - 45*a^5*c*d*x^2*e^12 - 15*(a^4*c^2*d^2*x^3 + 6*a^5*c*d^2*x)*e^11 + 3*(
2*a^3*c^3*d^3*x^4 - 15*a^5*c*d^3)*e^10 + 3*(8*a^3*c^3*d^4*x^3 + 15*a^4*c^2*
d^4*x)*e^9 - 6*(3*a^2*c^4*d^5*x^4 - 11*a^3*c^3*d^5*x^2 - 5*a^4*c^2*d^5)*e^8
 + 6*(3*a^2*c^4*d^6*x^3 + 14*a^3*c^3*d^6*x)*e^7 + 6*(3*a*c^5*d^7*x^4 + 2*a^
2*c^4*d^7*x^2 + 6*a^3*c^3*d^7)*e^6 - 6*(8*a*c^5*d^8*x^3 + 37*a^2*c^4*d^8*x)
*e^5 - (6*c^6*d^9*x^4 + 237*a*c^5*d^9*x^2 + 190*a^2*c^4*d^9)*e^4 + (21*c^6*
d^10*x^3 - 50*a*c^5*d^10*x)*e^3 + 35*(4*c^6*d^11*x^2 + 3*a*c^5*d^11)*e^2)*s
qrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(c^8*d^13*x*e^5 - a^4*c^4*d^4*x
^2*e^14 - (a^3*c^5*d^5*x^3 + 2*a^4*c^4*d^5*x)*e^13 + (a^3*c^5*d^6*x^2 - a^4
*c^4*d^6)*e^12 + (3*a^2*c^6*d^7*x^3 + 5*a^3*c^5*d^7*x)*e^11 + 3*(a^2*c^6*d^
8*x^2 + a^3*c^5*d^8)*e^10 - 3*(a*c^7*d^9*x^3 + a^2*c^6*d^9*x)*e^9 - (5*a*c^
7*d^10*x^2 + 3*a^2*c^6*d^10)*e^8 + (c^8*d^11*x^3 - a*c^7*d^11*x)*e^7 + (2*c
```

```
^8*d^12*x^2 + a*c^7*d^12)*e^6), -1/24*(15*(7*c^6*d^13*x - 3*a^6*x^2*e^13 -
3*(a^5*c*d*x^3 + 2*a^6*d*x)*e^12 - 3*(a^5*c*d^2*x^2 + a^6*d^2)*e^11 + 3*(a^
4*c^2*d^3*x^3 + a^5*c*d^3*x)*e^10 + (8*a^4*c^2*d^4*x^2 + 3*a^5*c*d^4)*e^9 +
 (2*a^3*c^3*d^5*x^3 + 7*a^4*c^2*d^5*x)*e^8 + 2*(5*a^3*c^3*d^6*x^2 + a^4*c^2
*d^6)*e^7 + 2*(3*a^2*c^4*d^7*x^3 + 7*a^3*c^3*d^7*x)*e^6 - 3*(a^2*c^4*d^8*x^
2 - 2*a^3*c^3*d^8)*e^5 - 3*(5*a*c^5*d^9*x^3 + 8*a^2*c^4*d^9*x)*e^4 - (23*a*
c^5*d^10*x^2 + 15*a^2*c^4*d^10)*e^3 + (7*c^6*d^11*x^3 - a*c^5*d^11*x)*e^2 +
 7*(2*c^6*d^12*x^2 + a*c^5*d^12)*e)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*
d^3*x*e + a*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(105*c^6*d^12*x*e
 - 45*a^5*c*d*x^2*e^12 - 15*(a^4*c^2*d^2*x^3 + 6*a^5*c*d^2*x)*e^11 + 3*(2*a
^3*c^3*d^3*x^4 - 15*a^5*c*d^3)*e^10 + 3*(8*a^3*c^3*d^4*x^3 + 15*a^4*c^2*d^4
*x)*e^9 - 6*(3*a^2*c^4*d^5*x^4 - 11*a^3*c^3*d^5*x^2 - 5*a^4*c^2*d^5)*e^8 +
6*(3*a^2*c^4*d^6*x^3 + 14*a^3*c^3*d^6*x)*e^7 + 6*(3*a*c^5*d^7*x^4 + 2*a^2*c
^4*d^7*x^2 + 6*a^3*c^3*d^7)*e^6 - 6*(8*a*c^5*d^8*x^3 + 37*a^2*c^4*d^8*x)*e^
5 - (6*c^6*d^9*x^4 + 237*a*c^5*d^9*x^2 + 190*a^2*c^4*d^9)*e^4 + (21*c^6*d^1
0*x^3 - 50*a*c^5*d^10*x)*e^3 + 35*(4*c^6*d^11*x^2 + 3*a*c^5*d^11)*e^2)*sqrt
(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(c^8*d^13*x*e^5 - a^4*c^4*d^4*x^2*
e^14 - (a^3*c^5*d^5*x^3 + 2*a^4*c^4*d^5*x)*e^13 + (a^3*c^5*d^6*x^2 - a^4*c^
4*d^6)*e^12 + (3*a^2*c^6*d^7*x^3 + 5*a^3*c^5*d^7*x)*e^11 + 3*(a^2*c^6*d^8*x
^2 + a^3*c^5*d^8)*e^10 - 3*(a*c^7*d^9*x^3 + a^2*c^6*d^9*x)*e^9 - (5*a*c^7*d
^10*x^2 + 3*a^2*c^6*d^10)*e^8 + (c^8*d^11*x^3 - a*c^7*d^11*x)*e^7 + (2*c^8*
d^12*x^2 + a*c^7*d^12)*e^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(x^5/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.478 $\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=438

$$-\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\,(cd^2-ae^2)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}-\frac{2x(ade(cd^2-ae^2)\,(5c^2d^4-10acd^2e^2-3a^2e^4)+(cd^2-}{3cde^2\,(cd^2-ae^2)^4\,\sqrt{ade+(c}}$$

```
[Out] -2/3*d*x^3*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(3*a*e^2+5*c*d^2)*arctanh(1/2*(2*c
*d*e*x+a*e^2+c*d^2)/c^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)-2/3*x*(a*d*e*(-a*e^2+c*d^2)*(-3*a^2*e^4-1
0*a*c*d^2*e^2+5*c^2*d^4)+(-a*e^2+c*d^2)*(-3*a^3*e^6-a^2*c*d^2*e^4-9*a*c^2*d
^4*e^2+5*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+1/3*(-9*a^3*e^6+9*a^2*c*d^2*e^4-31*a*c^2*d^4*e^2+15*c^3*d^6)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3/(-a*e^2+c*d^2)^3
```

Rubi [A]
time = 0.36, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 832, 654, 635, 212}

$$-\frac{2x(ade(cd^2-ae^2)\,(-3a^2e^4-10acd^2e^2+5c^2d^4)+x(cd^2-ae^2)\,(-3a^3e^6-a^2cd^2e^4-9ac^2d^4e^2+5c^3d^6))}{3cde^2\,(cd^2-ae^2)^4\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}+\frac{(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2e^3\,(cd^2-ae^2)^3}-\frac{(3ae^2+5cd^2)\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{5/2}d^{5/2}e^{7/2}}-\frac{2dx^3(cdx(cd^2-ae^2)+ae(cd^2-ae^2))}{3e\,(cd^2-ae^2)^2\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^3*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*x*(a*d*e*(c*d^2 -
a*e^2)*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(5*c^3*d^
6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*
e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^3*d^6 - 31*a*c
^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(3*c^2*d^2*e^3*(c*d^2 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*ArcTan
h[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2]])])/(2*c^(5/2)*d^(5/2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4(ae+cdx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2\int}{}$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x(}{}$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x(}{}$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x(}{}$$

$$= -\frac{2dx^3(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x(}{}$$

Mathematica [A]
time = 0.55, size = 300, normalized size = 0.68

$$\frac{\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{e}\,(ae+cdx)(-9a^4e^7(d+ex)^2+3a^3cde^5(3d-ex)(d+ex)^2+c^4d^7x(15d^2+20dex+3e^2x^2)+ac^3d^5e(15d^3-11d^2ex-39de^2x^2-9e^3x^3)+a^2c^2d^3e^3(-31d^3-33d^2ex+9de^2x^2+9e^3x^3))}{(cd^2-ae^2)^3} - 3(5cd^2+3ae^2)\,(ae+cdx)^{3/2}(d+ex)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{ae+cdx}}\right)}{3c^{5/2}d^{5/2}e^{7/2}((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] ((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-9*a^4*e^7*(d + e*x)^2 + 3*a^3*c*d
*e^5*(3*d - e*x)*(d + e*x)^2 + c^4*d^7*x*(15*d^2 + 20*d*e*x + 3*e^2*x^2) +
a*c^3*d^5*e*(15*d^3 - 11*d^2*e*x - 39*d*e^2*x^2 - 9*e^3*x^3) + a^2*c^2*d^3*
e^3*(-31*d^3 - 33*d^2*e*x + 9*d*e^2*x^2 + 9*e^3*x^3)))/(c*d^2 - a*e^2)^3 -
3*(5*c*d^2 + 3*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[c]*
Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(3*c^(5/2)*d^(5/2)*e^(
7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(408) = 816$.
time = 0.09, size = 1112, normalized size = 2.54

method	result

default	$\frac{\frac{x^2}{cde\sqrt{ade+(a\,e^2+c\,d^2)\,x+cde\,x^2}}-\frac{3(a\,e^2+c\,d^2)\left(-\frac{x}{cde\sqrt{ade+(a\,e^2+c\,d^2)\,x+cde\,x^2}}-\frac{(a\,e^2+c\,d^2)\left(-\frac{}{cde}\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(x^2/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/c/
d/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d
/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-2*a/c*(-1/c/d/e/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c
*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)))-d/e^2*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d
^2)/c/d/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c
/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)
^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))+1/e^3*d^2*(-
1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*
e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2))-2*d^3/e^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/e^5*d^4*(-2/3/(a*e^2-c*d^2)
/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d
^2)^3*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(393) = 786$.
time = 7.73, size = 1817, normalized size = 4.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")

[Out] [1/12*(3*(5*c^5*d^11*x + 6*a^3*c^2*d^6*e^5 - 3*a^5*x^2*e^11 - 3*(a^4*c*d*x^
3 + 2*a^5*d*x)*e^10 - (2*a^4*c*d^2*x^2 + 3*a^5*d^2)*e^9 + (4*a^3*c^2*d^3*x^
3 + 5*a^4*c*d^3*x)*e^8 + 2*(7*a^3*c^2*d^4*x^2 + 2*a^4*c*d^4)*e^7 + 2*(3*a^2
*c^3*d^5*x^3 + 8*a^3*c^2*d^5*x)*e^6 - 6*(2*a*c^4*d^7*x^3 + 3*a^2*c^3*d^7*x)
*e^4 - (19*a*c^4*d^8*x^2 + 12*a^2*c^3*d^8)*e^3 + (5*c^5*d^9*x^3 - 2*a*c^4*d
^9*x)*e^2 + 5*(2*c^5*d^10*x^2 + a*c^4*d^10)*e)*sqrt(c*d)*e^(1/2)*log(8*c^2*
d^3*x*e + c^2*d^4 + 8*a*c*d*x*e^3 + a^2*e^4 - 4*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d
^2*x^2 + 3*a*c*d^2)*e^2) + 4*(15*c^5*d^10*x*e - 9*a^4*c*d*x^2*e^10 - 3*(a^3
*c^2*d^2*x^3 + 6*a^4*c*d^2*x)*e^9 + 3*(a^3*c^2*d^3*x^2 - 3*a^4*c*d^3)*e^8 +
 3*(3*a^2*c^3*d^4*x^3 + 5*a^3*c^2*d^4*x)*e^7 + 9*(a^2*c^3*d^5*x^2 + a^3*c^2
*d^5)*e^6 - 3*(3*a*c^4*d^6*x^3 + 11*a^2*c^3*d^6*x)*e^5 - (39*a*c^4*d^7*x^2
+ 31*a^2*c^3*d^7)*e^4 + (3*c^5*d^8*x^3 - 11*a*c^4*d^8*x)*e^3 + 5*(4*c^5*d^9
*x^2 + 3*a*c^4*d^9)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(c^7*
d^12*x*e^4 - a^4*c^3*d^3*x^2*e^13 - (a^3*c^4*d^4*x^3 + 2*a^4*c^3*d^4*x)*e^1
2 + (a^3*c^4*d^5*x^2 - a^4*c^3*d^5)*e^11 + (3*a^2*c^5*d^6*x^3 + 5*a^3*c^4*d
^6*x)*e^10 + 3*(a^2*c^5*d^7*x^2 + a^3*c^4*d^7)*e^9 - 3*(a*c^6*d^8*x^3 + a^2
*c^5*d^8*x)*e^8 - (5*a*c^6*d^9*x^2 + 3*a^2*c^5*d^9)*e^7 + (c^7*d^10*x^3 - a
*c^6*d^10*x)*e^6 + (2*c^7*d^11*x^2 + a*c^6*d^11)*e^5), 1/6*(3*(5*c^5*d^11*x
 + 6*a^3*c^2*d^6*e^5 - 3*a^5*x^2*e^11 - 3*(a^4*c*d*x^3 + 2*a^5*d*x)*e^10 -
(2*a^4*c*d^2*x^2 + 3*a^5*d^2)*e^9 + (4*a^3*c^2*d^3*x^3 + 5*a^4*c*d^3*x)*e^8
 + 2*(7*a^3*c^2*d^4*x^2 + 2*a^4*c*d^4)*e^7 + 2*(3*a^2*c^3*d^5*x^3 + 8*a^3*c
^2*d^5*x)*e^6 - 6*(2*a*c^4*d^7*x^3 + 3*a^2*c^3*d^7*x)*e^4 - (19*a*c^4*d^8*x
^2 + 12*a^2*c^3*d^8)*e^3 + (5*c^5*d^9*x^3 - 2*a*c^4*d^9*x)*e^2 + 5*(2*c^5*d
^10*x^2 + a*c^4*d^10)*e)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (
c*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a
*c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(15*c^5*d^10*x*e - 9*a^4*c*d
*x^2*e^10 - 3*(a^3*c^2*d^2*x^3 + 6*a^4*c*d^2*x)*e^9 + 3*(a^3*c^2*d^3*x^2 -
3*a^4*c*d^3)*e^8 + 3*(3*a^2*c^3*d^4*x^3 + 5*a^3*c^2*d^4*x)*e^7 + 9*(a^2*c^3
*d^5*x^2 + a^3*c^2*d^5)*e^6 - 3*(3*a*c^4*d^6*x^3 + 11*a^2*c^3*d^6*x)*e^5 -
(39*a*c^4*d^7*x^2 + 31*a^2*c^3*d^7)*e^4 + (3*c^5*d^8*x^3 - 11*a*c^4*d^8*x)*
e^3 + 5*(4*c^5*d^9*x^2 + 3*a*c^4*d^9)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^
```

```
2 + a*d)*e))/(c^7*d^12*x*e^4 - a^4*c^3*d^3*x^2*e^13 - (a^3*c^4*d^4*x^3 + 2*
a^4*c^3*d^4*x)*e^12 + (a^3*c^4*d^5*x^2 - a^4*c^3*d^5)*e^11 + (3*a^2*c^5*d^6
*x^3 + 5*a^3*c^4*d^6*x)*e^10 + 3*(a^2*c^5*d^7*x^2 + a^3*c^4*d^7)*e^9 - 3*(a
*c^6*d^8*x^3 + a^2*c^5*d^8*x)*e^8 - (5*a*c^6*d^9*x^2 + 3*a^2*c^5*d^9)*e^7 +
 (c^7*d^10*x^3 - a*c^6*d^10*x)*e^6 + (2*c^7*d^11*x^2 + a*c^6*d^11)*e^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(x^4/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.479 $\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=297

$$-\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2(ade(cd^2-3ae^2)\left(3cd^2+ae^2\right)+(3c^3d^6-7ac^2d^4e^2-a}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

```
[Out] -2/3*d*x^2*(a*e*(-a*e^2+c*d^2)+c*d*(-a*e^2+c*d^2)*x)/e/(-a*e^2+c*d^2)^2/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+arctanh(1/2*(2*c*d*e*x+a*e^2+c*d^2)/c^
(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3
/2)/e^(5/2)-2/3*(a*d*e*(-3*a*e^2+c*d^2)*(a*e^2+3*c*d^2)+(-3*a^3*e^6-a^2*c*d
^2*e^4-7*a*c^2*d^4*e^2+3*c^3*d^6)*x)/c/d/e^2/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {863, 832, 791, 635, 212}

$$-\frac{2(x(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6)+ade(cd^2-3ae^2)\left(ae^2+3cd^2\right))}{3cde^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}-\frac{2dx^2(cdx(cd^2-ae^2)+ae(cd^2-ae^2))}{3e\left(cd^2-ae^2\right)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^
2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) - (2*(a*d*e*(c*d^2 - 3*
a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3
*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^(3/2)*d^(3/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 863

```
Int[((x_)^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_
)), x_Symbol] :> Int[x^n*(a/d + c*(x/e))*(a + b*x + c*x^2)^(p - 1), x] /; F
reeQ[{a, b, c, d, e, n, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e
+ a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !IntegerQ[2*p] || IGtQ[n
, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rubi steps

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3(ae+cdx)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} + \frac{2\int}{}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{2(a}{}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{2(a}{}$$

$$= -\frac{2dx^2(ae(cd^2-ae^2)+cd(cd^2-ae^2)\,x)}{3e\left(cd^2-ae^2\right)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{2(a}{}$$

Mathematica [A]
time = 0.34, size = 218, normalized size = 0.73

$$\frac{2\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{e}\,(ae+cdx)\left(-3a^3e^5(d+ex)^2+c^3d^6x(3d+4ex)-a^2cd^3e^3(8d+9ex)+ac^2d^4e\left(3d^2-4dex-9e^2x^2\right)\right)}{(cd^2-ae^2)^3}+3(ae+cdx)^{3/2}(d+ex)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)\right)}{3c^{3/2}d^{3/2}e^{5/2}((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-3*a^3*e^5*(d + e*x)^2 + c^3*
d^6*x*(3*d + 4*e*x) - a^2*c*d^3*e^3*(8*d + 9*e*x) + a*c^2*d^4*e*(3*d^2 - 4*
d*e*x - 9*e^2*x^2)))/(c*d^2 - a*e^2)^3) + 3*(a*e + c*d*x)^(3/2)*(d + e*x)^(
3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])]))
/(3*c^(3/2)*d^(3/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(273) = 546$.
time = 0.08, size = 639, normalized size = 2.15

method	result
default	$-\frac{x}{cde\sqrt{ade+(a\,e^2+c\,d^2)\,x+cde\,x^2}} - \frac{(a\,e^2+c\,d^2)\left(-\frac{1}{cde\sqrt{ade+(a\,e^2+c\,d^2)\,x+cde\,x^2}} - \frac{}{cde\left(4ac\,d^2e^2-(\right.}\right.}{2cde}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-x/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*e^2+c*d^2)/c/d
/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))+1/c/d/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2))-d/e^2*(-1/c/d/e/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(2*c*d*e*x+a*e^2+c
*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))+2*d^2/e^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-d^3/e^4*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d
*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*
e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(265) = 530$.
time = 6.31, size = 1471, normalized size = 4.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(c^4*d^9*x - a^4*x^2*e^9 - (a^3*c*d*x^3 + 2*a^4*d*x)*e^8 + (a^3*c*d
^2*x^2 - a^4*d^2)*e^7 + (3*a^2*c^2*d^3*x^3 + 5*a^3*c*d^3*x)*e^6 + 3*(a^2*c^
2*d^4*x^2 + a^3*c*d^4)*e^5 - 3*(a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e^4 - (5*a*c
```

```
^3*d^6*x^2 + 3*a^2*c^2*d^6)*e^3 + (c^4*d^7*x^3 - a*c^3*d^7*x)*e^2 + (2*c^4*
d^8*x^2 + a*c^3*d^8)*e)*sqrt(c*d)*e^(1/2)*log(8*c^2*d^3*x*e + c^2*d^4 + 8*a
*c*d*x*e^3 + a^2*e^4 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d
*x*e + c*d^2 + a*e^2)*sqrt(c*d)*e^(1/2) + 2*(4*c^2*d^2*x^2 + 3*a*c*d^2)*e^2
) - 4*(3*c^4*d^8*x*e - 4*a*c^3*d^6*x*e^3 - 9*a^2*c^2*d^4*x*e^5 - 3*a^3*c*d*
x^2*e^8 - 6*a^3*c*d^2*x*e^7 - 3*a^3*c*d^3*e^6 - (9*a*c^3*d^5*x^2 + 8*a^2*c^
2*d^5)*e^4 + (4*c^4*d^7*x^2 + 3*a*c^3*d^7)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e))/(c^6*d^11*x*e^3 - a^4*c^2*d^2*x^2*e^12 - (a^3*c^3*d^3*x^3
 + 2*a^4*c^2*d^3*x)*e^11 + (a^3*c^3*d^4*x^2 - a^4*c^2*d^4)*e^10 + (3*a^2*c^
4*d^5*x^3 + 5*a^3*c^3*d^5*x)*e^9 + 3*(a^2*c^4*d^6*x^2 + a^3*c^3*d^6)*e^8 -
3*(a*c^5*d^7*x^3 + a^2*c^4*d^7*x)*e^7 - (5*a*c^5*d^8*x^2 + 3*a^2*c^4*d^8)*e
^6 + (c^6*d^9*x^3 - a*c^5*d^9*x)*e^5 + (2*c^6*d^10*x^2 + a*c^5*d^10)*e^4),
-1/3*(3*(c^4*d^9*x - a^4*x^2*e^9 - (a^3*c*d*x^3 + 2*a^4*d*x)*e^8 + (a^3*c*d
^2*x^2 - a^4*d^2)*e^7 + (3*a^2*c^2*d^3*x^3 + 5*a^3*c*d^3*x)*e^6 + 3*(a^2*c^
2*d^4*x^2 + a^3*c*d^4)*e^5 - 3*(a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e^4 - (5*a*c
^3*d^6*x^2 + 3*a^2*c^2*d^6)*e^3 + (c^4*d^7*x^3 - a*c^3*d^7*x)*e^2 + (2*c^4*
d^8*x^2 + a*c^3*d^8)*e)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*(2*c*d*x*e + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^3*x*e + a*
c*d*x*e^3 + (c^2*d^2*x^2 + a*c*d^2)*e^2)) + 2*(3*c^4*d^8*x*e - 4*a*c^3*d^6*
x*e^3 - 9*a^2*c^2*d^4*x*e^5 - 3*a^3*c*d*x^2*e^8 - 6*a^3*c*d^2*x*e^7 - 3*a^3
*c*d^3*e^6 - (9*a*c^3*d^5*x^2 + 8*a^2*c^2*d^5)*e^4 + (4*c^4*d^7*x^2 + 3*a*c
^3*d^7)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(c^6*d^11*x*e^3 -
 a^4*c^2*d^2*x^2*e^12 - (a^3*c^3*d^3*x^3 + 2*a^4*c^2*d^3*x)*e^11 + (a^3*c^3
*d^4*x^2 - a^4*c^2*d^4)*e^10 + (3*a^2*c^4*d^5*x^3 + 5*a^3*c^3*d^5*x)*e^9 +
3*(a^2*c^4*d^6*x^2 + a^3*c^3*d^6)*e^8 - 3*(a*c^5*d^7*x^3 + a^2*c^4*d^7*x)*e
^7 - (5*a*c^5*d^8*x^2 + 3*a^2*c^4*d^8)*e^6 + (c^6*d^9*x^3 - a*c^5*d^9*x)*e^
5 + (2*c^6*d^10*x^2 + a*c^5*d^10)*e^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(x^3/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.480 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8ae\left(2ade+\left(cd^2+ae^2\right)x\right)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 2/3*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*
a*e*(2*a*d*e+(a*e^2+c*d^2)*x)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {868, 12, 650}

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 868

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(2*c*d - b*e))*(f + g*x)^n*((a + b*x
 + c*x^2)^(p + 1)/(e*p*(b^2 - 4*a*c)*(d + e*x))), x] - Dist[1/(d*e*p*(b^2 -
 4*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f
*(2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p +
1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &&
```

```
NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && I
GtQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{2}{\ldots}$$

$$= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{(4}{\ldots}$$

$$= \frac{2x^2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{}{3\ldots}$$

Mathematica [A]
time = 0.15, size = 95, normalized size = 0.75

$$\frac{2(ae+cdx)^3\left(d^2-\frac{6ade(d+ex)}{ae+cdx}-\frac{3a^2e^2(d+ex)^2}{(ae+cdx)^2}\right)}{3\left(cd^2-ae^2\right)^3((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*(d^2 - (6*a*d*e*(d + e*x))/(a*e + c*d*x) - (3*a^2*e^2*(d
 + e*x)^2)/(a*e + c*d*x)^2))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x))
^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(118) = 236.
time = 0.10, size = 365, normalized size = 2.90

method	result
gosper	$\frac{2(cdx+ae)\left(3a^2e^4x^2+6ac\,d^2e^2x^2-c^2d^4x^2+12a^2d\,e^3x+4ac\,d^3ex+8a^2d^2e^2\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$
trager	$\frac{2\left(3a^2e^4x^2+6ac\,d^2e^2x^2-c^2d^4x^2+12a^2d\,e^3x+4ac\,d^3ex+8a^2d^2e^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)(ex+d)^2\left(a\,e^2-c\,d^2\right)(cdx+ae)}$
default	$\frac{-\frac{1}{cde\sqrt{ade+(a\,e^2+c\,d^2)x+cde\,x^2}}-\frac{\left(a\,e^2+c\,d^2\right)\left(2cdex+a\,e^2+c\,d^2\right)}{cde\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cde\,x^2}}}{e} - \frac{}{e^2\left(4\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-1/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/c/d/e*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))-2*d/e^2*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/e^3*d^2*(-2/3/(a*e^2-c*
d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2
-c*d^2)^3*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d
/e))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(120) = 240$.
time = 6.51, size = 313, normalized size = 2.48

$$\frac{2\left(c^{2} d^{4} x^{2}-4 a c d^{3} x e-3 a^{2} x^{2} e^{4}-12 a^{2} d x e^{3}-2\left(3 a c d^{2} x^{2}+4 a^{2} d^{2}\right) e^{2}\right) \sqrt{c d^{2} x+a x e^{2}+\left(c d x^{2}+a d\right) e}}{3\left(c^{4} d^{9} x-a^{4} x^{2} e^{9}-\left(a^{3} c d x^{3}+2 a^{4} d x\right) e^{8}+\left(a^{3} c d^{2} x^{2}-a^{4} d^{2}\right) e^{7}+\left(3 a^{2} c^{2} d^{3} x^{3}+5 a^{3} c d^{3} x\right) e^{6}+3\left(a^{2} c^{2} d^{4} x^{2}+a^{3} c d^{4}\right) e^{5}-3\left(a c^{3} d^{5} x^{3}+a^{2} c^{2} d^{5} x\right) e^{4}-\left(5 a c^{3} d^{6} x^{2}+3 a^{2} c^{2} d^{6}\right) e^{3}+\left(c^{4} d^{7} x^{3}-a c^{3} d^{7} x\right) e^{2}+\left(2 c^{4} d^{8} x^{2}+a c^{3} d^{8}\right) e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] 2/3*(c^2*d^4*x^2 - 4*a*c*d^3*x*e - 3*a^2*x^2*e^4 - 12*a^2*d*x*e^3 - 2*(3*a*
c*d^2*x^2 + 4*a^2*d^2)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)/(c^
4*d^9*x - a^4*x^2*e^9 - (a^3*c*d*x^3 + 2*a^4*d*x)*e^8 + (a^3*c*d^2*x^2 - a^
4*d^2)*e^7 + (3*a^2*c^2*d^3*x^3 + 5*a^3*c*d^3*x)*e^6 + 3*(a^2*c^2*d^4*x^2 +
 a^3*c*d^4)*e^5 - 3*(a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e^4 - (5*a*c^3*d^6*x^2
+ 3*a^2*c^2*d^6)*e^3 + (c^4*d^7*x^3 - a*c^3*d^7*x)*e^2 + (2*c^4*d^8*x^2 + a
*c^3*d^8)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
```

```
[Out] integrate(x^2/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)), x)
```

Mupad [B]
time = 3.60, size = 1071, normalized size = 8.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (4*c*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(a^3*d*e^7 - c^3
*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*d^3*e^5 - c^3*d^6*e^2*x + 3*
a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) - (2*d^2*(a*d*e + a*e^2*x + c*d^2*x +
 c*d*e*x^2)^(1/2))/(3*c^2*d^6*e + 3*a^2*d^2*e^5 + 3*a^2*e^7*x^2 + 6*c^2*d^5
*e^2*x + 3*c^2*d^4*e^3*x^2 - 6*a*c*d^4*e^3 + 6*a^2*d*e^6*x - 12*a*c*d^3*e^4
*x - 6*a*c*d^2*e^5*x^2) - (4*a*d*e^2*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2
)^(1/2))/(3*(a^3*d*e^7 - c^3*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*
d^3*e^5 - c^3*d^6*e^2*x + 3*a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) + (2*c^4*
d^7*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a*c^4*
d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) + (22*a^3*c
*d^2*e^5)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a
*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) - (28*
a^2*c^2*d^4*e^3)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*
e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (2*a*c^3*d^6*e)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*
```

```
e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (10*a^2*c^2*d^3*e^4*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*
(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*
c*d*e^9)) + (2*a^3*c*d*e^6*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2
)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^
4*c*d*e^9)) - (22*a*c^3*d^5*e^2*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^
2)^(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e
^7 + a^4*c*d*e^9))
```

3.481 $\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\left(cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2/3*d/e/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2/3
*(3*a*e^2+c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {806, 627}

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 - a
*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$
$$= -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

Mathematica [A]
time = 0.13, size = 100, normalized size = 0.72

$$\frac{2(c^2d^3x(3d+2ex)+a^2e^3(2d+3ex)+2acde(3d^2+5dex+3e^2x^2))}{3\left(cd^2-ae^2\right)^3(d+ex)\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*(c^2*d^3*x*(3*d + 2*e*x) + a^2*e^3*(2*d + 3*e*x) + 2*a*c*d*e*(3*d^2 + 5*
d*e*x + 3*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d +
 e*x)])
```

Maple [A]
time = 0.10, size = 226, normalized size = 1.64

method	result
gosper	$-\frac{2(cdx+ae)\left(6acd\,e^3x^2+2c^2d^3e\,x^2+3a^2e^4x+10ac\,d^2e^2x+3c^2d^4x+2d\,e^3a^2+6ac\,d^3e\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$
trager	$-\frac{2\left(6acd\,e^3x^2+2c^2d^3e\,x^2+3a^2e^4x+10ac\,d^2e^2x+3c^2d^4x+2d\,e^3a^2+6ac\,d^3e\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)(ex+d)^2\left(a\,e^2-c\,d^2\right)(cdx+ae)}$
default	$\frac{4cdex+2a\,e^2+2c\,d^2}{e\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2}}-\frac{d\left(-\frac{2}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^2+(a\,e^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)-d/e^2*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+a*
e^2-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(128) = 256$.
time = 7.42, size = 318, normalized size = 2.30

$$\frac{2\,(3\,c^2d^4x + 10\,acd^2xe^2 + 3\,a^2xe^4 + 2\,(3\,acdx^2 + a^2d)e^3 + 2\,(c^2d^3x^2 + 3\,acd^3)e)\,\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}}{3\,(c^4d^9x - a^4x^2e^9 - (a^3cdx^3 + 2\,a^4dx)e^8 + (a^3cd^2x^2 - a^4d^2)e^7 + (3\,a^2c^2d^3x^3 + 5\,a^3cd^3x)e^6 + 3\,(a^2c^2d^4x^2 + a^3cd^4)e^5 - 3\,(ac^3d^5x^3 + a^2c^2d^5x)e^4 - (5\,ac^3d^6x^2 + 3\,a^2c^2d^6)e^3 + (c^4d^7x^3 - ac^3d^7x)e^2 + (2\,c^4d^8x^2 + ac^3d^8)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(3*c^2*d^4*x + 10*a*c*d^2*x*e^2 + 3*a^2*x*e^4 + 2*(3*a*c*d*x^2 + a^2*d)
*e^3 + 2*(c^2*d^3*x^2 + 3*a*c*d^3)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)/(c^4*d^9*x - a^4*x^2*e^9 - (a^3*c*d*x^3 + 2*a^4*d*x)*e^8 + (a^3*c*d^
2*x^2 - a^4*d^2)*e^7 + (3*a^2*c^2*d^3*x^3 + 5*a^3*c*d^3*x)*e^6 + 3*(a^2*c^2
*d^4*x^2 + a^3*c*d^4)*e^5 - 3*(a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e^4 - (5*a*c^
3*d^6*x^2 + 3*a^2*c^2*d^6)*e^3 + (c^4*d^7*x^3 - a*c^3*d^7*x)*e^2 + (2*c^4*d
^8*x^2 + a*c^3*d^8)*e)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{((d + ex)\,(ae + cdx))^{\frac{3}{2}}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(x/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)), x)
```

Mupad [B]
time = 3.32, size = 499, normalized size = 3.62

$$\frac{4\,a^2\,d\,e^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,a^2\,e^4\,x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,c^2\,d^4\,x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+4\,c^2\,d^3\,e\,x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+12\,a\,c\,d^3\,e\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+20\,a\,c\,d^2\,e^2\,x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+12\,a\,c\,d\,e^3\,x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{-3\,a^4\,d^2\,e^7-6\,a^4\,d\,e^8\,x-3\,a^4\,e^9\,x^2+9\,a^3\,c\,d^4\,e^5+15\,a^3\,c\,d^3\,e^6\,x+3\,a^3\,c\,d^2\,e^7\,x^2-3\,a^3\,c\,d\,e^8\,x^3-9\,a^2\,c^2\,d^6\,e^3-9\,a^2\,c^2\,d^5\,e^4\,x+9\,a^2\,c^2\,d^4\,e^5\,x^2+9\,a^2\,c^2\,d^3\,e^6\,x^3+3\,a\,c^3\,d^8\,e-3\,a\,c^3\,d^7\,e^2\,x-15\,a\,c^3\,d^6\,e^3\,x^2-9\,a\,c^3\,d^5\,e^4\,x^3+3\,c^4\,d^9\,x+6\,c^4\,d^8\,e\,x^2+3\,c^4\,d^7\,e^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (4*a^2*d*e^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*a^2*e^4*x*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*c^2*d^4*x*(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^(1/2) + 4*c^2*d^3*e*x^2*(x*(a*e^2 + c*d^2) + a*d*e +
 c*d*e*x^2)^(1/2) + 12*a*c*d^3*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2) + 20*a*c*d^2*e^2*x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 12*a
*c*d*e^3*x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*c^4*d^9*x -
3*a^4*d^2*e^7 - 3*a^4*e^9*x^2 + 9*a^3*c*d^4*e^5 + 6*c^4*d^8*e*x^2 - 9*a^2*c
^2*d^6*e^3 + 3*c^4*d^7*e^2*x^3 + 3*a*c^3*d^8*e - 6*a^4*d*e^8*x + 9*a^2*c^2*
d^4*e^5*x^2 + 9*a^2*c^2*d^3*e^6*x^3 - 3*a*c^3*d^7*e^2*x + 15*a^3*c*d^3*e^6*
x - 3*a^3*c*d*e^8*x^3 - 9*a^2*c^2*d^5*e^4*x - 15*a*c^3*d^6*e^3*x^2 + 3*a^3*
c*d^2*e^7*x^2 - 9*a*c^3*d^5*e^4*x^3)
```

3.482 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8cd(cd^2+ae^2+2cdex)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 2/3/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*c*d*
(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {672, 627}

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd(ae^2+cd^2+2cdex)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] 2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} +$$

$$= \frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} -$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 0.80

$$-\frac{2(ae+cdx)^3\left(-e^2+\frac{6cde(d+ex)}{ae+cdx}+\frac{3c^2d^2(d+ex)^2}{(ae+cdx)^2}\right)}{3\left(cd^2-ae^2\right)^3((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*(a*e + c*d*x)^3*(-e^2 + (6*c*d*e*(d + e*x))/(a*e + c*d*x) + (3*c^2*d^2*
(d + e*x)^2)/(a*e + c*d*x)^2))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x
))^(3/2))
```

Maple [A]
time = 0.09, size = 146, normalized size = 1.21

method	result
gosper	$-\frac{2(cdx+ae)\left(-8c^2d^2e^2x^2-4acd\,e^3x-12c^2d^3ex+a^2e^4-6ac\,d^2e^2-3c^2d^4\right)}{3\left(a^3e^6-3a^2c\,d^2e^4+3a\,c^2d^4e^2-c^3d^6\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}$
default	$\frac{-\frac{2}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}+\frac{8cde\left(2cde\left(x+\frac{d}{e}\right)+a\,e^2-c\,d^2\right)}{3\left(a\,e^2-c\,d^2\right)^3\sqrt{cde\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)}}}{e}$
trager	$-\frac{2\left(-8c^2d^2e^2x^2-4acd\,e^3x-12c^2d^3ex+a^2e^4-6ac\,d^2e^2-3c^2d^4\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)(ex+d)^2\left(a\,e^2-c\,d^2\right)(cdx+ae)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(114) = 228$.
time = 8.47, size = 308, normalized size = 2.55

$$-\frac{2\,(12\,c^2d^3xe+3\,c^2d^4+4\,acdxe^3-a^2e^4+2\,(4\,c^2d^2x^2+3\,acd^2)e^2)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}{3\,(c^4d^9x-a^4x^2e^9-(a^3cdx^3+2\,a^4dx)e^8+(a^3cd^2x^2-a^4d^2)e^7+(3\,a^2c^2d^3x^3+5\,a^3cd^3x)e^6+3\,(a^2c^2d^4x^2+a^3cd^4)e^5-3\,(ac^3d^5x^3+a^2c^2d^5x)e^4-(5\,ac^3d^6x^2+3\,a^2c^2d^6)e^3+(c^4d^7x^3-ac^3d^7x)e^2+(2\,c^4d^8x^2+ac^3d^8)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2/3*(12*c^2*d^3*x*e + 3*c^2*d^4 + 4*a*c*d*x*e^3 - a^2*e^4 + 2*(4*c^2*d^2*x
^2 + 3*a*c*d^2)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)/(c^4*d^9*x
 - a^4*x^2*e^9 - (a^3*c*d*x^3 + 2*a^4*d*x)*e^8 + (a^3*c*d^2*x^2 - a^4*d^2)*
e^7 + (3*a^2*c^2*d^3*x^3 + 5*a^3*c*d^3*x)*e^6 + 3*(a^2*c^2*d^4*x^2 + a^3*c*
d^4)*e^5 - 3*(a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e^4 - (5*a*c^3*d^6*x^2 + 3*a^2
*c^2*d^6)*e^3 + (c^4*d^7*x^3 - a*c^3*d^7*x)*e^2 + (2*c^4*d^8*x^2 + a*c^3*d^
8)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)), x)
```

Mupad [B]
time = 2.88, size = 120, normalized size = 0.99

$$\frac{2\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+4\,a\,c\,d\,e^3\,x+3\,c^2\,d^4+12\,c^2\,d^3\,e\,x+8\,c^2\,d^2\,e^2\,x^2\right)}{3\,\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-c\,d^2\right)^3\,\left(d+e\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] (2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*c^2*d^4 - a^2*e^4 + 8*c
^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 12*c^2*d^3*e*x + 4*a*c*d*e^3*x))/(3*(a*e +
 c*d*x)*(a*e^2 - c*d^2)^3*(d + e*x)^2)
```

3.483 $\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6+ac^2d^4e^2+7a^2cd^2e^4-3a^3e^6+cde\left(3cd^2-ae^2\right)\left(\right.\right.}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
-arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)+2/3*(3*c^3*d^6+a*c^2*d
^4*e^2+7*a^2*c*d^2*e^4-3*a^3*e^6+c*d*e*(-a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*x)/
a/d^2/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {865, 836, 12, 738, 212}

$$-\frac{\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}+\frac{2\left(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex\left(3cd^2-ae^2\right)\left(3ae^2+cd^2\right)+3c^3d^6\right)}{3ad^2e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6
 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)
^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2
 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])]/(a^(3/2)*d^(5/2)*e^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\int}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(3c}{}$$

Mathematica [A]
time = 0.35, size = 217, normalized size = 0.80

$$\frac{2\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(-3c^3d^5(d+ex)^2+a^3e^6(4d+3ex)-ac^2d^3e^3x(9d+8ex)+a^2cde^4\left(-9d^2-4dex+3e^2x^2\right)\right)}{\left(-cd^2+ae^2\right)^3}-3(ae+cdx)^{3/2}(d+ex)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)\right)}{3a^{3/2}d^{5/2}e^{3/2}((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-3*c^3*d^5*(d + e*x)^2 + a^3*e^
6*(4*d + 3*e*x) - a*c^2*d^3*e^3*x*(9*d + 8*e*x) + a^2*c*d*e^4*(-9*d^2 - 4*d
*e*x + 3*e^2*x^2)))/(-(c*d^2) + a*e^2)^3 - 3*(a*e + c*d*x)^(3/2)*(d + e*x)^
(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])
)/(3*a^(3/2)*d^(5/2)*e^(3/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A]
time = 0.09, size = 360, normalized size = 1.33

method	result

default	$-\frac{-\frac{2}{3\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}+\frac{8cde\left(2cde\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)}{3\left(a\,e^{2}-c\,d^{2}\right)^{3}\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d\right.}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/d*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1
/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(1/2))+1/d*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln
((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. 2(243) = 486.
time = 8.95, size = 1483, normalized size = 5.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"fricas")
```

```
[Out] [1/6*(3*(c^4*d^9*x - a^4*x^2*e^9 - (a^3*c*d*x^3 + 2*a^4*d*x)*e^8 + (a^3*c*d
^2*x^2 - a^4*d^2)*e^7 + (3*a^2*c^2*d^3*x^3 + 5*a^3*c*d^3*x)*e^6 + 3*(a^2*c^
2*d^4*x^2 + a^3*c*d^4)*e^5 - 3*(a*c^3*d^5*x^3 + a^2*c^2*d^5*x)*e^4 - (5*a*c
^3*d^6*x^2 + 3*a^2*c^2*d^6)*e^3 + (c^4*d^7*x^3 - a*c^3*d^7*x)*e^2 + (2*c^4*
d^8*x^2 + a*c^3*d^8)*e)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e
+ a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*
```

```
x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a
^2*d^2)*e^2)/x^2) + 4*(3*a*c^3*d^6*x^2*e^3 + 6*a*c^3*d^7*x*e^2 + 3*a*c^3*d^
8*e + 9*a^2*c^2*d^5*x*e^4 + 4*a^3*c*d^3*x*e^6 - 3*a^4*d*x*e^8 - (3*a^3*c*d^
2*x^2 + 4*a^4*d^2)*e^7 + (8*a^2*c^2*d^4*x^2 + 9*a^3*c*d^4)*e^5)*sqrt(c*d^2*
x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(a^2*c^4*d^12*x*e^2 - a^6*d^3*x^2*e^11 -
(a^5*c*d^4*x^3 + 2*a^6*d^4*x)*e^10 + (a^5*c*d^5*x^2 - a^6*d^5)*e^9 + (3*a^4
*c^2*d^6*x^3 + 5*a^5*c*d^6*x)*e^8 + 3*(a^4*c^2*d^7*x^2 + a^5*c*d^7)*e^7 - 3
*(a^3*c^3*d^8*x^3 + a^4*c^2*d^8*x)*e^6 - (5*a^3*c^3*d^9*x^2 + 3*a^4*c^2*d^9
)*e^5 + (a^2*c^4*d^10*x^3 - a^3*c^3*d^10*x)*e^4 + (2*a^2*c^4*d^11*x^2 + a^3
*c^3*d^11)*e^3), 1/3*(3*(c^4*d^9*x - a^4*x^2*e^9 - (a^3*c*d*x^3 + 2*a^4*d*x
)*e^8 + (a^3*c*d^2*x^2 - a^4*d^2)*e^7 + (3*a^2*c^2*d^3*x^3 + 5*a^3*c*d^3*x)
*e^6 + 3*(a^2*c^2*d^4*x^2 + a^3*c*d^4)*e^5 - 3*(a*c^3*d^5*x^3 + a^2*c^2*d^5
*x)*e^4 - (5*a*c^3*d^6*x^2 + 3*a^2*c^2*d^6)*e^3 + (c^4*d^7*x^3 - a*c^3*d^7*
x)*e^2 + (2*c^4*d^8*x^2 + a*c^3*d^8)*e)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x +
a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)
/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(3*a*c^3*d^
6*x^2*e^3 + 6*a*c^3*d^7*x*e^2 + 3*a*c^3*d^8*e + 9*a^2*c^2*d^5*x*e^4 + 4*a^3
*c*d^3*x*e^6 - 3*a^4*d*x*e^8 - (3*a^3*c*d^2*x^2 + 4*a^4*d^2)*e^7 + (8*a^2*c
^2*d^4*x^2 + 9*a^3*c*d^4)*e^5)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))
/(a^2*c^4*d^12*x*e^2 - a^6*d^3*x^2*e^11 - (a^5*c*d^4*x^3 + 2*a^6*d^4*x)*e^1
0 + (a^5*c*d^5*x^2 - a^6*d^5)*e^9 + (3*a^4*c^2*d^6*x^3 + 5*a^5*c*d^6*x)*e^8
 + 3*(a^4*c^2*d^7*x^2 + a^5*c*d^7)*e^7 - 3*(a^3*c^3*d^8*x^3 + a^4*c^2*d^8*x
)*e^6 - (5*a^3*c^3*d^9*x^2 + 3*a^4*c^2*d^9)*e^5 + (a^2*c^4*d^10*x^3 - a^3*c
^3*d^10*x)*e^4 + (2*a^2*c^4*d^11*x^2 + a^3*c^3*d^11)*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm=
"giac")
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)*x), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.484 $\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=394

$$-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2(3c^3d^6+ac^2d^4e^2+9a^2cd^2e^4-5a^3e^6+cde(3c^2d^4+10\cdots}{3ad^2e\left(cd^2-ae^2\right)^3x\sqrt{ade+\left(cd^2+ae^2\right)x+\cdots}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)+1/2*(5*a*e^2+3*c*d^2)*arctanh(1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1
/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2
)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+9*a^2*c*d^2*e^4-5*a^3*e^6+c*d*e*(-5*a^2*e^4+
10*a*c*d^2*e^2+3*c^2*d^4)*x)/a/d^2/e/(-a*e^2+c*d^2)^3/x/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-1/3*(-15*a^3*e^6+31*a^2*c*d^2*e^4-9*a*c^2*d^4*e^2+9*c^
3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)^3
/x
```

Rubi [A]

time = 0.36, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {865, 836, 820, 738, 212}

$$\frac{\left(5ae^2+3cd^2\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{5/2}d^{7/2}e^{5/2}}+\frac{2(-5a^3e^6+cdex(-5a^2e^4+10acd^2e^2+3c^2d^4)+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)}{3ad^2ex\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x\left(cd^2-ae^2\right)^3}-\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^3*e
^6 + c*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 -
 a*e^2)^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((9*c^3*d^6 - 9*
a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x) + ((3*c*d^2 + 5*a*e^2)
*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(5/2)*d^(7/2)*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(}{}$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(}{}$$

Mathematica [A]
time = 0.55, size = 303, normalized size = 0.77

$$\frac{-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\,(ae+cdx)\left(-9c^4d^7x(d+ex)^2-3ac^3d^5e(d-3ex)(d+ex)^2+a^4e^7\left(3d^2+20dex+15e^2x^2\right)+a^2c^2d^3e^3\left(9d^3+9d^2ex-33de^2x^2-31e^3x^3\right)-a^3cde^5\left(9d^3+39d^2ex+11de^2x^2-15e^3x^3\right)\right)}{\left(-cd^2+ae^2\right)^3x}+3\left(3cd^2+5ae^2\right)(ae+cdx)^{3/2}(d+ex)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{3a^{5/2}d^{7/2}e^{5/2}((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] (-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-9*c^4*d^7*x*(d + e*x)^2 - 3*a*c
^3*d^5*e*(d - 3*e*x)*(d + e*x)^2 + a^4*e^7*(3*d^2 + 20*d*e*x + 15*e^2*x^2)
+ a^2*c^2*d^3*e^3*(9*d^3 + 9*d^2*e*x - 33*d*e^2*x^2 - 31*e^3*x^3) - a^3*c*d
*e^5*(9*d^3 + 39*d^2*e*x + 11*d*e^2*x^2 - 15*e^3*x^3)))/((-(c*d^2) + a*e^2)
^3*x)) + 3*(3*c*d^2 + 5*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[
(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(3*a^(5/2)*d^
(7/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A]
time = 0.10, size = 716, normalized size = 1.82

method	result

default	$\frac{e\left(-\frac{2}{3\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}+\frac{8cde\left(2cde\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)}{3\left(a\,e^{2}-c\,d^{2}\right)^{3}\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d\right.}}\right.}{d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] e/d^2*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))+1/d*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*
ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-e/d^2*(1/a/d/e/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^
2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d
*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)*x^2),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(355) = 710$.
time = 32.92, size = 1889, normalized size = 4.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/12*(3*(3*c^5*d^11*x^2 - 6*a^2*c^3*d^5*x^4*e^6 - 5*a^5*x^3*e^11 - 5*(a^4*
c*d*x^4 + 2*a^5*d*x^2)*e^10 + (2*a^4*c*d^2*x^3 - 5*a^5*d^2*x)*e^9 + (12*a^3
*c^2*d^3*x^4 + 19*a^4*c*d^3*x^2)*e^8 + 6*(3*a^3*c^2*d^4*x^3 + 2*a^4*c*d^4*x
)*e^7 - 2*(8*a^2*c^3*d^6*x^3 + 3*a^3*c^2*d^6*x)*e^5 - 2*(2*a*c^4*d^7*x^4 +
7*a^2*c^3*d^7*x^2)*e^4 - (5*a*c^4*d^8*x^3 + 4*a^2*c^3*d^8*x)*e^3 + (3*c^5*d
^9*x^4 + 2*a*c^4*d^9*x^2)*e^2 + 3*(2*c^5*d^10*x^3 + a*c^4*d^10*x)*e)*sqrt(a
*d)*e^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3
+ 4*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*
e)*sqrt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 4*(9*a*c^4
*d^10*x*e - 15*a^5*d*x^2*e^10 - 5*(3*a^4*c*d^2*x^3 + 4*a^5*d^2*x)*e^9 + (11
*a^4*c*d^3*x^2 - 3*a^5*d^3)*e^8 + (31*a^3*c^2*d^4*x^3 + 39*a^4*c*d^4*x)*e^7
 + 3*(11*a^3*c^2*d^5*x^2 + 3*a^4*c*d^5)*e^6 - 9*(a^2*c^3*d^6*x^3 + a^3*c^2*
d^6*x)*e^5 - 3*(5*a^2*c^3*d^7*x^2 + 3*a^3*c^2*d^7)*e^4 + 3*(3*a*c^4*d^8*x^3
 - a^2*c^3*d^8*x)*e^3 + 3*(6*a*c^4*d^9*x^2 + a^2*c^3*d^9)*e^2)*sqrt(c*d^2*x
 + a*x*e^2 + (c*d*x^2 + a*d)*e))/(a^3*c^4*d^13*x^2*e^3 - a^7*d^4*x^3*e^12 -
 (a^6*c*d^5*x^4 + 2*a^7*d^5*x^2)*e^11 + (a^6*c*d^6*x^3 - a^7*d^6*x)*e^10 +
(3*a^5*c^2*d^7*x^4 + 5*a^6*c*d^7*x^2)*e^9 + 3*(a^5*c^2*d^8*x^3 + a^6*c*d^8*
x)*e^8 - 3*(a^4*c^3*d^9*x^4 + a^5*c^2*d^9*x^2)*e^7 - (5*a^4*c^3*d^10*x^3 +
3*a^5*c^2*d^10*x)*e^6 + (a^3*c^4*d^11*x^4 - a^4*c^3*d^11*x^2)*e^5 + (2*a^3*
c^4*d^12*x^3 + a^4*c^3*d^12*x)*e^4), -1/6*(3*(3*c^5*d^11*x^2 - 6*a^2*c^3*d^
5*x^4*e^6 - 5*a^5*x^3*e^11 - 5*(a^4*c*d*x^4 + 2*a^5*d*x^2)*e^10 + (2*a^4*c*
d^2*x^3 - 5*a^5*d^2*x)*e^9 + (12*a^3*c^2*d^3*x^4 + 19*a^4*c*d^3*x^2)*e^8 +
6*(3*a^3*c^2*d^4*x^3 + 2*a^4*c*d^4*x)*e^7 - 2*(8*a^2*c^3*d^6*x^3 + 3*a^3*c^
2*d^6*x)*e^5 - 2*(2*a*c^4*d^7*x^4 + 7*a^2*c^3*d^7*x^2)*e^4 - (5*a*c^4*d^8*x
^3 + 4*a^2*c^3*d^8*x)*e^3 + (3*c^5*d^9*x^4 + 2*a*c^4*d^9*x^2)*e^2 + 3*(2*c^
5*d^10*x^3 + a*c^4*d^10*x)*e)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 +
2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*
x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(9*a*c^4*d^10*x*e - 1
5*a^5*d*x^2*e^10 - 5*(3*a^4*c*d^2*x^3 + 4*a^5*d^2*x)*e^9 + (11*a^4*c*d^3*x^
2 - 3*a^5*d^3)*e^8 + (31*a^3*c^2*d^4*x^3 + 39*a^4*c*d^4*x)*e^7 + 3*(11*a^3*
c^2*d^5*x^2 + 3*a^4*c*d^5)*e^6 - 9*(a^2*c^3*d^6*x^3 + a^3*c^2*d^6*x)*e^5 -
3*(5*a^2*c^3*d^7*x^2 + 3*a^3*c^2*d^7)*e^4 + 3*(3*a*c^4*d^8*x^3 - a^2*c^3*d^
8*x)*e^3 + 3*(6*a*c^4*d^9*x^2 + a^2*c^3*d^9)*e^2)*sqrt(c*d^2*x + a*x*e^2 +
(c*d*x^2 + a*d)*e))/(a^3*c^4*d^13*x^2*e^3 - a^7*d^4*x^3*e^12 - (a^6*c*d^5*x
^4 + 2*a^7*d^5*x^2)*e^11 + (a^6*c*d^6*x^3 - a^7*d^6*x)*e^10 + (3*a^5*c^2*d^
7*x^4 + 5*a^6*c*d^7*x^2)*e^9 + 3*(a^5*c^2*d^8*x^3 + a^6*c*d^8*x)*e^8 - 3*(a
^4*c^3*d^9*x^4 + a^5*c^2*d^9*x^2)*e^7 - (5*a^4*c^3*d^10*x^3 + 3*a^5*c^2*d^1
0*x)*e^6 + (a^3*c^4*d^11*x^4 - a^4*c^3*d^11*x^2)*e^5 + (2*a^3*c^4*d^12*x^3
+ a^4*c^3*d^12*x)*e^4)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}} \left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)*x^2),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.485 $\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=522

$$-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2(3c^3d^6+ac^2d^4e^2+11a^2cd^2e^4-7a^3e^6+cde(3c^2d^4+1}{3ad^2e\left(cd^2-ae^2\right)^3x^2\sqrt{ade+\left(cd^2+ae^2\right)x}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)-5/8*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(1/2*(2*a*d*e+(a*e^2+c*
d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^
(7/2)/d^(9/2)/e^(7/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+11*a^2*c*d^2*e^4-7*a^3*e
^6+c*d*e*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)*x)/a/d^2/e/(-a*e^2+c*d^2)^3/
x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/6*(-35*a^3*e^6+61*a^2*c*d^2*e
^4-9*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/
d^3/e^2/(-a*e^2+c*d^2)^3/x^2+1/12*(-105*a^4*e^8+190*a^3*c*d^2*e^6-36*a^2*c^
2*d^4*e^4-30*a*c^3*d^6*e^2+45*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^3/d^4/e^3/(-a*e^2+c*d^2)^3/x
```

Rubi [A]
time = 0.50, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {865, 836, 848, 820, 738, 212}

$$-\frac{5\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}}+\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+11a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{3ad^2ex^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-35a^3e^6+61a^2cd^2e^4-9ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6a^2d^3e^2x^2\left(cd^2-ae^2\right)^3}+\frac{\left(-105a^4e^8+190a^3cd^2e^6-36a^2c^2d^4e^4-30ac^3d^6e^2+45c^4d^8\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12a^3d^4e^3x\left(cd^2-ae^2\right)^3}-\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((15*c^3*d^
6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(6*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^2) + ((45*c^4*d^8
 - 30*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^
2)^3*x) - (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])])/(8*a^(7/2)*d^(9/2)*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{ae+cdx}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - $$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + $$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + $$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + $$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + $$

$$= -\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + $$

Mathematica [A]
time = 0.80, size = 390, normalized size = 0.75

$$\frac{\frac{\sqrt{a}\sqrt{d}\sqrt{e}\,(ae+cdx)\left(-45c^5d^9x^2(d+ex)^2-15ac^4d^7ex(d-2ex)(d+ex)^2+6a^2c^3d^5e^2(d+ex)^2\left(d^2+2dex+6e^2x^2\right)+a^5e^8\left(-6d^3+21d^2ex+140de^2x^2+105e^3x^3\right)-2a^3c^2d^3e^4\left(9d^4-9d^3ex-6d^2e^2x^2+111de^3x^3+95e^4x^4\right)+a^4cde^6\left(18d^4-48d^3ex-237d^2e^2x^2-50de^3x^3+105e^4x^4\right)\right)}{\left(-cd^2+ae^2\right)^3x^2}-15\left(3c^2d^4+6acd^2e^2+7a^2e^4\right)(ae+cdx)^{3/2}(d+ex)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{12a^{7/2}d^{9/2}e^{7/2}((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-45*c^5*d^9*x^2*(d + e*x)^2 - 15*a
*c^4*d^7*e*x*(d - 2*e*x)*(d + e*x)^2 + 6*a^2*c^3*d^5*e^2*(d + e*x)^2*(d^2 +
```

```
 2*d*e*x + 6*e^2*x^2) + a^5*e^8*(-6*d^3 + 21*d^2*e*x + 140*d*e^2*x^2 + 105*
e^3*x^3) - 2*a^3*c^2*d^3*e^4*(9*d^4 - 9*d^3*e*x - 6*d^2*e^2*x^2 + 111*d*e^3
*x^3 + 95*e^4*x^4) + a^4*c*d*e^6*(18*d^4 - 48*d^3*e*x - 237*d^2*e^2*x^2 - 5
0*d*e^3*x^3 + 105*e^4*x^4)))/((-(c*d^2) + a*e^2)^3*x^2) - 15*(3*c^2*d^4 + 6
*a*c*d^2*e^2 + 7*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt
[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(12*a^(7/2)*d^(9/2
)*e^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(488) = 976$.
time = 0.08, size = 1354, normalized size = 2.59

method	result
default	$-\frac{e^{2}\left(-\frac{2}{3\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}+\frac{8cde\left(2cde\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)}{3\left(a\,e^{2}-c\,d^{2}\right)^{3}\sqrt{cde\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-\right.}}\right.}{d^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -e^2/d^3*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d
/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))+1/d*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*
e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e
)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)
/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/
a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))-e/d^2*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1
```

```
/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2
)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+e^2/d^3*(1/a/d/e/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*
a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/
e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)*x^3),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(473) = 946$.
time = 73.55, size = 2291, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/48*(15*(3*c^6*d^13*x^3 - 7*a^6*x^4*e^13 - 7*(a^5*c*d*x^5 + 2*a^6*d*x^3)*
e^12 + (a^5*c*d^2*x^4 - 7*a^6*d^2*x^2)*e^11 + (15*a^4*c^2*d^3*x^5 + 23*a^5*
c*d^3*x^3)*e^10 + 3*(8*a^4*c^2*d^4*x^4 + 5*a^5*c*d^4*x^2)*e^9 - 3*(2*a^3*c^
3*d^5*x^5 - a^4*c^2*d^5*x^3)*e^8 - 2*(7*a^3*c^3*d^6*x^4 + 3*a^4*c^2*d^6*x^2
)*e^7 - 2*(a^2*c^4*d^7*x^5 + 5*a^3*c^3*d^7*x^3)*e^6 - (7*a^2*c^4*d^8*x^4 +
2*a^3*c^3*d^8*x^2)*e^5 - (3*a*c^5*d^9*x^5 + 8*a^2*c^4*d^9*x^3)*e^4 - 3*(a*c
^5*d^10*x^4 + a^2*c^4*d^10*x^2)*e^3 + 3*(c^6*d^11*x^5 + a*c^5*d^11*x^3)*e^2
 + 3*(2*c^6*d^12*x^4 + a*c^5*d^12*x^2)*e)*sqrt(a*d)*e^(1/2)*log((c^2*d^4*x^
2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 - 4*(c*d^2*x + a*x*e^2 + 2*
a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(a*d)*e^(1/2) + 2*(3
*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) + 4*(45*a*c^5*d^12*x^2*e - 105*a^6*d*x^
3*e^12 - 35*(3*a^5*c*d^2*x^4 + 4*a^6*d^2*x^2)*e^11 + (50*a^5*c*d^3*x^3 - 21
*a^6*d^3*x)*e^10 + (190*a^4*c^2*d^4*x^4 + 237*a^5*c*d^4*x^2 + 6*a^6*d^4)*e^
9 + 6*(37*a^4*c^2*d^5*x^3 + 8*a^5*c*d^5*x)*e^8 - 6*(6*a^3*c^3*d^6*x^4 + 2*a
^4*c^2*d^6*x^2 + 3*a^5*c*d^6)*e^7 - 6*(14*a^3*c^3*d^7*x^3 + 3*a^4*c^2*d^7*x
)*e^6 - 6*(5*a^2*c^4*d^8*x^4 + 11*a^3*c^3*d^8*x^2 - 3*a^4*c^2*d^8)*e^5 - 3*
```

```
(15*a^2*c^4*d^9*x^3 + 8*a^3*c^3*d^9*x)*e^4 + 3*(15*a*c^5*d^10*x^4 - 2*a^3*c
^3*d^10)*e^3 + 15*(6*a*c^5*d^11*x^3 + a^2*c^4*d^11*x)*e^2)*sqrt(c*d^2*x + a
*x*e^2 + (c*d*x^2 + a*d)*e))/(a^4*c^4*d^14*x^3*e^4 - a^8*d^5*x^4*e^13 - (a^
7*c*d^6*x^5 + 2*a^8*d^6*x^3)*e^12 + (a^7*c*d^7*x^4 - a^8*d^7*x^2)*e^11 + (3
*a^6*c^2*d^8*x^5 + 5*a^7*c*d^8*x^3)*e^10 + 3*(a^6*c^2*d^9*x^4 + a^7*c*d^9*x
^2)*e^9 - 3*(a^5*c^3*d^10*x^5 + a^6*c^2*d^10*x^3)*e^8 - (5*a^5*c^3*d^11*x^4
 + 3*a^6*c^2*d^11*x^2)*e^7 + (a^4*c^4*d^12*x^5 - a^5*c^3*d^12*x^3)*e^6 + (2
*a^4*c^4*d^13*x^4 + a^5*c^3*d^13*x^2)*e^5), 1/24*(15*(3*c^6*d^13*x^3 - 7*a^
6*x^4*e^13 - 7*(a^5*c*d*x^5 + 2*a^6*d*x^3)*e^12 + (a^5*c*d^2*x^4 - 7*a^6*d^
2*x^2)*e^11 + (15*a^4*c^2*d^3*x^5 + 23*a^5*c*d^3*x^3)*e^10 + 3*(8*a^4*c^2*d
^4*x^4 + 5*a^5*c*d^4*x^2)*e^9 - 3*(2*a^3*c^3*d^5*x^5 - a^4*c^2*d^5*x^3)*e^8
 - 2*(7*a^3*c^3*d^6*x^4 + 3*a^4*c^2*d^6*x^2)*e^7 - 2*(a^2*c^4*d^7*x^5 + 5*a
^3*c^3*d^7*x^3)*e^6 - (7*a^2*c^4*d^8*x^4 + 2*a^3*c^3*d^8*x^2)*e^5 - (3*a*c^
5*d^9*x^5 + 8*a^2*c^4*d^9*x^3)*e^4 - 3*(a*c^5*d^10*x^4 + a^2*c^4*d^10*x^2)*
e^3 + 3*(c^6*d^11*x^5 + a*c^5*d^11*x^3)*e^2 + 3*(2*c^6*d^12*x^4 + a*c^5*d^1
2*x^2)*e)*sqrt(-a*d*e)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*
x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 +
(a*c*d^2*x^2 + a^2*d^2)*e^2)) + 2*(45*a*c^5*d^12*x^2*e - 105*a^6*d*x^3*e^12
 - 35*(3*a^5*c*d^2*x^4 + 4*a^6*d^2*x^2)*e^11 + (50*a^5*c*d^3*x^3 - 21*a^6*d
^3*x)*e^10 + (190*a^4*c^2*d^4*x^4 + 237*a^5*c*d^4*x^2 + 6*a^6*d^4)*e^9 + 6*
(37*a^4*c^2*d^5*x^3 + 8*a^5*c*d^5*x)*e^8 - 6*(6*a^3*c^3*d^6*x^4 + 2*a^4*c^2
*d^6*x^2 + 3*a^5*c*d^6)*e^7 - 6*(14*a^3*c^3*d^7*x^3 + 3*a^4*c^2*d^7*x)*e^6
- 6*(5*a^2*c^4*d^8*x^4 + 11*a^3*c^3*d^8*x^2 - 3*a^4*c^2*d^8)*e^5 - 3*(15*a^
2*c^4*d^9*x^3 + 8*a^3*c^3*d^9*x)*e^4 + 3*(15*a*c^5*d^10*x^4 - 2*a^3*c^3*d^1
0)*e^3 + 15*(6*a*c^5*d^11*x^3 + a^2*c^4*d^11*x)*e^2)*sqrt(c*d^2*x + a*x*e^2
 + (c*d*x^2 + a*d)*e))/(a^4*c^4*d^14*x^3*e^4 - a^8*d^5*x^4*e^13 - (a^7*c*d^
6*x^5 + 2*a^8*d^6*x^3)*e^12 + (a^7*c*d^7*x^4 - a^8*d^7*x^2)*e^11 + (3*a^6*c
^2*d^8*x^5 + 5*a^7*c*d^8*x^3)*e^10 + 3*(a^6*c^2*d^9*x^4 + a^7*c*d^9*x^2)*e^
9 - 3*(a^5*c^3*d^10*x^5 + a^6*c^2*d^10*x^3)*e^8 - (5*a^5*c^3*d^11*x^4 + 3*a
^6*c^2*d^11*x^2)*e^7 + (a^4*c^4*d^12*x^5 - a^5*c^3*d^12*x^3)*e^6 + (2*a^4*c
^4*d^13*x^4 + a^5*c^3*d^13*x^2)*e^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)*x^3),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.486 $\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=664

$$-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2(3c^3d^6+ac^2d^4e^2+13a^2cd^2e^4-9a^3e^6+cde(3c^2d^4+}{3ad^2e\left(cd^2-ae^2\right)^3x^3\sqrt{ade+\left(cd^2+ae^2\right)}}$$

```
[Out] -2/3*e*(c*d*x+a*e)/d/(-a*e^2+c*d^2)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)+5/16*(21*a^3*e^6+21*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(
1/2*(2*a*d*e+(a*e^2+c*d^2)*x)/a^(1/2)/d^(1/2)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/a^(9/2)/d^(11/2)/e^(9/2)+2/3*(3*c^3*d^6+a*c^2*d^4*e^2+1
3*a^2*c*d^2*e^4-9*a^3*e^6+c*d*e*(-9*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^4)*x)/a/
d^2/e/(-a*e^2+c*d^2)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/3*(-21
*a^3*e^6+33*a^2*c*d^2*e^4-3*a*c^2*d^4*e^2+7*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/(-a*e^2+c*d^2)^3/x^3+1/12*(-105*a^4*e^8+168*a
^3*c*d^2*e^6-18*a^2*c^2*d^4*e^4-16*a*c^3*d^6*e^2+35*c^4*d^8)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3/(-a*e^2+c*d^2)^3/x^2-1/24*(-315*a^5*e
^10+525*a^4*c*d^2*e^8-78*a^3*c^2*d^4*e^6-54*a^2*c^3*d^6*e^4-55*a*c^4*d^8*e^
2+105*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^5/e^4/(-a*e^2
+c*d^2)^3/x
```

Rubi [A]

time = 0.71, antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {865, 836, 848, 820, 738, 212}

$$\frac{2(-9a^3e^6+cdex(-9a^2e^4+14acd^2e^2+3c^2d^4)+13a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)}{3ad^2ex^3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2+7c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2d^3e^2x^3\left(cd^2-ae^2\right)^3}+\frac{5\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\tanh^{-1}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{16a^{9/2}d^{11/2}e^{9/2}}+\frac{\left(-105a^4e^8+168a^3cd^2e^6-18a^2c^2d^4e^4-16ac^3d^6e^2+35c^4d^8\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{12a^3d^4e^3x^2\left(cd^2-ae^2\right)^3}-\frac{\left(-315a^5e^{10}+525a^4cd^2e^8-78a^3c^2d^4e^6-54a^2c^3d^6e^4-55ac^4d^8e^2+105c^5d^{10}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{24a^4d^5e^4x\left(cd^2-ae^2\right)^3}-\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^4 - 9*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9*a^2*e^4)*x))/(3*a*d^2*e*(c*d^
2 - a*e^2)^3*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((7*c^3*d^6
 - 3*a*c^2*d^4*e^2 + 33*a^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^3) + ((35*c^4*d^8
- 16*a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*e^8)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^2
)^3*x^2) - ((105*c^5*d^10 - 55*a*c^4*d^8*e^2 - 54*a^2*c^3*d^6*e^4 - 78*a^3*
c^2*d^4*e^6 + 525*a^4*c*d^2*e^8 - 315*a^5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(24*a^4*d^5*e^4*(c*d^2 - a*e^2)^3*x) + (5*(7*c^3*d^6 + 15
*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 +
```

```
a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*a^(9/2)*d^(11/2)*e^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
```

```
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 865

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[((f + g*x)^n*(a + b*x + c*x^2)^(m +
p))/(a/d + c*(x/e))^m, x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !I
ntegerQ[p] && ILtQ[m, 0] && IntegerQ[n] && (LtQ[n, 0] || GtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{ae+cdx}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+$$

$$=-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+$$

Mathematica [A]

time = 1.02, size = 493, normalized size = 0.74

$$\frac{\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-105c^6d^{11}x^3(d+ex)^2-5ac^5d^9ex^2(7d-11ex)(d+ex)^2+a^2c^4d^7e^2x(d+ex)^2\left(14d^2+23dex+54e^2x^2\right)-2a^3c^3d^5e^3(d+ex)^2\left(4d^3+4d^2ex-9de^2x^2-39e^3x^3\right)+a^6e^9\left(8d^4-18d^3ex+63d^2e^2x^2+420de^3x^3+315e^4x^4\right)+a^4c^2d^3e^5\left(24d^5-12d^4ex+62d^3e^2x^2+3d^2e^3x^3-636de^4x^4-525e^5x^5\right)-a^5cde^7\left(24d^5-40d^4ex+135d^3e^2x^2+651d^2e^3x^3+105de^4x^4-315e^5x^5\right)\right)}{(cd^2-ae^2)^3x^3(d+ex)}+15(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{24a^{9/2}d^{11/2}e^{9/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x
]
```

```
[Out] ((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^6*d^11*x^3*(d + e*x)^2 - 5*a*c^5*d^9*e*x^
2*(7*d - 11*e*x)*(d + e*x)^2 + a^2*c^4*d^7*e^2*x*(d + e*x)^2*(14*d^2 + 23*d
*e*x + 54*e^2*x^2) - 2*a^3*c^3*d^5*e^3*(d + e*x)^2*(4*d^3 + 4*d^2*e*x - 9*d
*e^2*x^2 - 39*e^3*x^3) + a^6*e^9*(8*d^4 - 18*d^3*e*x + 63*d^2*e^2*x^2 + 420
*d*e^3*x^3 + 315*e^4*x^4) + a^4*c^2*d^3*e^5*(24*d^5 - 12*d^4*e*x + 62*d^3*e
^2*x^2 + 3*d^2*e^3*x^3 - 636*d*e^4*x^4 - 525*e^5*x^5) - a^5*c*d*e^7*(24*d^5
 - 40*d^4*e*x + 135*d^3*e^2*x^2 + 651*d^2*e^3*x^3 + 105*d*e^4*x^4 - 315*e^5
*x^5)))/((c*d^2 - a*e^2)^3*x^3*(d + e*x)) + 15*(7*c^3*d^6 + 15*a*c^2*d^4*e^
2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[
(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(24*a^(9/2)*d
^(11/2)*e^(9/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2408 vs. $2(626) = 1252$.
time = 0.10, size = 2409, normalized size = 3.63

method	result	size
default	Expression too large to display	2409

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] e^3/d^4*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)+8/3*c*d*e/(a*e^2-c*d^2)^3*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))-e/d^2*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2
*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*
e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2
)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1
/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))+e^2/d^3*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2
```

```
)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e
^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)
^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/d*(-1/3/a/d/e/x^3/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-7/6*(a*e^2+c*d^2)/a/d/e*(-1/2/a/d/e/x^2/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+
c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-3/2*c/a
*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*
d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)))-4/3*c/a*(-1/a/d/e/x/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*
d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*
d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)))-e^3/d^4*
(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d
*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)*x^4),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1335 vs. 2(604) = 1208.
time = 156.13, size = 2695, normalized size = 4.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
```

```
[Out] [1/96*(15*(7*c^7*d^15*x^4 - 21*a^7*x^5*e^15 - 21*a^7*d^2*x^3*e^13 - 21*(a^6
*c*d*x^6 + 2*a^7*d*x^4)*e^14 + 21*(2*a^5*c^2*d^3*x^6 + 3*a^6*c*d^3*x^4)*e^1
2 + 3*(23*a^5*c^2*d^4*x^5 + 14*a^6*c*d^4*x^3)*e^11 - 3*(5*a^4*c^3*d^5*x^6 -
 4*a^5*c^2*d^5*x^4)*e^10 - (34*a^4*c^3*d^6*x^5 + 15*a^5*c^2*d^6*x^3)*e^9 -
(4*a^3*c^4*d^7*x^6 + 23*a^4*c^3*d^7*x^4)*e^8 - (11*a^3*c^4*d^8*x^5 + 4*a^4*
c^3*d^8*x^3)*e^7 - (3*a^2*c^5*d^9*x^6 + 10*a^3*c^4*d^9*x^4)*e^6 - 3*(4*a^2*
c^5*d^10*x^5 + a^3*c^4*d^10*x^3)*e^5 - 3*(2*a*c^6*d^11*x^6 + 5*a^2*c^5*d^11
*x^4)*e^4 - (5*a*c^6*d^12*x^5 + 6*a^2*c^5*d^12*x^3)*e^3 + (7*c^7*d^13*x^6 +
 8*a*c^6*d^13*x^4)*e^2 + 7*(2*c^7*d^14*x^5 + a*c^6*d^14*x^3)*e)*sqrt(a*d)*e
^(1/2)*log((c^2*d^4*x^2 + 8*a*c*d^3*x*e + a^2*x^2*e^4 + 8*a^2*d*x*e^3 + 4*(
c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sq
rt(a*d)*e^(1/2) + 2*(3*a*c*d^2*x^2 + 4*a^2*d^2)*e^2)/x^2) - 4*(105*a*c^6*d^
14*x^3*e - 315*a^7*d*x^4*e^14 - 105*(3*a^6*c*d^2*x^5 + 4*a^7*d^2*x^3)*e^13
+ 21*(5*a^6*c*d^3*x^4 - 3*a^7*d^3*x^2)*e^12 + 3*(175*a^5*c^2*d^4*x^5 + 217*
a^6*c*d^4*x^3 + 6*a^7*d^4*x)*e^11 + (636*a^5*c^2*d^5*x^4 + 135*a^6*c*d^5*x^
2 - 8*a^7*d^5)*e^10 - (78*a^4*c^3*d^6*x^5 + 3*a^5*c^2*d^6*x^3 + 40*a^6*c*d^
6*x)*e^9 - 2*(87*a^4*c^3*d^7*x^4 + 31*a^5*c^2*d^7*x^2 - 12*a^6*c*d^7)*e^8 -
 2*(27*a^3*c^4*d^8*x^5 + 53*a^4*c^3*d^8*x^3 - 6*a^5*c^2*d^8*x)*e^7 - (131*a
^3*c^4*d^9*x^4 - 6*a^4*c^3*d^9*x^2 + 24*a^5*c^2*d^9)*e^6 - (55*a^2*c^5*d^10
*x^5 + 114*a^3*c^4*d^10*x^3 - 24*a^4*c^3*d^10*x)*e^5 - (75*a^2*c^5*d^11*x^4
 + 51*a^3*c^4*d^11*x^2 - 8*a^4*c^3*d^11)*e^4 + (105*a*c^6*d^12*x^5 + 15*a^2
*c^5*d^12*x^3 - 14*a^3*c^4*d^12*x)*e^3 + 35*(6*a*c^6*d^13*x^4 + a^2*c^5*d^1
3*x^2)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e))/(a^5*c^4*d^15*x^4*
e^5 - a^9*d^6*x^5*e^14 - (a^8*c*d^7*x^6 + 2*a^9*d^7*x^4)*e^13 + (a^8*c*d^8*
x^5 - a^9*d^8*x^3)*e^12 + (3*a^7*c^2*d^9*x^6 + 5*a^8*c*d^9*x^4)*e^11 + 3*(a
^7*c^2*d^10*x^5 + a^8*c*d^10*x^3)*e^10 - 3*(a^6*c^3*d^11*x^6 + a^7*c^2*d^11
*x^4)*e^9 - (5*a^6*c^3*d^12*x^5 + 3*a^7*c^2*d^12*x^3)*e^8 + (a^5*c^4*d^13*x
^6 - a^6*c^3*d^13*x^4)*e^7 + (2*a^5*c^4*d^14*x^5 + a^6*c^3*d^14*x^3)*e^6),
-1/48*(15*(7*c^7*d^15*x^4 - 21*a^7*x^5*e^15 - 21*a^7*d^2*x^3*e^13 - 21*(a^6
*c*d*x^6 + 2*a^7*d*x^4)*e^14 + 21*(2*a^5*c^2*d^3*x^6 + 3*a^6*c*d^3*x^4)*e^1
2 + 3*(23*a^5*c^2*d^4*x^5 + 14*a^6*c*d^4*x^3)*e^11 - 3*(5*a^4*c^3*d^5*x^6 -
 4*a^5*c^2*d^5*x^4)*e^10 - (34*a^4*c^3*d^6*x^5 + 15*a^5*c^2*d^6*x^3)*e^9 -
(4*a^3*c^4*d^7*x^6 + 23*a^4*c^3*d^7*x^4)*e^8 - (11*a^3*c^4*d^8*x^5 + 4*a^4*
c^3*d^8*x^3)*e^7 - (3*a^2*c^5*d^9*x^6 + 10*a^3*c^4*d^9*x^4)*e^6 - 3*(4*a^2*
c^5*d^10*x^5 + a^3*c^4*d^10*x^3)*e^5 - 3*(2*a*c^6*d^11*x^6 + 5*a^2*c^5*d^11
*x^4)*e^4 - (5*a*c^6*d^12*x^5 + 6*a^2*c^5*d^12*x^3)*e^3 + (7*c^7*d^13*x^6 +
 8*a*c^6*d^13*x^4)*e^2 + 7*(2*c^7*d^14*x^5 + a*c^6*d^14*x^3)*e)*sqrt(-a*d*e
)*arctan(1/2*(c*d^2*x + a*x*e^2 + 2*a*d*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^
2 + a*d)*e)*sqrt(-a*d*e)/(a*c*d^3*x*e + a^2*d*x*e^3 + (a*c*d^2*x^2 + a^2*d^
2)*e^2)) + 2*(105*a*c^6*d^14*x^3*e - 315*a^7*d*x^4*e^14 - 105*(3*a^6*c*d^2*
```

```
x^5 + 4*a^7*d^2*x^3)*e^13 + 21*(5*a^6*c*d^3*x^4 - 3*a^7*d^3*x^2)*e^12 + 3*(
175*a^5*c^2*d^4*x^5 + 217*a^6*c*d^4*x^3 + 6*a^7*d^4*x)*e^11 + (636*a^5*c^2*
d^5*x^4 + 135*a^6*c*d^5*x^2 - 8*a^7*d^5)*e^10 - (78*a^4*c^3*d^6*x^5 + 3*a^5
*c^2*d^6*x^3 + 40*a^6*c*d^6*x)*e^9 - 2*(87*a^4*c^3*d^7*x^4 + 31*a^5*c^2*d^7
*x^2 - 12*a^6*c*d^7)*e^8 - 2*(27*a^3*c^4*d^8*x^5 + 53*a^4*c^3*d^8*x^3 - 6*a
^5*c^2*d^8*x)*e^7 - (131*a^3*c^4*d^9*x^4 - 6*a^4*c^3*d^9*x^2 + 24*a^5*c^2*d
^9)*e^6 - (55*a^2*c^5*d^10*x^5 + 114*a^3*c^4*d^10*x^3 - 24*a^4*c^3*d^10*x)*
e^5 - (75*a^2*c^5*d^11*x^4 + 51*a^3*c^4*d^11*x^2 - 8*a^4*c^3*d^11)*e^4 + (1
05*a*c^6*d^12*x^5 + 15*a^2*c^5*d^12*x^3 - 14*a^3*c^4*d^12*x)*e^3 + 35*(6*a*
c^6*d^13*x^4 + a^2*c^5*d^13*x^2)*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e))/(a^5*c^4*d^15*x^4*e^5 - a^9*d^6*x^5*e^14 - (a^8*c*d^7*x^6 + 2*a^9*d
^7*x^4)*e^13 + (a^8*c*d^8*x^5 - a^9*d^8*x^3)*e^12 + (3*a^7*c^2*d^9*x^6 + 5*
a^8*c*d^9*x^4)*e^11 + 3*(a^7*c^2*d^10*x^5 + a^8*c*d^10*x^3)*e^10 - 3*(a^6*c
^3*d^11*x^6 + a^7*c^2*d^11*x^4)*e^9 - (5*a^6*c^3*d^12*x^5 + 3*a^7*c^2*d^12*
x^3)*e^8 + (a^5*c^4*d^13*x^6 - a^6*c^3*d^13*x^4)*e^7 + (2*a^5*c^4*d^14*x^5
+ a^6*c^3*d^14*x^3)*e^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
[Out] Integral(1/(x**4*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
[Out] integrate(1/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(x*e + d)*x^4),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
[Out] int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

3.487 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=259

$$\frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{8\left(ade\left(cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(c^3d^6+a^2cd^2e^4-2\right.\right.}{15e\left(cd^2-ae^2\right)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}$$

```
[Out] 2/5*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8/15
*(a*d*e*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)+(-2*a^3*e^6+a^2*c*d^2*e^4+c^3*d^6)*x
)/e/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+8/15*(5*a^2*e^
4+10*a*c*d^2*e^2+c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {868, 791, 627}

$$\frac{8\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8\left(x\left(-2a^3e^6+a^2cd^2e^4+c^3d^6\right)+ade\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)}{15e\left(cd^2-ae^2\right)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{2x^2}{5(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*x^2)/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)) - (8*(a*d*e*(c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (c^3*d^6 + a^2*c*d
^2*e^4 - 2*a^3*e^6)*x))/(15*e*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2)) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*
e^2 + 2*c*d*e*x))/(15*e*(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 868

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(2*c*d - b*e))*(f + g*x)^n*((a + b*x
 + c*x^2)^(p + 1)/(e*p*(b^2 - 4*a*c)*(d + e*x))), x] - Dist[1/(d*e*p*(b^2 -
 4*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f
*(2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p +
1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && I
GtQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} +$$

$$= \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} -$$

$$= \frac{2x^2}{5\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} -$$

Mathematica [**A**]
time = 0.22, size = 235, normalized size = 0.91

$$\frac{2\left(c^4d^6x^2\left(15d^2+20dex+8e^2x^2\right)+a^4e^6\left(8d^2+20dex+15e^2x^2\right)+4a^3cde^4\left(20d^3+53d^2ex+45de^2x^2+15e^3x^3\right)+4ac^3d^4ex\left(15d^3+45d^2ex+53de^2x^2+20e^3x^3\right)+2a^2c^2d^2e^2\left(20d^4+110d^3ex+189d^2e^2x^2+110de^3x^3+20e^4x^4\right)\right)}{15\left(cd^2-ae^2\right)^5(d+ex)((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

```
[Out] (2*(c^4*d^6*x^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2) + a^4*e^6*(8*d^2 + 20*d*e*x
 + 15*e^2*x^2) + 4*a^3*c*d*e^4*(20*d^3 + 53*d^2*e*x + 45*d*e^2*x^2 + 15*e^3
*x^3) + 4*a*c^3*d^4*e*x*(15*d^3 + 45*d^2*e*x + 53*d*e^2*x^2 + 20*e^3*x^3) +
 2*a^2*c^2*d^2*e^2*(20*d^4 + 110*d^3*e*x + 189*d^2*e^2*x^2 + 110*d*e^3*x^3
+ 20*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d + e*x))^(
3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(247) = 494.
time = 0.09, size = 621, normalized size = 2.40

method	result

gosper	$-\frac{2(cdx+ae)(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+180a}{15(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2}$
trager	$-\frac{2(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+180a^3c\,d^2e^6x^2}{15(a^4e^8-4a^3c\,d}$
default	$\frac{-\frac{1}{3cde\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}-\frac{\left(a\,e^2+c\,d^2\right)\left(\frac{\frac{4}{3}cdex+\frac{2}{3}a\,e^2+\frac{2}{3}c\,d^2}{\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}+\frac{16cde}{3\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)^2\sqrt{}}\right)}{2cde}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-1/3/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/2*(a*e^2+c*d^2)/c
/d/e*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+16/3*c*d*e/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2
*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)))-d/e^2*(2
/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)+16/3*c*d*e/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d
*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/e^3*d^2*(-2/5/
(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-8/5*c*d
*e/(a*e^2-c*d^2)*(-2/3*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(a*e^2-c*d^2)^2/(c*d*e
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+16/3*c*d*e/(a*e^2-c*d^2)^4*(2*c*d*e
*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(241) = 482$.

time = 53.61, size = 841, normalized size = 3.25

$$\frac{2\left(15c^4d^8x^2+15a^4x^2e^8+20\left(3a^3cdx^3+a^4dx\right)e^7+4\left(10a^2c^2d^2x^4+45a^3cd^2x^2+2a^4d^2\right)e^6+4\left(55a^2c^2d^3x^3+53a^3cd^3x\right)e^5+2\left(40ac^3d^4x^4+189a^2c^2d^4x^2+40a^3cd^4\right)e^4+4\left(53ac^3d^5x^3+55a^2c^2d^5x\right)e^3+4\left(2c^4d^6x^4+45ac^3d^6x^2+10a^2c^2d^6\right)e^2+20\left(c^4d^7x^3+3ac^3d^7x\right)e\right)\sqrt{cd^2x+axe^2+\left(cdx^2+ad\right)e}}{15\left(c^7d^{15}x^2-a^7x^3e^{15}-\left(2a^6cdx^4+3a^7dx^2\right)e^{14}-\left(a^5c^2d^2x^5+a^6cd^2x^3+3a^7d^2x\right)e^{13}+\left(7a^5c^2d^3x^4+9a^6cd^3x^2-a^7d^3\right)e^{12}+\left(5a^4c^3d^4x^5+17a^5c^2d^4x^3+13a^6cd^4x\right)e^{11}-\left(5a^4c^3d^5x^4+a^5c^2d^5x^2-5a^6cd^5\right)e^{10}-5\left(2a^3c^4d^6x^5+7a^4c^3d^6x^3+4a^5c^2d^6x\right)e^9-5\left(2a^3c^4d^7x^4+5a^4c^3d^7x^2+2a^5c^2d^7\right)e^8+5\left(2a^2c^5d^8x^5+5a^3c^4d^8x^3+2a^4c^3d^8x\right)e^7+5\left(4a^2c^5d^9x^4+7a^3c^4d^9x^2+2a^4c^3d^9\right)e^6-\left(5ac^6d^{10}x^5-a^2c^5d^{10}x^3-5a^3c^4d^{10}x\right)e^5-\left(13ac^6d^{11}x^4+17a^2c^5d^{11}x^2+5a^3c^4d^{11}\right)e^4+\left(c^7d^{12}x^5-9ac^6d^{12}x^3-7a^2c^5d^{12}x\right)e^3+\left(3c^7d^{13}x^4+ac^6d^{13}x^2+a^2c^5d^{13}\right)e^2+\left(3c^7d^{14}x^3+2ac^6d^{14}x\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"fricas")
```

```
[Out] 2/15*(15*c^4*d^8*x^2 + 15*a^4*x^2*e^8 + 20*(3*a^3*c*d*x^3 + a^4*d*x)*e^7 +
4*(10*a^2*c^2*d^2*x^4 + 45*a^3*c*d^2*x^2 + 2*a^4*d^2)*e^6 + 4*(55*a^2*c^2*d
^3*x^3 + 53*a^3*c*d^3*x)*e^5 + 2*(40*a*c^3*d^4*x^4 + 189*a^2*c^2*d^4*x^2 +
40*a^3*c*d^4)*e^4 + 4*(53*a*c^3*d^5*x^3 + 55*a^2*c^2*d^5*x)*e^3 + 4*(2*c^4*
d^6*x^4 + 45*a*c^3*d^6*x^2 + 10*a^2*c^2*d^6)*e^2 + 20*(c^4*d^7*x^3 + 3*a*c^
3*d^7*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)/(c^7*d^15*x^2 - a^7
*x^3*e^15 - (2*a^6*c*d*x^4 + 3*a^7*d*x^2)*e^14 - (a^5*c^2*d^2*x^5 + a^6*c*d
^2*x^3 + 3*a^7*d^2*x)*e^13 + (7*a^5*c^2*d^3*x^4 + 9*a^6*c*d^3*x^2 - a^7*d^3
)*e^12 + (5*a^4*c^3*d^4*x^5 + 17*a^5*c^2*d^4*x^3 + 13*a^6*c*d^4*x)*e^11 - (
5*a^4*c^3*d^5*x^4 + a^5*c^2*d^5*x^2 - 5*a^6*c*d^5)*e^10 - 5*(2*a^3*c^4*d^6*
x^5 + 7*a^4*c^3*d^6*x^3 + 4*a^5*c^2*d^6*x)*e^9 - 5*(2*a^3*c^4*d^7*x^4 + 5*a
^4*c^3*d^7*x^2 + 2*a^5*c^2*d^7)*e^8 + 5*(2*a^2*c^5*d^8*x^5 + 5*a^3*c^4*d^8*
x^3 + 2*a^4*c^3*d^8*x)*e^7 + 5*(4*a^2*c^5*d^9*x^4 + 7*a^3*c^4*d^9*x^2 + 2*a
^4*c^3*d^9)*e^6 - (5*a*c^6*d^10*x^5 - a^2*c^5*d^10*x^3 - 5*a^3*c^4*d^10*x)*
e^5 - (13*a*c^6*d^11*x^4 + 17*a^2*c^5*d^11*x^2 + 5*a^3*c^4*d^11)*e^4 + (c^7
*d^12*x^5 - 9*a*c^6*d^12*x^3 - 7*a^2*c^5*d^12*x)*e^3 + (3*c^7*d^13*x^4 + a*
c^6*d^13*x^2 + a^2*c^5*d^13)*e^2 + (3*c^7*d^14*x^3 + 2*a*c^6*d^14*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm=
"giac")
```

```
[Out] integrate(x^2/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(x*e + d)), x)
```

Mupad [B]
time = 4.33, size = 3099, normalized size = 11.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

```
[Out] (((6*a*e^2 - 10*c*d^2)/(15*(a*e^2 - c*d^2)^4) - (4*c*d^2)/(5*(a*e^2 - c*d^2
)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((e*(
2*a*e^3 - 2*c*d^2*e))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (4*c*d^
2*e^2)/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e + (e*(2*c*d^3 + 2*a*
d*e^2))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*
x^2)^(1/2)*(x*((((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2
*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^
2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(a*e^2 + c*d^2))/(c*d*e)
 - (6*c^2*d^2*e*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) + (8*a*c^3*d^4*e^3)/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^2*d^2*e*(46*a^2*e^4 + 4*c^2*d^4 +
 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5))) + (a*((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e - 2*a*c^2
*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^
2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - (c*d*(a*e^2 + c*d^2
)*(46*a^2*e^4 + 4*c^2*d^4 + 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5
*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d + e*x)) + ((x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*(((a*e^2 + c*d^2)*((4*c^4*d^4
*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^
3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d
^2)*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5))))/c + ((a*e^2 + c*d^2)*((a*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/
(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4
*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*
e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*((4*c^4*d^4*e^
3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2
*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c^2*d^2*e^2*(10*c^3*d
^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*(a*e^2 - c*d^2)^3*(c
^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3*e^2*(a*e^2 + c*d^2)
*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^
```

```
2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(12*a^3*e^5 - 36*a^2*c*d^2*e^3))/(15
*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (c*d*e*(a
*e^2 + c*d^2)*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 -
c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (8*a^3*c^
2*d^3*e^6)/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)
) - (c*d*e*(12*a^3*e^5 - 36*a^2*c*d^2*e^3)*(a*e^2 + c*d^2))/(15*(a*e^2 - c*
d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) + (a*((a*((4*c^4*d^4*e
^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^
2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*
d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c
*d^2)*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2
*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^4*d^4*e^3*(5*a*e^2 - c*d^2))/(15*(a*e
^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (2*c
^2*d^2*e^2*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d*e^4))/(15*(a*e^2 - c*d
^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^4*d^5*e^4)/(15*
(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^3*d^3
*e^2*(a*e^2 + c*d^2)*(5*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e -
2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(12*a^3*e^5 - 36
*a^2*c*d^2*e^3))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c
*d*e^5)) - (c*d*e*(a*e^2 + c*d^2)*(10*c^3*d^5 + 6*a*c^2*d^3*e^2 - 8*a^2*c*d
*e^4))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))
/c + (4*a^3*c*d^2*e^5*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)^2*(d + e*x)^2) - (2*d^2*e*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*x)^3*(5*a^3*e^7 - 5*c^
3*d^6*e + 15*a*c^2*d^4*e^3 - 15*a^2*c*d^2*e^5))
```

3.488 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}} \, dx$

Optimal. Leaf size=341

$$\frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}-\frac{8\left(2ade\left(cd^2+2ae^2\right)+\left(2c^2d^4+acd^2e^2+3a^2e^4\right)x\right)}{35e\left(cd^2-ae^2\right)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}+$$

```
[Out] 2/7*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-8/35
*(2*a*d*e*(2*a*e^2+c*d^2)+(3*a^2*e^4+a*c*d^2*e^2+2*c^2*d^4)*x)/e/(-a*e^2+c*
d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+16/105*(7*a^2*e^4+14*a*c*d^2
*e^2+3*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(3/2)-128/105*c*d*(7*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^4)*(2
*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)
```

Rubi [A]
time = 0.18, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {868, 791, 628, 627}

$$-\frac{128cd\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105\left(cd^2-ae^2\right)^7\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16\left(7a^2e^4+14acd^2e^2+3c^2d^4\right)\left(ae^2+cd^2+2cdex\right)}{105e\left(cd^2-ae^2\right)^5\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{8\left(x\left(3a^2e^4+acd^2e^2+2c^2d^4\right)+2ade\left(2ae^2+cd^2\right)\right)}{35e\left(cd^2-ae^2\right)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}+\frac{2x^2}{7(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (2*x^2)/(7*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)) - (8*(2*a*d*e*(c*d^2 + 2*a*e^2) + (2*c^2*d^4 + a*c*d^2*e^2 + 3*a^2
*e^4)*x))/(35*e*(c*d^2 - a*e^2)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)) + (16*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d
*e*x))/(105*e*(c*d^2 - a*e^2)^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2)) - (128*c*d*(3*c^2*d^4 + 14*a*c*d^2*e^2 + 7*a^2*e^4)*(c*d^2 + a*e^2 + 2*
c*d*e*x))/(105*(c*d^2 - a*e^2)^7*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 868

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(2*c*d - b*e))*(f + g*x)^n*((a + b*x
 + c*x^2)^(p + 1)/(e*p*(b^2 - 4*a*c)*(d + e*x))), x] - Dist[1/(d*e*p*(b^2 -
 4*a*c)), Int[(f + g*x)^(n - 1)*(a + b*x + c*x^2)^p*Simp[b*(a*e*g*n - c*d*f
*(2*p + 1)) - 2*a*c*(d*g*n - e*f*(2*p + 1)) - c*g*(b*d - 2*a*e)*(n + 2*p +
1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && I
GtQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}\,dx = \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} +$$
$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} -$$
$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} -$$
$$= \frac{2x^2}{7\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}} -$$

Mathematica [A]
time = 0.34, size = 440, normalized size = 1.29

$$-\frac{2\sqrt{(ae+cdx)(d+ex)}\left(-15d^2e^4(ae+cdx)^6+84cd^3e^3(ae+cdx)^5(d+ex)+42ade^5(ae+cdx)^5(d+ex)-210c^2d^4e^2(ae+cdx)^4(d+ex)^2-280acd^2e^4(ae+cdx)^4(d+ex)^2-35a^2e^6(ae+cdx)^4(d+ex)^2+420c^3d^5e(ae+cdx)^3(d+ex)^3+1260ac^2d^3e^3(ae+cdx)^3(d+ex)^3+420a^2cde^5(ae+cdx)^3(d+ex)^3+105c^4d^6(ae+cdx)^2(d+ex)^4+840ac^3d^4e^2(ae+cdx)^2(d+ex)^4+630a^2c^2d^2e^4(ae+cdx)^2(d+ex)^4-70ac^4d^5e(ae+cdx)(d+ex)^5-140a^2c^3d^3e^3(ae+cdx)(d+ex)^5+21a^2c^4d^4e^2(d+ex)^6\right)}{105\left(cd^2-ae^2\right)^7(ae+cdx)^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]
```

```
[Out] (-2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-15*d^2*e^4*(a*e + c*d*x)^6 + 84*c*d^3*e
^3*(a*e + c*d*x)^5*(d + e*x) + 42*a*d*e^5*(a*e + c*d*x)^5*(d + e*x) - 210*c
^2*d^4*e^2*(a*e + c*d*x)^4*(d + e*x)^2 - 280*a*c*d^2*e^4*(a*e + c*d*x)^4*(d
 + e*x)^2 - 35*a^2*e^6*(a*e + c*d*x)^4*(d + e*x)^2 + 420*c^3*d^5*e*(a*e + c
*d*x)^3*(d + e*x)^3 + 1260*a*c^2*d^3*e^3*(a*e + c*d*x)^3*(d + e*x)^3 + 420*
a^2*c*d*e^5*(a*e + c*d*x)^3*(d + e*x)^3 + 105*c^4*d^6*(a*e + c*d*x)^2*(d +
e*x)^4 + 840*a*c^3*d^4*e^2*(a*e + c*d*x)^2*(d + e*x)^4 + 630*a^2*c^2*d^2*e^
4*(a*e + c*d*x)^2*(d + e*x)^4 - 70*a*c^4*d^5*e*(a*e + c*d*x)*(d + e*x)^5 -
140*a^2*c^3*d^3*e^3*(a*e + c*d*x)*(d + e*x)^5 + 21*a^2*c^4*d^4*e^2*(d + e*x
)^6))/(105*(c*d^2 - a*e^2)^7*(a*e + c*d*x)^3*(d + e*x)^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(325) = 650$.
time = 0.09, size = 933, normalized size = 2.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-1/5/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-1/2*(a*e^2+c*d^2)/c
/d/e*(2/5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)+16/5*c*d*e/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)*(
2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(3/2)+16/3*c*d*e/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*
d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))))-d/e^2*(2/5*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(5/2)+16/5*c*d*e/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)*(2/3*(2*c*d*e
*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+16/3*c*d*e/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+
c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)))+1/e^3*d^2*(-2/7/(a*e^2-c*d
^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-12/7*c*d*e/(a*e^2
-c*d^2)*(-2/5*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(a*e^2-c*d^2)^2/(c*d*e*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-16/5*c*d*e/(a*e^2-c*d^2)^2*(-2/3*(2*c*d*e*(x
+d/e)+a*e^2-c*d^2)/(a*e^2-c*d^2)^2/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(3/2)+16/3*c*d*e/(a*e^2-c*d^2)^4*(2*c*d*e*(x+d/e)+a*e^2-c*d^2)/(c*d*e*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e^2*a>0)', see `assume?` for
 more d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(325) = 650$.
time = 99.68, size = 1540, normalized size = 4.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"fricas")
```

```
[Out] -2/105*(56*a^2*c^4*d^10*e^2 + 1120*a^3*c^3*d^8*e^4 + 1680*a^4*c^2*d^6*e^6 +
 224*a^5*c*d^4*e^8 - 8*a^6*d^2*e^10 + 128*(3*c^6*d^8*e^4 + 14*a*c^5*d^6*e^6
 + 7*a^2*c^4*d^4*e^8)*x^6 + 64*(21*c^6*d^9*e^3 + 113*a*c^5*d^7*e^5 + 119*a^
2*c^4*d^5*e^7 + 35*a^3*c^3*d^3*e^9)*x^5 + 80*(21*c^6*d^10*e^2 + 140*a*c^5*d
^8*e^4 + 254*a^2*c^4*d^6*e^6 + 140*a^3*c^3*d^4*e^8 + 21*a^4*c^2*d^2*e^10)*x
^4 + 40*(21*c^6*d^11*e + 203*a*c^5*d^9*e^3 + 602*a^2*c^4*d^7*e^5 + 542*a^3*
c^3*d^5*e^7 + 161*a^4*c^2*d^3*e^9 + 7*a^5*c*d*e^11)*x^3 + 5*(21*c^6*d^12 +
518*a*c^5*d^10*e^2 + 2639*a^2*c^4*d^8*e^4 + 4004*a^3*c^3*d^6*e^6 + 1859*a^4
*c^2*d^4*e^8 + 182*a^5*c*d^2*e^10 - 7*a^6*e^12)*x^2 + 4*(35*a*c^5*d^11*e +
749*a^2*c^4*d^9*e^3 + 2030*a^3*c^3*d^7*e^5 + 1610*a^4*c^2*d^5*e^7 + 191*a^5
*c*d^3*e^9 - 7*a^6*d*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(
a^3*c^7*d^18*e^3 - 7*a^4*c^6*d^16*e^5 + 21*a^5*c^5*d^14*e^7 - 35*a^6*c^4*d^
12*e^9 + 35*a^7*c^3*d^10*e^11 - 21*a^8*c^2*d^8*e^13 + 7*a^9*c*d^6*e^15 - a^
10*d^4*e^17 + (c^10*d^17*e^4 - 7*a*c^9*d^15*e^6 + 21*a^2*c^8*d^13*e^8 - 35*
a^3*c^7*d^11*e^10 + 35*a^4*c^6*d^9*e^12 - 21*a^5*c^5*d^7*e^14 + 7*a^6*c^4*d
^5*e^16 - a^7*c^3*d^3*e^18)*x^7 + (4*c^10*d^18*e^3 - 25*a*c^9*d^16*e^5 + 63
*a^2*c^8*d^14*e^7 - 77*a^3*c^7*d^12*e^9 + 35*a^4*c^6*d^10*e^11 + 21*a^5*c^5
*d^8*e^13 - 35*a^6*c^4*d^6*e^15 + 17*a^7*c^3*d^4*e^17 - 3*a^8*c^2*d^2*e^19)
*x^6 + 3*(2*c^10*d^19*e^2 - 10*a*c^9*d^17*e^4 + 15*a^2*c^8*d^15*e^6 + 7*a^3
*c^7*d^13*e^8 - 49*a^4*c^6*d^11*e^10 + 63*a^5*c^5*d^9*e^12 - 35*a^6*c^4*d^7
*e^14 + 5*a^7*c^3*d^5*e^16 + 3*a^8*c^2*d^3*e^18 - a^9*c*d*e^20)*x^5 + (4*c^
10*d^20*e - 10*a*c^9*d^18*e^3 - 30*a^2*c^8*d^16*e^5 + 155*a^3*c^7*d^14*e^7
- 245*a^4*c^6*d^12*e^9 + 147*a^5*c^5*d^10*e^11 + 35*a^6*c^4*d^8*e^13 - 95*a
^7*c^3*d^6*e^15 + 45*a^8*c^2*d^4*e^17 - 5*a^9*c*d^2*e^19 - a^10*e^21)*x^4 +
 (c^10*d^21 + 5*a*c^9*d^19*e^2 - 45*a^2*c^8*d^17*e^4 + 95*a^3*c^7*d^15*e^6
- 35*a^4*c^6*d^13*e^8 - 147*a^5*c^5*d^11*e^10 + 245*a^6*c^4*d^9*e^12 - 155*
a^7*c^3*d^7*e^14 + 30*a^8*c^2*d^5*e^16 + 10*a^9*c*d^3*e^18 - 4*a^10*d*e^20)
*x^3 + 3*(a*c^9*d^20*e - 3*a^2*c^8*d^18*e^3 - 5*a^3*c^7*d^16*e^5 + 35*a^4*c
```

```
^6*d^14*e^7 - 63*a^5*c^5*d^12*e^9 + 49*a^6*c^4*d^10*e^11 - 7*a^7*c^3*d^8*e^
13 - 15*a^8*c^2*d^6*e^15 + 10*a^9*c*d^4*e^17 - 2*a^10*d^2*e^19)*x^2 + (3*a^
2*c^8*d^19*e^2 - 17*a^3*c^7*d^17*e^4 + 35*a^4*c^6*d^15*e^6 - 21*a^5*c^5*d^1
3*e^8 - 35*a^6*c^4*d^11*e^10 + 77*a^7*c^3*d^9*e^12 - 63*a^8*c^2*d^7*e^14 +
25*a^9*c*d^5*e^16 - 4*a^10*d^3*e^18)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x, algorithm=
"giac")
```

```
[Out] integrate(x^2/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(7/2)*(x*e + d)), x)
```

Mupad [**B**]
time = 7.72, size = 2500, normalized size = 7.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(7/2)),x)
```

```
[Out] ((6*c^3*d^5 + 36*a*c^2*d^3*e^2 - 10*a^2*c*d*e^4)/(105*(a*e^2 - c*d^2)^6) -
x*((16*c^2*d^2*e)/(105*(a*e^2 - c*d^2)^5) - (8*c^2*d^2*e*(a*e^2 + c*d^2))/(
105*(a*e^2 - c*d^2)^6)) + (8*a*c^2*d^3*e^2)/(105*(a*e^2 - c*d^2)^6))/(x*(a*
e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + (x*((a*((64*c^5*d^5*e^4*(a*e^2 +
c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))
 - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e -
 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d^2)*(
(64*c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^
2*d^3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^
2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c
^4*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6
```

```
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*e^5)/(105*(a
*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (32*c^4*d^4*
e^3*(a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(60*c^4*d^7 -
204*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/(105*(a*e^2 - c*
d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (16*c^3*d^3*e^2*(a*e^
2 + c*d^2)*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*(((a*e^2 + c*d^2)*((64*
c^5*d^5*e^4*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^
3*e^3 + a^2*c*d*e^5)) - (64*c^5*d^5*e^4*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 -
c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (32*c^4*d
^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 18*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6*(c^
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (128*a*c^5*d^6*e^5)/(105*(a*e^2
 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (32*c^4*d^4*e^3*
(a*e^2 + c*d^2)*(5*a*e^2 - 3*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*
a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c*d*e*(a*e^2 + c*d^2)*(60*c^4*d^7 - 20
4*a*c^3*d^5*e^2 - 156*a^2*c^2*d^3*e^4 + 44*a^3*c*d*e^6))/(105*(a*e^2 - c*d^
2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))/(x*(a*e^2 + c*d^2) + a*d
*e + c*d*e*x^2)^(1/2) + (x*((a*((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2
 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6)))/
c + (36*c^4*d^7*e - 76*a*c^3*d^5*e^3 - 36*a^2*c^2*d^3*e^5 + 12*a^3*c*d*e^7)
/(105*e*(a*e^2 - c*d^2)^6) + ((a*e^2 + c*d^2)*((8*a*c^3*d^4*e^3)/(105*(a*e^
2 - c*d^2)^6) - (((8*c^3*d^3*e^2*(a*e^2 + c*d^2))/(105*(a*e^2 - c*d^2)^6) -
 (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 - c*d^2)^6))*(a*e^2 + c*d^2)
)/(c*d*e) + (2*c^2*d^2*e*(11*c^2*d^4 - 13*a^2*e^4 + 14*a*c*d^2*e^2))/(105*(
a*e^2 - c*d^2)^6)))/(c*d*e)) + (30*a^4*e^8 - 22*c^4*d^8 + 20*a*c^3*d^6*e^2
- 132*a^3*c*d^2*e^6 + 72*a^2*c^2*d^4*e^4)/(105*e*(a*e^2 - c*d^2)^6) + (a*((
8*a*c^3*d^4*e^3)/(105*(a*e^2 - c*d^2)^6) - (((8*c^3*d^3*e^2*(a*e^2 + c*d^2)
)/(105*(a*e^2 - c*d^2)^6) - (8*c^3*d^3*e^2*(3*a*e^2 - c*d^2))/(105*(a*e^2 -
 c*d^2)^6))*(a*e^2 + c*d^2))/(c*d*e) + (2*c^2*d^2*e*(11*c^2*d^4 - 13*a^2*e^
4 + 14*a*c*d^2*e^2))/(105*(a*e^2 - c*d^2)^6)))/c)/(x*(a*e^2 + c*d^2) + a*d*
e + c*d*e*x^2)^(3/2) - (((d*((e*(2*a*e^4 - 2*c*d^2*e^2))/(7*(a*e^2 - c*d^2)
^4*(5*a*e^3 - 5*c*d^2*e)) - (4*c*d^2*e^3)/(7*(a*e^2 - c*d^2)^4*(5*a*e^3 - 5
*c*d^2*e))))/e + (e*(2*a*d*e^3 + 2*c*d^3*e))/(7*(a*e^2 - c*d^2)^4*(5*a*e^3
- 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 +
 (((e*(10*a*e^3 - 14*c*d^2*e))/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e))
 - (4*c*d^2*e^2)/(7*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c
*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*((a*((a*((4*c^5*d^5*e^4
*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*
c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*(((a*e^2 + c*d
^2)*((4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a
*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2
 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (4*c^4
*d^4*e^3*(7*c^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^
```

```
3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (8*a*c^5*d^6*e^5)/(35*(a*e^2 -
c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2*c^4*d^4*e^3*(a*e
^2 + c*d^2)*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(14*c^4*d^7 - 56*a*c^3*d^
5*e^2 - 12*a^2*c^2*d^3*e^4 + 10*a^3*c*d*e^6))/(35*(a*e^2 - c*d^2)^4*(c^3*d^
5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^3*d^3*e^2*(a*e^2 + c*d^2)*(7*c
^2*d^4 - 9*a^2*e^4 + 4*a*c*d^2*e^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a
*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - ((a*e^2 + c*d^2)*((a*(((a*e^2 + c*d^2)*(
(4*c^5*d^5*e^4*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*
d^3*e^3 + a^2*c*d*e^5)) - (4*c^5*d^5*e^4*(7*a*e^2 - c*d^2))/(35*(a*e^2 - c*
d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e...
```

3.489 $\int x^3\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=170

$$\frac{6}{55}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{11}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\,\sqrt{\frac{1-\cdots}{(1+\cdots}}}{55\sqrt{\frac{1+x}{(1+\sqrt{3}+\cdots}}}$$

```
[Out] 6/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)-4/5
5*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*
(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)
/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {929, 285, 327, 224}

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{6}{55}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x+\frac{2}{11}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (6*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/11 - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[
(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 +
x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int x^3\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx=\frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x^3\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$

$$=\frac{2}{11}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int\frac{x^3}{\sqrt{1+x^3}}\,dx}{11\sqrt{1+x^3}}$$

$$=\frac{6}{55}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{11}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}-\frac{\left(6\sqrt{1+x}\,\sqrt{1-\right.}{5}$$

$$=\frac{6}{55}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{11}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 40.63, size = 221, normalized size = 1.30

$$\frac{2\left(x\sqrt{1+x}\,(3-3x+3x^2+5x^3-5x^4+5x^5)+\sqrt{-\frac{6i}{3i+\sqrt{3}}}\,\left(3i+\sqrt{3}\right)(1+x)\sqrt{\frac{3i+\sqrt{3}+\left(-3i+\sqrt{3}\right)x}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{-3i+\sqrt{3}+\left(3i+\sqrt{3}\right)x}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\right)}{55\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*(x*Sqrt[1 + x]*(3 - 3*x + 3*x^2 + 5*x^3 - 5*x^4 + 5*x^5) + Sqrt[(-6*I)/(
3*I + Sqrt[3])]*(3*I + Sqrt[3])*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I + Sqrt[
3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt[3])*x
)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3]
)]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(55*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.24, size = 257, normalized size = 1.51

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^4\sqrt{x^3+1}}{11}+\frac{6x\sqrt{x^3+1}}{55}-\frac{12\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+}{-\frac{3}{2}+}}}{55\sqrt{x^3+}}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x(5x^3+3)\sqrt{1+x}\sqrt{x^2-x+1}}{55}-\frac{12\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\right.}{55\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2}}$
default	$\frac{2\sqrt{1+x}\sqrt{x^2-x+1}\left(5x^7+3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x}{-3+i\sqrt{}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/55*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(5*x^7+3*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)
))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3
^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(
I*3^(1/2)+3))^(1/2))-9*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(
I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*
(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+8*x^4+3*
x)/(x^3+1)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 33, normalized size = 0.19

$$\frac{2}{55}\left(5\,x^{4}+3\,x\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}-\frac{12}{55}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/55*(5*x^4 + 3*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 12/55*weierstrassPInvers
e(0, -4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
[Out] Integral(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3}\,\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
[Out] int(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.490 $\int x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=23

$$\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

```
[Out] 2/9*(1+x)^(3/2)*(x^2-x+1)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{927\}$

$$\frac{2}{9}(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx=\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

Mathematica [A]
time = 10.04, size = 23, normalized size = 1.00

$$\frac{2}{9}(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
[Out] (2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))/9
```

Maple [A]
time = 0.10, size = 23, normalized size = 1.00

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}}{9}$	18
default	$\frac{2\left(x^{3}+1\right)\sqrt{1+x}\,\sqrt{x^{2}-x+1}}{9}$	23
risch	$\frac{2\left(x^{3}+1\right)\sqrt{1+x}\,\sqrt{x^{2}-x+1}}{9}$	23
elliptic	$\frac{\sqrt{(1+x)\left(x^{2}-x+1\right)}\,\left(\frac{2x^{3}\sqrt{x^{3}+1}}{9}+\frac{2\sqrt{x^{3}+1}}{9}\right)}{\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

Maxima [A]
time = 0.86, size = 22, normalized size = 0.96

$$\frac{2}{9}\left(x^{3}+1\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Fricas [A]
time = 3.11, size = 22, normalized size = 0.96

$$\frac{2}{9}\left(x^{3}+1\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{x+1}\,\sqrt{x^{2}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.
time = 0.99, size = 67, normalized size = 2.91

$$\frac{2}{315}\left((5(7x-23)(x+1)+258)(x+1)-213\right)\sqrt{(x+1)^2-3x}\,\sqrt{x+1}+\frac{2}{105}\left(3(5x-12)(x+1)+71\right)\sqrt{(x+1)^2-3x}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*((5*(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sq
rt(x + 1) + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x
+ 1)
```

Mupad [B]
time = 2.62, size = 22, normalized size = 0.96

$$\frac{2\left(x^3+1\right)\sqrt{x+1}\,\sqrt{x^2-x+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

```
[Out] (2*(x^3 + 1)*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/9
```

3.491 $\int x\sqrt{1+x}\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=294

$$\frac{2}{7}x^2\sqrt{1+x}\sqrt{1-x+x^2}+\frac{6\sqrt{1+x}\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)^{3/2}\sqrt{1-x+x^2}\sqrt{\frac{1-x+\cdots}{\left(1+\sqrt{3}+\cdots\right.}}}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\cdots}$$

```
[Out] 2/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+6/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^
(1/2))+2/7*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1
/2)+2*I)*2^(1/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)-3/7*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2
))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {823, 285, 309, 224, 1891}

$$\frac{2\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}(x^3+1)}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}(x^3+1)}+\frac{2}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2+\frac{6\sqrt{x+1}\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (6*Sqrt[1 + x]*Sqrt[1 - x + x^2])
/(7*(1 + Sqrt[3] + x)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1
- x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3
] + x)^2]*(1 + x^3)) + (2*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1
 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\sqrt{1+x}\,\sqrt{1-x+x^2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x\sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}}$$

$$= \frac{2}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{7\sqrt{1+x^3}} + \frac{(3 \cdots}{\cdots}$$

$$= \frac{2}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{6\sqrt{1+x}\,\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1 - \cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.34, size = 347, normalized size = 1.18

$$\frac{\sqrt{1+x}\left(4x^2\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\,(1-x+x^2)-3\sqrt{2}\left(-3i+\sqrt{3}\right)\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)+3\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\right)}{14\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\,\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (Sqrt[1 + x]*(4*x^2*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]*(1 - x + x^2) - 3*
Sqrt[2]*(-3*I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt
[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])] + 3*Sq
rt[2]*(-I + Sqrt[3])*Sqrt[(I + Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I
 + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((
-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/(14*Sqrt
[((-I)*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.12, size = 361, normalized size = 1.23

method	result

elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2\sqrt{x^3+1}}{7}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\right.\right.}{}\right.}{\sqrt{1+x}\sqrt{x^2-}}$
risch	$\frac{2x^2\sqrt{1+x}\sqrt{x^2-x+1}}{7}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{Ellip}\right.}{}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2
)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2))
)^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/
2)+3))^(1/2))+2*x^5+9*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I
*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(
1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-18*(-2*(1
+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/
2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(
-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+2*x^2)/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 30, normalized size = 0.10

$$\frac{2}{7}\sqrt{x^2-x+1}\,\sqrt{x+1}\,x^2-\frac{6}{7}\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2 - 6/7*weierstrassZeta(0, -4, weierstr
assPInverse(0, -4, x))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
```

```
[Out] int(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.492 $\int \sqrt{1+x}\,\sqrt{1-x+x^2}\,dx$

Optimal. Leaf size=144

$$\frac{2}{5}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+}{1+\sqrt{3}+}\right.\right.}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

```
[Out] 2/5*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/5*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^
(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2
))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {727, 201, 224}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{2}{5}x\sqrt{x^2-x+1}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x
)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{1+x}\,\sqrt{1-x+x^2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{5}x\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{5\sqrt{1+x^3}}$$

$$= \frac{2}{5}x\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\,\sqrt{\frac{1-\cdots}{\left(1+\cdots\right.}}}{5\sqrt{\frac{1+x}{\left(1+\sqrt{3}+\cdots\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.02, size = 169, normalized size = 1.17

$$\frac{2x\sqrt{1+x}\,(1-x+x^2)+\frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\,\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{5\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2) + (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3]
)*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[
```

```
Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]
)/Sqrt[(-I)/(3*I + Sqrt[3])]])/(5*Sqrt[1 - x + x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(117) = 234$.
time = 0.11, size = 252, normalized size = 1.75

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\cdots\right.}{5\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$\frac{2x\sqrt{1+x}\,\sqrt{x^2-x+1}}{5}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots\right.}{5\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+\cdots}}$
default	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/
2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)
))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1
/2)+3))^(1/2))-9*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1
/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/
(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x^4-2*x)/(x^
3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 25, normalized size = 0.17

$$\frac{2}{5}\sqrt{x^2-x+1}\,\sqrt{x+1}\,x+\frac{6}{5}\,\text{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/5*sqrt(x^2 - x + 1)*sqrt(x + 1)*x + 6/5*weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2),x)
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{x+1}\,\sqrt{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2),x)
[Out] int((x + 1)^(1/2)*(x^2 - x + 1)^(1/2), x)
```

3.493 $\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x}\,dx$

Optimal. Leaf size=66

$$\frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}-\frac{2\sqrt{1+x}\,\sqrt{1-x+x^2}\,\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2
-x+1)^(1/2)/(x^3+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {929, 272, 52, 65, 213}

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcT
anh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x^3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x}\,dx &= \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x}\,dx}{\sqrt{1+x^3}}\\
&= \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x,\right.}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{\left(2\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x,\right.}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2} - \frac{2\sqrt{1+x}\,\sqrt{1-x+x^2}\,\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}
\end{aligned}
$$

Mathematica [A]
time = 15.33, size = 48, normalized size = 0.73

$$
\frac{2}{3}\left(\sqrt{1+x}\,\sqrt{1-x+x^2} - \tanh^{-1}\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x,x]
```

```
[Out] (2*(Sqrt[1 + x]*Sqrt[1 - x + x^2] - ArcTanh[Sqrt[1 + x]*Sqrt[1 - x + x^2]])
)/3
```

Maple [A]
time = 0.11, size = 43, normalized size = 0.65

method	result	size
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\,\left(-\sqrt{x^3+1}+\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\right)}{3\sqrt{x^3+1}}$	43
elliptic	$\frac{\sqrt{(1+x)\,(x^2-x+1)}\,\left(\frac{2\sqrt{x^3+1}}{3}-\frac{2\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-(x^3+1)^(1/2)+arctanh((x^3+1)^(1/2)))/(x
^3+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

Fricas [A]
time = 2.01, size = 60, normalized size = 0.91

$$\frac{2}{3}\sqrt{x^2-x+1}\,\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) +
 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x,x)
```

```
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x, x)
```

3.494 $\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x^2}\,dx$

Optimal. Leaf size=287

$$-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x}+\frac{3\sqrt{1+x}\,\sqrt{1-x+x^2}}{1+\sqrt{3}+x}-\frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\cdots}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\cdots}$$

```
[Out] -(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+3*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+
3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^
(1/2)*(x^2-x+1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x
+3^(1/2))^2)^(1/2)-3/2*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(
1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {929, 283, 309, 224, 1891}

$$\frac{\sqrt{2}\,3^{3/4}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}-\frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}-\frac{\sqrt{x^2-x+1}\,\sqrt{x+1}}{x}+\frac{3\sqrt{x^2-x+1}\,\sqrt{x+1}}{x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + x]*Sqrt[1 - x + x^2])/(1
 + Sqrt[3] + x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*(1 + x^3)) + (Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x^2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x^2}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x} + \frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x} + \frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x^3}} + \frac{\Big(3\cdots}{\cdots}$$

$$= -\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+x}\,\sqrt{1-x+x^2}}{1+\sqrt{3}+x} - \frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.26, size = 349, normalized size = 1.22

$$-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x} + \frac{3\sqrt{1+\frac{2i(1+x)}{-3i+\sqrt{3}}}\,\sqrt{1-\frac{2i(1+x)}{3i+\sqrt{3}}}\left(-\frac{\left(-3i+\sqrt{3}\right)\sqrt{-\frac{i}{3i+\sqrt{3}}}\,\sqrt{1+x}\,E\left(i\sinh^{-1}\left(\sqrt{2}\,\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}} + \frac{\left(-i+\sqrt{3}\right)\sqrt{-\frac{i}{3i+\sqrt{3}}}\,\sqrt{1+x}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right)}{2\sqrt{2}\,\sqrt{-\frac{i}{3i+\sqrt{3}}}\,\sqrt{3-3(1+x)+(1+x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + ((2*I)*(1 + x))/(-3*I +
Sqrt[3])]*Sqrt[1 - ((2*I)*(1 + x))/(3*I + Sqrt[3])]*(-(((-3*I + Sqrt[3])*Sq
rt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 + x]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((-I)
*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(
1 + x))/(3*I + Sqrt[3])]) + ((-I + Sqrt[3])*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt
[1 + x]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]],
(3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(2
*Sqrt[2]*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

Maple [A]
time = 0.12, size = 363, normalized size = 1.26

method	result

elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{}}{2}\right.\right.}{}\right.}{\sqrt{1+x}\sqrt{x^2-x}}$
risch	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}}{x}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{Elliptic}\right.}{}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2
)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2))
)^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/
2)+3))^(1/2))*x-18*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^
(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x
)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x+9*(-2*(1+x
)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)
+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(
-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/x/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 32, normalized size = 0.11

$$-\frac{3\,x\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))+\sqrt{x^2-x+1}\,\sqrt{x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(3*x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^2 - x
+ 1)*sqrt(x + 1))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2,x)
```

```
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^2, x)
```

3.495 $\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x^3}\,dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{2x^2}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|\right.}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,(1+x^3)}$$

```
[Out] -1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2+1/2*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x
-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(
1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A]

time = 0.03, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {929, 283, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

```
[Out] -1/2*(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2 + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 +
x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elliptic
F[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x^3}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{\sqrt{1+x^3}}{x^3}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{2x^2} + \frac{\left(3\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{4\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\,\sqrt{\frac{1-\cdots}{(1+\cdots}}}{2\sqrt{\frac{1+x}{(1+\sqrt{3}+\cdots}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.30, size = 185, normalized size = 1.27

$$\frac{\sqrt{1+x}\left(-\frac{2(1-x+x^2)}{x^2}-\frac{3i\sqrt{2}\,\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\,\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right)}{4\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^3,x]
```

```
[Out] (Sqrt[1 + x]*((-2*(1 - x + x^2))/x^2 - ((3*I)*Sqrt[2]*Sqrt[(I + Sqrt[3] - (
2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I + Sqrt[3])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqr
t[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))/(4*Sqrt[1 -
x + x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(119) = 238$.
time = 0.12, size = 259, normalized size = 1.77

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}{2\sqrt{x^3+1}}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}}{2x^2}+\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.}{2\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/
2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)
))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1
/2)+3))^(1/2))*x^2-9*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*
3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1
+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2+2*x^3+
2)/(x^3+1)/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 32, normalized size = 0.22

$$\frac{3\,x^2\mathrm{weierstrassPInverse}(0,-4,x)-\sqrt{x^2-x+1}\,\sqrt{x+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*x^2*weierstrassPInverse(0, -4, x) - sqrt(x^2 - x + 1)*sqrt(x + 1))/x
^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3,x)
```

```
[Out] int(((x + 1)^(1/2)*(x^2 - x + 1)^(1/2))/x^3, x)
```

3.496 $\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=201

$$\frac{54}{935}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{18}{187}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{17}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)-\frac{36\;3^{3/4}\sqrt{2}}{}$$

```
[Out] 54/935*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+18/187*x^4*(1+x)^(1/2)*(x^2-x+1)^(1/2)
+2/17*x^4*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-36/935*3^(3/4)*(1+x)^(3/2)*El
lipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(
1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(
1/2))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {929, 285, 327, 224}

$$-\frac{36\;3^{3/4}\sqrt{2+\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{935\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{54}{935}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x+\frac{18}{187}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^4+\frac{2}{17}\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)\,x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (54*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/935 + (18*x^4*Sqrt[1 + x]*Sqrt[1 - x +
 x^2])/187 + (2*x^4*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/17 - (36*3^(3/
4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(935*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x^3\left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{17}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right) + \frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x^3$$

$$= \frac{18}{187}x^4\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right) +$$

$$= \frac{54}{935}x\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{18}{187}x^4\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1$$

$$= \frac{54}{935}x\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{18}{187}x^4\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2}{17}x^4\sqrt{1$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.59, size = 235, normalized size = 1.17

$$\frac{2\left(x\sqrt{1+x}\,(27-27x+27x^2+100x^3-100x^4+100x^5+55x^6-55x^7+55x^8)-\frac{9i\sqrt{6}\,(1+x)\sqrt{\frac{3i+\sqrt{3}+\left(-3i+\sqrt{3}\right)x}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{\frac{-3i+\sqrt{3}+\left(3i+\sqrt{3}\right)x}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}\right)}{935\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (2*(x*Sqrt[1 + x]*(27 - 27*x + 27*x^2 + 100*x^3 - 100*x^4 + 100*x^5 + 55*x^
6 - 55*x^7 + 55*x^8) - ((9*I)*Sqrt[6]*(1 + x)*Sqrt[(3*I + Sqrt[3] + (-3*I +
 Sqrt[3])*x)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[(-3*I + Sqrt[3] + (3*I + Sqrt
[3])*x)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + S
qrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + S
qrt[3])]))/(935*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.11, size = 262, normalized size = 1.30

method	result
risch	$\frac{2x(55x^6+100x^3+27)\sqrt{1+x}\,\sqrt{x^2-x+1}}{935}-\frac{108\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{Elli}}{935\sqrt{x^3+1}\ \sqrt{1+}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^7\sqrt{x^3+1}}{17}+\frac{40x^4\sqrt{x^3+1}}{187}+\frac{54x\sqrt{x^3+1}}{935}-\frac{108\left(\frac{3}{2}-\frac{i\sqrt{}}{}\right.}{}\right.}{x^3+1}$
default	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\left(55x^{10}+155x^7+27i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\ \mathrm{EllipticF}\left(\sqrt{-}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/935*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(55*x^10+155*x^7+27*I*3^(1/2)*(-2*(1+x)/(
-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*
```

```
x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+
I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-81*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(
1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*E
llipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1
/2))+127*x^4+27*x)/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 38, normalized size = 0.19

$$\frac{2}{935}\left(55\,x^{7}+100\,x^{4}+27\,x\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}-\frac{108}{935}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/935*(55*x^7 + 100*x^4 + 27*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 108/935*wei
erstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}(x+1)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(x+1)^{3/2}\,\left(x^2-x+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
[Out] int(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.497 $\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

```
[Out] 2/15*(1+x)^(5/2)*(x^2-x+1)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {927}

$$\frac{2}{15}(x+1)^{5/2}\left(x^2-x+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

Mathematica [A]
time = 10.04, size = 23, normalized size = 1.00

$$\frac{2}{15}(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
[Out] (2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2))/15
```

Maple [A]
time = 0.11, size = 28, normalized size = 1.22

method	result	size
gosper	$\frac{2(1+x)^{\frac{5}{2}}\left(x^{2}-x+1\right)^{\frac{5}{2}}}{15}$	18
default	$\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\left(x^{6}+2x^{3}+1\right)}{15}$	28
risch	$\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\left(x^{6}+2x^{3}+1\right)}{15}$	28
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\sqrt{(1+x)\left(x^{2}-x+1\right)}\,\left(\frac{2x^{6}\sqrt{x^{3}+1}}{15}+\frac{4x^{3}\sqrt{x^{3}+1}}{15}+\frac{2\sqrt{x^{3}+1}}{15}\right)}{x^{3}+1}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(x^6+2*x^3+1)
```

Maxima [A]
time = 0.50, size = 27, normalized size = 1.17

$$\frac{2}{15}\left(x^{6}+2\,x^{3}+1\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Fricas [A]
time = 2.17, size = 27, normalized size = 1.17

$$\frac{2}{15}\left(x^{6}+2\,x^{3}+1\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] 2/15*(x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}(x+1)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)

[Out] Integral(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(17) = 34$.
time = 1.12, size = 173, normalized size = 7.52

$$\frac{2}{45045}\,\left(\left(\left(7\,\left(3\,\left(11\,\left(13\,x-80\right)\left(x+1\right)+3165\right)\left(x+1\right)-16442\right)\left(x+1\right)+121227\right)\left(x+1\right)-80187\right)\left(x+1\right)+34077\right)\sqrt{\left(x+1\right)^{2}-3\,x}\,\sqrt{x+1}+\frac{2}{45045}\,\left(\left(5\,\left(7\,\left(9\,\left(11\,x-57\right)\left(x+1\right)+1601\right)\left(x+1\right)-15837\right)\left(x+1\right)+65172\right)\left(x+1\right)-34077\right)\sqrt{\left(x+1\right)^{2}-3\,x}\,\sqrt{x+1}+\frac{2}{315}\,\left(\left(5\,\left(7\,x-23\right)\left(x+1\right)+258\right)\left(x+1\right)-213\right)\sqrt{\left(x+1\right)^{2}-3\,x}\,\sqrt{x+1}+\frac{2}{105}\,\left(3\,\left(5\,x-12\right)\left(x+1\right)+71\right)\sqrt{\left(x+1\right)^{2}-3\,x}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")

[Out] 2/45045*(((7*(3*(11*(13*x - 80)*(x + 1) + 3165)*(x + 1) - 16442)*(x + 1) + 121227)*(x + 1) - 80187)*(x + 1) + 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/45045*((5*(7*(9*(11*x - 57)*(x + 1) + 1601)*(x + 1) - 15837)*(x + 1) + 65172)*(x + 1) - 34077)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/315*((5*(7*x - 23)*(x + 1) + 258)*(x + 1) - 213)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1) + 2/105*(3*(5*x - 12)*(x + 1) + 71)*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)

Mupad [B]
time = 0.12, size = 25, normalized size = 1.09

$$\frac{2\,\sqrt{x+1}\,{\left(x^2-x+1\right)}^{5/2}\,\left(x^2+2\,x+1\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)

[Out] (2*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2)*(2*x + x^2 + 1))/15

3.498 $\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx$

Optimal. Leaf size=325

$$\frac{18}{91}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{54\sqrt{1+x}\,\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)}+\frac{2}{13}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)-\frac{27\sqrt[4]{3}\,\sqrt{2-\sqrt{\ldots}}}{\ldots}$$

```
[Out] 18/91*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/13*x^2*(x^3+1)*(1+x)^(1/2)*(x^2-x+1
)^(1/2)+54/91*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+18/91*3^(3/4)*(1+x)
^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(x^2-x+
1)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^
(1/2)-27/91*3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(
1/2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2)
)^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {823, 285, 309, 224, 1891}

$$\frac{18\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}-\frac{27\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(x^3+1\right)}+\frac{18}{91}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^2+\frac{54\sqrt{x+1}\,\sqrt{x^2-x+1}}{91\left(x+\sqrt{3}+1\right)}+\frac{2}{13}\sqrt{x+1}\,\sqrt{x^2-x+1}\left(x^3+1\right)x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (18*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/91 + (54*Sqrt[1 + x]*Sqrt[1 - x + x^
2])/(91*(1 + Sqrt[3] + x)) + (2*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3)
)/13 - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(
1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + S
qrt[3] + x)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x
^3)) + (18*Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x(1+x^3)^{3/2}\,dx}{\sqrt{1+x^3}}$$

$$= \frac{2}{13}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3) + \frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x\sqrt{1}}{13\sqrt{1+x^3}}$$

$$= \frac{18}{91}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2}{13}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3) + \frac{(27}{}$$

$$= \frac{18}{91}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{2}{13}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3) + \frac{(27}{}$$

$$= \frac{18}{91}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{54\sqrt{1+x}\,\sqrt{1-x+x^2}}{91\left(1+\sqrt{3}+x\right)} + \frac{2}{13}x^2\sqrt{1+x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.33, size = 244, normalized size = 0.75

$$\frac{\sqrt{1+x}\left(4x^2(1-x+x^2)(16+7x^3) - \frac{27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\left(\left(-3i+\sqrt{3}\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right) - \left(-i+\sqrt{3}\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\right)}{\sqrt{-\frac{i(1+x)}{i+\sqrt{3}-2ix}}}\right)}{182\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (Sqrt[1 + x]*(4*x^2*(1 - x + x^2)*(16 + 7*x^3) - (27*Sqrt[2]*Sqrt[(-I + Sqr
t[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + S
qrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I + Sqrt
[3] - (2*I)*x)]))/(182*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.10, size = 366, normalized size = 1.13

method	result

risch	$\frac{2x^2(7x^3+16)\sqrt{1+x}\,\sqrt{x^2-x+1}}{91}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{}}{}\right.\right.}{}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^5\sqrt{x^3+1}}{13}+\frac{32x^2\sqrt{x^3+1}}{91}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}\right.}{}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(14x^8+27i\sqrt{3}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+}{-3+i}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/91*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(14*x^8+27*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/
2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I
*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))
/(I*3^(1/2)+3))^(1/2))+46*x^5+81*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2
)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*Elli
pticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)
)-162*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/
2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/
2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+32*x^2)/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 38, normalized size = 0.12

$$\frac{2}{91}\left(7\,x^5+16\,x^2\right)\sqrt{x^2-x+1}\,\sqrt{x+1}-\frac{54}{91}\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/91*(7*x^5 + 16*x^2)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 54/91*weierstrassZeta
(0, -4, weierstrassPInverse(0, -4, x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(x+1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(x+1\right)}^{3/2}\,{\left(x^2-x+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

```
[Out] int(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.499 $\int (1+x)^{3/2} \left(1-x+x^2\right)^{3/2} \, dx$

Optimal. Leaf size=173

$$\frac{18}{55}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{11}x\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}}{55\sqrt{\frac{\cdots}{(1\cdots}}}$$

[Out] 18/55*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/11*x*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1
/2)+18/55*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/
2)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^
2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$,
Rules used = {727, 201, 224}

$$\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{55\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{18}{55}x\sqrt{x^2-x+1}\,\sqrt{x+1}+\frac{2}{11}x\sqrt{x^2-x+1}\,(x^3+1)\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (18*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/55 + (2*x*Sqrt[1 + x]*Sqrt[1 - x + x^2
]*(1 + x^3))/11 + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x +
x^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3]
+ x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*(1 + x^3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (1+x)^{3/2}\left(1-x+x^2\right)^{3/2}\,dx=\frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \left(1+x^3\right)^{3/2}\,dx}{\sqrt{1+x^3}}$$

$$=\frac{2}{11}x\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \sqrt{1+x}$$

$$=\frac{18}{55}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{11}x\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(27\sqrt{1}\right.}{}$$

$$18\ 3^{3/4}$$

$$=\frac{18}{55}x\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{11}x\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)+$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.44, size = 176, normalized size = 1.02

$$\frac{2x\sqrt{1+x}\left(1-x+x^2\right)\left(14+5x^3\right)+\frac{9i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{55\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)*(1 - x + x^2)^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2)*(14 + 5*x^3) + ((9*I)*(1 + x)*Sqrt[1 + (6*I)
/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*Ell
ipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(55*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.11, size = 257, normalized size = 1.49

method	result
risch	$\frac{2x(5x^3+14)\sqrt{1+x}\,\sqrt{x^2-x+1}}{55}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\cdots\right.}{55\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x\cdots}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^4\sqrt{x^3+1}}{11}+\frac{28x\sqrt{x^3+1}}{55}+\frac{54\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}\right.}{x^3+1}$
default	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(-10x^7+27i\sqrt{3}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2\cdots}{-3\cdots}}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/55*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-10*x^7+27*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(
1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3
+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2
))/(I*3^(1/2)+3))^(1/2))-81*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x
+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF
((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-38*
x^4-28*x)/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 33, normalized size = 0.19

$$\frac{2}{55}\left(5 x^{4}+14 x\right) \sqrt{x^{2}-x+1}\, \sqrt{x+1}+\frac{54}{55} \operatorname{weierstrassPInverse}(0,-4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/55*(5*x^4 + 14*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 54/55*weierstrassPInver
se(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (x+1)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (x+1)^{3/2}\left(x^{2}-x+1\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2),x)
```

```
[Out] int((x + 1)^(3/2)*(x^2 - x + 1)^(3/2), x)
```

3.500 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)-\frac{2\sqrt{1+x}\,\sqrt{1-x+x^2}\,\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x^3}}$$

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)+2/9*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)-2/3
*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {929, 272, 52, 65, 213}

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}+\frac{2}{9}\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)-\frac{2\sqrt{x+1}\,\sqrt{x^2-x+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3 + (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 +
 x^3))/9 - (2*Sqrt[1 + x]*Sqrt[1 - x + x^2]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt
[1 + x^3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x}\,dx &= \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x}\,dx}{\sqrt{1+x^3}}\\
&= \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x}\,dx, x, x^3\right)}{3\sqrt{1+x^3}}\\
&= \frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1}}{}\right.}{3\sqrt{1+x^3}}\\
&= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(\sqrt{1+x}\,\sqrt{}\right.}{}\\
&= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)+\frac{\left(2\sqrt{1+x}\right.}{}\\
&= \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{2}{9}\sqrt{1+x}\,\sqrt{1-x+x^2}\left(1+x^3\right)-\frac{2\sqrt{1+x}\,\sqrt{}}{}
\end{aligned}$$

Mathematica [A]
time = 10.05, size = 53, normalized size = 0.56

$$\frac{2}{9}\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\left(4+x^3\right)-3\tanh^{-1}\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x,x]
```

```
[Out] (2*(Sqrt[1 + x]*Sqrt[1 - x + x^2]*(4 + x^3) - 3*ArcTanh[Sqrt[1 + x]*Sqrt[1
- x + x^2]]))/9
```

Maple [A]
time = 0.10, size = 57, normalized size = 0.61

method	result	s
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\left(-x^{3}\sqrt{x^{3}+1}+3\,\mathrm{arctanh}\left(\sqrt{x^{3}+1}\right)-4\sqrt{x^{3}+1}\right)}{9\sqrt{x^{3}+1}}$	5
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\sqrt{\left(1+x\right)\left(x^{2}-x+1\right)}\,\left(\frac{2x^{3}\sqrt{x^{3}+1}}{9}+\frac{8\sqrt{x^{3}+1}}{9}-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}\right)}{x^{3}+1}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-x^3*(x^3+1)^(1/2)+3*arctanh((x^3+1)^(1/2
))-4*(x^3+1)^(1/2))/(x^3+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

Fricas [A]
time = 2.69, size = 65, normalized size = 0.69

$$\frac{2}{9}\left(x^{3}+4\right)\sqrt{x^{2}-x+1}\,\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^{2}-x+1}\,\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^{2}-x+1}\,\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x+1\right)^{\frac{3}{2}}\left(x^{2}-x+1\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{3/2}\,(x^2-x+1)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x,x)
```

```
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x, x)
```

3.501 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=323

$$\frac{9}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2}+\frac{27\sqrt{1+x}\,\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)}-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}{x}-\frac{27\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1}{}$$

```
[Out] 9/7*x^2*(1+x)^(1/2)*(x^2-x+1)^(1/2)-(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)/x+2
7/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(1+x+3^(1/2))+9/7*3^(3/4)*(1+x)^(3/2)*Ellip
ticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(x^2-x+1)^(1/2)*((x
^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-27/14*
3^(1/4)*(1+x)^(3/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(x
^2-x+1)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(
x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {929, 283, 285, 309, 224, 1891}

$$\frac{9\sqrt{2}\,3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}-\frac{27\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(x+1)^{3/2}\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{9}{7}\sqrt{x+1}\,\sqrt{x^2-x+1}\,x^2+\frac{27\sqrt{x+1}\,\sqrt{x^2-x+1}}{7\left(x+\sqrt{3}+1\right)}-\frac{\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

```
[Out] (9*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/7 + (27*Sqrt[1 + x]*Sqrt[1 - x + x^2]
)/(7*(1 + Sqrt[3] + x)) - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1 + x^3))/x - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x
)], -7 - 4*Sqrt[3]])/(14*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3)) + (9*
Sqrt[2]*3^(3/4)*(1 + x)^(3/2)*Sqrt[1 - x + x^2]*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(7*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*(1 + x^3))
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^2}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x^2}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}{x} + \frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int x\sqrt{1+x}}{2\sqrt{1+x^3}}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}{x} + \frac{\left(27\sqrt{1+\right.}{}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}{x} + \frac{\left(27\sqrt{1+\right.}{}$$

$$= \frac{9}{7}x^2\sqrt{1+x}\,\sqrt{1-x+x^2} + \frac{27\sqrt{1+x}\,\sqrt{1-x+x^2}}{7\left(1+\sqrt{3}+x\right)} - \frac{\sqrt{1+x}\,\sqrt{1-x}}{x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.34, size = 244, normalized size = 0.76

$$\frac{\sqrt{1+x}\left(\frac{4(1-x+x^2)(-7+2x^3)}{x} - \frac{27\sqrt{2}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}\left(\left(-3i+\sqrt{3}\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right) - \left(-i+\sqrt{3}\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\right)}{\sqrt{-\frac{i(1+x)}{i+\sqrt{3}-2ix}}}\right)}{28\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[1 + x]*((4*(1 - x + x^2)*(-7 + 2*x^3))/x - (27*Sqrt[2]*Sqrt[(-I + Sqr
t[3] + (2*I)*x)/(-3*I + Sqrt[3])]*((-3*I + Sqrt[3])*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])
] - (-I + Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + S
qrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])]))/Sqrt[((-I)*(1 + x))/(I + Sqrt
[3] - (2*I)*x)]))/(28*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.12, size = 368, normalized size = 1.14

method	result

risch	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,(2x^3-7)}{7x}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\right.}{}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2\sqrt{x^3+1}}{7}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}\right.}{}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(27i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(27*I*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*
3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2
)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))
^(1/2))*3^(1/2)*x+4*x^6+81*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+
1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF(
(-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-16
2*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*(
(I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))
^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-10*x^3-14)/x/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 39, normalized size = 0.12

$$\frac{(2x^3-7)\sqrt{x^2-x+1}\,\sqrt{x+1}-27\,x\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))}{7\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*((2*x^3 - 7)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 27*x*weierstrassZeta(0, -4
, weierstrassPInverse(0, -4, x)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**2,x)
```

```
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x+1)^{3/2}\,(x^2-x+1)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2,x)
```

```
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^2, x)
```

3.502 $\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=175

$$\frac{9}{10}x\sqrt{1+x}\,\sqrt{1-x+x^2}-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}{2x^2}+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,(1+x)^{3/2}\sqrt{1-x+x^2}\,\sqrt{\frac{1}{\left(1-\right.}}}{10\sqrt{\frac{1+}{\left(1+\sqrt{3}\right.}}}$$

```
[Out] 9/10*x*(1+x)^(1/2)*(x^2-x+1)^(1/2)-1/2*(x^3+1)*(1+x)^(1/2)*(x^2-x+1)^(1/2)/
x^2+9/10*3^(3/4)*(1+x)^(3/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2
)+2*I)*(x^2-x+1)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2
)^(1/2)/(x^3+1)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {929, 283, 201, 224}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,\sqrt{x^2-x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x+1)^{3/2}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{10\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(x^3+1)}+\frac{9}{10}x\sqrt{x^2-x+1}\,\sqrt{x+1}-\frac{\sqrt{x^2-x+1}\,(x^3+1)\sqrt{x+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

```
[Out] (9*x*Sqrt[1 + x]*Sqrt[1 - x + x^2])/10 - (Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
+ x^3))/(2*x^2) + (9*3^(3/4)*Sqrt[2 + Sqrt[3]]*(1 + x)^(3/2)*Sqrt[1 - x + x
^2]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
 x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(10*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*(1 + x^3))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}{x^3}\,dx = \frac{\left(\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \frac{\left(1+x^3\right)^{3/2}}{x^3}\,dx}{\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)}{2x^2} + \frac{\left(9\sqrt{1+x}\,\sqrt{1-x+x^2}\right)\int \sqrt{1+x^3}}{4\sqrt{1+x^3}}$$

$$= \frac{9}{10}x\sqrt{1+x}\,\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)}{2x^2} + \frac{\left(27\sqrt{1+\right.}{}$$

$$= \frac{9}{10}x\sqrt{1+x}\,\sqrt{1-x+x^2} - \frac{\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)}{2x^2} + \frac{9\ 3^{3/4}\sqrt{2}}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.27, size = 192, normalized size = 1.10

$$\frac{\sqrt{1+x}\left(\frac{2(1-x+x^2)(-5+4x^3)}{x^2}-\frac{27i\sqrt{2}\sqrt{\frac{i+\sqrt{3}-2ix}{3i+\sqrt{3}}}\sqrt{\frac{-i+\sqrt{3}+2ix}{-3i+\sqrt{3}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i(1+x)}{3i+\sqrt{3}}}}\right)}{20\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(3/2)*(1 - x + x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[1 + x]*((2*(1 - x + x^2)*(-5 + 4*x^3))/x^2 - ((27*I)*Sqrt[2]*Sqrt[(I
+ Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])]*Sqrt[(-I + Sqrt[3] + (2*I)*x)/(-3*I +
 Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]
], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]))
/(20*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.11, size = 264, normalized size = 1.51

method	result
risch	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,(4x^3-5)}{10x^2}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{-\ldots}\right.}{10\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-\ldots}}$
elliptic	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5}+\frac{27\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ldots}{\ldots}\right.}{x^3+1}$
default	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(27i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\ldots\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(27*I*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I
*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/
2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3)
)^(1/2))*3^(1/2)*x^2-81*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/
```

```
(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2
*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2-8*x
^6+2*x^3+10)/(x^3+1)/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 38, normalized size = 0.22

$$\frac{27\,x^2\text{weierstrassPInverse}(0,-4,x)+(4\,x^3-5)\sqrt{x^2-x+1}\,\sqrt{x+1}}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="fricas")
[Out] 1/10*(27*x^2*weierstrassPInverse(0, -4, x) + (4*x^3 - 5)*sqrt(x^2 - x + 1)*
sqrt(x + 1))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**3,x)
[Out] Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{3/2} (x^2-x+1)^{3/2}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3,x)
[Out] int(((x + 1)^(3/2)*(x^2 - x + 1)^(3/2))/x^3, x)
```

3.503 $\int \frac{x^3}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=142

$$\frac{2x(1+x^3)}{5\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{4\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

```
[Out] 2/5*x*(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-4/15*EllipticF((1+x-3^(1/2))/(1+x
+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {929, 327, 224}

$$\frac{2x(x^3+1)}{5\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{4\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*x*(1 + x^3))/(5*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (4*Sqrt[2 + Sqrt[3]]*Sq
rt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 +
 Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x^3}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2x(1+x^3)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{5\sqrt{1+x}\sqrt{1-x+x^2}}$$
$$= \frac{2x(1+x^3)}{5\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\Big(\sin^{-1}}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.46, size = 169, normalized size = 1.19

$$\frac{6x\sqrt{1+x}\,(1-x+x^2) - \frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{15\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (6*x*Sqrt[1 + x]*(1 - x + x^2) - ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqr
t[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcS
inh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[
3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(15*Sqrt[1 - x + x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(115) = 230$.
time = 0.11, size = 248, normalized size = 1.75

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x\sqrt{x^3+1}}{5} - \frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\cdots\right.}{5\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$\frac{2x\sqrt{1+x}\,\sqrt{x^2-x+1}}{5} - \frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots\right.}{5\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+\cdots}}$
default	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*
((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^
(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)
+3))^(1/2))-3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)
+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3
+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+x^4+x)/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 25, normalized size = 0.18

$$\frac{2}{5}\sqrt{x^2-x+1}\,\sqrt{x+1}\,x-\frac{4}{5}\,\text{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/5*sqrt(x^2 - x + 1)*sqrt(x + 1)*x - 4/5*weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(x**3/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(x^3/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.504 $\int \frac{x^2}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}$$

```
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {927}

$$\frac{2}{3}\sqrt{x+1}\,\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx = \frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}$$

Mathematica [A]
time = 10.05, size = 23, normalized size = 1.00

$$\frac{2}{3}\sqrt{1+x}\,\sqrt{1-x+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[1 + x]*Sqrt[1 - x + x^2])/3
```

Maple [A]
time = 0.10, size = 18, normalized size = 0.78

method	result	size
gosper	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
default	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
risch	$\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3}$	18
elliptic	$\frac{2\sqrt{(1+x)\,(x^2-x+1)}\,\sqrt{x^3+1}}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)
```

Maxima [A]
time = 0.50, size = 22, normalized size = 0.96

$$\frac{2\,(x^3+1)}{3\,\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x^3 + 1)/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Fricas [A]
time = 1.23, size = 17, normalized size = 0.74

$$\frac{2}{3}\,\sqrt{x^2-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [A]
time = 1.24, size = 18, normalized size = 0.78

$$\frac{2}{3}\sqrt{(x+1)^2-3\,x}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt((x + 1)^2 - 3*x)*sqrt(x + 1)
```

Mupad [B]
time = 0.15, size = 9, normalized size = 0.39

$$\frac{2\sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] (2*(x^3 + 1)^(1/2))/3
```

3.505 $\int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=253

$$\frac{2(1+x^3)}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 2*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/3*EllipticF((1+x-3^(1
/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(1/4)*
EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/
2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {823, 309, 224, 1891}

$$\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{2(x^3+1)}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*(1 + x^3))/(Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (2*Sqrt[2]*Sqrt[1 + x]*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 +
 Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2(1+x^3)}{\sqrt{1+x}\,\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-\ldots}{(1+\sqrt{\ldots}}}}{\sqrt{\frac{1\ldots}{(1+\sqrt{\ldots}}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.66, size = 375, normalized size = 1.48

$$\frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\,(1-x+x^2)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}+\frac{i\sqrt{2}\left(3i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}\right)}{6\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (
3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[
3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*Ar
cSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqr
t[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I
)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + S
qrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*
I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(6*Sqrt[(-I)/(3*I + Sqrt[3])]*
Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.10, size = 275, normalized size = 1.09

method	result
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.\right.}{\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x}}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(-3+i\sqrt{3}\right)\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\left(i\,\mathrm{EllipticE}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-3+I*3^(1/2))*(-2*(1+x)/(-3+I*3^(1/2)))^(1
/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2
)))^(1/2)*(I*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*
3^(1/2)+3))^(1/2))*3^(1/2)-I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-
3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)+3*EllipticE((-2*(1+x)/(-3+I*3^(1
/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-EllipticF((-2*(1+x)/(-3+
I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)))/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.37, size = 9, normalized size = 0.04

$$-2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] -2*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(x/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(x/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.506 $\int \frac{1}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=110

$$\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

```
[Out] 2/3*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {727, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$
$$= \frac{2\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.11, size = 148, normalized size = 1.35

$$\frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\,\sqrt{\frac{2}{3}-\frac{4i}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}\,\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[2/3 - (4*I)/((3*
I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqr
t[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/(Sqrt[(-I)/(3*I + Sqrt[3])]*Sq
rt[1 - x + x^2])
```

Maple [A]
time = 0.10, size = 137, normalized size = 1.25

method	result

default	$\frac{(3-i\sqrt{3})\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\text{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\right.}{x^3+1}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\sqrt{(1+x)(}{\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (3-I*3^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*(
(I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(
1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+
3))^(1/2))/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 6, normalized size = 0.05

$$2\,\text{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(1/((x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.507 $\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {929, 272, 65, 213}

$$-\frac{2\sqrt{x^3+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] (-2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
```

```
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{1}{x\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{\sqrt{1+x^3}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= \frac{\left(2\sqrt{1+x^3}\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}\\
&= -\frac{2\sqrt{1+x^3}\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\sqrt{1-x+x^2}}
\end{aligned}$$

Mathematica [**A**]
time = 5.31, size = 29, normalized size = 0.69

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1+x}\sqrt{3-3(1+x)+(1+x)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
[Out] (-2*ArcTanh[Sqrt[1 + x]*Sqrt[3 - 3*(1 + x) + (1 + x)^2]])/3
```

Maple [**A**]
time = 0.10, size = 33, normalized size = 0.79

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{1+x}\,\sqrt{x^2-x+1}}{3\sqrt{x^3+1}}$	33
elliptic	$-\frac{2\sqrt{(1+x)\left(x^2-x+1\right)}\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

Fricas [A]
time = 1.76, size = 43, normalized size = 1.02

$$-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqr
t(x + 1) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
[Out] Integral(1/(x*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
[Out] int(1/(x*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.508 $\int \frac{1}{x^2\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=282

$$-\frac{1+x^3}{x\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{1+x^3}{\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\cdots}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\cdots}$$

```
[Out] (-x^3-1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x
^2-x+1)^(1/2)+1/3*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1
/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/(
(1+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/
2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^
(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {929, 331, 309, 224, 1891}

$$\frac{\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{x^3+1}{x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{x^3+1}{\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -((1 + x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2])) + (1 + x^3)/(Sqrt[1 + x]*(1
+ Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 - x + x^2]) + (Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]
])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= -\frac{1+x^3}{x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= -\frac{1+x^3}{x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{2\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\right.}{\sqrt{1+}}$$

$$\sqrt[4]{3}$$

$$= -\frac{1+x^3}{x\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{1+x^3}{\sqrt{1+x}\,\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.38, size = 400, normalized size = 1.42

$$-\frac{\sqrt{1+x}\,\sqrt{1-x+x^2}}{x} + \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\,(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}\right)}{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\,\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + ((1 + x)^(3/2)*((12*Sqrt[(-I)/(3*I +
 Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I
+ Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1
 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/S
qrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]*(3*
I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3
*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-
6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[
1 + x]))/(12*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.11, size = 363, normalized size = 1.29

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{}}{}\right.\right.}{}\right.}{\sqrt{1+x}\,\sqrt{x^2-x}}$
risch	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{Ellipti}\right.}{}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1
/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*El
lipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/
2))*3^(1/2)*x+3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/
2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(
-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-6*(-2*(1+x)/(
-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*
x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+
I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/x/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 31, normalized size = 0.11

$$-\frac{x\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))+\sqrt{x^2-x+1}\,\sqrt{x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^2 - x +
1)*sqrt(x + 1))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] int(1/(x^2*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.509 $\int \frac{1}{x^3\sqrt{1+x}\sqrt{1-x+x^2}}\,dx$

Optimal. Leaf size=146

$$\frac{-1-x^3}{2x^2\sqrt{1+x}\sqrt{1-x+x^2}}-\frac{\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] 1/2*(-x^3-1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-1/6*EllipticF((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A]

time = 0.03, antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {929, 331, 224}

$$-\frac{\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{x^3+1}{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] -1/2*(1 + x^3)/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (Sqrt[2 + Sqrt[3]]*Sqr
t[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt
[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 +
Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{1+x}\,\sqrt{1-x+x^2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3\sqrt{1+x^3}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= -\frac{1+x^3}{2x^2\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{4\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= -\frac{1+x^3}{2x^2\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.42, size = 171, normalized size = 1.17

$$\frac{-\frac{6\sqrt{1+x}\,(1-x+x^2)}{x^2}-\frac{i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\,\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{12\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]
```

```
[Out] ((-6*Sqrt[1 + x]*(1 - x + x^2))/x^2 - (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sq
rt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*Arc
Sinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt
[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(12*Sqrt[1 - x + x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(119) = 238.
time = 0.11, size = 259, normalized size = 1.77

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\right.}{2\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$-\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}}{2x^2}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.}{2\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x+}}$
default	$\frac{\sqrt{1+x}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1
/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*El
lipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/
2))*3^(1/2)*x^2-3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(
1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)
/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2-2*x^3-2)/
(x^3+1)/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 30, normalized size = 0.21

$$-\frac{x^2\text{weierstrassPInverse}(0,-4,x)+\sqrt{x^2-x+1}\,\sqrt{x+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*weierstrassPInverse(0, -4, x) + sqrt(x^2 - x + 1)*sqrt(x + 1))/x^
2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)),x)
```

```
[Out] int(1/(x^3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2)), x)
```

3.510 $\int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-x+x^2}}$$

```
[Out] -2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+4/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)
),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1
/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {929, 294, 224}

$$\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (-2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x^3}{\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{3\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{1+x}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.41, size = 161, normalized size = 1.18

$$\frac{-\frac{6x}{\sqrt{1+x}}+\frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{9\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] ((-6*x)/Sqrt[1 + x] + ((2*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 +
x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-
6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[
(-I)/(3*I + Sqrt[3])])/(9*Sqrt[1 - x + x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(110) = 220$.
time = 0.11, size = 245, normalized size = 1.79

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{2x}{3\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\right.}{3\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
risch	$-\frac{2x}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{}}{2}}}\right.}{3\sqrt{x^3+1}\,\sqrt{1+x}\,\sqrt{x^2-x}}$
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)
*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))
^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2
)+3))^(1/2))-3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2
)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-
3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+x)/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 38, normalized size = 0.28

$$-\frac{2\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,x-2\,(x^3+1)\text{weierstrassPInverse}(0,-4,x)\right)}{3\,(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x - 2*(x^3 + 1)*weierstrassPInverse(0,
-4, x))/(x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**3/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(x+1)^{3/2}\,(x^2-x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(x^3/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.511 $\int \frac{x^2}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

```
[Out] -2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{927\}$

$$-\frac{2}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x)^{3/2}\,(1-x+x^2)^{3/2}}\,dx = -\frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

Mathematica [A]
time = 10.03, size = 23, normalized size = 1.00

$$-\frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.10, size = 25, normalized size = 1.09

method	result	size
gosper	$-\frac{2}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	18
risch	$-\frac{2}{3\sqrt{1+x}\,\sqrt{x^2-x+1}}$	18
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^2-x+1}}{3(x^3+1)}$	25
elliptic	$-\frac{2\sqrt{(1+x)\,(x^2-x+1)}}{3\sqrt{1+x}\,\sqrt{x^2-x+1}\,\sqrt{x^3+1}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)
```

Maxima [A]
time = 0.48, size = 17, normalized size = 0.74

$$-\frac{2}{3\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] -2/3/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Fricas [A]
time = 2.13, size = 24, normalized size = 1.04

$$-\frac{2\sqrt{x^2-x+1}\,\sqrt{x+1}}{3\,(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] -2/3*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x**2/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [B]
time = 2.69, size = 17, normalized size = 0.74

$$-\frac{2}{3\,\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] -2/(3*(x + 1)^(1/2)*(x^2 - x + 1)^(1/2))
```

3.512 $\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{2x^2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{2(1+x^3)}{3\sqrt{1+x}\,\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}+\frac{\sqrt{2-\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

```
[Out] 2/3*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(
x^2-x+1)^(1/2)-2/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(
1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)+1/3*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1
/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3
^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {823, 296, 309, 224, 1891}

$$-\frac{2\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{2(x^3+1)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*(1 + x^3))/(3*Sqrt[1 + x]*(1
 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) + (Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1
 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sq
rt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 - x + x^2]) - (2*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] +
 x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3
]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x}{(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2x^2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{x}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2x^2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{\sqrt{1+x^3}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\right.}{3\sqrt{1+x}}$$

$$\sqrt{2}$$

$$= \frac{2x^2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{2(1+x^3)}{3\sqrt{1+x}\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.42, size = 402, normalized size = 1.43

$$\frac{2x^2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\,(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}\right)}{18\sqrt{-\frac{i}{3i+\sqrt{3}}}\,\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x^2)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - ((1 + x)^(3/2)*((12*Sqrt[(-I)/(
3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqrt[3])*Sqrt[
(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*
I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3
])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2
]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqr
t[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sq
rt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/
Sqrt[1 + x]))/(18*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.12, size = 356, normalized size = 1.26

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2}{3\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{}}{}\right.\right.}{}\right.}{\sqrt{1+x}\sqrt{x^2-x}}$
risch	$\frac{2x^2}{3\sqrt{1+x}\sqrt{x^2-x+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{Ellipti}\right.}{3}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)
*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))
^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2
)+3))^(1/2))+3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2
)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-
3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-6*(-2*(1+x)/(-3+
I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1
)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3
^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x^2)/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 42, normalized size = 0.15

$$\frac{2\left(\sqrt{x^2-x+1}\sqrt{x+1}\,x^2+(x^3+1)\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))\right)}{3\,(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2 + (x^3 + 1)*weierstrassZeta(0, -4, w
eierstrassPInverse(0, -4, x)))/(x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(x+1)^{3/2}\,(x^2-x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(x/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.513 $\int \frac{1}{(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2x}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x+x^2}}$$

```
[Out] 2/3*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2))
,I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {727, 205, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{2x}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] (2*x)/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
 x)^2]*Sqrt[1 - x + x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{\left(1+x^3\right)^{3/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2x}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\int \frac{1}{\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2x}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(\text{si}}{3\sqrt[4]{3}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.24, size = 216, normalized size = 1.58

$$\sqrt{3-3(1+x)+(1+x)^2}\left(-\frac{2}{9\sqrt{1+x}}+\frac{2(1+x)^{3/2}}{9\left(3-3(1+x)+(1+x)^2\right)}\right)+\frac{i\sqrt{\frac{2}{3}}\,(1+x)\sqrt{1-\frac{6}{\left(3-i\sqrt{3}\right)(1+x)}}\,\sqrt{1-\frac{6}{\left(3+i\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6}{3-i\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3-i\sqrt{3}}{3+i\sqrt{3}}\right)}{3\sqrt{-\frac{1}{3-i\sqrt{3}}}\,\sqrt{3-3(1+x)+(1+x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] Sqrt[3 - 3*(1 + x) + (1 + x)^2]*(-2/(9*Sqrt[1 + x]) + (2*(1 + x)^(3/2))/(9*
(3 - 3*(1 + x) + (1 + x)^2))) + ((I/3)*Sqrt[2/3]*(1 + x)*Sqrt[1 - 6/((3 - I
*Sqrt[3])*(1 + x))]*Sqrt[1 - 6/((3 + I*Sqrt[3])*(1 + x))]*EllipticF[I*ArcSi
```

```
nh[Sqrt[-6/(3 - I*Sqrt[3])]/Sqrt[1 + x]], (3 - I*Sqrt[3])/(3 + I*Sqrt[3])])
/(Sqrt[-(3 - I*Sqrt[3])^(-1)]*Sqrt[3 - 3*(1 + x) + (1 + x)^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(110) = 220$.
time = 0.10, size = 247, normalized size = 1.80

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x}{3\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\right.}{3\sqrt{x^3+1}}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$\frac{2x}{3\sqrt{1+x}\sqrt{x^2-x+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.}{3\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+}}$
default	$-\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)
*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))
^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2
)+3))^(1/2))-3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2
)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-
3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x)/(x^3+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 37, normalized size = 0.27

$$\frac{2\left(\sqrt{x^2-x+1}\,\sqrt{x+1}\,x+(x^3+1)\text{weierstrassPInverse}(0,-4,x)\right)}{3\,(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(x^2 - x + 1)*sqrt(x + 1)*x + (x^3 + 1)*weierstrassPInverse(0, -4,
 x))/(x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{(x+1)^{3/2}\,(x^2-x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(1/((x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.514 $\int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{2\sqrt{1+x^3}\,\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

```
[Out] 2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1
+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {929, 272, 53, 65, 213}

$$\frac{2}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{2\sqrt{x^3+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3
]])/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{1}{x(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx, x, x^3\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{\left(2\sqrt{1+x^3}\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} - \frac{2\sqrt{1+x^3}\,\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}
\end{aligned}
$$

Mathematica [A]
time = 11.21, size = 81, normalized size = 1.23

$$
\frac{2\left(\sqrt{1+x}-(1+x)^2\sqrt{\frac{1-x+x^2}{(1+x)^2}}\,\tanh^{-1}\left(\frac{1}{(1+x)^{3/2}\sqrt{\frac{1-x+x^2}{(1+x)^2}}}\right)\right)}{3(1+x)\sqrt{1-x+x^2}}
$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]

[Out] (2*(Sqrt[1 + x] - (1 + x)^2*Sqrt[(1 - x + x^2)/(1 + x)^2]*ArcTanh[1/((1 + x)^(3/2)*Sqrt[(1 - x + x^2)/(1 + x)^2])]))/(3*(1 + x)*Sqrt[1 - x + x^2])

Maple [A]
time = 0.13, size = 43, normalized size = 0.65

method	result	size
default	$-\frac{2\sqrt{1+x}\,\sqrt{x^{2}-x+1}\,\left(\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)\sqrt{x^{3}+1}-1\right)}{3\left(x^{3}+1\right)}$	43
elliptic	$\frac{\sqrt{\left(1+x\right)\left(x^{2}-x+1\right)}\,\left(\frac{2}{3\sqrt{x^{3}+1}}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	51
risch	$\frac{2}{3\sqrt{1+x}\,\sqrt{x^{2}-x+1}}-\frac{2\sqrt{\left(1+x\right)\left(x^{2}-x+1\right)}\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3\sqrt{1+x}\,\sqrt{x^{2}-x+1}}$	58

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)-1)/(x^3+1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)

Fricas [A]
time = 1.46, size = 78, normalized size = 1.18

$$-\frac{\left(x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\,\sqrt{x+1}+1\right)-\left(x^{3}+1\right)\log\left(\sqrt{x^{2}-x+1}\,\sqrt{x+1}-1\right)-2\,\sqrt{x^{2}-x+1}\,\sqrt{x+1}}{3\,\left(x^{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")

```
[Out] -1/3*((x^3 + 1)*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) - (x^3 + 1)*log(sqrt
(x^2 - x + 1)*sqrt(x + 1) - 1) - 2*sqrt(x^2 - x + 1)*sqrt(x + 1))/(x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(1/(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x(x+1)^{3/2}(x^2-x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(1/(x*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.515 $\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{2}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{5(1+x^3)}{3x\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{5(1+x^3)}{3\sqrt{1+x}\,\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}-\frac{5\sqrt{2-\sqrt{3}}}{}$$

```
[Out] 2/3/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-5/3*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)
+5/3*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)+5/9*EllipticF((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^
(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-5/6*3
^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/
2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((
1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {929, 296, 331, 309, 224, 1891}

$$\frac{5\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}-\frac{5\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^2-x+1}}+\frac{2}{3x\sqrt{x+1}\sqrt{x^2-x+1}}-\frac{5(x^3+1)}{3x\sqrt{x+1}\sqrt{x^2-x+1}}+\frac{5(x^3+1)}{3\sqrt{x+1}\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (5*(1 + x^3))/(3*x*Sqrt[1 + x]*Sqrt
[1 - x + x^2]) + (5*(1 + x^3))/(3*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x
+ x^2]) - (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3]
+ x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]])/(2*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (5*S
qrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1
+ x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^2(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)}{6\sqrt{1+x}\sqrt{}}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(5\sqrt{1+x^3}\right)}{6\sqrt{1+x}\sqrt{}}$$

$$= \frac{2}{3x\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{5(1+x^3)}{3x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{}{3\sqrt{1+x}\left(1-\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.43, size = 409, normalized size = 1.29

$$-\frac{3+5x^3}{3x\sqrt{1+x}\sqrt{1-x+x^2}}+\frac{5(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\left(1-x+x^2\right)}{(1+x)^2}+\frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}+\frac{i\sqrt{2}\left(3i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}\right)}{36\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] -1/3*(3 + 5*x^3)/(x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (5*(1 + x)^(3/2)*((12*
Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1 - I*Sqr
t[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sq
rt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3
*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]
+ (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sq
rt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
Sqrt[3])])/Sqrt[1 + x]))/(36*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.10, size = 363, normalized size = 1.15

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}-\frac{2x^2}{3\sqrt{x^3+1}}+\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}\right.}{\sqrt{1+x}\sqrt{x\cdots}}$
risch	$-\frac{5x^3+3}{3x\sqrt{1+x}\sqrt{x^2-x+1}}+\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\mathrm{Ellip}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(5i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(5*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1
/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*x*((I*3^(1/2)-2*x+1)/(I*
3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*(-2*(1+x)/(-3+I*
3^(1/2)))^(1/2)+15*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^
(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x
)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-30*(-2*(1+
x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2
)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-
(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-10*x^3-6)/(x^3+1)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 47, normalized size = 0.15

$$-\frac{(5x^3+3)\sqrt{x^2-x+1}\sqrt{x+1}+5(x^4+x)\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))}{3(x^4+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*((5*x^3 + 3)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 5*(x^4 + x)*weierstrassZe
ta(0, -4, weierstrassPInverse(0, -4, x)))/(x^4 + x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (x+1)^{\frac{3}{2}} (x^2-x+1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(x+1\right)}^{3/2}\,{\left(x^2-x+1\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
```

```
[Out] int(1/(x^2*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.516 $\int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{2}{3x^2\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{7(1+x^3)}{6x^2\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{7\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(\sin^{-1}\Big(}{6\sqrt[4]{3}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1-x}}$$

```
[Out] 2/3/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-7/6*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(
1/2)-7/18*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+
1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {929, 296, 331, 224}

$$-\frac{7\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{2}{3x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{7(x^3+1)}{6x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] 2/(3*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (7*(1 + x^3))/(6*x^2*Sqrt[1 + x]*
Sqrt[1 - x + x^2]) - (7*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -
7 - 4*Sqrt[3]])/(6*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3(1+x^3)^{3/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{x^3\sqrt{1+x^3}}\,dx}{3\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7(1+x^3)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{\left(7\sqrt{1+x^3}\right.}{12\sqrt{1+x}}$$

$$= \frac{2}{3x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7(1+x^3)}{6x^2\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{7\sqrt{2+\sqrt{3}}}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.25, size = 170, normalized size = 1.00

$$\frac{-\frac{6(3+7x^3)}{x^2\sqrt{1+x}}-\frac{7i(1+x)\sqrt{1+\frac{6i}{(-3i+\sqrt{3})(1+x)}}\sqrt{6-\frac{36i}{(3i+\sqrt{3})(1+x)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{36\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)),x]
```

```
[Out] ((-6*(3 + 7*x^3))/(x^2*Sqrt[1 + x]) - ((7*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I
+ Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I
*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I -
Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(36*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.11, size = 259, normalized size = 1.52

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{2x}{3\sqrt{x^3+1}}-\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{6\sqrt{x^3+1}}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}}$
risch	$-\frac{7x^3+3}{6x^2\sqrt{1+x}\sqrt{x^2-x+1}}-\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.}{6\sqrt{x^3+1}\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$\frac{\sqrt{1+x}\sqrt{x^2-x+1}\left(7i\sqrt{3}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(7*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(
1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*x^2*((I*3^(1/2)+2*x-1)/
(-3+I*3^(1/2)))^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I
```

```
*3^(1/2)+3))^(1/2)-21*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I
*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(
1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2-14*x^
3-6)/(x^3+1)/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 48, normalized size = 0.28

$$-\frac{(7\,x^3+3)\sqrt{x^2-x+1}\,\sqrt{x+1}\,+7\,(x^5+x^2)\mathrm{weierstrassPInverse}(0,-4,x)}{6\,(x^5+x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="fricas")
[Out] -1/6*((7*x^3 + 3)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 7*(x^5 + x^2)*weierstrass
PInverse(0, -4, x))/(x^5 + x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\,(x+1)^{\frac{3}{2}}\,(x^2-x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)
[Out] Integral(1/(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,(x+1)^{3/2}\,(x^2-x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)),x)
[Out] int(1/(x^3*(x + 1)^(3/2)*(x^2 - x + 1)^(3/2)), x)
```

3.517 $\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$\frac{4x}{27\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}+\frac{4\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(\mathrm{s}}{27\sqrt[4]{3}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

```
[Out] 4/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/9*x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2
)+4/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^
(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {929, 294, 205, 224}

$$\frac{4\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{4x}{27\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{2x}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (4*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) - (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
+ x^2]*(1 + x^3)) + (4*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x
^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x^3}{(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= -\frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right)\int \frac{1}{(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(2\sqrt{1+x^3}\right.}{27\sqrt{1+}}$$

$$= \frac{4x}{27\sqrt{1+x}\sqrt{1-x+x^2}} - \frac{2x}{9\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{4\sqrt{2+\sqrt{3}}}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 30.31, size = 178, normalized size = 1.06

$$\frac{\frac{6x(-1+2x^3)}{(1+x)^{3/2}(1-x+x^2)}+\frac{2i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((6*x*(-1 + 2*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((2*I)*(1 + x)*Sqrt[1 +
 (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x)
)]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sq
rt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(135) = 270.
time = 0.10, size = 467, normalized size = 2.78

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{2x}{9(x^3+1)^{\frac{3}{2}}}+\frac{4x}{27\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{27\sqrt{x^3+1}}\right.}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$
default	$-\frac{2\left(i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^3\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}-3\,\mathrm{Elli}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27*(I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1
/2)+3))^(1/2))*3^(1/2)*x^3*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1
/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)-3*Elliptic
F((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^
3*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2))
)^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)+I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))
^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(
```

```
1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*
3^(1/2)+3))^(1/2))-3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*
3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1
+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x^4+x)/(
x^2-x+1)^(3/2)/(1+x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 56, normalized size = 0.33

$$\frac{2\left((2x^4-x)\sqrt{x^2-x+1}\sqrt{x+1}+2(x^6+2x^3+1)\mathrm{weierstrassPInverse}(0,-4,x)\right)}{27(x^6+2x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
[Out] 2/27*((2*x^4 - x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 2*(x^6 + 2*x^3 + 1)*weier
strassPInverse(0, -4, x))/(x^6 + 2*x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x+1)^{\frac{5}{2}}(x^2-x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(x**3/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(x+1)^{5/2}\,(x^2-x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(x^3/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.518 $\int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

```
[Out] -2/9/(1+x)^(3/2)/(x^2-x+1)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {927}

$$-\frac{2}{9(x+1)^{3/2}\left(x^2-x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))
```

Rule 927

```
Int[(x_)^2*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(c*e*(
m + 2*p + 3))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*e*(m + p + 2
) + 2*c*d*(p + 1), 0] && EqQ[b*d*(p + 1) + a*e*(m + 1), 0] && NeQ[m + 2*p +
 3, 0]
```

Rubi steps

$$\int \frac{x^2}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = -\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

Mathematica [A]
time = 10.03, size = 23, normalized size = 1.00

$$-\frac{2}{9(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

[Out] -2/(9*(1 + x)^(3/2)*(1 - x + x^2)^(3/2))

Maple [A]
time = 0.12, size = 25, normalized size = 1.09

method	result	size
gosper	$-\frac{2}{9(1+x)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}}$	18
default	$-\frac{2}{9(x^3+1)\sqrt{1+x}\,\sqrt{x^2-x+1}}$	25
elliptic	$-\frac{2\sqrt{(1+x)(x^2-x+1)}}{9\sqrt{1+x}\,\sqrt{x^2-x+1}\,(x^3+1)^{\frac{3}{2}}}$	39

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/9/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)

Maxima [A]
time = 0.48, size = 24, normalized size = 1.04

$$-\frac{2}{9\,(x^3+1)\sqrt{x^2-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")

[Out] -2/9/((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1))

Fricas [A]
time = 1.01, size = 29, normalized size = 1.26

$$-\frac{2\sqrt{x^2-x+1}\,\sqrt{x+1}}{9\,(x^6+2\,x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")

[Out] -2/9*sqrt(x^2 - x + 1)*sqrt(x + 1)/(x^6 + 2*x^3 + 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x+1)^{\frac{5}{2}}\,(x^2-x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(x**2/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [B]
time = 2.88, size = 82, normalized size = 3.57

$$\frac{18\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}-18\,x\,\sqrt{x+1}\,\left(x^2-x+1\right)^{5/2}}{(x+1)\,\left(81\,x\,\left(x^2-x+1\right)^4-162\,\left(x^2-x+1\right)^4+81\,\left(x^2-x+1\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] (18*(x + 1)^(1/2)*(x^2 - x + 1)^(5/2) - 18*x*(x + 1)^(1/2)*(x^2 - x + 1)^(5
/2))/((x + 1)*(81*x*(x^2 - x + 1)^4 - 162*(x^2 - x + 1)^4 + 81*(x^2 - x + 1
)^5))
```

3.519 $\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=318

$$\frac{10x^2}{27\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{2x^2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}-\frac{10(1+x^3)}{27\sqrt{1+x}\,\left(1+\sqrt{3}+x\right)\sqrt{1-x+x^2}}+\frac{5\sqrt{\ldots}}{\ldots}$$

```
[Out] 10/27*x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x^2/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)
^(1/2)-10/27*(x^3+1)/(1+x+3^(1/2))/(1+x)^(1/2)/(x^2-x+1)^(1/2)-10/81*Ellipt
icF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(
1/2)+5/27*3^(1/4)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x
)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+
1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {823, 296, 309, 224, 1891}

$$-\frac{10\sqrt{2}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{5\sqrt{2-\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{10x^2}{27\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2x^2}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)}-\frac{10(x^3+1)}{27\sqrt{x+1}\,\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (10*x^2)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x^2)/(9*Sqrt[1 + x]*Sqrt[1
 - x + x^2]*(1 + x^3)) - (10*(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*S
qrt[1 - x + x^2]) + (5*Sqrt[2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(9*3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^
2]) - (10*Sqrt[2]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (b_.)*(x_) + (c_
.)*(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)
^FracPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(f + g*x)*(a*d + c*e*x^3)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[m, p] && EqQ[b*d +
a*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{x}{(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2x^2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(5\sqrt{1+x^3}\right)\int \frac{x}{(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{10x^2}{27\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} - \frac{\left(5\sqrt{1+\right.}{27\sqrt{1-}$$

$$= \frac{10x^2}{27\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} - \frac{\left(5\sqrt{1+\right.}{27\sqrt{1-}$$

$$= \frac{10x^2}{27\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2x^2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} - \frac{}{27\sqrt{1+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.49, size = 409, normalized size = 1.29

$$\frac{2x^2(8+5x^3)}{27(1+x)^{3/2}(1-x+x^2)^{3/2}} - \frac{5(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}(1-x+x^2)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}\right)}{162\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (2*x^2*(8 + 5*x^3))/(27*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) - (5*(1 + x)^(3/
2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]*(1
 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])]*Sqrt[(-
3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(
-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt
[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(
3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*Ell
ipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])
/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(162*Sqrt[(-I)/(3*I + Sqrt[3])]*Sqrt[1 - x
 + x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(252) = 504$.
time = 0.11, size = 688, normalized size = 2.16

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x^2}{9(x^3+1)^{\frac{3}{2}}}+\frac{10x^2}{27\sqrt{x^3+1}}-\frac{10\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\cdots}\right.}{\sqrt{1+x}\,\sqrt{x^2\cdots}}$
default	$-\frac{5i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^3\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}+15\,\mathrm{Ellipti}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*(5*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^
(1/2)+3))^(1/2))*3^(1/2)*x^3*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*(-2*(
1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)+15*Ellip
ticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))
*x^3*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/
2)))^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)-30*EllipticE((-2*(1+x)/(-3+I*3^(
1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^3*((I*3^(1/2)+2*x-1)/
(-3+I*3^(1/2)))^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I
*3^(1/2)+3))^(1/2)+5*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^
(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/
2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)-
10*x^5+15*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))
^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3
^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-30*(-2*(1+x)/(-3+I*3^
(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-
3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/
2))/(I*3^(1/2)+3))^(1/2))-16*x^2)/(x^2-x+1)^(3/2)/(1+x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 61, normalized size = 0.19

$$\frac{2\left((5\,x^5+8\,x^2)\sqrt{x^2-x+1}\,\sqrt{x+1}\,+5\,(x^6+2\,x^3+1)\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))\right)}{27\,(x^6+2\,x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*((5*x^5 + 8*x^2)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 5*(x^6 + 2*x^3 + 1)*w
eierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^6 + 2*x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x+1)^{\frac{5}{2}}\,(x^2-x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(x/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(x+1)^{5/2}\,(x^2-x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(x/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.520 $\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=168

$$\frac{14x}{27\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}+\frac{14\sqrt{2+\sqrt{3}}\,\sqrt{1+x}\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(\text{s}}{27\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{}}$$

```
[Out] 14/27*x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9*x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/
2)+14/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1
)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {727, 205, 224}

$$\frac{14\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{14x}{27\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2x}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] (14*x)/(27*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (2*x)/(9*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) + (14*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
-7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x +
 x^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 727

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^FracPart[p]/(a*d + c
*e*x^3)^FracPart[p]), Int[(d + e*x)^(m - p)*(a*d + c*e*x^3)^p, x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*d + a*e, 0] && EqQ[c*d + b*e, 0] &&
IGtQ[m - p + 1, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(7\sqrt{1+x^3}\right)\int \frac{1}{(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

$$= \frac{14x}{27\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(7\sqrt{1+\right.}{27\sqrt{1-}}$$

$$= \frac{14x}{27\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2x}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} + \frac{14\sqrt{2+}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.33, size = 178, normalized size = 1.06

$$\frac{\frac{6x(10+7x^3)}{(1+x)^{3/2}(1-x+x^2)} + \frac{7i(1+x)\sqrt{1+\frac{6i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{6-\frac{36i}{\left(3i+\sqrt{3}\right)(1+x)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{81\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]

[Out] ((6*x*(10 + 7*x^3))/((1 + x)^(3/2)*(1 - x + x^2)) + ((7*I)*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(81*Sqrt[1 - x + x^2])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(135) = 270$.
time = 0.12, size = 469, normalized size = 2.79

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2x}{9(x^3+1)^{\frac{3}{2}}}+\frac{14x}{27\sqrt{x^3+1}}+\frac{14\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{27\sqrt{x^3+1}}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$-\frac{7i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^3\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}-21\,\mathrm{Ellipti}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/27*(7*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*x^3*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)-21*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^3*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)+7*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)-21*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-14*x^4-20*x)/(x^2-x+1)^(3/2)/(1+x)^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 56, normalized size = 0.33

$$\frac{2\left((7x^4+10x)\sqrt{x^2-x+1}\sqrt{x+1}+7(x^6+2x^3+1)\text{weierstrassPInverse}(0,-4,x)\right)}{27(x^6+2x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*((7*x^4 + 10*x)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 7*(x^6 + 2*x^3 + 1)*we
ierstrassPInverse(0, -4, x))/(x^6 + 2*x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)^{\frac{5}{2}}(x^2-x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x+1)^{5/2}(x^2-x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(1/((x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.521 $\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,\left(1+x^3\right)}-\frac{2\sqrt{1+x^3}\,\tanh^{-1}\left(\sqrt{1+x^3}\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}}$$

```
[Out] 2/3/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/2)-2/3
*arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)/(1+x)^(1/2)/(x^2-x+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {929, 272, 53, 65, 213}

$$\frac{2}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2}{9\sqrt{x+1}\,\sqrt{x^2-x+1}\,\left(x^3+1\right)}-\frac{2\sqrt{x^3+1}\,\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\,\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 2/(3*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*Sqrt[1 + x]*Sqrt[1 - x + x^2]*(1
 + x^3)) - (2*Sqrt[1 + x^3]*ArcTanh[Sqrt[1 + x^3]])/(3*Sqrt[1 + x]*Sqrt[1 -
 x + x^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx &= \frac{\sqrt{1+x^3}\int \frac{1}{x(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{5/2}}\,dx, x, x^3\right)}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\sqrt{1+x^3}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx, x, x^3\right.}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\sqrt{1+x^3}}{3} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(2\sqrt{1+\right.}{} \\
&= \frac{2}{3\sqrt{1+x}\,\sqrt{1-x+x^2}} + \frac{2}{9\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)} - \frac{2\sqrt{1+x}}{3\sqrt{1}}
\end{aligned}$$

Mathematica [A]

time = 10.09, size = 95, normalized size = 0.99

$$\frac{\frac{2(4+3x^3)}{3(1+x)^{3/2}(1-x+x^2)} - 2(1+x)\sqrt{\frac{1-x+x^2}{(1+x)^2}}\tanh^{-1}\left(\frac{1}{(1+x)^{3/2}\sqrt{\frac{1-x+x^2}{(1+x)^2}}}\right)}{3\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((2*(4 + 3*x^3))/(3*(1 + x)^(3/2)*(1 - x + x^2)) - 2*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + x)^2]*ArcTanh[1/((1 + x)^(3/2)*Sqrt[(1 - x + x^2)/(1 + x)^2])])/(
3*Sqrt[1 - x + x^2])
```

Maple [A]
time = 0.11, size = 69, normalized size = 0.72

method	result	size
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(\frac{2}{9(x^3+1)^{\frac{3}{2}}}+\frac{2}{3\sqrt{x^3+1}}-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}\right)}{\sqrt{1+x}\,\sqrt{x^2-x+1}}$	60
default	$-\frac{2\left(3\sqrt{x^3+1}\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)x^3-3x^3+3\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}-4\right)}{9(x^3+1)\sqrt{x^2-x+1}\,\sqrt{1+x}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(3*(x^3+1)^(1/2)*arctanh((x^3+1)^(1/2))*x^3-3*x^3+3*arctanh((x^3+1)^(1
/2))*(x^3+1)^(1/2)-4)/(x^3+1)/(x^2-x+1)^(1/2)/(1+x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

Fricas [A]
time = 1.41, size = 101, normalized size = 1.05

$$\frac{2\left(3x^3+4\right)\sqrt{x^2-x+1}\,\sqrt{x+1}-3\left(x^6+2x^3+1\right)\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}+1\right)+3\left(x^6+2x^3+1\right)\log\left(\sqrt{x^2-x+1}\,\sqrt{x+1}-1\right)}{9\left(x^6+2x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(3*x^3 + 4)*sqrt(x^2 - x + 1)*sqrt(x + 1) - 3*(x^6 + 2*x^3 + 1)*log(
sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 3*(x^6 + 2*x^3 + 1)*log(sqrt(x^2 - x +
 1)*sqrt(x + 1) - 1))/(x^6 + 2*x^3 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(x+1)^{\frac{5}{2}}\,(x^2-x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/(x*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,(x+1)^{5/2}\,(x^2-x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(1/(x*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.522 $\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=349

$$\frac{22}{27x\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{2}{9x\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}-\frac{55(1+x^3)}{27x\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{55}{27\sqrt{1+x}\,\left(1+\right.}$$

```
[Out] 22/27/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)^(1/
2)-55/27*(x^3+1)/x/(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/27*(x^3+1)/(1+x+3^(1/2))/
(1+x)^(1/2)/(x^2-x+1)^(1/2)+55/81*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3
^(1/2)+2*I)*2^(1/2)*(1+x)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(
x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-55/54*3^(1/4)*EllipticE((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*
((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(
1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {929, 296, 331, 309, 224, 1891}

$$\frac{55\sqrt{2}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}-\frac{55\sqrt{2-\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{22}{27x\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{55(x^3+1)}{27x\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{55(x^3+1)}{27\sqrt{x+1}\,\left(x+\sqrt{3}+1\right)\sqrt{x^2-x+1}}+\frac{2}{9x\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 22/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x*Sqrt[1 + x]*Sqrt[1 - x + x
^2]*(1 + x^3)) - (55*(1 + x^3))/(27*x*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + (55*
(1 + x^3))/(27*Sqrt[1 + x]*(1 + Sqrt[3] + x)*Sqrt[1 - x + x^2]) - (55*Sqrt[
2 - Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2]) + (55*Sqrt[2]*Sqrt[1 +
x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] +
x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^2\left(1+x^3\right)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(11\sqrt{1+x^3}\right)\int \frac{1}{x^2\left(1+x^3\right)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} + \frac{\left(55\sqrt{\ }\right.}{27}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{}{27x\sqrt{\ }}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{}{27x\sqrt{\ }}$$

$$= \frac{22}{27x\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x\sqrt{1+x}\sqrt{1-x+x^2}\left(1+x^3\right)} - \frac{}{27x\sqrt{\ }}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.50, size = 414, normalized size = 1.19

$$-\frac{27+88x^3+55x^6}{27x(1+x)^{3/2}\left(1-x+x^2\right)^{3/2}} + \frac{55(1+x)^{3/2}\left(\frac{12\sqrt{-\frac{i}{3i+\sqrt{3}}}\left(1-x+x^2\right)}{(1+x)^2} + \frac{3\sqrt{2}\left(1-i\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}} + \frac{i\sqrt{2}\left(3i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}-\frac{6i}{1+x}}{3i+\sqrt{3}}}\sqrt{\frac{-3i+\sqrt{3}+\frac{6i}{1+x}}{-3i+\sqrt{3}}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{1+x}}\right)}{324\sqrt{-\frac{i}{3i+\sqrt{3}}}\sqrt{1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] -1/27*(27 + 88*x^3 + 55*x^6)/(x*(1 + x)^(3/2)*(1 - x + x^2)^(3/2)) + (55*(1
 + x)^(3/2)*((12*Sqrt[(-I)/(3*I + Sqrt[3])]*(1 - x + x^2))/(1 + x)^2 + (3*S
qrt[2]*(1 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3])
```

```
]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSi
nh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3
])]])/Sqrt[1 + x] + (I*Sqrt[2]*(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(
1 + x))/(3*I + Sqrt[3])]*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt
[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I +
 Sqrt[3])/(3*I - Sqrt[3])]])/Sqrt[1 + x]))/(324*Sqrt[(-I)/(3*I + Sqrt[3])]*S
qrt[1 - x + x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(277) = 554$.
time = 0.12, size = 695, normalized size = 1.99

method	result
elliptic	$\frac{\sqrt{(1+x)(x^2-x+1)}\left(-\frac{\sqrt{x^3+1}}{x}-\frac{2x^2}{9(x^3+1)^{\frac{3}{2}}}-\frac{28x^2}{27\sqrt{x^3+1}}+\frac{55\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}\right.}{\sqrt{1+\cdots}}$
default	$\frac{55i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^4\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}+165\,\mathrm{Ellip}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(55*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^
(1/2)+3))^(1/2))*3^(1/2)*x^4*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*
x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)+165*Elli
pticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)
)*x^4*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/
2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)-330*EllipticE((-2*(1+x)/(-3+I*3
^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^4*(-2*(1+x)/(-3+I*3
^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(
-3+I*3^(1/2)))^(1/2)+55*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I
*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*x*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*(
(I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(
1/2)-110*x^6+165*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1
/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/
(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-330*(-2*(1+x
)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)
+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(
```

```
-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-176*x^3-54)/x/(x^2-x+1)^(3/2)/(1+x)^(
3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.30, size = 62, normalized size = 0.18

$$-\frac{(55\,x^6+88\,x^3+27)\sqrt{x^2-x+1}\,\sqrt{x+1}\,+55\,(x^7+2\,x^4+x)\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))}{27\,(x^7+2\,x^4+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/27*((55*x^6 + 88*x^3 + 27)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 55*(x^7 + 2*x
^4 + x)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)))/(x^7 + 2*x^4
 + x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\,(x+1)^{\frac{5}{2}}\,(x^2-x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 (x+1)^{5/2} (x^2-x+1)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
[Out] int(1/(x^2*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.523 $\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$\frac{26}{27x^2\sqrt{1+x}\,\sqrt{1-x+x^2}}+\frac{2}{9x^2\sqrt{1+x}\,\sqrt{1-x+x^2}\,(1+x^3)}-\frac{91(1+x^3)}{54x^2\sqrt{1+x}\,\sqrt{1-x+x^2}}-\frac{91\sqrt{2+\sqrt{3}}}{\ldots}$$

```
[Out] 26/27/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)+2/9/x^2/(x^3+1)/(1+x)^(1/2)/(x^2-x+1)
^(1/2)-91/54*(x^3+1)/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2)-91/162*EllipticF((1+x-
3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1+x)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))
*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^2-x+1)^(1/2)/((1+x)/(1+x+3^(1
/2))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {929, 296, 331, 224}

$$-\frac{91\sqrt{2+\sqrt{3}}\,\sqrt{x+1}\,\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{54\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^2-x+1}}+\frac{26}{27x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}-\frac{91(x^3+1)}{54x^2\sqrt{x+1}\,\sqrt{x^2-x+1}}+\frac{2}{9x^2\sqrt{x+1}\,\sqrt{x^2-x+1}\,(x^3+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] 26/(27*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]) + 2/(9*x^2*Sqrt[1 + x]*Sqrt[1 - x
 + x^2]*(1 + x^3)) - (91*(1 + x^3))/(54*x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2])
- (91*Sqrt[2 + Sqrt[3]]*Sqrt[1 + x]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]
*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5
4*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 - x + x^2])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 929

```
Int[((g_.)*(x_))^(n_)*((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*
(x_)^2)^(p_), x_Symbol] :> Dist[(d + e*x)^FracPart[p]*((a + b*x + c*x^2)^Fr
acPart[p]/(a*d + c*e*x^3)^FracPart[p]), Int[(g*x)^n*(a*d + c*e*x^3)^p, x],
x] /; FreeQ[{a, b, c, d, e, g, m, n, p}, x] && EqQ[m - p, 0] && EqQ[b*d + a
*e, 0] && EqQ[c*d + b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3(1+x)^{5/2}\left(1-x+x^2\right)^{5/2}}\,dx = \frac{\sqrt{1+x^3}\int \frac{1}{x^3(1+x^3)^{5/2}}\,dx}{\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{\left(13\sqrt{1+x^3}\right)\int \frac{1}{x^3(1+x^3)^{3/2}}\,dx}{9\sqrt{1+x}\sqrt{1-x+x^2}}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} + \frac{(9 \ldots}{\ldots}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{\ldots}{54 \ldots}$$

$$= \frac{26}{27x^2\sqrt{1+x}\sqrt{1-x+x^2}} + \frac{2}{9x^2\sqrt{1+x}\sqrt{1-x+x^2}\,(1+x^3)} - \frac{\ldots}{54 \ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.43, size = 183, normalized size = 0.90

$$-\frac{\frac{6(27+130x^3+91x^6)}{x^2(1+x)^{3/2}}-\frac{91i(1+x)(1-x+x^2)\sqrt{6+\frac{36i}{\left(-3i+\sqrt{3}\right)(1+x)}}\sqrt{1-\frac{6i}{\left(3i+\sqrt{3}\right)(1+x)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{1+x}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{3i+\sqrt{3}}}}}{324\left(1-x+x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x)^(5/2)*(1 - x + x^2)^(5/2)),x]
```

```
[Out] ((-6*(27 + 130*x^3 + 91*x^6))/(x^2*(1 + x)^(3/2)) - ((91*I)*(1 + x)*(1 - x
+ x^2)*Sqrt[6 + (36*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[1 - (6*I)/((3*I + S
qrt[3])*(1 + x))]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 +
 x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[(-I)/(3*I + Sqrt[3])])/(324*(1
 - x + x^2)^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(164) = 328.
time = 0.13, size = 481, normalized size = 2.37

method	result
elliptic	$\frac{\sqrt{(1+x)\left(x^2-x+1\right)}\left(-\frac{\sqrt{x^3+1}}{2x^2}-\frac{2x}{9\left(x^3+1\right)^{\frac{3}{2}}}-\frac{32x}{27\sqrt{x^3+1}}-\frac{91\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{5}\right.}{\sqrt{1+x}\sqrt{x^2-x+1}}$
default	$91i\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\,x^5\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}-273\,\mathrm{Ellipti}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/108*(91*I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3
^(1/2)+3))^(1/2))*3^(1/2)*x^5*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2
*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)-273*Ell
ipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2
))*x^5*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1
```

```
/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)+91*I*EllipticF((-2*(1+x)/(-3+I
*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*x^2*(-2*(1+
x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2
)+2*x-1)/(-3+I*3^(1/2)))^(1/2)-273*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1
/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*El
lipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/
2))*x^2-182*x^6-260*x^3-54)/x^2/(x^2-x+1)^(3/2)/(1+x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 63, normalized size = 0.31

$$-\frac{(91\,x^6+130\,x^3+27)\sqrt{x^2-x+1}\,\sqrt{x+1}\,+91\,(x^8+2\,x^5+x^2)\mathrm{weierstrassPInverse}(0,-4,x)}{54\,(x^8+2\,x^5+x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="fricas")
[Out] -1/54*((91*x^6 + 130*x^3 + 27)*sqrt(x^2 - x + 1)*sqrt(x + 1) + 91*(x^8 + 2*
x^5 + x^2)*weierstrassPInverse(0, -4, x))/(x^8 + 2*x^5 + x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\,(x+1)^{\frac{5}{2}}\,(x^2-x+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)
[Out] Integral(1/(x**3*(x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(x+1\right)}^{5/2}\,{\left(x^2-x+1\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)),x)
```

```
[Out] int(1/(x^3*(x + 1)^(5/2)*(x^2 - x + 1)^(5/2)), x)
```

3.524 $\int \frac{x}{(-1+x)^3(3+5x+4x^2)^2} dx$

Optimal. Leaf size=97

$$-\frac{21}{736(1-x)^2}-\frac{97}{4416(1-x)}+\frac{39+44x}{276(1-x)^2(3+5x+4x^2)}+\frac{6023\tan^{-1}\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}}+\frac{11\log(1-x)}{2304}-\frac{11\log(3+$$

```
[Out] -21/736/(1-x)^2-97/4416/(1-x)+1/276*(39+44*x)/(1-x)^2/(4*x^2+5*x+3)+11/2304
*ln(1-x)-11/4608*ln(4*x^2+5*x+3)+6023/1218816*arctan(1/23*(5+8*x)*23^(1/2))
*23^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {836, 814, 648, 632, 210, 642}

$$\frac{6023\text{ArcTan}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}+\frac{44x+39}{276(1-x)^2(4x^2+5x+3)}-\frac{11\log(4x^2+5x+3)}{4608}-\frac{97}{4416(1-x)}-\frac{21}{736(1-x)^2}+\frac{11\log(1-x)}{2304}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -21/(736*(1 - x)^2) - 97/(4416*(1 - x)) + (39 + 44*x)/(276*(1 - x)^2*(3 + 5
*x + 4*x^2)) + (6023*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23]) + (11*Log
[1 - x])/2304 - (11*Log[3 + 5*x + 4*x^2])/4608
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{x}{(-1+x)^3 (3+5x+4x^2)^2} \, dx = \frac{39+44x}{276(1-x)^2 (3+5x+4x^2)} + \frac{1}{276} \int \frac{57+132x}{(-1+x)^3 (3+5x+4x^2)} \, dx$$

$$= \frac{39+44x}{276(1-x)^2 (3+5x+4x^2)} + \frac{1}{276} \int \left(\frac{63}{4(-1+x)^3} - \frac{97}{16(-1+x)^2} + \text{1!}\right.$$

$$= -\frac{21}{736(1-x)^2} - \frac{97}{4416(1-x)} + \frac{39+44x}{276(1-x)^2 (3+5x+4x^2)} + \frac{11 \log(1}{2304}$$

$$= -\frac{21}{736(1-x)^2} - \frac{97}{4416(1-x)} + \frac{39+44x}{276(1-x)^2 (3+5x+4x^2)} + \frac{11 \log(1}{2304}$$

$$= -\frac{21}{736(1-x)^2} - \frac{97}{4416(1-x)} + \frac{39+44x}{276(1-x)^2 (3+5x+4x^2)} + \frac{11 \log(1}{2304}$$

$$= -\frac{21}{736(1-x)^2} - \frac{97}{4416(1-x)} + \frac{39+44x}{276(1-x)^2 (3+5x+4x^2)} + \frac{6023 \tan}{5299}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.80

$$\frac{-\frac{25392}{(-1+x)^2} + \frac{59248}{-1+x} + \frac{184(975+2204x)}{3+5x+4x^2} + 36138\sqrt{23} \tan^{-1}\left(\frac{5+8x}{\sqrt{23}}\right) + 34914 \log(1-x) - 17457 \log(3+5x+4x^2)}{7312896}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] (-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4*x^2
) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 17457*Lo
g[3 + 5*x + 4*x^2])/7312896
```

Maple [A]
time = 0.20, size = 68, normalized size = 0.70

method	result
default	$-\frac{1}{288(-1+x)^2} + \frac{7}{864(-1+x)} + \frac{11 \ln(-1+x)}{2304} - \frac{-\frac{2204x}{23} - \frac{975}{23}}{6912(x^2+\frac{5}{4}x+\frac{3}{4})} - \frac{11 \ln(4x^2+5x+3)}{4608} + \frac{6023 \arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{2}}{1218816}$
risch	$\frac{\frac{97}{1104}x^3 - \frac{407}{4416}x^2 - \frac{5}{184}x - \frac{15}{1472}}{(-1+x)^2(4x^2+5x+3)} - \frac{11 \ln(64x^2+80x+48)}{4608} + \frac{6023 \arctan\left(\frac{(5+8x)\sqrt{23}}{23}\right)\sqrt{23}}{1218816} + \frac{11 \ln(-1+x)}{2304}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-1+x)^3/(4*x^2+5*x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/288/(-1+x)^2+7/864/(-1+x)+11/2304*ln(-1+x)-1/6912*(-2204/23*x-975/23)/(x
^2+5/4*x+3/4)-11/4608*ln(4*x^2+5*x+3)+6023/1218816*arctan(1/23*(5+8*x)*23^(
1/2))*23^(1/2)
```

Maxima [A]
time = 0.50, size = 75, normalized size = 0.77

$$\frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(8\,x+5)\right)+\frac{388\,x^3-407\,x^2-120\,x-45}{4416\,(4\,x^4-3\,x^3-3\,x^2-x+3)}-\frac{11}{4608}\log\left(4\,x^2+5\,x+3\right)+\frac{11}{2304}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

```
[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 11/4608*log(4*x^2 +
5*x + 3) + 11/2304*log(x - 1)
```

Fricas [A]
time = 1.14, size = 134, normalized size = 1.38

$$\frac{214176\,x^3+12046\,\sqrt{23}\,(4\,x^4-3\,x^3-3\,x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}\,(8\,x+5)\right)-224664\,x^2-5819\,(4\,x^4-3\,x^3-3\,x^2-x+3)\log\left(4\,x^2+5\,x+3\right)+11638\,(4\,x^4-3\,x^3-3\,x^2-x+3)\log\left(x-1\right)-66240\,x-24840}{2437632\,(4\,x^4-3\,x^3-3\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arct
an(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 - x
+ 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x - 1
) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

Sympy [A]
time = 0.10, size = 88, normalized size = 0.91

$$\frac{388x^3-407x^2-120x-45}{17664x^4-13248x^3-13248x^2-4416x+13248}+\frac{11\log\left(x-1\right)}{2304}-\frac{11\log\left(x^2+\frac{5x}{4}+\frac{3}{4}\right)}{4608}+\frac{6023\sqrt{23}\operatorname{atan}\left(\frac{8\sqrt{23}\,x}{23}+\frac{5\sqrt{23}}{23}\right)}{1218816}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] (388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2 -
4416*x + 13248) + 11*log(x - 1)/2304 - 11*log(x**2 + 5*x/4 + 3/4)/4608 + 60
23*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

Giac [A]
time = 1.53, size = 71, normalized size = 0.73

$$\frac{6023}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(8\,x+5)\right)+\frac{388\,x^3-407\,x^2-120\,x-45}{4416\,(4\,x^2+5\,x+3)(x-1)^2}-\frac{11}{4608}\log\left(4\,x^2+5\,x+3\right)+\frac{11}{2304}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")

[Out] 6023/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 1/4416*(388*x^3 - 4
07*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 11/4608*log(4*x^2 + 5*
x + 3) + 11/2304*log(abs(x - 1))

Mupad [B]
time = 0.13, size = 84, normalized size = 0.87

$$\frac{11\ln(x-1)}{2304}+\frac{-\frac{97x^3}{4416}+\frac{407x^2}{17664}+\frac{5x}{736}+\frac{15}{5888}}{-x^4+\frac{3x^3}{4}+\frac{3x^2}{4}+\frac{x}{4}-\frac{3}{4}}-\ln\left(x+\frac{5}{8}-\frac{\sqrt{23}\,1\mathrm{i}}{8}\right)\left(\frac{11}{4608}+\frac{\sqrt{23}\,6023\mathrm{i}}{2437632}\right)+\ln\left(x+\frac{5}{8}+\frac{\sqrt{23}\,1\mathrm{i}}{8}\right)\left(-\frac{11}{4608}+\frac{\sqrt{23}\,6023\mathrm{i}}{2437632}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x - 1)^3*(5*x + 4*x^2 + 3)^2),x)

[Out] (11*log(x - 1))/2304 + ((5*x)/736 + (407*x^2)/17664 - (97*x^3)/4416 + 15/58
88)/(x/4 + (3*x^2)/4 + (3*x^3)/4 - x^4 - 3/4) - log(x - (23^(1/2)*1i)/8 + 5
/8)*((23^(1/2)*6023i)/2437632 + 11/4608) + log(x + (23^(1/2)*1i)/8 + 5/8)*(
(23^(1/2)*6023i)/2437632 - 11/4608)

3.525 $\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=490

$$-\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))\,(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+ex)^{5/2}}{5c^2e^3}+\frac{2(d+ex)^{7/2}}{7ce^3}+$$

```
[Out] 2/3*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*(e*x+d)^(3/2)/c^3/e^3-2/5*(b*e+2*c*d)*
(e*x+d)^(5/2)/c^2/e^3+2/7*(e*x+d)^(7/2)/c/e^3-2*b*(-2*a*c+b^2)*(e*x+d)^(1/2
)/c^4+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2))*2^(1/2)*(b^3*c*d-2*a*b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(5*a^2*b*
c^2*e-2*a^2*c^3*d-5*a*b^3*c*e+4*a*b^2*c^2*d+b^5*e-b^4*c*d)/(-4*a*c+b^2)^(1/
2))/c^(9/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*c*d-2*a*
b*c^2*d-b^4*e+3*a*b^2*c*e-a^2*c^2*e+(-5*a^2*b*c^2*e+2*a^2*c^3*d+5*a*b^3*c*e
-4*a*b^2*c^2*d-b^5*e+b^4*c*d)/(-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-e*(b+(-4*a
*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 12.35, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {911, 1301, 1180, 214}

$$\frac{\sqrt{2}\left(-\frac{-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{-5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}}-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4}+\frac{2(d+ex)^{3/2}(ce(bd-ae)+b^2e^2+c^2d^2)}{3c^3e^3}-\frac{2(d+ex)^{5/2}(be+2cd)}{5c^2e^3}+\frac{2(d+ex)^{7/2}}{7ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*(b^2 - 2*a*c)*Sqrt[d + e*x])/c^4 + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d -
 a*e))*(d + e*x)^(3/2))/(3*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^(5/2))/(5*
c^2*e^3) + (2*(d + e*x)^(7/2))/(7*c*e^3) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d
- b^4*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d
- b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b^3*c*d - 2*a*b*c^2*d -
 b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d -
 b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{(b^3-2abc)e}{c^4}+\frac{(c^2d^2+b^2e^2+ce(bd-ae))x^2}{c^3e^2}-\frac{(2cd+be)x^4}{c^2e^2}+\frac{x^6}{ce^2}+\frac{b(b^2-2ac)(cd^2-bde+ae^2)}{c^4e\left(\frac{cd^2-bd}{e}\right.}\right.\right.}{e}$$

$$= -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+}{5c^2e^3}$$

$$= -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+}{5c^2e^3}$$

$$= -\frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))(d+ex)^{3/2}}{3c^3e^3}-\frac{2(2cd+be)(d+}{5c^2e^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.89, size = 627, normalized size = 1.28

$$\frac{2\sqrt{d+ex}\left(-105b^3e^3-7c^2e(d+ex)(-2bd+5ae+3bex)+c^3(8d^3-4d^2ex+3de^2x^2+15e^3x^3)+35bce^2(6ae+b(d+ex))\right)}{105c^4e^3}+\frac{\left(ib^5e-b^3c\left(\sqrt{-b^2+4ac}\,d+5iae\right)+abc^2\left(2\sqrt{-b^2+4ac}\,d+5iae\right)+ab^2c\left(4icd-3\sqrt{-b^2+4ac}\,e\right)+b^4\left(-icd+\sqrt{-b^2+4ac}\,e\right)+a^2c^2\left(-2icd+\sqrt{-b^2+4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{c^{9/2}\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(-ib^5e+abc^2\left(2\sqrt{-b^2+4ac}\,d-5iae\right)+b^3c\left(-\sqrt{-b^2+4ac}\,d+5iae\right)+ab^2c\left(-4icd-3\sqrt{-b^2+4ac}\,e\right)+b^4\left(icd+\sqrt{-b^2+4ac}\,e\right)+a^2c^2\left(2icd+\sqrt{-b^2+4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{c^{9/2}\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-105*b^3*e^3 - 7*c^2*e*(d + e*x)*(-2*b*d + 5*a*e + 3*b*e*
x) + c^3*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) + 35*b*c*e^2*(6*a*e
 + b*(d + e*x))))/(105*c^4*e^3) + ((I*b^5*e - b^3*c*(Sqrt[-b^2 + 4*a*c]*d +
 (5*I)*a*e) + a*b*c^2*(2*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e) + a*b^2*c*((4*I)
*c*d - 3*Sqrt[-b^2 + 4*a*c]*e) + b^4*((-I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + a^
2*c^2*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-1/2*b^2
 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((-I)*b^5*e + a*b
*c^2*(2*Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e) + b^3*c*(-(Sqrt[-b^2 + 4*a*c]*d)
+ (5*I)*a*e) + a*b^2*c*((-4*I)*c*d - 3*Sqrt[-b^2 + 4*a*c]*e) + b^4*(I*c*d +
 Sqrt[-b^2 + 4*a*c]*e) + a^2*c^2*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e))*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e
```

```
]])/(c^(9/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c]
)*e])
```

Maple [A]
time = 0.25, size = 708, normalized size = 1.44

method	result
derivativedivides	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^{3}}{7}-\frac{b\,c^{2}e(ex+d)^{\frac{5}{2}}}{5}-\frac{2c^{3}d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b^{2}c\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b\,c^{2}de(ex+d)^{\frac{3}{2}}}{3}+\frac{c^{3}d^{2}(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^{3}\sqrt{ex}\right.}{c^{4}}$
default	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^{3}}{7}-\frac{b\,c^{2}e(ex+d)^{\frac{5}{2}}}{5}-\frac{2c^{3}d(ex+d)^{\frac{5}{2}}}{5}-\frac{a\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b^{2}c\,e^{2}(ex+d)^{\frac{3}{2}}}{3}+\frac{b\,c^{2}de(ex+d)^{\frac{3}{2}}}{3}+\frac{c^{3}d^{2}(ex+d)^{\frac{3}{2}}}{3}+2abc\,e^{3}\sqrt{ex}\right.}{c^{4}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(1/c^4*(1/7*(e*x+d)^(7/2)*c^3-1/5*b*c^2*e*(e*x+d)^(5/2)-2/5*c^3*d*(e*
x+d)^(5/2)-1/3*a*c^2*e^2*(e*x+d)^(3/2)+1/3*b^2*c*e^2*(e*x+d)^(3/2)+1/3*b*c^
2*d*e*(e*x+d)^(3/2)+1/3*c^3*d^2*(e*x+d)^(3/2)+2*a*b*c*e^3*(e*x+d)^(1/2)-b^3
*e^3*(e*x+d)^(1/2))-4*e^3/c^3*(1/8*(-5*a^2*b*c^2*e^2+2*a^2*c^3*d*e+5*a*b^3*
e^2*c-4*a*b^2*c^2*d*e-b^5*e^2+b^4*c*d*e-(-e^2*(4*a*c-b^2))^(1/2)*a^2*c^2*e+
3*(-e^2*(4*a*c-b^2))^(1/2)*a*b^2*c*e-2*(-e^2*(4*a*c-b^2))^(1/2)*a*b*c^2*d-(
-e^2*(4*a*c-b^2))^(1/2)*b^4*e+(-e^2*(4*a*c-b^2))^(1/2)*b^3*c*d)/c/(-e^2*(4*
a*c-b^2))^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
an(c*(e*x+d)^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-
1/8*(5*a^2*b*c^2*e^2-2*a^2*c^3*d*e-5*a*b^3*e^2*c+4*a*b^2*c^2*d*e+b^5*e^2-b^
4*c*d*e-(-e^2*(4*a*c-b^2))^(1/2)*a^2*c^2*e+3*(-e^2*(4*a*c-b^2))^(1/2)*a*b^2
*c*e-2*(-e^2*(4*a*c-b^2))^(1/2)*a*b*c^2*d-(-e^2*(4*a*c-b^2))^(1/2)*b^4*e+(-
e^2*(4*a*c-b^2))^(1/2)*b^3*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-
e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*x^4/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5512 vs. $2(457) = 914$.
time = 2.60, size = 5512, normalized size = 11.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*c^4*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3
*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^
3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 +
 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*
a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^
9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d
*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8
*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(
b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*e^3*log(sqrt(2)*((b^12*c - 12*
a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^
2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4
*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e - (b^6*c^9 - 8*a*b^4*c^10 + 18*a
^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4
 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2
 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*
b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c
^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4
*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)
*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4
 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2
 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*
b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c
```

```
^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c
^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*
c^3 + a^8*c^4)*e)*sqrt(x*e + d)) - 105*sqrt(2)*c^4*sqrt(((b^8*c - 8*a*b^6*c
^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27
*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(
((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^
6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 +
67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*
a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 23
0*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a
^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*e^
3*log(-sqrt(2)*((b^12*c - 12*a*b^10*c^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4
+ 104*a^4*b^4*c^5 - 32*a^5*b^2*c^6)*d - (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^
2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e - (
b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^12)*sqrt(((b^14*c^2 - 12
*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*
b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3
 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4
*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
+ 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^
8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4
*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 -
30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e + (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12
*a*b^12*c^3 + 56*a^2*b^10*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*
b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3
 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4
*a^7*b*c^8)*d*e + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3
+ 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^
8*c^8)*e^2)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^4*b^7*c -
 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c +
 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e)*sqrt(x*e + d)) + 105*sqrt(2)
*c^4*sqrt(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c
^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e
 - (b^2*c^9 - 4*a*c^10)*sqrt(((b^14*c^2 - 12*a*b^12*c^3 + 56*a^2*b^10*c^4 -
 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 -
 2*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^
7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^16 - 14*a*
b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6
*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^18 - 4*a*c^1
9)))/(b^2*c^9 - 4*a*c^10))*e^3*log(sqrt(2)*((b^12*c - 12*a*b^10*c^2 + 54*a^
2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1171 vs. $2(457) = 914$.
time = 1.70, size = 1171, normalized size = 2.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d*e - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*
c^2 - 2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^2*c
^3)*sqrt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^2
)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d +
 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^
2 - (3*b^5*c^3 - 14*a*b^3*c^4 + 12*a^2*b*c^5)*d*e + (b^6*c^2 - 5*a*b^4*c^3
+ 5*a^2*b^2*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^24
 - b*c^7*e^25 + sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 + a*c^7*e^26)*c^8*e^24
 + (2*c^8*d*e^24 - b*c^7*e^25)^2))*e^(-24)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^
2 - sqrt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 1/4*(
sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^5*c - 6*a*b^3*c^2 + 8*
a^2*b*c^3)*d*e - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*c^2 +
2*((b^3*c^3 - 2*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^4*c^2 - 2*a*b^2*c^3)*sq
rt(b^2 - 4*a*c)*d*e + (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt
(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*
c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*d^2 - (3
*b^5*c^3 - 14*a*b^3*c^4 + 12*a^2*b*c^5)*d*e + (b^6*c^2 - 5*a*b^4*c^3 + 5*a^
2*b^2*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^24 - b*c
^7*e^25 - sqrt(-4*(c^8*d^2*e^24 - b*c^7*d*e^25 + a*c^7*e^26)*c^8*e^24 + (2*
c^8*d*e^24 - b*c^7*e^25)^2))*e^(-24)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sq
rt(b^2 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 2/105*(15*(
x*e + d)^(7/2)*c^6*e^18 - 42*(x*e + d)^(5/2)*c^6*d*e^18 + 35*(x*e + d)^(3/2
)*c^6*d^2*e^18 - 21*(x*e + d)^(5/2)*b*c^5*e^19 + 35*(x*e + d)^(3/2)*b*c^5*d
*e^19 + 35*(x*e + d)^(3/2)*b^2*c^4*e^20 - 35*(x*e + d)^(3/2)*a*c^5*e^20 - 1
05*sqrt(x*e + d)*b^3*c^3*e^21 + 210*sqrt(x*e + d)*a*b*c^4*e^21)*e^(-21)/c^7
```

Mupad [B]
time = 4.86, size = 2500, normalized size = 5.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)

[Out] (d + e*x)^(3/2)*((4*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(3*c^2*e^6) + (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(3*c*e^3)) - atan(((((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 - 50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2 - 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i - (((8*(a*b^5*c^5*e^4 + 8*a^3*b*c^7*e^4 - b^6*c^5*d*e^3 - 6*a^2*b^3*c^6*e^4 + b^5*c^6*d^2*e^2 + 6*a*b^4*c^6*d*e^3 - 6*a*b^3*c^7*d^2*e^2 + 8*a^2*b*c^8*d^2*e^2 - 8*a^2*b^2*c^7*d*e^3))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e

```
+ 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^
3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e*(-(
4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b
^3*c^9*e^3 - 2*b^2*c^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b
^11*e + 8*a^5*c^6*d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^
6*c^3*d + 96*a^3*b^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*
b^5*c^3*e + 129*a^4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b
^9*c*e + 12*a*b^8*c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 7*a*b^6*c*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^
(1/2) + 4*a^3*b*c^4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c
- b^2)^3)^(1/2) + 15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^
3*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(
1/2) + (8*(d + e*x)^(1/2)*(b^10*e^4 - 2*a^5*c^5*e^4 + 35*a^2*b^6*c^2*e^4 -
50*a^3*b^4*c^3*e^4 + 25*a^4*b^2*c^4*e^4 + 2*a^4*c^6*d^2*e^2 + b^8*c^2*d^2*e
^2 - 10*a*b^8*c*e^4 - 2*b^9*c*d*e^3 + 20*a^2*b^4*c^4*d^2*e^2 - 16*a^3*b^2*c
^5*d^2*e^2 + 18*a*b^7*c^2*d*e^3 - 18*a^4*b*c^5*d*e^3 - 8*a*b^6*c^3*d^2*e^2
- 54*a^2*b^5*c^3*d*e^3 + 60*a^3*b^3*c^4*d*e^3))/c^7)*(-(b^11*e + 8*a^5*c^6*
d + b^8*e*(-(4*a*c - b^2)^3)^(1/2) - b^10*c*d - 52*a^2*b^6*c^3*d + 96*a^3*b
^4*c^4*d - 66*a^4*b^2*c^5*d + 63*a^2*b^7*c^2*e - 138*a^3*b^5*c^3*e + 129*a^
4*b^3*c^4*e + a^4*c^4*e*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e + 12*a*b^8*
c^2*d - 36*a^5*b*c^5*e - b^7*c*d*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e*(-(
4*a*c - b^2)^3)^(1/2) + 6*a*b^5*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 4*a^3*b*c^
4*d*(-(4*a*c - b^2)^3)^(1/2) - 10*a^2*b^3*c^3*d*(-(4*a*c - b^2)^3)^(1/2) +
15*a^2*b^4*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*...
```

3.526 $\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=326

$$\frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}+\frac{\left(b^3-3abc-\sqrt{b^2-4ac}\,(b^2-ac)\right)\sqrt{2cd-(}}{\sqrt{2}}$$

```
[Out] -2/3*(b*e+c*d)*(e*x+d)^(3/2)/c^2/e^2+2/5*(e*x+d)^(5/2)/c/e^2+2*(-a*c+b^2)*(
e*x+d)^(1/2)/c^3+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*
a*c+b^2)^(1/2)))^(1/2))*(b^3-3*a*b*c-(-a*c+b^2)*(-4*a*c+b^2)^(1/2))*(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)-1/2*arct
anh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(
b^3-3*a*b*c+(-a*c+b^2)*(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
^(1/2)/c^(7/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 4.96, antiderivative size = 397, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {911, 1301, 1180, 214}

$$-\frac{\sqrt{2}\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2(b^2-ac)\sqrt{d+ex}}{c^3}-\frac{2(d+ex)^{3/2}(be+cd)}{3c^2e^2}+\frac{2(d+ex)^{5/2}}{5ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2 - a*c)*Sqrt[d + e*x])/c^3 - (2*(c*d + b*e)*(d + e*x)^(3/2))/(3*c^2*
e^2) + (2*(d + e*x)^(5/2))/(5*c*e^2) - (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e
+ 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/S
qrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- (Sqrt[2]*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d
- b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sq
rt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3} - \frac{(cd+be)x^2}{c^2e} + \frac{x^4}{ce} - \frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right) dx,\right.}{e}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3} - \frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2} + \frac{2(d+ex)^{5/2}}{5ce^2} - \frac{2\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-}{c\ldots}\right.}{}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3} - \frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2} + \frac{2(d+ex)^{5/2}}{5ce^2} + \frac{\left(b^2cd-ac^2d-b^3e+\right.}{}$$

$$= \frac{2(b^2-ac)\sqrt{d+ex}}{c^3} - \frac{2(cd+be)(d+ex)^{3/2}}{3c^2e^2} + \frac{2(d+ex)^{5/2}}{5ce^2} - \frac{\sqrt{2}\left(b^2cd-ac^2d-b\right.}{}$$

<u>Mathematica</u> [A]
time = 1.47, size = 465, normalized size = 1.43

$$\frac{\frac{2\sqrt{c}\sqrt{d+ex}\,(15b^2e^2+c^2(-2d^2+dex+3e^2x^2)-5ce(3ae+b(d+ex)))}{e^2} - \frac{15\sqrt{2}\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}\,d+4ae\right)+b^3\left(cd+\sqrt{b^2-4ac}\,e\right)-abc\left(3cd+2\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{15\sqrt{2}\left(b^4e+ac^2\left(\sqrt{b^2-4ac}\,d+2ae\right)-b^2c\left(\sqrt{b^2-4ac}\,d+4ae\right)+abc\left(3cd-2\sqrt{b^2-4ac}\,e\right)+b^3\left(-cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}}{15c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x + 3*e^2*x^2) -
5*c*e*(3*a*e + b*(d + e*x))))/e^2 - (15*Sqrt[2]*(-(b^4*e) + a*c^2*(Sqrt[b^2
 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(c*d +
Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[
2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2]*(b^4*e
 + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a*e
) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*c]
*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4
*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]))/(
15*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(273) = 546$.
time = 0.15, size = 559, normalized size = 1.71

method	result
derivativedivides	$-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{c^2d(ex+d)^{\frac{3}{2}}}{3}+ace^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}\right)}{c^3}+\frac{8e^2\left(\frac{\left(-2a^2c^2e^2+4ab^2e^2c-3abc^2de-\right.}{}\right.}{}$
default	$-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^2}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{c^2d(ex+d)^{\frac{3}{2}}}{3}+ace^2\sqrt{ex+d}-b^2e^2\sqrt{ex+d}\right)}{c^3}+\frac{8e^2\left(\frac{\left(-2a^2c^2e^2+4ab^2e^2c-3abc^2de-\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-1/c^3*(-1/5*(e*x+d)^(5/2)*c^2+1/3*b*c*e*(e*x+d)^(3/2)+1/3*c^2*d*(e*
x+d)^(3/2)+a*c*e^2*(e*x+d)^(1/2)-b^2*e^2*(e*x+d)^(1/2))+4/c^2*e^2*(1/8*(-2*
a^2*c^2*e^2+4*a*b^2*e^2*c-3*a*b*c^2*d*e-b^4*e^2+b^3*c*d*e+2*(-e^2*(4*a*c-b^
2))^(1/2)*a*b*c*e-(-e^2*(4*a*c-b^2))^(1/2)*a*c^2*d-(-e^2*(4*a*c-b^2))^(1/2)
*b^3*e+(-e^2*(4*a*c-b^2))^(1/2)*b^2*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1
/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(2*a^2*c^2*e^2-4*a*
b^2*e^2*c+3*a*b*c^2*d*e+b^4*e^2-b^3*c*d*e+2*(-e^2*(4*a*c-b^2))^(1/2)*a*b*c*
e-(-e^2*(4*a*c-b^2))^(1/2)*a*c^2*d-(-e^2*(4*a*c-b^2))^(1/2)*b^3*e+(-e^2*(4*
a*c-b^2))^(1/2)*b^2*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2*c*d+(-
e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-e*b+2*c
*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*x^3/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4258 vs. $2(281) = 562$.
time = 1.58, size = 4258, normalized size = 13.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*c^3*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4
)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4*a*c
^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4
*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 +
 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14
 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*e^2*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 +
 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 3
5*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e - (b^5*c^7 -
 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b
^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*
b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e
 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*
a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3
 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c
 + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*
a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6
*c^3)*e)*sqrt(x*e + d)) - 15*sqrt(2)*c^3*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2
*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*
e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24
*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^
3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6
*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*e^2*log(-sqrt(2)*((
b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (
b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^
5*c^5)*e - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(((b^10*c^2 - 8*a*b^8
```

```
*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9
*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6
)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^6*c -
 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e + (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3 +
 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^9
*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e
+ (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))
 + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c
+ 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(x*e + d)) + 15*sqrt(2)*c^3*sqrt(((b^6*c
- 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^
3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10*c^2 - 8*a*b^8*c^3
+ 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^11*c - 9*a*b^
9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e
 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)
)*e^2*log(sqrt(2)*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - 31*a^3*b^3*c^4 +
 12*a^4*b*c^5)*d - (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 2
9*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*sqrt(
((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)
*d^2 - 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b
^3*c^5 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^
15)))*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7*
a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^10
*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 -
 2*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5
 - 3*a^5*b*c^6)*d*e + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^14 - 4*a*c^15)))/
(b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5*b*c^3)*d - (a^
3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e)*sqrt(x*e + d)) - 15*sqrt(
2)*c^3*sqrt(((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d - (b^7 - 7
*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - (b^2*c^7 - 4*a*c^8)*sqrt(((b^1
0*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(281) = 562$.
time = 4.05, size = 1045, normalized size = 3.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^4*c - 5*a*b^2*c^2
 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^2 - 2*((b^2*c^3
- a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*c)*d*e
+ (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqr
t(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*(2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d
*e + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*
e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e
^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/
((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(b^2 - 4*a*
c)*a*c^5*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*
((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^
2)*c^2 + 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*s
qrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4
*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c -
 sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b
^2*c^4 + 4*a^2*c^5)*d*e + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2))*arcta
n(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6
*d^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^1
3)^2))*e^(-12)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*
d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2) + 2/15*(3*(x*e + d)^(5/2)*c^4*e^8 -
 5*(x*e + d)^(3/2)*c^4*d*e^8 - 5*(x*e + d)^(3/2)*b*c^3*e^9 + 15*sqrt(x*e +
d)*b^2*c^2*e^10 - 15*sqrt(x*e + d)*a*c^3*e^10)*e^(-10)/c^5
```

Mupad [B]
time = 4.37, size = 2500, normalized size = 7.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e
^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*
d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5
*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b
```

^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6*d^2*e^2))/c^5 + (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (8*(d + e*x)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e

```
+ 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((8*(4*a^3*c
^6*e^4 + a*b^4*c^4*e^4 - b^5*c^4*d*e^3 - 5*a^2*b^2*c^5*e^4 + 4*a^2*c^7*d^2*
e^2 + b^4*c^5*d^2*e^2 + 5*a*b^3*c^5*d*e^3 - 4*a^2*b*c^6*d*e^3 - 5*a*b^2*c^6
*d^2*e^2))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c
 - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b
^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7
*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) +
 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1
/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b
^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3
- 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c
^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3
*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2
)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^...
```

3.527 $\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=316

$$-\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{\sqrt{2}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4a\right.}}}\right.}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2/3*(e*x+d)^(3/2)/c/e-2*b*(e*x+d)^(1/2)/c^2+arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+a*c*e+(
-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^
2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))^(1/2)
```

Rubi [A]
time = 2.19, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {911, 1301, 1180, 214}

$$\frac{\sqrt{2}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (-2*b*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c*e) + (Sqrt[2]*(b*c*d -
b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(b*c*
d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b(cd^2-bde+ae^2)-(bcd-b^2e+ace)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{2\text{Subst}\left(\int \frac{b(cd^2-bde+ae^2)+(-bcd+b^2e-ace)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right.}{c^2e^2}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \right.}{c^2e^2}$$

$$= -\frac{2b\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3ce}+\frac{\sqrt{2}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\right.}{c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4a\right.}}}$$

Mathematica [A]
time = 1.10, size = 375, normalized size = 1.19

$$\frac{\frac{2\sqrt{c}\sqrt{d+ex}(-3be+c(d+ex))}{e}+\frac{3\sqrt{2}\left(-b^3e+bc\left(-\sqrt{b^2-4ac}\,d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}\,e\right)-ac\left(2cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{3\sqrt{2}\left(b^3e-bc\left(\sqrt{b^2-4ac}\,d+3ae\right)+ac\left(2cd-\sqrt{b^2-4ac}\,e\right)+b^2\left(-cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}}{3c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x]*(-3*b*e + c*(d + e*x)))/e + (3*Sqrt[2]*(-(b^3*e)
+ b*c*(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) -
a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d +
(b - Sqrt[b^2 - 4*a*c])*e]) + (3*Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a*c]*d
+ 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2 - 4*a
*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2
- 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e])
)/(3*c^(5/2))
```

Maple [A]
time = 0.16, size = 430, normalized size = 1.36

method	result
derivativedivides	$-\frac{2\left(-\frac{(ex+d)^{\frac{3}{2}}c}{3}+be\sqrt{ex+d}\right)}{c^2}+\frac{8e\left(\frac{\left(-3ab\,e^2c+2a\,c^2de+b^3e^2-b^2dec-\sqrt{-e^2\,(4ac-b^2)}\,ace+\sqrt{-e^2\,(4ac-\right.}{8c\sqrt{-e^2\,(4ac-b^2}}\right.}{}$
default	$-\frac{2\left(-\frac{(ex+d)^{\frac{3}{2}}c}{3}+be\sqrt{ex+d}\right)}{c^2}+\frac{8e\left(\frac{\left(-3ab\,e^2c+2a\,c^2de+b^3e^2-b^2dec-\sqrt{-e^2\,(4ac-b^2)}\,ace+\sqrt{-e^2\,(4ac-\right.}{8c\sqrt{-e^2\,(4ac-b^2}}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/c^2*(-1/3*(e*x+d)^(3/2)*c+b*e*(e*x+d)^(1/2))+4/c*e*(1/8*(-3*a*b*e^2
*c+2*a*c^2*d*e+b^3*e^2-b^2*d*e*c-(-e^2*(4*a*c-b^2))^(1/2)*a*c*e+(-e^2*(4*a*
c-b^2))^(1/2)*b^2*e-(-e^2*(4*a*c-b^2))^(1/2)*b*c*d)/c/(-e^2*(4*a*c-b^2))^(1
/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)
^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(3*a*b*e
^2*c-2*a*c^2*d*e-b^3*e^2+b^2*d*e*c-(-e^2*(4*a*c-b^2))^(1/2)*a*c*e+(-e^2*(4*
a*c-b^2))^(1/2)*b^2*e-(-e^2*(4*a*c-b^2))^(1/2)*b*c*d)/c/(-e^2*(4*a*c-b^2))^
(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*
x+d)^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*x^2/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2992 vs. $2(285) = 570$.
time = 2.70, size = 2992, normalized size = 9.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*c^2*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b
^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 +
4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*
d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^
2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*e*log(sqrt(2)*((b^6*c - 6*a*b^4*c
^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e
- (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*
b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10
 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^
2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a
^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(x*e + d)) - 3*sqrt(2)*c^2*sqrt(((b^
4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2
*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*
c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*
a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^
5 - 4*a*c^6))*e*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e - (b^4*c^5 - 6*a*b^2*c^6 +
8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5
*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c
 - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^
5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c -
 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 -
 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*e)*sqrt(x*e + d)) + 3*sqrt(2)*c^2*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c
^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*
c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3
*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*e*log(sqrt(2)
*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c
^2 - 4*a^3*b*c^3)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 -
```

```
4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*
d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2
- 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c -
 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(x*e + d)) - 3*s
qrt(2)*c^2*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5
*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^
2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 -
 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*e*log(-sqrt(2)*((b^6*c - 6*a*b^4*c^2 + 8*
a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e + (b^4*
c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4
)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*
c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^
2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*
a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4
- 3*a^3*b^2*c + a^4*c^2)*e)*sqrt(x*e + d)) + 4*(c*d + (c*x - 3*b)*e)*sqrt(x
*e + d))*e^(-1)/c^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 868 vs. 2(285) = 570.
time = 2.21, size = 868, normalized size = 2.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^2)
*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^3*
```

```
d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b*c
^4)*d*e + (b^4*c^2 - 3*a*b^2*c^3)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sq
rt(-(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e
^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((sqrt(b^2 - 4*a*c
)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2)
 + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^3*c - 4*a*b*c^
2)*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(sqrt(b^2 - 4*a*c)*b*c^
3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*sqrt
(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + sqrt(-4*c^2*d + 2*(b*
c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b
*c^4)*d*e + (b^4*c^2 - 3*a*b^2*c^3)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/
sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 - sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3
*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((sqrt(b^2 - 4*a
*c)*c^5*d^2 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^
2) + 2/3*((x*e + d)^(3/2)*c^2*e^2 - 3*sqrt(x*e + d)*b*c*e^3)*e^(-3)/c^3
```

Mupad [B]
time = 3.91, size = 2500, normalized size = 7.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(3/2))/(3*c*e) - atan(((((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 -
 b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/
c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^
(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-
(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e
*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*
(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^
7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c
^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
+ 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*
(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4
- 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 -
 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 -
4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3
)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*
(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
```

e(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*1i - (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*
e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*(-(b
^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*
c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e
 + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*
b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2
*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^
2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(
4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2
*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^
3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/
2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c
^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^
3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-
(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^
2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c
*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*
a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/
(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c*e^5 - a^3*b
^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*
d^2*e^3))/c^3 + (((8*(a*b^3*c^3*e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3
*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2 + 4*a*b^2*c^4*d*e^3))/c^3 - (8*(d + e*x)^(
1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18
*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/
2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^
(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2
*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d +
b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^
2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20
*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*
a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b
^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2)
)/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d -
18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3
)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((8*(a*b^3*c^3*
e^4 - 4*a^2*b*c^4*e^4 - b^4*c^3*d*e^3 + b^3*c^4*d^2*e^2 - 4*a*b*c^5*d^2*e^2
 + 4*a*b^2*c^4*d*e^3))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^
4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*

```
e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a
^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2...
```

3.528 $\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=287

$$\frac{2\sqrt{d+ex}}{c}+\frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\sqrt{2}\ldots}{\ldots}$$

```
[Out] 2*(e*x+d)^(1/2)/c+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*c*d-b^2*e+2*a*c*e-(-b*e+c*d)*(-4*a*c+b^2)^(
1/2))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arc
tanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*
2^(1/2)*(b*c*d-b^2*e+2*a*c*e+(-b*e+c*d)*(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 2.20, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {838, 840, 1180, 214}

$$\frac{\sqrt{2}\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x])/c + (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]
*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d
- b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
- 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx &= \frac{2\sqrt{d+ex}}{c}+\frac{\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx}{c}\\
&= \frac{2\sqrt{d+ex}}{c}+\frac{2\text{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)}{c}\\
&= \frac{2\sqrt{d+ex}}{c}-\frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(- \cdots}\right.}{c\sqrt{b^2-4ac}}\\
&= \frac{2\sqrt{d+ex}}{c}+\frac{\sqrt{2}\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{2cd-\left(b-\cdots\right.}}\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.96, size = 341, normalized size = 1.19

$$\frac{2\sqrt{c}\,\sqrt{d+ex}-\frac{\left(-ibcd-c\sqrt{-b^2+4ac}\,d+ib^2e-2iace+b\sqrt{-b^2+4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\,\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}-\frac{\left(ibcd-c\sqrt{-b^2+4ac}\,d-ib^2e+2iace+b\sqrt{-b^2+4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\,\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d + e*x])/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x] - (((-I)*b*c*d - c*Sqrt[-b^2 + 4*a*c]*d + I*b^2*e
- (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*S
qrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - ((I*b*c*d - c*Sqrt[-b^2 + 4*a
*c]*d - I*b^2*e + (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/
2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/c^(3/2)
```

Maple [A]
time = 0.15, size = 331, normalized size = 1.15

method	result
derivativedivides	$\frac{2\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^2-b^2e^2+bcde-\sqrt{-e^2\left(4ac-b^2\right)}\,be+\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\ldots\right)}{c\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-\ldots\right.}\right.}}$
default	$\frac{2\sqrt{ex+d}}{c}+\frac{\left(2ac\,e^2-b^2e^2+bcde-\sqrt{-e^2\left(4ac-b^2\right)}\,be+\sqrt{-e^2\left(4ac-b^2\right)}\,cd\right)\sqrt{2}\,\arctan\left(\ldots\right)}{c\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-\ldots\right.}\right.}}$
risch	$\frac{2\sqrt{ex+d}}{c}+\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(-eb+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)a\,e^2}{\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\left(-eb+2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\ldots\right.}{c\sqrt{-e^2\left(4ac\ldots\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(e*x+d)^(1/2)/c+(2*a*c*e^2-b^2*e^2+b*c*d*e-(-e^2*(4*a*c-b^2))^(1/2)*b*e+(
-e^2*(4*a*c-b^2))^(1/2)*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((e*b-2*c*d
+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((e*b-2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-(-2*a*c*e^2+b^2*e^2-b*c*d*e-(-e^2*(
4*a*c-b^2))^(1/2)*b*e+(-e^2*(4*a*c-b^2))^(1/2)*c*d)/c/(-e^2*(4*a*c-b^2))^(1
/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+
d)^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*x/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(254) = 508$.
time = 2.38, size = 1742, normalized size = 6.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4
*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c
- 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sq
rt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2
)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^
2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b
^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*
d - (a*b^2 - a^2*c)*e)*sqrt(x*e + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d
 - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a
*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d
*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c -
2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*
(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7
)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(x*e + d)) +
 sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c
^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
```

```
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*((b^3*c - 4*
a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((
b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b
^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^
3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c
 + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d -
(a*b^2 - a^2*c)*e)*sqrt(x*e + d)) - sqrt(2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (
b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c
^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e +
 (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(((b^2*c - 2*a*
c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3
*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/
(b^2*c^3 - 4*a*c^4)) + 4*(a*b*c*d - (a*b^2 - a^2*c)*e)*sqrt(x*e + d)) + 4*s
qrt(x*e + d))/c
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(254) = 508.
time = 1.33, size = 753, normalized size = 2.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(
(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 - 2*(sqrt(b^2 - 4*a*c)*c^3
*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^4*d^2 - (3*b^2*c^3
- 4*a*c^4)*d*e + (b^3*c^2 - 2*a*b*c^3)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b
^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e
+ sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/(
(sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c
)*a*c^3*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(
(b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 + 2*(sqrt(b^2 - 4*a*c)*c^3
```

```
*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*sqrt(-4*c
^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^4*d^2 - (3*b^2*c^3
- 4*a*c^4)*d*e + (b^3*c^2 - 2*a*b*c^3)*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b
^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e
- sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/(
(sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c
)*a*c^3*e^2)*c^2)
```

Mupad [B]
time = 3.82, size = 2500, normalized size = 8.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^(1/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4
*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d +
e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d
 + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^
(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b
^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a
^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e +
 a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*
c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i
- (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b
^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*((8*a^2*c^3*d - b
^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*
a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a
^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 -
2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e
- b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c -
 b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*
c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)
*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e
^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(
4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1
/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(
16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^3*e^4 - a*b^
2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d
```

e^3))/c - (8(d + e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (16*(a*c^2*d^3*e^2 - a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c + (8*(d + e*x)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((8*(4*a^2*c^3*e^4 - a*b^2*c^2*e^4 + 4*a*c^4*d^2*e^2 + b^3*c^2*d*e^3 - b^2*c^3*d^2*e^2 - 4*a*b*c^3*d*e^3))/c - (8*(d + e*x)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^...

3.529 $\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{2}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right.}}{}$$

```
[Out] -arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2))*2^(1/2)*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/
2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2))*2^(1/2)*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/c^(1/2)/(-4*a*c+b^2)^(
1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {713, 1144, 214}

$$\frac{\sqrt{2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2
 - 4*a*c])) + (Sqrt[2]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx = (2e)\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)$$

$$= -\left(\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d\right.\right.$$

$$= -\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{2}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 252, normalized size = 1.27

$$\frac{\sqrt{2}\left(\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (Sqrt[2]*((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/Sqrt[-2*c*
d + (b - I*Sqrt[-b^2 + 4*a*c])*e] + (((2*I)*c*d + ((-I)*b + Sqrt[-b^2 + 4*a
*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-
b^2 + 4*a*c]*e]])/Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/(Sqrt[c]*Sq
rt[-b^2 + 4*a*c])
```

Maple [A]
time = 0.14, size = 251, normalized size = 1.27

method	result
derivativedivides	$8ec\left(\frac{\left(eb-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8\sqrt{-e^2\left(4ac-b^2\right)}\,c\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}} - \cdots\right.$
default	$8ec\left(\frac{\left(eb-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}}\right)}{8\sqrt{-e^2\left(4ac-b^2\right)}\,c\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right)c}} - \cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 8*e*c*(1/8/(-e^2*(4*a*c-b^2))^(1/2)*(e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))/c*
2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/
2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(-e*b+2*c*d+
(-e^2*(4*a*c-b^2))^(1/2))/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2*c*d+(
-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-e*b+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(164) = 328$.
time = 2.35, size = 683, normalized size = 3.45

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+\frac{(b^2c-4\,ac^2)e}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{2}\,(b^2c-4\,ac^2)\sqrt{\frac{2\,cd-be+\frac{(b^2c-4\,ac^2)e}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\,e}{\sqrt{b^2c^2-4\,ac^3}}+2\sqrt{xe+d}\,e\right)+\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be+\frac{(b^2c-4\,ac^2)e}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{2}\,(b^2c-4\,ac^2)\sqrt{\frac{2\,cd-be+\frac{(b^2c-4\,ac^2)e}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\,e}{\sqrt{b^2c^2-4\,ac^3}}+2\sqrt{xe+d}\,e\right)+\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be-\frac{(b^2c-4\,ac^2)e}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{2}\,(b^2c-4\,ac^2)\sqrt{\frac{2\,cd-be-\frac{(b^2c-4\,ac^2)e}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\,e}{\sqrt{b^2c^2-4\,ac^3}}+2\sqrt{xe+d}\,e\right)-\frac{1}{2}\sqrt{2}\sqrt{\frac{2\,cd-be-\frac{(b^2c-4\,ac^2)e}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{2}\,(b^2c-4\,ac^2)\sqrt{\frac{2\,cd-be-\frac{(b^2c-4\,ac^2)e}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\,e}{\sqrt{b^2c^2-4\,ac^3}}+2\sqrt{xe+d}\,e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3
))/(b^2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt((2*c*d - b*e + (b^
2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*e/sqrt(b^2*c^2
 - 4*a*c^3) + 2*sqrt(x*e + d)*e) + 1/2*sqrt(2)*sqrt((2*c*d - b*e + (b^2*c -
 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c
 - 4*a*c^2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3)
)/(b^2*c - 4*a*c^2))*e/sqrt(b^2*c^2 - 4*a*c^3) + 2*sqrt(x*e + d)*e) + 1/2*s
qrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^
2*c - 4*a*c^2))*log(sqrt(2)*(b^2*c - 4*a*c^2)*sqrt((2*c*d - b*e - (b^2*c -
4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*e/sqrt(b^2*c^2 - 4*a
*c^3) + 2*sqrt(x*e + d)*e) - 1/2*sqrt(2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c
^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(-sqrt(2)*(b^2*c - 4*a
*c^2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2
*c - 4*a*c^2))*e/sqrt(b^2*c^2 - 4*a*c^3) + 2*sqrt(x*e + d)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.84, size = 223, normalized size = 1.13

$$-\frac{\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,\sqrt{xe+d}}{\sqrt{-\frac{2\,cd-be+\sqrt{(2\,cd-be)^2-4\,(cd^2-bde+ae^2)c}}{c}}}\right)}{\sqrt{b^2-4\,ac}\,|c|}+\frac{\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,\sqrt{xe+d}}{\sqrt{-\frac{2\,cd-be-\sqrt{(2\,cd-be)^2-4\,(cd^2-bde+ae^2)c}}{c}}}\right)}{\sqrt{b^2-4\,ac}\,|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x
*e + d)/sqrt(-(2*c*d - b*e + sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^
2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)) + sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 -
 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c*d - b*e - sqrt((2
*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)
)
```

Mupad [B]
time = 2.99, size = 709, normalized size = 3.58

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)+\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,d\,c^4\,e^2\right)\,\left(b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e\right)}{2\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{-\frac{b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e}{2\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{16\,c^2\,d^2\,e^3-16\,b\,c\,d\,e^4+16\,a\,c\,e^5}\right)\,\sqrt{-\frac{b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e}{2\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{d+e\,x}\,\left(-8\,b^2\,c\,e^4+16\,b\,c^2\,d\,e^3-16\,c^3\,d^2\,e^2+16\,a\,c^2\,e^4\right)-\frac{\sqrt{d+e\,x}\,\left(8\,b^3\,c^2\,e^3-16\,d\,b^2\,c^3\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,d\,c^4\,e^2\right)\,\left(e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e\right)}{2\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{\frac{e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e}{2\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{16\,c^2\,d^2\,e^3-16\,b\,c\,d\,e^4+16\,a\,c\,e^5}\right)\,\sqrt{\frac{e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e}{2\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(a + b*x + c*x^2),x)
```

```
[Out] - 2*atanh((2*((d + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2
+ 16*b*c^2*d*e^3) + ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32
*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^
2*d - 2*b^2*c*d - 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^
3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16
*b*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d
- 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh((2*((d
 + e*x)^(1/2)*(16*a*c^2*e^4 - 8*b^2*c*e^4 - 16*c^3*d^2*e^2 + 16*b*c^2*d*e^3
) - ((d + e*x)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 6
4*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d
+ 4*a*b*c*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a*c - b^2)^3
)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b^4*c + 16*a^2*c^3
 - 8*a*b^2*c^2)))^(1/2))/(16*c^2*d^2*e^3 + 16*a*c*e^5 - 16*b*c*d*e^4))*((e*
(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.530 $\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)} dx$

Optimal. Leaf size=275

$$-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}+\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -2*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b*d-2*a*e+d
*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2))*2^(1/2)*c^(1/2)*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1
/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {911, 1301, 212, 1180, 214}

$$\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a + (Sqrt[2]*Sqrt[c]*(b*d + Sqr
t[b^2 - 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]])/(a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{de}{a(d-x^2)}+\frac{e(cd^2-bde+ae^2-cdx^2)}{a(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{(2d)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x,\right.}{a}$$

$$= -\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{}{\frac{1}{2}\sqrt{b^2-}}\right.}{a\sqrt{b^2-4ac}}$$

$$= -\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{}{\sqrt{2c}}\right.}{a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-\right.}}}$$

Mathematica [A]
time = 0.84, size = 266, normalized size = 0.97

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-bd+\sqrt{b^2-4ac}\,d+2ae\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x*(a + b*x + c*x^2)),x]
```

```
[Out] -(((Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 -
4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b*d)
 + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt
[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b
+ Sqrt[b^2 - 4*a*c])*e]) + 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a)
```

Maple [A]
time = 0.12, size = 291, normalized size = 1.06

method	result

derivativedivides	$2e^2 \left(\frac{4c \left(\frac{\left(-2a\,e^2+bde-\sqrt{-e^2(4ac-b^2)}\,d\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(eb-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}\right)}{8\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(eb-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}} \right.}{a} \right.$
default	$2e^2 \left(\frac{4c \left(\frac{\left(-2a\,e^2+bde-\sqrt{-e^2(4ac-b^2)}\,d\right)\sqrt{2}\,\arctan\left(\frac{c\sqrt{ex+d}\,\sqrt{2}}{\sqrt{\left(eb-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}}\right)}{8\sqrt{-e^2(4ac-b^2)}\,\sqrt{\left(eb-2cd+\sqrt{-e^2(4ac-b^2)}\right)c}} \right.}{a} \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e^2*(4/a/e^2*c*(1/8*(-2*a*e^2+b*d*e-(-e^2*(4*a*c-b^2))^(1/2)*d)/(-e^2*(4*
a*c-b^2))^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arct
an(c*(e*x+d)^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-
1/8*(2*a*e^2-b*d*e-(-e^2*(4*a*c-b^2))^(1/2)*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(
1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2
)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))-d^(1/2)/a/e^2*a
rctanh((e*x+d)^(1/2)/d^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/((c*x^2 + b*x + a)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(234) = 468$.
time = 2.73, size = 2489, normalized size = 9.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*l
og(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(x*e + d
)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)
*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e -
(b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(x*e + d
)) + sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*lo
g(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(x*e + d)
) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^
2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log
(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*
sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(x*e + d)
) + 2*sqrt(d)*log((x*e - 2*sqrt(x*e + d)*sqrt(d) + 2*d)/x))/a, 1/2*(sqrt(2)
*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a
```

```
*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(x*e + d)) - sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((
b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)
*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*
a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(x*e + d)) + sqrt(2)*
a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*
b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(x*e + d)) - sqrt(2)*a
*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((b
^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*
d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a
^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(x*e + d)) + 4*sqrt(-d
)*arctan(sqrt(x*e + d)*sqrt(-d)/d))/a]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(234) = 468.
time = 1.98, size = 712, normalized size = 2.59

$$\frac{2\,d\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}-\frac{\left(\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,(b^2-4\,ac)a^2de-2\left(\sqrt{b^2-4\,ac}\,acd^2-\sqrt{b^2-4\,ac}\,abde+\sqrt{b^2-4\,ac}\,a^2e^2\right)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,|a|-(2\,a^2bcd^2+2\,a^3be^2-(a^2b^2+4\,a^3c)de)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,acd-abe+\sqrt{-4\,(acd^2-abde+a^2e^2)ac+(2\,acd-abe)^2}}{ac}}}\right)}{4\left(\sqrt{b^2-4\,ac}\,a^2cd^2-\sqrt{b^2-4\,ac}\,a^2bde+\sqrt{b^2-4\,ac}\,a^3e^2\right)|a||c|}+\frac{\left(\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\,(b^2-4\,ac)a^2de+2\left(\sqrt{b^2-4\,ac}\,acd^2-\sqrt{b^2-4\,ac}\,abde+\sqrt{b^2-4\,ac}\,a^2e^2\right)\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\,|a|-(2\,a^2bcd^2+2\,a^3be^2-(a^2b^2+4\,a^3c)de)\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,acd-abe-\sqrt{-4\,(acd^2-abde+a^2e^2)ac+(2\,acd-abe)^2}}{ac}}}\right)}{4\left(\sqrt{b^2-4\,ac}\,a^2cd^2-\sqrt{b^2-4\,ac}\,a^2bde+\sqrt{b^2-4\,ac}\,a^3e^2\right)|a||c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*d*arctan(sqrt(x*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e - 2*(sqrt(b^2 - 4*a*c)*a
```

```
*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2
*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2
 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d
^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*
a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*a
bs(a)*abs(c)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^2
- 4*a*c)*a^2*d*e + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e
 + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d
)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a
*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)
*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c))
```

Mupad [B]
time = 7.41, size = 2500, normalized size = 9.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x*(a + b*x + c*x^2)),x)
```

```
[Out] - atan(((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b
*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a
^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*
a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*
c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4
*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c -
b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3
*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e
^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^
8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 3
84*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b
^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)
^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576
*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d
*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e
^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*
c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*
e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*
a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^
3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10
) + (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 +
 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e
```

+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(96*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^4*c^4*d*e^10 + 384*a^3*c^5*d^3*e^8 - 96*a^2*b^2*c^4*d^3*e^8 + 96*a^2*b^3*c^3*d^2*e^9 - 384*a^3*b*c^4*d^2*e^9 - 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^3*e^9 - 32*a*b^3*c^2*d*e^11 + 160*a^2*b*c^3*d*e^11 - 192*a*b^2*c^3*d^2*e^10) - (d + e*x)^(1/2)*(32*a^2*c^3*e^12 + 96*c^5*d^4*e^8 - 128*b*c^4*d^3*e^9 + 64*b^2*c^3*d^2*e^10 - 64*a*b*c^3*d*e^11))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i)/((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^4*c^4*d*e^10 - 384*a^3*c^5*d^3*e^8 + 96*a^2*b^2*c^4*d^3*e^8 - 96*a^2*b^3*c^3*d^2*e^9 + 384*a^3*b*c^4*d^2*e^9 + 96*a^3*b^2*c^3*d*e^10) - (d + e*x)^(1/2)*(128*a^3*b*c^3*e^11 + 192*a^3*c^4*d*e^10 - 32*a^2*b^3*c^2*e^11 + 576*a^2*c^5*d^3*e^8 + 64*b^4*c^3*d^3*e^8 - 64*b^5*c^2*d^2*e^9 + 64*a*b^4*c^2*d*e^10 - 384*a*b^2*c^4*d^3*e^8 + 384*a*b^3*c^3*d^2*e^9 - 576*a^2*b*c^4*d^2*e^9 - 288*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 96*a*c^5*d^4*e^8 + 96*a^2*c^4*d^2*e^10 - 32*b^2*c^4*d^4*e^8 + 32*b^4*c^2*d^2*e^10 + 64*a*b*c^4*d^...

3.531 $\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=368

$$-\frac{\sqrt{d+ex}}{ax}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}}+\frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2\right.}{a^2\sqrt{b^2-}}$$

```
[Out] e*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)+2*(-a*e+b*d)*arctanh((e*x+d)^(1/
2)/d^(1/2))/a^2/d^(1/2)-(e*x+d)^(1/2)/a/x-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(
1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d-2*a*c*d
-a*b*e+(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d-2*a*c*d-a*b*e-(-a*e+b*d)
*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)
```

Rubi [A]
time = 2.43, antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {911, 1301, 205, 212, 1180, 214}

$$-\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\sqrt{c}\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{d+ex}}{ax}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -(Sqrt[d + e*x]/(a*x)) + (e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a*Sqrt[d]) + (
2*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt[2]*Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*S
qrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c
]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqr
t[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)}\,dx = \frac{2\mathrm{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\mathrm{Subst}\left(\int \left(\frac{de^2}{a(d-x^2)^2}-\frac{e(-bd+ae)}{a^2(d-x^2)}+\frac{e(-b(cd^2-bde+ae^2)+c(bd-ae)x^2)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\mathrm{Subst}\left(\int \frac{-b(cd^2-bde+ae^2)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2} + \frac{(2de)\mathrm{Subst}\left(\int \frac{1}{(d-x^2)^2}\,dx\right.}{a}$$

$$= -\frac{\sqrt{d+ex}}{ax} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\mathrm{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex}\right.}{a}$$

$$= -\frac{\sqrt{d+ex}}{ax} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{2}\ \cdots}{\cdots}$$

Mathematica [A]
time = 1.33, size = 337, normalized size = 0.92

$$\frac{-\frac{a\sqrt{d+ex}}{x}+\frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd+b\sqrt{b^2-4ac}\,d-abe-a\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2d+2acd+b\sqrt{b^2-4ac}\,d+abe-a\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+\frac{(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*Sqrt[d + e*x])/x) + (Sqrt[2]*Sqrt[c]*(b^2*d - 2*a*c*d + b*Sqrt[b^2 -
4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2
*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 2*a*c*d +
 b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 -
4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((2*b*d - a*e)*ArcTanh[S
qrt[d + e*x]/Sqrt[d]])/Sqrt[d])/a^2
```

Maple [A]
time = 0.14, size = 373, normalized size = 1.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e^3*(4/a^2/e^3*c*(1/8*(a*b*e^2+2*a*c*d*e-b^2*d*e-(-e^2*(4*a*c-b^2))^(1/2)
*a*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((e*b-2
*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((e*
b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(-a*b*e^2-2*a*c*d*e+b^2*d*e
-(-e^2*(4*a*c-b^2))^(1/2)*a*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d)/(-e^2*(4*a*c-b^
2))^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c
*(e*x+d)^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))+1/
a^2/e^3*(-1/2*a*(e*x+d)^(1/2)/x-1/2*(a*e-2*b*d)/d^(1/2)*arctanh((e*x+d)^(1/
2)/d^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/((c*x^2 + b*x + a)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2451 vs. $2(313) = 626$.
time = 14.12, size = 4909, normalized size = 13.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^
2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2
- 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*
c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*
a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b
^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 -
 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c
^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b
^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2
 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(x*e + d)) - sqrt(2)*a^2*d*x
*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 -
 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^
3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 -
```

4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(x*e + d)) + sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(x*e + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e + (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e - (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt(x*e + d)) - 2*sqrt(x*e + d)*a*d - (2*b*d*x - a*x*e)*sqrt(d)*log((x*e - 2*sqrt(x*e + d)*sqrt(d) + 2*d)/x))/(a^2*d*x), 1/2*(sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e - (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*sqrt

```
(x*e + d)) - sqrt(2)*a^2*d*x*sqrt(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3
 - 3*a^2*b*c)*e + (a^4*b^2 - 4*a^5*c)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-sqrt(2)*((
b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*e
- (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*sqrt(((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [**B**]
time = 6.81, size = 2500, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] (atan((((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((a*e
 - 2*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8
*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^
4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b
^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d
```

^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^
11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e - 2*b*d)*(
(8*(d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e
^11 - 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*
a^2*b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108
*a^4*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*
a^4*b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 -
(((8*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c
^5*d^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4
*d^2*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^
8 + 24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*
c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 +
8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^
6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a
^2*d^(1/2))))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)))*1i)/(2*a^2*d^(1/2)) + ((a*
e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*
c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d
*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 + ((a*e - 2*b*d)*(
(8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^
3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e
^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*
e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3
*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4
*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 + ((a*e - 2*b*d)*((8*(d + e*x
)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11 - 35*a^
5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*b^7*c^2
*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4*b^2*c^
5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*b^4*c^3
*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 + (((8*(32*a^
8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d^2*e^9
+ 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2*e^9 -
64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 + 24*a^6*
b^3*c^3*d*e^10))/a^4 + (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*e^10 +
4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^6*b^4*c
^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^5*c^2*d
*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/2)))*(a*e - 2*b*d))/(2*a^2*d^(1/2))
))/(2*a^2*d^(1/2))))/(2*a^2*d^(1/2)))*1i)/(2*a^2*d^(1/2)))/((16*(a^3*c^5*e^
13 + 2*a*c^7*d^4*e^9 - 4*b*c^7*d^5*e^8 + 3*a^2*c^6*d^2*e^11 + 4*b^2*c^6*d^4
*e^9 - 8*a*b*c^6*d^3*e^10 - 3*a^2*b*c^5*d*e^12 + 2*a*b^2*c^5*d^2*e^11))/a^4
 - ((a*e - 2*b*d)*((8*(d + e*x)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 +
 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*
b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/a^4 - ((a*e - 2
*b*d)*((8*(16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4
*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^
4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c

```
^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e
^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 -
 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11))/a^4 - ((a*e - 2*b*d)*((8*(
d + e*x)^(1/2)*(60*a^6*b*c^4*e^11 + 16*a^6*c^5*d*e^10 + 5*a^4*b^5*c^2*e^11
- 35*a^5*b^3*c^3*e^11 + 40*a^5*c^6*d^3*e^8 - 8*a^2*b^6*c^3*d^3*e^8 + 8*a^2*
b^7*c^2*d^2*e^9 + 56*a^3*b^4*c^4*d^3*e^8 - 52*a^3*b^5*c^3*d^2*e^9 - 108*a^4
*b^2*c^5*d^3*e^8 + 68*a^4*b^3*c^4*d^2*e^9 - 12*a^3*b^6*c^2*d*e^10 + 87*a^4*
b^4*c^3*d*e^10 + 56*a^5*b*c^5*d^2*e^9 - 162*a^5*b^2*c^4*d*e^10))/a^4 - (((8
*(32*a^8*c^4*e^11 + 2*a^6*b^4*c^2*e^11 - 16*a^7*b^2*c^3*e^11 + 32*a^7*c^5*d
^2*e^9 + 8*a^5*b^3*c^4*d^3*e^8 - 6*a^5*b^4*c^3*d^2*e^9 + 16*a^6*b^2*c^4*d^2
*e^9 - 64*a^7*b*c^4*d*e^10 - 2*a^5*b^5*c^2*d*e^10 - 32*a^6*b*c^5*d^3*e^8 +
24*a^6*b^3*c^3*d*e^10))/a^4 - (4*(a*e - 2*b*d)*(d + e*x)^(1/2)*(64*a^9*c^4*
e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2*e^8 + 8*a^
6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9 - 8*a^6*b^
5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/(a^6*d^(1/...
```

3.532 $\int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)}\,dx$

Optimal. Leaf size=531

$$-\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}$$

```
[Out] -3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(3/2)-e*(-a*e+b*d)*arctanh((e*x
+d)^(1/2)/d^(1/2))/a^2/d^(3/2)-2*(-a*b*e-a*c*d+b^2*d)*arctanh((e*x+d)^(1/2)
/d^(1/2))/a^3/d^(1/2)-1/2*(e*x+d)^(1/2)/a/x^2+3/4*e*(e*x+d)^(1/2)/a/d/x+(-a
*e+b*d)*(e*x+d)^(1/2)/a^2/d/x+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^3*d-a*c*(-2*a*e+d*(-4*a
*c+b^2)^(1/2))+b^2*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c*d+e*(-4*a*c+b^2)^(1
/2)))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh
(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1
/2)*c^(1/2)*(b^3*d-b^2*(a*e+d*(-4*a*c+b^2)^(1/2))+a*c*(2*a*e+d*(-4*a*c+b^2)
^(1/2))-a*b*(3*c*d-e*(-4*a*c+b^2)^(1/2)))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 2.43, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {911, 1301, 205, 212, 1180, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(-abe-acd+b^2d)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)-ab\left(3cd-e\sqrt{b^2-4ac}\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}}+\frac{\sqrt{d+ex}(bd-ae)}{a^2dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*Sqrt[d + e*x]/(a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*d*x) + ((b*d - a*e)*S
qrt[d + e*x])/(a^2*d*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*d^(3/
2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*d^(3/2)) - (2*(b^
2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[
2]*Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a
*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*
Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d - b
^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3
*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2
```

```
*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{de^3}{a(d-x^2)^3}+\frac{e^2(-bd+ae)}{a^2(d-x^2)^2}+\frac{e(-b^2d+acd+abe)}{a^3(d-x^2)}+\frac{e((b^2-ac)(cd^2-bde+ae^2)-c(b^2d-a}{a^3(cd^2-bde+ae^2-(2cd-be)x^2}\right.\right.}{e}$$

$$= \frac{2\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-bde+ae^2)-c(b^2d-acd-abe)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^3} - \frac{(2de^2)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{2(b^2d-acd-abe)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right.}{a^2d^{3/2}}$$

$$= -\frac{\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4adx}+\frac{(bd-ae)\sqrt{d+ex}}{a^2dx}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}}-$$

Mathematica [A]
time = 1.88, size = 433, normalized size = 0.82

$$\frac{\frac{a\sqrt{d+ex}\,(4bdx-a(2d+ex))}{dx^2}+\frac{4\sqrt{2}\sqrt{c}\left(-b^3d+ac\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2\left(-\sqrt{b^2-4ac}\,d+ae\right)+ab\left(3cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{4\sqrt{2}\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}\,d+ae\right)+ac\left(\sqrt{b^2-4ac}\,d+2ae\right)+ab\left(-3cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+\frac{\left(-8b^2d^2+4abde+a\left(8cd^2+ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] ((a*Sqrt[d + e*x]*(4*b*d*x - a*(2*d + e*x)))/(d*x^2) + (4*Sqrt[2]*Sqrt[c]*(
-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*a*c]*d)
+ a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b^3*d - b^2*(Sqr
```

```
t[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d +
 Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d +
 (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^
2 - 4*a*c])*e]) + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*ArcTanh[S
qrt[d + e*x]/Sqrt[d]])/d^(3/2))/(4*a^3)
```

Maple [A]

time = 0.15, size = 507, normalized size = 0.95

method	result
derivativedivides	$2e^4\left(\frac{4c\left(\frac{\left(2a^2ce^2-ab^2e^2-3abcde+b^3de+\sqrt{-e^2(4ac-b^2)}\,abe+\sqrt{-e^2(4ac-b^2)}\,acd-\sqrt{-e^2(4ac-\right.}{8\sqrt{-e^2(4ac-b^2)}\sqrt{\left(eb-2cd+\sqrt{-e\right.}}\right.}{}\right.$
default	$2e^4\left(\frac{4c\left(\frac{\left(2a^2ce^2-ab^2e^2-3abcde+b^3de+\sqrt{-e^2(4ac-b^2)}\,abe+\sqrt{-e^2(4ac-b^2)}\,acd-\sqrt{-e^2(4ac-\right.}{8\sqrt{-e^2(4ac-b^2)}\sqrt{\left(eb-2cd+\sqrt{-e\right.}}\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e^4*(4/a^3/e^4*c*(1/8*(2*a^2*c*e^2-a*b^2*e^2-3*a*b*c*d*e+b^3*d*e+(-e^2*(4
*a*c-b^2))^(1/2)*a*b*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d-(-e^2*(4*a*c-b^2))^(1
/2)*b^2*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^
(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2
))^(1/2))*c)^(1/2))-1/8*(-2*a^2*c*e^2+a*b^2*e^2+3*a*b*c*d*e-b^3*d*e+(-e^2*(
4*a*c-b^2))^(1/2)*a*b*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d-(-e^2*(4*a*c-b^2))^(
1/2)*b^2*d)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2)
)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c
-b^2))^(1/2))*c)^(1/2)))-1/a^3/e^4*((1/8*a*e*(a*e-4*b*d)/d*(e*x+d)^(3/2)+(1
/2*a*b*d*e+1/8*a^2*e^2)*(e*x+d)^(1/2))/e^2/x^2-1/8*(a^2*e^2+4*a*b*d*e+8*a*c
*d^2-8*b^2*d^2)/d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/((c*x^2 + b*x + a)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3736 vs. 2(472) = 944.
time = 168.19, size = 7479, normalized size = 14.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*
c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4*a^7*c)*sqrt((
(b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 -
2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e
 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/
(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(sqrt(2)*((b^9 - 9*a*b^7*c
+ 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8*a^2*b^6*c
+ 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e - (a^6*b^5 - 7*a^7*b^3*c +
 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 +
9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c
^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2
*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((b^6 - 6*a*b^4*c + 9*a^2
*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2
```

- 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*sqrt(x*e + d)) - 4*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(-sqrt(2)*((b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e - (a^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*sqrt(x*e + d)) + 4*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c))*log(sqrt(2)*((b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e + (a^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*sqrt(x*e + d)) - 4*sqrt(2)*a^3*d^2*x^2*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e - (a^6*b^2 - 4*a^7*c)*sqrt(((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5*c^2 -

```
 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^
2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*c)))/(a^6*b^2 - 4*a^7*
c))*log(-sqrt(2)*((b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a
^4*b*c^4)*d - (a*b^8 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^
5*c^4)*e + (a^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*sqrt(((b^10 - 8*a*b^8*c +
 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^
7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3
*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^12*b^2 - 4*a^13*
c)))*sqrt(((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(472) = 944$.
time = 1.38, size = 1041, normalized size = 1.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*a^2 - 2*((a*b^2*c - a^2*c^2)*sqrt
(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^
3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*abs(a) - sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c -
 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^
4*b*c)*e^2))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e +
sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))
/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sq
rt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/4*(sqrt(-4*c^2*d + 2*(b*c + sqr
t(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*
c)*e^2)*a^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b
*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-
4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - sqrt(-4*c^2*d + 2*(b*c
+ sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3
*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^4*b*c)*e^2))*arctan(2*sqrt(1/2)*sq
rt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a
```

```
^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4
*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*ab
s(c)) + 1/4*(8*b^2*d^2 - 8*a*c*d^2 - 4*a*b*d*e - a^2*e^2)*arctan(sqrt(x*e +
 d)/sqrt(-d))/(a^3*sqrt(-d)*d) + 1/4*(4*(x*e + d)^(3/2)*b*d*e - 4*sqrt(x*e
+ d)*b*d^2*e - (x*e + d)^(3/2)*a*e^2 - sqrt(x*e + d)*a*d*e^2)*e^(-2)/(a^2*d
*x^2)
```

Mupad [B]
time = 8.09, size = 2500, normalized size = 4.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x^3*(a + b*x + c*x^2)),x)
```

```
[Out] atan(((((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^
10 + 128*a^8*b^4*c^4*d^5*e^8 - 96*a^8*b^5*c^3*d^4*e^9 - 32*a^8*b^6*c^2*d^3*
e^10 - 704*a^9*b^2*c^5*d^5*e^8 + 448*a^9*b^3*c^4*d^4*e^9 + 392*a^9*b^4*c^3*
d^3*e^10 + 24*a^9*b^5*c^2*d^2*e^11 - 1280*a^10*b^2*c^4*d^3*e^10 - 192*a^10*
b^3*c^3*d^2*e^11 - 256*a^10*b*c^5*d^4*e^9 + 8*a^10*b^4*c^2*d*e^12 + 384*a^1
1*b*c^4*d^2*e^11 - 64*a^11*b^2*c^3*d*e^12)/(2*a^8*d^2) - ((d + e*x)^(1/2)*(
(b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^
4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +
 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1536*a^12*c^5*d^4*e^8 + 1024*a^13*c^4*
d^2*e^10 + 128*a^10*b^4*c^3*d^4*e^8 - 128*a^10*b^5*c^2*d^3*e^9 - 896*a^11*b
^2*c^4*d^4*e^8 + 960*a^11*b^3*c^3*d^3*e^9 + 64*a^11*b^4*c^2*d^2*e^10 - 512*
a^12*b^2*c^3*d^2*e^10 - 1792*a^12*b*c^4*d^3*e^9))/(2*a^8*d^2))*((b^8*d + 8*
a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 3
8*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c
^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)
^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^
2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x)^(1/2)*(8*a^10*c^5*d*e^12 - 12*a^10*b*
c^4*e^13 - a^8*b^5*c^2*e^13 + 7*a^9*b^3*c^3*e^13 + 1152*a^8*c^7*d^5*e^8 + 5
12*a^9*c^6*d^3*e^10 + 128*a^4*b^8*c^3*d^5*e^8 - 128*a^4*b^9*c^2*d^4*e^9 - 1
152*a^5*b^6*c^4*d^5*e^8 + 1088*a^5*b^7*c^3*d^4*e^9 + 192*a^5*b^8*c^2*d^3*e^
10 + 3520*a^6*b^4*c^5*d^5*e^8 - 2816*a^6*b^5*c^4*d^4*e^9 - 1728*a^6*b^6*c^3
*d^3*e^10 - 64*a^6*b^7*c^2*d^2*e^11 - 4096*a^7*b^2*c^6*d^5*e^8 + 1792*a^7*b
^3*c^5*d^4*e^9 + 4944*a^7*b^4*c^4*d^3*e^10 + 568*a^7*b^5*c^3*d^2*e^11 - 451
2*a^8*b^2*c^5*d^3*e^10 - 1536*a^8*b^3*c^4*d^2*e^11 - 8*a^7*b^6*c^2*d*e^12 +
 896*a^8*b*c^6*d^4*e^9 + 57*a^8*b^4*c^3*d*e^12 + 1152*a^9*b*c^5*d^2*e^11 -
102*a^9*b^2*c^4*d*e^12))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*
```

c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (4*a^9*c^5*e^14 - a^6*b^6*c^2*e^14 + 7*a^7*b^4*c^3*e^14 - 13*a^8*b^2*c^4*e^14 - 192*a^6*c^8*d^6*e^8 - 192*a^7*c^7*d^4*e^10 + 4*a^8*c^6*d^2*e^12 - 128*a^2*b^8*c^4*d^6*e^8 + 96*a^2*b^9*c^3*d^5*e^9 + 32*a^2*b^10*c^2*d^4*e^10 + 960*a^3*b^6*c^5*d^6*e^8 - 512*a^3*b^7*c^4*d^5*e^9 - 552*a^3*b^8*c^3*d^4*e^10 - 56*a^3*b^9*c^2*d^3*e^11 - 2176*a^4*b^4*c^6*d^6*e^8 + 224*a^4*b^5*c^5*d^5*e^9 + 2688*a^4*b^6*c^4*d^4*e^10 + 672*a^4*b^7*c^3*d^3*e^11 + 24*a^4*b^8*c^2*d^2*e^12 + 1600*a^5*b^2*c^7*d^6*e^8 + 1408*a^5*b^3*c^6*d^5*e^9 - 4536*a^5*b^4*c^5*d^4*e^10 - 2616*a^5*b^5*c^4*d^3*e^11 - 209*a^5*b^6*c^3*d^2*e^12 + 2336*a^6*b^2*c^6*d^4*e^10 + 3648*a^6*b^3*c^5*d^3*e^11 + 559*a^6*b^4*c^4*d^2*e^12 - 429*a^7*b^2*c^5*d^2*e^12 - 132*a^8*b*c^5*d*e^13 + a^5*b^7*c^2*d*e^13 - 1088*a^6*b*c^7*d^5*e^9 - 23*a^6*b^5*c^3*d*e^13 - 1408*a^7*b*c^6*d^3*e^11 + 109*a^7*b^3*c^4*d*e^13)/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(2*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((128*a^12*c^4*d*e^12 + 768*a^10*c^6*d^5*e^8 + 896*a^11*c^5*d^3*e^10 + 128*a^8*b^4*...

3.533 $\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=650

$$-\frac{2(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\sqrt{d+ex}}{c^5}-\frac{2b(b^2-2ac)\,(d+ex)^{3/2}}{3c^4}+\frac{2(c^2d^2+b^2e^2+ce(bd-ae))}{5c^3e^3}$$

```
[Out] -2/3*b*(-2*a*c+b^2)*(e*x+d)^(3/2)/c^4+2/5*(c^2*d^2+b^2*e^2+c*e*(-a*e+b*d))*
(e*x+d)^(5/2)/c^3/e^3-2/7*(b*e+2*c*d)*(e*x+d)^(7/2)/c^2/e^3+2/9*(e*x+d)^(9/
2)/c/e^3-2*(-a^2*c^2*e+3*a*b^2*c*e-2*a*b*c^2*d-b^4*e+b^3*c*d)*(e*x+d)^(1/2)
/c^5+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2))*2^(1/2)*((a*c*e-b^2*e+b*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(2
*b^5*c*d*e-10*a*b^3*c^2*d*e+10*a^2*b*c^3*d*e-b^6*e^2+a*b^2*c^2*(-9*a*e^2+4*
c*d^2)-b^4*c*(-6*a*e^2+c*d^2)-2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))
/c^(11/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e
*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*((a*c*e-b^2*e+b
*c*d)*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+(-2*b^5*c*d*e+10*a*b^3*c^2*d*e-10
*a^2*b*c^3*d*e+b^6*e^2-a*b^2*c^2*(-9*a*e^2+4*c*d^2)+b^4*c*(-6*a*e^2+c*d^2)+
2*a^2*c^3*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))/c^(11/2)/(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 1.86, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {911, 1301, 1180, 214}

$$-\frac{2\sqrt{d+ex}\,(-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd)}{c^5}+\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3(cd^2-ae^2)-b^4c(cd^2-6ae^2)-10ab^3c^2de+ab^2c^2(4cd^2-9ae^2)+b^6(-e^2)+2b^5cde}{\sqrt{b^2-4ac}}+(ace+b^2(-e)+bcd)\,(3abce-2ac^2d+b^3(-e)+b^2cd)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{11/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left((ace+b^2(-e)+bcd)\,(3abce-2ac^2d+b^3(-e)+b^2cd)-\frac{10a^2bc^3de-2a^2c^3(cd^2-ae^2)-b^4c(cd^2-6ae^2)-10ab^3c^2de+ab^2c^2(4cd^2-9ae^2)+b^6(-e^2)+2b^5cde}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{11/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2b(b^2-2ac)\,(d+ex)^{3/2}}{3c^4}+\frac{2(d+ex)^{5/2}\,(ce(bd-ae)+b^2e^2+c^2d^2)}{5c^3e^3}-\frac{2(d+ex)^{7/2}(be+2cd)}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*Sqrt[d + e*x]
)/c^5 - (2*b*(b^2 - 2*a*c)*(d + e*x)^(3/2))/(3*c^4) + (2*(c^2*d^2 + b^2*e^2
 + c*e*(b*d - a*e))*(d + e*x)^(5/2))/(5*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*
x)^(7/2))/(7*c^2*e^3) + (2*(d + e*x)^(9/2))/(9*c*e^3) + (Sqrt[2]*((b*c*d -
b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 1
0*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2
) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e]])/(c^(11/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((b*c
```

```
*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*c*d*
e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*
a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*
a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]])/(c^(11/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^4}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{e(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)}{c^5} - \frac{b(b^2-2ac)ex^2}{c^4} + \frac{(c^2d^2+b^2e^2+ce(bd-ae))x^4}{c^3e^2} - \frac{(2cd-}{c}\right.\right.}{}$$

$$= -\frac{2(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\sqrt{d+ex}}{c^5} - \frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4} + \frac{2(c}{}$$

$$= -\frac{2(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\sqrt{d+ex}}{c^5} - \frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4} + \frac{2(c}{}$$

$$= -\frac{2(b^3cd-2abc^2d-b^4e+3ab^2ce-a^2c^2e)\sqrt{d+ex}}{c^5} - \frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4} + \frac{2(c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.03, size = 901, normalized size = 1.39

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(315*b^4*e^4 - 105*b^2*c*e^3*(4*b*d + 9*a*e + b*e*x) - 9*c
^3*e*(d + e*x)^2*(-2*b*d + 7*a*e + 5*b*e*x) + c^4*(d + e*x)^2*(8*d^2 - 20*d
*e*x + 35*e^2*x^2) + 21*c^2*e^2*(15*a^2*e^2 + 3*b^2*(d + e*x)^2 + 10*a*b*e*
(4*d + e*x))))/(315*c^5*e^3) - ((I*b^6*e^2 + b^5*e*((-2*I)*c*d + Sqrt[-b^2
+ 4*a*c]*e) + I*b^4*c*(c*d^2 + (2*I)*Sqrt[-b^2 + 4*a*c]*d*e - 6*a*e^2) + a*
b^2*c^2*((-4*I)*c*d^2 + 6*Sqrt[-b^2 + 4*a*c]*d*e + (9*I)*a*e^2) + a*b*c^2*(
3*a*Sqrt[-b^2 + 4*a*c]*e^2 - 2*c*d*(Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^
3*c*(-4*a*Sqrt[-b^2 + 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d + (10*I)*a*e))
 - (2*I)*a^2*c^3*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]]
)/(c^(11/2)*Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])
*e]) - (((-I)*b^6*e^2 + b^5*e*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + b^4*c*((
```

```
-I)*c*d^2 - 2*Sqrt[-b^2 + 4*a*c]*d*e + (6*I)*a*e^2) + a*b^2*c^2*((4*I)*c*d^
2 + 6*Sqrt[-b^2 + 4*a*c]*d*e - (9*I)*a*e^2) + a*b*c^2*(3*a*Sqrt[-b^2 + 4*a*
c]*e^2 - 2*c*d*(Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e)) + b^3*c*(-4*a*Sqrt[-b^2
+ 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d - (10*I)*a*e)) + (2*I)*a^2*c^3*(-(
c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(11/2)*Sqrt[-1/2*b
^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
```

Maple [A]
time = 0.17, size = 1094, normalized size = 1.68 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(1/c^5*(1/9*(e*x+d)^(9/2)*c^4-1/7*b*c^3*e*(e*x+d)^(7/2)-2/7*c^4*d*(e*
x+d)^(7/2)-1/5*a*c^3*e^2*(e*x+d)^(5/2)+1/5*b^2*c^2*e^2*(e*x+d)^(5/2)+1/5*b*
c^3*d*e*(e*x+d)^(5/2)+1/5*c^4*d^2*(e*x+d)^(5/2)+2/3*a*b*c^2*e^3*(e*x+d)^(3/
2)-1/3*b^3*c*e^3*(e*x+d)^(3/2)+a^2*c^2*e^4*(e*x+d)^(1/2)-3*a*b^2*c*e^4*(e*x
+d)^(1/2)+2*a*b*c^2*d*e^3*(e*x+d)^(1/2)+b^4*e^4*(e*x+d)^(1/2)-b^3*c*d*e^3*(
e*x+d)^(1/2))-4/c^4*e^3*(1/8*(-2*a^3*c^3*e^3+9*a^2*b^2*c^2*e^3-10*a^2*b*c^3
*d*e^2+2*a^2*c^4*d^2*e-6*a*b^4*e^3*c+10*a*b^3*c^2*d*e^2-4*a*b^2*c^3*d^2*e+b
^6*e^3-2*b^5*d*e^2*c+b^4*c^2*d^2*e+3*(-e^2*(4*a*c-b^2))^(1/2)*a^2*b*c^2*e^2
-2*(-e^2*(4*a*c-b^2))^(1/2)*a^2*c^3*d*e-4*(-e^2*(4*a*c-b^2))^(1/2)*a*b^3*c*
e^2+6*(-e^2*(4*a*c-b^2))^(1/2)*a*b^2*c^2*d*e-2*(-e^2*(4*a*c-b^2))^(1/2)*a*b
*c^3*d^2+(-e^2*(4*a*c-b^2))^(1/2)*b^5*e^2-2*(-e^2*(4*a*c-b^2))^(1/2)*b^4*c*
d*e+(-e^2*(4*a*c-b^2))^(1/2)*b^3*c^2*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2
)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(
1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(2*a^3*c^3*e^3-9*a
^2*b^2*c^2*e^3+10*a^2*b*c^3*d*e^2-2*a^2*c^4*d^2*e+6*a*b^4*e^3*c-10*a*b^3*c^
2*d*e^2+4*a*b^2*c^3*d^2*e-b^6*e^3+2*b^5*d*e^2*c-b^4*c^2*d^2*e+3*(-e^2*(4*a*
c-b^2))^(1/2)*a^2*b*c^2*e^2-2*(-e^2*(4*a*c-b^2))^(1/2)*a^2*c^3*d*e-4*(-e^2*
(4*a*c-b^2))^(1/2)*a*b^3*c*e^2+6*(-e^2*(4*a*c-b^2))^(1/2)*a*b^2*c^2*d*e-2*(
-e^2*(4*a*c-b^2))^(1/2)*a*b*c^3*d^2+(-e^2*(4*a*c-b^2))^(1/2)*b^5*e^2-2*(-e^
2*(4*a*c-b^2))^(1/2)*b^4*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b^3*c^2*d^2)/c/(-e^
2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)
^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

[Out] integrate((x*e + d)^(3/2)*x^4/(c*x^2 + b*x + a), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14252 vs. $2(611) = 1222$.
time = 25.09, size = 14252, normalized size = 21.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")

[Out] -1/630*(315*sqrt(2)*c^5*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2 - 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/(b^2*c^22 - 4*a*c^23)))/(b^2*c^11 - 4*a*c^12))*e^3*log(sqrt(2)*((b^12*c^4 - 12*a*b^10*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^13*c^3 - 52*a*b^11*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2*b^14*c^2 - 28*a*b^12*c^3 + 154*a^2*b^10*c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^15*c - 60*a*b^13*c^2 + 360*a^2*b^11*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^16 - 16*a*b^14*c + 104*a^2*b^12*c^2 - 352*a^3*b^10*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 - ((b^6*c^12 - 8*a*b^4*c^13 + 18*a^2*b^2*c^14 - 8*a^3*c^15)*d - (b^7*c^11 - 9*a*b^5*c^12 + 25*a^2*b^3*c^13 - 20*a^3*b*c^14)*e)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d

```
^6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*
a^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*
(5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a
^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8
*c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*
a^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700
*a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 53
0*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 +
 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e
^4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068
*a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*
a^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a*b^18*c + 137*a^2*b^16*c^2
- 574*a^3*b^14*c^3 + 1444*a^4*b^12*c^4 - 2232*a^5*b^10*c^5 + 2083*a^6*b^8*c
^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^10*c^10)*e^6)/
(b^2*c^22 - 4*a*c^23)))*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*
a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 -
30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^10*c - 10*a*b^8*c^2 + 35*a^2*b^6
*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^11 - 11*a*b^
9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3
+ (b^2*c^11 - 4*a*c^12)*sqrt(((b^14*c^6 - 12*a*b^12*c^7 + 56*a^2*b^10*c^8 -
 128*a^3*b^8*c^9 + 148*a^4*b^6*c^10 - 80*a^5*b^4*c^11 + 16*a^6*b^2*c^12)*d^
6 - 6*(b^15*c^5 - 13*a*b^13*c^6 + 67*a^2*b^11*c^7 - 174*a^3*b^9*c^8 + 239*a
^4*b^7*c^9 - 166*a^5*b^5*c^10 + 50*a^6*b^3*c^11 - 4*a^7*b*c^12)*d^5*e + 3*(
5*b^16*c^4 - 70*a*b^14*c^5 + 395*a^2*b^12*c^6 - 1150*a^3*b^10*c^7 + 1835*a^
4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^10 - 100*a^7*b^2*c^11 + 3*a^8*
c^12)*d^4*e^2 - 2*(10*b^17*c^3 - 150*a*b^15*c^4 + 920*a^2*b^13*c^5 - 2970*a
^3*b^11*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*
a^7*b^3*c^10 + 49*a^8*b*c^11)*d^3*e^3 + 3*(5*b^18*c^2 - 80*a*b^16*c^3 + 530
*a^2*b^14*c^4 - 1880*a^3*b^12*c^5 + 3855*a^4*b^10*c^6 - 4600*a^5*b^8*c^7 +
3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^10 - 2*a^9*c^11)*d^2*e^
4 - 6*(b^19*c - 17*a*b^17*c^2 + 121*a^2*b^15*c^3 - 468*a^3*b^13*c^4 + 1068*
a^4*b^11*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a
^8*b^3*c^9 - 5*a^9*b*c^10)*d*e^5 + (b^20 - 18*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1577 vs. $2(611) = 1222$.

time = 1.52, size = 1577, normalized size = 2.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2*e - 2*(b^6*c - 7*a*b^4*c^2
 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^2 + (b^7 - 8*a*b^5*c + 19*a^2*b^3*c^2 -
12*a^3*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 - 2
*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*a*b^2*c^4 +
a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2*b*c^4)*sqrt
(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqrt(b^2 - 4*a*
c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(b^4*c
^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3 - (5*b^5*c^4 - 24*a*b^3*c^5 + 22*a^2*b*c^
6)*d^2*e + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5 - 2*a^3*c^6)*d*e^2
- (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5)*e^3)*sqrt(-4*c^2*d
+ 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-
(2*c^10*d*e^30 - b*c^9*e^31 + sqrt(-4*(c^10*d^2*e^30 - b*c^9*d*e^31 + a*c^9
*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))*e^(-30)/c^10))/((sqrt(b
^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e + sqrt(b^2 - 4*a*c)*a*c^7
*e^2)*c^2) + 1/4*(((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^2*e - 2*(b^6*c -
 7*a*b^4*c^2 + 13*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^2 + (b^7 - 8*a*b^5*c + 19*a^
2*b^3*c^2 - 12*a^3*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c
)*e)*c^2 + 2*((b^3*c^4 - 2*a*b*c^5)*sqrt(b^2 - 4*a*c)*d^3 - (2*b^4*c^3 - 5*
a*b^2*c^4 + a^2*c^5)*sqrt(b^2 - 4*a*c)*d^2*e + (b^5*c^2 - 2*a*b^3*c^3 - a^2
*b*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*sqr
t(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c)
 + (2*(b^4*c^5 - 4*a*b^2*c^6 + 2*a^2*c^7)*d^3 - (5*b^5*c^4 - 24*a*b^3*c^5 +
 22*a^2*b*c^6)*d^2*e + 2*(2*b^6*c^3 - 11*a*b^4*c^4 + 14*a^2*b^2*c^5 - 2*a^3
*c^6)*d*e^2 - (b^7*c^2 - 6*a*b^5*c^3 + 9*a^2*b^3*c^4 - 2*a^3*b*c^5)*e^3)*sq
rt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e
 + d)/sqrt(-(2*c^10*d*e^30 - b*c^9*e^31 - sqrt(-4*(c^10*d^2*e^30 - b*c^9*d*
e^31 + a*c^9*e^32)*c^10*e^30 + (2*c^10*d*e^30 - b*c^9*e^31)^2))*e^(-30)/c^1
0))/((sqrt(b^2 - 4*a*c)*c^8*d^2 - sqrt(b^2 - 4*a*c)*b*c^7*d*e + sqrt(b^2 -
4*a*c)*a*c^7*e^2)*c^2) + 2/315*(35*(x*e + d)^(9/2)*c^8*e^24 - 90*(x*e + d)^
(7/2)*c^8*d*e^24 + 63*(x*e + d)^(5/2)*c^8*d^2*e^24 - 45*(x*e + d)^(7/2)*b*c
^7*e^25 + 63*(x*e + d)^(5/2)*b*c^7*d*e^25 + 63*(x*e + d)^(5/2)*b^2*c^6*e^26
 - 63*(x*e + d)^(5/2)*a*c^7*e^26 - 105*(x*e + d)^(3/2)*b^3*c^5*e^27 + 210*(
x*e + d)^(3/2)*a*b*c^6*e^27 - 315*sqrt(x*e + d)*b^3*c^5*d*e^27 + 630*sqrt(x
*e + d)*a*b*c^6*d*e^27 + 315*sqrt(x*e + d)*b^4*c^4*e^28 - 945*sqrt(x*e + d)
*a*b^2*c^5*e^28 + 315*sqrt(x*e + d)*a^2*c^6*e^28)*e^(-27)/c^9
```

Mupad [B]
time = 7.97, size = 2500, normalized size = 3.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] (d + e*x)^(1/2)*((2*d^4)/(c*e^3) - ((a*e^5 + c*d^2*e^3 - b*d*e^4)*((12*d^2)
/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((8*d)/(c*e^3) +
(2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e^3)))/(c*e^3) +
 ((b*e^4 - 2*c*d*e^3)*((8*d^3)/(c*e^3) - (((8*d)/(c*e^3) + (2*(b*e^4 - 2*c*
d*e^3))/(c^2*e^6))*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c*e^3) + ((b*e^4 - 2*c*d
*e^3)*((12*d^2)/(c*e^3) - (2*(a*e^5 + c*d^2*e^3 - b*d*e^4))/(c^2*e^6) + (((
8*d)/(c*e^3) + (2*(b*e^4 - 2*c*d*e^3))/(c^2*e^6))*(b*e^4 - 2*c*d*e^3))/(c*e
^3)))/(c*e^3)))/(c*e^3)) - atan(((((8*(4*a^4*c^9*e^5 - a*b^6*c^6*e^5 + b^7*
c^6*d*e^4 + 7*a^2*b^4*c^7*e^5 - 13*a^3*b^2*c^8*e^5 + 4*a^3*c^10*d^2*e^3 + b
^5*c^8*d^3*e^2 - 2*b^6*c^7*d^2*e^3 - 21*a^2*b^2*c^9*d^2*e^3 - 6*a*b^5*c^7*d
*e^4 + 4*a^3*b*c^9*d*e^4 - 6*a*b^3*c^9*d^3*e^2 + 13*a*b^4*c^8*d^2*e^3 + 8*a
^2*b*c^10*d^3*e^2 + 7*a^2*b^3*c^8*d*e^4))/c^9 - (8*(d + e*x)^(1/2)*(-(b^13*
e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12
*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 5
2*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^
2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a
^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 28*a^2*b^6*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 15*a^4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*
b^8*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e
^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5
*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e +
 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 30
6*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2
*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^
13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2)*(b^3*c^11*e^3 - 2*b^2*c^12*d*e^2 - 4*
a*b*c^12*e^3 + 8*a*c^13*d*e^2))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3
*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*
e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4
*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 +
363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^
(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d
*e^2 + 10*a^2*b^3*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) - 28*a^2*b^6*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 35*a^3*b^4*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a^
4*b^2*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^8*c*e^3*(-(4*a*c - b^2)^3)^(
```

1/2) - 39*a*b^9*c^3*d^2*e + 42*a*b^10*c^2*d*e^2 - 108*a^5*b*c^7*d^2*e + 3*b^9*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^5*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c^6*d^3*(-(4*a*c - b^2)^3)^(1/2) + 189*a^2*b^7*c^4*d^2*e - 225*a^2*b^8*c^3*d*e^2 - 414*a^3*b^5*c^5*d^2*e + 570*a^3*b^6*c^4*d*e^2 + 387*a^4*b^3*c^6*d^2*e - 675*a^4*b^4*c^5*d*e^2 + 306*a^5*b^2*c^6*d*e^2 - 3*a^4*c^6*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^8*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 21*a*b^6*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^7*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 15*a^4*b*c^5*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 45*a^2*b^4*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 63*a^2*b^5*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a^3*b^2*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 60*a^3*b^3*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^13 + b^4*c^11 - 8*a*b^2*c^12)))^(1/2) - (8*(d + e*x)^(1/2)*(b^12*e^6 + 2*a^6*c^6*e^6 + 54*a^2*b^8*c^2*e^6 - 112*a^3*b^6*c^3*e^6 + 105*a^4*b^4*c^4*e^6 - 36*a^5*b^2*c^5*e^6 + 2*a^4*c^8*d^4*e^2 - 12*a^5*c^7*d^2*e^4 + b^8*c^4*d^4*e^2 - 4*b^9*c^3*d^3*e^3 + 6*b^10*c^2*d^2*e^4 - 12*a*b^10*c*e^6 - 4*b^11*c*d*e^5 + 20*a^2*b^4*c^6*d^4*e^2 - 108*a^2*b^5*c^5*d^3*e^3 + 210*a^2*b^6*c^4*d^2*e^4 - 16*a^3*b^2*c^7*d^4*e^2 + 120*a^3*b^3*c^6*d^3*e^3 - 300*a^3*b^4*c^5*d^2*e^4 + 150*a^4*b^2*c^6*d^2*e^4 + 44*a*b^9*c^2*d*e^5 + 44*a^5*b*c^6*d*e^5 - 8*a*b^6*c^5*d^4*e^2 + 36*a*b^7*c^4*d^3*e^3 - 60*a*b^8*c^3*d^2*e^4 - 176*a^2*b^7*c^3*d*e^5 + 308*a^3*b^5*c^4*d*e^5 - 36*a^4*b*c^7*d^3*e^3 - 220*a^4*b^3*c^5*d*e^5))/c^9)*(-(b^13*e^3 + 8*a^5*c^8*d^3 - b^10*c^3*d^3 - b^10*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^8*c^4*d^3 + 44*a^6*b*c^6*e^3 - 24*a^6*c^7*d*e^2 + 3*b^11*c^2*d^2*e - 52*a^2*b^6*c^5*d^3 + 96*a^3*b^4*c^6*d^3 - 66*a^4*b^2*c^7*d^3 + 88*a^2*b^9*c^2*e^3 - 253*a^3*b^7*c^3*e^3 + 363*a^4*b^5*c^4*e^3 - 231*a^5*b^3*c^5*e^3 + a^5*c^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^7*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^11*c*e^3 - 3*b^12*c*d*e^2 + 10*a^2*b^...

3.534 $\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=581

$$\frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4}+\frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+ex)^{5/2}}{5c^2e^2}+\frac{2(d+ex)^{7/2}}{7ce^2}+\frac{\sqrt{2}}{}$$

```
[Out] 2/3*(-a*c+b^2)*(e*x+d)^(3/2)/c^3-2/5*(b*e+c*d)*(e*x+d)^(5/2)/c^2/e^2+2/7*(e
*x+d)^(7/2)/c/e^2+2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d)*(e*x+d)^(1/2)/c^4+arc
tanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*
2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^4*e^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a
*e^2+c*d^2)+(-2*b^4*c*d*e+8*a*b^2*c^2*d*e-4*a^2*c^3*d*e+b^5*e^2+b^3*c*(-5*a
*e^2+c*d^2)-a*b*c^2*(-5*a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*b^3*c*d*e-4*a*b*c^2*d*e-b^4*e
^2-b^2*c*(-3*a*e^2+c*d^2)+a*c^2*(-a*e^2+c*d^2)+(2*b^4*c*d*e-8*a*b^2*c^2*d*e
+4*a^2*c^3*d*e-b^5*e^2-b^3*c*(-5*a*e^2+c*d^2)+a*b*c^2*(-5*a*e^2+3*c*d^2))/(
-4*a*c+b^2)^(1/2))/c^(9/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 13.80, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {911, 1301, 1180, 214}

$$\frac{\sqrt{2}\left(-\frac{4a^2c^3de-b^3c(cd^2-5ae^2)-8ab^2c^2de+abc^2(3cd^2-5ae^2)+b^5(-e^2)+2b^4cde}{\sqrt{b^2-4ac}}-b^2c(cd^2-3ae^2)-4abc^2de+ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(\frac{4a^2c^3de-b^3c(cd^2-5ae^2)-8ab^2c^2de+abc^2(3cd^2-5ae^2)+b^5(-e^2)+2b^4cde}{\sqrt{b^2-4ac}}-b^2c(cd^2-3ae^2)-4abc^2de+ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{9/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3}+\frac{2\sqrt{d+ex}(2abce-ac^2d+b^3(-e)+b^2cd)}{c^4}-\frac{2(d+ex)^{5/2}(be+cd)}{5c^2e^2}+\frac{2(d+ex)^{7/2}}{7ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Sqrt[d + e*x])/c^4 + (2*(b^2 - a
*c)*(d + e*x)^(3/2))/(3*c^3) - (2*(c*d + b*e)*(d + e*x)^(5/2))/(5*c^2*e^2)
+ (2*(d + e*x)^(7/2))/(7*c*e^2) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b
^4*e^2 - b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) - (2*b^4*c*d*e - 8
*a*b^2*c^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^
2*(3*c*d^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*b^3*c*d*e - 4*a*b*c^2*d*e - b^4*e^2 -
 b^2*c*(c*d^2 - 3*a*e^2) + a*c^2*(c*d^2 - a*e^2) + (2*b^4*c*d*e - 8*a*b^2*c
^2*d*e + 4*a^2*c^3*d*e - b^5*e^2 - b^3*c*(c*d^2 - 5*a*e^2) + a*b*c^2*(3*c*d
^2 - 5*a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
```

```
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(9/2)*Sqrt[2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{e(b^2cd-ac^2d-b^3e+2abce)}{c^4}+\frac{(b^2-ac)ex^2}{c^3}-\frac{(cd+be)x^4}{c^2e}+\frac{x^6}{ce}-\frac{(b^2cd-ac^2d-b^3e+2abce)(ca\right.\right.}{e}$$

$$= \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4}+\frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+e}{5c^2e^2}$$

$$= \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4}+\frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+e}{5c^2e^2}$$

$$= \frac{2(b^2cd-ac^2d-b^3e+2abce)\sqrt{d+ex}}{c^4}+\frac{2(b^2-ac)(d+ex)^{3/2}}{3c^3}-\frac{2(cd+be)(d+e}{5c^2e^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.46, size = 755, normalized size = 1.30

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(105*b^3*e^3 + 3*c^3*(2*d - 5*e*x)*(d + e*x)^2 - 35*b*c*e
^2*(4*b*d + 6*a*e + b*e*x) + 7*c^2*e*(3*b*(d + e*x)^2 + 5*a*e*(4*d + e*x)))
)/(105*c^4*e^2) + ((I*b^5*e^2 + b^4*e*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) +
 I*b^3*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 + 4*a*c]*d - 5*a*e)) + a*c^2*(a*Sqrt[-
b^2 + 4*a*c]*e^2 - c*d*(Sqrt[-b^2 + 4*a*c]*d + (4*I)*a*e)) + a*b*c^2*((-3*I
)*c*d^2 + e*(4*Sqrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^2*c*(-3*a*Sqrt[-b^2 +
 4*a*c]*e^2 + c*d*(Sqrt[-b^2 + 4*a*c]*d + (8*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt
[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*S
qrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((-I)
*b^5*e^2 + b^4*e*((2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + a*c^2*(a*Sqrt[-b^2 +
4*a*c]*e^2 + c*d*(-(Sqrt[-b^2 + 4*a*c]*d) + (4*I)*a*e)) + a*b*c^2*((3*I)*c*
d^2 + e*(4*Sqrt[-b^2 + 4*a*c]*d - (5*I)*a*e)) + b^3*c*((-I)*c*d^2 + e*(-2*S
```

```
qrt[-b^2 + 4*a*c]*d + (5*I)*a*e)) + b^2*c*(-3*a*Sqrt[-b^2 + 4*a*c]*e^2 + c*
d*(Sqrt[-b^2 + 4*a*c]*d - (8*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(c^(9/2)*Sqrt[-1/2*b^2 + 2
*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
```

Maple [A]
time = 0.16, size = 893, normalized size = 1.54

method	result
derivativedivides	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^3}{7}-\frac{b c^2 e (ex+d)^{\frac{5}{2}}}{5}-\frac{c^3 d (ex+d)^{\frac{5}{2}}}{5}-\frac{a c^2 e^2 (ex+d)^{\frac{3}{2}}}{3}+\frac{b^2 c e^2 (ex+d)^{\frac{3}{2}}}{3}+2abc e^3\sqrt{ex+d}-a c^2 d e^2\sqrt{ex+d}-b^3\right.}{c^4}$
default	$\frac{2\left(\frac{(ex+d)^{\frac{7}{2}}c^3}{7}-\frac{b c^2 e (ex+d)^{\frac{5}{2}}}{5}-\frac{c^3 d (ex+d)^{\frac{5}{2}}}{5}-\frac{a c^2 e^2 (ex+d)^{\frac{3}{2}}}{3}+\frac{b^2 c e^2 (ex+d)^{\frac{3}{2}}}{3}+2abc e^3\sqrt{ex+d}-a c^2 d e^2\sqrt{ex+d}-b^3\right.}{c^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(1/c^4*(1/7*(e*x+d)^(7/2)*c^3-1/5*b*c^2*e*(e*x+d)^(5/2)-1/5*c^3*d*(e*
x+d)^(5/2)-1/3*a*c^2*e^2*(e*x+d)^(3/2)+1/3*b^2*c*e^2*(e*x+d)^(3/2)+2*a*b*c*
e^3*(e*x+d)^(1/2)-a*c^2*d*e^2*(e*x+d)^(1/2)-b^3*e^3*(e*x+d)^(1/2)+b^2*c*d*e
^2*(e*x+d)^(1/2))-4*e^2/c^3*(1/8*(-5*a^2*b*c^2*e^3+4*a^2*c^3*d*e^2+5*a*b^3*
e^3*c-8*a*b^2*c^2*d*e^2+3*a*b*c^3*d^2*e-b^5*e^3+2*b^4*d*e^2*c-b^3*c^2*d^2*e
-(-e^2*(4*a*c-b^2))^(1/2)*a^2*c^2*e^2+3*(-e^2*(4*a*c-b^2))^(1/2)*a*b^2*c*e^
2-4*(-e^2*(4*a*c-b^2))^(1/2)*a*b*c^2*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c^3*d^2
-(-e^2*(4*a*c-b^2))^(1/2)*b^4*e^2+2*(-e^2*(4*a*c-b^2))^(1/2)*b^3*c*d*e-(-e^
2*(4*a*c-b^2))^(1/2)*b^2*c^2*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((e*b-
2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((e
*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(5*a^2*b*c^2*e^3-4*a^2*c^3
*d*e^2-5*a*b^3*e^3*c+8*a*b^2*c^2*d*e^2-3*a*b*c^3*d^2*e+b^5*e^3-2*b^4*d*e^2*
```

```
c+b^3*c^2*d^2*e-(-e^2*(4*a*c-b^2))^(1/2)*a^2*c^2*e^2+3*(-e^2*(4*a*c-b^2))^(
1/2)*a*b^2*c*e^2-4*(-e^2*(4*a*c-b^2))^(1/2)*a*b*c^2*d*e+(-e^2*(4*a*c-b^2))^
(1/2)*a*c^3*d^2-(-e^2*(4*a*c-b^2))^(1/2)*b^4*e^2+2*(-e^2*(4*a*c-b^2))^(1/2)
*b^3*c*d*e-(-e^2*(4*a*c-b^2))^(1/2)*b^2*c^2*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)
*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^
(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)*x^3/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11377 vs. 2(536) = 1072.
time = 21.02, size = 11377, normalized size = 19.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/210*(105*sqrt(2)*c^4*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^
3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e
 + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*
e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3
 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24
*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7
*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^
4 - 50*a*b^10*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 6
0*a^5*b^2*c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460
*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6
*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^
3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)
*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 +
239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (
b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 -
 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^
18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*e^2*log(sqrt(2)*((b^9*c^4 - 9*a*b^7*
c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^10*c^3 - 4
0*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*
c^8)*d^3*e + 3*(2*b^11*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^
```

5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 2
16*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^
6*c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181
*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 - ((b^5*c^10 - 7*a*b^3*c^1
1 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 + 18*a^2*b^2*c^11 - 8*a^3*c^
12)*e)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*
a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5
*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5
 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3
*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 91
0*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 +
 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a
^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^1
5*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 -
 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*
c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5
+ 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))*
sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2
- 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^
2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c +
 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*
sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2
*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 +
22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*
a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^
10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b
^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b
^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*
c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 1
3*a*b^13*c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^
5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*
a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a
^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19)))/(b^2*c^
9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^
3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b
^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*
b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3
+ (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d
*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*
e^5)*sqrt(x*e + d)) - 105*sqrt(2)*c^4*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*
b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^
3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 +
 2*a^4*c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*
a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*sqrt(((b^10*c^6 - 8*a*b^8*c^7 + 22*a^
2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11*c^5 - 9*a*b^9*c^

```
6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e
+ 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185*a^2*b^8*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

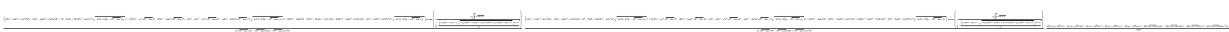
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. 2(536) = 1072.
time = 1.09, size = 1362, normalized size = 2.34



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e - 2*(b^5*c - 6*a*b^3*c^2 +
8*a^2*b*c^3)*d*e^2 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*sq
rt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 - 2*((b^2*c^4 - a*c^5)*s
qrt(b^2 - 4*a*c)*d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e + (b
^4*c^2 - a*b^2*c^3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2*a^2*
b*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)
*e)*abs(c) + (2*(b^3*c^5 - 3*a*b*c^6)*d^3 - (5*b^4*c^4 - 19*a*b^2*c^5 + 8*a
^2*c^6)*d^2*e + 2*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5)*d*e^2 - (b^6*c^2
- 5*a*b^4*c^3 + 5*a^2*b^2*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a
*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^16 - b*c^7*e^1
7 + sqrt(-4*(c^8*d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18)*c^8*e^16 + (2*c^8*d*
e^16 - b*c^7*e^17)^2))*e^(-16)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2
 - 4*a*c)*b*c^6*d*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) - 1/4*(((b^4*c^2 -
5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^
2 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*sqrt(-4*c^2*d + 2*(
b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*((b^2*c^4 - a*c^5)*sqrt(b^2 - 4*a*c)*
d^3 - (2*b^3*c^3 - 3*a*b*c^4)*sqrt(b^2 - 4*a*c)*d^2*e + (b^4*c^2 - a*b^2*c^
3 - a^2*c^4)*sqrt(b^2 - 4*a*c)*d*e^2 - (a*b^3*c^2 - 2*a^2*b*c^3)*sqrt(b^2 -
 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(
b^3*c^5 - 3*a*b*c^6)*d^3 - (5*b^4*c^4 - 19*a*b^2*c^5 + 8*a^2*c^6)*d^2*e + 2
*(2*b^5*c^3 - 9*a*b^3*c^4 + 7*a^2*b*c^5)*d*e^2 - (b^6*c^2 - 5*a*b^4*c^3 + 5
*a^2*b^2*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan
(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^8*d*e^16 - b*c^7*e^17 - sqrt(-4*(c^8*
d^2*e^16 - b*c^7*d*e^17 + a*c^7*e^18)*c^8*e^16 + (2*c^8*d*e^16 - b*c^7*e^17
```

```
)^2))*e^(-16)/c^8))/((sqrt(b^2 - 4*a*c)*c^7*d^2 - sqrt(b^2 - 4*a*c)*b*c^6*d
*e + sqrt(b^2 - 4*a*c)*a*c^6*e^2)*c^2) + 2/105*(15*(x*e + d)^(7/2)*c^6*e^12
 - 21*(x*e + d)^(5/2)*c^6*d*e^12 - 21*(x*e + d)^(5/2)*b*c^5*e^13 + 35*(x*e
+ d)^(3/2)*b^2*c^4*e^14 - 35*(x*e + d)^(3/2)*a*c^5*e^14 + 105*sqrt(x*e + d)
*b^2*c^4*d*e^14 - 105*sqrt(x*e + d)*a*c^5*d*e^14 - 105*sqrt(x*e + d)*b^3*c^
3*e^15 + 210*sqrt(x*e + d)*a*b*c^4*e^15)*e^(-14)/c^7
```

Mupad [B]
time = 7.14, size = 2500, normalized size = 4.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] atan(((((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e
^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^
2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12
*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 - (8*(d + e*x)^(1/2)*(-(b^11
*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*
a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*
a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3
*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^
4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*
b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
 + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c
^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*
b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2
))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*(b^3*c^9*e^3 - 2*b^2*c
^10*d*e^2 - 4*a*b*c^10*e^3 + 8*a*c^11*d*e^2))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*
d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 3
6*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 +
 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^
3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*
c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 8
4*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^
3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126
```

*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2) - (8*(d + e*x)^(1/2)*(b^10*e^6 - 2*a^5*c^5*e^6 + 35*a^2*b^6*c^2*e^6 - 50*a^3*b^4*c^3*e^6 + 25*a^4*b^2*c^4*e^6 - 2*a^3*c^7*d^4*e^2 + 12*a^4*c^6*d^2*e^4 + b^6*c^4*d^4*e^2 - 4*b^7*c^3*d^3*e^3 + 6*b^8*c^2*d^2*e^4 - 10*a*b^8*c*e^6 - 4*b^9*c*d*e^5 + 9*a^2*b^2*c^6*d^4*e^2 - 56*a^2*b^3*c^5*d^3*e^3 + 120*a^2*b^4*c^4*d^2*e^4 - 96*a^3*b^2*c^5*d^2*e^4 + 36*a*b^7*c^2*d*e^5 - 36*a^4*b*c^5*d*e^5 - 6*a*b^4*c^5*d^4*e^2 + 28*a*b^5*c^4*d^3*e^3 - 48*a*b^6*c^3*d^2*e^4 - 108*a^2*b^5*c^3*d*e^5 + 28*a^3*b*c^6*d^3*e^3 + 120*a^3*b^3*c^4*d*e^5))/c^7)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4*e^3 + a^4*c^4*e^3*(-(4*a*c - b^2)^3)^(1/2) - b^5*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 13*a*b^9*c*e^3 - 3*b^10*c*d*e^2 + 15*a^2*b^4*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a^3*b^2*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^6*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 33*a*b^7*c^3*d^2*e + 36*a*b^8*c^2*d*e^2 + 84*a^4*b*c^6*d^2*e - 3*b^7*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b^3*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 126*a^2*b^5*c^4*d^2*e - 156*a^2*b^6*c^3*d*e^2 - 189*a^3*b^3*c^5*d^2*e + 288*a^3*b^4*c^4*d*e^2 - 198*a^4*b^2*c^5*d*e^2 - 3*a^3*c^5*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^6*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 15*a*b^4*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^5*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^3*b*c^4*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a^2*b^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 30*a^2*b^3*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^11 + b^4*c^9 - 8*a*b^2*c^10)))^(1/2)*1i - (((8*(4*a^3*c^8*d*e^4 - 8*a^3*b*c^7*e^5 - a*b^5*c^5*e^5 + b^6*c^5*d*e^4 + 6*a^2*b^3*c^6*e^5 + 4*a^2*c^9*d^3*e^2 + b^4*c^7*d^3*e^2 - 2*b^5*c^6*d^2*e^3 - 5*a*b^4*c^6*d*e^4 - 5*a*b^2*c^8*d^3*e^2 + 11*a*b^3*c^7*d^2*e^3 - 12*a^2*b*c^8*d^2*e^3 + 3*a^2*b^2*c^7*d*e^4))/c^7 + (8*(d + e*x)^(1/2)*(-(b^11*e^3 - 8*a^4*c^7*d^3 - b^8*c^3*d^3 + b^8*e^3*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^6*c^4*d^3 - 36*a^5*b*c^5*e^3 + 24*a^5*c^6*d*e^2 + 3*b^9*c^2*d^2*e - 33*a^2*b^4*c^5*d^3 + 38*a^3*b^2*c^6*d^3 + 63*a^2*b^7*c^2*e^3 - 138*a^3*b^5*c^3*e^3 + 129*a^4*b^3*c^4...

3.535 $\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2} dx$

Optimal. Leaf size=441

$$-\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{\sqrt{2}\left(\left((cd-be)\left(bcd-b^2e+2ace\right)+\frac{2b^3cde-6ab\cdots}{\cdots}\right.\right.}{c^{\cdots}}$$

```
[Out] -2/3*b*(e*x+d)^(3/2)/c^2+2/5*(e*x+d)^(5/2)/c/e-2*(a*c*e-b^2*e+b*c*d)*(e*x+d
)^(1/2)/c^3+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2))*2^(1/2)*((-b*e+c*d)*(2*a*c*e-b^2*e+b*c*d)+(2*b^3*c*d*e-6*a*b
*c^2*d*e-b^4*e^2-b^2*c*(-4*a*e^2+c*d^2)+2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2
)^(1/2))/c^(7/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(
1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*((-b*e+c
*d)*(2*a*c*e-b^2*e+b*c*d)+(-2*b^3*c*d*e+6*a*b*c^2*d*e+b^4*e^2+b^2*c*(-4*a*e
^2+c*d^2)-2*a*c^2*(-a*e^2+c*d^2))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 1.52, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {911, 1301, 1180, 214}

$$-\frac{2\sqrt{d+ex}\left(ace+b^2(-e)+bcd\right)}{c^3}+\frac{\sqrt{2}\left((cd-be)\left(2ace+b^2(-e)+bcd\right)+\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+b^4\left(-e^2\right)+2b^3cde}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left((cd-be)\left(2ace+b^2(-e)+bcd\right)-\frac{-b^2c\left(cd^2-4ae^2\right)-6abc^2de+2ac^2\left(cd^2-ae^2\right)+b^4\left(-e^2\right)+2b^3cde}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (-2*(b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x])/c^3 - (2*b*(d + e*x)^(3/2))/(3*c
^2) + (2*(d + e*x)^(5/2))/(5*c*e) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e +
2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2)
 + 2*a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c
*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*
a*c^2*(c*d^2 - a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(7/2)*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{e(bcd-b^2e+ace)}{c^3}-\frac{bex^2}{c^2}+\frac{x^4}{c}+\frac{(bcd-b^2e+ace)(cd^2-bde+ae^2)-(cd-be)(bcd-b^2e+2ace)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)} \right.\right.}{e}$$

$$= -\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{2\text{Subst}\left(\int \frac{(bcd-b^2e+a}{}\right.}{}$$

$$= -\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}-\frac{\left(\left(cd-be\right)\left(bcd-b^2\right.\right.}{}$$

$$= -\frac{2(bcd-b^2e+ace)\sqrt{d+ex}}{c^3}-\frac{2b(d+ex)^{3/2}}{3c^2}+\frac{2(d+ex)^{5/2}}{5ce}+\frac{\sqrt{2}\left(\left(cd-be\right)\left(bcd\right.\right.}{}$$

Mathematica [A]
time = 1.84, size = 537, normalized size = 1.22

$$\frac{\frac{2\sqrt{c}\sqrt{d+ex}\,(15b^2e^2+3c^2(d+ex)^2-5ce(4bd+3ae+bex))}{e}-\frac{15\sqrt{2}\left(-b^4e^2+b^3e\left(2cd+\sqrt{b^2-4ac}\,e\right)+bc\left(-2a\sqrt{b^2-4ac}\,e^2+cd\left(\sqrt{b^2-4ac}\,d-6ae\right)\right)-b^2c\left(cd^2+2e\left(\sqrt{b^2-4ac}\,d-2ae\right)\right)+2ac^2\left(cd^2+e\left(\sqrt{b^2-4ac}\,d-ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{15\sqrt{2}\left(b^4e^2+b^3e\left(-2cd+\sqrt{b^2-4ac}\,e\right)+2ac^2\left(-cd^2+e\left(\sqrt{b^2-4ac}\,d+ae\right)\right)+b^2c\left(cd^2-2e\left(\sqrt{b^2-4ac}\,d+2ae\right)\right)+bc\left(-2a\sqrt{b^2-4ac}\,e^2+cd\left(\sqrt{b^2-4ac}\,d+6ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}}{15c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x]*(15*b^2*e^2 + 3*c^2*(d + e*x)^2 - 5*c*e*(4*b*d +
3*a*e + b*e*x)))/e - (15*Sqrt[2]*(-(b^4*e^2) + b^3*e*(2*c*d + Sqrt[b^2 - 4*
a*c]*e) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 6*a*
e)) - b^2*c*(c*d^2 + 2*e*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + 2*a*c^2*(c*d^2 +
e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt
[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b
- Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2]*(b^4*e^2 + b^3*e*(-2*c*d + Sqrt[b^2
- 4*a*c]*e) + 2*a*c^2*(-(c*d^2) + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b^2*c*(c
*d^2 - 2*e*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + b*c*(-2*a*Sqrt[b^2 - 4*a*c]*e^2
 + c*d*(Sqrt[b^2 - 4*a*c]*d + 6*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*
d + (b + Sqrt[b^2 - 4*a*c])*e]))/(15*c^(7/2))
```

Maple [A]

time = 0.14, size = 674, normalized size = 1.53

method	result
derivativedivides	$-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^{2}}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+ace^{2}\sqrt{ex+d}-b^{2}e^{2}\sqrt{ex+d}+bdec\sqrt{ex+d}\right)}{c^{3}}+\frac{8e\left(\frac{\left(-2a^{2}c^{2}e^{3}+4a\,b^{2}e^{3}c-6a\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2\left(-\frac{(ex+d)^{\frac{5}{2}}c^{2}}{5}+\frac{bce(ex+d)^{\frac{3}{2}}}{3}+ace^{2}\sqrt{ex+d}-b^{2}e^{2}\sqrt{ex+d}+bdec\sqrt{ex+d}\right)}{c^{3}}+\frac{8e\left(\frac{\left(-2a^{2}c^{2}e^{3}+4a\,b^{2}e^{3}c-6a\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/c^3*(-1/5*(e*x+d)^(5/2)*c^2+1/3*b*c*e*(e*x+d)^(3/2)+a*c*e^2*(e*x+d)
^(1/2)-b^2*e^2*(e*x+d)^(1/2)+b*d*e*c*(e*x+d)^(1/2))+4*e/c^2*(1/8*(-2*a^2*c^
2*e^3+4*a*b^2*e^3*c-6*a*b*c^2*d*e^2+2*a*c^3*d^2*e-b^4*e^3+2*b^3*d*e^2*c-b^2
*c^2*d^2*e+2*(-e^2*(4*a*c-b^2))^(1/2)*a*b*c*e^2-2*(-e^2*(4*a*c-b^2))^(1/2)*
a*c^2*d*e-(-e^2*(4*a*c-b^2))^(1/2)*b^3*e^2+2*(-e^2*(4*a*c-b^2))^(1/2)*b^2*c
*d*e-(-e^2*(4*a*c-b^2))^(1/2)*b*c^2*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1
/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(2*a^2*c^2*e^3-4*a*
b^2*e^3*c+6*a*b*c^2*d*e^2-2*a*c^3*d^2*e+b^4*e^3-2*b^3*d*e^2*c+b^2*c^2*d^2*e
+2*(-e^2*(4*a*c-b^2))^(1/2)*a*b*c*e^2-2*(-e^2*(4*a*c-b^2))^(1/2)*a*c^2*d*e-
(-e^2*(4*a*c-b^2))^(1/2)*b^3*e^2+2*(-e^2*(4*a*c-b^2))^(1/2)*b^2*c*d*e-(-e^2
*(4*a*c-b^2))^(1/2)*b*c^2*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-e
*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)*x^2/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8462 vs. $2(405) = 810$.
time = 7.60, size = 8462, normalized size = 19.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*c^3*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^
5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b
^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d
^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b
^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^
2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a
^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^
3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 +
 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b
^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a
^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*e*l
og(sqrt(2)*((b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a
*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c
^3 + 39*a^2*b^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a
*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10
- 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5
)*e^4 - ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 +
12*a^2*b*c^9)*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7
*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30
*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b
^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d
^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 +
 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7
*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^
```

```
2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a
*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7
+ 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c
^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 +
 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*
a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^
2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11
*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5
*b*c^6)*d*e^5 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c
^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^5 - (4*a^2*b^4*c^3 - 11*a
^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b
*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d
^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3
*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*sqrt(x*e + d)) - 15*sqrt
(2)*c^3*sqrt(((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^
3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3
*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 + (b^2*c
^7 - 4*a*c^8)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^
5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*
b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*
c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*
e^3 + 3*(5*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45
*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^
3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)*e^6)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*e*log(-sqrt(2)*((
b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53
*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b
^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 10
7*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 - ((b^4
*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)
*e)*sqrt(((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^
5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 +
55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a
*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5
*b^10*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c
^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^11*c - 9*a*b^9*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(405) = 810$.
time = 1.09, size = 1160, normalized size = 2.63

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^3*c^2 - 4*a*b*c^3)*d^2*e - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*
e^2 + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^
2 - 4*a*c)*c)*e)*c^2 - 2*(sqrt(b^2 - 4*a*c)*b*c^4*d^3 + sqrt(b^2 - 4*a*c)*b
^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2*e - (a*b^2*c^2 - a
^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c
)*e)*abs(c) + (2*(b^2*c^5 - 2*a*c^6)*d^3 - (5*b^3*c^4 - 14*a*b*c^5)*d^2*e +
 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^2 - (b^5*c^2 - 4*a*b^3*c^3 + 2
*a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2
*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^6 - b*c^5*e^7 + sqrt(-4*(c^6*d^2*
e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^6*d*e^6 - b*c^5*e^7)^2))*e^(-
6)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^2 - 4*a*c)*b*c^5*d*e + sqrt(b
^2 - 4*a*c)*a*c^5*e^2)*c^2) + 1/4*(((b^3*c^2 - 4*a*b*c^3)*d^2*e - 2*(b^4*c
- 5*a*b^2*c^2 + 4*a^2*c^3)*d*e^2 + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^3)*sqr
t(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*(sqrt(b^2 - 4*a*c)*b*
c^4*d^3 + sqrt(b^2 - 4*a*c)*b^3*c^2*d*e^2 - (2*b^2*c^3 - a*c^4)*sqrt(b^2 -
4*a*c)*d^2*e - (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^3)*sqrt(-4*c^2*d +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*(b^2*c^5 - 2*a*c^6)*d^3 - (5*
b^3*c^4 - 14*a*b*c^5)*d^2*e + 2*(2*b^4*c^3 - 7*a*b^2*c^4 + 2*a^2*c^5)*d*e^2
 - (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt
(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^6*d*e^6 -
b*c^5*e^7 - sqrt(-4*(c^6*d^2*e^6 - b*c^5*d*e^7 + a*c^5*e^8)*c^6*e^6 + (2*c^
6*d*e^6 - b*c^5*e^7)^2))*e^(-6)/c^6))/((sqrt(b^2 - 4*a*c)*c^6*d^2 - sqrt(b^
2 - 4*a*c)*b*c^5*d*e + sqrt(b^2 - 4*a*c)*a*c^5*e^2)*c^2) + 2/15*(3*(x*e + d
)^(5/2)*c^4*e^4 - 5*(x*e + d)^(3/2)*b*c^3*e^5 - 15*sqrt(x*e + d)*b*c^3*d*e^
5 + 15*sqrt(x*e + d)*b^2*c^2*e^6 - 15*sqrt(x*e + d)*a*c^3*e^6)*e^(-5)/c^5
```

Mupad [B]
time = 5.72, size = 2500, normalized size = 5.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)

[Out] atan((((((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3 + 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 - (8*(d + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (8*(d + e*x)^(1/2)*(b^8*e^6 + 2*a^4*c^4*e^6 + 20*a^2*b^4*c^2*e^6 - 16*a^3*b^2*c^3*e^6 + 2*a^2*c^6*d^4*e^2 - 12*a^3*c^5*d^2*e^4 + b^4*c^4*d^4*e^2 - 4*b^5*c^3*d^3*e^3 + 6*b^6*c^2*d^2*e^4 - 8*a*b^6*c*e^6 - 4*b^7*c*d*e^5 + 54*a^2*b^2*c^4*d^2*e^4 + 28*a*b^5*c^2*d*e^5 + 28*a^3*b*c^4*d*e^5 - 4*a*b^2*c^5*d^4*e^2 + 20*a*b^3*c^4*d^3*e^3 - 36*a*b^4*c^3*d^2*e^4 - 20*a^2*b*c^5*d^3*e^3 - 56*a^2*b^3*c^3*d*e^5))/c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*

(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*
a^2*b*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*1i - (((8*(4*a^3*c^6*e^5 + a*b^4*c^4*e^5 - b^5*c^4*d*e^4 - 5
*a^2*b^2*c^5*e^5 + 4*a^2*c^7*d^2*e^3 - b^3*c^6*d^3*e^2 + 2*b^4*c^5*d^2*e^3
+ 4*a*b*c^7*d^3*e^2 + 4*a*b^3*c^5*d*e^4 - 9*a*b^2*c^6*d^2*e^3))/c^5 + (8*(d
 + e*x)^(1/2)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*
b^7*c^2*d^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^
3 + a^3*c^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 11*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^3*(-(4*a*c -
 b^2)^3)^(1/2) - 27*a*b^5*c^3*d^2*e + 30*a*b^6*c^2*d*e^2 - 60*a^3*b*c^5*d^2
*e + 3*b^5*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 75*a^2*b^3*c^4*d^2*e - 99*a^2
*b^4*c^3*d*e^2 + 114*a^3*b^2*c^4*d*e^2 - 3*a^2*c^4*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 3*b^4*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^2*c^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 12*a*b^3*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8
)))^(1/2)*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2 - 4*a*b*c^8*e^3 + 8*a*c^9*d*e^2))/
c^5)*(-(b^9*e^3 + 8*a^3*c^6*d^3 - b^6*c^3*d^3 - b^6*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^4*c^4*d^3 + 28*a^4*b*c^4*e^3 - 24*a^4*c^5*d*e^2 + 3*b^7*c^2*d
^2*e - 18*a^2*b^2*c^5*d^3 + 42*a^2*b^5*c^2*e^3 - 63*a^3*b^3*c^3*e^3 + a^3*c
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + b^3*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*e^3 - 3*b^8*c*d*e^2 - 6*a^2*b^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 2*a*b*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b...

3.536 $\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2} \, dx$

Optimal. Leaf size=453

$$\frac{2(cd-be)\sqrt{d+ex}}{c^2}+\frac{2(d+ex)^{3/2}}{3c}+\frac{\sqrt{2}\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-\right.\right.\right.}{c^{5/2}\sqrt{b^2-}}$$

```
[Out] 2/3*(e*x+d)^(3/2)/c+2*(-b*e+c*d)*(e*x+d)^(1/2)/c^2+arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*e^2-b^2*
e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-
4*a*c+b^2)^(1/2)))+b*c*(c*d^2+e*(-3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/2)/(
-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*c^
(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b^3*e^
2-b^2*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e
+d*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2-e*(3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/
2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 2.94, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {838, 840, 1180, 214}

$$\frac{\sqrt{2}\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)-c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2e\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\sqrt{d+ex}(cd-be)}{c^2}+\frac{2(d+ex)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*(c*d - b*e)*Sqrt[d + e*x])/c^2 + (2*(d + e*x)^(3/2))/(3*c) + (Sqrt[2]*(b
^3*e^2 - b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d
 - 3*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - (Sqrt[2]*(b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 -
 c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{\sqrt{d+ex}\,(-ae+(cd-be)x)}{a+bx+cx^2}\,dx}{c}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\int \frac{-ae(2cd-be)+(c^2d^2+b^2e^2-ce(2bd+ae))x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{2\text{Subst}\left(\int \frac{-ae^2(2cd-be)-d(c^2d^2+b^2e^2-ce(2bd+ae))+(c^2}{cd^2-bde+ae^2+(-2cd+be)x^2}\right.}{c^2}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} - \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-\right.\right.}{}$$

$$= \frac{2(cd-be)\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3c} + \frac{\sqrt{2}\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.73, size = 493, normalized size = 1.09

$$\frac{2\sqrt{c}\sqrt{d+ex}\,(4cd-3be+cex)+\frac{3\left(ib^3e^2+b^2e\left(-2icd+\sqrt{-b^2+4ac}\,e\right)+ibc\left(cd^2+e\left(2i\sqrt{-b^2+4ac}\,d-3ae\right)\right)+c\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d+4iae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{3\left(-ib^3e^2+b^2e\left(2icd+\sqrt{-b^2+4ac}\,e\right)+bc\left(-icd^2+e\left(-2\sqrt{-b^2+4ac}\,d+3iae\right)\right)+c\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d-4iae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{3c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x]*(4*c*d - 3*b*e + c*e*x) + (3*(I*b^3*e^2 + b^2*e*((
-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + I*b*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 + 4*a
*c]*d - 3*a*e)) + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*
d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e -
 I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqr
t[-b^2 + 4*a*c])*e]) + (3*((-I)*b^3*e^2 + b^2*e*((2*I)*c*d + Sqrt[-b^2 + 4*
a*c]*e) + b*c*((-I)*c*d^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (3*I)*a*e)) + c*(-
(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e)))*ArcTa
n[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*
e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/
(3*c^(5/2))
```

Maple [A]
time = 0.17, size = 521, normalized size = 1.15

method	result
derivativedivides	$-\frac{2\left(-\frac{(ex+d)^{\frac{3}{2}}c}{3}+be\sqrt{ex+d}-cd\sqrt{ex+d}\right)}{c^2}+$ $\left(-3abce^3+4a\,c^2d\,e^2+b^3e^3-2b^2cd\,e^2+b\,c^2d^2e-\sqrt{-e^2\,(4ac-}\right.$
default	$-\frac{2\left(-\frac{(ex+d)^{\frac{3}{2}}c}{3}+be\sqrt{ex+d}-cd\sqrt{ex+d}\right)}{c^2}+$ $\left(-3abce^3+4a\,c^2d\,e^2+b^3e^3-2b^2cd\,e^2+b\,c^2d^2e-\sqrt{-e^2\,(4ac-}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^2*(-1/3*(e*x+d)^(3/2)*c+b*e*(e*x+d)^(1/2)-c*d*(e*x+d)^(1/2))+8/c*(1/8*
(-3*a*b*c*e^3+4*a*c^2*d*e^2+b^3*e^3-2*b^2*c*d*e^2+b*c^2*d^2*e-(-e^2*(4*a*c-
b^2))^(1/2)*a*c*e^2+(-e^2*(4*a*c-b^2))^(1/2)*b^2*e^2-2*(-e^2*(4*a*c-b^2))^(
1/2)*b*c*d*e+(-e^2*(4*a*c-b^2))^(1/2)*c^2*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2
^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2
)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(3*a*b*c*e^3-
4*a*c^2*d*e^2-b^3*e^3+2*b^2*c*d*e^2-b*c^2*d^2*e-(-e^2*(4*a*c-b^2))^(1/2)*a*
c*e^2+(-e^2*(4*a*c-b^2))^(1/2)*b^2*e^2-2*(-e^2*(4*a*c-b^2))^(1/2)*b*c*d*e+(
-e^2*(4*a*c-b^2))^(1/2)*c^2*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+
2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((
-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)*x/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5507 vs. $2(406) = 812$.

time = 3.90, size = 5507, normalized size = 12.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)
*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^
6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3
 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*
a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(sqrt(2)*((b^3
*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(
2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2
+ 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4
*a^3*b*c^3)*e^4 - ((b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2
*c^7)*e)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 1
0*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*
c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^
2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*
a*c^11)))*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3
*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*
e^3 + (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e +
 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^
3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^
4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c
^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^
6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2
*c^3 - 3*a^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^
4*c - 3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2
*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*sqrt(x*e + d)) - 3*sqrt(2)*c^2*sqrt(((b^
2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*
a*c^6)*sqrt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*
a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^
5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*
e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(-sqrt(2)*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b
^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 1
2*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4
```

```
)*d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 - ((b^3*c^6
- 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*sqrt((b^2*c^6*d^6 -
 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4
*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2
- 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c
^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^2*c^3 - 2
*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^
2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*sq
rt((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5
 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^
3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(
b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(
b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2*
(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 - a^3*c^
3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2
)*e^5)*sqrt(x*e + d)) + 3*sqrt(2)*c^2*sqrt(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^
3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5
 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*sqrt((b^2*c^6*d^6 - 6
*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e
^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 -
20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2
 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
*log(sqrt(2)*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^
2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6
*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c +
13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 + ((b^3*c^6 -...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 978 vs. 2(406) = 812.
time = 2.15, size = 978, normalized size = 2.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(((b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^
2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b
^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-
4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^5*d^3 - (5*b^2*c
^4 - 8*a*c^5)*d^2*e + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 - (b^4*c^2 - 3*a*b^2*
c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1
/2)*sqrt(x*e + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(-4*(c^4*d^2 - b*c^3*d*e +
 a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((sqrt(b^2 - 4*a*c)*c^5*d^2
 - sqrt(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2) - 1/4*((
(b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^2*c
+ 4*a^2*c^2)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2 + 2*
(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4
*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(c) + (2*b*c^5*d^3 - (5*b^2*c^4 - 8
*a*c^5)*d^2*e + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 - (b^4*c^2 - 3*a*b^2*c^3)*e
^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sq
rt(x*e + d)/sqrt(-(2*c^4*d - b*c^3*e - sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3
*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((sqrt(b^2 - 4*a*c)*c^5*d^2 - sqr
t(b^2 - 4*a*c)*b*c^4*d*e + sqrt(b^2 - 4*a*c)*a*c^4*e^2)*c^2) + 2/3*((x*e +
d)^(3/2)*c^2 + 3*sqrt(x*e + d)*c^2*d - 3*sqrt(x*e + d)*b*c*e)/c^3
```

Mupad [B]
time = 4.72, size = 2500, normalized size = 5.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x)^(3/2))/(a + b*x + c*x^2),x)
```

```
[Out] (2*(d + e*x)^(3/2))/(3*c) - ((2*d)/c + (2*(b*e - 2*c*d))/c^2)*(d + e*x)^(1/
2) - atan(((((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*
c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5
*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 - (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*
c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3
- 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2
+ 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54
*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d
*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1
/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(
-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2)
```

+ 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((8*(a*b^3*c^3*e^5 - 4*a^2*b*c^4*e^5 + 4*a*c^6*d^3*e^2 + 4*a^2*c^5*d*e^4 - b^4*c^3*d*e^4 - b^2*c^5*d^3*e^2 + 2*b^3*c^4*d^2*e^3 - 8*a*b*c^5*d^2*e^3 + 3*a*b^2*c^4*d*e^4))/c^3 + (8*(d + e*x)^(1/2)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2 - 4*a*b*c^6*e^3 + 8*a*c^7*d*e^2))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (8*(d + e*x)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 + 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(b^7*e^3 - 8*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^4*d^3 - 20*a^3*b*c^3*e^3

```
- b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e +
 25*a^2*b^3*c^2*e^3 + a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^3
- 3*b^6*c*d*e^2 - 3*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 21*a*b^3*c^3*d^2
*e + 24*a*b^4*c^2*d*e^2 + 36*a^2*b*c^4*d^2*e - 3*a*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 54*a^2*b^2*c^3*d*e^2
+ 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b*c^2*d*e^2*(-(4*a*c - b^2
)^3)^(1/2))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((16*(a^4*c
*e^8 - a^3*b^2*e^8 - a*b^4*d^2*e^6 + 2*a^2*b^3*...
```

3.537 $\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx$

Optimal. Leaf size=322

$$\frac{2e\sqrt{d+ex}}{c}-\frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,\sqrt{}}{\sqrt{2cd-\left(b-\sqrt{}\right.}}\right.}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2*e*(e*x+d)^(1/2)/c-arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b-(-4*a
*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*
e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))
-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c^(3/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.80, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {717, 840, 1180, 214}

$$-\frac{\sqrt{2}\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2e\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*e*Sqrt[d + e*x])/c - (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2
 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(2*c^2*d^2 + b*(b +
Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c
^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}}{c} + \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{2\text{Subst}\left(\int \frac{-de(2cd-be)+e(cd^2-ae^2)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{c}$$

$$= \frac{2e\sqrt{d+ex}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subs}}{c\sqrt{b^2-4ac}}$$

$$= \frac{2e\sqrt{d+ex}}{c} - \frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)}{c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.27, size = 364, normalized size = 1.13

$$\frac{2\sqrt{c}\,e\sqrt{d+ex}+\frac{\left(-2ic^2d^2-b\left(ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(ibd+\sqrt{-b^2+4ac}\,d+iae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\,\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(2ic^2d^2-b\left(-ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(-ibd+\sqrt{-b^2+4ac}\,d-iae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\,\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x] + (((-2*I)*c^2*d^2 - b*(I*b + Sqrt[-b^2 + 4*a*c]
)*e^2 + 2*c*e*(I*b*d + Sqrt[-b^2 + 4*a*c]*d + I*a*e))*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*
b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((2*I)*c^2*d^2
 - b*((-I)*b + Sqrt[-b^2 + 4*a*c])*e^2 + 2*c*e*((-I)*b*d + Sqrt[-b^2 + 4*a*
c]*d - I*a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I
*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[
-b^2 + 4*a*c])*e]))/c^(3/2)
```

Maple [A]
time = 0.14, size = 353, normalized size = 1.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e*((e*x+d)^(1/2)/c+1/2*(2*a*c*e^2-b^2*e^2+2*b*c*d*e-2*c^2*d^2-(-e^2*(4*a*
c-b^2))^(1/2)*b*e+2*(-e^2*(4*a*c-b^2))^(1/2)*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2
)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(
1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/2*(-2*a*c*e^
2+b^2*e^2-2*b*c*d*e+2*c^2*d^2-(-e^2*(4*a*c-b^2))^(1/2)*b*e+2*(-e^2*(4*a*c-b
^2))^(1/2)*c*d)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*
c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*
(4*a*c-b^2))^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/(c*x^2 + b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2688 vs. 2(282) = 564.
time = 2.47, size = 2688, normalized size = 8.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas")

[Out] -1/2*(sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(x*e + d)) - sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(x*e + d)) + sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(x*

```
e + d)) - sqrt(2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d
*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b
*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4
))*log(-sqrt(2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^
3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3
 - 4*a*b*c^4)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*
a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
6)/(b^2*c^6 - 4*a*c^7)))*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c
^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 -
 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d
*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*
a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 -
(b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*sqrt(x*e + d)) - 4*sqrt(x*e +
d)*e)/c
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 783 vs. 2(282) = 564.
time = 0.90, size = 783, normalized size = 2.43

$$\frac{2\sqrt{xe+d}\,e}{c}+\frac{\left(\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,(2(b^2c-4ac^2)de^2-(b^3-4abc)e^3)c^2-2\left(\sqrt{b^2-4ac}\,c^3d^2e-\sqrt{b^2-4ac}\,bc^2de^2+\sqrt{b^2-4ac}\,ac^2e^3\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|c|-(4c^5d^3-6bc^4d^2e+4(b^2c^3-ac^4)de^2-(b^3c^2-2abc^3)e^3)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2c^2d-bce+\sqrt{-4(c^2d^2-bcde+ace^2)c^2+(2c^2d-bce)^2}}{c^2}}}\right)}{4\left(\sqrt{b^2-4ac}\,c^4d^2-\sqrt{b^2-4ac}\,bc^3de+\sqrt{b^2-4ac}\,ac^3e^2\right)c^2}-\frac{\left(\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,(2(b^2c-4ac^2)de^2-(b^3-4abc)e^3)c^2+2\left(\sqrt{b^2-4ac}\,c^3d^2e-\sqrt{b^2-4ac}\,bc^2de^2+\sqrt{b^2-4ac}\,ac^2e^3\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|c|-(4c^5d^3-6bc^4d^2e+4(b^2c^3-ac^4)de^2-(b^3c^2-2abc^3)e^3)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2c^2d-bce-\sqrt{-4(c^2d^2-bcde+ace^2)c^2+(2c^2d-bce)^2}}{c^2}}}\right)}{4\left(\sqrt{b^2-4ac}\,c^4d^2-\sqrt{b^2-4ac}\,bc^3de+\sqrt{b^2-4ac}\,ac^3e^2\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*e/c + 1/4*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)
*(2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*c^2 - 2*(sqrt(b^2 - 4*a*
c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)
*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(c) - (4*c^5*d^3 - 6*b
*c^4*d^2*e + 4*(b^2*c^3 - a*c^4)*d*e^2 - (b^3*c^2 - 2*a*b*c^3)*e^3)*sqrt(-4
*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)
/sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^
2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 - sqrt(b^2 - 4*a*c)*b*c^
3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2) - 1/4*(sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*(2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*c^
2 + 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b
```

```
^2 - 4*a*c)*a*c^2*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs
(c) - (4*c^5*d^3 - 6*b*c^4*d^2*e + 4*(b^2*c^3 - a*c^4)*d*e^2 - (b^3*c^2 - 2
*a*b*c^3)*e^3)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*s
qrt(1/2)*sqrt(x*e + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e
+ a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((sqrt(b^2 - 4*a*c)*c^4*d^2 -
sqrt(b^2 - 4*a*c)*b*c^3*d*e + sqrt(b^2 - 4*a*c)*a*c^3*e^2)*c^2)
```

Mupad [B]
time = 4.44, size = 2500, normalized size = 7.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(a + b*x + c*x^2),x)
```

```
[Out] (2*e*(d + e*x)^(1/2))/c - atan(((((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c
^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c - (8*(d
+ e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(
b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*
b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (8*(d + e*
x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*
c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^
2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c
^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1
/2)*1i - (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*
e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*(d + e*x)^(1/2)*(-(b^5*e^3
 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e
^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2
*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^
2*c^4*d*e^2 - 4*a*b*c^4*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 -
 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a
^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a
```

```
*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*
d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a
^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2
*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3
 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e
^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2
*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((8*(4*a^2*c^
3*e^5 - a*b^2*c^2*e^5 + 4*a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 -
 4*a*b*c^3*d*e^4))/c - (8*(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*
c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*
d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*
e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e +
 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c^4
*e^3 + 8*a*c^5*d*e^2))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^
2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (8*(d + e*x)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*d^4*
e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*c*e^
6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*
c^3*d^3 - b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*
d*e^2 + 3*b^3*c^2*d^2*e - 3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*
e^3 + a*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e +
 3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((8*(4*a^2*c^3*e^5 - a*b^2*c^2*e^5 + 4*
a*c^4*d^2*e^3 + b^3*c^2*d*e^4 - b^2*c^3*d^2*e^3 - 4*a*b*c^3*d*e^4))/c + (8*
(d + e*x)^(1/2)*(-(b^5*e^3 + 8*a*c^4*d^3 - 2*b^2*c^3*d^3 - b^2*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^3 - 24*a^2*c^3*d*e^2 + 3*b^3*c^2*d^2*e -
3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c*e^3 + a*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 3*b^4*c*d*e^2 - 12*a*b*c^3*d^2*e + 3*b*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 18*a*b^2*c^2*d*e^2)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2)*(b^3*c^3*e^3 - 2*b^2*c^4*d*e^2 - 4*a*b*c...
```

3.538 $\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx$

Optimal. Leaf size=340

$$-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}-\frac{\sqrt{2}\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b(cd^2+ae^2)\right)\tanh^{-1}\left(\frac{\quad}{\sqrt{\quad}}\right.}{a\sqrt{c}\,\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/a-arctanh(2^(1/2)*c^(1/2)*(e*x+d)
^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(-b*(a*e^2+c*d^2)+a*
e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-4*a*c+b^2)^(1/2)))/a/c^(1/2)/(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*c^(1/2)*
(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(b*(a*e^2+c*d
^2)+a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))/a/c^(1/2)/(-
4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.99, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {911, 1301, 212, 1180, 214}

$$-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b(ae^2+cd^2)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b(ae^2+cd^2)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*
c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*S
qrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2
]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 +
 a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{d^2e}{a(d-x^2)}+\frac{e\left(d(cd^2-bde+ae^2)-(cd^2-ae^2)x^2\right)}{a(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \frac{d(cd^2-bde+ae^2)+(-cd^2+ae^2)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x\right.}{a}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b(c\right.}{}$$

$$= -\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} - \frac{\sqrt{2}\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-\right.}{a\sqrt{c}\,\sqrt{b^2-4ac}\,\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.31, size = 371, normalized size = 1.09

$$-\frac{\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d+4iae\right)-ib(cd^2+ae^2)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d-4iae\right)+ib(cd^2+ae^2)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}+2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x]
```

```
[Out] -(((Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d + (4*I
)*a*e) - I*b*(c*d^2 + a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-
2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2
*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^
2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e) + I*b*(c*d^2 + a*e^2))*ArcTan[(
Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]]
)/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])
+ 2*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a)
```

Maple [A]
time = 0.14, size = 370, normalized size = 1.09

method	result

derivativedivides	$2e^2\left(\frac{4c\left(\frac{\left(ab\,e^3-4acd\,e^2+bc\,d^2e+\sqrt{-e^2\left(4ac-b^2\right)}\,a\,e^2-\sqrt{-e^2\left(4ac-b^2\right)}\,c\,d^2\right)\sqrt{2}\,\arctan\left(\frac{}{\sqrt{\left(eb-\right.}}\right.}{8c\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{}\right.$
default	$2e^2\left(\frac{4c\left(\frac{\left(ab\,e^3-4acd\,e^2+bc\,d^2e+\sqrt{-e^2\left(4ac-b^2\right)}\,a\,e^2-\sqrt{-e^2\left(4ac-b^2\right)}\,c\,d^2\right)\sqrt{2}\,\arctan\left(\frac{}{\sqrt{\left(eb-\right.}}\right.}{8c\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-b^2\right)}\right.}}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e^2*(4/a/e^2*c*(1/8*(a*b*e^3-4*a*c*d*e^2+b*c*d^2*e+(-e^2*(4*a*c-b^2))^(1/
2)*a*e^2-(-e^2*(4*a*c-b^2))^(1/2)*c*d^2)/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)
/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1
/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(-a*b*e^3+4*a*c*d*e
^2-b*c*d^2*e+(-e^2*(4*a*c-b^2))^(1/2)*a*e^2-(-e^2*(4*a*c-b^2))^(1/2)*c*d^2)
/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*
```

```
c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1
/2))*c)^(1/2)))-d^(3/2)/a/e^2*arctanh((e*x+d)^(1/2)/d^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*x^2 + b*x + a)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2534 vs. $2(291) = 582$.
time = 13.13, size = 5074, normalized size = 14.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 - (b^2*c - 2*a*c^2)*d
^3 + a^2*b*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*
e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4
*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*
b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((
a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6
 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^
4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e
^2 - (b^2*c - 2*a*c^2)*d^3 + a^2*b*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*
c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c
*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) +
4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - (b^2*c + 3*a*c^2)*d^4*e
- a^2*b*d*e^4 + a^3*e^5)*sqrt(x*e + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e -
6*a^2*c*d*e^2 - (b^2*c - 2*a*c^2)*d^3 + a^2*b*e^3 + (a^2*b^2*c - 4*a^3*c^2)
*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^
3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a
^3*c^2))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^
3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2
*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2
+ 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3))
)*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 - (b^2*c - 2*a*c^2)*d^3 + a^2*b*e^3
+ (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d
^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^
5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*
```

```
c*d^2*e^3 - (b^2*c + 3*a*c^2)*d^4*e - a^2*b*d*e^4 + a^3*e^5)*sqrt(x*e + d))
 + sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 - (b^2*c - 2*a*c^2)*d^3 +
 a^2*b*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e +
9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2
*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^
2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*
b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6
*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 +
a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 -
 (b^2*c - 2*a*c^2)*d^3 + a^2*b*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*
d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2
*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b
*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - (b^2*c + 3*a*c^2)*d^4*e - a^
2*b*d*e^4 + a^3*e^5)*sqrt(x*e + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^
2*c*d*e^2 - (b^2*c - 2*a*c^2)*d^3 + a^2*b*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqr
t((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 -
6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c
^2))*log(-sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e
+ (a^2*b^2 - 4*a^3*c)*d*e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c -
 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*
a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))*sq
rt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 - (b^2*c - 2*a*c^2)*d^3 + a^2*b*e^3 - (a
^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e
^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^
3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^
2*e^3 - (b^2*c + 3*a*c^2)*d^4*e - a^2*b*d*e^4 + a^3*e^5)*sqrt(x*e + d)) - 2
*d^(3/2)*log((x*e - 2*sqrt(x*e + d)*sqrt(d) + 2*d)/x))/a, -1/2*(sqrt(2)*a*s
qrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 - (b^2*c - 2*a*c^2)*d^3 + a^2*b*e^3 + (
a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*
e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c
^3)))/(a^2*b^2*c - 4*a^3*c^2))*log(sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*
b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d*e^3 + ((a^2*b^3*c - 4*a^3*
b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e
 + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*
b^2*c^2 - 4*a^5*c^3)))*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 - (b^2*c - 2*a*
c^2)*d^3 + a^2*b*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6 - 6*a*b*c^
2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6
)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) + 4*(b*c^2*d^5 + 4*a
*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - (b^2*c + 3*a*c^2)*d^4*e - a^2*b*d*e^4 + a^
3*e^5)*sqrt(x*e + d)) - sqrt(2)*a*sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d*e^2 - (b
^2*c - 2*a*c^2)*d^3 + a^2*b*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt((b^2*c^2*d^6
 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^
4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b...
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. 2(291) = 582.
time = 0.99, size = 822, normalized size = 2.42

$$\frac{2\,d^2\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}}-\frac{\left(\left((b^2c-4ac^2)d^2e-(ab^2-4a^2c)e^3\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,a^2-2\left(\sqrt{b^2-4ac}\,ac^2d^3-\sqrt{b^2-4ac}\,abcd^2e+\sqrt{b^2-4ac}\,a^2cde^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|a|-\left(2a^2bc^2d^3+6a^3bcde^2-a^3b^2e^3-(a^2b^2c+8a^3c^2)d^2e\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2acd-abe+\sqrt{-4(acd^2-abde+a^2e^2)ac+(2acd-abe)^2}}{ac}}}\right)}{4\left(\sqrt{b^2-4ac}\,a^2c^2d^2-\sqrt{b^2-4ac}\,a^2bcde+\sqrt{b^2-4ac}\,a^3ce^2\right)|a||c|}+\frac{\left(\left((b^2c-4ac^2)d^2e-(ab^2-4a^2c)e^3\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,a^2+2\left(\sqrt{b^2-4ac}\,ac^2d^3-\sqrt{b^2-4ac}\,abcd^2e+\sqrt{b^2-4ac}\,a^2cde^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|a|-\left(2a^2bc^2d^3+6a^3bcde^2-a^3b^2e^3-(a^2b^2c+8a^3c^2)d^2e\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2acd-abe-\sqrt{-4(acd^2-abde+a^2e^2)ac+(2acd-abe)^2}}{ac}}}\right)}{4\left(\sqrt{b^2-4ac}\,a^2c^2d^2-\sqrt{b^2-4ac}\,a^2bcde+\sqrt{b^2-4ac}\,a^3ce^2\right)|a||c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 2*d^2*arctan(sqrt(x*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/4*(((b^2*c - 4*a*c^2)
*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e
+ sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 + 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2
*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*ar
ctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d^2 -
a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*
a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*
abs(a)*abs(c)) + 1/4*(((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqr
t(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 + 2*(sqrt(b^2 - 4*a*c)*a*
c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sq
rt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 +
6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*
d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt
(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d -
 a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2
*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c))
```

Mupad [B]
time = 8.16, size = 2500, normalized size = 7.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a + b*x + c*x^2)),x)
```

```
[Out] atan(((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c -
8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c
*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*
c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2
*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d
^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e
 + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(
512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5
*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4
*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 384*a^3*c^5*d^4*e^
8 - 384*a^4*c^4*d^2*e^10 + 96*a^2*b^2*c^4*d^4*e^8 - 128*a^2*b^3*c^3*d^3*e^9
 + 32*a^2*b^4*c^2*d^2*e^10 - 32*a^3*b^2*c^3*d^2*e^10 + 128*a^4*b*c^3*d*e^11
 + 512*a^3*b*c^4*d^3*e^9 - 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3
*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8
 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2
*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*
b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*
b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d
^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b
^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))
^(1/2) + 96*a*c^5*d^7*e^8 + 32*a^4*c^2*d*e^14 - 672*a^2*c^4*d^5*e^10 - 736*
a^3*c^3*d^3*e^12 - 32*b^2*c^4*d^7*e^8 - 32*b^3*c^3*d^6*e^9 + 64*b^4*c^2*d^5
*e^10 - 96*a^2*b^2*c^2*d^3*e^12 + 256*a*b*c^4*d^6*e^9 - 32*a^3*b^2*c*d*e^14
 - 288*a*b^2*c^3*d^5*e^10 - 160*a*b^3*c^2*d^4*e^11 + 1280*a^2*b*c^3*d^4*e^1
1 + 32*a^2*b^3*c*d^2*e^13 + 128*a^3*b*c^2*d^2*e^13) + (d + e*x)^(1/2)*(32*a
^4*c*e^16 + 96*c^5*d^8*e^8 - 256*a*c^4*d^6*e^10 - 256*b*c^4*d^7*e^9 + 64*b^
4*c*d^4*e^12 + 256*a^2*c^3*d^4*e^12 + 128*a^3*c^2*d^2*e^14 + 384*b^2*c^3*d^
6*e^10 - 256*b^3*c^2*d^5*e^11 - 128*a^3*b*c*d*e^15 - 128*a*b^3*c*d^3*e^13 +
 256*a*b^2*c^2*d^4*e^12 - 384*a^2*b*c^2*d^3*e^13 + 192*a^2*b^2*c*d^2*e^14))
*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*
b^2*c^2)))^(1/2)*1i + (((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e
^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^
```

3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*((d + e*x)^(1/2)*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 + 4*a^3*b*c*e^3 + b*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*c*d^2*e - 3*a*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2*e + 6*a^2*b^2*c*d*e^2)/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 384*a^3*c^5*d^4*e^8 + 384*a^4*c^4*d^2*e^10 - 96*a^2*b^2*c^4*d^4*e^8 + 128*a^2*b^3*c^3*d^3*e^9 - 32*a^2*b^4*c^2*d^2*e^10 + 32*a^3*b^2*c^3*d^2*e^10 - 128*a^4*b*c^3*d*e^11 - 512*a^3*b*c^4*d^3*e^9 + 32*a^3*b^3*c^2*d*e^11) + (d + e*x)^(1/2)*(32*a^3*b^3*c*e^13 - 128*a^4*b*c^2*e^13 + 704*a^4*c^3*d*e^12 - 576*a^2*c^5*d^5*e^8 + 896*a^3*c^4*d^3*e^10 - 64*b^4*c^3*d^5*e^8 + 64*b^5*c^2*d^4*e^9 + 192*a^2*b^2*c^3*d^3*e^10 + 448*a^2*b^3*c^2*d^2*e^11 - 64*a^2*b^4*c*d*e^12 + 384*a*b^2*c^4*d^5*e^8 - 320*a*b^3*c^3*d^4*e^9 - 128*a*b^4*c^2*d^3*e^10 + 384*a^2*b*c^4*d^4*e^9 - 1664*a^3*b*c^3*d^2*e^11 + 64*a^3*b^2*c^2*d*e^12))*((b^4*c*d^3 - a^2*b^3*e^3 + 8*a^2*c^3*d^3 + a^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d^3 - 24*a^3*c^2*d*e^2 +...

3.539 $\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\, dx$

Optimal. Leaf size=403

$$-\frac{d\sqrt{d+ex}}{ax}+\frac{\sqrt{d}\, e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}+\frac{2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2}-\frac{\sqrt{2}\,\sqrt{c}\,\left(b^2d^2+bd\left(\sqrt{b^2\right.\right.}{}$$

```
[Out] e*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a+2*(-2*a*e+b*d)*arctanh((e*x+d)^(
1/2)/d^(1/2))*d^(1/2)/a^2-d*(e*x+d)^(1/2)/a/x-arctanh(2^(1/2)*c^(1/2)*(e*x+
d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^2*d^2+b
*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2+e*(-a*e+d*(-4*a*c+b^2)^(1/2))))
/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctanh(2^(1
/2)*c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c
^(1/2)*(b^2*d^2-b*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2-e*(a*e+d*(-4*a*
c+b^2)^(1/2))))/a^2/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)
```

Rubi [A]

time = 2.03, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {911, 1301, 205, 212, 1180, 214}

$$-\frac{\sqrt{2}\,\sqrt{c}\,\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^2\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{2}\,\sqrt{c}\,\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2ae\left(d\sqrt{b^2-4ac}+ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^2\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2}-\frac{d\sqrt{d+ex}}{ax}+\frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((d*Sqrt[d + e*x])/(a*x)) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a +
 (2*Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2 - (Sqrt[2]*Sq
rt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*(Sqr
t[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(Sqrt[
b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a^2*Sqrt
[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x^2\,(a+bx+cx^2)}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{d^2e^2}{a(d-x^2)^2}-\frac{de(-bd+2ae)}{a^2(d-x^2)}+\frac{e(-(bd-ae)(cd^2-bde+ae^2)+cd(bd-2ae)x^2)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)\,dx, x, \sqrt{d+\right.}{e}$$

$$= \frac{2\text{Subst}\left(\int \frac{-(bd-ae)(cd^2-bde+ae^2)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex}\right)}{a^2} + \frac{(2d^2e)\,\text{Subst}\left(\int \frac{1}{(d-x\right.}{}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2} + \frac{(de)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \right.}{a}$$

$$= -\frac{d\sqrt{d+ex}}{ax} + \frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a} + \frac{2\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right.}{a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.60, size = 416, normalized size = 1.03

$$\frac{-\frac{ad\sqrt{d+ex}}{x}+\frac{\sqrt{2}\sqrt{c}\left(-ib^2d^2+bd\left(\sqrt{-b^2+4ac}\,d+2iae\right)-2ia\left(-cd^2+e\left(-i\sqrt{-b^2+4ac}\,d+ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(ib^2d^2+bd\left(\sqrt{-b^2+4ac}\,d-2iae\right)+2ia\left(-cd^2+e\left(i\sqrt{-b^2+4ac}\,d+ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}+\sqrt{d}\,(2bd-3ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x])/x) + (Sqrt[2]*Sqrt[c]*((-I)*b^2*d^2 + b*d*(Sqrt[-b^2
 + 4*a*c]*d + (2*I)*a*e) - (2*I)*a*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*c]*d
 + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e - I*Sqrt
[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*
a*c])*e]) + (Sqrt[2]*Sqrt[c]*(I*b^2*d^2 + b*d*(Sqrt[-b^2 + 4*a*c]*d - (2*I)
*a*e) + (2*I)*a*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(S
qrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]) + Sqrt[d]*(2
*b*d - 3*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a^2
```

Maple [A]
time = 0.16, size = 409, normalized size = 1.01

method	result
derivativedivides	$2e^3\left(\frac{4c\left(\frac{\left(-2a^2e^3+2abd\,e^2+2ac\,d^2e-b^2d^2e-2\sqrt{-e^2\left(4ac-b^2\right)}\,ade+\sqrt{-e^2\left(4ac-b^2\right)}\,b\,d^2\right)\sqrt{2}\,\arctan\left(\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-\right.}}\right.}{}\right.$
default	$2e^3\left(\frac{4c\left(\frac{\left(-2a^2e^3+2abd\,e^2+2ac\,d^2e-b^2d^2e-2\sqrt{-e^2\left(4ac-b^2\right)}\,ade+\sqrt{-e^2\left(4ac-b^2\right)}\,b\,d^2\right)\sqrt{2}\,\arctan\left(\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\sqrt{\left(eb-2cd+\sqrt{-e^2\left(4ac-\right.}}\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e^3*(4/a^2/e^3*c*(1/8*(-2*a^2*e^3+2*a*b*d*e^2+2*a*c*d^2*e-b^2*d^2*e-2*(-e
^2*(4*a*c-b^2))^(1/2)*a*d*e+(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/(-e^2*(4*a*c-b^
2))^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(
e*x+d)^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))-1/8*(2
```

```
*a^2*e^3-2*a*b*d*e^2-2*a*c*d^2*e+b^2*d^2*e-2*(-e^2*(4*a*c-b^2))^(1/2)*a*d*e
+(-e^2*(4*a*c-b^2))^(1/2)*b*d^2)/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-e*b+2*
c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-e
*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))-d/a^2/e^3*(1/2*a*(e*x+d)^(1/2
)/x+1/2*(3*a*e-2*b*d)/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*x^2 + b*x + a)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4260 vs. $2(361) = 722$.
time = 54.11, size = 8526, normalized size = 21.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 +
 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a
^5*c)*sqrt(-(6*a^5*b*d*e^5 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 - a^6*e^
6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2
*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^
2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(sqrt(2
)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3
*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c +
8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - (b^6 - 4*a*b
^4*c + 4*a^2*b^2*c^2)*d^6 - a^6*e^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)
*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 -
 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)
))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*
b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5
*b*d*e^5 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 - a^6*e^6 + 6*(a*b^5 - 3*a
^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^
4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e
^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 + (b^3*
c^2 - 2*a*b*c^3)*d^5 - a^4*c*e^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e +
2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*sqrt
```

(x*e + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
d^3 + 3(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2
- 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 -
a^6*e^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a
^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*
a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(
-sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c +
20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*
(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b
^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 - (b^6
- 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 - a^6*e^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3
*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^
3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4
*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 -
3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*sqrt(
-(6*a^5*b*d*e^5 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 - a^6*e^6 + 6*(a*b^
5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*
c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c
)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4
+ (b^3*c^2 - 2*a*b*c^3)*d^5 - a^4*c*e^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d
^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^
3)*sqrt(x*e + d)) + sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (
a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*d^6 - a^6*e^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4
- 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3
- 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c
))*log(sqrt(2)*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b
^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^
2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6
*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*sqrt(-(6*a^5*b*d*e^5 -
(b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 - a^6*e^6 + 6*(a*b^5 - 3*a^2*b^3*c +
2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*
(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b
^2 - 4*a^9*c)))*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a
*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)
*sqrt(-(6*a^5*b*d*e^5 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 - a^6*e^6 + 6
*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c +
3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2
*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*
d*e^4 + (b^3*c^2 - 2*a*b*c^3)*d^5 - a^4*c*e^5 - (b^4*c + a*b^2*c^2 - 3*a^2*
c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*
d^2*e^3)*sqrt(x*e + d)) - sqrt(2)*a^2*x*sqrt(-(a^3*b*e^3 - (b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e
^2 - (a^4*b^2 - 4*a^5*c)*sqrt(-(6*a^5*b*d*e^5 - (b^6 - 4*a*b^4*c + 4*a^2*b^

```
2*c^2)*d^6 - a^6*e^6 + 6*(a*b^5 - 3*a^2*b^3*c +...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.78, size = 425, normalized size = 1.05

$$-\frac{\sqrt{xe+d}\,d}{ax}-\frac{(2\,bd^2-3\,ade)\arctan\left(\frac{\sqrt{xe+d}}{\sqrt{-d}}\right)}{a^2\sqrt{-d}}-\frac{\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2-2\,ac+\sqrt{b^2-4\,ac}\,b\right)d-\left(ab+\sqrt{b^2-4\,ac}\,a\right)e\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,a^2cd-a^2be+\sqrt{-4\,(a^2cd^2-a^2bde+a^3e^2)a^2c+(2\,a^2cd-a^2be)^2}}{a^2c}}}\right)}{2\sqrt{b^2-4\,ac}\,a^2|c|}+\frac{\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2-2\,ac-\sqrt{b^2-4\,ac}\,b\right)d-\left(ab-\sqrt{b^2-4\,ac}\,a\right)e\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{xe+d}}{\sqrt{-\frac{2\,a^2cd-a^2be-\sqrt{-4\,(a^2cd^2-a^2bde+a^3e^2)a^2c+(2\,a^2cd-a^2be)^2}}{a^2c}}}\right)}{2\sqrt{b^2-4\,ac}\,a^2|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(x*e + d)*d/(a*x) - (2*b*d^2 - 3*a*d*e)*arctan(sqrt(x*e + d)/sqrt(-d))
/(a^2*sqrt(-d)) - 1/2*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^
2 - 2*a*c + sqrt(b^2 - 4*a*c)*b)*d - (a*b + sqrt(b^2 - 4*a*c)*a)*e)*arctan(
2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 -
 a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2
 - 4*a*c)*a^2*abs(c)) + 1/2*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e
)*((b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*b)*d - (a*b - sqrt(b^2 - 4*a*c)*a)*e)*a
rctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c
*d^2 - a^2*b*d*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sq
rt(b^2 - 4*a*c)*a^2*abs(c))
```

Mupad [B]
time = 7.36, size = 2500, normalized size = 6.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] (d^(1/2)*atan(((d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8
*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b
^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2
*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*
a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3
*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*
```

d^3*e^13))/a^4 - (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 1
00*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11
*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 +
 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e
^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*
d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9
 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252
*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 + (d^(1/2)*((8*(d + e*x)^(
1/2)*(16*a^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c
^6*d^5*e^8 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d
^4*e^9 - 56*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d
^3*e^10 + 108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^
3*d^3*e^10 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b
^3*c^3*d^2*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b
*c^4*d^2*e^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*((8*
(80*a^8*c^4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4
*c^3*d^3*e^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*
c^3*d^2*e^10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*
d^2*e^10 - 28*a^7*b^2*c^3*d*e^11))/a^4 - (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*
x)^(1/2)*(64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a
^8*c^5*d^2*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b
*c^4*d*e^9 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3
*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d)*1i)/(2*a^
2) + (d^(1/2)*((8*(d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a
^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8
*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d
^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^
3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e
^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))
/a^4 + (d^(1/2)*((8*(56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4
*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c
^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b
^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*
a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 -
 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*
c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4
*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14))/a^4 - (d^(1/2)*((8*(d + e*x)^(1/2)*(16*a
^7*b*c^3*e^13 + 88*a^7*c^4*d*e^12 - 4*a^6*b^3*c^2*e^13 - 40*a^5*c^6*d^5*e^8
 + 184*a^6*c^5*d^3*e^10 + 8*a^2*b^6*c^3*d^5*e^8 - 8*a^2*b^7*c^2*d^4*e^9 - 5
6*a^3*b^4*c^4*d^5*e^8 + 36*a^3*b^5*c^3*d^4*e^9 + 28*a^3*b^6*c^2*d^3*e^10 +
108*a^4*b^2*c^5*d^5*e^8 + 36*a^4*b^3*c^4*d^4*e^9 - 179*a^4*b^4*c^3*d^3*e^10
 - 33*a^4*b^5*c^2*d^2*e^11 + 234*a^5*b^2*c^4*d^3*e^10 + 215*a^5*b^3*c^3*d^2
*e^11 - 224*a^5*b*c^5*d^4*e^9 + 16*a^5*b^4*c^2*d*e^12 - 348*a^6*b*c^4*d^2*e
^11 - 84*a^6*b^2*c^3*d*e^12))/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*((8*(80*a^8*c^
4*d*e^11 + 80*a^7*c^5*d^3*e^9 + 8*a^5*b^3*c^4*d^4*e^8 - 6*a^5*b^4*c^3*d^3*e

```
^9 - 2*a^5*b^5*c^2*d^2*e^10 + 4*a^6*b^2*c^4*d^3*e^9 + 36*a^6*b^3*c^3*d^2*e^
10 - 32*a^6*b*c^5*d^4*e^8 + 2*a^6*b^4*c^2*d*e^11 - 112*a^7*b*c^4*d^2*e^10 -
 28*a^7*b^2*c^3*d*e^11))/a^4 + (4*d^(1/2)*(3*a*e - 2*b*d)*(d + e*x)^(1/2)*(
64*a^9*c^4*e^10 + 4*a^7*b^4*c^2*e^10 - 32*a^8*b^2*c^3*e^10 + 96*a^8*c^5*d^2
*e^8 + 8*a^6*b^4*c^3*d^2*e^8 - 56*a^7*b^2*c^4*d^2*e^8 - 112*a^8*b*c^4*d*e^9
 - 8*a^6*b^5*c^2*d*e^9 + 60*a^7*b^3*c^3*d*e^9))/a^6))/(2*a^2))*(3*a*e - 2*b
*d))/(2*a^2))*(3*a*e - 2*b*d))/(2*a^2))*(3*a*e - 2*b*d)*1i)/(2*a^2))/((16*(
6*a*c^7*d^9*e^9 + 6*a^5*c^3*d*e^17 - 4*b*c^7*d^10*e^8 + 6*a^2*c^6*d^7*e^11
+ 6*a^4*c^4*d^3*e^15 + 8*b^2*c^6*d^9*e^9 - 4*b^3*c^5*d^8*e^10 + 4*a^2*b^2*c
^4*d^5*e^13 - 11*a^2*b^3*c^3*d^4*e^14 + 22*a^3*b^2*c^3*d^3*e^15 - 16*a*b*c^
6*d^8*e^10 + 8*a*b^2*c^5*d^7*e^11 + 2*a*b^4*c^3*d^5*e^13 - 3*a^2*b*c^5*d^6*
e^12 - 10*a^3*b*c^4*d^4*e^14 - 19*a^4*b*c^3*d^2*e^16))/a^4 - (d^(1/2)*((8*(
d + e*x)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 1
32*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*
c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^
2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a...
```

3.540 $\int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)}\,dx$

Optimal. Leaf size=607

$$-\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}-\frac{e(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+e}}{\sqrt{d}}\right.}{a^2\sqrt{d}}$$

```
[Out] -3/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(1/2)-e*(-2*a*e+b*d)*arctanh((e
*x+d)^(1/2)/d^(1/2))/a^2/d^(1/2)-2*(b^2*d^2-2*a*b*d*e-a*(-a*e^2+c*d^2))*arc
tanh((e*x+d)^(1/2)/d^(1/2))/a^3/d^(1/2)-1/2*d*(e*x+d)^(1/2)/a/x^2+3/4*e*(e*
x+d)^(1/2)/a/x+(-2*a*e+b*d)*(e*x+d)^(1/2)/a^2/x+arctanh(2^(1/2)*c^(1/2)*(e*
x+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/2)*(b^3*d^2
+b^2*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))+a*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*
e+d*(-4*a*c+b^2)^(1/2)))-a*b*(3*c*d^2+e*(-a*e+2*d*(-4*a*c+b^2)^(1/2))))/a^3
/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-arctanh(2^(1/2)*
c^(1/2)*(e*x+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*c^(1/
2)*(b^3*d^2-b^2*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-a*(a*e^2*(-4*a*c+b^2)^(1/2)-
c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))-a*b*(3*c*d^2-e*(a*e+2*d*(-4*a*c+b^2)^(1/2
))))/a^3/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 3.19, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {911, 1301, 205, 212, 1180, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(-2abde-a(cd^2-ae^2)+b^2d^2\right)}{a^3\sqrt{d}}+\frac{\sqrt{2}\sqrt{c}\left(-ab\left(e\left(2d\sqrt{b^2-4ac}-ae\right)+3cd^2\right)+a\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)+b^2d\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{2}\sqrt{c}\left(-ab\left(3cd^2-e\left(2d\sqrt{b^2-4ac}+ae\right)\right)-a\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2d\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex}(bd-2ae)}{a^2x}-\frac{e(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}-\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(d*Sqrt[d + e*x])/(a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*x) + ((b*d - 2*a*
e)*Sqrt[d + e*x])/(a^2*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*Sqr
t[d]) - (e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (2
*(b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/
(a^3*Sqrt[d]) + (Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*
a*e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - a*
b*(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqr
t[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 - 4*a*c
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^
2*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d
```

```
 + a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))
*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{x^3\left(a+bx+cx^2\right)}\,dx = \frac{2\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{d^2e^3}{a(d-x^2)^3}+\frac{de^2(-bd+2ae)}{a^2(d-x^2)^2}+\frac{e(-b^2d^2+2abde+a(cd^2-ae^2))}{a^3(d-x^2)}+\frac{e((b^2d-acd-abe)(cd^2}{a^3(cd^2}\right.\right.}{e}$$

$$= \frac{2\text{Subst}\left(\int \frac{(b^2d-acd-abe)(cd^2-bde+ae^2)-c(b^2d^2-2abde-a(cd^2-ae^2))x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx,x,\sqrt{d+ex}\right)}{a^3} - \frac{(2}{}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{2(b^2d^2-2abde-a(cd^2-ae^2))\tanh^{-1}\left(\right.}{a^3\sqrt{d}}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{e(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{}}\right.}{a^2\sqrt{d}}$$

$$= -\frac{d\sqrt{d+ex}}{2ax^2}+\frac{3e\sqrt{d+ex}}{4ax}+\frac{(bd-2ae)\sqrt{d+ex}}{a^2x}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.65, size = 560, normalized size = 0.92

$$\frac{\frac{a\sqrt{d+ex}\,(-2ad+4bdx-5aex)}{x^2}+\frac{4\sqrt{2}\sqrt{c}\left(ib^3d^2-b^2d\left(\sqrt{-b^2+4ac}\,d+2iae\right)+ab\left(-3icd^2+e\left(2\sqrt{-b^2+4ac}\,d+iae\right)\right)+a\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d+4iae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}-\frac{4\sqrt{2}\sqrt{c}\left(ib^3d^2+b^2d\left(\sqrt{-b^2+4ac}\,d-2iae\right)+a\left(a\sqrt{-b^2+4ac}\,e^2+cd\left(-\sqrt{-b^2+4ac}\,d+4iae\right)\right)+iab\left(-3cd^2+e\left(2i\sqrt{-b^2+4ac}\,d+ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(-8b^2d^2+12abde+a(8cd^2-3ae^2)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x]
```

```
[Out] ((a*Sqrt[d + e*x]*(-2*a*d + 4*b*d*x - 5*a*e*x))/x^2 + (4*Sqrt[2]*Sqrt[c]*(I
*b^3*d^2 - b^2*d*(Sqrt[-b^2 + 4*a*c]*d + (2*I)*a*e) + a*b*((-3*I)*c*d^2 + e
*(2*Sqrt[-b^2 + 4*a*c]*d + I*a*e)) + a*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(
Sqrt[-b^2 + 4*a*c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/
Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c
```

```
*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(I*b^3*d^2 + b^2*d
*(Sqrt[-b^2 + 4*a*c]*d - (2*I)*a*e) + a*(a*Sqrt[-b^2 + 4*a*c]*e^2 + c*d*(-(
Sqrt[-b^2 + 4*a*c]*d) + (4*I)*a*e)) + I*a*b*(-3*c*d^2 + e*((2*I)*Sqrt[-b^2
+ 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[-2*c*d + b*
e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt
[-b^2 + 4*a*c])*e]) + ((-8*b^2*d^2 + 12*a*b*d*e + a*(8*c*d^2 - 3*a*e^2))*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/(4*a^3)
```

Maple [A]

time = 0.17, size = 601, normalized size = 0.99

method	result
derivativedivides	$2e^4 \left(\frac{4c \left(\frac{\left(a^2 b e^3+4a^2 c d e^2-2a b^2 d e^2-3abc d^2 e+b^3 d^2 e-\sqrt{-e^2\left(4ac-b^2\right)}\, a^2e^2+2\sqrt{-e^2\left(4ac-b^2\right)}\, abde+\sqrt{\cdots}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\cdots}} \cdots \right)}{\cdots} \right.$
default	$2e^4 \left(\frac{4c \left(\frac{\left(a^2 b e^3+4a^2 c d e^2-2a b^2 d e^2-3abc d^2 e+b^3 d^2 e-\sqrt{-e^2\left(4ac-b^2\right)}\, a^2e^2+2\sqrt{-e^2\left(4ac-b^2\right)}\, abde+\sqrt{\cdots}\right.}{8\sqrt{-e^2\left(4ac-b^2\right)}\,\sqrt{\cdots}} \cdots \right)}{\cdots} \right.$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*e^4*(4/e^4/a^3*c*(1/8*(a^2*b*e^3+4*a^2*c*d*e^2-2*a*b^2*d*e^2-3*a*b*c*d^2*
e+b^3*d^2*e-(-e^2*(4*a*c-b^2))^(1/2)*a^2*e^2+2*(-e^2*(4*a*c-b^2))^(1/2)*a*b
*d*e+(-e^2*(4*a*c-b^2))^(1/2)*a*c*d^2-(-e^2*(4*a*c-b^2))^(1/2)*b^2*d^2)/(-e
^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2
)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((e*b-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(
1/2))-1/8*(-a^2*b*e^3-4*a^2*c*d*e^2+2*a*b^2*d*e^2+3*a*b*c*d^2*e-b^3*d^2*e-(
-e^2*(4*a*c-b^2))^(1/2)*a^2*e^2+2*(-e^2*(4*a*c-b^2))^(1/2)*a*b*d*e+(-e^2*(4
*a*c-b^2))^(1/2)*a*c*d^2-(-e^2*(4*a*c-b^2))^(1/2)*b^2*d^2)/(-e^2*(4*a*c-b^2
))^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*
(e*x+d)^(1/2)*2^(1/2)/((-e*b+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)))-1/e
^4/a^3*((1/8*a*e*(5*a*e-4*b*d)*(e*x+d)^(3/2)+(1/2*a*b*d^2*e-3/8*d*a^2*e^2)*
(e*x+d)^(1/2))/e^2/x^2+1/8*(3*a^2*e^2-12*a*b*d*e-8*a*c*d^2+8*b^2*d^2)/d^(1/
2)*arctanh((e*x+d)^(1/2)/d^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*x^2 + b*x + a)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1121 vs. $2(543) = 1086$.
time = 1.15, size = 1121, normalized size = 1.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e - 2*(a*b^3 - 4*a^2*b*c)*d*e^2 +
(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*a
^2 + 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2*e + sqrt(b^2 - 4*a*c)*a^3*b*e^3 - (a*b^
2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^3 - (2*a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c
)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) + (a^4*b^2
*e^3 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*d^3 + (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*
d^2*e - 2*(a^3*b^3 - a^4*b*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*
a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e + s
qrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/
(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqr
t(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/4*(((b^4 - 5*a*b^2*c + 4*a^2*c^2
)*d^2*e - 2*(a*b^3 - 4*a^2*b*c)*d*e^2 + (a^2*b^2 - 4*a^3*c)*e^3)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*b^3*d^2
*e + sqrt(b^2 - 4*a*c)*a^3*b*e^3 - (a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^
3 - (2*a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*d*e^2)*sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*abs(a) + (a^4*b^2*e^3 - 2*(a^2*b^3*c - 3*a^3*b*c^2)*
d^3 + (a^2*b^4 + a^3*b^2*c - 8*a^4*c^2)*d^2*e - 2*(a^3*b^3 - a^4*b*c)*d*e^2
)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt
(x*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4
*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c
*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(
c)) + 1/4*(8*b^2*d^2 - 8*a*c*d^2 - 12*a*b*d*e + 3*a^2*e^2)*arctan(sqrt(x*e
+ d)/sqrt(-d))/(a^3*sqrt(-d)) + 1/4*(4*(x*e + d)^(3/2)*b*d*e - 4*sqrt(x*e +
 d)*b*d^2*e - 5*(x*e + d)^(3/2)*a*e^2 + 3*sqrt(x*e + d)*a*d*e^2)*e^(-2)/(a^
2*x^2)
```

Mupad [B]
time = 8.19, size = 2500, normalized size = 4.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)),x)
```

[Out] (((3*a*d*e^2 - 4*b*d^2*e)*(d + e*x)^(1/2))/(4*a^2) - ((5*a*e^2 - 4*b*d*e)*(d + e*x)^(3/2))/(4*a^2))/((d + e*x)^2 - 2*d*(d + e*x) + d^2) + atan((((((192*a^11*b^2*c^3*e^12 - 24*a^10*b^4*c^2*e^12 - 384*a^12*c^4*e^12 + 768*a^10*c^6*d^4*e^8 + 384*a^11*c^5*d^2*e^10 + 128*a^8*b^4*c^4*d^4*e^8 - 96*a^8*b^5*c^3*d^3*e^9 - 32*a^8*b^6*c^2*d^2*e^10 - 704*a^9*b^2*c^5*d^4*e^8 + 320*a^9*b^3*c^4*d^3*e^9 + 488*a^9*b^4*c^3*d^2*e^10 - 1536*a^10*b^2*c^4*d^2*e^10 + 1408*a^11*b*c^4*d*e^11 + 56*a^9*b^5*c^2*d*e^11 + 256*a^10*b*c^5*d^3*e^9 - 576*a^10*b^3*c^3*d*e^11)/(2*a^8) - ((d + e*x)^(1/2)*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(1024*a^13*c^4*e^10 + 64*a^11*b^4*c^2*e^10 - 512*a^12*b^2*c^3*e^10 + 1536*a^12*c^5*d^2*e^8 + 128*a^10*b^4*c^3*d^2*e^8 - 896*a^11*b^2*c^4*d^2*e^8 - 1792*a^12*b*c^4*d*e^9 - 128*a^10*b^5*c^2*d*e^9 + 960*a^11*b^3*c^3*d*e^9))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x)^(1/2)*(876*a^10*b*c^4*e^13 + 1336*a^10*c^5*d*e^12 + 73*a^8*b^5*c^2*e^13 - 511*a^9*b^3*c^3*e^13 - 1152*a^8*c^7*d^5*e^8 + 2176*a^9*c^6*d^3*e^10 - 128*a^4*b^8*c^3*d^5*e^8 + 128*a^4*b^9*c^2*d^4*e^9 + 1152*a^5*b^6*c^4*d^5*e^8 - 832*a^5*b^7*c^3*d^4*e^9 - 448*a^5*b^8*c^2*d^3*e^10 - 3520*a^6*b^4*c^5*d^5*e^8 + 768*a^6*b^5*c^4*d^4*e^9 + 3520*a^6*b^6*c^3*d^3*e^10 + 576*a^6*b^7*c^2*d^2*e^11 + 4096*a^7*b^2*c^6*d^5*e^8 + 3328*a^7*b^3*c^5*d^4*e^9 - 7824*a^7*b^4*c^4*d^3*e^10 - 4520*a^7*b^5*c^3*d^2*e^11 + 2912*a^8*b^2*c^5*d^3*e^10 + 10016*a^8*b^3*c^4*d^2*e^11 - 328*a^7*b^6*c^2*d*e^12 - 4864*a^8*b*c^6*d^4*e^9 + 2479*a^8*b^4*c^3*d*e^12 - 4352*a^9*b*c^5*d^2*e^11 - 5034*a^9*b^2*c^4*d*e^12))/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 4*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e

*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^5*c*d^2*e - 24*a^3*b^4*c*d*e^2 + 60*a^4*b*c^3*d^2*e + 3*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 75*a^3*b^3*c^2*d^2*e + 54*a^4*b^2*c^2*d*e^2 - 3*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - (216*a^9*b*c^4*e^15 + 604*a^9*c^5*d*e^14 + 15*a^7*b^5*c^2*e^15 - 114*a^8*b^3*c^3*e^15 + 192*a^6*c^8*d^7*e^8 - 1344*a^7*c^7*d^5*e^10 - 932*a^8*c^6*d^3*e^12 + 128*a^2*b^8*c^4*d^7*e^8 - 96*a^2*b^9*c^3*d^6*e^9 - 32*a^2*b^10*c^2*d^5*e^10 - 960*a^3*b^6*c^5*d^7*e^8 + 128*a^3*b^7*c^4*d^6*e^9 + 840*a^3*b^8*c^3*d^5*e^10 + 152*a^3*b^9*c^2*d^4*e^11 + 2176*a^4*b^4*c^6*d^7*e^8 + 2336*a^4*b^5*c^5*d^6*e^9 - 3648*a^4*b^6*c^4*d^5*e^10 - 2496*a^4*b^7*c^3*d^4*e^11 - 280*a^4*b^8*c^2*d^3*e^12 - 1600*a^5*b^2*c^7*d^7*e^8 - 6016*a^5*b^3*c^6*d^6*e^9 + 2328*a^5*b^4*c^5*d^5*e^10 + 10216*a^5*b^5*c^4*d^4*e^11 + 3497*a^5*b^6*c^3*d^3*e^12 + 247*a^5*b^7*c^2*d^2*e^13 + 3744*a^6*b^2*c^6*d^5*e^10 - 10912*a^6*b^3*c^5*d^4*e^11 - 12151*a^6*b^4*c^4*d^3*e^12 - 2498*a^6*b^5*c^3*d^2*e^13 + 10885*a^7*b^2*c^5*d^3*e^12 + 7081*a^7*b^3*c^4*d^2*e^13 + 3200*a^6*b*c^7*d^6*e^9 - 102*a^6*b^6*c^2*d*e^14 + 1024*a^7*b*c^6*d^4*e^11 + 867*a^7*b^4*c^3*d*e^14 - 4292*a^8*b*c^5*d^2*e^13 - 1971*a^8*b^2*c^4*d*e^14)/(2*a^8))*((b^8*d^3 - a^3*b^5*e^3 + 8*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 7*a^4*b^3*c*e^3 - 12*a^5*b*c^2*e^3 + a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 24*a^5*c^3*d*e^2 + 33*a^2*b^4*c^2*d^3 - 38*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(...

3.541 $\int \frac{x^m(e+fx)^n}{a+bx+cx^2} \, dx$

Optimal. Leaf size=201

$$\frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)}-\frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-}}{\sqrt{b^2-4}}$$

[Out] 2*c*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-2*c*x/(b-(-4*a*c+b^2)^(1/2)),-f*x/e)/(1+m)/((1+f*x/e)^n)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-2*c*x/(b+(-4*a*c+b^2)^(1/2)),-f*x/e)/(1+m)/((1+f*x/e)^n)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))

Rubi [A]
time = 0.34, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {925, 140, 138}

$$\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

[In] Int[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]

[Out] (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + m)*(1 + (f*x)/e)^n) - (2*c*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*(1 + (f*x)/e)^n)

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx &= \int \left(\frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)}-\frac{2cx^m(e+fx)^n}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) dx\\
&= \frac{(2c)\int \frac{x^m(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{x^m(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\left(2c(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}-\frac{\left(2c(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x}{b+\sqrt{b}}}{\sqrt{b^2-4ac}}\\
&= \frac{2cx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)}-\frac{2cx^{1+}}{}\end{aligned}$$

Mathematica [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^m(e+fx)^n}{a+bx+cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2),x]
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x + c*x^2), x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^m(fx+e)^n}{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
[Out] int(x^m*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,{\left(e+f\,x\right)}^n}{c\,x^2+b\,x+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e + f*x)^n)/(a + b*x + c*x^2),x)
[Out] int((x^m*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.542 $\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=290

$$-\frac{(ce+bf)(e+fx)^{1+n}}{c^2f^2(1+n)}+\frac{(e+fx)^{2+n}}{cf^2(2+n)}+\frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e-}{2ce-\left(b-\sqrt{b}\right.}\right.}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

```
[Out] -(b*f+c*e)*(f*x+e)^(1+n)/c^2/f^2/(1+n)+(f*x+e)^(2+n)/c/f^2/(2+n)+(f*x+e)^(1
+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*
(a-b^2/c+b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+n)/(2*c*e-f*(b-(-4*a*c+b
^2)^(1/2)))+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+
(-4*a*c+b^2)^(1/2))))*(a-b^2/c-b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+n)
/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.53, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1642, 70}

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(bf+ce)(e+fx)^{n+1}}{c^2f^2(n+1)}+\frac{(e+fx)^{n+2}}{cf^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] -(((c*e + b*f)*(e + f*x)^(1 + n))/(c^2*f^2*(1 + n))) + (e + f*x)^(2 + n)/(c
*f^2*(2 + n)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e +
 f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (
b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))
 + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(e + f*x)^(1 + n)
*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2
- 4*a*c])*f)])/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{(-ce-bf)(e+fx)^n}{c^2 f} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}-\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b(b}{c^2\sqrt{b}}\right.}{b+\sqrt{b^2}}\right.$$

$$= -\frac{(ce+bf)(e+fx)^{1+n}}{c^2 f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)\int \frac{(e+f}{b+\sqrt{b^2-4a}}$$

$$= -\frac{(ce+bf)(e+fx)^{1+n}}{c^2 f^2(1+n)} + \frac{(e+fx)^{2+n}}{cf^2(2+n)} + \frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(\right.}{c\left(2ce-\left(b-\sqrt{b^2-\right.\right.}}$$

Mathematica [A]
time = 0.60, size = 353, normalized size = 1.22

$$\frac{2^{-1-n}(e+fx)^n\left(\left(-b^3f+3abcf+b^2\sqrt{(b^2-4ac)f^2}-ac\sqrt{(b^2-4ac)f^2}\right)\left(\frac{c(e+fx)}{bf-\sqrt{(b^2-4ac)f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{2ce-bf+\sqrt{(b^2-4ac)f^2}}{-bf+\sqrt{(b^2-4ac)f^2}-2cfx}\right)+\left(b^3f-3abcf+b^2\sqrt{(b^2-4ac)f^2}-ac\sqrt{(b^2-4ac)f^2}\right)\left(\frac{c(e+fx)}{bf+\sqrt{(b^2-4ac)f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{-2ce+bf+\sqrt{(b^2-4ac)f^2}}{bf+\sqrt{(b^2-4ac)f^2}+2cfx}\right)\right)}{c^3\sqrt{(b^2-4ac)f^2}\,n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (2^(-1 - n)*(e + f*x)^n*(((-(b^3*f) + 3*a*b*c*f + b^2*Sqrt[(b^2 - 4*a*c)*f^
2] - a*c*Sqrt[(b^2 - 4*a*c)*f^2])*Hypergeometric2F1[-n, -n, 1 - n, (2*c*e -
 b*f + Sqrt[(b^2 - 4*a*c)*f^2])/(-(b*f) + Sqrt[(b^2 - 4*a*c)*f^2] - 2*c*f*x
)])/((c*(e + f*x))/(b*f - Sqrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n + ((b^3*f -
 3*a*b*c*f + b^2*Sqrt[(b^2 - 4*a*c)*f^2] - a*c*Sqrt[(b^2 - 4*a*c)*f^2])*Hyp
ergeometric2F1[-n, -n, 1 - n, (-2*c*e + b*f + Sqrt[(b^2 - 4*a*c)*f^2])/(b*f
 + Sqrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x)])/((c*(e + f*x))/(b*f + Sqrt[(b^2 - 4
*a*c)*f^2] + 2*c*f*x))^n))/(c^3*Sqrt[(b^2 - 4*a*c)*f^2]*n)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3(fx+e)^n}{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x^3*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(e+f\,x\right)}^n}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(e + f*x)^n)/(a + b*x + c*x^2),x)
[Out] int((x^3*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.543 $\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=237

$$\frac{(e+fx)^{1+n}}{cf(1+n)}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4a\cdots}}\right.\cdots}{\cdots}$$

```
[Out] (f*x+e)^(1+n)/c/f/(1+n)+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/
(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/(1+n
)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))+(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2
*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2))))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(
1/2))/c/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.27, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1642, 70}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{(e+fx)^{n+1}}{cf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (e + f*x)^(1 + n)/(c*f*(1 + n)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e -
 (b - Sqrt[b^2 - 4*a*c])*f)])/(c*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n
)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])
/(c*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{(e+fx)^n}{c} + \frac{\left(-\frac{b}{c}+\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx} \cdots\right.$$

$$= \frac{(e+fx)^{1+n}}{cf(1+n)} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\cdots}{b+\sqrt{\cdots}}}{c}$$

$$= \frac{(e+fx)^{1+n}}{cf(1+n)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\cdots\right.}\cdots\right.}{c\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

Mathematica [A]
time = 0.78, size = 346, normalized size = 1.46

$$\frac{2^{-1-n}(e+fx)^n\left(\frac{2^{1+n}c\sqrt{(b^2-4ac)f^2}\,n(e+fx)}{f(1+n)}-\left(-b^2f+2acf+b\sqrt{(b^2-4ac)f^2}\right)\left(\frac{c(e+fx)}{bf-\sqrt{(b^2-4ac)f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{2ce-bf+\sqrt{(b^2-4ac)f^2}}{-bf+\sqrt{(b^2-4ac)f^2}-2cfx}\right)-\left(b^2f-2acf+b\sqrt{(b^2-4ac)f^2}\right)\left(\frac{c(e+fx)}{bf+\sqrt{(b^2-4ac)f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{-2ce+bf+\sqrt{(b^2-4ac)f^2}}{bf+\sqrt{(b^2-4ac)f^2}+2cfx}\right)\right)}{c^2\sqrt{(b^2-4ac)f^2}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] (2^(-1 - n)*(e + f*x)^n*((2^(1 + n)*c*Sqrt[(b^2 - 4*a*c)*f^2]*n*(e + f*x))/
(f*(1 + n)) - ((-(b^2*f) + 2*a*c*f + b*Sqrt[(b^2 - 4*a*c)*f^2])*Hypergeomet
ric2F1[-n, -n, 1 - n, (2*c*e - b*f + Sqrt[(b^2 - 4*a*c)*f^2])/(-(b*f) + Sqr
t[(b^2 - 4*a*c)*f^2] - 2*c*f*x)])/((c*(e + f*x))/(b*f - Sqrt[(b^2 - 4*a*c)*
f^2] + 2*c*f*x))^n - ((b^2*f - 2*a*c*f + b*Sqrt[(b^2 - 4*a*c)*f^2])*Hyperge
ometric2F1[-n, -n, 1 - n, (-2*c*e + b*f + Sqrt[(b^2 - 4*a*c)*f^2])/(b*f + S
qrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x)])/((c*(e + f*x))/(b*f + Sqrt[(b^2 - 4*a*c
)*f^2] + 2*c*f*x))^n))/(c^2*Sqrt[(b^2 - 4*a*c)*f^2]*n)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^2(fx+e)^n}{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(e+fx)^n}{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Integral(x**2*(e + f*x)**n/(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(e+f\,x\right)}^n}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(e + f*x)^n)/(a + b*x + c*x^2),x)
[Out] int((x^2*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.544 $\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx$

Optimal. Leaf size=198

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}-\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{\ldots}}{\left(2ce-\left(\ldots\right.\right.}$$

```
[Out] -(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2
)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))-
(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)
^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.14, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {844, 70}

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] -(((1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/((2*c*e - (b -
 Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - ((1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1
+ n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)])/((2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\right)dx$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(e+ }{b+\sqrt{b^2-4}$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}-\frac{\left(1-\right.}{}$$

Mathematica [**A**]
time = 0.37, size = 183, normalized size = 0.92

$$\frac{(e+fx)^{1+n}\left(-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}\right)}{2ce+\left(-b+\sqrt{b^2-4ac}\right)f}-\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/(a + b*x + c*x^2),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(((1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + n
, 2 + n, (2*c*(e + f*x))/(2*c*e + (-b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*e + (-
b + Sqrt[b^2 - 4*a*c])*f)) - ((1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1
, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(2*c*
e - (b + Sqrt[b^2 - 4*a*c])*f)))/(1 + n)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x(fx+e)^n}{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int(x*(f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(e+fx)^n}{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(c*x**2+b*x+a),x)
[Out] Integral(x*(e + f*x)**n/(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(e+f\,x\right)}^n}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(e + f*x)^n)/(a + b*x + c*x^2),x)
[Out] int((x*(e + f*x)^n)/(a + b*x + c*x^2), x)
```

3.545 $\int \frac{(e+fx)^n}{a+bx+cx^2} \, dx$

Optimal. Leaf size=191

$$-\frac{2c(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}+\frac{2c(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b+\sqrt{b^2-4ac}\right)f\right)(1+n)}$$

```
[Out] -2*c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c
+b^2)^(1/2))))/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+2*
c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^
2)^(1/2))))/(1+n)/(-4*a*c+b^2)^(1/2)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.18, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {725, 70}

$$\frac{2c(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{2c(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(a + b*x + c*x^2),x]
```

```
[Out] (-2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/
(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2 - 4*a*c]*(2*c*e - (b - Sqrt
[b^2 - 4*a*c])*f)*(1 + n)) + (2*c*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1
+ n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(Sqrt[b^2
 - 4*a*c]*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 725

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx = \int \left(\frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(e+fx)^n}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) dx$$
$$= \frac{(2c)\int \frac{(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(e+fx)^n}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$
$$= -\frac{2c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{\sqrt{b^2-4ac}\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} + \frac{2c(e+fx)^{1+n}\,{}_2F_1\left(1,1\right.}{\sqrt{b^2-4ac}\left(2ce-\right.}$$

Mathematica [A]
time = 0.24, size = 245, normalized size = 1.28

$$\frac{2^{-n}f(e+fx)^n\left(\left(\frac{c(e+fx)}{bf-\sqrt{(b^2-4ac)f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{2ce-bf+\sqrt{(b^2-4ac)f^2}}{-bf+\sqrt{(b^2-4ac)f^2}-2cfx}\right)-\left(\frac{c(e+fx)}{bf+\sqrt{(b^2-4ac)f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{-2ce+bf+\sqrt{(b^2-4ac)f^2}}{bf+\sqrt{(b^2-4ac)f^2}+2cfx}\right)\right)}{\sqrt{(b^2-4ac)f^2}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(a + b*x + c*x^2),x]
```

```
[Out] (f*(e + f*x)^n*(Hypergeometric2F1[-n, -n, 1 - n, (2*c*e - b*f + Sqrt[(b^2 -
 4*a*c)*f^2])/(-(b*f) + Sqrt[(b^2 - 4*a*c)*f^2] - 2*c*f*x)]/((c*(e + f*x))/
(b*f - Sqrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n - Hypergeometric2F1[-n, -n, 1
- n, (-2*c*e + b*f + Sqrt[(b^2 - 4*a*c)*f^2])/(b*f + Sqrt[(b^2 - 4*a*c)*f^2
] + 2*c*f*x)]/((c*(e + f*x))/(b*f + Sqrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n))
/(2^n*Sqrt[(b^2 - 4*a*c)*f^2]*n)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/(c*x^2+b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^2 + b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^n}{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(c*x**2+b*x+a),x)
```

```
[Out] Integral((e + f*x)**n/(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e+f\,x)^n}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(a + b*x + c*x^2),x)
```

```
[Out] int((e + f*x)^n/(a + b*x + c*x^2), x)
```

3.546 $\int \frac{(e+fx)^n}{x(a+bx+cx^2)}\,dx$

Optimal. Leaf size=242

$$\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)}{a\left(2ce-\left(\right.\right.}$$

```
[Out] -(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a/e/(1+n)+c*(f*x+e)^(1+n)*
hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(1+b
/(-4*a*c+b^2)^(1/2))/a/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))+c*(f*x+e)^(1+
n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2))))*(
1-b/(-4*a*c+b^2)^(1/2))/a/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.28, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {974, 67, 844, 70}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{ae(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b
 - Sqrt[b^2 - 4*a*c])*f)*(1 + n)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(e + f*x)^
(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + Sq
rt[b^2 - 4*a*c])*f)])/(a*(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)*(1 + n)) - ((e
 + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 +
n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(e+fx)^n}{x\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{(e+fx)^n}{ax}+\frac{(-b-cx)(e+fx)^n}{a\left(a+bx+cx^2\right)}\right)\,dx$$

$$= \frac{\int \frac{(e+fx)^n}{x}\,dx}{a}+\frac{\int \frac{(-b-cx)(e+fx)^n}{a+bx+cx^2}\,dx}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)}+\frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(e+fx)^n}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(-c+ \cdots\right.}{b\cdots}\right.}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(e+f\cdots}{b+\sqrt{b^2-4\cdots}}}{a}$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}+\frac{c\cdots}{\cdots}$$

Mathematica [A]

time = 1.17, size = 331, normalized size = 1.37

$$\frac{(e+fx)^n\left(2\left(1+\frac{e}{fx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{e}{fx}\right)+\frac{2^{-n}\left(-\left(\left(bf+\sqrt{(b^2-4ac)f^2}\right)\left(\frac{c(e+fx)}{bf-\sqrt{(b^2-4ac)f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{2ce-bf+\sqrt{(b^2-4ac)f^2}}{-bf+\sqrt{(b^2-4ac)f^2}-2cfx}\right)\right)-\left(-bf+\sqrt{(b^2-4ac)f^2}\right)\left(\frac{c(e+fx)}{bf+\sqrt{(b^2-4ac)f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{-2ce+bf+\sqrt{(b^2-4ac)f^2}}{bf+\sqrt{(b^2-4ac)f^2}+2cfx}\right)\right)}{\sqrt{(b^2-4ac)f^2}}\right)}{2an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x + c*x^2)),x]
```

```
[Out] ((e + f*x)^n*((2*Hypergeometric2F1[-n, -n, 1 - n, -(e/(f*x))])/(1 + e/(f*x)
)^n + (-(((b*f + Sqrt[(b^2 - 4*a*c)*f^2])*Hypergeometric2F1[-n, -n, 1 - n,
(2*c*e - b*f + Sqrt[(b^2 - 4*a*c)*f^2])/(-(b*f) + Sqrt[(b^2 - 4*a*c)*f^2] -
 2*c*f*x)])/((c*(e + f*x))/(b*f - Sqrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n) -
((-(b*f) + Sqrt[(b^2 - 4*a*c)*f^2])*Hypergeometric2F1[-n, -n, 1 - n, (-2*c*
e + b*f + Sqrt[(b^2 - 4*a*c)*f^2])/(b*f + Sqrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x
)])/((c*(e + f*x))/(b*f + Sqrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n)/(2^n*Sqrt[
(b^2 - 4*a*c)*f^2])))/(2*a*n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{x\,(c\,x^2+bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/x/(c*x^2+b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^3 + b*x^2 + a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^n}{x\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(c*x**2+b*x+a),x)
```

```
[Out] Integral((e + f*x)**n/(x*(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{x\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x*(a + b*x + c*x^2)),x)
```

```
[Out] int((e + f*x)^n/(x*(a + b*x + c*x^2)), x)
```

3.547 $\int \frac{(e+fx)^n}{x^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=296

$$-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx}{a^2\left(2ce-\right.}$$

```
[Out] b*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e)/a^2/e/(1+n)+f*(f*x+e)^(1+
n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a/e^2/(1+n)-c*(f*x+e)^(1+n)*hypergeom(
[1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2))))*(b+(-2*a*c+b^2
)/(-4*a*c+b^2)^(1/2))/a^2/(1+n)/(2*c*e-f*(b-(-4*a*c+b^2)^(1/2)))-c*(f*x+e)^
(1+n)*hypergeom([1, 1+n],[2+n],2*c*(f*x+e)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/2)))
)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+n)/(2*c*e-f*(b+(-4*a*c+b^2)^(1/
2)))
```

Rubi [A]
time = 0.31, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {974, 67, 844, 70}

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{2c(e+fx)}{2ce-\left(b+\sqrt{b^2-4ac}\right)f}\right)}{a^2(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{b(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{a^2e(n+1)}+\frac{f(e+fx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)])/
(a^2*(2*c*e - (b - Sqrt[b^2 - 4*a*c])*f)*(1 + n))) - (c*(b - (b^2 - 2*a*c)/
Sqrt[b^2 - 4*a*c])*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (2*
c*(e + f*x))/(2*c*e - (b + Sqrt[b^2 - 4*a*c])*f)])/(a^2*(2*c*e - (b + Sqrt[
b^2 - 4*a*c])*f)*(1 + n)) + (b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, 1 + (f*x)/e])/(a^2*e*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometri
c2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(e+fx)^n}{x^2\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{(e+fx)^n}{ax^2}-\frac{b(e+fx)^n}{a^2x}+\frac{\left(b^2-ac+bcx\right)(e+fx)^n}{a^2\left(a+bx+cx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(b^2-ac+bcx\right)(e+fx)^n}{a+bx+cx^2}\,dx}{a^2}+\frac{\int \frac{(e+fx)^n}{x^2}\,dx}{a}-\frac{b\int \frac{(e+fx)^n}{x}\,dx}{a^2}$$

$$= \frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)}+\frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\right.}{ae^2(1+n)}$$

$$= \frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{a^2e(1+n)}+\frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\right.}{ae^2(1+n)}$$

$$= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{2c(e+fx)}{2ce-\left(b-\sqrt{b^2-4ac}\right)f}\right)}{a^2\left(2ce-\left(b-\sqrt{b^2-4ac}\right)f\right)(1+n)} -$$

Mathematica [A]

time = 1.39, size = 431, normalized size = 1.46

$$\frac{\left(1+\frac{e}{fx}\right)^{-n}(e+fx)^n\left(\frac{2a\,{}_2F_1\left(1-n,-n;2-n;-\frac{e}{fx}\right)}{(-1+n)x}-\frac{2b\,{}_2F_1\left(-n,-n;1-n;-\frac{e}{fx}\right)}{n}+\frac{2^{-n}\left(b^2f-2acf+b\sqrt{(b^2-4ac)\,f^2}\right)\left(1+\frac{e}{fx}\right)^n\left(\frac{c(e+fx)}{bf-\sqrt{(b^2-4ac)\,f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{2ce-bf+\sqrt{(b^2-4ac)\,f^2}}{-bf+\sqrt{(b^2-4ac)\,f^2}-2cfx}\right)}{\sqrt{(b^2-4ac)\,f^2}\,n}+\frac{2^{-n}\left(-b^2f+2acf+b\sqrt{(b^2-4ac)\,f^2}\right)\left(1+\frac{e}{fx}\right)^n\left(\frac{c(e+fx)}{bf+\sqrt{(b^2-4ac)\,f^2}+2cfx}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{-2ce+bf+\sqrt{(b^2-4ac)\,f^2}}{bf+\sqrt{(b^2-4ac)\,f^2}+2cfx}\right)}{\sqrt{(b^2-4ac)\,f^2}\,n}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((e + f*x)^n*((2*a*Hypergeometric2F1[1 - n, -n, 2 - n, -(e/(f*x))])/((-1 +
n)*x) - (2*b*Hypergeometric2F1[-n, -n, 1 - n, -(e/(f*x))])/n + ((b^2*f - 2*
a*c*f + b*Sqrt[(b^2 - 4*a*c)*f^2])*(1 + e/(f*x))^n*Hypergeometric2F1[-n, -n
, 1 - n, (2*c*e - b*f + Sqrt[(b^2 - 4*a*c)*f^2])/(-(b*f) + Sqrt[(b^2 - 4*a*
c)*f^2] - 2*c*f*x)])/(2^n*Sqrt[(b^2 - 4*a*c)*f^2]*n*((c*(e + f*x))/(b*f - S
qrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n) + ((-(b^2*f) + 2*a*c*f + b*Sqrt[(b^2
- 4*a*c)*f^2])*(1 + e/(f*x))^n*Hypergeometric2F1[-n, -n, 1 - n, (-2*c*e + b
*f + Sqrt[(b^2 - 4*a*c)*f^2])/(b*f + Sqrt[(b^2 - 4*a*c)*f^2] + 2*c*f*x)])/(
2^n*Sqrt[(b^2 - 4*a*c)*f^2]*n*((c*(e + f*x))/(b*f + Sqrt[(b^2 - 4*a*c)*f^2]
 + 2*c*f*x))^n)))/(2*a^2*(1 + e/(f*x))^n)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{x^2\,(c\,x^2+bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

```
[Out] int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(c*x^4 + b*x^3 + a*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{x^2\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x^2*(a + b*x + c*x^2)),x)
```

```
[Out] int((e + f*x)^n/(x^2*(a + b*x + c*x^2)), x)
```

3.548 $\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=141

$$-\frac{d^2(7e^2f^2+16defg+8d^2g^2)\,x}{e^2}-\frac{d(2e^2f^2+7defg+4d^2g^2)\,x^2}{e}-\frac{1}{3}(ef+dg)(ef+7dg)x^3-\frac{1}{2}eg(ef+2dg)x^4-\frac{1}{5}$$

```
[Out] -d^2*(8*d^2*g^2+16*d*e*f*g+7*e^2*f^2)*x/e^2-d*(4*d^2*g^2+7*d*e*f*g+2*e^2*f^
2)*x^2/e-1/3*(d*g+e*f)*(7*d*g+e*f)*x^3-1/2*e*g*(2*d*g+e*f)*x^4-1/5*e^2*g^2*
x^5-8*d^3*(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 90}

$$-\frac{8d^3(dg+ef)^2\log(d-ex)}{e^3}-\frac{dx^2(4d^2g^2+7defg+2e^2f^2)}{e}-\frac{d^2x(8d^2g^2+16defg+7e^2f^2)}{e^2}-\frac{1}{2}egx^4(2dg+ef)-\frac{1}{3}x^3(dg+ef)(7dg+ef)-\frac{1}{5}e^2g^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d^2*(7*e^2*f^2 + 16*d*e*f*g + 8*d^2*g^2)*x)/e^2) - (d*(2*e^2*f^2 + 7*d*e
*f*g + 4*d^2*g^2)*x^2)/e - ((e*f + d*g)*(e*f + 7*d*g)*x^3)/3 - (e*g*(e*f +
2*d*g)*x^4)/2 - (e^2*g^2*x^5)/5 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2}\,dx = \int \frac{(d+ex)^3(f+gx)^2}{d-ex}\,dx$$
$$= \int \left(-\frac{d^2(7e^2f^2+16defg+8d^2g^2)}{e^2} - \frac{2d(2e^2f^2+7defg+4d^2g^2)\,x}{e} + (-ef -\right.$$
$$= -\frac{d^2(7e^2f^2+16defg+8d^2g^2)\,x}{e^2} - \frac{d(2e^2f^2+7defg+4d^2g^2)\,x^2}{e} - \frac{1}{3}(ef+dg)$$

Mathematica [A]
time = 0.06, size = 134, normalized size = 0.95

$$-\frac{x(240d^4g^2+120d^3eg(4f+gx)+70d^2e^2(3f^2+3fgx+g^2x^2)+10de^3x(6f^2+8fgx+3g^2x^2)+e^4x^2(10f^2+15fgx+6g^2x^2))}{30e^2} - \frac{8d^3(ef+dg)^2\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/30*(x*(240*d^4*g^2 + 120*d^3*e*g*(4*f + g*x) + 70*d^2*e^2*(3*f^2 + 3*f*g
*x + g^2*x^2) + 10*d*e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2) + e^4*x^2*(10*f^2
+ 15*f*g*x + 6*g^2*x^2)))/e^2 - (8*d^3*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Maple [A]
time = 0.08, size = 179, normalized size = 1.27

method	result
norman	$\left(-\frac{7}{3}d^2g^2-\frac{8}{3}defg-\frac{1}{3}e^2f^2\right)x^3-\frac{e^2g^2x^5}{5}-\frac{d(4d^2g^2+7defg+2e^2f^2)x^2}{e}-\frac{d^2(8d^2g^2+16defg+7e^2f^2)x}{e^2}-\frac{eg(2dg+}{2}$
default	$-\frac{\frac{1}{5}g^2e^4x^5+d\,e^3g^2x^4+\frac{1}{2}e^4fg\,x^4+\frac{7}{3}d^2e^2g^2x^3+\frac{8}{3}d\,e^3fg\,x^3+\frac{1}{3}e^4f^2x^3+4d^3e\,g^2x^2+7d^2e^2fg\,x^2+2d\,e^3f^2x^2+8d^4g^2x+16d^3efgx+7d^2}{e^2}$
risch	$-\frac{e^2g^2x^5}{5}-ed\,g^2x^4-\frac{e^2fg\,x^4}{2}-\frac{7d^2g^2x^3}{3}-\frac{8edfg\,x^3}{3}-\frac{e^2f^2x^3}{3}-\frac{4d^3g^2x^2}{e}-7d^2fg\,x^2-2ed\,f^2x^2-\frac{8d^4g^2x}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2*(1/5*g^2*e^4*x^5+d*e^3*g^2*x^4+1/2*e^4*f*g*x^4+7/3*d^2*e^2*g^2*x^3+8
/3*d*e^3*f*g*x^3+1/3*e^4*f^2*x^3+4*d^3*e*g^2*x^2+7*d^2*e^2*f*g*x^2+2*d*e^3*
f^2*x^2+8*d^4*g^2*x+16*d^3*e*f*g*x+7*d^2*e^2*f^2*x)-8*d^3*(d^2*g^2+2*d*e*f*
g+e^2*f^2)/e^3*ln(-e*x+d)
```

Maxima [A]
time = 0.29, size = 167, normalized size = 1.18

$$-8\left(d^5g^2+2\,d^4fge+d^3f^2e^2\right)e^{(-3)}\log\left(xe-d\right)-\frac{1}{30}\left(6\,g^2x^5e^4+15\left(2\,dg^2e^3+fge^4\right)x^4+10\left(7\,d^2g^2e^2+8\,dfge^3+f^2e^4\right)x^3+30\left(4\,d^3g^2e+7\,d^2fge^2+2\,df^2e^3\right)x^2+30\left(8\,d^4g^2+16\,d^3fge+7\,d^2f^2e^2\right)x\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -8*(d^5*g^2 + 2*d^4*f*g*e + d^3*f^2*e^2)*e^(-3)*log(x*e - d) - 1/30*(6*g^2*
x^5*e^4 + 15*(2*d*g^2*e^3 + f*g*e^4)*x^4 + 10*(7*d^2*g^2*e^2 + 8*d*f*g*e^3
+ f^2*e^4)*x^3 + 30*(4*d^3*g^2*e + 7*d^2*f*g*e^2 + 2*d*f^2*e^3)*x^2 + 30*(8
*d^4*g^2 + 16*d^3*f*g*e + 7*d^2*f^2*e^2)*x)*e^(-2)
```

Fricas [A]
time = 3.78, size = 168, normalized size = 1.19

$$-\frac{1}{30}\left(240\,d^{4}g^{2}xe+\left(6\,g^{2}x^{5}+15\,fgx^{4}+10\,f^{2}x^{3}\right)e^{5}+10\left(3\,dg^{2}x^{4}+8\,dfgx^{3}+6\,df^{2}x^{2}\right)e^{4}+70\left(d^{2}g^{2}x^{3}+3\,d^{2}fgx^{2}+3\,d^{2}f^{2}x\right)e^{3}+120\left(d^{3}g^{2}x^{2}+4\,d^{3}fgx\right)e^{2}+240\left(d^{5}g^{2}+2\,d^{4}fge+d^{3}f^{2}e^{2}\right)\log\left(xe-d\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/30*(240*d^4*g^2*x*e + (6*g^2*x^5 + 15*f*g*x^4 + 10*f^2*x^3)*e^5 + 10*(3*
d*g^2*x^4 + 8*d*f*g*x^3 + 6*d*f^2*x^2)*e^4 + 70*(d^2*g^2*x^3 + 3*d^2*f*g*x^
2 + 3*d^2*f^2*x)*e^3 + 120*(d^3*g^2*x^2 + 4*d^3*f*g*x)*e^2 + 240*(d^5*g^2 +
 2*d^4*f*g*e + d^3*f^2*e^2)*log(x*e - d))*e^(-3)
```

Sympy [A]
time = 0.27, size = 150, normalized size = 1.06

$$-\frac{8d^{3}(dg+ef)^{2}\log\left(-d+ex\right)}{e^{3}}-\frac{e^{2}g^{2}x^{5}}{5}-x^{4}\left(deg^{2}+\frac{e^{2}fg}{2}\right)-x^{3}\cdot\left(\frac{7d^{2}g^{2}}{3}+\frac{8defg}{3}+\frac{e^{2}f^{2}}{3}\right)-x^{2}\cdot\left(\frac{4d^{3}g^{2}}{e}+7d^{2}fg+2def^{2}\right)-x\left(\frac{8d^{4}g^{2}}{e^{2}}+\frac{16d^{3}fg}{e}+7d^{2}f^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -8*d**3*(d*g + e*f)**2*log(-d + e*x)/e**3 - e**2*g**2*x**5/5 - x**4*(d*e*g*
*2 + e**2*f*g/2) - x**3*(7*d**2*g**2/3 + 8*d*e*f*g/3 + e**2*f**2/3) - x**2*
(4*d**3*g**2/e + 7*d**2*f*g + 2*d*e*f**2) - x*(8*d**4*g**2/e**2 + 16*d**3*f
*g/e + 7*d**2*f**2)
```

Giac [A]
time = 1.07, size = 177, normalized size = 1.26

$$-8\left(d^{5}g^{2}+2\,d^{4}fge+d^{3}f^{2}e^{2}\right)e^{(-3)}\log\left(|xe-d|\right)-\frac{1}{30}\left(6\,g^{2}x^{5}e^{7}+30\,dg^{2}x^{4}e^{6}+70\,d^{2}g^{2}x^{3}e^{5}+120\,d^{3}g^{2}x^{2}e^{4}+240\,d^{4}g^{2}xe^{3}+15\,fgx^{4}e^{7}+80\,dfgx^{3}e^{6}+210\,d^{2}fgx^{2}e^{5}+480\,d^{3}fgxe^{4}+10\,f^{2}x^{3}e^{7}+60\,df^{2}x^{2}e^{6}+210\,d^{2}f^{2}xe^{5}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -8*(d^5*g^2 + 2*d^4*f*g*e + d^3*f^2*e^2)*e^(-3)*log(abs(x*e - d)) - 1/30*(6
*g^2*x^5*e^7 + 30*d*g^2*x^4*e^6 + 70*d^2*g^2*x^3*e^5 + 120*d^3*g^2*x^2*e^4
+ 240*d^4*g^2*x*e^3 + 15*f*g*x^4*e^7 + 80*d*f*g*x^3*e^6 + 210*d^2*f*g*x^2*e
^5 + 480*d^3*f*g*x*e^4 + 10*f^2*x^3*e^7 + 60*d*f^2*x^2*e^6 + 210*d^2*f^2*x*
e^5)*e^(-5)
```

Mupad [B]
time = 0.11, size = 351, normalized size = 2.49

$$-x^2\left(\frac{d^3g^2+6d^2efg+3de^2f^2}{2e}+\frac{d\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{e}+\frac{d\left(eg\left(3dg+2ef\right)+deg^2\right)}{e}\right)}{2e}\right)-x^3\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{3e}+\frac{d\left(eg\left(3dg+2ef\right)+deg^2\right)}{3e}\right)-x^4\left(\frac{eg\left(3dg+2ef\right)}{4}+\frac{deg^2}{4}\right)-x\left(\frac{d\left(\frac{d^3g^2+6d^2efg+3de^2f^2}{e}+\frac{d\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{e}+\frac{d\left(eg\left(3dg+2ef\right)+deg^2\right)}{e}\right)}{e}\right)}{e}+\frac{d^2f\left(2dg+3ef\right)}{e}\right)-\frac{\ln\left(ex-d\right)\left(8d^5g^2+16d^4efg+8d^3e^2f^2\right)}{e^3}-\frac{e^2g^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^2*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/(2*e) + (d*((e^3*f^2 + 3*d^2*e
*g^2 + 6*d*e^2*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/(2*e)) - x^
3*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/(3*e) + (d*(e*g*(3*d*g + 2*e*f) +
d*e*g^2))/(3*e)) - x^4*((e*g*(3*d*g + 2*e*f))/4 + (d*e*g^2)/4) - x*((d*((d^
3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e + (d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2
*f*g)/e + (d*(e*g*(3*d*g + 2*e*f) + d*e*g^2))/e))/e))/e + (d^2*f*(2*d*g + 3
*e*f))/e) - (log(e*x - d)*(8*d^5*g^2 + 8*d^3*e^2*f^2 + 16*d^4*e*f*g))/e^3 -
 (e^2*g^2*x^5)/5
```

3.549 $\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{d(ef+2dg)(3ef+2dg)x}{e^2}-\frac{(e^2f^2+6defg+4d^2g^2)\,x^2}{2e}-\frac{1}{3}g(2ef+3dg)x^3-\frac{1}{4}eg^2x^4-\frac{4d^2(ef+dg)^2\log(d-e$$

```
[Out] -d*(2*d*g+e*f)*(2*d*g+3*e*f)*x/e^2-1/2*(4*d^2*g^2+6*d*e*f*g+e^2*f^2)*x^2/e-
1/3*g*(3*d*g+2*e*f)*x^3-1/4*e*g^2*x^4-4*d^2*(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 90}

$$-\frac{4d^2(dg+ef)^2\log(d-ex)}{e^3}-\frac{x^2(4d^2g^2+6defg+e^2f^2)}{2e}-\frac{dx(2dg+ef)(2dg+3ef)}{e^2}-\frac{1}{3}gx^3(3dg+2ef)-\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((d*(e*f + 2*d*g)*(3*e*f + 2*d*g)*x)/e^2) - ((e^2*f^2 + 6*d*e*f*g + 4*d^2*
g^2)*x^2)/(2*e) - (g*(2*e*f + 3*d*g)*x^3)/3 - (e*g^2*x^4)/4 - (4*d^2*(e*f +
 d*g)^2*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(d+ex)^2(f+gx)^2}{d-ex}\,dx\\ &= \int \left(\frac{d(-3ef-2dg)(ef+2dg)}{e^2}-\frac{(e^2f^2+6defg+4d^2g^2)\,x}{e}-g(2ef+3dg)x^2-\right.\\ &= -\frac{d(ef+2dg)(3ef+2dg)x}{e^2}-\frac{(e^2f^2+6defg+4d^2g^2)\,x^2}{2e}-\frac{1}{3}g(2ef+3dg)x^3-\end{aligned}$$

Mathematica [A]
time = 0.04, size = 103, normalized size = 0.94

$$-\frac{ex(48d^3g^2+24d^2eg(4f+gx)+12de^2(3f^2+3fgx+g^2x^2)+e^3x(6f^2+8fgx+3g^2x^2))+48d^2(ef+dg)^2\log(d-ex)}{12e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/12*(e*x*(48*d^3*g^2 + 24*d^2*e*g*(4*f + g*x) + 12*d*e^2*(3*f^2 + 3*f*g*x
 + g^2*x^2) + e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2)) + 48*d^2*(e*f + d*g)^2*L
og[d - e*x])/e^3
```

Maple [A]
time = 0.08, size = 149, normalized size = 1.37

method	result
norman	$-\frac{e\,g^2x^4}{4}-\frac{g(3dg+2ef)x^3}{3}-\frac{(4d^2g^2+6defg+e^2f^2)x^2}{2e}-\frac{d(4d^2g^2+8defg+3e^2f^2)x}{e^2}-\frac{4d^2(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{e^3}$
risch	$-\frac{e\,g^2x^4}{4}-x^3d\,g^2-\frac{2e\,x^3fg}{3}-\frac{2x^2d^2g^2}{e}-3x^2dfg-\frac{e\,x^2f^2}{2}-\frac{4d^3g^2x}{e^2}-\frac{8d^2fgx}{e}-3d\,f^2x-\frac{4d^4\ln(-ex+d)g^2}{e^3}$
default	$-\frac{\frac{g^2e^3x^4}{4}+\frac{((2dg+ef)e^2g+eg(gde+e^2f))x^3}{3}+\frac{((2dg+ef)(gde+e^2f)+eg(2d^2g+3def))x^2}{2}+(2dg+ef)(2d^2g+3def)x}{e^2}-\frac{4d^2(d^2g^2+2def}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2*(1/4*g^2*e^3*x^4+1/3*((2*d*g+e*f)*e^2*g+e*g*(d*e*g+e^2*f))*x^3+1/2*(
(2*d*g+e*f)*(d*e*g+e^2*f)+e*g*(2*d^2*g+3*d*e*f))*x^2+(2*d*g+e*f)*(2*d^2*g+3
*d*e*f)*x)-4*d^2*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3*ln(-e*x+d)
```

Maxima [A]
time = 0.28, size = 133, normalized size = 1.22

$$-4\left(d^4g^2+2\,d^3fge+d^2f^2e^2\right)e^{(-3)}\log\left(xe-d\right)-\frac{1}{12}\left(3\,g^2x^4e^3+4\left(3\,dg^2e^2+2\,fge^3\right)x^3+6\left(4\,d^2g^2e+6\,dfge^2+f^2e^3\right)x^2+12\left(4\,d^3g^2+8\,d^2fge+3\,df^2e^2\right)x\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -4*(d^4*g^2 + 2*d^3*f*g*e + d^2*f^2*e^2)*e^(-3)*log(x*e - d) - 1/12*(3*g^2*
x^4*e^3 + 4*(3*d*g^2*e^2 + 2*f*g*e^3)*x^3 + 6*(4*d^2*g^2*e + 6*d*f*g*e^2 +
f^2*e^3)*x^2 + 12*(4*d^3*g^2 + 8*d^2*f*g*e + 3*d*f^2*e^2)*x)*e^(-2)
```

Fricas [A]
time = 2.31, size = 131, normalized size = 1.20

$$-\frac{1}{12}\left(48\,d^3g^2xe+\left(3\,g^2x^4+8\,fgx^3+6\,f^2x^2\right)e^4+12\left(dg^2x^3+3\,dfgx^2+3\,df^2x\right)e^3+24\left(d^2g^2x^2+4\,d^2fgx\right)e^2+48\left(d^4g^2+2\,d^3fge+d^2f^2e^2\right)\log\left(xe-d\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/12*(48*d^3*g^2*x*e + (3*g^2*x^4 + 8*f*g*x^3 + 6*f^2*x^2)*e^4 + 12*(d*g^2
*x^3 + 3*d*f*g*x^2 + 3*d*f^2*x)*e^3 + 24*(d^2*g^2*x^2 + 4*d^2*f*g*x)*e^2 +
48*(d^4*g^2 + 2*d^3*f*g*e + d^2*f^2*e^2)*log(x*e - d))*e^(-3)
```

Sympy [A]
time = 0.22, size = 109, normalized size = 1.00

$$-\frac{4d^2(dg+ef)^2\log{(-d+ex)}}{e^3}-\frac{eg^2x^4}{4}-x^3\left(dg^2+\frac{2efg}{3}\right)-x^2\cdot\left(\frac{2d^2g^2}{e}+3dfg+\frac{ef^2}{2}\right)-x\left(\frac{4d^3g^2}{e^2}+\frac{8d^2fg}{e}+3df^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -4*d**2*(d*g + e*f)**2*log(-d + e*x)/e**3 - e*g**2*x**4/4 - x**3*(d*g**2 +
2*e*f*g/3) - x**2*(2*d**2*g**2/e + 3*d*f*g + e*f**2/2) - x*(4*d**3*g**2/e**
2 + 8*d**2*f*g/e + 3*d*f**2)
```

Giac [A]
time = 1.12, size = 139, normalized size = 1.28

$$-4\left(d^4g^2+2d^3fge+d^2f^2e^2\right)e^{(-3)}\log\left(|xe-d|\right)-\frac{1}{12}\left(3g^2x^4e^5+12dg^2x^3e^4+24d^2g^2x^2e^3+48d^3g^2xe^2+8fgx^3e^5+36dfgx^2e^4+96d^2fgxe^3+6f^2x^2e^5+36df^2xe^4\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -4*(d^4*g^2 + 2*d^3*f*g*e + d^2*f^2*e^2)*e^(-3)*log(abs(x*e - d)) - 1/12*(3
*g^2*x^4*e^5 + 12*d*g^2*x^3*e^4 + 24*d^2*g^2*x^2*e^3 + 48*d^3*g^2*x*e^2 + 8
*f*g*x^3*e^5 + 36*d*f*g*x^2*e^4 + 96*d^2*f*g*x*e^3 + 6*f^2*x^2*e^5 + 36*d*f
^2*x*e^4)*e^(-4)
```

Mupad [B]
time = 2.59, size = 197, normalized size = 1.81

$$-x^3\left(\frac{2g\left(dg+ef\right)}{3}+\frac{dg^2}{3}\right)-x^2\left(\frac{d^2g^2+4defg+e^2f^2}{2e}+\frac{d\left(2g\left(dg+ef\right)+dg^2\right)}{2e}\right)-x\left(\frac{d\left(\frac{d^2g^2+4defg+e^2f^2}{e}+\frac{d\left(2g\left(dg+ef\right)+dg^2\right)}{e}\right)}{e}+\frac{2df\left(dg+ef\right)}{e}\right)-\frac{\ln\left(ex-d\right)\left(4d^4g^2+8d^3efg+4d^2e^2f^2\right)}{e^3}-\frac{eg^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^3*((2*g*(d*g + e*f))/3 + (d*g^2)/3) - x^2*((d^2*g^2 + e^2*f^2 + 4*d*e*f
*g)/(2*e) + (d*(2*g*(d*g + e*f) + d*g^2))/(2*e)) - x*((d*((d^2*g^2 + e^2*f^
2 + 4*d*e*f*g)/e + (d*(2*g*(d*g + e*f) + d*g^2))/e))/e + (2*d*f*(d*g + e*f)
)/e) - (log(e*x - d)*(4*d^4*g^2 + 4*d^2*e^2*f^2 + 8*d^3*e*f*g))/e^3 - (e*g^
2*x^4)/4
```

3.550 $\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{2dg(ef+dg)x}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}-\frac{2d(ef+dg)^2\log(d-ex)}{e^3}$$

```
[Out] -2*d*g*(d*g+e*f)*x/e^2-d*(g*x+f)^2/e-1/3*(g*x+f)^3/g-2*d*(d*g+e*f)^2*ln(-e*
x+d)/e^3
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 78}

$$-\frac{2d(dg+ef)^2\log(d-ex)}{e^3}-\frac{2dgx(dg+ef)}{e^2}-\frac{d(f+gx)^2}{e}-\frac{(f+gx)^3}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] (-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*(
e*f + d*g)^2*Log[d - e*x])/e^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2}\,dx = \int \frac{(d+ex)(f+gx)^2}{d-ex}\,dx$$
$$= \int \left(-\frac{2dg(ef+dg)}{e^2} - \frac{2d(ef+dg)^2}{e^2(-d+ex)} - \frac{2dg(f+gx)}{e} - (f+gx)^2\right) dx$$
$$= -\frac{2dg(ef+dg)x}{e^2} - \frac{d(f+gx)^2}{e} - \frac{(f+gx)^3}{3g} - \frac{2d(ef+dg)^2\log(d-ex)}{e^3}$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 1.12

$$-\frac{ex(6d^2g^2+3deg(4f+gx)+e^2(3f^2+3fgx+g^2x^2))+6d(ef+dg)^2\log(d-ex)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -1/3*(e*x*(6*d^2*g^2 + 3*d*e*g*(4*f + g*x) + e^2*(3*f^2 + 3*f*g*x + g^2*x^2
)) + 6*d*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Maple [A]
time = 0.07, size = 95, normalized size = 1.46

method	result	si
norman	$-\frac{g^2x^3}{3}-\frac{(2d^2g^2+4defg+e^2f^2)x}{e^2}-\frac{g(dg+ef)x^2}{e}-\frac{2d(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{e^3}$	88
default	$-\frac{\frac{1}{3}g^2x^3e^2+de\,g^2x^2+e^2fg\,x^2+2d^2g^2x+4defgx+e^2f^2x}{e^2}-\frac{2d(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{e^3}$	95
risch	$-\frac{g^2x^3}{3}-\frac{d\,g^2x^2}{e}-fg\,x^2-\frac{2d^2g^2x}{e^2}-\frac{4dfgx}{e}-f^2x-\frac{2d^3\ln(-ex+d)g^2}{e^3}-\frac{4d^2\ln(-ex+d)fg}{e^2}-\frac{2d\ln(-ex+d)f^2}{e}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2*(1/3*g^2*x^3*e^2+d*e*g^2*x^2+e^2*f*g*x^2+2*d^2*g^2*x+4*d*e*f*g*x+e^2
*f^2*x)-2*d*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3*ln(-e*x+d)
```

Maxima [A]
time = 0.30, size = 95, normalized size = 1.46

$$-2\left(d^3g^2+2\,d^2fge+df^2e^2\right)e^{(-3)}\log\left(xe-d\right)-\frac{1}{3}\left(g^2x^3e^2+3\left(dg^2e+fge^2\right)x^2+3\left(2\,d^2g^2+4\,dfge+f^2e^2\right)x\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -2*(d^3*g^2 + 2*d^2*f*g*e + d*f^2*e^2)*e^(-3)*log(x*e - d) - 1/3*(g^2*x^3*e
^2 + 3*(d*g^2*e + f*g*e^2)*x^2 + 3*(2*d^2*g^2 + 4*d*f*g*e + f^2*e^2)*x)*e^(
-2)
```

Fricas [A]
time = 2.56, size = 94, normalized size = 1.45

$$-\frac{1}{3}\left(6\,d^2g^2xe+\left(g^2x^3+3\,fgx^2+3\,f^2x\right)e^3+3\left(dg^2x^2+4\,dfgx\right)e^2+6\left(d^3g^2+2\,d^2fge+df^2e^2\right)\log\left(xe-d\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*d^2*g^2*x*e + (g^2*x^3 + 3*f*g*x^2 + 3*f^2*x)*e^3 + 3*(d*g^2*x^2 +
4*d*f*g*x)*e^2 + 6*(d^3*g^2 + 2*d^2*f*g*e + d*f^2*e^2)*log(x*e - d))*e^(-3)
```

Sympy [A]
time = 0.17, size = 70, normalized size = 1.08

$$-\frac{2d(dg+ef)^2\log\left(-d+ex\right)}{e^3}-\frac{g^2x^3}{3}-x^2\left(\frac{dg^2}{e}+fg\right)-x\left(\frac{2d^2g^2}{e^2}+\frac{4dfg}{e}+f^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -2*d*(d*g + e*f)**2*log(-d + e*x)/e**3 - g**2*x**3/3 - x**2*(d*g**2/e + f*g
) - x*(2*d**2*g**2/e**2 + 4*d*f*g/e + f**2)
```

Giac [A]
time = 0.80, size = 100, normalized size = 1.54

$$-2\left(d^3g^2+2\,d^2fge+df^2e^2\right)e^{(-3)}\log\left(|xe-d|\right)-\frac{1}{3}\left(g^2x^3e^3+3\,dg^2x^2e^2+6\,d^2g^2xe+3\,fgx^2e^3+12\,dfgxe^2+3\,f^2xe^3\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -2*(d^3*g^2 + 2*d^2*f*g*e + d*f^2*e^2)*e^(-3)*log(abs(x*e - d)) - 1/3*(g^2*
x^3*e^3 + 3*d*g^2*x^2*e^2 + 6*d^2*g^2*x*e + 3*f*g*x^2*e^3 + 12*d*f*g*x*e^2
+ 3*f^2*x*e^3)*e^(-3)
```

Mupad [B]
time = 0.07, size = 127, normalized size = 1.95

$$-x^2\left(\frac{d\,g^2+2\,e\,f\,g}{2\,e}+\frac{d\,g^2}{2\,e}\right)-x\left(\frac{e\,f^2+2\,d\,g\,f}{e}+\frac{d\left(\frac{d\,g^2+2\,e\,f\,g}{e}+\frac{d\,g^2}{e}\right)}{e}\right)-\frac{g^2\,x^3}{3}-\frac{\ln\left(e\,x-d\right)\,\left(2\,d^3\,g^2+4\,d^2\,e\,f\,g+2\,d\,e^2\,f^2\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2),x)
```

```
[Out] - x^2*((d*g^2 + 2*e*f*g)/(2*e) + (d*g^2)/(2*e)) - x*((e*f^2 + 2*d*f*g)/e +
(d*((d*g^2 + 2*e*f*g)/e + (d*g^2)/e))/e) - (g^2*x^3)/3 - (log(e*x - d)*(2*d
^3*g^2 + 2*d*e^2*f^2 + 4*d^2*e*f*g))/e^3
```

3.551 $\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=50

$$-\frac{g(ef+dg)x}{e^2}-\frac{(f+gx)^2}{2e}-\frac{(ef+dg)^2\log(d-ex)}{e^3}$$

```
[Out] -g*(d*g+e*f)*x/e^2-1/2*(g*x+f)^2/e-(d*g+e*f)^2*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {813, 45}

$$-\frac{(dg+ef)^2\log(d-ex)}{e^3}-\frac{gx(dg+ef)}{e^2}-\frac{(f+gx)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
```

```
[Out] -((g*(e*f + d*g)*x)/e^2) - (f + g*x)^2/(2*e) - ((e*f + d*g)^2*Log[d - e*x])
/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 813

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c/g)*x)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}\,dx &= \int \frac{(f+gx)^2}{d-ex}\,dx\\ &= \int \left(-\frac{g(ef+dg)}{e^2}+\frac{(ef+dg)^2}{e^2(d-ex)}-\frac{g(f+gx)}{e}\right)dx\\ &= -\frac{g(ef+dg)x}{e^2}-\frac{(f+gx)^2}{2e}-\frac{(ef+dg)^2\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.86

$$-\frac{egx(4ef+2dg+egx)+2(ef+dg)^2\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2),x]
[Out] -1/2*(e*g*x*(4*e*f + 2*d*g + e*g*x) + 2*(e*f + d*g)^2*Log[d - e*x])/e^3
```

Maple [A]
time = 0.08, size = 59, normalized size = 1.18

method	result	size
default	$-\frac{g\left(\frac{1}{2}eg\,x^2+dgx+2efx\right)}{e^2}+\frac{\left(-d^2g^2-2defg-e^2f^2\right)\ln\left(-ex+d\right)}{e^3}$	59
norman	$-\frac{g^2x^2}{2e}-\frac{g(dg+2ef)x}{e^2}-\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln\left(-ex+d\right)}{e^3}$	61
risch	$-\frac{g^2x^2}{2e}-\frac{g^2dx}{e^2}-\frac{2gfx}{e}-\frac{\ln(-ex+d)d^2g^2}{e^3}-\frac{2\ln(-ex+d)dfg}{e^2}-\frac{\ln(-ex+d)f^2}{e}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
[Out] -g/e^2*(1/2*e*g*x^2+d*g*x+2*e*f*x)+(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3*ln(-e*x
+d)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 1.28

$$-\left(d^2g^2+2\,dfge+f^2e^2\right)e^{(-3)}\log\left(xe-d\right)-\frac{1}{2}\left(g^2x^2e+2\left(dg^2+2\,fge\right)x\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
[Out] -(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e - d) - 1/2*(g^2*x^2*e + 2*(
d*g^2 + 2*f*g*e)*x)*e^(-2)
```

Fricas [A]
time = 2.06, size = 62, normalized size = 1.24

$$-\frac{1}{2}\left(2\,dg^2xe+\left(g^2x^2+4\,fgx\right)e^2+2\left(d^2g^2+2\,dfge+f^2e^2\right)\log\left(xe-d\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*g^2*x*e + (g^2*x^2 + 4*f*g*x)*e^2 + 2*(d^2*g^2 + 2*d*f*g*e + f^2*
e^2)*log(x*e - d))*e^(-3)
```

Sympy [A]
time = 0.11, size = 46, normalized size = 0.92

$$-x\left(\frac{dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{g^2x^2}{2e}-\frac{(dg+ef)^2\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -x*(d*g**2/e**2 + 2*f*g/e) - g**2*x**2/(2*e) - (d*g + e*f)**2*log(-d + e*x)
/e**3
```

Giac [A]
time = 1.35, size = 64, normalized size = 1.28

$$-\left(d^2g^2+2\,dfge+f^2e^2\right)e^{(-3)}\log\left(|xe-d|\right)-\frac{1}{2}\left(g^2x^2e+2\,dg^2x+4\,fgxe\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e - d)) - 1/2*(g^2*x^2*e
+ 2*d*g^2*x + 4*f*g*x*e)*e^(-2)
```

Mupad [B]
time = 2.61, size = 65, normalized size = 1.30

$$-x\left(\frac{d\,g^2}{e^2}+\frac{2\,f\,g}{e}\right)-\frac{\ln\left(e\,x-d\right)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{e^3}-\frac{g^2\,x^2}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2),x)
```

```
[Out] - x*((d*g^2)/e^2 + (2*f*g)/e) - (log(e*x - d)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*
g))/e^3 - (g^2*x^2)/(2*e)
```

3.552 $\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx$

Optimal. Leaf size=62

$$-\frac{g^2x}{e^2}-\frac{(ef+dg)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}$$

```
[Out] -g^2*x/e^2-1/2*(d*g+e*f)^2*ln(-e*x+d)/d/e^3+1/2*(-d*g+e*f)^2*ln(e*x+d)/d/e^
3
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {716, 647, 31}

$$-\frac{(dg+ef)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}-\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] -((g^2*x)/e^2) - ((e*f + d*g)^2*Log[d - e*x])/(2*d*e^3) + ((e*f - d*g)^2*Lo
g[d + e*x])/(2*d*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\int \frac{(f+gx)^2}{d^2-e^2x^2}\,dx = \int \left(-\frac{g^2}{e^2}+\frac{e^2f^2+d^2g^2+2e^2fgx}{e^2(d^2-e^2x^2)}\right)dx$$
$$= -\frac{g^2x}{e^2}+\frac{\int \frac{e^2f^2+d^2g^2+2e^2fgx}{d^2-e^2x^2}\,dx}{e^2}$$
$$= -\frac{g^2x}{e^2}-\frac{(ef-dg)^2\int \frac{1}{-de-e^2x}\,dx}{2de}+\frac{(ef+dg)^2\int \frac{1}{de-e^2x}\,dx}{2de}$$
$$= -\frac{g^2x}{e^2}-\frac{(ef+dg)^2\log(d-ex)}{2de^3}+\frac{(ef-dg)^2\log(d+ex)}{2de^3}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.89

$$\frac{(e^2f^2+d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)-deg(gx+f\log(d^2-e^2x^2))}{de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2),x]
```

```
[Out] ((e^2*f^2 + d^2*g^2)*ArcTanh[(e*x)/d] - d*e*g*(g*x + f*Log[d^2 - e^2*x^2]))
/(d*e^3)
```

Maple [A]
time = 0.09, size = 84, normalized size = 1.35

method	result	size
norman	$-\frac{g^2x}{e^2}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{2e^3d}-\frac{(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{2d\,e^3}$	82
default	$-\frac{g^2x}{e^2}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{2e^3d}+\frac{(-d^2g^2-2defg-e^2f^2)\ln(-ex+d)}{2d\,e^3}$	84
risch	$-\frac{g^2x}{e^2}-\frac{d\ln(ex-d)g^2}{2e^3}-\frac{\ln(ex-d)fg}{e^2}-\frac{\ln(ex-d)f^2}{2ed}+\frac{d\ln(-ex-d)g^2}{2e^3}-\frac{\ln(-ex-d)fg}{e^2}+\frac{\ln(-ex-d)f^2}{2ed}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -g^2*x/e^2+1/2/e^3*(d^2*g^2-2*d*e*f*g+e^2*f^2)/d*ln(e*x+d)+1/2*(-d^2*g^2-2*
d*e*f*g-e^2*f^2)/d/e^3*ln(-e*x+d)
```

Maxima [A]
time = 0.28, size = 81, normalized size = 1.31

$$-g^2xe^{(-2)}+\frac{(d^2g^2-2\,dfge+f^2e^2)e^{(-3)}\log(xe+d)}{2\,d}-\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log(xe-d)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -g^2*x*e^(-2) + 1/2*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e + d)/d -
 1/2*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e - d)/d
```

Fricas [A]
time = 1.80, size = 87, normalized size = 1.40

$$-\frac{\left(2\,dg^2xe^2+2\,dfge^2\log\left(x^2e^2-d^2\right)-\left(d^2g^2+f^2e^2\right)e\log\left(\frac{x^2e^2+2\,dxe+d^2}{x^2e^2-d^2}\right)\right)e^{(-4)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*g^2*x*e^2 + 2*d*f*g*e^2*log(x^2*e^2 - d^2) - (d^2*g^2 + f^2*e^2)*
e*log((x^2*e^2 + 2*d*x*e + d^2)/(x^2*e^2 - d^2)))*e^(-4)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.
time = 0.29, size = 112, normalized size = 1.81

$$-\frac{g^2x}{e^2}+\frac{(dg-ef)^2\log\left(x+\frac{2d^2fg+\frac{d(dg-ef)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}-\frac{(dg+ef)^2\log\left(x+\frac{2d^2fg-\frac{d(dg+ef)^2}{e}}{d^2g^2+e^2f^2}\right)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -g**2*x/e**2 + (d*g - e*f)**2*log(x + (2*d**2*f*g + d*(d*g - e*f)**2/e)/(d*
*2*g**2 + e**2*f**2))/(2*d*e**3) - (d*g + e*f)**2*log(x + (2*d**2*f*g - d*(
d*g + e*f)**2/e)/(d**2*g**2 + e**2*f**2))/(2*d*e**3)
```

Giac [A]
time = 0.92, size = 81, normalized size = 1.31

$$-g^2xe^{(-2)}-fge^{(-2)}\log\left(\left|x^2e^2-d^2\right|\right)-\frac{\left(d^2g^2+f^2e^2\right)e^{(-3)}\log\left(\frac{\left|2\,xe^2-2\,|d|e\right|}{\left|2\,xe^2+2\,|d|e\right|}\right)}{2\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -g^2*x*e^(-2) - f*g*e^(-2)*log(abs(x^2*e^2 - d^2)) - 1/2*(d^2*g^2 + f^2*e^2
)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

Mupad [B]
time = 0.15, size = 81, normalized size = 1.31

$$\frac{\ln (d+e x) \left(d^2 g^2-2 d e f g+e^2 f^2\right)}{2 d e^3}-\frac{g^2 x}{e^2}-\frac{\ln (d-e x) \left(d^2 g^2+2 d e f g+e^2 f^2\right)}{2 d e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2),x)
[Out] (log(d + e*x)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))/(2*d*e^3) - (g^2*x)/e^2 - (l
og(d - e*x)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(2*d*e^3)
```

3.553 $\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)} \, dx$

Optimal. Leaf size=86

$$-\frac{(ef-dg)^2}{2de^3(d+ex)}-\frac{(ef+dg)^2\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)(ef+3dg)\log(d+ex)}{4d^2e^3}$$

```
[Out] -1/2*(-d*g+e*f)^2/d/e^3/(e*x+d)-1/4*(d*g+e*f)^2*ln(-e*x+d)/d^2/e^3+1/4*(-d*
g+e*f)*(3*d*g+e*f)*ln(e*x+d)/d^2/e^3
```

Rubi [A]

time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 90}

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3}-\frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}-\frac{(ef-dg)^2}{2de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/2*(e*f - d*g)^2/(d*e^3*(d + e*x)) - ((e*f + d*g)^2*Log[d - e*x])/(4*d^2*
e^3) + ((e*f - d*g)*(e*f + 3*d*g)*Log[d + e*x])/(4*d^2*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)} \, dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^2} \, dx \\
&= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)}+\frac{(-ef+dg)^2}{2de^2(d+ex)^2}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)}\right) dx \\
&= -\frac{(ef-dg)^2}{2de^3(d+ex)}-\frac{(ef+dg)^2\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)(ef+3dg)\log(d+ex)}{4d^2e^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 0.95

$$\frac{-(ef+dg)^2(d+ex)\log(d-ex)+(ef-dg)(2d(-ef+dg)+(ef+3dg)(d+ex)\log(d+ex))}{4d^2e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)),x]
```

```
[Out] (-((e*f + d*g)^2*(d + e*x)*Log[d - e*x]) + (e*f - d*g)*(2*d*(-(e*f) + d*g)
+ (e*f + 3*d*g)*(d + e*x)*Log[d + e*x]))/(4*d^2*e^3*(d + e*x))
```

Maple [A]
time = 0.08, size = 112, normalized size = 1.30

method	result
default	$\frac{(-3d^2g^2+2defg+e^2f^2)\ln(ex+d)}{4d^2e^3}-\frac{d^2g^2-2defg+e^2f^2}{2e^3d(ex+d)}+\frac{(-d^2g^2-2defg-e^2f^2)\ln(-ex+d)}{4d^2e^3}$
norman	$\frac{-d^2g^2+2defg-e^2f^2}{2d\,e^3(ex+d)}-\frac{(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{4e^3d^2}-\frac{(3d^2g^2-2defg-e^2f^2)\ln(ex+d)}{4e^3d^2}$
risch	$-\frac{d\,g^2}{2e^3(ex+d)}+\frac{fg}{e^2(ex+d)}-\frac{f^2}{2ed(ex+d)}-\frac{3\ln(-ex-d)g^2}{4e^3}+\frac{\ln(-ex-d)fg}{2e^2d}+\frac{\ln(-ex-d)f^2}{4e\,d^2}-\frac{\ln(ex-d)g^2}{4e^3}-\frac{\ln(ex-d)fg}{2e^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-3*d^2*g^2+2*d*e*f*g+e^2*f^2)/d^2/e^3*ln(e*x+d)-1/2/e^3*(d^2*g^2-2*d*e
*f*g+e^2*f^2)/d/(e*x+d)+1/4*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/d^2/e^3*ln(-e*x+d)
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.30

$$-\frac{(3\,d^2g^2-2\,dfge-f^2e^2)e^{(-3)}\log(xe+d)}{4\,d^2}-\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log(xe-d)}{4\,d^2}-\frac{d^2g^2-2\,dfge+f^2e^2}{2\,(dxe^4+d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*d^2*g^2 - 2*d*f*g*e - f^2*e^2)*e^(-3)*log(x*e + d)/d^2 - 1/4*(d^2*g
^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e - d)/d^2 - 1/2*(d^2*g^2 - 2*d*f*g*
e + f^2*e^2)/(d*x*e^4 + d^2*e^3)
```

Fricas [A]
time = 1.73, size = 164, normalized size = 1.91

$$-\frac{2\,d^3g^2-4\,d^2fge+2\,df^2e^2+(3\,d^3g^2-f^2xe^3-(2\,dfgx+df^2)e^2+(3\,d^2g^2x-2\,d^2fg)e)\log(xe+d)+(d^3g^2+f^2xe^3+(2\,dfgx+df^2)e^2+(d^2g^2x+2\,d^2fg)e)\log(xe-d)}{4\,(d^2xe^4+d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^3*g^2 - 4*d^2*f*g*e + 2*d*f^2*e^2 + (3*d^3*g^2 - f^2*x*e^3 - (2*d
*f*g*x + d*f^2)*e^2 + (3*d^2*g^2*x - 2*d^2*f*g)*e)*log(x*e + d) + (d^3*g^2
+ f^2*x*e^3 + (2*d*f*g*x + d*f^2)*e^2 + (d^2*g^2*x + 2*d^2*f*g)*e)*log(x*e
- d))/(d^2*x*e^4 + d^3*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(75) = 150$.
time = 0.51, size = 182, normalized size = 2.12

$$-\frac{d^2g^2-2defg+e^2f^2}{2d^2e^3+2de^4x}-\frac{(dg-ef)\,(3dg+ef)\log\left(x+\frac{-2d^3g^2+d(dg-ef)(3dg+ef)}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3}-\frac{(dg+ef)^2\log\left(x+\frac{-2d^3g^2+d(dg+ef)^2}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2),x)
```

```
[Out] -(d**2*g**2 - 2*d*e*f*g + e**2*f**2)/(2*d**2*e**3 + 2*d*e**4*x) - (d*g - e*
f)*(3*d*g + e*f)*log(x + (-2*d**3*g**2 + d*(d*g - e*f)*(3*d*g + e*f))/(d**2
*e*g**2 - 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3) - (d*g + e*f)**2*log(x +
 (-2*d**3*g**2 + d*(d*g + e*f)**2)/(d**2*e*g**2 - 2*d*e**2*f*g - e**3*f**2)
)/(4*d**2*e**3)
```

Giac [A]
time = 1.01, size = 116, normalized size = 1.35

$$-\frac{(3\,d^2g^2-2\,dfge-f^2e^2)e^{(-3)}\log\left(|xe+d|\right)}{4\,d^2}-\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log\left(|xe-d|\right)}{4\,d^2}-\frac{(d^3g^2-2\,d^2fge+df^2e^2)e^{(-3)}}{2\,(xe+d)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] -1/4*(3*d^2*g^2 - 2*d*f*g*e - f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^2 - 1/4*(
d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^2 - 1/2*(d^3*g^2
- 2*d^2*f*g*e + d*f^2*e^2)*e^(-3)/((x*e + d)*d^2)
```

Mupad [B]
time = 2.70, size = 109, normalized size = 1.27

$$\frac{\ln\left(d+e\,x\right)\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}-\frac{\ln\left(d-e\,x\right)\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^3}-\frac{d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2}{2\,d\,e^3\,\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d^2*e^3) - (log(d - e*x
)*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^3) - (d^2*g^2 + e^2*f^2 - 2*d*e
*f*g)/(2*d*e^3*(d + e*x))
```

3.554 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)}\,dx$

Optimal. Leaf size=87

$$-\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

```
[Out] -1/4*(-d*g+e*f)^2/d/e^3/(e*x+d)^2-1/4*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+d
)+1/4*(d*g+e*f)^2*arctanh(e*x/d)/d^3/e^3
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}-\frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)}-\frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/4*(e*f - d*g)^2/(d*e^3*(d + e*x)^2) - ((e*f - d*g)*(e*f + 3*d*g))/(4*d^2
*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^2\,(d^2-e^2x^2)}\,dx &= \int \frac{(f+gx)^2}{(d-ex)(d+ex)^3}\,dx \\
&= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^3}+\frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^2}+\frac{(ef+dg)^2}{4d^2e^2\,(d^2-e^2x^2)}\right)dx \\
&= -\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2} \\
&= -\frac{(ef-dg)^2}{4de^3(d+ex)^2}-\frac{(ef-dg)(ef+3dg)}{4d^2e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 87, normalized size = 1.00

$$
\frac{\frac{2d(-ef+dg)(2d^2g+e^2fx+de(2f+3gx))}{(d+ex)^2}-(ef+dg)^2\log(d-ex)+(ef+dg)^2\log(d+ex)}{8d^3e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)),x]
```

```
[Out] ((2*d*(-(e*f) + d*g)*(2*d^2*g + e^2*f*x + d*e*(2*f + 3*g*x)))/(d + e*x)^2 -
 (e*f + d*g)^2*Log[d - e*x] + (e*f + d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

Maple [A]
time = 0.09, size = 148, normalized size = 1.70

method	result
norman	$\frac{\frac{d^2g^2-e^2f^2}{2d\,e^3}+\frac{\left(3d^2g^2-2defg-e^2f^2\right)x}{4d^2e^2}}{\left(ex+d\right)^2}-\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(-ex+d)}{8e^3d^3}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(ex+d)}{8e^3d^3}$
default	$-\frac{-3d^2g^2+2defg+e^2f^2}{4d^2e^3(ex+d)}-\frac{d^2g^2-2defg+e^2f^2}{4e^3d(ex+d)^2}+\frac{\left(d^2g^2+2defg+e^2f^2\right)\ln(ex+d)}{8e^3d^3}+\frac{\left(-d^2g^2-2defg-e^2f^2\right)\ln(-ex+d)}{8e^3d^3}$
risch	$\frac{\frac{d^2g^2-e^2f^2}{2d\,e^3}+\frac{\left(3d^2g^2-2defg-e^2f^2\right)x}{4d^2e^2}}{\left(ex+d\right)^2}-\frac{\ln(-ex+d)g^2}{8e^3d}-\frac{\ln(-ex+d)fg}{4e^2d^2}-\frac{\ln(-ex+d)f^2}{8e\,d^3}+\frac{\ln(ex+d)g^2}{8e^3d}+\frac{\ln(ex+d)fg}{4e^2d^2}+$ ln(e

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-3*d^2*g^2+2*d*e*f*g+e^2*f^2)/d^2/e^3/(e*x+d)-1/4/e^3*(d^2*g^2-2*d*e*
f*g+e^2*f^2)/d/(e*x+d)^2+1/8*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^3*ln(e*x+d)+
1/8*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3/d^3*ln(-e*x+d)
```

Maxima [A]
time = 0.29, size = 145, normalized size = 1.67

$$
\frac{2\,d^3g^2-2\,df^2e^2+(3\,d^2g^2e-2\,dfge^2-f^2e^3)x}{4\,(d^2x^2e^5+2\,d^3xe^4+d^4e^3)}+\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log(xe+d)}{8\,d^3}-\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log(xe-d)}{8\,d^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^3*g^2 - 2*d*f^2*e^2 + (3*d^2*g^2*e - 2*d*f*g*e^2 - f^2*e^3)*x)/(d^
2*x^2*e^5 + 2*d^3*x*e^4 + d^4*e^3) + 1/8*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^
(-3)*log(x*e + d)/d^3 - 1/8*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e
- d)/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(85) = 170$.
time = 2.93, size = 261, normalized size = 3.00

$$\frac{6\,d^3g^2xe+4\,d^4g^2-2\,df^2xe^3-4\,(d^2fgx+d^2f^2)e^2+(d^4g^2+f^2x^2e^4+2\,(dfgx^2+df^2x)e^3+(d^2g^2x^2+4\,d^2fgx+d^2f^2)e^2+2\,(d^3g^2x+d^3fg)e)\log\left(xe+d\right)-(d^4g^2+f^2x^2e^4+2\,(dfgx^2+df^2x)e^3+(d^2g^2x^2+4\,d^2fgx+d^2f^2)e^2+2\,(d^3g^2x+d^3fg)e)\log\left(xe-d\right)}{8\,(d^3x^2e^5+2\,d^4xe^4+d^5e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] 1/8*(6*d^3*g^2*x*e + 4*d^4*g^2 - 2*d*f^2*x*e^3 - 4*(d^2*f*g*x + d^2*f^2)*e^
2 + (d^4*g^2 + f^2*x^2*e^4 + 2*(d*f*g*x^2 + d*f^2*x)*e^3 + (d^2*g^2*x^2 + 4
*d^2*f*g*x + d^2*f^2)*e^2 + 2*(d^3*g^2*x + d^3*f*g)*e)*log(x*e + d) - (d^4*
g^2 + f^2*x^2*e^4 + 2*(d*f*g*x^2 + d*f^2*x)*e^3 + (d^2*g^2*x^2 + 4*d^2*f*g*
x + d^2*f^2)*e^2 + 2*(d^3*g^2*x + d^3*f*g)*e)*log(x*e - d))/(d^3*x^2*e^5 +
2*d^4*x*e^4 + d^5*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(75) = 150$.
time = 0.49, size = 185, normalized size = 2.13

$$-\frac{-2d^3g^2+2de^2f^2+x(-3d^2eg^2+2de^2fg+e^3f^2)}{4d^4e^3+8d^3e^4x+4d^2e^5x^2}-\frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{8d^3e^3}+\frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{8d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**3*g**2 + 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 + 2*d*e**2*f*g + e**3*f*
*2))/(4*d**4*e**3 + 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g + e*f)**2*log(
-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3
) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f*
*2)) + x)/(8*d**3*e**3)
```

Giac [A]
time = 1.28, size = 151, normalized size = 1.74

$$\frac{\left(\frac{3\,d^2g^2e^3}{xe+d}-\frac{d^3g^2e^3}{(xe+d)^2}-\frac{2\,dfge^4}{xe+d}+\frac{2\,d^2fge^4}{(xe+d)^2}-\frac{f^2e^5}{xe+d}-\frac{df^2e^5}{(xe+d)^2}\right)e^{(-6)}}{4\,d^2}-\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log\left(\left|-\frac{2\,d}{xe+d}+1\right|\right)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/4*(3*d^2*g^2*e^3/(x*e + d) - d^3*g^2*e^3/(x*e + d)^2 - 2*d*f*g*e^4/(x*e +
 d) + 2*d^2*f*g*e^4/(x*e + d)^2 - f^2*e^5/(x*e + d) - d*f^2*e^5/(x*e + d)^2
)*e^(-6)/d^2 - 1/8*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(-2*d/(x*e
 + d) + 1))/d^3
```

Mupad [B]
time = 0.13, size = 100, normalized size = 1.15

$$\frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,\left(-3\,d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^2\,e^2}}{d^2+2\,d\,e\,x+e^2\,x^2}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{4\,d^3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - e^2*f^2)/(2*d*e^3) - (x*(e^2*f^2 - 3*d^2*g^2 + 2*d*e*f*g))/(4*d
^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*x) + (atanh((e*x)/d)*(d*g + e*f)^2)/(4*d^3*
e^3)
```

3.555 $\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx$

Optimal. Leaf size=113

$$-\frac{(ef-dg)^2}{6de^3(d+ex)^3}-\frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2}-\frac{(ef+dg)^2}{8d^3e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

```
[Out] -1/6*(-d*g+e*f)^2/d/e^3/(e*x+d)^3-1/8*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+d
)^2-1/8*(d*g+e*f)^2/d^3/e^3/(e*x+d)+1/8*(d*g+e*f)^2*arctanh(e*x/d)/d^4/e^3
```

Rubi [A]
time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(dg+ef)^2}{8d^3e^3(d+ex)}-\frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2}-\frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/6*(e*f - d*g)^2/(d*e^3*(d + e*x)^3) - ((e*f - d*g)*(e*f + 3*d*g))/(8*d^2
*e^3*(d + e*x)^2) - (e*f + d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f + d*g)^2*Ar
cTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)}\,dx = \int \frac{(f+gx)^2}{(d-ex)(d+ex)^4}\,dx$$
$$= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^4} + \frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^3} + \frac{(ef+dg)^2}{8d^3e^2(d+ex)^2} + \frac{(ef+dg)}{8d^3e^2(d^2-e\ldots}\right.$$
$$= -\frac{(ef-dg)^2}{6de^3(d+ex)^3} - \frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2} - \frac{(ef+dg)^2}{8d^3e^3(d+ex)} + \frac{(ef+dg)^2\int\frac{\ldots}{d^2-\ldots}}{8d^3e^2}$$
$$= -\frac{(ef-dg)^2}{6de^3(d+ex)^3} - \frac{(ef-dg)(ef+3dg)}{8d^2e^3(d+ex)^2} - \frac{(ef+dg)^2}{8d^3e^3(d+ex)} + \frac{(ef+dg)^2\tanh\ldots}{8d^4e^3}$$

Mathematica [A]

time = 0.04, size = 122, normalized size = 1.08

$$\frac{-\frac{8d^3(ef-dg)^2}{(d+ex)^3} + \frac{6d^2(-e^2f^2-2defg+3d^2g^2)}{(d+ex)^2} - \frac{6d(ef+dg)^2}{d+ex} - 3(ef+dg)^2\log(d-ex) + 3(ef+dg)^2\log(d+ex)}{48d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)),x]
```

```
[Out] ((-8*d^3*(e*f - d*g)^2)/(d + e*x)^3 + (6*d^2*(-(e^2*f^2) - 2*d*e*f*g + 3*d^
2*g^2))/(d + e*x)^2 - (6*d*(e*f + d*g)^2)/(d + e*x) - 3*(e*f + d*g)^2*Log[d
 - e*x] + 3*(e*f + d*g)^2*Log[d + e*x])/(48*d^4*e^3)
```

Maple [A]

time = 0.09, size = 184, normalized size = 1.63

method	result
norman	$\frac{-\frac{(d^2g^2-2defg-5e^2f^2)x^3}{12d^4}-\frac{(d^2g^2+2defg-7e^2f^2)x}{8d^2e^2}-\frac{(3d^2g^2-2defg-9e^2f^2)x^2}{8d^3e}}{(ex+d)^3} - \frac{(d^2g^2+2defg+e^2f^2)\ln(-ex+d)}{16e^3d^4} + \frac{(d^2g^2+2defg\ldots}{1\ldots}$
default	$-\frac{-3d^2g^2+2defg+e^2f^2}{8d^2e^3(ex+d)^2} - \frac{d^2g^2-2defg+e^2f^2}{6e^3d(ex+d)^3} + \frac{(d^2g^2+2defg+e^2f^2)\ln(ex+d)}{16e^3d^4} - \frac{d^2g^2+2defg+e^2f^2}{8e^3d^3(ex+d)} + \frac{(-d^2g^2-2defg-e\ldots}{16e^3\ldots}$
risch	$\frac{-\frac{(d^2g^2+2defg+e^2f^2)x^2}{8d^3e}+\frac{(d^2g^2-6defg-3e^2f^2)x}{8d^2e^2}+\frac{d^2g^2-2defg-5e^2f^2}{12de^3}}{(ex+d)^3} - \frac{\ln(-ex+d)g^2}{16e^3d^2} - \frac{\ln(-ex+d)fg}{8e^2d^3} - \frac{\ln(-ex+d)f^2}{16e\,d^4} + \frac{\ln(\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-3*d^2*g^2+2*d*e*f*g+e^2*f^2)/d^2/e^3/(e*x+d)^2-1/6/e^3*(d^2*g^2-2*d*
e*f*g+e^2*f^2)/d/(e*x+d)^3+1/16*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^4*ln(e*x+
d)-1/8*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^3/(e*x+d)+1/16*(-d^2*g^2-2*d*e*f*g
-e^2*f^2)/e^3/d^4*ln(-e*x+d)
```

Maxima [A]
time = 0.29, size = 197, normalized size = 1.74

$$\frac{2\,d^4g^2-4\,d^3fge-10\,d^2f^2e^2-3\,(d^2g^2e^2+2\,dfge^3+f^2e^4)x^2+3\,(d^3g^2e-6\,d^2fge^2-3\,df^2e^3)x}{24\,(d^3x^3e^6+3\,d^4x^2e^5+3\,d^5xe^4+d^6e^3)}+\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log{(xe+d)}}{16\,d^4}-\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log{(xe-d)}}{16\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] 1/24*(2*d^4*g^2 - 4*d^3*f*g*e - 10*d^2*f^2*e^2 - 3*(d^2*g^2*e^2 + 2*d*f*g*e
^3 + f^2*e^4)*x^2 + 3*(d^3*g^2*e - 6*d^2*f*g*e^2 - 3*d*f^2*e^3)*x)/(d^3*x^3
*e^6 + 3*d^4*x^2*e^5 + 3*d^5*x*e^4 + d^6*e^3) + 1/16*(d^2*g^2 + 2*d*f*g*e +
 f^2*e^2)*e^(-3)*log(x*e + d)/d^4 - 1/16*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^
(-3)*log(x*e - d)/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(110) = 220$.
time = 2.29, size = 397, normalized size = 3.51

$$\frac{4\,d^5g^2-6\,df^2x^2e^4-6\,(2\,d^2fgx^2+3\,d^2f^2x)e^3-2\,(3\,d^3g^2x^2+18\,d^3fgx+10\,d^3f^2)e^2+2\,(3\,d^4g^2x-4\,d^4fg)e+3\,(d^5g^2+f^2x^3e^5+(2\,dfgx^3+3\,df^2x^2)e^4+(d^2g^2x^3+6\,d^2fgx^2+3\,d^2f^2x)e^3+(3\,d^3g^2x^2+6\,d^3fgx+d^3f^2)e^2+(3\,d^4g^2x+2\,d^4fg)e)\log{(xe+d)}-3\,(d^5g^2+f^2x^3e^5+(2\,dfgx^3+3\,df^2x^2)e^4+(d^2g^2x^3+6\,d^2fgx^2+3\,d^2f^2x)e^3+(3\,d^3g^2x^2+6\,d^3fgx+d^3f^2)e^2+(3\,d^4g^2x+2\,d^4fg)e)\log{(xe-d)}}{48\,(d^4x^3e^6+3\,d^5x^2e^5+3\,d^6xe^4+d^7e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] 1/48*(4*d^5*g^2 - 6*d*f^2*x^2*e^4 - 6*(2*d^2*f*g*x^2 + 3*d^2*f^2*x)*e^3 - 2
*(3*d^3*g^2*x^2 + 18*d^3*f*g*x + 10*d^3*f^2)*e^2 + 2*(3*d^4*g^2*x - 4*d^4*f
*g)*e + 3*(d^5*g^2 + f^2*x^3*e^5 + (2*d*f*g*x^3 + 3*d*f^2*x^2)*e^4 + (d^2*g
^2*x^3 + 6*d^2*f*g*x^2 + 3*d^2*f^2*x)*e^3 + (3*d^3*g^2*x^2 + 6*d^3*f*g*x +
d^3*f^2)*e^2 + (3*d^4*g^2*x + 2*d^4*f*g)*e)*log(x*e + d) - 3*(d^5*g^2 + f^2
*x^3*e^5 + (2*d*f*g*x^3 + 3*d*f^2*x^2)*e^4 + (d^2*g^2*x^3 + 6*d^2*f*g*x^2 +
 3*d^2*f^2*x)*e^3 + (3*d^3*g^2*x^2 + 6*d^3*f*g*x + d^3*f^2)*e^2 + (3*d^4*g^
2*x + 2*d^4*f*g)*e)*log(x*e - d))/(d^4*x^3*e^6 + 3*d^5*x^2*e^5 + 3*d^6*x*e^
4 + d^7*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(99) = 198$.
time = 0.63, size = 248, normalized size = 2.19

$$-\frac{-2d^4g^2+4d^3efg+10d^2e^2f^2+x^2\cdot(3d^2e^2g^2+6de^3fg+3e^4f^2)+x(-3d^3eg^2+18d^2e^2fg+9de^3f^2)}{24d^6e^3+72d^5e^4x+72d^4e^5x^2+24d^3e^6x^3}-\frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{16d^4e^3}+\frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{16d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2),x)
```

```
[Out] -(-2*d**4*g**2 + 4*d**3*e*f*g + 10*d**2*e**2*f**2 + x**2*(3*d**2*e**2*g**2
+ 6*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 18*d**2*e**2*f*g + 9*d*
e**3*f**2))/(24*d**6*e**3 + 72*d**5*e**4*x + 72*d**4*e**5*x**2 + 24*d**3*e*
```

```
*6*x**3) - (d*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g +
 e**2*f**2)) + x)/(16*d**4*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(
d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(16*d**4*e**3)
```

Giac [A]
time = 1.39, size = 181, normalized size = 1.60

$$\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log\left(|xe+d|\right)}{16\,d^4}-\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}\log\left(|xe-d|\right)}{16\,d^4}+\frac{(2\,d^5g^2-4\,d^4fge-10\,d^3f^2e^2-3\,(d^3g^2e^2+2\,d^2fge^3+df^2e^4)x^2+3\,(d^4g^2e-6\,d^3fge^2-3\,d^2f^2e^3)x)e^{(-3)}}{24\,(xe+d)^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/16*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^4 - 1/16*(d
^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^4 + 1/24*(2*d^5*g^
2 - 4*d^4*f*g*e - 10*d^3*f^2*e^2 - 3*(d^3*g^2*e^2 + 2*d^2*f*g*e^3 + d*f^2*e
^4)*x^2 + 3*(d^4*g^2*e - 6*d^3*f*g*e^2 - 3*d^2*f^2*e^3)*x)*e^(-3)/((x*e + d
)^3*d^4)
```

Mupad [B]
time = 2.65, size = 152, normalized size = 1.35

$$\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{8\,d^4\,e^3}-\frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2}{12\,d\,e^3}+\frac{x\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{8\,d^3\,e}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^3),x)
```

```
[Out] (atanh((e*x)/d)*(d*g + e*f)^2)/(8*d^4*e^3) - ((5*e^2*f^2 - d^2*g^2 + 2*d*e*
f*g)/(12*d*e^3) + (x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(8*d^2*e^2) + (x^2*
(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 +
3*d^2*e*x)
```

3.556 $\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx$

Optimal. Leaf size=139

$$-\frac{(ef-dg)^2}{8de^3(d+ex)^4}-\frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3}-\frac{(ef+dg)^2}{16d^3e^3(d+ex)^2}-\frac{(ef+dg)^2}{16d^4e^3(d+ex)}+\frac{(ef+dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}$$

```
[Out] -1/8*(-d*g+e*f)^2/d/e^3/(e*x+d)^4-1/12*(-d*g+e*f)*(3*d*g+e*f)/d^2/e^3/(e*x+
d)^3-1/16*(d*g+e*f)^2/d^3/e^3/(e*x+d)^2-1/16*(d*g+e*f)^2/d^4/e^3/(e*x+d)+1/
16*(d*g+e*f)^2*arctanh(e*x/d)/d^5/e^3
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\frac{(dg+ef)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3}-\frac{(dg+ef)^2}{16d^4e^3(d+ex)}-\frac{(dg+ef)^2}{16d^3e^3(d+ex)^2}-\frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3}-\frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/8*(e*f - d*g)^2/(d*e^3*(d + e*x)^4) - ((e*f - d*g)*(e*f + 3*d*g))/(12*d^
2*e^3*(d + e*x)^3) - (e*f + d*g)^2/(16*d^3*e^3*(d + e*x)^2) - (e*f + d*g)^2
/(16*d^4*e^3*(d + e*x)) + ((e*f + d*g)^2*ArcTanh[(e*x)/d])/(16*d^5*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)}\,dx = \int \frac{(f+gx)^2}{(d-ex)(d+ex)^5}\,dx$$
$$= \int \left(\frac{(-ef+dg)^2}{2de^2(d+ex)^5} + \frac{(ef-dg)(ef+3dg)}{4d^2e^2(d+ex)^4} + \frac{(ef+dg)^2}{8d^3e^2(d+ex)^3} + \frac{(ef+dg)^2}{16d^4e^2(d+e\ldots}\right.$$
$$= -\frac{(ef-dg)^2}{8de^3(d+ex)^4} - \frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3} - \frac{(ef+dg)^2}{16d^3e^3(d+ex)^2} - \frac{(ef+dg)^2}{16d^4e^3(d+ex\ldots}$$
$$= -\frac{(ef-dg)^2}{8de^3(d+ex)^4} - \frac{(ef-dg)(ef+3dg)}{12d^2e^3(d+ex)^3} - \frac{(ef+dg)^2}{16d^3e^3(d+ex)^2} - \frac{(ef+dg)^2}{16d^4e^3(d+ex\ldots}$$

Mathematica [A]

time = 0.06, size = 142, normalized size = 1.02

$$-\frac{\frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{8d^3(e^2f^2+2defg-3d^2g^2)}{(d+ex)^3} + \frac{6d^2(ef+dg)^2}{(d+ex)^2} + \frac{6d(ef+dg)^2}{d+ex} + 3(ef+dg)^2\log(d-ex) - 3(ef+dg)^2\log(d+ex)}{96d^5e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)),x]
```

```
[Out] -1/96*((12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(e^2*f^2 + 2*d*e*f*g - 3
*d^2*g^2))/(d + e*x)^3 + (6*d^2*(e*f + d*g)^2)/(d + e*x)^2 + (6*d*(e*f + d*
g)^2)/(d + e*x) + 3*(e*f + d*g)^2*Log[d - e*x] - 3*(e*f + d*g)^2*Log[d + e*
x])/(d^5*e^3)
```

Maple [A]

time = 0.10, size = 220, normalized size = 1.58

method	result
norman	$\frac{-\frac{(3d^2g^2-26defg-61e^2f^2)x^3}{48d^4}-\frac{(d^2g^2-2defg-7e^2f^2)x^2}{4e\,d^3}+\frac{e^2(dfg+2e\,f^2)x^4}{6d^5}-\frac{(d^2g^2+2defg-15e^2f^2)x}{16d^2e^2}}{(ex+d)^4} - \frac{(d^2g^2+2defg+e^2f^2)\ln(-ex\ldots}{32e^3d^5}$
default	$-\frac{-3d^2g^2+2defg+e^2f^2}{12d^2e^3(ex+d)^3} - \frac{d^2g^2-2defg+e^2f^2}{8e^3d(ex+d)^4} + \frac{(d^2g^2+2defg+e^2f^2)\ln(ex+d)}{32e^3d^5} - \frac{d^2g^2+2defg+e^2f^2}{16e^3d^4(ex+d)} - \frac{d^2g^2+2defg+e^2f^2\ldots}{16e^3d^3(ex+d)^2}$
risch	$\frac{-\frac{(d^2g^2+2defg+e^2f^2)x^3}{16d^4}-\frac{(d^2g^2+2defg+e^2f^2)x^2}{4d^3e}-\frac{(3d^2g^2+38defg+19e^2f^2)x}{48d^2e^2}-\frac{f(dg+2ef)}{6e^2d}}{(ex+d)^4} - \frac{\ln(-ex+d)g^2}{32e^3d^3} - \frac{\ln(-ex+d)fg}{16e^2d^4} - \ln\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-3*d^2*g^2+2*d*e*f*g+e^2*f^2)/d^2/e^3/(e*x+d)^3-1/8/e^3*(d^2*g^2-2*d
*e*f*g+e^2*f^2)/d/(e*x+d)^4+1/32*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^5*ln(e*x
+d)-1/16*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^4/(e*x+d)-1/16*(d^2*g^2+2*d*e*f*
```

```
g+e^2*f^2)/e^3/d^3/(e*x+d)^2+1/32*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3/d^5*ln(-
e*x+d)
```

Maxima [A]
time = 0.30, size = 228, normalized size = 1.64

$$-\frac{8\,d^{4}fg+16\,d^{3}f^{2}e+3\,(d^{2}g^{2}e^{2}+2\,dfge^{3}+f^{2}e^{4})x^{3}+12\,(d^{3}g^{2}e+2\,d^{2}fge^{2}+df^{2}e^{3})x^{2}+(3\,d^{4}g^{2}+38\,d^{3}fge+19\,d^{2}f^{2}e^{2})x}{48\,(d^{4}x^{4}e^{6}+4\,d^{5}x^{3}e^{5}+6\,d^{6}x^{2}e^{4}+4\,d^{7}xe^{3}+d^{8}e^{2})}+\frac{(d^{2}g^{2}+2\,dfge+f^{2}e^{2})e^{(-3)}\log\left(xe+d\right)}{32\,d^{5}}-\frac{(d^{2}g^{2}+2\,dfge+f^{2}e^{2})e^{(-3)}\log\left(xe-d\right)}{32\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -1/48*(8*d^4*f*g + 16*d^3*f^2*e + 3*(d^2*g^2*e^2 + 2*d*f*g*e^3 + f^2*e^4)*x
^3 + 12*(d^3*g^2*e + 2*d^2*f*g*e^2 + d*f^2*e^3)*x^2 + (3*d^4*g^2 + 38*d^3*f
*g*e + 19*d^2*f^2*e^2)*x)/(d^4*x^4*e^6 + 4*d^5*x^3*e^5 + 6*d^6*x^2*e^4 + 4*
d^7*x*e^3 + d^8*e^2) + 1/32*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e
+ d)/d^5 - 1/32*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e - d)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(135) = 270$.
time = 3.30, size = 505, normalized size = 3.63

$$-\frac{6\,df^{2}x^{3}e^{5}+12\,(d^{2}fgx^{3}+2\,d^{2}f^{2}x^{2})e^{4}+2\,(3\,d^{3}g^{2}x^{3}+24\,d^{3}fgx^{2}+19\,d^{3}f^{2}x)e^{3}+4\,(6\,d^{4}g^{2}x^{2}+19\,d^{4}fgx+8\,d^{4}f^{2})e^{2}+2\,(3\,d^{5}g^{2}x+8\,d^{5}fg)e-3\,(d^{6}g^{2}+f^{2}x^{4}e^{6}+2\,(dfgx^{4}+2\,df^{2}x^{3})e^{5}+(d^{2}g^{2}x^{4}+8\,d^{2}fgx^{3}+6\,d^{2}f^{2}x^{2})e^{4}+4\,(d^{3}g^{2}x^{3}+3\,d^{3}fgx^{2}+d^{3}f^{2}x)e^{3}+(6\,d^{4}g^{2}x^{2}+8\,d^{4}fgx+d^{4}f^{2})e^{2}+2\,(2\,d^{5}g^{2}x+d^{5}fg)e)\log\left(xe+d\right)+3\,(d^{6}g^{2}+f^{2}x^{4}e^{6}+2\,(dfgx^{4}+2\,df^{2}x^{3})e^{5}+(d^{2}g^{2}x^{4}+8\,d^{2}fgx^{3}+6\,d^{2}f^{2}x^{2})e^{4}+4\,(d^{3}g^{2}x^{3}+3\,d^{3}fgx^{2}+d^{3}f^{2}x)e^{3}+(6\,d^{4}g^{2}x^{2}+8\,d^{4}fgx+d^{4}f^{2})e^{2}+2\,(2\,d^{5}g^{2}x+d^{5}fg)e)\log\left(xe-d\right)}{96\,(d^{5}x^{4}e^{7}+4\,d^{6}x^{3}e^{6}+6\,d^{7}x^{2}e^{5}+4\,d^{8}xe^{4}+d^{9}e^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="fricas")
```

```
[Out] -1/96*(6*d*f^2*x^3*e^5 + 12*(d^2*f*g*x^3 + 2*d^2*f^2*x^2)*e^4 + 2*(3*d^3*g^
2*x^3 + 24*d^3*f*g*x^2 + 19*d^3*f^2*x)*e^3 + 4*(6*d^4*g^2*x^2 + 19*d^4*f*g*
x + 8*d^4*f^2)*e^2 + 2*(3*d^5*g^2*x + 8*d^5*f*g)*e - 3*(d^6*g^2 + f^2*x^4*e
^6 + 2*(d*f*g*x^4 + 2*d*f^2*x^3)*e^5 + (d^2*g^2*x^4 + 8*d^2*f*g*x^3 + 6*d^2
*f^2*x^2)*e^4 + 4*(d^3*g^2*x^3 + 3*d^3*f*g*x^2 + d^3*f^2*x)*e^3 + (6*d^4*g^
2*x^2 + 8*d^4*f*g*x + d^4*f^2)*e^2 + 2*(2*d^5*g^2*x + d^5*f*g)*e)*log(x*e +
 d) + 3*(d^6*g^2 + f^2*x^4*e^6 + 2*(d*f*g*x^4 + 2*d*f^2*x^3)*e^5 + (d^2*g^2
*x^4 + 8*d^2*f*g*x^3 + 6*d^2*f^2*x^2)*e^4 + 4*(d^3*g^2*x^3 + 3*d^3*f*g*x^2
+ d^3*f^2*x)*e^3 + (6*d^4*g^2*x^2 + 8*d^4*f*g*x + d^4*f^2)*e^2 + 2*(2*d^5*g
^2*x + d^5*f*g)*e)*log(x*e - d))/(d^5*x^4*e^7 + 4*d^6*x^3*e^6 + 6*d^7*x^2*e
^5 + 4*d^8*x*e^4 + d^9*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(122) = 244$.
time = 0.74, size = 282, normalized size = 2.03

$$-\frac{8d^{4}fg+16d^{3}ef^{2}+x^{3}\cdot(3d^{2}e^{2}g^{2}+6de^{3}fg+3e^{4}f^{2})+x^{2}\cdot(12d^{3}eg^{2}+24d^{2}e^{2}fg+12de^{3}f^{2})+x(3d^{4}g^{2}+38d^{3}efg+19d^{2}e^{2}f^{2})}{48d^{8}e^{2}+192d^{7}e^{3}x+288d^{6}e^{4}x^{2}+192d^{5}e^{5}x^{3}+48d^{4}e^{6}x^{4}}-\frac{(dg+ef)^{2}\log\left(-\frac{d(dg+ef)^{2}}{e(d^{2}g^{2}+2defg+e^{2}f^{2})}+x\right)}{32d^{5}e^{3}}+\frac{(dg+ef)^{2}\log\left(\frac{d(dg+ef)^{2}}{e(d^{2}g^{2}+2defg+e^{2}f^{2})}+x\right)}{32d^{5}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2),x)
```

```
[Out] -(8*d**4*f*g + 16*d**3*e*f**2 + x**3*(3*d**2*e**2*g**2 + 6*d*e**3*f*g + 3*e
**4*f**2) + x**2*(12*d**3*e*g**2 + 24*d**2*e**2*f*g + 12*d*e**3*f**2) + x*(
3*d**4*g**2 + 38*d**3*e*f*g + 19*d**2*e**2*f**2))/(48*d**8*e**2 + 192*d**7*
e**3*x + 288*d**6*e**4*x**2 + 192*d**5*e**5*x**3 + 48*d**4*e**6*x**4) - (d*
g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) +
 x)/(32*d**5*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*
d*e*f*g + e**2*f**2)) + x)/(32*d**5*e**3)
```

Giac [A]
time = 1.25, size = 206, normalized size = 1.48

$$\frac{(d^2g^2 + 2\,dfge + f^2e^2)e^{(-3)}\log\left(|xe + d|\right)}{32\,d^5} - \frac{(d^2g^2 + 2\,dfge + f^2e^2)e^{(-3)}\log\left(|xe - d|\right)}{32\,d^5} - \frac{(8\,d^5fge + 16\,d^4f^2e^2 + 3\,(d^3g^2e^3 + 2\,d^2fge^4 + df^2e^5)x^3 + 12\,(d^4g^2e^2 + 2\,d^3fge^3 + d^2f^2e^4)x^2 + (3\,d^5g^2e + 38\,d^4fge^2 + 19\,d^3f^2e^3)x)e^{(-3)}}{48\,(xe + d)^4d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x, algorithm="giac")
```

```
[Out] 1/32*(d^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^5 - 1/32*(d
^2*g^2 + 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^5 - 1/48*(8*d^5*f*
g*e + 16*d^4*f^2*e^2 + 3*(d^3*g^2*e^3 + 2*d^2*f*g*e^4 + d*f^2*e^5)*x^3 + 12
*(d^4*g^2*e^2 + 2*d^3*f*g*e^3 + d^2*f^2*e^4)*x^2 + (3*d^5*g^2*e + 38*d^4*f*
g*e^2 + 19*d^3*f^2*e^3)*x)*e^(-3)/((x*e + d)^4*d^5)
```

Mupad [B]
time = 0.15, size = 180, normalized size = 1.29

$$\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g+e\,f\right)}^2}{16\,d^5\,e^3}-\frac{\frac{x^3\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{16\,d^4}+\frac{2\,e\,f^2+d\,g\,f}{6\,d\,e^2}+\frac{x\,\left(3\,d^2\,g^2+38\,d\,e\,f\,g+19\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2\right)}{4\,d^3\,e}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)*(d + e*x)^4),x)
```

```
[Out] (atanh((e*x)/d)*(d*g + e*f)^2)/(16*d^5*e^3) - ((x^3*(d^2*g^2 + e^2*f^2 + 2*
d*e*f*g))/(16*d^4) + (2*e*f^2 + d*f*g)/(6*d*e^2) + (x*(3*d^2*g^2 + 19*e^2*f
^2 + 38*d*e*f*g))/(48*d^2*e^2) + (x^2*(d^2*g^2 + e^2*f^2 + 2*d*e*f*g))/(4*d
^3*e))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

3.557 $\int \frac{(d+ex)^7(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{d^3(49e^2f^2+160defg+112d^2g^2)\,x}{e^2}+\frac{d^2(23e^2f^2+98defg+80d^2g^2)\,x^2}{2e}+\frac{1}{3}d\left(7e^2f^2+46defg+49d^2g^2\right)x^3+\frac{1}{4}$$

```
[Out] d^3*(112*d^2*g^2+160*d*e*f*g+49*e^2*f^2)*x/e^2+1/2*d^2*(80*d^2*g^2+98*d*e*f
*g+23*e^2*f^2)*x^2/e+1/3*d*(49*d^2*g^2+46*d*e*f*g+7*e^2*f^2)*x^3+1/4*e*(23*
d^2*g^2+14*d*e*f*g+e^2*f^2)*x^4+1/5*e^2*g*(7*d*g+2*e*f)*x^5+1/6*e^3*g^2*x^6
+32*d^5*(d*g+e*f)^2/e^3/(-e*x+d)+16*d^4*(d*g+e*f)*(9*d*g+5*e*f)*ln(-e*x+d)/
e^3
```

Rubi [A]
time = 0.18, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 90}

$$\frac{32d^5(dg+ef)^2}{e^3(d-ex)}+\frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3}+\frac{1}{4}ex^4\left(23d^2g^2+14defg+e^2f^2\right)+\frac{1}{3}dx^3\left(49d^2g^2+46defg+7e^2f^2\right)+\frac{d^2x^2(80d^2g^2+98defg+23e^2f^2)}{2e}+\frac{d^3x(112d^2g^2+160defg+49e^2f^2)}{e^2}+\frac{1}{5}e^2gx^5(7dg+2ef)+\frac{1}{6}e^3g^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x)) +
 (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \int \frac{(d+ex)^5(f+gx)^2}{(d-ex)^2}\,dx$$
$$= \int \left(\frac{d^3(49e^2f^2+160defg+112d^2g^2)}{e^2} + \frac{d^2(23e^2f^2+98defg+80d^2g^2)\,x}{e} + d(\right.$$
$$= \frac{d^3(49e^2f^2+160defg+112d^2g^2)\,x}{e^2} + \frac{d^2(23e^2f^2+98defg+80d^2g^2)\,x^2}{2e} + \frac{1}{3}d($$

Mathematica [A]
time = 0.09, size = 226, normalized size = 1.04

$$\frac{d^3(49e^2f^2+160defg+112d^2g^2)\,x}{e^2} + \frac{d^2(23e^2f^2+98defg+80d^2g^2)\,x^2}{2e} + \frac{1}{3}d\left(7e^2f^2+46defg+49d^2g^2\right)x^3 + \frac{1}{4}e\left(e^2f^2+14defg+23d^2g^2\right)x^4 + \frac{1}{5}e^2g(2ef+7dg)x^5 + \frac{1}{6}e^3g^2x^6 - \frac{32d^5(ef+dg)^2}{e^3(-d+ex)} + \frac{16d^4(5e^2f^2+14defg+9d^2g^2)\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 9
8*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^
2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f +
 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 - (32*d^5*(e*f + d*g)^2)/(e^3*(-d + e*x))
+ (16*d^4*(5*e^2*f^2 + 14*d*e*f*g + 9*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A]
time = 0.08, size = 259, normalized size = 1.19

method	result
default	$\frac{\frac{1}{6}g^2e^5x^6+\frac{7}{5}de^4g^2x^5+\frac{2}{5}e^5fgx^5+\frac{23}{4}d^2e^3g^2x^4+\frac{7}{2}de^4fgx^4+\frac{1}{4}e^5f^2x^4+\frac{49}{3}d^3e^2g^2x^3+\frac{46}{3}d^2e^3fgx^3+\frac{7}{3}de^4f^2x^3+40d^4eg^2x^2+49d^3e^2}{e^2}$
risch	$\frac{e^3g^2x^6}{6}+\frac{7e^2dg^2x^5}{5}+\frac{2e^3fgx^5}{5}+\frac{23ed^2g^2x^4}{4}+\frac{7e^2dfgx^4}{2}+\frac{e^3f^2x^4}{4}+\frac{49d^3g^2x^3}{3}+\frac{46ed^2fgx^3}{3}+\frac{7e^2df^2x^3}{3}+\frac{40d^4}{e}$
norman	$\frac{\left(-\frac{287}{3}d^5g^2-\frac{434}{3}d^4efg-\frac{140}{3}d^3e^2f^2\right)x^3+\left(-\frac{224}{15}g^2d^3e^2-\frac{224}{15}fge^3d^2-\frac{7}{3}f^2de^4\right)x^5+\left(-\frac{137}{4}g^2ed^4-\frac{91}{2}fgd^3e^2-\frac{45}{4}f^2e^3d^2\right)x^4+\left(-\frac{67}{12}g\right.}{-e^2x^2+d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/6*g^2*e^5*x^6+7/5*d*e^4*g^2*x^5+2/5*e^5*f*g*x^5+23/4*d^2*e^3*g^2*x
^4+7/2*d*e^4*f*g*x^4+1/4*e^5*f^2*x^4+49/3*d^3*e^2*g^2*x^3+46/3*d^2*e^3*f*g*
x^3+7/3*d*e^4*f^2*x^3+40*d^4*e*g^2*x^2+49*d^3*e^2*f*g*x^2+23/2*d^2*e^3*f^2*
x^2+112*d^5*g^2*x+160*d^4*e*f*g*x+49*d^3*e^2*f^2*x)+16/e^3*d^4*(9*d^2*g^2+1
4*d*e*f*g+5*e^2*f^2)*ln(-e*x+d)+32*d^5*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*
x+d)
```

Maxima [A]
time = 0.28, size = 245, normalized size = 1.12

$$16\left(9\,d^{6}g^{2}+14\,d^{5}fge+5\,d^{4}f^{2}e^{2}\right)e^{(-3)}\log\left(xe-d\right)+\frac{1}{60}\left(10\,g^{2}x^{6}e^{5}+12\left(7\,dg^{2}e^{4}+2\,fge^{5}\right)x^{5}+15\left(23\,d^{2}g^{2}e^{3}+14\,dfge^{4}+f^{2}e^{5}\right)x^{4}+20\left(49\,d^{3}g^{2}e^{2}+46\,d^{2}fge^{3}+7\,df^{2}e^{4}\right)x^{3}+30\left(80\,d^{4}g^{2}e+98\,d^{3}fge^{2}+23\,d^{2}f^{2}e^{3}\right)x^{2}+60\left(112\,d^{5}g^{2}+160\,d^{4}fge+49\,d^{3}f^{2}e^{2}\right)x\right)e^{(-2)}-\frac{32\left(d^{7}g^{2}+2\,d^{6}fge+d^{5}f^{2}e^{2}\right)}{xe^{4}-de^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 16*(9*d^6*g^2 + 14*d^5*f*g*e + 5*d^4*f^2*e^2)*e^(-3)*log(x*e - d) + 1/60*(1
0*g^2*x^6*e^5 + 12*(7*d*g^2*e^4 + 2*f*g*e^5)*x^5 + 15*(23*d^2*g^2*e^3 + 14*
d*f*g*e^4 + f^2*e^5)*x^4 + 20*(49*d^3*g^2*e^2 + 46*d^2*f*g*e^3 + 7*d*f^2*e^
4)*x^3 + 30*(80*d^4*g^2*e + 98*d^3*f*g*e^2 + 23*d^2*f^2*e^3)*x^2 + 60*(112*
d^5*g^2 + 160*d^4*f*g*e + 49*d^3*f^2*e^2)*x)*e^(-2) - 32*(d^7*g^2 + 2*d^6*f
*g*e + d^5*f^2*e^2)/(x*e^4 - d*e^3)
```

Fricas [A]
time = 2.59, size = 317, normalized size = 1.45

$$-\frac{1920\,d^{7}g^{2}-(10\,g^{2}x^{7}+24\,fgx^{6}+15\,f^{2}x^{5})e^{7}-(74\,dg^{2}x^{6}+186\,dfgx^{5}+125\,df^{2}x^{4})e^{6}-(261\,d^{2}g^{2}x^{5}+710\,d^{2}fgx^{4}+550\,d^{2}f^{2}x^{3})e^{5}-5\,(127\,d^{3}g^{2}x^{4}+404\,d^{3}fgx^{3}+450\,d^{3}f^{2}x^{2})e^{4}-20\,(71\,d^{4}g^{2}x^{3}+333\,d^{4}fgx^{2}-147\,d^{4}f^{2}x)e^{3}-480\,(9\,d^{5}g^{2}x^{2}-20\,d^{5}fgx-4\,d^{5}f^{2})e^{2}+960\,(7\,d^{6}g^{2}x+4\,d^{6}fg)e+960\,(9\,d^{7}g^{2}-5\,d^{4}f^{2}xe^{3}-(14\,d^{5}fgx-5\,d^{5}f^{2})e^{2}-(9\,d^{6}g^{2}x-14\,d^{6}fg)e)\log\left(xe-d\right)}{60\,(xe^{4}-de^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/60*(1920*d^7*g^2 - (10*g^2*x^7 + 24*f*g*x^6 + 15*f^2*x^5)*e^7 - (74*d*g^
2*x^6 + 186*d*f*g*x^5 + 125*d*f^2*x^4)*e^6 - (261*d^2*g^2*x^5 + 710*d^2*f*g
*x^4 + 550*d^2*f^2*x^3)*e^5 - 5*(127*d^3*g^2*x^4 + 404*d^3*f*g*x^3 + 450*d^
3*f^2*x^2)*e^4 - 20*(71*d^4*g^2*x^3 + 333*d^4*f*g*x^2 - 147*d^4*f^2*x)*e^3
- 480*(9*d^5*g^2*x^2 - 20*d^5*f*g*x - 4*d^5*f^2)*e^2 + 960*(7*d^6*g^2*x + 4
*d^6*f*g)*e + 960*(9*d^7*g^2 - 5*d^4*f^2*x*e^3 - (14*d^5*f*g*x - 5*d^5*f^2)
*e^2 - (9*d^6*g^2*x - 14*d^6*f*g)*e)*log(x*e - d))/(x*e^4 - d*e^3)
```

Sympy [A]
time = 0.54, size = 250, normalized size = 1.15

$$\frac{16d^{4}(dg+ef)\,(9dg+5ef)\log\left(-d+ex\right)}{e^{3}}+\frac{e^{3}g^{2}x^{6}}{6}+x^{5}\cdot\left(\frac{7de^{2}g^{2}}{5}+\frac{2e^{3}fg}{5}\right)+x^{4}\cdot\left(\frac{23d^{2}eg^{2}}{4}+\frac{7de^{2}fg}{2}+\frac{e^{3}f^{2}}{4}\right)+x^{3}\cdot\left(\frac{49d^{3}g^{2}}{3}+\frac{46d^{2}efg}{3}+\frac{7de^{2}f^{2}}{3}\right)+x^{2}\cdot\left(\frac{40d^{4}g^{2}}{e}+49d^{3}fg+\frac{23d^{2}ef^{2}}{2}\right)+x\left(\frac{112d^{5}g^{2}}{e^{2}}+\frac{160d^{4}fg}{e}+49d^{3}f^{2}\right)+\frac{-32d^{7}g^{2}-64d^{6}efg-32d^{5}e^{2}f^{2}}{-de^{3}+e^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 16*d**4*(d*g + e*f)*(9*d*g + 5*e*f)*log(-d + e*x)/e**3 + e**3*g**2*x**6/6 +
 x**5*(7*d*e**2*g**2/5 + 2*e**3*f*g/5) + x**4*(23*d**2*e*g**2/4 + 7*d*e**2*
f*g/2 + e**3*f**2/4) + x**3*(49*d**3*g**2/3 + 46*d**2*e*f*g/3 + 7*d*e**2*f*
*2/3) + x**2*(40*d**4*g**2/e + 49*d**3*f*g + 23*d**2*e*f**2/2) + x*(112*d**
5*g**2/e**2 + 160*d**4*f*g/e + 49*d**3*f**2) + (-32*d**7*g**2 - 64*d**6*e*f
*g - 32*d**5*e**2*f**2)/(-d*e**3 + e**4*x)
```

Giac [A]
time = 1.28, size = 257, normalized size = 1.18

$$16\,(9\,d^6g^2+14\,d^5fge+5\,d^4f^2e^2)e^{(-3)}\log\left(|xe-d|\right)+\frac{1}{60}\,(10\,g^2x^6e^{15}+84\,dg^2x^5e^{14}+345\,d^2g^2x^4e^{13}+980\,d^3g^2x^3e^{12}+2400\,d^4g^2x^2e^{11}+6720\,d^5g^2xe^{10}+24\,fgx^5e^{15}+210\,dfgx^4e^{14}+920\,d^2fgx^3e^{13}+2940\,d^3fgx^2e^{12}+9600\,d^4fgxe^{11}+15\,f^2x^4e^{15}+140\,df^2x^3e^{14}+690\,d^2f^2x^2e^{13}+2940\,d^3f^2xe^{12})e^{(-12)}-\frac{32\,(d^7g^2+2\,d^6fge+d^5f^2e^2)e^{(-3)}}{xe-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 16*(9*d^6*g^2 + 14*d^5*f*g*e + 5*d^4*f^2*e^2)*e^(-3)*log(abs(x*e - d)) + 1/
60*(10*g^2*x^6*e^15 + 84*d*g^2*x^5*e^14 + 345*d^2*g^2*x^4*e^13 + 980*d^3*g^
2*x^3*e^12 + 2400*d^4*g^2*x^2*e^11 + 6720*d^5*g^2*x*e^10 + 24*f*g*x^5*e^15
+ 210*d*f*g*x^4*e^14 + 920*d^2*f*g*x^3*e^13 + 2940*d^3*f*g*x^2*e^12 + 9600*
d^4*f*g*x*e^11 + 15*f^2*x^4*e^15 + 140*d*f^2*x^3*e^14 + 690*d^2*f^2*x^2*e^1
3 + 2940*d^3*f^2*x*e^12)*e^(-12) - 32*(d^7*g^2 + 2*d^6*f*g*e + d^5*f^2*e^2)
*e^(-3)/(x*e - d)
```

Mupad [B]
time = 2.64, size = 1029, normalized size = 4.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^5*((e^2*g*(5*d*g + 2*e*f))/5 + (2*d*e^2*g^2)/5) + x^3*((5*d*(2*d^2*g^2 +
e^2*f^2 + 4*d*e*f*g))/3 + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e
^2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(3*e) - (d
^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/(3*e^2)) + x^4*((e^5*f^2 + 10*d^2
*e^3*g^2 + 10*d*e^4*f*g)/(4*e^2) - (d^2*e*g^2)/4 + (d*(e^2*g*(5*d*g + 2*e*f
) + 2*d*e^2*g^2))/(2*e)) + x^2*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f*g))/(
2*e) - (d^2*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2
*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/(2*e^2) + (d*(5*d*(2*d^2*g^2
+ e^2*f^2 + 4*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^
2 - d^2*e*g^2 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e
^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e^2))/e) + x*((d^5*g^2 + 10*d^3*e^2*f^
2 + 10*d^4*e*f*g)/e^2 - (d^2*(5*d*(2*d^2*g^2 + e^2*f^2 + 4*d*e*f*g) + (2*d*
((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(5
*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2
*g^2))/e^2))/e^2 + (2*d*((5*d^2*(d^2*g^2 + 2*e^2*f^2 + 4*d*e*f*g))/e - (d^2
*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2 + (2*d*(e^2*g*(
5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e^2 + (2*d*(5*d*(2*d^2*g^2 + e^2*f^2 + 4
*d*e*f*g) + (2*d*((e^5*f^2 + 10*d^2*e^3*g^2 + 10*d*e^4*f*g)/e^2 - d^2*e*g^2
 + (2*d*(e^2*g*(5*d*g + 2*e*f) + 2*d*e^2*g^2))/e))/e - (d^2*(e^2*g*(5*d*g +
 2*e*f) + 2*d*e^2*g^2))/e^2))/e))/e) + (log(e*x - d)*(144*d^6*g^2 + 80*d^4*
e^2*f^2 + 224*d^5*e*f*g))/e^3 + (32*(d^7*g^2 + d^5*e^2*f^2 + 2*d^6*e*f*g))/
(e*(d*e^2 - e^3*x)) + (e^3*g^2*x^6)/6
```

3.558 $\int \frac{(d+ex)^6(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=177

$$\frac{d^2(17e^2f^2+64defg+48d^2g^2)\,x}{e^2}+\frac{d(3e^2f^2+17defg+16d^2g^2)\,x^2}{e}+\frac{1}{3}\left(e^2f^2+12defg+17d^2g^2\right)x^3+\frac{1}{2}eg(ef-$$

```
[Out] d^2*(48*d^2*g^2+64*d*e*f*g+17*e^2*f^2)*x/e^2+d*(16*d^2*g^2+17*d*e*f*g+3*e^2
*f^2)*x^2/e+1/3*(17*d^2*g^2+12*d*e*f*g+e^2*f^2)*x^3+1/2*e*g*(3*d*g+e*f)*x^4
+1/5*e^2*g^2*x^5+16*d^4*(d*g+e*f)^2/e^3/(-e*x+d)+32*d^3*(d*g+e*f)*(2*d*g+e*
f)*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 90}

$$\frac{16d^4(dg+ef)^2}{e^3(d-ex)}+\frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3}+\frac{1}{3}x^3(17d^2g^2+12defg+e^2f^2)+\frac{dx^2(16d^2g^2+17defg+3e^2f^2)}{e}+\frac{d^2x(48d^2g^2+64defg+17e^2f^2)}{e^2}+\frac{1}{2}egx^4(3dg+ef)+\frac{1}{5}e^2g^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3*(d
 - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^2}\,dx$$
$$= \int \left(\frac{d^2(17e^2f^2+64defg+48d^2g^2)}{e^2} + \frac{2d(3e^2f^2+17defg+16d^2g^2)\,x}{e} + (e^2f^2$$
$$= \frac{d^2(17e^2f^2+64defg+48d^2g^2)\,x}{e^2} + \frac{d(3e^2f^2+17defg+16d^2g^2)\,x^2}{e} + \frac{1}{3}(e^2f^2$$

Mathematica [A]
time = 0.08, size = 185, normalized size = 1.05

$$\frac{d^2(17e^2f^2+64defg+48d^2g^2)\,x}{e^2} + \frac{d(3e^2f^2+17defg+16d^2g^2)\,x^2}{e} + \frac{1}{3}(e^2f^2+12defg+17d^2g^2)\,x^3 + \frac{1}{2}eg(ef+3dg)x^4 + \frac{1}{5}e^2g^2x^5 - \frac{16d^4(ef+dg)^2}{e^3(-d+ex)} + \frac{32d^3(e^2f^2+3defg+2d^2g^2)\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e
*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (
e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 - (16*d^4*(e*f + d*g)^2)/(e^3*(-
d + e*x)) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A]
time = 0.07, size = 217, normalized size = 1.23

method	result
default	$\frac{\frac{1}{5}g^2e^4x^5+\frac{3}{2}de^3g^2x^4+\frac{1}{2}e^4fgx^4+\frac{17}{3}d^2e^2g^2x^3+4de^3fgx^3+\frac{1}{3}e^4f^2x^3+16d^3eg^2x^2+17d^2e^2fgx^2+3de^3f^2x^2+48d^4g^2x+64d^3efgx+}{e^2}$
risch	$\frac{e^2g^2x^5}{5}+\frac{3edg^2x^4}{2}+\frac{e^2fgx^4}{2}+\frac{17d^2g^2x^3}{3}+4edfgx^3+\frac{e^2f^2x^3}{3}+\frac{16d^3g^2x^2}{e}+17d^2fgx^2+3edf^2x^2+\frac{48d^4}{e^2}$
norman	$\frac{(-\frac{127}{3}d^4g^2-60fgd^3e-\frac{50}{3}d^2e^2f^2)x^3+(-\frac{82}{15}g^2d^2e^2-4fgde^3-\frac{1}{3}f^2e^4)x^5+(-\frac{29}{2}g^2d^3e-\frac{33}{2}fgd^2e^2-3f^2de^3)x^4+\frac{d^2(32d^5g^2+49d^4efg}{e^3}}{-e^2x^2+d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/5*g^2*e^4*x^5+3/2*d*e^3*g^2*x^4+1/2*e^4*f*g*x^4+17/3*d^2*e^2*g^2*x
^3+4*d*e^3*f*g*x^3+1/3*e^4*f^2*x^3+16*d^3*e*g^2*x^2+17*d^2*e^2*f*g*x^2+3*d*
e^3*f^2*x^2+48*d^4*g^2*x+64*d^3*e*f*g*x+17*d^2*e^2*f^2*x)+16*d^4*(d^2*g^2+2
*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)+32*d^3/e^3*(2*d^2*g^2+3*d*e*f*g+e^2*f^2)*ln(
-e*x+d)
```

Maxima [A]
time = 0.29, size = 208, normalized size = 1.18

$$32\,(2\,d^5g^2+3\,d^4fge+d^3f^2e^2)e^{(-3)}\log(xe-d)+\frac{1}{30}\,(6\,g^2x^5e^4+15\,(3\,dg^2e^3+fge^4)x^4+10\,(17\,d^2g^2e^2+12\,dfge^3+f^2e^4)x^3+30\,(16\,d^3g^2e+17\,d^2fge^2+3\,df^2e^3)x^2+30\,(48\,d^4g^2+64\,d^3fge+17\,d^2f^2e^2)x)e^{(-2)}-\frac{16\,(d^6g^2+2\,d^5fge+d^4f^2e^2)}{xe^4-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 32*(2*d^5*g^2 + 3*d^4*f*g*e + d^3*f^2*e^2)*e^(-3)*log(x*e - d) + 1/30*(6*g^
2*x^5*e^4 + 15*(3*d*g^2*e^3 + f*g*e^4)*x^4 + 10*(17*d^2*g^2*e^2 + 12*d*f*g*
e^3 + f^2*e^4)*x^3 + 30*(16*d^3*g^2*e + 17*d^2*f*g*e^2 + 3*d*f^2*e^3)*x^2 +
 30*(48*d^4*g^2 + 64*d^3*f*g*e + 17*d^2*f^2*e^2)*x)*e^(-2) - 16*(d^6*g^2 +
2*d^5*f*g*e + d^4*f^2*e^2)/(x*e^4 - d*e^3)
```

Fricas [A]
time = 2.21, size = 280, normalized size = 1.58

$$-\frac{480\,d^6g^2-(6\,g^2x^6+15\,fgx^5+10\,f^2x^4)e^6-(39\,dg^2x^5+105\,dfgx^4+80\,df^2x^3)e^5-5\,(25\,d^2g^2x^4+78\,d^2fgx^3+84\,d^2f^2x^2)e^4-10\,(31\,d^3g^2x^3+141\,d^3fgx^2-51\,d^3f^2x)e^3-480\,(2\,d^4g^2x^2-4\,d^4fgx-d^4f^2)e^2+480\,(3\,d^5g^2x+2\,d^5fg)e+960\,(2\,d^6g^2-d^3f^2xe^3-(3\,d^4fgx-d^4f^2)e^2-(2\,d^5g^2x-3\,d^5fg)e)\log(xe-d)}{30\,(xe^4-de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(480*d^6*g^2 - (6*g^2*x^6 + 15*f*g*x^5 + 10*f^2*x^4)*e^6 - (39*d*g^2*
x^5 + 105*d*f*g*x^4 + 80*d*f^2*x^3)*e^5 - 5*(25*d^2*g^2*x^4 + 78*d^2*f*g*x^
3 + 84*d^2*f^2*x^2)*e^4 - 10*(31*d^3*g^2*x^3 + 141*d^3*f*g*x^2 - 51*d^3*f^2
*x)*e^3 - 480*(2*d^4*g^2*x^2 - 4*d^4*f*g*x - d^4*f^2)*e^2 + 480*(3*d^5*g^2*
x + 2*d^5*f*g)*e + 960*(2*d^6*g^2 - d^3*f^2*x*e^3 - (3*d^4*f*g*x - d^4*f^2)
*e^2 - (2*d^5*g^2*x - 3*d^5*f*g)*e)*log(x*e - d))/(x*e^4 - d*e^3)
```

Sympy [A]
time = 0.47, size = 199, normalized size = 1.12

$$\frac{32d^3(dg+ef)(2dg+ef)\log(-d+ex)}{e^3}+\frac{e^2g^2x^5}{5}+x^4\cdot\left(\frac{3deg^2}{2}+\frac{e^2fg}{2}\right)+x^3\cdot\left(\frac{17d^2g^2}{3}+4defg+\frac{e^2f^2}{3}\right)+x^2\cdot\left(\frac{16d^3g^2}{e}+17d^2fg+3def^2\right)+x\left(\frac{48d^4g^2}{e^2}+\frac{64d^3fg}{e}+17d^2f^2\right)+\frac{-16d^6g^2-32d^5efg-16d^4e^2f^2}{-de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 32*d**3*(d*g + e*f)*(2*d*g + e*f)*log(-d + e*x)/e**3 + e**2*g**2*x**5/5 + x
**4*(3*d*e*g**2/2 + e**2*f*g/2) + x**3*(17*d**2*g**2/3 + 4*d*e*f*g + e**2*f
**2/3) + x**2*(16*d**3*g**2/e + 17*d**2*f*g + 3*d*e*f**2) + x*(48*d**4*g**2
/e**2 + 64*d**3*f*g/e + 17*d**2*f**2) + (-16*d**6*g**2 - 32*d**5*e*f*g - 16
*d**4*e**2*f**2)/(-d*e**3 + e**4*x)
```

Giac [A]
time = 1.22, size = 218, normalized size = 1.23

$$32\,(2\,d^5g^2+3\,d^4fge+d^3f^2e^2)e^{(-3)}\log(|xe-d|)+\frac{1}{30}\,(6\,g^2x^5e^{12}+45\,dg^2x^4e^{11}+170\,d^2g^2x^3e^{10}+480\,d^3g^2x^2e^9+1440\,d^4g^2xe^8+15\,fgx^4e^{12}+120\,dfgx^3e^{11}+510\,d^2fgx^2e^{10}+1920\,d^3fgxe^9+10\,f^2x^3e^{12}+90\,df^2x^2e^{11}+510\,d^2f^2xe^{10})e^{(-10)}-\frac{16\,(d^6g^2+2\,d^5fge+d^4f^2e^2)e^{(-3)}}{xe-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 32*(2*d^5*g^2 + 3*d^4*f*g*e + d^3*f^2*e^2)*e^(-3)*log(abs(x*e - d)) + 1/30*
(6*g^2*x^5*e^12 + 45*d*g^2*x^4*e^11 + 170*d^2*g^2*x^3*e^10 + 480*d^3*g^2*x^
2*e^9 + 1440*d^4*g^2*x*e^8 + 15*f*g*x^4*e^12 + 120*d*f*g*x^3*e^11 + 510*d^2
*f*g*x^2*e^10 + 1920*d^3*f*g*x*e^9 + 10*f^2*x^3*e^12 + 90*d*f^2*x^2*e^11 +
510*d^2*f^2*x*e^10)*e^(-10) - 16*(d^6*g^2 + 2*d^5*f*g*e + d^4*f^2*e^2)*e^(-
3)/(x*e - d)
```

Mupad [B]

time = 2.61, size = 565, normalized size = 3.19

$$x^2\left(\frac{2d\left(d^2g^2+3defg+e^2f^2\right)}{e}-\frac{d^2\left(2eg\left(2dg+ef\right)+2deg^2\right)}{2e^2}+\frac{d\left(\frac{6d^2e^2g^2+8de^3fg+e^4f^2}{e^2}-d^2g^2+\frac{2d\left(2eg\left(2dg+ef\right)+2deg^2\right)}{e}\right)}{e}\right)+x^4\left(\frac{eg\left(2dg+ef\right)}{2}+\frac{deg^2}{2}\right)+x\left(\frac{d^4g^2+8d^3efg+6d^2e^2f^2}{e^2}-\frac{d^2\left(\frac{6d^2e^2g^2+8de^3fg+e^4f^2}{e^2}-d^2g^2+\frac{2d\left(2eg\left(2dg+ef\right)+2deg^2\right)}{e}\right)}{e^2}+\frac{2d\left(\frac{4d\left(d^2g^2+3defg+e^2f^2\right)}{e}-\frac{d^2\left(2eg\left(2dg+ef\right)+2deg^2\right)}{e^2}+\frac{2d\left(\frac{6d^2e^2g^2+8de^3fg+e^4f^2}{e^2}-d^2g^2+\frac{2d\left(2eg\left(2dg+ef\right)+2deg^2\right)}{e}\right)}{e}\right)}{e}\right)+x^3\left(\frac{6d^2e^2g^2+8de^3fg+e^4f^2}{3e^2}-\frac{d^2g^2}{3}+\frac{2d\left(2eg\left(2dg+ef\right)+2deg^2\right)}{3e}\right)+\frac{\ln\left(ex-d\right)\left(64d^5g^2+96d^4efg+32d^3e^2f^2\right)}{e^3}+\frac{16\left(d^6g^2+2d^5efg+d^4e^2f^2\right)}{e\left(de^2-e^3x\right)}+\frac{e^2g^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^2*((2*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e - (d^2*(2*e*g*(2*d*g + e*f) +
2*d*e*g^2))/(2*e^2) + (d*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2
*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e))/e) + x^4*((e*g*(2*d*g +
e*f))/2 + (d*e*g^2)/2) + x*((d^4*g^2 + 6*d^2*e^2*f^2 + 8*d^3*e*f*g)/e^2 - (
d^2*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2
*d*g + e*f) + 2*d*e*g^2))/e))/e^2 + (2*d*((4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f
*g))/e - (d^2*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/e^2 + (2*d*((e^4*f^2 + 6*d
^2*e^2*g^2 + 8*d*e^3*f*g)/e^2 - d^2*g^2 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e
*g^2))/e))/e))/e) + x^3*((e^4*f^2 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(3*e^2) -
(d^2*g^2)/3 + (2*d*(2*e*g*(2*d*g + e*f) + 2*d*e*g^2))/(3*e)) + (log(e*x - d
)*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g))/e^3 + (16*(d^6*g^2 + d^4*e^
2*f^2 + 2*d^5*e*f*g))/(e*(d*e^2 - e^3*x)) + (e^2*g^2*x^5)/5
```

3.559 $\int \frac{(d+ex)^5(f+gx)^2}{\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{d(5e^2f^2+24defg+20d^2g^2)\,x}{e^2}+\frac{(e^2f^2+10defg+12d^2g^2)\,x^2}{2e}+\frac{1}{3}g(2ef+5dg)x^3+\frac{1}{4}eg^2x^4+\frac{8d^3(ef+dg)^2}{e^3(d-ex)}+$$

```
[Out] d*(20*d^2*g^2+24*d*e*f*g+5*e^2*f^2)*x/e^2+1/2*(12*d^2*g^2+10*d*e*f*g+e^2*f^
2)*x^2/e+1/3*g*(5*d*g+2*e*f)*x^3+1/4*e*g^2*x^4+8*d^3*(d*g+e*f)^2/e^3/(-e*x+
d)+4*d^2*(d*g+e*f)*(7*d*g+3*e*f)*ln(-e*x+d)/e^3
```

Rubi [A]

time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\frac{8d^3(dg+ef)^2}{e^3(d-ex)}+\frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3}+\frac{x^2(12d^2g^2+10defg+e^2f^2)}{2e}+\frac{dx(20d^2g^2+24defg+5e^2f^2)}{e^2}+\frac{1}{3}gx^3(5dg+2ef)+\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d^3
*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*Log[d
- e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^2}\,dx$$
$$= \int \left(\frac{d(5e^2f^2+24defg+20d^2g^2)}{e^2} + \frac{(e^2f^2+10defg+12d^2g^2)\,x}{e} + g(2ef+5d\ldots\right.$$
$$= \frac{d(5e^2f^2+24defg+20d^2g^2)\,x}{e^2} + \frac{(e^2f^2+10defg+12d^2g^2)\,x^2}{2e} + \frac{1}{3}g(2ef+5d\ldots$$

Mathematica [A]

time = 0.06, size = 154, normalized size = 1.05

$$\frac{d(5e^2f^2+24defg+20d^2g^2)\,x}{e^2} + \frac{(e^2f^2+10defg+12d^2g^2)\,x^2}{2e} + \frac{1}{3}g(2ef+5dg)x^3 + \frac{1}{4}eg^2x^4 - \frac{8d^3(ef+dg)^2}{e^3(-d+ex)} + \frac{4d^2(3e^2f^2+10defg+7d^2g^2)\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g +
12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 - (8*d^3
*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d^2*(3*e^2*f^2 + 10*d*e*f*g + 7*d^2*g
^2)*Log[d - e*x])/e^3
```

Maple [A]

time = 0.09, size = 177, normalized size = 1.21

method	result
default	$\frac{\frac{1}{4}g^2e^3x^4+\frac{5}{3}de^2g^2x^3+\frac{2}{3}e^3fgx^3+6d^2eg^2x^2+5de^2fgx^2+\frac{1}{2}e^3f^2x^2+20d^3g^2x+24d^2efgx+5de^2f^2x}{e^2}+\frac{8d^3(d^2g^2+2defg+e^2f^2)}{e^3(-ex+d)}+$
risch	$\frac{eg^2x^4}{4}+\frac{5x^3dg^2}{3}+\frac{2ex^3fg}{3}+\frac{6x^2d^2g^2}{e}+5x^2dfg+\frac{ex^2f^2}{2}+\frac{20d^3g^2x}{e^2}+\frac{24d^2fgx}{e}+5df^2x+\frac{8d^5g^2}{e^3(-ex+d)}+\ldots$
norman	$\frac{(-\frac{55}{3}d^3g^2-\frac{70}{3}d^2efg-5de^2f^2)x^3+(-\frac{23}{4}g^2d^2e-5fgde^2-\frac{1}{2}f^2e^3)x^4+\frac{d^3(28d^2g^2+40defg+13e^2f^2)x}{e^2}+\frac{d^2(28d^4g^2+42fgd^3e+17d^2e^2f^2)\ldots}{2e^3}}{-e^2x^2+d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/4*g^2*e^3*x^4+5/3*d*e^2*g^2*x^3+2/3*e^3*f*g*x^3+6*d^2*e*g^2*x^2+5*
d*e^2*f*g*x^2+1/2*e^3*f^2*x^2+20*d^3*g^2*x+24*d^2*e*f*g*x+5*d*e^2*f^2*x)+8*
d^3*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)+4*d^2/e^3*(7*d^2*g^2+10*d*e*f*
g+3*e^2*f^2)*ln(-e*x+d)
```

Maxima [A]

time = 0.29, size = 175, normalized size = 1.20

$$4\left(7d^4g^2+10d^3fge+3d^2f^2e^2\right)e^{(-3)}\log\left(xe-d\right)+\frac{1}{12}\left(3g^2x^4e^3+4\left(5dg^2e^2+2fge^3\right)x^3+6\left(12d^2g^2e+10dfge^2+f^2e^3\right)x^2+12\left(20d^3g^2+24d^2fge+5df^2e^2\right)x\right)e^{(-2)}-\frac{8\left(d^5g^2+2d^4fge+d^3f^2e^2\right)}{xe^4-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 4*(7*d^4*g^2 + 10*d^3*f*g*e + 3*d^2*f^2*e^2)*e^(-3)*log(x*e - d) + 1/12*(3*
g^2*x^4*e^3 + 4*(5*d*g^2*e^2 + 2*f*g*e^3)*x^3 + 6*(12*d^2*g^2*e + 10*d*f*g*
e^2 + f^2*e^3)*x^2 + 12*(20*d^3*g^2 + 24*d^2*f*g*e + 5*d*f^2*e^2)*x)*e^(-2)
 - 8*(d^5*g^2 + 2*d^4*f*g*e + d^3*f^2*e^2)/(x*e^4 - d*e^3)
```

Fricas [**A**]
time = 3.63, size = 243, normalized size = 1.66

$$-\frac{96\,d^5g^2-(3\,g^2x^5+8\,fgx^4+6\,f^2x^3)e^5-(17\,dg^2x^4+52\,dfgx^3+54\,df^2x^2)e^4-4\,(13\,d^2g^2x^3+57\,d^2fgx^2-15\,d^2f^2x)e^3-24\,(7\,d^3g^2x^2-12\,d^3fgx-4\,d^3f^2)e^2+48\,(5\,d^4g^2x+4\,d^4fg)e+48\,(7\,d^5g^2-3\,d^2f^2xe^3-(10\,d^3fgx-3\,d^3f^2)e^2-(7\,d^4g^2x-10\,d^4fg)e)\log\left(xe-d\right)}{12\,(xe^4-de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(96*d^5*g^2 - (3*g^2*x^5 + 8*f*g*x^4 + 6*f^2*x^3)*e^5 - (17*d*g^2*x^4
 + 52*d*f*g*x^3 + 54*d*f^2*x^2)*e^4 - 4*(13*d^2*g^2*x^3 + 57*d^2*f*g*x^2 -
15*d^2*f^2*x)*e^3 - 24*(7*d^3*g^2*x^2 - 12*d^3*f*g*x - 4*d^3*f^2)*e^2 + 48*
(5*d^4*g^2*x + 4*d^4*f*g)*e + 48*(7*d^5*g^2 - 3*d^2*f^2*x*e^3 - (10*d^3*f*g
*x - 3*d^3*f^2)*e^2 - (7*d^4*g^2*x - 10*d^4*f*g)*e)*log(x*e - d))/(x*e^4 -
d*e^3)
```

Sympy [**A**]
time = 0.41, size = 162, normalized size = 1.11

$$\frac{4d^2(dg+ef)\,(7dg+3ef)\log\left(-d+ex\right)}{e^3}+\frac{eg^2x^4}{4}+x^3\cdot\left(\frac{5dg^2}{3}+\frac{2efg}{3}\right)+x^2\cdot\left(\frac{6d^2g^2}{e}+5dfg+\frac{ef^2}{2}\right)+x\left(\frac{20d^3g^2}{e^2}+\frac{24d^2fg}{e}+5df^2\right)+\frac{-8d^5g^2-16d^4efg-8d^3e^2f^2}{-de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d**2*(d*g + e*f)*(7*d*g + 3*e*f)*log(-d + e*x)/e**3 + e*g**2*x**4/4 + x**
3*(5*d*g**2/3 + 2*e*f*g/3) + x**2*(6*d**2*g**2/e + 5*d*f*g + e*f**2/2) + x*
(20*d**3*g**2/e**2 + 24*d**2*f*g/e + 5*d*f**2) + (-8*d**5*g**2 - 16*d**4*e*
f*g - 8*d**3*e**2*f**2)/(-d*e**3 + e**4*x)
```

Giac [**A**]
time = 1.92, size = 181, normalized size = 1.24

$$4\left(7\,d^4g^2+10\,d^3fge+3\,d^2f^2e^2\right)e^{(-3)}\log\left(|xe-d|\right)+\frac{1}{12}\left(3\,g^2x^4e^9+20\,dg^2x^3e^8+72\,d^2g^2x^2e^7+240\,d^3g^2xe^6+8\,fgx^3e^9+60\,dfgx^2e^8+288\,d^2fgxe^7+6\,f^2x^2e^9+60\,df^2xe^8\right)e^{(-8)}-\frac{8\,(d^5g^2+2\,d^4fge+d^3f^2e^2)e^{(-3)}}{xe-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 4*(7*d^4*g^2 + 10*d^3*f*g*e + 3*d^2*f^2*e^2)*e^(-3)*log(abs(x*e - d)) + 1/1
2*(3*g^2*x^4*e^9 + 20*d*g^2*x^3*e^8 + 72*d^2*g^2*x^2*e^7 + 240*d^3*g^2*x*e^
```

```
6 + 8*f*g*x^3*e^9 + 60*d*f*g*x^2*e^8 + 288*d^2*f*g*x*e^7 + 6*f^2*x^2*e^9 +
60*d*f^2*x*e^8)*e^(-8) - 8*(d^5*g^2 + 2*d^4*f*g*e + d^3*f^2*e^2)*e^(-3)/(x*
e - d)
```

Mupad [B]
time = 0.09, size = 316, normalized size = 2.16

$$x\left(\frac{d^3g^2+6d^2efg+3de^2f^2}{e^2}-\frac{d^2\left(g\left(3dg+2ef\right)+2dg^2\right)}{e^2}+\frac{2d\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{e^2}-\frac{d^2g^2}{e}+\frac{2d\left(g\left(3dg+2ef\right)+2dg^2\right)}{e}\right)}{e}\right)+x^2\left(\frac{3d^2eg^2+6de^2fg+e^3f^2}{2e^2}-\frac{d^2g^2}{2e}+\frac{d\left(g\left(3dg+2ef\right)+2dg^2\right)}{e}\right)+x^3\left(\frac{g\left(3dg+2ef\right)}{3}+\frac{2dg^2}{3}\right)+\frac{\ln\left(ex-d\right)\left(28d^4g^2+40d^3efg+12d^2e^2f^2\right)}{e^3}+\frac{8\left(d^5g^2+2d^4efg+d^3e^2f^2\right)}{e\left(de^2-e^3x\right)}+\frac{eg^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x*((d^3*g^2 + 3*d*e^2*f^2 + 6*d^2*e*f*g)/e^2 - (d^2*(g*(3*d*g + 2*e*f) + 2*
d*g^2))/e^2 + (2*d*((e^3*f^2 + 3*d^2*e*g^2 + 6*d*e^2*f*g)/e^2 - (d^2*g^2)/e
 + (2*d*(g*(3*d*g + 2*e*f) + 2*d*g^2))/e))/e) + x^2*((e^3*f^2 + 3*d^2*e*g^2
 + 6*d*e^2*f*g)/(2*e^2) - (d^2*g^2)/(2*e) + (d*(g*(3*d*g + 2*e*f) + 2*d*g^2
))/e) + x^3*((g*(3*d*g + 2*e*f))/3 + (2*d*g^2)/3) + (log(e*x - d)*(28*d^4*g
^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g))/e^3 + (8*(d^5*g^2 + d^3*e^2*f^2 + 2*d^
4*e*f*g))/(e*(d*e^2 - e^3*x)) + (e*g^2*x^4)/4
```

3.560 $\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

Optimal. Leaf size=107

$$\frac{(e^2f^2+8defg+8d^2g^2)\,x}{e^2}+\frac{g(ef+2dg)x^2}{e}+\frac{g^2x^3}{3}+\frac{4d^2(ef+dg)^2}{e^3(d-ex)}+\frac{4d(ef+dg)(ef+3dg)\log(d-ex)}{e^3}$$

```
[Out] (8*d^2*g^2+8*d*e*f*g+e^2*f^2)*x/e^2+g*(2*d*g+e*f)*x^2/e+1/3*g^2*x^3+4*d^2*(
d*g+e*f)^2/e^3/(-e*x+d)+4*d*(d*g+e*f)*(3*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 90}

$$\frac{4d^2(dg+ef)^2}{e^3(d-ex)}+\frac{x(8d^2g^2+8defg+e^2f^2)}{e^2}+\frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3}+\frac{gx^2(2dg+ef)}{e}+\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 + (4*d^2*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d*(e*f + d*g)*(e*f + 3*
d*g)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^2}\,dx$$
$$= \int \left(\frac{e^2f^2+8defg+8d^2g^2}{e^2} + \frac{2g(ef+2dg)x}{e} + g^2x^2 + \frac{4d(-ef-3dg)(ef+dg}{e^2(d-ex)}\right.$$
$$= \frac{(e^2f^2+8defg+8d^2g^2)\,x}{e^2} + \frac{g(ef+2dg)x^2}{e} + \frac{g^2x^3}{3} + \frac{4d^2(ef+dg)^2}{e^3(d-ex)} + \frac{4d(ef\ }{}$$

Mathematica [A]
time = 0.06, size = 115, normalized size = 1.07

$$\frac{(e^2f^2+8defg+8d^2g^2)\,x}{e^2} + \frac{g(ef+2dg)x^2}{e} + \frac{g^2x^3}{3} - \frac{4d^2(ef+dg)^2}{e^3(-d+ex)} + \frac{4d(e^2f^2+4defg+3d^2g^2)\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((e^2*f^2 + 8*d*e*f*g + 8*d^2*g^2)*x)/e^2 + (g*(e*f + 2*d*g)*x^2)/e + (g^2*
x^3)/3 - (4*d^2*(e*f + d*g)^2)/(e^3*(-d + e*x)) + (4*d*(e^2*f^2 + 4*d*e*f*g
 + 3*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A]
time = 0.07, size = 133, normalized size = 1.24

method	result
default	$\frac{\frac{1}{3}g^2x^3e^2+2de\,g^2x^2+e^2fg\,x^2+8d^2g^2x+8defgx+e^2f^2x}{e^2}+\frac{4d^2\left(d^2g^2+2defg+e^2f^2\right)}{e^3(-ex+d)}+\frac{4d\left(3d^2g^2+4defg+e^2f^2\right)\ln(-ex+d)}{e^3}$
risch	$\frac{g^2x^3}{3}+\frac{2d\,g^2x^2}{e}+fg\,x^2+\frac{8d^2g^2x}{e^2}+\frac{8dfgx}{e}+f^2x+\frac{4d^4g^2}{e^3(-ex+d)}+\frac{8d^3fg}{e^2(-ex+d)}+\frac{4d^2f^2}{e(-ex+d)}+\frac{12d^3\ln(-ex+d)g^2}{e^3}$
norman	$\frac{\left(-\frac{23}{3}d^2g^2-8defg-e^2f^2\right)x^3+\frac{d^2\left(6d^3g^2+9d^2efg+4d\,e^2f^2\right)}{e^3}+\frac{d^2\left(12d^2g^2+16defg+5e^2f^2\right)x}{e^2}-\frac{e^2g^2x^5}{3}-eg(2dg+ef)x^4}{-e^2x^2+d^2}+\frac{4d(3d^2g^2+4de}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/3*g^2*x^3*e^2+2*d*e*g^2*x^2+e^2*f*g*x^2+8*d^2*g^2*x+8*d*e*f*g*x+e^
2*f^2*x)+4*d^2*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)+4*d/e^3*(3*d^2*g^2+
4*d*e*f*g+e^2*f^2)*ln(-e*x+d)
```

Maxima [A]
time = 0.28, size = 137, normalized size = 1.28

$$4\left(3d^3g^2+4d^2fge+df^2e^2\right)e^{(-3)}\log\left(xe-d\right)+\frac{1}{3}\left(g^2x^3e^2+3\left(2dg^2e+fge^2\right)x^2+3\left(8d^2g^2+8dfge+f^2e^2\right)x\right)e^{(-2)}-\frac{4\left(d^4g^2+2d^3fge+d^2f^2e^2\right)}{xe^4-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 4*(3*d^3*g^2 + 4*d^2*f*g*e + d*f^2*e^2)*e^(-3)*log(x*e - d) + 1/3*(g^2*x^3*
e^2 + 3*(2*d*g^2*e + f*g*e^2)*x^2 + 3*(8*d^2*g^2 + 8*d*f*g*e + f^2*e^2)*x)*
e^(-2) - 4*(d^4*g^2 + 2*d^3*f*g*e + d^2*f^2*e^2)/(x*e^4 - d*e^3)
```

Fricas [A]
time = 2.76, size = 201, normalized size = 1.88

$$-\frac{12\,d^4g^2-(g^2x^4+3\,fgx^3+3\,f^2x^2)e^4-(5\,dg^2x^3+21\,dfgx^2-3\,df^2x)e^3-6\,(3\,d^2g^2x^2-4\,d^2fgx-2\,d^2f^2)e^2+24\,(d^3g^2x+d^3fg)e+12\,(3\,d^4g^2-df^2xe^3-(4\,d^2fgx-d^2f^2)e^2-(3\,d^3g^2x-4\,d^3fg)e)\log\left(xe-d\right)}{3\,(xe^4-de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*d^4*g^2 - (g^2*x^4 + 3*f*g*x^3 + 3*f^2*x^2)*e^4 - (5*d*g^2*x^3 + 2
1*d*f*g*x^2 - 3*d*f^2*x)*e^3 - 6*(3*d^2*g^2*x^2 - 4*d^2*f*g*x - 2*d^2*f^2)*
e^2 + 24*(d^3*g^2*x + d^3*f*g)*e + 12*(3*d^4*g^2 - d*f^2*x*e^3 - (4*d^2*f*g
*x - d^2*f^2)*e^2 - (3*d^3*g^2*x - 4*d^3*f*g)*e)*log(x*e - d))/(x*e^4 - d*e
^3)
```

Sympy [A]
time = 0.33, size = 119, normalized size = 1.11

$$\frac{4d(dg+ef)\,(3dg+ef)\log\left(-d+ex\right)}{e^3}+\frac{g^2x^3}{3}+x^2\cdot\left(\frac{2dg^2}{e}+fg\right)+x\left(\frac{8d^2g^2}{e^2}+\frac{8dfg}{e}+f^2\right)+\frac{-4d^4g^2-8d^3efg-4d^2e^2f^2}{-de^3+e^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] 4*d*(d*g + e*f)*(3*d*g + e*f)*log(-d + e*x)/e**3 + g**2*x**3/3 + x**2*(2*d*
g**2/e + f*g) + x*(8*d**2*g**2/e**2 + 8*d*f*g/e + f**2) + (-4*d**4*g**2 - 8
*d**3*e*f*g - 4*d**2*e**2*f**2)/(-d*e**3 + e**4*x)
```

Giac [A]
time = 1.37, size = 141, normalized size = 1.32

$$4\left(3\,d^3g^2+4\,d^2fge+df^2e^2\right)e^{(-3)}\log\left(|xe-d|\right)+\frac{1}{3}\left(g^2x^3e^6+6\,dg^2x^2e^5+24\,d^2g^2xe^4+3\,fgx^2e^6+24\,dfgxe^5+3\,f^2xe^6\right)e^{(-6)}-\frac{4\,(d^4g^2+2\,d^3fge+d^2f^2e^2)e^{(-3)}}{xe-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 4*(3*d^3*g^2 + 4*d^2*f*g*e + d*f^2*e^2)*e^(-3)*log(abs(x*e - d)) + 1/3*(g^2
*x^3*e^6 + 6*d*g^2*x^2*e^5 + 24*d^2*g^2*x*e^4 + 3*f*g*x^2*e^6 + 24*d*f*g*x*
e^5 + 3*f^2*x*e^6)*e^(-6) - 4*(d^4*g^2 + 2*d^3*f*g*e + d^2*f^2*e^2)*e^(-3)/
(x*e - d)
```

Mupad [B]
time = 0.07, size = 185, normalized size = 1.73

$$x^2\left(\frac{g\,(d\,g+e\,f)}{e}+\frac{d\,g^2}{e}\right)+x\,\left(\frac{d^2\,g^2+4\,d\,e\,f\,g+e^2\,f^2}{e^2}+\frac{2\,d\,\left(\frac{2\,g\,(d\,g+e\,f)}{e}+\frac{2\,d\,g^2}{e}\right)}{e}-\frac{d^2\,g^2}{e^2}\right)+\frac{g^2\,x^3}{3}+\frac{4\,(d^4\,g^2+2\,d^3\,e\,f\,g+d^2\,e^2\,f^2)}{e\,(d\,e^2-e^3\,x)}+\frac{\ln(e\,x-d)\,\left(12\,d^3\,g^2+16\,d^2\,e\,f\,g+4\,d\,e^2\,f^2\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x^2*((g*(d*g + e*f))/e + (d*g^2)/e) + x*((d^2*g^2 + e^2*f^2 + 4*d*e*f*g)/e^
2 + (2*d*((2*g*(d*g + e*f))/e + (2*d*g^2)/e))/e - (d^2*g^2)/e^2) + (g^2*x^3
)/3 + (4*(d^4*g^2 + d^2*e^2*f^2 + 2*d^3*e*f*g))/(e*(d*e^2 - e^3*x)) + (log(
e*x - d)*(12*d^3*g^2 + 4*d*e^2*f^2 + 16*d^2*e*f*g))/e^3
```

3.561 $\int \frac{(d+ex)^3(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=78

$$\frac{g(2ef+3dg)x}{e^2}+\frac{g^2x^2}{2e}+\frac{2d(ef+dg)^2}{e^3(d-ex)}+\frac{(ef+dg)(ef+5dg)\log(d-ex)}{e^3}$$

```
[Out] g*(3*d*g+2*e*f)*x/e^2+1/2*g^2*x^2/e+2*d*(d*g+e*f)^2/e^3/(-e*x+d)+(d*g+e*f)*
(5*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 78}

$$\frac{2d(dg+ef)^2}{e^3(d-ex)}+\frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3}+\frac{gx(3dg+2ef)}{e^2}+\frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g*(2*e*f + 3*d*g)*x)/e^2 + (g^2*x^2)/(2*e) + (2*d*(e*f + d*g)^2)/(e^3*(d -
 e*x)) + ((e*f + d*g)*(e*f + 5*d*g)*Log[d - e*x])/e^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \int \frac{(d+ex)(f+gx)^2}{(d-ex)^2}\,dx$$
$$= \int \left(\frac{g(2ef+3dg)}{e^2} + \frac{g^2x}{e} + \frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)} + \frac{2d(ef+dg)^2}{e^2(-d+ex)^2}\right)dx$$
$$= \frac{g(2ef+3dg)x}{e^2} + \frac{g^2x^2}{2e} + \frac{2d(ef+dg)^2}{e^3(d-ex)} + \frac{(ef+dg)(ef+5dg)\log(d-ex)}{e^3}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 1.06

$$\frac{2eg(2ef+3dg)x + e^2g^2x^2 + \frac{4d(ef+dg)^2}{d-ex} + 2(e^2f^2+6defg+5d^2g^2)\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (2*e*g*(2*e*f + 3*d*g)*x + e^2*g^2*x^2 + (4*d*(e*f + d*g)^2)/(d - e*x) + 2*
(e^2*f^2 + 6*d*e*f*g + 5*d^2*g^2)*Log[d - e*x])/(2*e^3)
```

Maple [A]
time = 0.09, size = 93, normalized size = 1.19

method	result
default	$\frac{g\left(\frac{1}{2}eg\,x^2+3dgx+2efx\right)}{e^2}+\frac{\left(5d^2g^2+6defg+e^2f^2\right)\ln(-ex+d)}{e^3}+\frac{2d\left(d^2g^2+2defg+e^2f^2\right)}{e^3(-ex+d)}$
risch	$\frac{g^2x^2}{2e}+\frac{3g^2dx}{e^2}+\frac{2gfx}{e}+\frac{5\ln(-ex+d)d^2g^2}{e^3}+\frac{6\ln(-ex+d)dfg}{e^2}+\frac{\ln(-ex+d)f^2}{e}+\frac{2d^3g^2}{e^3(-ex+d)}+\frac{4d^2fg}{e^2(-ex+d)}+\frac{2d\,f^2}{e(-ex+d)}$
norman	$\frac{\frac{d\left(5d^2g^2+6defg+2e^2f^2\right)x}{e^2}+\frac{d^2\left(5d^2g^2+8defg+4e^2f^2\right)}{2e^3}-\frac{e\,g^2x^4}{2}-g(3dg+2ef)x^3}{-e^2x^2+d^2}+\frac{\left(5d^2g^2+6defg+e^2f^2\right)\ln(-ex+d)}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] g/e^2*(1/2*e*g*x^2+3*d*g*x+2*e*f*x)+1/e^3*(5*d^2*g^2+6*d*e*f*g+e^2*f^2)*ln(
-e*x+d)+2*d*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)
```

Maxima [A]
time = 0.28, size = 103, normalized size = 1.32

$$(5\,d^2g^2+6\,dfge+f^2e^2)e^{(-3)}\log(xe-d)+\frac{1}{2}\left(g^2x^2e+2\left(3\,dg^2+2\,fge\right)x\right)e^{(-2)}-\frac{2\left(d^3g^2+2\,d^2fge+df^2e^2\right)}{xe^4-de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] (5*d^2*g^2 + 6*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e - d) + 1/2*(g^2*x^2*e + 2*
(3*d*g^2 + 2*f*g*e)*x)*e^(-2) - 2*(d^3*g^2 + 2*d^2*f*g*e + d*f^2*e^2)/(x*e^
4 - d*e^3)
```

Fricas [A]
time = 1.77, size = 155, normalized size = 1.99

$$-\frac{4\,d^3g^2-(g^2x^3+4\,fgx^2)e^3-(5\,dg^2x^2-4\,dfgx-4\,df^2)e^2+2\,(3\,d^2g^2x+4\,d^2fg)e+2\,(5\,d^3g^2-f^2xe^3-(6\,dfgx-df^2)e^2-(5\,d^2g^2x-6\,d^2fg)e)\log\left(xe-d\right)}{2\,(xe^4-de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^3*g^2 - (g^2*x^3 + 4*f*g*x^2)*e^3 - (5*d*g^2*x^2 - 4*d*f*g*x - 4*
d*f^2)*e^2 + 2*(3*d^2*g^2*x + 4*d^2*f*g)*e + 2*(5*d^3*g^2 - f^2*x*e^3 - (6*
d*f*g*x - d*f^2)*e^2 - (5*d^2*g^2*x - 6*d^2*f*g)*e)*log(x*e - d))/(x*e^4 -
d*e^3)
```

Sympy [A]
time = 0.28, size = 94, normalized size = 1.21

$$x\left(\frac{3dg^2}{e^2}+\frac{2fg}{e}\right)+\frac{-2d^3g^2-4d^2efg-2de^2f^2}{-de^3+e^4x}+\frac{g^2x^2}{2e}+\frac{(dg+ef)\,(5dg+ef)\log\left(-d+ex\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] x*(3*d*g**2/e**2 + 2*f*g/e) + (-2*d**3*g**2 - 4*d**2*e*f*g - 2*d*e**2*f**2)
/(-d*e**3 + e**4*x) + g**2*x**2/(2*e) + (d*g + e*f)*(5*d*g + e*f)*log(-d +
e*x)/e**3
```

Giac [A]
time = 2.00, size = 104, normalized size = 1.33

$$(5\,d^2g^2+6\,dfge+f^2e^2)e^{(-3)}\log\left(|xe-d|\right)+\frac{1}{2}\,(g^2x^2e^3+6\,dg^2xe^2+4\,fgxe^3)e^{(-4)}-\frac{2\,(d^3g^2+2\,d^2fge+df^2e^2)e^{(-3)}}{xe-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] (5*d^2*g^2 + 6*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e - d)) + 1/2*(g^2*x^2*e
^3 + 6*d*g^2*x*e^2 + 4*f*g*x*e^3)*e^(-4) - 2*(d^3*g^2 + 2*d^2*f*g*e + d*f^2
*e^2)*e^(-3)/(x*e - d)
```

Mupad [B]
time = 2.53, size = 116, normalized size = 1.49

$$x\left(\frac{d\,g^2+2\,e\,f\,g}{e^2}+\frac{2\,d\,g^2}{e^2}\right)+\frac{\ln\left(e\,x-d\right)\,\left(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2\right)}{e^3}+\frac{g^2\,x^2}{2\,e}+\frac{2\,\left(d^3\,g^2+2\,d^2\,e\,f\,g+d\,e^2\,f^2\right)}{e\,\left(d\,e^2-e^3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] x*((d*g^2 + 2*e*f*g)/e^2 + (2*d*g^2)/e^2) + (log(e*x - d)*(5*d^2*g^2 + e^2*
f^2 + 6*d*e*f*g))/e^3 + (g^2*x^2)/(2*e) + (2*(d^3*g^2 + d*e^2*f^2 + 2*d^2*e
*f*g))/(e*(d*e^2 - e^3*x))
```

3.562 $\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx$

Optimal. Leaf size=50

$$\frac{g^2x}{e^2}+\frac{(ef+dg)^2}{e^3(d-ex)}+\frac{2g(ef+dg)\log(d-ex)}{e^3}$$

```
[Out] g^2*x/e^2+(d*g+e*f)^2/e^3/(-e*x+d)+2*g*(d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 45}

$$\frac{(dg+ef)^2}{e^3(d-ex)}+\frac{2g(dg+ef)\log(d-ex)}{e^3}+\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (g^2*x)/e^2 + (e*f + d*g)^2/(e^3*(d - e*x)) + (2*g*(e*f + d*g)*Log[d - e*x]
)/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2}\,dx\\ &= \int \left(\frac{g^2}{e^2}+\frac{(ef+dg)^2}{e^2(-d+ex)^2}+\frac{2g(ef+dg)}{e^2(-d+ex)}\right)dx\\ &= \frac{g^2x}{e^2}+\frac{(ef+dg)^2}{e^3(d-ex)}+\frac{2g(ef+dg)\log(d-ex)}{e^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.92

$$\frac{eg^2x + \frac{(ef+dg)^2}{d-ex} + 2g(ef+dg)\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
[Out] (e*g^2*x + (e*f + d*g)^2/(d - e*x) + 2*g*(e*f + d*g)*Log[d - e*x])/e^3
```

Maple [A]
time = 0.07, size = 63, normalized size = 1.26

method	result	size
default	$\frac{g^2x}{e^2}+\frac{2g(dg+ef)\ln(-ex+d)}{e^3}+\frac{d^2g^2+2defg+e^2f^2}{e^3(-ex+d)}$	63
risch	$\frac{g^2x}{e^2}+\frac{2g^2\ln(-ex+d)d}{e^3}+\frac{2g\ln(-ex+d)f}{e^2}+\frac{d^2g^2}{e^3(-ex+d)}+\frac{2dfg}{e^2(-ex+d)}+\frac{f^2}{e(-ex+d)}$	89
norman	$\frac{\frac{d\left(d^2g^2+2defg+e^2f^2\right)}{e^3}+\frac{\left(2d^2g^2+2defg+e^2f^2\right)x}{e^2}-g^2x^3}{-e^2x^2+d^2}+\frac{2g(dg+ef)\ln(-ex+d)}{e^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
[Out] g^2*x/e^2+2*g*(d*g+e*f)*ln(-e*x+d)/e^3+(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*
x+d)
```

Maxima [A]
time = 0.29, size = 67, normalized size = 1.34

$$g^2xe^{(-2)} + 2\left(dg^2 + fge\right)e^{(-3)}\log\left(xe - d\right) - \frac{d^2g^2 + 2\,dfge + f^2e^2}{xe^4 - de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
[Out] g^2*x*e^(-2) + 2*(d*g^2 + f*g*e)*e^(-3)*log(x*e - d) - (d^2*g^2 + 2*d*f*g*e
 + f^2*e^2)/(x*e^4 - d*e^3)
```

Fricas [A]
time = 1.92, size = 96, normalized size = 1.92

$$-\frac{d^2g^2 - (g^2x^2 - f^2)e^2 + (dg^2x + 2\,dfg)e + 2\left(d^2g^2 - fgxe^2 - (dg^2x - dfg)e\right)\log\left(xe - d\right)}{xe^4 - de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -(d^2*g^2 - (g^2*x^2 - f^2)*e^2 + (d*g^2*x + 2*d*f*g)*e + 2*(d^2*g^2 - f*g*
x*e^2 - (d*g^2*x - d*f*g)*e)*log(x*e - d))/(x*e^4 - d*e^3)
```

Sympy [A]
time = 0.18, size = 61, normalized size = 1.22

$$\frac{-d^2g^2 - 2defg - e^2f^2}{-de^3 + e^4x} + \frac{g^2x}{e^2} + \frac{2g(dg + ef)\log(-d + ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-d*e**3 + e**4*x) + g**2*x/e**2 + 2*g
*(d*g + e*f)*log(-d + e*x)/e**3
```

Giac [A]
time = 1.59, size = 68, normalized size = 1.36

$$g^2xe^{(-2)} + 2\left(dg^2 + fge\right)e^{(-3)}\log\left(|xe - d|\right) - \frac{(d^2g^2 + 2\,dfge + f^2e^2)e^{(-3)}}{xe - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] g^2*x*e^(-2) + 2*(d*g^2 + f*g*e)*e^(-3)*log(abs(x*e - d)) - (d^2*g^2 + 2*d*
f*g*e + f^2*e^2)*e^(-3)/(x*e - d)
```

Mupad [B]
time = 2.56, size = 72, normalized size = 1.44

$$\frac{d^2\,g^2 + 2\,d\,e\,f\,g + e^2\,f^2}{e\,\left(d\,e^2 - e^3\,x\right)} + \frac{g^2\,x}{e^2} + \frac{\ln\left(e\,x - d\right)\,\left(2\,d\,g^2 + 2\,e\,f\,g\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^2,x)
```

```
[Out] (d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(e*(d*e^2 - e^3*x)) + (g^2*x)/e^2 + (log(e*
x - d)*(2*d*g^2 + 2*e*f*g))/e^3
```

3.563 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=86

$$\frac{(ef+dg)^2}{2de^3(d-ex)}-\frac{(ef-3dg)(ef+dg)\log(d-ex)}{4d^2e^3}+\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}$$

```
[Out] 1/2*(d*g+e*f)^2/d/e^3/(-e*x+d)-1/4*(-3*d*g+e*f)*(d*g+e*f)*ln(-e*x+d)/d^2/e^
3+1/4*(-d*g+e*f)^2*ln(e*x+d)/d^2/e^3
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {813, 90}

$$\frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}-\frac{(ef-3dg)(dg+ef)\log(d-ex)}{4d^2e^3}+\frac{(dg+ef)^2}{2de^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (e*f + d*g)^2/(2*d*e^3*(d - e*x)) - ((e*f - 3*d*g)*(e*f + d*g)*Log[d - e*x]
)/(4*d^2*e^3) + ((e*f - d*g)^2*Log[d + e*x])/(4*d^2*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 813

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c/g)*x)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2}\,dx = \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^2} + \frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)} + \frac{(-ef+dg)^2}{4d^2e^2(d+ex)}\right) dx$$
$$= \frac{(ef+dg)^2}{2de^3(d-ex)} - \frac{(ef-3dg)(ef+dg)\log(d-ex)}{4d^2e^3} + \frac{(ef-dg)^2\log(d+ex)}{4d^2e^3}$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 1.06

$$\frac{2d(ef+dg)^2 + (-e^2f^2+2defg+3d^2g^2)(d-ex)\log(d-ex) + (ef-dg)^2(d-ex)\log(d+ex)}{4d^2e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (2*d*(e*f + d*g)^2 + (-(e^2*f^2) + 2*d*e*f*g + 3*d^2*g^2)*(d - e*x)*Log[d -
 e*x] + (e*f - d*g)^2*(d - e*x)*Log[d + e*x])/(4*d^2*e^3*(d - e*x))
```

Maple [A]
time = 0.10, size = 112, normalized size = 1.30

method	result
default	$\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{4e^3d^2}+\frac{d^2g^2+2defg+e^2f^2}{2d\,e^3(-ex+d)}+\frac{(3d^2g^2+2defg-e^2f^2)\ln(-ex+d)}{4e^3d^2}$
norman	$\frac{-\frac{-d^2g^2-2defg-e^2f^2}{2e^3}+\frac{(d^2g^2+2defg+e^2f^2)x}{2e^2d}}{-e^2x^2+d^2}+\frac{(d^2g^2-2defg+e^2f^2)\ln(ex+d)}{4e^3d^2}+\frac{(3d^2g^2+2defg-e^2f^2)\ln(-ex+d)}{4e^3d^2}$
risch	$\frac{dg^2}{2e^3(-ex+d)}+\frac{fg}{e^2(-ex+d)}+\frac{f^2}{2de(-ex+d)}+\frac{3\ln(ex-d)g^2}{4e^3}+\frac{\ln(ex-d)fg}{2e^2d}-\frac{\ln(ex-d)f^2}{4e\,d^2}+\frac{\ln(-ex-d)g^2}{4e^3}-\frac{\ln(-ex-d)fg}{2e^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^2*ln(e*x+d)+1/2*(d^2*g^2+2*d*e*f*g+e^
2*f^2)/d/e^3/(-e*x+d)+1/4*(3*d^2*g^2+2*d*e*f*g-e^2*f^2)/e^3/d^2*ln(-e*x+d)
```

Maxima [A]
time = 0.30, size = 113, normalized size = 1.31

$$\frac{(d^2g^2-2\,dfge+f^2e^2)e^{(-3)}\log(xe+d)}{4\,d^2} + \frac{(3\,d^2g^2+2\,dfge-f^2e^2)e^{(-3)}\log(xe-d)}{4\,d^2} - \frac{d^2g^2+2\,dfge+f^2e^2}{2\,(dxe^4-d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e + d)/d^2 + 1/4*(3*d^2*g^
2 + 2*d*f*g*e - f^2*e^2)*e^(-3)*log(x*e - d)/d^2 - 1/2*(d^2*g^2 + 2*d*f*g*e
 + f^2*e^2)/(d*x*e^4 - d^2*e^3)
```

Fricas [A]
time = 3.00, size = 167, normalized size = 1.94

$$-\frac{2\,d^3g^2+4\,d^2fge+2\,df^2e^2+(d^3g^2-f^2xe^3+(2\,dfgx+df^2)e^2-(d^2g^2x+2\,d^2fg)e)\log\left(xe+d\right)+(3\,d^3g^2+f^2xe^3-(2\,dfgx+df^2)e^2-(3\,d^2g^2x-2\,d^2fg)e)\log\left(xe-d\right)}{4\,(d^2xe^4-d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^3*g^2 + 4*d^2*f*g*e + 2*d*f^2*e^2 + (d^3*g^2 - f^2*x*e^3 + (2*d*f
*g*x + d*f^2)*e^2 - (d^2*g^2*x + 2*d^2*f*g)*e)*log(x*e + d) + (3*d^3*g^2 +
f^2*x*e^3 - (2*d*f*g*x + d*f^2)*e^2 - (3*d^2*g^2*x - 2*d^2*f*g)*e)*log(x*e
- d))/(d^2*x*e^4 - d^3*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(75) = 150$.
time = 0.51, size = 182, normalized size = 2.12

$$\frac{-d^2g^2-2defg-e^2f^2}{-2d^2e^3+2de^4x}+\frac{(dg-ef)^2\log\left(x+\frac{2d^3g^2-d(dg-ef)^2}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}+\frac{(dg+ef)\,(3dg-ef)\log\left(x+\frac{2d^3g^2-d(dg+ef)(3dg-ef)}{d^2eg^2+2de^2fg-e^3f^2}\right)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-d**2*g**2 - 2*d*e*f*g - e**2*f**2)/(-2*d**2*e**3 + 2*d*e**4*x) + (d*g - e
*f)**2*log(x + (2*d**3*g**2 - d*(d*g - e*f)**2)/(d**2*e*g**2 + 2*d*e**2*f*g
 - e**3*f**2))/(4*d**2*e**3) + (d*g + e*f)*(3*d*g - e*f)*log(x + (2*d**3*g*
*2 - d*(d*g + e*f)*(3*d*g - e*f))/(d**2*e*g**2 + 2*d*e**2*f*g - e**3*f**2))
/(4*d**2*e**3)
```

Giac [A]
time = 2.11, size = 118, normalized size = 1.37

$$\frac{(d^2g^2-2\,dfge+f^2e^2)e^{(-3)}\log\left(|xe+d|\right)}{4\,d^2}+\frac{(3\,d^2g^2+2\,dfge-f^2e^2)e^{(-3)}\log\left(|xe-d|\right)}{4\,d^2}-\frac{(d^3g^2+2\,d^2fge+df^2e^2)e^{(-3)}}{2\,(xe-d)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^2 + 1/4*(3*d
^2*g^2 + 2*d*f*g*e - f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^2 - 1/2*(d^3*g^2 +
 2*d^2*f*g*e + d*f^2*e^2)*e^(-3)/((x*e - d)*d^2)
```

Mupad [B]
time = 2.64, size = 111, normalized size = 1.29

$$\frac{d^2 g^2 + 2 d e f g + e^2 f^2}{2 d e^3 (d - e x)} + \frac{\ln (d + e x) (d^2 g^2 - 2 d e f g + e^2 f^2)}{4 d^2 e^3} + \frac{\ln (d - e x) (3 d^2 g^2 + 2 d e f g - e^2 f^2)}{4 d^2 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^2,x)
```

```
[Out] (d^2*g^2 + e^2*f^2 + 2*d*e*f*g)/(2*d*e^3*(d - e*x)) + (log(d + e*x)*(d^2*g^
2 + e^2*f^2 - 2*d*e*f*g))/(4*d^2*e^3) + (log(d - e*x)*(3*d^2*g^2 - e^2*f^2
+ 2*d*e*f*g))/(4*d^2*e^3)
```

3.564 $\int \frac{(f+gx)^2}{\left(d^2-e^2x^2\right)^2} dx$

Optimal. Leaf size=74

$$\frac{(d^2g+e^2fx)(f+gx)}{2d^2e^2(d^2-e^2x^2)}+\frac{(ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

```
[Out] 1/2*(e^2*f*x+d^2*g)*(g*x+f)/d^2/e^2/(-e^2*x^2+d^2)+1/2*(-d*g+e*f)*(d*g+e*f)
*arctanh(e*x/d)/d^3/e^3
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {737, 214}

$$\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}+\frac{(f+gx)(d^2g+e^2fx)}{2d^2e^2(d^2-e^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(2*d^2*e^2*(d^2 - e^2*x^2)) + ((e*f - d*g)*(e
*f + d*g)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(f+gx)^2}{(d^2-e^2x^2)^2}\,dx &= \frac{(d^2g+e^2fx)(f+gx)}{2d^2e^2(d^2-e^2x^2)}-\frac{1}{2}\left(-\frac{f^2}{d^2}+\frac{g^2}{e^2}\right)\int \frac{1}{d^2-e^2x^2}\,dx\\ &= \frac{(d^2g+e^2fx)(f+gx)}{2d^2e^2(d^2-e^2x^2)}+\frac{(ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 85, normalized size = 1.15

$$\frac{-2d^2fg-e^2f^2x-d^2g^2x}{2d^2e^2\left(-d^2+e^2x^2\right)}-\frac{\left(-e^2f^2+d^2g^2\right)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^2,x]
```

```
[Out] (-2*d^2*f*g - e^2*f^2*x - d^2*g^2*x)/(2*d^2*e^2*(-d^2 + e^2*x^2)) - ((-(e^2
*f^2) + d^2*g^2)*ArcTanh[(e*x)/d])/(2*d^3*e^3)
```

Maple [A]
time = 0.08, size = 136, normalized size = 1.84

method	result	size
norman	$\frac{\frac{fg}{e^2}+\frac{\left(d^2g^2+e^2f^2\right)x}{2e^2d^2}}{-e^2x^2+d^2}+\frac{\left(d^2g^2-e^2f^2\right)\ln\left(-ex+d\right)}{4e^3d^3}-\frac{\left(d^2g^2-e^2f^2\right)\ln\left(ex+d\right)}{4e^3d^3}$	109
risch	$\frac{\frac{fg}{e^2}+\frac{\left(d^2g^2+e^2f^2\right)x}{2e^2d^2}}{-e^2x^2+d^2}+\frac{\ln\left(ex-d\right)g^2}{4e^3d}-\frac{\ln\left(ex-d\right)f^2}{4e\,d^3}-\frac{\ln\left(-ex-d\right)g^2}{4e^3d}+\frac{\ln\left(-ex-d\right)f^2}{4e\,d^3}$	126
default	$\frac{\left(-d^2g^2+e^2f^2\right)\ln\left(ex+d\right)}{4e^3d^3}-\frac{d^2g^2-2defg+e^2f^2}{4e^3d^2\left(ex+d\right)}+\frac{\left(d^2g^2-e^2f^2\right)\ln\left(-ex+d\right)}{4e^3d^3}+\frac{d^2g^2+2defg+e^2f^2}{4e^3d^2\left(-ex+d\right)}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/e^3/d^3*(-d^2*g^2+e^2*f^2)*ln(e*x+d)-1/4*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^
3/d^2/(e*x+d)+1/4/e^3*(d^2*g^2-e^2*f^2)/d^3*ln(-e*x+d)+1/4*(d^2*g^2+2*d*e*f
*g+e^2*f^2)/e^3/d^2/(-e*x+d)
```

Maxima [A]
time = 0.29, size = 106, normalized size = 1.43

$$-\frac{2\,d^2fg+\left(d^2g^2+f^2e^2\right)x}{2\left(d^2x^2e^4-d^4e^2\right)}-\frac{\left(d^2g^2-f^2e^2\right)e^{(-3)}\log\left(xe+d\right)}{4\,d^3}+\frac{\left(d^2g^2-f^2e^2\right)e^{(-3)}\log\left(xe-d\right)}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*d^2*f*g + (d^2*g^2 + f^2*e^2)*x)/(d^2*x^2*e^4 - d^4*e^2) - 1/4*(d^2
*g^2 - f^2*e^2)*e^(-3)*log(x*e + d)/d^3 + 1/4*(d^2*g^2 - f^2*e^2)*e^(-3)*lo
g(x*e - d)/d^3
```

Fricas [A]
time = 1.58, size = 126, normalized size = 1.70

$$-\frac{2\,df^2xe^4-\left(d^4g^2+f^2x^2e^4-\left(d^2g^2x^2+d^2f^2\right)e^2\right)e\log\left(\frac{x^2e^2+2\,dxe+d^2}{x^2e^2-d^2}\right)+2\left(d^3g^2x+2\,d^3fg\right)e^2}{4\left(d^3x^2e^6-d^5e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*f^2*x*e^4 - (d^4*g^2 + f^2*x^2*e^4 - (d^2*g^2*x^2 + d^2*f^2)*e^2)
*e*log((x^2*e^2 + 2*d*x*e + d^2)/(x^2*e^2 - d^2)) + 2*(d^3*g^2*x + 2*d^3*f*
g)*e^2)/(d^3*x^2*e^6 - d^5*e^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(66) = 132$.

time = 0.35, size = 156, normalized size = 2.11

$$\frac{-2d^2fg + x(-d^2g^2 - e^2f^2)}{-2d^4e^2 + 2d^2e^4x^2} + \frac{(dg - ef)(dg + ef)\log\left(-\frac{d(dg-ef)(dg+ef)}{e(d^2g^2-e^2f^2)} + x\right)}{4d^3e^3} - \frac{(dg - ef)(dg + ef)\log\left(\frac{d(dg-ef)(dg+ef)}{e(d^2g^2-e^2f^2)} + x\right)}{4d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**2*f*g + x*(-d**2*g**2 - e**2*f**2))/(-2*d**4*e**2 + 2*d**2*e**4*x**2
) + (d*g - e*f)*(d*g + e*f)*log(-d*(d*g - e*f)*(d*g + e*f)/(e*(d**2*g**2 -
e**2*f**2)) + x)/(4*d**3*e**3) - (d*g - e*f)*(d*g + e*f)*log(d*(d*g - e*f)*
(d*g + e*f)/(e*(d**2*g**2 - e**2*f**2)) + x)/(4*d**3*e**3)
```

Giac [A]

time = 2.05, size = 101, normalized size = 1.36

$$\frac{(d^2g^2 - f^2e^2)e^{(-3)}\log\left(\frac{|2\,xe^2-2\,|d|e|}{|2\,xe^2+2\,|d|e|}\right)}{4\,d^2|d|} - \frac{(d^2g^2x + 2\,d^2fg + f^2xe^2)e^{(-2)}}{2\,(x^2e^2 - d^2)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^2*g^2 - f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 +
2*abs(d)*e))/(d^2*abs(d)) - 1/2*(d^2*g^2*x + 2*d^2*f*g + f^2*x*e^2)*e^(-2)/
((x^2*e^2 - d^2)*d^2)
```

Mupad [B]

time = 2.61, size = 115, normalized size = 1.55

$$\frac{\frac{f\,g}{e^2} + \frac{x\,(d^2\,g^2+e^2\,f^2)}{2\,d^2\,e^2}}{d^2 - e^2\,x^2} - \frac{2\,\mathrm{atanh}\left(\frac{4\,e\,x\,\left(\frac{d^2\,g^2}{4}-\frac{e^2\,f^2}{4}\right)}{d\,(d^2\,g^2-e^2\,f^2)}\right)\,\left(\frac{d^2\,g^2}{4} - \frac{e^2\,f^2}{4}\right)}{d^3\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2)^2,x)
```

```
[Out] ((f*g)/e^2 + (x*(d^2*g^2 + e^2*f^2))/(2*d^2*e^2))/(d^2 - e^2*x^2) - (2*atan
h((4*e*x*((d^2*g^2)/4 - (e^2*f^2)/4))/(d*(d^2*g^2 - e^2*f^2)))*((d^2*g^2)/4
 - (e^2*f^2)/4))/(d^3*e^3)
```

3.565 $\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^2}\,dx$

Optimal. Leaf size=121

$$\frac{(ef+dg)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}+\frac{(3ef-dg)(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

```
[Out] 1/8*(d*g+e*f)^2/d^3/e^3/(-e*x+d)-1/8*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^2+1/4*(d^
2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)+1/8*(-d*g+3*e*f)*(d*g+e*f)*arctanh(e*x/d)/d^
4/e^3
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}+\frac{(dg+ef)^2}{8d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^2e^3(d+ex)^2}-\frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(8*d^3*e^3*(d - e*x)) - (e*f - d*g)^2/(8*d^2*e^3*(d + e*x)^2)
 - (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d + e*x)) + ((3*e*f - d*g)*(e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^2}\,dx = \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^3}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{8d^3e^2(d-ex)^2} + \frac{(-ef+dg)^2}{4d^2e^2(d+ex)^3} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^2} + \frac{(3ef-dg)(ef+\cdots}{8d^3e^2\left(d^2-e^2x^{\cdots}\right.}\right.$$
$$= \frac{(ef+dg)^2}{8d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^2e^3(d+ex)^2} - \frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)} + \frac{((3ef-dg)(ef+dg))\int\cdots}{8d^3e^2}$$
$$= \frac{(ef+dg)^2}{8d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^2e^3(d+ex)^2} - \frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)} + \frac{(3ef-dg)(ef+dg)\tan\cdots}{8d^4e^3}$$

Mathematica [A]
time = 0.08, size = 139, normalized size = 1.15

$$\frac{\frac{2d(ef+dg)^2}{d-ex} - \frac{2d^2(ef-dg)^2}{(d+ex)^2} + \frac{4d(-e^2f^2+d^2g^2)}{d+ex} + (-3e^2f^2-2defg+d^2g^2)\log(d-ex) + (3e^2f^2+2defg-d^2g^2)\log(d+ex)}{16d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((2*d*(e*f + d*g)^2)/(d - e*x) - (2*d^2*(e*f - d*g)^2)/(d + e*x)^2 + (4*d*(
-(e^2*f^2) + d^2*g^2))/(d + e*x) + (-3*e^2*f^2 - 2*d*e*f*g + d^2*g^2)*Log[d
 - e*x] + (3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

Maple [A]
time = 0.09, size = 180, normalized size = 1.49

method	result
default	$-\frac{-d^2g^2+e^2f^2}{4e^3d^3(ex+d)} + \frac{(-d^2g^2+2defg+3e^2f^2)\ln(ex+d)}{16e^3d^4} - \frac{d^2g^2-2defg+e^2f^2}{8e^3d^2(ex+d)^2} + \frac{(d^2g^2-2defg-3e^2f^2)\ln(-ex+d)}{16e^3d^4} + \frac{d^2g^2+2de\cdots}{8e^3d^3(-\cdots}$
norman	$\frac{-\frac{-d^2g^2-2defg+e^2f^2}{4d\,e^3} - \frac{\left(-3d^2g^2-2defg-3e^2f^2\right)x}{8e^2d^2} + \frac{\left(-d^2g^2+2defg+3e^2f^2\right)x^2}{8e\,d^3}}{(ex+d)^2(-ex+d)} + \frac{(d^2g^2-2defg-3e^2f^2)\ln(-ex+d)}{16e^3d^4} - \frac{(d^2g^2-2def\cdots}{}$
risch	$\frac{-\frac{\left(d^2g^2-2defg-3e^2f^2\right)x^2}{8e\,d^3} + \frac{\left(3d^2g^2+2defg+3e^2f^2\right)x}{8e^2d^2} + \frac{d^2g^2+2defg-e^2f^2}{4d\,e^3}}{(ex+d)(-e^2x^2+d^2)} + \frac{\ln(ex-d)g^2}{16e^3d^2} - \frac{\ln(ex-d)fg}{8e^2d^3} - \frac{3\ln(ex-d)f^2}{16e\,d^4} - \frac{\ln(-e\cdots}{16\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-d^2*g^2+e^2*f^2)/e^3/d^3/(e*x+d)+1/16*(-d^2*g^2+2*d*e*f*g+3*e^2*f^2)
/e^3/d^4*ln(e*x+d)-1/8*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^2/(e*x+d)^2+1/16/e
^3*(d^2*g^2-2*d*e*f*g-3*e^2*f^2)/d^4*ln(-e*x+d)+1/8*(d^2*g^2+2*d*e*f*g+e^2*
f^2)/e^3/d^3/(-e*x+d)
```

Maxima [A]
time = 0.29, size = 200, normalized size = 1.65

$$-\frac{2\,d^4g^2+4\,d^3fge-2\,d^2f^2e^2-(d^2g^2e^2-2\,dfge^3-3\,f^2e^4)x^2+(3\,d^3g^2e+2\,d^2fge^2+3\,df^2e^3)x}{8\,(d^3x^3e^6+d^4x^2e^5-d^5xe^4-d^6e^3)}-\frac{(d^2g^2-2\,dfge-3\,f^2e^2)e^{(-3)}\log{(xe+d)}}{16\,d^4}+\frac{(d^2g^2-2\,dfge-3\,f^2e^2)e^{(-3)}\log{(xe-d)}}{16\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*d^4*g^2 + 4*d^3*f*g*e - 2*d^2*f^2*e^2 - (d^2*g^2*e^2 - 2*d*f*g*e^3
- 3*f^2*e^4)*x^2 + (3*d^3*g^2*e + 2*d^2*f*g*e^2 + 3*d*f^2*e^3)*x)/(d^3*x^3*
e^6 + d^4*x^2*e^5 - d^5*x*e^4 - d^6*e^3) - 1/16*(d^2*g^2 - 2*d*f*g*e - 3*f^
2*e^2)*e^(-3)*log(x*e + d)/d^4 + 1/16*(d^2*g^2 - 2*d*f*g*e - 3*f^2*e^2)*e^(
-3)*log(x*e - d)/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(116) = 232$.
time = 2.07, size = 399, normalized size = 3.30

$$-\frac{4\,d^5g^2+6\,df^2x^2e^4+2\,(2\,d^2fgx^2+3\,d^2f^2x)e^3-2\,(d^3g^2x^2-2\,d^3fgx+2\,d^3f^2)e^2+2\,(3\,d^4g^2x+4\,d^4fg)e-(d^5g^2+3\,f^2x^3e^5+(2\,dfgx^3+3\,df^2x^2)e^4-(d^2g^2x^3-2\,d^2fgx^2+3\,d^2f^2x)e^3-(d^3g^2x^2+2\,d^3fgx+3\,d^3f^2)e^2+(d^4g^2x-2\,d^4fg)e)\log{(xe+d)}+(d^5g^2+3\,f^2x^3e^5+(2\,dfgx^3+3\,df^2x^2)e^4-(d^2g^2x^3-2\,d^2fgx^2+3\,d^2f^2x)e^3-(d^3g^2x^2+2\,d^3fgx+3\,d^3f^2)e^2+(d^4g^2x-2\,d^4fg)e)\log{(xe-d)}}{16\,(d^4x^3e^6+d^5x^2e^5-d^6xe^4-d^7e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*d^5*g^2 + 6*d*f^2*x^2*e^4 + 2*(2*d^2*f*g*x^2 + 3*d^2*f^2*x)*e^3 -
2*(d^3*g^2*x^2 - 2*d^3*f*g*x + 2*d^3*f^2)*e^2 + 2*(3*d^4*g^2*x + 4*d^4*f*g)
*e - (d^5*g^2 + 3*f^2*x^3*e^5 + (2*d*f*g*x^3 + 3*d*f^2*x^2)*e^4 - (d^2*g^2*
x^3 - 2*d^2*f*g*x^2 + 3*d^2*f^2*x)*e^3 - (d^3*g^2*x^2 + 2*d^3*f*g*x + 3*d^3
*f^2)*e^2 + (d^4*g^2*x - 2*d^4*f*g)*e)*log(x*e + d) + (d^5*g^2 + 3*f^2*x^3*
e^5 + (2*d*f*g*x^3 + 3*d*f^2*x^2)*e^4 - (d^2*g^2*x^3 - 2*d^2*f*g*x^2 + 3*d^
2*f^2*x)*e^3 - (d^3*g^2*x^2 + 2*d^3*f*g*x + 3*d^3*f^2)*e^2 + (d^4*g^2*x - 2
*d^4*f*g)*e)*log(x*e - d))/(d^4*x^3*e^6 + d^5*x^2*e^5 - d^6*x*e^4 - d^7*e^3
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(105) = 210$.
time = 0.63, size = 279, normalized size = 2.31

$$\frac{-2d^4g^2-4d^3efg+2d^2e^2f^2+x^2(d^2e^2g^2-2de^3fg-3e^4f^2)+x(-3d^3eg^2-2d^2e^2fg-3de^3f^2)}{-8d^6e^3-8d^5e^4x+8d^4e^5x^2+8d^3e^6x^3}+\frac{(dg-3ef)\,(dg+ef)\log{\left(-\frac{d(dg-3ef)(dg+ef)}{e(d^2g^2-2defg-3e^2f^2)}+x\right)}}{16d^4e^3}-\frac{(dg-3ef)\,(dg+ef)\log{\left(\frac{d(dg-3ef)(dg+ef)}{e(d^2g^2-2defg-3e^2f^2)}+x\right)}}{16d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**4*g**2 - 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 - 2*
d*e**3*f*g - 3*e**4*f**2) + x*(-3*d**3*e*g**2 - 2*d**2*e**2*f*g - 3*d*e**3*
f**2))/(-8*d**6*e**3 - 8*d**5*e**4*x + 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
```

```
 + (d*g - 3*e*f)*(d*g + e*f)*log(-d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2
 - 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - 3*e*f)*(d*g + e*f
)*log(d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2 - 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

Giac [A]
time = 1.99, size = 194, normalized size = 1.60

$$-\frac{(d^2g^2 - 2\,dfge - 3\,f^2e^2)e^{(-3)}\log\left(|xe + d|\right)}{16\,d^4} + \frac{(d^2g^2 - 2\,dfge - 3\,f^2e^2)e^{(-3)}\log\left(|xe - d|\right)}{16\,d^4} - \frac{(2\,d^5g^2 + 4\,d^4fge - 2\,d^3f^2e^2 - (d^3g^2e^2 - 2\,d^2fge^3 - 3\,df^2e^4)x^2 + (3\,d^4g^2e + 2\,d^3fge^2 + 3\,d^2f^2e^3)x)e^{(-3)}}{8\,(xe + d)^2(xe - d)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] -1/16*(d^2*g^2 - 2*d*f*g*e - 3*f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^4 + 1/16
*(d^2*g^2 - 2*d*f*g*e - 3*f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^4 - 1/8*(2*d^
5*g^2 + 4*d^4*f*g*e - 2*d^3*f^2*e^2 - (d^3*g^2*e^2 - 2*d^2*f*g*e^3 - 3*d*f^
2*e^4)*x^2 + (3*d^4*g^2*e + 2*d^3*f*g*e^2 + 3*d^2*f^2*e^3)*x)*e^(-3)/((x*e
+ d)^2*(x*e - d)*d^4)
```

Mupad [B]
time = 0.15, size = 198, normalized size = 1.64

$$\frac{\frac{d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3+d^2\,e\,x-d\,e^2\,x^2-e^3\,x^3}+\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g+e\,f\right)\,\left(d\,g-3\,e\,f\right)}{d\,\left(-d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g-3\,e\,f\right)}{8\,d^4\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)),x)
```

```
[Out] ((d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 + 2*
d*e*f*g))/(8*d^2*e^2) + (x^2*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(8*d^3*e))/
(d^3 - e^3*x^3 - d*e^2*x^2 + d^2*e*x) + (atanh((e*x*(d*g + e*f)*(d*g - 3*e*
f))/(d*(3*e^2*f^2 - d^2*g^2 + 2*d*e*f*g)))*(d*g + e*f)*(d*g - 3*e*f))/(8*d^
4*e^3)
```

3.566 $\int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2} dx$

Optimal. Leaf size=146

$$\frac{(ef+dg)^2}{16d^4e^3(d-ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\frac{(3ef-dg)(ef+dg)}{16d^4e^3(d+ex)}+\frac{f(ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}$$

```
[Out] 1/16*(d*g+e*f)^2/d^4/e^3/(-e*x+d)-1/12*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^3+1/8*(
d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^2-1/16*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*x
+d)+1/4*f*(d*g+e*f)*arctanh(e*x/d)/d^5/e^2
```

Rubi [A]
time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2}+\frac{(dg+ef)^2}{16d^4e^3(d-ex)}-\frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(16*d^4*e^3*(d - e*x)) - (e*f - d*g)^2/(12*d^2*e^3*(d + e*x)^
3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e*x)^2) - ((3*e*f - d*g)*(e*f + d*
g))/(16*d^4*e^3*(d + e*x)) + (f*(e*f + d*g)*ArcTanh[(e*x)/d])/(4*d^5*e^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^4}\,dx$$
$$=\int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^4}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^3}+\cdots\right)$$
$$=\frac{(ef+dg)^2}{16d^4e^3(d-ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\cdots$$
$$=\frac{(ef+dg)^2}{16d^4e^3(d-ex)}-\frac{(ef-dg)^2}{12d^2e^3(d+ex)^3}-\frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}-\cdots$$

Mathematica [A]
time = 0.07, size = 171, normalized size = 1.17

$$\frac{2d(2d^5g^2+3e^5f^2x^3+d^3e^2f(-4f+gx)+3de^4fx^2(2f+gx)+2d^4eg(f+2gx)+d^2e^3fx(f+6gx))+3ef(ef+dg)(-d+ex)(d+ex)^3\log(d-ex)+3ef(ef+dg)(d-ex)(d+ex)^3\log(d+ex)}{24d^5e^3(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (2*d*(2*d^5*g^2 + 3*e^5*f^2*x^3 + d^3*e^2*f*(-4*f + g*x) + 3*d*e^4*f*x^2*(2
*f + g*x) + 2*d^4*e*g*(f + 2*g*x) + d^2*e^3*f*x*(f + 6*g*x)) + 3*e*f*(e*f +
 d*g)*(-d + e*x)*(d + e*x)^3*Log[d - e*x] + 3*e*f*(e*f + d*g)*(d - e*x)*(d
+ e*x)^3*Log[d + e*x])/(24*d^5*e^3*(d - e*x)*(d + e*x)^3)
```

Maple [A]
time = 0.10, size = 189, normalized size = 1.29

method	result
norman	$\frac{\frac{dfg-3e f^2}{8d e^2}-\frac{\left(-d^2g^2-defg-e^2f^2\right)x^3}{3d^4}+\frac{f(dg+ef)x^2}{2d^3}-\frac{e\left(-4d^2g^2-defg-e^2f^2\right)x^4}{24d^5}}{(ex+d)^3(-ex+d)}-\frac{f(dg+ef)\ln(-ex+d)}{8d^5e^2}+\frac{f(dg+ef)\ln(ex+d)}{8d^5e^2}$
risch	$\frac{\frac{ef(dg+ef)x^3}{4d^4}+\frac{f(dg+ef)x^2}{2d^3}+\frac{\left(4d^2g^2+defg+e^2f^2\right)x}{12e^2d^2}+\frac{d^2g^2+defg-2e^2f^2}{6e^3d}}{(ex+d)^2(-e^2x^2+d^2)}-\frac{\ln(-ex+d)fg}{8e^2d^4}-\frac{\ln(-ex+d)f^2}{8e d^5}+\frac{\ln(ex+d)fg}{8e^2d^4}+\cdots$
default	$-\frac{-d^2g^2+e^2f^2}{8e^3d^3(ex+d)^2}-\frac{-d^2g^2+2defg+3e^2f^2}{16e^3d^4(ex+d)}-\frac{d^2g^2-2defg+e^2f^2}{12e^3d^2(ex+d)^3}+\frac{f(dg+ef)\ln(ex+d)}{8d^5e^2}+\frac{d^2g^2+2defg+e^2f^2}{16e^3d^4(-ex+d)}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-d^2*g^2+e^2*f^2)/e^3/d^3/(e*x+d)^2-1/16*(-d^2*g^2+2*d*e*f*g+3*e^2*f^
2)/e^3/d^4/(e*x+d)-1/12*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^2/(e*x+d)^3+1/8*f
*(d*g+e*f)/d^5/e^2*ln(e*x+d)+1/16*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/d^4/(-e*x
+d)-1/8*f*(d*g+e*f)/d^5/e^2*ln(-e*x+d)
```

Maxima [A]
time = 0.30, size = 189, normalized size = 1.29

$$-\frac{2\,d^5g^2+2\,d^4fge-4\,d^3f^2e^2+3\,(dfge^4+f^2e^5)x^3+6\,(d^2fge^3+df^2e^4)x^2+(4\,d^4g^2e+d^3fge^2+d^2f^2e^3)x}{12\,(d^4x^4e^7+2\,d^5x^3e^6-2\,d^7xe^4-d^8e^3)}+\frac{(dfg+f^2e)e^{(-2)}\log{(xe+d)}}{8\,d^5}-\frac{(dfg+f^2e)e^{(-2)}\log{(xe-d)}}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(2*d^5*g^2 + 2*d^4*f*g*e - 4*d^3*f^2*e^2 + 3*(d*f*g*e^4 + f^2*e^5)*x^
3 + 6*(d^2*f*g*e^3 + d*f^2*e^4)*x^2 + (4*d^4*g^2*e + d^3*f*g*e^2 + d^2*f^2*
e^3)*x)/(d^4*x^4*e^7 + 2*d^5*x^3*e^6 - 2*d^7*x*e^4 - d^8*e^3) + 1/8*(d*f*g
+ f^2*e)*e^(-2)*log(x*e + d)/d^5 - 1/8*(d*f*g + f^2*e)*e^(-2)*log(x*e - d)/
d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(140) = 280$.
time = 4.62, size = 331, normalized size = 2.27

$$-\frac{4\,d^6g^2+6\,df^2x^3e^5+6\,(d^2fgx^3+2\,d^2f^2x^2)e^4+2\,(6\,d^3fgx^2+d^3f^2x)e^3+2\,(d^4fgx-4\,d^4f^2)e^2+4\,(2\,d^5g^2x+d^5fg)e-3\,(2\,d^2fgx^3e^4-d^5fge+f^2x^4e^6-2\,d^3f^2xe^3+(dfgx^4+2\,df^2x^3)e^5-(2\,d^4fgx+d^4f^2)e^2)\log{(xe+d)}+3\,(2\,d^2fgx^3e^4-d^5fge+f^2x^4e^6-2\,d^3f^2xe^3+(dfgx^4+2\,df^2x^3)e^5-(2\,d^4fgx+d^4f^2)e^2)\log{(xe-d)}}{24\,(d^5x^4e^7+2\,d^6x^3e^6-2\,d^8xe^4-d^9e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(4*d^6*g^2 + 6*d*f^2*x^3*e^5 + 6*(d^2*f*g*x^3 + 2*d^2*f^2*x^2)*e^4 +
2*(6*d^3*f*g*x^2 + d^3*f^2*x)*e^3 + 2*(d^4*f*g*x - 4*d^4*f^2)*e^2 + 4*(2*d^
5*g^2*x + d^5*f*g)*e - 3*(2*d^2*f*g*x^3*e^4 - d^5*f*g*e + f^2*x^4*e^6 - 2*d
^3*f^2*x*e^3 + (d*f*g*x^4 + 2*d*f^2*x^3)*e^5 - (2*d^4*f*g*x + d^4*f^2)*e^2)
*log(x*e + d) + 3*(2*d^2*f*g*x^3*e^4 - d^5*f*g*e + f^2*x^4*e^6 - 2*d^3*f^2*
x*e^3 + (d*f*g*x^4 + 2*d*f^2*x^3)*e^5 - (2*d^4*f*g*x + d^4*f^2)*e^2)*log(x*
e - d))/(d^5*x^4*e^7 + 2*d^6*x^3*e^6 - 2*d^8*x*e^4 - d^9*e^3)
```

Sympy [A]
time = 0.68, size = 241, normalized size = 1.65

$$\frac{-2d^5g^2-2d^4efg+4d^3e^2f^2+x^3(-3de^4fg-3e^5f^2)+x^2(-6d^2e^3fg-6de^4f^2)+x(-4d^4eg^2-d^3e^2fg-d^2e^3f^2)}{-12d^8e^3-24d^7e^4x+24d^5e^6x^3+12d^4e^7x^4}-\frac{f(dg+ef)\log{\left(-\frac{df(dg+ef)}{e(dfg+ef^2)}+x\right)}}{8d^5e^2}+\frac{f(dg+ef)\log{\left(\frac{df(dg+ef)}{e(dfg+ef^2)}+x\right)}}{8d^5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-2*d**5*g**2 - 2*d**4*e*f*g + 4*d**3*e**2*f**2 + x**3*(-3*d*e**4*f*g - 3*e
**5*f**2) + x**2*(-6*d**2*e**3*f*g - 6*d*e**4*f**2) + x*(-4*d**4*e*g**2 - d
**3*e**2*f*g - d**2*e**3*f**2))/(-12*d**8*e**3 - 24*d**7*e**4*x + 24*d**5*e
**6*x**3 + 12*d**4*e**7*x**4) - f*(d*g + e*f)*log(-d*f*(d*g + e*f)/(e*(d*f*
g + e*f**2)) + x)/(8*d**5*e**2) + f*(d*g + e*f)*log(d*f*(d*g + e*f)/(e*(d*f
*g + e*f**2)) + x)/(8*d**5*e**2)
```

Giac [A]
time = 1.64, size = 227, normalized size = 1.55

$$-\frac{(dfg+f^2e)e^{(-2)}\log\left(\left|-\frac{2d}{xe+d}+1\right|\right)}{8\,d^5}+\frac{(d^2g^2+2\,dfge+f^2e^2)e^{(-3)}}{32\,d^5\left(\frac{2d}{xe+d}-1\right)}+\frac{\left(\frac{3\,d^4g^2e^3}{xe+d}+\frac{6\,d^5g^2e^3}{(xe+d)^2}-\frac{4\,d^6g^2e^3}{(xe+d)^3}-\frac{6\,d^3fge^4}{xe+d}+\frac{8\,d^5fge^4}{(xe+d)^3}-\frac{9\,d^2f^2e^5}{xe+d}-\frac{6\,d^3f^2e^5}{(xe+d)^2}-\frac{4\,d^4f^2e^5}{(xe+d)^3}\right)e^{(-6)}}{48\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(d*f*g + f^2*e)*e^(-2)*log(abs(-2*d/(x*e + d) + 1))/d^5 + 1/32*(d^2*g^
2 + 2*d*f*g*e + f^2*e^2)*e^(-3)/(d^5*(2*d/(x*e + d) - 1)) + 1/48*(3*d^4*g^2
*e^3/(x*e + d) + 6*d^5*g^2*e^3/(x*e + d)^2 - 4*d^6*g^2*e^3/(x*e + d)^3 - 6*
d^3*f*g*e^4/(x*e + d) + 8*d^5*f*g*e^4/(x*e + d)^3 - 9*d^2*f^2*e^5/(x*e + d)
 - 6*d^3*f^2*e^5/(x*e + d)^2 - 4*d^4*f^2*e^5/(x*e + d)^3)*e^(-6)/d^6
```

Mupad [B]
time = 2.63, size = 148, normalized size = 1.01

$$\frac{\frac{d^2\,g^2+d\,e\,f\,g-2\,e^2\,f^2}{6\,d\,e^3}+\frac{f\,x^2\,\left(d\,g+e\,f\right)}{2\,d^3}+\frac{x\,\left(4\,d^2\,g^2+d\,e\,f\,g+e^2\,f^2\right)}{12\,d^2\,e^2}+\frac{e\,f\,x^3\,\left(d\,g+e\,f\right)}{4\,d^4}}{d^4+2\,d^3\,e\,x-2\,d\,e^3\,x^3-e^4\,x^4}+\frac{f\,\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(d\,g+e\,f\right)}{4\,d^5\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - 2*e^2*f^2 + d*e*f*g)/(6*d*e^3) + (f*x^2*(d*g + e*f))/(2*d^3) +
(x*(4*d^2*g^2 + e^2*f^2 + d*e*f*g))/(12*d^2*e^2) + (e*f*x^3*(d*g + e*f))/(4
*d^4))/(d^4 - e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x) + (f*atanh((e*x)/d)*(d*g +
 e*f))/(4*d^5*e^2)
```

3.567 $\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx$

Optimal. Leaf size=178

$$\frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(3ef-dg)(ef+dg)}{32d^4e^3(d+ex)^2}-\frac{f(ef+dg)}{8d^5e^2(d+ex)}+\frac{(ef+dg)(5ef-}{32}$$

```
[Out] 1/32*(d*g+e*f)^2/d^5/e^3/(-e*x+d)-1/16*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^4+1/12*
(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^3-1/32*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*
x+d)^2-1/8*f*(d*g+e*f)/d^5/e^2/(e*x+d)+1/32*(d*g+e*f)*(d*g+5*e*f)*arctanh(e
*x/d)/d^6/e^3
```

Rubi [A]
time = 0.13, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\frac{(dg+ef)(dg+5ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^6e^3}+\frac{(dg+ef)^2}{32d^5e^3(d-ex)}-\frac{f(dg+ef)}{8d^5e^2(d+ex)}-\frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(32*d^5*e^3*(d - e*x)) - (e*f - d*g)^2/(16*d^2*e^3*(d + e*x)^
4) - (e^2*f^2 - d^2*g^2)/(12*d^3*e^3*(d + e*x)^3) - ((3*e*f - d*g)*(e*f + d
*g))/(32*d^4*e^3*(d + e*x)^2) - (f*(e*f + d*g))/(8*d^5*e^2*(d + e*x)) + ((e
*f + d*g)*(5*e*f + d*g)*ArcTanh[(e*x)/d])/(32*d^6*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[m, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)^3\left(d^2-e^2x^2\right)^2}\,dx=\int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^5}\,dx$$
$$=\int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^5}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^4}+\frac{(3ef-dg)(ef\ }{16d^4e^2(d+e\ }\right.$$
$$=\frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(3ef-dg)(ef+d\ }{32d^4e^3(d+ex)^2}$$
$$=\frac{(ef+dg)^2}{32d^5e^3(d-ex)}-\frac{(ef-dg)^2}{16d^2e^3(d+ex)^4}-\frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}-\frac{(3ef-dg)(ef+d\ }{32d^4e^3(d+ex)^2}$$

Mathematica [A]

time = 0.10, size = 195, normalized size = 1.10

$$\frac{\frac{6d(ef+dg)^2}{d-ex}-\frac{12d^4(ef-dg)^2}{(d+ex)^4}+\frac{16d^3\left(-e^2f^2+d^2g^2\right)}{(d+ex)^3}+\frac{6d^2\left(-3e^2f^2-2defg+d^2g^2\right)}{(d+ex)^2}-\frac{24def(ef+dg)}{d+ex}-3\left(5e^2f^2+6defg+d^2g^2\right)\log(d-ex)+3\left(5e^2f^2+6defg+d^2g^2\right)\log(d+ex)}{192d^6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^3*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((6*d*(e*f + d*g)^2)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (16*d
^3*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^3 + (6*d^2*(-3*e^2*f^2 - 2*d*e*f*g + d
^2*g^2))/(d + e*x)^2 - (24*d*e*f*(e*f + d*g))/(d + e*x) - 3*(5*e^2*f^2 + 6*
d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*Log[d
 + e*x])/(192*d^6*e^3)
```

Maple [A]

time = 0.10, size = 241, normalized size = 1.35

method	result
default	$\frac{\left(d^2g^2+6defg+5e^2f^2\right)\ln(ex+d)}{64e^3d^6}-\frac{-d^2g^2+e^2f^2}{12e^3d^3(ex+d)^3}-\frac{-d^2g^2+2defg+3e^2f^2}{32e^3d^4(ex+d)^2}-\frac{d^2g^2-2defg+e^2f^2}{16e^3d^2(ex+d)^4}-\frac{f(dg+ef)}{8d^5e^2(ex+d)}+\frac{(-d^2g}{}$
norman	$\frac{\frac{\left(25d^2g^2+54defg-19e^2f^2\right)x^3}{96d^4}-\frac{\left(3d^2g^2-14defg-33e^2f^2\right)x^2}{32e\,d^3}+\frac{3e\left(3d^2g^2+2defg-9e^2f^2\right)x^4}{32d^5}+\frac{e^2\left(d^2g^2-4e^2f^2\right)x^5}{12d^6}-\frac{\left(d^2g^2+6defg-27e^2f^2\right)x}{32d^2e^2}}{(ex+d)^4(-ex+d)}$
risch	$\frac{\frac{e\left(d^2g^2+6defg+5e^2f^2\right)x^4}{32d^5}+\frac{3\left(d^2g^2+6defg+5e^2f^2\right)x^3}{32d^4}+\frac{7\left(d^2g^2+6defg+5e^2f^2\right)x^2}{96d^3e}+\frac{\left(7d^2g^2-6defg-5e^2f^2\right)x}{32d^2e^2}+\frac{d^2g^2-4e^2f^2}{12e^3d}}{(ex+d)^3\left(-e^2x^2+d^2\right)}-\frac{\ln(-ex+d}{64e^3d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/e^3*(d^2*g^2+6*d*e*f*g+5*e^2*f^2)/d^6*ln(e*x+d)-1/12*(-d^2*g^2+e^2*f^2
)/e^3/d^3/(e*x+d)^3-1/32*(-d^2*g^2+2*d*e*f*g+3*e^2*f^2)/e^3/d^4/(e*x+d)^2-1
```

```
/16*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^2/(e*x+d)^4-1/8*f*(d*g+e*f)/d^5/e^2/(
e*x+d)+1/64*(-d^2*g^2-6*d*e*f*g-5*e^2*f^2)/e^3/d^6*ln(-e*x+d)+1/32*(d^2*g^2
+2*d*e*f*g+e^2*f^2)/e^3/d^5/(-e*x+d)
```

Maxima [A]
time = 0.31, size = 281, normalized size = 1.58

$$-\frac{8\,d^{6}g^{2}-32\,d^{4}f^{2}e^{2}+3\,(d^{2}g^{2}e^{4}+6\,dfge^{5}+5\,f^{2}e^{6})x^{4}+9\,(d^{3}g^{2}e^{3}+6\,d^{2}fge^{4}+5\,df^{2}e^{5})x^{3}+7\,(d^{4}g^{2}e^{2}+6\,d^{3}fge^{3}+5\,d^{2}f^{2}e^{4})x^{2}+3\,(7\,d^{5}g^{2}e-6\,d^{4}fge^{2}-5\,d^{3}f^{2}e^{3})x}{96\,(d^{5}x^{5}e^{8}+3\,d^{6}x^{4}e^{7}+2\,d^{7}x^{3}e^{6}-2\,d^{8}x^{2}e^{5}-3\,d^{9}xe^{4}-d^{10}e^{3})}+\frac{(d^{2}g^{2}+6\,dfge+5\,f^{2}e^{2})e^{(-3)}\log(xe+d)}{64\,d^{6}}-\frac{(d^{2}g^{2}+6\,dfge+5\,f^{2}e^{2})e^{(-3)}\log(xe-d)}{64\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/96*(8*d^6*g^2 - 32*d^4*f^2*e^2 + 3*(d^2*g^2*e^4 + 6*d*f*g*e^5 + 5*f^2*e^
6)*x^4 + 9*(d^3*g^2*e^3 + 6*d^2*f*g*e^4 + 5*d*f^2*e^5)*x^3 + 7*(d^4*g^2*e^2
 + 6*d^3*f*g*e^3 + 5*d^2*f^2*e^4)*x^2 + 3*(7*d^5*g^2*e - 6*d^4*f*g*e^2 - 5*
d^3*f^2*e^3)*x)/(d^5*x^5*e^8 + 3*d^6*x^4*e^7 + 2*d^7*x^3*e^6 - 2*d^8*x^2*e^
5 - 3*d^9*x*e^4 - d^10*e^3) + 1/64*(d^2*g^2 + 6*d*f*g*e + 5*f^2*e^2)*e^(-3)
*log(x*e + d)/d^6 - 1/64*(d^2*g^2 + 6*d*f*g*e + 5*f^2*e^2)*e^(-3)*log(x*e -
 d)/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(172) = 344$.
time = 3.53, size = 638, normalized size = 3.58

$$-\frac{42\,d^{6}g^{2}xe+16\,d^{7}g^{2}+30\,df^{2}x^{4}e^{6}+18\,(2\,d^{2}fgx^{4}+5\,d^{2}f^{2}x^{3})e^{5}+2\,(3\,d^{3}g^{2}x^{4}+54\,d^{3}fgx^{3}+35\,d^{3}f^{2}x^{2})e^{4}+6\,(3\,d^{4}g^{2}x^{3}+14\,d^{4}fgx^{2}-5\,d^{4}f^{2}x)e^{3}+2\,(7\,d^{5}g^{2}x^{2}-18\,d^{5}fgx-32\,d^{5}f^{2})e^{2}+3\,(d^{7}g^{2}-5\,f^{2}x^{5}e^{7}-3\,(2\,dfgx^{5}+5\,df^{2}x^{4})e^{6}-(d^{2}g^{2}x^{5}+18\,d^{2}fgx^{4}+10\,d^{2}f^{2}x^{3})e^{5}-(3\,d^{3}g^{2}x^{4}+12\,d^{3}fgx^{3}-10\,d^{3}f^{2}x^{2})e^{4}-(2\,d^{4}g^{2}x^{3}-12\,d^{4}fgx^{2}-15\,d^{4}f^{2}x)e^{3}+(2\,d^{5}g^{2}x^{2}+18\,d^{5}fgx+5\,d^{5}f^{2})e^{2}+3\,(d^{6}g^{2}x+2\,d^{6}fg)e)\log(xe+d)-3\,(d^{7}g^{2}-5\,f^{2}x^{5}e^{7}-3\,(2\,dfgx^{5}+5\,df^{2}x^{4})e^{6}-(d^{2}g^{2}x^{5}+18\,d^{2}fgx^{4}+10\,d^{2}f^{2}x^{3})e^{5}-(3\,d^{3}g^{2}x^{4}+12\,d^{3}fgx^{3}-10\,d^{3}f^{2}x^{2})e^{4}-(2\,d^{4}g^{2}x^{3}-12\,d^{4}fgx^{2}-15\,d^{4}f^{2}x)e^{3}+(2\,d^{5}g^{2}x^{2}+18\,d^{5}fgx+5\,d^{5}f^{2})e^{2}+3\,(d^{6}g^{2}x+2\,d^{6}fg)e)\log(xe-d)}{192\,(d^{6}x^{5}e^{8}+3\,d^{7}x^{4}e^{7}+2\,d^{8}x^{3}e^{6}-2\,d^{9}x^{2}e^{5}-3\,d^{10}xe^{4}-d^{11}e^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(42*d^6*g^2*x*e + 16*d^7*g^2 + 30*d*f^2*x^4*e^6 + 18*(2*d^2*f*g*x^4
+ 5*d^2*f^2*x^3)*e^5 + 2*(3*d^3*g^2*x^4 + 54*d^3*f*g*x^3 + 35*d^3*f^2*x^2)*
e^4 + 6*(3*d^4*g^2*x^3 + 14*d^4*f*g*x^2 - 5*d^4*f^2*x)*e^3 + 2*(7*d^5*g^2*x
^2 - 18*d^5*f*g*x - 32*d^5*f^2)*e^2 + 3*(d^7*g^2 - 5*f^2*x^5*e^7 - 3*(2*d*f
*g*x^5 + 5*d*f^2*x^4)*e^6 - (d^2*g^2*x^5 + 18*d^2*f*g*x^4 + 10*d^2*f^2*x^3)
*e^5 - (3*d^3*g^2*x^4 + 12*d^3*f*g*x^3 - 10*d^3*f^2*x^2)*e^4 - (2*d^4*g^2*x
^3 - 12*d^4*f*g*x^2 - 15*d^4*f^2*x)*e^3 + (2*d^5*g^2*x^2 + 18*d^5*f*g*x + 5
*d^5*f^2)*e^2 + 3*(d^6*g^2*x + 2*d^6*f*g)*e)*log(x*e + d) - 3*(d^7*g^2 - 5*
f^2*x^5*e^7 - 3*(2*d*f*g*x^5 + 5*d*f^2*x^4)*e^6 - (d^2*g^2*x^5 + 18*d^2*f*g
*x^4 + 10*d^2*f^2*x^3)*e^5 - (3*d^3*g^2*x^4 + 12*d^3*f*g*x^3 - 10*d^3*f^2*x
^2)*e^4 - (2*d^4*g^2*x^3 - 12*d^4*f*g*x^2 - 15*d^4*f^2*x)*e^3 + (2*d^5*g^2*
x^2 + 18*d^5*f*g*x + 5*d^5*f^2)*e^2 + 3*(d^6*g^2*x + 2*d^6*f*g)*e)*log(x*e
- d))/(d^6*x^5*e^8 + 3*d^7*x^4*e^7 + 2*d^8*x^3*e^6 - 2*d^9*x^2*e^5 - 3*d^10
*x*e^4 - d^11*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(162) = 324$.

time = 0.91, size = 376, normalized size = 2.11

$$\frac{-8d^6g^2+32d^4e^2f^2+x^4(-3d^2e^4g^2-18de^5fg-15e^6f^2)+x^3(-9d^3e^3g^2-54d^2e^4fg-45de^5f^2)+x^2(-7d^4e^2g^2-42d^3e^3fg-35d^2e^4f^2)+x(-21d^5eg^2+18d^4e^2fg+15d^3e^3f^2)}{-96d^{10}e^3-288d^9e^4x-192d^8e^5x^2+192d^7e^6x^3+288d^6e^7x^4+96d^5e^8x^5}-\frac{(dg+ef)\,(dg+5ef)\log\left(-\frac{d(dg+ef)(dg+5ef)}{e(d^2g^2+6defg+5e^2f^2)}+x\right)}{64d^6e^3}+\frac{(dg+ef)\,(dg+5ef)\log\left(\frac{d(dg+ef)(dg+5ef)}{e(d^2g^2+6defg+5e^2f^2)}+x\right)}{64d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-8*d**6*g**2 + 32*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2 - 18*d*e**5*f*g
 - 15*e**6*f**2) + x**3*(-9*d**3*e**3*g**2 - 54*d**2*e**4*f*g - 45*d*e**5*f
**2) + x**2*(-7*d**4*e**2*g**2 - 42*d**3*e**3*f*g - 35*d**2*e**4*f**2) + x*
(-21*d**5*e*g**2 + 18*d**4*e**2*f*g + 15*d**3*e**3*f**2))/(-96*d**10*e**3 -
 288*d**9*e**4*x - 192*d**8*e**5*x**2 + 192*d**7*e**6*x**3 + 288*d**6*e**7*
x**4 + 96*d**5*e**8*x**5) - (d*g + e*f)*(d*g + 5*e*f)*log(-d*(d*g + e*f)*(d
*g + 5*e*f)/(e*(d**2*g**2 + 6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3) +
 (d*g + e*f)*(d*g + 5*e*f)*log(d*(d*g + e*f)*(d*g + 5*e*f)/(e*(d**2*g**2 +
6*d*e*f*g + 5*e**2*f**2)) + x)/(64*d**6*e**3)
```

Giac [A]
time = 2.22, size = 254, normalized size = 1.43

$$\frac{(d^2g^2+6\,dfge+5\,f^2e^2)e^{(-3)}\log\left(|xe+d|\right)}{64\,d^6}-\frac{(d^2g^2+6\,dfge+5\,f^2e^2)e^{(-3)}\log\left(|xe-d|\right)}{64\,d^6}-\frac{(8\,d^7g^2-32\,d^5f^2e^2+3\,(d^3g^2e^4+6\,d^2fge^5+5\,df^2e^6)x^4+9\,(d^4g^2e^3+6\,d^3fge^4+5\,d^2f^2e^5)x^3+7\,(d^5g^2e^2+6\,d^4fge^3+5\,d^3f^2e^4)x^2+3\,(7\,d^6g^2e-6\,d^5fge^2-5\,d^4f^2e^3)x)e^{(-3)}}{96\,(xe+d)^4(xe-d)d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(d^2*g^2 + 6*d*f*g*e + 5*f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^6 - 1/64*
(d^2*g^2 + 6*d*f*g*e + 5*f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^6 - 1/96*(8*d^
7*g^2 - 32*d^5*f^2*e^2 + 3*(d^3*g^2*e^4 + 6*d^2*f*g*e^5 + 5*d*f^2*e^6)*x^4
+ 9*(d^4*g^2*e^3 + 6*d^3*f*g*e^4 + 5*d^2*f^2*e^5)*x^3 + 7*(d^5*g^2*e^2 + 6*
d^4*f*g*e^3 + 5*d^3*f^2*e^4)*x^2 + 3*(7*d^6*g^2*e - 6*d^5*f*g*e^2 - 5*d^4*f
^2*e^3)*x)*e^(-3)/((x*e + d)^4*(x*e - d)*d^6)
```

Mupad [B]
time = 2.70, size = 274, normalized size = 1.54

$$\frac{\frac{d^2\,g^2-4\,e^2\,f^2}{12\,d\,e^3}+\frac{3\,x^3\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^4}+\frac{e\,x^4\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^5}-\frac{x\,\left(-7\,d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{32\,d^2\,e^2}+\frac{7\,x^2\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}{96\,d^3\,e}}{d^5+3\,d^4\,e\,x+2\,d^3\,e^2\,x^2-2\,d^2\,e^3\,x^3-3\,d\,e^4\,x^4-e^5\,x^5}+\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g+e\,f\right)\,\left(d\,g+5\,e\,f\right)}{d\,\left(d^2\,g^2+6\,d\,e\,f\,g+5\,e^2\,f^2\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g+5\,e\,f\right)}{32\,d^6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^3),x)
```

```
[Out] ((d^2*g^2 - 4*e^2*f^2)/(12*d*e^3) + (3*x^3*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g
))/(32*d^4) + (e*x^4*(d^2*g^2 + 5*e^2*f^2 + 6*d*e*f*g))/(32*d^5) - (x*(5*e^
2*f^2 - 7*d^2*g^2 + 6*d*e*f*g))/(32*d^2*e^2) + (7*x^2*(d^2*g^2 + 5*e^2*f^2
+ 6*d*e*f*g))/(96*d^3*e))/(d^5 - e^5*x^5 - 3*d*e^4*x^4 + 2*d^3*e^2*x^2 - 2*
d^2*e^3*x^3 + 3*d^4*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 5*e*f))/(d*(d^2*g
^2 + 5*e^2*f^2 + 6*d*e*f*g)))*(d*g + e*f)*(d*g + 5*e*f))/(32*d^6*e^3)
```

3.568 $\int \frac{(f+gx)^2}{(d+ex)^4 \left(d^2-e^2x^2\right)^2} \, dx$

Optimal. Leaf size=210

$$\frac{(ef+dg)^2}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(3ef-dg)(ef+dg)}{48d^4e^3(d+ex)^3}-\frac{f(ef+dg)}{16d^5e^2(d+ex)^2}-\frac{(ef+dg)(5e}{64d^6e^3(d+}$$

```
[Out] 1/64*(d*g+e*f)^2/d^6/e^3/(-e*x+d)-1/20*(-d*g+e*f)^2/d^2/e^3/(e*x+d)^5+1/16*
(d^2*g^2-e^2*f^2)/d^3/e^3/(e*x+d)^4-1/48*(-d*g+3*e*f)*(d*g+e*f)/d^4/e^3/(e*
x+d)^3-1/16*f*(d*g+e*f)/d^5/e^2/(e*x+d)^2-1/64*(d*g+e*f)*(d*g+5*e*f)/d^6/e^
3/(e*x+d)+1/32*(d*g+e*f)*(d*g+3*e*f)*arctanh(e*x/d)/d^7/e^3
```

Rubi [A]
time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\frac{(dg+ef)(dg+3ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^7e^3}+\frac{(dg+ef)^2}{64d^6e^3(d-ex)}-\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)}-\frac{f(dg+ef)}{16d^5e^2(d+ex)^2}-\frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] (e*f + d*g)^2/(64*d^6*e^3*(d - e*x)) - (e*f - d*g)^2/(20*d^2*e^3*(d + e*x)^
5) - (e^2*f^2 - d^2*g^2)/(16*d^3*e^3*(d + e*x)^4) - ((3*e*f - d*g)*(e*f + d
*g))/(48*d^4*e^3*(d + e*x)^3) - (f*(e*f + d*g))/(16*d^5*e^2*(d + e*x)^2) -
((e*f + d*g)*(5*e*f + d*g))/(64*d^6*e^3*(d + e*x)) + ((e*f + d*g)*(3*e*f +
d*g)*ArcTanh[(e*x)/d])/(32*d^7*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
```

```
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^4\left(d^2-e^2x^2\right)^2}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^2(d+ex)^6}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{64d^6e^2(d-ex)^2}+\frac{(-ef+dg)^2}{4d^2e^2(d+ex)^6}+\frac{e^2f^2-d^2g^2}{4d^3e^2(d+ex)^5}+\frac{(3ef-dg)(ef-}{16d^4e^2(d+e}\right. \\
&= \frac{(ef+dg)^2}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(3ef-dg)(ef+d}{48d^4e^3(d+ex)^3} \\
&= \frac{(ef+dg)^2}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{20d^2e^3(d+ex)^5}-\frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}-\frac{(3ef-dg)(ef+d}{48d^4e^3(d+ex)^3}
\end{aligned}
$$

Mathematica [A]

time = 0.12, size = 229, normalized size = 1.09

$$
\frac{\frac{15d(ef+dg)^2}{d-ex}-\frac{48d^5(ef-dg)^2}{(d+ex)^5}+\frac{60d^4(-e^2f^2+d^2g^2)}{(d+ex)^4}+\frac{20d^3(-3e^2f^2-2defg+d^2g^2)}{(d+ex)^3}-\frac{60d^2ef(ef+dg)}{(d+ex)^2}-\frac{15d(5e^2f^2+6defg+d^2g^2)}{d+ex}-15(3e^2f^2+4defg+d^2g^2)\log(d-ex)+15(3e^2f^2+4defg+d^2g^2)\log(d+ex)}{960d^7e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^4*(d^2 - e^2*x^2)^2),x]
```

```
[Out] ((15*d*(e*f + d*g)^2)/(d - e*x) - (48*d^5*(e*f - d*g)^2)/(d + e*x)^5 + (60*
d^4*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^4 + (20*d^3*(-3*e^2*f^2 - 2*d*e*f*g +
 d^2*g^2))/(d + e*x)^3 - (60*d^2*e*f*(e*f + d*g))/(d + e*x)^2 - (15*d*(5*e^
2*f^2 + 6*d*e*f*g + d^2*g^2))/(d + e*x) - 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g
^2)*Log[d - e*x] + 15*(3*e^2*f^2 + 4*d*e*f*g + d^2*g^2)*Log[d + e*x])/(960*
d^7*e^3)
```

Maple [A]

time = 0.11, size = 278, normalized size = 1.32

method	result
default	$\frac{\left(d^2g^2+4defg+3e^2f^2\right)\ln(ex+d)}{64e^3d^7}-\frac{d^2g^2+6defg+5e^2f^2}{64e^3d^6(ex+d)}-\frac{-d^2g^2+e^2f^2}{16e^3d^3(ex+d)^4}-\frac{-d^2g^2+2defg+3e^2f^2}{48e^3d^4(ex+d)^3}-\frac{d^2g^2-2defg+e^2f^2}{20e^3d^2(ex+d)^5}-$
norman	$\frac{\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^3}{6d^4}-\frac{\left(d^2g^2-4defg-13e^2f^2\right)x^2}{8e\,d^3}+\frac{e\left(7d^2g^2+4defg-27e^2f^2\right)x^4}{24d^5}+\frac{e^2\left(79d^2g^2-68defg-531e^2f^2\right)x^5}{480d^6}+\frac{e^3\left(d^2g^2-2defg-9e\right.}{30d^7}}{(ex+d)^5(-ex+d)}$
risch	$\frac{\frac{e^2\left(d^2g^2+4defg+3e^2f^2\right)x^5}{32d^6}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)e\,x^4}{8d^5}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^3}{6d^4}+\frac{\left(d^2g^2+4defg+3e^2f^2\right)x^2}{24d^3e}+\frac{\left(49d^2g^2-188defg-141e^2f^2\right)}{480e^2d^2}}{(ex+d)^4(-e^2x^2+d^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/e^3*(d^2*g^2+4*d*e*f*g+3*e^2*f^2)/d^7*ln(e*x+d)-1/64/e^3*(d^2*g^2+6*d*
e*f*g+5*e^2*f^2)/d^6/(e*x+d)-1/16*(-d^2*g^2+e^2*f^2)/e^3/d^3/(e*x+d)^4-1/48
*(-d^2*g^2+2*d*e*f*g+3*e^2*f^2)/e^3/d^4/(e*x+d)^3-1/20*(d^2*g^2-2*d*e*f*g+e
^2*f^2)/e^3/d^2/(e*x+d)^5-1/16*f*(d*g+e*f)/d^5/e^2/(e*x+d)^2+1/64*(-d^2*g^2
-4*d*e*f*g-3*e^2*f^2)/e^3/d^7*ln(-e*x+d)+1/64*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e
^3/d^6/(-e*x+d)
```

Maxima [A]
time = 0.31, size = 323, normalized size = 1.54

$$-\frac{16\,d^{7}g^{2}-32\,d^{6}fge-144\,d^{5}f^{2}e^{2}+15\,(d^{2}g^{2}e^{5}+4\,dfge^{6}+3\,f^{2}e^{7})x^{5}+60\,(d^{3}g^{2}e^{4}+4\,d^{2}fge^{5}+3\,df^{2}e^{6})x^{4}+80\,(d^{4}g^{2}e^{3}+4\,d^{3}fge^{4}+3\,d^{2}f^{2}e^{5})x^{3}+20\,(d^{5}g^{2}e^{2}+4\,d^{4}fge^{3}+3\,d^{3}f^{2}e^{4})x^{2}+(49\,d^{6}g^{2}e-188\,d^{5}fge^{2}-141\,d^{4}f^{2}e^{3})x}{480\,(d^{6}x^{6}e^{9}+4\,d^{7}x^{5}e^{8}+5\,d^{8}x^{4}e^{7}-5\,d^{10}x^{2}e^{5}-4\,d^{11}xe^{4}-d^{12}e^{3})}+\frac{(d^{2}g^{2}+4\,dfge+3\,f^{2}e^{2})e^{(-3)}\log(xe+d)}{64\,d^{7}}-\frac{(d^{2}g^{2}+4\,dfge+3\,f^{2}e^{2})e^{(-3)}\log(xe-d)}{64\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="maxima")
```

```
[Out] -1/480*(16*d^7*g^2 - 32*d^6*f*g*e - 144*d^5*f^2*e^2 + 15*(d^2*g^2*e^5 + 4*d
*f*g*e^6 + 3*f^2*e^7)*x^5 + 60*(d^3*g^2*e^4 + 4*d^2*f*g*e^5 + 3*d*f^2*e^6)*
x^4 + 80*(d^4*g^2*e^3 + 4*d^3*f*g*e^4 + 3*d^2*f^2*e^5)*x^3 + 20*(d^5*g^2*e^
2 + 4*d^4*f*g*e^3 + 3*d^3*f^2*e^4)*x^2 + (49*d^6*g^2*e - 188*d^5*f*g*e^2 -
141*d^4*f^2*e^3)*x)/(d^6*x^6*e^9 + 4*d^7*x^5*e^8 + 5*d^8*x^4*e^7 - 5*d^10*x
^2*e^5 - 4*d^11*x*e^4 - d^12*e^3) + 1/64*(d^2*g^2 + 4*d*f*g*e + 3*f^2*e^2)*
e^(-3)*log(x*e + d)/d^7 - 1/64*(d^2*g^2 + 4*d*f*g*e + 3*f^2*e^2)*e^(-3)*log
(x*e - d)/d^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(204) = 408.
time = 4.15, size = 685, normalized size = 3.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="fricas")
```

```
[Out] -1/960*(32*d^8*g^2 + 90*d*f^2*x^5*e^7 + 120*(d^2*f*g*x^5 + 3*d^2*f^2*x^4)*e
^6 + 30*(d^3*g^2*x^5 + 16*d^3*f*g*x^4 + 16*d^3*f^2*x^3)*e^5 + 40*(3*d^4*g^2
*x^4 + 16*d^4*f*g*x^3 + 3*d^4*f^2*x^2)*e^4 + 2*(80*d^5*g^2*x^3 + 80*d^5*f*g
*x^2 - 141*d^5*f^2*x)*e^3 + 8*(5*d^6*g^2*x^2 - 47*d^6*f*g*x - 36*d^6*f^2)*e
^2 + 2*(49*d^7*g^2*x - 32*d^7*f*g)*e + 15*(d^8*g^2 - 3*f^2*x^6*e^8 - 4*(d*f
*g*x^6 + 3*d*f^2*x^5)*e^7 - (d^2*g^2*x^6 + 16*d^2*f*g*x^5 + 15*d^2*f^2*x^4)
*e^6 - 4*(d^3*g^2*x^5 + 5*d^3*f*g*x^4)*e^5 - 5*(d^4*g^2*x^4 - 3*d^4*f^2*x^2
)*e^4 + 4*(5*d^5*f*g*x^2 + 3*d^5*f^2*x)*e^3 + (5*d^6*g^2*x^2 + 16*d^6*f*g*x
 + 3*d^6*f^2)*e^2 + 4*(d^7*g^2*x + d^7*f*g)*e)*log(x*e + d) - 15*(d^8*g^2 -
 3*f^2*x^6*e^8 - 4*(d*f*g*x^6 + 3*d*f^2*x^5)*e^7 - (d^2*g^2*x^6 + 16*d^2*f*
g*x^5 + 15*d^2*f^2*x^4)*e^6 - 4*(d^3*g^2*x^5 + 5*d^3*f*g*x^4)*e^5 - 5*(d^4*
```

```
g^2*x^4 - 3*d^4*f^2*x^2)*e^4 + 4*(5*d^5*f*g*x^2 + 3*d^5*f^2*x)*e^3 + (5*d^6
*g^2*x^2 + 16*d^6*f*g*x + 3*d^6*f^2)*e^2 + 4*(d^7*g^2*x + d^7*f*g)*e)*log(x
*e - d))/(d^7*x^6*e^9 + 4*d^8*x^5*e^8 + 5*d^9*x^4*e^7 - 5*d^11*x^2*e^5 - 4*
d^12*x*e^4 - d^13*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(192) = 384$.
time = 1.06, size = 427, normalized size = 2.03

$$\frac{-16d^7g^2+32d^6efg+144d^5e^2f^2+x^5(-15d^2e^5g^2-60de^6fg-45e^7f^2)+x^4(-60d^3e^4g^2-240d^2e^5fg-180de^6f^2)+x^3(-80d^4e^3g^2-320d^3e^4fg-240d^2e^5f^2)+x^2(-20d^5e^2g^2-80d^4e^3fg-60d^3e^4f^2)+x(-49d^6eg^2+188d^5e^2fg+141d^4e^3f^2)}{-480d^{12}e^3-1920d^{11}e^4x-2400d^{10}e^5x^2+2400d^8e^7x^4+1920d^7e^8x^5+480d^6e^9x^6} - \frac{(dg+ef)(dg+3ef)\log\left(-\frac{d(dg+ef)(dg+3ef)}{e(d^2g^2+4defg+3e^2f^2)}+x\right)}{64d^7e^3} + \frac{(dg+ef)(dg+3ef)\log\left(\frac{d(dg+ef)(dg+3ef)}{e(d^2g^2+4defg+3e^2f^2)}+x\right)}{64d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**4/(-e**2*x**2+d**2)**2,x)
```

```
[Out] (-16*d**7*g**2 + 32*d**6*e*f*g + 144*d**5*e**2*f**2 + x**5*(-15*d**2*e**5*g
**2 - 60*d*e**6*f*g - 45*e**7*f**2) + x**4*(-60*d**3*e**4*g**2 - 240*d**2*e
**5*f*g - 180*d*e**6*f**2) + x**3*(-80*d**4*e**3*g**2 - 320*d**3*e**4*f*g -
 240*d**2*e**5*f**2) + x**2*(-20*d**5*e**2*g**2 - 80*d**4*e**3*f*g - 60*d**
3*e**4*f**2) + x*(-49*d**6*e*g**2 + 188*d**5*e**2*f*g + 141*d**4*e**3*f**2)
)/(-480*d**12*e**3 - 1920*d**11*e**4*x - 2400*d**10*e**5*x**2 + 2400*d**8*e
**7*x**4 + 1920*d**7*e**8*x**5 + 480*d**6*e**9*x**6) - (d*g + e*f)*(d*g + 3
*e*f)*log(-d*(d*g + e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f
**2)) + x)/(64*d**7*e**3) + (d*g + e*f)*(d*g + 3*e*f)*log(d*(d*g + e*f)*(d*
g + 3*e*f)/(e*(d**2*g**2 + 4*d*e*f*g + 3*e**2*f**2)) + x)/(64*d**7*e**3)
```

Giac [A]
time = 2.00, size = 296, normalized size = 1.41

$$\frac{(d^2g^2+4\,dfge+3\,f^2e^2)e^{(-3)}\log\left(|xe+d|\right)}{64\,d^7} - \frac{(d^2g^2+4\,dfge+3\,f^2e^2)e^{(-3)}\log\left(|xe-d|\right)}{64\,d^7} - \frac{(16\,d^8g^2-32\,d^7fge-144\,d^6f^2e^2+15\,(d^3g^2e^5+4\,d^2fge^6+3\,df^2e^7)x^5+60\,(d^4g^2e^4+4\,d^3fge^5+3\,d^2f^2e^6)x^4+80\,(d^5g^2e^3+4\,d^4fge^4+3\,d^3f^2e^5)x^3+20\,(d^6g^2e^2+4\,d^5fge^3+3\,d^4f^2e^4)x^2+(49\,d^7g^2e-188\,d^6fge^2-141\,d^5f^2e^3)x)e^{(-3)}}{480\,(xe+d)^5(xe-d)d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(d^2*g^2 + 4*d*f*g*e + 3*f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^7 - 1/64*
(d^2*g^2 + 4*d*f*g*e + 3*f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^7 - 1/480*(16*
d^8*g^2 - 32*d^7*f*g*e - 144*d^6*f^2*e^2 + 15*(d^3*g^2*e^5 + 4*d^2*f*g*e^6
+ 3*d*f^2*e^7)*x^5 + 60*(d^4*g^2*e^4 + 4*d^3*f*g*e^5 + 3*d^2*f^2*e^6)*x^4 +
 80*(d^5*g^2*e^3 + 4*d^4*f*g*e^4 + 3*d^3*f^2*e^5)*x^3 + 20*(d^6*g^2*e^2 + 4
*d^5*f*g*e^3 + 3*d^4*f^2*e^4)*x^2 + (49*d^7*g^2*e - 188*d^6*f*g*e^2 - 141*d
^5*f^2*e^3)*x)*e^(-3)/((x*e + d)^5*(x*e - d)*d^7)
```

Mupad [B]
time = 2.72, size = 314, normalized size = 1.50

$$\frac{\frac{x^3\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}{6\,d^4}-\frac{-d^2\,g^2+2\,d\,e\,f\,g+9\,e^2\,f^2}{30\,d\,e^3}+\frac{e\,x^4\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^5}-\frac{x\,\left(-49\,d^2\,g^2+188\,d\,e\,f\,g+141\,e^2\,f^2\right)}{480\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}{24\,d^3\,e}+\frac{e^2\,x^5\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}{32\,d^6}}{d^6+4\,d^5\,e\,x+5\,d^4\,e^2\,x^2-5\,d^2\,e^4\,x^4-4\,d\,e^5\,x^5-e^6\,x^6}+\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g+e\,f\right)\,\left(d\,g+3\,e\,f\right)}{d\,\left(d^2\,g^2+4\,d\,e\,f\,g+3\,e^2\,f^2\right)}\right)\,\left(d\,g+e\,f\right)\,\left(d\,g+3\,e\,f\right)}{32\,d^7\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^2*(d + e*x)^4),x)

[Out] ((x^3*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(6*d^4) - (9*e^2*f^2 - d^2*g^2 + 2
*d*e*f*g)/(30*d*e^3) + (e*x^4*(d^2*g^2 + 3*e^2*f^2 + 4*d*e*f*g))/(8*d^5) -
(x*(141*e^2*f^2 - 49*d^2*g^2 + 188*d*e*f*g))/(480*d^2*e^2) + (x^2*(d^2*g^2
+ 3*e^2*f^2 + 4*d*e*f*g))/(24*d^3*e) + (e^2*x^5*(d^2*g^2 + 3*e^2*f^2 + 4*d*
e*f*g))/(32*d^6))/(d^6 - e^6*x^6 - 4*d*e^5*x^5 + 5*d^4*e^2*x^2 - 5*d^2*e^4*
x^4 + 4*d^5*e*x) + (atanh((e*x*(d*g + e*f)*(d*g + 3*e*f))/(d*(d^2*g^2 + 3*e
^2*f^2 + 4*d*e*f*g)))*(d*g + e*f)*(d*g + 3*e*f))/(32*d^7*e^3)
```

3.569 $\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal. Leaf size=179

$$-\frac{d(7e^2f^2+48defg+56d^2g^2)\,x}{e^2}-\frac{(ef+2dg)(ef+12dg)x^2}{2e}-\frac{1}{3}g(2ef+7dg)x^3-\frac{1}{4}eg^2x^4+\frac{8d^4(ef+dg)^2}{e^3(d-ex)^2}-32$$

```
[Out] -d*(56*d^2*g^2+48*d*e*f*g+7*e^2*f^2)*x/e^2-1/2*(2*d*g+e*f)*(12*d*g+e*f)*x^2
/e-1/3*g*(7*d*g+2*e*f)*x^3-1/4*e*g^2*x^4+8*d^4*(d*g+e*f)^2/e^3/(-e*x+d)^2-3
2*d^3*(d*g+e*f)*(2*d*g+e*f)/e^3/(-e*x+d)-8*d^2*(13*d^2*g^2+14*d*e*f*g+3*e^2
*f^2)*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.16, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 90}

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2}-\frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)}-\frac{dx(56d^2g^2+48defg+7e^2f^2)}{e^2}-\frac{8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)}{e^3}-\frac{1}{3}gx^3(7dg+2ef)-\frac{x^2(2dg+ef)(12dg+ef)}{2e}-\frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e*f + 2*d*g)*(e*f +
12*d*g)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*d^4*(e*
f + d*g)^2)/(e^3*(d - e*x)^2) - (32*d^3*(e*f + d*g)*(e*f + 2*d*g))/(e^3*(d
- e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \int \frac{(d+ex)^4(f+gx)^2}{(d-ex)^3}\,dx$$
$$= \int \left(-\frac{d(7e^2f^2+48defg+56d^2g^2)}{e^2} + \frac{(-ef-12dg)(ef+2dg)x}{e} - g(2ef+7dg\right.$$
$$= -\frac{d(7e^2f^2+48defg+56d^2g^2)\,x}{e^2} - \frac{(ef+2dg)(ef+12dg)x^2}{2e} - \frac{1}{3}g(2ef+7dg)x$$

Mathematica [A]
time = 0.06, size = 193, normalized size = 1.08

$$-\frac{d(7e^2f^2+48defg+56d^2g^2)\,x}{e^2} - \frac{(e^2f^2+14defg+24d^2g^2)\,x^2}{2e} - \frac{1}{3}g(2ef+7dg)x^3 - \frac{1}{4}eg^2x^4 + \frac{8d^4(ef+dg)^2}{e^3(d-ex)^2} + \frac{32d^3(e^2f^2+3defg+2d^2g^2)}{e^3(-d+ex)} - \frac{8d^2(3e^2f^2+14defg+13d^2g^2)\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^7*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((d*(7*e^2*f^2 + 48*d*e*f*g + 56*d^2*g^2)*x)/e^2) - ((e^2*f^2 + 14*d*e*f*g
 + 24*d^2*g^2)*x^2)/(2*e) - (g*(2*e*f + 7*d*g)*x^3)/3 - (e*g^2*x^4)/4 + (8*
d^4*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (32*d^3*(e^2*f^2 + 3*d*e*f*g + 2*d^2
*g^2))/(e^3*(-d + e*x)) - (8*d^2*(3*e^2*f^2 + 14*d*e*f*g + 13*d^2*g^2)*Log[
d - e*x])/e^3
```

Maple [A]
time = 0.09, size = 216, normalized size = 1.21

method	result
default	$-\frac{\frac{1}{4}g^2e^3x^4+\frac{7}{3}de^2g^2x^3+\frac{2}{3}e^3fgx^3+12d^2eg^2x^2+7de^2fgx^2+\frac{1}{2}e^3f^2x^2+56d^3g^2x+48d^2efgx+7de^2f^2x}{e^2} - \frac{32d^3(2d^2g^2+3defg+e^2f^2}{e^3(-ex+d)}$
risch	$-\frac{eg^2x^4}{4} - \frac{7x^3dg^2}{3} - \frac{2ex^3fg}{3} - \frac{12x^2d^2g^2}{e} - 7x^2dfg - \frac{ex^2f^2}{2} - \frac{56d^3g^2x}{e^2} - \frac{48d^2fgx}{e} - 7df^2x + \frac{(64d^5g^2+96d^4ef}{}$
norman	$\frac{(\frac{521}{3}d^5g^2+\frac{574}{3}d^4efg+46d^3e^2f^2)x^3+(-\frac{154}{3}g^2d^3e^2-\frac{140}{3}fge^3d^2-7f^2de^4)x^5+(-\frac{23}{2}g^2e^3d^2-7fgde^4-\frac{1}{2}f^2e^5)x^6-\frac{d^4(319g^2ed^4+376fg}{4e}}{(-e^2x^2+d^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2*(1/4*g^2*e^3*x^4+7/3*d*e^2*g^2*x^3+2/3*e^3*f*g*x^3+12*d^2*e*g^2*x^2+
7*d*e^2*f*g*x^2+1/2*e^3*f^2*x^2+56*d^3*g^2*x+48*d^2*e*f*g*x+7*d*e^2*f^2*x)-
32*d^3/e^3*(2*d^2*g^2+3*d*e*f*g+e^2*f^2)/(-e*x+d)+8*d^4*(d^2*g^2+2*d*e*f*g+
e^2*f^2)/e^3/(-e*x+d)^2-8*d^2*(13*d^2*g^2+14*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/
e^3
```

Maxima [A]
time = 0.29, size = 218, normalized size = 1.22

$$-8\left(13\,d^4g^2+14\,d^3fge+3\,d^2f^2e^2\right)e^{(-3)}\log\left(xe-d\right)-\frac{1}{12}\left(3\,g^2x^4e^3+4\left(7\,dg^2e^2+2\,fge^3\right)x^3+6\left(24\,d^2g^2e+14\,dfge^2+f^2e^3\right)x^2+12\left(56\,d^3g^2+48\,d^2fge+7\,df^2e^2\right)x\right)e^{(-2)}-\frac{8\left(7\,d^6g^2+10\,d^5fge+3\,d^4f^2e^2-4\left(2\,d^5g^2e+3\,d^4fge^2+d^3f^2e^3\right)x\right)}{x^2e^5-2\,dxe^4+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -8*(13*d^4*g^2 + 14*d^3*f*g*e + 3*d^2*f^2*e^2)*e^(-3)*log(x*e - d) - 1/12*(
3*g^2*x^4*e^3 + 4*(7*d*g^2*e^2 + 2*f*g*e^3)*x^3 + 6*(24*d^2*g^2*e + 14*d*f*
g*e^2 + f^2*e^3)*x^2 + 12*(56*d^3*g^2 + 48*d^2*f*g*e + 7*d*f^2*e^2)*x)*e^(-
2) - 8*(7*d^6*g^2 + 10*d^5*f*g*e + 3*d^4*f^2*e^2 - 4*(2*d^5*g^2*e + 3*d^4*f
*g*e^2 + d^3*f^2*e^3)*x)/(x^2*e^5 - 2*d*x*e^4 + d^2*e^3)
```

Fricas [A]
time = 4.08, size = 322, normalized size = 1.80

$$-\frac{672\,d^6g^2+\left(3\,g^2x^6+8\,fgx^5+6\,f^2x^4\right)e^6+2\left(11\,dg^2x^5+34\,dfgx^4+36\,df^2x^3\right)e^5+\left(91\,d^2g^2x^4+416\,d^2fgx^3-162\,d^2f^2x^2\right)e^4+4\left(103\,d^3g^2x^3-267\,d^3fgx^2-75\,d^3f^2x\right)e^3-48\left(25\,d^4g^2x^2+12\,d^4fgx-6\,d^4f^2\right)e^2-96\left(d^5g^2x-10\,d^5fg\right)e+96\left(13\,d^6g^2+3\,d^2f^2x^2e^4+2\left(7\,d^3fgx^2-3\,d^3f^2x\right)e^3+\left(13\,d^4g^2x^2-28\,d^4fgx+3\,d^4f^2\right)e^2-2\left(13\,d^5g^2x-7\,d^5fg\right)e\right)\log\left(xe-d\right)}{12\left(x^2e^5-2\,dxe^4+d^2e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(672*d^6*g^2 + (3*g^2*x^6 + 8*f*g*x^5 + 6*f^2*x^4)*e^6 + 2*(11*d*g^2*
x^5 + 34*d*f*g*x^4 + 36*d*f^2*x^3)*e^5 + (91*d^2*g^2*x^4 + 416*d^2*f*g*x^3
- 162*d^2*f^2*x^2)*e^4 + 4*(103*d^3*g^2*x^3 - 267*d^3*f*g*x^2 - 75*d^3*f^2*
x)*e^3 - 48*(25*d^4*g^2*x^2 + 12*d^4*f*g*x - 6*d^4*f^2)*e^2 - 96*(d^5*g^2*x
 - 10*d^5*f*g)*e + 96*(13*d^6*g^2 + 3*d^2*f^2*x^2*e^4 + 2*(7*d^3*f*g*x^2 -
3*d^3*f^2*x)*e^3 + (13*d^4*g^2*x^2 - 28*d^4*f*g*x + 3*d^4*f^2)*e^2 - 2*(13*
d^5*g^2*x - 7*d^5*f*g)*e)*log(x*e - d))/(x^2*e^5 - 2*d*x*e^4 + d^2*e^3)
```

Sympy [A]
time = 0.75, size = 219, normalized size = 1.22

$$-\frac{8d^2\cdot\left(13d^2g^2+14defg+3e^2f^2\right)\log\left(-d+ex\right)}{e^3}-\frac{eg^2x^4}{4}-x^3\cdot\left(\frac{7dg^2}{3}+\frac{2efg}{3}\right)-x^2\cdot\left(\frac{12d^2g^2}{e}+7dfg+\frac{ef^2}{2}\right)-x\left(\frac{56d^3g^2}{e^2}+\frac{48d^2fg}{e}+7df^2\right)-\frac{56d^6g^2+80d^5efg+24d^4e^2f^2+x(-64d^5eg^2-96d^4e^2fg-32d^3e^3f^2)}{d^2e^3-2de^4x+e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -8*d**2*(13*d**2*g**2 + 14*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - e*g*
*2*x**4/4 - x**3*(7*d*g**2/3 + 2*e*f*g/3) - x**2*(12*d**2*g**2/e + 7*d*f*g
+ e*f**2/2) - x*(56*d**3*g**2/e**2 + 48*d**2*f*g/e + 7*d*f**2) - (56*d**6*g
**2 + 80*d**5*e*f*g + 24*d**4*e**2*f**2 + x*(-64*d**5*e*g**2 - 96*d**4*e**2
*f*g - 32*d**3*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

Giac [A]
time = 1.80, size = 215, normalized size = 1.20

$$-8\left(13\,d^4g^2+14\,d^3fge+3\,d^2f^2e^2\right)e^{(-3)}\log\left(|xe-d|\right)-\frac{1}{12}\left(3\,g^2x^4e^{13}+28\,dg^2x^3e^{12}+144\,d^2g^2x^2e^{11}+672\,d^3g^2xe^{10}+8\,fgx^3e^{13}+84\,dfgx^2e^{12}+576\,d^2fgxe^{11}+6\,f^2x^2e^{13}+84\,df^2xe^{12}\right)e^{(-12)}-\frac{8\left(7\,d^6g^2+10\,d^5fge+3\,d^4f^2e^2-4\left(2\,d^5g^2e+3\,d^4fge^2+d^3f^2e^3\right)x\right)e^{(-3)}}{\left(xe-d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^7*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -8*(13*d^4*g^2 + 14*d^3*f*g*e + 3*d^2*f^2*e^2)*e^(-3)*log(abs(x*e - d)) - 1
/12*(3*g^2*x^4*e^13 + 28*d*g^2*x^3*e^12 + 144*d^2*g^2*x^2*e^11 + 672*d^3*g^
2*x*e^10 + 8*f*g*x^3*e^13 + 84*d*f*g*x^2*e^12 + 576*d^2*f*g*x*e^11 + 6*f^2*
x^2*e^13 + 84*d*f^2*x*e^12)*e^(-12) - 8*(7*d^6*g^2 + 10*d^5*f*g*e + 3*d^4*f
^2*e^2 - 4*(2*d^5*g^2*e + 3*d^4*f*g*e^2 + d^3*f^2*e^3)*x)*e^(-3)/(x*e - d)^
2
```

Mupad [B]
time = 0.14, size = 375, normalized size = 2.09

$$\frac{x\,\left(64\,d^5\,g^2+96\,d^4\,e\,f\,g+32\,d^3\,e^2\,f^2\right)-\frac{8\,\left(7\,d^6\,g^2+10\,d^5\,e\,f\,g+3\,d^4\,e^2\,f^2\right)}{e}}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-x^2\,\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{2\,e^3}-\frac{3\,d^2\,g^2}{2\,e}+\frac{3\,d\,\left(2\,g\,\left(2\,d\,g+e\,f\right)+3\,d\,g^2\right)}{2\,e}\right)-x\,\left(\frac{d^3\,g^2}{e^2}-\frac{3\,d^2\,\left(2\,g\,\left(2\,d\,g+e\,f\right)+3\,d\,g^2\right)}{e^2}+\frac{4\,d\,\left(d^2\,g^2+3\,d\,e\,f\,g+e^2\,f^2\right)}{e^2}+\frac{3\,d\,\left(\frac{6\,d^2\,e^2\,g^2+8\,d\,e^3\,f\,g+e^4\,f^2}{e^3}-\frac{3\,d^2\,g^2}{e}+\frac{3\,d\,\left(2\,g\,\left(2\,d\,g+e\,f\right)+3\,d\,g^2\right)}{e}\right)}{e}\right)-x^3\,\left(\frac{2\,g\,\left(2\,d\,g+e\,f\right)}{3}+d\,g^2\right)-\frac{\ln\left(e\,x-d\right)\,\left(104\,d^4\,g^2+112\,d^3\,e\,f\,g+24\,d^2\,e^2\,f^2\right)}{e^3}-\frac{e\,g^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^7)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (x*(64*d^5*g^2 + 32*d^3*e^2*f^2 + 96*d^4*e*f*g) - (8*(7*d^6*g^2 + 3*d^4*e^2
*f^2 + 10*d^5*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x^2*((e^4*f^2 +
6*d^2*e^2*g^2 + 8*d*e^3*f*g)/(2*e^3) - (3*d^2*g^2)/(2*e) + (3*d*(2*g*(2*d*g
 + e*f) + 3*d*g^2))/(2*e)) - x*((d^3*g^2)/e^2 - (3*d^2*(2*g*(2*d*g + e*f) +
 3*d*g^2))/e^2 + (4*d*(d^2*g^2 + e^2*f^2 + 3*d*e*f*g))/e^2 + (3*d*((e^4*f^2
 + 6*d^2*e^2*g^2 + 8*d*e^3*f*g)/e^3 - (3*d^2*g^2)/e + (3*d*(2*g*(2*d*g + e*
f) + 3*d*g^2))/e))/e) - x^3*((2*g*(2*d*g + e*f))/3 + d*g^2) - (log(e*x - d)
*(104*d^4*g^2 + 24*d^2*e^2*f^2 + 112*d^3*e*f*g))/e^3 - (e*g^2*x^4)/4
```

3.570 $\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal. Leaf size=149

$$-\frac{(e^2f^2+12defg+18d^2g^2)\,x}{e^2}-\frac{g(ef+3dg)x^2}{e}-\frac{g^2x^3}{3}+\frac{4d^3(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d^2(ef+dg)(3ef+7dg)}{e^3(d-ex)}-\frac{2d(3e^2f$$

```
[Out] -(18*d^2*g^2+12*d*e*f*g+e^2*f^2)*x/e^2-g*(3*d*g+e*f)*x^2/e-1/3*g^2*x^3+4*d^
3*(d*g+e*f)^2/e^3/(-e*x+d)^2-4*d^2*(d*g+e*f)*(7*d*g+3*e*f)/e^3/(-e*x+d)-2*d
*(19*d^2*g^2+18*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 90}

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2}-\frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)}-\frac{x(18d^2g^2+12defg+e^2f^2)}{e^2}-\frac{2d(19d^2g^2+18defg+3e^2f^2)\log(d-ex)}{e^3}-\frac{gx^2(3dg+ef)}{e}-\frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - (4*d^2*(e*f + d*g)*
(3*e*f + 7*d*g))/(e^3*(d - e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g + 19*d^2*g^
2)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \int \frac{(d+ex)^3(f+gx)^2}{(d-ex)^3}\,dx$$
$$= \int \left(\frac{-e^2f^2-12defg-18d^2g^2}{e^2} - \frac{2g(ef+3dg)x}{e} - g^2x^2 + \frac{4d^2(-3ef-7dg)(ef}{e^2(d-ex)^2}\right.$$
$$= -\frac{(e^2f^2+12defg+18d^2g^2)\,x}{e^2} - \frac{g(ef+3dg)x^2}{e} - \frac{g^2x^3}{3} + \frac{4d^3(ef+dg)^2}{e^3(d-ex)^2} - \frac{4d^2(}{}$$

Mathematica [A]

time = 0.06, size = 157, normalized size = 1.05

$$-\frac{(e^2f^2+12defg+18d^2g^2)\,x}{e^2} - \frac{g(ef+3dg)x^2}{e} - \frac{g^2x^3}{3} + \frac{4d^3(ef+dg)^2}{e^3(d-ex)^2} + \frac{4d^2(3e^2f^2+10defg+7d^2g^2)}{e^3(-d+ex)} - \frac{2d(3e^2f^2+18defg+19d^2g^2)\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^6*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -(((e^2*f^2 + 12*d*e*f*g + 18*d^2*g^2)*x)/e^2) - (g*(e*f + 3*d*g)*x^2)/e -
(g^2*x^3)/3 + (4*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)^2) + (4*d^2*(3*e^2*f^2 +
 10*d*e*f*g + 7*d^2*g^2))/(e^3*(-d + e*x)) - (2*d*(3*e^2*f^2 + 18*d*e*f*g +
 19*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A]

time = 0.09, size = 174, normalized size = 1.17

method	result
default	$-\frac{\frac{1}{3}g^2x^3e^2+3de\,g^2x^2+e^2fg\,x^2+18d^2g^2x+12defgx+e^2f^2x}{e^2}+\frac{4d^3\left(d^2g^2+2defg+e^2f^2\right)}{e^3(-ex+d)^2}-\frac{2d\left(19d^2g^2+18defg+3e^2f^2\right)\ln(-ex+d)}{e^3}$
risch	$-\frac{g^2x^3}{3}-\frac{3d\,g^2x^2}{e}-fg\,x^2-\frac{18d^2g^2x}{e^2}-\frac{12dfgx}{e}-f^2x-\frac{(-28d^4g^2-40fg\,d^3e-12d^2e^2f^2)x+\frac{8d^3\left(3d^2g^2+4defg+e^2f^2\right)}{e}}{e^2(-ex+d)^2}-$
norman	$\frac{(\frac{191}{3}d^4g^2+64fg\,d^3e+14d^2e^2f^2)x^3+(-\frac{52}{3}g^2d^2e^2-12fgd\,e^3-f^2e^4)x^5+\frac{d^2\left(41g^2d^3e+51fg\,d^2e^2+16f^2d\,e^3\right)x^2}{e^2}-\frac{d^4\left(30g^2d^3e+34fg\,d^2e^2+8f\right.}{e^4}}{(-e^2x^2+d^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2*(1/3*g^2*x^3*e^2+3*d*e*g^2*x^2+e^2*f*g*x^2+18*d^2*g^2*x+12*d*e*f*g*x
+e^2*f^2*x)+4*d^3*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)^2-2*d*(19*d^2*g^
2+18*d*e*f*g+3*e^2*f^2)*ln(-e*x+d)/e^3-4*d^2/e^3*(7*d^2*g^2+10*d*e*f*g+3*e^
2*f^2)/(-e*x+d)
```

Maxima [A]

time = 0.29, size = 182, normalized size = 1.22

$$-2\,(19\,d^3g^2+18\,d^2fge+3\,df^2e^2)e^{(-3)}\log(xe-d) - \frac{1}{3}\,(g^2x^3e^2+3\,(3\,dg^2e+fge^2)x^2+3\,(18\,d^2g^2+12\,dfge+f^2e^2)x)e^{(-2)} - \frac{4\,(6\,d^5g^2+8\,d^4fge+2\,d^3f^2e^2-(7\,d^4g^2e+10\,d^3fge^2+3\,d^2f^2e^3)x)}{x^2e^5-2\,dxe^4+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -2*(19*d^3*g^2 + 18*d^2*f*g*e + 3*d*f^2*e^2)*e^(-3)*log(x*e - d) - 1/3*(g^2
*x^3*e^2 + 3*(3*d*g^2*e + f*g*e^2)*x^2 + 3*(18*d^2*g^2 + 12*d*f*g*e + f^2*e
^2)*x)*e^(-2) - 4*(6*d^5*g^2 + 8*d^4*f*g*e + 2*d^3*f^2*e^2 - (7*d^4*g^2*e +
 10*d^3*f*g*e^2 + 3*d^2*f^2*e^3)*x)/(x^2*e^5 - 2*d*x*e^4 + d^2*e^3)
```

Fricas [A]
time = 9.63, size = 282, normalized size = 1.89

$$-\frac{72\,d^5g^2+(g^2x^5+3\,fgx^4+3\,f^2x^3)e^5+(7\,dg^2x^4+30\,dfgx^3-6\,df^2x^2)e^4+(37\,d^2g^2x^3-69\,d^2fgx^2-33\,d^2f^2x)e^3-3\,(33\,d^3g^2x^2+28\,d^3fgx-8\,d^3f^2)e^2-6\,(5\,d^4g^2x-16\,d^4fg)e+6\,(19\,d^5g^2+3\,df^2x^2e^4+6\,(3\,d^2fgx^2-d^2f^2x)e^3+(19\,d^3g^2x^2-36\,d^3fgx+3\,d^3f^2)e^2-2\,(19\,d^4g^2x-9\,d^4fg)e)\log(xe-d)}{3\,(x^2e^5-2\,dxe^4+d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(72*d^5*g^2 + (g^2*x^5 + 3*f*g*x^4 + 3*f^2*x^3)*e^5 + (7*d*g^2*x^4 + 3
0*d*f*g*x^3 - 6*d*f^2*x^2)*e^4 + (37*d^2*g^2*x^3 - 69*d^2*f*g*x^2 - 33*d^2*
f^2*x)*e^3 - 3*(33*d^3*g^2*x^2 + 28*d^3*f*g*x - 8*d^3*f^2)*e^2 - 6*(5*d^4*g
^2*x - 16*d^4*f*g)*e + 6*(19*d^5*g^2 + 3*d*f^2*x^2*e^4 + 6*(3*d^2*f*g*x^2 -
 d^2*f^2*x)*e^3 + (19*d^3*g^2*x^2 - 36*d^3*f*g*x + 3*d^3*f^2)*e^2 - 2*(19*d
^4*g^2*x - 9*d^4*f*g)*e)*log(x*e - d))/(x^2*e^5 - 2*d*x*e^4 + d^2*e^3)
```

Sympy [A]
time = 0.64, size = 178, normalized size = 1.19

$$-\frac{2d(19d^2g^2+18defg+3e^2f^2)\log(-d+ex)}{e^3}-\frac{g^2x^3}{3}-x^2\cdot\left(\frac{3dg^2}{e}+fg\right)-x\left(\frac{18d^2g^2}{e^2}+\frac{12dfg}{e}+f^2\right)-\frac{24d^5g^2+32d^4efg+8d^3e^2f^2+x(-28d^4eg^2-40d^3e^2fg-12d^2e^3f^2)}{d^2e^3-2de^4x+e^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -2*d*(19*d**2*g**2 + 18*d*e*f*g + 3*e**2*f**2)*log(-d + e*x)/e**3 - g**2*x*
*3/3 - x**2*(3*d*g**2/e + f*g) - x*(18*d**2*g**2/e**2 + 12*d*f*g/e + f**2)
- (24*d**5*g**2 + 32*d**4*e*f*g + 8*d**3*e**2*f**2 + x*(-28*d**4*e*g**2 - 4
0*d**3*e**2*f*g - 12*d**2*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2)
```

Giac [A]
time = 2.23, size = 177, normalized size = 1.19

$$-2\,(19\,d^3g^2+18\,d^2fge+3\,df^2e^2)e^{(-3)}\log(|xe-d|)-\frac{1}{3}\,(g^2x^3e^9+9\,dg^2x^2e^8+54\,d^2g^2xe^7+3\,fgx^2e^9+36\,dfgxe^8+3\,f^2xe^9)e^{(-9)}-\frac{4\,(6\,d^5g^2+8\,d^4fge+2\,d^3f^2e^2-(7\,d^4g^2e+10\,d^3fge^2+3\,d^2f^2e^3)x)e^{(-3)}}{(xe-d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^6*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -2*(19*d^3*g^2 + 18*d^2*f*g*e + 3*d*f^2*e^2)*e^(-3)*log(abs(x*e - d)) - 1/3
*(g^2*x^3*e^9 + 9*d*g^2*x^2*e^8 + 54*d^2*g^2*x*e^7 + 3*f*g*x^2*e^9 + 36*d*f
```

```
*g*x*e^8 + 3*f^2*x*e^9)*e^(-9) - 4*(6*d^5*g^2 + 8*d^4*f*g*e + 2*d^3*f^2*e^2
 - (7*d^4*g^2*e + 10*d^3*f*g*e^2 + 3*d^2*f^2*e^3)*x)*e^(-3)/(x*e - d)^2
```

Mupad [B]
time = 0.10, size = 240, normalized size = 1.61

$$\frac{x\left(28\,d^4\,g^2+40\,d^3\,e\,f\,g+12\,d^2\,e^2\,f^2\right)-\frac{8\left(3\,d^5\,g^2+4\,d^4\,e\,f\,g+d^3\,e^2\,f^2\right)}{e}}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-x\left(\frac{3\,d^2\,e\,g^2+6\,d\,e^2\,f\,g+e^3\,f^2}{e^3}+\frac{3\,d\left(\frac{g\,(3\,d\,g+2\,e\,f)}{e}+\frac{3\,d\,g^2}{e}\right)}{e}-\frac{3\,d^2\,g^2}{e^2}\right)-x^2\left(\frac{g\,(3\,d\,g+2\,e\,f)}{2\,e}+\frac{3\,d\,g^2}{2\,e}\right)-\frac{g^2\,x^3}{3}-\frac{\ln\left(e\,x-d\right)\,\left(38\,d^3\,g^2+36\,d^2\,e\,f\,g+6\,d\,e^2\,f^2\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^6)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (x*(28*d^4*g^2 + 12*d^2*e^2*f^2 + 40*d^3*e*f*g) - (8*(3*d^5*g^2 + d^3*e^2*f
^2 + 4*d^4*e*f*g))/e)/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((e^3*f^2 + 3*d^2
*e*g^2 + 6*d*e^2*f*g)/e^3 + (3*d*((g*(3*d*g + 2*e*f))/e + (3*d*g^2)/e))/e -
 (3*d^2*g^2)/e^2) - x^2*((g*(3*d*g + 2*e*f))/(2*e) + (3*d*g^2)/(2*e)) - (g^
2*x^3)/3 - (log(e*x - d)*(38*d^3*g^2 + 6*d*e^2*f^2 + 36*d^2*e*f*g))/e^3
```

3.571 $\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{g(2ef+5dg)x}{e^2}-\frac{g^2x^2}{2e}+\frac{2d^2(ef+dg)^2}{e^3(d-ex)^2}-\frac{4d(ef+dg)(ef+3dg)}{e^3(d-ex)}-\frac{(e^2f^2+10defg+13d^2g^2)\log(d-ex)}{e^3}$$

```
[Out] -g*(5*d*g+2*e*f)*x/e^2-1/2*g^2*x^2/e+2*d^2*(d*g+e*f)^2/e^3/(-e*x+d)^2-4*d*(
d*g+e*f)*(3*d*g+e*f)/e^3/(-e*x+d)-(13*d^2*g^2+10*d*e*f*g+e^2*f^2)*ln(-e*x+d
)/e^3
```

Rubi [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {862, 90}

$$\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2}-\frac{(13d^2g^2+10defg+e^2f^2)\log(d-ex)}{e^3}-\frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)}-\frac{gx(5dg+2ef)}{e^2}-\frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g*(2*e*f + 5*d*g)*x)/e^2) - (g^2*x^2)/(2*e) + (2*d^2*(e*f + d*g)^2)/(e^3
*(d - e*x)^2) - (4*d*(e*f + d*g)*(e*f + 3*d*g))/(e^3*(d - e*x)) - ((e^2*f^2
 + 10*d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \int \frac{(d+ex)^2(f+gx)^2}{(d-ex)^3}\,dx$$
$$= \int \left(-\frac{g(2ef+5dg)}{e^2} - \frac{g^2x}{e} + \frac{4d(-ef-3dg)(ef+dg)}{e^2(d-ex)^2} - \frac{4d^2(ef+dg)^2}{e^2(-d+ex)^3} + \frac{-e^2 \ldots}{}\right.$$
$$= -\frac{g(2ef+5dg)x}{e^2} - \frac{g^2x^2}{2e} + \frac{2d^2(ef+dg)^2}{e^3(d-ex)^2} - \frac{4d(ef+dg)(ef+3dg)}{e^3(d-ex)} - \frac{(e^2f^2+\ldots}{}$$

Mathematica [A]
time = 0.06, size = 118, normalized size = 1.00

$$-\frac{2eg(2ef+5dg)x+e^2g^2x^2-\frac{4d^2(ef+dg)^2}{(d-ex)^2}+\frac{8d(e^2f^2+4defg+3d^2g^2)}{d-ex}+2(e^2f^2+10defg+13d^2g^2)\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^5*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -1/2*(2*e*g*(2*e*f + 5*d*g)*x + e^2*g^2*x^2 - (4*d^2*(e*f + d*g)^2)/(d - e*
x)^2 + (8*d*(e^2*f^2 + 4*d*e*f*g + 3*d^2*g^2))/(d - e*x) + 2*(e^2*f^2 + 10*
d*e*f*g + 13*d^2*g^2)*Log[d - e*x])/e^3
```

Maple [A]
time = 0.07, size = 133, normalized size = 1.13

method	result
default	$-\frac{g\left(\frac{1}{2}eg\,x^2+5dgx+2efx\right)}{e^2}-\frac{4d\left(3d^2g^2+4defg+e^2f^2\right)}{e^3(-ex+d)}+\frac{2d^2\left(d^2g^2+2defg+e^2f^2\right)}{e^3(-ex+d)^2}+\frac{\left(-13d^2g^2-10defg-e^2f^2\right)\ln(-ex+d)}{e^3}$
risch	$-\frac{g^2x^2}{2e}-\frac{5g^2dx}{e^2}-\frac{2gfx}{e}+\frac{(12d^3g^2+16d^2efg+4d\,e^2f^2)x-\frac{2d^2\left(5d^2g^2+6defg+e^2f^2\right)}{e}}{e^2(-ex+d)^2}-\frac{13\ln(-ex+d)d^2g^2}{e^3}-\frac{10\ln(-ex+d)dfg \ldots}{e^2}$
norman	$\frac{(22d^3g^2+20d^2efg+4d\,e^2f^2)x^3-\frac{d^4\left(11g^2d^2e+12fgd\,e^2+2f^2e^3\right)}{e^4}-\frac{e^3g^2x^6}{2}+\frac{d^2\left(31g^2d^2e+40fgd\,e^2+12f^2e^3\right)x^2}{2e^2}-e^2g(5dg+2ef)x^5-\frac{d^4g(13d \ldots}{}}{(-e^2x^2+d^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -g/e^2*(1/2*e*g*x^2+5*d*g*x+2*e*f*x)-4*d/e^3*(3*d^2*g^2+4*d*e*f*g+e^2*f^2)/
(-e*x+d)+2*d^2*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)^2+(-13*d^2*g^2-10*d
*e*f*g-e^2*f^2)/e^3*ln(-e*x+d)
```

Maxima [A]
time = 0.29, size = 146, normalized size = 1.24

$$-(13\,d^2g^2+10\,dfge+f^2e^2)e^{(-3)}\log(xe-d)-\frac{1}{2}\left(g^2x^2e+2\,(5\,dg^2+2\,fge)x\right)e^{(-2)}-\frac{2\,(5\,d^4g^2+6\,d^3fge+d^2f^2e^2-2\,(3\,d^3g^2e+4\,d^2fge^2+df^2e^3)x)}{x^2e^5-2\,dxe^4+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -(13*d^2*g^2 + 10*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e - d) - 1/2*(g^2*x^2*e +
 2*(5*d*g^2 + 2*f*g*e)*x)*e^(-2) - 2*(5*d^4*g^2 + 6*d^3*f*g*e + d^2*f^2*e^2
 - 2*(3*d^3*g^2*e + 4*d^2*f*g*e^2 + d*f^2*e^3)*x)/(x^2*e^5 - 2*d*x*e^4 + d^
2*e^3)
```

Fricas [A]
time = 3.51, size = 231, normalized size = 1.96

$$-\frac{20\,d^4g^2+(g^2x^4+4\,fgx^3)e^4+8\,(dg^2x^3-dfgx^2-df^2x)e^3-(19\,d^2g^2x^2+28\,d^2fgx-4\,d^2f^2)e^2-2\,(7\,d^3g^2x-12\,d^3fg)e+2\,(13\,d^4g^2+f^2x^2e^4+2\,(5\,dfgx^2-df^2x)e^3+(13\,d^2g^2x^2-20\,d^2fgx+d^2f^2)e^2-2\,(13\,d^3g^2x-5\,d^3fg)e)\log(xe-d)}{2\,(x^2e^5-2\,dxe^4+d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(20*d^4*g^2 + (g^2*x^4 + 4*f*g*x^3)*e^4 + 8*(d*g^2*x^3 - d*f*g*x^2 - d
*f^2*x)*e^3 - (19*d^2*g^2*x^2 + 28*d^2*f*g*x - 4*d^2*f^2)*e^2 - 2*(7*d^3*g^
2*x - 12*d^3*f*g)*e + 2*(13*d^4*g^2 + f^2*x^2*e^4 + 2*(5*d*f*g*x^2 - d*f^2*
x)*e^3 + (13*d^2*g^2*x^2 - 20*d^2*f*g*x + d^2*f^2)*e^2 - 2*(13*d^3*g^2*x -
5*d^3*f*g)*e)*log(x*e - d))/(x^2*e^5 - 2*d*x*e^4 + d^2*e^3)
```

Sympy [A]
time = 0.57, size = 151, normalized size = 1.28

$$-x\left(\frac{5dg^2}{e^2}+\frac{2fg}{e}\right)-\frac{10d^4g^2+12d^3efg+2d^2e^2f^2+x(-12d^3eg^2-16d^2e^2fg-4de^3f^2)}{d^2e^3-2de^4x+e^5x^2}-\frac{g^2x^2}{2e}-\frac{(13d^2g^2+10defg+e^2f^2)\log(-d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -x*(5*d*g**2/e**2 + 2*f*g/e) - (10*d**4*g**2 + 12*d**3*e*f*g + 2*d**2*e**2*
f**2 + x*(-12*d**3*e*g**2 - 16*d**2*e**2*f*g - 4*d*e**3*f**2))/(d**2*e**3 -
 2*d*e**4*x + e**5*x**2) - g**2*x**2/(2*e) - (13*d**2*g**2 + 10*d*e*f*g + e
**2*f**2)*log(-d + e*x)/e**3
```

Giac [A]
time = 2.02, size = 138, normalized size = 1.17

$$-(13\,d^2g^2+10\,dfge+f^2e^2)e^{(-3)}\log(|xe-d|)-\frac{1}{2}\,(g^2x^2e^5+10\,dg^2xe^4+4\,fgxe^5)e^{(-6)}-\frac{2\,(5\,d^4g^2+6\,d^3fge+d^2f^2e^2-2\,(3\,d^3g^2e+4\,d^2fge^2+df^2e^3)x)e^{(-3)}}{(xe-d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -(13*d^2*g^2 + 10*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e - d)) - 1/2*(g^2*x^
2*e^5 + 10*d*g^2*x*e^4 + 4*f*g*x*e^5)*e^(-6) - 2*(5*d^4*g^2 + 6*d^3*f*g*e +
```

```
d^2*f^2*e^2 - 2*(3*d^3*g^2*e + 4*d^2*f*g*e^2 + d*f^2*e^3)*x)*e^(-3)/(x*e -
d)^2
```

Mupad [B]
time = 2.60, size = 161, normalized size = 1.36

$$-\frac{\frac{2\,\left(5\,d^4\,g^2+6\,d^3\,e\,f\,g+d^2\,e^2\,f^2\right)}{e}-x\,\left(12\,d^3\,g^2+16\,d^2\,e\,f\,g+4\,d\,e^2\,f^2\right)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-x\,\left(\frac{2\,g\,\left(d\,g+e\,f\right)}{e^2}+\frac{3\,d\,g^2}{e^2}\right)-\frac{\ln\left(e\,x-d\right)\,\left(13\,d^2\,g^2+10\,d\,e\,f\,g+e^2\,f^2\right)}{e^3}-\frac{g^2\,x^2}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^5)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((2*(5*d^4*g^2 + d^2*e^2*f^2 + 6*d^3*e*f*g))/e - x*(12*d^3*g^2 + 4*d*e^2*
f^2 + 16*d^2*e*f*g))/(d^2*e^2 + e^4*x^2 - 2*d*e^3*x) - x*((2*g*(d*g + e*f))
/e^2 + (3*d*g^2)/e^2) - (log(e*x - d)*(13*d^2*g^2 + e^2*f^2 + 10*d*e*f*g))/
e^3 - (g^2*x^2)/(2*e)
```

3.572 $\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal. Leaf size=81

$$-\frac{g^2x}{e^2}+\frac{d(ef+dg)^2}{e^3(d-ex)^2}-\frac{(ef+dg)(ef+5dg)}{e^3(d-ex)}-\frac{2g(ef+2dg)\log(d-ex)}{e^3}$$

```
[Out] -g^2*x/e^2+d*(d*g+e*f)^2/e^3/(-e*x+d)^2-(d*g+e*f)*(5*d*g+e*f)/e^3/(-e*x+d)-
2*g*(2*d*g+e*f)*ln(-e*x+d)/e^3
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 78}

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}+\frac{d(dg+ef)^2}{e^3(d-ex)^2}-\frac{2g(2dg+ef)\log(d-ex)}{e^3}-\frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] -((g^2*x)/e^2) + (d*(e*f + d*g)^2)/(e^3*(d - e*x)^2) - ((e*f + d*g)*(e*f +
5*d*g))/(e^3*(d - e*x)) - (2*g*(e*f + 2*d*g)*Log[d - e*x])/e^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \int \frac{(d+ex)(f+gx)^2}{(d-ex)^3}\,dx$$
$$= \int \left(-\frac{g^2}{e^2} + \frac{(-ef-5dg)(ef+dg)}{e^2(d-ex)^2} - \frac{2d(ef+dg)^2}{e^2(-d+ex)^3} - \frac{2g(ef+2dg)}{e^2(-d+ex)}\right) dx$$
$$= -\frac{g^2x}{e^2} + \frac{d(ef+dg)^2}{e^3(d-ex)^2} - \frac{(ef+dg)(ef+5dg)}{e^3(d-ex)} - \frac{2g(ef+2dg)\log(d-ex)}{e^3}$$

Mathematica [A]
time = 0.03, size = 93, normalized size = 1.15

$$\frac{-4d^3g^2+4d^2eg(-f+gx)+2de^2gx(3f+gx)+e^3x(f^2-g^2x^2)-2g(ef+2dg)(d-ex)^2\log(d-ex)}{e^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (-4*d^3*g^2 + 4*d^2*e*g*(-f + g*x) + 2*d*e^2*g*x*(3*f + g*x) + e^3*x*(f^2 -
 g^2*x^2) - 2*g*(e*f + 2*d*g)*(d - e*x)^2*Log[d - e*x])/(e^3*(d - e*x)^2)
```

Maple [A]
time = 0.08, size = 101, normalized size = 1.25

method	result
risch	$-\frac{g^2x}{e^2} - \frac{(-5d^2g^2-6defg-e^2f^2)x+\frac{4d^2g(dg+ef)}{e}}{e^2(-ex+d)^2} - \frac{4g^2\ln(-ex+d)d}{e^3} - \frac{2g\ln(-ex+d)f}{e^2}$
default	$-\frac{g^2x}{e^2} + \frac{d(d^2g^2+2defg+e^2f^2)}{e^3(-ex+d)^2} + \frac{-5d^2g^2-6defg-e^2f^2}{e^3(-ex+d)} - \frac{2g(2dg+ef)\ln(-ex+d)}{e^3}$
norman	$\frac{(7d^2g^2+6defg+e^2f^2)x^3-\frac{d^4(4deg^2+4e^2fg)}{e^4}-e^2g^2x^5+\frac{2d(3g^2d^2e+4fgde^2+f^2e^3)x^2}{e^2}-\frac{d^2(4d^2g^2+2defg-e^2f^2)x}{e^2}}{(-e^2x^2+d^2)^2} - \frac{2g(2dg+ef)\ln(-e}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -g^2*x/e^2+d*(d^2*g^2+2*d*e*f*g+e^2*f^2)/e^3/(-e*x+d)^2+(-5*d^2*g^2-6*d*e*f
*g-e^2*f^2)/e^3/(-e*x+d)-2*g*(2*d*g+e*f)*ln(-e*x+d)/e^3
```

Maxima [A]
time = 0.29, size = 102, normalized size = 1.26

$$-g^2xe^{(-2)} - 2\left(2\,dg^2+fge\right)e^{(-3)}\log\left(xe-d\right) - \frac{4\,d^3g^2+4\,d^2fge-(5\,d^2g^2e+6\,dfge^2+f^2e^3)x}{x^2e^5-2\,dxe^4+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -g^2*x*e^(-2) - 2*(2*d*g^2 + f*g*e)*e^(-3)*log(x*e - d) - (4*d^3*g^2 + 4*d^
2*f*g*e - (5*d^2*g^2*e + 6*d*f*g*e^2 + f^2*e^3)*x)/(x^2*e^5 - 2*d*x*e^4 + d
^2*e^3)
```

Fricas [A]
time = 3.09, size = 156, normalized size = 1.93

$$-\frac{4\,d^3g^2+(g^2x^3-f^2x)e^3-2\,(dg^2x^2+3\,dfgx)e^2-4\,(d^2g^2x-d^2fg)e+2\,(2\,d^3g^2+fgx^2e^3+2\,(dg^2x^2-dfgx)e^2-(4\,d^2g^2x-d^2fg)e)\log(xe-d)}{x^2e^5-2\,dxe^4+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -(4*d^3*g^2 + (g^2*x^3 - f^2*x)*e^3 - 2*(d*g^2*x^2 + 3*d*f*g*x)*e^2 - 4*(d^
2*g^2*x - d^2*f*g)*e + 2*(2*d^3*g^2 + f*g*x^2*e^3 + 2*(d*g^2*x^2 - d*f*g*x)
*e^2 - (4*d^2*g^2*x - d^2*f*g)*e)*log(x*e - d))/(x^2*e^5 - 2*d*x*e^4 + d^2*
e^3)
```

Sympy [A]
time = 0.44, size = 102, normalized size = 1.26

$$-\frac{4d^3g^2+4d^2efg+x(-5d^2eg^2-6de^2fg-e^3f^2)}{d^2e^3-2de^4x+e^5x^2}-\frac{g^2x}{e^2}-\frac{2g(2dg+ef)\log(-d+ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(4*d**3*g**2 + 4*d**2*e*f*g + x*(-5*d**2*e*g**2 - 6*d*e**2*f*g - e**3*f**2
))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - g**2*x/e**2 - 2*g*(2*d*g + e*f)*l
og(-d + e*x)/e**3
```

Giac [A]
time = 2.97, size = 94, normalized size = 1.16

$$-g^2xe^{(-2)}-2\left(2\,dg^2+fge\right)e^{(-3)}\log(|xe-d|)-\frac{(4\,d^3g^2+4\,d^2fge-(5\,d^2g^2e+6\,dfge^2+f^2e^3)x)e^{(-3)}}{(xe-d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -g^2*x*e^(-2) - 2*(2*d*g^2 + f*g*e)*e^(-3)*log(abs(x*e - d)) - (4*d^3*g^2 +
 4*d^2*f*g*e - (5*d^2*g^2*e + 6*d*f*g*e^2 + f^2*e^3)*x)*e^(-3)/(x*e - d)^2
```

Mupad [B]
time = 2.60, size = 107, normalized size = 1.32

$$-\frac{\frac{4\,(d^3\,g^2+e\,f\,d^2\,g)}{e}-x\,(5\,d^2\,g^2+6\,d\,e\,f\,g+e^2\,f^2)}{d^2\,e^2-2\,d\,e^3\,x+e^4\,x^2}-\frac{g^2\,x}{e^2}-\frac{\ln(e\,x-d)\,\left(4\,d\,g^2+2\,e\,f\,g\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^4)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((4*(d^3*g^2 + d^2*e*f*g))/e - x*(5*d^2*g^2 + e^2*f^2 + 6*d*e*f*g))/(d^2*
e^2 + e^4*x^2 - 2*d*e^3*x) - (g^2*x)/e^2 - (log(e*x - d)*(4*d*g^2 + 2*e*f*g
))/e^3
```

3.573 $\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx$

Optimal. Leaf size=61

$$\frac{(ef+dg)^2}{2e^3(d-ex)^2}-\frac{2g(ef+dg)}{e^3(d-ex)}-\frac{g^2\log(d-ex)}{e^3}$$

```
[Out] 1/2*(d*g+e*f)^2/e^3/(-e*x+d)^2-2*g*(d*g+e*f)/e^3/(-e*x+d)-g^2*ln(-e*x+d)/e^
3
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {862, 45}

$$-\frac{2g(dg+ef)}{e^3(d-ex)}+\frac{(dg+ef)^2}{2e^3(d-ex)^2}-\frac{g^2\log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(2*e^3*(d - e*x)^2) - (2*g*(e*f + d*g))/(e^3*(d - e*x)) - (g^
2*Log[d - e*x])/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \int \frac{(f+gx)^2}{(d-ex)^3}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{e^2(d-ex)^3} - \frac{2g(ef+dg)}{e^2(d-ex)^2} + \frac{g^2}{e^2(d-ex)}\right) dx$$
$$= \frac{(ef+dg)^2}{2e^3(d-ex)^2} - \frac{2g(ef+dg)}{e^3(d-ex)} - \frac{g^2\log(d-ex)}{e^3}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.80

$$\frac{\frac{(ef+dg)(-3dg+e(f+4gx))}{(d-ex)^2} - 2g^2\log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (((e*f + d*g)*(-3*d*g + e*(f + 4*g*x)))/(d - e*x)^2 - 2*g^2*Log[d - e*x])/(
2*e^3)
```

Maple [A]
time = 0.08, size = 74, normalized size = 1.21

method	result	size
risch	$\frac{\frac{2g(dg+ef)x}{e^2} - \frac{3d^2g^2+2defg-e^2f^2}{2e^3}}{(-ex+d)^2} - \frac{g^2\ln(-ex+d)}{e^3}$	69
default	$-\frac{2g(dg+ef)}{e^3(-ex+d)} - \frac{g^2\ln(-ex+d)}{e^3} - \frac{-d^2g^2-2defg-e^2f^2}{2e^3(-ex+d)^2}$	74
norman	$\frac{(2d\,g^2+2efg)x^3 - \frac{d^2\left(3g^2d^2e+2fgd\,e^2-f^2e^3\right)}{2e^4} + \frac{\left(5g^2d^2e+6fgd\,e^2+f^2e^3\right)x^2}{2e^2} - \frac{d\left(d^2g^2-e^2f^2\right)x}{e^2}}{(-e^2x^2+d^2)^2} - \frac{g^2\ln(-ex+d)}{e^3}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*g*(d*g+e*f)/e^3/(-e*x+d)-g^2*ln(-e*x+d)/e^3-1/2*(-d^2*g^2-2*d*e*f*g-e^2*
f^2)/e^3/(-e*x+d)^2
```

Maxima [A]
time = 0.28, size = 79, normalized size = 1.30

$$-g^2e^{(-3)}\log\left(xe-d\right) - \frac{3\,d^2g^2 + 2\,dfge - f^2e^2 - 4\,(dg^2e + fge^2)x}{2\,(x^2e^5 - 2\,dxe^4 + d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -g^2*e^(-3)*log(x*e - d) - 1/2*(3*d^2*g^2 + 2*d*f*g*e - f^2*e^2 - 4*(d*g^2*
e + f*g*e^2)*x)/(x^2*e^5 - 2*d*x*e^4 + d^2*e^3)
```

Fricas [A]
time = 3.22, size = 99, normalized size = 1.62

$$-\frac{3\,d^2g^2 - (4\,fgx + f^2)e^2 - 2\,(2\,dg^2x - dfg)e + 2\,(g^2x^2e^2 - 2\,dg^2xe + d^2g^2)\log{(xe - d)}}{2\,(x^2e^5 - 2\,dxe^4 + d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*d^2*g^2 - (4*f*g*x + f^2)*e^2 - 2*(2*d*g^2*x - d*f*g)*e + 2*(g^2*x^
2*e^2 - 2*d*g^2*x*e + d^2*g^2)*log(x*e - d))/(x^2*e^5 - 2*d*x*e^4 + d^2*e^3
)
```

Sympy [A]
time = 0.26, size = 83, normalized size = 1.36

$$-\frac{3d^2g^2 + 2defg - e^2f^2 + x(-4deg^2 - 4e^2fg)}{2d^2e^3 - 4de^4x + 2e^5x^2} - \frac{g^2\log{(-d + ex)}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(3*d**2*g**2 + 2*d*e*f*g - e**2*f**2 + x*(-4*d*e*g**2 - 4*e**2*f*g))/(2*d*
*2*e**3 - 4*d*e**4*x + 2*e**5*x**2) - g**2*log(-d + e*x)/e**3
```

Giac [A]
time = 1.46, size = 74, normalized size = 1.21

$$-g^2e^{(-3)}\log{(|xe - d|)} + \frac{(4\,(dg^2 + fge)x - (3\,d^2g^2 + 2\,dfge - f^2e^2)e^{(-1)})e^{(-2)}}{2\,(xe - d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -g^2*e^(-3)*log(abs(x*e - d)) + 1/2*(4*(d*g^2 + f*g*e)*x - (3*d^2*g^2 + 2*d
*f*g*e - f^2*e^2)*e^(-1))*e^(-2)/(x*e - d)^2
```

Mupad [B]
time = 0.07, size = 80, normalized size = 1.31

$$-\frac{\frac{3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2}{2\,e^3} - \frac{2\,g\,x\,(d\,g+e\,f)}{e^2}}{d^2 - 2\,d\,e\,x + e^2\,x^2} - \frac{g^2\,\ln{(e\,x - d)}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] - ((3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g)/(2*e^3) - (2*g*x*(d*g + e*f))/e^2)/(d^
2 + e^2*x^2 - 2*d*e*x) - (g^2*log(e*x - d))/e^3
```

3.574 $\int \frac{(d+ex)^2(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=88

$$\frac{(ef+dg)^2}{4de^3(d-ex)^2}+\frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

```
[Out] 1/4*(d*g+e*f)^2/d/e^3/(-e*x+d)^2+1/4*(-3*d*g+e*f)*(d*g+e*f)/d^2/e^3/(-e*x+d
)+1/4*(-d*g+e*f)^2*arctanh(e*x/d)/d^3/e^3
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {862, 90, 214}

$$\frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}+\frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)}+\frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(4*d*e^3*(d - e*x)^2) + ((e*f - 3*d*g)*(e*f + d*g))/(4*d^2*e^
3*(d - e*x)) + ((e*f - d*g)^2*ArcTanh[(e*x)/d])/(4*d^3*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)}\,dx$$
$$= \int \left(\frac{(ef+dg)^2}{2de^2(d-ex)^3} + \frac{(ef-3dg)(ef+dg)}{4d^2e^2(d-ex)^2} + \frac{(-ef+dg)^2}{4d^2e^2\left(d^2-e^2x^2\right)}\right)dx$$
$$= \frac{(ef+dg)^2}{4de^3(d-ex)^2} + \frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)} + \frac{(ef-dg)^2\int \frac{1}{d^2-e^2x^2}\,dx}{4d^2e^2}$$
$$= \frac{(ef+dg)^2}{4de^3(d-ex)^2} + \frac{(ef-3dg)(ef+dg)}{4d^2e^3(d-ex)} + \frac{(ef-dg)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

Mathematica [A]
time = 0.06, size = 90, normalized size = 1.02

$$\frac{-\frac{2d(ef+dg)\left(2d^2g+e^2fx-de(2f+3gx)\right)}{(d-ex)^2} - (ef-dg)^2\log(d-ex) + (ef-dg)^2\log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((-2*d*(e*f + d*g)*(2*d^2*g + e^2*f*x - d*e*(2*f + 3*g*x)))/(d - e*x)^2 - (
e*f - d*g)^2*Log[d - e*x] + (e*f - d*g)^2*Log[d + e*x])/(8*d^3*e^3)
```

Maple [A]
time = 0.08, size = 152, normalized size = 1.73

method	result
default	$\frac{\left(d^2g^2-2defg+e^2f^2\right)\ln(ex+d)}{8e^3d^3} + \frac{-3d^2g^2-2defg+e^2f^2}{4e^3d^2(-ex+d)} - \frac{-d^2g^2-2defg-e^2f^2}{4e^3d(-ex+d)^2} + \frac{\left(-d^2g^2+2defg-e^2f^2\right)\ln(-ex+d)}{8e^3d^3}$
risch	$\frac{\frac{\left(3d^2g^2+2defg-e^2f^2\right)x}{4e^2d^2} - \frac{d^2g^2-e^2f^2}{2d\,e^3}}{(-ex+d)^2} - \frac{\ln(-ex+d)g^2}{8e^3d} + \frac{\ln(-ex+d)fg}{4e^2d^2} - \frac{\ln(-ex+d)f^2}{8e\,d^3} + \frac{\ln(ex+d)g^2}{8e^3d} - \frac{\ln(ex+d)fg}{4e^2d^2} + \frac{\ln(ex\text{-}}{8e}$
norman	$\frac{\frac{d\left(-g^2d^2e+f^2e^3\right)}{2e^4} - \frac{\left(d^2g^2-2defg-3e^2f^2\right)x}{4e^2} + \frac{\left(3d^2g^2+2defg-e^2f^2\right)x^3}{4d^2} - \frac{\left(-de\,g^2-e^2fg\right)x^2}{e^2}}{\left(-e^2x^2+d^2\right)^2} - \frac{\left(d^2g^2-2defg+e^2f^2\right)\ln(-ex+d)}{8e^3d^3} + \frac{(d^2g^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^3*ln(e*x+d)+1/4*(-3*d^2*g^2-2*d*e*f*g
+e^2*f^2)/e^3/d^2/(-e*x+d)-1/4*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3/d/(-e*x+d)^
2+1/8*(-d^2*g^2+2*d*e*f*g-e^2*f^2)/e^3/d^3*ln(-e*x+d)
```

Maxima [A]
time = 0.28, size = 146, normalized size = 1.66

$$-\frac{2\,d^3g^2-2\,df^2e^2-(3\,d^2g^2e+2\,dfge^2-f^2e^3)x}{4\,(d^2x^2e^5-2\,d^3xe^4+d^4e^3)}+\frac{(d^2g^2-2\,dfge+f^2e^2)e^{(-3)}\log{(xe+d)}}{8\,d^3}-\frac{(d^2g^2-2\,dfge+f^2e^2)e^{(-3)}\log{(xe-d)}}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*d^3*g^2 - 2*d*f^2*e^2 - (3*d^2*g^2*e + 2*d*f*g*e^2 - f^2*e^3)*x)/(d
^2*x^2*e^5 - 2*d^3*x*e^4 + d^4*e^3) + 1/8*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*e
^(-3)*log(x*e + d)/d^3 - 1/8*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*e^(-3)*log(x*e
 - d)/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(89) = 178.
time = 3.32, size = 261, normalized size = 2.97

$$\frac{6\,d^3g^2xe-4\,d^4g^2-2\,df^2xe^3+4\,(d^2fgx+d^2f^2)e^2+(d^4g^2+f^2x^2e^4-2\,(dfgx^2+df^2x)e^3+(d^2g^2x^2+4\,d^2fgx+d^2f^2)e^2-2\,(d^3g^2x+d^3fg)e)\log{(xe+d)}-(d^4g^2+f^2x^2e^4-2\,(dfgx^2+df^2x)e^3+(d^2g^2x^2+4\,d^2fgx+d^2f^2)e^2-2\,(d^3g^2x+d^3fg)e)\log{(xe-d)}}{8\,(d^3x^2e^5-2\,d^4xe^4+d^5e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(6*d^3*g^2*x*e - 4*d^4*g^2 - 2*d*f^2*x*e^3 + 4*(d^2*f*g*x + d^2*f^2)*e^
2 + (d^4*g^2 + f^2*x^2*e^4 - 2*(d*f*g*x^2 + d*f^2*x)*e^3 + (d^2*g^2*x^2 + 4
*d^2*f*g*x + d^2*f^2)*e^2 - 2*(d^3*g^2*x + d^3*f*g)*e)*log(x*e + d) - (d^4*
g^2 + f^2*x^2*e^4 - 2*(d*f*g*x^2 + d*f^2*x)*e^3 + (d^2*g^2*x^2 + 4*d^2*f*g*
x + d^2*f^2)*e^2 - 2*(d^3*g^2*x + d^3*f*g)*e)*log(x*e - d))/(d^3*x^2*e^5 -
2*d^4*x*e^4 + d^5*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(75) = 150.
time = 0.51, size = 185, normalized size = 2.10

$$-\frac{2d^3g^2-2de^2f^2+x(-3d^2eg^2-2de^2fg+e^3f^2)}{4d^4e^3-8d^3e^4x+4d^2e^5x^2}-\frac{(dg-ef)^2\log{\left(-\frac{d(dg-ef)^2}{e(d^2g^2-2defg+e^2f^2)}+x\right)}}{8d^3e^3}+\frac{(dg-ef)^2\log{\left(\frac{d(dg-ef)^2}{e(d^2g^2-2defg+e^2f^2)}+x\right)}}{8d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(2*d**3*g**2 - 2*d*e**2*f**2 + x*(-3*d**2*e*g**2 - 2*d*e**2*f*g + e**3*f**
2))/(4*d**4*e**3 - 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g - e*f)**2*log(-
d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3)
 + (d*g - e*f)**2*log(d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**
2)) + x)/(8*d**3*e**3)
```

Giac [A]
time = 1.74, size = 142, normalized size = 1.61

$$\frac{(d^2g^2 - 2\,dfge + f^2e^2)e^{(-3)}\log\left(|xe + d|\right)}{8\,d^3} - \frac{(d^2g^2 - 2\,dfge + f^2e^2)e^{(-3)}\log\left(|xe - d|\right)}{8\,d^3} - \frac{(2\,d^4g^2 - 2\,d^2f^2e^2 - (3\,d^3g^2e + 2\,d^2fge^2 - df^2e^3)x)e^{(-3)}}{4\,(xe - d)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^3 - 1/8*(d^2
*g^2 - 2*d*f*g*e + f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^3 - 1/4*(2*d^4*g^2 -
 2*d^2*f^2*e^2 - (3*d^3*g^2*e + 2*d^2*f*g*e^2 - d*f^2*e^3)*x)*e^(-3)/((x*e
- d)^2*d^3)
```

Mupad [B]
time = 0.13, size = 103, normalized size = 1.17

$$\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,{\left(d\,g-e\,f\right)}^2}{4\,d^3\,e^3}-\frac{\frac{d^2\,g^2-e^2\,f^2}{2\,d\,e^3}-\frac{x\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g-e^2\,f^2\right)}{4\,d^2\,e^2}}{d^2-2\,d\,e\,x+e^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^2)/(d^2 - e^2*x^2)^3,x)
```

```
[Out] (atanh((e*x)/d)*(d*g - e*f)^2)/(4*d^3*e^3) - ((d^2*g^2 - e^2*f^2)/(2*d*e^3)
 - (x*(3*d^2*g^2 - e^2*f^2 + 2*d*e*f*g))/(4*d^2*e^2))/(d^2 + e^2*x^2 - 2*d*
e*x)
```

3.575 $\int \frac{(d+ex)(f+gx)^2}{\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=122

$$\frac{(ef+dg)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(ef-dg)(3ef+dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

```
[Out] 1/8*(d*g+e*f)^2/d^2/e^3/(-e*x+d)^2+1/4*(-d^2*g^2+e^2*f^2)/d^3/e^3/(-e*x+d)-
1/8*(-d*g+e*f)^2/d^3/e^3/(e*x+d)+1/8*(-d*g+e*f)*(d*g+3*e*f)*arctanh(e*x/d)/
d^4/e^3
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {813, 90, 214}

$$\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3}-\frac{(ef-dg)^2}{8d^3e^3(d+ex)}+\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2}+\frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (e*f + d*g)^2/(8*d^2*e^3*(d - e*x)^2) + (e^2*f^2 - d^2*g^2)/(4*d^3*e^3*(d -
 e*x)) - (e*f - d*g)^2/(8*d^3*e^3*(d + e*x)) + ((e*f - d*g)*(3*e*f + d*g)*A
rcTanh[(e*x)/d])/(8*d^4*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 813

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^m*(f + g*x)^(p + 1)*(a/f + (c/g)*x)^p, x] /
; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*f^2 + a*g^2, 0] && (IntegerQ[p]
|| (GtQ[a, 0] && GtQ[f, 0] && EqQ[p, -1]))
```

Rubi steps

$$\int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^2}\,dx$$

$$= \int \left(\frac{(ef+dg)^2}{4d^2e^2(d-ex)^3} + \frac{e^2f^2-d^2g^2}{4d^3e^2(d-ex)^2} + \frac{(-ef+dg)^2}{8d^3e^2(d+ex)^2} + \frac{(ef-dg)(3ef+dg)}{8d^3e^2(d^2-e^2x^2)}\right)$$

$$= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2} + \frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^3e^3(d+ex)} + \frac{((ef-dg)(3ef+dg))\int \frac{1}{d^2-\cdots}}{8d^3e^2}$$

$$= \frac{(ef+dg)^2}{8d^2e^3(d-ex)^2} + \frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^3e^3(d+ex)} + \frac{(ef-dg)(3ef+dg)\tanh^{-1}}{8d^4e^3}$$

Mathematica [A]

time = 0.07, size = 140, normalized size = 1.15

$$\frac{\frac{2d^2(ef+dg)^2}{(d-ex)^2} + \frac{4de^2f^2-4d^3g^2}{d-ex} - \frac{2d(ef-dg)^2}{d+ex} + (-3e^2f^2+2defg+d^2g^2)\log(d-ex) + (3e^2f^2-2defg-d^2g^2)\log(d+ex)}{16d^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x)^2)/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((2*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (4*d*e^2*f^2 - 4*d^3*g^2)/(d - e*x) -
(2*d*(e*f - d*g)^2)/(d + e*x) + (-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2)*Log[d -
e*x] + (3*e^2*f^2 - 2*d*e*f*g - d^2*g^2)*Log[d + e*x])/(16*d^4*e^3)
```

Maple [A]

time = 0.09, size = 183, normalized size = 1.50

method	result
default	$\frac{(-d^2g^2-2defg+3e^2f^2)\ln(ex+d)}{16e^3d^4} - \frac{d^2g^2-2defg+e^2f^2}{8e^3d^3(ex+d)} + \frac{-d^2g^2+e^2f^2}{4d^3e^3(-ex+d)} - \frac{-d^2g^2-2defg-e^2f^2}{8e^3d^2(-ex+d)^2} + \frac{(d^2g^2+2defg-3e^2f^2)\ln(}{16e^3d^4}$
norman	$\frac{\frac{-g^2d^2e+2fgde^2+f^2e^3}{4e^4} + \frac{(d^2g^2+2defg-3e^2f^2)x^3}{8d^3} + \frac{g^2x^2}{2e} + \frac{(d^2g^2+2defg+5e^2f^2)x}{8e^2d}}{(-e^2x^2+d^2)^2} + \frac{(d^2g^2+2defg-3e^2f^2)\ln(-ex+d)}{16e^3d^4} - \frac{(d^2g^2+2de}{}$
risch	$\frac{\frac{(d^2g^2+2defg-3e^2f^2)x^2}{8d^3e} + \frac{(3d^2g^2-2defg+3e^2f^2)x}{8e^2d^2} - \frac{d^2g^2-2defg-e^2f^2}{4de^3}}{(-ex+d)(-e^2x^2+d^2)} + \frac{\ln(ex-d)g^2}{16e^3d^2} + \frac{\ln(ex-d)fg}{8e^2d^3} - \frac{3\ln(ex-d)f^2}{16ed^4} - \frac{\ln(-ex-}{16e^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-d^2*g^2-2*d*e*f*g+3*e^2*f^2)/e^3/d^4*ln(e*x+d)-1/8*(d^2*g^2-2*d*e*f*
g+e^2*f^2)/e^3/d^3/(e*x+d)+1/4*(-d^2*g^2+e^2*f^2)/d^3/e^3/(-e*x+d)-1/8*(-d^
2*g^2-2*d*e*f*g-e^2*f^2)/e^3/d^2/(-e*x+d)^2+1/16/e^3*(d^2*g^2+2*d*e*f*g-3*e
^2*f^2)/d^4*ln(-e*x+d)
```

Maxima [A]
time = 0.29, size = 201, normalized size = 1.65

$$-\frac{2\,d^4g^2-4\,d^3fge-2\,d^2f^2e^2-(d^2g^2e^2+2\,dfge^3-3\,f^2e^4)x^2-(3\,d^3g^2e-2\,d^2fge^2+3\,df^2e^3)x}{8\,(d^3x^3e^6-d^4x^2e^5-d^5xe^4+d^6e^3)}-\frac{(d^2g^2+2\,dfge-3\,f^2e^2)e^{(-3)}\log{(xe+d)}}{16\,d^4}+\frac{(d^2g^2+2\,dfge-3\,f^2e^2)e^{(-3)}\log{(xe-d)}}{16\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*d^4*g^2 - 4*d^3*f*g*e - 2*d^2*f^2*e^2 - (d^2*g^2*e^2 + 2*d*f*g*e^3
- 3*f^2*e^4)*x^2 - (3*d^3*g^2*e - 2*d^2*f*g*e^2 + 3*d*f^2*e^3)*x)/(d^3*x^3*
e^6 - d^4*x^2*e^5 - d^5*x*e^4 + d^6*e^3) - 1/16*(d^2*g^2 + 2*d*f*g*e - 3*f^
2*e^2)*e^(-3)*log(x*e + d)/d^4 + 1/16*(d^2*g^2 + 2*d*f*g*e - 3*f^2*e^2)*e^(
-3)*log(x*e - d)/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(119) = 238$.
time = 2.94, size = 399, normalized size = 3.27

$$-\frac{4\,d^5g^2+6\,df^2x^2e^4-2\,(2\,d^2fgx^2+3\,d^2f^2x)e^3-2\,(d^3g^2x^2-2\,d^3fgx+2\,d^3f^2)e^2-2\,(3\,d^4g^2x+4\,d^4fg)e+(d^5g^2-3\,f^2x^3e^5+(2\,dfgx^3+3\,df^2x^2)e^4+(d^2g^2x^3-2\,d^2fgx^2+3\,d^2f^2x)e^3-(d^3g^2x^2+2\,d^3fgx+3\,d^3f^2)e^2-(d^4g^2x-2\,d^4fg)e)\log{(xe+d)}-(d^5g^2-3\,f^2x^3e^5+(2\,dfgx^3+3\,df^2x^2)e^4+(d^2g^2x^3-2\,d^2fgx^2+3\,d^2f^2x)e^3-(d^3g^2x^2+2\,d^3fgx+3\,d^3f^2)e^2-(d^4g^2x-2\,d^4fg)e)\log{(xe-d)}}{16\,(d^4x^3e^6-d^5x^2e^5-d^6xe^4+d^7e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*d^5*g^2 + 6*d*f^2*x^2*e^4 - 2*(2*d^2*f*g*x^2 + 3*d^2*f^2*x)*e^3 -
2*(d^3*g^2*x^2 - 2*d^3*f*g*x + 2*d^3*f^2)*e^2 - 2*(3*d^4*g^2*x + 4*d^4*f*g)
*e + (d^5*g^2 - 3*f^2*x^3*e^5 + (2*d*f*g*x^3 + 3*d*f^2*x^2)*e^4 + (d^2*g^2*
x^3 - 2*d^2*f*g*x^2 + 3*d^2*f^2*x)*e^3 - (d^3*g^2*x^2 + 2*d^3*f*g*x + 3*d^3
*f^2)*e^2 - (d^4*g^2*x - 2*d^4*f*g)*e)*log(x*e + d) - (d^5*g^2 - 3*f^2*x^3*
e^5 + (2*d*f*g*x^3 + 3*d*f^2*x^2)*e^4 + (d^2*g^2*x^3 - 2*d^2*f*g*x^2 + 3*d^
2*f^2*x)*e^3 - (d^3*g^2*x^2 + 2*d^3*f*g*x + 3*d^3*f^2)*e^2 - (d^4*g^2*x - 2
*d^4*f*g)*e)*log(x*e - d))/(d^4*x^3*e^6 - d^5*x^2*e^5 - d^6*x*e^4 + d^7*e^3
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(105) = 210$.
time = 0.64, size = 277, normalized size = 2.27

$$-\frac{2d^4g^2-4d^3efg-2d^2e^2f^2+x^2(-d^2e^2g^2-2de^3fg+3e^4f^2)+x(-3d^3eg^2+2d^2e^2fg-3de^3f^2)}{8d^6e^3-8d^5e^4x-8d^4e^5x^2+8d^3e^6x^3}+\frac{(dg-ef)\,(dg+3ef)\log{\left(-\frac{d(dg-ef)(dg+3ef)}{e(d^2g^2+2defg-3e^2f^2)}+x\right)}}{16d^4e^3}-\frac{(dg-ef)\,(dg+3ef)\log{\left(\frac{d(dg-ef)(dg+3ef)}{e(d^2g^2+2defg-3e^2f^2)}+x\right)}}{16d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(2*d**4*g**2 - 4*d**3*e*f*g - 2*d**2*e**2*f**2 + x**2*(-d**2*e**2*g**2 - 2
*d*e**3*f*g + 3*e**4*f**2) + x*(-3*d**3*e*g**2 + 2*d**2*e**2*f*g - 3*d*e**3
*f**2))/(8*d**6*e**3 - 8*d**5*e**4*x - 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3)
```

```
 + (d*g - e*f)*(d*g + 3*e*f)*log(-d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2
 + 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - e*f)*(d*g + 3*e*f
)*log(d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 2*d*e*f*g - 3*e**2*f**2))
 + x)/(16*d**4*e**3)
```

Giac [A]
time = 1.61, size = 195, normalized size = 1.60

$$-\frac{(d^2g^2+2\,dfge-3\,f^2e^2)e^{(-3)}\log\left(|xe+d|\right)}{16\,d^4}+\frac{(d^2g^2+2\,dfge-3\,f^2e^2)e^{(-3)}\log\left(|xe-d|\right)}{16\,d^4}-\frac{(2\,d^5g^2-4\,d^4fge-2\,d^3f^2e^2-(d^3g^2e^2+2\,d^2fge^3-3\,df^2e^4)x^2-(3\,d^4g^2e-2\,d^3fge^2+3\,d^2f^2e^3)x)e^{(-3)}}{8\,(xe+d)(xe-d)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(d^2*g^2 + 2*d*f*g*e - 3*f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^4 + 1/16
*(d^2*g^2 + 2*d*f*g*e - 3*f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^4 - 1/8*(2*d^
5*g^2 - 4*d^4*f*g*e - 2*d^3*f^2*e^2 - (d^3*g^2*e^2 + 2*d^2*f*g*e^3 - 3*d*f^
2*e^4)*x^2 - (3*d^4*g^2*e - 2*d^3*f*g*e^2 + 3*d^2*f^2*e^3)*x)*e^(-3)/((x*e
+ d)*(x*e - d)^2*d^4)
```

Mupad [B]
time = 2.64, size = 198, normalized size = 1.62

$$\frac{\frac{-d^2\,g^2+2\,d\,e\,f\,g+e^2\,f^2}{4\,d\,e^3}+\frac{x\,\left(3\,d^2\,g^2-2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{8\,d^2\,e^2}+\frac{x^2\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}{8\,d^3\,e}}{d^3-d^2\,e\,x-d\,e^2\,x^2+e^3\,x^3}-\frac{\mathrm{atanh}\left(\frac{e\,x\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{d\,\left(d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2\right)}\right)\,\left(d\,g-e\,f\right)\,\left(d\,g+3\,e\,f\right)}{8\,d^4\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x))/(d^2 - e^2*x^2)^3,x)
```

```
[Out] ((e^2*f^2 - d^2*g^2 + 2*d*e*f*g)/(4*d*e^3) + (x*(3*d^2*g^2 + 3*e^2*f^2 - 2*
d*e*f*g))/(8*d^2*e^2) + (x^2*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g))/(8*d^3*e))/
(d^3 + e^3*x^3 - d*e^2*x^2 - d^2*e*x) - (atanh((e*x*(d*g - e*f)*(d*g + 3*e*
f))/(d*(d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)))*(d*g - e*f)*(d*g + 3*e*f))/(8*d^
4*e^3)
```

3.576 $\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3} \, dx$

Optimal. Leaf size=127

$$\frac{(d^2g+e^2fx)\,(f+gx)}{4d^2e^2\,(d^2-e^2x^2)^2}+\frac{2d^2fg+(3e^2f^2-d^2g^2)\,x}{8d^4e^2\,(d^2-e^2x^2)}+\frac{(3e^2f^2-d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}$$

```
[Out] 1/4*(e^2*f*x+d^2*g)*(g*x+f)/d^2/e^2/(-e^2*x^2+d^2)^2+1/8*(2*d^2*f*g+(-d^2*g
^2+3*e^2*f^2)*x)/d^4/e^2/(-e^2*x^2+d^2)+1/8*(-d^2*g^2+3*e^2*f^2)*arctanh(e*
x/d)/d^5/e^3
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {753, 653, 214}

$$\frac{(f+gx)\,(d^2g+e^2fx)}{4d^2e^2\,(d^2-e^2x^2)^2}+\frac{(3e^2f^2-d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}+\frac{x(3e^2f^2-d^2g^2)+2d^2fg}{8d^4e^2\,(d^2-e^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] ((d^2*g + e^2*f*x)*(f + g*x))/(4*d^2*e^2*(d^2 - e^2*x^2)^2) + (2*d^2*f*g +
(3*e^2*f^2 - d^2*g^2)*x)/(8*d^4*e^2*(d^2 - e^2*x^2)) + ((3*e^2*f^2 - d^2*g^
2)*ArcTanh[(e*x)/d])/(8*d^5*e^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
```

ntQuadraticQ[a, 0, c, d, e, m, p, x]

Rubi steps

$$\int \frac{(f+gx)^2}{(d^2-e^2x^2)^3}\,dx = \frac{(d^2g+e^2fx)(f+gx)}{4d^2e^2(d^2-e^2x^2)^2} - \frac{\int \frac{-3e^2f^2+d^2g^2-2e^2fgx}{(d^2-e^2x^2)^2}\,dx}{4d^2e^2}$$

$$= \frac{(d^2g+e^2fx)(f+gx)}{4d^2e^2(d^2-e^2x^2)^2} + \frac{2d^2fg+(3e^2f^2-d^2g^2)x}{8d^4e^2(d^2-e^2x^2)} - \frac{\left(-\frac{3e^2f^2}{d^2}+g^2\right)\int \frac{1}{d^2-e^2x^2}\,dx}{8d^2e^2}$$

$$= \frac{(d^2g+e^2fx)(f+gx)}{4d^2e^2(d^2-e^2x^2)^2} + \frac{2d^2fg+(3e^2f^2-d^2g^2)x}{8d^4e^2(d^2-e^2x^2)} + \frac{(3e^2f^2-d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3}$$

Mathematica [A]
time = 0.03, size = 110, normalized size = 0.87

$$\frac{-3de^5f^2x^3+d^5eg(4f+gx)+d^3e^3x(5f^2+g^2x^2)+(3e^2f^2-d^2g^2)(d^2-e^2x^2)^2\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3(d^2-e^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/(d^2 - e^2*x^2)^3,x]
```

```
[Out] (-3*d*e^5*f^2*x^3 + d^5*e*g*(4*f + g*x) + d^3*e^3*x*(5*f^2 + g^2*x^2) + (3*
e^2*f^2 - d^2*g^2)*(d^2 - e^2*x^2)^2*ArcTanh[(e*x)/d])/(8*d^5*e^3*(d^2 - e^
2*x^2)^2)
```

Maple [A]
time = 0.08, size = 216, normalized size = 1.70

method	result
norman	$\frac{\frac{fg}{2e^2}+\frac{(d^2g^2-3e^2f^2)x^3}{8d^4}+\frac{(d^2g^2+5e^2f^2)x}{8e^2d^2}}{(-e^2x^2+d^2)^2}+\frac{(d^2g^2-3e^2f^2)\ln(-ex+d)}{16d^5e^3}-\frac{(d^2g^2-3e^2f^2)\ln(ex+d)}{16d^5e^3}$
risch	$\frac{\frac{fg}{2e^2}+\frac{(d^2g^2-3e^2f^2)x^3}{8d^4}+\frac{(d^2g^2+5e^2f^2)x}{8e^2d^2}}{(-e^2x^2+d^2)^2}-\frac{\ln(-ex-d)g^2}{16d^3e^3}+\frac{3\ln(-ex-d)f^2}{16d^5e}+\frac{\ln(ex-d)g^2}{16d^3e^3}-\frac{3\ln(ex-d)f^2}{16d^5e}$
default	$\frac{(-d^2g^2+3e^2f^2)\ln(ex+d)}{16e^3d^5}-\frac{-d^2g^2-2defg+3e^2f^2}{16e^3d^4(ex+d)}-\frac{d^2g^2-2defg+e^2f^2}{16e^3d^3(ex+d)^2}+\frac{(d^2g^2-3e^2f^2)\ln(-ex+d)}{16d^5e^3}-\frac{-d^2g^2-2defg-e^2f^2}{16e^3d^3(-ex+d)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-d^2*g^2+3*e^2*f^2)/e^3/d^5*ln(e*x+d)-1/16*(-d^2*g^2-2*d*e*f*g+3*e^2*
f^2)/e^3/d^4/(e*x+d)-1/16*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^3/(e*x+d)^2+1/1
```

```
6*(d^2*g^2-3*e^2*f^2)/d^5/e^3*ln(-e*x+d)-1/16*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/
e^3/d^3/(-e*x+d)^2+1/16*(-d^2*g^2+2*d*e*f*g+3*e^2*f^2)/e^3/d^4/(-e*x+d)
```

Maxima [A]
time = 0.29, size = 140, normalized size = 1.10

$$\frac{4\,d^4fg + (d^2g^2e^2 - 3\,f^2e^4)x^3 + (d^4g^2 + 5\,d^2f^2e^2)x}{8\,(d^4x^4e^6 - 2\,d^6x^2e^4 + d^8e^2)} - \frac{(d^2g^2 - 3\,f^2e^2)e^{(-3)}\log(xe+d)}{16\,d^5} + \frac{(d^2g^2 - 3\,f^2e^2)e^{(-3)}\log(xe-d)}{16\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*d^4*f*g + (d^2*g^2*e^2 - 3*f^2*e^4)*x^3 + (d^4*g^2 + 5*d^2*f^2*e^2)*
x)/(d^4*x^4*e^6 - 2*d^6*x^2*e^4 + d^8*e^2) - 1/16*(d^2*g^2 - 3*f^2*e^2)*e^(
-3)*log(x*e + d)/d^5 + 1/16*(d^2*g^2 - 3*f^2*e^2)*e^(-3)*log(x*e - d)/d^5
```

Fricas [A]
time = 2.49, size = 188, normalized size = 1.48

$$-\frac{6\,df^2x^3e^6 + (d^6g^2 - 3\,f^2x^4e^6 + (d^2g^2x^4 + 6\,d^2f^2x^2)e^4 - (2\,d^4g^2x^2 + 3\,d^4f^2)e^2)e\log\left(\frac{x^2e^2+2\,dxe+d^2}{x^2e^2-d^2}\right) - 2\,(d^3g^2x^3 + 5\,d^3f^2x)e^4 - 2\,(d^5g^2x + 4\,d^5fg)e^2}{16\,(d^5x^4e^8 - 2\,d^7x^2e^6 + d^9e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*d*f^2*x^3*e^6 + (d^6*g^2 - 3*f^2*x^4*e^6 + (d^2*g^2*x^4 + 6*d^2*f^
2*x^2)*e^4 - (2*d^4*g^2*x^2 + 3*d^4*f^2)*e^2)*e*log((x^2*e^2 + 2*d*x*e + d^
2)/(x^2*e^2 - d^2)) - 2*(d^3*g^2*x^3 + 5*d^3*f^2*x)*e^4 - 2*(d^5*g^2*x + 4*
d^5*f*g)*e^2)/(d^5*x^4*e^8 - 2*d^7*x^2*e^6 + d^9*e^4)
```

Sympy [A]
time = 0.48, size = 144, normalized size = 1.13

$$-\frac{-4d^4fg + x^3(-d^2e^2g^2 + 3e^4f^2) + x(-d^4g^2 - 5d^2e^2f^2)}{8d^8e^2 - 16d^6e^4x^2 + 8d^4e^6x^4} + \frac{(d^2g^2 - 3e^2f^2)\log\left(-\frac{d}{e} + x\right)}{16d^5e^3} - \frac{(d^2g^2 - 3e^2f^2)\log\left(\frac{d}{e} + x\right)}{16d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-4*d**4*f*g + x**3*(-d**2*e**2*g**2 + 3*e**4*f**2) + x*(-d**4*g**2 - 5*d*
*2*e**2*f**2))/(8*d**8*e**2 - 16*d**6*e**4*x**2 + 8*d**4*e**6*x**4) + (d**2
*g**2 - 3*e**2*f**2)*log(-d/e + x)/(16*d**5*e**3) - (d**2*g**2 - 3*e**2*f**
2)*log(d/e + x)/(16*d**5*e**3)
```

Giac [A]
time = 2.88, size = 127, normalized size = 1.00

$$\frac{(d^2g^2 - 3\,f^2e^2)e^{(-3)}\log\left(\frac{|2\,xe^2-2\,|d|e|}{|2\,xe^2+2\,|d|e|}\right)}{16\,d^4|d|} + \frac{(d^2g^2x^3e^2 + d^4g^2x + 4\,d^4fg - 3\,f^2x^3e^4 + 5\,d^2f^2xe^2)e^{(-2)}}{8\,(x^2e^2 - d^2)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(d^2*g^2 - 3*f^2*e^2)*e^(-3)*log(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2
 + 2*abs(d)*e))/(d^4*abs(d)) + 1/8*(d^2*g^2*x^3*e^2 + d^4*g^2*x + 4*d^4*f*g
 - 3*f^2*x^3*e^4 + 5*d^2*f^2*x*e^2)*e^(-2)/((x^2*e^2 - d^2)^2*d^4)
```

Mupad [B]
time = 0.10, size = 114, normalized size = 0.90

$$\frac{\frac{x^3\,\left(d^2\,g^2-3\,e^2\,f^2\right)}{8\,d^4}+\frac{f\,g}{2\,e^2}+\frac{x\,\left(d^2\,g^2+5\,e^2\,f^2\right)}{8\,d^2\,e^2}}{d^4-2\,d^2\,e^2\,x^2+e^4\,x^4}-\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(d^2\,g^2-3\,e^2\,f^2\right)}{8\,d^5\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/(d^2 - e^2*x^2)^3,x)
```

```
[Out] ((x^3*(d^2*g^2 - 3*e^2*f^2))/(8*d^4) + (f*g)/(2*e^2) + (x*(d^2*g^2 + 5*e^2*
f^2))/(8*d^2*e^2))/(d^4 + e^4*x^4 - 2*d^2*e^2*x^2) - (atanh((e*x)/d)*(d^2*g
^2 - 3*e^2*f^2))/(8*d^5*e^3)
```

3.577 $\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx$

Optimal. Leaf size=188

$$\frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2}-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}+\frac{(5e^2f^2+2def}{}$$

```
[Out] 1/32*(d*g+e*f)^2/d^4/e^3/(-e*x+d)^2+1/8*f*(d*g+e*f)/d^5/e^2/(-e*x+d)-1/24*(
-d*g+e*f)^2/d^3/e^3/(e*x+d)^3-1/32*(-d*g+e*f)*(d*g+3*e*f)/d^4/e^3/(e*x+d)^2
+1/16*(d^2*g^2-3*e^2*f^2)/d^5/e^3/(e*x+d)+1/16*(-d^2*g^2+2*d*e*f*g+5*e^2*f^
2)*arctanh(e*x/d)/d^6/e^3
```

Rubi [A]

time = 0.14, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\frac{f(dg+ef)}{8d^5e^2(d-ex)}-\frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2}+\frac{(dg+ef)^2}{32d^4e^3(d-ex)^2}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}+\frac{(-d^2g^2+2defg+5e^2f^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3}-\frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(32*d^4*e^3*(d - e*x)^2) + (f*(e*f + d*g))/(8*d^5*e^2*(d - e*
x)) - (e*f - d*g)^2/(24*d^3*e^3*(d + e*x)^3) - ((e*f - d*g)*(3*e*f + d*g))/
(32*d^4*e^3*(d + e*x)^2) - (3*e^2*f^2 - d^2*g^2)/(16*d^5*e^3*(d + e*x)) + (
(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(16*d^6*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] && (IntegerQ[p] || (GtQ[m, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^4}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{16d^4e^2(d-ex)^3}+\frac{f(ef+dg)}{8d^5e(d-ex)^2}+\frac{(-ef+dg)^2}{8d^3e^2(d+ex)^4}+\frac{(ef-dg)(3ef+d
\right. \\
&= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2} \\
&= \frac{(ef+dg)^2}{32d^4e^3(d-ex)^2}+\frac{f(ef+dg)}{8d^5e^2(d-ex)}-\frac{(ef-dg)^2}{24d^3e^3(d+ex)^3}-\frac{(ef-dg)(3ef+dg)}{32d^4e^3(d+ex)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 197, normalized size = 1.05

$$
\frac{\frac{3d^2(ef+dg)^2}{(d-ex)^2}+\frac{12def(ef+dg)}{d-ex}-\frac{4d^3(ef-dg)^2}{(d+ex)^3}+\frac{3d^2\left(-3e^2f^2+2defg+d^2g^2\right)}{(d+ex)^2}+\frac{6d\left(-3e^2f^2+d^2g^2\right)}{d+ex}+3\left(-5e^2f^2-2defg+d^2g^2\right)\log(d-ex)+3\left(5e^2f^2+2defg-d^2g^2\right)\log(d+ex)}{96d^6e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((3*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (12*d*e*f*(e*f + d*g))/(d - e*x) - (4*
d^3*(e*f - d*g)^2)/(d + e*x)^3 + (3*d^2*(-3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))
/(d + e*x)^2 + (6*d*(-3*e^2*f^2 + d^2*g^2))/(d + e*x) + 3*(-5*e^2*f^2 - 2*d
*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*Log[d
+ e*x])/(96*d^6*e^3)
```

Maple [A]
time = 0.12, size = 245, normalized size = 1.30

method	result
default	$-\frac{-d^2g^2+3e^2f^2}{16e^3d^5(ex+d)}-\frac{-d^2g^2-2defg+3e^2f^2}{32e^3d^4(ex+d)^2}+\frac{\left(-d^2g^2+2defg+5e^2f^2\right)\ln(ex+d)}{32e^3d^6}-\frac{d^2g^2-2defg+e^2f^2}{24e^3d^3(ex+d)^3}-\frac{-d^2g^2-2defg-e^2f^2}{32e^3d^4(-ex+d)^2}$
norman	$\frac{\frac{\left(11d^2g^2+26defg-31e^2f^2\right)x^3}{48d^4}+\frac{\left(d^2g^2+14defg+3e^2f^2\right)x^2}{16e\,d^3}-\frac{e\left(d^2g^2+22defg+7e^2f^2\right)x^4}{48d^5}-\frac{e^2\left(d^2g^2+4defg-2e^2f^2\right)x^5}{12d^6}+\frac{\left(d^2g^2-2defg+11e^2f^2\right)}{16e^2d^2}}{(ex+d)^3(-ex+d)^2}$
risch	$\frac{\frac{\left(d^2g^2-2defg-5e^2f^2\right)e\,x^4}{16d^5}+\frac{\left(d^2g^2-2defg-5e^2f^2\right)x^3}{16d^4}-\frac{5\left(d^2g^2-2defg-5e^2f^2\right)x^2}{48d^3e}+\frac{\left(7d^2g^2+10defg+25e^2f^2\right)x}{48e^2d^2}+\frac{d^2g^2+4defg-2e^2f^2}{12d\,e^3}}{(ex+d)\left(-e^2x^2+d^2\right)^2}+\frac{\ln(e}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-d^2*g^2+3*e^2*f^2)/e^3/d^5/(e*x+d)-1/32*(-d^2*g^2-2*d*e*f*g+3*e^2*f
^2)/e^3/d^4/(e*x+d)^2+1/32*(-d^2*g^2+2*d*e*f*g+5*e^2*f^2)/e^3/d^6*ln(e*x+d)
```

```
-1/24*(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3/d^3/(e*x+d)^3-1/32*(-d^2*g^2-2*d*e*f*
g-e^2*f^2)/e^3/d^4/(-e*x+d)^2+1/32*(d^2*g^2-2*d*e*f*g-5*e^2*f^2)/d^6/e^3*ln
(-e*x+d)+1/8*f*(d*g+e*f)/d^5/e^2/(-e*x+d)
```

Maxima [A]
time = 0.30, size = 286, normalized size = 1.52

$$\frac{4\,d^6g^2+16\,d^5fge-8\,d^4f^2e^2+3\,(d^2g^2e^4-2\,dfge^5-5\,f^2e^6)x^4+3\,(d^3g^2e^3-2\,d^2fge^4-5\,df^2e^5)x^3-5\,(d^4g^2e^2-2\,d^3fge^3-5\,d^2f^2e^4)x^2+(7\,d^5g^2e+10\,d^4fge^2+25\,d^3f^2e^3)x}{48\,(d^5x^5e^8+d^6x^4e^7-2\,d^7x^3e^6-2\,d^8x^2e^5+d^9xe^4+d^{10}e^3)}-\frac{(d^2g^2-2\,dfge-5\,f^2e^2)e^{(-3)}\log(xe+d)}{32\,d^6}+\frac{(d^2g^2-2\,dfge-5\,f^2e^2)e^{(-3)}\log(xe-d)}{32\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(4*d^6*g^2 + 16*d^5*f*g*e - 8*d^4*f^2*e^2 + 3*(d^2*g^2*e^4 - 2*d*f*g*e
^5 - 5*f^2*e^6)*x^4 + 3*(d^3*g^2*e^3 - 2*d^2*f*g*e^4 - 5*d*f^2*e^5)*x^3 - 5
*(d^4*g^2*e^2 - 2*d^3*f*g*e^3 - 5*d^2*f^2*e^4)*x^2 + (7*d^5*g^2*e + 10*d^4*
f*g*e^2 + 25*d^3*f^2*e^3)*x)/(d^5*x^5*e^8 + d^6*x^4*e^7 - 2*d^7*x^3*e^6 - 2
*d^8*x^2*e^5 + d^9*x*e^4 + d^10*e^3) - 1/32*(d^2*g^2 - 2*d*f*g*e - 5*f^2*e^
2)*e^(-3)*log(x*e + d)/d^6 + 1/32*(d^2*g^2 - 2*d*f*g*e - 5*f^2*e^2)*e^(-3)*
log(x*e - d)/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(180) = 360$.
time = 2.86, size = 639, normalized size = 3.40

$$\frac{8\,d^7g^2-30\,df^2x^4e^6-6\,(2\,d^2fgx^4+5\,d^2f^2x^3)e^5+2\,(3\,d^3g^2x^4-6\,d^3fgx^3+25\,d^3f^2x^2)e^4+2\,(3\,d^4g^2x^3+10\,d^4fgx^2+25\,d^4f^2x)e^3-2\,(5\,d^5g^2x^2-10\,d^5fgx+8\,d^5f^2)e^2+2\,(7\,d^6g^2x+16\,d^6fg)e-3\,(d^7g^2-5\,f^2x^5e^7-(2\,dfgx^5+5\,df^2x^4)e^6+(d^2g^2x^5-2\,d^2fgx^4+10\,d^2f^2x^3)e^5+(d^3g^2x^4+4\,d^3fgx^3+10\,d^3f^2x^2)e^4-(2\,d^4g^2x^3-4\,d^4fgx^2+5\,d^4f^2x)e^3-(2\,d^5g^2x^2+2\,d^5fgx+5\,d^5f^2)e^2+(d^6g^2x-2\,d^6fg)e)\log(xe+d)+3\,(d^7g^2-5\,f^2x^5e^7-(2\,dfgx^5+5\,df^2x^4)e^6+(d^2g^2x^5-2\,d^2fgx^4+10\,d^2f^2x^3)e^5+(d^3g^2x^4+4\,d^3fgx^3+10\,d^3f^2x^2)e^4-(2\,d^4g^2x^3-4\,d^4fgx^2+5\,d^4f^2x)e^3-(2\,d^5g^2x^2+2\,d^5fgx+5\,d^5f^2)e^2+(d^6g^2x-2\,d^6fg)e)\log(xe-d)}{96\,(d^6x^5e^8+d^7x^4e^7-2\,d^8x^3e^6-2\,d^9x^2e^5+d^{10}xe^4+d^{11}e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(8*d^7*g^2 - 30*d*f^2*x^4*e^6 - 6*(2*d^2*f*g*x^4 + 5*d^2*f^2*x^3)*e^5
+ 2*(3*d^3*g^2*x^4 - 6*d^3*f*g*x^3 + 25*d^3*f^2*x^2)*e^4 + 2*(3*d^4*g^2*x^3
 + 10*d^4*f*g*x^2 + 25*d^4*f^2*x)*e^3 - 2*(5*d^5*g^2*x^2 - 10*d^5*f*g*x + 8
*d^5*f^2)*e^2 + 2*(7*d^6*g^2*x + 16*d^6*f*g)*e - 3*(d^7*g^2 - 5*f^2*x^5*e^7
 - (2*d*f*g*x^5 + 5*d*f^2*x^4)*e^6 + (d^2*g^2*x^5 - 2*d^2*f*g*x^4 + 10*d^2*
f^2*x^3)*e^5 + (d^3*g^2*x^4 + 4*d^3*f*g*x^3 + 10*d^3*f^2*x^2)*e^4 - (2*d^4*
g^2*x^3 - 4*d^4*f*g*x^2 + 5*d^4*f^2*x)*e^3 - (2*d^5*g^2*x^2 + 2*d^5*f*g*x +
 5*d^5*f^2)*e^2 + (d^6*g^2*x - 2*d^6*f*g)*e)*log(x*e + d) + 3*(d^7*g^2 - 5*
f^2*x^5*e^7 - (2*d*f*g*x^5 + 5*d*f^2*x^4)*e^6 + (d^2*g^2*x^5 - 2*d^2*f*g*x^
4 + 10*d^2*f^2*x^3)*e^5 + (d^3*g^2*x^4 + 4*d^3*f*g*x^3 + 10*d^3*f^2*x^2)*e^
4 - (2*d^4*g^2*x^3 - 4*d^4*f*g*x^2 + 5*d^4*f^2*x)*e^3 - (2*d^5*g^2*x^2 + 2*
d^5*f*g*x + 5*d^5*f^2)*e^2 + (d^6*g^2*x - 2*d^6*f*g)*e)*log(x*e - d))/(d^6*
x^5*e^8 + d^7*x^4*e^7 - 2*d^8*x^3*e^6 - 2*d^9*x^2*e^5 + d^10*x*e^4 + d^11*e
^3)
```

Sympy [A]
time = 0.90, size = 321, normalized size = 1.71

$$-\frac{-4d^6g^2-16d^5efg+8d^4e^2f^2+x^4(-3d^2e^4g^2+6de^5fg+15e^6f^2)+x^3(-3d^3e^3g^2+6d^2e^4fg+15de^5f^2)+x^2\cdot(5d^4e^2g^2-10d^3e^3fg-25d^2e^4f^2)+x(-7d^5eg^2-10d^4e^2fg-25d^3e^3f^2)}{48d^{10}e^3+48d^9e^4x-96d^8e^5x^2-96d^7e^6x^3+48d^6e^7x^4+48d^5e^8x^5}+\frac{(d^2g^2-2defg-5e^2f^2)\log\left(-\frac{d}{e}+x\right)}{32d^6e^3}-\frac{(d^2g^2-2defg-5e^2f^2)\log\left(\frac{d}{e}+x\right)}{32d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-4*d**6*g**2 - 16*d**5*e*f*g + 8*d**4*e**2*f**2 + x**4*(-3*d**2*e**4*g**2
 + 6*d*e**5*f*g + 15*e**6*f**2) + x**3*(-3*d**3*e**3*g**2 + 6*d**2*e**4*f*g
 + 15*d*e**5*f**2) + x**2*(5*d**4*e**2*g**2 - 10*d**3*e**3*f*g - 25*d**2*e*
*4*f**2) + x*(-7*d**5*e*g**2 - 10*d**4*e**2*f*g - 25*d**3*e**3*f**2))/(48*d
**10*e**3 + 48*d**9*e**4*x - 96*d**8*e**5*x**2 - 96*d**7*e**6*x**3 + 48*d**
6*e**7*x**4 + 48*d**5*e**8*x**5) + (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*lo
g(-d/e + x)/(32*d**6*e**3) - (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*log(d/e
+ x)/(32*d**6*e**3)
```

Giac [A]
time = 2.03, size = 262, normalized size = 1.39

$$-\frac{(d^2g^2 - 2\,dfge - 5\,f^2e^2)e^{(-3)}\log\left(|xe + d|\right)}{32\,d^6} + \frac{(d^2g^2 - 2\,dfge - 5\,f^2e^2)e^{(-3)}\log\left(|xe - d|\right)}{32\,d^6} + \frac{(4\,d^7g^2 + 16\,d^6fge - 8\,d^5f^2e^2 + 3\,(d^3g^2e^4 - 2\,d^2fge^5 - 5\,df^2e^6)x^4 + 3\,(d^4g^2e^3 - 2\,d^3fge^4 - 5\,d^2f^2e^5)x^3 - 5\,(d^5g^2e^2 - 2\,d^4fge^3 - 5\,d^3f^2e^4)x^2 + (7\,d^6g^2e + 10\,d^5fge^2 + 25\,d^4f^2e^3)x)e^{(-3)}}{48\,(xe + d)^3(xe - d)^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] -1/32*(d^2*g^2 - 2*d*f*g*e - 5*f^2*e^2)*e^(-3)*log(abs(x*e + d))/d^6 + 1/32
*(d^2*g^2 - 2*d*f*g*e - 5*f^2*e^2)*e^(-3)*log(abs(x*e - d))/d^6 + 1/48*(4*d
^7*g^2 + 16*d^6*f*g*e - 8*d^5*f^2*e^2 + 3*(d^3*g^2*e^4 - 2*d^2*f*g*e^5 - 5*
d*f^2*e^6)*x^4 + 3*(d^4*g^2*e^3 - 2*d^3*f*g*e^4 - 5*d^2*f^2*e^5)*x^3 - 5*(d
^5*g^2*e^2 - 2*d^4*f*g*e^3 - 5*d^3*f^2*e^4)*x^2 + (7*d^6*g^2*e + 10*d^5*f*g
*e^2 + 25*d^4*f^2*e^3)*x)*e^(-3)/((x*e + d)^3*(x*e - d)^2*d^6)
```

Mupad [B]
time = 2.68, size = 249, normalized size = 1.32

$$\frac{\frac{d^2\,g^2+4\,d\,e\,f\,g-2\,e^2\,f^2}{12\,d\,e^3}-\frac{x^3\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^4}-\frac{e\,x^4\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^5}+\frac{x\,\left(7\,d^2\,g^2+10\,d\,e\,f\,g+25\,e^2\,f^2\right)}{48\,d^2\,e^2}+\frac{5\,x^2\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{48\,d^3\,e}}{d^5+d^4\,e\,x-2\,d^3\,e^2\,x^2-2\,d^2\,e^3\,x^3+d\,e^4\,x^4+e^5\,x^5}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(-d^2\,g^2+2\,d\,e\,f\,g+5\,e^2\,f^2\right)}{16\,d^6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)),x)
```

```
[Out] ((d^2*g^2 - 2*e^2*f^2 + 4*d*e*f*g)/(12*d*e^3) - (x^3*(5*e^2*f^2 - d^2*g^2 +
 2*d*e*f*g))/(16*d^4) - (e*x^4*(5*e^2*f^2 - d^2*g^2 + 2*d*e*f*g))/(16*d^5)
+ (x*(7*d^2*g^2 + 25*e^2*f^2 + 10*d*e*f*g))/(48*d^2*e^2) + (5*x^2*(5*e^2*f^
2 - d^2*g^2 + 2*d*e*f*g))/(48*d^3*e))/(d^5 + e^5*x^5 + d*e^4*x^4 - 2*d^3*e^
2*x^2 - 2*d^2*e^3*x^3 + d^4*e*x) + (atanh((e*x)/d)*(5*e^2*f^2 - d^2*g^2 + 2
*d*e*f*g))/(16*d^6*e^3)
```

3.578 $\int \frac{(f+gx)^2}{(d+ex)^2 (d^2-e^2x^2)^3} \, dx$

Optimal. Leaf size=235

$$\frac{(ef+dg)^2}{64d^5e^3(d-ex)^2}+\frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}-\frac{(ef-dg)(3ef+dg)}{48d^4e^3(d+ex)^3}-\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}-\frac{5e^2f^2 \cdots}{32 \cdots}$$

```
[Out] 1/64*(d*g+e*f)^2/d^5/e^3/(-e*x+d)^2+1/64*(d*g+e*f)*(d*g+5*e*f)/d^6/e^3/(-e*
x+d)-1/32*(-d*g+e*f)^2/d^3/e^3/(e*x+d)^4-1/48*(-d*g+e*f)*(d*g+3*e*f)/d^4/e^
3/(e*x+d)^3+1/32*(d^2*g^2-3*e^2*f^2)/d^5/e^3/(e*x+d)^2+1/32*(d^2*g^2-2*d*e*
f*g-5*e^2*f^2)/d^6/e^3/(e*x+d)+1/64*(-d^2*g^2+10*d*e*f*g+15*e^2*f^2)*arctan
h(e*x/d)/d^7/e^3
```

Rubi [A]

time = 0.17, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {862, 90, 214}

$$\frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)}+\frac{(dg+ef)^2}{64d^5e^3(d-ex)^2}-\frac{(dg+3ef)(ef-dg)}{48d^4e^3(d+ex)^3}-\frac{(ef-dg)^2}{32d^3e^3(d+ex)^4}+\frac{(-d^2g^2+10defg+15e^2f^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{64d^7e^3}-\frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)}-\frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] (e*f + d*g)^2/(64*d^5*e^3*(d - e*x)^2) + ((e*f + d*g)*(5*e*f + d*g))/(64*d^
6*e^3*(d - e*x)) - (e*f - d*g)^2/(32*d^3*e^3*(d + e*x)^4) - ((e*f - d*g)*(3
*e*f + d*g))/(48*d^4*e^3*(d + e*x)^3) - (3*e^2*f^2 - d^2*g^2)/(32*d^5*e^3*(
d + e*x)^2) - (5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)/(32*d^6*e^3*(d + e*x)) + ((
15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*ArcTanh[(e*x)/d])/(64*d^7*e^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx)^2}{(d+ex)^2\left(d^2-e^2x^2\right)^3}\,dx &= \int \frac{(f+gx)^2}{(d-ex)^3(d+ex)^5}\,dx \\
&= \int \left(\frac{(ef+dg)^2}{32d^5e^2(d-ex)^3} + \frac{(ef+dg)(5ef+dg)}{64d^6e^2(d-ex)^2} + \frac{(-ef+dg)^2}{8d^3e^2(d+ex)^5} + \frac{(ef-dg)(3\cdots}{16d^4e^2(d\cdots}\right. \\
&= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2} + \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{(ef-dg)(3ef\cdots}{48d^4e^3(d+e\cdots} \\
&= \frac{(ef+dg)^2}{64d^5e^3(d-ex)^2} + \frac{(ef+dg)(5ef+dg)}{64d^6e^3(d-ex)} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} - \frac{(ef-dg)(3ef\cdots}{48d^4e^3(d+e\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 244, normalized size = 1.04

$$
\frac{\frac{6d^2(ef+dg)^2}{(d-ex)^2} + \frac{6d\left(5e^2f^2+6defg+d^2g^2\right)}{d-ex} - \frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{8d^3\left(-3e^2f^2+2defg+d^2g^2\right)}{(d+ex)^3} + \frac{12d^2\left(-3e^2f^2+d^2g^2\right)}{(d+ex)^2} + \frac{12d\left(-5e^2f^2-2defg+d^2g^2\right)}{d+ex} + 3\left(-15e^2f^2-10defg+d^2g^2\right)\log(d-ex) + 3\left(15e^2f^2+10defg-d^2g^2\right)\log(d+ex)}{384d^7e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)^2*(d^2 - e^2*x^2)^3),x]
```

```
[Out] ((6*d^2*(e*f + d*g)^2)/(d - e*x)^2 + (6*d*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)
)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (8*d^3*(-3*e^2*f^2 + 2*d
*e*f*g + d^2*g^2))/(d + e*x)^3 + (12*d^2*(-3*e^2*f^2 + d^2*g^2))/(d + e*x)^
2 + (12*d*(-5*e^2*f^2 - 2*d*e*f*g + d^2*g^2))/(d + e*x) + 3*(-15*e^2*f^2 -
10*d*e*f*g + d^2*g^2)*Log[d - e*x] + 3*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*
Log[d + e*x])/(384*d^7*e^3)
```

Maple [A]
time = 0.12, size = 297, normalized size = 1.26

method	result
norman	$\frac{\frac{\left(31d^2g^2+74defg-81e^2f^2\right)x^3}{96d^4} + \frac{\left(d^2g^2+22defg+17e^2f^2\right)x^2}{32e\,d^3} + \frac{e\left(11d^2g^2-14defg-69e^2f^2\right)x^4}{96d^5} - \frac{e^2\left(29d^2g^2+94defg-51e^2f^2\right)x^5}{192d^6} - \frac{e^3\left(d^2g^2+2defg\cdots\right.}{12d\cdots}}{(ex+d)^4(-ex+d)^2}$
default	$-\frac{-d^2g^2+3e^2f^2}{32e^3d^5(ex+d)^2} - \frac{-d^2g^2-2defg+3e^2f^2}{48e^3d^4(ex+d)^3} - \frac{-d^2g^2+2defg+5e^2f^2}{32e^3d^6(ex+d)} + \frac{\left(-d^2g^2+10defg+15e^2f^2\right)\ln(ex+d)}{128e^3d^7} - \frac{d^2g^2-2defg+e\cdots}{32e^3d^3(ex+d)\cdots}$
risch	$\frac{\frac{\left(d^2g^2-10defg-15e^2f^2\right)e^2x^5}{64d^6} + \frac{\left(d^2g^2-10defg-15e^2f^2\right)e\,x^4}{32d^5} - \frac{\left(d^2g^2-10defg-15e^2f^2\right)x^3}{96d^4} - \frac{5\left(d^2g^2-10defg-15e^2f^2\right)x^2}{96d^3e} + \frac{\left(35d^2g^2+34defg+5\cdots\right.}{192d^2e^2}}{(ex+d)^2\left(-e^2x^2+d^2\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(-d^2*g^2+3*e^2*f^2)/e^3/d^5/(e*x+d)^2-1/48*(-d^2*g^2-2*d*e*f*g+3*e^2
*f^2)/e^3/d^4/(e*x+d)^3-1/32*(-d^2*g^2+2*d*e*f*g+5*e^2*f^2)/e^3/d^6/(e*x+d)
+1/128*(-d^2*g^2+10*d*e*f*g+15*e^2*f^2)/e^3/d^7*ln(e*x+d)-1/32*(d^2*g^2-2*d
*e*f*g+e^2*f^2)/e^3/d^3/(e*x+d)^4+1/64/e^3*(d^2*g^2+6*d*e*f*g+5*e^2*f^2)/d^
6/(-e*x+d)-1/64*(-d^2*g^2-2*d*e*f*g-e^2*f^2)/e^3/d^5/(-e*x+d)^2+1/128*(d^2*
g^2-10*d*e*f*g-15*e^2*f^2)/d^7/e^3*ln(-e*x+d)
```

Maxima [A]
time = 0.32, size = 332, normalized size = 1.41

$$\frac{16\,d^7g^2+32\,d^6fge-48\,d^5f^2e^2+3\,(d^2g^2e^5-10\,dfge^6-15\,f^2e^7)x^5+6\,(d^3g^2e^4-10\,d^2fge^5-15\,df^2e^6)x^4-2\,(d^4g^2e^3-10\,d^3fge^4-15\,d^2f^2e^5)x^3-10\,(d^5g^2e^2-10\,d^4fge^3-15\,d^3f^2e^4)x^2+(35\,d^6g^2e+34\,d^5fge^2+51\,d^4f^2e^3)x}{192\,(d^6x^6e^9+2\,d^7x^5e^8-d^8x^4e^7-4\,d^9x^3e^6-d^{10}x^2e^5+2\,d^{11}xe^4+d^{12}e^3)}-\frac{(d^2g^2-10\,dfge-15\,f^2e^2)e^{(-3)}\log(xe+d)}{128\,d^7}+\frac{(d^2g^2-10\,dfge-15\,f^2e^2)e^{(-3)}\log(xe-d)}{128\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(16*d^7*g^2 + 32*d^6*f*g*e - 48*d^5*f^2*e^2 + 3*(d^2*g^2*e^5 - 10*d*f
*g*e^6 - 15*f^2*e^7)*x^5 + 6*(d^3*g^2*e^4 - 10*d^2*f*g*e^5 - 15*d*f^2*e^6)*
x^4 - 2*(d^4*g^2*e^3 - 10*d^3*f*g*e^4 - 15*d^2*f^2*e^5)*x^3 - 10*(d^5*g^2*e
^2 - 10*d^4*f*g*e^3 - 15*d^3*f^2*e^4)*x^2 + (35*d^6*g^2*e + 34*d^5*f*g*e^2
+ 51*d^4*f^2*e^3)*x)/(d^6*x^6*e^9 + 2*d^7*x^5*e^8 - d^8*x^4*e^7 - 4*d^9*x^3
*e^6 - d^10*x^2*e^5 + 2*d^11*x*e^4 + d^12*e^3) - 1/128*(d^2*g^2 - 10*d*f*g*
e - 15*f^2*e^2)*e^(-3)*log(x*e + d)/d^7 + 1/128*(d^2*g^2 - 10*d*f*g*e - 15*
f^2*e^2)*e^(-3)*log(x*e - d)/d^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 758 vs. 2(225) = 450.
time = 2.04, size = 758, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(32*d^8*g^2 - 90*d*f^2*x^5*e^7 - 60*(d^2*f*g*x^5 + 3*d^2*f^2*x^4)*e^6
 + 6*(d^3*g^2*x^5 - 20*d^3*f*g*x^4 + 10*d^3*f^2*x^3)*e^5 + 4*(3*d^4*g^2*x^4
 + 10*d^4*f*g*x^3 + 75*d^4*f^2*x^2)*e^4 - 2*(2*d^5*g^2*x^3 - 100*d^5*f*g*x^
2 - 51*d^5*f^2*x)*e^3 - 4*(5*d^6*g^2*x^2 - 17*d^6*f*g*x + 24*d^6*f^2)*e^2 +
 2*(35*d^7*g^2*x + 32*d^7*f*g)*e - 3*(d^8*g^2 - 15*f^2*x^6*e^8 - 10*(d*f*g*
x^6 + 3*d*f^2*x^5)*e^7 + (d^2*g^2*x^6 - 20*d^2*f*g*x^5 + 15*d^2*f^2*x^4)*e^
6 + 2*(d^3*g^2*x^5 + 5*d^3*f*g*x^4 + 30*d^3*f^2*x^3)*e^5 - (d^4*g^2*x^4 - 4
```

```
0*d^4*f*g*x^3 - 15*d^4*f^2*x^2)*e^4 - 2*(2*d^5*g^2*x^3 - 5*d^5*f*g*x^2 + 15
*d^5*f^2*x)*e^3 - (d^6*g^2*x^2 + 20*d^6*f*g*x + 15*d^6*f^2)*e^2 + 2*(d^7*g^
2*x - 5*d^7*f*g)*e)*log(x*e + d) + 3*(d^8*g^2 - 15*f^2*x^6*e^8 - 10*(d*f*g*
x^6 + 3*d*f^2*x^5)*e^7 + (d^2*g^2*x^6 - 20*d^2*f*g*x^5 + 15*d^2*f^2*x^4)*e^
6 + 2*(d^3*g^2*x^5 + 5*d^3*f*g*x^4 + 30*d^3*f^2*x^3)*e^5 - (d^4*g^2*x^4 - 4
0*d^4*f*g*x^3 - 15*d^4*f^2*x^2)*e^4 - 2*(2*d^5*g^2*x^3 - 5*d^5*f*g*x^2 + 15
*d^5*f^2*x)*e^3 - (d^6*g^2*x^2 + 20*d^6*f*g*x + 15*d^6*f^2)*e^2 + 2*(d^7*g^
2*x - 5*d^7*f*g)*e)*log(x*e - d))/(d^7*x^6*e^9 + 2*d^8*x^5*e^8 - d^9*x^4*e^
7 - 4*d^10*x^3*e^6 - d^11*x^2*e^5 + 2*d^12*x*e^4 + d^13*e^3)
```

Sympy [**A**]
time = 1.01, size = 372, normalized size = 1.58

$$- \frac{-16d^7g^2 - 32d^6efg + 48d^5e^2f^2 + x^5(-3d^2e^5g^2 + 30de^6fg + 45e^7f^2) + x^4(-6d^3e^4g^2 + 60d^2e^5fg + 90de^6f^2) + x^3 \cdot (2d^4e^3g^2 - 20d^3e^4fg - 30d^2e^5f^2) + x^2 \cdot (10d^5e^2g^2 - 100d^4e^3fg - 150d^3e^4f^2) + x(-35d^6eg^2 - 34d^5e^2fg - 51d^4e^3f^2)}{192d^{12}e^3 + 384d^{11}e^4x - 192d^{10}e^5x^2 - 768d^9e^6x^3 - 192d^8e^7x^4 + 384d^7e^8x^5 + 192d^6e^9x^6} + \frac{(d^2g^2 - 10defg - 15e^2f^2)\log\left(-\frac{d}{e} + x\right)}{128d^7e^3} - \frac{(d^2g^2 - 10defg - 15e^2f^2)\log\left(\frac{d}{e} + x\right)}{128d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] -(-16*d**7*g**2 - 32*d**6*e*f*g + 48*d**5*e**2*f**2 + x**5*(-3*d**2*e**5*g*
*2 + 30*d*e**6*f*g + 45*e**7*f**2) + x**4*(-6*d**3*e**4*g**2 + 60*d**2*e**5
*f*g + 90*d*e**6*f**2) + x**3*(2*d**4*e**3*g**2 - 20*d**3*e**4*f*g - 30*d**
2*e**5*f**2) + x**2*(10*d**5*e**2*g**2 - 100*d**4*e**3*f*g - 150*d**3*e**4*
f**2) + x*(-35*d**6*e*g**2 - 34*d**5*e**2*f*g - 51*d**4*e**3*f**2))/(192*d*
*12*e**3 + 384*d**11*e**4*x - 192*d**10*e**5*x**2 - 768*d**9*e**6*x**3 - 19
2*d**8*e**7*x**4 + 384*d**7*e**8*x**5 + 192*d**6*e**9*x**6) + (d**2*g**2 -
10*d*e*f*g - 15*e**2*f**2)*log(-d/e + x)/(128*d**7*e**3) - (d**2*g**2 - 10*
d*e*f*g - 15*e**2*f**2)*log(d/e + x)/(128*d**7*e**3)
```

Giac [**A**]
time = 1.78, size = 332, normalized size = 1.41

$$\frac{(d^2g^2 - 10\,dfge - 15\,f^2e^2)e^{(-3)}\log\left(\left|-\frac{2\,d}{xe+d} + 1\right|\right)}{128\,d^7} - \frac{\left(3\,d^2g^2 + 14\,dfge + 11\,f^2e^2 - \frac{8\,(d^3g^2e + 4\,d^2fge^2 + 3\,df^2e^3)e^{(-1)}}{xe+d}\right)e^{(-3)}}{256\,d^7\left(\frac{2\,d}{xe+d} - 1\right)^2} + \frac{\left(\frac{3\,d^8g^2e^9}{xe+d} + \frac{3\,d^9g^2e^9}{(xe+d)^2} + \frac{2\,d^{10}g^2e^9}{(xe+d)^3} - \frac{3\,d^{11}g^2e^9}{(xe+d)^4} - \frac{6\,d^7fge^{10}}{xe+d} + \frac{4\,d^9fge^{10}}{(xe+d)^3} + \frac{6\,d^{10}fge^{10}}{(xe+d)^4} - \frac{15\,d^6f^2e^{11}}{xe+d} - \frac{9\,d^7f^2e^{11}}{(xe+d)^2} - \frac{6\,d^8f^2e^{11}}{(xe+d)^3} - \frac{3\,d^9f^2e^{11}}{(xe+d)^4}\right)e^{(-12)}}{96\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

```
[Out] 1/128*(d^2*g^2 - 10*d*f*g*e - 15*f^2*e^2)*e^(-3)*log(abs(-2*d/(x*e + d) + 1
))/d^7 - 1/256*(3*d^2*g^2 + 14*d*f*g*e + 11*f^2*e^2 - 8*(d^3*g^2*e + 4*d^2*
f*g*e^2 + 3*d*f^2*e^3)*e^(-1)/(x*e + d))*e^(-3)/(d^7*(2*d/(x*e + d) - 1)^2)
 + 1/96*(3*d^8*g^2*e^9/(x*e + d) + 3*d^9*g^2*e^9/(x*e + d)^2 + 2*d^10*g^2*e
^9/(x*e + d)^3 - 3*d^11*g^2*e^9/(x*e + d)^4 - 6*d^7*f*g*e^10/(x*e + d) + 4*
d^9*f*g*e^10/(x*e + d)^3 + 6*d^10*f*g*e^10/(x*e + d)^4 - 15*d^6*f^2*e^11/(x
*e + d) - 9*d^7*f^2*e^11/(x*e + d)^2 - 6*d^8*f^2*e^11/(x*e + d)^3 - 3*d^9*f
^2*e^11/(x*e + d)^4)*e^(-12)/d^12
```

Mupad [B]
time = 2.64, size = 296, normalized size = 1.26

$$\frac{\frac{d^2\,g^2+2\,d\,e\,f\,g-3\,e^2\,f^2}{12\,d\,e^3}+\frac{x^3\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{96\,d^4}-\frac{e\,x^4\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{32\,d^5}+\frac{x\,\left(35\,d^2\,g^2+34\,d\,e\,f\,g+51\,e^2\,f^2\right)}{192\,d^2\,e^2}+\frac{5\,x^2\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{96\,d^3\,e}-\frac{e^2\,x^5\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{64\,d^6}}{d^6+2\,d^5\,e\,x-d^4\,e^2\,x^2-4\,d^3\,e^3\,x^3-d^2\,e^4\,x^4+2\,d\,e^5\,x^5+e^6\,x^6}+\frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)\,\left(-d^2\,g^2+10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{64\,d^7\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d^2 - e^2*x^2)^3*(d + e*x)^2),x)
```

```
[Out] ((d^2*g^2 - 3*e^2*f^2 + 2*d*e*f*g)/(12*d*e^3) + (x^3*(15*e^2*f^2 - d^2*g^2
+ 10*d*e*f*g))/(96*d^4) - (e*x^4*(15*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(32*d
^5) + (x*(35*d^2*g^2 + 51*e^2*f^2 + 34*d*e*f*g))/(192*d^2*e^2) + (5*x^2*(15
*e^2*f^2 - d^2*g^2 + 10*d*e*f*g))/(96*d^3*e) - (e^2*x^5*(15*e^2*f^2 - d^2*g
^2 + 10*d*e*f*g))/(64*d^6))/(d^6 + e^6*x^6 + 2*d*e^5*x^5 - d^4*e^2*x^2 - 4*
d^3*e^3*x^3 - d^2*e^4*x^4 + 2*d^5*e*x) + (atanh((e*x)/d)*(15*e^2*f^2 - d^2*
g^2 + 10*d*e*f*g))/(64*d^7*e^3)
```

3.579 $\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=269

$$\frac{(ef+dg)^5(d+ex)^3}{5de^6(d^2-e^2x^2)^{5/2}}+\frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6(d^2-e^2x^2)^{3/2}}+\frac{(ef+dg)^3(2e^2f^2-21defg+127d^2g^2)(d+ex)}{15d^3e^6\sqrt{d^2-e^2x^2}}+$$

```
[Out] 1/5*(d*g+e*f)^5*(e*x+d)^3/d/e^6/(-e^2*x^2+d^2)^(5/2)+1/15*(-23*d*g+2*e*f)*(
d*g+e*f)^4*(e*x+d)^2/d^2/e^6/(-e^2*x^2+d^2)^(3/2)-1/2*g^3*(13*d^2*g^2+30*d*
e*f*g+20*e^2*f^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^6+1/15*(d*g+e*f)^3*(12
7*d^2*g^2-21*d*e*f*g+2*e^2*f^2)*(e*x+d)/d^3/e^6/(-e^2*x^2+d^2)^(1/2)+g^4*(3
*d*g+5*e*f)*(-e^2*x^2+d^2)^(1/2)/e^6+1/2*g^5*x*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A]
time = 0.60, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1649, 1829, 655, 223, 209}

$$-\frac{g^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(13d^2g^2+30defg+20e^2f^2)}{2e^6}+\frac{g^4\sqrt{d^2-e^2x^2}(3dg+5ef)}{e^6}+\frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{15d^2e^6(d^2-e^2x^2)^{3/2}}+\frac{(d+ex)^3(dg+ef)^5}{5de^6(d^2-e^2x^2)^{5/2}}+\frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5}+\frac{(d+ex)(dg+ef)^3(127d^2g^2-21defg+2e^2f^2)}{15d^3e^6\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 23*
d*g)*(e*f + d*g)^4*(d + e*x)^2)/(15*d^2*e^6*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(15*d^3*e^6*Sqrt
[d^2 - e^2*x^2]) + (g^4*(5*e*f + 3*d*g)*Sqrt[d^2 - e^2*x^2])/e^6 + (g^5*x*S
qrt[d^2 - e^2*x^2])/(2*e^5) - (g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*A
rcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^5f^5-15de^4f^4g-30d^2e^3f^3g^2-30d^3e^2f^2g^3-15d^4efg^4-3d^5g^5}{e^5}+\frac{5d}{}\cdots\right.}{}}{}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^5f^5-15de^4f^4g}{}\cdots\right.}{}}{}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-\cdots\right.}{15d^3e\cdots}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-\cdots\right.}{15d^3e\cdots}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-\cdots\right.}{15d^3e\cdots}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-\cdots\right.}{15d^3e\cdots}$$

$$= \frac{(ef+dg)^5(d+ex)^3}{5de^6\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-23dg)(ef+dg)^4(d+ex)^2}{15d^2e^6\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)^3\left(2e^2f^2-\cdots\right.}{15d^3e\cdots}$$

Mathematica [A]
time = 1.46, size = 311, normalized size = 1.16

$$\frac{\frac{e^3\sqrt{d^2-e^2x^2}\left(304d^7g^5+4e^7f^5x^2+3d^6eg^4(240f-239gx)-6de^6f^4x(2f+5gx)+2d^2e^5f^3\left(7f^2+45fgx+70g^2x^2\right)+d^5e^2g^3\left(440f^2-1710fgx+479g^2x^2\right)+5d^4e^3g^2\left(8f^3-204f^2gx+234fg^2x^2-9g^3x^3\right)-5d^3e^4g\left(6f^4+24f^3gx-128f^2g^2x^2+30fg^3x^3+3g^4x^4\right)\right)}{d^3(d-ex)^3}+15\left(-e^2\right)^{3/2}g^3\left(20e^2f^2+30defg+13d^2g^2\right)\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{30e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e^3*Sqrt[d^2 - e^2*x^2]*(304*d^7*g^5 + 4*e^7*f^5*x^2 + 3*d^6*e*g^4*(240*f
 - 239*g*x) - 6*d*e^6*f^4*x*(2*f + 5*g*x) + 2*d^2*e^5*f^3*(7*f^2 + 45*f*g*x
 + 70*g^2*x^2) + d^5*e^2*g^3*(440*f^2 - 1710*f*g*x + 479*g^2*x^2) + 5*d^4*e
^3*g^2*(8*f^3 - 204*f^2*g*x + 234*f*g^2*x^2 - 9*g^3*x^3) - 5*d^3*e^4*g*(6*f
^4 + 24*f^3*g*x - 128*f^2*g^2*x^2 + 30*f*g^3*x^3 + 3*g^4*x^4)))/(d^3*(d - e
*x)^3) + 15*(-e^2)^(3/2)*g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*Log[-(S
qrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(30*e^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(247) = 494$.

time = 0.14, size = 1029, normalized size = 3.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*g^5*(-1/2*x^7/e^2/(-e^2*x^2+d^2)^(5/2)+7/2*d^2/e^2*(1/5*x^5/e^2/(-e^2*x
^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^
2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))
))+(3*d*e^2*g^5+5*e^3*f*g^4)*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/
e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d
^2/e^4/(-e^2*x^2+d^2)^(5/2))))+(3*d^2*e*g^5+15*d*e^2*f*g^4+10*e^3*f^2*g^3)*
(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1
/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e
^2*x^2+d^2)^(1/2)))))+(d^3*g^5+15*d^2*e*f*g^4+30*d*e^2*f^2*g^3+10*e^3*f^3*g
^2)*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/
2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+(5*d^3*f*g^4+30*d^2*e*f^2*g^3+30*d*e
^2*f^3*g^2+5*e^3*f^4*g)*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*
x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/
d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+(10*
d^3*f^2*g^3+30*d^2*e*f^3*g^2+15*d*e^2*f^4*g+e^3*f^5)*(1/3*x^2/e^2/(-e^2*x^2
+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+(10*d^3*f^3*g^2+15*d^2*e*f^4
*g+3*d*e^2*f^5)*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^
2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^
2+d^2)^(1/2))))+1/5*(5*d^3*f^4*g+3*d^2*e*f^5)/e^2/(-e^2*x^2+d^2)^(5/2)+d^3*
f^5*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)
+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. $2(244) = 488$.

time = 0.52, size = 1489, normalized size = 5.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*g^5*x^7*e/(-x^2*e^2 + d^2)^(5/2) + 7/30*(15*x^4*e^(-2)/(-x^2*e^2 + d^2
)^(5/2) - 20*d^2*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8*d^4*e^(-6)/(-x^2*e^2
 + d^2)^(5/2))*d^2*g^5*x*e - 7/6*(3*x^2*e^(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d
^2*e^(-4)/(-x^2*e^2 + d^2)^(3/2))*d^2*g^5*x*e^(-1) + 14/15*d^4*g^5*x*e^(-5)
/(-x^2*e^2 + d^2)^(3/2) - 7/2*d^2*g^5*arcsin(x*e/d)*e^(-6) - 49/30*d^2*g^5*
x*e^(-5)/sqrt(-x^2*e^2 + d^2) + d^3*f^4*g*e^(-2)/(-x^2*e^2 + d^2)^(5/2) + 3
/5*d^2*f^5*e^(-1)/(-x^2*e^2 + d^2)^(5/2) - (3*d*g^5*e^2 + 5*f*g^4*e^3)*x^6*
e^(-2)/(-x^2*e^2 + d^2)^(5/2) + 6*(3*d*g^5*e^2 + 5*f*g^4*e^3)*d^2*x^4*e^(-4
```

```
)/(-x^2*e^2 + d^2)^(5/2) - 8*(3*d*g^5*e^2 + 5*f*g^4*e^3)*d^4*x^2*e^(-6)/(-x
^2*e^2 + d^2)^(5/2) + 16/5*(3*d*g^5*e^2 + 5*f*g^4*e^3)*d^6*e^(-8)/(-x^2*e^2
 + d^2)^(5/2) + 1/5*d*f^5*x/(-x^2*e^2 + d^2)^(5/2) - 1/3*(3*d^2*g^5*e + 15*
d*f*g^4*e^2 + 10*f^2*g^3*e^3)*(3*x^2*e^(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d^2*
e^(-4)/(-x^2*e^2 + d^2)^(3/2))*x*e^(-2) + 4/15*f^5*x/((-x^2*e^2 + d^2)^(3/2
)*d) + (d^3*g^5 + 15*d^2*f*g^4*e + 30*d*f^2*g^3*e^2 + 10*f^3*g^2*e^3)*x^4*e
^(-2)/(-x^2*e^2 + d^2)^(5/2) - 4/3*(d^3*g^5 + 15*d^2*f*g^4*e + 30*d*f^2*g^3
*e^2 + 10*f^3*g^2*e^3)*d^2*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8/15*(d^3*g^
5 + 15*d^2*f*g^4*e + 30*d*f^2*g^3*e^2 + 10*f^3*g^2*e^3)*d^4*e^(-6)/(-x^2*e^
2 + d^2)^(5/2) + 4/15*(3*d^2*g^5*e + 15*d*f*g^4*e^2 + 10*f^2*g^3*e^3)*d^2*x
*e^(-6)/(-x^2*e^2 + d^2)^(3/2) + 1/15*(3*d^2*g^5*e + 15*d*f*g^4*e^2 + 10*f^
2*g^3*e^3)*(15*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2*e^(-4)/(-x^2*
e^2 + d^2)^(5/2) + 8*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2))*x - (3*d^2*g^5*e +
15*d*f*g^4*e^2 + 10*f^2*g^3*e^3)*arcsin(x*e/d)*e^(-7) + 8/15*f^5*x/(sqrt(-x
^2*e^2 + d^2)*d^3) + 5/2*(d^3*f*g^4 + 6*d^2*f^2*g^3*e + 6*d*f^3*g^2*e^2 + f
^4*g*e^3)*x^3*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 3/2*(d^3*f*g^4 + 6*d^2*f^2*g^
3*e + 6*d*f^3*g^2*e^2 + f^4*g*e^3)*d^2*x*e^(-4)/(-x^2*e^2 + d^2)^(5/2) - 7/
15*(3*d^2*g^5*e + 15*d*f*g^4*e^2 + 10*f^2*g^3*e^3)*x*e^(-6)/sqrt(-x^2*e^2 +
 d^2) + 1/3*(10*d^3*f^2*g^3 + 30*d^2*f^3*g^2*e + 15*d*f^4*g*e^2 + f^5*e^3)*
x^2*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 2/15*(10*d^3*f^2*g^3 + 30*d^2*f^3*g^2*e
 + 15*d*f^4*g*e^2 + f^5*e^3)*d^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 1/2*(d^3*f
*g^4 + 6*d^2*f^2*g^3*e + 6*d*f^3*g^2*e^2 + f^4*g*e^3)*x*e^(-4)/(-x^2*e^2 +
d^2)^(3/2) + 1/5*(10*d^3*f^3*g^2 + 15*d^2*f^4*g*e + 3*d*f^5*e^2)*x*e^(-2)/(
-x^2*e^2 + d^2)^(5/2) + (d^3*f*g^4 + 6*d^2*f^2*g^3*e + 6*d*f^3*g^2*e^2 + f^
4*g*e^3)*x*e^(-4)/(sqrt(-x^2*e^2 + d^2)*d^2) - 1/15*(10*d^3*f^3*g^2 + 15*d^
2*f^4*g*e + 3*d*f^5*e^2)*x*e^(-2)/((-x^2*e^2 + d^2)^(3/2)*d^2) - 2/15*(10*d
^3*f^3*g^2 + 15*d^2*f^4*g*e + 3*d*f^5*e^2)*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d
^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(244) = 488$.
time = 3.08, size = 781, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(304*d^8*g^5 - 14*f^5*x^3*e^8 + 30*(13*d^8*g^5 - 20*d^3*f^2*g^3*x^3*e
^5 - 30*(d^4*f*g^4*x^3 - 2*d^4*f^2*g^3*x^2)*e^4 - (13*d^5*g^5*x^3 - 90*d^5*
f*g^4*x^2 + 60*d^5*f^2*g^3*x)*e^3 + (39*d^6*g^5*x^2 - 90*d^6*f*g^4*x + 20*d
^6*f^2*g^3)*e^2 - 3*(13*d^7*g^5*x - 10*d^7*f*g^4)*e)*arctan(-(d - sqrt(-x^2
*e^2 + d^2))*e^(-1)/x) + 6*(5*d*f^4*g*x^3 + 7*d*f^5*x^2)*e^7 - 2*(20*d^2*f^
3*g^2*x^3 + 45*d^2*f^4*g*x^2 + 21*d^2*f^5*x)*e^6 - 2*(220*d^3*f^2*g^3*x^3 -
 60*d^3*f^3*g^2*x^2 - 45*d^3*f^4*g*x - 7*d^3*f^5)*e^5 - 30*(24*d^4*f*g^4*x^
```

```
3 - 44*d^4*f^2*g^3*x^2 + 4*d^4*f^3*g^2*x + d^4*f^4*g)*e^4 - 8*(38*d^5*g^5*x
^3 - 270*d^5*f*g^4*x^2 + 165*d^5*f^2*g^3*x - 5*d^5*f^3*g^2)*e^3 + 8*(114*d^
6*g^5*x^2 - 270*d^6*f*g^4*x + 55*d^6*f^2*g^3)*e^2 - 48*(19*d^7*g^5*x - 15*d
^7*f*g^4)*e + (304*d^7*g^5 + 4*f^5*x^2*e^7 - 6*(5*d*f^4*g*x^2 + 2*d*f^5*x)*
e^6 + 2*(70*d^2*f^3*g^2*x^2 + 45*d^2*f^4*g*x + 7*d^2*f^5)*e^5 - 5*(3*d^3*g^
5*x^4 + 30*d^3*f*g^4*x^3 - 128*d^3*f^2*g^3*x^2 + 24*d^3*f^3*g^2*x + 6*d^3*f
^4*g)*e^4 - 5*(9*d^4*g^5*x^3 - 234*d^4*f*g^4*x^2 + 204*d^4*f^2*g^3*x - 8*d^
4*f^3*g^2)*e^3 + (479*d^5*g^5*x^2 - 1710*d^5*f*g^4*x + 440*d^5*f^2*g^3)*e^2
 - 3*(239*d^6*g^5*x - 240*d^6*f*g^4)*e)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3*e^9
- 3*d^4*x^2*e^8 + 3*d^5*x*e^7 - d^6*e^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3 (f+gx)^5}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**5/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**5/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 934 vs. 2(244) = 488.
time = 1.33, size = 934, normalized size = 3.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/2*(13*d^2*g^5 + 30*d*f*g^4*e + 20*f^2*g^3*e^2)*arcsin(x*e/d)*e^(-6)*sgn(
d) + 1/2*(g^5*x*e^(-5) + 2*(3*d*g^5*e^10 + 5*f*g^4*e^11)*e^(-16))*sqrt(-x^2
*e^2 + d^2) - 2/15*(445*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^5*g^5*e^(-2)/x - 6
65*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^5*g^5*e^(-4)/x^2 + 405*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^3*d^5*g^5*e^(-6)/x^3 - 90*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4
*d^5*g^5*e^(-8)/x^4 - 107*d^5*g^5 - 285*d^4*f*g^4*e + 1200*(d*e + sqrt(-x^2
*e^2 + d^2)*e)*d^4*f*g^4*e^(-1)/x - 1800*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d
^4*f*g^4*e^(-3)/x^2 + 1050*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*f*g^4*e^(-5
)/x^3 - 225*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*f*g^4*e^(-7)/x^4 - 220*d^3
*f^2*g^3*e^2 - 1450*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*f^2*g^3*e^(-2)/x^2
 + 750*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*f^2*g^3*e^(-4)/x^3 - 150*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)^4*d^3*f^2*g^3*e^(-6)/x^4 + 950*(d*e + sqrt(-x^2*e^
2 + d^2)*e)*d^3*f^2*g^3/x - 20*d^2*f^3*g^2*e^3 + 100*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)*d^2*f^3*g^2*e/x - 200*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*f^3*g^2
```

```
*e^(-1)/x^2 + 15*d*f^4*g*e^4 - 75*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*f^4*g*e^
2/x - 75*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*f^4*g*e^(-2)/x^3 + 75*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^2*d*f^4*g/x^2 - 7*f^5*e^5 + 20*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)*f^5*e^3/x - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*f^5*e/x^2 + 30*(d*
e + sqrt(-x^2*e^2 + d^2)*e)^3*f^5*e^(-1)/x^3 - 15*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^4*f^5*e^(-3)/x^4)*e^(-6)/(d^3*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/
x - 1)^5)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^5\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] int(((f + g*x)^5*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.580 $\int \frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=215

$$\frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(ef+dg)^2\left(e^2f^2-8defg+36d^2g^2\right)(d+ex)}{15d^3e^5\sqrt{d^2-e^2x^2}}+\frac{g^4}{}$$

```
[Out] 1/5*(d*g+e*f)^4*(e*x+d)^3/d/e^5/(-e^2*x^2+d^2)^(5/2)+2/15*(-9*d*g+e*f)*(d*g
+e*f)^3*(e*x+d)^2/d^2/e^5/(-e^2*x^2+d^2)^(3/2)-g^3*(3*d*g+4*e*f)*arctan(e*x
/(-e^2*x^2+d^2)^(1/2))/e^5+2/15*(d*g+e*f)^2*(36*d^2*g^2-8*d*e*f*g+e^2*f^2)*
(e*x+d)/d^3/e^5/(-e^2*x^2+d^2)^(1/2)+g^4*(-e^2*x^2+d^2)^(1/2)/e^5
```

Rubi [A]
time = 0.41, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1649, 655, 223, 209}

$$-\frac{g^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(3dg+4ef)}{e^5}+\frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^4}{5de^5\left(d^2-e^2x^2\right)^{5/2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{e^5}+\frac{2(d+ex)(dg+ef)^2\left(36d^2g^2-8defg+e^2f^2\right)}{15d^3e^5\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 9*d
*g)*(e*f + d*g)^3*(d + e*x)^2)/(15*d^2*e^5*(d^2 - e^2*x^2)^(3/2)) + (2*(e*f
 + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(15*d^3*e^5*Sqrt[d^
2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/e^5 - (g^3*(4*e*f + 3*d*g)*ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^4}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^4f^4-12de^3f^3g-18d^2e^2f^2g^2-12d^3efg^3-3d^4g^4}{e^4}+\frac{5dg^2\left(6e^2f^2+4d\cdots\right)}{e^3}\cdots\right)}{\left(d^2-e^2x^2\right)^{5/2}}}{5d}\cdots$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^4f^4-12de^3f^3g+\cdots}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-\cdots\right)}{15d^3e^5\sqrt{\cdots}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-\cdots\right)}{15d^3e^5\sqrt{\cdots}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-\cdots\right)}{15d^3e^5\sqrt{\cdots}}$$

$$= \frac{(ef+dg)^4(d+ex)^3}{5de^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-9dg)(ef+dg)^3(d+ex)^2}{15d^2e^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{2(ef+dg)^2\left(e^2f^2-\cdots\right)}{15d^3e^5\sqrt{\cdots}}$$

Mathematica [A]
time = 1.29, size = 240, normalized size = 1.12

$$\frac{\sqrt{d^2-e^2x^2}\left(72d^6g^4+2e^6f^4x^2+d^5eg^3(88f-171gx)-6de^5f^3x(f+2gx)+3d^4e^2g^2(4f^2-68fgx+39g^2x^2)+d^2e^4f^2(7f^2+36fgx+42g^2x^2)-d^3e^3g(12f^3+36f^2gx-128fg^2x^2+15g^3x^3)\right)}{15d^3e^5(d-ex)^3}+\frac{g^3(4ef+3dg)\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^4\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^4)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(72*d^6*g^4 + 2*e^6*f^4*x^2 + d^5*e*g^3*(88*f - 171*g*
x) - 6*d*e^5*f^3*x*(f + 2*g*x) + 3*d^4*e^2*g^2*(4*f^2 - 68*f*g*x + 39*g^2*x
^2) + d^2*e^4*f^2*(7*f^2 + 36*f*g*x + 42*g^2*x^2) - d^3*e^3*g*(12*f^3 + 36*
f^2*g*x - 128*f*g^2*x^2 + 15*g^3*x^3)))/(15*d^3*e^5*(d - e*x)^3) + (g^3*(4*
e*f + 3*d*g)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(e^4*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(199) = 398$.
time = 0.11, size = 842, normalized size = 3.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*g^4*(-x^6/e^2/(-e^2*x^2+d^2)^(5/2)+6*d^2/e^2*(x^4/e^2/(-e^2*x^2+d^2)^(5
/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)
^(5/2))))+(3*d*e^2*g^4+4*e^3*f*g^3)*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2
*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+(3*d^2*e*g^4+12*d
*e^2*f*g^3+6*e^3*f^2*g^2)*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/
e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+(d^3*g^4+12*d^
2*e*f*g^3+18*d*e^2*f^2*g^2+4*e^3*f^3*g)*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3
/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2
+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2
)^(1/2)))))+(4*d^3*f*g^3+18*d^2*e*f^2*g^2+12*d*e^2*f^3*g+e^3*f^4)*(1/3*x^2/
e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+(6*d^3*f^2*g^2+
12*d^2*e*f^3*g+3*d*e^2*f^4)*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/
5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/
d^4/(-e^2*x^2+d^2)^(1/2))))+1/5*(4*d^3*f^3*g+3*d^2*e*f^4)/e^2/(-e^2*x^2+d^2
)^(5/2)+d^3*f^4*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^
2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(199) = 398$.
time = 0.55, size = 1107, normalized size = 5.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -g^4*x^6*e/(-x^2*e^2 + d^2)^(5/2) + 6*d^2*g^4*x^4*e^(-1)/(-x^2*e^2 + d^2)^(
5/2) - 8*d^4*g^4*x^2*e^(-3)/(-x^2*e^2 + d^2)^(5/2) + 16/5*d^6*g^4*e^(-5)/(-
x^2*e^2 + d^2)^(5/2) + 4/5*d^3*f^3*g*e^(-2)/(-x^2*e^2 + d^2)^(5/2) + 3/5*d^
2*f^4*e^(-1)/(-x^2*e^2 + d^2)^(5/2) - 1/3*(3*d*g^4*e^2 + 4*f*g^3*e^3)*(3*x^
2*e^(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d^2*e^(-4)/(-x^2*e^2 + d^2)^(3/2))*x*e^
```

```
(-2) + 1/5*d*f^4*x/(-x^2*e^2 + d^2)^(5/2) + 3*(d^2*g^4*e + 4*d*f*g^3*e^2 +
2*f^2*g^2*e^3)*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 4*(d^2*g^4*e + 4*d*f*g^3
*e^2 + 2*f^2*g^2*e^3)*d^2*x^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8/5*(d^2*g^4*
e + 4*d*f*g^3*e^2 + 2*f^2*g^2*e^3)*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2) + 4/15
*(3*d*g^4*e^2 + 4*f*g^3*e^3)*d^2*x*e^(-6)/(-x^2*e^2 + d^2)^(3/2) + 1/15*(3*
d*g^4*e^2 + 4*f*g^3*e^3)*(15*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2
*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2))*x - (
3*d*g^4*e^2 + 4*f*g^3*e^3)*arcsin(x*e/d)*e^(-7) + 4/15*f^4*x/((-x^2*e^2 + d
^2)^(3/2)*d) + 1/2*(d^3*g^4 + 12*d^2*f*g^3*e + 18*d*f^2*g^2*e^2 + 4*f^3*g*e
^3)*x^3*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 3/10*(d^3*g^4 + 12*d^2*f*g^3*e + 18
*d*f^2*g^2*e^2 + 4*f^3*g*e^3)*d^2*x*e^(-4)/(-x^2*e^2 + d^2)^(5/2) - 7/15*(3
*d*g^4*e^2 + 4*f*g^3*e^3)*x*e^(-6)/sqrt(-x^2*e^2 + d^2) + 8/15*f^4*x/(sqrt(
-x^2*e^2 + d^2)*d^3) + 1/3*(4*d^3*f*g^3 + 18*d^2*f^2*g^2*e + 12*d*f^3*g*e^2
 + f^4*e^3)*x^2*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 2/15*(4*d^3*f*g^3 + 18*d^2*
f^2*g^2*e + 12*d*f^3*g*e^2 + f^4*e^3)*d^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 1
/10*(d^3*g^4 + 12*d^2*f*g^3*e + 18*d*f^2*g^2*e^2 + 4*f^3*g*e^3)*x*e^(-4)/(-
x^2*e^2 + d^2)^(3/2) + 3/5*(2*d^3*f^2*g^2 + 4*d^2*f^3*g*e + d*f^4*e^2)*x*e^
(-2)/(-x^2*e^2 + d^2)^(5/2) + 1/5*(d^3*g^4 + 12*d^2*f*g^3*e + 18*d*f^2*g^2*
e^2 + 4*f^3*g*e^3)*x*e^(-4)/(sqrt(-x^2*e^2 + d^2)*d^2) - 1/5*(2*d^3*f^2*g^2
 + 4*d^2*f^3*g*e + d*f^4*e^2)*x*e^(-2)/((-x^2*e^2 + d^2)^(3/2)*d^2) - 2/5*(
2*d^3*f^2*g^2 + 4*d^2*f^3*g*e + d*f^4*e^2)*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d
^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(199) = 398$.
time = 2.65, size = 603, normalized size = 2.80

$$-\frac{72\,d^7g^4 - 7\,f^4x^3e^7 + 30\,(3\,d^7g^4 - 4\,d^3fg^3x^3e^4 - 3\,(d^4g^4x^3 - 4\,d^4fg^3x^2)e^3 + 3\,(3\,d^5g^4x^2 - 4\,d^5fg^3x)e^2 - (9\,d^6g^4x - 4\,d^6fg^3)e)\arctan\left(-\frac{\left(d - \sqrt{-x^2e^2 + d^2}\right)e^{(-1)}}{x}\right) + 3\,(4\,df^3gx^3 + 7\,df^4x^2)e^6 - 3\,(4\,d^2f^2g^2x^3 + 12\,d^2f^3gx^2 + 7\,d^2f^4x)e^5 - (88\,d^3fg^3x^3 - 36\,d^3f^2g^2x^2 - 36\,d^3f^3gx - 7\,d^3f^4)e^4 - 12\,(6\,d^4g^4x^3 - 22\,d^4fg^3x^2 + 3\,d^4f^2g^2x + d^4f^3g)e^3 + 12\,(18\,d^5g^4x^2 - 22\,d^5fg^3x + d^5f^2g^2)e^2 - 8\,(27\,d^6g^4x - 11\,d^6fg^3)e + (72\,d^6g^4 + 2\,f^4x^2e^6 - 6\,(2\,df^3gx^2 + df^4x)e^5 + (42\,d^2f^2g^2x^2 + 36\,d^2f^3gx + 7\,d^2f^4)e^4 - (15\,d^3g^4x^3 - 128\,d^3fg^3x^2 + 36\,d^3f^2g^2x + 12\,d^3f^3g)e^3 + 3\,(39\,d^4g^4x^2 - 68\,d^4fg^3x + 4\,d^4f^2g^2)e^2 - (171\,d^5g^4x - 88\,d^5fg^3)e)\sqrt{-x^2e^2 + d^2}}{15\,(d^3x^3e^8 - 3\,d^4x^2e^7 + 3\,d^5xe^6 - d^6e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(72*d^7*g^4 - 7*f^4*x^3*e^7 + 30*(3*d^7*g^4 - 4*d^3*f*g^3*x^3*e^4 - 3
*(d^4*g^4*x^3 - 4*d^4*f*g^3*x^2)*e^3 + 3*(3*d^5*g^4*x^2 - 4*d^5*f*g^3*x)*e^
2 - (9*d^6*g^4*x - 4*d^6*f*g^3)*e)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1
)/x) + 3*(4*d*f^3*g*x^3 + 7*d*f^4*x^2)*e^6 - 3*(4*d^2*f^2*g^2*x^3 + 12*d^2*
f^3*g*x^2 + 7*d^2*f^4*x)*e^5 - (88*d^3*f*g^3*x^3 - 36*d^3*f^2*g^2*x^2 - 36*
d^3*f^3*g*x - 7*d^3*f^4)*e^4 - 12*(6*d^4*g^4*x^3 - 22*d^4*f*g^3*x^2 + 3*d^4
*f^2*g^2*x + d^4*f^3*g)*e^3 + 12*(18*d^5*g^4*x^2 - 22*d^5*f*g^3*x + d^5*f^2
*g^2)*e^2 - 8*(27*d^6*g^4*x - 11*d^6*f*g^3)*e + (72*d^6*g^4 + 2*f^4*x^2*e^6
 - 6*(2*d*f^3*g*x^2 + d*f^4*x)*e^5 + (42*d^2*f^2*g^2*x^2 + 36*d^2*f^3*g*x +
 7*d^2*f^4)*e^4 - (15*d^3*g^4*x^3 - 128*d^3*f*g^3*x^2 + 36*d^3*f^2*g^2*x +
12*d^3*f^3*g)*e^3 + 3*(39*d^4*g^4*x^2 - 68*d^4*f*g^3*x + 4*d^4*f^2*g^2)*e^2
 - (171*d^5*g^4*x - 88*d^5*f*g^3)*e)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3*e^8 - 3
*d^4*x^2*e^7 + 3*d^5*x*e^6 - d^6*e^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\,(f+gx)^4}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**4/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. 2(199) = 398.
time = 2.17, size = 729, normalized size = 3.39

$$\sqrt{-x^2e^2+d^2}\,g^4e^{(-5)} - (3\,dg^4+4\,fg^3e)\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\mathrm{sgn}(d) - \frac{2\left(\frac{240\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^4g^4e^{(-2)}}{x} - \frac{360\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^4g^4e^{(-4)}}{x^2} + \frac{210\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^4g^4e^{(-6)}}{x^3} - \frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^4g^4e^{(-8)}}{x^4} - 57\,d^4g^4 - 88\,d^3fg^3e + \frac{380\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3fg^3e^{(-1)}}{x} - \frac{580\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^3fg^3e^{(-3)}}{x^2} + \frac{300\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^3fg^3e^{(-5)}}{x^3} - \frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3fg^3e^{(-7)}}{x^4} - 12\,d^2f^2g^2e^2 - \frac{120\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2f^2g^2e^{(-2)}}{x^2} + \frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2f^2g^2}{x} + 12\,df^3ge^3 - \frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)df^3ge}{x} + \frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2df^3ge^{(-1)}}{x^2} - \frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3df^3ge^{(-3)}}{x^3} - 7\,f^4e^4 + \frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)f^4e^2}{x} + \frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3f^4e^{(-2)}}{x^3} - \frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4f^4e^{(-4)}}{x^4} - \frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2f^4}{x^2}\right)e^{(-5)}}{15\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] sqrt(-x^2*e^2 + d^2)*g^4*e^(-5) - (3*d*g^4 + 4*f*g^3*e)*arcsin(x*e/d)*e^(-5
)*sgn(d) - 2/15*(240*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*g^4*e^(-2)/x - 360*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*g^4*e^(-4)/x^2 + 210*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^3*d^4*g^4*e^(-6)/x^3 - 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^
4*g^4*e^(-8)/x^4 - 57*d^4*g^4 - 88*d^3*f*g^3*e + 380*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)*d^3*f*g^3*e^(-1)/x - 580*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*f*g^
3*e^(-3)/x^2 + 300*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*f*g^3*e^(-5)/x^3 -
60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^3*f*g^3*e^(-7)/x^4 - 12*d^2*f^2*g^2*e
^2 - 120*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*f^2*g^2*e^(-2)/x^2 + 60*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)*d^2*f^2*g^2/x + 12*d*f^3*g*e^3 - 60*(d*e + sqrt(-
x^2*e^2 + d^2)*e)*d*f^3*g*e/x + 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*f^3*g
*e^(-1)/x^2 - 60*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*f^3*g*e^(-3)/x^3 - 7*f^
4*e^4 + 20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*f^4*e^2/x + 30*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^3*f^4*e^(-2)/x^3 - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*f^4*e^(
-4)/x^4 - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*f^4/x^2)*e^(-5)/(d^3*((d*e +
sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^4\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int(((f + g*x)^4*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.581 $\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=183

$$\frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(ef+dg)\left(2e^2f^2-11defg+32d^2g^2\right)(d+ex)}{15d^3e^4\sqrt{d^2-e^2x^2}}-\frac{g^3}{}$$

```
[Out] 1/5*(d*g+e*f)^3*(e*x+d)^3/d/e^4/(-e^2*x^2+d^2)^(5/2)+1/15*(-13*d*g+2*e*f)*(
d*g+e*f)^2*(e*x+d)^2/d^2/e^4/(-e^2*x^2+d^2)^(3/2)-g^3*arctan(e*x/(-e^2*x^2+
d^2)^(1/2))/e^4+1/15*(d*g+e*f)*(32*d^2*g^2-11*d*e*f*g+2*e^2*f^2)*(e*x+d)/d^
3/e^4/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1649, 792, 223, 209}

$$-\frac{g^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}+\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)^3(dg+ef)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{(d+ex)(dg+ef)\left(32d^2g^2-11defg+2e^2f^2\right)}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^(5/2)) + ((2*e*f - 13*
d*g)*(e*f + d*g)^2*(d + e*x)^2)/(15*d^2*e^4*(d^2 - e^2*x^2)^(3/2)) + ((e*f
+ d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(15*d^3*e^4*Sqrt[d^
2 - e^2*x^2]) - (g^3*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-\frac{2e^3f^3-9de^2f^2g-9d^2efg^2-3d^3g^3}{e^3}+\frac{5dg^2(3ef+dg)x}{e^2}+\frac{5dg^3x^2}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,d}{5d}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{(d+ex)\left(\frac{2e^3f^3-9de^2f^2}{}\right)}{(d}}{}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-\right.}{15d^3e}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-\right.}{15d^3e}$$

$$= \frac{(ef+dg)^3(d+ex)^3}{5de^4\left(d^2-e^2x^2\right)^{5/2}} + \frac{(2ef-13dg)(ef+dg)^2(d+ex)^2}{15d^2e^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{(ef+dg)\left(2e^2f^2-\right.}{15d^3e}$$

Mathematica [A]
time = 0.86, size = 165, normalized size = 0.90

$$\frac{(ef+dg)\sqrt{d^2-e^2x^2}\left(22d^4g^2+2e^4f^2x^2-de^3fx(6f+11gx)-d^3eg(16f+51gx)+d^2e^2(7f^2+33fgx+32g^2x^2)\right)}{15d^3e^4(d-ex)^3}+\frac{g^3\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^3\sqrt{-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^3)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)*Sqrt[d^2 - e^2*x^2]*(22*d^4*g^2 + 2*e^4*f^2*x^2 - d*e^3*f*x*(6
*f + 11*g*x) - d^3*e*g*(16*f + 51*g*x) + d^2*e^2*(7*f^2 + 33*f*g*x + 32*g^2
```

```
*x^2)))/(15*d^3*e^4*(d - e*x)^3) + (g^3*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^
2*x^2]])/(e^3*Sqrt[-e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(169) = 338$.
time = 0.10, size = 688, normalized size = 3.76

method	result
default	$e^{3} g^{3} \left(\frac{x^{5}}{5 e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{\frac{x^{3}}{3 e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}}{e^{2}}}{e^{2}}\right)+\left(3 e^{2} d\, g^{3}+3 e^{3} f\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*g^3*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)
^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2)))))+(3*d*e^2*g^3+3*e^3*f*g^2)*(x^4/e^2/(-e^2*x^2+
d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x
^2+d^2)^(5/2)))+(3*d^2*e*g^3+9*d*e^2*f*g^2+3*e^3*f^2*g)*(1/2*x^3/e^2/(-e^2*
x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5
*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d
^4/(-e^2*x^2+d^2)^(1/2)))))+(d^3*g^3+9*d^2*e*f*g^2+9*d*e^2*f^2*g+e^3*f^3)*(
1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+(3*d^3*
f*g^2+9*d^2*e*f^2*g+3*d*e^2*f^3)*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^
2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2
/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+1/5*(3*d^3*f^2*g+3*d^2*e*f^3)/e^2/(-e^2*x^
2+d^2)^(5/2)+d^3*f^3*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e
^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(169) = 338$.
time = 0.51, size = 834, normalized size = 4.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 20*d^2*x^2*e^(-4)/(-x^2*e^2 +
d^2)^(5/2) + 8*d^4*e^(-6)/(-x^2*e^2 + d^2)^(5/2))*g^3*x*e^3 - 1/3*(3*x^2*e^
(-2)/(-x^2*e^2 + d^2)^(3/2) - 2*d^2*e^(-4)/(-x^2*e^2 + d^2)^(3/2))*g^3*x*e
+ 4/15*d^2*g^3*x*e^(-3)/(-x^2*e^2 + d^2)^(3/2) - g^3*arcsin(x*e/d)*e^(-4) +
 3/5*d^3*f^2*g*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 7/15*g^3*x*e^(-3)/sqrt(-x^2*
e^2 + d^2) + 3/5*d^2*f^3*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 3*(d*g^3*e^2 + f*g
^2*e^3)*x^4*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 4*(d*g^3*e^2 + f*g^2*e^3)*d^2*x
^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 8/5*(d*g^3*e^2 + f*g^2*e^3)*d^4*e^(-6)/(
-x^2*e^2 + d^2)^(5/2) + 1/5*d*f^3*x/(-x^2*e^2 + d^2)^(5/2) + 3/2*(d^2*g^3*e
 + 3*d*f*g^2*e^2 + f^2*g*e^3)*x^3*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 9/10*(d^2
*g^3*e + 3*d*f*g^2*e^2 + f^2*g*e^3)*d^2*x*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 4
/15*f^3*x/((-x^2*e^2 + d^2)^(3/2)*d) + 1/3*(d^3*g^3 + 9*d^2*f*g^2*e + 9*d*f
^2*g*e^2 + f^3*e^3)*x^2*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 2/15*(d^3*g^3 + 9*d
^2*f*g^2*e + 9*d*f^2*g*e^2 + f^3*e^3)*d^2*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 3
/10*(d^2*g^3*e + 3*d*f*g^2*e^2 + f^2*g*e^3)*x*e^(-4)/(-x^2*e^2 + d^2)^(3/2)
 + 8/15*f^3*x/(sqrt(-x^2*e^2 + d^2)*d^3) + 3/5*(d^3*f*g^2 + 3*d^2*f^2*g*e +
 d*f^3*e^2)*x*e^(-2)/(-x^2*e^2 + d^2)^(5/2) + 3/5*(d^2*g^3*e + 3*d*f*g^2*e^
2 + f^2*g*e^3)*x*e^(-4)/(sqrt(-x^2*e^2 + d^2)*d^2) - 1/5*(d^3*f*g^2 + 3*d^2
*f^2*g*e + d*f^3*e^2)*x*e^(-2)/((-x^2*e^2 + d^2)^(3/2)*d^2) - 2/5*(d^3*f*g^
2 + 3*d^2*f^2*g*e + d*f^3*e^2)*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(169) = 338$.
time = 2.76, size = 442, normalized size = 2.42

$$-\frac{22\,d^6g^3 - 7\,f^3x^3e^6 - 30\,(d^3g^3x^3e^3 - 3\,d^4g^3x^2e^2 + 3\,d^5g^3xe - d^6g^3)\arctan\left(-\frac{\left(d - \sqrt{-x^2e^2 + d^2}\right)e^{(-1)}}{x}\right) + 3\,(3\,df^2gx^3 + 7\,df^3x^2)e^5 - 3\,(2\,d^2fg^2x^3 + 9\,d^2f^2gx^2 + 7\,d^2f^3x)e^4 - (22\,d^3g^3x^3 - 18\,d^3fg^2x^2 - 27\,d^3f^2gx - 7\,d^3f^3)e^3 + 3\,(22\,d^4g^3x^2 - 6\,d^4fg^2x - 3\,d^4f^2g)e^2 - 6\,(11\,d^5g^3x - d^5fg^2)e + (22\,d^5g^3 + 2\,f^3x^2e^5 - 3\,(3\,df^2gx^2 + 2\,df^3x)e^4 + (21\,d^2fg^2x^2 + 27\,d^2f^2gx + 7\,d^2f^3)e^3 + (32\,d^3g^3x^2 - 18\,d^3fg^2x - 9\,d^3f^2g)e^2 - 3\,(17\,d^4g^3x - 2\,d^4fg^2)e)\sqrt{-x^2e^2 + d^2}}{15\,(d^3x^3e^7 - 3\,d^4x^2e^6 + 3\,d^5xe^5 - d^6e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(22*d^6*g^3 - 7*f^3*x^3*e^6 - 30*(d^3*g^3*x^3*e^3 - 3*d^4*g^3*x^2*e^2
 + 3*d^5*g^3*x*e - d^6*g^3)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) +
3*(3*d*f^2*g*x^3 + 7*d*f^3*x^2)*e^5 - 3*(2*d^2*f*g^2*x^3 + 9*d^2*f^2*g*x^2
+ 7*d^2*f^3*x)*e^4 - (22*d^3*g^3*x^3 - 18*d^3*f*g^2*x^2 - 27*d^3*f^2*g*x -
7*d^3*f^3)*e^3 + 3*(22*d^4*g^3*x^2 - 6*d^4*f*g^2*x - 3*d^4*f^2*g)*e^2 - 6*(
11*d^5*g^3*x - d^5*f*g^2)*e + (22*d^5*g^3 + 2*f^3*x^2*e^5 - 3*(3*d*f^2*g*x^
2 + 2*d*f^3*x)*e^4 + (21*d^2*f*g^2*x^2 + 27*d^2*f^2*g*x + 7*d^2*f^3)*e^3 +
(32*d^3*g^3*x^2 - 18*d^3*f*g^2*x - 9*d^3*f^2*g)*e^2 - 3*(17*d^4*g^3*x - 2*d
^4*f*g^2)*e)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3*e^7 - 3*d^4*x^2*e^6 + 3*d^5*x*e
^5 - d^6*e^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\,(f+gx)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(169) = 338.
time = 3.45, size = 537, normalized size = 2.93

$$-g^3\arcsin\left(\frac{xe}{d}\right)e^{(-4)}\mathrm{sgn}(d)-\frac{2\left(\frac{95\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^3g^3e^{(-2)}}{x}-\frac{145\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^3g^3e^{(-4)}}{x^2}+\frac{75\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^3g^3e^{(-6)}}{x^3}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^3g^3e^{(-8)}}{x^4}-22\,d^3g^3-6\,d^2fg^2e+\frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2fg^2e^{(-1)}}{x}-\frac{60\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2fg^2e^{(-3)}}{x^2}+9\,df^2ge^2+\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2df^2ge^{(-2)}}{x^2}-\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3df^2ge^{(-4)}}{x^3}-\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)df^2g}{x}-7\,f^3e^3+\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)f^3e}{x}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2f^3e^{(-1)}}{x^2}+\frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3f^3e^{(-3)}}{x^3}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4f^3e^{(-5)}}{x^4}\right)e^{(-4)}}{15\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -g^3*arcsin(x*e/d)*e^(-4)*sgn(d) - 2/15*(95*(d*e + sqrt(-x^2*e^2 + d^2)*e)*
d^3*g^3*e^(-2)/x - 145*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*g^3*e^(-4)/x^2
+ 75*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*g^3*e^(-6)/x^3 - 15*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^4*d^3*g^3*e^(-8)/x^4 - 22*d^3*g^3 - 6*d^2*f*g^2*e + 30*(d
*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*f*g^2*e^(-1)/x - 60*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*d^2*f*g^2*e^(-3)/x^2 + 9*d*f^2*g*e^2 + 45*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^2*d*f^2*g*e^(-2)/x^2 - 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*f^2*g
*e^(-4)/x^3 - 45*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*f^2*g/x - 7*f^3*e^3 + 20*
(d*e + sqrt(-x^2*e^2 + d^2)*e)*f^3*e/x - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^
2*f^3*e^(-1)/x^2 + 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*f^3*e^(-3)/x^3 - 15*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*f^3*e^(-5)/x^4)*e^(-4)/(d^3*((d*e + sqrt(-
x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^3\,(d+e\,x)^3}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] int(((f + g*x)^3*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2), x)
```

3.582 $\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{\left(2e^2f^2-6defg+7d^2g^2\right)(d+ex)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(d*g+e*f)^2*(e*x+d)^3/d/e^3/(-e^2*x^2+d^2)^(5/2)+2/15*(-4*d*g+e*f)*(d*g
+e*f)*(e*x+d)^2/d^2/e^3/(-e^2*x^2+d^2)^(3/2)+1/15*(7*d^2*g^2-6*d*e*f*g+2*e^
2*f^2)*(e*x+d)/d^3/e^3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1649, 803, 651}

$$\frac{(d+ex)^3(dg+ef)^2}{5de^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{(d+ex)\left(7d^2g^2-6defg+2e^2f^2\right)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^(5/2)) + (2*(e*f - 4*d
*g)*(e*f + d*g)*(d + e*x)^2)/(15*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) + ((2*e^2*f
^2 - 6*d*e*f*g + 7*d^2*g^2)*(d + e*x))/(15*d^3*e^3*Sqrt[d^2 - e^2*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
```

```
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{\int \frac{(d+ex)^2\left(-2f^2+\frac{6dfg}{e}+\frac{3d^2g^2}{e^2}+\frac{5dg^2x}{e}\right)}{(d^2-e^2x^2)^{5/2}}\,dx}{5d}$$
$$= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{(2e^2f^2-6defg+7d^2g}{15d^2e}$$
$$= \frac{(ef+dg)^2(d+ex)^3}{5de^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(ef-4dg)(ef+dg)(d+ex)^2}{15d^2e^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{(2e^2f^2-6defg+7d^2g}{15d^3e^3\sqrt{d^2-e}}$$

Mathematica [A]
time = 0.61, size = 105, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}\left(2d^4g^2+2e^4f^2x^2-6d^3eg(f+gx)-6de^3fx(f+gx)+d^2e^2(7f^2+18fgx+7g^2x^2)\right)}{15d^3e^3(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x)^2)/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(2*d^4*g^2 + 2*e^4*f^2*x^2 - 6*d^3*e*g*(f + g*x) - 6*d
*e^3*f*x*(f + g*x) + d^2*e^2*(7*f^2 + 18*f*g*x + 7*g^2*x^2)))/(15*d^3*e^3*(
d - e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(133) = 266.
time = 0.09, size = 532, normalized size = 3.67

method	result
trager	$\frac{\left(7d^2e^2g^2x^2-6d\,e^3fg\,x^2+2e^4f^2x^2-6d^3e\,g^2x+18d^2e^2fgx-6d\,e^3f^2x+2d^4g^2-6fg\,d^3e+7d^2e^2f^2\right)\sqrt{-e^2x^2+d^2}}{15d^3e^3(-ex+d)^3}$
gosper	$\frac{(ex+d)^4(-ex+d)\left(7d^2e^2g^2x^2-6d\,e^3fg\,x^2+2e^4f^2x^2-6d^3e\,g^2x+18d^2e^2fgx-6d\,e^3f^2x+2d^4g^2-6fg\,d^3e+7d^2e^2f^2\right)}{15d^3e^3(-e^2x^2+d^2)^{\frac{7}{2}}}$

default	$g^2 e^3 \left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)}{e^2}\right)+\left(3e^2 d\, g^2+2e^3 f g\right)\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] g^2*e^3*(x^4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)
^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+(3*d*e^2*g^2+2*e^3*f*g)*(1/2*x^3
/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d
^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3
/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+(3*d^2*e*g^2+6*d*e^2*f*g+e^3*f^2)*(1
/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+(d^3*g^2
+6*d^2*e*f*g+3*d*e^2*f^2)*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*
x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^
4/(-e^2*x^2+d^2)^(1/2))))+1/5*(2*d^3*f*g+3*d^2*e*f^2)/e^2/(-e^2*x^2+d^2)^(5
/2)+d^3*f^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^
2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(134) = 268$.
time = 0.32, size = 544, normalized size = 3.75

$$\frac{g^2x^4e}{(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{4\,d^2g^2x^2e^{(-1)}}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{8\,d^4g^2e^{(-3)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{2\,d^3fge^{(-2)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{3\,d^2f^2e^{(-1)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{(3\,dg^2e^2+2\,fge^3)x^3e^{(-2)}}{2\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{3\,(3\,dg^2e^2+2\,fge^3)d^2xe^{(-4)}}{10\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{df^2x}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{(3\,d^2g^2e+6\,dfge^2+f^2e^3)x^2e^{(-2)}}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}-\frac{2\,(3\,d^2g^2e+6\,dfge^2+f^2e^3)d^2e^{(-4)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{(3\,dg^2e^2+2\,fge^3)xe^{(-4)}}{10\,(-x^2e^2+d^2)^{\frac{3}{2}}}+\frac{4\,f^2x}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d}+\frac{(d^3g^2+6\,d^2fge+3\,df^2e^2)xe^{(-2)}}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{(3\,dg^2e^2+2\,fge^3)xe^{(-4)}}{5\,\sqrt{-x^2e^2+d^2}\,d^2}+\frac{8\,f^2x}{15\,\sqrt{-x^2e^2+d^2}\,d^3}-\frac{(d^3g^2+6\,d^2fge+3\,df^2e^2)xe^{(-2)}}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d^2}-\frac{2\,(d^3g^2+6\,d^2fge+3\,df^2e^2)xe^{(-2)}}{15\,\sqrt{-x^2e^2+d^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] g^2*x^4*e/(-x^2*e^2 + d^2)^(5/2) - 4/3*d^2*g^2*x^2*e^(-1)/(-x^2*e^2 + d^2)^
(5/2) + 8/15*d^4*g^2*e^(-3)/(-x^2*e^2 + d^2)^(5/2) + 2/5*d^3*f*g*e^(-2)/(-x
^2*e^2 + d^2)^(5/2) + 3/5*d^2*f^2*e^(-1)/(-x^2*e^2 + d^2)^(5/2) + 1/2*(3*d*
g^2*e^2 + 2*f*g*e^3)*x^3*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 3/10*(3*d*g^2*e^2
+ 2*f*g*e^3)*d^2*x*e^(-4)/(-x^2*e^2 + d^2)^(5/2) + 1/5*d*f^2*x/(-x^2*e^2 +
d^2)^(5/2) + 1/3*(3*d^2*g^2*e + 6*d*f*g*e^2 + f^2*e^3)*x^2*e^(-2)/(-x^2*e^2
 + d^2)^(5/2) - 2/15*(3*d^2*g^2*e + 6*d*f*g*e^2 + f^2*e^3)*d^2*e^(-4)/(-x^2
*e^2 + d^2)^(5/2) + 1/10*(3*d*g^2*e^2 + 2*f*g*e^3)*x*e^(-4)/(-x^2*e^2 + d^2
```

```
)^(3/2) + 4/15*f^2*x/((-x^2*e^2 + d^2)^(3/2)*d) + 1/5*(d^3*g^2 + 6*d^2*f*g*
e + 3*d*f^2*e^2)*x*e^(-2)/(-x^2*e^2 + d^2)^(5/2) + 1/5*(3*d*g^2*e^2 + 2*f*g
*e^3)*x*e^(-4)/(sqrt(-x^2*e^2 + d^2)*d^2) + 8/15*f^2*x/(sqrt(-x^2*e^2 + d^2
)*d^3) - 1/15*(d^3*g^2 + 6*d^2*f*g*e + 3*d*f^2*e^2)*x*e^(-2)/((-x^2*e^2 + d
^2)^(3/2)*d^2) - 2/15*(d^3*g^2 + 6*d^2*f*g*e + 3*d*f^2*e^2)*x*e^(-2)/(sqrt(
-x^2*e^2 + d^2)*d^4)
```

Fricas [A]
time = 1.84, size = 266, normalized size = 1.83

$$-\frac{2\,d^5g^2 - 7\,f^2x^3e^5 + 3\,(2\,dfgx^3 + 7\,df^2x^2)e^4 - (2\,d^2g^2x^3 + 18\,d^2fgx^2 + 21\,d^2f^2x)e^3 + (6\,d^3g^2x^2 + 18\,d^3fgx + 7\,d^3f^2)e^2 - 6\,(d^4g^2x + d^4fg)e + (2\,d^4g^2 + 2\,f^2x^2e^4 - 6\,(dfgx^2 + df^2x)e^3 + (7\,d^2g^2x^2 + 18\,d^2fgx + 7\,d^2f^2)e^2 - 6\,(d^3g^2x + d^3fg)e)\sqrt{-x^2e^2 + d^2}}{15\,(d^3x^3e^6 - 3\,d^4x^2e^5 + 3\,d^5xe^4 - d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*d^5*g^2 - 7*f^2*x^3*e^5 + 3*(2*d*f*g*x^3 + 7*d*f^2*x^2)*e^4 - (2*d
^2*g^2*x^3 + 18*d^2*f*g*x^2 + 21*d^2*f^2*x)*e^3 + (6*d^3*g^2*x^2 + 18*d^3*f
*g*x + 7*d^3*f^2)*e^2 - 6*(d^4*g^2*x + d^4*f*g)*e + (2*d^4*g^2 + 2*f^2*x^2*
e^4 - 6*(d*f*g*x^2 + d*f^2*x)*e^3 + (7*d^2*g^2*x^2 + 18*d^2*f*g*x + 7*d^2*f
^2)*e^2 - 6*(d^3*g^2*x + d^3*f*g)*e)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3*e^6 - 3
*d^4*x^2*e^5 + 3*d^5*x*e^4 - d^6*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3\,(f + gx)^2}{(-(-d + ex)\,(d + ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(134) = 268.
time = 1.99, size = 356, normalized size = 2.46

$$-\frac{2\left(\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2g^2e^{(-2)}}{x} - \frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2g^2e^{(-4)}}{x^2} - 2\,d^2g^2 + 6\,dfge - \frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)dfge^{(-1)}}{x} + \frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2dfge^{(-3)}}{x^2} - \frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3dfge^{(-5)}}{x^3} - 7\,f^2e^2 - \frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2f^2e^{(-2)}}{x^2} + \frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3f^2e^{(-4)}}{x^3} - \frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4f^2e^{(-6)}}{x^4} + \frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)f^2}{x}\right)e^{(-3)}}{15\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x} - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(10*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*g^2*e^(-2)/x - 20*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^2*d^2*g^2*e^(-4)/x^2 - 2*d^2*g^2 + 6*d*f*g*e - 30*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)*d*f*g*e^(-1)/x + 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)
```

```
^2*d*f*g*e^(-3)/x^2 - 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*f*g*e^(-5)/x^3
- 7*f^2*e^2 - 40*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*f^2*e^(-2)/x^2 + 30*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^3*f^2*e^(-4)/x^3 - 15*(d*e + sqrt(-x^2*e^2 + d^2)
*e)^4*f^2*e^(-6)/x^4 + 20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*f^2/x)*e^(-3)/(d^3
*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [B]
time = 2.87, size = 125, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,d^4\,g^2-6\,d^3\,e\,f\,g-6\,d^3\,e\,g^2\,x+7\,d^2\,e^2\,f^2+18\,d^2\,e^2\,f\,g\,x+7\,d^2\,e^2\,g^2\,x^2-6\,d\,e^3\,f^2\,x-6\,d\,e^3\,f\,g\,x^2+2\,e^4\,f^2\,x^2\right)}{15\,d^3\,e^3\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)
[Out] ((d^2 - e^2*x^2)^(1/2)*(2*d^4*g^2 + 7*d^2*e^2*f^2 + 2*e^4*f^2*x^2 - 6*d^3*e
*f*g + 7*d^2*e^2*g^2*x^2 - 6*d*e^3*f^2*x - 6*d^3*e*g^2*x + 18*d^2*e^2*f*g*x
 - 6*d*e^3*f*g*x^2))/(15*d^3*e^3*(d - e*x)^3)
```

3.583 $\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=117

$$\frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{(2ef-3dg)x}{15d^3e\sqrt{d^2-e^2x^2}}$$

```
[Out] 1/5*(d*g+e*f)*(e*x+d)^3/d/e^2/(-e^2*x^2+d^2)^(5/2)+2/15*(-3*d*g+2*e*f)*(e*x
+d)/d/e^2/(-e^2*x^2+d^2)^(3/2)+1/15*(-3*d*g+2*e*f)*x/d^3/e/(-e^2*x^2+d^2)^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {803, 667, 197}

$$\frac{(d+ex)^3(dg+ef)}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(d+ex)(2ef-3dg)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] ((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^(5/2)) + (2*(2*e*f - 3*d
*g)*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^(3/2)) + ((2*e*f - 3*d*g)*x)/(15*d
^3*e*Sqrt[d^2 - e^2*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^3(f+gx)}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{(-5ef+3(ef+dg))\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx}{5de}\\ &= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{(-5ef+3(ef+dg))\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}\,dx}{15de}\\ &= \frac{(ef+dg)(d+ex)^3}{5de^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{2(2ef-3dg)(d+ex)}{15de^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{(2ef-3dg)x}{15d^3e\sqrt{d^2-e^2x^2}}\end{aligned}$$

Mathematica [A]
time = 0.48, size = 77, normalized size = 0.66

$$\frac{\sqrt{d^2-e^2x^2}\left(-3d^3g+2e^3fx^2-3de^2x(2f+gx)+d^2e(7f+9gx)\right)}{15d^3e^2(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x))/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-3*d^3*g + 2*e^3*f*x^2 - 3*d*e^2*x*(2*f + g*x) + d^2*
e*(7*f + 9*g*x)))/(15*d^3*e^2*(d - e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(105) = 210.
time = 0.07, size = 409, normalized size = 3.50

method	result
trager	$-\frac{\left(3d\,e^{2}g\,x^{2}-2e^{3}f\,x^{2}-9d^{2}egx+6d\,e^{2}fx+3d^{3}g-7d^{2}ef\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{3}\left(-ex+d\right)^{3}e^{2}}$
gosper	$-\frac{\left(ex+d\right)^{4}\left(-ex+d\right)\left(3d\,e^{2}g\,x^{2}-2e^{3}f\,x^{2}-9d^{2}egx+6d\,e^{2}fx+3d^{3}g-7d^{2}ef\right)}{15d^{3}e^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$

default	$e^{3} g \left(\frac{x^{3}}{2 e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{3 d^{2} \left(\frac{x}{4 e^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{d^{2} \left(\frac{x}{5 d^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4 x}{15 d^{2} \left(-e^{2} x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8 x}{15 d^{4} \sqrt{-e^{2} x^{2}+d^{2}}}}{d^{2}}\right)}{4 e^{2}}\right)}{2 e^{2}}\right) +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*g*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^
2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e
^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+(3*d*e^2*g+e^3*f)*(1/3
*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+(3*d^2*e*g
+3*d*e^2*f)*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^
2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^
2)^(1/2))))+1/5*(d^3*g+3*d^2*e*f)/e^2/(-e^2*x^2+d^2)^(5/2)+d^3*f*(1/5*x/d^2
/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e
^2*x^2+d^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(104) = 208$.
time = 0.29, size = 346, normalized size = 2.96

$$\frac{gx^{3}e}{2\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{3\,d^{2}gxe^{(-1)}}{10\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{d^{3}ge^{(-2)}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{3\,d^{2}fe^{(-1)}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{gxe^{(-1)}}{10\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{(3\,dge^{2}+fe^{3})x^{2}e^{(-2)}}{3\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}-\frac{2\,(3\,dge^{2}+fe^{3})d^{2}e^{(-4)}}{15\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{dfx}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{gxe^{(-1)}}{5\sqrt{-x^{2}e^{2}+d^{2}}\,d^{2}}+\frac{3\,(d^{2}ge+dfe^{2})xe^{(-2)}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{4\,fx}{15\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d}-\frac{(d^{2}ge+dfe^{2})xe^{(-2)}}{5\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}d^{2}}+\frac{8\,fx}{15\sqrt{-x^{2}e^{2}+d^{2}}\,d^{3}}-\frac{2\,(d^{2}ge+dfe^{2})xe^{(-2)}}{5\sqrt{-x^{2}e^{2}+d^{2}}\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*g*x^3*e/(-x^2*e^2 + d^2)^(5/2) - 3/10*d^2*g*x*e^(-1)/(-x^2*e^2 + d^2)^(
5/2) + 1/5*d^3*g*e^(-2)/(-x^2*e^2 + d^2)^(5/2) + 3/5*d^2*f*e^(-1)/(-x^2*e^2
 + d^2)^(5/2) + 1/10*g*x*e^(-1)/(-x^2*e^2 + d^2)^(3/2) + 1/3*(3*d*g*e^2 + f
*e^3)*x^2*e^(-2)/(-x^2*e^2 + d^2)^(5/2) - 2/15*(3*d*g*e^2 + f*e^3)*d^2*e^(-
4)/(-x^2*e^2 + d^2)^(5/2) + 1/5*d*f*x/(-x^2*e^2 + d^2)^(5/2) + 1/5*g*x*e^(-
1)/(sqrt(-x^2*e^2 + d^2)*d^2) + 3/5*(d^2*g*e + d*f*e^2)*x*e^(-2)/(-x^2*e^2
+ d^2)^(5/2) + 4/15*f*x/((-x^2*e^2 + d^2)^(3/2)*d) - 1/5*(d^2*g*e + d*f*e^2
)*x*e^(-2)/((-x^2*e^2 + d^2)^(3/2)*d^2) + 8/15*f*x/(sqrt(-x^2*e^2 + d^2)*d^
3) - 2/5*(d^2*g*e + d*f*e^2)*x*e^(-2)/(sqrt(-x^2*e^2 + d^2)*d^4)
```

Fricas [A]
time = 2.61, size = 174, normalized size = 1.49

$$\frac{3\,d^4g+7\,fx^3e^4-3\,(dgx^3+7\,dfx^2)e^3+3\,(3\,d^2gx^2+7\,d^2fx)e^2-(9\,d^3gx+7\,d^3f)e+(3\,d^3g-2\,fx^2e^3+3\,(dgx^2+2\,dfx)e^2-(9\,d^2gx+7\,d^2f)e)\sqrt{-x^2e^2+d^2}}{15\,(d^3x^3e^5-3\,d^4x^2e^4+3\,d^5xe^3-d^6e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*d^4*g + 7*f*x^3*e^4 - 3*(d*g*x^3 + 7*d*f*x^2)*e^3 + 3*(3*d^2*g*x^2
+ 7*d^2*f*x)*e^2 - (9*d^3*g*x + 7*d^3*f)*e + (3*d^3*g - 2*f*x^2*e^3 + 3*(d*
g*x^2 + 2*d*f*x)*e^2 - (9*d^2*g*x + 7*d^2*f)*e)*sqrt(-x^2*e^2 + d^2))/(d^3*
x^3*e^5 - 3*d^4*x^2*e^4 + 3*d^5*x*e^3 - d^6*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\,(f+gx)}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3*(f + g*x)/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(104) = 208.
time = 1.44, size = 264, normalized size = 2.26

$$\frac{2\left(\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)dge^{(-2)}}{x}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2dge^{(-4)}}{x^2}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3dge^{(-6)}}{x^3}-3\,dg+7\,fe-\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)fe^{(-1)}}{x}+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2fe^{(-3)}}{x^2}-\frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3fe^{(-5)}}{x^3}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4fe^{(-7)}}{x^4}\right)e^{(-2)}}{15\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d*g*e^(-2)/x - 15*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^2*d*g*e^(-4)/x^2 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*g*e^
(-6)/x^3 - 3*d*g + 7*f*e - 20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*f*e^(-1)/x + 4
0*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*f*e^(-3)/x^2 - 30*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)^3*f*e^(-5)/x^3 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*f*e^(-7)/x^4)
*e^(-2)/(d^3*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [B]
time = 2.79, size = 79, normalized size = 0.68

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,g\,d^3-9\,g\,d^2\,e\,x-7\,f\,d^2\,e+3\,g\,d\,e^2\,x^2+6\,f\,d\,e^2\,x-2\,f\,e^3\,x^2\right)}{15\,d^3\,e^2\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)*(d + e*x)^3)/(d^2 - e^2*x^2)^(7/2),x)

[Out] -((d^2 - e^2*x^2)^(1/2)*(3*d^3*g - 2*e^3*f*x^2 - 7*d^2*e*f + 6*d*e^2*f*x - 9*d^2*e*g*x + 3*d*e^2*g*x^2))/(15*d^3*e^2*(d - e*x)^3)

3.584 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

```
[Out] 1/5*(-e^2*x^2+d^2)^(1/2)/d/e/(-e*x+d)^3+2/15*(-e^2*x^2+d^2)^(1/2)/d^2/e/(-e
*x+d)^2+2/15*(-e^2*x^2+d^2)^(1/2)/d^3/e/(-e*x+d)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {669, 673, 665}

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^2*e
*(d - e*x)^2) + (2*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d - e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 669

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(2*m)/a^m, Int[(a + c*x^2)^(m + p)/(d - e*x)^m, x], x] /; FreeQ[{a, c, d,
 e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IntegerQ[m] && R
ationalQ[p] && (LtQ[0, -m, p] || LtQ[p, -m, 0]) && NeQ[m, 2] && NeQ[m, -1]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}\,dx\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}\,dx}{5d}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}\,dx}{15d^2}\\
&= \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2}+\frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.51

$$\frac{\sqrt{d^2-e^2x^2}\,\left(7d^2-6dex+2e^2x^2\right)}{15d^3e(d-ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(91) = 182.
time = 0.08, size = 246, normalized size = 2.39

method	result
trager	$\frac{\left(2e^2x^2-6dex+7d^2\right)\sqrt{-e^2x^2+d^2}}{15d^3\left(-ex+d\right)^3 e}$
gosper	$\frac{\left(ex+d\right)^4\left(-ex+d\right)\left(2e^2x^2-6dex+7d^2\right)}{15d^3e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
default	$e^3\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)+3d\,e^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\cdots}{d^2 \cdots}\right.}{4e^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e^4/(-e^2*x^2+d^2)^(5/2))+3*
d*e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)
^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/
2))))+3/5*d^2/e/(-e^2*x^2+d^2)^(5/2)+d^3*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/
5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [A]
time = 0.29, size = 96, normalized size = 0.93

$$\frac{x^2 e}{3\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{7\,d^2e^{(-1)}}{15\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{4\,dx}{5\,(-x^2e^2+d^2)^{\frac{5}{2}}}+\frac{x}{15\,(-x^2e^2+d^2)^{\frac{3}{2}}d}+\frac{2\,x}{15\,\sqrt{-x^2e^2+d^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^2*e/(-x^2*e^2 + d^2)^(5/2) + 7/15*d^2*e^(-1)/(-x^2*e^2 + d^2)^(5/2) +
 4/5*d*x/(-x^2*e^2 + d^2)^(5/2) + 1/15*x/((-x^2*e^2 + d^2)^(3/2)*d) + 2/15*
x/(sqrt(-x^2*e^2 + d^2)*d^3)
```

Fricas [A]
time = 2.24, size = 102, normalized size = 0.99

$$\frac{7\,x^3e^3-21\,dx^2e^2+21\,d^2xe-7\,d^3-(2\,x^2e^2-6\,dxe+7\,d^2)\sqrt{-x^2e^2+d^2}}{15\,(d^3x^3e^4-3\,d^4x^2e^3+3\,d^5xe^2-d^6e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*x^3*e^3 - 21*d*x^2*e^2 + 21*d^2*x*e - 7*d^3 - (2*x^2*e^2 - 6*d*x*e
+ 7*d^2)*sqrt(-x^2*e^2 + d^2))/(d^3*x^3*e^4 - 3*d^4*x^2*e^3 + 3*d^5*x*e^2 -
 d^6*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Giac [A]
time = 1.87, size = 158, normalized size = 1.53

$$-\frac{2\left(\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2e^{(-4)}}{x^2}+\frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3e^{(-6)}}{x^3}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4e^{(-8)}}{x^4}-7\right)e^{(-1)}}{15\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 40*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*e^(-4)/x^2 + 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*e^(-6)/x^3 -
15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^(-8)/x^4 - 7)*e^(-1)/(d^3*((d*e + sqr
t(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [B]
time = 2.70, size = 49, normalized size = 0.48

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,{\left(d-e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^3
)
```

3.585 $\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=242

$$\frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}}-\frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{15d^3g^2+e\left(2e^2f^2+9defg+22d^2g^2\right)x}{15d^3(ef+dg)^3\sqrt{d^2-e^2x^2}}+\frac{g^3\tan^{-}}{}$$

```
[Out] 4/5*d*(e*x+d)/(d*g+e*f)/(-e^2*x^2+d^2)^(5/2)+1/15*(-5*d*(-d*g+e*f)+e*(11*d*
g+e*f)*x)/d/(d*g+e*f)^2/(-e^2*x^2+d^2)^(3/2)+g^3*arctan((e^2*f*x+d^2*g)/(-d
^2*g^2+e^2*f^2)^(1/2)/(-e^2*x^2+d^2)^(1/2))/(d*g+e*f)^3/(-d^2*g^2+e^2*f^2)^
(1/2)+1/15*(15*d^3*g^2+e*(22*d^2*g^2+9*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^
3/(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1661, 837, 12, 739, 210}

$$\frac{g^3\text{ArcTan}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}}-\frac{5d(ef-dg)-ex(11dg+ef)}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^2}+\frac{4d(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)}+\frac{15d^3g^2+ex\left(22d^2g^2+9defg+2e^2f^2\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*(d + e*x))/(5*(e*f + d*g)*(d^2 - e^2*x^2)^(5/2)) - (5*d*(e*f - d*g) -
e*(e*f + 11*d*g)*x)/(15*d*(e*f + d*g)^2*(d^2 - e^2*x^2)^(3/2)) + (15*d^3*g^
2 + e*(2*e^2*f^2 + 9*d*e*f*g + 22*d^2*g^2)*x)/(15*d^3*(e*f + d*g)^3*Sqrt[d^
2 - e^2*x^2]) + (g^3*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/((e*f + d*g)^3*Sqrt[e^2*f^2 - d^2*g^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2(ef+5dg)}{ef+dg}-\frac{d^2e^3(5ef-11dg)x}{ef+dg}}{(f+gx)(d^2-e^2x^2)^{5/2}}\,dx}{5d^2e^2}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{\int -\frac{d^3e^4(ef-dg)(\ldots}{\ldots}}{\ldots}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e(2\ldots}{15d^3(ef\ldots}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e(2\ldots}{15d^3(ef\ldots}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e(2\ldots}{15d^3(ef\ldots}$$

$$= \frac{4d(d+ex)}{5(ef+dg)\left(d^2-e^2x^2\right)^{5/2}} - \frac{5d(ef-dg)-e(ef+11dg)x}{15d(ef+dg)^2\left(d^2-e^2x^2\right)^{3/2}} + \frac{15d^3g^2+e(2\ldots}{15d^3(ef\ldots}$$

Mathematica [A]
time = 10.29, size = 225, normalized size = 0.93

$$\frac{\frac{(-e^2f^2+d^2g^2)(d+ex)(32d^4g^2+2e^4f^2x^2+3d^3eg(8f-17gx)+3de^3fx(-2f+3gx)+d^2e^2(7f^2-27fgx+22g^2x^2))}{d^3(d-ex)^2\sqrt{d^2-e^2x^2}} - 15g^3\sqrt{e^2f^2-d^2g^2}\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{15(-ef+dg)(ef+dg)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((-(e^2*f^2) + d^2*g^2)*(d + e*x)*(32*d^4*g^2 + 2*e^4*f^2*x^2 + 3*d^3*e*g*
(8*f - 17*g*x) + 3*d*e^3*f*x*(-2*f + 3*g*x) + d^2*e^2*(7*f^2 - 27*f*g*x + 2
2*g^2*x^2)))/(d^3*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]) - 15*g^3*Sqrt[e^2*f^2 -
d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x
^2])])/(15*(-(e*f) + d*g)*(e*f + d*g)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1666 vs. $2(223) = 446$.
time = 0.10, size = 1667, normalized size = 6.89

method	result	size

default	Expression too large to display	1667

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e/g^3*(g^2*e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e^2
*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2
+d^2)^(1/2))))+1/5*(3*d*e*g^2-e^2*f*g)/e^2/(-e^2*x^2+d^2)^(5/2)+3*d^2*g^2*(
1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*
x/d^4/(-e^2*x^2+d^2)^(1/2)))-3*d*e*f*g*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/
d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+e^2*f^
2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2
/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f
^3)/g^4*(1/5/(d^2*g^2-e^2*f^2)*g^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g
^2-e^2*f^2)/g^2)^(5/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/5*(-2*e^2*(x+f/g)+2*e^2
*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/
g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-16/5*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g
^2-4*e^4*f^2/g^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)
/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2
)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+
f/g)+2*e^2*f/g)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1
/2)))+1/(d^2*g^2-e^2*f^2)*g^2*(1/3/(d^2*g^2-e^2*f^2)*g^2/(-(x+f/g)^2*e^2+2*
e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/3
*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-
(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e
^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-(x+f
/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))+1/(d^2*g^2-e^2*f^
2)*g^2*(1/(d^2*g^2-e^2*f^2)*g^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-
e^2*f^2)/g^2)^(1/2)-2*e^2*f*g/(d^2*g^2-e^2*f^2)*(-2*e^2*(x+f/g)+2*e^2*f/g)/
(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f
/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)-1/(d^2*g^2-e^2*f^2)*g^2/((d^2*g^2-e^2*f^2)
/g^2)^(1/2)*ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f
^2)/g^2)^(1/2)*(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/
2))/(x+f/g)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(225) = 450$.
time = 3.75, size = 1742, normalized size = 7.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(32*d^7*g^4 + 7*f^4*x^3*e^7 - 15*(d^3*g^3*x^3*e^3 - 3*d^4*g^3*x^2*e^2
 + 3*d^5*g^3*x*e - d^6*g^3)*sqrt(d^2*g^2 - f^2*e^2)*log((d^3*g^2 + d*f*g*x*
e^2 - sqrt(d^2*g^2 - f^2*e^2)*(d^2*g + f*x*e^2 + sqrt(-x^2*e^2 + d^2)*d*g)
+ (d^2*g^2 - f^2*e^2)*sqrt(-x^2*e^2 + d^2))/(g*x + f)) + 3*(8*d*f^3*g*x^3 -
 7*d*f^4*x^2)*e^6 + (25*d^2*f^2*g^2*x^3 - 72*d^2*f^3*g*x^2 + 21*d^2*f^4*x)*
e^5 - (24*d^3*f*g^3*x^3 + 75*d^3*f^2*g^2*x^2 - 72*d^3*f^3*g*x + 7*d^3*f^4)*
e^4 - (32*d^4*g^4*x^3 - 72*d^4*f*g^3*x^2 - 75*d^4*f^2*g^2*x + 24*d^4*f^3*g)
*e^3 + (96*d^5*g^4*x^2 - 72*d^5*f*g^3*x - 25*d^5*f^2*g^2)*e^2 - 24*(4*d^6*g
^4*x - d^6*f*g^3)*e + (32*d^6*g^4 - 2*f^4*x^2*e^6 - 3*(3*d*f^3*g*x^2 - 2*d*
f^4*x)*e^5 - (20*d^2*f^2*g^2*x^2 - 27*d^2*f^3*g*x + 7*d^2*f^4)*e^4 + 3*(3*d
^3*f*g^3*x^2 + 15*d^3*f^2*g^2*x - 8*d^3*f^3*g)*e^3 + (22*d^4*g^4*x^2 - 27*d
^4*f*g^3*x - 25*d^4*f^2*g^2)*e^2 - 3*(17*d^5*g^4*x - 8*d^5*f*g^3)*e)*sqrt(-
x^2*e^2 + d^2))/(d^11*g^5 + d^3*f^5*x^3*e^8 + 3*(d^4*f^4*g*x^3 - d^4*f^5*x^
2)*e^7 + (2*d^5*f^3*g^2*x^3 - 9*d^5*f^4*g*x^2 + 3*d^5*f^5*x)*e^6 - (2*d^6*f
^2*g^3*x^3 + 6*d^6*f^3*g^2*x^2 - 9*d^6*f^4*g*x + d^6*f^5)*e^5 - 3*(d^7*f*g^
4*x^3 - 2*d^7*f^2*g^3*x^2 - 2*d^7*f^3*g^2*x + d^7*f^4*g)*e^4 - (d^8*g^5*x^3
 - 9*d^8*f*g^4*x^2 + 6*d^8*f^2*g^3*x + 2*d^8*f^3*g^2)*e^3 + (3*d^9*g^5*x^2
- 9*d^9*f*g^4*x + 2*d^9*f^2*g^3)*e^2 - 3*(d^10*g^5*x - d^10*f*g^4)*e), 1/15
*(32*d^7*g^4 + 7*f^4*x^3*e^7 - 30*(d^3*g^3*x^3*e^3 - 3*d^4*g^3*x^2*e^2 + 3*
d^5*g^3*x*e - d^6*g^3)*sqrt(-d^2*g^2 + f^2*e^2)*arctan(sqrt(-d^2*g^2 + f^2*
e^2)*(d*g*x + d*f - sqrt(-x^2*e^2 + d^2)*f)/(d^2*g^2*x - f^2*x*e^2)) + 3*(8
*d*f^3*g*x^3 - 7*d*f^4*x^2)*e^6 + (25*d^2*f^2*g^2*x^3 - 72*d^2*f^3*g*x^2 +
21*d^2*f^4*x)*e^5 - (24*d^3*f*g^3*x^3 + 75*d^3*f^2*g^2*x^2 - 72*d^3*f^3*g*x
 + 7*d^3*f^4)*e^4 - (32*d^4*g^4*x^3 - 72*d^4*f*g^3*x^2 - 75*d^4*f^2*g^2*x +
 24*d^4*f^3*g)*e^3 + (96*d^5*g^4*x^2 - 72*d^5*f*g^3*x - 25*d^5*f^2*g^2)*e^2
 - 24*(4*d^6*g^4*x - d^6*f*g^3)*e + (32*d^6*g^4 - 2*f^4*x^2*e^6 - 3*(3*d*f^
3*g*x^2 - 2*d*f^4*x)*e^5 - (20*d^2*f^2*g^2*x^2 - 27*d^2*f^3*g*x + 7*d^2*f^4
)*e^4 + 3*(3*d^3*f*g^3*x^2 + 15*d^3*f^2*g^2*x - 8*d^3*f^3*g)*e^3 + (22*d^4*
g^4*x^2 - 27*d^4*f*g^3*x - 25*d^4*f^2*g^2)*e^2 - 3*(17*d^5*g^4*x - 8*d^5*f*
g^3)*e)*sqrt(-x^2*e^2 + d^2))/(d^11*g^5 + d^3*f^5*x^3*e^8 + 3*(d^4*f^4*g*x^
3 - d^4*f^5*x^2)*e^7 + (2*d^5*f^3*g^2*x^3 - 9*d^5*f^4*g*x^2 + 3*d^5*f^5*x)*
e^6 - (2*d^6*f^2*g^3*x^3 + 6*d^6*f^3*g^2*x^2 - 9*d^6*f^4*g*x + d^6*f^5)*e^5
 - 3*(d^7*f*g^4*x^3 - 2*d^7*f^2*g^3*x^2 - 2*d^7*f^3*g^2*x + d^7*f^4*g)*e^4
```

```
- (d^8*g^5*x^3 - 9*d^8*f*g^4*x^2 + 6*d^8*f^2*g^3*x + 2*d^8*f^3*g^2)*e^3 + (
3*d^9*g^5*x^2 - 9*d^9*f*g^4*x + 2*d^9*f^2*g^3)*e^2 - 3*(d^10*g^5*x - d^10*f
*g^4)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\left(-(-d+ex)(d+ex)\right)^{\frac{7}{2}}(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 607 vs. 2(225) = 450.
time = 2.00, size = 607, normalized size = 2.51

$$-\frac{2\,g^3\arctan\left(\frac{dg+\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)fe^{(-1)}}{x}}{\sqrt{-d^2g^2+f^2e^2}}\right)}{\left(d^3g^3+3\,d^2fg^2e+3\,df^2ge^2+f^3e^3\right)\sqrt{-d^2g^2+f^2e^2}}-\frac{2\left(\frac{115\left(de+\sqrt{-x^2e^2+d^2}\,e\right)d^2g^2e^{(-2)}}{x}-\frac{185\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2d^2g^2e^{(-4)}}{x^2}+\frac{135\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3d^2g^2e^{(-6)}}{x^3}-\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4d^2g^2e^{(-8)}}{x^4}-32\,d^2g^2-24\,dfge+\frac{75\left(de+\sqrt{-x^2e^2+d^2}\,e\right)dfge^{(-1)}}{x}-\frac{135\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2dfge^{(-3)}}{x^2}+\frac{105\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3dfge^{(-5)}}{x^3}-\frac{45\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4dfge^{(-7)}}{x^4}-7\,f^2e^2-\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2f^2e^{(-2)}}{x^2}+\frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3f^2e^{(-4)}}{x^3}-\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4f^2e^{(-6)}}{x^4}+\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)f^2}{x}\right)}{15\left(d^6g^3+3\,d^5fg^2e+3\,d^4f^2ge^2+d^3f^3e^3\right)\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -2*g^3*arctan((d*g + (d*e + sqrt(-x^2*e^2 + d^2)*e)*f*e^(-1)/x)/sqrt(-d^2*g
^2 + f^2*e^2))/((d^3*g^3 + 3*d^2*f*g^2*e + 3*d*f^2*g*e^2 + f^3*e^3)*sqrt(-d
^2*g^2 + f^2*e^2)) - 2/15*(115*(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^2*g^2*e^(-2
)/x - 185*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*g^2*e^(-4)/x^2 + 135*(d*e +
sqrt(-x^2*e^2 + d^2)*e)^3*d^2*g^2*e^(-6)/x^3 - 45*(d*e + sqrt(-x^2*e^2 + d^
2)*e)^4*d^2*g^2*e^(-8)/x^4 - 32*d^2*g^2 - 24*d*f*g*e + 75*(d*e + sqrt(-x^2*
e^2 + d^2)*e)*d*f*g*e^(-1)/x - 135*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*f*g*e
^(-3)/x^2 + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*f*g*e^(-5)/x^3 - 45*(d*e
 + sqrt(-x^2*e^2 + d^2)*e)^4*d*f*g*e^(-7)/x^4 - 7*f^2*e^2 - 40*(d*e + sqrt(
-x^2*e^2 + d^2)*e)^2*f^2*e^(-2)/x^2 + 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*f
^2*e^(-4)/x^3 - 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*f^2*e^(-6)/x^4 + 20*(d*
e + sqrt(-x^2*e^2 + d^2)*e)*f^2/x)/((d^6*g^3 + 3*d^5*f*g^2*e + 3*d^4*f^2*g*
e^2 + d^3*f^3*e^3)*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x - 1)^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{(f+g\,x)\,\left(d^2-e^2\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)*(d^2 - e^2*x^2)^(7/2)), x)
```

3.586 $\int \frac{(d+ex)^3}{(f+gx)^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=311

$$\frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}}-\frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{e\left(45d^3g^2+e\left(2e^2f^2+14defg+57d^2g^2\right)x\right)}{15d^3(ef+dg)^4\sqrt{d^2-e^2x^2}}$$

```
[Out] 4/5*d*e*(e*x+d)/(d*g+e*f)^2/(-e^2*x^2+d^2)^(5/2)-1/15*e*(5*d*(-3*d*g+e*f)-e
*(21*d*g+e*f)*x)/d/(d*g+e*f)^3/(-e^2*x^2+d^2)^(3/2)+e*g^3*(-3*d*g+4*e*f)*ar
ctan((e^2*f*x+d^2*g)/(-d^2*g^2+e^2*f^2)^(1/2)/(-e^2*x^2+d^2)^(1/2))/(-d*g+e
*f)/(d*g+e*f)^4/(-d^2*g^2+e^2*f^2)^(1/2)+1/15*e*(45*d^3*g^2+e*(57*d^2*g^2+1
4*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^4/(-e^2*x^2+d^2)^(1/2)+g^4*(-e^2*x^2+
d^2)^(1/2)/(-d*g+e*f)/(d*g+e*f)^4/(g*x+f)
```

Rubi [A]
time = 1.01, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1661, 821, 739, 210}

$$\frac{eg^3(4ef-3dg)\text{ArcTan}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}}+\frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4}-\frac{e(5d(ef-3dg)-ex(21dg+ef))}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^3}+\frac{4de(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^2}+\frac{e\left(45d^3g^2+ex\left(57d^2g^2+14defg+2e^2f^2\right)\right)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*e*(d + e*x))/(5*(e*f + d*g)^2*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d*(e*f -
3*d*g) - e*(e*f + 21*d*g)*x))/(15*d*(e*f + d*g)^3*(d^2 - e^2*x^2)^(3/2)) +
(e*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(15*d^3*(e*f +
 d*g)^4*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/((e*f - d*g)*(e*f
+ d*g)^4*(f + g*x)) + (e*g^3*(4*e*f - 3*d*g)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt
[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/((e*f - d*g)*(e*f + d*g)^4*Sqrt[
e^2*f^2 - d^2*g^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^2f^2+10defg+5d^2g^2\right)}{(ef+dg)^2} - \frac{d^2e^3(ef-5dg)(5ef+3dg)x}{(ef+dg)^2} + \frac{16d^3e\ldots}{(ef+\ldots}}{(f+gx)^2(d^2-e^2x^2)^{5/2}}}{5d^2e^2}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{d^3e^4\left(2e^3\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(45d^3g^2\ldots}{1\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(45d^3g^2\ldots}{1\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(45d^3g^2\ldots}{1\ldots}$$

$$= \frac{4de(d+ex)}{5(ef+dg)^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{e(5d(ef-3dg)-e(ef+21dg)x)}{15d(ef+dg)^3\left(d^2-e^2x^2\right)^{3/2}} + \frac{e(45d^3g^2\ldots}{1\ldots}$$

Mathematica [A]

time = 10.44, size = 341, normalized size = 1.10

$$\frac{\frac{(e^2f^2-d^2g^2)(d+ex)(15d^6g^4+2e^6f^3x^2(f+gx)-9d^5eg^3(8f+13gx)+6de^5f^2x(-f^2+fgx+2g^2x^2)+d^4e^2g^2(38f^2+164fgx+171g^2x^2)-3d^3e^3g(-9f^3+19f^2gx+47fg^2x^2+24g^3x^3)+d^2e^4f(7f^3-29f^2gx+7fg^2x^2+43g^3x^3))}{d^3(d-ex)^2(f+gx)\sqrt{d^2-e^2x^2}}+15eg^3(4ef-3dg)\sqrt{e^2f^2-d^2g^2}\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}}\right)}{15(ef-dg)^2(ef+dg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (((e^2*f^2 - d^2*g^2)*(d + e*x)*(15*d^6*g^4 + 2*e^6*f^3*x^2*(f + g*x) - 9*d
^5*e*g^3*(8*f + 13*g*x) + 6*d*e^5*f^2*x*(-f^2 + f*g*x + 2*g^2*x^2) + d^4*e^
2*g^2*(38*f^2 + 164*f*g*x + 171*g^2*x^2) - 3*d^3*e^3*g*(-9*f^3 + 19*f^2*g*x
 + 47*f*g^2*x^2 + 24*g^3*x^3) + d^2*e^4*f*(7*f^3 - 29*f^2*g*x + 7*f*g^2*x^2
 + 43*g^3*x^3)))/(d^3*(d - e*x)^2*(f + g*x)*Sqrt[d^2 - e^2*x^2]) + 15*e*g^3
*(4*e*f - 3*d*g)*Sqrt[e^2*f^2 - d^2*g^2]*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2
*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2])])/(15*(e*f - d*g)^2*(e*f + d*g)^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3288 vs. $2(291) = 582$.
time = 0.10, size = 3289, normalized size = 10.58

method	result	size
default	Expression too large to display	3289

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/g^3*(1/5/e*g/(-e^2*x^2+d^2)^(5/2)+3*d*g*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)
+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))-2
*e*f*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2
)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+3*e/g^4*(d^2*g^2-2*d*e*f*g+e^2*f^2)*(1/
5/(d^2*g^2-e^2*f^2)*g^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)
/g^2)^(5/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/5*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e
^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(
d^2*g^2-e^2*f^2)/g^2)^(5/2)-16/5*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^
2/g^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*
f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/
3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*
f/g)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)))+1/(d^
2*g^2-e^2*f^2)*g^2*(1/3/(d^2*g^2-e^2*f^2)*g^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+
f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2/3*(-2*e^2*(x
+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e
^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2
-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-(x+f/g)^2*e^2+2
*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))+1/(d^2*g^2-e^2*f^2)*g^2*(1/(
d^2*g^2-e^2*f^2)*g^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^
```

```
2)^(1/2)-2*e^2*f*g/(d^2*g^2-e^2*f^2)*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^
2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^
2-e^2*f^2)/g^2)^(1/2)-1/(d^2*g^2-e^2*f^2)*g^2/((d^2*g^2-e^2*f^2)/g^2)^(1/2)
*ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1
/2)*(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g)
))))+1/g^5*(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)*(-1/(d^2*g^2-e^2*f
^2)*g^2/(x+f/g)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5
/2)-7*e^2*f*g/(d^2*g^2-e^2*f^2)*(1/5/(d^2*g^2-e^2*f^2)*g^2/(-(x+f/g)^2*e^2+
2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-e^2*f*g/(d^2*g^2-e^2*f^2)*(2
/5*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/
(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-16/5*e^2/(-4
*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(
-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/
g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^
4*f^2/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(
d^2*g^2-e^2*f^2)/g^2)^(1/2)))+1/(d^2*g^2-e^2*f^2)*g^2*(1/3/(d^2*g^2-e^2*f^2
)*g^2/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(3/2)-e^2*f*
g/(d^2*g^2-e^2*f^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^
2)/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g
^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)^2*(-2*e^2*(
x+f/g)+2*e^2*f/g)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^
(1/2))+1/(d^2*g^2-e^2*f^2)*g^2*(1/(d^2*g^2-e^2*f^2)*g^2/(-(x+f/g)^2*e^2+2*e
^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)-2*e^2*f*g/(d^2*g^2-e^2*f^2)*(-2
*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+f
/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(1/2)-1/(d^2*g^2-e^2*f^2
)*g^2/((d^2*g^2-e^2*f^2)/g^2)^(1/2)*ln((2*(d^2*g^2-e^2*f^2)/g^2+2*e^2*f/g*(
x+f/g)+2*((d^2*g^2-e^2*f^2)/g^2)^(1/2)*(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d
^2*g^2-e^2*f^2)/g^2)^(1/2))/(x+f/g)))))+6*e^2/(d^2*g^2-e^2*f^2)*g^2*(2/5*(-
2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+
f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g^2-e^2*f^2)/g^2)^(5/2)-16/5*e^2/(-4*e^2*
(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)*(2/3*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-4*e^
2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2/g^2)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d
^2*g^2-e^2*f^2)/g^2)^(3/2)-16/3*e^2/(-4*e^2*(d^2*g^2-e^2*f^2)/g^2-4*e^4*f^2
/g^2)^2*(-2*e^2*(x+f/g)+2*e^2*f/g)/(-(x+f/g)^2*e^2+2*e^2*f/g*(x+f/g)+(d^2*g
^2-e^2*f^2)/g^2)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1607 vs. $2(301) = 602$.

time = 5.30, size = 3252, normalized size = 10.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/15*(15*d^9*g^7*x + 15*d^9*f*g^6 - 15*sqrt(d^2*g^2 - f^2*e^2)*(4*(d^3*f^
2*g^4*x^4 + d^3*f^3*g^3*x^3)*e^5 - 3*(d^4*f*g^5*x^4 + 5*d^4*f^2*g^4*x^3 + 4
*d^4*f^3*g^3*x^2)*e^4 + 3*(3*d^5*f*g^5*x^3 + 7*d^5*f^2*g^4*x^2 + 4*d^5*f^3*
g^3*x)*e^3 - (9*d^6*f*g^5*x^2 + 13*d^6*f^2*g^4*x + 4*d^6*f^3*g^3)*e^2 + 3*(
d^7*f*g^5*x + d^7*f^2*g^4)*e)*log((d^3*g^2 + d*f*g*x*e^2 - sqrt(d^2*g^2 - f
^2*e^2)*(d^2*g + f*x*e^2 + sqrt(-x^2*e^2 + d^2)*d*g) + (d^2*g^2 - f^2*e^2)*
sqrt(-x^2*e^2 + d^2))/(g*x + f)) + 7*(f^6*g*x^4 + f^7*x^3)*e^9 + 3*(9*d*f^5
*g^2*x^4 + 2*d*f^6*g*x^3 - 7*d*f^7*x^2)*e^8 + (31*d^2*f^4*g^3*x^4 - 50*d^2*
f^5*g^2*x^3 - 60*d^2*f^6*g*x^2 + 21*d^2*f^7*x)*e^7 - (99*d^3*f^3*g^4*x^4 +
192*d^3*f^4*g^3*x^3 + 12*d^3*f^5*g^2*x^2 - 74*d^3*f^6*g*x + 7*d^3*f^7)*e^6
- (23*d^4*f^2*g^5*x^4 - 274*d^4*f^3*g^4*x^3 - 390*d^4*f^4*g^3*x^2 - 66*d^4*
f^5*g^2*x + 27*d^4*f^6*g)*e^5 + (72*d^5*f*g^6*x^4 + 141*d^5*f^2*g^5*x^3 - 2
28*d^5*f^3*g^4*x^2 - 328*d^5*f^4*g^3*x - 31*d^5*f^5*g^2)*e^4 - 3*(5*d^6*g^7
*x^4 + 77*d^6*f*g^6*x^3 + 95*d^6*f^2*g^5*x^2 - 10*d^6*f^3*g^4*x - 33*d^6*f^
4*g^3)*e^3 + (45*d^7*g^7*x^3 + 261*d^7*f*g^6*x^2 + 239*d^7*f^2*g^5*x + 23*d
^7*f^3*g^4)*e^2 - 9*(5*d^8*g^7*x^2 + 13*d^8*f*g^6*x + 8*d^8*f^2*g^5)*e + (1
5*d^8*f*g^6 - 2*(f^6*g*x^3 + f^7*x^2)*e^8 - 6*(2*d*f^5*g^2*x^3 + d*f^6*g*x^
2 - d*f^7*x)*e^7 - (41*d^2*f^4*g^3*x^3 + 5*d^2*f^5*g^2*x^2 - 29*d^2*f^6*g*x
 + 7*d^2*f^7)*e^6 + 3*(28*d^3*f^3*g^4*x^3 + 49*d^3*f^4*g^3*x^2 + 17*d^3*f^5
*g^2*x - 9*d^3*f^6*g)*e^5 + (43*d^4*f^2*g^5*x^3 - 164*d^4*f^3*g^4*x^2 - 193
*d^4*f^4*g^3*x - 31*d^4*f^5*g^2)*e^4 - 3*(24*d^5*f*g^6*x^3 + 47*d^5*f^2*g^5
*x^2 - 20*d^5*f^3*g^4*x - 33*d^5*f^4*g^3)*e^3 + (171*d^6*f*g^6*x^2 + 164*d^
6*f^2*g^5*x + 23*d^6*f^3*g^4)*e^2 - 9*(13*d^7*f*g^6*x + 8*d^7*f^2*g^5)*e)*s
qrt(-x^2*e^2 + d^2))/(d^13*f*g^8*x + d^13*f^2*g^7 - (d^3*f^8*g*x^4 + d^3*f^
9*x^3)*e^10 - 3*(d^4*f^7*g^2*x^4 - d^4*f^9*x^2)*e^9 - (d^5*f^6*g^3*x^4 - 8*
d^5*f^7*g^2*x^3 - 6*d^5*f^8*g*x^2 + 3*d^5*f^9*x)*e^8 + (5*d^6*f^5*g^4*x^4 +
 8*d^6*f^6*g^3*x^3 - 6*d^6*f^7*g^2*x^2 - 8*d^6*f^8*g*x + d^6*f^9)*e^7 + (5*
d^7*f^4*g^5*x^4 - 10*d^7*f^5*g^4*x^3 - 18*d^7*f^6*g^3*x^2 + 3*d^7*f^8*g)*e^
6 - (d^8*f^3*g^6*x^4 + 16*d^8*f^4*g^5*x^3 - 16*d^8*f^6*g^3*x - d^8*f^7*g^2)
*e^5 - (3*d^9*f^2*g^7*x^4 - 18*d^9*f^4*g^5*x^2 - 10*d^9*f^5*g^4*x + 5*d^9*f
^6*g^3)*e^4 - (d^10*f*g^8*x^4 - 8*d^10*f^2*g^7*x^3 - 6*d^10*f^3*g^6*x^2 + 8
*d^10*f^4*g^5*x + 5*d^10*f^5*g^4)*e^3 + (3*d^11*f*g^8*x^3 - 6*d^11*f^2*g^7*
x^2 - 8*d^11*f^3*g^6*x + d^11*f^4*g^5)*e^2 - 3*(d^12*f*g^8*x^2 - d^12*f^3*g
```

```
^6)*e), -1/15*(15*d^9*g^7*x + 15*d^9*f*g^6 - 30*sqrt(-d^2*g^2 + f^2*e^2)*(4
*(d^3*f^2*g^4*x^4 + d^3*f^3*g^3*x^3)*e^5 - 3*(d^4*f*g^5*x^4 + 5*d^4*f^2*g^4
*x^3 + 4*d^4*f^3*g^3*x^2)*e^4 + 3*(3*d^5*f*g^5*x^3 + 7*d^5*f^2*g^4*x^2 + 4*
d^5*f^3*g^3*x)*e^3 - (9*d^6*f*g^5*x^2 + 13*d^6*f^2*g^4*x + 4*d^6*f^3*g^3)*e
^2 + 3*(d^7*f*g^5*x + d^7*f^2*g^4)*e)*arctan(sqrt(-d^2*g^2 + f^2*e^2)*(d*g*
x + d*f - sqrt(-x^2*e^2 + d^2)*f)/(d^2*g^2*x - f^2*x*e^2)) + 7*(f^6*g*x^4 +
 f^7*x^3)*e^9 + 3*(9*d*f^5*g^2*x^4 + 2*d*f^6*g*x^3 - 7*d*f^7*x^2)*e^8 + (31
*d^2*f^4*g^3*x^4 - 50*d^2*f^5*g^2*x^3 - 60*d^2*f^6*g*x^2 + 21*d^2*f^7*x)*e^
7 - (99*d^3*f^3*g^4*x^4 + 192*d^3*f^4*g^3*x^3 + 12*d^3*f^5*g^2*x^2 - 74*d^3
*f^6*g*x + 7*d^3*f^7)*e^6 - (23*d^4*f^2*g^5*x^4 - 274*d^4*f^3*g^4*x^3 - 390
*d^4*f^4*g^3*x^2 - 66*d^4*f^5*g^2*x + 27*d^4*f^6*g)*e^5 + (72*d^5*f*g^6*x^4
 + 141*d^5*f^2*g^5*x^3 - 228*d^5*f^3*g^4*x^2 - 328*d^5*f^4*g^3*x - 31*d^5*f
^5*g^2)*e^4 - 3*(5*d^6*g^7*x^4 + 77*d^6*f*g^6*x^3 + 95*d^6*f^2*g^5*x^2 - 10
*d^6*f^3*g^4*x - 33*d^6*f^4*g^3)*e^3 + (45*d^7*g^7*x^3 + 261*d^7*f*g^6*x^2
+ 239*d^7*f^2*g^5*x + 23*d^7*f^3*g^4)*e^2 - 9*(5*d^8*g^7*x^2 + 13*d^8*f*g^6
*x + 8*d^8*f^2*g^5)*e + (15*d^8*f*g^6 - 2*(f^6*g*x^3 + f^7*x^2)*e^8 - 6*(2*
d*f^5*g^2*x^3 + d*f^6*g*x^2 - d*f^7*x)*e^7 - (41*d^2*f^4*g^3*x^3 + 5*d^2*f^
5*g^2*x^2 - 29*d^2*f^6*g*x + 7*d^2*f^7)*e^6 + 3*(28*d^3*f^3*g^4*x^3 + 49*d^
3*f^4*g^3*x^2 + 17*d^3*f^5*g^2*x - 9*d^3*f^6*g)*e^5 + (43*d^4*f^2*g^5*x^3 -
 164*d^4*f^3*g^4*x^2 - 193*d^4*f^4*g^3*x - 31*d^4*f^5*g^2)*e^4 - 3*(24*d^5*
f*g^6*x^3 + 47*d^5*f^2*g^5*x^2 - 20*d^5*f^3*g^4*x - 33*d^5*f^4*g^3)*e^3 + (
171*d^6*f*g^6*x^2 + 164*d^6*f^2*g^5*x + 23*d^6*f^3*g^4)*e^2 - 9*(13*d^7*f*g
^6*x + 8*d^7*f^2*g^5)*e)*sqrt(-x^2*e^2 + d^2))/(d^13*f*g^8*x + d^13*f^2*g^7
 - (d^3*f^8*g*x^4 + d^3*f^9*x^3)*e^10 - 3*(d^4*f^7*g^2*x^4 - d^4*f^9*x^2)*e
^9 - (d^5*f^6*g^3*x^4 - 8*d^5*f^7*g^2*x^3 - 6*d^5*f^8*g*x^2 + 3*d^5*f^9*x)*
e^8 + (5*d^6*f^5*g^4*x^4 + 8*d^6*f^6*g^3*x^3 - 6*d^6*f^7*g^2*x^2 - 8*d^6*f^
8*g*x + d^6*f^9)*e^7 + (5*d^7*f^4*g^5*x^4 - 10*d^7*f^5*g^4*x^3 - 18*d^7*f^6
*g^3*x^2 + 3*d^7*f^8*g)*e^6 - (d^8*f^3*g^6*x^4 + 16*d^8*f^4*g^5*x^3 - 16*d^
8*f^6*g^3*x - d^8*f^7*g^2)*e^5 - (3*d^9*f^2*g^7*x^4 - 18*d^9*f^4*g^5*x^2 -
10*d^9*f^5*g^4*x + 5*d^9*f^6*g^3)*e^4 - (d^10*f*g^8*x^4 - 8*d^10*f^2*g^7*x^
3 - 6*d^10*f^3*g^6*x^2 + 8*d^10*f^4*g^5*x + 5*d^10*f^5*g^4)*e^3 + (3*d^11*f
*g^8*x^3 - 6*d^11*f^2*g^7*x^2 - 8*d^11*f^3*g^6*x + d^11*f^4*g^5)*e^2 - 3*(d
^12*f*g^8*x^2 - d^12*f^3*g^6)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(f+gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{(f+g\,x)^2\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x)
[Out] int((d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)), x)
```

3.587 $\int \frac{(d+ex)^3}{(f+gx)^3 \left(d^2-e^2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=398

$$\frac{4de^2(d+ex)}{5(ef+dg)^3 \left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4 \left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d^3g^2+e(2e^2f^2+19defg+107d^2g^2)\,x}{15d^3(ef+dg)^5 \sqrt{d^2-e^2x^2}}$$

```
[Out] 4/5*d*e^2*(e*x+d)/(d*g+e*f)^3/(-e^2*x^2+d^2)^(5/2)-1/15*e^2*(5*d*(-5*d*g+e*
f)-e*(31*d*g+e*f)*x)/d/(d*g+e*f)^4/(-e^2*x^2+d^2)^(3/2)+1/2*e^2*g^3*(13*d^2
*g^2-30*d*e*f*g+20*e^2*f^2)*arctan((e^2*f*x+d^2*g)/(-d^2*g^2+e^2*f^2)^(1/2)
/(-e^2*x^2+d^2)^(1/2))/(-d*g+e*f)^2/(d*g+e*f)^5/(-d^2*g^2+e^2*f^2)^(1/2)+1/
15*e^2*(90*d^3*g^2+e*(107*d^2*g^2+19*d*e*f*g+2*e^2*f^2)*x)/d^3/(d*g+e*f)^5/
(-e^2*x^2+d^2)^(1/2)+1/2*g^4*(-e^2*x^2+d^2)^(1/2)/(-d*g+e*f)/(d*g+e*f)^4/(g
*x+f)^2+3/2*e*g^4*(-2*d*g+3*e*f)*(-e^2*x^2+d^2)^(1/2)/(-d*g+e*f)^2/(d*g+e*f
)^5/(g*x+f)
```

Rubi [A]
time = 2.84, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1661, 1665, 821, 739, 210}

$$\frac{e^2g^3(13d^2g^2-30defg+20e^2f^2)\,\text{ArcTan}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}} + \frac{3eg^4\sqrt{d^2-e^2x^2}\,(3ef-2dg)}{2(f+gx)(ef-dg)^2(dg+ef)^5} + \frac{g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4} - \frac{e^2(5d(ef-5dg)-ex(31dg+ef))}{15d\left(d^2-e^2x^2\right)^{3/2}(dg+ef)^4} + \frac{4de^2(d+ex)}{5\left(d^2-e^2x^2\right)^{5/2}(dg+ef)^3} + \frac{e^2(90d^3g^2+ex(107d^2g^2+19defg+2e^2f^2))}{15d^3\sqrt{d^2-e^2x^2}\,(dg+ef)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (4*d*e^2*(d + e*x))/(5*(e*f + d*g)^3*(d^2 - e^2*x^2)^(5/2)) - (e^2*(5*d*(e*
f - 5*d*g) - e*(e*f + 31*d*g)*x))/(15*d*(e*f + d*g)^4*(d^2 - e^2*x^2)^(3/2)
) + (e^2*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x))/(15*d^3
*(e*f + d*g)^5*Sqrt[d^2 - e^2*x^2]) + (g^4*Sqrt[d^2 - e^2*x^2])/(2*(e*f - d
*g)*(e*f + d*g)^4*(f + g*x)^2) + (3*e*g^4*(3*e*f - 2*d*g)*Sqrt[d^2 - e^2*x^
2])/(2*(e*f - d*g)^2*(e*f + d*g)^5*(f + g*x)) + (e^2*g^3*(20*e^2*f^2 - 30*d
*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(Sqrt[e^2*f^2 - d^2*g^2]*Sqrt
[d^2 - e^2*x^2])])/(2*(e*f - d*g)^2*(e*f + d*g)^5*Sqrt[e^2*f^2 - d^2*g^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(f+gx)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} + \frac{\int \frac{\frac{d^3e^2\left(e^3f^3+15de^2f^2g+15d^2efg^2+5d^3g^3\right)}{(ef+dg)^3} - \frac{d^2e^3\left(5e^3f^3-33de^2 \cdots\right.}{(e \cdots}}{(f+gx)^3(d \cdots} }{5d \cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{\int \frac{d^3e^4\left(2 \cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d^3 \cdots}{\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d^3 \cdots}{\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d^3 \cdots}{\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d^3 \cdots}{\cdots}$$

$$= \frac{4de^2(d+ex)}{5(ef+dg)^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{e^2(5d(ef-5dg)-e(ef+31dg)x)}{15d(ef+dg)^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{e^2(90d^3 \cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.96, size = 387, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{6e^2(ef+dg)^2}{d(d-ex)^3}+\frac{2e^2(ef+dg)(2ef+17dg)}{d^2(d-ex)^2}+\frac{2e^2\left(2e^2f^2+19defg+107d^2g^2\right)}{d^3(d-ex)}+\frac{15g^4(ef+dg)}{(ef-dg)(f+gx)^2}+\frac{45eg^4(3ef-2dg)}{(ef-dg)^2(f+gx)}\right)-\frac{15ie^2g^3\left(20e^2f^2-30defg+13d^2g^2\right)\log\left(\frac{4(ef-dg)^2(ef+dg)^5\left(id^2g+ie^2fx+\sqrt{e^2f^2-d^2g^2}\sqrt{d^2-e^2x^2}\right)}{e^2g^2\sqrt{e^2f^2-d^2g^2}\left(20e^2f^2-30defg+13d^2g^2\right)(f+gx)}\right)}{(ef-dg)^2\sqrt{e^2f^2-d^2g^2}}}{30(ef+dg)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*((6*e^2*(e*f + d*g)^2)/(d*(d - e*x)^3) + (2*e^2*(e*f +
 d*g)*(2*e*f + 17*d*g))/(d^2*(d - e*x)^2) + (2*e^2*(2*e^2*f^2 + 19*d*e*f*g
+ 107*d^2*g^2))/(d^3*(d - e*x)) + (15*g^4*(e*f + d*g))/((e*f - d*g)*(f + g*
x)^2) + (45*e*g^4*(3*e*f - 2*d*g))/((e*f - d*g)^2*(f + g*x))) - ((15*I)*e^2
*g^3*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*Log[(4*(e*f - d*g)^2*(e*f + d*g
)^5*(I*d^2*g + I*e^2*f*x + Sqrt[e^2*f^2 - d^2*g^2]*Sqrt[d^2 - e^2*x^2]))/(e
```

```
^2*g^2*Sqrt[e^2*f^2 - d^2*g^2]*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*(f +
g*x))])/((e*f - d*g)^2*Sqrt[e^2*f^2 - d^2*g^2]))/(30*(e*f + d*g)^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6395 vs. $2(370) = 740$.
time = 0.09, size = 6396, normalized size = 16.07

method	result	size
default	Expression too large to display	6396

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(373) = 746$.
time = 27.97, size = 5268, normalized size = 13.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*d^11*g^10*x^2 + 30*d^11*f*g^9*x + 15*d^11*f^2*g^8 + 15*sqrt(d^2*
g^2 - f^2*e^2)*(20*(d^3*f^4*g^5*x^5 + 2*d^3*f^5*g^4*x^4 + d^3*f^6*g^3*x^3)*
e^7 - 30*(d^4*f^3*g^6*x^5 + 4*d^4*f^4*g^5*x^4 + 5*d^4*f^5*g^4*x^3 + 2*d^4*f
^6*g^3*x^2)*e^6 + (13*d^5*f^2*g^7*x^5 + 116*d^5*f^3*g^6*x^4 + 253*d^5*f^4*g
^5*x^3 + 210*d^5*f^5*g^4*x^2 + 60*d^5*f^6*g^3*x)*e^5 - (39*d^6*f^2*g^7*x^4
+ 168*d^6*f^3*g^6*x^3 + 239*d^6*f^4*g^5*x^2 + 130*d^6*f^5*g^4*x + 20*d^6*f^
6*g^3)*e^4 + 3*(13*d^7*f^2*g^7*x^3 + 36*d^7*f^3*g^6*x^2 + 33*d^7*f^4*g^5*x
+ 10*d^7*f^5*g^4)*e^3 - 13*(d^8*f^2*g^7*x^2 + 2*d^8*f^3*g^6*x + d^8*f^4*g^5
)*e^2)*log((d^3*g^2 + d*f*g*x*e^2 - sqrt(d^2*g^2 - f^2*e^2)*(d^2*g + f*x*e^
```

2 + sqrt(-x^2*e^2 + d^2)*d*g) + (d^2*g^2 - f^2*e^2)*sqrt(-x^2*e^2 + d^2))/(
g*x + f)) - 14*(f^8*g^2*x^5 + 2*f^9*g*x^4 + f^10*x^3)*e^11 - 6*(10*d*f^7*g^
3*x^5 + 13*d*f^8*g^2*x^4 - 4*d*f^9*g*x^3 - 7*d*f^10*x^2)*e^10 - 6*(13*d^2*f
^6*g^4*x^5 - 4*d^2*f^7*g^3*x^4 - 40*d^2*f^8*g^2*x^3 - 16*d^2*f^9*g*x^2 + 7*
d^2*f^10*x)*e^9 + 2*(240*d^3*f^5*g^5*x^5 + 597*d^3*f^6*g^4*x^4 + 384*d^3*f^
7*g^3*x^3 - 56*d^3*f^8*g^2*x^2 - 76*d^3*f^9*g*x + 7*d^3*f^10)*e^8 - 6*(52*d
^4*f^4*g^6*x^5 + 344*d^4*f^5*g^5*x^4 + 571*d^4*f^6*g^4*x^3 + 308*d^4*f^7*g^
3*x^2 + 19*d^4*f^8*g^2*x - 10*d^4*f^9*g)*e^7 - 6*(55*d^5*f^3*g^7*x^5 - 46*d
^5*f^4*g^6*x^4 - 497*d^5*f^5*g^5*x^3 - 649*d^5*f^6*g^4*x^2 - 266*d^5*f^7*g^
3*x - 13*d^5*f^8*g^2)*e^6 + (419*d^6*f^2*g^8*x^5 + 1828*d^6*f^3*g^7*x^4 + 1
463*d^6*f^4*g^6*x^3 - 1362*d^6*f^5*g^5*x^2 - 1896*d^6*f^6*g^4*x - 480*d^6*f
^7*g^3)*e^5 - 3*(30*d^7*f*g^9*x^5 + 479*d^7*f^2*g^8*x^4 + 1198*d^7*f^3*g^7*
x^3 + 975*d^7*f^4*g^6*x^2 + 122*d^7*f^5*g^5*x - 104*d^7*f^6*g^4)*e^4 - 3*(5
*d^8*g^10*x^5 - 80*d^8*f*g^9*x^4 - 594*d^8*f^2*g^8*x^3 - 1038*d^8*f^3*g^7*x
^2 - 639*d^8*f^4*g^6*x - 110*d^8*f^5*g^5)*e^3 + (45*d^9*g^10*x^4 - 180*d^9*
f*g^9*x^3 - 914*d^9*f^2*g^8*x^2 - 1108*d^9*f^3*g^7*x - 419*d^9*f^4*g^6)*e^2
 - 45*(d^10*g^10*x^3 - 3*d^10*f^2*g^8*x - 2*d^10*f^3*g^7)*e + (15*d^10*f^2*
g^8 + 4*(f^8*g^2*x^4 + 2*f^9*g*x^3 + f^10*x^2)*e^10 + 6*(5*d*f^7*g^3*x^4 +
8*d*f^8*g^2*x^3 + d*f^9*g*x^2 - 2*d*f^10*x)*e^9 + 2*(69*d^2*f^6*g^4*x^4 + 9
3*d^2*f^7*g^3*x^3 - 14*d^2*f^8*g^2*x^2 - 31*d^2*f^9*g*x + 7*d^2*f^10)*e^8 -
 3*(185*d^3*f^5*g^5*x^4 + 408*d^3*f^6*g^4*x^3 + 276*d^3*f^7*g^3*x^2 + 38*d^
3*f^8*g^2*x - 20*d^3*f^9*g)*e^7 + 3*(54*d^4*f^4*g^6*x^4 + 513*d^4*f^5*g^5*x
^3 + 800*d^4*f^6*g^4*x^2 + 352*d^4*f^7*g^3*x + 26*d^4*f^8*g^2)*e^6 + 3*(175
*d^5*f^3*g^7*x^4 + 153*d^5*f^4*g^6*x^3 - 399*d^5*f^5*g^5*x^2 - 542*d^5*f^6*
g^4*x - 160*d^5*f^7*g^3)*e^5 - (304*d^6*f^2*g^8*x^4 + 1733*d^6*f^3*g^7*x^3
+ 1897*d^6*f^4*g^6*x^2 + 81*d^6*f^5*g^5*x - 312*d^6*f^6*g^4)*e^4 + 3*(239*d
^7*f^2*g^8*x^3 + 673*d^7*f^3*g^7*x^2 + 569*d^7*f^4*g^6*x + 110*d^7*f^5*g^5)
*e^3 - (479*d^8*f^2*g^8*x^2 + 913*d^8*f^3*g^7*x + 419*d^8*f^4*g^6)*e^2 + 45
*(d^9*f^2*g^8*x + 2*d^9*f^3*g^7)*e)*sqrt(-x^2*e^2 + d^2))/(d^15*f^2*g^11*x^
2 + 2*d^15*f^3*g^10*x + d^15*f^4*g^9 + (d^3*f^11*g^2*x^5 + 2*d^3*f^12*g*x^4
 + d^3*f^13*x^3)*e^12 + 3*(d^4*f^10*g^3*x^5 + d^4*f^11*g^2*x^4 - d^4*f^12*g
*x^3 - d^4*f^13*x^2)*e^11 - 3*(3*d^5*f^10*g^3*x^4 + 5*d^5*f^11*g^2*x^3 + d^
5*f^12*g*x^2 - d^5*f^13*x)*e^10 - (8*d^6*f^8*g^5*x^5 + 16*d^6*f^9*g^4*x^4 -
 d^6*f^10*g^3*x^3 - 17*d^6*f^11*g^2*x^2 - 7*d^6*f^12*g*x + d^6*f^13)*e^9 -
3*(2*d^7*f^7*g^6*x^5 - 4*d^7*f^8*g^5*x^4 - 14*d^7*f^9*g^4*x^3 - 7*d^7*f^10*
g^3*x^2 + 2*d^7*f^11*g^2*x + d^7*f^12*g)*e^8 + 6*(d^8*f^6*g^7*x^5 + 5*d^8*f
^7*g^6*x^4 + 3*d^8*f^8*g^5*x^3 - 5*d^8*f^9*g^4*x^2 - 4*d^8*f^10*g^3*x)*e^7
+ 2*(4*d^9*f^5*g^8*x^5 - d^9*f^6*g^7*x^4 - 23*d^9*f^7*g^6*x^3 - 23*d^9*f^8*
g^5*x^2 - d^9*f^9*g^4*x + 4*d^9*f^10*g^3)*e^6 - 6*(4*d^10*f^5*g^8*x^4 + 5*d
^10*f^6*g^7*x^3 - 3*d^10*f^7*g^6*x^2 - 5*d^10*f^8*g^5*x - d^10*f^9*g^4)*e^5
 - 3*(d^11*f^3*g^10*x^5 + 2*d^11*f^4*g^9*x^4 - 7*d^11*f^5*g^8*x^3 - 14*d^11
*f^6*g^7*x^2 - 4*d^11*f^7*g^6*x + 2*d^11*f^8*g^5)*e^4 - (d^12*f^2*g^11*x^5
- 7*d^12*f^3*g^10*x^4 - 17*d^12*f^4*g^9*x^3 - d^12*f^5*g^8*x^2 + 16*d^12*f^
6*g^7*x + 8*d^12*f^7*g^6)*e^3 + 3*(d^13*f^2*g^11*x^4 - d^13*f^3*g^10*x^3 -
5*d^13*f^4*g^9*x^2 - 3*d^13*f^5*g^8*x)*e^2 - 3*(d^14*f^2*g^11*x^3 + d^14*f^

```
3*g^10*x^2 - d^14*f^4*g^9*x - d^14*f^5*g^8)*e), -1/30*(15*d^11*g^10*x^2 + 3
0*d^11*f*g^9*x + 15*d^11*f^2*g^8 + 30*sqrt(-d^2*g^2 + f^2*e^2)*(20*(d^3*f^4
*g^5*x^5 + 2*d^3*f^5*g^4*x^4 + d^3*f^6*g^3*x^3)*e^7 - 30*(d^4*f^3*g^6*x^5 +
 4*d^4*f^4*g^5*x^4 + 5*d^4*f^5*g^4*x^3 + 2*d^4*f^6*g^3*x^2)*e^6 + (13*d^5*f
^2*g^7*x^5 + 116*d^5*f^3*g^6*x^4 + 253*d^5*f^4*g^5*x^3 + 210*d^5*f^5*g^4*x^
2 + 60*d^5*f^6*g^3*x)*e^5 - (39*d^6*f^2*g^7*x^4 + 168*d^6*f^3*g^6*x^3 + 239
*d^6*f^4*g^5*x^2 + 130*d^6*f^5*g^4*x + 20*d^6*f^6*g^3)*e^4 + 3*(13*d^7*f^2*
g^7*x^3 + 36*d^7*f^3*g^6*x^2 + 33*d^7*f^4*g^5*x + 10*d^7*f^5*g^4)*e^3 - 13*
(d^8*f^2*g^7*x^2 + 2*d^8*f^3*g^6*x + d^8*f^4*g^5)*e^2)*arctan(sqrt(-d^2*g^2
 + f^2*e^2)*(d*g*x + d*f - sqrt(-x^2*e^2 + d^2)*f)/(d^2*g^2*x - f^2*x*e^2))
 - 14*(f^8*g^2*x^5 + 2*f^9*g*x^4 + f^10*x^3)*e^11 - 6*(10*d*f^7*g^3*x^5 + 1
3*d*f^8*g^2*x^4 - 4*d*f^9*g*x^3 - 7*d*f^10*x^2)*e^10 - 6*(13*d^2*f^6*g^4*x^
5 - 4*d^2*f^7*g^3*x^4 - 40*d^2*f^8*g^2*x^3 - 16...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(f+gx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**3/((-(-d + e*x)*(d + e*x))**(7/2)*(f + g*x)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1314 vs. 2(373) = 746.
time = 1.41, size = 1314, normalized size = 3.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -(13*d^2*g^5*e^2 - 30*d*f*g^4*e^3 + 20*f^2*g^3*e^4)*arctan((d*g + (d*e + sq
rt(-x^2*e^2 + d^2)*e)*f*e^(-1)/x)/sqrt(-d^2*g^2 + f^2*e^2))/((d^7*g^7 + 3*d
^6*f*g^6*e + d^5*f^2*g^5*e^2 - 5*d^4*f^3*g^4*e^3 - 5*d^3*f^4*g^3*e^4 + d^2*
f^5*g^2*e^5 + 3*d*f^6*g*e^6 + f^7*e^7)*sqrt(-d^2*g^2 + f^2*e^2)) + (2*(d*e
+ sqrt(-x^2*e^2 + d^2)*e)^2*d^5*g^8*e^(-4)/x^2 + 2*(d*e + sqrt(-x^2*e^2 + d
^2)*e)*d^4*f*g^7*e^(-1)/x + 12*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^4*f*g^7*e
^(-3)/x^2 + 2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^4*f*g^7*e^(-5)/x^3 + d^3*f
^2*g^6*e^2 - 19*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^3*f^2*g^6*e^(-2)/x^2 + 6
*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^3*f^2*g^6*e^(-4)/x^3 + 18*(d*e + sqrt(-
x^2*e^2 + d^2)*e)*d^3*f^2*g^6/x + 6*d^2*f^3*g^5*e^3 - 29*(d*e + sqrt(-x^2*e
^2 + d^2)*e)*d^2*f^3*g^5*e/x + 6*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d^2*f^3*g
```

```
^5*e^(-1)/x^2 - 11*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*f^3*g^5*e^(-3)/x^3
- 10*d*f^4*g^4*e^4 - 10*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*f^4*g^4/x^2)/((d
^7*f^2*g^7 + 3*d^6*f^3*g^6*e + d^5*f^4*g^5*e^2 - 5*d^4*f^5*g^4*e^3 - 5*d^3*
f^6*g^3*e^4 + d^2*f^7*g^2*e^5 + 3*d*f^8*g*e^6 + f^9*e^7)*(2*(d*e + sqrt(-x^
2*e^2 + d^2)*e)*d*g*e^(-2)/x + f*e + (d*e + sqrt(-x^2*e^2 + d^2)*e)^2*f*e^(
-3)/x^2)^2) + 2/15*(127*d^2*g^2*e^2 + 745*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*
d^2*g^2*e^(-2)/x^2 - 525*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d^2*g^2*e^(-4)/x^
3 + 150*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^2*g^2*e^(-6)/x^4 - 485*(d*e + sq
rt(-x^2*e^2 + d^2)*e)*d^2*g^2/x + 44*d*f*g*e^3 - 145*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)*d*f*g*e/x + 245*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*d*f*g*e^(-1)/x^2
- 195*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*d*f*g*e^(-3)/x^3 + 75*(d*e + sqrt(-x
^2*e^2 + d^2)*e)^4*d*f*g*e^(-5)/x^4 + 7*f^2*e^4 - 20*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)*f^2*e^2/x - 30*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*f^2*e^(-2)/x^3 + 1
5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*f^2*e^(-4)/x^4 + 40*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*f^2/x^2)/((d^8*g^5 + 5*d^7*f*g^4*e + 10*d^6*f^2*g^3*e^2 + 10*d
^5*f^3*g^2*e^3 + 5*d^4*f^4*g*e^4 + d^3*f^5*e^5)*((d*e + sqrt(-x^2*e^2 + d^2
)*e)*e^(-2)/x - 1)^5)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{(f+g\,x)^3\,(d^2-e^2\,x^2)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)),x)
```

```
[Out] int((d + e*x)^3/((f + g*x)^3*(d^2 - e^2*x^2)^(7/2)), x)
```

3.588 $\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=112

$$-\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}-\frac{2(cf^2+ag^2)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

```
[Out] -2*(a*g^2+c*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/g^(3/2)/(-d
*g+e*f)^(3/2)-2*(a*e^2+c*d^2)/e^2/(-d*g+e*f)/(e*x+d)^(1/2)+2*c*(e*x+d)^(1/2
)/e^2/g
```

Rubi [A]
time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {912, 1275, 211}

$$-\frac{2(ag^2+cf^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}-\frac{2(ae^2+cd^2)}{e^2\sqrt{d+ex}\,(ef-dg)}+\frac{2c\sqrt{d+ex}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*(c*d^2 + a*e^2))/(e^2*(e*f - d*g)*Sqrt[d + e*x]) + (2*c*Sqrt[d + e*x])/
(e^2*g) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]
])/(g^(3/2)*(e*f - d*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cd^2+ae^2}{e^2}-\frac{2cdx^2}{e^2}+\frac{cx^4}{e^2}}{x^2\left(\frac{ef-dg}{e}+\frac{gx^2}{e}\right)}\,dx,x,\sqrt{d+ex}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cd^2+ae^2}{e(ef-dg)x^2}-\frac{e(cf^2+ag^2)}{g(-ef+dg)(-ef+dg-gx^2)}\right)dx,x,\sqrt{d+ex}\right)}{e}$$

$$= -\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}+\frac{(2(cf^2+ag^2))\,\text{Subst}\left(\int \frac{1}{-ef+dg-gx^2}\,dx\right.}{g(ef-dg)}$$

$$= -\frac{2(cd^2+ae^2)}{e^2(ef-dg)\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^2g}-\frac{2(cf^2+ag^2)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

Mathematica [A]
time = 0.25, size = 118, normalized size = 1.05

$$-\frac{2(cd^2g+ae^2g-cef(d+ex)+cdg(d+ex))}{e^2g(ef-dg)\sqrt{d+ex}}-\frac{2(cf^2+ag^2)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*(c*d^2*g + a*e^2*g - c*e*f*(d + e*x) + c*d*g*(d + e*x)))/(e^2*g*(e*f -
d*g)*Sqrt[d + e*x]) - (2*(c*f^2 + a*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqr
t[e*f - d*g]])/(g^(3/2)*(e*f - d*g)^(3/2))
```

Maple [A]
time = 0.10, size = 114, normalized size = 1.02

method	result	size
derivativedivides	$\frac{\frac{2c\sqrt{ex+d}}{g}-\frac{2\left(-a\,e^2-c\,d^2\right)}{(dg-ef)\sqrt{ex+d}}-\frac{2e^2\left(a\,g^2+c\,f^2\right)\text{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)\,g}}\right)}{(dg-ef)g\sqrt{(dg-ef)\,g}}}{e^2}$	114

default	$\frac{\frac{2c\sqrt{ex+d}}{g}-\frac{2\left(-a\,e^{2}-c\,d^{2}\right)}{(dg-ef)\sqrt{ex+d}}-\frac{2e^{2}\left(a\,g^{2}+c\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)\,g}}\right)}{(dg-ef)g\sqrt{(dg-ef)\,g}}}{e^{2}}$	114
risch	$\frac{2c\sqrt{ex+d}}{e^{2}g}-\frac{2\left(-\frac{\left(a\,e^{2}+c\,d^{2}\right)g}{(dg-ef)\sqrt{ex+d}}+\frac{e^{2}\left(a\,g^{2}+c\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)\,g}}\right)}{(dg-ef)\sqrt{(dg-ef)\,g}}\right)}{g\,e^{2}}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(c/g*(e*x+d)^(1/2)-(-a*e^2-c*d^2)/(d*g-e*f)/(e*x+d)^(1/2)-e^2*(a*g^2+
c*f^2)/(d*g-e*f)/g/((d*g-e*f)*g)^(1/2)*arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g
)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(101) = 202$.
time = 3.34, size = 493, normalized size = 4.40

$$\left[-\frac{\sqrt{dg^{2}-fge}\,((cf^{2}+ag^{2})xe^{3}+(cdf^{2}+adg^{2})e^{2})\log\left(\frac{2\,dg+(gx-f)e+2\,\sqrt{dg^{2}-fge}\,\sqrt{xe+d}}{gx+f}\right)-2\,(2\,cd^{3}g^{3}+(cf^{2}gx-afg^{2})e^{3}-(2\,cdfg^{2}x-cdf^{2}g-adg^{3})e^{2}+(cd^{2}g^{3}x-3\,cd^{2}fg^{2})e)\sqrt{xe+d}}{d^{3}g^{4}e^{2}+f^{2}g^{2}xe^{5}-(2\,dfg^{3}x-df^{2}g^{2})e^{4}+(d^{2}g^{4}x-2\,d^{2}fg^{3})e^{3}},\ \frac{2\left(\sqrt{-dg^{2}+fge}\,((cf^{2}+ag^{2})xe^{3}+(cdf^{2}+adg^{2})e^{2})\arctan\left(\frac{\sqrt{-dg^{2}+fge}\,\sqrt{xe+d}}{gxe+dg}\right)+(2\,cd^{3}g^{3}+(cf^{2}gx-afg^{2})e^{3}-(2\,cdfg^{2}x-cdf^{2}g-adg^{3})e^{2}+(cd^{2}g^{3}x-3\,cd^{2}fg^{2})e)\sqrt{xe+d}\right)}{d^{3}g^{4}e^{2}+f^{2}g^{2}xe^{5}-(2\,dfg^{3}x-df^{2}g^{2})e^{4}+(d^{2}g^{4}x-2\,d^{2}fg^{3})e^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="fricas")
```

```
[Out] [-(sqrt(d*g^2 - f*g*e)*((c*f^2 + a*g^2)*x*e^3 + (c*d*f^2 + a*d*g^2)*e^2)*lo
g((2*d*g + (g*x - f)*e + 2*sqrt(d*g^2 - f*g*e)*sqrt(x*e + d))/(g*x + f)) -
2*(2*c*d^3*g^3 + (c*f^2*g*x - a*f*g^2)*e^3 - (2*c*d*f*g^2*x - c*d*f^2*g - a
*d*g^3)*e^2 + (c*d^2*g^3*x - 3*c*d^2*f*g^2)*e)*sqrt(x*e + d))/(d^3*g^4*e^2
+ f^2*g^2*x*e^5 - (2*d*f*g^3*x - d*f^2*g^2)*e^4 + (d^2*g^4*x - 2*d^2*f*g^3)
*e^3), 2*(sqrt(-d*g^2 + f*g*e)*((c*f^2 + a*g^2)*x*e^3 + (c*d*f^2 + a*d*g^2)
```

```
*e^2)*arctan(sqrt(-d*g^2 + f*g*e)*sqrt(x*e + d)/(g*x*e + d*g)) + (2*c*d^3*g
^3 + (c*f^2*g*x - a*f*g^2)*e^3 - (2*c*d*f*g^2*x - c*d*f^2*g - a*d*g^3)*e^2
+ (c*d^2*g^3*x - 3*c*d^2*f*g^2)*e)*sqrt(x*e + d))/(d^3*g^4*e^2 + f^2*g^2*x*
e^5 - (2*d*f*g^3*x - d*f^2*g^2)*e^4 + (d^2*g^4*x - 2*d^2*f*g^3)*e^3)]
```

Sympy [A]
time = 16.96, size = 107, normalized size = 0.96

$$\frac{2c\sqrt{d+ex}}{e^2g}+\frac{2(ag^2+cf^2)\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{g^2\sqrt{-\frac{dg-ef}{g}}\,(dg-ef)}+\frac{2(ae^2+cd^2)}{e^2\sqrt{d+ex}\,(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(3/2)/(g*x+f),x)
```

```
[Out] 2*c*sqrt(d + e*x)/(e**2*g) + 2*(a*g**2 + c*f**2)*atan(sqrt(d + e*x)/sqrt(-(
d*g - e*f)/g))/(g**2*sqrt(-(d*g - e*f)/g)*(d*g - e*f)) + 2*(a*e**2 + c*d**2
)/(e**2*sqrt(d + e*x)*(d*g - e*f))
```

Giac [A]
time = 0.99, size = 116, normalized size = 1.04

$$\frac{2\sqrt{xe+d}\,ce^{(-2)}}{g}+\frac{2\left(cf^2+ag^2\right)\arctan\left(\frac{\sqrt{xe+d}\,g}{\sqrt{-dg^2+fge}}\right)}{(dg^2-fge)\sqrt{-dg^2+fge}}+\frac{2\left(cd^2+ae^2\right)}{(dge^2-fe^3)\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] 2*sqrt(x*e + d)*c*e^(-2)/g + 2*(c*f^2 + a*g^2)*arctan(sqrt(x*e + d)*g/sqrt(
-d*g^2 + f*g*e))/((d*g^2 - f*g*e)*sqrt(-d*g^2 + f*g*e)) + 2*(c*d^2 + a*e^2)
/((d*g*e^2 - f*e^3)*sqrt(x*e + d))
```

Mupad [B]
time = 0.23, size = 124, normalized size = 1.11

$$\frac{2\,c\,\sqrt{d+e\,x}}{e^2\,g}+\frac{2\,\left(c\,g\,d^2+a\,g\,e^2\right)}{e^2\,g\,\left(d\,g-e\,f\right)\,\sqrt{d+e\,x}}+\frac{\mathrm{atan}\left(\frac{d\,g^{3/2}\,\sqrt{d+e\,x}\,1{}\mathrm{i}-e\,f\,\sqrt{g}\,\sqrt{d+e\,x}\,1{}\mathrm{i}}{{\left(d\,g-e\,f\right)}^{3/2}}\right)\,\left(c\,f^2+a\,g^2\right)\,2{}\mathrm{i}}{g^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)*(d + e*x)^(3/2)),x)
```

```
[Out] (atan((d*g^(3/2)*(d + e*x)^(1/2)*1i - e*f*g^(1/2)*(d + e*x)^(1/2)*1i)/(d*g
- e*f)^(3/2))*(a*g^2 + c*f^2)*2i)/(g^(3/2)*(d*g - e*f)^(3/2)) + (2*c*(d + e
*x)^(1/2))/(e^2*g) + (2*(a*e^2*g + c*d^2*g))/(e^2*g*(d*g - e*f)*(d + e*x)^(
1/2))
```

3.589 $\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=240

$$-\frac{2(ef-dg)^3\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)(f+gx)^{3/2}}{3g^6}-\frac{2(ef-dg)\left(3ae^2g^2\right.}{}$$

```
[Out] 2/3*(-d*g+e*f)^2*(3*a*e*g^2+c*f*(-2*d*g+5*e*f))*(g*x+f)^(3/2)/g^6-2/5*(-d*g
+e*f)*(3*a*e^2*g^2+c*(d^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6+2/7*
e*(a*e^2*g^2+c*(3*d^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*c*e
^2*(-3*d*g+5*e*f)*(g*x+f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6-2*(-d*g+e
*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^6
```

Rubi [A]
time = 0.23, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {912, 1167}

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c(d^2g^2-8defg+10e^2f^2)\right)}{5g^6}-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^3}{g^6}+\frac{2(f+gx)^{3/2}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{3g^6}-\frac{2ce^2(f+gx)^{9/2}(5ef-3dg)}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3*
a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^6) - (2*(e*f - d*g)*(3
*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6)
 + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/
2))/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c*e^3*
(f + g*x)^(11/2))/(11*g^6)
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3 \left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(-ef+dg)^3\,(cf^2+ag^2)}{g^5}+\frac{(ef-dg)^2\,(3aeg^2+cf(5ef-2dg))x^2}{g^5}+\frac{(ef-dg)(-3ae^2g^2-c(10\ldots}{g\ldots}\right.\right.}{}$$

$$= -\frac{2(ef-dg)^3\,(cf^2+ag^2)\,\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2\,(3aeg^2+cf(5ef-2dg))\,(\ldots}{3g^6}$$

Mathematica [A]

time = 0.19, size = 282, normalized size = 1.18

$$\frac{2\sqrt{f+gx}\,(99ag^2(35d^3g^3+35d^2eg^2(-2f+gx)+7de^2g(8f^2-4fgx+3g^2x^2)+e^3(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3))+c(231d^3g^3(8f^2-4fgx+3g^2x^2)+297d^2eg^2(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3)+33de^2g(128f^4-64f^3gx+48f^2g^2x^2-40fg^3x^3+35g^4x^4)-5e^3(256f^5-128f^4gx+96f^3g^2x^2-80f^2g^3x^3+70fg^4x^4-63g^5x^5)))}{3465g^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(99*a*g^2*(35*d^3*g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e^
2*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2
+ 5*g^3*x^3)) + c*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 297*d^2*e*g^
2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 33*d*e^2*g*(128*f^4 - 6
4*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) - 5*e^3*(256*f^5 -
128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5*x^5))
))/(3465*g^6)
```

Maple [A]

time = 0.08, size = 243, normalized size = 1.01

method	result
derivativedivides	$\frac{2c\,e^3(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^2c-2fc\,e^3\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^3c-6\ldots\right.}{}$
default	$\frac{2c\,e^3(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^2c-2fc\,e^3\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^3c-6\ldots\right.}{}$
gosper	$\frac{2\sqrt{gx+f}\,(315c\,e^3x^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400c\,e^3f\ldots}{}$
trager	$\frac{2\sqrt{gx+f}\,(315c\,e^3x^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400c\,e^3f\ldots}{}$
risch	$\frac{2\sqrt{gx+f}\,(315c\,e^3x^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400c\,e^3f\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^6*(1/11*c*e^3*(g*x+f)^(11/2)+1/9*(3*(d*g-e*f)*e^2*c-2*f*c*e^3)*(g*x+f)^
(9/2)+1/7*(3*(d*g-e*f)^2*e*c-6*(d*g-e*f)*e^2*c*f+e^3*(a*g^2+c*f^2))*(g*x+f)
^(7/2)+1/5*((d*g-e*f)^3*c-6*(d*g-e*f)^2*e*c*f+3*(d*g-e*f)*e^2*(a*g^2+c*f^2)
)*(g*x+f)^(5/2)+1/3*(-2*(d*g-e*f)^3*c*f+3*(d*g-e*f)^2*e*(a*g^2+c*f^2))*(g*x
+f)^(3/2)+(d*g-e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2))
```

Maxima [A]
time = 0.30, size = 314, normalized size = 1.31

$$\frac{2\left(315\,(gx+f)^{\frac{11}{2}}ce^3+385\,(3\,cdge^2-5\,cfe^3)(gx+f)^{\frac{9}{2}}-495\,(12\,cdfge^2-10\,cf^2e^3-(3\,cd^2e+ae^3)g^2)(gx+f)^{\frac{7}{2}}+693\,(18\,cdf^2ge^2-10\,cf^3e^3-3\,(3\,cd^2e+ae^3)fg^2+(cd^3+3\,ade^2)g^3)(gx+f)^{\frac{5}{2}}+1155\,(3\,ad^2g^4e-12\,cdf^3ge^2+5\,cf^4e^3+3\,(3\,cd^2e+ae^3)f^2g^2-2\,(cd^3+3\,ade^2)fg^3)(gx+f)^{\frac{3}{2}}+3465\,(ad^3g^5-3\,ad^2fg^4e+3\,cdf^4ge^2-cf^5e^3-(3\,cd^2e+ae^3)f^3g^2+(cd^3+3\,ade^2)f^2g^3)\sqrt{gx+f}\right)}{3465\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 + 385*(3*c*d*g*e^2 - 5*c*f*e^3)*(g*x + f
)^(9/2) - 495*(12*c*d*f*g*e^2 - 10*c*f^2*e^3 - (3*c*d^2*e + a*e^3)*g^2)*(g*
x + f)^(7/2) + 693*(18*c*d*f^2*g*e^2 - 10*c*f^3*e^3 - 3*(3*c*d^2*e + a*e^3)
*f*g^2 + (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(3*a*d^2*g^4*e - 1
2*c*d*f^3*g*e^2 + 5*c*f^4*e^3 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*a*d*e^2)*f*g^3)*(g*x + f)^(3/2) + 3465*(a*d^3*g^5 - 3*a*d^2*f*g^4*e + 3*c
*d*f^4*g*e^2 - c*f^5*e^3 - (3*c*d^2*e + a*e^3)*f^3*g^2 + (c*d^3 + 3*a*d*e^2
)*f^2*g^3)*sqrt(g*x + f))/g^6
```

Fricas [A]
time = 2.63, size = 323, normalized size = 1.35

$$\frac{2\,(693\,cd^3g^5x^2-924\,cd^3fg^4x+1848\,cd^3f^2g^3+3465\,ad^3g^5+(315\,cg^5x^5-350\,cfg^4x^4-1280\,cf^5-1584\,af^3g^2+5\,(80\,cf^2g^3+99\,ag^5)x^3-6\,(80\,cf^3g^2+99\,afg^4)x^2+8\,(80\,cf^4g+99\,af^2g^3)x)e^3+33\,(35\,cdg^5x^4-40\,cdfg^4x^3+128\,cdf^4g+168\,adf^2g^3+3\,(16\,cdf^2g^3+21\,adg^5)x^2-4\,(16\,cdf^3g^2+21\,adfg^4)x)e^2+99\,(15\,cd^2g^5x^3-18\,cd^2fg^4x^2-48\,cd^2f^3g^2-70\,ad^2fg^4+(24\,cd^2f^2g^3+35\,ad^2g^5)x)e)\sqrt{gx+f}}{3465\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(693*c*d^3*g^5*x^2 - 924*c*d^3*f*g^4*x + 1848*c*d^3*f^2*g^3 + 3465*a
*d^3*g^5 + (315*c*g^5*x^5 - 350*c*f*g^4*x^4 - 1280*c*f^5 - 1584*a*f^3*g^2 +
 5*(80*c*f^2*g^3 + 99*a*g^5)*x^3 - 6*(80*c*f^3*g^2 + 99*a*f*g^4)*x^2 + 8*(8
0*c*f^4*g + 99*a*f^2*g^3)*x)*e^3 + 33*(35*c*d*g^5*x^4 - 40*c*d*f*g^4*x^3 +
128*c*d*f^4*g + 168*a*d*f^2*g^3 + 3*(16*c*d*f^2*g^3 + 21*a*d*g^5)*x^2 - 4*(
16*c*d*f^3*g^2 + 21*a*d*f*g^4)*x)*e^2 + 99*(15*c*d^2*g^5*x^3 - 18*c*d^2*f*g
^4*x^2 - 48*c*d^2*f^3*g^2 - 70*a*d^2*f*g^4 + (24*c*d^2*f^2*g^3 + 35*a*d^2*g
^5)*x)*e)*sqrt(g*x + f)/g^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. 2(241) = 482.
time = 50.13, size = 1040, normalized size = 4.33

$$\begin{cases} \frac{-\frac{2ad^3f}{\sqrt{f+gx}}-2ad^3\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)-\frac{6ad^2ef\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}-\frac{6ad^2e\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}-\frac{6ade^2f\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2}-\frac{6ade^2\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}-\frac{2ae^3f\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^3}-\frac{2ae^3\left(\frac{f^4}{\sqrt{f+gx}}+4f^3\sqrt{f+gx}-2f^2(f+gx)^{\frac{3}{2}}+\frac{4f(f+gx)^{\frac{5}{2}}}{5}-\frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^3}-\frac{2cd^3f\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2}-\frac{2cd^3\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}-\frac{6cd^2ef\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^3}-\frac{6cd^2e\left(\frac{f^4}{\sqrt{f+gx}}+4f^3\sqrt{f+gx}-2f^2(f+gx)^{\frac{3}{2}}+\frac{4f(f+gx)^{\frac{5}{2}}}{5}-\frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^3}-\frac{6cde^2f\left(\frac{f^4}{\sqrt{f+gx}}+4f^3\sqrt{f+gx}-2f^2(f+gx)^{\frac{3}{2}}+\frac{4f(f+gx)^{\frac{5}{2}}}{5}-\frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^4}-\frac{6cde^2\left(-\frac{f^5}{\sqrt{f+gx}}-5f^4\sqrt{f+gx}+\frac{10f^3(f+gx)^{\frac{3}{2}}}{3}-2f^2(f+gx)^{\frac{5}{2}}+\frac{5f(f+gx)^{\frac{7}{2}}}{7}-\frac{(f+gx)^{\frac{9}{2}}}{9}\right)}{g^4}-\frac{2ce^3f\left(-\frac{f^5}{\sqrt{f+gx}}-5f^4\sqrt{f+gx}+\frac{10f^3(f+gx)^{\frac{3}{2}}}{3}-2f^2(f+gx)^{\frac{5}{2}}+\frac{5f(f+gx)^{\frac{7}{2}}}{7}-\frac{(f+gx)^{\frac{9}{2}}}{9}\right)}{g^5}-\frac{2ce^3\left(\frac{f^6}{\sqrt{f+gx}}+6f^5\sqrt{f+gx}-5f^4(f+gx)^{\frac{3}{2}}+4f^3(f+gx)^{\frac{5}{2}}-\frac{15f^2(f+gx)^{\frac{7}{2}}}{7}+\frac{2f(f+gx)^{\frac{9}{2}}}{3}-\frac{(f+gx)^{\frac{11}{2}}}{11}\right)}{g^5}}{g} & \text{for}\: g \neq 0 \\ \frac{ad^3x+\frac{3ad^2ex^2}{2}+\frac{3cde^2x^5}{5}+\frac{ce^3x^6}{6}+\frac{x^4\left(ae^3+3cd^2e\right)}{4}+\frac{x^3\left(3ade^2+cd^3\right)}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d**3*f/sqrt(f + g*x) - 2*a*d**3*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 6*a*d**2*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 6*a*d**2*e*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 6*a*d*e**2
*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 6*a
*d*e**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
(f + g*x)**(5/2)/5)/g**2 - 2*a*e**3*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f
+ g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 2*a*e**3*(f**4/sqr
t(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)
**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*c*d**3*f*(f**2/sqrt(f + g*x) + 2*f
*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d**3*(-f**3/sqrt(f + g*x) -
 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 6*c
*d**2*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2)
- (f + g*x)**(5/2)/5)/g**3 - 6*c*d**2*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f
 + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/
2)/7)/g**3 - 6*c*d*e**2*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f*
*2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 6
*c*d*e**2*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**
(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/
2)/9)/g**4 - 2*c*e**3*f*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f*
*3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 -
(f + g*x)**(9/2)/9)/g**5 - 2*c*e**3*(f**6/sqrt(f + g*x) + 6*f**5*sqrt(f + g
*x) - 5*f**4*(f + g*x)**(3/2) + 4*f**3*(f + g*x)**(5/2) - 15*f**2*(f + g*x)
**(7/2)/7 + 2*f*(f + g*x)**(9/2)/3 - (f + g*x)**(11/2)/11)/g**5)/g, Ne(g, 0
)), ((a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x*
*4*(a*e**3 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + c*d**3)/3)/sqrt(f), True))
```

Giac [A]
time = 1.60, size = 378, normalized size = 1.58

$$\frac{2\left(3465\sqrt{gx+f}\,ad^3+\frac{3465\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ad^2e}{g}+\frac{231\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)cd^3}{g^2}+\frac{693\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)ade^2}{g^2}+\frac{297\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)cd^2e}{g^3}+\frac{99\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)ae^3}{g^3}+\frac{33\left(35(gx+f)^{\frac{9}{2}}-180(gx+f)^{\frac{7}{2}}f+378(gx+f)^{\frac{5}{2}}f^2-420(gx+f)^{\frac{3}{2}}f^3+315\sqrt{gx+f}\,f^4\right)cde^2}{g^4}+\frac{5\left(63(gx+f)^{\frac{11}{2}}-385(gx+f)^{\frac{9}{2}}f+990(gx+f)^{\frac{7}{2}}f^2-1386(gx+f)^{\frac{5}{2}}f^3+1155(gx+f)^{\frac{3}{2}}f^4-693\sqrt{gx+f}\,f^5\right)ce^3}{g^5}\right)}{3465\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*a*d^2*e/g + 231*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
```

```
+ f)*f^2)*c*d^3/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sq
rt(g*x + f)*f^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*
f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 99*(5*(g*x
 + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x +
 f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(
g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*d*e
^2/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/
2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x
 + f)*f^5)*c*e^3/g^5)/g
```

Mupad [B]
time = 0.12, size = 222, normalized size = 0.92

$$\frac{(f+gx)^{7/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2)}{7\,g^6}+\frac{2\,\sqrt{f+g\,x}\,\left(c\,f^2+a\,g^2\right)\,{\left(d\,g-e\,f\right)}^3}{g^6}+\frac{2\,c\,e^3\,{\left(f+g\,x\right)}^{11/2}}{11\,g^6}+\frac{2\,{\left(f+g\,x\right)}^{3/2}\,{\left(d\,g-e\,f\right)}^2\,\left(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2\right)}{3\,g^6}+\frac{2\,{\left(f+g\,x\right)}^{5/2}\,\left(d\,g-e\,f\right)\,\left(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{5\,g^6}+\frac{2\,c\,e^2\,{\left(f+g\,x\right)}^{9/2}\,\left(3\,d\,g-5\,e\,f\right)}{9\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f
*g))/(7*g^6) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^3)/g^6 + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6) + (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g
^2 + 5*c*e*f^2 - 2*c*d*f*g))/(3*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a*
e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g))/(5*g^6) + (2*c*e^2*(f +
g*x)^(9/2)*(3*d*g - 5*e*f))/(9*g^6)
```

3.590 $\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=175

$$\frac{2(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)(f+gx)^{3/2}}{3g^5}+\frac{2(ae^2g^2+c(6e^2f^2-6\ldots}{}$$

```
[Out] -4/3*(-d*g+e*f)*(a*e*g^2+c*f*(-d*g+2*e*f))*(g*x+f)^(3/2)/g^5+2/5*(a*e^2*g^2
+c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)^(5/2)/g^5-4/7*c*e*(-d*g+2*e*f)*(g
*x+f)^(7/2)/g^5+2/9*c*e^2*(g*x+f)^(9/2)/g^5+2*(-d*g+e*f)^2*(a*g^2+c*f^2)*(g
*x+f)^(1/2)/g^5
```

Rubi [A]
time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {912, 1167}

$$\frac{2(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{3g^5}-\frac{4ce(f+gx)^{7/2}(2ef-dg)}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*g
^2 + c*f*(2*e*f - d*g))*(f + g*x)^(3/2))/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e^2
*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (4*c*e*(2*e*f - d*g
)*(f + g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2 (a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2 \left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(-ef+dg)^2(cf^2+ag^2)}{g^4}+\frac{2(ef-dg)(-aeg^2-cf(2ef-dg))x^2}{g^4}+\frac{(ae^2g^2+c(6e^2f^2-6defg+c\dots}{g^4}\right.\right.}{g}$$

$$= \frac{2(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}}{g^5} - \frac{4(ef-dg)(aeg^2+cf(2ef-dg))(f+gx\dots}{3g^5}$$

Mathematica [A]
time = 0.12, size = 177, normalized size = 1.01

$$\frac{2\sqrt{f+gx}\,(21ag^2(15d^2g^2+10deg(-2f+gx)+e^2(8f^2-4fgx+3g^2x^2))+c(21d^2g^2(8f^2-4fgx+3g^2x^2)+18deg(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3)+e^2(128f^4-64f^3gx+48f^2g^2x^2-40fg^3x^3+35g^4x^4)))}{315g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(21*a*g^2*(15*d^2*g^2 + 10*d*e*g*(-2*f + g*x) + e^2*(8*f^2
 - 4*f*g*x + 3*g^2*x^2)) + c*(21*d^2*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 18
*d*e*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^2*(128*f^4 - 64*
f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4))))/(315*g^5)
```

Maple [A]
time = 0.07, size = 174, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{2c\,e^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec-2fce^2\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c-4(dg-ef)ecf+e^2\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)^2cf+2(dg\dots\right.}{}}{g^5}$
default	$\frac{\frac{2c\,e^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec-2fce^2\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c-4(dg-ef)ecf+e^2\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)^2cf+2(dg\dots\right.}{}}{g^5}$
gosper	$\frac{2\sqrt{gx+f}\,(35c\,e^2x^4g^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2g^2x^2+210ade\,g\dots}{}$
trager	$\frac{2\sqrt{gx+f}\,(35c\,e^2x^4g^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2g^2x^2+210ade\,g\dots}{}$
risch	$\frac{2\sqrt{gx+f}\,(35c\,e^2x^4g^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2g^2x^2+210ade\,g\dots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 2/g^5*(1/9*c*e^2*(g*x+f)^(9/2)+1/7*(2*(d*g-e*f)*e*c-2*f*c*e^2)*(g*x+f)^(7/2
)+1/5*((d*g-e*f)^2*c-4*(d*g-e*f)*e*c*f+e^2*(a*g^2+c*f^2))*(g*x+f)^(5/2)+1/3
(-2(d*g-e*f)^2*c*f+2*(d*g-e*f)*e*(a*g^2+c*f^2))*(g*x+f)^(3/2)+(d*g-e*f)^2
*(a*g^2+c*f^2)*(g*x+f)^(1/2))

Maxima [A]
time = 0.32, size = 195, normalized size = 1.11

$$\frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^2+90\,(cdge-2\,cfe^2)(gx+f)^{\frac{7}{2}}-63\,(6\,cdfge-6\,cf^2e^2-(cd^2+ae^2)g^2)(gx+f)^{\frac{5}{2}}+210\,(3\,cdf^2ge+adg^3e-2\,cf^3e^2-(cd^2+ae^2)fg^2)(gx+f)^{\frac{3}{2}}+315\,(ad^2g^4-2\,cdf^3ge-2\,adfg^3e+cf^4e^2+(cd^2+ae^2)f^2g^2)\sqrt{gx+f}\right)}{315\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")

[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 + 90*(c*d*g*e - 2*c*f*e^2)*(g*x + f)^(7/2)
- 63*(6*c*d*f*g*e - 6*c*f^2*e^2 - (c*d^2 + a*e^2)*g^2)*(g*x + f)^(5/2) + 21
0*(3*c*d*f^2*g*e + a*d*g^3*e - 2*c*f^3*e^2 - (c*d^2 + a*e^2)*f*g^2)*(g*x +
f)^(3/2) + 315*(a*d^2*g^4 - 2*c*d*f^3*g*e - 2*a*d*f*g^3*e + c*f^4*e^2 + (c*
d^2 + a*e^2)*f^2*g^2)*sqrt(g*x + f))/g^5

Fricas [A]
time = 3.30, size = 196, normalized size = 1.12

$$\frac{2\,(63\,cd^2g^4x^2-84\,cd^2fg^3x+168\,cd^2f^2g^2+315\,ad^2g^4+(35\,cg^4x^4-40\,cfg^3x^3+128\,cf^4+168\,af^2g^2+3\,(16\,cf^2g^2+21\,ag^4)x^2-4\,(16\,cf^3g+21\,afg^3)x)e^2+6\,(15\,cdg^4x^3-18\,cdfg^3x^2-48\,cdf^3g-70\,adfg^3+(24\,cdf^2g^2+35\,adg^4)x)e)\sqrt{gx+f}}{315\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")

[Out] 2/315*(63*c*d^2*g^4*x^2 - 84*c*d^2*f*g^3*x + 168*c*d^2*f^2*g^2 + 315*a*d^2*
g^4 + (35*c*g^4*x^4 - 40*c*f*g^3*x^3 + 128*c*f^4 + 168*a*f^2*g^2 + 3*(16*c*
f^2*g^2 + 21*a*g^4)*x^2 - 4*(16*c*f^3*g + 21*a*f*g^3)*x)*e^2 + 6*(15*c*d*g^
4*x^3 - 18*c*d*f*g^3*x^2 - 48*c*d*f^3*g - 70*a*d*f*g^3 + (24*c*d*f^2*g^2 +
35*a*d*g^4)*x)*e)*sqrt(g*x + f)/g^5

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(175) = 350$.
time = 31.54, size = 673, normalized size = 3.85

$$\begin{cases} \frac{-\frac{2ad^2f}{\sqrt{f+gx}}-2ad^2\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)-\frac{4adef\left(-\frac{f}{\sqrt{f+gx}}-\sqrt{f+gx}\right)}{g}-\frac{4ade\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g}-\frac{2ae^2f\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2}-\frac{2ae^2\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}-\frac{2cd^2f\left(\frac{f^2}{\sqrt{f+gx}}+2f\sqrt{f+gx}-\frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2}-\frac{2cd^2\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}-\frac{4cdef\left(-\frac{f^3}{\sqrt{f+gx}}-3f^2\sqrt{f+gx}+f(f+gx)^{\frac{3}{2}}-\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^3}-\frac{4cde\left(\frac{f^4}{\sqrt{f+gx}}+4f^3\sqrt{f+gx}-2f^2(f+gx)^{\frac{3}{2}}+\frac{4f(f+gx)^{\frac{5}{2}}}{5}-\frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^3}-\frac{2ce^2f\left(\frac{f^4}{\sqrt{f+gx}}+4f^3\sqrt{f+gx}-2f^2(f+gx)^{\frac{3}{2}}+\frac{4f(f+gx)^{\frac{5}{2}}}{5}-\frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^4}-\frac{2ce^2\left(-\frac{f^5}{\sqrt{f+gx}}-5f^4\sqrt{f+gx}+\frac{10f^3(f+gx)^{\frac{3}{2}}}{3}-2f^2(f+gx)^{\frac{5}{2}}+\frac{5f(f+gx)^{\frac{7}{2}}}{7}-\frac{(f+gx)^{\frac{9}{2}}}{9}\right)}{g^4}}{g} & \text{for}\: g \neq 0 \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3(ae^2+cd^2)}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(1/2),x)

[Out] Piecewise(((-2*a*d**2*f/sqrt(f + g*x) - 2*a*d**2*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 4*a*d*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 4*a*d*e*(f**2/s
qrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*a*e**2*f*(f**2
/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*a*e**2*(-

```
f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)*
*(5/2)/5)/g**2 - 2*c*d**2*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f +
g*x)**(3/2)/3)/g**2 - 2*c*d**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x)
+ f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 4*c*d*e*f*(-f**3/sqrt(f +
 g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**
3 - 4*c*d*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**
(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*c*e**2*f*(f**
4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f +
 g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*c*e**2*(-f**5/sqrt(f + g*x) -
 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2
) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4)/g, Ne(g, 0)), ((a*d*
*2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3
)/sqrt(f), True))
```

Giac [A]
time = 1.01, size = 243, normalized size = 1.39

$$\frac{2\left(315\,\sqrt{gx+f}\,ad^2+\frac{210\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}\,f\right)ade}{g}+\frac{21\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^2\right)cd^2}{g^2}+\frac{21\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^2\right)ae^2}{g^2}+\frac{18\left(5\,(gx+f)^{\frac{7}{2}}-21\,(gx+f)^{\frac{5}{2}}f+35\,(gx+f)^{\frac{3}{2}}f^2-35\,\sqrt{gx+f}\,f^3\right)cde}{g^3}+\frac{\left(35\,(gx+f)^{\frac{9}{2}}-180\,(gx+f)^{\frac{7}{2}}f+378\,(gx+f)^{\frac{5}{2}}f^2-420\,(gx+f)^{\frac{3}{2}}f^3+315\,\sqrt{gx+f}\,f^4\right)ce^2}{g^4}\right)}{315\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
a*d*e/g + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*c*d^2/g^2 + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*a*e^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*
x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + (35*(g*x + f)^(9/2) -
180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 +
 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

Mupad [B]
time = 2.58, size = 159, normalized size = 0.91

$$\frac{(f+g\,x)^{5/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2\right)}{5\,g^5}+\frac{2\,\sqrt{f+g\,x}\,\left(c\,f^2+a\,g^2\right)\,{\left(d\,g-e\,f\right)}^2}{g^5}+\frac{4\,{(f+g\,x)}^{3/2}\,\left(d\,g-e\,f\right)\,\left(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2\right)}{3\,g^5}+\frac{2\,c\,e^2\,{(f+g\,x)}^{9/2}}{9\,g^5}+\frac{4\,c\,e\,{(f+g\,x)}^{7/2}\,\left(d\,g-2\,e\,f\right)}{7\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g))
/(5*g^5) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2)/g^5 + (4*(f +
g*x)^(3/2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/(3*g^5) + (2*c*e^2*
(f + g*x)^(9/2))/(9*g^5) + (4*c*e*(f + g*x)^(7/2)*(d*g - 2*e*f))/(7*g^5)
```

3.591 $\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=113

$$-\frac{2(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{3/2}}{3g^4}-\frac{2c(3ef-dg)(f+gx)^{5/2}}{5g^4}+\frac{2ce$$

```
[Out] 2/3*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(3/2)/g^4-2/5*c*(-d*g+3*e*f)*(g*x+
f)^(5/2)/g^4+2/7*c*e*(g*x+f)^(7/2)/g^4-2*(-d*g+e*f)*(a*g^2+c*f^2)*(g*x+f)^(
1/2)/g^4
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {786}

$$-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}\left(aeg^2+cf(3ef-2dg)\right)}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*e
*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^(5/2))
/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7*g^4)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3\sqrt{f+gx}}+\frac{\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^3}+\frac{c(-3ef+\right.\\ &= -\frac{2(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{3/2}}{3g^4}-\end{aligned}$$

Mathematica [A]
time = 0.07, size = 94, normalized size = 0.83

$$\frac{2\sqrt{f+gx}\left(35ag^2(-2ef+3dg+egx)+7cdg\left(8f^2-4fgx+3g^2x^2\right)-3ce\left(16f^3-8f^2gx+6fg^2x^2-5g^3x^3\right)\right)}{105g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(35*a*g^2*(-2*e*f + 3*d*g + e*g*x) + 7*c*d*g*(8*f^2 - 4*f*
g*x + 3*g^2*x^2) - 3*c*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))/(
105*g^4)
```

Maple [A]
time = 0.06, size = 105, normalized size = 0.93

method	result
gosper	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+56cd\,f^2g-48c$
trager	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+56cd\,f^2g-48c$
risch	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+56cd\,f^2g-48c$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c-2cef)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)cf+e\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^2+c\,f^2\right)\sqrt{gx+f}}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c-2cef)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)cf+e\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^2+c\,f^2\right)\sqrt{gx+f}}{g^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^4*(1/7*c*e*(g*x+f)^(7/2)+1/5*((d*g-e*f)*c-2*c*e*f)*(g*x+f)^(5/2)+1/3*(-
2*(d*g-e*f)*c*f+e*(a*g^2+c*f^2))*(g*x+f)^(3/2)+(d*g-e*f)*(a*g^2+c*f^2)*(g*x
+f)^(1/2))
```

Maxima [A]
time = 0.29, size = 110, normalized size = 0.97

$$\frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce+21\,(cdg-3\,cfe)(gx+f)^{\frac{5}{2}}-35\,(2\,cdfg-3\,cf^2e-ag^2e)(gx+f)^{\frac{3}{2}}+105\,(cdf^2g+adg^3-cf^3e-afg^2e)\sqrt{gx+f}\right)}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e + 21*(c*d*g - 3*c*f*e)*(g*x + f)^(5/2) - 35*(
2*c*d*f*g - 3*c*f^2*e - a*g^2*e)*(g*x + f)^(3/2) + 105*(c*d*f^2*g + a*d*g^3
 - c*f^3*e - a*f*g^2*e)*sqrt(g*x + f))/g^4
```

Fricas [A]
time = 2.85, size = 99, normalized size = 0.88

$$\frac{2\,(21\,cdg^3x^2-28\,cdfg^2x+56\,cdf^2g+105\,adg^3+(15\,cg^3x^3-18\,cfg^2x^2-48\,cf^3-70\,afg^2+(24\,cf^2g+35\,ag^3)x)e)\sqrt{gx+f}}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(21*c*d*g^3*x^2 - 28*c*d*f*g^2*x + 56*c*d*f^2*g + 105*a*d*g^3 + (15*c
*g^3*x^3 - 18*c*f*g^2*x^2 - 48*c*f^3 - 70*a*f*g^2 + (24*c*f^2*g + 35*a*g^3)
*x)*e)*sqrt(g*x + f)/g^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(114) = 228$.
time = 16.63, size = 374, normalized size = 3.31

$$\begin{cases} \frac{-\frac{2adf}{\sqrt{f+gx}} - 2ad\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2aef\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2ae\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2cdf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2cd\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2} - \frac{2cef\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^3} - \frac{2ce\left(\frac{f^4}{\sqrt{f+gx}} + 4f^3\sqrt{f+gx} - 2f^2(f+gx)^{\frac{3}{2}} + \frac{4f(f+gx)^{\frac{5}{2}}}{5} - \frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^3}}{g} & \text{for}\: g \neq 0 \\ \frac{adx + \frac{aex^2}{2} + \frac{cdx^3}{3} + \frac{cex^4}{4}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d*f/sqrt(f + g*x) - 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x
)) - 2*a*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*a*e*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*c*d*f*(f**2/sqrt(f + g*x
) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d*(-f**3/sqrt(f + g*
x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 -
 2*c*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
 (f + g*x)**(5/2)/5)/g**3 - 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x
) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/
g**3)/g, Ne(g, 0)), ((a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/sqrt(f)
, True))
```

Giac [A]
time = 1.65, size = 134, normalized size = 1.19

$$\frac{2\left(105\sqrt{gx+f}\,ad + \frac{35\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}\,f\right)ae}{g} + \frac{7\left(3(gx+f)^{\frac{5}{2}} - 10(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}\,f^2\right)cd}{g^2} + \frac{3\left(5(gx+f)^{\frac{7}{2}} - 21(gx+f)^{\frac{5}{2}}f + 35(gx+f)^{\frac{3}{2}}f^2 - 35\sqrt{gx+f}\,f^3\right)ce}{g^3}\right)}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e
/g + 7*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*
d/g^2 + 3*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^
2 - 35*sqrt(g*x + f)*f^3)*c*e/g^3)/g
```

Mupad [B]
time = 0.07, size = 100, normalized size = 0.88

$$\frac{(f+g\,x)^{3/2}\,(6\,c\,e\,f^2 - 4\,c\,d\,f\,g + 2\,a\,e\,g^2)}{3\,g^4} + \frac{2\,c\,e\,(f+g\,x)^{7/2}}{7\,g^4} + \frac{2\,c\,(f+g\,x)^{5/2}\,(d\,g - 3\,e\,f)}{5\,g^4} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2 + a\,g^2)\,(d\,g - e\,f)}{g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/(3*g^4) + (2*c*e*(f +
 g*x)^(7/2))/(7*g^4) + (2*c*(f + g*x)^(5/2)*(d*g - 3*e*f))/(5*g^4) + (2*(f
+ g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f))/g^4
```

3.592 $\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=61

$$\frac{2(cf^2+ag^2)\sqrt{f+gx}}{g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

```
[Out] -4/3*c*f*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^(5/2)/g^3+2*(a*g^2+c*f^2)*(g*x+f)^
(1/2)/g^3
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {711}

$$\frac{2\sqrt{f+gx}\,(ag^2+cf^2)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^3 - (4*c*f*(f + g*x)^(3/2))/(3*g^3) + (
2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx &= \int \left(\frac{cf^2+ag^2}{g^2\sqrt{f+gx}}-\frac{2cf\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx\\ &= \frac{2(cf^2+ag^2)\sqrt{f+gx}}{g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.72

$$\frac{2\sqrt{f+gx}\,(15ag^2+c(8f^2-4fgx+3g^2x^2))}{15g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(15*a*g^2 + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)
```

Maple [A]
time = 0.07, size = 52, normalized size = 0.85

method	result	size
gosper	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
trager	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
risch	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
derivativedivides	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^{2}\sqrt{gx+f}+2c\,f^{2}\sqrt{gx+f}}{g^{3}}$	52
default	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^{2}\sqrt{gx+f}+2c\,f^{2}\sqrt{gx+f}}{g^{3}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^3*(1/5*c*(g*x+f)^(5/2)-2/3*c*f*(g*x+f)^(3/2)+a*g^2*(g*x+f)^(1/2)+c*f^2*
(g*x+f)^(1/2))
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.87

$$\frac{2\left(15\sqrt{gx+f}\,a+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}\,f^{2}\right)c}{g^{2}}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

Fricas [A]
time = 3.28, size = 40, normalized size = 0.66

$$\frac{2\left(3\,cg^{2}x^{2}-4\,cfgx+8\,cf^{2}+15\,ag^{2}\right)\sqrt{gx+f}}{15\,g^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*g^2*x^2 - 4*c*f*g*x + 8*c*f^2 + 15*a*g^2)*sqrt(g*x + f)/g^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(60) = 120.
time = 3.34, size = 150, normalized size = 2.46

$$\begin{cases} \frac{-\frac{2af}{\sqrt{f+gx}} - 2a\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2cf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2c\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g \neq 0 \\ \frac{ax + \frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*f/sqrt(f + g*x) - 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) -
 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 -
 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f
+ g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + c*x**3/3)/sqrt(f), True))
```

Giac [A]
time = 1.24, size = 53, normalized size = 0.87

$$\frac{2\left(15\sqrt{gx+f}\,a + \frac{\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*c/g^2)/g
```

Mupad [B]
time = 2.56, size = 44, normalized size = 0.72

$$\frac{2\sqrt{f+g\,x}\,\left(3\,c\,{\left(f+g\,x\right)}^2+15\,a\,g^2+15\,c\,f^2-10\,c\,f\,\left(f+g\,x\right)\right)}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(f + g*x)^(1/2),x)
```

```
[Out] (2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 - 10*c*f*(f + g*x
)))/(15*g^3)
```

3.593 $\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=104

$$-\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d
*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1/2)-2*c*(d*g+e*f)*(g*x+f)^(1/2)/e^2/g^2
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {912, 1167, 214}

$$-\frac{2(ae^2+cd^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{2c\sqrt{f+gx}\,(dg+ef)}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*c*(e*f + d*g)*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))/(3*e*g^2
) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e
^(5/2)*Sqrt[e*f - d*g])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\right)\text{Subst}\left(\int \frac{1}{d-\frac{ef}{g}+\frac{ex^2}{g}}\,d\right.}{g}$$

$$= -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+g}}{\sqrt{ef-dg}}\right.}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A]
time = 0.21, size = 92, normalized size = 0.88

$$\frac{2c\sqrt{f+gx}\,(-2ef-3dg+egx)}{3e^2g^2}+\frac{2(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}\sqrt{-ef+dg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x]*(-2*e*f - 3*d*g + e*g*x))/(3*e^2*g^2) + (2*(c*d^2 + a*e^
2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)*Sqrt[-(e*f)
 + d*g])
```

Maple [A]
time = 0.08, size = 96, normalized size = 0.92

method	result

derivativedivides	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{e^2\sqrt{(dg-ef)\,e}}}{g^2}$	9
default	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{e^2\sqrt{(dg-ef)\,e}}}{g^2}$	9
risch	$-\frac{2c(-egx+3dg+2ef)\sqrt{gx+f}}{3g^2e^2}+\frac{2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)a}{\sqrt{(dg-ef)\,e}}+\frac{2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)c\,d^2}{e^2\sqrt{(dg-ef)\,e}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^2*(-c/e^2*(-1/3*e*(g*x+f)^(3/2)+d*g*(g*x+f)^(1/2)+e*f*(g*x+f)^(1/2))+g^
2*(a*e^2+c*d^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e
)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [A]
time = 3.10, size = 292, normalized size = 2.81

$$\left[-\frac{3\left(cd^2g^2+ag^2e^2\right)\sqrt{-dge+fe^2}\log\left(-\frac{dg-(gx+2f)e+2\sqrt{-dge+fe^2}\sqrt{gx+f}}{xe+d}\right)+2\left(3cd^2g^2e+(cfgx-2cf^2)e^3-(cdg^2x+cdfg)e^2\right)\sqrt{gx+f}}{3\left(dg^3e^3-fg^2e^4\right)},-\frac{2\left(3\left(cd^2g^2+ag^2e^2\right)\sqrt{dge-fe^2}\arctan\left(-\frac{\sqrt{dge-fe^2}\sqrt{gx+f}}{dg-fe}\right)+\left(3cd^2g^2e+(cfgx-2cf^2)e^3-(cdg^2x+cdfg)e^2\right)\sqrt{gx+f}\right)}{3\left(dg^3e^3-fg^2e^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(c*d^2*g^2 + a*g^2*e^2)*sqrt(-d*g*e + f*e^2)*log(-(d*g - (g*x + 2*
f)*e + 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e + d)) + 2*(3*c*d^2*g^2*e
+ (c*f*g*x - 2*c*f^2)*e^3 - (c*d*g^2*x + c*d*f*g)*e^2)*sqrt(g*x + f))/(d*g^
```

```
3*e^3 - f*g^2*e^4), -2/3*(3*(c*d^2*g^2 + a*g^2*e^2)*sqrt(d*g*e - f*e^2)*arc
tan(-sqrt(d*g*e - f*e^2)*sqrt(g*x + f)/(d*g - f*e)) + (3*c*d^2*g^2*e + (c*f
*g*x - 2*c*f^2)*e^3 - (c*d*g^2*x + c*d*f*g)*e^2)*sqrt(g*x + f))/(d*g^3*e^3
- f*g^2*e^4)]
```

Sympy [A]
time = 10.03, size = 100, normalized size = 0.96

$$\frac{2c(f+gx)^{\frac{3}{2}}}{3eg^2}-\frac{2c\sqrt{f+gx}\,(dg+ef)}{e^2g^2}-\frac{2(ae^2+cd^2)\,\text{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{e^2\sqrt{\frac{e}{dg-ef}}\,(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*e*g**2) - 2*c*sqrt(f + g*x)*(d*g + e*f)/(e**2*g**2)
 - 2*(a*e**2 + c*d**2)*atan(1/(sqrt(e/(d*g - e*f))*sqrt(f + g*x)))/(e**2*sq
rt(e/(d*g - e*f))*(d*g - e*f))
```

Giac [A]
time = 1.21, size = 107, normalized size = 1.03

$$\frac{2\left(cd^2+ae^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)e^{(-2)}}{\sqrt{dge-fe^2}}-\frac{2\left(3\sqrt{gx+f}\,cdg^5e-(gx+f)^{\frac{3}{2}}cg^4e^2+3\sqrt{gx+f}\,cfg^4e^2\right)e^{(-3)}}{3\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))*e^(-2)/sqrt(d
*g*e - f*e^2) - 2/3*(3*sqrt(g*x + f)*c*d*g^5*e - (g*x + f)^(3/2)*c*g^4*e^2
+ 3*sqrt(g*x + f)*c*f*g^4*e^2)*e^(-3)/g^6
```

Mupad [B]
time = 0.11, size = 107, normalized size = 1.03

$$\frac{2\,\text{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}}-\sqrt{f+g\,x}\,\left(\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}+\frac{4\,c\,f}{e\,g^2}\right)+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

```
[Out] (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2))/(e^(5
/2)*(d*g - e*f)^(1/2)) - (f + g*x)^(1/2)*((2*c*(d*g^3 - e*f*g^2))/(e^2*g^4)
 + (4*c*f)/(e*g^2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

3.594 $\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=122

$$\frac{2c\sqrt{f+gx}}{e^2g}-\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)}+\frac{(ae^2g+cd(4ef-3dg))\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

```
[Out] (a*e^2*g+c*d*(-3*d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)
)/e^(5/2)/(-d*g+e*f)^(3/2)+2*c*(g*x+f)^(1/2)/e^2/g-(a+c*d^2/e^2)*(g*x+f)^(1
/2)/(-d*g+e*f)/(e*x+d)
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {912, 1171, 396, 214}

$$-\frac{\sqrt{f+gx}\,(ae^2+cd^2)}{e^2(d+ex)(ef-dg)}+\frac{(ae^2g+cd(4ef-3dg))\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((c*d^2 + a*e^2)*Sqrt[f + g*x])/(e^2*(e*f - d
*g)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f +
 g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
```

```
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{\text{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cf^2}{g^2}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx, x, \sqrt{f+gx}\right)}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} - \frac{\left(a+\frac{cd(4ef-3dg)}{e^2g}\right)\text{Subst}\left(\int \frac{1}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\right.}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{(ae^2g+cd(4ef-3dg))\tanh^{-1}\left(\frac{\sqrt{e}}{\sqrt{e}}\right.}{e^{5/2}(ef-dg)^{3/2}}$$

Mathematica [A]
time = 0.43, size = 133, normalized size = 1.09

$$\frac{\sqrt{f+gx}\,(-ae^2g+c(-3d^2g+2e^2fx+2de(f-gx)))}{e^2g(ef-dg)(d+ex)} + \frac{(ae^2g+cd(4ef-3dg))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}(-ef+dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(-(a*e^2*g) + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x))))/(
e^2*g*(e*f - d*g)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTan[(Sqr
t[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)*(-(e*f) + d*g)^(3/2))
```

Maple [A]
time = 0.09, size = 139, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(a e^2+c d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a e^2 g-3c d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^2}}{g}$
default	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(a e^2+c d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a e^2 g-3c d^2 g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^2}}{g}$
risch	$\frac{2c\sqrt{gx+f}}{e^2 g}+\frac{g\sqrt{gx+f}\,a}{(dg-ef)(egx+dg)}+\frac{g\sqrt{gx+f}\,c d^2}{e^2(dg-ef)(egx+dg)}+\frac{\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)ag}{(dg-ef)\sqrt{(dg-ef)e}}-\frac{3\arctan\left(\frac{e}{\sqrt{}}\right.}{e^2(dg-ef)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g*(c/e^2*(g*x+f)^(1/2)+g/e^2*(1/2*g*(a*e^2+c*d^2)/(d*g-e*f)*(g*x+f)^(1/2)
/(e*(g*x+f)+d*g-e*f)+1/2*(a*e^2*g-3*c*d^2*g+4*c*d*e*f)/(d*g-e*f)/((d*g-e*f)
*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(109) = 218$.

time = 2.49, size = 525, normalized size = 4.30

$$\left[-\frac{(3\,cd^3g^2 - ag^2xe^3 - (4\,cdfgx + adg^2)e^2 + (3\,cd^2g^2x - 4\,cd^2fg)e)\sqrt{-dge + fe^2}\log\left(-\frac{dg-(gx+2\,f)e-2\,\sqrt{-dge+fe^2}\,\sqrt{gx+f}}{xe+d}\right) - 2\,(3\,cd^3g^2e + (2\,cf^2x - afg)e^4 - (4\,cdfgx - 2\,cdf^2 - adg^2)e^3 + (2\,cd^2g^2x - 5\,cd^2fg)e^2)\sqrt{gx+f}}{2\,(d^3g^3e^3 + f^2gxe^6 - (2\,dfg^2x - df^2g)e^5 + (d^2g^3x - 2\,d^2fg^2)e^4)}, \frac{(3\,cd^3g^2 - ag^2xe^3 - (4\,cdfgx + adg^2)e^2 + (3\,cd^2g^2x - 4\,cd^2fg)e)\sqrt{dge - fe^2}\arctan\left(-\frac{\sqrt{dge-fe^2}\,\sqrt{gx+f}}{dg-fe}\right) + (3\,cd^3g^2e + (2\,cf^2x - afg)e^4 - (4\,cdfgx - 2\,cdf^2 - adg^2)e^3 + (2\,cd^2g^2x - 5\,cd^2fg)e^2)\sqrt{gx+f}}{d^3g^3e^3 + f^2gxe^6 - (2\,dfg^2x - df^2g)e^5 + (d^2g^3x - 2\,d^2fg^2)e^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*c*d^3*g^2 - a*g^2*x*e^3 - (4*c*d*f*g*x + a*d*g^2)*e^2 + (3*c*d^2*
g^2*x - 4*c*d^2*f*g)*e)*sqrt(-d*g*e + f*e^2)*log(-(d*g - (g*x + 2*f)*e - 2*
sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e + d)) - 2*(3*c*d^3*g^2*e + (2*c*f^
2*x - a*f*g)*e^4 - (4*c*d*f*g*x - 2*c*d*f^2 - a*d*g^2)*e^3 + (2*c*d^2*g^2*x
 - 5*c*d^2*f*g)*e^2)*sqrt(g*x + f))/(d^3*g^3*e^3 + f^2*g*x*e^6 - (2*d*f*g^2
*x - d*f^2*g)*e^5 + (d^2*g^3*x - 2*d^2*f*g^2)*e^4), ((3*c*d^3*g^2 - a*g^2*x
*e^3 - (4*c*d*f*g*x + a*d*g^2)*e^2 + (3*c*d^2*g^2*x - 4*c*d^2*f*g)*e)*sqrt(
d*g*e - f*e^2)*arctan(-sqrt(d*g*e - f*e^2)*sqrt(g*x + f)/(d*g - f*e)) + (3*
c*d^3*g^2*e + (2*c*f^2*x - a*f*g)*e^4 - (4*c*d*f*g*x - 2*c*d*f^2 - a*d*g^2)
*e^3 + (2*c*d^2*g^2*x - 5*c*d^2*f*g)*e^2)*sqrt(g*x + f))/(d^3*g^3*e^3 + f^2
*g*x*e^6 - (2*d*f*g^2*x - d*f^2*g)*e^5 + (d^2*g^3*x - 2*d^2*f*g^2)*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + cx^2}{(d + ex)^2 \sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Giac [A]
time = 1.07, size = 148, normalized size = 1.21

$$\frac{2\,\sqrt{gx+f}\,ce^{(-2)}}{g} - \frac{(3\,cd^2g - 4\,cdfe - age^2)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{(dge^2 - fe^3)\sqrt{dge-fe^2}} + \frac{\sqrt{gx+f}\,cd^2g + \sqrt{gx+f}\,age^2}{(dge^2 - fe^3)(dg + (gx+f)e - fe)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(g*x + f)*c*e^(-2)/g - (3*c*d^2*g - 4*c*d*f*e - a*g*e^2)*arctan(sqrt(
g*x + f)*e/sqrt(d*g*e - f*e^2))/((d*g*e^2 - f*e^3)*sqrt(d*g*e - f*e^2)) + (
sqrt(g*x + f)*c*d^2*g + sqrt(g*x + f)*a*g*e^2)/((d*g*e^2 - f*e^3)*(d*g + (g
*x + f)*e - f*e))
```

Mupad [B]
time = 2.68, size = 128, normalized size = 1.05

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(-3\,c\,g\,d^2+4\,c\,f\,d\,e+a\,g\,e^2\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}}+\frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)}+\frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 3*c*d^2*g + 4
*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(1/2)*(a*e^2*g + c*d^2*
g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d*e^2*g)) + (2*c*(f + g*x)^(1/2))
/(e^2*g)
```

3.595 $\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=178

$$-\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{(3ae^2g+cd(8ef-5dg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)}-\frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\tanh^{-1}}{4e^{5/2}(ef-dg)^{5/2}}$$

```
[Out] -1/4*(3*a*e^2*g^2+c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f
)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)-1/2*(a+c*d^2/e^2)*(g*x+f
)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d*(-5*d*g+8*e*f))*(g*x+f)^(1/
2)/e^2/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A]

time = 0.19, antiderivative size = 182, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {912, 1171, 393, 214}

$$-\frac{\sqrt{f+gx}\,(ae^2+cd^2)}{2e^2(d+ex)^2(ef-dg)}-\frac{(3ae^2g^2+c(3d^2g^2-8defg+8e^2f^2))\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}+\frac{\sqrt{f+gx}\,(3ae^2g+cd(8ef-5dg))}{4e^2(d+ex)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/2*((c*d^2 + a*e^2)*Sqrt[f + g*x])/(e^2*(e*f - d*g)*(d + e*x)^2) + ((3*a*
e^2*g + c*d*(8*e*f - 5*d*g))*Sqrt[f + g*x])/(4*e^2*(e*f - d*g)^2*(d + e*x))
 - ((3*a*e^2*g^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*
Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
```

```
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{4cf^2}{g^2}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{2(ef-dg)}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(8ef-5dg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} + \frac{(3ae^2g^2+c(8e}{}$$

$$= -\frac{\left(a+\frac{cd^2}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(8ef-5dg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{(3ae^2g^2+c(8e}{}$$

Mathematica [A]
time = 0.72, size = 166, normalized size = 0.93

$$\frac{\frac{\sqrt{e}\sqrt{f+gx}\,(ae^2(-2ef+5dg+3egx)+cd(-3d^2g+8e^2fx+de(6f-5gx)))}{(ef-dg)^2(d+ex)^2}+\frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{(-ef+dg)^{5/2}}}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] ((Sqrt[e]*Sqrt[f + g*x]*(a*e^2*(-2*e*f + 5*d*g + 3*e*g*x) + c*d*(-3*d^2*g +
 8*e^2*f*x + d*e*(6*f - 5*g*x))))/((e*f - d*g)^2*(d + e*x)^2) + ((3*a*e^2*g
^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/
Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(5/2))/(4*e^(5/2))
```

Maple [A]
time = 0.10, size = 220, normalized size = 1.24

method	result
derivativedivides	$\frac{\frac{g\left(3a\,e^{2}g-5c\,d^{2}g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^{2}g^{2}-2defg+e^{2}f^{2}\right)}+\frac{\left(5a\,e^{2}g-3c\,d^{2}g+8cdef\right)g\sqrt{gx+f}}{4e^{2}(dg-ef)}}{(e(gx+f)+dg-ef)^{2}}+\frac{\left(3a\,e^{2}g^{2}+3c\,d^{2}g^{2}-8cdefg+8c\,e^{2}f^{2}\right)\operatorname{arctan}\cdots}{4(d^{2}g^{2}-2defg+e^{2}f^{2})e^{2}\sqrt{(d\cdots}}$
default	$\frac{\frac{g\left(3a\,e^{2}g-5c\,d^{2}g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^{2}g^{2}-2defg+e^{2}f^{2}\right)}+\frac{\left(5a\,e^{2}g-3c\,d^{2}g+8cdef\right)g\sqrt{gx+f}}{4e^{2}(dg-ef)}}{(e(gx+f)+dg-ef)^{2}}+\frac{\left(3a\,e^{2}g^{2}+3c\,d^{2}g^{2}-8cdefg+8c\,e^{2}f^{2}\right)\operatorname{arctan}\cdots}{4(d^{2}g^{2}-2defg+e^{2}f^{2})e^{2}\sqrt{(d\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/8*g*(3*a*e^2*g-5*c*d^2*g+8*c*d*e*f)/e/(d^2*g^2-2*d*e*f*g+e^2*f^2)*(g*x
+f)^(3/2)+1/8*(5*a*e^2*g-3*c*d^2*g+8*c*d*e*f)/e^2*g/(d*g-e*f)*(g*x+f)^(1/2)
)/(e*(g*x+f)+d*g-e*f)^2+1/4*(3*a*e^2*g^2+3*c*d^2*g^2-8*c*d*e*f*g+8*c*e^2*f^
2)/(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/
2)/((d*g-e*f)*e)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(159) = 318$.
time = 3.34, size = 870, normalized size = 4.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((3*c*d^4*g^2 + (8*c*f^2 + 3*a*g^2)*x^2*e^4 - 2*(4*c*d*f*g*x^2 - (8*c
*d*f^2 + 3*a*d*g^2)*x)*e^3 + (3*c*d^2*g^2*x^2 - 16*c*d^2*f*g*x + 8*c*d^2*f^
2 + 3*a*d^2*g^2)*e^2 + 2*(3*c*d^3*g^2*x - 4*c*d^3*f*g)*e)*sqrt(-d*g*e + f*e
^2)*log(-(d*g - (g*x + 2*f)*e + 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e
+ d)) + 2*(3*c*d^4*g^2*e + (3*a*f*g*x - 2*a*f^2)*e^5 + (7*a*d*f*g + (8*c*d*
f^2 - 3*a*d*g^2)*x)*e^4 - (13*c*d^2*f*g*x - 6*c*d^2*f^2 + 5*a*d^2*g^2)*e^3
+ (5*c*d^3*g^2*x - 9*c*d^3*f*g)*e^2)*sqrt(g*x + f))/(d^5*g^3*e^3 - f^3*x^2*
e^8 + (3*d*f^2*g*x^2 - 2*d*f^3*x)*e^7 - (3*d^2*f*g^2*x^2 - 6*d^2*f^2*g*x +
d^2*f^3)*e^6 + (d^3*g^3*x^2 - 6*d^3*f*g^2*x + 3*d^3*f^2*g)*e^5 + (2*d^4*g^3
*x - 3*d^4*f*g^2)*e^4), -1/4*((3*c*d^4*g^2 + (8*c*f^2 + 3*a*g^2)*x^2*e^4 -
2*(4*c*d*f*g*x^2 - (8*c*d*f^2 + 3*a*d*g^2)*x)*e^3 + (3*c*d^2*g^2*x^2 - 16*c
*d^2*f*g*x + 8*c*d^2*f^2 + 3*a*d^2*g^2)*e^2 + 2*(3*c*d^3*g^2*x - 4*c*d^3*f*
g)*e)*sqrt(d*g*e - f*e^2)*arctan(-sqrt(d*g*e - f*e^2)*sqrt(g*x + f)/(d*g -
f*e)) + (3*c*d^4*g^2*e + (3*a*f*g*x - 2*a*f^2)*e^5 + (7*a*d*f*g + (8*c*d*f^
2 - 3*a*d*g^2)*x)*e^4 - (13*c*d^2*f*g*x - 6*c*d^2*f^2 + 5*a*d^2*g^2)*e^3 +
(5*c*d^3*g^2*x - 9*c*d^3*f*g)*e^2)*sqrt(g*x + f))/(d^5*g^3*e^3 - f^3*x^2*e^
8 + (3*d*f^2*g*x^2 - 2*d*f^3*x)*e^7 - (3*d^2*f*g^2*x^2 - 6*d^2*f^2*g*x + d^
2*f^3)*e^6 + (d^3*g^3*x^2 - 6*d^3*f*g^2*x + 3*d^3*f^2*g)*e^5 + (2*d^4*g^3*x
 - 3*d^4*f*g^2)*e^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.36, size = 278, normalized size = 1.56

$$\frac{\left(3\,cd^2g^2-8\,cdfge+8\,cf^2e^2+3\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\,\left(d^2g^2e^2-2\,dfge^3+f^2e^4\right)\sqrt{dge-fe^2}}-\frac{3\,\sqrt{gx+f}\,cd^3g^3+5\,(gx+f)^{\frac{3}{2}}cd^2g^2e-11\,\sqrt{gx+f}\,cd^2fg^2e-8\,(gx+f)^{\frac{3}{2}}cdfge^2+8\,\sqrt{gx+f}\,cdf^2ge^2-5\,\sqrt{gx+f}\,adg^3e^2-3\,(gx+f)^{\frac{3}{2}}ag^2e^3+5\,\sqrt{gx+f}\,afg^2e^3}{4\,\left(d^2g^2e^2-2\,dfge^3+f^2e^4\right)\left(dg+(gx+f)e-fe\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*g^2 - 8*c*d*f*g*e + 8*c*f^2*e^2 + 3*a*g^2*e^2)*arctan(sqrt(g*x
 + f)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*sqrt(d*
g*e - f*e^2)) - 1/4*(3*sqrt(g*x + f)*c*d^3*g^3 + 5*(g*x + f)^(3/2)*c*d^2*g^
2*e - 11*sqrt(g*x + f)*c*d^2*f*g^2*e - 8*(g*x + f)^(3/2)*c*d*f*g*e^2 + 8*sq
rt(g*x + f)*c*d*f^2*g*e^2 - 5*sqrt(g*x + f)*a*d*g^3*e^2 - 3*(g*x + f)^(3/2)
*a*g^2*e^3 + 5*sqrt(g*x + f)*a*f*g^2*e^3)/((d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2
*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

Mupad [B]
time = 2.91, size = 224, normalized size = 1.26

$$\frac{\frac{\sqrt{f+g\,x}\,\left(-3\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+5\,a\,e^2\,g^2\right)}{4\,e^2\,\left(d\,g-e\,f\right)}+\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] (((f + g*x)^(1/2)*(5*a*e^2*g^2 - 3*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e^2*(d*g -
e*f)) + ((f + g*x)^(3/2)*(3*a*e^2*g^2 - 5*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e*(d
*g - e*f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 +
e^2*f^2 - 2*d*e*f*g) + (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(
3*a*e^2*g^2 + 3*c*d^2*g^2 + 8*c*e^2*f^2 - 8*c*d*e*f*g))/(4*e^(5/2)*(d*g - e
*f)^(5/2))
```

3.596 $\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=238

$$\frac{2(ef-dg)^3\,(cf^2+ag^2)}{g^6\sqrt{f+gx}}+\frac{2(ef-dg)^2\,(3aeg^2+cf(5ef-2dg))\,\sqrt{f+gx}}{g^6}-\frac{2(ef-dg)\,(3ae^2g^2+c(10e^2f^2-}{3g^6}$$

```
[Out] -2/3*(-d*g+e*f)*(3*a*e^2*g^2+c*(d^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3/2
)/g^6+2/5*e*(a*e^2*g^2+c*(3*d^2*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g
^6-2/7*c*e^2*(-3*d*g+5*e*f)*(g*x+f)^(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^6+2
*(-d*g+e*f)^3*(a*g^2+c*f^2)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(3*a*e*g^2+c*f
*(-2*d*g+5*e*f))*(g*x+f)^(1/2)/g^6
```

Rubi [A]
time = 0.17, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {912, 1275}

$$\frac{2e(f+gx)^{5/2}\,(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2))}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\,(3ae^2g^2+c(d^2g^2-8defg+10e^2f^2))}{3g^6}+\frac{2(ag^2+cf^2)\,(ef-dg)^3}{g^6\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\,(ef-dg)^2\,(3aeg^2+cf(5ef-2dg))}{g^6}-\frac{2ce^2(f+gx)^{7/2}(5ef-3dg)}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d*g)^2*(3
*a*e*g^2 + c*f*(5*e*f - 2*d*g))*Sqrt[f + g*x])/g^6 - (2*(e*f - d*g)*(3*a*e^
2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) + (2
*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(
5*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*(f +
g*x)^(9/2))/(9*g^6)
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(ef-dg)^2(3aeg^2+cf(5ef-2dg))}{g^5}+\frac{(-ef+dg)^3(cf^2+ag^2)}{g^5x^2}+\frac{(ef-dg)(-3ae^2g^2-c(10e^2}{g^5}\right.\right.}{}$$

$$= \frac{2(ef-dg)^3\left(cf^2+ag^2\right)}{g^6\sqrt{f+gx}}+\frac{2(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)\sqrt{f+gx}}{g^6} -$$

Mathematica [A]
time = 0.21, size = 278, normalized size = 1.17

$$\frac{2(63ag^2(-5d^3g^3+15d^2eg^2(2f+gx)+5de^2g(-8f^2-4fgx+g^2x^2)+e^3(16f^3+8f^2gx-2fg^2x^2+g^3x^3))+c(105d^3g^3(-8f^2-4fgx+g^2x^2)+189d^2eg^2(16f^3+8f^2gx-2fg^2x^2+g^3x^3)+27de^2g(-128f^4-64f^3gx+16f^2g^2x^2-8fg^3x^3+5g^4x^4)+5e^3(256f^5+128f^4gx-32f^3g^2x^2+16f^2g^3x^3-10fg^4x^4+7g^5x^5)))}{315g^6\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(63*a*g^2*(-5*d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 - 4
*f*g*x + g^2*x^2) + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + c*(
105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16*f^3 + 8*f^2*g*
x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2
*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x - 32*f^3*g^2
*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5))))/(315*g^6*Sqrt[f + g*x]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(218) = 436$.
time = 0.10, size = 438, normalized size = 1.84 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^6*(1/9*c*e^3*(g*x+f)^(9/2)+3/7*c*d*e^2*g*(g*x+f)^(7/2)-5/7*c*e^3*f*(g*x
+f)^(7/2)+1/5*a*e^3*g^2*(g*x+f)^(5/2)+3/5*c*d^2*e*g^2*(g*x+f)^(5/2)-12/5*c*
d*e^2*f*g*(g*x+f)^(5/2)+2*c*e^3*f^2*(g*x+f)^(5/2)+a*d*e^2*g^3*(g*x+f)^(3/2)
-a*e^3*f*g^2*(g*x+f)^(3/2)+1/3*c*d^3*g^3*(g*x+f)^(3/2)-3*c*d^2*e*f*g^2*(g*x
+f)^(3/2)+6*c*d*e^2*f^2*g*(g*x+f)^(3/2)-10/3*c*e^3*f^3*(g*x+f)^(3/2)+3*a*d^
2*e*g^4*(g*x+f)^(1/2)-6*a*d*e^2*f*g^3*(g*x+f)^(1/2)+3*a*e^3*f^2*g^2*(g*x+f)
^(1/2)-2*c*d^3*f*g^3*(g*x+f)^(1/2)+9*c*d^2*e*f^2*g^2*(g*x+f)^(1/2)-12*c*d*e
^2*f^3*g*(g*x+f)^(1/2)+5*c*e^3*f^4*(g*x+f)^(1/2)-(a*d^3*g^5-3*a*d^2*e*f*g^4
```

```
+3*a*d*e^2*f^2*g^3-a*e^3*f^3*g^2+c*d^3*f^2*g^3-3*c*d^2*e*f^3*g^2+3*c*d*e^2*
f^4*g-c*e^3*f^5)/(g*x+f)^(1/2))
```

Maxima [A]
time = 0.30, size = 322, normalized size = 1.35

$$\frac{2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^3+45\,(3\,cdge^2-5\,cfe^3)(gx+f)^{\frac{7}{2}}-63\,(12\,cdfge^2-10\,cf^2e^3-(3\,cd^2e+ae^3)g^2)(gx+f)^{\frac{5}{2}}+105\,(18\,cdf^2ge^2-10\,cf^3e^3-3\,(3\,cd^2e+ae^3)fg^2+(cd^3+3\,ade^2)g^3)(gx+f)^{\frac{3}{2}}+315\,(3\,ad^2g^4e-12\,cdf^3ge^2+5\,cf^4e^3+3\,(3\,cd^2e+ae^3)f^2g^2-2\,(cd^3+3\,ade^2)fg^3)\sqrt{gx+f}}{g^5}-\frac{315\,(ad^3g^5-3\,ad^2fg^4e+3\,cdf^4ge^2-cf^5e^3-(3\,cd^2e+ae^3)f^3g^2+(cd^3+3\,ade^2)f^2g^3)}{\sqrt{gx+f}\,g^5}\right)}{315\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 + 45*(3*c*d*g*e^2 - 5*c*f*e^3)*(g*x + f)^(
7/2) - 63*(12*c*d*f*g*e^2 - 10*c*f^2*e^3 - (3*c*d^2*e + a*e^3)*g^2)*(g*x +
f)^(5/2) + 105*(18*c*d*f^2*g*e^2 - 10*c*f^3*e^3 - 3*(3*c*d^2*e + a*e^3)*f*g
^2 + (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(3*a*d^2*g^4*e - 12*c*d
*f^3*g*e^2 + 5*c*f^4*e^3 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*a*d
*e^2)*f*g^3)*sqrt(g*x + f))/g^5 - 315*(a*d^3*g^5 - 3*a*d^2*f*g^4*e + 3*c*d*
f^4*g*e^2 - c*f^5*e^3 - (3*c*d^2*e + a*e^3)*f^3*g^2 + (c*d^3 + 3*a*d*e^2)*f
^2*g^3)/(sqrt(g*x + f)*g^5))/g
```

Fricas [A]
time = 3.15, size = 330, normalized size = 1.39

$$\frac{2\,(105\,cd^3g^5x^2-420\,cd^3fg^4x-840\,cd^3f^2g^3-315\,ad^3g^5+(35\,cg^5x^5-50\,cfg^4x^4+1280\,cf^5+1008\,af^3g^2+(80\,cf^2g^3+63\,ag^5)x^3-2\,(80\,cf^3g^2+63\,afg^4)x^2+8\,(80\,cf^4g+63\,af^2g^3)x)e^3+9\,(15\,cdg^5x^4-24\,cdfg^4x^3-384\,cdf^4g-280\,adf^2g^3+(48\,cdf^2g^3+35\,adg^5)x^2-4\,(48\,cdf^3g^2+35\,adfg^4)x)e^2+189\,(cd^2g^5x^3-2\,cd^2fg^4x^2+16\,cd^2f^3g^2+10\,ad^2fg^4+(8\,cd^2f^2g^3+5\,ad^2g^5)x)e)\sqrt{gx+f}}{315\,(g^7x+fg^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(105*c*d^3*g^5*x^2 - 420*c*d^3*f*g^4*x - 840*c*d^3*f^2*g^3 - 315*a*d^
3*g^5 + (35*c*g^5*x^5 - 50*c*f*g^4*x^4 + 1280*c*f^5 + 1008*a*f^3*g^2 + (80*
c*f^2*g^3 + 63*a*g^5)*x^3 - 2*(80*c*f^3*g^2 + 63*a*f*g^4)*x^2 + 8*(80*c*f^4
*g + 63*a*f^2*g^3)*x)*e^3 + 9*(15*c*d*g^5*x^4 - 24*c*d*f*g^4*x^3 - 384*c*d*
f^4*g - 280*a*d*f^2*g^3 + (48*c*d*f^2*g^3 + 35*a*d*g^5)*x^2 - 4*(48*c*d*f^3
*g^2 + 35*a*d*f*g^4)*x)*e^2 + 189*(c*d^2*g^5*x^3 - 2*c*d^2*f*g^4*x^2 + 16*c
*d^2*f^3*g^2 + 10*a*d^2*f*g^4 + (8*c*d^2*f^2*g^3 + 5*a*d^2*g^5)*x)*e)*sqrt(
g*x + f)/(g^7*x + f*g^6)
```

Sympy [A]
time = 25.73, size = 328, normalized size = 1.38

$$\frac{2ce^3(f+gx)^{\frac{9}{2}}}{9g^6}+\frac{(f+gx)^{\frac{7}{2}}\cdot(6cde^2g-10ce^3f)}{7g^6}+\frac{(f+gx)^{\frac{5}{2}}\cdot(2ae^3g^2+6cd^2eg^2-24cde^2fg+20ce^3f^2)}{5g^6}+\frac{(f+gx)^{\frac{3}{2}}\cdot(6ade^2g^3-6ae^3fg^2+2cd^3g^3-18cd^2efg^2+36cde^2f^2g-20ce^3f^3)}{3g^6}+\frac{\sqrt{f+gx}\,(6ad^2eg^4-12ade^2fg^3+6ae^3f^2g^2-4cd^3fg^3+18cd^2ef^2g^2-24cde^2f^3g+10ce^3f^4)}{g^6}-\frac{2(ag^2+cf^2)\,(dg-ef)^3}{g^6\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**3*(f + g*x)**(9/2)/(9*g**6) + (f + g*x)**(7/2)*(6*c*d*e**2*g - 10*c*
e**3*f)/(7*g**6) + (f + g*x)**(5/2)*(2*a*e**3*g**2 + 6*c*d**2*e*g**2 - 24*c
```

```
*d*e**2*f*g + 20*c*e**3*f**2)/(5*g**6) + (f + g*x)**(3/2)*(6*a*d*e**2*g**3
- 6*a*e**3*f*g**2 + 2*c*d**3*g**3 - 18*c*d**2*e*f*g**2 + 36*c*d*e**2*f**2*g
 - 20*c*e**3*f**3)/(3*g**6) + sqrt(f + g*x)*(6*a*d**2*e*g**4 - 12*a*d*e**2*
f*g**3 + 6*a*e**3*f**2*g**2 - 4*c*d**3*f*g**3 + 18*c*d**2*e*f**2*g**2 - 24*
c*d*e**2*f**3*g + 10*c*e**3*f**4)/g**6 - 2*(a*g**2 + c*f**2)*(d*g - e*f)**3
/(g**6*sqrt(f + g*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(222) = 444.
time = 1.02, size = 453, normalized size = 1.90

$$-\frac{2\,(cd^3f^2g^3+ad^3g^5-3\,cd^2f^3g^2e-3\,ad^2fg^4e+3\,cdf^4ge^2+3\,adf^2g^3e^2-cf^5e^3-af^3g^2e^3)}{\sqrt{gx+f}\,g^6}+\frac{2\,\left(105\,(gx+f)^{\frac{3}{2}}cd^3g^{51}-630\,\sqrt{gx+f}\,cd^3fg^{51}+189\,(gx+f)^{\frac{5}{2}}cd^2g^{50}e-945\,(gx+f)^{\frac{3}{2}}cd^2fg^{50}e+2835\,\sqrt{gx+f}\,cd^2f^2g^{50}e+945\,\sqrt{gx+f}\,ad^2g^{52}e+135\,(gx+f)^{\frac{7}{2}}cdg^{49}e^2-756\,(gx+f)^{\frac{5}{2}}cdfg^{49}e^2+1890\,(gx+f)^{\frac{3}{2}}cdf^2g^{49}e^2-3780\,\sqrt{gx+f}\,cdf^3g^{49}e^2+315\,(gx+f)^{\frac{3}{2}}adg^{51}e^2-1890\,\sqrt{gx+f}\,adfg^{51}e^2+35\,(gx+f)^{\frac{9}{2}}cg^{48}e^3-225\,(gx+f)^{\frac{7}{2}}cfg^{48}e^3+630\,(gx+f)^{\frac{5}{2}}cf^2g^{48}e^3-1050\,(gx+f)^{\frac{3}{2}}cf^3g^{48}e^3+1575\,\sqrt{gx+f}\,cf^4g^{48}e^3+63\,(gx+f)^{\frac{5}{2}}ag^{50}e^3-315\,(gx+f)^{\frac{3}{2}}afg^{50}e^3+945\,\sqrt{gx+f}\,af^2g^{50}e^3\right)}{315\,g^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^3*f^2*g^3 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e - 3*a*d^2*f*g^4*e + 3*c*d
*f^4*g*e^2 + 3*a*d*f^2*g^3*e^2 - c*f^5*e^3 - a*f^3*g^2*e^3)/(sqrt(g*x + f)*
g^6) + 2/315*(105*(g*x + f)^(3/2)*c*d^3*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^
51 + 189*(g*x + f)^(5/2)*c*d^2*g^50*e - 945*(g*x + f)^(3/2)*c*d^2*f*g^50*e
+ 2835*sqrt(g*x + f)*c*d^2*f^2*g^50*e + 945*sqrt(g*x + f)*a*d^2*g^52*e + 13
5*(g*x + f)^(7/2)*c*d*g^49*e^2 - 756*(g*x + f)^(5/2)*c*d*f*g^49*e^2 + 1890*
(g*x + f)^(3/2)*c*d*f^2*g^49*e^2 - 3780*sqrt(g*x + f)*c*d*f^3*g^49*e^2 + 31
5*(g*x + f)^(3/2)*a*d*g^51*e^2 - 1890*sqrt(g*x + f)*a*d*f*g^51*e^2 + 35*(g*
x + f)^(9/2)*c*g^48*e^3 - 225*(g*x + f)^(7/2)*c*f*g^48*e^3 + 630*(g*x + f)^
(5/2)*c*f^2*g^48*e^3 - 1050*(g*x + f)^(3/2)*c*f^3*g^48*e^3 + 1575*sqrt(g*x
+ f)*c*f^4*g^48*e^3 + 63*(g*x + f)^(5/2)*a*g^50*e^3 - 315*(g*x + f)^(3/2)*a
*f*g^50*e^3 + 945*sqrt(g*x + f)*a*f^2*g^50*e^3)/g^54
```

Mupad [B]
time = 0.09, size = 292, normalized size = 1.23

$$\frac{(f+g\,x)^{5/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2)}{5\,g^6}-\frac{2\,c\,d^3\,f^2\,g^3+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2-6\,a\,d^2\,e\,f\,g^4+6\,c\,d\,e^2\,f^4\,g+6\,a\,d\,e^2\,f^2\,g^3-2\,c\,e^3\,f^5-2\,a\,e^3\,f^3\,g^2}{g^6\,\sqrt{f+g\,x}}+\frac{2\,c\,e^3\,(f+g\,x)^{9/2}}{9\,g^6}+\frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^2\,(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2)}{g^6}+\frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{3\,g^6}+\frac{2\,c\,e^2\,(f+g\,x)^{7/2}\,(3\,d\,g-5\,e\,f)}{7\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f
*g))/(5*g^6) - (2*a*d^3*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g
^3 - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*c*d^2*e*f^3*
g^2)/(g^6*(f + g*x)^(1/2)) + (2*c*e^3*(f + g*x)^(9/2))/(9*g^6) + (2*(f + g*
x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/g^6 + (2*(f + g
*x)^(3/2)*(d*g - e*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g
))/(3*g^6) + (2*c*e^2*(f + g*x)^(7/2)*(3*d*g - 5*e*f))/(7*g^6)
```

3.597 $\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{2(ef-dg)^2\left(cf^2+ag^2\right)}{g^5\sqrt{f+gx}}-\frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)\sqrt{f+gx}}{g^5}+\frac{2(ae^2g^2+c(6e^2f^2-6defg+d^2g}{3g^5}$$

```
[Out] 2/3*(a*e^2*g^2+c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)^(3/2)/g^5-4/5*c*e*(
-d*g+2*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(g*x+f)^(7/2)/g^5-2*(-d*g+e*f)^2*(a
*g^2+c*f^2)/g^5/(g*x+f)^(1/2)-4*(-d*g+e*f)*(a*e*g^2+c*f*(-d*g+2*e*f))*(g*x+
f)^(1/2)/g^5
```

Rubi [A]

time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {912, 1275}

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}\,(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{g^5}-\frac{4ce(f+gx)^{5/2}(2ef-dg)}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*Sqrt[f + g*x]) - (4*(e*f - d*g)*(a*
e*g^2 + c*f*(2*e*f - d*g))*Sqrt[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*f^
2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (4*c*e*(2*e*f - d*g)*(
f + g*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$
$$= \frac{2\text{Subst}\left(\int \left(\frac{2(ef-dg)(-aeg^2-cf(2ef-dg))}{g^4}+\frac{(-ef+dg)^2(cf^2+ag^2)}{g^4x^2}+\frac{(ae^2g^2+c(6e^2f^2-6defg+d\right.}{g^4}}{g}$$
$$= -\frac{2(ef-dg)^2\,(cf^2+ag^2)}{g^5\sqrt{f+gx}} - \frac{4(ef-dg)\,(aeg^2+cf(2ef-dg))\,\sqrt{f+gx}}{g^5} + \frac{2(}{}$$

Mathematica [A]
time = 0.15, size = 177, normalized size = 1.02

$$\frac{-70ag^2(3d^2g^2-6deg(2f+gx)+e^2(8f^2+4fgx-g^2x^2))+2c(35d^2g^2(-8f^2-4fgx+g^2x^2)+42deg(16f^3+8f^2gx-2fg^2x^2+g^3x^3)-3e^2(128f^4+64f^3gx-16f^2g^2x^2+8fg^3x^3-5g^4x^4))}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-70*a*g^2*(3*d^2*g^2 - 6*d*e*g*(2*f + g*x) + e^2*(8*f^2 + 4*f*g*x - g^2*x^
2)) + 2*c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3 + 8*f
^2*g*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g*x - 16*f^2*g^2*
x^2 + 8*f*g^3*x^3 - 5*g^4*x^4)))/(105*g^5*Sqrt[f + g*x])
```

Maple [A]
time = 0.09, size = 256, normalized size = 1.48

method	result
risch	$\frac{2(15ce^2x^3g^3+42cdeg^3x^2-39ce^2fg^2x^2+35ae^2g^3x+35cd^2g^3x-126cdefg^2x+87ce^2f^2gx+210adeg^3-175ae^2fg^2-1}{105g^5}$
gosper	$-\frac{2(-15ce^2x^4g^4-42cdeg^4x^3+24ce^2fg^3x^3-35ae^2g^4x^2-35cd^2g^4x^2+84cdefg^3x^2-48ce^2f^2g^2x^2-210adeg^4x+140a}{105}$
trager	$-\frac{2(-15ce^2x^4g^4-42cdeg^4x^3+24ce^2fg^3x^3-35ae^2g^4x^2-35cd^2g^4x^2+84cdefg^3x^2-48ce^2f^2g^2x^2-210adeg^4x+140a}{105}$
derivativedivides	$\frac{\frac{2ce^2(gx+f)^{\frac{7}{2}}}{7}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8ce^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2ae^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{2cd^2g^2(gx+f)^{\frac{3}{2}}}{3}-4cdefg(gx+f)^{\frac{3}{2}}+4ce^2f^2(gx+f)^{\frac{3}{2}}-}{}$
default	$\frac{\frac{2ce^2(gx+f)^{\frac{7}{2}}}{7}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8ce^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2ae^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{2cd^2g^2(gx+f)^{\frac{3}{2}}}{3}-4cdefg(gx+f)^{\frac{3}{2}}+4ce^2f^2(gx+f)^{\frac{3}{2}}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^5*(1/7*c*e^2*(g*x+f)^(7/2)+2/5*c*d*e*g*(g*x+f)^(5/2)-4/5*c*e^2*f*(g*x+f
)^(5/2)+1/3*a*e^2*g^2*(g*x+f)^(3/2)+1/3*c*d^2*g^2*(g*x+f)^(3/2)-2*c*d*e*f*g
*(g*x+f)^(3/2)+2*c*e^2*f^2*(g*x+f)^(3/2)+2*a*d*e*g^3*(g*x+f)^(1/2)-2*a*e^2*
f*g^2*(g*x+f)^(1/2)-2*c*d^2*f*g^2*(g*x+f)^(1/2)+6*c*d*e*f^2*g*(g*x+f)^(1/2)
-4*c*e^2*f^3*(g*x+f)^(1/2)-(a*d^2*g^4-2*a*d*e*f*g^3+a*e^2*f^2*g^2+c*d^2*f^2
*g^2-2*c*d*e*f^3*g+c*e^2*f^4)/(g*x+f)^(1/2))
```

Maxima [A]
time = 0.31, size = 203, normalized size = 1.17

$$\frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}ce^2+42\,(cdge-2\,cfe^2)(gx+f)^{\frac{5}{2}}-35\,(6\,cdfge-6\,cf^2e^2-(cd^2+ae^2)g^2)(gx+f)^{\frac{3}{2}}+210\,(3\,cdf^2ge+adg^3e-2\,cf^3e^2-(cd^2+ae^2)fg^2)\sqrt{gx+f}}{g^4}-\frac{105\,(ad^2g^4-2\,cdf^3ge-2\,adfg^3e+cf^4e^2+(cd^2+ae^2)f^2g^2)}{\sqrt{gx+f}\,g^4}\right)}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 + 42*(c*d*g*e - 2*c*f*e^2)*(g*x + f)^(5/2)
 - 35*(6*c*d*f*g*e - 6*c*f^2*e^2 - (c*d^2 + a*e^2)*g^2)*(g*x + f)^(3/2) + 2
10*(3*c*d*f^2*g*e + a*d*g^3*e - 2*c*f^3*e^2 - (c*d^2 + a*e^2)*f*g^2)*sqrt(g
*x + f))/g^4 - 105*(a*d^2*g^4 - 2*c*d*f^3*g*e - 2*a*d*f*g^3*e + c*f^4*e^2 +
 (c*d^2 + a*e^2)*f^2*g^2)/(sqrt(g*x + f)*g^4))/g
```

Fricas [A]
time = 3.97, size = 204, normalized size = 1.18

$$\frac{2\,(35\,cd^2g^4x^2-140\,cd^2fg^3x-280\,cd^2f^2g^2-105\,ad^2g^4+(15\,cg^4x^4-24\,cfg^3x^3-384\,cf^4-280\,af^2g^2+(48\,cf^2g^2+35\,ag^4)x^2-4\,(48\,cf^3g+35\,afg^3)x)e^2+42\,(cdg^4x^3-2\,cdfg^3x^2+16\,cdf^3g+10\,adfg^3+(8\,cdf^2g^2+5\,adg^4)x)e)\sqrt{gx+f}}{105\,(g^6x+fg^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(35*c*d^2*g^4*x^2 - 140*c*d^2*f*g^3*x - 280*c*d^2*f^2*g^2 - 105*a*d^2
*g^4 + (15*c*g^4*x^4 - 24*c*f*g^3*x^3 - 384*c*f^4 - 280*a*f^2*g^2 + (48*c*f
^2*g^2 + 35*a*g^4)*x^2 - 4*(48*c*f^3*g + 35*a*f*g^3)*x)*e^2 + 42*(c*d*g^4*x
^3 - 2*c*d*f*g^3*x^2 + 16*c*d*f^3*g + 10*a*d*f*g^3 + (8*c*d*f^2*g^2 + 5*a*d
*g^4)*x)*e)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

Sympy [A]
time = 15.17, size = 204, normalized size = 1.18

$$\frac{2ce^2(f+gx)^{\frac{7}{2}}}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\cdot(4cdeg-8ce^2f)}{5g^5}+\frac{(f+gx)^{\frac{3}{2}}\cdot(2ae^2g^2+2cd^2g^2-12cdefg+12ce^2f^2)}{3g^5}+\frac{\sqrt{f+gx}\,(4adeg^3-4ae^2fg^2-4cd^2fg^2+12cdef^2g-8ce^2f^3)}{g^5}-\frac{2(ag^2+cf^2)\,(dg-ef)^2}{g^5\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + (f + g*x)**(5/2)*(4*c*d*e*g - 8*c*e**2
*f)/(5*g**5) + (f + g*x)**(3/2)*(2*a*e**2*g**2 + 2*c*d**2*g**2 - 12*c*d*e*f
```

```
*g + 12*c*e**2*f**2)/(3*g**5) + sqrt(f + g*x)*(4*a*d*e*g**3 - 4*a*e**2*f*g*
*2 - 4*c*d**2*f*g**2 + 12*c*d*e*f**2*g - 8*c*e**2*f**3)/g**5 - 2*(a*g**2 +
c*f**2)*(d*g - e*f)**2/(g**5*sqrt(f + g*x))
```

Giac [A]
time = 1.70, size = 275, normalized size = 1.59

$$-\frac{2\,(cd^2f^2g^2 + ad^2g^4 - 2\,cdf^3ge - 2\,adfg^3e + cf^4e^2 + af^2g^2e^2)}{\sqrt{gx+f}\,g^5} + \frac{2\left(35\,(gx+f)^{\frac{3}{2}}cd^2g^{32} - 210\,\sqrt{gx+f}\,cd^2fg^{32} + 42\,(gx+f)^{\frac{5}{2}}cdg^{31}e - 210\,(gx+f)^{\frac{3}{2}}cdfg^{31}e + 630\,\sqrt{gx+f}\,cdf^2g^{31}e + 210\,\sqrt{gx+f}\,adg^{33}e + 15\,(gx+f)^{\frac{7}{2}}cg^{30}e^2 - 84\,(gx+f)^{\frac{5}{2}}cfg^{30}e^2 + 210\,(gx+f)^{\frac{3}{2}}cf^2g^{30}e^2 - 420\,\sqrt{gx+f}\,cf^3g^{30}e^2 + 35\,(gx+f)^{\frac{3}{2}}ag^{32}e^2 - 210\,\sqrt{gx+f}\,afg^{32}e^2\right)}{105\,g^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*f^2*g^2 + a*d^2*g^4 - 2*c*d*f^3*g*e - 2*a*d*f*g^3*e + c*f^4*e^2 +
 a*f^2*g^2*e^2)/(sqrt(g*x + f)*g^5) + 2/105*(35*(g*x + f)^(3/2)*c*d^2*g^32
- 210*sqrt(g*x + f)*c*d^2*f*g^32 + 42*(g*x + f)^(5/2)*c*d*g^31*e - 210*(g*x
 + f)^(3/2)*c*d*f*g^31*e + 630*sqrt(g*x + f)*c*d*f^2*g^31*e + 210*sqrt(g*x
+ f)*a*d*g^33*e + 15*(g*x + f)^(7/2)*c*g^30*e^2 - 84*(g*x + f)^(5/2)*c*f*g^
30*e^2 + 210*(g*x + f)^(3/2)*c*f^2*g^30*e^2 - 420*sqrt(g*x + f)*c*f^3*g^30*
e^2 + 35*(g*x + f)^(3/2)*a*g^32*e^2 - 210*sqrt(g*x + f)*a*f*g^32*e^2)/g^35
```

Mupad [B]
time = 2.66, size = 199, normalized size = 1.15

$$\frac{{(f+g\,x)}^{3/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{3\,g^5}-\frac{2\,c\,d^2\,f^2\,g^2+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g-4\,a\,d\,e\,f\,g^3+2\,c\,e^2\,f^4+2\,a\,e^2\,f^2\,g^2}{g^5\,\sqrt{f+g\,x}}+\frac{4\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2)}{g^5}+\frac{2\,c\,e^2\,{(f+g\,x)}^{7/2}}{7\,g^5}+\frac{4\,c\,e\,{(f+g\,x)}^{5/2}\,(d\,g-2\,e\,f)}{5\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g))
/(3*g^5) - (2*a*d^2*g^4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 -
 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^(1/2)) + (4*(f + g*x)^(1/2)*
(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/g^5 + (2*c*e^2*(f + g*x)^(7/2)
)/(7*g^5) + (4*c*e*(f + g*x)^(5/2)*(d*g - 2*e*f))/(5*g^5)
```

3.598 $\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2(ef-dg)\left(cf^2+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^4}-\frac{2c(3ef-dg)(f+gx)^{3/2}}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

```
[Out] -2/3*c*(-d*g+3*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(5/2)/g^4+2*(-d*g+e*f
)*(a*g^2+c*f^2)/g^4/(g*x+f)^(1/2)+2*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(1
/2)/g^4
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{786\}$

$$\frac{2\left(ag^2+cf^2\right)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\left(aeg^2+cf(3ef-2dg)\right)}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(a*e*g^2 + c*f*(3*
e*f - 2*d*g))*Sqrt[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^(3/2))/(3*g
^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(a+cx^2\right)}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2+ag^2\right)}{g^3(f+gx)^{3/2}}+\frac{aeg^2+cf(3ef-2dg)}{g^3\sqrt{f+gx}}+\frac{c(-3ef+dg)\sqrt{f+gx}}{g^3}\right.\\ &= \frac{2(ef-dg)\left(cf^2+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^4}-\frac{2c(3ef-dg)(f}{3g^4}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 92, normalized size = 0.83

$$\frac{30ag^2(2ef-dg+egx)+10cdg\left(-8f^2-4fgx+g^2x^2\right)+6ce\left(16f^3+8f^2gx-2fg^2x^2+g^3x^3\right)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (30*a*g^2*(2*e*f - d*g + e*g*x) + 10*c*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 6
*c*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))/(15*g^4*Sqrt[f + g*x])
```

Maple [A]
time = 0.10, size = 120, normalized size = 1.08

method	result
gosper	$-\frac{2\left(-3ce\,x^{3}g^{3}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30aef\,g^{2}+40cd\,f^{2}g-48ce\,f^{3}\right)}{15\sqrt{gx+f}\,g^{4}}$
trager	$-\frac{2\left(-3ce\,x^{3}g^{3}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30aef\,g^{2}+40cd\,f^{2}g-48ce\,f^{3}\right)}{15\sqrt{gx+f}\,g^{4}}$
risch	$\frac{2\left(3ce\,x^{2}g^{2}+5cdx\,g^{2}-9cefgx+15ae\,g^{2}-25cdfg+33ce\,f^{2}\right)\sqrt{gx+f}}{15g^{4}}-\frac{2\left(ad\,g^{3}-aef\,g^{2}+cd\,f^{2}g-ce\,f^{3}\right)}{g^{4}\sqrt{gx+f}}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2a\,g^{2}e\sqrt{gx+f}-4cdfg\sqrt{gx+f}+6c\,f^{2}e\sqrt{gx+f}-\frac{2\left(ad\,g^{3}-a\right.}{\sqrt{}}}{g^{4}}$
default	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2a\,g^{2}e\sqrt{gx+f}-4cdfg\sqrt{gx+f}+6c\,f^{2}e\sqrt{gx+f}-\frac{2\left(ad\,g^{3}-a\right.}{\sqrt{}}}{g^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^4*(1/5*c*e*(g*x+f)^(5/2)+1/3*c*d*g*(g*x+f)^(3/2)-c*e*f*(g*x+f)^(3/2)+a*
g^2*e*(g*x+f)^(1/2)-2*c*d*f*g*(g*x+f)^(1/2)+3*c*f^2*e*(g*x+f)^(1/2)-(a*d*g^
3-a*e*f*g^2+c*d*f^2*g-c*e*f^3)/(g*x+f)^(1/2))
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.06

$$\frac{2\left(\frac{3\,(gx+f)^{\frac{5}{2}}ce+5\,(cdg-3\,cfe)(gx+f)^{\frac{3}{2}}-15\,(2\,cdfg-3\,cf^{2}e-ag^{2}e)\sqrt{gx+f}}{g^{3}}-\frac{15\,(cdf^{2}g+adg^{3}-cf^{3}e-afg^{2}e)}{\sqrt{gx+f}\,g^{3}}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e + 5*(c*d*g - 3*c*f*e)*(g*x + f)^(3/2) - 15*(2*
c*d*f*g - 3*c*f^2*e - a*g^2*e)*sqrt(g*x + f))/g^3 - 15*(c*d*f^2*g + a*d*g^3
 - c*f^3*e - a*f*g^2*e)/(sqrt(g*x + f)*g^3))/g
```

Fricas [A]
time = 3.17, size = 109, normalized size = 0.98

$$\frac{2\,(5\,cdg^3x^2-20\,cdfg^2x-40\,cdf^2g-15\,adg^3+3\,(cg^3x^3-2\,cfg^2x^2+16\,cf^3+10\,afg^2+(8\,cf^2g+5\,ag^3)x)e)\sqrt{gx+f}}{15\,(g^5x+fg^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*c*d*g^3*x^2 - 20*c*d*f*g^2*x - 40*c*d*f^2*g - 15*a*d*g^3 + 3*(c*g^3
*x^3 - 2*c*f*g^2*x^2 + 16*c*f^3 + 10*a*f*g^2 + (8*c*f^2*g + 5*a*g^3)*x)*e)*
sqrt(g*x + f)/(g^5*x + f*g^4)
```

Sympy [A]
time = 8.33, size = 112, normalized size = 1.01

$$\frac{2ce(f+gx)^{\frac{5}{2}}}{5g^4}+\frac{(f+gx)^{\frac{3}{2}}\cdot(2cdg-6cef)}{3g^4}+\frac{\sqrt{f+gx}\,(2aeg^2-4cdfg+6cef^2)}{g^4}-\frac{2(ag^2+cf^2)\,(dg-ef)}{g^4\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e*(f + g*x)**(5/2)/(5*g**4) + (f + g*x)**(3/2)*(2*c*d*g - 6*c*e*f)/(3*g
**4) + sqrt(f + g*x)*(2*a*e*g**2 - 4*c*d*f*g + 6*c*e*f**2)/g**4 - 2*(a*g**2
 + c*f**2)*(d*g - e*f)/(g**4*sqrt(f + g*x))
```

Giac [A]
time = 1.47, size = 143, normalized size = 1.29

$$-\frac{2\,(cdf^2g+adg^3-cf^3e-afg^2e)}{\sqrt{gx+f}\,g^4}+\frac{2\left(5\,(gx+f)^{\frac{3}{2}}cdg^{17}-30\,\sqrt{gx+f}\,cdfg^{17}+3\,(gx+f)^{\frac{5}{2}}cg^{16}e-15\,(gx+f)^{\frac{3}{2}}cfg^{16}e+45\,\sqrt{gx+f}\,cf^2g^{16}e+15\,\sqrt{gx+f}\,ag^{18}e\right)}{15\,g^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d*f^2*g + a*d*g^3 - c*f^3*e - a*f*g^2*e)/(sqrt(g*x + f)*g^4) + 2/15*(
5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f*g^17 + 3*(g*x + f)^(5/2
)*c*g^16*e - 15*(g*x + f)^(3/2)*c*f*g^16*e + 45*sqrt(g*x + f)*c*f^2*g^16*e
+ 15*sqrt(g*x + f)*a*g^18*e)/g^20
```

Mupad [B]
time = 0.07, size = 111, normalized size = 1.00

$$\frac{\sqrt{f+g\,x}\,\left(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2\right)}{g^4}-\frac{-2\,c\,e\,f^3+2\,c\,d\,f^2\,g-2\,a\,e\,f\,g^2+2\,a\,d\,g^3}{g^4\,\sqrt{f+g\,x}}+\frac{2\,c\,e\,{\left(f+g\,x\right)}^{5/2}}{5\,g^4}+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}\,\left(d\,g-3\,e\,f\right)}{3\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(d + e*x))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(1/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/g^4 - (2*a*d*g^3 - 2*
c*e*f^3 - 2*a*e*f*g^2 + 2*c*d*f^2*g)/(g^4*(f + g*x)^(1/2)) + (2*c*e*(f + g*
x)^(5/2))/(5*g^4) + (2*c*(f + g*x)^(3/2)*(d*g - 3*e*f))/(3*g^4)
```

3.599 $\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2(cf^2+ag^2)}{g^3\sqrt{f+gx}}-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/g^3-2*(a*g^2+c*f^2)/g^3/(g*x+f)^(1/2)-4*c*f*(g*x+f)^(1/
2)/g^3
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{711\}$

$$-\frac{2(ag^2+cf^2)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g^3*Sqrt[f + g*x]) - (4*c*f*Sqrt[f + g*x])/g^3 + (2*c
*(f + g*x)^(3/2))/(3*g^3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{cf^2+ag^2}{g^2(f+gx)^{3/2}}-\frac{2cf}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx\\ &= -\frac{2(cf^2+ag^2)}{g^3\sqrt{f+gx}}-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.73

$$\frac{2(-3ag^2+c(-8f^2-4fgx+g^2x^2))}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (2*(-3*a*g^2 + c*(-8*f^2 - 4*f*g*x + g^2*x^2)))/(3*g^3*Sqrt[f + g*x])
```

Maple [A]
time = 0.06, size = 48, normalized size = 0.81

method	result	size
gosper	$-\frac{2\left(-c\,x^{2} g^{2}+4 c f x g+3 a\,g^{2}+8 c\,f^{2}\right)}{3 \sqrt{g x +f}\, g^{3}}$	41
trager	$-\frac{2\left(-c\,x^{2} g^{2}+4 c f x g+3 a\,g^{2}+8 c\,f^{2}\right)}{3 \sqrt{g x +f}\, g^{3}}$	41
risch	$-\frac{2 c \left(-g x +5 f \right) \sqrt{g x +f}}{3 g^{3}}-\frac{2 \left(a\,g^{2}+c\,f^{2}\right)}{g^{3} \sqrt{g x +f}}$	46
derivativedivides	$\frac{\frac{2 c \left(g x +f \right)^{\frac{3}{2}}}{3}-4 c f \sqrt{g x +f}-\frac{2 \left(a\,g^{2}+c\,f^{2}\right)}{\sqrt{g x +f}}}{g^{3}}$	48
default	$\frac{\frac{2 c \left(g x +f \right)^{\frac{3}{2}}}{3}-4 c f \sqrt{g x +f}-\frac{2 \left(a\,g^{2}+c\,f^{2}\right)}{\sqrt{g x +f}}}{g^{3}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^3*(1/3*c*(g*x+f)^(3/2)-2*c*f*(g*x+f)^(1/2)-(a*g^2+c*f^2)/(g*x+f)^(1/2))
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.92

$$\frac{2 \left(\frac{(gx+f)^{\frac{3}{2}} c-6 \sqrt{gx+f}\, cf}{g^2}-\frac{3\,(cf^2+ag^2)}{\sqrt{gx+f}\, g^2}\right)}{3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 6*sqrt(g*x + f)*c*f)/g^2 - 3*(c*f^2 + a*g^2)/(sqr
t(g*x + f)*g^2))/g
```

Fricas [A]
time = 2.84, size = 49, normalized size = 0.83

$$\frac{2\,(cg^2x^2-4\,cfgx-8\,cf^2-3\,ag^2)\sqrt{gx+f}}{3\,(g^4x+fg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 4*c*f*g*x - 8*c*f^2 - 3*a*g^2)*sqrt(g*x + f)/(g^4*x + f*g^
3)
```

Sympy [A]
time = 3.54, size = 58, normalized size = 0.98

$$-\frac{4cf\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{\frac{3}{2}}}{3g^3}-\frac{2(ag^2+cf^2)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(g*x+f)**(3/2),x)
```

```
[Out] -4*c*f*sqrt(f + g*x)/g**3 + 2*c*(f + g*x)**(3/2)/(3*g**3) - 2*(a*g**2 + c*f
**2)/(g**3*sqrt(f + g*x))
```

Giac [A]
time = 1.39, size = 56, normalized size = 0.95

$$-\frac{2\,(cf^2+ag^2)}{\sqrt{gx+f}\,g^3}+\frac{2\left((gx+f)^{\frac{3}{2}}cg^6-6\,\sqrt{gx+f}\,cfg^6\right)}{3\,g^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6 - 6*sqr
t(g*x + f)*c*f*g^6)/g^9
```

Mupad [B]
time = 0.05, size = 44, normalized size = 0.75

$$-\frac{6\,a\,g^2-2\,c\,{\left(f+g\,x\right)}^2+6\,c\,f^2+12\,c\,f\,\left(f+g\,x\right)}{3\,g^3\,\sqrt{f+g\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/(f + g*x)^(3/2),x)
```

```
[Out] -(6*a*g^2 - 2*c*(f + g*x)^2 + 6*c*f^2 + 12*c*f*(f + g*x))/(3*g^3*(f + g*x)^
(1/2))
```

3.600 $\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

```
[Out] -2*(a*e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-
d*g+e*f)^(3/2)+2*(a*g^2+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2*c*(g*x+f)^(1/
2)/e/g^2
```

Rubi [A]
time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {912, 1275, 214}

$$-\frac{2(ae^2+cd^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2(ag^2+cf^2)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f + g*x])/(
e*g^2) - (2*(c*d^2 + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]
])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{(2(cd^2+ae^2))\,\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}\,dx,\right.}{e(ef-dg)}$$

$$= \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{ef-dg}}\right.}{e^{3/2}(ef-dg)^{3/2}}$$

Mathematica [A]
time = 0.33, size = 114, normalized size = 1.02

$$-\frac{2(aeg^2-cdg(f+gx)+cef(2f+gx))}{eg^2(-ef+dg)\sqrt{f+gx}}-\frac{2(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
[Out] (-2*(a*e*g^2 - c*d*g*(f + g*x) + c*e*f*(2*f + g*x)))/(e*g^2*(-(e*f) + d*g)*
Sqrt[f + g*x]) - (2*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e
*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(3/2))
```

Maple [A]
time = 0.08, size = 112, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2(a g^2+c f^2)}{(dg-ef)\sqrt{gx+f}}-\frac{2g^2(a e^2+c d^2)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)e\sqrt{(dg-ef)e}}}{g^2}$

default	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}-\frac{2g^{2}\left(a\,e^{2}+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{(dg-ef)e\sqrt{(dg-ef)\,e}}}{g^{2}}$
risch	$\frac{2c\sqrt{gx+f}}{e\,g^{2}}-\frac{2a}{(dg-ef)\sqrt{gx+f}}-\frac{2c\,f^{2}}{g^{2}(dg-ef)\sqrt{gx+f}}-\frac{2e\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)a}{(dg-ef)\sqrt{(dg-ef)\,e}}-\frac{2\,\mathrm{arc}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^2*(c/e*(g*x+f)^(1/2)-(a*g^2+c*f^2)/(d*g-e*f)/(g*x+f)^(1/2)-g^2*(a*e^2+c
*d^2)/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^
(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(97) = 194$.
time = 3.33, size = 475, normalized size = 4.24

$$\left[\frac{(cd^2g^3x+cd^2fg^2+(ag^3x+afg^2)e^2)\sqrt{-dge+fe^2}\log\left(-\frac{dg-(gx+2f)e+2\sqrt{-dge+fe^2}\sqrt{gx+f}}{xe+d}\right)+2\sqrt{gx+f}\,((cf^2gx+2cf^3+afg^2)e^3-(2cdfg^2x+3cdf^2g+adg^3)e^2+(cd^2g^3x+cd^2fg^2)e)}{(f^2g^3x+f^3g^2)e^4-2(dfg^4x+df^2g^3)e^3+(d^2g^5x+d^2fg^4)e^2},\ \frac{2\left((cd^2g^3x+cd^2fg^2+(ag^3x+afg^2)e^2)\sqrt{dge-fe^2}\arctan\left(-\frac{\sqrt{dge-fe^2}\sqrt{gx+f}}{dg-fe}\right)+\sqrt{gx+f}\,((cf^2gx+2cf^3+afg^2)e^3-(2cdfg^2x+3cdf^2g+adg^3)e^2+(cd^2g^3x+cd^2fg^2)e)\right)}{(f^2g^3x+f^3g^2)e^4-2(dfg^4x+df^2g^3)e^3+(d^2g^5x+d^2fg^4)e^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [((c*d^2*g^3*x + c*d^2*f*g^2 + (a*g^3*x + a*f*g^2)*e^2)*sqrt(-d*g*e + f*e^2
)*log(-(d*g - (g*x + 2*f)*e + 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e +
d)) + 2*sqrt(g*x + f)*((c*f^2*g*x + 2*c*f^3 + a*f*g^2)*e^3 - (2*c*d*f*g^2*x
 + 3*c*d*f^2*g + a*d*g^3)*e^2 + (c*d^2*g^3*x + c*d^2*f*g^2)*e))/((f^2*g^3*x
 + f^3*g^2)*e^4 - 2*(d*f*g^4*x + d*f^2*g^3)*e^3 + (d^2*g^5*x + d^2*f*g^4)*e
^2), 2*((c*d^2*g^3*x + c*d^2*f*g^2 + (a*g^3*x + a*f*g^2)*e^2)*sqrt(d*g*e -
f*e^2)*arctan(-sqrt(d*g*e - f*e^2)*sqrt(g*x + f)/(d*g - f*e)) + sqrt(g*x +
```

```
f)*((c*f^2*g*x + 2*c*f^3 + a*f*g^2)*e^3 - (2*c*d*f*g^2*x + 3*c*d*f^2*g + a*
d*g^3)*e^2 + (c*d^2*g^3*x + c*d^2*f*g^2)*e))/((f^2*g^3*x + f^3*g^2)*e^4 - 2
*(d*f*g^4*x + d*f^2*g^3)*e^3 + (d^2*g^5*x + d^2*f*g^4)*e^2)]
```

Sympy [A]
time = 13.72, size = 104, normalized size = 0.93

$$\frac{2c\sqrt{f+gx}}{eg^2} - \frac{2(ag^2+cf^2)}{g^2\sqrt{f+gx}\,(dg-ef)} - \frac{2(ae^2+cd^2)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}\,(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*sqrt(f + g*x)/(e*g**2) - 2*(a*g**2 + c*f**2)/(g**2*sqrt(f + g*x)*(d*g -
 e*f)) - 2*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**2*
sqrt((d*g - e*f)/e)*(d*g - e*f))
```

Giac [A]
time = 1.09, size = 101, normalized size = 0.90

$$-\frac{2\,(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{(dge-fe^2)^{\frac{3}{2}}} + \frac{2\,\sqrt{gx+f}\,ce^{(-1)}}{g^2} - \frac{2\,(cf^2+ag^2)}{(dg^3-fg^2e)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/(d*g*e - f*e
^2)^(3/2) + 2*sqrt(g*x + f)*c*e^(-1)/g^2 - 2*(c*f^2 + a*g^2)/((d*g^3 - f*g^
2*e)*sqrt(g*x + f))
```

Mupad [B]
time = 0.14, size = 141, normalized size = 1.26

$$\frac{2\operatorname{atan}\left(\frac{2\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)\,(e^2\,f-d\,e\,g)}{\sqrt{e}\,(2\,c\,d^2+2\,a\,e^2)\,(d\,g-e\,f)^{3/2}}\right)(c\,d^2+a\,e^2)}{e^{3/2}\,(d\,g-e\,f)^{3/2}} + \frac{2\,c\,\sqrt{f+g\,x}}{e\,g^2} - \frac{2\,(c\,e\,f^2+a\,e\,g^2)}{e\,g^2\,\sqrt{f+g\,x}\,(d\,g-e\,f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

```
[Out] (2*atan((2*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(e^2*f - d*e*g))/(e^(1/2)*(2*a*e
^2 + 2*c*d^2)*(d*g - e*f)^(3/2)))*(a*e^2 + c*d^2))/(e^(3/2)*(d*g - e*f)^(3/
2)) + (2*c*(f + g*x)^(1/2))/(e*g^2) - (2*(a*e*g^2 + c*e*f^2))/(e*g^2*(f + g
*x)^(1/2)*(d*g - e*f))
```

3.601 $\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{2(cf^2+ag^2)}{g(ef-dg)^2\sqrt{f+gx}}-\frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)}+\frac{(3ae^2g+cd(4ef-dg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

```
[Out] (3*a*e^2*g+c*d*(-d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2)
)/e^(3/2)/(-d*g+e*f)^(5/2)-2*(a*g^2+c*f^2)/g/(-d*g+e*f)^2/(g*x+f)^(1/2)-(a*
e^2+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A]
time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {912, 1273, 464, 214}

$$-\frac{\sqrt{f+gx}\,(ae^2+cd^2)}{e(d+ex)(ef-dg)^2}+\frac{(3ae^2g+cd(4ef-dg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}-\frac{2(ag^2+cf^2)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqr
t[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((3*a*e^2*g + c*d*(4*e*f - d*g))*
ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\text{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2+ag^2\right)}{g^5}+\frac{e\left(ae^2g^2-c\left(2e^2f^2-4defg+d^2g\right.\right.}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\right.}{e^2(ef-dg)^2}$$

$$= -\frac{2(cf^2+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{(3ae^2g+cd(4ef-dg))}{}$$

$$= -\frac{2(cf^2+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(3ae^2g+cd(4ef-dg))}{e^{3/2}(ef}$$

Mathematica [A]
time = 0.53, size = 148, normalized size = 1.03

$$\frac{-c(2def^2+2e^2f^2x+d^2g(f+gx))-aeg(2dg+e(f+3gx))}{eg(ef-dg)^2(d+ex)\sqrt{f+gx}}+\frac{(-3ae^2g+cd(-4ef+dg))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-(c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x))) - a*e*g*(2*d*g + e*(f + 3
*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f + g*x]) + ((-3*a*e^2*g + c*d*(-
4*e*f + d*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/2)*
(-(e*f) + d*g)^(5/2))
```

Maple [A]
time = 0.07, size = 152, normalized size = 1.06

method	result	size
derivativedivides	$\frac{-\frac{2g\left(\frac{g\left(a\,e^{2}+c\,d^{2}\right)\sqrt{gx+f}}{2e\left(e\left(gx+f\right)+dg-ef\right)}+\frac{\left(3a\,e^{2}g-c\,d^{2}g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{\left(dg-ef\right)e}}\right)}{2e\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^{2}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{\left(dg-ef\right)^{2}\sqrt{gx+f}}}{g}$	152
default	$\frac{-\frac{2g\left(\frac{g\left(a\,e^{2}+c\,d^{2}\right)\sqrt{gx+f}}{2e\left(e\left(gx+f\right)+dg-ef\right)}+\frac{\left(3a\,e^{2}g-c\,d^{2}g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{\left(dg-ef\right)e}}\right)}{2e\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^{2}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{\left(dg-ef\right)^{2}\sqrt{gx+f}}}{g}$	152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g*(-g/(d*g-e*f)^2*(1/2*g*(a*e^2+c*d^2)/e*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*f
)+1/2*(3*a*e^2*g-c*d^2*g+4*c*d*e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^
(1/2)/((d*g-e*f)*e)^(1/2)))-(a*g^2+c*f^2)/(d*g-e*f)^2/(g*x+f)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(131) = 262$.

time = 3.04, size = 890, normalized size = 6.18

$$\left[-\frac{\left(cd^{3}g^{3}x+cd^{3}fg^{2}-3\left(ag^{3}x^{2}+afg^{2}x\right)e^{3}-\left(4\,cdfg^{2}x^{2}+3\,adfg^{2}+\left(4\,cdf^{2}g+3\,adg^{3}\right)x\right)e^{2}+\left(cd^{2}g^{3}x^{2}-3\,cd^{2}fg^{2}x-4\,cd^{2}f^{2}g\right)e\right)\sqrt{-dge+fe^{2}}\log\left(-\frac{dg-(gx+2f)e-2\sqrt{-dge+fe^{2}}\sqrt{gx+f}}{xe+d}\right)+2\sqrt{gx+f}\left(\left(af^{2}g+\left(2\,cf^{3}+3\,afg^{2}\right)x\right)e^{4}+\left(2\,cdf^{3}+adfg^{2}-\left(2\,cdf^{2}g+3\,adg^{3}\right)x\right)e^{3}+\left(cd^{2}fg^{2}x-cd^{2}f^{2}g-2\,ad^{2}g^{3}\right)e^{2}-\left(cd^{3}g^{3}x+cd^{3}fg^{2}\right)e\right)}{2\left(\left(f^{3}g^{2}x^{2}+f^{4}gx\right)e^{6}-\left(3\,df^{2}g^{3}x^{2}+2\,df^{3}g^{2}x-df^{4}g\right)e^{5}+3\left(d^{2}fg^{4}x^{2}-d^{2}f^{3}g^{2}\right)e^{4}-\left(d^{3}g^{5}x^{2}-2\,d^{3}fg^{4}x-3\,d^{3}f^{2}g^{3}\right)e^{3}-\left(d^{4}g^{5}x+d^{4}fg^{4}\right)e^{2}\right)},\ \frac{\left(cd^{3}g^{3}x+cd^{3}fg^{2}-3\left(ag^{3}x^{2}+afg^{2}x\right)e^{3}-\left(4\,cdfg^{2}x^{2}+3\,adfg^{2}+\left(4\,cdf^{2}g+3\,adg^{3}\right)x\right)e^{2}+\left(cd^{2}g^{3}x^{2}-3\,cd^{2}fg^{2}x-4\,cd^{2}f^{2}g\right)e\right)\sqrt{dge-fe^{2}}\arctan\left(-\frac{\sqrt{dge-fe^{2}}\sqrt{gx+f}}{dg-fe}\right)-\sqrt{gx+f}\left(\left(af^{2}g+\left(2\,cf^{3}+3\,afg^{2}\right)x\right)e^{4}+\left(2\,cdf^{3}+adfg^{2}-\left(2\,cdf^{2}g+3\,adg^{3}\right)x\right)e^{3}+\left(cd^{2}fg^{2}x-cd^{2}f^{2}g-2\,ad^{2}g^{3}\right)e^{2}-\left(cd^{3}g^{3}x+cd^{3}fg^{2}\right)e\right)}{\left(f^{3}g^{2}x^{2}+f^{4}gx\right)e^{6}-\left(3\,df^{2}g^{3}x^{2}+2\,df^{3}g^{2}x-df^{4}g\right)e^{5}+3\left(d^{2}fg^{4}x^{2}-d^{2}f^{3}g^{2}\right)e^{4}-\left(d^{3}g^{5}x^{2}-2\,d^{3}fg^{4}x-3\,d^{3}f^{2}g^{3}\right)e^{3}-\left(d^{4}g^{5}x+d^{4}fg^{4}\right)e^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^3*g^3*x + c*d^3*f*g^2 - 3*(a*g^3*x^2 + a*f*g^2*x)*e^3 - (4*c*d*
f*g^2*x^2 + 3*a*d*f*g^2 + (4*c*d*f^2*g + 3*a*d*g^3)*x)*e^2 + (c*d^2*g^3*x^2
 - 3*c*d^2*f*g^2*x - 4*c*d^2*f^2*g)*e)*sqrt(-d*g*e + f*e^2)*log(-(d*g - (g*
x + 2*f)*e - 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e + d)) + 2*sqrt(g*x
+ f)*((a*f^2*g + (2*c*f^3 + 3*a*f*g^2)*x)*e^4 + (2*c*d*f^3 + a*d*f*g^2 - (2
*c*d*f^2*g + 3*a*d*g^3)*x)*e^3 + (c*d^2*f*g^2*x - c*d^2*f^2*g - 2*a*d^2*g^3
)*e^2 - (c*d^3*g^3*x + c*d^3*f*g^2)*e))/((f^3*g^2*x^2 + f^4*g*x)*e^6 - (3*d
*f^2*g^3*x^2 + 2*d*f^3*g^2*x - d*f^4*g)*e^5 + 3*(d^2*f*g^4*x^2 - d^2*f^3*g^
2)*e^4 - (d^3*g^5*x^2 - 2*d^3*f*g^4*x - 3*d^3*f^2*g^3)*e^3 - (d^4*g^5*x + d
^4*f*g^4)*e^2), ((c*d^3*g^3*x + c*d^3*f*g^2 - 3*(a*g^3*x^2 + a*f*g^2*x)*e^3
 - (4*c*d*f*g^2*x^2 + 3*a*d*f*g^2 + (4*c*d*f^2*g + 3*a*d*g^3)*x)*e^2 + (c*d
^2*g^3*x^2 - 3*c*d^2*f*g^2*x - 4*c*d^2*f^2*g)*e)*sqrt(d*g*e - f*e^2)*arctan
(-sqrt(d*g*e - f*e^2)*sqrt(g*x + f)/(d*g - f*e)) - sqrt(g*x + f)*((a*f^2*g
+ (2*c*f^3 + 3*a*f*g^2)*x)*e^4 + (2*c*d*f^3 + a*d*f*g^2 - (2*c*d*f^2*g + 3*
a*d*g^3)*x)*e^3 + (c*d^2*f*g^2*x - c*d^2*f^2*g - 2*a*d^2*g^3)*e^2 - (c*d^3*
g^3*x + c*d^3*f*g^2)*e))/((f^3*g^2*x^2 + f^4*g*x)*e^6 - (3*d*f^2*g^3*x^2 +
2*d*f^3*g^2*x - d*f^4*g)*e^5 + 3*(d^2*f*g^4*x^2 - d^2*f^3*g^2)*e^4 - (d^3*g
^5*x^2 - 2*d^3*f*g^4*x - 3*d^3*f^2*g^3)*e^3 - (d^4*g^5*x + d^4*f*g^4)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+cx^{2}}{\left(d+ex\right)^{2}\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Integral((a + c*x**2)/((d + e*x)**2*(f + g*x)**(3/2)), x)
```

Giac [A]
time = 1.37, size = 225, normalized size = 1.56

$$\frac{\left(cd^{2}g-4\,cdfe-3\,age^{2}\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^{2}}}\right)}{\left(d^{2}g^{2}e-2\,dfge^{2}+f^{2}e^{3}\right)\sqrt{dge-fe^{2}}}-\frac{(gx+f)cd^{2}g^{2}+2\,cdf^{2}ge+2\,adg^{3}e+2\,(gx+f)cf^{2}e^{2}-2\,cf^{3}e^{2}+3\,(gx+f)ag^{2}e^{2}-2\,afg^{2}e^{2}}{\left(d^{2}g^{3}e-2\,dfg^{2}e^{2}+f^{2}ge^{3}\right)\left(\sqrt{gx+f}\,dg+(gx+f)^{\frac{3}{2}}e-\sqrt{gx+f}\,fe\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] (c*d^2*g - 4*c*d*f*e - 3*a*g*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2
))/((d^2*g^2*e - 2*d*f*g*e^2 + f^2*e^3)*sqrt(d*g*e - f*e^2)) - ((g*x + f)*c
*d^2*g^2 + 2*c*d*f^2*g*e + 2*a*d*g^3*e + 2*(g*x + f)*c*f^2*e^2 - 2*c*f^3*e^
2 + 3*(g*x + f)*a*g^2*e^2 - 2*a*f*g^2*e^2)/((d^2*g^3*e - 2*d*f*g^2*e^2 + f^
2*g*e^3)*(sqrt(g*x + f)*d*g + (g*x + f)^(3/2)*e - sqrt(g*x + f)*f*e))
```

Mupad [B]
time = 3.29, size = 187, normalized size = 1.30

$$-\frac{\frac{2\,\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+2\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{e\,{\left(d\,g-e\,f\right)}^2}}{\sqrt{f+g\,x}\,\left(d\,g^2-e\,f\,g\right)+e\,g\,{\left(f+g\,x\right)}^{3/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{5/2}}\right)\,\left(-c\,g\,d^2+4\,c\,f\,d\,e+3\,a\,g\,e^2\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

```
[Out] - ((2*(a*g^2 + c*f^2))/(d*g - e*f) + ((f + g*x)*(3*a*e^2*g^2 + c*d^2*g^2 +
2*c*e^2*f^2))/(e*(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f
+ g*x)^(3/2)) - (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))
/(e^(1/2)*(d*g - e*f)^(5/2)))*(3*a*e^2*g - c*d^2*g + 4*c*d*e*f))/(e^(3/2)*(
d*g - e*f)^(5/2))
```

3.602 $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2(cf^2+ag^2)}{(ef-dg)^3\sqrt{f+gx}}-\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2}+\frac{(7ae^2g+cd(8ef-dg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)}-\frac{(15ae^2g^2+c(8e^2f^2+8\ldots}{}$$

```
[Out] -1/4*(15*a*e^2*g^2+c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f
)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)+2*(a*g^2+c*f^2)/(-d*g+e*
f)^3/(g*x+f)^(1/2)-1/2*(a*e^2+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d)^2
+1/4*(7*a*e^2*g+c*d*(-d*g+8*e*f))*(g*x+f)^(1/2)/e/(-d*g+e*f)^3/(e*x+d)
```

Rubi [A]

time = 0.31, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {912, 1273, 467, 464, 214}

$$-\frac{\sqrt{f+gx}\,(ae^2+cd^2)}{2e(d+ex)^2(ef-dg)^2}-\frac{(15ae^2g^2+c(-d^2g^2+8defg+8e^2f^2))\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}+\frac{\sqrt{f+gx}\,(7ae^2g+cd(8ef-dg))}{4e(d+ex)(ef-dg)^3}+\frac{2(ag^2+cf^2)}{\sqrt{f+gx}\,(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 + a*e^2)*Sqrt[f
 + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((7*a*e^2*g + c*d*(8*e*f - d*g))
*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d + e*x)) - ((15*a*e^2*g^2 + c*(8*e^2*f
^2 + 8*d*e*f*g - d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]]
)/(4*e^(3/2)*(e*f - d*g)^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2+ag^2}{g^2}-\frac{2cfx^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)(cf^2+ag^2)}{g^5}+\frac{e\left(3ae^2g^2-c\left(4e^2f^2-8defg+d\right.\right.}{g^5}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\right.}{2e^2(ef-dg)^2}$$

$$= -\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(7ae^2g+cd(8ef-dg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} + \frac{g^3\text{Subst}\left(\int\right.}{}$$

$$= \frac{2(cf^2+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(7ae^2g+cd(8ef-dg))\sqrt{\cdot}}{4e(ef-dg)^3(d+ex}$$

$$= \frac{2(cf^2+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(7ae^2g+cd(8ef-dg))\sqrt{\cdot}}{4e(ef-dg)^3(d+ex}$$

Mathematica [A]
time = 1.01, size = 230, normalized size = 1.07

$$\frac{\frac{\sqrt{e}\,(c(8e^3f^2x^2+d^3g(f+gx)+8de^2fx(3f+gx)+d^2e(14f^2+5fgx-g^2x^2))+ae(8d^2g^2+deg(9f+25gx)+e^2(-2f^2+5fgx+15g^2x^2)))}{(ef-dg)^3(d+ex)^2\sqrt{f+gx}} - \frac{(15ae^2g^2+c(8e^2f^2+8defg-d^2g^2))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{(-ef+dg)^{7/2}}}{4e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] ((Sqrt[e]*(c*(8*e^3*f^2*x^2 + d^3*g*(f + g*x) + 8*d*e^2*f*x*(3*f + g*x) + d
^2*e*(14*f^2 + 5*f*g*x - g^2*x^2)) + a*e*(8*d^2*g^2 + d*e*g*(9*f + 25*g*x)
+ e^2*(-2*f^2 + 5*f*g*x + 15*g^2*x^2))))/((e*f - d*g)^3*(d + e*x)^2*Sqrt[f
+ g*x]) - ((15*a*e^2*g^2 + c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2))*ArcTan[(Sqr
t[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(7/2))/(4*e^(3/2))
```

Maple [A]
time = 0.09, size = 230, normalized size = 1.07

method	result

derivativedivides	$-\frac{2\left(\frac{\left(\frac{7}{8}a e^2g^2-\frac{1}{8}c d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad e^2g^2-9a e^3fg+c d^3g^2+7c d^2efg-8cd e^2f^2\right)\sqrt{gx+f}}{8e}}{(e(gx+f)+dg-ef)^2}+\frac{\left(15a e^2g^2-c d^2g^2 \cdots}{\cdots}\right.}{(dg-ef)^3}$
default	$-\frac{2\left(\frac{\left(\frac{7}{8}a e^2g^2-\frac{1}{8}c d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad e^2g^2-9a e^3fg+c d^3g^2+7c d^2efg-8cd e^2f^2\right)\sqrt{gx+f}}{8e}}{(e(gx+f)+dg-ef)^2}+\frac{\left(15a e^2g^2-c d^2g^2 \cdots}{\cdots}\right.}{(dg-ef)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(d*g-e*f)^3*(((7/8*a*e^2*g^2-1/8*c*d^2*g^2+c*d*e*f*g)*(g*x+f)^(3/2)+1/8*
g*(9*a*d*e^2*g^2-9*a*e^3*f*g+c*d^3*g^2+7*c*d^2*e*f*g-8*c*d*e^2*f^2)/e*(g*x+
f)^(1/2))/(e*(g*x+f)+d*g-e*f)^2+1/8*(15*a*e^2*g^2-c*d^2*g^2+8*c*d*e*f*g+8*c
*e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
)-2*(a*g^2+c*f^2)/(d*g-e*f)^3/(g*x+f)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(192) = 384.
time = 1.84, size = 1507, normalized size = 7.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((c*d^4*g^3*x + c*d^4*f*g^2 - ((8*c*f^2*g + 15*a*g^3)*x^3 + (8*c*f^3
+ 15*a*f*g^2)*x^2)*e^4 - 2*(4*c*d*f*g^2*x^3 + 3*(4*c*d*f^2*g + 5*a*d*g^3)*x
```

```
^2 + (8*c*d*f^3 + 15*a*d*f*g^2)*x)*e^3 + (c*d^2*g^3*x^3 - 15*c*d^2*f*g^2*x^
2 - 8*c*d^2*f^3 - 15*a*d^2*f*g^2 - 3*(8*c*d^2*f^2*g + 5*a*d^2*g^3)*x)*e^2 +
 2*(c*d^3*g^3*x^2 - 3*c*d^3*f*g^2*x - 4*c*d^3*f^2*g)*e)*sqrt(-d*g*e + f*e^2
)*log(-(d*g - (g*x + 2*f)*e + 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e +
d)) - 2*sqrt(g*x + f)*((5*a*f^2*g*x - 2*a*f^3 + (8*c*f^3 + 15*a*f*g^2)*x^2)
*e^5 - (15*a*d*g^3*x^2 - 11*a*d*f^2*g - 4*(6*c*d*f^3 + 5*a*d*f*g^2)*x)*e^4
- (9*c*d^2*f*g^2*x^2 - 14*c*d^2*f^3 + a*d^2*f*g^2 + (19*c*d^2*f^2*g + 25*a*
d^2*g^3)*x)*e^3 + (c*d^3*g^3*x^2 - 4*c*d^3*f*g^2*x - 13*c*d^3*f^2*g - 8*a*d
^3*g^3)*e^2 - (c*d^4*g^3*x + c*d^4*f*g^2)*e))/((f^4*g*x^3 + f^5*x^2)*e^8 -
2*(2*d*f^3*g^2*x^3 + d*f^4*g*x^2 - d*f^5*x)*e^7 + (6*d^2*f^2*g^3*x^3 - 2*d^
2*f^3*g^2*x^2 - 7*d^2*f^4*g*x + d^2*f^5)*e^6 - 4*(d^3*f*g^4*x^3 - 2*d^3*f^2
*g^3*x^2 - 2*d^3*f^3*g^2*x + d^3*f^4*g)*e^5 + (d^4*g^5*x^3 - 7*d^4*f*g^4*x^
2 - 2*d^4*f^2*g^3*x + 6*d^4*f^3*g^2)*e^4 + 2*(d^5*g^5*x^2 - d^5*f*g^4*x - 2
*d^5*f^2*g^3)*e^3 + (d^6*g^5*x + d^6*f*g^4)*e^2), -1/4*((c*d^4*g^3*x + c*d^
4*f*g^2 - ((8*c*f^2*g + 15*a*g^3)*x^3 + (8*c*f^3 + 15*a*f*g^2)*x^2)*e^4 - 2
*(4*c*d*f*g^2*x^3 + 3*(4*c*d*f^2*g + 5*a*d*g^3)*x^2 + (8*c*d*f^3 + 15*a*d*f
*g^2)*x)*e^3 + (c*d^2*g^3*x^3 - 15*c*d^2*f*g^2*x^2 - 8*c*d^2*f^3 - 15*a*d^2
*f*g^2 - 3*(8*c*d^2*f^2*g + 5*a*d^2*g^3)*x)*e^2 + 2*(c*d^3*g^3*x^2 - 3*c*d^
3*f*g^2*x - 4*c*d^3*f^2*g)*e)*sqrt(d*g*e - f*e^2)*arctan(-sqrt(d*g*e - f*e^
2)*sqrt(g*x + f)/(d*g - f*e)) - sqrt(g*x + f)*((5*a*f^2*g*x - 2*a*f^3 + (8*
c*f^3 + 15*a*f*g^2)*x^2)*e^5 - (15*a*d*g^3*x^2 - 11*a*d*f^2*g - 4*(6*c*d*f^
3 + 5*a*d*f*g^2)*x)*e^4 - (9*c*d^2*f*g^2*x^2 - 14*c*d^2*f^3 + a*d^2*f*g^2 +
 (19*c*d^2*f^2*g + 25*a*d^2*g^3)*x)*e^3 + (c*d^3*g^3*x^2 - 4*c*d^3*f*g^2*x
- 13*c*d^3*f^2*g - 8*a*d^3*g^3)*e^2 - (c*d^4*g^3*x + c*d^4*f*g^2)*e))/((f^4
*g*x^3 + f^5*x^2)*e^8 - 2*(2*d*f^3*g^2*x^3 + d*f^4*g*x^2 - d*f^5*x)*e^7 + (
6*d^2*f^2*g^3*x^3 - 2*d^2*f^3*g^2*x^2 - 7*d^2*f^4*g*x + d^2*f^5)*e^6 - 4*(d
^3*f*g^4*x^3 - 2*d^3*f^2*g^3*x^2 - 2*d^3*f^3*g^2*x + d^3*f^4*g)*e^5 + (d^4*
g^5*x^3 - 7*d^4*f*g^4*x^2 - 2*d^4*f^2*g^3*x + 6*d^4*f^3*g^2)*e^4 + 2*(d^5*g
^5*x^2 - d^5*f*g^4*x - 2*d^5*f^2*g^3)*e^3 + (d^6*g^5*x + d^6*f*g^4)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.11, size = 361, normalized size = 1.69

$$\frac{(cd^2g^2 - 8\,cdfge - 8\,cf^2e^2 - 15\,ag^2e^2)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\,(d^3g^3e - 3\,d^2fg^2e^2 + 3\,df^2ge^3 - f^3e^4)\sqrt{dge-fe^2}} - \frac{2\,(cf^2+ag^2)}{(d^3g^3 - 3\,d^2fg^2e + 3\,df^2ge^2 - f^3e^3)\sqrt{gx+f}} - \frac{\sqrt{gx+f}\,cd^3g^3 - (gx+f)^{\frac{3}{2}}cd^2g^2e + 7\,\sqrt{gx+f}\,cd^2fg^2e + 8\,(gx+f)^{\frac{3}{2}}cdfge^2 - 8\,\sqrt{gx+f}\,cdf^2ge^2 + 9\,\sqrt{gx+f}\,adg^3e^2 + 7\,(gx+f)^{\frac{3}{2}}ag^2e^3 - 9\,\sqrt{gx+f}\,afg^2e^3}{4\,(d^3g^3e - 3\,d^2fg^2e^2 + 3\,df^2ge^3 - f^3e^4)(dg+(gx+f)e-fe)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^2*g^2 - 8*c*d*f*g*e - 8*c*f^2*e^2 - 15*a*g^2*e^2)*arctan(sqrt(g*x
+ f)*e/sqrt(d*g*e - f*e^2))/((d^3*g^3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^3 -
 f^3*e^4)*sqrt(d*g*e - f*e^2)) - 2*(c*f^2 + a*g^2)/((d^3*g^3 - 3*d^2*f*g^2*
e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(g*x + f)) - 1/4*(sqrt(g*x + f)*c*d^3*g^3
- (g*x + f)^(3/2)*c*d^2*g^2*e + 7*sqrt(g*x + f)*c*d^2*f*g^2*e + 8*(g*x + f)
^(3/2)*c*d*f*g*e^2 - 8*sqrt(g*x + f)*c*d*f^2*g*e^2 + 9*sqrt(g*x + f)*a*d*g^
3*e^2 + 7*(g*x + f)^(3/2)*a*g^2*e^3 - 9*sqrt(g*x + f)*a*f*g^2*e^3)/((d^3*g^
3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^3 - f^3*e^4)*(d*g + (g*x + f)*e - f*e)^
2)
```

Mupad [B]
time = 3.37, size = 310, normalized size = 1.45

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+15\,a\,e^2\,g^2\right)}{4\,e^{3/2}\,{\left(d\,g-e\,f\right)}^{7/2}}-\frac{\frac{2\,\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f}+\frac{{\left(f+g\,x\right)}^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+15\,a\,e^2\,g^2\right)}{4\,{\left(d\,g-e\,f\right)}^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g+16\,c\,e^2\,f^2+25\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^{5/2}-{\left(f+g\,x\right)}^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^2
*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 +
 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) - ((2*(a*g^2 + c*f^2))/(d*g -
e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 + 8*c*d*e*f*g))
/(4*(d*g - e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 +
8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2) - (f + g*x)^(3/2)*(
2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g))
```

3.603 $\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=147

$$-\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\tanh^{-1}}{4e^{5/2}g^{5/2}}$$

```
[Out] 1/4*(8*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2))*arctanh(g^(1/2)*(e*x+d)
^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)^(3/2)*(g*x+f)^(
1/2)/e^2/g-1/4*c*(5*d*g+3*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^2
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {966, 81, 65, 223, 212}

$$\frac{(8ae^2g^2+c(3d^2g^2+2defg+3e^2f^2))\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*(c*(3*e*f + 5*d*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g^2) + (c*(d + e*
x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*
g + 3*d^2*g^2))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(
4*e^(5/2)*g^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
 !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+cx^2}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}ce(3ef+5dg)x}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{1}{8}\left(8a+\frac{c(3e^2f\right.$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{\left(8a+\frac{c(3e^2f^2+2c}{e}\right.}{}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{\left(8a+\frac{c(3e^2f^2+2c}{e}\right.}{}$$

$$= -\frac{c(3ef+5dg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(8ae^2g^2+c(3e^2}{}$$

Mathematica [A]

time = 0.34, size = 123, normalized size = 0.84

$$\frac{c\sqrt{d+ex}\,\sqrt{f+gx}\,(-3ef-3dg+2egx)}{4e^2g^2}+\frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (c*Sqrt[d + e*x]*Sqrt[f + g*x]*(-3*e*f - 3*d*g + 2*e*g*x))/(4*e^2*g^2) + ((
8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[
f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5/2)*g^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(121) = 242.
time = 0.08, size = 306, normalized size = 2.08

method	result
default	$\frac{\left(4\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cegx+3\ln\left(\frac{2egx+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)cd^2g^2+2\ln\left(\frac{2egx+2\sqrt{(e}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*g*x+3*ln(1/2*(2*e*g*x+2*((e*
x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2
*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*
g+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1
/2))*c*e^2*f^2+8*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+
e*f)/(e*g)^(1/2))*a*e^2*g^2-6*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d*g-6*(
e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)
^(1/2)/g^2/e^2/((e*x+d)*(g*x+f))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 2.77, size = 338, normalized size = 2.30

$$\left[\frac{\left(\left(3\,cd^2g^2+2\,cdfge+(3\,cf^2+8\,ag^2)e^2\right)\sqrt{g}\,e^{\frac{1}{2}}\log\left(d^2g^2+4\,(dg+(2\,gx+f)e)\sqrt{gx+f}\,\sqrt{xe+d}\,\sqrt{g}\,e^{\frac{1}{2}}+(8\,g^2x^2+8\,fgx+f^2)e^2+2\,(4\,dg^2x+3\,dfg)e\right)-4\,(3\,cdg^2e-(2\,cg^2x-3\,cfg)e^2)\sqrt{gx+f}\,\sqrt{xe+d}\right)e^{(-3)}}{16\,g^3},\ -\frac{\left(\left(3\,cd^2g^2+2\,cdfge+(3\,cf^2+8\,ag^2)e^2\right)\sqrt{-ge}\,\arctan\left(\frac{(dg+(2\,gx+f)e)\sqrt{gx+f}\,\sqrt{-ge}\,\sqrt{xe+d}}{2\,((g^2x^2+fgx)e^2+(dg^2x+dfg)e)}\right)+2\,(3\,cdg^2e-(2\,cg^2x-3\,cfg)e^2)\sqrt{gx+f}\,\sqrt{xe+d}\right)e^{(-3)}}{8\,g^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^2*g^2 + 2*c*d*f*g*e + (3*c*f^2 + 8*a*g^2)*e^2)*sqrt(g)*e^(1/2
)*log(d^2*g^2 + 4*(d*g + (2*g*x + f)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(g)
*e^(1/2) + (8*g^2*x^2 + 8*f*g*x + f^2)*e^2 + 2*(4*d*g^2*x + 3*d*f*g)*e) - 4
*(3*c*d*g^2*e - (2*c*g^2*x - 3*c*f*g)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))*e^(
-3)/g^3, -1/8*((3*c*d^2*g^2 + 2*c*d*f*g*e + (3*c*f^2 + 8*a*g^2)*e^2)*sqrt(-
g*e)*arctan(1/2*(d*g + (2*g*x + f)*e)*sqrt(g*x + f)*sqrt(-g*e)*sqrt(x*e + d
)/((g^2*x^2 + f*g*x)*e^2 + (d*g^2*x + d*f*g)*e)) + 2*(3*c*d*g^2*e - (2*c*g^
2*x - 3*c*f*g)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))*e^(-3)/g^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+cx^2}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [A]
time = 1.16, size = 160, normalized size = 1.09

$$\frac{\left(\sqrt{dg^2+(gx+f)ge-fge}\,\sqrt{gx+f}\left(\frac{2\,(gx+f)ce^{(-1)}}{g^3}-\frac{(3\,cdg^6e+5\,cfg^5e^2)e^{(-3)}}{g^8}\right)-\frac{(3\,cd^2g^2+2\,cdfge+3\,cf^2e^2+8\,ag^2e^2)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{gx+f}\,\sqrt{g}\,e^{\frac{1}{2}}+\sqrt{dg^2+(gx+f)ge-fge}\right|\right)}{g^{\frac{5}{2}}}\right)g}{4\,|g|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)*sqrt(g*x + f)*(2*(g*x + f)*c*e^(-1
)/g^3 - (3*c*d*g^6*e + 5*c*f*g^5*e^2)*e^(-3)/g^8) - (3*c*d^2*g^2 + 2*c*d*f*
g*e + 3*c*f^2*e^2 + 8*a*g^2*e^2)*e^(-5/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^
(1/2) + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))/g^(5/2))*g/abs(g)
```

Mupad [B]

time = 20.13, size = 569, normalized size = 3.87

$$\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\,\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}}-\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}}-\frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\,\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\,\left(\sqrt{f+g\,x}-\sqrt{f}\right)}-\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^3\,\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^3}+\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^7\,\left(\frac{3\,c\,d^2\,g^2}{2}+c\,d\,e\,f\,g+\frac{3\,c\,e^2\,f^2}{2}\right)}{e^2\,g^3\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^7}-\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^5\,\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{e\,g^4\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^5}+\frac{\sqrt{d}\,\sqrt{f}\,\left(32\,c\,d\,g+32\,c\,e\,f\right)\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^4}{g^4\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^4}}{\frac{{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^8}{{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^8}+\frac{e^4}{g^4}-\frac{4\,e\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^6}{g\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^6}-\frac{4\,e^3\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^2}{g^3\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^2}+\frac{6\,e^2\,{\left(\sqrt{d+e\,x}-\sqrt{d}\right)}^4}{g^2\,{\left(\sqrt{f+g\,x}-\sqrt{f}\right)}^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) -
f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*g))/(2*e^(5/2)*g^(5/2)) - (4*a*
atan((e*((f + g*x)^(1/2) - f^(1/2)))/((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/
2)))))/(-e*g)^(1/2) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*e^3*f^2)/2 + (3*c
*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*
x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))
/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^7*((3*c
*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f + g*x)^(1/2) - f^(
1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2
)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (d^(1/2)*f^(1/
2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/
2) - f^(1/2))^4))/((((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x)^(1/2) - f^(1/2)
)^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f + g*x)^(1/2) - f
^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((f + g*x)^(1/2) -
f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*((f + g*x)^(1/2) -
 f^(1/2))^4))
```

3.604 $\int \frac{-1+2x^2}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx$

Optimal. Leaf size=16

$$\sqrt{-1+x}\,x\sqrt{1+x}$$

```
[Out] x*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{392\}$

$$\sqrt{x-1}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Rule 392

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*x*(a1 + b1*x^(n/2))^(p + 1
)*((a2 + b2*x^(n/2))^(p + 1)/(a1*a2)), x] /; FreeQ[{a1, b1, a2, b2, c, d, n
, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d - b1*b2*
c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx = \sqrt{-1+x}\,x\sqrt{1+x}$$

Mathematica [A]
time = 0.04, size = 16, normalized size = 1.00

$$\sqrt{-1+x}\,x\sqrt{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.81

method	result	size
gosper	$x\sqrt{-1+x}\,\sqrt{1+x}$	13
default	$x\sqrt{-1+x}\,\sqrt{1+x}$	13
risch	$x\sqrt{-1+x}\,\sqrt{1+x}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(-1+x)^(1/2)*(1+x)^(1/2)
```

Maxima [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.28, size = 9, normalized size = 0.56

$$\sqrt{x^2-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 1)*x
```

Fricas [A]
time = 2.48, size = 12, normalized size = 0.75

$$\sqrt{x+1}\,\sqrt{x-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(-1+x)**(1/2)/(1+x)**(1/2),x)
[Out] Timed out
```

Giac [A]
time = 1.31, size = 12, normalized size = 0.75

$$\sqrt{x+1}\,\sqrt{x-1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*sqrt(x - 1)*x
```

Mupad [B]
time = 2.80, size = 16, normalized size = 1.00

$$\frac{(x^2 + x)\ \sqrt{x - 1}}{\sqrt{x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 1)/((x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] ((x + x^2)*(x - 1)^(1/2))/(x + 1)^(1/2)
```

3.605 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal. Leaf size=411

$$\frac{e\sqrt{d+ex}\,\sqrt{f+gx}}{c}+\frac{\sqrt{e}\,(ef+3dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}}+\frac{\left(\frac{a(ae^2g-cd(2ef+dg))}{\sqrt{c}}-\sqrt{-a}\,(cd^2f-ae(e\right.}{ac\sqrt{\sqrt{c}\,d-\sqrt{}}}$$

```
[Out] (3*d*g+e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)/c/
g^(1/2)+e*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2
)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(-(c*d^2*
f-a*e*(2*d*g+e*f))*(-a)^(1/2)+a*(a*e^2*g-c*d*(d*g+2*e*f))/c^(1/2))/a/c/(-e*
(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)+arctanh((e*x+d)
^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)
)^(1/2))*((c*d^2*f-a*e*(2*d*g+e*f))*(-a)^(1/2)+a*(a*e^2*g-c*d*(d*g+2*e*f))/
c^(1/2))/a/c/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]

time = 1.58, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {918, 81, 65, 223, 212, 6857, 95, 214}

$$\frac{\left(\frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}}-\sqrt{-a}\,(cd^2f-ae(2dg+ef))\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{ac\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}+\frac{\left(\sqrt{-a}\,(cd^2f-ae(2dg+ef))+\frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}}\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{ac\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}+\frac{e\sqrt{d+ex}\,\sqrt{f+gx}}{c}+\frac{\sqrt{e}\,(3dg+ef)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (e*Sqrt[d + e*x]*Sqrt[f + g*x])/c + (Sqrt[e]*(e*f + 3*d*g)*ArcTanh[(Sqrt[g]
*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqrt[g]) + (((a*(a*e^2*g - c*d
*(2*e*f + d*g)))/Sqrt[c] - Sqrt[-a]*(c*d^2*f - a*e*(e*f + 2*d*g)))*ArcTanh[
(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*
Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a
]*g]) + (((a*(a*e^2*g - c*d*(2*e*f + d*g)))/Sqrt[c] + Sqrt[-a]*(c*d^2*f - a
*e*(e*f + 2*d*g)))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(a*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*
e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 918

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[g/c, Int[Simp[2*e*f + d*g + e*g*x, x]*(d + e*x)^(m
- 1)*(f + g*x)^(n - 2), x], x] + Dist[1/c, Int[Simp[c*d*f^2 - 2*a*e*f*g - a
*d*g^2 + (c*e*f^2 + 2*c*d*f*g - a*e*g^2)*x, x]*(d + e*x)^(m - 1)*((f + g*x)
^(n - 2)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2
+ a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\int \frac{cd^2f-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx}{c} + \frac{e\int \frac{ef+2dg+egx}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\int \left(\frac{-\frac{a\left(-ae^2g+cd(2ef+dg)\right)}{\sqrt{c}}+\sqrt{-a}\,(cd^2f-ae(ef+2dg))}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\left(-ae\right.}{2a\left(\right.}\right.}{c}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{(ef+3dg)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d+e\right.}{c}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{(ef+3dg)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} + \frac{\left(\frac{a(}{}\right.}{}$$

$$= \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\sqrt{e}\,(ef+3dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(\frac{a(ae^2g-}{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.33, size = 438, normalized size = 1.07

$$\frac{\sqrt{c}\,e\sqrt{d+ex}\sqrt{f+gx} + \frac{\left(i\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}} + \frac{\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}} + \frac{\sqrt{c}\sqrt{e}\,(ef+3dg)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{\sqrt{g}}}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[f + g*x] + ((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*
d^2 + a*e^2]*(Sqrt[c]*f - I*Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f +
 g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d
 + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]
*g))]) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*(Sqrt[c]*f + I*S
qrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I
```

```
*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((S
qrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (Sqrt[c]*Sqrt[e]*(e*
f + 3*d*g)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/Sqrt[g
])/c^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2384 vs. $2(331) = 662$.
time = 0.12, size = 2385, normalized size = 5.80

method	result	size
default	Expression too large to display	2385

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(
1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c
*d*f)/c)^(1/2)*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*
f)/(e*g)^(1/2))*c*d*e*g+(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*
e*g+c*d*f)/c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1
/2)*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1
/2))*c*e^2*f-2*a*c*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*
d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(
g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-
(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*d*e*g-a*c*(e*g)^(1/2
)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e
*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)
*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)
*e*f)/(c*x+(-a*c)^(1/2)))*e^2*f+(-a*c)^(1/2)*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g
+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*
e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1
/2)))*a*e^2*g+(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/
c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f
))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c
)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c^2*d^2*f-(-a*c)^(1/2)*(e
*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((c*d
*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*
c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*
c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d^2*g-2*(-a*c)^(1/2)*(e*g)^(1/2)*(((-a*
c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(
-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a
*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c
*x+(-a*c)^(1/2)))*c*d*e*f+2*a*c*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)
```

```
^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*d*e*g+a
*c*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*l
n((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2
*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c
)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*e^2*f+(-a*c)^(1/2)*(e*g)^(1/2)*(-((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*
g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(
1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(
c*x-(-a*c)^(1/2)))*a*e^2*g-(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d^2*f-(-
a*c)^(1/2)*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)
^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+
c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d^2*g-2*(-a*c)^(1/2)*(e*g)
^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a
*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d
)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*
c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*e*f+2*(-a*c)^(1/2)*((e*x+d)*(g*x+f))^(1/
2)*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-
((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c*e)/((e*x+d)*(g*x
+f))^(1/2)/(-a*c)^(1/2)/c^2/(e*g)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
-a*e*g+c*d*f)/c)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)*(x*e + d)^(3/2)/(c*x^2 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/(a + c*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.69index.cc index_m
 i_lex_is_greater Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}{c\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2), x)
```

3.606 $\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal. Leaf size=342

$$\frac{2\sqrt{e}\,\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c}+\frac{\left(cdf-aeg-\sqrt{-a}\,\sqrt{c}\,(ef+dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+\cdots}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+\cdots}}\right.}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}$$

```
[Out] 2*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)*g^(1/2)/c+ar
ctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^
(1/2)+d*c^(1/2))^(1/2))*(c*d*f-a*e*g-(d*g+e*f)*(-a)^(1/2)*c^(1/2))/c/(-a)^(
1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctan
h((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+
d*c^(1/2))^(1/2))*(c*d*f-a*e*g+(d*g+e*f)*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]

time = 1.26, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {920, 65, 223, 212, 6857, 95, 214}

$$\frac{\left(-\sqrt{-a}\,\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(\sqrt{-a}\,\sqrt{c}\,(dg+ef)-aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}+\frac{2\sqrt{e}\,\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] (2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])]
)/c + ((c*d*f - a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x]
)])/(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])
- ((c*d*f - a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]*f +
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 920

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[e*(g/c), Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1), x
], x] + Dist[1/c, Int[Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x)^
(m - 1)*((f + g*x)^(n - 1)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0
] && GtQ[n, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+cx^2)}\,dx}{c} + \frac{(eg)\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{-a\sqrt{c}\,(ef+dg)+\sqrt{-a}\,(cdf-aeg)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{a\sqrt{c}\,(ef+dg)+\sqrt{-a}\,(cdf-aeg)}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+g}\right.}{c}$$

$$= \frac{(2g)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\,\sqrt{c}\,(ef+dg)\right)\int}{2\sqrt{}}$$

$$= \frac{2\sqrt{e}\,\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c} - \frac{\left(cdf-aeg-\sqrt{-a}\,\sqrt{c}\,(ef+dg)\right)\text{Sub}}{}$$

$$= \frac{2\sqrt{e}\,\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c} + \frac{\left(cdf-aeg-\sqrt{-a}\,\sqrt{c}\,(ef+dg)\right)\tanh}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.91, size = 363, normalized size = 1.06

$$\frac{\frac{\sqrt{cd^2+ae^2}\left(i\sqrt{c}\,f+\sqrt{a}\,g\right)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\,\sqrt{d+ex}}\right)}{\sqrt{a}\,\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}} + \frac{\sqrt{cd^2+ae^2}\left(-i\sqrt{c}\,f+\sqrt{a}\,g\right)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\,\sqrt{d+ex}}\right)}{\sqrt{a}\,\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}} + 2\sqrt{e}\,\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{d+ex}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

```
[Out] ((Sqrt[c*d^2 + a*e^2]*(I*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]
*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g)
)]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f -
I*Sqrt[a]*g))]) + (Sqrt[c*d^2 + a*e^2]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[
(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt
[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sqrt
[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + 2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[e]*Sq
rt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1494 vs. $2(262) = 524$.
time = 0.09, size = 1495, normalized size = 4.37

method	result
default	$-\frac{\sqrt{ex+d}\,\sqrt{gx+f}\,\left(\sqrt{eg}\,\sqrt{\frac{\sqrt{-ac}\,dg+\sqrt{-ac}\,ef-aeg+cdf}{c}}\,\ln\left(\frac{cdgx+cefx-2\sqrt{-ac}\,egx+2cdf+2\sqrt{(ex+d)}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*((e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*d*g+(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*(-a*c)^(1/2)*e*f+(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*e*g-(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d*f+(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*d*g+(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*e*f-(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*a*e*g+(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c*d*f-2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*(-a*c)^(1/2)*e*g)/((e*x+d)*(g*x+f))^(1/2)/(-a*c)^(1/2)/c/(e*g)^(1/

```
2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)/(-((-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)*sqrt(x*e + d)/(c*x^2 + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{a+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/(a + c*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.52index.cc index_m
 i_lex_is_greater Error: Bad Argument Value
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(a + c*x^2),x)
[Out] \text{Hanged}
```

3.607 $\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-}}}{\sqrt{\sqrt{c}\,d+\sqrt{-}}}\right.}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2
)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(
1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(g*(-a)^(1/2)+f*c
^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {924, 95, 214}

$$\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*d - Sqrt[-a]*e]) - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \left(\frac{\sqrt{-a}\,f-\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\sqrt{-a}\,f+\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{\right.$$

$$=\frac{1}{2}\left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx+\frac{1}{2}\left(\frac{af}{(-a)^{3/2}}\right.$$

$$=\left(\frac{af}{(-a)^{3/2}}-\frac{g}{\sqrt{c}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c}\,d+\sqrt{-a}\,e-\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)x^2}\,dx,x,\frac{\sqrt{d}}{\sqrt{f}}\right.$$

$$=\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}}}{}$$

Mathematica [A]
time = 10.33, size = 229, normalized size = 0.95

$$\frac{\frac{\sqrt{-\sqrt{c}\,f+\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{-\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{-\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-\sqrt{c}\,d+\sqrt{-a}\,e}}-\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}}}{\sqrt{-a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

```
[Out] ((Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*
Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(Sqr
t[c]*d) + Sqrt[-a]*e] - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x]
)])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/(Sqrt[-a]*Sqrt[c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(176) = 352$.

time = 0.11, size = 1387, normalized size = 5.78

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{ex+d}\left(\sqrt{\frac{\sqrt{-ac}\,dg+\sqrt{-ac}\,ef-aeg+cdf}{c}}\ \ln\left(\frac{cdgx+cefx-2\sqrt{-ac}\,egx+2cdf+2\sqrt{(ex+d)\,(gx}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*((((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+
c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)
*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*
c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f-(((-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((c*d*g*x+
c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1
/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*e^2*g+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*
e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*
x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1
/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c^2*d^2*f-(((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((c*d*
g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c
)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d^2*g-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f-
(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln(
(2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(
(e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^
(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e^2*g-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d^
2*f-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)
*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)
/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d^2*g)/((e*x+d)*(g*x+f))^(1/2)/(
(-a*c)^(1/2)*e+c*d)/(c*d-(-a*c)^(1/2)*e)/(-a*c)^(1/2)/(((-a*c)^(1/2)*d*g+(-
```

```
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a
*e*g-c*d*f)/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*sqrt(x*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1865 vs. $2(182) = 364$.
time = 19.26, size = 1865, normalized size = 7.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f + a*g*e + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(d^2*g^2 - 2*d*f*
g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a
^2*c*e^2))*log(-(2*d*g^2*x*e + d^2*g^2 + 2*(c*d^2*g - c*d*f*e + (a*c^2*d^2*
e + a^2*c*e^3)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2
*d^2*e^2 + a^3*c*e^4)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-(c*d*f + a*g*e +
(a*c^2*d^2 + a^2*c*e^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) - (2*f*g*x + f^2)
*e^2 - (c^2*d^3*g*x + c^2*d^2*f*x*e + 2*c^2*d^3*f + a*c*f*x*e^3 + (a*c*d*g*
x + 2*a*c*d*f)*e^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^
2*c^2*d^2*e^2 + a^3*c*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*g*e + (a*c^2*d^2 + a
^2*c*e^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*
e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(2*d*g^2*x*e + d^2*g^2 - 2
*(c*d^2*g - c*d*f*e + (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-(d^2*g^2 - 2*d*f*g*e
+ f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(g*x + f)*sqrt
(x*e + d)*sqrt(-(c*d*f + a*g*e + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(d^2*g^2 - 2
*d*f*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^
2 + a^2*c*e^2)) - (2*f*g*x + f^2)*e^2 - (c^2*d^3*g*x + c^2*d^2*f*x*e + 2*c^
2*d^3*f + a*c*f*x*e^3 + (a*c*d*g*x + 2*a*c*d*f)*e^2)*sqrt(-(d^2*g^2 - 2*d*f
*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x) - 1/4*sqrt
(-(c*d*f + a*g*e - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2
*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)
)*log(-(2*d*g^2*x*e + d^2*g^2 + 2*(c*d^2*g - c*d*f*e - (a*c^2*d^2*e + a^2*c
*e^3)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-(c*d*f + a*g*e - (a*c^2*d^
```

```
2 + a^2*c*e^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2
*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) - (2*f*g*x + f^2)*e^2 + (c
^2*d^3*g*x + c^2*d^2*f*x*e + 2*c^2*d^3*f + a*c*f*x*e^3 + (a*c*d*g*x + 2*a*c
*d*f)*e^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2
*e^2 + a^3*c*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*g*e - (a*c^2*d^2 + a^2*c*e^2)
*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(2*d*g^2*x*e + d^2*g^2 - 2*(c*d^2*g
 - c*d*f*e - (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2
)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(g*x + f)*sqrt(x*e + d)
*sqrt(-(c*d*f + a*g*e - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e
+ f^2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c
*e^2)) - (2*f*g*x + f^2)*e^2 + (c^2*d^3*g*x + c^2*d^2*f*x*e + 2*c^2*d^3*f +
 a*c*f*x*e^3 + (a*c*d*g*x + 2*a*c*d*f)*e^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^
2*e^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)\,\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+a),x)
```

```
[Out] Integral(sqrt(f + g*x)/((a + c*x**2)*sqrt(d + e*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.608 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx$

Optimal. Leaf size=351

$$-\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}}+\frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}\,(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,(cd^2+ae^2)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{(cdf$$

```
[Out] -2*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c
*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(-e*(-a)
^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/
2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1
/2))*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]

time = 1.36, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {922, 37, 6857, 95, 214}

$$-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,(ae^2+cd^2)}+\frac{\left(\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,(ae^2+cd^2)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(-\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,(ae^2+cd^2)\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + ((c*d*f + a*e*g + Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f +
a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 922

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(-g)*((e*f - d*g)/(c*f^2 + a*g^2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[Simp[c*d*f + a*e*g
 + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)), x
], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+cx^2)}\,dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} + \frac{\int \left(\frac{a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{-a\sqrt{\ldots}}{2a\left(\sqrt{-a}\ldots\right.}\ldots\right.}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\,\sqrt{c}\,(ef-dg)\right)\int \frac{\ldots}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\ldots}}{2\sqrt{-a}\,(cd^2+ae^2)\ldots}$$

$$= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\sqrt{-a}\,\sqrt{c}\,(ef-dg)\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{c}\,d+\ldots}\right.}{\sqrt{-a}\,(cd^2+\ldots}$$

$$= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} + \frac{\left(cdf+aeg+\sqrt{-a}\,\sqrt{c}\,(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\ldots}}{\sqrt{\sqrt{c}\ldots}}\right.}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,(cd^2+ae^2)\sqrt{\sqrt{c}\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.23, size = 361, normalized size = 1.03

$$-\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} - \frac{i\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\,\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\sqrt{cd^2+ae^2}} + \frac{i\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\,\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\sqrt{cd^2+ae^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) - (I*Sqrt[-((Sqrt[c]*d
 + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqr
t[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*S
qrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]) + (
I*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt
[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[
c*d^2 + a*e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5382 vs. $2(279) = 558$.
time = 0.10, size = 5383, normalized size = 15.34

method	result	size
default	Expression too large to display	5383

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(x*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5816 vs. $2(279) = 558$.
time = 30.94, size = 5816, normalized size = 16.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2
)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*
d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2
 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*
c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a
^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10
 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))
*log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4 - 3*a*
d^2*e^2)*g^2 + 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*
a*c*d^3*e^2 + 3*a^2*d*e^4)*g - 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c
*d^3*e^5 + a^4*d*e^7)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*
f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6
*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 1
5*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e
^10 + a^7*e^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2
)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*
d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2
```

- 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)) + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x + (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/x) - (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))*log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4 - 3*a*d^2*e^2)*g^2 - 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d*e^4)*g - 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d*e^7)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)) + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x + (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/x) + (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a

```
^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))*log(((3*c*d^2*e^2 -
a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4 - 3*a*d^2*e^2)*g^2 + 2*((3*
c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d
*e^4)*g + 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{(c\,x^2+a)\,(d+e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.609 $\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx$

Optimal. Leaf size=613

$$-\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}}+\frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}}+\frac{e\left(cdf+aeg-\sqrt{-a}\,\sqrt{c}\,(ef-dg)\right)\sqrt{f}}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)\left(cd^2+ae^2\right)(ef-dg}$$

```
[Out] -2/3*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(3/2)+4/3*e*g*(g*x+f)^(1/2)/(a*e
^2+c*d^2)/(-d*g+e*f)/(e*x+d)^(1/2)+e*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(
1/2))*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(
1/2))/(e*x+d)^(1/2)-e*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*(g*x+f)^(
1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^
(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(
-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*
c^(1/2))/(a*e^2+c*d^2)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(-g*(-a)^
(1/2)+f*c^(1/2))^(1/2)+arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)*(c*d*f*(-a)^(1/2)+a*
e*g*(-a)^(1/2)+a*(-d*g+e*f)*c^(1/2))/a/(a*e^2+c*d^2)/(e*(-a)^(1/2)+d*c^(1/2
))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]

time = 1.99, antiderivative size = 613, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {922, 47, 37, 6857, 98, 95, 214}

$$\frac{e\sqrt{f+gx}\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\,\sqrt{d+ex}\,\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)(ef-dg)}-\frac{e\sqrt{f+gx}\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)}{\sqrt{-a}\,\sqrt{d+ex}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(ae^2+cd^2\right)(ef-dg)}+\frac{4eg\sqrt{f+gx}}{3\sqrt{d+ex}\,\left(ae^2+cd^2\right)(ef-dg)}-\frac{2e\sqrt{f+gx}}{3(d+ex)^{3/2}\left(ae^2+cd^2\right)}+\frac{\sqrt{c}\,\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\left(ae^2+cd^2\right)\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}+\frac{\sqrt{c}\,\left(a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,cdf+\sqrt{-a}\,aeg\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{a\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}\left(ae^2+cd^2\right)\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (-2*e*Sqrt[f + g*x])/(3*(c*d^2 + a*e^2)*(d + e*x)^(3/2)) + (4*e*g*Sqrt[f +
g*x])/(3*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (e*(c*d*f + a*e*g - S
qrt[-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]
*e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) - (e*(c*d*f + a*e*g + Sqrt[-
a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(
c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[c]*(c*d*f + a*e*g + Sqrt[
-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x
])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sq
rt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) + (Sqrt[c]*(S
qrt[-a]*c*d*f + Sqrt[-a]*a*e*g + a*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[
c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*
```

```
x])])/(a*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(c*d^2 + a*e^2)*Sqrt[Sqrt[c]*f + Sq
rt[-a]*g])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 922

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(-g)*((e*f - d*g)/(c*f^2 + a*g^2)), Int[(d + e*x)^(
```

```
m - 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[Simp[c*d*f + a*e*g
 + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)), x
], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\left(a+cx^2\right)}\,dx = \frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx}{cd^2+ae^2} + \frac{(e(ef-dg))\int \frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{\int \left(\frac{a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{-a\sqrt{\ldots}}{2a\left(\sqrt{-\ldots}\right.}\right.}{cd^2+ae^2}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} - \frac{\left(cdf+aeg-\ldots\right.}{\ldots}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+\ldots\right.}{\sqrt{-a}\,\left(\sqrt{c}\,d\ldots\right.}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+\ldots\right.}{\sqrt{-a}\,\left(\sqrt{c}\,d\ldots\right.}$$

$$= -\frac{2e\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(d+ex)^{3/2}} + \frac{4eg\sqrt{f+gx}}{3\left(cd^2+ae^2\right)(ef-dg)\sqrt{d+ex}} + \frac{e\left(cdf+\ldots\right.}{\sqrt{-a}\,\left(\sqrt{c}\,d\ldots\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.68, size = 422, normalized size = 0.69

$$\frac{2\sqrt{f+gx}\,\left(ae^4(f+gx)+cde\left(-6d^2g+6e^2fx+de(7f-5gx)\right)\right)}{3\left(cd^2+ae^2\right)^2(-ef+dg)(d+ex)^{3/2}} - \frac{i\sqrt{c}\,\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\,\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\,\sqrt{d+ex}}\right)}{\sqrt{a}\,\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)^2\sqrt{cd^2+ae^2}} + \frac{i\sqrt{c}\,\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\,\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\,\sqrt{d+ex}}\right)}{\sqrt{a}\,\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)^2\sqrt{cd^2+ae^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

```
[Out] (2*Sqrt[f + g*x]*(a*e^4*(f + g*x) + c*d*e*(-6*d^2*g + 6*e^2*f*x + d*e*(7*f
- 5*g*x))))/(3*(c*d^2 + a*e^2)^2*(-(e*f) + d*g)*(d + e*x)^(3/2)) - (I*Sqrt[
c]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqr
t[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*
f - I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d - I*Sqrt[a]*e)^2*Sq
rt[c*d^2 + a*e^2]) + (I*Sqrt[c]*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[
c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(
Sqrt[c]*d + I*Sqrt[a]*e)^2*Sqrt[c*d^2 + a*e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 14860 vs. $2(501) = 1002$.
time = 0.08, size = 14861, normalized size = 24.24

method	result	size
default	Expression too large to display	14861

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/((c*x^2 + a)*(x*e + d)^(5/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)),x)
```

```
[Out] int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)), x)
```

3.610 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal. Leaf size=337

$$\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}+\frac{\left(cd^2-2\sqrt{-a}\sqrt{c}\,de-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{f+gx}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{(c\ldots}{\ldots}$$

```
[Out] 2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)+ar
ctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^
(1/2)+d*c^(1/2))^(1/2))*(c*d^2-a*e^2-2*d*e*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e
*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^
(1/2))^(1/2))*(c*d^2-a*e^2+2*d*e*(-a)^(1/2)*c^(1/2))/c/(-a)^(1/2)/(e*(-a)^(
1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]
time = 1.63, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {924, 65, 223, 212, 6857, 95, 214}

$$\frac{\left(-2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\left(2\sqrt{-a}\sqrt{c}\,de-ae^2+cd^2\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,c\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}+\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) + ((c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f -
 Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/
(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((
c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*
g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*
c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \left(\frac{e^2}{c\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{cd^2-ae^2+2cdex}{c\sqrt{d+ex}\,\sqrt{f+gx}\,(a+cx^2)}\right)dx$$

$$= \frac{\int \frac{cd^2-ae^2+2cdex}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+cx^2)}\,dx}{c} + \frac{e^2\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{-2a\sqrt{c}\,de+\sqrt{-a}\,(cd^2-ae^2)}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{2a\sqrt{c}\,de+\sqrt{-a}\,(cd^2-ae^2)}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+g\cdots}} \cdots\right.}{c}$$

$$= \frac{(2e)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} - \frac{\left(cd^2-2\sqrt{-a}\,\sqrt{c}\,de-ae^2\right)\int \frac{\cdots}{\left(\sqrt{-\cdots}\right.}}{2\sqrt{-a}\cdots}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{\left(cd^2-2\sqrt{-a}\,\sqrt{c}\,de-ae^2\right)\text{Subst}\left(\int \frac{\cdots}{-\sqrt{c}\cdots}\right.}{\sqrt{\cdots}}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(cd^2-2\sqrt{-a}\,\sqrt{c}\,de-ae^2\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{\cdots}}}{\sqrt{\sqrt{\cdots}}}\right.}{\sqrt{-a}\,c\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.02, size = 363, normalized size = 1.08

$$\frac{\frac{\left(i\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{cd^2+ae^2}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\,\sqrt{d+ex}}\right)}{\sqrt{a}\,\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}} + \frac{\left(-i\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{cd^2+ae^2}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\,\sqrt{d+ex}}\right)}{\sqrt{a}\,\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}} + \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{d+ex}}\right)}{\sqrt{g}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*ArcTan[(Sqrt[c*d^2 + a*e^2]
*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g)
)]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f -
I*Sqrt[a]*g))]) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*ArcTan[
(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt
[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sqrt
[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[d + e*x])])/Sqrt[g])/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2335 vs. $2(257) = 514$.

time = 0.09, size = 2336, normalized size = 6.93

method	result	size
default	Expression too large to display	2336

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+
2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*
g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*
a^2*e^2*g^2*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)
^(1/2)-ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))
^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^
(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(1/2)*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x-
2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)
/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f-a*e*g+c*d*f)/c)^(1/2)+2*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*
d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*
d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*d*
e*g^2*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)
*(-a*c)^(1/2)-ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(
g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-
(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c^2*d^2*f^2*(e*g)^(1
/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)+2*ln((c*d*g*x
+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(
1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*d*e*f^2*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*
d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(
((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-
a*c)^(1/2)))*a^2*e^2*g^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/
c)^(1/2)*(e*g)^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*d^2*g^2*(-
((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)-ln((2*
(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*
x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/
2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1
/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)+2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*
```

```
x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)
^(1/2)))*a*d*e*g^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/
2)*(e*g)^(1/2)*(-a*c)^(1/2)+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d^2
*f^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)
+2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*
f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*d*e*f^2*(-((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(e*g)^(1/2)*(-a*c)^(1/2)-2*ln(1/2*(2*e
*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^2*g^2*
(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)-2*ln(1/2*(2*e*g*x+2*
((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2*(-((-a*
c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*
c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2))/((e*x+d)*(g*x+f))^(1/2)/(g
*(-a*c)^(1/2)+c*f)/(-a*c)^(1/2)/(e*g)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)
*e*f-a*e*g+c*d*f)/c)^(1/2)/(c*f-g*(-a*c)^(1/2))/(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate((x*e + d)^(3/2)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/((a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.7index.cc index_m
i_lex_is_greater Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)), x)
```

3.611 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-\cdots}}}{\sqrt{\sqrt{c}\,d+\sqrt{-a\cdots}}}\right.}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))*(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2
)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(
1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(e*(-a)^(1/2)+d*c
^(1/2))^(1/2)/(-a)^(1/2)/c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {924, 95, 214}

$$\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sq
rt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
+ g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \left(\frac{\sqrt{-a}\,d-\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{\sqrt{-a}\,d+\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{\vphantom{f}}}\right.$$

$$=\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\int\frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx+\frac{1}{2}\left(\frac{ad}{(-a)^{3/2}}\right.$$

$$=\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c}\,d+\sqrt{-a}\,e-\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)x^2}\,dx,x,\frac{\sqrt{d}}{\sqrt{f}}\right.$$

$$=\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{c}\,d+\sqrt{-a}}}{}$$

Mathematica [A]
time = 10.26, size = 229, normalized size = 0.95

$$\frac{\frac{\sqrt{-\sqrt{c}\,d+\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{-\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{-\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-\sqrt{c}\,f+\sqrt{-a}\,g}}-\frac{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}}{\sqrt{-a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*
Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(Sqr
t[c]*f) + Sqrt[-a]*g] - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x]
)]])/Sqrt[Sqrt[c]*f + Sqrt[-a]*g])/(Sqrt[-a]*Sqrt[c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(176) = 352$.
time = 0.07, size = 1387, normalized size = 5.78

method	result
default	$\frac{\sqrt{ex+d}\,\sqrt{gx+f}\left(\sqrt{\frac{\sqrt{-ac}\,dg+\sqrt{-ac}\,ef-aeg+cdf}{c}}\,\ln\left(\frac{cdgx+cefx-2\sqrt{-ac}\,egx+2cdf+2\sqrt{(ex+d)\,(gx}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*((((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+
c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)
*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*
c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*c*d*g^2-(((-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((c*d*g*x+
c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1
/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*e*g^2+(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*
e*g+c*d*f)/c)^(1/2)*ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*
x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1
/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c^2*d*f^2-(((-
a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln((c*d*
g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c
)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*c*e*f^2-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1
/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*d*g^2-
(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)*ln(
(2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*(
(e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^
(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*e*g^2-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c^2*d*
f^2-(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*(-a*c)^(1/2)
*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)
/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*c*e*f^2)/((e*x+d)*(g*x+f))^(1/2)/(
g*(-a*c)^(1/2)+c*f)/(-a*c)^(1/2)/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+
```

```
c*d*f)/c)^(1/2)/(c*f-g*(-a*c)^(1/2))/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a
*e*g-c*d*f)/c)^(1/2)
```

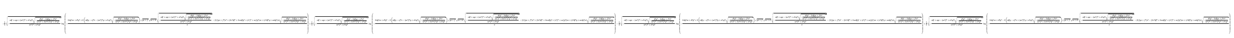
Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x*e + d)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1929 vs. $2(182) = 364$.
time = 19.96, size = 1929, normalized size = 8.04



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(-(c*d*f + a*g*e + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(d^2*g^2 - 2*d*f*
g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a
^2*c*g^2))*log(-(2*d*g^2*x*e + d^2*g^2 + 2*(c*d*f*g - c*f^2*e - (a*c^2*f^2*
g + a^2*c*g^3)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2
*f^2*g^2 + a^3*c*g^4)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-(c*d*f + a*g*e +
(a*c^2*f^2 + a^2*c*g^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*f^4 +
2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) - (2*f*g*x + f^2)
*e^2 + (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*f^3 + a*c*f*g^2)*x*e + (c^2*d*f^
2*g + a*c*d*g^3)*x)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^
2*c^2*f^2*g^2 + a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*g*e + (a*c^2*f^2 + a
^2*c*g^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*
g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(2*d*g^2*x*e + d^2*g^2 - 2
*(c*d*f*g - c*f^2*e - (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(d^2*g^2 - 2*d*f*g*e
+ f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(g*x + f)*sqrt
(x*e + d)*sqrt(-(c*d*f + a*g*e + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(d^2*g^2 - 2
*d*f*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^
2 + a^2*c*g^2)) - (2*f*g*x + f^2)*e^2 + (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2
*f^3 + a*c*f*g^2)*x*e + (c^2*d*f^2*g + a*c*d*g^3)*x)*sqrt(-(d^2*g^2 - 2*d*f
*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x) - 1/4*sqrt
(-(c*d*f + a*g*e - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2
*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)
)*log(-(2*d*g^2*x*e + d^2*g^2 + 2*(c*d*f*g - c*f^2*e + (a*c^2*f^2*g + a^2*c
*g^3)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2
+ a^3*c*g^4)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-(c*d*f + a*g*e - (a*c^2*f^
```

```
2 + a^2*c*g^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2
*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) - (2*f*g*x + f^2)*e^2 - (2
*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*f^3 + a*c*f*g^2)*x*e + (c^2*d*f^2*g + a*c
*d*g^3)*x)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2
*g^2 + a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*g*e - (a*c^2*f^2 + a^2*c*g^2)
*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3
*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(2*d*g^2*x*e + d^2*g^2 - 2*(c*d*f*g
 - c*f^2*e + (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^2*e^2
)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(g*x + f)*sqrt(x*e + d)
*sqrt(-(c*d*f + a*g*e - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(d^2*g^2 - 2*d*f*g*e
+ f^2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c
*g^2)) - (2*f*g*x + f^2)*e^2 - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*f^3 + a*
c*f*g^2)*x*e + (c^2*d*f^2*g + a*c*d*g^3)*x)*sqrt(-(d^2*g^2 - 2*d*f*g*e + f^
2*e^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(d + e*x)/((a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
[Out] \text{Hanged}
```

3.612 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal. Leaf size=230

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}}$$

```
[Out] arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a
)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1
/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(e*(-a)^(1/2)+
d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {926, 95, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt
[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]
*f - Sqrt[-a]*g]) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(S
qrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
& !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{\sqrt{-}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right)}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+}}}{2\sqrt{-a}}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{-\sqrt{c}\,d+\sqrt{-a}\,e-\left(-\sqrt{c}\,f+\sqrt{-a}\,g\right)x^2}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{\sqrt{-a}} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-}}{\sqrt{\sqrt{c}\,d+}}\right.}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d+\sqrt{-}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.86, size = 286, normalized size = 1.24

$$\frac{\sqrt[4]{-1}\left(-\frac{\sqrt{-i\sqrt{c}\,d+\sqrt{a}\,e}\,\tan^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-i\sqrt{c}\,d+\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}\,\sqrt{d+ex}}\right)}{\sqrt{\sqrt{c}\,f-i\sqrt{a}\,g}}+\frac{\sqrt{i\sqrt{c}\,d+\sqrt{a}\,e}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{i\sqrt{c}\,d+\sqrt{a}\,e}\,\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}\,\sqrt{d+ex}}\right)}{\sqrt{\sqrt{c}\,f+i\sqrt{a}\,g}}\right)}{\sqrt{a}\,\sqrt{cd^2+ae^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] ((-1)^(1/4)*(-((Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*ArcTan[((-1)^(1/4)*Sqrt[c*
d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c]*
f - I*Sqrt[a]*g]*Sqrt[d + e*x])])/Sqrt[Sqrt[c]*f - I*Sqrt[a]*g]) + (Sqrt[I*
```

Sqrt[c]*d + Sqrt[a]*e]*ArcTanh[((-1)^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x
])/(Sqrt[I*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c]*f + I*Sqrt[a]*g]*Sqrt[d + e*
x])]])/Sqrt[Sqrt[c]*f + I*Sqrt[a]*g]))/(Sqrt[a]*Sqrt[c*d^2 + a*e^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(170) = 340$.
time = 0.10, size = 1415, normalized size = 6.15

method	result
default	$\frac{c^2\left(\ln\left(\frac{cdgx+cefx-2\sqrt{-ac}\,egx+2cdf+2\sqrt{(ex+d)(gx+f)}\,\sqrt{-\frac{\sqrt{-ac}\,dg+\sqrt{-ac}\,ef+aeg-cdf}{c}}\,c-\sqrt{-ac}\,dg-\cdots}{cx+\sqrt{-ac}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*c^2*(ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f
))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c
)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a^2*e^2*g^2*(((-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x-2*(-a*c)^(
1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/
2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*
c)^(1/2)))*a*c*d^2*g^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^
(1/2)+ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^
(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(
1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(((-a*c)^(1/2)*d
*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)+ln((c*d*g*x+c*e*f*x-2*(-a*c)^(1/2
)*e*g*x+2*c*d*f+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*
e*f+a*e*g-c*d*f)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f)/(c*x+(-a*c)^
(1/2)))*c^2*d^2*f^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/
2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
f+2((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*
f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2*(-((-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e
*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2
)))*a*c*d^2*g^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)-
ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+
2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/
c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(-((-a*c)^(1/2)*d*g+(-a
*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*
x+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*d*f)/c)^(1/2)*c+2*c*d*f)/(c*x-(-a*c)^(1/2)))

```
*c^2*d^2*f^2*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)/c)^(1/2))*(g
*x+f)^(1/2)*(e*x+d)^(1/2)/(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-c*d*f)
/c)^(1/2)/(c*f-g*(-a*c)^(1/2))/(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+c*
d*f)/c)^(1/2)/(g*(-a*c)^(1/2)+c*f)/(-a*c)^(1/2)/(c*d-(-a*c)^(1/2)*e)/((-a*c
)^(1/2)*e+c*d)/((e*x+d)*(g*x+f))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*sqrt(g*x + f)*sqrt(x*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4509 vs. $2(176) = 352$.
time = 61.51, size = 4509, normalized size = 19.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(c*d*f - a*g*e + (a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a
^3*g^2)*e^2)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2
*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a
^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e
^2)))/(a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2))*log((d^2
*g^2 + 2*(c*d^2*f*g - a*f*g*e^2 + (c*d*f^2 - a*d*g^2)*e - (a*c^2*d^3*f^2*g
+ a^2*c*d^3*g^3 + (a^2*c*f^3 + a^3*f*g^2)*e^3 + (a^2*c*d*f^2*g + a^3*d*g^3)
*e^2 + (a*c^2*d^2*f^3 + a^2*c*d^2*f*g^2)*e)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e
+ c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^
3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2
*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-(c*d
*f - a*g*e + (a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2)*sq
rt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^
2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2
*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))/(a*c^2*d^
2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2)) + (2*f*g*x + f^2)*e^2 +
 2*(d*g^2*x + d*f*g)*e + (2*c^2*d^3*f^3 + 2*a*c*d^3*f*g^2 + (a*c*f^3 + a^2*
f*g^2)*x*e^3 + (c^2*d^2*f^3 + a*c*d^2*f*g^2)*x*e + (c^2*d^3*f^2*g + a*c*d^3
*g^3)*x + (2*a*c*d*f^3 + 2*a^2*d*f*g^2 + (a*c*d*f^2*g + a^2*d*g^3)*x)*e^2)*
sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*
f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 +
```

2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))/x) + 1/4*sqrt(-(c*d*f - a*g*e + (a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))/(a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2))*log((d^2*g^2 - 2*(c*d^2*f*g - a*f*g*e^2 + (c*d*f^2 - a*d*g^2)*e - (a*c^2*d^3*f^2*g + a^2*c*d^3*g^3 + (a^2*c*f^3 + a^3*f*g^2)*e^3 + (a^2*c*d*f^2*g + a^3*d*g^3)*e^2 + (a*c^2*d^2*f^3 + a^2*c*d^2*f*g^2)*e)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-(c*d*f - a*g*e + (a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))/(a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2)) + (2*f*g*x + f^2)*e^2 + 2*(d*g^2*x + d*f*g)*e + (2*c^2*d^3*f^3 + 2*a*c*d^3*f*g^2 + (a*c*f^3 + a^2*f*g^2)*x*e^3 + (c^2*d^2*f^3 + a*c*d^2*f*g^2)*x*e + (c^2*d^3*f^2*g + a*c*d^3*g^3)*x + (2*a*c*d*f^3 + 2*a^2*d*f*g^2 + (a*c*d*f^2*g + a^2*d*g^3)*x)*e^2)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))/x) - 1/4*sqrt(-(c*d*f - a*g*e - (a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))/(a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2))*log((d^2*g^2 + 2*(c*d^2*f*g - a*f*g*e^2 + (c*d*f^2 - a*d*g^2)*e + (a*c^2*d^3*f^2*g + a^2*c*d^3*g^3 + (a^2*c*f^3 + a^3*f*g^2)*e^3 + (a^2*c*d*f^2*g + a^3*d*g^3)*e^2 + (a*c^2*d^2*f^3 + a^2*c*d^2*f*g^2)*e)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-(c*d*f - a*g*e - (a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))/(a*c^2*d^2*f^2 + a^2*c*d^2*g^2 + (a^2*c*f^2 + a^3*g^2)*e^2)) + (2*f*g*x + f^2)*e^2 + 2*(d*g^2*x + d*f*g)*e - (2*c^2*d^3*f^3 + 2*a*c*d^3*f*g^2 + (a*c*f^3 + a^2*f*g^2)*x*e^3 + (c^2*d^2*f^3 + a*c*d^2*f*g^2)*x*e + (c^2*d^3*f^2*g + a*c*d^3*g^3)*x + (2*a*c*d*f^3 + 2*a^2*d*f*g^2 + (a*c*d*f^2*g + a^2*d*g^3)*x)*e^2)*sqrt(-(c*d^2*g^2 + 2*c*d*f*g*e + c*f^2*e^2)/(a*c^4*d^4*f^4 + 2*a^2*c^3*d^4*f^2*g^2 + a^3*c^2*d^4*g^4 + (a^3*c^2*f^4 + 2*a^4*c*f^2*g^2 + a^5*g^4)*e^4 + 2*(a^2*c^3*d^2*f^4 + 2*a^3*c^2*d^2*f^2*g^2 + a^4*c*d^2*g^4)*e^2)))/x) + 1/4*sqrt

```
(-(c*d*f - a*g*e - (a*c^2*d^2*f^2 + a^2*c*d^2*g...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.613 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal. Leaf size=354

$$-\frac{e\sqrt{f+gx}}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}}+\frac{\sqrt{c}\,\tanh^{-1}\Bigg(}{\sqrt{-a}\,\left(\sqrt{c}\,d-\right.}$$

```
[Out] -e*(g*x+f)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1
/2)+e*(g*x+f)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^
(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(
-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2)
)^(3/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)
+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a
)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {926, 98, 95, 214}

$$-\frac{e\sqrt{f+gx}}{\sqrt{-a}\,\sqrt{d+ex}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\,\sqrt{d+ex}\,\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ef-dg)}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] -((e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d +
 e*x])) + (e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*
Sqrt[d + e*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*
x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - S
qrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqrt[Sqr
t[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])]])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*
g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}} + \frac{}{2a\left(\sqrt{-a}+\sqrt{c}\,\right.}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/}}}{2\sqrt{-a}}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{}\right.}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{}\right.}$$

$$= -\frac{e\sqrt{f+gx}}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}} + \frac{e}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.96, size = 383, normalized size = 1.08

$$\frac{2e^2\sqrt{f+gx}}{(cd^2+ae^2)(-ef+dg)\sqrt{d+ex}}+\frac{i\sqrt{c}\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)^2\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\,\sqrt{d+ex}}\right)}{\sqrt{a}\,(cd^2+ae^2)^{3/2}\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}}-\frac{i\sqrt{c}\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)^2\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\,\sqrt{d+ex}}\right)}{\sqrt{a}\,(cd^2+ae^2)^{3/2}\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x])/((c*d^2 + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*x]) + (I*S
qrt[c]*(Sqrt[c]*d + I*Sqrt[a]*e)^2*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x
])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d + e
*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqr
t[c]*f - I*Sqrt[a]*g))]) - (I*Sqrt[c]*(Sqrt[c]*d - I*Sqrt[a]*e)^2*ArcTan[(S
qrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c
]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-
((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. 2(270) = 540.
time = 0.09, size = 10977, normalized size = 31.01

method	result	size
default	Expression too large to display	10977

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11846 vs. 2(270) = 540.
time = 50.32, size = 11846, normalized size = 33.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(8*sqrt(e*x + d)*sqrt(g*x + f)*e^2 + ((c*d^3*e + a*d*e^3)*f - (c*d^4 +
 a*d^2*e^2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)*sqrt(-((
c^3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g + ((a*c^4*d^6 +
3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a
^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*c^5*d^4*e^2 - 6*
a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^
2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a
*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 +
15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^
12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*
c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*
a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4
*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2
+ 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3
*a^4*c*d^2*e^4 + a^5*e^6)*g^2))*log(-((3*c^3*d^2*e^2 - a*c^2*e^4)*f^2 + 4*(
c^3*d^3*e - a*c^2*d*e^3)*f*g + (c^3*d^4 - 3*a*c^2*d^2*e^2)*g^2 + 2*((3*c^4*
d^4*e - 4*a*c^3*d^2*e^3 + a^2*c^2*e^5)*f^2 + (c^4*d^5 - 10*a*c^3*d^3*e^2 +
5*a^2*c^2*d*e^4)*f*g - 2*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*g^2 - (2*(a*c^5*
d^7*e + 3*a^2*c^4*d^5*e^3 + 3*a^3*c^3*d^3*e^5 + a^4*c^2*d*e^7)*f^3 + (a*c^5
*d^8 + 2*a^2*c^4*d^6*e^2 - 2*a^4*c^2*d^2*e^6 - a^5*c*e^8)*f^2*g + 2*(a^2*c^
4*d^7*e + 3*a^3*c^3*d^5*e^3 + 3*a^4*c^2*d^3*e^5 + a^5*c*d*e^7)*f*g^2 + (a^2
*c^4*d^8 + 2*a^3*c^3*d^6*e^2 - 2*a^5*c*d^2*e^6 - a^6*e^8)*g^3)*sqrt(-((9*c^
5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*
d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2
*e^4)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*
c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 +
 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6
*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3
*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 +
15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))*sqrt(e*x + d)*sqrt
(g*x + f)*sqrt(-((c^3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*
g + ((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 +
(a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((
9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*
c^4*d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3
*d^2*e^4)*g^2)/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*
a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f
^4 + 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4
*d^6*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 +
(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^
6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/((a*c^4*d^6 +
```

```
3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a
^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)) + 2*((3*c^3*d^2*e^2 - a*c
^2*e^4)*f*g + (c^3*d^3*e - 3*a*c^2*d*e^3)*g^2)*x + (2*(c^5*d^7 + 3*a*c^4*d^
5*e^2 + 3*a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^3 + 2*(a*c^4*d^7 + 3*a^2*c^3*d
^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*f*g^2 + ((c^5*d^6*e + 3*a*c^4*d^4
*e^3 + 3*a^2*c^3*d^2*e^5 + a^3*c^2*e^7)*f^3 + (c^5*d^7 + 3*a*c^4*d^5*e^2 +
3*a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^2*g + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3
 + 3*a^3*c^2*d^2*e^5 + a^4*c*e^7)*f*g^2 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 +
3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*g^3)*x)*sqrt(-((9*c^5*d^4*e^2 - 6*a*c^4*d^
2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d*
e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^1
2 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c
^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6*a
^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*
e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^5*
d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6
*a^8*c*d^2*e^10 + a^9*e^12)*g^4)))/x) - ((c*d^3*e + a*d*e^3)*f - (c*d^4 + a
*d^2*e^2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)*sqrt(-((c^
3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g + ((a*c^4*d^6 + 3*
a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3
*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*sqrt(-((9*c^5*d^4*e^2 - 6*a*
c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*
c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c
^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15
*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12
 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+cx^2)\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.614 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal. Leaf size=625

$$\frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}}-\frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)}-\frac{\sqrt{e}\,\left(cdf+aeg-\sqrt{-a}\,\sqrt{c}\,(ef-dg)\right)\text{ta}}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,(cf^2+}$$

```
[Out] -2*(-d*g+e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*e^(1/2)/
(a*g^2+c*f^2)/g^(1/2)-arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*
(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*e^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2
)/c^(1/2)/g^(1/2)+arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))*(c*d
*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*e^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^
(1/2)/g^(1/2)+2*(-d*g+e*f)*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+arctan
h((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2
)+d*c^(1/2))^(1/2))*(c*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))*(-e*(-a)^(1
/2)+d*c^(1/2))^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^(1/2)/(-g*(-a)^(1/2)+f*c^(1
/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/
2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/
2))*(e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(a*g^2+c*f^2)/(-a)^(1/2)/c^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]

time = 1.59, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {922, 49, 65, 223, 212, 6857, 132, 12, 95, 214}

$$\frac{2\sqrt{d+ex}\,(ef-dg)}{\sqrt{f+gx}\,(ag^2+cf^2)}-\frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(ag^2+cf^2)}-\frac{\sqrt{e}\,\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,(ag^2+cf^2)}+\frac{\sqrt{e}\,\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{g}\,(ag^2+cf^2)}+\frac{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(-\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,(ag^2+cf^2)}-\frac{\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\left(\sqrt{-a}\,\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{c}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,(ag^2+cf^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*(e*f - d*g)*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[e]*
(e*f - d*g)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt
[g]*(c*f^2 + a*g^2)) - (Sqrt[e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*
g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqr
t[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*
(e*f - d*g))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqr
t[-a]*Sqrt[c]*Sqrt[g]*(c*f^2 + a*g^2)) + (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d
*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-
a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-
a]*Sqrt[c]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - (Sqrt[Sqrt[c]*d
```

```
+ Sqrt[-a]*e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[
Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
 + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 922

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(-g)*((e*f - d*g)/(c*f^2 + a*g^2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[Simp[c*d*f + a*e*g
 + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)), x
], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{\int \frac{\sqrt{d+ex}\,(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}\,(a+cx^2)}\,dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}}\,dx}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\int \left(\frac{\left(-a\sqrt{c}\,(ef-dg)+\sqrt{-a}\,(cdf+aeg)\right)\sqrt{d+ex}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{f+gx}} + \frac{\left(a\sqrt{\cdots}\right.}{\cdots}\right.}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{(2(ef-dg))\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{dg}{e}+\frac{gx^2}{e}}}\,dx, x, \sqrt{d}\cdots\right.}{cf^2+ag^2}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{(2(ef-dg))\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{cf^2+ag^2} - \cdots$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)} - \frac{(cdf+\cdots}{\cdots}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)} + \frac{\sqrt{\sqrt{c}\cdots}}{\cdots}$$

$$= \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{2\sqrt{e}\,(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}\,(cf^2+ag^2)} - \frac{\sqrt{e}\,(c\cdots}{\cdots}$$

Mathematica [A]

time = 10.55, size = 336, normalized size = 0.54

$$-\left(\left(\frac{d}{\sqrt{-a}}-\frac{e}{\sqrt{c}}\right)\left(\frac{\sqrt{d+ex}}{\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)\sqrt{f+gx}}+\frac{\sqrt{-\sqrt{c}\,d+\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{-\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{-\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\left(-\sqrt{c}\,f+\sqrt{-a}\,g\right)^{3/2}}\right)\right)-\left(\frac{ad}{(-a)^{3/2}}-\frac{e}{\sqrt{c}}\right)\left(\frac{\sqrt{d+ex}}{\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)\sqrt{f+gx}}-\frac{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f+\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d+\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -((d/Sqrt[-a] - e/Sqrt[c])*(Sqrt[d + e*x]/((Sqrt[c]*f - Sqrt[-a]*g)*Sqrt[f
+ g*x]) + (Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqr
t[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])])/(
-(Sqrt[c]*f) + Sqrt[-a]*g)^(3/2))) - ((a*d)/(-a)^(3/2) - e/Sqrt[c])*(Sqrt[d
 + e*x]/((Sqrt[c]*f + Sqrt[-a]*g)*Sqrt[f + g*x]) - (Sqrt[Sqrt[c]*d + Sqrt[-
a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d
+ Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8263 vs. $2(497) = 994$.
time = 0.08, size = 8264, normalized size = 13.22

method	result	size
default	Expression too large to display	8264

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^{3/2}\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

3.615 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal. Leaf size=351

$$-\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}}+\frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}\,(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,\sqrt{d+ex}}{\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{f+gx}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,(cf^2+ag^2)}-\frac{(cd}{}$$

```
[Out] -2*g*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+arctanh((e*x+d)^(1/2)*(-g*(-
a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2))*(c
*d*f+a*e*g-(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*g^2+c*f^2)/(-a)^(1/2)/(-e*(-a)
^(1/2)+d*c^(1/2))^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/
2)*(g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1
/2))*(c*d*f+a*e*g+(-d*g+e*f)*(-a)^(1/2)*c^(1/2))/(a*g^2+c*f^2)/(-a)^(1/2)/(
e*(-a)^(1/2)+d*c^(1/2))^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(1/2)
```

Rubi [A]
time = 1.16, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {922, 37, 6857, 95, 214}

$$-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}+\frac{\left(-\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}\,(ag^2+cf^2)}-\frac{\left(\sqrt{-a}\sqrt{c}\,(ef-dg)+aeg+cdf\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}\,(ag^2+cf^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + ((c*d*f + a*e*g - Sq
rt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*(c*f^2 + a*g^2)) - ((c*d*f +
a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sq
rt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*(c*f^2 + a*g^2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 922

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Dist[(-g)*((e*f - d*g)/(c*f^2 + a*g^2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n, x], x] + Dist[1/(c*f^2 + a*g^2), Int[Simp[c*d*f + a*e*g
 + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)), x
], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)}\,dx}{cf^2+ag^2} - \frac{(g(ef-dg))\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}}\,dx}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\int \left(\frac{-a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}} + \frac{a\sqrt{\ldots}}{2a\left(\sqrt{-\ldots}\right.}\ldots\right)}{cf^2+ag^2}$$

$$= -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\int \frac{\ldots}{\left(\sqrt{-a}+\sqrt{c}x\ldots\right.}}{2\sqrt{-a}(cf^2+ag^2)}$$

$$= -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} - \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\text{Subst}\left(\int \frac{\ldots}{-\sqrt{c}\ldots}\right.}{\sqrt{-a}(cf^{2}\ldots}$$

$$= -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{c}\ldots}}{\sqrt{\sqrt{c}\ldots}}\right.}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.24, size = 361, normalized size = 1.03

$$-\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{i\sqrt{-\left((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g)\right)}\tan^{-1}\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}(\sqrt{c}f+i\sqrt{a}g)\sqrt{cf^2+ag^2}} - \frac{i\sqrt{-\left((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g)\right)}\tan^{-1}\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}(\sqrt{c}f-i\sqrt{a}g)\sqrt{cf^2+ag^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (I*Sqrt[-((Sqrt[c]*d
 + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqr
t[d + e*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*S
qrt[f + g*x])])/(Sqrt[a]*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[c*f^2 + a*g^2]) - (
I*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt
[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*Sqrt[f + g*x])])/(Sqrt[a]*(Sqrt[c]*f - I*Sqrt[a]*g)*Sqrt[
c*f^2 + a*g^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5382 vs. 2(279) = 558.
time = 0.08, size = 5383, normalized size = 15.34

method	result	size
default	Expression too large to display	5383

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5929 vs. 2(290) = 580.
time = 121.11, size = 5929, normalized size = 16.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((c*f^3 + a*f*g^2 + (c*f^2*g + a*g^3)*x)*sqrt(-(c^2*d*f^3 - 3*a*c*d*f*
g^2 + (3*a*c*f^2*g - a^2*g^3)*e + (a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*
f^2*g^4 + a^4*g^6)*sqrt(-(9*c^3*d^2*f^4*g^2 - 6*a*c^2*d^2*f^2*g^4 + a^2*c*d
^2*g^6 + (c^3*f^6 - 6*a*c^2*f^4*g^2 + 9*a^2*c*f^2*g^4)*e^2 - 2*(3*c^3*d*f^5
*g - 10*a*c^2*d*f^3*g^3 + 3*a^2*c*d*f*g^5)*e)/(a*c^6*f^12 + 6*a^2*c^5*f^10*
g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*
c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 +
 a^4*g^6))*log((3*c*d^2*f^2*g^2 - a*d^2*g^4 + 2*(3*c^2*d*f^4*g - 4*a*c*d*f^
2*g^3 + a^2*d*g^5 - (c^2*f^5 - 4*a*c*f^3*g^2 + 3*a^2*f*g^4)*e - 2*(a*c^3*f^
7*g + 3*a^2*c^2*f^5*g^3 + 3*a^3*c*f^3*g^5 + a^4*f*g^7)*sqrt(-(9*c^3*d^2*f^4
*g^2 - 6*a*c^2*d^2*f^2*g^4 + a^2*c*d^2*g^6 + (c^3*f^6 - 6*a*c^2*f^4*g^2 + 9
*a^2*c*f^2*g^4)*e^2 - 2*(3*c^3*d*f^5*g - 10*a*c^2*d*f^3*g^3 + 3*a^2*c*d*f*g
^5)*e)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f
^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))*sqrt(g*x + f)*
sqrt(x*e + d)*sqrt(-(c^2*d*f^3 - 3*a*c*d*f*g^2 + (3*a*c*f^2*g - a^2*g^3)*e
+ (a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(9*c^3*
d^2*f^4*g^2 - 6*a*c^2*d^2*f^2*g^4 + a^2*c*d^2*g^6 + (c^3*f^6 - 6*a*c^2*f^4*
```

g^2 + 9*a^2*c*f^2*g^4)*e^2 - 2*(3*c^3*d*f^5*g - 10*a*c^2*d*f^3*g^3 + 3*a^2*
c*d*f*g^5)*e)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^
4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*
f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)) - (c*f^4 - 3*a*f^2*g^
2 + 2*(c*f^3*g - 3*a*f*g^3)*x)*e^2 + 2*(c*d*f^3*g + a*d*f*g^3 + (3*c*d*f^2*
g^2 - a*d*g^4)*x)*e + (2*c^3*d*f^7 + 6*a*c^2*d*f^5*g^2 + 6*a^2*c*d*f^3*g^4
+ 2*a^3*d*f*g^6 + (c^3*f^7 + 3*a*c^2*f^5*g^2 + 3*a^2*c*f^3*g^4 + a^3*f*g^6)
*x*e + (c^3*d*f^6*g + 3*a*c^2*d*f^4*g^3 + 3*a^2*c*d*f^2*g^5 + a^3*d*g^7)*x)
*sqrt(-(9*c^3*d^2*f^4*g^2 - 6*a*c^2*d^2*f^2*g^4 + a^2*c*d^2*g^6 + (c^3*f^6
- 6*a*c^2*f^4*g^2 + 9*a^2*c*f^2*g^4)*e^2 - 2*(3*c^3*d*f^5*g - 10*a*c^2*d*f^
3*g^3 + 3*a^2*c*d*f*g^5)*e)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f
^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g
^12)))/x) - (c*f^3 + a*f*g^2 + (c*f^2*g + a*g^3)*x)*sqrt(-(c^2*d*f^3 - 3*a*
c*d*f*g^2 + (3*a*c*f^2*g - a^2*g^3)*e + (a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*
a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(9*c^3*d^2*f^4*g^2 - 6*a*c^2*d^2*f^2*g^4 + a
^2*c*d^2*g^6 + (c^3*f^6 - 6*a*c^2*f^4*g^2 + 9*a^2*c*f^2*g^4)*e^2 - 2*(3*c^3
*d*f^5*g - 10*a*c^2*d*f^3*g^3 + 3*a^2*c*d*f*g^5)*e)/(a*c^6*f^12 + 6*a^2*c^5
*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 +
6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2
*g^4 + a^4*g^6))*log((3*c*d^2*f^2*g^2 - a*d^2*g^4 - 2*(3*c^2*d*f^4*g - 4*a*
c*d*f^2*g^3 + a^2*d*g^5 - (c^2*f^5 - 4*a*c*f^3*g^2 + 3*a^2*f*g^4)*e - 2*(a*
c^3*f^7*g + 3*a^2*c^2*f^5*g^3 + 3*a^3*c*f^3*g^5 + a^4*f*g^7)*sqrt(-(9*c^3*d
^2*f^4*g^2 - 6*a*c^2*d^2*f^2*g^4 + a^2*c*d^2*g^6 + (c^3*f^6 - 6*a*c^2*f^4*g
^2 + 9*a^2*c*f^2*g^4)*e^2 - 2*(3*c^3*d*f^5*g - 10*a*c^2*d*f^3*g^3 + 3*a^2*c
*d*f*g^5)*e)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4
*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))*sqrt(g*x
+ f)*sqrt(x*e + d)*sqrt(-(c^2*d*f^3 - 3*a*c*d*f*g^2 + (3*a*c*f^2*g - a^2*g
^3)*e + (a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(
9*c^3*d^2*f^4*g^2 - 6*a*c^2*d^2*f^2*g^4 + a^2*c*d^2*g^6 + (c^3*f^6 - 6*a*c^
2*f^4*g^2 + 9*a^2*c*f^2*g^4)*e^2 - 2*(3*c^3*d*f^5*g - 10*a*c^2*d*f^3*g^3 +
3*a^2*c*d*f*g^5)*e)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 +
20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(
a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)) - (c*f^4 - 3*a*
f^2*g^2 + 2*(c*f^3*g - 3*a*f*g^3)*x)*e^2 + 2*(c*d*f^3*g + a*d*f*g^3 + (3*c*
d*f^2*g^2 - a*d*g^4)*x)*e + (2*c^3*d*f^7 + 6*a*c^2*d*f^5*g^2 + 6*a^2*c*d*f^
3*g^4 + 2*a^3*d*f*g^6 + (c^3*f^7 + 3*a*c^2*f^5*g^2 + 3*a^2*c*f^3*g^4 + a^3*
f*g^6)*x*e + (c^3*d*f^6*g + 3*a*c^2*d*f^4*g^3 + 3*a^2*c*d*f^2*g^5 + a^3*d*g
^7)*x)*sqrt(-(9*c^3*d^2*f^4*g^2 - 6*a*c^2*d^2*f^2*g^4 + a^2*c*d^2*g^6 + (c^
3*f^6 - 6*a*c^2*f^4*g^2 + 9*a^2*c*f^2*g^4)*e^2 - 2*(3*c^3*d*f^5*g - 10*a*c^
2*d*f^3*g^3 + 3*a^2*c*d*f*g^5)*e)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3
*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 +
a^7*g^12)))/x) + (c*f^3 + a*f*g^2 + (c*f^2*g + a*g^3)*x)*sqrt(-(c^2*d*f^3
- 3*a*c*d*f*g^2 + (3*a*c*f^2*g - a^2*g^3)*e - (a*c^3*f^6 + 3*a^2*c^2*f^4*g^
2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*sqrt(-(9*c^3*d^2*f^4*g^2 - 6*a*c^2*d^2*f^2*g
^4 + a^2*c*d^2*g^6 + (c^3*f^6 - 6*a*c^2*f^4*g^2 + 9*a^2*c*f^2*g^4)*e^2 - 2*

```
(3*c^3*d*f^5*g - 10*a*c^2*d*f^3*g^3 + 3*a^2*c*d*f*g^5)*e)/(a*c^6*f^12 + 6*a
^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*
g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3
*c*f^2*g^4 + a^4*g^6))*log((3*c*d^2*f^2*g^2 - a*d^2*g^4 + 2*(3*c^2*d*f^4*g
- 4*a*c*d*f^2*g^3 + a^2*d*g^5 - (c^2*f^5 - 4*a*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
[Out] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

3.616 $\int \frac{1}{\sqrt{d+ex}\,(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal. Leaf size=354

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\,\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\,\left(\sqrt{c}\,f+\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}}+\frac{\sqrt{c}\,\tanh^{-1}\Bigg(\ldots}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d\ldots}}$$

```
[Out] g*(e*x+d)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/
2)-g*(e*x+d)^(1/2)/(-d*g+e*f)/(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(
1/2)+arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-
e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)^(1/2)/(-g*(-a)^(1/2)+f*c^(1/2))
^(3/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(1/2)-arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+
f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))*c^(1/2)/(-a)
^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))^(3/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {926, 98, 95, 214}

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\,\sqrt{f+gx}\,\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\,\sqrt{f+gx}\,\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)(ef-dg)}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}\,\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)^{3/2}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}\,\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f + g
*x]) - (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)*Sqr
t[f + g*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])
/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d -
Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c
]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x
])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\,(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+gx)^{3/2}}+\frac{\sqrt{}}{2a\left(\sqrt{-a}+\sqrt{c}\,x\right.}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)\sqrt{d+e}}}{2\sqrt{-a}}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\,\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g}{\sqrt{-a}\,\left(\sqrt{c}\,f+\sqrt{-}\right.}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\,\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g}{\sqrt{-a}\,\left(\sqrt{c}\,f+\sqrt{-}\right.}$$

$$= \frac{g\sqrt{d+ex}}{\sqrt{-a}\,\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)\sqrt{f+gx}} - \frac{g}{\sqrt{-a}\,\left(\sqrt{c}\,f+\sqrt{-}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.91, size = 383, normalized size = 1.08

$$\frac{2g^2\sqrt{d+ex}}{(ef-dg)\,(cf^2+ag^2)\,\sqrt{f+gx}}-\frac{i\sqrt{c}\,\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)^2\tan^{-1}\left(\frac{\sqrt{cf^2+ag^2}\,\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\,\sqrt{f+gx}}\right)}{\sqrt{a}\,\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\,(cf^2+ag^2)^{3/2}}+\frac{i\sqrt{c}\,\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)^2\tan^{-1}\left(\frac{\sqrt{cf^2+ag^2}\,\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\,\sqrt{f+gx}}\right)}{\sqrt{a}\,\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\,(cf^2+ag^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (2*g^2*Sqrt[d + e*x])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) - (I*Sqrt
[c]*(Sqrt[c]*f - I*Sqrt[a]*g)^2*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/
(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[f + g*x]
)])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*(
c*f^2 + a*g^2)^(3/2)) + (I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)^2*ArcTan[(Sqrt
[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*Sqrt[f + g*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e
)*(Sqrt[c]*f + I*Sqrt[a]*g))]*(c*f^2 + a*g^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. 2(270) = 540.
time = 0.08, size = 10977, normalized size = 31.01

method	result	size
default	Expression too large to display	10977

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*(g*x + f)^(3/2)*sqrt(x*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12761 vs. 2(280) = 560.
time = 191.77, size = 12761, normalized size = 36.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")

[Out] -1/4*(8*sqrt(g*x + f)*sqrt(x*e + d)*g^2 + (c*d*f^3*g + a*d*f*g^3 + (c*d*f^2*g^2 + a*d*g^4)*x - (c*f^4 + a*f^2*g^2 + (c*f^3*g + a*f*g^3)*x)*e)*sqrt(-(c^3*d*f^3 - 3*a*c^2*d*f*g^2 - (3*a*c^2*f^2*g - a^2*c*g^3)*e + (a*c^4*d^2*f^6 + 3*a^2*c^3*d^2*f^4*g^2 + 3*a^3*c^2*d^2*f^2*g^4 + a^4*c*d^2*g^6 + (a^2*c^3*f^6 + 3*a^3*c^2*f^4*g^2 + 3*a^4*c*f^2*g^4 + a^5*g^6)*e^2)*sqrt(-(9*c^5*d^2*f^4*g^2 - 6*a*c^4*d^2*f^2*g^4 + a^2*c^3*d^2*g^6 + (c^5*f^6 - 6*a*c^4*f^4*g^2 + 9*a^2*c^3*f^2*g^4)*e^2 + 2*(3*c^5*d*f^5*g - 10*a*c^4*d*f^3*g^3 + 3*a^2*c^3*d*f*g^5)*e)/(a*c^8*d^4*f^12 + 6*a^2*c^7*d^4*f^10*g^2 + 15*a^3*c^6*d^4*f^8*g^4 + 20*a^4*c^5*d^4*f^6*g^6 + 15*a^5*c^4*d^4*f^4*g^8 + 6*a^6*c^3*d^4*f^2*g^10 + a^7*c^2*d^4*g^12 + (a^3*c^6*f^12 + 6*a^4*c^5*f^10*g^2 + 15*a^5*c^4*f^8*g^4 + 20*a^6*c^3*f^6*g^6 + 15*a^7*c^2*f^4*g^8 + 6*a^8*c*f^2*g^10 + a^9*g^12)*e^4 + 2*(a^2*c^7*d^2*f^12 + 6*a^3*c^6*d^2*f^10*g^2 + 15*a^4*c^5*d^2*f^8*g^4 + 20*a^5*c^4*d^2*f^6*g^6 + 15*a^6*c^3*d^2*f^4*g^8 + 6*a^7*c^2*d^2*f^2*g^10 + a^8*c*d^2*g^12)*e^2)))/(a*c^4*d^2*f^6 + 3*a^2*c^3*d^2*f^4*g^2 + 3*a^3*c^2*d^2*f^2*g^4 + a^4*c*d^2*g^6 + (a^2*c^3*f^6 + 3*a^3*c^2*f^4*g^2 + 3*a^4*c*f^2*g^4 + a^5*g^6)*e^2))*log(-(3*c^3*d^2*f^2*g^2 - a*c^2*d^2*g^4 + 2*(3*c^4*d^2*f^4*g - 4*a*c^3*d^2*f^2*g^3 + a^2*c^2*d^2*g^5 - 2*(a*c^3*f^4*g - 3*a^2*c^2*f^2*g^3)*e^2 + (c^4*d*f^5 - 10*a*c^3*d*f^3*g^2 + 5*a^2*c^2*d*f*g^4)*e - (2*a*c^5*d^3*f^7*g + 6*a^2*c^4*d^3*f^5*g^3 + 6*a^3*c^3*d^3*f^3*g^5 + 2*a^4*c^2*d^3*f*g^7 + (a^2*c^4*f^8 + 2*a^3*c^3*f^6*g^2 - 2*a^5*c*f^2*g^6 - a^6*g^8)*e^3 + 2*(a^2*c^4*d*f^7*g + 3*a^3*c^3*d*f^5*g^3 + 3*a^4*c^2*d*f^3*g^5 + a^5*c*d*f*g^7)*e^2 + (a*c^5*d^2*f^8 + 2*a^2*c^4*d^2*f^6*g^2 - 2*a^4*c^2*d^2*f^2*g^6 - a^5*c*d^2*g^8)*e)*sqrt(-(9*c^5*d^2*f^4*g^2 - 6*a*c^4*d^2*f^2*g^4 + a^2*c^3*d^2*g^6 + (c^5*f^6 - 6*a*c^4*f^4*g^2 + 9*a^2*c^3*f^2*g^4)*e^2 + 2*(3*c^5*d*f^5*g - 10*a*c^4*d*f^3*g^3 + 3*a^2*c^3*d*f*g^5)*e)/(a*c^8*d^4*f^12 + 6*a^2*c^7*d^4*f^10*g^2 + 15*a^3*c^6*d^4*f^8*g^4 + 20*a^4*c^5*d^4*f^6*g^6 + 15*a^5*c^4*d^4*f^4*g^8 + 6*a^6*c^3*d^4*f^2*g^10 + a^7*c^2*d^4*g^12 + (a^3*c^6*f^12 + 6*a^4*c^5*f^10*g^2 + 15*a^5*c^4*f^8*g^4 + 20*a^6*c^3*f^6*g^6 + 15*a^7*c^2*f^4*g^8 + 6*a^8*c*f^2*g^10 + a^9*g^12)*e^4 + 2*(a^2*c^7*d^2*f^12 + 6*a^3*c^6*d^2*f^10*g^2 + 15*a^4*c^5*d^2*f^8*g^4 + 20*a^5*c^4*d^2*f^6*g^6 + 15*a^6*c^3*d^2*f^4*g^8 + 6*a^7*c^2*d^2*f^2*g^10 + a^8*c*d^2*g^12)*e^2)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-(c^3*d*f^3 - 3*a*c^2*d*f*g^2 - (3*a*c^2*f^2*g - a^2*c*g^3)*e + (a*c^4*d^2*f^6 + 3*a^2*c^3*d^2*f^4*g^2 + 3*a^3*c^2*d^2*f^2*g^4 + a^4*c*d^2*g^6 + (a^2*c^3*f^6 + 3*a^3*c^2*f^4*g^2 + 3*a^4*c*f^2*g^4 + a^5*g^6)*e^2)*sqrt(-(9*c^5*d^2*f^4*g^2 - 6*a*c^4*d^2*f^2*g^4 + a^2*c^3*d^2*g^6 + (c^5*f^6 - 6*a*c^4*f^4*g^2 + 9*a^2*c^3*f^2*g^4)*e^2 + 2*(3*c^5*d*f^5*g - 10*a*c^4*d*f^3*g^3 + 3*a^2*c^3*d*f*g^5)*e)/(a*c^8*d^4*f^12 + 6*a^2*c^7*d^4*f^10*g^2 + 15*a^3*c^6*d^4*f^8*g^4 + 20*a^4*c^5*d^4*f^6*g^6 + 15*a^5*c^4*d^4*f^4*g^8 + 6*a^6*c^3*d^4*f^2*g^10 + a^7*c^2*d^4*g^12 + (a^3*c^6*f^12 + 6*a^4*c^5*f^10*g^2 + 15*a^5*c^4*f^8*g^4 + 20*a^6*c^3*f^6*g^6 + 15*a^7*c^2*f^4*g^8 + 6*a^8*c*f^2*g^10 + a^9*g^12)*e^4 + 2*(a^2*c^7*d

```
^2*f^12 + 6*a^3*c^6*d^2*f^10*g^2 + 15*a^4*c^5*d^2*f^8*g^4 + 20*a^5*c^4*d^2*
f^6*g^6 + 15*a^6*c^3*d^2*f^4*g^8 + 6*a^7*c^2*d^2*f^2*g^10 + a^8*c*d^2*g^12)
*e^2)))/(a*c^4*d^2*f^6 + 3*a^2*c^3*d^2*f^4*g^2 + 3*a^3*c^2*d^2*f^2*g^4 + a^
4*c*d^2*g^6 + (a^2*c^3*f^6 + 3*a^3*c^2*f^4*g^2 + 3*a^4*c*f^2*g^4 + a^5*g^6)
*e^2)) + (c^3*f^4 - 3*a*c^2*f^2*g^2 + 2*(c^3*f^3*g - 3*a*c^2*f*g^3)*x)*e^2
+ 2*(2*c^3*d*f^3*g - 2*a*c^2*d*f*g^3 + (3*c^3*d*f^2*g^2 - a*c^2*d*g^4)*x)*e
 + (2*c^5*d^3*f^7 + 6*a*c^4*d^3*f^5*g^2 + 6*a^2*c^3*d^3*f^3*g^4 + 2*a^3*c^2
*d^3*f*g^6 + (a*c^4*f^7 + 3*a^2*c^3*f^5*g^2 + 3*a^3*c^2*f^3*g^4 + a^4*c*f*g
^6)*x*e^3 + (c^5*d^2*f^7 + 3*a*c^4*d^2*f^5*g^2 + 3*a^2*c^3*d^2*f^3*g^4 + a^
3*c^2*d^2*f*g^6)*x*e + (c^5*d^3*f^6*g + 3*a*c^4*d^3*f^4*g^3 + 3*a^2*c^3*d^3
*f^2*g^5 + a^3*c^2*d^3*g^7)*x + (2*a*c^4*d*f^7 + 6*a^2*c^3*d*f^5*g^2 + 6*a^
3*c^2*d*f^3*g^4 + 2*a^4*c*d*f*g^6 + (a*c^4*d*f^6*g + 3*a^2*c^3*d*f^4*g^3 +
3*a^3*c^2*d*f^2*g^5 + a^4*c*d*g^7)*x)*e^2)*sqrt(-(9*c^5*d^2*f^4*g^2 - 6*a*c
^4*d^2*f^2*g^4 + a^2*c^3*d^2*g^6 + (c^5*f^6 - 6*a*c^4*f^4*g^2 + 9*a^2*c^3*f
^2*g^4)*e^2 + 2*(3*c^5*d*f^5*g - 10*a*c^4*d*f^3*g^3 + 3*a^2*c^3*d*f*g^5)*e)
/(a*c^8*d^4*f^12 + 6*a^2*c^7*d^4*f^10*g^2 + 15*a^3*c^6*d^4*f^8*g^4 + 20*a^4
*c^5*d^4*f^6*g^6 + 15*a^5*c^4*d^4*f^4*g^8 + 6*a^6*c^3*d^4*f^2*g^10 + a^7*c^
2*d^4*g^12 + (a^3*c^6*f^12 + 6*a^4*c^5*f^10*g^2 + 15*a^5*c^4*f^8*g^4 + 20*a
^6*c^3*f^6*g^6 + 15*a^7*c^2*f^4*g^8 + 6*a^8*c*f^2*g^10 + a^9*g^12)*e^4 + 2*
(a^2*c^7*d^2*f^12 + 6*a^3*c^6*d^2*f^10*g^2 + 15*a^4*c^5*d^2*f^8*g^4 + 20*a^
5*c^4*d^2*f^6*g^6 + 15*a^6*c^3*d^2*f^4*g^8 + 6*a^7*c^2*d^2*f^2*g^10 + a^8*c
*d^2*g^12)*e^2)))/x) - (c*d*f^3*g + a*d*f*g^3 + (c*d*f^2*g^2 + a*d*g^4)*x -
 (c*f^4 + a*f^2*g^2 + (c*f^3*g + a*f*g^3)*x)*e)*sqrt(-(c^3*d*f^3 - 3*a*c^2*
d*f*g^2 - (3*a*c^2*f^2*g - a^2*c*g^3)*e + (a*c^4*d^2*f^6 + 3*a^2*c^3*d^2*f^
4*g^2 + 3*a^3*c^2*d^2*f^2*g^4 + a^4*c*d^2*g^6 + (a^2*c^3*f^6 + 3*a^3*c^2*f^
4*g^2 + 3*a^4*c*f^2*g^4 + a^5*g^6)*e^2)*sqrt(-(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f+g\,x)^{3/2}\,\left(c\,x^2+a\right)\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)), x)
```

3.617 $\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal. Leaf size=549

$$-\frac{e}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\,\sqrt{f+gx}}+\frac{e}{\sqrt{-a}\,\left(\sqrt{c}\,d+\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\,\sqrt{f+}$$

```
[Out] c*arctanh((e*x+d)^(1/2)*(-g*(-a)^(1/2)+f*c^(1/2))^(1/2)/(g*x+f)^(1/2)/(-e*(
-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(-e*(-a)^(1/2)+d*c^(1/2))^(3/2)/(-g*
(-a)^(1/2)+f*c^(1/2))^(3/2)-c*arctanh((e*x+d)^(1/2)*(g*(-a)^(1/2)+f*c^(1/2)
)^(1/2)/(g*x+f)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2))^(1/2))/(-a)^(1/2)/(e*(-a)^(1
/2)+d*c^(1/2))^(3/2)/(g*(-a)^(1/2)+f*c^(1/2))^(3/2)-e/(-d*g+e*f)/(-a)^(1/2)
/(-e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1/2)/(g*x+f)^(1/2)+e/(-d*g+e*f)/(-a)^(1
/2)/(e*(-a)^(1/2)+d*c^(1/2))/(e*x+d)^(1/2)/(g*x+f)^(1/2)+g*(2*e*g*(-a)^(1/2
)-(d*g+e*f)*c^(1/2))*(e*x+d)^(1/2)/(-d*g+e*f)^2/(-a)^(1/2)/(-e*(-a)^(1/2)+d
*c^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/2)+g*(2*e*g*(-a)^(1/2)+(d*g+
e*f)*c^(1/2))*(e*x+d)^(1/2)/(-d*g+e*f)^2/(-a)^(1/2)/(e*(-a)^(1/2)+d*c^(1/2)
)/(g*(-a)^(1/2)+f*c^(1/2))/(g*x+f)^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 543, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {926, 106, 157, 12, 95, 214}

$$-\frac{e}{\sqrt{-a}\,\sqrt{d+ex}\,\sqrt{f+gx}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\,\sqrt{d+ex}\,\sqrt{f+gx}\,\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)(ef-dg)}+\frac{g\sqrt{d+ex}\,\left(2aeg-\sqrt{-a}\,\sqrt{c}\,(dg+ef)\right)}{a\sqrt{f+gx}\,\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)(ef-dg)^2}+\frac{g\sqrt{d+ex}\,\left(\sqrt{-a}\,\sqrt{c}\,(dg+ef)+2aeg\right)}{a\sqrt{f+gx}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)(ef-dg)^2}+\frac{c\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{c}\,f-\sqrt{-a}\,g}}{\sqrt{f+gx}\,\sqrt{\sqrt{c}\,d-\sqrt{-a}\,e}}\right)}{\sqrt{-a}\,\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)^{3/2}\left(\sqrt{c}\,f-\sqrt{-a}\,g\right)^{3/2}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{\sqrt{-a}\,g+\sqrt{c}\,f}}{\sqrt{f+gx}\,\sqrt{\sqrt{-a}\,e+\sqrt{c}\,d}}\right)}{\sqrt{-a}\,\left(\sqrt{-a}\,e+\sqrt{c}\,d\right)^{3/2}\left(\sqrt{-a}\,g+\sqrt{c}\,f\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] -(e/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])) + e/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt
[f + g*x]) + (g*(2*a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*
(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*
x]) + (g*(2*a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*(Sqrt[c
]*d - Sqrt[-a]*e)*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*x]) + (
c*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sq
rt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2)*(Sqrt[c
]*f - Sqrt[-a]*g)^(3/2)) - (c*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d
+ e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d
 + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}(f+gx)^{3/2}} + \frac{}{2a\left(\sqrt{-a}+\sqrt{c}\right.}\right.$$

$$= -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{c}\,x\right)(d+ex)^{3/2}(f+gx)^{3/2}}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{c}\,x\right)(d+ex)^{3/2}(f}}{2\sqrt{-a}}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\ (}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\ (}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\ (}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\ (}$$

$$= -\frac{e}{\sqrt{-a}\left(\sqrt{c}\,d-\sqrt{-a}\,e\right)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} + \frac{}{\sqrt{-a}\ (}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 1.92, size = 477, normalized size = 0.87

$$-\frac{2\left(c\left(d^3g^3+d^2eg^3x+e^3f^2(f+gx)\right)+ae^2g^2(dg+e(f+2gx))\right)}{\left(cd^2+ae^2\right)(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{d+ex}\sqrt{f+gx}} - \frac{ic\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)^2} + \frac{ic\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}\,d+i\sqrt{a}\,e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

```
[Out] (-2*(c*(d^3*g^3 + d^2*e*g^3*x + e^3*f^2*(f + g*x)) + a*e^2*g^2*(d*g + e*(f
+ 2*g*x))))/((c*d^2 + a*e^2)*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[d + e*x]*Sq
rt[f + g*x]) - (I*c*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]
*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqr
t[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d -
I*Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*(Sqrt[c]*f - I*Sqrt[a]*g)^2) + (I*c*Sqrt[-
```

```
((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 +
 a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqr
t[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt[c*d^2 +
a*e^2]*(Sqrt[c]*f + I*Sqrt[a]*g)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 30647 vs. $2(433) = 866$.
time = 0.14, size = 30648, normalized size = 55.83

method	result	size
default	Expression too large to display	30648

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)*(g*x + f)^(3/2)*(x*e + d)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + cx^2)(d + ex)^{\frac{3}{2}}(f + gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*(f + g*x)**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
```

```
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

3.618 $\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{1+x}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{1+x}}\right)$$

```
[Out] -1/2*(1-I)^(3/2)*arctanh((1-I)^(1/2)*x^(1/2)/(1+x)^(1/2))-1/2*(1+I)^(3/2)*a
rctanh((1+I)^(1/2)*x^(1/2)/(1+x)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {924, 95, 214}

$$-\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{x+1}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -1/2*((1 - I)^(3/2)*ArcTanh[(Sqrt[1 - I]*Sqrt[x])/Sqrt[1 + x]]) - ((1 + I)^
(3/2)*ArcTanh[(Sqrt[1 + I]*Sqrt[x])/Sqrt[1 + x]])/2
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 924

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[c*d^2 + a*e^2, 0] && IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{1+x}\,(1+x^2)}\,dx = \int \left(-\frac{1}{2(i-x)\sqrt{x}\,\sqrt{1+x}}+\frac{1}{2\sqrt{x}\,(i+x)\sqrt{1+x}}\right)dx$$
$$= -\left(\frac{1}{2}\int \frac{1}{(i-x)\sqrt{x}\,\sqrt{1+x}}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{x}\,(i+x)\sqrt{1+x}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{i-(1+i)x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+x}}\right)+\text{Subst}\left(\int \frac{1}{i+(1-i)x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+}}\right.$$
$$= -\frac{1}{2}(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1-i}\,\sqrt{x}}{\sqrt{1+x}}\right)-\frac{1}{2}(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+i}\,\sqrt{x}}{\sqrt{1+x}}\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.07, size = 59, normalized size = 0.91

$$-\text{RootSum}\left[16+32\#1+16\#1^2+\#1^4\&, \frac{\log\left(-2x+2\sqrt{x}\,\sqrt{1+x}+\#1\right)\#1^2}{8+8\#1+\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] -RootSum[16 + 32*#1 + 16*#1^2 + #1^4 & , (Log[-2*x + 2*Sqrt[x]*Sqrt[1 + x]
+ #1]*#1^2)/(8 + 8*#1 + #1^3) & ]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(45) = 90.
time = 0.18, size = 305, normalized size = 4.69

method	result
default	$\sqrt{\frac{x(1+x)}{\left(\sqrt{2}-1+x\right)^2}}\left(\sqrt{2}-1+x\right)\left(\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{\frac{\left(3\sqrt{2}-4\right)x(1+x)\left(4+3\sqrt{2}\right)}{\left(\sqrt{2}-1+x\right)^2}}\sqrt{-2+2\sqrt{2}}\left(3+\right.}{4x(1+x)}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+1)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/x^(1/2)/(1+x)^(1/2)*(x*(1+x)/(2^(1/2)-1+x)^2)^(1/2)*(2^(1/2)-1+x)*((-2+
2*2^(1/2))^(1/2)*arctan(1/4*((3*2^(1/2)-4)*x*(1+x)*(4+3*2^(1/2))/(2^(1/2)-1
+x)^2)^(1/2)*(-2+2*2^(1/2))^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)
*(2^(1/2)-1+x)/x/(1+x))*(1+2^(1/2))^(1/2)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*ar
ctan(1/4*((3*2^(1/2)-4)*x*(1+x)*(4+3*2^(1/2))/(2^(1/2)-1+x)^2)^(1/2)*(-2+2*
2^(1/2))^(1/2)*(3+2*2^(1/2))*(2^(1/2)+1-x)*(3*2^(1/2)-4)*(2^(1/2)-1+x)/x/(1
+x))*(1+2^(1/2))^(1/2)+4*arctanh(2^(1/2)*(x*(1+x)/(2^(1/2)-1+x)^2)^(1/2)/(1
+2^(1/2))^(1/2))*2^(1/2)-6*arctanh(2^(1/2)*(x*(1+x)/(2^(1/2)-1+x)^2)^(1/2)/
(1+2^(1/2))^(1/2)))*2^(1/2)/(3*2^(1/2)-4)/(1+2^(1/2))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/((x^2 + 1)*sqrt(x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(37) = 74$.
time = 2.72, size = 744, normalized size = 11.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-8*sqrt(x + 1)*x^(3/2) +
8*x^2 + 2*(2^(1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x
+ 1) - 2*x))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/8*2^(1/4)*sqrt(
2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-8*sqrt(x + 1)*x^(3/2) + 8*x^2 - 2*(2^(1/4
)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x + 1) - 2*x))*sqrt
(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*ar
ctan(1/7*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*sqrt(x + 1)*sqrt(x) - 1/
7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1/28*sqrt(-8*sqrt(x + 1)*x^(3/2)
+ 8*x^2 - 2*(2^(1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(
x + 1) - 2*x))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(2*sqrt(2)*(5*sqr
t(2) + 6) - (2^(3/4)*(3*sqrt(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(2*sqrt
(2) + 4) + 16*sqrt(2) + 8) - 1/7*sqrt(2)*(8*x + 3) - 1/14*((2^(3/4)*(3*sqrt
(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(x + 1)*sqrt(x) - 2^(3/4)*(sqrt(2)*
(3*x + 2) + 5*x + 1) - 2*2^(1/4)*(sqrt(2)*(x + 3) + 4*x - 2))*sqrt(2*sqrt(2
) + 4) - 4/7*x - 5/7) - 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*(sqrt(2
)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*sqrt(x + 1)*sqrt(x) + 1/7*sqrt(2)*(sqrt(
2)*(5*x + 1) + 6*x + 4) + 1/28*sqrt(-8*sqrt(x + 1)*x^(3/2) + 8*x^2 + 2*(2^(
```

```
1/4)*sqrt(x + 1)*sqrt(x)*(sqrt(2) - 2) - 2^(1/4)*(sqrt(2)*(x + 1) - 2*x))*s
qrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(
3/4)*(3*sqrt(2) + 5) + 2*2^(1/4)*(sqrt(2) + 4))*sqrt(2*sqrt(2) + 4) + 16*sq
rt(2) + 8) + 1/7*sqrt(2)*(8*x + 3) - 1/14*((2^(3/4)*(3*sqrt(2) + 5) + 2*2^(
1/4)*(sqrt(2) + 4))*sqrt(x + 1)*sqrt(x) - 2^(3/4)*(sqrt(2)*(3*x + 2) + 5*x
+ 1) - 2*2^(1/4)*(sqrt(2)*(x + 3) + 4*x - 2))*sqrt(2*sqrt(2) + 4) + 4/7*x +
 5/7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x+1}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+1)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/(sqrt(x + 1)*(x**2 + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(37) = 74.
time = 4.03, size = 375, normalized size = 5.77

$$\frac{1}{4}\left(\sqrt{2\sqrt{2}+2}+\sqrt{2\sqrt{2}-2}\right)\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{-\frac{1}{x+1}+1}\right)}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\left(\sqrt{2\sqrt{2}+2}+\sqrt{2\sqrt{2}-2}\right)\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{-\frac{1}{x+1}+1}\right)}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\left(\sqrt{2\sqrt{2}+2}-\sqrt{2\sqrt{2}-2}\right)\log\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{-\frac{1}{x+1}+1}+\sqrt{\frac{1}{2}}-\frac{1}{x+1}+1\right)+\frac{1}{8}\left(\sqrt{2\sqrt{2}+2}-\sqrt{2\sqrt{2}-2}\right)\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{-\frac{1}{x+1}+1}+\sqrt{\frac{1}{2}}-\frac{1}{x+1}+1\right)-\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\right)}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\right)}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+\sqrt{\frac{1}{2}}+1\right)+\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+\sqrt{\frac{1}{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2*sqrt(2) + 2) + sqrt(2*sqrt(2) - 2))*arctan(2*(1/2)^(3/4)*((1/2)
^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(-1/(x + 1) + 1))/sqrt(-sqrt(2) + 2)) + 1/
4*(sqrt(2*sqrt(2) + 2) + sqrt(2*sqrt(2) - 2))*arctan(-2*(1/2)^(3/4)*((1/2)^
(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(-1/(x + 1) + 1))/sqrt(-sqrt(2) + 2)) - 1/8
*(sqrt(2*sqrt(2) + 2) - sqrt(2*sqrt(2) - 2))*log((1/2)^(1/4)*sqrt(sqrt(2) +
 2)*sqrt(-1/(x + 1) + 1) + sqrt(1/2) - 1/(x + 1) + 1) + 1/8*(sqrt(2*sqrt(2)
 + 2) - sqrt(2*sqrt(2) - 2))*log(-(1/2)^(1/4)*sqrt(sqrt(2) + 2)*sqrt(-1/(x
+ 1) + 1) + sqrt(1/2) - 1/(x + 1) + 1) - 1/4*sqrt(2*sqrt(2) + 2)*arctan(2*(
1/2)^(3/4)*((1/2)^(1/4)*sqrt(sqrt(2) + 2) + 2)/sqrt(-sqrt(2) + 2)) - 1/4*sq
rt(2*sqrt(2) + 2)*arctan(-2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(sqrt(2) + 2) - 2)
/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log((1/2)^(1/4)*sqrt(sqrt(2)
 + 2) + sqrt(1/2) + 1) + 1/8*sqrt(2*sqrt(2) - 2)*log(-(1/2)^(1/4)*sqrt(sqrt
(2) + 2) + sqrt(1/2) + 1)
```

Mupad [B]
time = 8.49, size = 1610, normalized size = 24.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/((x^2 + 1)*(x + 1)^(1/2)),x)

```
[Out] - atan(((((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454
158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/
2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) - ((53150
2202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/
2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16
 - 1/16)^(1/2)) - (12079595520*x)/((x + 1)^(1/2) - 1)^2 + 68451041280))*((-
 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (13555990528*x)/((
x + 1)^(1/2) - 1)^2 + 9529458688) + (3556769792*x^(1/2))/((x + 1)^(1/2) - 1
))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i - (((- 2^(1
/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*((13555990528*x)/((x + 1)
^(1/2) - 1)^2 - ((28454158336*x^(1/2))/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16
 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((112742891520*x^(1/2))/((x +
1)^(1/2) - 1) + ((531502202880*x)/((x + 1)^(1/2) - 1)^2 - 241591910400)*((-
 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/
16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + (12079595520*x)/((x + 1)^(1/2) - 1
)^2 - 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2
)) + 9529458688) - (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16
 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*1i)/((((- 2^(1/2)/16 - 1/16)^(1
/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((28454158336*x^(1/2))/((x + 1)^(1/2) - 1
) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2))*(((1127428915
20*x^(1/2))/((x + 1)^(1/2) - 1) - ((531502202880*x)/((x + 1)^(1/2) - 1)^2 -
 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)))*(
(- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) - (12079595520*x)/
((x + 1)^(1/2) - 1)^2 + 68451041280))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/
2)/16 - 1/16)^(1/2)) + (13555990528*x)/((x + 1)^(1/2) - 1)^2 + 9529458688)
+ (3556769792*x^(1/2))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) -
(2^(1/2)/16 - 1/16)^(1/2)) + (((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 -
1/16)^(1/2))*((13555990528*x)/((x + 1)^(1/2) - 1)^2 - ((28454158336*x^(1/2)
)/((x + 1)^(1/2) - 1) + ((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^
(1/2))*(((112742891520*x^(1/2))/((x + 1)^(1/2) - 1) + ((531502202880*x)/((x
 + 1)^(1/2) - 1)^2 - 241591910400)*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/
16 - 1/16)^(1/2)))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)
) + (12079595520*x)/((x + 1)^(1/2) - 1)^2 - 68451041280))*((- 2^(1/2)/16 -
1/16)^(1/2) - (2^(1/2)/16 - 1/16)^(1/2)) + 9529458688) - (3556769792*x^(1/2
))/((x + 1)^(1/2) - 1))*((- 2^(1/2)/16 - 1/16)^(1/2) - (2^(1/2)/16 - 1/16)^
(1/2)) + (7549747200*x)/((x + 1)^(1/2) - 1)^2 + 503316480))*((- 2^(1/2)/16
- 1/16)^(1/2)*2i - (2^(1/2)/16 - 1/16)^(1/2)*2i) - atan(((x^(1/2)*(- 2^(1/2
)/16 - 1/16)^(1/2)*848i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)
^(1/2)*848i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(- 2^(1/2)/16 - 1/16)^(3/2)*678
4i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(3/2)*6784i)/((x + 1
)^(1/2) - 1) + (x^(1/2)*(- 2^(1/2)/16 - 1/16)^(5/2)*26880i)/((x + 1)^(1/2)
```

```
- 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(5/2)*26880i)/((x + 1)^(1/2) - 1) + (x^
(1/2)*(2^(1/2)/16 - 1/16)^2*(- 2^(1/2)/16 - 1/16)^(1/2)*134400i)/((x + 1)^(
1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(2^(1/2)/16 + 1/16)^2*134400
i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)*(- 2^(1/2)/16 - 1/16)
^(1/2)*20352i)/((x + 1)^(1/2) - 1) - (x^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(2^
(1/2)/16 + 1/16)*20352i)/((x + 1)^(1/2) - 1) + (x^(1/2)*(2^(1/2)/16 - 1/16)
*(- 2^(1/2)/16 - 1/16)^(3/2)*268800i)/((x + 1)^(1/2) - 1) - (x^(1/2)*(2^(1/
2)/16 - 1/16)^(3/2)*(2^(1/2)/16 + 1/16)*268800i)/((x + 1)^(1/2) - 1))/(4544
*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 65280*(2^(1/2)/16
- 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(3/2) + 65280*(2^(1/2)/16 - 1/16)^(3/2)
*(- 2^(1/2)/16 - 1/16)^(1/2) + 345600*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/
16 - 1/16)^(5/2) + 1152000*(2^(1/2)/16 - 1/16)^(3/2)*(- 2^(1/2)/16 - 1/16)^
(3/2) + 345600*(2^(1/2)/16 - 1/16)^(5/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + x/((
x + 1)^(1/2) - 1)^2 + (6464*x*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/1
6)^(1/2))/((x + 1)^(1/2) - 1)^2 - (11520*x*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(
1/2)/16 - 1/16)^(3/2))/((x + 1)^(1/2) - 1)^2 - (11520*x*(2^(1/2)/16 - 1/16)
^(3/2)*(- 2^(1/2)/16 - 1/16)^(1/2))/((x + 1)^(1/2) - 1)^2 - (760320*x*(2^(1
/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(5/2))/((x + 1)^(1/2) - 1)^2 - (
2534400*x*(2^(1/2)/16 - 1/16)^(3/2)*(- 2^(1/2)/16 - 1/16)^(3/2))/((x + 1)^(
1/2) - 1)^2 - (760320*x*(2^(1/2)/16 - 1/16)^(5/2)*(- 2^(1/2)/16 - 1/16)^(1/
2))/((x + 1)^(1/2) - 1)^2 + 1))*((- 2^(1/2)/16 - 1/16)^(1/2)*2i + (2^(1/2)/
16 - 1/16)^(1/2)*2i)
```

3.619 $\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx$

Optimal. Leaf size=80

$$\frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}}+\frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(1+x)}{\sqrt{1-x^2}}-g^2\sin^{-1}(x)$$

```
[Out] 1/5*(f+g)^2*(1+x)^4/(-x^2+1)^(5/2)+1/15*(f-9*g)*(f+g)*(1+x)^3/(-x^2+1)^(3/2
)-g^2*arcsin(x)+2*g^2*(1+x)/(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {867, 1649, 803, 667, 222}

$$-g^2\text{ArcSin}(x)+\frac{(x+1)^4(f+g)^2}{5\left(1-x^2\right)^{5/2}}+\frac{(x+1)^3(f-9g)(f+g)}{15\left(1-x^2\right)^{3/2}}+\frac{2g^2(x+1)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] ((f + g)^2*(1 + x)^4)/(5*(1 - x^2)^(5/2)) + ((f - 9*g)*(f + g)*(1 + x)^3)/(
15*(1 - x^2)^(3/2)) + (2*g^2*(1 + x))/Sqrt[1 - x^2] - g^2*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)^2\sqrt{1-x^2}}{(1-x)^4}\,dx &= \int \frac{(1+x)^4(f+gx)^2}{(1-x^2)^{7/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} - \frac{1}{5}\int \frac{(1+x)^3\left(-f^2+8fg+4g^2+5g^2x\right)}{\left(1-x^2\right)^{5/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + g^2\int \frac{(1+x)^2}{\left(1-x^2\right)^{3/2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{(f+g)^2(1+x)^4}{5\left(1-x^2\right)^{5/2}} + \frac{(f-9g)(f+g)(1+x)^3}{15\left(1-x^2\right)^{3/2}} + \frac{2g^2(1+x)}{\sqrt{1-x^2}} - g^2\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.35, size = 86, normalized size = 1.08

$$\frac{\sqrt{1-x^2}\left(f^2(-4-3x+x^2)-2fg(-1+3x+4x^2)-3g^2(8-19x+13x^2)\right)}{15(-1+x)^3}+2g^2\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[1 - x^2])/(1 - x)^4,x]
```

```
[Out] (Sqrt[1 - x^2]*(f^2*(-4 - 3*x + x^2) - 2*f*g*(-1 + 3*x + 4*x^2) - 3*g^2*(8
- 19*x + 13*x^2)))/(15*(-1 + x)^3) + 2*g^2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.09, size = 125, normalized size = 1.56

method	result
risch	$-\frac{(1+x)\left(f^{2} x^{2}-8 f g \,x^{2}-39 g^{2} x^{2}-3 f^{2} x-6 f g x+57 g^{2} x-4 f^{2}+2 f g-24 g^{2}\right)}{15 \left(-1+x\right)^{2} \sqrt{-x^{2}+1}}-g^{2} \arcsin \left(x \right)$
trager	$\frac{\left(f^{2} x^{2}-8 f g \,x^{2}-39 g^{2} x^{2}-3 f^{2} x-6 f g x+57 g^{2} x-4 f^{2}+2 f g-24 g^{2}\right) \sqrt{-x^{2}+1}}{15 \left(-1+x\right)^{3}}+g^{2} \mathrm{RootOf}\left(\_Z^{2}+1\right) \ln \left(-\mathrm{RootOf}\right.$
default	$\left(f^{2}+2 f g+g^{2}\right) \left(\frac{\left(-\left(-1+x\right)^{2}+2-2 x\right)^{\frac{3}{2}}}{5 \left(-1+x\right)^{4}}-\frac{\left(-\left(-1+x\right)^{2}+2-2 x\right)^{\frac{3}{2}}}{15 \left(-1+x\right)^{3}}\right)+g^{2} \left(\frac{\left(-\left(-1+x\right)^{2}+2-2 x\right)^{\frac{3}{2}}}{\left(-1+x\right)^{2}}+\sqrt{-\left(-1+\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (f^2+2*f*g+g^2)*(1/5/(-1+x)^4*(-(-1+x)^2+2-2*x)^(3/2)-1/15/(-1+x)^3*(-(-1+x
)^2+2-2*x)^(3/2))+g^2*(1/(-1+x)^2*(-(-1+x)^2+2-2*x)^(3/2)+(-(-1+x)^2+2-2*x)
^(1/2)-arcsin(x))+2/3*g*(f+g)/(-1+x)^3*(-(-1+x)^2+2-2*x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*sqrt(-x^2 + 1)/(x - 1)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(70) = 140$.
time = 2.42, size = 193, normalized size = 2.41

$$\frac{2\,(2\,f^2-fg+12\,g^2)x^3-6\,(2\,f^2-fg+12\,g^2)x^2-4\,f^2+2\,fg-24\,g^2+6\,(2\,f^2-fg+12\,g^2)x+30\,(g^2x^3-3\,g^2x^2+3\,g^2x-g^2)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+((f^2-8\,fg-39\,g^2)x^2-4\,f^2+2\,fg-24\,g^2-3\,(f^2+2\,fg-19\,g^2)x)\sqrt{-x^2+1}}{15\,(x^3-3\,x^2+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(2*(2*f^2 - f*g + 12*g^2)*x^3 - 6*(2*f^2 - f*g + 12*g^2)*x^2 - 4*f^2 +
 2*f*g - 24*g^2 + 6*(2*f^2 - f*g + 12*g^2)*x + 30*(g^2*x^3 - 3*g^2*x^2 + 3*
g^2*x - g^2)*arctan((sqrt(-x^2 + 1) - 1)/x) + ((f^2 - 8*f*g - 39*g^2)*x^2 -
 4*f^2 + 2*f*g - 24*g^2 - 3*(f^2 + 2*f*g - 19*g^2)*x)*sqrt(-x^2 + 1))/(x^3
- 3*x^2 + 3*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}\,(f+gx)^2}{(x-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-x**2+1)**(1/2)/(1-x)**4,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))*(f + g*x)**2/(x - 1)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(70) = 140.
time = 3.38, size = 266, normalized size = 3.32

$$-g^2\arcsin(x)+\frac{2\left(4f^2-2fg+24g^2+\frac{5f^2\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{10fg\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{105g^2\left(\sqrt{-x^2+1}-1\right)}{x}+\frac{25f^2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{10fg\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{165g^2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{15f^2\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-\frac{30fg\left(\sqrt{-x^2+1}-1\right)^3}{x^3}+\frac{75g^2\left(\sqrt{-x^2+1}-1\right)^3}{x^3}+\frac{15f^2\left(\sqrt{-x^2+1}-1\right)^4}{x^4}+\frac{15g^2\left(\sqrt{-x^2+1}-1\right)^4}{x^4}\right)}{15\left(\frac{\sqrt{-x^2+1}-1}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x, algorithm="giac")
```

```
[Out] -g^2*arcsin(x) + 2/15*(4*f^2 - 2*f*g + 24*g^2 + 5*f^2*(sqrt(-x^2 + 1) - 1)/
x - 10*f*g*(sqrt(-x^2 + 1) - 1)/x + 105*g^2*(sqrt(-x^2 + 1) - 1)/x + 25*f^2
*(sqrt(-x^2 + 1) - 1)^2/x^2 + 10*f*g*(sqrt(-x^2 + 1) - 1)^2/x^2 + 165*g^2*(
sqrt(-x^2 + 1) - 1)^2/x^2 + 15*f^2*(sqrt(-x^2 + 1) - 1)^3/x^3 - 30*f*g*(sqr
t(-x^2 + 1) - 1)^3/x^3 + 75*g^2*(sqrt(-x^2 + 1) - 1)^3/x^3 + 15*f^2*(sqrt(-
x^2 + 1) - 1)^4/x^4 + 15*g^2*(sqrt(-x^2 + 1) - 1)^4/x^4)/((sqrt(-x^2 + 1) -
 1)/x + 1)^5
```

Mupad [B]
time = 2.96, size = 164, normalized size = 2.05

$$\sqrt{1-x^2}\left(\frac{\frac{f^2}{3}+2fg+\frac{5g^2}{3}}{x-1}-\frac{\frac{f^2}{3}+2fg+\frac{5g^2}{3}}{(x-1)^2}\right)-\sqrt{1-x^2}\left(\frac{\frac{2f^2}{5}+\frac{4fg}{5}+\frac{2g^2}{5}}{(x-1)^3}+\frac{\frac{4f^2}{15}+\frac{8fg}{15}+\frac{4g^2}{15}}{x-1}-\frac{\frac{4f^2}{15}+\frac{8fg}{15}+\frac{4g^2}{15}}{(x-1)^2}\right)-g^2\operatorname{asin}(x)-\frac{\sqrt{1-x^2}\,(4g^2+2fg)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(1 - x^2)^(1/2))/(x - 1)^4,x)
```

```
[Out] (1 - x^2)^(1/2)*((2*f*g + f^2/3 + (5*g^2)/3)/(x - 1) - (2*f*g + f^2/3 + (5*
g^2)/3)/(x - 1)^2) - (1 - x^2)^(1/2)*(((4*f*g)/5 + (2*f^2)/5 + (2*g^2)/5)/(
x - 1)^3 + ((8*f*g)/15 + (4*f^2)/15 + (4*g^2)/15)/(x - 1) - ((8*f*g)/15 + (
4*f^2)/15 + (4*g^2)/15)/(x - 1)^2) - g^2*asin(x) - ((1 - x^2)^(1/2)*(2*f*g
+ 4*g^2))/(x - 1)
```

3.620 $\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}+\frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}$$

```
[Out] -(a*c-2*d)*arcsin(a*x)/d^2+(a*c-d)^2*arctan((a^2*c*x+d)/(a^2*c^2-d^2)^(1/2)
/(-a^2*x^2+1)^(1/2))/d^2/(a^2*c^2-d^2)^(1/2)-(-a^2*x^2+1)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {867, 1668, 858, 222, 739, 210}

$$\frac{(ac-d)^2\text{ArcTan}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{\text{ArcSin}(ax)(ac-2d)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)}\,dx &= \int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2} + \frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2} - \frac{(ac-d)^2\text{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx, x, \frac{d+\cdots}{\sqrt{1-\cdots}}\right)}{d^2} \\
&= -\frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2} + \frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 501 vs. 2(107) = 214.

time = 1.79, size = 501, normalized size = 4.68

$$-\frac{ad^3(ac+d)\sqrt{1-a^2x^2}-a(ac-d)\sqrt{2a^2c^2-d^2-2ac\sqrt{a^2c^2-d^2}}\left(a^2c^2-d^2+ac\sqrt{a^2c^2-d^2}\right)\tan^{-1}\left(\frac{d\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)}{\sqrt{2a^2c^2-d^2-2ac\sqrt{a^2c^2-d^2}}}\right)+a(ac-d)\left(-a^2c^2+d^2+ac\sqrt{a^2c^2-d^2}\right)\sqrt{2a^2c^2-d^2+2ac\sqrt{a^2c^2-d^2}}\tan^{-1}\left(\frac{d\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)}{\sqrt{2a^2c^2-d^2+2ac\sqrt{a^2c^2-d^2}}}\right)+\sqrt{-a^2}\,(ac-d)d^2\sqrt{-a^2c^2+d^2}\tan^{-1}\left(\frac{-\sqrt{-a^2}\,d^2x\sqrt{1-a^2x^2}+a^2(c^2-d^2x^2)}{ac\sqrt{-a^2c^2+d^2}}\right)+\sqrt{-a^2}\,(ac-2d)d^2(ac+d)\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{ad^4(ac+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)/((1 - a*x)^2*(c + d*x)),x]
```

```
[Out] -((a*d^3*(a*c + d)*Sqrt[1 - a^2*x^2] - a*(a*c - d)*Sqrt[2*a^2*c^2 - d^2 - 2
*a*c*Sqrt[a^2*c^2 - d^2]]*(a^2*c^2 - d^2 + a*c*Sqrt[a^2*c^2 - d^2])*ArcTan[
(d*(Sqrt[-a^2]*x - Sqrt[1 - a^2*x^2]))/Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^
2*c^2 - d^2]]] + a*(a*c - d)*(-(a^2*c^2) + d^2 + a*c*Sqrt[a^2*c^2 - d^2])*S
qrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]]*ArcTan[(d*(Sqrt[-a^2]*x -
Sqrt[1 - a^2*x^2]))/Sqrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]]] + Sq
rt[-a^2]*(a*c - d)*d^2*Sqrt[-(a^2*c^2) + d^2]*ArcTan[(-(Sqrt[-a^2]*d^2*x*Sq
rt[1 - a^2*x^2]) + a^2*(c^2 - d^2*x^2))/(a*c*Sqrt[-(a^2*c^2) + d^2])] + Sqr
t[-a^2]*(a*c - 2*d)*d^2*(a*c + d)*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]])
/(a*d^4*(a*c + d)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(99) = 198$.

time = 0.10, size = 866, normalized size = 8.09

method	result
risch	$\frac{a^2x^2-1}{d\sqrt{-a^2x^2+1}}-\frac{a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)c}{d^2\sqrt{a^2}}+\frac{2a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{d\sqrt{a^2}}-\frac{\ln\left(\frac{-\frac{2\left(a^2c^2-d^2\right)}{d^2}+\frac{2a^2c\left(x+\frac{c}{d}\right)}{d}+}{}\right.}{}$
default	$\frac{-\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{1}{a}\right)^2}-3a\left(\frac{\left(-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)\right)^{\frac{3}{2}}}{3}-a\left(-\frac{\left(-2a^2\left(x-\frac{1}{a}\right)-2a\right)\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}{4a^2}+\right.\right.}{a(ac+d)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(a*c+d)*(-1/a/(x-1/a)^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(5/2)-3*a*(1/3*(-a
^2*(x-1/a)^2-2*a*(x-1/a))^(3/2)-a*(-1/4*(-2*a^2*(x-1/a)-2*a)/a^2*(-a^2*(x-1
/a)^2-2*a*(x-1/a))^(1/2)+1/2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)
^2-2*a*(x-1/a))^(1/2)))))-d/(a*c+d)^2*(1/3*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(3/
2)-a*(-1/4*(-2*a^2*(x-1/a)-2*a)/a^2*(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)+1/2/
(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2))))+d/(a
*c+d)^2*(1/3*(-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(3/2)+a^2
*c/d*(-1/4*(-2*a^2*(x+c/d)+2*a^2*c/d)/a^2*(-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)
-(a^2*c^2-d^2)/d^2)^(1/2)-1/8*(4*a^2*(a^2*c^2-d^2)/d^2-4*a^4*c^2/d^2)/a^2/(
a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-
d^2)/d^2)^(1/2)))-(a^2*c^2-d^2)/d^2*((-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)-(a^2
*c^2-d^2)/d^2)^(1/2)+a^2*c/d/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*(x+c/d)
^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))+(a^2*c^2-d^2)/d^2/(-(a^2*c^2
-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^
2)/d^2)^(1/2)*(-a^2*(x+c/d)^2+2*a^2*c/d*(x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(
x+c/d))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="maxima")
[Out] integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)), x)
```

Fricas [A]
time = 3.34, size = 318, normalized size = 2.97

$$\left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}\,(acd+d^2)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2\,(ac-2\,d)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\,d}{d^2}, \frac{2\,(ac-d)\sqrt{\frac{ac-d}{ac+d}}\arctan\left(\frac{\left(dx-\sqrt{-a^2x^2+1}\,c+c\right)\sqrt{\frac{ac-d}{ac+d}}}{(ac-d)x}\right)+2\,(ac-2\,d)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}\,d}{d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="fricas")
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}}{(c+dx)(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/(-a*x+1)**2/(d*x+c),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)/((c + d*x)*(a*x - 1)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(99) = 198.
time = 3.97, size = 208, normalized size = 1.94

$$-\left(\frac{(ax-1)\sqrt{-\frac{2}{ax-1}-1}\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)}{ad}-\frac{2\left(ac\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)-2d\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)\right)\arctan\left(\sqrt{-\frac{2}{ax-1}-1}\right)}{ad^2}+\frac{2\left(a^2c^2\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)-2acd\,\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)+d^2\mathrm{sgn}\left(\frac{1}{ax-1}\right)\mathrm{sgn}(a)\right)\arctan\left(\frac{ac\sqrt{-\frac{2}{ax-1}-1}+d\sqrt{-\frac{2}{ax-1}-1}}{\sqrt{a^2c^2-d^2}}\right)}{\sqrt{a^2c^2-d^2}\,ad^2}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -((a*x - 1)*sqrt(-2/(a*x - 1) - 1)*sgn(1/(a*x - 1))*sgn(a)/(a*d) - 2*(a*c*s
gn(1/(a*x - 1))*sgn(a) - 2*d*sgn(1/(a*x - 1))*sgn(a))*arctan(sqrt(-2/(a*x -
 1) - 1))/(a*d^2) + 2*(a^2*c^2*sgn(1/(a*x - 1))*sgn(a) - 2*a*c*d*sgn(1/(a*x
 - 1))*sgn(a) + d^2*sgn(1/(a*x - 1))*sgn(a))*arctan((a*c*sqrt(-2/(a*x - 1)
- 1) + d*sqrt(-2/(a*x - 1) - 1))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*
a*d^2))*abs(a)
```

Mupad [B]
time = 0.29, size = 148, normalized size = 1.38

$$-\frac{\sqrt{1-a^2\,x^2}}{d}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\left(2\,a\,\sqrt{-a^2}-\frac{a^2\,c\,\sqrt{-a^2}}{d}\right)}{a^2\,d}-\frac{\left(\ln\left(\sqrt{1-\frac{a^2\,c^2}{d^2}}\,\sqrt{1-a^2\,x^2}+\frac{a^2\,c\,x}{d}+1\right)-\ln\left(c+d\,x\right)\right)\,\left(a^2\,c^2-2\,a\,c\,d+d^2\right)}{d^3\,\sqrt{1-\frac{a^2\,c^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - a^2*x^2)^(3/2)/((a*x - 1)^2*(c + d*x)),x)
```

```
[Out] - (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*c
*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2)^
(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3*(1
 - (a^2*c^2)/d^2)^(1/2))
```

3.621 $\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}+\frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2x^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}$$

```
[Out] -(a*c-2*d)*arcsin(a*x)/d^2+(a*c-d)^2*arctan((a^2*c*x+d)/(a^2*c^2-d^2)^(1/2)
/(-a^2*x^2+1)^(1/2))/d^2/(a^2*c^2-d^2)^(1/2)-(-a^2*x^2+1)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1668, 858, 222, 739, 210}

$$\frac{(ac-d)^2\text{ArcTan}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}}-\frac{\sqrt{1-a^2x^2}}{d}-\frac{\text{ArcSin}(ax)(ac-2d)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/d) - ((a*c - 2*d)*ArcSin[a*x])/d^2 + ((a*c - d)^2*ArcTa
n[(d + a^2*c*x)/(Sqrt[a^2*c^2 - d^2]*Sqrt[1 - a^2*x^2])])/(d^2*Sqrt[a^2*c^2
 - d^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{\int \frac{-a^2d^2+a^3(ac-2d)dx}{(c+dx)\sqrt{1-a^2x^2}}\,dx}{a^2d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(a(ac-2d))\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{d^2}+\frac{(ac-d)^2\int \frac{1}{(c+dx)\sqrt{1-a^2x^2}}}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}-\frac{(ac-d)^2\text{Subst}\left(\int \frac{1}{-a^2c^2+d^2-x^2}\,dx,x,-\right.}{d^2}\\
&= -\frac{\sqrt{1-a^2x^2}}{d}-\frac{(ac-2d)\sin^{-1}(ax)}{d^2}+\frac{(ac-d)^2\tan^{-1}\left(\frac{d+a^2cx}{\sqrt{a^2c^2-d^2}\sqrt{1-a^2}}\right.}{d^2\sqrt{a^2c^2-d^2}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 501 vs. 2(107) = 214.
time = 1.14, size = 501, normalized size = 4.68

$$
-\frac{ad^3(ac+d)\sqrt{1-a^2x^2}-a(ac-d)\sqrt{2a^2c^2-d^2-2ac\sqrt{a^2c^2-d^2}}\left(a^2c^2-d^2+ac\sqrt{a^2c^2-d^2}\right)\tan^{-1}\left(\frac{d\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)}{\sqrt{2a^2c^2-d^2-2ac\sqrt{a^2c^2-d^2}}}\right)+a(ac-d)\left(-a^2c^2+d^2+ac\sqrt{a^2c^2-d^2}\right)\sqrt{2a^2c^2-d^2+2ac\sqrt{a^2c^2-d^2}}\tan^{-1}\left(\frac{d\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)}{\sqrt{2a^2c^2-d^2+2ac\sqrt{a^2c^2-d^2}}}\right)+\sqrt{-a^2}\,(ac-d)d^2\sqrt{-a^2c^2+d^2}\tan^{-1}\left(\frac{-\sqrt{-a^2}\,d^2x\sqrt{1-a^2x^2}+a^2(c^2-d^2x^2)}{ac\sqrt{-a^2c^2+d^2}}\right)+\sqrt{-a^2}\,(ac-2d)d^2(ac+d)\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{ad^4(ac+d)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^2/((c + d*x)*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((a*d^3*(a*c + d)*Sqrt[1 - a^2*x^2] - a*(a*c - d)*Sqrt[2*a^2*c^2 - d^2 - 2
*a*c*Sqrt[a^2*c^2 - d^2]]*(a^2*c^2 - d^2 + a*c*Sqrt[a^2*c^2 - d^2])*ArcTan[
(d*(Sqrt[-a^2]*x - Sqrt[1 - a^2*x^2]))/Sqrt[2*a^2*c^2 - d^2 - 2*a*c*Sqrt[a^
2*c^2 - d^2]]] + a*(a*c - d)*(-(a^2*c^2) + d^2 + a*c*Sqrt[a^2*c^2 - d^2])*S
qrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]]*ArcTan[(d*(Sqrt[-a^2]*x -
Sqrt[1 - a^2*x^2]))/Sqrt[2*a^2*c^2 - d^2 + 2*a*c*Sqrt[a^2*c^2 - d^2]]] + Sq
rt[-a^2]*(a*c - d)*d^2*Sqrt[-(a^2*c^2) + d^2]*ArcTan[(-(Sqrt[-a^2]*d^2*x*Sq
rt[1 - a^2*x^2]) + a^2*(c^2 - d^2*x^2))/(a*c*Sqrt[-(a^2*c^2) + d^2])] + Sqr
t[-a^2]*(a*c - 2*d)*d^2*(a*c + d)*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]])
/(a*d^4*(a*c + d)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(99) = 198$.

time = 0.10, size = 242, normalized size = 2.26

method	result
default	$-\frac{a\left(\frac{d\sqrt{-a^2x^2+1}}{a}+\frac{ac\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{2d\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}\right)}{d^2}-\frac{(a^2c^2-2acd+d^2)\ln\left(\frac{-\frac{2(a^2c^2-d^2)}{d^2}\cdots}{\cdots}\right.}{\cdots}$
risch	$\frac{a^2x^2-1}{d\sqrt{-a^2x^2+1}}-\frac{a^2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)c}{d^2\sqrt{a^2}}+\frac{2a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{d\sqrt{a^2}}-\frac{\ln\left(\frac{-\frac{2(a^2c^2-d^2)}{d^2}+\frac{2a^2c\left(x+\frac{c}{d}\right)}{d}+2\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d^2*(1/a*d*(-a^2*x^2+1)^(1/2)+a*c/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2
*x^2+1)^(1/2))-2*d/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)))-(a
^2*c^2-2*a*c*d+d^2)/d^3/(-(a^2*c^2-d^2)/d^2)^(1/2)*ln((-2*(a^2*c^2-d^2)/d^2
+2*a^2*c/d*(x+c/d)+2*(-(a^2*c^2-d^2)/d^2)^(1/2)*(-a^2*(x+c/d)^2+2*a^2*c/d*(
x+c/d)-(a^2*c^2-d^2)/d^2)^(1/2))/(x+c/d))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a*c>0)', see `assume?` for more d
etails)
```

Fricas [A]
time = 4.41, size = 318, normalized size = 2.97

$$\left[-\frac{(ac-d)\sqrt{-\frac{ac-d}{ac+d}}\log\left(\frac{a^2cdx+d^2-(a^2c^2-d^2)\sqrt{-a^2x^2+1}-\left(acd+d^2+(a^3c^2+a^2cd)x+\sqrt{-a^2x^2+1}(acd+d^2)\right)\sqrt{-\frac{ac-d}{ac+d}}}{dx+c}\right)-2(ac-2d)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}d}{d^2}, \frac{2(ac-d)\sqrt{\frac{ac-d}{ac+d}}\arctan\left(\frac{\left(dx-\sqrt{-a^2x^2+1}c+c\right)\sqrt{\frac{ac-d}{ac+d}}}{(ac-d)x}\right)+2(ac-2d)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)-\sqrt{-a^2x^2+1}d}{d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-((a*c - d)*sqrt(-(a*c - d)/(a*c + d))*log((a^2*c*d*x + d^2 - (a^2*c^2 - d
^2)*sqrt(-a^2*x^2 + 1) - (a*c*d + d^2 + (a^3*c^2 + a^2*c*d)*x + sqrt(-a^2*x
^2 + 1)*(a*c*d + d^2))*sqrt(-(a*c - d)/(a*c + d)))/(d*x + c)) - 2*(a*c - 2*
d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2 + 1)*d)/d^2, (2*(
a*c - d)*sqrt((a*c - d)/(a*c + d))*arctan((d*x - sqrt(-a^2*x^2 + 1)*c + c)*
sqrt((a*c - d)/(a*c + d))/((a*c - d)*x)) + 2*(a*c - 2*d)*arctan((sqrt(-a^2*
x^2 + 1) - 1)/(a*x)) - sqrt(-a^2*x^2 + 1)*d)/d^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^2}{\sqrt{-(ax-1)(ax+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**2/(d*x+c)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a*x + 1)**2/(sqrt(-(a*x - 1)*(a*x + 1))*(c + d*x)), x)
```

Giac [A]
time = 3.90, size = 131, normalized size = 1.22

$$-\frac{(a^2c-2\,ad)\arcsin(ax)\,\mathrm{sgn}(a)}{d^2|a|}-\frac{\sqrt{-a^2x^2+1}}{d}-\frac{2\left(a^3c^2-2\,a^2cd+ad^2\right)\arctan\left(\frac{d+\frac{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)c}{ax}}{\sqrt{a^2c^2-d^2}}\right)}{\sqrt{a^2c^2-d^2}\,d^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^2/(d*x+c)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a^2*c - 2*a*d)*arcsin(a*x)*sgn(a)/(d^2*abs(a)) - sqrt(-a^2*x^2 + 1)/d - 2
*(a^3*c^2 - 2*a^2*c*d + a*d^2)*arctan((d + (sqrt(-a^2*x^2 + 1)*abs(a) + a)*
c/(a*x))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*d^2*abs(a))
```

Mupad [B]
time = 0.12, size = 148, normalized size = 1.38

$$-\frac{\sqrt{1-a^2\,x^2}}{d}-\frac{\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)\,\left(2\,a\,\sqrt{-a^2}-\frac{a^2\,c\,\sqrt{-a^2}}{d}\right)}{a^2\,d}-\frac{\left(\ln\left(\sqrt{1-\frac{a^2\,c^2}{d^2}}\,\sqrt{1-a^2\,x^2}+\frac{a^2\,c\,x}{d}+1\right)-\ln\left(c+d\,x\right)\right)\,\left(a^2\,c^2-2\,a\,c\,d+d^2\right)}{d^3\,\sqrt{1-\frac{a^2\,c^2}{d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((1 - a^2*x^2)^(1/2)*(c + d*x)),x)
```

```
[Out] - (1 - a^2*x^2)^(1/2)/d - (asinh(x*(-a^2)^(1/2))*(2*a*(-a^2)^(1/2) - (a^2*c
*(-a^2)^(1/2))/d))/(a^2*d) - ((log((1 - (a^2*c^2)/d^2)^(1/2)*(1 - a^2*x^2)^
(1/2) + (a^2*c*x)/d + 1) - log(c + d*x))*(d^2 + a^2*c^2 - 2*a*c*d))/(d^3*(1
 - (a^2*c^2)/d^2)^(1/2))
```

3.622 $\int (d+ex)^3 \sqrt{f+gx} \sqrt{a+cx^2} \, dx$

Optimal. Leaf size=851

$$-\frac{2(150a^2e^4g^4 - 6ace^2g^2(2e^2f^2 - 33defg + 165d^2g^2) + c^2(187e^4f^4 - 732de^3f^3g + 1098d^2e^2f^2g^2 - 798d^3ef}{3465c^2eg^4}$$

```
[Out] -2/3465*(2*a*e^2*g^2*(-231*d*g+74*e*f)-c*(-567*d^3*g^3+1107*d^2*e*f*g^2-843
*d*e^2*f^2*g+233*e^3*f^3))*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^4+2/693*e*(18*
a*e^2*g^2-c*(81*d^2*g^2-96*d*e*f*g+29*e^2*f^2))*(g*x+f)^(5/2)*(c*x^2+a)^(1/
2)/c/g^4+2/99*e^2*(-3*d*g+e*f)*(g*x+f)^(7/2)*(c*x^2+a)^(1/2)/g^4-2/3465*(15
0*a^2*e^4*g^4-6*a*c*e^2*g^2*(165*d^2*g^2-33*d*e*f*g+2*e^2*f^2)+c^2*(315*d^4
*g^4-798*d^3*e*f*g^3+1098*d^2*e^2*f^2*g^2-732*d*e^3*f^3*g+187*e^4*f^4))*(g*
x+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/e/g^4+2/11*(e*x+d)^4*(g*x+f)^(1/2)*(c*x^2+a)
^(1/2)/e+4/3465*(3*a^2*e^2*g^4*(231*d*g+26*e*f)-c^2*f^2*(-231*d^3*g^3+396*d
^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)-9*a*c*g^2*(77*d^3*g^3+88*d^2*e*f*g^2
-33*d*e^2*f^2*g+6*e^3*f^3))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^
(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*
(c*x^2/a+1)^(1/2)/c^(3/2)/g^5/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2
)+f*c^(1/2)))^(1/2)-4/3465*(a*g^2+c*f^2)*(75*a^2*e^3*g^4-3*a*c*e*g^2*(165*d
^2*g^2-33*d*e*f*g+2*e^2*f^2)-c^2*f*(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*
f^2*g+64*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2
*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(5/2)/g^5/(g*x+f)^(1/2)/(c*x^
2+a)^(1/2)
```

Rubi [A]

time = 2.50, antiderivative size = 851, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {933, 1668, 858, 733, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2)
 + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*
g^3 + 315*d^4*g^4))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3465*c^2*e*g^4) + (2*(d
```

```
 + e*x)^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(11*e) - (2*(2*a*e^2*g^2*(74*e*f -
 231*d*g) - c*(233*e^3*f^3 - 843*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g
^3))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(3465*c*g^4) + (2*e*(18*a*e^2*g^2 - c
*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(
693*c*g^4) + (2*e^2*(e*f - 3*d*g)*(f + g*x)^(7/2)*Sqrt[a + c*x^2])/(99*g^4)
 + (4*Sqrt[-a]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g) - c^2*f^2*(64*e^3*f^3 - 26
4*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) - 9*a*c*g^2*(6*e^3*f^3 - 33*
d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(3465*c^(3/2)*g^5*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(75*a^2*e^
3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) - c^2*f*(64*e^3*
f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3))*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3
465*c^(5/2)*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m/(Sqrt[f + g*x
]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3
*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}\,dx=\frac{2(d+ex)^4\sqrt{f+gx}\,\sqrt{a+cx^2}}{11e}+\frac{\int \frac{(d+ex)^3(a(3ef-dg)-2(cdf-aeg)x+c(ef-3d}{\sqrt{f+gx}\,\sqrt{a+cx^2}}}{11e}$$

$$=\frac{2(d+ex)^4\sqrt{f+gx}\,\sqrt{a+cx^2}}{11e}+\frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{a+cx^2}}{99g^4}$$

$$=\frac{2(d+ex)^4\sqrt{f+gx}\,\sqrt{a+cx^2}}{11e}+\frac{2e(18ae^2g^2-c(29e^2f^2-96defg+}{693cg}$$

$$=\frac{2(d+ex)^4\sqrt{f+gx}\,\sqrt{a+cx^2}}{11e}-\frac{2(2ae^2g^2(74ef-231dg)-c(233e^3}{}$$

$$=-\frac{2(150a^2e^4g^4-6ace^2g^2(2e^2f^2-33defg+165d^2g^2)+c^2(187e^4f^4-}{34}$$

$$=-\frac{2(150a^2e^4g^4-6ace^2g^2(2e^2f^2-33defg+165d^2g^2)+c^2(187e^4f^4-}{34}$$

$$=-\frac{2(150a^2e^4g^4-6ace^2g^2(2e^2f^2-33defg+165d^2g^2)+c^2(187e^4f^4-}{34}$$

$$=-\frac{2(150a^2e^4g^4-6ace^2g^2(2e^2f^2-33defg+165d^2g^2)+c^2(187e^4f^4-}{34}$$

Mathematica [C] Result contains complex when optimal does not.
time = 26.98, size = 1034, normalized size = 1.22

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*((2*g^2*(-3*a^2*e^2*g^4*(26*e*f + 231*d*g) + c^2*f^2*(64*e
^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 9*a*c*g^2*(6*e^
3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*(a + c*x^2))/(f + g*
x) - g^2*(a + c*x^2)*(150*a^2*e^3*g^4 - 2*a*c*e*g^2*(495*d^2*g^2 + 33*d*e*g
*(4*f + 7*g*x) + e^2*(-23*f^2 + 16*f*g*x + 45*g^2*x^2)) + c^2*(-231*d^3*g^3
*(f + 3*g*x) - 99*d^2*e*g^2*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) - 33*d*e^2*g*(8
*f^3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3) + e^3*(64*f^4 - 48*f^3*g*x + 4
0*f^2*g^2*x^2 - 35*f*g^3*x^3 - 315*g^4*x^4))) + (2*Sqrt[c]*((-I)*Sqrt[c]*f
+ Sqrt[a]*g)*(-3*a^2*e^2*g^4*(26*e*f + 231*d*g) + c^2*f^2*(64*e^3*f^3 - 264
*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 9*a*c*g^2*(6*e^3*f^3 - 33*d
*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x
))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x
]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqr
t[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]] + (2*Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(75*a^2*e^3*g^4 - (3*I)*a^
(3/2)*Sqrt[c]*e^2*g^3*(e*f + 231*d*g) - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g
 + 165*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 264*d*e^2*f^2*g - 396*d^2*e*f*g^2 +
231*d^3*g^3) + (3*I)*Sqrt[a]*c^(3/2)*g*(16*e^3*f^3 - 66*d*e^2*f^2*g + 99*d^
2*e*f*g^2 + 231*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3465*c
^2*g^6*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6456 vs. $2(761) = 1522$.

time = 0.19, size = 6457, normalized size = 7.59

method	result	size
elliptic	Expression too large to display	1824
risch	Expression too large to display	2571
default	Expression too large to display	6457

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 753, normalized size = 0.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/10395*(2*(231*c^3*d^3*f^3*g^3 + 2079*a*c^2*d^3*f*g^5 - (64*c^3*f^6 + 102*
a*c^2*f^4*g^2 - 51*a^2*c*f^2*g^4 - 225*a^3*g^6)*e^3 + 33*(8*c^3*d*f^5*g + 1
5*a*c^2*d*f^3*g^3 - 33*a^2*c*d*f*g^5)*e^2 - 99*(4*c^3*d^2*f^4*g^2 + 11*a*c^
2*d^2*f^2*g^4 + 15*a^2*c*d^2*g^6)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f
^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g
) + 6*(231*c^3*d^3*f^2*g^4 - 693*a*c^2*d^3*g^6 - 2*(32*c^3*f^5*g + 27*a*c^2
*f^3*g^3 - 39*a^2*c*f*g^5)*e^3 + 33*(8*c^3*d*f^4*g^2 + 9*a*c^2*d*f^2*g^4 +
21*a^2*c*d*g^6)*e^2 - 396*(c^3*d^2*f^3*g^3 + 2*a*c^2*d^2*f*g^5)*e)*sqrt(c*g
)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/
(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 +
9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*(693*c^3*d^3*g^6*x + 231*c^3*d^
3*f*g^5 + (315*c^3*g^6*x^4 + 35*c^3*f*g^5*x^3 - 64*c^3*f^4*g^2 - 46*a*c^2*f
^2*g^4 - 150*a^2*c*g^6 - 10*(4*c^3*f^2*g^4 - 9*a*c^2*g^6)*x^2 + 16*(3*c^3*f
^3*g^3 + 2*a*c^2*f*g^5)*x)*e^3 + 33*(35*c^3*d*g^6*x^3 + 5*c^3*d*f*g^5*x^2 +
 8*c^3*d*f^3*g^3 + 8*a*c^2*d*f*g^5 - 2*(3*c^3*d*f^2*g^4 - 7*a*c^2*d*g^6)*x)
*e^2 + 99*(15*c^3*d^2*g^6*x^2 + 3*c^3*d^2*f*g^5*x - 4*c^3*d^2*f^2*g^4 + 10*
a*c^2*d^2*g^6)*e)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^3*g^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+cx^2}\,(d+ex)^3\,\sqrt{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**3*sqrt(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3,x)
```

```
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3, x)
```

3.623 $\int (d+ex)^2 \sqrt{f+gx} \sqrt{a+cx^2} \, dx$

Optimal. Leaf size=635

$$-\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{315ceg^3}+\frac{2(d+ex)^3\sqrt{f}}{9}$$

[Out] 4/315*(7*a*e^2*g^2-c*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^3+2/63*e*(-3*d*g+e*f)*(g*x+f)^(5/2)*(c*x^2+a)^(1/2)/g^3-2/315*(6*a*e^2*g^2*(-10*d*g+e*f)-c*(-35*d^3*g^3+63*d^2*e*f*g^2-57*d*e^2*f^2*g+19*e^3*f^3))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e/g^3+2/9*(e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e+4/315*(21*a^2*e^2*g^4+3*a*c*g^2*(-21*d^2*g^2-16*d*e*f*g+3*e^2*f^2)+c^2*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/c^(3/2)/g^4/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/315*(a*g^2+c*f^2)*(3*a*e*g^2*(-10*d*g+e*f)+c*f*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^4/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)

Rubi [A]

time = 1.07, antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {933, 1668, 858, 733, 435, 430}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,(21a^2e^2g^4+3acg^2(-21d^2g^2-16defg+3e^2f^2)+c^2f^2(21d^2g^2-24defg+8e^2f^2))\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(3aeg^2(ef-10dg)+cf(21d^2g^2-24defg+8e^2f^2))\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{a+cx^2}\,(f+gx)^{3/2}\,(7ae^2g^2-c(21d^2g^2-24defg+8e^2f^2))}{315cg^3}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}\,(6ae^2g^2(ef-10dg)-c(-35d^3g^3+63d^2efg^2-57de^2f^2g+19e^3f^3))}{315ceg^3}+\frac{2e\sqrt{a+cx^2}\,(f+gx)^{5/2}(ef-3dg)}{63g^3}+\frac{2\sqrt{a+cx^2}\,(d+ex)^3\sqrt{f+gx}}{9e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]

[Out] (-2*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*e*g^3) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*e) + (4*(7*a*e^2*g^2 - c*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*c*g^3) + (2*e*(e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^3) + (4*Sqrt[-a]*(21*a^2*e^2*g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt

```
[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^4*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(3*a*e*g^2*
(e*f - 10*d*g) + c*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*Sqrt[(Sqrt[c]*(
f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]
)/(315*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m/(Sqrt[f + g*x
]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3
*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
```

```
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9e} + \frac{\int \frac{(d+ex)^2(a(3ef-dg)-2(cdf-aeg)x+c(ef-}{\sqrt{f+gx}\,\sqrt{a+cx^2}}}{9e}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9e} + \frac{2e(ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^3}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9e} + \frac{4(7ae^2g^2-c(8e^2f^2-24defg+2}{315cg}$$

$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3}{315ceg^3}$$

$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3}{315ceg^3}$$

$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3}{315ceg^3}$$

$$= -\frac{2(6ae^2g^2(ef-10dg)-c(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3}{315ceg^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.78, size = 809, normalized size = 1.27

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e*g^2*(4*e*f + 30*d*g + 7*e*g*x) + c*(2
1*d^2*g^2*(f + 3*g*x) + 6*d*e*g*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) + e^2*(8*f^
3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3))))/(c*g^3) - (4*(g^2*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21
*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2))*(a + c*x^2) -
 I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 -
 24*d*e*f*g + 21*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2
))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt
[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)*a^(3/2)*e^
2*g^3 - 3*a*Sqrt[c]*e*g^2*(e*f - 10*d*g) + c^(3/2)*f*(-8*e^2*f^2 + 24*d*e*f
*g - 21*d^2*g^2) - (3*I)*Sqrt[a]*c*g*(-2*e^2*f^2 + 6*d*e*f*g + 21*d^2*g^2))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)]))/(c^2*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(315*Sqrt
[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4350 vs. $2(551) = 1102$.

time = 0.10, size = 4351, normalized size = 6.85

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^2x^3\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{9}+\frac{2\left(2cdeg+\frac{1}{9}fce^2\right)x^2\sqrt{cg\,x^3+cf\,x^2+agx+f}}{7cg}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-108*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*
c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c
*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a
*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a
*c*d*e*f^2*g^4-2*a*c^2*e^2*f^3*g^3*x-60*a^2*c*d*e*f*g^5+24*a*c^2*d*e*f^3*g^
3-84*a*c^2*d^2*f*g^5*x-62*a*c^2*e^2*f*g^5*x^3+6*c^3*d*e*f^2*g^4*x^3-7*a*c^2
*e^2*f^2*g^4*x^2+24*c^3*d*e*f^3*g^3*x^2-60*a^2*c*d*e*g^6*x-22*a^2*c*e^2*f*g
^5*x-108*c^3*d*e*f*g^5*x^4-150*a*c^2*d*e*g^6*x^3-35*c^3*e^2*g^6*x^6-63*c^3*
d^2*g^6*x^4+84*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))
*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^
(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c
*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c^2*d^2*f^2*g^4-34*(-(g*x+f)*c/(g*(-a*c)
^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x
+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c
)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c
^2*e^2*f^4*g^2+48*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/
2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f
))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2
)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^3*d*e*f^5*g+42*a^3*(-(g*x+f)*c/(g*(-a
*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((
c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-
a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*
e^2*g^6-42*a^3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))
*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^
(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c
*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^2*g^6-16*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f
))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1
/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*
f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^3*e^2*f^6-1
68*a*c^2*d*e*f*g^5*x^2+6*a*c^2*d*e*f^2*g^4*x-63*a*c^2*d^2*g^6*x^2-21*c^3*d^
2*f^2*g^4*x^2-8*c^3*e^2*f^4*g^2*x^2-8*a^2*c*e^2*f^2*g^4-21*a*c^2*d^2*f^2*g^
4-8*a*c^2*e^2*f^4*g^2-84*c^3*d^2*f*g^5*x^3-2*c^3*e^2*f^3*g^3*x^3-14*a^2*c*e
^2*g^6*x^2-90*c^3*d*e*g^6*x^5-40*c^3*e^2*f*g^5*x^5-49*a*c^2*e^2*g^6*x^4+c^3
*e^2*f^2*g^4*x^4-126*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(
-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^
(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-
a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d^2*g^6+126*a^2*c*(-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c
/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(
1/2))*d^2*g^6-42*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2
))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f)
)^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)
-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^3*d^2*f^4*g^2-60*(-a*c)^(1/2)*(-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f)
```

```
)^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f
))^(1/2))*a^2*d*e*g^6+6*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2
)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/
(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/
2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*e^2*f*g^5+42*(-a
*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g
*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)
*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(
g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d^2*f^3*g^3+16*(-a*c)^(1/2)*(-(g*x+f)*c/(g*
(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)
*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g
*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2
))*c^2*e^2*f^5*g+54*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c
*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1
/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*c*e^2*f^2*g^4-126*(-(g*x+f)*c/(g*
(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)
*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g
*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2
))*a*c^2*d^2*f^2*g^4+12*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*
c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 503, normalized size = 0.79

$$\frac{2\left(2\left(21c^2d^2f^3g^2+189acd^2fg^4+(8c^2f^5+15acf^3g^2-33a^2fg^4)e^2-6(4c^2df^4g+11acdf^2g^3+15a^2dg^5)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\frac{3gx+f}{3g}\right)+6\left(21c^2d^2f^2g^3-63acd^2g^5+(8c^2f^4g+9acf^2g^3+21a^2g^5)e^2-24(c^2df^3g^2+2acdfg^4)e\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\mathrm{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\frac{3gx+f}{3g}\right)\right)+3\left(63c^2d^2g^5x+21c^2d^2fg^4+(35c^2g^5x^3+5c^2fg^4x^2+8c^2f^3g^2+8acfg^4-2(3c^2f^2g^3-7acg^5)x)e^2+6(15c^2dg^5x^2+3c^2dfg^4x-4c^2df^2g^3+10acdg^5)e\right)\sqrt{cx^2+a}\,\sqrt{gx+f}\right)}{945c^2g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(2*(21*c^2*d^2*f^3*g^2 + 189*a*c*d^2*f*g^4 + (8*c^2*f^5 + 15*a*c*f^3*
g^2 - 33*a^2*f*g^4)*e^2 - 6*(4*c^2*d*f^4*g + 11*a*c*d*f^2*g^3 + 15*a^2*d*g^
5)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c
*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(21*c^2*d^2*f^2*g^3 - 63*
a*c*d^2*g^5 + (8*c^2*f^4*g + 9*a*c*f^2*g^3 + 21*a^2*g^5)*e^2 - 24*(c^2*d*f^
```

```
3*g^2 + 2*a*c*d*f*g^4)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(
c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 -
 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) +
 3*(63*c^2*d^2*g^5*x + 21*c^2*d^2*f*g^4 + (35*c^2*g^5*x^3 + 5*c^2*f*g^4*x^2
 + 8*c^2*f^3*g^2 + 8*a*c*f*g^4 - 2*(3*c^2*f^2*g^3 - 7*a*c*g^5)*x)*e^2 + 6*(
15*c^2*d*g^5*x^2 + 3*c^2*d*f*g^4*x - 4*c^2*d*f^2*g^3 + 10*a*c*d*g^5)*e)*sqr
t(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + cx^2}\,(d + ex)^2\,\sqrt{f + gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2,x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2, x)
```

3.624 $\int (d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=434

$$4\sqrt{-a}\,(cf^2$$

$$-\frac{2\sqrt{f+gx}\,(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2}+\frac{2e\sqrt{f+gx}\,(a+cx^2)^{3/2}}{7c}-$$

```
[Out] 2/7*e*(c*x^2+a)^(3/2)*(g*x+f)^(1/2)/c-2/105*(5*a*e*g^2+c*f*(-7*d*g+4*e*f)-3
*c*g*(7*d*g+e*f)*x)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2-4/105*(c*f^2*(-7*d*
g+4*e*f)+a*g^2*(21*d*g+8*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/
2)*(c*x^2/a+1)^(1/2)/g^3/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(
1/2)+f*c^(1/2)))^(1/2)+4/105*(a*g^2+c*f^2)*(5*a*e*g^2+c*f*(-7*d*g+4*e*f))*E
llipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1
/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^
(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {847, 829, 858, 733, 435, 430}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(5aeg^2+cf(4ef-7dg))\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{105c^{3/2}g^3\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(ag^2(21dg+8ef)+cf^2(4ef-7dg))\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{105\sqrt{c}\,g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{2\sqrt{a+cx^2}\,\sqrt{f+gx}\,(5aeg^2-3cgx(7dg+ef)+cf(4ef-7dg))}{105cg^2}+\frac{2e(a+cx^2)^{3/2}\sqrt{f+gx}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)
*Sqrt[a + c*x^2])/(105*c*g^2) + (2*e*Sqrt[f + g*x]*(a + c*x^2)^(3/2))/(7*c)
 - (4*Sqrt[-a]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(c*f^2
+ a*g^2)*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c
]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^
3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}\,dx = \frac{2e\sqrt{f+gx}\,(a+cx^2)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}(7cdf-aeg)+\frac{1}{2}c(ef+7dg)x\right)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx}{7c}$$

$$= -\frac{2\sqrt{f+gx}\,(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} +$$

$$= -\frac{2\sqrt{f+gx}\,(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} +$$

$$= -\frac{2\sqrt{f+gx}\,(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} +$$

$$= -\frac{2\sqrt{f+gx}\,(5aeg^2+cf(4ef-7dg)-3cg(ef+7dg)x)\sqrt{a+cx^2}}{105cg^2} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.13, size = 610, normalized size = 1.41

$$\frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(10aeg^2+7cdg(f+3gx)+ce(-4f^2+3fgx+15g^2x^2))}{cg^2}+\frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(cf^2(4ef-7dg)+ag^2(8ef+21dg))(a+cx^2)+i\sqrt{c}\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)(cf^2(-4ef+7dg)-ag^2(8ef+21dg))\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)+\sqrt{a}\,g\left(i\sqrt{c}\,f-\sqrt{a}\,g\right)\left(5iaeg^2+icf(4ef-7dg)+3\sqrt{a}\,\sqrt{c}\,g(ef+7dg)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{cg^4\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+gx)}\right)}{105\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e*g^2 + 7*c*d*g*(f + 3*g*x) + c*e*(-4*
f^2 + 3*f*g*x + 15*g^2*x^2)))/(c*g^2) + (4*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*(a + c*x^2) + I*Sqrt
[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(c*f^2*(-4*e*f + 7*d*g) - a*g^2*(8*e*f + 21*d
*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sq
rt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] + Sqrt[a]*g*(I*Sqrt[c]*f - Sqrt[a]*g)*((5*I)*a*e*g^2 + I*c*f*(
4*e*f - 7*d*g) + 3*Sqrt[a]*Sqrt[c]*g*(e*f + 7*d*g))*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
+ g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. $2(362) = 724$.

time = 0.10, size = 2549, normalized size = 5.87

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2ex^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{7}+\frac{2\left(dgc+\frac{1}{7}cef\right)x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5cg}\right.}{}$

risch	$\frac{2(15ce\,x^2g^2+21cdx\,g^2+3cefgx+10ae\,g^2+7cdfg-4ce\,f^2)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{105c\,g^2} - \frac{2\left(-\frac{2\left(21ad\,g^3c+8aef\,g^2c-7c^2d\,f^2g+4c^2e\,f^3\right)}{\cdots}\right)}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/105*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(14*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*a*c*d*f*g^4-18*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*a*c*e*f^2*g^3-6*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*e*f*g^4-42*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*d*f^2*g^3+4*a*c^2*e*f^3*g^2-10*a^2*c*e*f*g^4-7*a*c^2*d*f^2*g^3-21*a*c^2*d*g^5*x^2-7*c^3*d*f^2*g^3*x^2+4*c^3*e*f^3*g^2*x^2-10*a^2*c*e*g^5*x-25*a*c^2*e*g^5*x^3-28*c^3*d*f*g^4*x^3+c^3*e*f^2*g^3*x^3-18*c^3*e*f*g^4*x^4-28*a*c^2*d*f*g^4*x+a*c^2*e*f^2*g^3*x-28*a*c^2*e*f*g^4*x^2-15*c^3*e*g^5*x^5-21*c^3*d*g^5*x^4+8*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*c^3*e*f^5-14*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*

(-a*c)^(1/2)+c*f))^(1/2)*c^3*d*f^4*g-42*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*d*g^5-10*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*a^2*e*g^5+42*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*d*g^5+28*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*d*f^2*g^3+24*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*e*f^3*g^2-6*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*e*f^3*g^2+14*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*c^2*d*f^3*g^2-8*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*c^2*e*f^4*g+16*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*e*f*g^4)/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^4/c^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.38, size = 346, normalized size = 0.80

$$\frac{2\left(2\left(7c^2df^3g+63acdfg^3-(4c^2f^4+11acf^2g^2+15a^2g^4)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\frac{3gx+f}{3g}\right)+6\left(7c^2df^2g^2-21acdg^4-4(c^2f^3g+2acfg^3)e\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\mathrm{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\frac{3gx+f}{3g}\right)\right)+3\left(21c^2dg^4x+7c^2dfg^3+(15c^2g^4x^2+3c^2fg^3x-4c^2f^2g^2+10acg^4)e\right)\sqrt{cx^2+a}\,\sqrt{gx+f}\right)}{315c^2g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(2*(7*c^2*d*f^3*g + 63*a*c*d*f*g^3 - (4*c^2*f^4 + 11*a*c*f^2*g^2 + 15
*a^2*g^4)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -
8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(7*c^2*d*f^2*g^2 -
 21*a*c*d*g^4 - 4*(c^2*f^3*g + 2*a*c*f*g^3)*e)*sqrt(c*g)*weierstrassZeta(4/
3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrass
PInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3),
1/3*(3*g*x + f)/g)) + 3*(21*c^2*d*g^4*x + 7*c^2*d*f*g^3 + (15*c^2*g^4*x^2 +
 3*c^2*f*g^3*x - 4*c^2*f^2*g^2 + 10*a*c*g^4)*e)*sqrt(c*x^2 + a)*sqrt(g*x +
f))/(c^2*g^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+cx^2}\,(d+ex)\,\sqrt{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x),x)
```

```
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x), x)
```

3.625 $\int \sqrt{f+gx}\,\sqrt{a+cx^2}\,dx$

Optimal. Leaf size=362

$$-\frac{4f\sqrt{f+gx}\,\sqrt{a+cx^2}}{15g}+\frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g}+\frac{4\sqrt{-a}\,(cf^2-3ag^2)\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\right.\right.}{15\sqrt{c}\,g^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}}$$

```
[Out] 2/5*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/g-4/15*f*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+
4/15*(-3*a*g^2+c*f^2)*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2
/a+1)^(1/2)/g^2/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^
(1/2)))^(1/2)-4/15*f*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(
1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/
a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/g^2/c^(1/2)/(g*
x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {749, 847, 858, 733, 435, 430}

$$-\frac{4\sqrt{-a}\,f\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{15\sqrt{c}\,g^2\sqrt{a+cx^2}\,\sqrt{f+gx}}+\frac{4\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(cf^2-3ag^2)\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{15\sqrt{c}\,g^2\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}+\frac{2\sqrt{a+cx^2}\,(f+gx)^{3/2}}{5g}-\frac{4f\sqrt{a+cx^2}\,\sqrt{f+gx}}{15g}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (-4*f*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*g) + (2*(f + g*x)^(3/2)*Sqrt[a + c
*x^2])/(5*g) + (4*Sqrt[-a]*(c*f^2 - 3*a*g^2)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqr
t[-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*
f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*f*(c*f^2 + a*g^2)*Sqrt[(Sqr
t[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f -
 a*g)])/(15*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{f+gx}\,\sqrt{a+cx^2}\,dx = \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{2\int \frac{(ag-cfx)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{5g}$$

$$= -\frac{4f\sqrt{f+gx}\,\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{4\int \frac{2acfg-\frac{1}{2}c(cf^2-3ag^2)x}{\sqrt{f+gx}\,\sqrt{a+cx}}}{15cg}$$

$$= -\frac{4f\sqrt{f+gx}\,\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{1}{15}\left(2\left(3a-\frac{cf^2}{g^2}\right)\right)\int$$

$$\left(4a\left(3a-\frac{cf^2}{g^2}\right)\sqrt{f+\right.$$

$$= -\frac{4f\sqrt{f+gx}\,\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} + \frac{}{}$$

$$4\sqrt{-a}\left(3a-\frac{cf^2}{g^2}\right)\sqrt{f}$$

$$= -\frac{4f\sqrt{f+gx}\,\sqrt{a+cx^2}}{15g} + \frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g} - \frac{}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 21.88, size = 536, normalized size = 1.48

$$\frac{\sqrt{f+gx}\left(\frac{2(f+3gx)(a+cx^2)}{g} - \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(-3a^2g^2+c^2f^2x^2+ac(f^2-3g^2x^2))+\sqrt{c}\left(-ic^{3/2}f^3+\sqrt{a}\,cf^2g+3ia\sqrt{c}\,fg^2-3a^{3/2}g^3\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)-\sqrt{a}\,\sqrt{c}\,g\left(cf^2+4i\sqrt{a}\,\sqrt{c}\,fg-3ag^2\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{cg^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+gx)}\right)}{15\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(f + 3*g*x)*(a + c*x^2))/g - (4*(g^2*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]]*(-3*a^2*g^2 + c^2*f^2*x^2 + a*c*(f^2 - 3*g^2*x^2)) + Sqrt[c]*
((-I)*c^(3/2)*f^3 + Sqrt[a]*c*f^2*g + (3*I)*a*Sqrt[c]*f*g^2 - 3*a^(3/2)*g^3
```

```
)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)] - Sqrt[a]*Sqrt[c]*g*(c*f^2 + (4*I)*Sqrt[a]*Sqrt[c]*f*g - 3*a*g^2)
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)]))/(c*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(15*Sqrt[a
+ c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(290) = 580$.

time = 0.10, size = 1162, normalized size = 3.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(6*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)
-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^
(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f
))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*g^4+6*((c*x+(-a
*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1
/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f
))^(1/2)*a*c*f^2*g^2-2*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*El
lipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(
-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/
2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*a*f*g^3-2*((c*x+(-a*c
)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2
)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1
/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)+c*f))^(1/2)*c*f^3*g-6*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(
1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*
f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)
*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*g^4-4*((c*x+(-a*c)^(1/2
))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f)
)^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-(g*x+f)*c/(g*
(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)
*a*c*f^2*g^2+2*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE(
(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1
/2)+c*f))^(1/2))*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2
))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*c^2*f^4+3*c^2*g^4*x^4+4*c^2*f*g^3*x^3+3*a*
c*g^4*x^2+c^2*f^2*g^2*x^2+4*a*c*f*g^3*x+a*c*f^2*g^2)/c/(c*g*x^3+c*f*x^2+a*g
*x+a*f)/g^3
```

Maple [F]

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 229, normalized size = 0.63

$$\frac{2\left(2\left(cf^{3}+9\,afg^{2}\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^{2}-3\,ag^{2}\right)}{3\,cg^{2}},-\frac{8\left(cf^{3}+9\,afg^{2}\right)}{27\,cg^{3}},\frac{3\,gx+f}{3\,g}\right)+6\left(cf^{2}g-3\,ag^{3}\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\left(cf^{2}-3\,ag^{2}\right)}{3\,cg^{2}},-\frac{8\left(cf^{3}+9\,afg^{2}\right)}{27\,cg^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^{2}-3\,ag^{2}\right)}{3\,cg^{2}},-\frac{8\left(cf^{3}+9\,afg^{2}\right)}{27\,cg^{3}},\frac{3\,gx+f}{3\,g}\right)\right)+3\left(3\,cg^{3}x+cfg^{2}\right)\sqrt{cx^{2}+a}\,\sqrt{gx+f}\right)}{45\,cg^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(2*(c*f^3 + 9*a*f*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(c*
f^2*g - 3*a*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -
8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)
/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*(3*c*g
^3*x + c*f*g^2)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+cx^{2}}\,\sqrt{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(a + c*x^2)^(1/2), x)
```

3.626 $\int \frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=683

$$\frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3e}-\frac{2\sqrt{-a}\,\sqrt{c}\,(ef-3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2a}{\sqrt{-a}\,\sqrt{\cdots}}\right.}{3e^2g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e-2/3*(-3*d*g+e*f)*EllipticE(1/2*(1-x*c^(
1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*
(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e^2/g/(c*x^2+a)^(1/2)/((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/3*(2*a*e^2*g-3*c*d*(-d*g+e
*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+2/3
*f*(-3*d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*
g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*
((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/g/(g*x+f)^(1/2)/(c*x^2
+a)^(1/2)-2*(a*e^2+c*d^2)*(-d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2)
)^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^
(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f
*c^(1/2)))^(1/2)/e^3/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 1.40, antiderivative size = 683, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {933, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\,(ae^2+cd^2)(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e^3\sqrt{a+cx^2}\,\sqrt{f+gx}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}+\frac{2\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{cx^2}{a}+1}\,(ef-3dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3e^2g\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(ef-3dg)E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3e^2g\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{2\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(2ae^2g-3cd(ef-dg))F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3\sqrt{c}\,e^3\sqrt{a+cx^2}\,\sqrt{f+gx}}+\frac{2\sqrt{a+cx^2}\,\sqrt{f+gx}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) - (2*Sqrt[-a]*Sqrt[c]*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqr
t[c]*f*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
```

```
(-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*e^2*g*Sqrt[f + g*x]*Sqrt[a + c*x^2
]) - (2*Sqrt[-a]*(2*a*e^2*g - 3*c*d*(e*f - d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(
Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c
]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-
a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 933

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m/(Sqrt[f + g*x
]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f - 3
*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[e*f - d*g
, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3e} + \frac{\int \frac{a(3ef-dg)-2(cdf-aeg)x+c(ef-3dg)x^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3e}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3e} + \frac{\int \left(\frac{2ae^2g-3cd(ef-dg)}{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{c(ef-3dg)x}{e\sqrt{f+gx}\,\sqrt{a+cx^2}} + \right.}{3e}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3e} + \frac{(c(ef-3dg))\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3e^2} + \frac{((cd^2+a}{}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3e} + \frac{(c(ef-3dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3e^2g} - \frac{(cf(ef-3dg))\int -}{3}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\,(2ae^2g-3cd(ef-dg))\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{}}{3\sqrt{c}\,e^3\sqrt{f+}}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\,\sqrt{c}\,(ef-3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\right.}{3e^2g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3e} - \frac{2\sqrt{-a}\,\sqrt{c}\,(ef-3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\right.}{3e^2g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.07, size = 1216, normalized size = 1.78

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) + ((f + g*x)^(3/2)*(2*c*e^2*f*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - 6*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] +
(2*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*c*d*e*f^2*g
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (2*a*e^2*f*g^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]])/(f + g*x)^2 - (4*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
g*x) + (12*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (2*Sqrt[
c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - 3*d*g)*Sqrt[1 - f/(f + g*x) - (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c
]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]
 + (2*e*(3*Sqrt[c]*d - I*Sqrt[a]*e)*g*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[1 -
 f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I
*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)])/Sqrt[f + g*x] + ((6*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*
x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((6*I)*a*e^2*g^2*Sqrt[1 - f
/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]
*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x
]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(
3*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + (c*(f + g*x)^2*(-1 + f/
(f + g*x))^2)/g^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2495 vs. $2(556) = 1112$.
time = 0.12, size = 2496, normalized size = 3.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(
-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*E
llipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*
```

(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3+3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3-(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g+3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*d*e*g^3-3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e^2*f*g^2-3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d^2*f*g^2+3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e*f^2*g-3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*d*e*g^3+(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e^2*f*g^2-3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e*f^2*g+(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e^2*f^3-3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3-3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3+3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-

```
a*c)^(1/2)-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(
g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*
f))^(1/2))*a*c*e^2*f*g^2+3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(
-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^
(1/2)-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a
*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(
1/2))*c^2*d^2*f*g^2-c^2*e^2*g^3*x^3-c^2*e^2*f*g^2*x^2-a*c*e^2*g^3*x-a*c*e^2
*f*g^2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e^3/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(x*e + d), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,\sqrt{f+gx}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(x*e + d), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x), x)
```

3.627 $\int \frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=650

$$-\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{e(d+ex)}-\frac{3\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}+$$

[Out] -(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)-3*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+3*f*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(-3*d*g+2*e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)

Rubi [A]
time = 1.08, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$,
Rules used = {931, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$-\frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{\frac{cx^2}{a}+1}\,(2ef-3dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e^3\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(ae^2g-cd(2ef-3dg))\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e^3\sqrt{a+cx^2}\,\sqrt{f+gx}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}+\frac{3\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e^2\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{3\sqrt{-a}\,\sqrt{c}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e^2\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{\sqrt{a+cx^2}\,\sqrt{f+gx}}{e(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]

[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x))) - (3*Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipt

```
icF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt
[c]*f - a*g)])/(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*(2*e
*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*
x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/
(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*
g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]
*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/
(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g
*x]*Sqrt[a + c*x^2]))*Simp[a*g + 2*c*f*x + 3*c*g*x^2, x], x], x] /; FreeQ[{
a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && Inte
gerQ[2*m] && LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{e(d+ex)} + \frac{\int \frac{ag+2cfx+3cgx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{e(d+ex)} + \frac{\int \left(\frac{c(2ef-3dg)}{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{3cgx}{e\sqrt{f+gx}\,\sqrt{a+cx^2}}\right.}{2e}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{e(d+ex)} + \frac{(3cg)\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2e^2} + \frac{(c(2ef-3dg))\int}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{e(d+ex)} + \frac{(3c)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2e^2} - \frac{(3cf)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}}{2e^2}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{e(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,(2ef-3dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}}{e^3\sqrt{f+gx}}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{e(d+ex)} - \frac{3\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{e^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+}}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{e(d+ex)} - \frac{3\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{e^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.78, size = 1331, normalized size = 2.05

$$\frac{\sqrt{f+gx}\left(-\frac{e^2(a+cx^2)}{d+ex}-\frac{-3ce^2f^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+3cdef^2g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}-3ae^2fg^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+3adeg^3\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}+6ce^2f^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}(f+gx)-6cdefg\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}(f+gx)-3ce^2f\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}(f+gx)^2+3cdeg\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}(f+gx)^2+3\sqrt{c}\,e\left(-i\sqrt{c}f+\sqrt{a}g\right)(-ef+dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+e\left(\sqrt{c}f+i\sqrt{a}g\right)\left(\sqrt{a}eg-i\sqrt{c}(2ef-3dg)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+2icdefg\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}\Pi\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+i\sqrt{a}g)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-3icd^2g^2\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}\Pi\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+i\sqrt{a}g)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-iae^2g^2\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}\Pi\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+i\sqrt{a}g)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-dg)(f+gx)}\right)}{e^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

```
[Out] (Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-3*c*e^2*f^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]] + 3*c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a
*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]] + 6*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) -
6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 3*c*e^2*f*Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]*(f + g*x)^2 + 3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g
)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)] + e*(Sqrt[c]*f + I*Sqrt[a]*g)*(Sqrt[a]*e*g - I*Sqrt[c]*(2*e*f - 3
*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
 I*Sqrt[a]*g)] + (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] - (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f
 + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(
f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*
EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(
Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*
(f + g*x))))/(e^3*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6043 vs. $2(531) = 1062$.
time = 0.10, size = 6044, normalized size = 9.30

method	result

elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{e(ex+d)}+\frac{2\left(-\frac{c(2dg-ef)}{e^3}+\frac{cdg}{2e^3}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}}{\ldots}\right.}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(x*e + d)^2, x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,\sqrt{f+gx}}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2,x)
```

```
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2, x)
```

3.628 $\int \frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=1205

$$-\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2e(d+ex)^2}-\frac{\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}{4e\left(cd^2+ae^2\right)(ef-dg)(d+ex)}-\frac{\sqrt{-a}\,\sqrt{c}\,\left(ae^2g-cd(2ef-3dg)\right)\sqrt{\cdots}}{4e^2\left(cd^2+a\cdots\right.}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)^2-1/4*(a*e^2*g-c*d*(-3*d*g+2*e
*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-1/4*(
a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^
(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f
)^(1/2)*(c*x^2/a+1)^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)/((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-3/2*g*EllipticF(1/2*(1-x*c^(1/
2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-
a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*f*(a*e^2*g-c*d*(-3*d*g+2*e*f
))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1
/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f)^(1
/2)/(c*x^2+a)^(1/2)-1/4*d*g*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*(1-x
*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/
2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c
^(1/2)))^(1/2)/e^3/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-c
*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+
d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(e
+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*(a*e^2*g-c*d*(-3*d
*g+2*e*f))^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d
*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))
*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(a*
e^2+c*d^2)/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2
)
```

Rubi [A]

time = 2.76, antiderivative size = 1205, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {931, 6874, 733, 430, 954, 858, 435, 947, 174, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

```
[Out] -1/2*(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x)^2) - ((a*e^2*g - c*d*(2*e
*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g
)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g
*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqr
t[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f -
d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) -
(3*Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
rt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
 (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4
*e^2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]
*Sqrt[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt
[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]
*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e^3*(c*d^2
 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (c*(e*f - 3*d*g)*Sqr
t[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipti
cPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]
/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt
[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))
^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^3*((Sqrt[c]
*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
)
```

<u>Rule 174</u>

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

<u>Rule 430</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

<u>Rule 435</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/
(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g
*x]*Sqrt[a + c*x^2]))*Simp[a*g + 2*c*f*x + 3*c*g*x^2, x], x], x] /; FreeQ[{
a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && Inte
gerQ[2*m] && LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
```

```
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{\int \frac{ag+2cfx+3cgx^2}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{\int \left(\frac{3cg}{e^2\sqrt{f+gx}\sqrt{a+cx^2}} + \frac{ae^2g-cd(2ef-3dg)}{e^2(d+ex)^2\sqrt{f+gx}\sqrt{a+\cdots}}\right.}{4e}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} + \frac{(3cg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{4e^3} + \frac{(c(ef-3dg))\int \cdots}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{(ae^2\cdots}{\cdots}$$

$3\sqrt{-\cdots}$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)} - \cdots$$

$3\sqrt{-\cdots}$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)} - \cdots$$

$3\sqrt{-\cdots}$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)} - \cdots$$

$3\sqrt{-\cdots}$

Mathematica [C] Result contains complex when optimal does not.
time = 27.49, size = 2526, normalized size = 2.10

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-2 + ((a*e^2*g + c*d*(-2*e*f + 3*d*g))*(d +
 e*x))/((c*d^2 + a*e^2)*(-(e*f) + d*g))))/(4*e*(d + e*x)^2) + (-2*c^2*d*e^3
*f^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*c^2*d^2*e^2*f^3*g*Sqrt[-f - (I*Sq
rt[a]*g)/Sqrt[c]] + a*c*e^4*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*c^2*
d^3*e*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*d*e^3*f^2*g^2*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*a*c*d^2*e^2*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]] + a^2*e^4*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*d^3*e*g^4*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a^2*d*e^3*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]] + 4*c^2*d*e^3*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 10*c^2
*d^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*a*c*e^4*f^2*g
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 6*c^2*d^3*e*f*g^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*a*c*d*e^3*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]*(f + g*x) - 2*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
 g*x)^2 + 5*c^2*d^2*e^2*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 +
a*c*e^4*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 3*c^2*d^3*e*g^2*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - a*c*d*e^3*g^2*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + I*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(-(e
*f) + d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(
3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*d - I*Sqrt
[a]*e)*g*(I*Sqrt[c]*f - Sqrt[a]*g)*(a*e^2*g + c*d*(4*e*f - 3*d*g) + (2*I)*S
qrt[a]*Sqrt[c]*e*(e*f - d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]
*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF
[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*
Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (4*I)*a*c*e^4*f^2*g*Sqrt[(g*((I*Sqr
t[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqr
t[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[
c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (4*I)*c^2*d^3*e*f*g^2*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c
]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*
x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (12*I)*a*c*d*e^
3*f*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
```

```
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (3*
I)*c^2*d^4*g^3*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqr
t[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f
 - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)
] + (6*I)*a*c*d^2*e^2*g^3*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(S
qrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] - I*a^2*e^4*g^3*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*
Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi
[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)])/(4*e^3*(c*d^2 + a*e^2)*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19180 vs. 2(1009) = 2018.

time = 0.10, size = 19181, normalized size = 15.92

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{2e(ex+d)^2}+\frac{\left(a\,e^2g+3c\,d^2g-2cdef\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{4e\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)(ex+d)}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(x*e + d)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}\,\sqrt{f + gx}}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(x*e + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f + g\,x}\,\sqrt{c\,x^2 + a}}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3,x)
```

```
[Out] int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3, x)
```

3.629 $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=666

$$-\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f+gx}\,\sqrt{a+cx^2}}{315cg^4}+\frac{2(d+ex)^3\sqrt{}}{}$$

```
[Out] 4/315*e*(7*a*e^2*g^2+c*(42*d^2*g^2-111*d*e*f*g+64*e^2*f^2))*(g*x+f)^(3/2)*(
c*x^2+a)^(1/2)/c/g^4-4/63*e^2*(-3*d*g+4*e*f)*(g*x+f)^(5/2)*(c*x^2+a)^(1/2)/
g^4-4/315*(9*a*e^2*g^2*(-5*d*g+2*e*f)+c*(-35*d^3*g^3+168*d^2*e*f*g^2-204*d*
e^2*f^2*g+76*e^3*f^3))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^4+2/9*(e*x+d)^3*(g
*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/315*(21*a^2*e^3*g^4-3*a*c*e*g^2*(63*d^2*g^2
-39*d*e*f*g+10*e^2*f^2)-c^2*f*(-105*d^3*g^3+252*d^2*e*f*g^2-216*d*e^2*f^2*g
+64*e^3*f^3))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/
(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1
/2)/c^(3/2)/g^5/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^
(1/2)-4/315*(a*g^2+c*f^2)*(9*a*e^2*g^2*(-5*d*g+2*e*f)-c*(-105*d^3*g^3+252*d
^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/
2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c
*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^
5/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 1.04, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {935, 1668, 858, 733, 435, 430}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,(21a^2e^3g^4-3aceg^2(63d^2g^2-39defg+10e^2f^2)-c^2f(-105d^3g^3+252d^2efg^2-216de^2f^2g+64e^3f^3))\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(9ae^2g^2(2ef-5dg)-c(-105d^3g^3+252d^2efg^2-216de^2f^2g+64e^3f^3))\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4e\sqrt{a+cx^2}\,(f+gx)^{3/2}(7ae^2g^2+c(42d^2g^2-111defg+64e^2f^2))}{315cg^4}-\frac{4\sqrt{a+cx^2}\sqrt{f+gx}\,(9ae^2g^2(2ef-5dg)+c(-35d^3g^3+168d^2efg^2-204de^2f^2g+76e^3f^3))}{315cg^4}-\frac{4e^2\sqrt{a+cx^2}\,(f+gx)^{5/2}(4ef-3dg)}{63g^4}+\frac{2\sqrt{a+cx^2}\,(d+ex)^3\sqrt{f+gx}}{9g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-4*(9*a*e^2*g^2*(2*e*f - 5*d*g) + c*(76*e^3*f^3 - 204*d*e^2*f^2*g + 168*d^
2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(315*c*g^4) + (2*(d
 + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*g) + (4*e*(7*a*e^2*g^2 + c*(64*
e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(315*
c*g^4) - (4*e^2*(4*e*f - 3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(63*g^4) +
 (4*Sqrt[-a]*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^
```

```
2*g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^
3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)*g^5
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*S
qrt[-a]*(c*f^2 + a*g^2)*(9*a*e^2*g^2*(2*e*f - 5*d*g) - c*(64*e^3*f^3 - 216*
d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqr
t[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(315*c^(3/2)
*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 935

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/(g
*(2*m + 3))), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)^(m - 1)/(Sqrt[f + g
*x]*Sqrt[a + c*x^2]))*Simp[2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)*
x - (2*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[
```

```
m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9g} - \frac{\int \frac{(d+ex)^2(2a(3ef-4dg)+2(cdf-aeg)x+2c(4ef-3dg)x^2)}{\sqrt{f+gx}\,\sqrt{a+cx^2}}}{9g}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9g} - \frac{4e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{a+cx^2}}{63g^4} - \frac{2\int}{}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}{9g} + \frac{4e(7ae^2g^2+c(64e^2f^2-111defg+42d^2g^2)}{315cg^4}$$

$$= -\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f}}{315cg^4}$$

$$= -\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f}}{315cg^4}$$

$$= -\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f}}{315cg^4}$$

$$= -\frac{4(9ae^2g^2(2ef-5dg)+c(76e^3f^3-204de^2f^2g+168d^2efg^2-35d^3g^3))\sqrt{f}}{315cg^4}$$

Mathematica [C] Result contains complex when optimal does not.
time = 25.14, size = 864, normalized size = 1.30

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-(c*g^2*(a + c*x^2)*(-2*a*e^2*g^2*(-11*e*f + 45*d*g + 7*e
*g*x) + c*(-105*d^3*g^3 + 63*d^2*e*g^2*(4*f - 3*g*x) - 27*d*e^2*g*(8*f^2 -
6*f*g*x + 5*g^2*x^2) + e^3*(64*f^3 - 48*f^2*g*x + 40*f*g^2*x^2 - 35*g^3*x^3
)))) - (2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^3*g^4 - 3*a*c*e*g
^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2*
f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g^3))*(a + c*x^2) - Sqrt[c]*(I*Sqrt[c]*f
- Sqrt[a]*g)*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^
2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2*f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g
^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sq
rt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)*a^(3/2)*
e^3*g^3 - 9*a*Sqrt[c]*e^2*g^2*(2*e*f - 5*d*g) - (3*I)*Sqrt[a]*c*e*g*(16*e^2
*f^2 - 54*d*e*f*g + 63*d^2*g^2) + c^(3/2)*(64*e^3*f^3 - 216*d*e^2*f^2*g + 2
52*d^2*e*f*g^2 - 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)
]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elliptic
F[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I
*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(f + g*x))))/(315*c^2*g^6*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5078 vs. $2(582) = 1164$.

time = 0.12, size = 5079, normalized size = 7.63

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2e^3x^3\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{9g}+\frac{2\left(3d\,e^2c-\frac{8cf\,e^3}{9g}\right)x^2\sqrt{cg\,x^3+cf\,x^2+agx+ \dots}}{7cg} \dots\right.}{\dots}$
risch	Expression too large to display

default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(x*e + d)^3/sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 568, normalized size = 0.85

$$\frac{2\left(2\left(105c^2d^3f^2g^3+315acd^3g^5-2\left(32c^2f^5+39acf^3g^2-6a^2fg^4\right)e^3+9\left(24c^2df^4g+31acdf^2g^3-15a^2dg^5\right)e^2-126\left(2c^2d^2f^3g^2+3acd^2fg^4\right)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^2-3ag^2\right)}{3cg^2},-\frac{8\left(cf^3+9afg^2\right)}{27cg^3},\frac{3gx+f}{3g}\right)+6\left(105c^2d^3fg^4-\left(64c^2f^4g+30acf^2g^3-21a^2g^5\right)e^3+9\left(24c^2df^3g^2+13acdfg^4\right)e^2-63\left(4c^2d^2f^2g^3+3acd^2g^5\right)e\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\left(cf^2-3ag^2\right)}{3cg^2},-\frac{8\left(cf^3+9afg^2\right)}{27cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^2-3ag^2\right)}{3cg^2},-\frac{8\left(cf^3+9afg^2\right)}{27cg^3},\frac{3gx+f}{3g}\right)\right)+3\left(105c^2d^3g^5+\left(35c^2g^5x^3-40c^2fg^4x^2-64c^2f^3g^2-22acfg^4+2\left(24c^2f^2g^3+7acg^5\right)x\right)e^3+9\left(15c^2dg^5x^2-18c^2dfg^4x+24c^2df^2g^3+10acdg^5\right)e^2+63\left(3c^2d^2g^5x-4c^2d^2fg^4\right)e\right)\sqrt{cx^2+a}\,\sqrt{gx+f}\right)}{945c^2g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(2*(105*c^2*d^3*f^2*g^3 + 315*a*c*d^3*g^5 - 2*(32*c^2*f^5 + 39*a*c*f^
3*g^2 - 6*a^2*f*g^4)*e^3 + 9*(24*c^2*d*f^4*g + 31*a*c*d*f^2*g^3 - 15*a^2*d*
g^5)*e^2 - 126*(2*c^2*d^2*f^3*g^2 + 3*a*c*d^2*f*g^4)*e)*sqrt(c*g)*weierstra
ssPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3)
, 1/3*(3*g*x + f)/g) + 6*(105*c^2*d^3*f*g^4 - (64*c^2*f^4*g + 30*a*c*f^2*g^
3 - 21*a^2*g^5)*e^3 + 9*(24*c^2*d*f^3*g^2 + 13*a*c*d*f*g^4)*e^2 - 63*(4*c^2
*d^2*f^2*g^3 + 3*a*c*d^2*g^5)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a
*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(
c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f
)/g)) + 3*(105*c^2*d^3*g^5 + (35*c^2*g^5*x^3 - 40*c^2*f*g^4*x^2 - 64*c^2*f^
3*g^2 - 22*a*c*f*g^4 + 2*(24*c^2*f^2*g^3 + 7*a*c*g^5)*x)*e^3 + 9*(15*c^2*d*
g^5*x^2 - 18*c^2*d*f*g^4*x + 24*c^2*d*f^2*g^3 + 10*a*c*d*g^5)*e^2 + 63*(3*c
^2*d^2*g^5*x - 4*c^2*d^2*f*g^4)*e)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,(d+ex)^3}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**3/sqrt(f + g*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(x*e + d)^3/sqrt(g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}{\sqrt{f+g\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2), x)
```

3.630 $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=508

$$\frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3}+\frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7g}-\frac{4e(3ef\ \cdots}{}$$

```
[Out] -4/35*e*(-2*d*g+3*e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/g^3+4/105*(5*a*e^2*g^2
+c*(10*d^2*g^2-34*d*e*f*g+21*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^3+
2/7*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/105*(a*e*g^2*(-42*d*g+13*e*
f)+c*f*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^
(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)
*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g^4/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/
2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+4/105*(a*g^2+c*f^2)*(5*a*e^2*g^2-c*(35*d
^2*g^2-56*d*e*f*g+24*e^2*f^2))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)
^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^4/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 503, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {935, 1668, 858, 733, 435, 430}

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(5ae^2g^2-c(35d^2g^2-56defg+24e^2f^2))\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{105c^{3/2}g^4\sqrt{a+cx^2}\,\sqrt{f+gx}}+\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(aeg^2(13ef-42dg)+cf(35d^2g^2-56defg+24e^2f^2))\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{105\sqrt{c}\,g^4\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}+\frac{4\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(e^2\left(\frac{5a}{c}+\frac{21f^2}{g^2}\right)+10d^2-\frac{34def}{g}\right)}{105g}-\frac{4e\sqrt{a+cx^2}\,(f+gx)^{3/2}(3ef-2dg)}{35g^3}+\frac{2\sqrt{a+cx^2}\,(d+ex)^2\sqrt{f+gx}}{7g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (4*(10*d^2 + e^2*((5*a)/c + (21*f^2)/g^2) - (34*d*e*f)/g)*Sqrt[f + g*x]*Sqr
t[a + c*x^2])/(105*g) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(7*g)
 - (4*e*(3*e*f - 2*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*g^3) + (4*Sqrt
[-a]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2
))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*Sqrt[c]*g^4*
Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sq
rt[-a]*(c*f^2 + a*g^2)*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g
```

```
^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]
*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-
a]*Sqrt[c]*f - a*g)])/(105*c^(3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 935

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/(g
*(2*m + 3))), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)^(m - 1)/(Sqrt[f + g
*x]*Sqrt[a + c*x^2]))*Simp[2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)*
x - (2*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[
m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7g} - \frac{\int \frac{(d+ex)\left(2a(2ef-3dg)+2(cdf-aeg)x+2c(3ef-2dg)x^2\right)}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,d}{7g}$$

$$= \frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7g} - \frac{4e(3ef-2dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35g^3} - \frac{2\int}{}$$

$$= \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2}{}$$

$$= \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2}{}$$

$$= \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2}{}$$

$$= \frac{4(5ae^2g^2+c(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105cg^3} + \frac{2(d+ex)^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.54, size = 712, normalized size = 1.40

$$\frac{2\sqrt{f+gx}\left(g^2(a+cx^2)\left(10ae^2g^2+c(35d^2g^2+14deg(-4f+3gx)+3e^2(8f^2-6fgx+5g^2x^2))\right)-\frac{2\left(g^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\left(a^2eg^2(13ef-42dg)+c^2f(24e^2f^2-56defg+35d^2g^2)x^2+ac\left(35d^2fg^2-14deg(4f^2+3g^2x^2)+e^2(24f^3+13fg^2x^2)\right)\right)-i\sqrt{c}\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\left(aeg^2(13ef-42dg)+cf(24e^2f^2-56defg+35d^2g^2)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)+\sqrt{a}\,g\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\left(5ae^2g^2+6i\sqrt{a}\sqrt{c}\,eg(3ef-7dg)+c(-24e^2f^2+56defg-35d^2g^2)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+gx)}\right)}{105cg^5\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(g^2*(a + c*x^2)*(10*a*e^2*g^2 + c*(35*d^2*g^2 + 14*d*e*g*
(-4*f + 3*g*x) + 3*e^2*(8*f^2 - 6*f*g*x + 5*g^2*x^2))) - (2*(g^2*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]*(a^2*e*g^2*(13*e*f - 42*d*g) + c^2*f*(24*e^2*f^2 - 5
6*d*e*f*g + 35*d^2*g^2)*x^2 + a*c*(35*d^2*f*g^2 - 14*d*e*g*(4*f^2 + 3*g^2*x
^2) + e^2*(24*f^3 + 13*f*g^2*x^2))) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(
a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] + Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(5*a*e^2*g^2 + (6*I)*Sqrt[a]*Sqr
t[c]*e*g*(3*e*f - 7*d*g) + c*(-24*e^2*f^2 + 56*d*e*f*g - 35*d^2*g^2))*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*c*g^5*Sqrt[a + c*x
^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3277 vs. $2(430) = 860$.
time = 0.11, size = 3278, normalized size = 6.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(112*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)
^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x
+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c
)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c
*d*e*f*g^4-196*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^
(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-
c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(
1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*e*f^2*g^3-38*(-a*c)^(1/2)*(-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f)
)^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f
))^(1/2))*a*c*e^2*f^2*g^3+112*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f)
```

```
)^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/
2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f
))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e*f^3*g^
2+84*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(
g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2
)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/
(g*(-a*c)^(1/2)+c*f))^(1/2))*d*e*f^2*g^3-56*c^3*d*e*f^2*g^3*x^2+6*a*c^2*e^2
*f^2*g^3*x-56*a*c^2*d*e*f^2*g^3+7*a*c^2*e^2*f*g^4*x^2-14*c^3*d*e*f*g^4*x^3+
42*a*c^2*d*e*g^5*x^2+15*c^3*e^2*g^5*x^5+35*c^3*d^2*g^5*x^3-112*(-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c
/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(
1/2))*c^3*d*e*f^4*g-36*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*
(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^2*f^3*g^2-84*a^2*c*(-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f
))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x
+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*
f))^(1/2))*d*e*g^5+10*a^2*c*e^2*g^5*x+35*a*c^2*d^2*g^5*x+10*a^2*c*e^2*f*g^4
+35*a*c^2*d^2*f*g^4+24*a*c^2*e^2*f^3*g^2+24*c^3*e^2*f^3*g^2*x^2-3*c^3*e^2*f
*g^4*x^4+25*a*c^2*e^2*g^5*x^3+6*c^3*e^2*f^2*g^3*x^3+35*c^3*d^2*f*g^4*x^2+42
*c^3*d*e*g^5*x^4+48*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c
*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1
/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^3*e^2*f^5-14*a*c^2*d*e*f*g^4*x+10*(
-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/
(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/
2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)
/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*e^2*g^5+70*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*
f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c
*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^3*d^2*f^3*
g^2+84*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g
/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1
/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f
)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*e*g^5-36*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c
)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2
)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^2*f*g^4
+26*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g
*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)
*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(
g*(-a*c)^(1/2)+c*f))^(1/2))*e^2*f*g^4+70*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-
c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)
```

```
^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d^2*f*g^4+
74*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*
(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*
EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g
*(-a*c)^(1/2)+c*f))^(1/2))*e^2*f^3*g^2-70*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c
)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*
c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*
c*d^2*g^5-70*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)...
```

Maximа [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(x*e + d)^2/sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 407, normalized size = 0.80

$$\frac{2\left(2\left(35c^2d^2f^2g^2+105acd^2g^4+(24c^2f^4+31acf^2g^2-15a^2g^4)e^2-28(2c^2df^3g+3acdfg^3)e\right)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\frac{3gx+f}{3g}\right)+6\left(35c^2d^2fg^3+(24c^2f^3g+13acfg^3)e^2-14(4c^2df^2g^2+3acdg^4)e\right)\sqrt{cg}\,\text{weierstrassZeta}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\text{weierstrassPInverse}\left(\frac{4(cf^2-3ag^2)}{3cg^2},-\frac{8(cf^3+9afg^2)}{27cg^3},\frac{3gx+f}{3g}\right)\right)+3\left(35c^2d^2g^4+(15c^2g^4x^2-18c^2fg^3x+24c^2f^2g^2+10acg^4)e^2+14(3c^2dg^4x-4c^2dfg^3)e\right)\sqrt{cx^2+a}\sqrt{gx+f}\right)}{315c^2g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] 2/315*(2*(35*c^2*d^2*f^2*g^2 + 105*a*c*d^2*g^4 + (24*c^2*f^4 + 31*a*c*f^2*g
^2 - 15*a^2*g^4)*e^2 - 28*(2*c^2*d*f^3*g + 3*a*c*d*f*g^3)*e)*sqrt(c*g)*weie
rstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c
*g^3), 1/3*(3*g*x + f)/g) + 6*(35*c^2*d^2*f*g^3 + (24*c^2*f^3*g + 13*a*c*f*
g^3)*e^2 - 14*(4*c^2*d*f^2*g^2 + 3*a*c*d*g^4)*e)*sqrt(c*g)*weierstrassZeta(
4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstra
ssPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3)
, 1/3*(3*g*x + f)/g)) + 3*(35*c^2*d^2*g^4 + (15*c^2*g^4*x^2 - 18*c^2*f*g^3*
x + 24*c^2*f^2*g^2 + 10*a*c*g^4)*e^2 + 14*(3*c^2*d*g^4*x - 4*c^2*d*f*g^3)*e
)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,(d+ex)^2}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)**2/sqrt(f + g*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(x*e + d)^2/sqrt(g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2), x)
```

3.631 $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=364

$$-\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2}-\frac{4\sqrt{-a}\,(3aeg^2+cf(4ef-5dg))\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^-\right.}{15\sqrt{c}\,g^3\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a}}$$

```
[Out] -2/15*(-3*e*g*x-5*d*g+4*e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g^2-4/15*(3*a*e*
g^2+c*f*(-5*d*g+4*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2
),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x
^2/a+1)^(1/2)/g^3/c^(1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2)+4/15*(-5*d*g+4*e*f)*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2
)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a
)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/
g^3/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {829, 858, 733, 435, 430}

$$\frac{4\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)(4ef-5dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{15\sqrt{c}\,g^3\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{4\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(3aeg^2+cf(4ef-5dg))\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{15\sqrt{c}\,g^3\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{2\sqrt{a+cx^2}\,\sqrt{f+gx}\,(-5dg+4ef-3egx)}{15g^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*Sqrt[a + c*x^2])/(15*g^2) - (4*
Sqrt[-a]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a
]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[
-a]*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
+ Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (4*Sqrt[-a]*(4*e*f - 5*d*g)*(c*f^2 + a*g^
2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/(15*Sqrt[c]*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} + \frac{4\int \frac{-\frac{1}{2}acg(ef-5dg)+\frac{1}{2}c\left(3aeg^2+cf(4ef-5d\right.}{\sqrt{f+gx}\,\sqrt{a+cx^2}}}{15cg^2}$$

$$= -\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{(2(4ef-5dg)\,(cf^2+ag^2))\int \frac{}{\sqrt{f}}}{15g^3}$$

$$= -\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} + \frac{\left(4a(3aeg^2+cf(4ef-5dg))\,\sqrt{f}\right.}{}$$

$$= -\frac{2\sqrt{f+gx}\,(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2} - \frac{4\sqrt{-a}\,(3aeg^2+cf(4ef-5dg))}{15}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.18, size = 545, normalized size = 1.50

$$\frac{\sqrt{f+gx}\left(\frac{2(-4ef+5dg+3egx)(a+cx^2)}{g^2}+\frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(3aeg^2+cf(4ef-5dg))(a+cx^2)-\sqrt{c}\left(i\sqrt{c}f-\sqrt{a}g\right)(3aeg^2+cf(4ef-5dg))\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+\sqrt{a}\sqrt{c}g\left(\sqrt{c}f+i\sqrt{a}g\right)\left(3i\sqrt{a}eg+\sqrt{c}(-4ef+5dg)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)\right)}{cg^4\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(f+gx)}\right)}{15\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(-4*e*f + 5*d*g + 3*e*g*x)*(a + c*x^2))/g^2 + (4*(g^2*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(a + c*x^2
) - Sqrt[c]*(I*Sqrt[c]*f - Sqrt[a]*g)*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
```

```
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((3*I)*Sqrt[a]*e*g + Sqr
t[c]*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-
(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
 + g*x))))/(15*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. $2(298) = 596$.

time = 0.14, size = 1828, normalized size = 5.02

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2ex\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5g}+\frac{2\left(cd-\frac{4cfe}{5g}\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\cdots\right.}{\cdots}$
risch	$\frac{2(3egx+5dg-4ef)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{15g^2}+\frac{2\left(\frac{2\left(3ae\,g^2-5cdfg+4ce\,f^2\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/15*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(6*a^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e*g^4+6*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e*f^2*g^2-10*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*d*g^4+8*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*f*g^3-10*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*f^2*g^2+8*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*e*f^3*g-6*a^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e*g^4+10*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*f*g^3-14*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e*f^2*g^2+10*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*f^3*g-8*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e*f^4+3*c^2*e*g^4*x^4+5*c^2*d*g^4*x^3-c^2*e*f*g^3*x^3+3*a*c*e*g^4*x^2+5*c^2*d*f*g^3*x^2-4*c^2*e*f^2*g^2*x^2+5*a*c*d*g^4*x-a*c*e*f*g^3*x+5*a*c*d*f*g^3-4*a*c*e*f^2*g^2)/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^4

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(x*e + d)/sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.34, size = 276, normalized size = 0.76

$$\frac{2\left(2\left(5\,cdf^2g+15\,adg^3-2\left(2\,cf^3+3\,afg^2\right)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)+6\left(5\,cdfg^2-\left(4\,cf^2g+3\,ag^3\right)e\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)+3\left(5\,cdg^3+\left(3\,cg^3x-4\,cfg^2\right)e\right)\sqrt{cx^2+a}\,\sqrt{gx+f}\right)}{45\,cg^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(2*(5*c*d*f^2*g + 15*a*d*g^3 - 2*(2*c*f^3 + 3*a*f*g^2)*e)*sqrt(c*g)*we
ierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/
(c*g^3), 1/3*(3*g*x + f)/g) + 6*(5*c*d*f*g^2 - (4*c*f^2*g + 3*a*g^3)*e)*sqr
t(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*
g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f
^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*(5*c*d*g^3 + (3*c*g^3*x -
4*c*f*g^2)*e)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c*g^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,(d+ex)}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x)/sqrt(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)*(x*e + d)/sqrt(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}{\sqrt{f+g\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2),x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2), x)
```

3.632 $\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=322

$$\frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g}+\frac{4\sqrt{-a}\,\sqrt{c}\,f\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3g^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/3*f*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(
1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*
c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2
)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/3*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1
/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(
-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2
)/g^2/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {749, 858, 733, 435, 430}

$$-\frac{4\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3\sqrt{c}\,g^2\sqrt{a+cx^2}\,\sqrt{f+gx}}+\frac{4\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{3g^2\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}+\frac{2\sqrt{a+cx^2}\,\sqrt{f+gx}}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) + (4*Sqrt[-a]*Sqrt[c]*f*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*g^2*Sqrt[(Sqrt[c]*(f + g*x))
/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqr
t[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g} + \frac{2\int \frac{ag-cfx}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3g}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g} + \frac{1}{3}\left(2\left(a+\frac{cf^2}{g^2}\right)\right)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx - \frac{(2cf)\int \frac{\sqrt{f+}}{\sqrt{a+}}}{3g^2}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g} - \frac{\left(4a\sqrt{c}\,f\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}}{\sqrt{-a}\,\left(cf\right.}}}{\sqrt{1-x}}\right.}{3\sqrt{-a}\,g^2\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}\,g}{\sqrt{-a}}}}\,\sqrt{a+}}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3g} + \frac{4\sqrt{-a}\,\sqrt{c}\,f\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\right.}{3g^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 21.28, size = 456, normalized size = 1.42

$$\frac{2\sqrt{f+gx}\left(g^2(a+cx^2)-\frac{2\left(fg^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(a+cx^2)+\sqrt{c}\,f\left(-i\sqrt{c}\,f+\sqrt{a}\,g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)-\sqrt{a}\,g\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\,\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(f+gx)}\right)}{3g^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(g^2*(a + c*x^2) - (2*(f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]*(a + c*x^2) + Sqrt[c]*f*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a
])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))
]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt
```

```
[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqrt[a]*
g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*S
qrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
+ g*x))))/(3*g^3*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(256) = 512$.

time = 0.12, size = 688, normalized size = 2.14

method	result
risch	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{3g}+\frac{2\left(-\frac{2cf\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\left(-\frac{f}{g}-\sqrt{}\right.}{\sqrt{}}\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)\,(c\,x^2+a)}\left(\frac{2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3g}+\frac{4a\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}}{3\sqrt{cg\,x^3}}\right.}{}$
default	$-\frac{2\sqrt{c\,x^2+a}\,\sqrt{gx+f}\left(2\sqrt{-ac}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\ \text{EllipticF}\left(\sqrt{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -2/3*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(2*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1
/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-
a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(
1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*g^3+
2*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))
*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^
(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c
*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*f^2*g-2*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-
c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)
^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*f*g^2-2*(-
(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2
)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((
-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/
2)+c*f))^(1/2))*c^2*f^3-c^2*g^3*x^3-c^2*f*g^2*x^2-a*c*g^3*x-a*c*f*g^2)/c/(c
*g*x^3+c*f*x^2+a*g*x+a*f)/g^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 208, normalized size = 0.65

$$\frac{2\left(6\sqrt{cg}\,cfg\,\text{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)+3\sqrt{cx^2+a}\,\sqrt{gx+f}\,cg^2+2\,(cf^2+3\,ag^2)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)}{9\,cg^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")

```
[Out] 2/9*(6*sqrt(c*g)*c*f*g*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27
*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*
g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*sqrt(c*x^2
 + a)*sqrt(g*x + f)*c*g^2 + 2*(c*f^2 + 3*a*g^2)*sqrt(c*g)*weierstrassPInver
se(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3
*g*x + f)/g))/(c*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{\sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/sqrt(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + a}}{\sqrt{f + g\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

3.633 $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=473

$$-\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{eg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}+\frac{2\sqrt{-a}\,\sqrt{c}\,(ef+dg)\sqrt{\ \ }}{}$$

```
[Out] -2*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)
/e/g/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*(d*
g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f
*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*
c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/g/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
-2*(a*e^2+c*d^2)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {937, 947, 174, 552, 551, 858, 733, 435, 430}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\,(ae^2+cd^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e^2\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}+\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{\frac{cx^2}{a}+1}\,(dg+ef)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e^2g\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{eg\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])
/(e*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*Sqrt[c]*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt
[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*g*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqr
t[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e),
```

```
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f +
 Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 937

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)
]), x_Symbol] :> Dist[(c*d^2 + a*e^2)/e^2, Int[1/((d + e*x)*Sqrt[f + g*x]*S
qrt[a + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d - c*e*x)/(Sqrt[f + g*x]*Sqrt
[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] &
& NeQ[c*d^2 + a*e^2, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx - \frac{\int \frac{cd-cex}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{e^2}$$

$$= \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{eg} - \frac{(c(ef+dg))\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{e^2 g} + \frac{\left(\left(a+\frac{cd^2}{e^2}\right)\sqrt{1+\cdots}\right.}{\cdots}$$

$$= -\frac{\left(2\left(a+\frac{cd^2}{e^2}\right)\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}\,g}{\sqrt{c}}-\cdots}}\cdots\right.}{\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\cdots\right.}{eg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\cdots\right.}{eg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.04, size = 1096, normalized size = 2.32

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]

[Out] (-2*(-(c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(e^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(386) = 772$.
time = 0.12, size = 1216, normalized size = 2.57

method	result

elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{2cd\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{-\ldots}\right.}{e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{\ldots}$
default	$\frac{2\left(\sqrt{-ac}\,\mathrm{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)c\,d^2g^3-\sqrt{-ac}\,\mathrm{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\ldots}{g\ldots}}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((-a*c)^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*
c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d^2*g^3-(-a*c)^(1/2)*EllipticF
((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(
1/2)+c*f))^(1/2))*c*e^2*f^2*g-(-a*c)^(1/2)*EllipticPi((-(g*x+f)*c/(g*(-a*c)
^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f
)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e^2*g^3-(-a*c)^(1/2)*EllipticPi((-(g*x+f)*
c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c
)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d^2*g^3+a*c*EllipticF((-(g*x+f)
*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))
^(1/2))*d*e*g^3-a*c*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*
(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^2*f*g^2-EllipticF((-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f)
)^(1/2))*c^2*d^2*f*g^2+EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-
(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e*f^2*g-a*c*Ellipti
cE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)
^(1/2)+c*f))^(1/2))*d*e*g^3+a*c*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))
^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^2*f*g^2-Ellipt
icE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c
)^(1/2)+c*f))^(1/2))*c^2*d*e*f^2*g+EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*
f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e^2*f^3+E
llipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/
```

```
(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e^2*f*g^2
+EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/
c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d^2*f*g
^2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*
g/(g*(-a*c)^(1/2)+c*f))^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*(g*x+
f)^(1/2)*(c*x^2+a)^(1/2)/g^2/c/e^2/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/(sqrt(g*x + f)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)\sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(sqrt(g*x + f)*(x*e + d)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

3.634 $\displaystyle \int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=694

$$-\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(ef-dg)(d+ex)}-\frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}+\cdots$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)-EllipticE(1/2*(1-x*c^(1/2
)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a
)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e/(-d*g+e*f)/(c*x^2+a)^(1/2
)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+f*EllipticF(1/2*(1-x*c^(
1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*
(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/
2)))^(1/2)/e/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(-d*g+2*e*f)*Elliptic
F(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(
1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)
^(1/2)+f*c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+(a*e^
2*g+c*d*(-d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2)
,2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2
)/e^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 1.10, antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {939, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$-\frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{\frac{cx^2}{a}+1}\,(2ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e^2\sqrt{a+cx^2}\,\sqrt{f+gx}\,(ef-dg)}+\frac{\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(ae^2g+cd(2ef-dg))\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e^2\sqrt{a+cx^2}\,\sqrt{f+gx}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)(ef-dg)}+\frac{\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e\sqrt{a+cx^2}\,\sqrt{f+gx}\,(ef-dg)}-\frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e\sqrt{a+cx^2}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{\sqrt{a+cx^2}\,\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x))) - (Sqrt[-a]*Sqrt
[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)
/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*S
qrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[
-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (
```

```
c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g
)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]
) - (Sqrt[-a]*Sqrt[c]*(2*e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e^2*(e*f - d*g)*Sqrt[
f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*(f
 + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((S
qrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2
*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*
f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 939

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/((m + 1)*(e*f - d*g))), x] - Dist[1/(2*(m + 1)*(e*f - d*g)), Int[((d + e*
x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*g*(2*m + 3) + 2*(c*f)*x
+ c*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f
- d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \frac{-ag+2cfx+cgx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \left(\frac{c(2ef-dg)}{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{cgx}{e\sqrt{f+gx}\,\sqrt{a+cx^2}} + \right.}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} + \frac{(c(2ef-dg))\int \frac{}{\sqrt{f}}}{2e^2(ef}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(ef-dg)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2e(ef-dg)} -$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,(2ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}\,F}{e^2(ef-dg)\sqrt{f+g}}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}}{\sqrt{-}}}}{\sqrt{2}}\right.\right.}{e(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a}}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}}{\sqrt{-}}}}{\sqrt{2}}\right.\right.}{e(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.48, size = 1336, normalized size = 1.93

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*((a + c*x^2)/(d + e*x) - (c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]] - c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*e^2*f*g^2*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] -
2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*c*d*e*f*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x)^2 - c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + I
*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
+ g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f
+ I*Sqrt[a]*g)*(Sqrt[a]*e*g + I*Sqrt[c]*(2*e*f - d*g))*Sqrt[(g*((I*Sqrt[a])
/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*
(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f
 + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (2*I)*c*d*
e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/S
qrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/
(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/S
qrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + I*c*d
^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))
/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a*
e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e^2
*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(f + g*x))))/((-(e*f) + d*g
)*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6033 vs. $2(575) = 1150$.
time = 0.14, size = 6034, normalized size = 8.69

method	result

elliptic	$\frac{\sqrt{(gx+f)(c x^2+a)}\left(\frac{\sqrt{cg x^3+cf x^2+agx+fa}}{(dg-ef)(ex+d)}+\frac{2\left(\frac{c}{e^2}-\frac{cdg}{2e^2(dg-ef)}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\cdots}}{\sqrt{\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(sqrt(g*x + f)*(x*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + a)/(sqrt(g*x + f)*(x*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

3.635 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1241

$$-\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2}+\frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)}+\frac{\sqrt{-a}\,\sqrt{c}\,(3ae^2g+cd(2ef+dg))\,\sqrt{\ }}{4e\left(cd^2+ae^{2}\right.}$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d*
(d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)^2/(e*x+
d)+1/4*(3*a*e^2*g+c*d*(d*g+2*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(
1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)
*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)^2/(c*x^2+a)^(1/
2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+1/2*g*EllipticF(1/2*(1-
x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1
/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2)/e^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*f*(3*a*e^2
*g+c*d*(d*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-
2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1
/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(-d*g+
e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*d*g*(3*a*e^2*g+c*d*(d*g+2*e*f))*El
lipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/
2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(
g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)^2/(g*x+f)^(1/2)
/(c*x^2+a)^(1/2)-c*(d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*
2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)
))^(1/2)/e^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1
/2)-1/4*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(3*a*e^2*g+c*d*(d*g+2*e*f))*EllipticPi
(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^
(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/(a*e^2+c*d^2)/(-d*g+e*f)^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 2.72, antiderivative size = 1241, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {939, 6874, 733, 430, 954, 858, 435, 947, 174, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/2*(Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*
g + c*d*(2*e*f + d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e^2)*(e
*f - d*g)^2*(d + e*x)) + (Sqrt[-a]*Sqrt[c]*(3*a*e^2*g + c*d*(2*e*f + d*g))*
Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)
*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a +
c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a
]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e^2*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*f*(3*a*e^2*g + c*d*(2*e*f + d*g))
*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*S
qrt[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a +
 c*x^2]) + (Sqrt[-a]*Sqrt[c]*d*g*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcS
in[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f -
a*g)])/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
- (c*(e*f + d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1
+ (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (
Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e
^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
((a*e^2*g - c*d*(2*e*f - 3*d*g))*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*
e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e^2*((Sqrt[c]*d)/Sqrt[-a]
+ e)*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 939

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/((m + 1)*(e*f - d*g))), x] - Dist[1/(2*(m + 1)*(e*f - d*g)), Int[((d + e*
x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*g*(2*m + 3) + 2*(c*f)*x
+ c*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f
- d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
```

```
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \frac{-3ag+2cfx-cgx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \left(-\frac{cg}{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{-3ae^2g-cd(2ef+dg)}{e^2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+}\right.}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4e^2(ef-dg)} + \frac{(c(ef+dg))\int \frac{}{(d+ex}}{2e^2(}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)} + \frac{(3ae^2}{}$$

$$\sqrt{-a}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)} + \frac{}{}$$

$$\sqrt{-a}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)} + \frac{}{}$$

$$\sqrt{-a}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)} + \frac{}{}$$

$$\sqrt{-a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.29, size = 2197, normalized size = 1.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] (c^2*d^2*f^3 - 3*a*c*e^2*f^3 - (2*c^2*d*e*f^4)/g + (c^2*d^3*f^2*g)/e + a*c*
d*e*f^2*g + a*c*d^2*f*g^2 - 3*a^2*e^2*f*g^2 + (a*c*d^3*g^3)/e + 3*a^2*d*e*g
^3 - 2*c^2*d^2*f^2*(f + g*x) + 6*a*c*e^2*f^2*(f + g*x) + (4*c^2*d*e*f^3*(f
+ g*x))/g - (2*c^2*d^3*f*g*(f + g*x))/e - 6*a*c*d*e*f*g*(f + g*x) + c^2*d^2
*f*(f + g*x)^2 - 3*a*c*e^2*f*(f + g*x)^2 - (2*c^2*d*e*f^2*(f + g*x)^2)/g +
(c^2*d^3*g*(f + g*x)^2)/e + 3*a*c*d*e*g*(f + g*x)^2 - ((e*f - d*g)*(f + g*x
)*(a + c*x^2)*(a*e^2*(2*e*f - 5*d*g - 3*e*g*x) - c*d*(3*d^2*g + 2*e^2*f*x +
 d*e*g*x)))/(d + e*x)^2 + (Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d
*g)*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f +
 g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*
f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*g*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]) + ((I*Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(-3*a*e^2*g
 - (6*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g) + c*d*(-4*e*f + d*g))*Sqrt[(g*((I*Sq
rt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g
*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + ((4*I)*a*c*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/S
qrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
 + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)
), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] +
 ((4*I)*c^2*d^3*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]
*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[
a]*g)])/(e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) - ((4*I)*a*c*d*e*f*g*Sqrt[(g*(
(I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(
f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f +
 I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]] + ((6*I)*a*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*
x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipt
icPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (I*c^2*d^4*g^2*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
```

```
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c
]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*
x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(e^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]) + ((3*I)*a^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(
3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]
*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(4*(c*d^2
 + a*e^2)*(e*f - d*g)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19169 vs. $2(1045) = 2090$.
time = 0.13, size = 19170, normalized size = 15.45

method	result
elliptic	$\dfrac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{2(dg-ef)(ex+d)^2}+\frac{(3a\,e^2g+c\,d^2g+2cdef)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{4(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef)(dg-ef)(ex+d)}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/(sqrt(g*x + f)*(x*e + d)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)^3 \sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)/(sqrt(g*x + f)*(x*e + d)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
[Out] int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

3.636 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=531

$$-\frac{2e(25ae^2g^2+c(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2}+\frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}{7c}+\frac{2e^2(e$$

```
[Out] 2/35*e^2*(11*d*g+e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^2-2/105*e*(25*a*e^2
*g^2+c*(-90*d^2*g^2+12*d*e*f*g+7*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^
2/g^2+2/7*e*(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c+2/105*(a*e^2*g^2*(189
*d*g+19*e*f)-c*(105*d^3*g^3+105*d^2*e*f*g^2-42*d*e^2*f^2*g+8*e^3*f^3))*Elli
pticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)
*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/c^(3/2)/g^3/(c
*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/105*e*(a*g
^2+c*f^2)*(25*a*e^2*g^2-c*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*EllipticF(1/2
*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2))
)^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1
/2)))^(1/2)/c^(5/2)/g^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 0.74, antiderivative size = 527, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {956, 1668, 858, 733, 435, 430}

$$-\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(25ae^2g^2-c(105d^2g^2-42defg+8e^2f^2))\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{105c^{5/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,(ae^2g^2(189dg+19ef)-c(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3))\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}\left(-e^2\left(\frac{25a}{c}+\frac{7f^2}{g^2}\right)+90d^2-\frac{12def}{g}\right)}{105c}+\frac{2e^2\sqrt{a+cx^2}\,(f+gx)^{3/2}(11dg+ef)}{35cg^2}+\frac{2e\sqrt{a+cx^2}\,(d+ex)^2\sqrt{f+gx}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*(90*d^2 - e^2*((25*a)/c + (7*f^2)/g^2) - (12*d*e*f)/g)*Sqrt[f + g*x]*S
qrt[a + c*x^2])/(105*c) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(
7*c) + (2*e^2*(e*f + 11*d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(35*c*g^2) +
(2*Sqrt[-a]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^3*f^3 - 42*d*e^2*f^2*g +
 105*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(105*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a
]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*e*(c*f^2 + a*g^2)*(25*a*e^2*g^2 - c*(8
```

```
*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqr
t[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(105*c^(5/2)*g^3*Sqr
t[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 956

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)/(Sqr
t[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
+ 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d
*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
```

```
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7c} - \frac{\int \frac{(d+ex)\left(-7cd^2f+ae(4ef+dg)+\left(5ae^2g-cd(12ef+7dg)\right)x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}}{7c}$$

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{7c} + \frac{2e^2(ef+11dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2} - \frac{2\int}{}$$

$$= -\frac{2e(25ae^2g^2+c(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex}{}$$

$$= -\frac{2e(25ae^2g^2+c(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex}{}$$

$$= -\frac{2e(25ae^2g^2+c(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex}{}$$

$$= -\frac{2e(25ae^2g^2+c(7e^2f^2+12defg-90d^2g^2))\sqrt{f+gx}\,\sqrt{a+cx^2}}{105c^2g^2} + \frac{2e(d+ex}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.11, size = 747, normalized size = 1.41

$$\frac{2\sqrt{f+gx}\left(-g^2(a+cx^2)\left(25ae^3g^2+ce\left(-105d^2g^2-21deg(f+3gx)+e^2\left(4f^2-3fgx-15g^2x^2\right)\right)\right)+\frac{g^2\left(-a^2e^2g^2(19ef+189dg)+c^2\left(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3g^3\right)x^2+ac\left(105d^2efg^2+105d^3g^3-21de^2g\left(2f^2+9g^2x^2\right)+e^3\left(8f^3-19fg^2x^2\right)\right)\right)}{f+gx}+ic\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\left(-ae^2g^2(19ef+189dg)+c\left(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3g^3\right)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+\frac{g\left(\sqrt{c}f+i\sqrt{a}g\right)\left(105ic^{3/2}d^3g^2+25a^{3/2}e^3g^2+3ia\sqrt{c}e^2g(2ef-63dg)+\sqrt{a}ce\left(-8e^2f^2+42defg-105d^2g^2\right)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}\right)}{105c^2g^4\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(-(g^2*(a + c*x^2)*(25*a*e^3*g^2 + c*e*(-105*d^2*g^2 - 21*
d*e*g*(f + 3*g*x) + e^2*(4*f^2 - 3*f*g*x - 15*g^2*x^2)))) + (g^2*(-(a^2*e^2
*g^2*(19*e*f + 189*d*g)) + c^2*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^
2 + 105*d^3*g^3)*x^2 + a*c*(105*d^2*e*f*g^2 + 105*d^3*g^3 - 21*d*e^2*g*(2*f
^2 + 9*g^2*x^2) + e^3*(8*f^3 - 19*f*g^2*x^2))))/(f + g*x) + I*c*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(-(a*e^2*g^2*(19*e*f + 189*d*g)) + c*(8*e^3*f^3 - 42*
d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f +
g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (g*(Sqrt[c]*f + I*Sqr
t[a]*g)*((105*I)*c^(3/2)*d^3*g^2 + 25*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2
*g*(2*e*f - 63*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 42*d*e*f*g - 105*d^2*g^2))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
 - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(105*c^2*g^4*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3921 vs. $2(453) = 906$.
time = 0.13, size = 3922, normalized size = 7.39

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2e^3x^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{7c}+\frac{2\left(3d\,e^2g+\frac{1}{7}f\,e^3\right)x\sqrt{cg\,x^3+cf\,x^2+agx+}}{5cg}\right.}{}$

risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-19*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c
)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2
)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^3*f*g^4
-105*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/
2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f
))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2
)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*d^2*e*g^5+17*(-a*c)^(1/2)*(-(g*x+f)
*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))
^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f)
)^(1/2))*a*c*e^3*f^2*g^3-105*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))
^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2
))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f)
)^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d^2*e*f^2*g
^3+42*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1
/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*
f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/
2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e^2*f^3*g^2+7*a*c^2*e^3*f*g^4*x^
2-105*c^3*d^2*e*f*g^4*x^2-21*c^3*d*e^2*f^2*g^3*x^2-105*a*c^2*d^2*e*g^5*x+a*
c^2*e^3*f^2*g^3*x-105*a*c^2*d^2*e*f*g^4-21*a*c^2*d*e^2*f^2*g^3-84*c^3*d*e^2
*f*g^4*x^3-63*a*c^2*d*e^2*g^5*x^2-15*c^3*e^3*g^5*x^5+189*a*c^2*(-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c
/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(
1/2))*d*e^2*f^2*g^3+189*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*
x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*
c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g
*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*e^2*g^5-63*c^3*d*e^2*g^5*
x^4-18*c^3*e^3*f*g^4*x^4+10*a*c^2*e^3*g^5*x^3-105*c^3*d^2*e*g^5*x^3+c^3*e^3
*f^2*g^3*x^3+4*c^3*e^3*f^3*g^2*x^2+25*a^2*c*e^3*g^5*x+25*a^2*c*e^3*f*g^4+4*
a*c^2*e^3*f^3*g^2+8*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c
*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1
/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^3*e^3*f^5-84*a*c^2*d*e^2*f*g^4*x+25
*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*
g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(
1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*
```

```
f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a^2*e^3*g^5-105*(-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c
)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2
)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^3*d^3*f
^2*g^3+105*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(
g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2
)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/
(g*(-a*c)^(1/2)+c*f))^(1/2))*c^3*d^3*f^2*g^3-105*a*c^2*(-(g*x+f)*c/(g*(-a*c
)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*
c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d^
3*g^5+105*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2)
)*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))
^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-
c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d^3*g^5-6*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1/
2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a
*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1
/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^3*f*g
^4-6*a*c^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(
g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2
)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/
(g*(-a*c)^(1/2)+c*f))^(1/2))*e^3*f^3*g^2-189*a^2*c*(-(g*x+f)*c/(g*(-a*c)^(1
/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-
a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(
1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*e^2*
g^5+105*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(
-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*E
llipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*
(-a*c)^(1/2)+c*f))^(1/2))*c^3*d^2*e*f^3*g^2-42*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^3/sqrt(c*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 436, normalized size = 0.82

$$\frac{2\left(\left(210\,c^2d^3fg^3-\left(8\,c^2f^4-13\,acf^2g^2-75\,a^2g^4\right)e^3+42\,\left(c^2df^3g-6\,acdfg^3\right)e^2-105\,\left(c^2d^2f^2g^2+3\,acd^2g^4\right)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\,\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\,\left(cf^3+9\,afg^2\right)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)-3\,\left(105\,c^2d^3g^4+105\,c^2d^2fg^3e+\left(8\,c^2f^3g-19\,acfg^3\right)e^3-21\,\left(2\,c^2df^2g^2+9\,acdg^4\right)e^2\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\,\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\,\left(cf^3+9\,afg^2\right)}{27\,cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\,\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\,\left(cf^3+9\,afg^2\right)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)+3\,\left(105\,c^2d^2g^4e+\left(15\,c^2g^4x^2+3\,c^2fg^3x-4\,c^2f^2g^2-25\,acg^4\right)e^3+21\,\left(3\,c^2dg^4x+c^2dfg^3\right)e^2\right)\sqrt{cx^2+a}\,\sqrt{gx+f}\right)}{315\,c^3g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*((210*c^2*d^3*f*g^3 - (8*c^2*f^4 - 13*a*c*f^2*g^2 - 75*a^2*g^4)*e^3 +
 42*(c^2*d*f^3*g - 6*a*c*d*f*g^3)*e^2 - 105*(c^2*d^2*f^2*g^2 + 3*a*c*d^2*g^
4)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c
*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*(105*c^2*d^3*g^4 + 105*c^
2*d^2*f*g^3*e + (8*c^2*f^3*g - 19*a*c*f*g^3)*e^3 - 21*(2*c^2*d*f^2*g^2 + 9*
a*c*d*g^4)*e^2)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8
/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/
(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*(105*c^
2*d^2*g^4*e + (15*c^2*g^4*x^2 + 3*c^2*f*g^3*x - 4*c^2*f^2*g^2 - 25*a*c*g^4)
*e^3 + 21*(3*c^2*d*g^4*x + c^2*d*f*g^3)*e^2)*sqrt(c*x^2 + a)*sqrt(g*x + f))
/(c^3*g^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^3/sqrt(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2), x)
```

3.637 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=410

$$\frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg}+\frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c}+\frac{2\sqrt{-a}\,(9ae^2g^2+c(2e^2f^2-10defg-}{}$$

```
[Out] 2/15*e*(7*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/5*e*(e*x+d)*(g*x+f)^
(1/2)*(c*x^2+a)^(1/2)/c+2/15*(9*a*e^2*g^2+c*(-15*d^2*g^2-10*d*e*f*g+2*e^2*f
^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/c^(3/
2)/g^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-4/1
5*e*(-5*d*g+e*f)*(a*g^2+c*f^2)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)
*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)
^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^2/(g*x+f)
^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {956, 1668, 858, 733, 435, 430}

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(9ae^2g^2+c(-15d^2g^2-10defg+2e^2f^2))\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{4\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\,(ef-5dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\,\sqrt{f+gx}}+\frac{2e\sqrt{a+cx^2}\,\sqrt{f+gx}\,(7dg+ef)}{15cg}+\frac{2e\sqrt{a+cx^2}\,(d+ex)\sqrt{f+gx}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*(e*f + 7*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g) + (2*e*(d + e*x)
*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c) + (2*Sqrt[-a]*(9*a*e^2*g^2 + c*(2*e^2
*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Elliptic
E[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(15*c^(3/2)*g^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]
*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*e*(e*f - 5*d*g)*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f
 - a*g)])/(15*c^(3/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 956

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)/(Sqr
t[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
+ 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d
*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
```

```
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - \frac{\int \frac{-5cd^2f+ae(2ef+dg)+(3ae^2g-cd(8ef+5dg))x-ce(ef+}{\sqrt{f+gx}\,\sqrt{a+cx^2}}}{5c}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - \frac{2\int \frac{-\frac{1}{2}}{}}{}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - \frac{1}{15}\Bigg(-$$

$$\Bigg(2a\Bigg($$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - $$

$$2\sqrt{-c}$$

$$= \frac{2e(ef+7dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5c} - $$

Mathematica [C] Result contains complex when optimal does not.
time = 22.91, size = 596, normalized size = 1.45

$$\frac{2\sqrt{f+gx}\left(ceg^2(a+cx^2)(10dg+e(f+3gx))+\frac{g^2(-9a^2e^2g^2+c^2(-2e^2f^2+10defg+15d^2g^2)x^2+ac(10defg+15d^2g^2-e^2(2f^2+9g^2x^2)))}{f+gx}-ic\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(9ae^2g^2+c(2e^2f^2-10defg-15d^2g^2))\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)+\frac{\sqrt{c}\,g\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\left(15icd^2g-9iae^2g+2\sqrt{a}\,\sqrt{c}\,e(ef-5dg)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}\right)}{15c^2g^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e*g^2*(a + c*x^2)*(10*d*g + e*(f + 3*g*x)) + (g^2*(-9*a
^2*e^2*g^2 + c^2*(-2*e^2*f^2 + 10*d*e*f*g + 15*d^2*g^2)*x^2 + a*c*(10*d*e*f
*g + 15*d^2*g^2 - e^2*(2*f^2 + 9*g^2*x^2))))/(f + g*x) - I*c*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]*(9*a*e^2*g^2 + c*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
 - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)] + (Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((15*I)*c*d^2*g - (9*I)*a*e^2*
g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(
f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*El
lipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]
*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]))/(15*c^2*g^3*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2469 vs. $2(338) = 676$.

time = 0.12, size = 2470, normalized size = 6.02

method	result
elliptic	$\frac{\sqrt{(gx+f)\,(c\,x^2+a)}\left(\frac{2e^2x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5c}+\frac{2\left(2gde+\frac{1}{5}e^2f\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\right.}{}$

risch	$\frac{2e(3egx+10dg+ef)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{15cg} - \frac{\left(2\left(9a\,e^2g^2-15c\,d^2g^2-10cdefg+2c\,e^2f^2\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-4*a*c*e^2*f*g^3*x-10*a*c*d*e*f*g^3-10
*c^2*d*e*f*g^3*x^2-10*a*c*d*e*g^4*x-3*c^2*e^2*g^4*x^4+9*(-(g*x+f)*c/(g*(-a*
c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c
*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a
*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a
*c*e^2*f^2*g^2-10*c^2*d*e*g^4*x^3-4*c^2*e^2*f*g^3*x^3-3*a*c*e^2*g^4*x^2-c^2
*e^2*f^2*g^2*x^2-10*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((
-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(
-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(
-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*e*f^2*g^2+9*a^2*(-(g
*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+
c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(
g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)
+c*f))^(1/2))*e^2*g^4-15*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*
(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d^2*g^4-15*(-(g*x+f)*c/(g*(-
a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*(
(c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(
-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))
*c^2*d^2*f^2*g^2+15*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*
c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/
2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c
)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d^2*g^4+15*(-(g*x+f)*c/(g*(-a*c)^
(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+
```

```
(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)
^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*
d^2*f^2*g^2-9*a^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/
2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f
))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2
)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^2*g^4-2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c
*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^
(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-
c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e^2*f^4
-10*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2
))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f)
)^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)
-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*d*e*g^4+2*(-a*c)^(1/2)*(-(g*x+f)*c/(g*
(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)
*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g
*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2
))*a*e^2*f*g^3+2*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*
x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*
c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g
*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*e^2*f^3*g-11*(-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c
/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(
1/2))*a*c*e^2*f^2*g^2+10*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a
*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1
/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*
c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e*f^3*g+10*a*c*(-(g*x+f)*c
/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(
1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*
c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^
(1/2))*d*e*f*g^3-a*c*e^2*f^2*g^2)/c^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^2/sqrt(c*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 300, normalized size = 0.73

$$\frac{2\left(2\left(15\,cd^2fg^2+(cf^3-6\,afg^2)e^2-5\,(cdf^2g+3\,adg^3)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)-3\,(15\,cd^2g^3+10\,cdfg^2e-(2\,cf^2g+9\,ag^3)e^2)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)+3\,(10\,cdg^3e+(3\,cg^3x+cfg^2)e^2)\sqrt{cx^2+a}\,\sqrt{gx+f}\right)}{45\,c^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(2*(15*c*d^2*f*g^2 + (c*f^3 - 6*a*f*g^2)*e^2 - 5*(c*d*f^2*g + 3*a*d*g^
3)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c
*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*(15*c*d^2*g^3 + 10*c*d*f*
g^2*e - (2*c*f^2*g + 9*a*g^3)*e^2)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3
*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3
*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x +
 f)/g)) + 3*(10*c*d*g^3*e + (3*c*g^3*x + c*f*g^2)*e^2)*sqrt(c*x^2 + a)*sqrt
(g*x + f))/(c^2*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^2/sqrt(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2), x)
```

3.638 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=331

$$\frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c}-\frac{2\sqrt{-a}\,(ef+3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-}\right.}{3\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{a+cx^2}}$$

```
[Out] 2/3*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c-2/3*(3*d*g+e*f)*EllipticE(1/2*(1-x*c^
(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))
*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g/c^(1/2)/(c*x^2+a)^(1/2)/((g*x
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2/3*e*(a*g^2+c*f^2)*EllipticF(1
/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2
)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^
(1/2)))^(1/2)/c^(3/2)/g/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {847, 858, 733, 435, 430}

$$\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,(ag^2+cf^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{3c^{3/2}g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,(3dg+ef)E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{3\sqrt{c}\,g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c) - (2*Sqrt[-a]*(e*f + 3*d*g)*Sqrt[
f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*e*(c*
f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*g*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\,\sqrt{a+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}(3cdf-aeg)+\frac{1}{2}c(ef+3dg)x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+cx^2}}{3c} + \frac{(ef+3dg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g} - \frac{(e(cf^2+ag^2))\int \frac{\quad}{\sqrt{f+}}}{3cg}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+cx^2}}{3c} + \frac{\left(2a(ef+3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1-}}{}\right.}{3\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{c(f+}{cf-\frac{a}{\sqrt{}}}}}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+cx^2}}{3c} - \frac{2\sqrt{-a}\,(ef+3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{}}{}\right.\right.}{3\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.21, size = 464, normalized size = 1.40

$$\frac{2\sqrt{f+gx}\left(e(a+cx^2)+\frac{(ef+3dg)(a+cx^2)}{f+gx}+\frac{ic\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(ef+3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)}{g^2}+\frac{i\left(3\sqrt{c}\,d+i\sqrt{a}\,e\right)\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)}{g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}\right)}{3c\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(e*(a + c*x^2) + ((e*f + 3*d*g)*(a + c*x^2))/(f + g*x) + (
I*c*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f + 3*d*g)*Sqrt[(g*((I*Sqrt[a])/Sqr
t[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt
[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x
]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/g^2 + (I*(3*Sqrt[
```

```
c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f +
g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (
Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a
]*g)/Sqrt[c]])))/(3*c*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1285 vs. $2(265) = 530$.
time = 0.10, size = 1286, normalized size = 3.89 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c
*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g
*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+
c*f))^(1/2))*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*a*e*g^3+(
(-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*
(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),
(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*(-(g*x+f)*
c/(g*(-a*c)^(1/2)-c*f))^(1/2)*c*e*f^2*g+3*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^
(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ellipti
cF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)
^(1/2)+c*f))^(1/2))*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*d*g^3+3*((-
c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-
a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-
(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-(g*x+f)*c/(g*(-a*c)^(1/
2)-c*f))^(1/2)*c^2*d*f^2*g-3*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(
1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*
c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^
(1/2))*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*d*g^3-((-c*x+(-a*c)^(1/2
))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f)
)^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)
-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1
/2)*e*f*g^2-3*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*
c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/
2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*c^2*d*f^2*g-((-c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ellip
ticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*
c)^(1/2)+c*f))^(1/2))*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*c^2*e*f^3+c^2
*e*g^3*x^3+c^2*e*f*g^2*x^2+a*c*e*g^3*x+a*c*e*f*g^2)/(c*g*x^3+c*f*x^2+a*g*x+
a*f)/c^2/g^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)/sqrt(c*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 231, normalized size = 0.70

$$\frac{2\left(3\sqrt{cx^2+a}\,\sqrt{gx+f}\,cg^2e+(6\,cdfg-(cf^2+3\,ag^2)e)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)-3\,(3\,cdg^2+cfge)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)\right)}{9\,c^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + a)*sqrt(g*x + f)*c*g^2*e + (6*c*d*f*g - (c*f^2 + 3*a*g^
2)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c
*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*(3*c*d*g^2 + c*f*g*e)*sqr
t(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*
g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f
^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)))/(c^2*g^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)/sqrt(c*x^2 + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\left(d+e\,x\right)}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2), x)
```

3.639 $\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{-a}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

```
[Out] -2*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/c^(1/2)
/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {733, 435}

$$-\frac{2\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c
]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{\left(2a\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}\,gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}\,g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, \frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right.}{\sqrt{-a}\sqrt{c}\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}\,g}{\sqrt{-a}}}}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{a+cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.39, size = 294, normalized size = 2.16

$$\frac{2i\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)\sqrt{\frac{g\left(\sqrt{a}+i\sqrt{c}\,x\right)}{-i\sqrt{c}\,f+\sqrt{a}\,g}}\sqrt{f+gx}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f-i\sqrt{a}\,g}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)-F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f-i\sqrt{a}\,g}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{g\left(i\sqrt{a}+\sqrt{c}\,x\right)}}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

```
[Out] ((2*I)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqr
t[c]*f + Sqrt[a]*g)]*Sqrt[f + g*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(f
+ g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f
```

```
 - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/
(Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt[a + c
*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(108) = 216$.
time = 0.09, size = 396, normalized size = 2.91

method	result
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\left(cf-g\sqrt{-ac}\right)\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\,\left(\sqrt{-ac}\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)\left(c\,x^2+a\right)}\left(\frac{2f\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\ldots}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(c*f-g*(-a*c)^(1/2))*(-(g*x+f)*c/(g*(-a*c)^
(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+
(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-a*c)^(1/2)*EllipticF((-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f
))^(1/2))*g-(-a*c)^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),
(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*g+f*EllipticF((-(g*x+f)
*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))
^(1/2))*c-EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/
2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*f)/g/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.28, size = 173, normalized size = 1.27

$$\frac{2\left(2\sqrt{cg}\,f\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)-3\sqrt{cg}\,g\mathrm{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)\right)}{3\,cg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*sqrt(c*g)*f*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27
*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*sqrt(c*g)*g*weierstras
sZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), wei
erstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(
c*g^3), 1/3*(3*g*x + f)/g)))/(c*g)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f + gx}}{\sqrt{a + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/(a + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)/(a + c*x^2)^(1/2), x)
```

3.640 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=319

$$-\frac{2\sqrt{-a}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e\sqrt{f+gx}\,\sqrt{a+cx^2}}-\frac{2(ef-dg)\sqrt{\frac{\sqrt{}}{\sqrt{c}}}}{}$$

```
[Out] -2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*(-d
*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1
/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x
^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(e+d*c^(1/
2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {958, 733, 430, 947, 174, 552, 551}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}-\frac{2\sqrt{-a}\,g\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e\sqrt{a+cx^2}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
(c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*
g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[a + c*x^2]) -
 (2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 +
 (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*
((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
```

```
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{e}$$

$$= \frac{\left((ef-dg)\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}\,x}{\sqrt{-a}}}\,(d+ex)\sqrt{f+gx}}\,dx}{e\sqrt{a+cx^2}} + \cdots$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2a\cdots}{\sqrt{-a}\,\sqrt{\cdots}}\right)}{\sqrt{c}\,e\sqrt{f+gx}\sqrt{a+cx^2}} \cdots$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2a\cdots}{\sqrt{-a}\,\sqrt{\cdots}}\right)}{\sqrt{c}\,e\sqrt{f+gx}\sqrt{a+cx^2}} \cdots$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2a\cdots}{\sqrt{-a}\,\sqrt{\cdots}}\right)}{\sqrt{c}\,e\sqrt{f+gx}\sqrt{a+cx^2}} \cdots$$

Mathematica [C] Result contains complex when optimal does not.

time = 20.51, size = 300, normalized size = 0.94

$$-\frac{2i\sqrt{\frac{g\left(\sqrt{a}+i\sqrt{c}\,x\right)}{-i\sqrt{c}\,f+\sqrt{a}\,g}}\,\sqrt{f+gx}\left(F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f-i\sqrt{a}\,g}}\right)|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)-\Pi\left(\frac{e\left(f-\frac{i\sqrt{a}\,g}{\sqrt{c}}\right)}{ef-dg};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f-i\sqrt{a}\,g}}\right)|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{e\sqrt{\frac{\sqrt{c}\,(f+gx)}{g\left(i\sqrt{a}+\sqrt{c}\,x\right)}}\,\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*Sqrt
[f + g*x]*(EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sq
rt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Ellipti
cPi[(e*(f - (I*Sqrt[a]*g)/Sqrt[c]))/(e*f - d*g), I*ArcSinh[Sqrt[-((Sqrt[c]*
(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)]))/(e*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x)
)]*Sqrt[a + c*x^2])
```

Maple [A]
time = 0.09, size = 439, normalized size = 1.38

method	result
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\left(f\,\mathrm{EllipticF}\left(\sqrt{-\frac{(gx}{g\sqrt{-}}}\right.\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)\left(c\,x^2+a\right)}\left(\frac{2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{}{\frac{f}{g}-}}\right.}{e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-
a*c)^(1/2)-c*f))^(1/2)*(f*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)
,(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c-(-a*c)^(1/2)*Ellipti
cF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)
^(1/2)+c*f))^(1/2))*g-EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g
*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f
))^(1/2))*c*f+(-a*c)^(1/2)*EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/
```

```
2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2
)+c*f))^(1/2))*g)/e/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f + gx}}{\sqrt{a + cx^2}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.641 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=698

$$-\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)}-\frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{(cd^2+ae^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}+\cdots$$

```
[Out] -e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)-EllipticE(1/2*(1-x*c
^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2)
)*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(a*e^2+c*d^2)/(c*x^2+a
)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+f*EllipticF(1/2*(1
-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(
1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f
*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-d*g*EllipticF(
1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(
1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-(a*e^2
*g+c*d*(-d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)
/e/(a*e^2+c*d^2)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 1.29, antiderivative size = 698, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {960, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\frac{\sqrt{-a}\,\sqrt{c}\,f\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{a+cx^2}\,\sqrt{f+gx}\,(ae^2+cd^2)}-\frac{\sqrt{-a}\,\sqrt{c}\,dg\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{e\sqrt{a+cx^2}\,\sqrt{f+gx}\,(ae^2+cd^2)}-\frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{a+cx^2}\,(ae^2+cd^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(ae^2g+cd(2ef-dg))\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e\sqrt{a+cx^2}\,\sqrt{f+gx}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)(ae^2+cd^2)}-\frac{e\sqrt{a+cx^2}\,\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) - (Sqrt[-a
]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 +
a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
+ (Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sq
```

```
rt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
 (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[
a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(e*(c*d^2 + a*e^2)*Sqr
t[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*
(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/(
(Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]],
(2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*
d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 960

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^
2]/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[2*c*d*f*(m + 1) -
 e*(a*g) + 2*c*(d*g*(m + 1) - e*f*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x], x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\int \frac{-2cdf-aeg-2cdgx-cegx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\int \left(-\frac{cdg}{e\sqrt{f+gx}\,\sqrt{a+cx^2}} - \frac{cgx}{\sqrt{f+gx}\,\sqrt{a+cx^2}} \right.}{2(cd^2+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)} + \frac{(cdg)\int \frac{}{\sqrt{f+gx}}}{2e(cd^2}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,dg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin\right.}{e(cd^2+ae^2)\sqrt{f+gx}}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-}}{\sqrt{}}\right.\right.}{(cd^2+ae^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-}}{\sqrt{}}\right.\right.}{(cd^2+ae^2)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.09, size = 1330, normalized size = 1.91

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-(c*e^2*f^3*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*e^2
*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c*d*
e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
+ g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((
I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f
 + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] +
e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a]
)/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]
*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[
f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (2*I)*c*d
*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))
/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + I*c*
d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a
*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g
))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(f + g*x))))/((c*d^2*e + a*e^3
)*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5742 vs. $2(579) = 1158$.
time = 0.11, size = 5743, normalized size = 8.23

method	result

elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{e\sqrt{cgx^3+cfx^2+agx+fa}}{(ae^2+cd^2)(ex+d)}+\frac{dgc\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-a}}{c}}{-\frac{f}{g}-\frac{\sqrt{-}}{}}}}{(ae^2+cd^2)e}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(x*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(x*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.642 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=1246

$$-\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2}-\frac{e(ae^2g+cd(6ef-5dg))\,\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}-\frac{\sqrt{-a}\,\sqrt{c}\,(ae^2g+cd(6ef-5dg)}{4\left(cd^2+\right.}$$

[Out] -1/2*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^2-1/4*e*(a*e^2*g+c*d*(-5*d*g+6*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)/(e*x+d)-1/4*(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+1/2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*f*(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-1/4*d*g*(a*e^2*g+c*d*(-5*d*g+6*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)^2/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+c*(-3*d*g+e*f)*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+1/4*(a*e^2*g+c*d*(-5*d*g+6*e*f))*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e/(a*e^2+c*d^2)^2/(-d*g+e*f)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)

Rubi [A]

time = 2.87, antiderivative size = 1246, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {960, 6874, 733, 430, 954, 858, 435, 947, 174, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*(e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)^2) - (e*(
a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a
*e^2)^2*(e*f - d*g)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g + c*d*(6*e*f -
5*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 +
a*e^2)^2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqr
t[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + S
qrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[
-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e*(c*d^2 + a*e^2)*S
qrt[f + g*x]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g + c*d*(6*e*f -
 5*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2
)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sq
rt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sq
rt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Elliptic
F[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c
]*f - a*g)])/(4*e*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^
2]) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*S
qrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt
[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g
)])/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*
x^2]) + ((a*e^2*g + c*d*(6*e*f - 5*d*g))*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sq
rt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Ellipt
icPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]
]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(4*e*((Sqrt[c]*d)/Sqr
t[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
```

```
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 960

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^
2]/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(c*d^2 + a*e^2)), Int
[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[2*c*d*f*(m + 1) -
 e*(a*g) + 2*c*(d*g*(m + 1) - e*f*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x], x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{\int \frac{-4cdf-aeg+2c(ef-2dg)x+cegx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{\int \left(\frac{cg}{e\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{-ae^2g-cd(6ef-5dg)}{e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+}}\right.}{4\left(cd^2+ae\right.}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4e\left(cd^2+ae^2\right)} - \frac{(c(ef-3dg))\int}{2}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} -$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} +$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} +$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.32, size = 2450, normalized size = 1.97

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (-11*c^2*d^2*e^2*f^3 + a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 5*c^2*d^3*e*f^2*
g + 5*a*c*d*e^3*f^2*g - 11*a*c*d^2*e^2*f*g^2 + a^2*e^4*f*g^2 + 5*a*c*d^3*e*
g^3 - a^2*d*e^3*g^3 + 22*c^2*d^2*e^2*f^2*(f + g*x) - 2*a*c*e^4*f^2*(f + g*x
) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 10*c^2*d^3*e*f*g*(f + g*x) + 2*a*c*d*e
^3*f*g*(f + g*x) - 11*c^2*d^2*e^2*f*(f + g*x)^2 + a*c*e^4*f*(f + g*x)^2 + (
6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 5*c^2*d^3*e*g*(f + g*x)^2 - a*c*d*e^3*g*(f
 + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d^2 + a*e^2)*(e*f
- d*g) + (a*e^2*g + c*d*(6*e*f - 5*d*g))*(d + e*x)))/(d + e*x)^2 + (Sqrt[c]
*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(6*e*f - 5*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + (e*(I*Sqrt[c]*d + Sqrt[a]*
e)*(Sqrt[c]*f + I*Sqrt[a]*g)*(a*e^2*g + (2*I)*Sqrt[a]*Sqrt[c]*e*(e*f - d*g)
 + c*d*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt
[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*Ar
cSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[
a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((8*I)
*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*S
qrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e
*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*
g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((4*I)*a*c*e^4*f^2*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x
))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt
[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c
]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]] - ((12*I)*c^2*d^3*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*
Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi
[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((12*I)*a*c*d*e^3*f*g*S
qrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
- g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqr
t[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + ((3*I)*c^2*d^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))
```

```
/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2
)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((10*I)*a*c
*d^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[
a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])
/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (I*a^2*e^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqr
t[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f +
 g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)),
 I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*
Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(4
*e*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 20358 vs. $2(1050) = 2100$.
time = 0.11, size = 20359, normalized size = 16.34

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{e\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{2\left(a\,e^2+c\,d^2\right)(ex+d)^2}+\frac{e\left(a\,e^2g-5c\,d^2g+6cdef\right)\sqrt{cg\,x^3+cf\,x^2+agx+}}{4\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)\left(a\,e^2+c\,d^2\right)(ex+d)}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(x*e + d)^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(x*e + d)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

3.643 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=600

$$\frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce}-\frac{2\sqrt{-a}\,g(7ef-3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1+\frac{a\sqrt{c}\,x}{(-a)^{3/2}}}}{\sqrt{2}}\right)|\frac{2a}{-\sqrt{-a}\ldots}\right.}{3\sqrt{c}\,e^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

```
[Out] 2/3*g^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e-2/3*g*(-3*d*g+7*e*f)*EllipticE(1/
2*(1+a*x*c^(1/2)/(-a)^(3/2))^(1/2)*2^(1/2),2^(1/2)*(a*g/(a*g-f*(-a)^(1/2)*c
^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e^2/c^(1/2)/(c*x
^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2/3*g*(a*e^2*g
^2+c*(-3*d^2*g^2+6*d*e*f*g-2*e^2*f^2))*EllipticF(1/2*(1+a*x*c^(1/2)/(-a)^(3
/2))^(1/2)*2^(1/2),2^(1/2)*(a*g/(a*g-f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/
2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/
2)/e^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*(-d*g+e*f)^2*EllipticPi((g*x+f)^(1/2
)*(c/(c*f+g*(-a)^(1/2)*c^(1/2)))^(1/2),e*(f+g*(-a)^(1/2)/c^(1/2))/(-d*g+e*f
),((g*(-a)^(1/2)+f*c^(1/2))/(-g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(g*((-a)^(1/2
)-x*c^(1/2))/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)*(-g*((-a)^(1/2)+x*c^(1/2))/(-g
*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^3/(c*x^2+a)^(1/2)/(c/(c*f+g*(-a)^(1/2)*c^(1
/2)))^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 808, normalized size of antiderivative = 1.35, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {972, 733, 430, 947, 174, 552, 551, 435, 757, 858}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) - (8*Sqrt[-a]*f*g*Sqrt[f + g*
x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*e*Sqrt[(Sqrt[c]*(f +
 g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*g*(e*f - d*
g)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
```

```
a]*g*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[
1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-
2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + (2*Sqrt[-a]*g*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3/2)*e*Sqrt[f +
 g*x]*Sqrt[a + c*x^2]) - (2*(e*f - d*g)^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]
*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a
] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqr
t[c]*f + Sqrt[-a]*g)])/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a
 + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{g(ef-dg)\sqrt{f}}{e^2\sqrt{a+cx}}\right.$$

$$= \frac{g\int \frac{(f+gx)^{3/2}}{\sqrt{a+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{e^2} + \frac{(g(ef-dg)^2)\int \frac{1}{\sqrt{f+gx}}}{e^3}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce} + \frac{(2g)\int \frac{\frac{1}{2}(3cf^2-ag^2)+2cfgx}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3ce} + \frac{\left((ef-dg)^3\sqrt{1+}\right.}{}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce} - \frac{2\sqrt{-a}\,g(ef-dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\right.\right.}{\sqrt{c}\,e^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce} - \frac{2\sqrt{-a}\,g(ef-dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\right.\right.}{\sqrt{c}\,e^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3ce} - \frac{8\sqrt{-a}\,fg\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{}}{\sqrt{}}}}{\sqrt{2}}\right.\right.}{3\sqrt{c}\,e\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.83, size = 1212, normalized size = 2.02

$$\frac{2\sqrt{f+gx}\left(-14ce^2f^2+6cdefg+\frac{7ce^2f^3}{f+gx}-\frac{3cdef^2g}{f+gx}+\frac{7ae^2fg^2}{f+gx}-\frac{3adeg^3}{f+gx}+7ce^2f(f+gx)-3cdeg(f+gx)+e^2g^2(a+cx^2)+ice\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}(7ef-3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+\frac{e\left(-i\sqrt{c}f+\sqrt{a}g\right)\left(-i\sqrt{a}eg+\sqrt{c}(-6ef+3dg)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}+\frac{3ice^2f^2\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,\Pi\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+i\sqrt{a}g)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}-\frac{6icdefg\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,\Pi\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+i\sqrt{a}g)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}+\frac{3icd^2g^2\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,\Pi\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+i\sqrt{a}g)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}\right)}{3ce^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(-14*c*e^2*f^2 + 6*c*d*e*f*g + (7*c*e^2*f^3)/(f + g*x) - (
3*c*d*e*f^2*g)/(f + g*x) + (7*a*e^2*f*g^2)/(f + g*x) - (3*a*d*e*g^3)/(f + g
*x) + 7*c*e^2*f*(f + g*x) - 3*c*d*e*g*(f + g*x) + e^2*g^2*(a + c*x^2) + I*c
*e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(7*e*f - 3*d*g)*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqr
t[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*
x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (e*((-I)*Sqrt[c
]*f + Sqrt[a]*g)*((-I)*Sqrt[a]*e*g + Sqrt[c]*(-6*e*f + 3*d*g))*Sqrt[(g*((I*
Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f +
 g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((3*I)*c*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
 + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f +
 g*x]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((6*I)*
c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*
g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticPi[(Sqrt[c]*(e*f - d*g)
)/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt
[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c
] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f
+ g*x]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*Ar
cSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[
a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3*c*e
^3*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3163 vs. 2(493) = 986.
time = 0.12, size = 3164, normalized size = 5.27

method	result

elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2g^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3ec}+\frac{2\left(\frac{g\left(d^2g^2-3defg+3e^2f^2\right)}{e^3}-\frac{g^3a}{3ce}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{}{\frac{f}{g}-}}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/
2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g
/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1
/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*d*e*g^3-6*(-(g*
x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c
*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g
*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+
c*f))^(1/2))*a*c*e^2*f*g^2-(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(
-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^
(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-
a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3-3*(-(g*
x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c
*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g
*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+
c*f))^(1/2))*c^2*d^2*f*g^2+9*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*
(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e*f^2*g-9*(-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c
/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(
1/2))*c^2*e^2*f^3+3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c
```

```
*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1
/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3-6*(-a*c)^(1/2)
*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(
1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Elliptic
F((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^
(1/2)+c*f))^(1/2))*c*d*e*f*g^2+2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((
-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(
-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(
-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*c*e^2*f^2*g
+3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)
^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ellipt
icPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-
e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d^2*f*g^2-6*(-
(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2
)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticPi(
(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),
(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e*f^2*g+3*(-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f
))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticPi((-(g*
x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*
(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e^2*f^3-3*(-(g*x+f)*c/(g
*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2
)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticPi((-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^
(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*c*d^2*g^3+6*(-a*c)^(1/
2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)
^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ellipt
icPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-
e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*e*f*g^2-3*(-a*
c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*
(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*
EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c
/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*e^2*f^2*g-
3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^
(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ellipti
cE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)
^(1/2)+c*f))^(1/2))*a*c*d*e*g^3+7*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*(
(-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*
(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),
(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e^2*f*g^2-3*(-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f
))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x
+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*
f))^(1/2))*c^2*d*e*f^2*g+7*(-(g*x+f)*c/(g*(-a*c...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^{\frac{5}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral((f + g*x)**(5/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{5/2}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.644 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=469

$$-\frac{2\sqrt{-a}\,g\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}-\frac{2\sqrt{-a}\,g(ef-dg)\sqrt{\frac{\sqrt{c}}{\sqrt{c}}}\cdots}{\cdots}$$

```
[Out] -2*g*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/e/c^(
1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2*g*(
-d*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*
g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/
2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2
)-2*(-d*g+e*f)^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/e^2
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {972, 733, 430, 947, 174, 552, 551, 435}

$$-\frac{2\sqrt{-a}\,g\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e^2\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{2\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{e^2\sqrt{a+cx^2}\,\sqrt{f+gx}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}-\frac{2\sqrt{-a}\,g\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) -
 (2*Sqrt[-a]*g*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g
)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*e^2*Sqrt[f + g*x]*Sqrt
[a + c*x^2]) - (2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[
-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), Ar
```

```
cSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + S
qrt[-a]*g)])/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \left(\frac{g(ef-dg)}{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{g\sqrt{f+gx}}{e\sqrt{a+cx^2}}\right) d$$

$$= \frac{g\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{e^2} + \frac{(ef-dg)^2\int \frac{}{(d+e}}{}$$

$$= \frac{\left((ef-dg)^2\sqrt{1+\frac{cx^2}{a}}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}\,\sqrt{1+\frac{\sqrt{c}\,x}{\sqrt{-a}}}\,(d+ex)\sqrt{f+gx}}\,dx}{e^2\sqrt{a+cx^2}} + \frac{\Big(}{}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\,g\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,e\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.75, size = 927, normalized size = 1.98

$$\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\left(\frac{2i\sqrt{a}fg\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)}{\sqrt{c}e}-\frac{i\sqrt{a}dg^2\sqrt{1+\frac{cx^2}{a}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)}{\sqrt{c}e^2}+\frac{g\sqrt{\frac{g(\sqrt{a}+i\sqrt{c}x)}{-i\sqrt{c}f+\sqrt{a}g}}(i\sqrt{a}+\sqrt{c}x)\left((\sqrt{c}f+i\sqrt{a}g)E\left(\sin^{-1}\left(\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-i\sqrt{a}gF\left(\sin^{-1}\left(\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)\right)}{ce\sqrt{\frac{g(\sqrt{a}-i\sqrt{c}x)}{i\sqrt{c}f+\sqrt{a}g}}}-\frac{\sqrt{a}f^2\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2\sqrt{a}e}{i\sqrt{c}d+\sqrt{a}e};\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)}{i\sqrt{c}d+\sqrt{a}e}+\frac{2\sqrt{a}dfg\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2\sqrt{a}e}{i\sqrt{c}d+\sqrt{a}e};\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)}{i\sqrt{c}de+\sqrt{a}e^2}-\frac{\sqrt{a}d^2g^2\sqrt{1+\frac{cx^2}{a}}\Pi\left(\frac{2\sqrt{a}e}{i\sqrt{c}d+\sqrt{a}e};\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)}{e^2(i\sqrt{c}d+\sqrt{a}e)}\right)}{\sqrt{f+gx}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g)]*(((2*I)*Sqrt[a]*f*g*
Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2
]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e) - (I*Sqrt[a]*d*g^
2*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt
[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e^2) + (g*Sqrt[(g*
(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*(I*Sqrt[a] + Sqrt[c]
*x)*((Sqrt[c]*f + I*Sqrt[a]*g)*EllipticE[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(S
qrt[c]*f - I*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)] - I*Sqrt[a]*g*EllipticF[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I
*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*e*
Sqrt[(g*(Sqrt[a] - I*Sqrt[c]*x))/(I*Sqrt[c]*f + Sqrt[a]*g)]) - (Sqrt[a]*f^2
*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), Ar
cSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f +
 Sqrt[a]*g)])/(I*Sqrt[c]*d + Sqrt[a]*e) + (2*Sqrt[a]*d*f*g*Sqrt[1 + (c*x^2)
/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1 - (I*
Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(I*
Sqrt[c]*d*e + Sqrt[a]*e^2) - (Sqrt[a]*d^2*g^2*Sqrt[1 + (c*x^2)/a]*EllipticP
i[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sq
rt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(e^2*(I*Sqrt[c]*
d + Sqrt[a]*e))))/(Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(382) = 764$.
time = 0.10, size = 959, normalized size = 2.04

method	result

elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{2g(dg-2ef)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}}{e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}+cf}}\,\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\left(\sqrt{-ac}\,\mathrm{EllipticF}\left(\sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-
c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-
a*c)^(1/2)-c*f))^(1/2)/c*((-a*c)^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2
)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*g^2-(-a
*c)^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/
2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e*f*g-(-a*c)^(1/2)*EllipticPi((-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a
*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*g^2+(-a*c)^(1/2)*EllipticPi((
-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(
-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e*f*g+EllipticF((-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f)
)^(1/2))*a*e*g^2-EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a
*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*f*g+2*EllipticF((-(g*x+f)*c
/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(
1/2))*c*e*f^2-EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)
^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2-EllipticE((-(g*x+f)*c/(g*(
-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))
*c*e*f^2+EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)
-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d
*f*g-EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f
)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*e*f^2
```

```
)/e^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^{\frac{3}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral((f + g*x)**(3/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.645 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=457

$$-\frac{8e^2(ef-3dg)\sqrt{f+gx}\,\sqrt{a+cx^2}}{15cg^2}+\frac{2e^2(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}{5cg}+\frac{2\sqrt{-a}\,e(9ae^2g^2-c(8e^2f^2-30def}{}$$

```
[Out] -8/15*e^2*(-3*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2+2/5*e^2*(e*x+d)*
(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/15*e*(9*a*e^2*g^2-c*(45*d^2*g^2-30*d*e*
f*g+8*e^2*f^2))*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*
g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^
(1/2)/c^(3/2)/g^3/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2)-2/15*(a*e^2*g^2*(-15*d*g+7*e*f)-c*(-15*d^3*g^3+45*d^2*e*f*g^2-30*d*
e^2*f^2*g+8*e^3*f^3))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((
g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^3/(g*x+f)^(1/2)/(c
*x^2+a)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {945, 1668, 858, 733, 435, 430}

$$\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,(9ae^2g^2-c(45d^2g^2-30defg+8e^2f^2))\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{2\sqrt{-a}\,\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,(ae^2g^2(7ef-15dg)-c(-15d^3g^3+45d^2efg^2-30de^2f^2g+8e^3f^3))\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\,\sqrt{f+gx}}-\frac{8e^2\sqrt{a+cx^2}\,\sqrt{f+gx}\,(ef-3dg)}{15cg^2}+\frac{2e^2\sqrt{a+cx^2}\,(d+ex)\sqrt{f+gx}}{5cg}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-8*e^2*(e*f - 3*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(15*c*g^2) + (2*e^2*(d
 + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c*g) + (2*Sqrt[-a]*e*(9*a*e^2*g^2
 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt
[-a]*Sqrt[c]*f - a*g)])/(15*c^(3/2)*g^3*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*(a*e^2*g^2*(7*e*f - 15*d*g)
- c*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d^3*g^3))*Sqrt[(Sqrt[
c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/(15*c^(3/2)*g^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
```

```
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{\int \frac{-5cd^3g+ae^2(2ef+dg)+e(3ae^2g+cd(2ef-15dg))x+4ce}{\sqrt{f+gx}\sqrt{a+cx^2}}}{5cg}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{2\int}{}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - \frac{(e(}{}$$

$$\Big(2$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} - $$

$$2\sqrt{}$$

$$= -\frac{8e^2(ef-3dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}{5cg} + $$

Mathematica [C] Result contains complex when optimal does not.
time = 22.85, size = 625, normalized size = 1.37

$$\frac{2\sqrt{f+gx}\left(ce^2g^2(-4ef+15dg+3egx)(a+cx^2)+\frac{eg^2\left(-9a^2e^2g^2+c^2(8e^2f^2-30defg+45d^2g^2)x^2+ac\left(-30defg+45d^2g^2+e^2(8f^2-9g^2x^2)\right)\right)}{f+gx}+ice\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\left(-9ae^2g^2+c(8e^2f^2-30defg+45d^2g^2)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)+\frac{\sqrt{c}\,g\left(15ic^{3/2}d^3g^2+9a^{3/2}e^3g^2-ia\sqrt{c}\,e^2g(2ef+15dg)+\sqrt{a}\,ce(-8e^2f^2+30defg-45d^2g^2)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}\right)}{15c^2g^4\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e^2*g^2*(-4*e*f + 15*d*g + 3*e*g*x)*(a + c*x^2) + (e*g^
2*(-9*a^2*e^2*g^2 + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2)*x^2 + a*c*(-3
0*d*e*f*g + 45*d^2*g^2 + e^2*(8*f^2 - 9*g^2*x^2))))/(f + g*x) + I*c*e*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(8*e^2*f^2 - 30*d*e*f*g + 45*
d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*
g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)] + (Sqrt[c]*g*((15*I)*c^(3/2)*d^3*g^2 + 9*a^(3/2)*e^3*g^2 -
I*a*Sqrt[c]*e^2*g*(2*e*f + 15*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 30*d*e*f*g -
 45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt
[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(15*c^2*g^4*Sqrt[a
+ c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2948 vs. $2(385) = 770$.

time = 0.10, size = 2949, normalized size = 6.45

method	result
elliptic	$\frac{\sqrt{(gx+f)\,(c\,x^2+a)}\left(\frac{2e^3x\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{5cg}+\frac{2\left(3d\,e^2-\frac{4f\,e^3}{5g}\right)\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}\right.}{}$

risch	$\frac{2e^2(3egx+15dg-4ef)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{15c\,g^2} - \frac{\left(\frac{2\left(9a\,e^3g^2-45c\,d^2e\,g^2+30cd\,e^2fg-8c\,e^3f^2\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}}{\ldots}\right.}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(-3*a*c*e^3*g^4*x^2+4*c^2*e^3*f^2*g^2*x^2-6*(-(g*x+f)*c/(g*(-a*c)^(1/
2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a
*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1
/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e^3
*f^2*g^2+45*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2)
)*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))
^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-
c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d^2*e*g^4-(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f
))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1
/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*
f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e^3*f^2*g
^2+45*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a
*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ell
ipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-
a*c)^(1/2)+c*f))^(1/2))*c^2*d^2*e*f^2*g^2-30*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*
f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c
*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e^2*f^
3*g-45*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c
*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1
/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d^2*e*f*g^3-30*a*c*(-(g*x+f)*c/(g*(
-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*
((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*
(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2)
```

```
)*d*e^2*f*g^3-3*e^3*x^4*g^4*c^2-15*c^2*d*e^2*f*g^3*x^2-15*a*d*e^2*f*g^3*c-1
5*a*c*d*e^2*g^4*x+a*c*e^3*f*g^3*x+4*a*c*e^3*f^2*g^2-15*(-a*c)^(1/2)*(-(g*x+
f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f
))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x
+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*
f))^(1/2))*a*d*e^2*g^4+30*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1
/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*
g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(
1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*e^2*f^2*g^2+45
*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(
1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Elliptic
F((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^
(1/2)+c*f))^(1/2))*a*c*d*e^2*f*g^3+15*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1
/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-
a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(
1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d^3*
g^4-15*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-
a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*El
lipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(
-a*c)^(1/2)+c*f))^(1/2))*c^2*d^3*f*g^3+9*a^2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*
f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c
*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*e^3*g^4-9*a^
2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^
(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ellipti
cE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)
^(1/2)+c*f))^(1/2))*e^3*g^4+8*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*
x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*
c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g
*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e^3*f^4+7*(-a*c)^(1/2)*
(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1
/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF
((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(
1/2)+c*f))^(1/2))*a*e^3*f*g^3-8*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*
f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c
*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*e^3*f^3*g-
45*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-
a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*El
lipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(
-a*c)^(1/2)+c*f))^(1/2))*d^2*e*g^4-15*c^2*d*e^2*g^4*x^3+c^2*e^3*f*g^3*x^3)*
(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/g^4/(c*g*x^3+...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 315, normalized size = 0.69

$$\frac{2\left(\left(45\,cd^3g^3-45\,cd^2fg^2e-(8\,cf^3-3\,afg^2)e^3+15\,(2\,cdf^2g-3\,adg^3)e^2\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)-3\,\left(45\,cd^2g^3e-30\,cdfg^2e^2+(8\,cf^2g-9\,ag^3)e^3\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)+3\,\left(15\,cdg^3e^2+(3\,cg^3x-4\,cfg^2)e^3\right)\sqrt{cx^2+a}\,\sqrt{gx+f}\right)}{45\,c^2g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*((45*c*d^3*g^3 - 45*c*d^2*f*g^2*e - (8*c*f^3 - 3*a*f*g^2)*e^3 + 15*(2*
c*d*f^2*g - 3*a*d*g^3)*e^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*(45
*c*d^2*g^3*e - 30*c*d*f*g^2*e^2 + (8*c*f^2*g - 9*a*g^3)*e^3)*sqrt(c*g)*weie
rstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3
), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*
g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*(15*c*d*g^3*e^2 + (3*c*g^3*x - 4*c*f*
g^2)*e^3)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^2}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.646 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=356

$$\frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg}+\frac{4\sqrt{-a}\,e(ef-3dg)\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}}\right.}{3\sqrt{c}\,g^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{a+cx^2}}$$

```
[Out] 2/3*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+4/3*e*(-3*d*g+e*f)*EllipticE(1/2*
(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))
^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g^2/c^(1/2)/(c*x^2+a)^(1
/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)-2/3*((-a*e^2+3*c*d^2)*
g^2+2*c*e*f*(-3*d*g+e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1
/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2
)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(3/2)/g^2/(g*x+f)^(1/2
)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {945, 24, 858, 733, 435, 430}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\left(g^2(3cd^2-ae^2)+2cef(ef-3dg)\right)F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{3c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{4\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,(ef-3dg)E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{3\sqrt{c}\,g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}+\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*g) + (4*Sqrt[-a]*e*(e*f - 3*d*g)
*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*Sqrt[c]*g^2*Sqrt
[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-
a]*((3*c*d^2 - a*e^2)*g^2 + 2*c*e*f*(e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))
/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (S
qrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(3*c^(3
/2)*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3cg} - \frac{\int \frac{-d(3cd^2-ae^2)g+e(ae^2g+cd(2ef-9dg))x+2ce^2(ef-3dg)x^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3cg}$$

$$= \frac{2e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3cg} - \frac{\int \frac{-e^2(3cd^2-ae^2)g+2ce^3(ef-3dg)x}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{3ce^2g}$$

$$= \frac{2e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3cg} - \frac{(2e(ef-3dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{3g^2} + \frac{1}{3}\Bigg(3d^2 - \frac{ae^2}{c} +$$

$$= \frac{2e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3cg} - \frac{\left(4ae(ef-3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\Bigg(\int -}{3\sqrt{-a}\,\sqrt{c}\,g^2\sqrt{\frac{c(}{cf}}}$$

$$= \frac{2e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{3cg} + \frac{4\sqrt{-a}\,e(ef-3dg)\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\Bigg(\sin^{-1}}{3\sqrt{c}\,g^2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.31, size = 473, normalized size = 1.33

$$\frac{2\sqrt{f+gx}\left(e^2g^2(a+cx^2)-\frac{2eg^2(ef-3dg)(a+cx^2)}{f+gx}-2ice\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(ef-3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)+\frac{g\left(3icd^2g-iae^2g+2\sqrt{a}\,\sqrt{c}\,e(ef-3dg)\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}\right)}{3cg^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(e^2*g^2*(a + c*x^2) - (2*e*g^2*(e*f - 3*d*g)*(a + c*x^2))
/(f + g*x) - (2*I)*c*e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - 3*d*g)*Sqrt[
```

```
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
 + (g*((3*I)*c*d^2*g - I*a*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 3*d*g))*Sqrt[
(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*
x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
)/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3*c*g^3*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1768 vs. $2(290) = 580$.

time = 0.10, size = 1769, normalized size = 4.97

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3cg}+\frac{2\left(d^2-\frac{a\,e^2}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
risch	$\frac{2e^2\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{3cg}-\frac{\left(-\frac{2\left(6cdeg-2fc\,e^2\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*
c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Elli
pticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a
*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*g^3-3*(-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c
)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2
)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1
/2)*c*d^2*g^3+6*(-a*c)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x
+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*
(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*e*f*g^2-2*(-(g*x+f)*c/(g
*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2
)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(
g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/
2))*(-a*c)^(1/2)*c*e^2*f^2*g+6*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c
*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a
*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(
g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*d*e*g^3-3*(-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-(g*x+f)*c
/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(
1/2))*a*c*e^2*f*g^2+3*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)
^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)
-c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^
(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d^2*f*g^2-6*(-(g*x+f)*c/(g*(-a*
c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c
*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a
*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a
*c*d*e*g^3+2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g
/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1
/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f
)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e^2*f*g^2-6*(-(g*x+f)*c/(g*(-a*c)^(1/2)-
c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)
^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*e*f^
2*g+2*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a
*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ell
ipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-
a*c)^(1/2)+c*f))^(1/2))*c^2*e^2*f^3+c^2*e^2*g^3*x^3+c^2*e^2*f*g^2*x^2+a*c*e
^2*g^3*x+a*c*e^2*f*g^2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^2/g^3/(c*g*x^3+c*f*
x^2+a*g*x+a*f)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 245, normalized size = 0.69

$$\frac{2\left(3\sqrt{cx^2+a}\sqrt{gx+f}\,cg^2e^2+\left(9\,cd^2g^2-6\,cdfge+\left(2\,cf^2-3\,ag^2\right)e^2\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)-6\left(3\,cdg^2e-cfge^2\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)\right)}{9\,c^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + a)*sqrt(g*x + f)*c*g^2*e^2 + (9*c*d^2*g^2 - 6*c*d*f*g*e
 + (2*c*f^2 - 3*a*g^2)*e^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 6*(3*
c*d*g^2*e - c*f*g*e^2)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g
^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*
a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)))/(c^
2*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^2}{\sqrt{a+cx^2}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.647 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=288

$$-\frac{2\sqrt{-a}\,e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{a+cx^2}}+\frac{2\sqrt{-a}\,(ef-dg)\sqrt{\frac{\sqrt{c}}{\sqrt{c}\,f}}}{}$$

```
[Out] -2*e*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(
-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/g/c^(
1/2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*(-d
*g+e*f)*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+
f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)
/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/g/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {858, 733, 435, 430}

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,g\sqrt{a+cx^2}\sqrt{f+gx}}-\frac{2\sqrt{-a}\,e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\,g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt
[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
 (2*Sqrt[-a]*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[a +
 c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = \frac{e\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{g} + \frac{(-ef+dg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{g}$$

$$= \frac{\left(2ae\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2a\sqrt{c}\,gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{c}\,g}{\sqrt{-a}}\right)}}}{\sqrt{1-x^2}}\,dx, x, - \ldots\right.}{\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}\,g}{\sqrt{-a}}}}\,\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\,e\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag \ldots}\right.}{\sqrt{c}\,g\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 21.05, size = 439, normalized size = 1.52

$$-\frac{2\left(-eg^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(a+cx^2)+i\sqrt{c}\,e(\sqrt{c}\,f+i\sqrt{a}\,g)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)+\sqrt{c}\,(-i\sqrt{c}\,d+\sqrt{a}\,e)\,g\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{cg^2\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,\sqrt{f+gx}\,\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*(-(e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a + c*x^2)) + I*Sqrt[c]*e*(S
qrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[
-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + Sqrt[c]*((-I)*Sqrt[c]*d + Sqrt[a]*e)*g*Sq
rt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)]))/(c*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[f + g*x]*Sqrt[a + c*x^
2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(232) = 464$.

time = 0.10, size = 520, normalized size = 1.81

method	result
default	$\frac{2\left(\text{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)ae\,g^2+\text{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)cdfg\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)\left(c\,x^2+a\right)}\left(\frac{2d\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\text{EllipticF}\left(\sqrt{\frac{x}{\frac{f}{g}-}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+fa}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)
/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2+EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*f*g-(-
a*c)^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1
/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*d*g^2+(-a*c)^(1/2)*EllipticF((-(g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f)
)^(1/2))*e*f*g-EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c
)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2-EllipticE((-(g*x+f)*c/(g*
(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2)
)*c*e*f^2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^
(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2
)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 184, normalized size = 0.64

$$-\frac{2\left(3\sqrt{cg}\,g e \mathrm{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)-\sqrt{cg}\,(3\,dg-fe)\mathrm{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)\right)}{3\,cg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(c*g)*g*e*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*
(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g
^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - sqrt(c*g)*(3*
d*g - f*e)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3
+ 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g))/(c*g^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{\sqrt{a+cx^2}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+e\,x}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

3.648 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

```
[Out] -2*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a
)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(
-a)^(1/2)+f*c^(1/2)))^(1/2)/c^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {733, 430}

$$-\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}\,f-ag}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[-a]*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)
/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(Sqrt[c]*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
```

```
e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{\left(2a\sqrt{\frac{c(f+gx)}{cf-\frac{a\sqrt{c}\,g}{\sqrt{-a}}}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2a\sqrt{c}\,gx^2}{\sqrt{-a}\left(cf-\frac{a\sqrt{\cdots}}{\sqrt{\cdots}}\right.}}}\right.}{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

$$= -\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{cx^2}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{\cdots}{\sqrt{-a}\cdots}\right.}{\sqrt{c}\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.19, size = 186, normalized size = 1.37

$$\frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)}{g\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + c*x^2])
```

Maple [A]
time = 0.08, size = 200, normalized size = 1.47

method	result
default	$\frac{2\left(cf-g\sqrt{-ac}\right)\text{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}+cf}}\sqrt{}}{gc(cg\,x^3+cf\,x^2+agx+fa)}$
elliptic	$\frac{2\sqrt{(gx+f)\left(c\,x^2+a\right)}\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\text{EllipticF}\left(\sqrt{\frac{x}{\frac{f}{g}-\sqrt{}}}\right.}{\sqrt{gx+f}\sqrt{c\,x^2+a}\sqrt{cg\,x^3+cf\,x^2+agx+fa}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(c*f-g*(-a*c)^(1/2))*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-
(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*((c*x+(-a*c)^(1/2))*g/(g*
(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)
*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g/c/
(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 66, normalized size = 0.49

$$\frac{2\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)}{cg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3
 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)/(c*g)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+cx^2}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)

[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)

[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)

3.649 $\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{1+\frac{cx^2}{a}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\,\sqrt{a+cx^2}}$$

```
[Out] -2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(
-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+
1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(e+d*c^(1/2)/(-a)
^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {947, 174, 552, 551}

$$-\frac{2\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*
EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sq
rt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/S
qrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{\sqrt{1+\frac{cx^2}{a}}\int \frac{1}{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{c}\,x}{\sqrt{-a}}}(d+ex)\sqrt{f+gx}}dx}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e-ex^2\right)\sqrt{f+\frac{\sqrt{-a}\,g}{\sqrt{c}}-}\right.}{\sqrt{a+cx^2}}$$

$$= -\frac{\left(2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{cx^2}{a}}\right)\text{Subst}\left(\int \frac{}{\sqrt{2-x^2}\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e-e\right.}\right.}{\sqrt{f+gx}\sqrt{a+c}}$$

$$= -\frac{2\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{cx^2}{a}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right.\right.}{\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{a+cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.68, size = 311, normalized size = 1.86

$$-\frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)-\Pi\left(\frac{\sqrt{c}\,(ef-dg)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(ef-dg)\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqr
```

```
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] - EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g))
, I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I
*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(e*f - d*g)*Sqrt[a + c*x^2])
```

Maple [A]
time = 0.09, size = 235, normalized size = 1.41

method	result
default	$\frac{2\left(cf-g\sqrt{-ac}\right)\mathrm{EllipticPi}\left(\sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\frac{\left(g\sqrt{-ac}-cf\right)e}{c(dg-ef)},\sqrt{-\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{g\sqrt{-ac}-cf}}\ \sqrt{\frac{(-}{g}}}{c(dg-ef)(cg\,x^3+cf\,x^2+agx+fa)}$
elliptic	$\frac{2\sqrt{(gx+f)\,(c\,x^2+a)}\,\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\ \sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\ \sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{}{\frac{f}{g}-}}\right.}{\sqrt{gx+f}\ \sqrt{c\,x^2+a}\ e\sqrt{cg\,x^3+cf\,x^2+agx+fa}\ \left(-\frac{f}{g}+\frac{d}{e}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(c*f-g*(-a*c)^(1/2))*EllipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(
g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*
f))^(1/2))*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^
(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2
)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.650 $\displaystyle \int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=746

$$-\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)}-\frac{\sqrt{-a}\sqrt{c}\,e\sqrt{f+gx}\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}\cdots}\right.}{(cd^2+ae^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{a+cx^2}}$$

```
[Out] -e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-e*Ellip
ticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*
c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(a*e^2+
c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2))
)^(1/2)+e*f*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-
a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*
x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+
f)^(1/2)/(c*x^2+a)^(1/2)-d*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2
^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^
2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)
/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+(a*e^2*g-c*d*(-3*d*g+2*e*f))*Elli
pticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/
2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2
)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)
/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]

time = 1.35, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {954, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\frac{\sqrt{-a}\sqrt{c}\,ef\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}(ae^2+cd^2)(ef-dg)}-\frac{\sqrt{-a}\sqrt{c}\,dg\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}(ae^2+cd^2)(ef-dg)}-\frac{\sqrt{-a}\sqrt{c}\,e\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}(ae^2+cd^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}}+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-a}g+\sqrt{c}f}}(ae^2g-cd(2ef-3dg))\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x
))) - (Sqrt[-a]*Sqrt[c]*e*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sq
rt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*e*f*Sqrt[(Sqrt[c]*(f + g*x)
```

```
)/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (
Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((c*d^
2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d
*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)])/((c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x
^2]) + ((a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a]
+ e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[
c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g
)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\int \frac{ae^2g-2cd(ef-dg)-2cdegx-ce^2gx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\int \left(-\frac{cdg}{\sqrt{f+gx}\,\sqrt{a+cx^2}} - \frac{\ldots}{\sqrt{f \ldots}}\right.}{2(c\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} + \frac{(cdg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)(ef-dg)} + \ldots$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} + \frac{(ce)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{2(cd^2+ae^2)(ef-dg)} - \frac{(cef)\int \ldots}{2(c\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,dg\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\ldots}{(cd^2+ae\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,e\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\ldots}{(cd^2+ae^2)(ef-d\ldots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)} - \frac{\sqrt{-a}\,\sqrt{c}\,e\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\ldots}{(cd^2+ae^2)(ef-d\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.55, size = 1349, normalized size = 1.81

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*((-2*e^2*(a + c*x^2))/(d + e*x) + (2*(-(c*e^2*f^3*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*
e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*c
*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*
(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g
*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)
/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
 + (Sqrt[c]*d - I*Sqrt[a]*e)*g*(Sqrt[a]*e*g + I*Sqrt[c]*(e*f - 2*d*g))*Sqrt
[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g
*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*
g)] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[
c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)] + (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sq
rt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(
Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)] + I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*S
qrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[
(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
 I*Sqrt[a]*g)]))/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-(e*f) + d*g)*(f + g*
x))))/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5737 vs. $2(627) = 1254$.
time = 0.10, size = 5738, normalized size = 7.69

method	result

elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)(ex+d)}-\frac{cdg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}}{\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d\right.}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+cx^2}\,(d+ex)^2\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

3.651 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=1257

$$-\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}+\frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\,\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}+\frac{3\sqrt{-a}\,\sqrt{c}\,e(ae^2g-c}{}$$

```
[Out] -1/2*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)^2+3
/4*e^2*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*
d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/4*e*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticE(1/
2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)
))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(a*e^2+c*d^2)^
2/(-d*g+e*f)^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(
1/2)+1/2*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a
*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x
+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(g*x+f
)^(1/2)/(c*x^2+a)^(1/2)-3/4*e*f*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*
(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))
^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)
+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/
2)+3/4*d*g*(a*e^2*g-c*d*(-3*d*g+2*e*f))*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/
2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^
(1/2)*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a
*e^2+c*d^2)^2/(-d*g+e*f)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)+c*(-3*d*g+e*f)*Ell
ipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1
/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/
2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f
)/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-3/4*(a*e^2*g-c*d*(
-3*d*g+2*e*f))^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/
(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1
/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(a*
e^2+c*d^2)^2/(-d*g+e*f)^2/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^
(1/2)
```

Rubi [A]
time = 2.87, antiderivative size = 1257, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$,

Rules used = {954, 6874, 733, 430, 858, 435, 947, 174, 552, 551}

$$\frac{3\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{cx^2+a}\,e^2}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}-\frac{\sqrt{f+gx}\sqrt{cx^2+a}\,e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}+\frac{3\sqrt{-a}\sqrt{c}\left(ae^2g-cd(2ef-3dg)\right)\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)e}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}-\frac{3\sqrt{-a}\sqrt{c}f\left(ae^2g-cd(2ef-3dg)\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)e}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{3\sqrt{-a}\sqrt{c}dg\left(ae^2g-cd(2ef-3dg)\right)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{2\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}-\frac{3\left(ae^2g-cd(2ef-3dg)\right)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{4\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)\left(cd^2+ae^2\right)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d +
e*x)^2) + (3*e^2*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x
^2])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*(d + e*x)) + (3*Sqrt[-a]*Sqrt[c]*e*
(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE
[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]
*f - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sq
rt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]
*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[
Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g
)])/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*Sqrt
[-a]*Sqrt[c]*e*f*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(
Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2
 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt
[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
 + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/S
qrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)
^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (c*(e*f - 3*d*g)*Sqrt[(Sq
rt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(
2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt
[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)
*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (3*(a*e^2*g -
 c*d*(2*e*f - 3*d*g))^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*
Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqr
t[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*
g)])/(4*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f +
 g*x]*Sqrt[a + c*x^2])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 954

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Dist[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2
]))*Simp[2*d*(c*e*f - c*d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*e*(c*d*g*(m +
1) - c*e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*
m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \frac{3ae^2g-4cd(ef-dg)+2ce(ef-2dg)x+ce^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+cx^2}}}{4\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \left(\frac{cg}{\sqrt{f+gx}\,\sqrt{a+cx^2}} + \frac{}{(d+e}\right.}{}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx}{4\left(cd^2+ae^2\right)(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f}}{4\left(cd^2+ae^2\right)^2(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f}}{4\left(cd^2+ae^2\right)^2(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f}}{4\left(cd^2+ae^2\right)^2(ef-dg)}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f}}{4\left(cd^2+ae^2\right)^2(ef-dg)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.95, size = 2491, normalized size = 1.98

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (-15*c^2*d^2*e^2*f^3 - 3*a*c*e^4*f^3 + (6*c^2*d*e^3*f^4)/g + 9*c^2*d^3*e*f^
2*g + 9*a*c*d*e^3*f^2*g - 15*a*c*d^2*e^2*f*g^2 - 3*a^2*e^4*f*g^2 + 9*a*c*d^
3*e*g^3 + 3*a^2*d*e^3*g^3 + 30*c^2*d^2*e^2*f^2*(f + g*x) + 6*a*c*e^4*f^2*(f
 + g*x) - (12*c^2*d*e^3*f^3*(f + g*x))/g - 18*c^2*d^3*e*f*g*(f + g*x) - 6*a
*c*d*e^3*f*g*(f + g*x) - 15*c^2*d^2*e^2*f*(f + g*x)^2 - 3*a*c*e^4*f*(f + g*
x)^2 + (6*c^2*d*e^3*f^2*(f + g*x)^2)/g + 9*c^2*d^3*e*g*(f + g*x)^2 + 3*a*c*
d*e^3*g*(f + g*x)^2 - (e^2*(e*f - d*g)*(f + g*x)*(a + c*x^2)*(2*(c*d^2 + a*
e^2)*(e*f - d*g) - 3*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*(d + e*x)))/(d + e*x)
^2 + (3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*(a*e^2*g + c*
d*(-2*e*f + 3*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((
I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)
/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + ((Sqrt[
c]*d - I*Sqrt[a]*e)*(3*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2*g*(e*f - 2*d*g
) - Sqrt[a]*c*e*(2*e^2*f^2 - 6*d*e*f*g + d^2*g^2) - I*c^(3/2)*d*(4*e^2*f^2
- 9*d*e*f*g + 8*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*A
rcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt
[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((8*I
)*c^2*d^2*e^2*f^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*
Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(
e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - ((4*I)*a*c*e^4*f^2*Sqrt[(g*((I*Sqr
t[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqr
t[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[
c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]] - ((20*I)*c^2*d^3*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]
*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticP
i[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
 + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((4*I)*a*c*d*e^3*f*g*S
qrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]
- g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqr
t[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*
```

```
Sqrt[a]*g)/Sqrt[c]] + ((15*I)*c^2*d^4*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x)
)/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/
2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSin
h[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g
)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((6*I)*a*c
*d^2*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[
a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])
/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + ((3*I)*a^2*e^4*g^2*Sqrt[(g*((I*Sqrt[a])
/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*
(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*
g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f
- I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
)/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 20365 vs. $2(1061) = 2122$.

time = 0.12, size = 20366, normalized size = 16.20

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{e^2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{2\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)(ex+d)^2}+\frac{3e^2\left(a\,e^2g+3c\,d^2g-2cdef\right)\sqrt{cg\,x^3+cf\,x^2+agx+}}{4\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)^2(ex+d)}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^3\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)**3*sqrt(f + g*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)*(x*e + d)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

3.652 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=387

$$\frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\,(cf^2+ag^2)\,\sqrt{f+gx}}+\frac{2\sqrt{-a}\,\sqrt{c}\,g\sqrt{f+gx}\,\sqrt{1+\frac{cx^2}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{\cdots}}\right.}{(ef-dg)\,(cf^2+ag^2)\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\,\sqrt{a+cx^2}}$$

```
[Out] 2*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+2*g*EllipticE(
1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/
2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(-d*g+e*f)/(
a*g^2+c*f^2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/
2)-2*e*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/
2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^
2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)/(e
+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {972, 759, 21, 733, 435, 947, 174, 552, 551}

$$\frac{2\sqrt{-a}\,\sqrt{c}\,g\sqrt{\frac{cx^2}{a}+1}\,\sqrt{f+gx}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\,\sqrt{c}\,f-ag}\right)}{\sqrt{a+cx^2}\,(ag^2+cf^2)\,(ef-dg)\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}}-\frac{2e\sqrt{\frac{cx^2}{a}+1}\,\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{-a}\,g+\sqrt{c}\,f}}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}\,d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}\,g}{\sqrt{c}\,f+\sqrt{-a}\,g}\right)}{\sqrt{a+cx^2}\,\sqrt{f+gx}\,\left(\frac{\sqrt{c}\,d}{\sqrt{-a}}+e\right)(ef-dg)}+\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}\,(ag^2+cf^2)\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) + (2*Sq
rt[-a]*Sqrt[c]*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
- (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e
*f - d*g)*(c*f^2 + a*g^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[a + c*x^2]) - (2*e*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[S
qrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a
]*g)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
```

```
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{3/2}\sqrt{a+cx^2}} + \frac{e}{(ef-dg)(d+ex)\sqrt{f+g}}\right.$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{(2cg)\int \frac{-\frac{f}{2}-\frac{gx}{2}}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{(ef-dg)\left(cf^2+ag^2\right)}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{(cg)\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx}{(ef-dg)\left(cf^2+ag^2\right)} - \frac{\left(2e\sqrt{\right.}}{}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} - \frac{\left(2e\sqrt{\frac{\sqrt{c}\,(f+gx)}{\sqrt{c}\,f+\sqrt{-a}\,g}}\sqrt{1+\frac{c}{}}\right.}{}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}} + \frac{2\sqrt{-a}\,\sqrt{c}\,g\sqrt{f+gx}\sqrt{1+\frac{cx}{a}}}{(ef-dg)\left(cf^2+\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.15, size = 468, normalized size = 1.21

$$\frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}\,g}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)\left(\sqrt{c}\,(ef-dg)E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)+\left(i\sqrt{a}\,eg+\sqrt{c}\,(-2ef+dg)\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)+e\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\Pi\left(\frac{\sqrt{c}\,(ef-dg)}{e\left(\sqrt{c}\,f+i\sqrt{a}\,g\right)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}\,f-i\sqrt{a}\,g}{\sqrt{c}\,f+i\sqrt{a}\,g}\right)\right)}{\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)\sqrt{-f-\frac{i\sqrt{a}\,g}{\sqrt{c}}}\,(ef-dg)^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(Sqrt[c]*(e*f - d*g)*EllipticE[I*ArcSi
nh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*
g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (I*Sqrt[a]*e*g + Sqrt[c]*(-2*e*f + d*g))*El
lipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]
*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f - I*Sqrt[a]*g)*
EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(
Sqrt[c]*f + I*Sqrt[a]*g)]))/((Sqrt[c]*f - I*Sqrt[a]*g)*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]*(e*f - d*g)^2*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. $2(324) = 648$.
time = 0.13, size = 2011, normalized size = 5.20

method	result
elliptic	$\frac{\sqrt{(gx+f)\,(c\,x^2+a)}\left(-\frac{2\left(cg\,x^2+ag\right)g}{\left(a\,g^2+c\,f^2\right)(dg-ef)\sqrt{\left(x+\frac{f}{g}\right)\left(cg\,x^2+ag\right)}}+\frac{2gfc\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\ \sqrt{\cdots}}{(dg-\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ellip
ticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g
-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e*f*g^2-(-a*c
)^(1/2)*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(
-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*E
```

```
llipticPi((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/
(d*g-e*f),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^3+(-(g*
x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c
*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticPi((-(
g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(-(
g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e*f^3-(-a*c)^(1/2)*(-(
g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)
+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticPi((
-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(g*(-a*c)^(1/2)-c*f)*e/c/(d*g-e*f),(
-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*e*f^2*g-((-c*x+(-a*c)^
(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-
c*f))^(1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(
1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f)
)^(1/2)*d*g^3+(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*
g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(
1/2)*EllipticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*
f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e*f*g^2-((-c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ellip
ticF((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*
c)^(1/2)+c*f))^(1/2))*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*c^2*d*f^2*g+(
-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/
2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF(
(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1
/2)+c*f))^(1/2))*c^2*e*f^3+((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/
2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1
/2))*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*d*g^3-((-c*x+(-a*c)^(1/2))
*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^
(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c
*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2
)*e*f*g^2+((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(
1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-(g*x+f)*c/(g*(-a*c)^(1/2)-c
*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-(g*x+f)*c/
(g*(-a*c)^(1/2)-c*f))^(1/2)*c^2*d*f^2*g-((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1
/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE
((-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(
1/2)+c*f))^(1/2))*(-(g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*c^2*e*f^3+c^2*d*g
^3*x^2-c^2*e*f*g^2*x^2+a*c*d*g^3-a*c*e*f*g^2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)
/c/(a*g^2+c*f^2)/(d*g-e*f)^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(g*x + f)^(3/2)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,(d + ex)\,(f + gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(g*x + f)^(3/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{c\,x^2+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.653 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=818

$$\frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\,(cf^2+ag^2)\,(f+gx)^{3/2}}+\frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\,(cf^2+ag^2)^2\,\sqrt{f+gx}}+\frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\,(cf^2+ag^2)\,\sqrt{f+gx}}$$

```
[Out] 2/3*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(3/2)+8/3*c*f*g^2*
(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)^2/(g*x+f)^(1/2)+2*e*g^2*(c*x^2+a)^
(1/2)/(-d*g+e*f)^2/(a*g^2+c*f^2)/(g*x+f)^(1/2)+8/3*c^(3/2)*f*g*EllipticE(1/
2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)
))^(1/2))*(-a)^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^
2)^2/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)+2*e*g
*EllipticE(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),(-2*a*g/(-a*g+f*(-a)^
(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(g*x+f)^(1/2)*(c*x^2/a+1)^(1/2)/(
-d*g+e*f)^2/(a*g^2+c*f^2)/(c*x^2+a)^(1/2)/((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*
c^(1/2)))^(1/2)-2/3*g*EllipticF(1/2*(1-x*c^(1/2)/(-a)^(1/2))^(1/2)*2^(1/2),
(-2*a*g/(-a*g+f*(-a)^(1/2)*c^(1/2)))^(1/2))*(-a)^(1/2)*c^(1/2)*(c*x^2/a+1)^
(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)/(a*g^2+c*
f^2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)-2*e^2*EllipticPi(1/2*(1-x*c^(1/2)/(-a)^(
1/2))^(1/2)*2^(1/2),2*e/(e+d*c^(1/2)/(-a)^(1/2)),2^(1/2)*(g*(-a)^(1/2)/(g*(
-a)^(1/2)+f*c^(1/2)))^(1/2))*(c*x^2/a+1)^(1/2)*((g*x+f)*c^(1/2)/(g*(-a)^(1/
2)+f*c^(1/2)))^(1/2)/(-d*g+e*f)^2/(e+d*c^(1/2)/(-a)^(1/2))/(g*x+f)^(1/2)/(c
*x^2+a)^(1/2)
```

Rubi [A]

time = 0.67, antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {972, 759, 849, 858, 733, 435, 430, 21, 947, 174, 552, 551}

$$-\frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,\Pi\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}\right)e^2}{\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{2\sqrt{-a}\sqrt{c}g\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)e}{(ef-dg)^2(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}+\frac{2g^2\sqrt{cx^2+a}\,e}{(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}}+\frac{8\sqrt{-a}c^{3/2}fg\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)(cf^2+ag^2)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{cx^2+a}}-\frac{2\sqrt{-a}\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\sqrt{\frac{cx^2}{a}+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3(ef-dg)(cf^2+ag^2)\sqrt{f+gx}\sqrt{cx^2+a}}+\frac{8cfg^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)^2\sqrt{f+gx}}+\frac{2g^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(3/2)) + (
8*c*f*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])
+ (2*e*g^2*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x]) +
```

```
 (8*Sqrt[-a]*c^(3/2)*f*g*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*
g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f +
 Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqrt[c]*e*g*Sqrt[f + g*x]*Sqrt
[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (
-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[(S
qrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*
Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^
2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(S
qrt[-a]*Sqrt[c]*f - a*g)])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]*Sqr
t[a + c*x^2]) - (2*e^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*S
qrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt
[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g
)])/(((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]
)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{5/2}\sqrt{a+cx^2}} - \frac{eg}{(ef-dg)^2(f+gx)^{3/2}\sqrt{a}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx}{(ef-dg)^2} -$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{2eg^2\sqrt{a+cx^2}}{(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{f+}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f-}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f-}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f-}}$$

$$= \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}} + \frac{8cfg^2\sqrt{a+cx^2}}{3(ef-dg)\left(cf^2+ag^2\right)^2\sqrt{f-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 25.90, size = 1917, normalized size = 2.34

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
[Out] (2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(a + c*x^2)*(a*g^2*(4*
e*f - d*g + 3*e*g*x) + c*f*(-(d*g*(5*f + 4*g*x)) + e*f*(8*f + 7*g*x))) - (f
 + g*x)*(7*c^2*e^2*f^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 11*c^2*d*e*f^4*g*
Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*c^2*d^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]] + 10*a*c*e^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*a*c*d
*e*f^2*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*a*c*d^2*f*g^4*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]] + 3*a^2*e^2*f*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*
a^2*d*e*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*c^2*e^2*f^4*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 22*c^2*d*e*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]*(f + g*x) - 8*c^2*d^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
g*x) - 6*a*c*e^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 6*a*c
*d*e*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 7*c^2*e^2*f^3*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 11*c^2*d*e*f^2*g*Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]*(f + g*x)^2 + 4*c^2*d^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x)^2 + 3*a*c*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*
x)^2 - 3*a*c*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]
*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(3*a*e*g^2 + c*f*(7*e*f - 4*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqr
t[a]*g)] + (Sqrt[c]*f + I*Sqrt[a]*g)*(3*a^(3/2)*e^2*g^3 + (3*I)*a*Sqrt[c]*e
*g^2*(2*e*f - d*g) + Sqrt[a]*c*g*(2*e^2*f^2 + 2*d*e*f*g - d^2*g^2) + (3*I)*
c^(3/2)*f*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] +
 x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]],
(Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3*I)*c^2*e^2*f^4*Sq
rt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt
[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (6*I)*a*c*e^2
*f^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g
))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3
*I)*a^2*e^2*g^4*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sq
rt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*
f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/
```

```
Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)])))/(3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)^3*(c*f^2 + a*g^2)^2*(
f + g*x)^(3/2)*Sqrt[a + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9414 vs. $2(683) = 1366$.
time = 0.13, size = 9415, normalized size = 11.51

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{2\sqrt{cg\,x^3+cf\,x^2+agx+fa}}{3\left(a\,g^2+c\,f^2\right)(dg-ef)\left(x+\frac{f}{g}\right)^2}+\frac{2\left(cg\,x^2+ag\right)g\left(3ae\,g^2-4cdfg+7ce\,f^2\right)}{3\left(a\,g^2+c\,f^2\right)^2(dg-ef)^2\sqrt{\left(x+\frac{f}{g}\right)(cg\,x^2+ag)}} \cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(g*x + f)^(5/2)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,(d + ex)\,(f + gx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + a)*(g*x + f)^(5/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f + g\,x)^{5/2}\,\sqrt{c\,x^2 + a}\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
[Out] int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

3.654 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2\sqrt{\frac{\sqrt{-c}\,(f+gx)}{\sqrt{-c}\,f+g}}\,\Pi\left(\frac{2e}{\sqrt{-c}\,d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}\,x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}\,f+g}\right)}{\left(\sqrt{-c}\,d+e\right)\sqrt{f+gx}}$$

```
[Out] -2*EllipticPi(1/2*(1-x*(-c)^(1/2))^(1/2)*2^(1/2),2*e/(e+d*(-c)^(1/2)),2^(1/
2)*(g/(g+f*(-c)^(1/2)))^(1/2))*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))^(1/2)/
(e+d*(-c)^(1/2))/(g*x+f)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {946, 174, 552, 551}

$$-\frac{2\sqrt{\frac{\sqrt{-c}\,(f+gx)}{\sqrt{-c}\,f+g}}\,\Pi\left(\frac{2e}{\sqrt{-c}\,d+e};\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c}\,x}}{\sqrt{2}}\right)|\frac{2g}{\sqrt{-c}\,f+g}\right)}{\left(\sqrt{-c}\,d+e\right)\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

```
[Out] (-2*Sqrt[(Sqrt[-c]*(f + g*x))/(Sqrt[-c]*f + g)]*EllipticPi[(2*e)/(Sqrt[-c]*
d + e), ArcSin[Sqrt[1 - Sqrt[-c]*x]/Sqrt[2]], (2*g)/(Sqrt[-c]*f + g)])/((Sq
rt[-c]*d + e)*Sqrt[f + g*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{1-\sqrt{-c}\,x}\,\sqrt{1+\sqrt{-c}\,x}\,(d+ex)\sqrt{f+gx}}\,dx$$

$$= -\left(2\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,\left(\sqrt{-c}\,d+e-ex^2\right)\sqrt{f+\frac{g}{\sqrt{-c}}-\frac{gx^2}{\sqrt{-c}}}}\right.\right.$$

$$= -\frac{\left(2\sqrt{1+\frac{g\left(-1+\sqrt{-c}\,x\right)}{\sqrt{-c}\,f+g}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\,\left(\sqrt{-c}\,d+e-ex^2\right)\sqrt{}}\right.}{\sqrt{f+gx}}$$

$$= -\frac{2\sqrt{1-\frac{g\left(1-\sqrt{-c}\,x\right)}{\sqrt{-c}\,f+g}}\,\Pi\left(\frac{2e}{\sqrt{-c}\,d+e};\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}\,x}}{\sqrt{2}}\right)|\right.}{\left(\sqrt{-c}\,d+e\right)\sqrt{f+gx}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 20.65, size = 261, normalized size = 2.37

$$-\frac{2i\sqrt{\frac{g\left(\frac{i}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{ig}{\sqrt{c}}-gx}{f+gx}}\,(f+gx)\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}\,f-ig}{\sqrt{c}\,f+ig}\right)-\Pi\left(\frac{\sqrt{c}\,(ef-dg)}{e\left(\sqrt{c}\,f+ig\right)};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right)|\frac{\sqrt{c}\,f-ig}{\sqrt{c}\,f+ig}\right)\right)}{\sqrt{-f-\frac{ig}{\sqrt{c}}}\,(ef-dg)\sqrt{1+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

```
[Out] ((-2*I)*Sqrt[(g*(I/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*g)/Sqrt[c] - g*x)/(f
 + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)] - EllipticPi[(Sqrt[c]*(e*f - d*
g))/(e*(Sqrt[c]*f + I*g)), I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f + g*x]
], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)]))/(Sqrt[-f - (I*g)/Sqrt[c]]*(e*f -
d*g)*Sqrt[1 + c*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(95) = 190$.
time = 0.13, size = 215, normalized size = 1.95

method	result
default	$\frac{2\left(g+f\sqrt{-c}\right)\text{EllipticPi}\left(\sqrt{\frac{(gx+f)\sqrt{-c}}{g+f\sqrt{-c}}},-\frac{\left(g+f\sqrt{-c}\right)e}{\sqrt{-c}\,(dg-ef)},\sqrt{\frac{g+f\sqrt{-c}}{f\sqrt{-c}-g}}\right)\sqrt{-\frac{\left(x\sqrt{-c}-1\right)g}{g+f\sqrt{-c}}}\sqrt{-\frac{\left(x\sqrt{-c}\right.}{f\sqrt{-c}}}}{\sqrt{-c}\,(dg-ef)(cg\,x^3+cf\,x^2+gx+f)}$
elliptic	$\frac{2\sqrt{(gx+f)\left(c\,x^2+1\right)}\left(\frac{f}{g}+\frac{1}{\sqrt{-c}}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\sqrt{\frac{x+\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\sqrt{\frac{x-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}}\text{EllipticPi}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^2+1}\,e\sqrt{cg\,x^3+cf\,x^2+gx+f}\,\left(-\frac{f}{g}+\frac{d}{e}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(g+f*(-c)^(1/2))/(-c)^(1/2)*EllipticPi(((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2
)))^(1/2),-(g+f*(-c)^(1/2))*e/(-c)^(1/2)/(d*g-e*f),((g+f*(-c)^(1/2))/(f*(-c
)^(1/2)-g))^(1/2))*(-(x*(-c)^(1/2)-1)*g/(g+f*(-c)^(1/2)))^(1/2)*(-(x*(-c)^(
1/2)+1)*g/(f*(-c)^(1/2)-g))^(1/2)*((g*x+f)*(-c)^(1/2)/(g+f*(-c)^(1/2)))^(1/
2)*(c*x^2+1)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+c*f*x^2+g*x+f)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + 1)*sqrt(g*x + f)*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(c*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + 1)*sqrt(g*x + f)*(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+1}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)), x)
```

3.655 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=454

$$-\frac{\sqrt[4]{cf^2+ag^2}\,(d+ex)\sqrt{\frac{(ef-dg)^2\,(a+cx^2)}{(cf^2+ag^2)\,(d+ex)^2}}\left(1+\frac{\sqrt{cd^2+ae^2}\,(f+gx)}{\sqrt{cf^2+ag^2}\,(d+ex)}\right)\sqrt{\frac{1-\frac{2(cdf+aeg)(f+gx)}{(cf^2+ag^2)(d+ex)}+\frac{(cd^2+ae^2)}{(cf^2+ag^2}}{\left(1+\frac{\sqrt{cd^2+ae^2}\,(f+gx)}{\sqrt{cf^2+ag^2}\,(d+ex)}\right.}}}{\sqrt[4]{cd^2+ae^2}\,(ef-dg)\sqrt{a+cx^2}\,\sqrt{1-\frac{2(cdf+aeg)(f}{(cf^2+ag^2)\,(d}}}$$

[Out] -(a*g^2+c*f^2)^(1/4)*(e*x+d)*(cos(2*arctan((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2)))^2)^(1/2)/cos(2*arctan((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2)))*EllipticF(sin(2*arctan((a*e^2+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4)/(e*x+d)^(1/2))),1/2*(2+2*(a*e*g+c*d*f)/(a*e^2+c*d^2)^(1/2)/(a*g^2+c*f^2)^(1/2))^(1/2))*(1+(g*x+f)*(a*e^2+c*d^2)^(1/2)/(e*x+d)/(a*g^2+c*f^2)^(1/2))*((-d*g+e*f)^2*(c*x^2+a)/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)*((1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)/(e*x+d)+(a*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)/(1+(g*x+f)*(a*e^2+c*d^2)^(1/2)/(e*x+d)/(a*g^2+c*f^2)^(1/2))^2)^(1/2)/(a*e^2+c*d^2)^(1/4)/(-d*g+e*f)/(c*x^2+a)^(1/2)/(1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)/(e*x+d)+(a*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)

Rubi [A]
time = 0.41, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {950, 1117}

$$-\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{cd^2+ae^2}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}\sqrt{d+ex}}\right)|\frac{1}{2}\left(\frac{cdf+aeg}{\sqrt{cd^2+ae^2}\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}\,(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]

[Out] -(((c*f^2 + a*g^2)^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/((c*f^2 + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 + a*g^2]*(d + e*x)))*Sqrt[(1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)*(d + e*x)) + ((c*d^2 + a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2))/(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 + a*g^2]*(d + e*x)))^2]*EllipticF[2*ArcTan[((c*d^2 + a*e^2)^(1/4)*Sqrt[f + g*x])/((c*f^2 + a*g^2)^(1/4)*Sqrt[d + e*x])], (1 + (c*d*f + a*e*g)/(Sqrt[c*d^2 + a*e^2]*Sqrt[c*f^2 + a*g^2]))/2])/((c*d^2 + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + c*x^2]*Sqrt[1 - (2

```
*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)*(d + e*x)) + ((c*d^2 + a*e^2)*
(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2)]))
```

Rule 950

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*
(x_)^2]), x_Symbol] :> Dist[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*((a + c*x^2)/(
(c*f^2 + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqrt[a + c*x^2])), Subst[Int[1/
Sqrt[1 - (2*c*d*f + 2*a*e*g)*(x^2/(c*f^2 + a*g^2)) + (c*d^2 + a*e^2)*(x^4/(
c*f^2 + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,\sqrt{a+cx^2}}\,dx = -\frac{\left(2(d+ex)\sqrt{\frac{(ef-dg)^2\,(a+cx^2)}{(cf^2+ag^2)\,(d+ex)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-\frac{(2cdf+2a}{cf^2+a}}}\right.}{(ef-dg)\sqrt{a+cx^2}}$$

$$= -\frac{\sqrt[4]{cf^2+ag^2}\,(d+ex)\sqrt{\frac{(ef-dg)^2\,(a+cx^2)}{(cf^2+ag^2)\,(d+ex)^2}}\left(1+\frac{\sqrt{cd^2+ae^2}\,(f}{\sqrt{cf^2+ag^2}\,(d}\right.}{\sqrt[4]{cd^2+ae^2}\,(ef}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.34, size = 344, normalized size = 0.76

$$\frac{\sqrt{2}\left(i\sqrt{a}+\sqrt{c}\,x\right)\sqrt{d+ex}\sqrt{\frac{d-\frac{i\sqrt{a}\,e}{\sqrt{c}}+\frac{i\sqrt{c}\,dx}{\sqrt{a}}+ex}{d+ex}}\sqrt{\frac{\left(i\sqrt{c}\,d+\sqrt{a}\,e\right)(f+gx)}{\left(i\sqrt{c}\,f+\sqrt{a}\,g\right)(d+ex)}}\,F\left(\sin^{-1}\left(\sqrt{\frac{(ef-dg)\left(i\sqrt{a}+\sqrt{c}\,x\right)}{\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)(d+ex)}}\right)\middle|-\frac{\frac{i\sqrt{c}\,df}{\sqrt{a}}-ef+dg+\frac{i\sqrt{a}\,eg}{\sqrt{c}}}{2ef-2dg}\right)}{\left(\sqrt{c}\,d-i\sqrt{a}\,e\right)\sqrt{\frac{(ef-dg)\left(i\sqrt{a}+\sqrt{c}\,x\right)}{\left(\sqrt{c}\,f-i\sqrt{a}\,g\right)(d+ex)}}\sqrt{f+gx}\,\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[2]*(I*Sqrt[a] + Sqrt[c]*x)*Sqrt[d + e*x]*Sqrt[(d - (I*Sqrt[a]*e)/Sqrt
[c] + (I*Sqrt[c]*d*x)/Sqrt[a] + e*x)/(d + e*x)]*Sqrt[((I*Sqrt[c]*d + Sqrt[a
]*e)*(f + g*x))/((I*Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*EllipticF[ArcSin[Sqr
t[((e*f - d*g)*(I*Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x
))]], -(((I*Sqrt[c]*d*f)/Sqrt[a] - e*f + d*g + (I*Sqrt[a]*e*g)/Sqrt[c])/(2*
e*f - 2*d*g))])/((Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[((e*f - d*g)*(I*Sqrt[a] + S
qrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x]*Sqrt[a + c*
x^2])
```

Maple [A]
time = 0.16, size = 396, normalized size = 0.87

method	result
default	$-\frac{2c\left(-cdg\,x^2+cef\,x^2+2\sqrt{-ac}\,dgx-2\sqrt{-ac}\,efx+adg-aef\right)\operatorname{EllipticF}\left(\sqrt{\frac{\left(g\sqrt{-ac}+cf\right)(ex+d)}{(dg-ef)\left(-cx+\sqrt{-ac}\right)}},\sqrt{2}\,\sqrt{-\frac{\cdots}{\left(\sqrt{\cdots}\right.}}\right.}{\sqrt{-\frac{(ex+d)\left(-cx+\sqrt{-ac}\right)\left(cx+\cdots\right.}{c}}}$
elliptic	$\frac{2\sqrt{(ex+d)\left(c\,x^2+a\right)(gx+f)}\left(-\frac{d}{e}+\frac{f}{g}\right)\sqrt{\frac{\left(-\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\left(x+\frac{d}{e}\right)}{\left(-\frac{f}{g}+\frac{d}{e}\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}}\left(x-\frac{\sqrt{-ac}}{c}\right)^2\sqrt{\frac{\left(\frac{\sqrt{-ac}}{c}+\frac{d}{e}\right)\left(x\cdots\right.}{\left(-\frac{\sqrt{-ac}}{c}+\frac{d}{e}\right)\left(\cdots\right.}}}{\sqrt{ex+d}\,\sqrt{c\,x^2+a}\,\sqrt{gx+f}\left(-\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\left(\frac{\sqrt{-a\cdots}}{c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-c*d*g*x^2+c*e*f*x^2+2*(-a*c)^(1/2)*d*g*x-2*(-a*c)^(1/2)*e*f*x+a*d*g-
a*e*f)*EllipticF(((g*(-a*c)^(1/2)+c*f)*(e*x+d)/(d*g-e*f)/(-c*x+(-a*c)^(1/2)
))^(1/2),2^(1/2)*(-(-a*c)^(1/2)*(d*g-e*f)*c/((-a*c)^(1/2)*e-c*d)/(g*(-a*c)^
(1/2)+c*f))^(1/2))*(-((-a*c)^(1/2)*e+c*d)*(g*x+f)/(d*g-e*f)/(-c*x+(-a*c)^(1
/2)))^(1/2)*(((-a*c)^(1/2)*e+c*d)*(c*x+(-a*c)^(1/2))/((-a*c)^(1/2)*e-c*d)/(
-c*x+(-a*c)^(1/2)))^(1/2)*((g*(-a*c)^(1/2)+c*f)*(e*x+d)/(d*g-e*f)/(-c*x+(-a
*c)^(1/2)))^(1/2)*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-1/c*(e*x+d)
*(-c*x+(-a*c)^(1/2))*(c*x+(-a*c)^(1/2))*(g*x+f))^(1/2)/((-a*c)^(1/2)*e+c*d)
/(g*(-a*c)^(1/2)+c*f)/((e*x+d)*(c*x^2+a)*(g*x+f))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)*sqrt(x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(c*x^2 + a)*sqrt(g*x + f)*sqrt(x*e + d)/(c*d*g*x^3 + c*d*f*x^2
 + a*d*g*x + a*d*f + (c*g*x^4 + c*f*x^3 + a*g*x^2 + a*f*x)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^2}\,\sqrt{d + ex}\,\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)*sqrt(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

3.656 $\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{1-2x^2}\,\sqrt{1-x^2}\,F(\sin^{-1}(x)|\,2)}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{-1+2x^2}}$$

```
[Out] EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)*(-x^2+1)^(1/2)/(-1+x)^(1/2)/(1+x)^(1/
2)/(2*x^2-1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {533, 432, 430}

$$\frac{\sqrt{1-2x^2}\,\sqrt{1-x^2}\,F(\text{ArcSin}(x)|2)}{\sqrt{x-1}\,\sqrt{x+1}\,\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]*Sqrt[
1 + x]*Sqrt[-1 + 2*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{-1+2x^2}}\,dx &= \frac{\sqrt{-1+x^2}\int \frac{1}{\sqrt{-1+x^2}\,\sqrt{-1+2x^2}}\,dx}{\sqrt{-1+x}\,\sqrt{1+x}}\\
&= \frac{\left(\sqrt{1-2x^2}\,\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{1-2x^2}\,\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{-1+2x^2}}\\
&= \frac{\left(\sqrt{1-2x^2}\,\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-2x^2}\,\sqrt{1-x^2}}\,dx}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{-1+2x^2}}\\
&= \frac{\sqrt{1-2x^2}\,\sqrt{1-x^2}\,F(\sin^{-1}(x)|\,2)}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{-1+2x^2}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(52) = 104.
time = 33.71, size = 107, normalized size = 2.06

$$-\frac{2(-1+x)^{3/2}\sqrt{\frac{1+x}{1-x}}\,\sqrt{\frac{1-2x^2}{(-1+x)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2+\sqrt{2}+\frac{1}{-1+x}}}{2^{3/4}}\right)|4\left(-4+3\sqrt{2}\right)\right)}{\sqrt{3+2\sqrt{2}}\,\sqrt{1+x}\,\sqrt{-1+2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] (-2*(-1 + x)^(3/2)*Sqrt[(1 + x)/(1 - x)]*Sqrt[(1 - 2*x^2)/(-1 + x)^2]*Ellip
ticF[ArcSin[Sqrt[2 + Sqrt[2] + (-1 + x)^(-1)]/2^(3/4)], 4*(-4 + 3*Sqrt[2])]
)/(Sqrt[3 + 2*Sqrt[2]]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2])
```

Maple [A]
time = 0.12, size = 58, normalized size = 1.12

method	result	size
default	$\frac{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{2x^2-1}\,\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,\mathrm{EllipticF}\left(x,\sqrt{2}\right)}{2x^4-3x^2+1}$	58
elliptic	$\frac{\sqrt{(2x^2-1)\,(x^2-1)}\,\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,\mathrm{EllipticF}\left(x,\sqrt{2}\right)}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{2x^2-1}\,\sqrt{2x^4-3x^2+1}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+x)^(1/2)*(1+x)^(1/2)*(2*x^2-1)^(1/2)/(2*x^4-3*x^2+1)*(-x^2+1)^(1/2)*(-2
*x^2+1)^(1/2)*EllipticF(x,2^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Fricas [**A**]
time = 0.92, size = 3, normalized size = 0.06

$$\text{ellipticF}(x, 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] ellipticF(x, 2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\,\sqrt{x+1}\,\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/(1+x)**(1/2)/(2*x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x - 1)*sqrt(x + 1)*sqrt(2*x**2 - 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^2-1}\,\sqrt{x-1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
[Out] int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)), x)
```

3.657 $\int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16(cdf-aeg)^2\,(2ae^2g-cd(3ef-dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^4d^4e\sqrt{d+ex}}+\frac{16g(cdf-aeg)^2\sqrt{d+ex}\,\sqrt{ade}}{35c^3d^3e}$$

```
[Out] -16/35*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^4/d^4/e/(e*x+d)^(1/2)+12/35*(-a*e*g+c*d*f)*(g*x+f)^2*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+2/7*(g*x+f)^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+16/35*g*(-a*e*g+c*
d*f)^2*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A]

time = 0.26, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{16\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^2\,(2ae^2g-cd(3ef-dg))}{35c^4d^4e\sqrt{d+ex}}+\frac{16g\sqrt{d+ex}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^2}{35c^3d^3e}+\frac{12(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)}{35c^2d^2\sqrt{d+ex}}+\frac{2(f+gx)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(35*c^4*d^4*e*Sqrt[d + e*x]) + (16*g*(c*d*f - a*e*
g)^2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3
*e) + (12*(c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(35*c^2*d^2*Sqrt[d + e*x]) + (2*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(7*c*d*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7cd\sqrt{d+ex}} + \frac{(6(cde^2f+cd^2eg-e(}{}$$

$$= \frac{12(cdf-aeg)(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^2d^2\sqrt{d+ex}} + \frac{2(f+gx)}{}$$

$$= \frac{16g(cdf-aeg)^2\sqrt{d+ex}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{35c^3d^3e} + \frac{12(cdf}{}$$

$$= -\frac{16(cdf-aeg)^2\,(2ae^2g-cd(3ef-dg))\,\sqrt{ade+(cd^2+ae^2)\,x+c}}{35c^4d^4e\sqrt{d+ex}}$$

Mathematica [A]
time = 0.11, size = 136, normalized size = 0.51

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(-16a^3e^3g^3+8a^2cde^2g^2(7f+gx)-2ac^2d^2eg(35f^2+14fgx+3g^2x^2)+c^3d^3(35f^3+35f^2gx+21fg^2x^2+5g^3x^3))}{35c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(7*f
+ g*x) - 2*a*c^2*d^2*e*g*(35*f^2 + 14*f*g*x + 3*g^2*x^2) + c^3*d^3*(35*f^3
+ 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3)))/(35*c^4*d^4*Sqrt[d + e*x])
```

Maple [A]
time = 0.14, size = 170, normalized size = 0.63

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-5g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e}{35\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-5g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-21c^3d^3f\,g^2x^2-8a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-35c^3d^3f^2gx+16a^3e^3g^3-56a^2cd\,e^2f\,g^2+7}{35c^4d^4\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/35/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-5*c^3*d^3*g^3*x^3+6*a*c^2
*d^2*e*g^3*x^2-21*c^3*d^3*f*g^2*x^2-8*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^
2*x-35*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-56*a^2*c*d*e^2*f*g^2+70*a*c^2*d^2*e*f
^2*g-35*c^3*d^3*f^3)/c^4/d^4
```

Maxima [A]
time = 0.33, size = 219, normalized size = 0.81

$$\frac{2\sqrt{cdx+ae}\,f^3}{cd}+\frac{2\,(c^2d^2x^2-acdxe-2\,a^2e^2)f^2g}{\sqrt{cdx+ae}\,c^2d^2}+\frac{2\,(3\,c^3d^3x^3-ac^2d^2x^2e+4\,a^2cdxe^2+8\,a^3e^3)fg^2}{5\sqrt{cdx+ae}\,c^3d^3}+\frac{2\,(5\,c^4d^4x^4-ac^3d^3x^3e+2\,a^2c^2d^2x^2e^2-8\,a^3cdxe^3-16\,a^4e^4)g^3}{35\sqrt{cdx+ae}\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^3/(c*d) + 2*(c^2*d^2*x^2 - a*c*d*x*e - 2*a^2*e^2)*f^2
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3 - a*c^2*d^2*x^2*e + 4*a
^2*c*d*x*e^2 + 8*a^3*e^3)*f*g^2/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(5*c^4*d
^4*x^4 - a*c^3*d^3*x^3*e + 2*a^2*c^2*d^2*x^2*e^2 - 8*a^3*c*d*x*e^3 - 16*a^4
*e^4)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Fricas [A]
time = 4.85, size = 194, normalized size = 0.72

$$\frac{2\,(5\,c^3d^3g^3x^3+21\,c^3d^3fg^2x^2+35\,c^3d^3f^2gx+35\,c^3d^3f^3-16\,a^3g^3e^3+8\,(a^2cdg^3x+7\,a^2cdfg^2)e^2-2\,(3\,ac^2d^2g^3x^2+14\,ac^2d^2fg^2x+35\,ac^2d^2f^2g)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{35\,(c^4d^4xe+c^4d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g^3*x^3 + 21*c^3*d^3*f*g^2*x^2 + 35*c^3*d^3*f^2*g*x + 35*c^
3*d^3*f^3 - 16*a^3*g^3*e^3 + 8*(a^2*c*d*g^3*x + 7*a^2*c*d*f*g^2)*e^2 - 2*(3
```

```
*a*c^2*d^2*g^3*x^2 + 14*a*c^2*d^2*f*g^2*x + 35*a*c^2*d^2*f^2*g)*e)*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^4*d^4*x*e + c^4*d^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,(f+gx)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(252) = 504.
time = 1.36, size = 605, normalized size = 2.25

$$\frac{2\,(c^3d^3f^3-3\,ac^2d^2f^2ge+3\,a^2cdfg^2e^2-a^3g^3e^3)\sqrt{(xe+d)cde-cd^2e+ae^3}\,e^{(-1)}}{c^4d^4}+\frac{2\left(5\sqrt{-cd^2e+ae^3}\,c^3d^6g^3-21\sqrt{-cd^2e+ae^3}\,c^3d^5fg^2e+35\sqrt{-cd^2e+ae^3}\,c^3d^4f^2ge^2+6\sqrt{-cd^2e+ae^3}\,ac^2d^4g^3e^2-35\sqrt{-cd^2e+ae^3}\,c^3d^3f^3e^3-28\sqrt{-cd^2e+ae^3}\,ac^2d^3fg^2e^3+70\sqrt{-cd^2e+ae^3}\,ac^2d^2f^2ge^4+8\sqrt{-cd^2e+ae^3}\,a^2cd^2g^3e^4-56\sqrt{-cd^2e+ae^3}\,a^2cdfg^2e^5+16\sqrt{-cd^2e+ae^3}\,a^3g^3e^6\right)e^{(-4)}}{35\,c^4d^4}+\frac{2\left(35\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^2d^2f^2ge^4-70\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}acdfg^2e^5+21\,((xe+d)cde-cd^2e+ae^3)^{\frac{5}{2}}cdfg^2e^2+35\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}a^2g^3e^6-21\,((xe+d)cde-cd^2e+ae^3)^{\frac{5}{2}}ag^3e^3+5\,((xe+d)cde-cd^2e+ae^3)^{\frac{7}{2}}g^3\right)e^{(-7)}}{35\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2*(c^3*d^3*f^3 - 3*a*c^2*d^2*f^2*g*e + 3*a^2*c*d*f*g^2*e^2 - a^3*g^3*e^3)*s
qrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*e^(-1)/(c^4*d^4) + 2/35*(5*sqrt(-c*d
^2*e + a*e^3)*c^3*d^6*g^3 - 21*sqrt(-c*d^2*e + a*e^3)*c^3*d^5*f*g^2*e + 35*
sqrt(-c*d^2*e + a*e^3)*c^3*d^4*f^2*g*e^2 + 6*sqrt(-c*d^2*e + a*e^3)*a*c^2*d
^4*g^3*e^2 - 35*sqrt(-c*d^2*e + a*e^3)*c^3*d^3*f^3*e^3 - 28*sqrt(-c*d^2*e +
 a*e^3)*a*c^2*d^3*f*g^2*e^3 + 70*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*f^2*g*e^4
 + 8*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*g^3*e^4 - 56*sqrt(-c*d^2*e + a*e^3)*a
^2*c*d*f*g^2*e^5 + 16*sqrt(-c*d^2*e + a*e^3)*a^3*g^3*e^6)*e^(-4)/(c^4*d^4)
+ 2/35*(35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*f^2*g*e^4 - 70
*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c*d*f*g^2*e^5 + 21*((x*e + d)*
c*d*e - c*d^2*e + a*e^3)^(5/2)*c*d*f*g^2*e^2 + 35*((x*e + d)*c*d*e - c*d^2*
e + a*e^3)^(3/2)*a^2*g^3*e^6 - 21*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)
*a*g^3*e^3 + 5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*g^3)*e^(-7)/(c^4*d
^4)
```

Mupad [B]
time = 3.66, size = 218, normalized size = 0.81

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-112\,a^2\,c\,d\,e^2\,f\,g^2+140\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,c^4\,d^4\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{7\,c\,d\,e}+\frac{6\,g^2\,x^2\,\left(2\,a\,e\,g-7\,c\,d\,f\right)\,\sqrt{d+e\,x}}{35\,c^2\,d^2\,e}-\frac{2\,g\,x\,\sqrt{d+e\,x}\,\left(8\,a^2\,e^2\,g^2-28\,a\,c\,d\,e\,f\,g+35\,c^2\,d^2\,f^2\right)}{35\,c^3\,d^3\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e
^3*g^3 - 70*c^3*d^3*f^3 + 140*a*c^2*d^2*e*f^2*g - 112*a^2*c*d*e^2*f*g^2))/(
35*c^4*d^4*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(7*c*d*e) + (6*g^2*x^2*(2*a*e*g
 - 7*c*d*f)*(d + e*x)^(1/2))/(35*c^2*d^2*e) - (2*g*x*(d + e*x)^(1/2)*(8*a^2
*e^2*g^2 + 35*c^2*d^2*f^2 - 28*a*c*d*e*f*g))/(35*c^3*d^3*e)))/(x + d/e)
```

3.658 $\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8(cdf-aeg)\,(2ae^2g-cd(3ef-dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^3d^3e\sqrt{d+ex}}+\frac{8g(cdf-aeg)\sqrt{d+ex}\,\sqrt{ade+(cd}{15c^2d^2e}$$

```
[Out] -8/15*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c^3/d^3/e/(e*x+d)^(1/2)+2/5*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+8/15*g*(-a*e*g+c*d*f)*(e*x+d)^(1/2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
```

Rubi [A]

time = 0.15, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{8\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)\,(2ae^2g-cd(3ef-dg))}{15c^3d^3e\sqrt{d+ex}}+\frac{8g\sqrt{d+ex}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)}{15c^2d^2e}+\frac{2(f+gx)^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) + (8*g*(c*d*f - a*e*g)*S
qrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) +
(2*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*Sqrt[d +
 e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cd\sqrt{d+ex}} + \frac{(4(cde^2f+cd^2eg-e$$

$$= \frac{8g(cdf-aeg)\sqrt{d+ex}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^2d^2e} + \frac{2(f+g$$

$$= -\frac{8(cdf-aeg)\,(2ae^2g-cd(3ef-dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cd}}{15c^3d^3e\sqrt{d+ex}}$$

Mathematica [A]
time = 0.07, size = 89, normalized size = 0.44

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(8a^2e^2g^2-4acdeg(5f+gx)+c^2d^2(15f^2+10fgx+3g^2x^2))}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(5*f + g*x) +
 c^2*d^2*(15*f^2 + 10*f*g*x + 3*g^2*x^2)))/(15*c^3*d^3*Sqrt[d + e*x])
```

Maple [A]
time = 0.14, size = 98, normalized size = 0.49

method	result	size
default	$\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,(3g^2x^2c^2d^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15f^2c^2d^2)}{15\sqrt{ex+d}\;c^3d^3}$	98

gosper	$\frac{2(cdx+ae)\left(3g^2x^2c^2d^2-4acde\,g^2x+10c^2d^2fgx+8a^2e^2g^2-20acdefg+15f^2c^2d^2\right)\sqrt{ex+d}}{15c^3d^3\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/15/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*g^2*x^2-4*a*c*d*e
*g^2*x+10*c^2*d^2*f*g*x+8*a^2*e^2*g^2-20*a*c*d*e*f*g+15*c^2*d^2*f^2)/c^3/d^
3
```

Maxima [A]
time = 0.33, size = 135, normalized size = 0.68

$$\frac{2\sqrt{cdx+ae}\,f^2}{cd}+\frac{4\,(c^2d^2x^2-acdxe-2\,a^2e^2)fg}{3\sqrt{cdx+ae}\,c^2d^2}+\frac{2\,(3\,c^3d^3x^3-ac^2d^2x^2e+4\,a^2cdxe^2+8\,a^3e^3)g^2}{15\sqrt{cdx+ae}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3*(c^2*d^2*x^2 - a*c*d*x*e - 2*a^2*e^2)*f
*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*x^3 - a*c^2*d^2*x^2*e + 4*
a^2*c*d*x*e^2 + 8*a^3*e^3)*g^2/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Fricas [A]
time = 3.34, size = 124, normalized size = 0.62

$$\frac{2\,(3\,c^2d^2g^2x^2+10\,c^2d^2fgx+15\,c^2d^2f^2+8\,a^2g^2e^2-4\,(acdg^2x+5\,acdfg)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{15\,(c^3d^3xe+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g^2*x^2 + 10*c^2*d^2*f*g*x + 15*c^2*d^2*f^2 + 8*a^2*g^2*e^2
 - 4*(a*c*d*g^2*x + 5*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d
)*e)*sqrt(x*e + d)/(c^3*d^3*x*e + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d+ex}\,(f+gx)^2}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

<u>Giac</u> [A]
time = 1.16, size = 344, normalized size = 1.72

$$\frac{2\,(c^2d^2f^2-2\,acdfge+a^2g^2e^2)\sqrt{(xe+d)cde-cd^2e+ae^3}\,e^{(-1)}}{c^3d^3}-\frac{2\left(3\sqrt{-cd^2e+ae^3}\,c^2d^4g^2-10\sqrt{-cd^2e+ae^3}\,c^2d^3fge+15\sqrt{-cd^2e+ae^3}\,c^2d^2f^2e^2+4\sqrt{-cd^2e+ae^3}\,acd^2g^2e^2-20\sqrt{-cd^2e+ae^3}\,acdfge^3+8\sqrt{-cd^2e+ae^3}\,a^2g^2e^4\right)e^{(-3)}}{15\,c^3d^3}+\frac{2\left(10\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}cdfge^2-10\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}ag^2e^3+3\,((xe+d)cde-cd^2e+ae^3)^{\frac{5}{2}}g^2\right)e^{(-5)}}{15\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2*(c^2*d^2*f^2 - 2*a*c*d*f*g*e + a^2*g^2*e^2)*sqrt((x*e + d)*c*d*e - c*d^2*
e + a*e^3)*e^(-1)/(c^3*d^3) - 2/15*(3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4*g^2 -
10*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*f*g*e + 15*sqrt(-c*d^2*e + a*e^3)*c^2*d^2
*f^2*e^2 + 4*sqrt(-c*d^2*e + a*e^3)*a*c*d^2*g^2*e^2 - 20*sqrt(-c*d^2*e + a*
e^3)*a*c*d*f*g*e^3 + 8*sqrt(-c*d^2*e + a*e^3)*a^2*g^2*e^4)*e^(-3)/(c^3*d^3)
 + 2/15*(10*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*f*g*e^2 - 10*((x*
e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*g^2*e^3 + 3*((x*e + d)*c*d*e - c*d^
2*e + a*e^3)^(5/2)*g^2)*e^(-5)/(c^3*d^3)
```

<u>Mupad</u> [B]
time = 3.40, size = 142, normalized size = 0.71

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,a^2\,e^2\,g^2-40\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2\right)}{15\,c^3\,d^3\,e}+\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{5\,c\,d\,e}-\frac{4\,g\,x\,\left(2\,a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{15\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e^
2*g^2 + 30*c^2*d^2*f^2 - 40*a*c*d*e*f*g))/(15*c^3*d^3*e) + (2*g^2*x^2*(d +
e*x)^(1/2))/(5*c*d*e) - (4*g*x*(2*a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*c^2
*d^2*e)))/(x + d/e)
```

3.659 $\int \frac{\sqrt{d+ex}\,(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2(2ae^2g-cd(3ef-dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e\sqrt{d+ex}}+\frac{2g\sqrt{d+ex}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cde}$$

```
[Out] -2/3*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^2/d^2/e/(e*x+d)^(1/2)+2/3*g*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/c/d/e
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {808, 662}

$$\frac{2g\sqrt{d+ex}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3cde}-\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(2ae^2g-cd(3ef-dg))}{3c^2d^2e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(3*c^2*d^2*e*Sqrt[d + e*x]) + (2*g*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(3*c*d*e)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2g\sqrt{d+ex}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cde} + \frac{1}{3}\bigg(3f-\frac{dg}{e}-\frac{2aeg}{cd}$$

$$= -\frac{2(2ae^2g-cd(3ef-dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2e\sqrt{d+ex}} + \frac{2g\sqrt{}}{}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.42

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(-2aeg+cd(3f+gx))}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(3*f + g*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

Maple [A]
time = 0.14, size = 49, normalized size = 0.39

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,(-cdgx+2aeg-3cdf)}{3\sqrt{ex+d}\,c^2d^2}$	49
gosper	$-\frac{2(cdx+ae)(-cdgx+2aeg-3cdf)\sqrt{ex+d}}{3c^2d^2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c*d*g*x+2*a*e*g-3*c*d*f)/c
^2/d^2
```

Maxima [A]
time = 0.31, size = 67, normalized size = 0.54

$$\frac{2\sqrt{cdx+ae}\,f}{cd} + \frac{2\,(c^2d^2x^2-acdxe-2\,a^2e^2)g}{3\sqrt{cdx+ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)*f/(c*d) + 2/3*(c^2*d^2*x^2 - a*c*d*x*e - 2*a^2*e^2)*g/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

Fricas [A]
time = 4.72, size = 74, normalized size = 0.59

$$\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(cdgx+3\,cdf-2\,age)\sqrt{xe+d}}{3\,(c^2d^2xe+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*g*x + 3*c*d*f - 2*a*g*
e)*sqrt(x*e + d)/(c^2*d^2*x*e + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d+ex}\,(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [A]
time = 1.43, size = 160, normalized size = 1.28

$$\frac{2\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}ge^{(-3)}}{3\,c^2d^2}+\frac{2\sqrt{(xe+d)cde-cd^2e+ae^3}\,(cdf-age)e^{(-1)}}{c^2d^2}+\frac{2\left(\sqrt{-cd^2e+ae^3}\,cd^2g-3\sqrt{-cd^2e+ae^3}\,cdfe+2\sqrt{-cd^2e+ae^3}\,age^2\right)e^{(-2)}}{3\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] 2/3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*g*e^(-3)/(c^2*d^2) + 2*sqrt((
x*e + d)*c*d*e - c*d^2*e + a*e^3)*(c*d*f - a*g*e)*e^(-1)/(c^2*d^2) + 2/3*(s
qrt(-c*d^2*e + a*e^3)*c*d^2*g - 3*sqrt(-c*d^2*e + a*e^3)*c*d*f*e + 2*sqrt(-
c*d^2*e + a*e^3)*a*g*e^2)*e^(-2)/(c^2*d^2)
```

Mupad [B]
time = 3.23, size = 88, normalized size = 0.70

$$-\frac{\left(\frac{(4\,a\,e\,g-6\,c\,d\,f)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}-\frac{2\,g\,x\,\sqrt{d+e\,x}}{3\,c\,d\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2),x)
```

```
[Out] -((((4*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*c^2*d^2*e) - (2*g*x*(d + e*x)^(
1/2))/(3*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

3.660 $\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{662\}$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cd\sqrt{d+ex}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.76

$$\frac{2\sqrt{(ae+cdx)(d+ex)}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/(c*d*Sqrt[d + e*x])
```

Maple [A]
time = 0.14, size = 32, normalized size = 0.70

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}}{\sqrt{ex+d}\, cd}$	32
gosper	$\frac{2(cdx+ae)\sqrt{ex+d}}{cd\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/c/d
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.41

$$\frac{2\sqrt{cdx+ae}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2*sqrt(c*d*x + a*e)/(c*d)
```

Fricas [A]
time = 2.56, size = 51, normalized size = 1.11

$$\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{cdxe+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d*x*e + c*d^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [A]
time = 1.60, size = 62, normalized size = 1.35

$$\frac{2\sqrt{(xe+d)cde-cd^2e+ae^3}\,e^{(-1)}}{cd}-\frac{2\sqrt{-cd^2e+ae^3}\,e^{(-1)}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] 2*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*e^(-1)/(c*d) - 2*sqrt(-c*d^2*e +
a*e^3)*e^(-1)/(c*d)
```

Mupad [B]
time = 3.20, size = 54, normalized size = 1.17

$$\frac{2\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c\,d\,e\,\left(x+\frac{d}{e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

```
[Out] (2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c*d*e*(x
 + d/e))
```

3.661 $\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=80

$$\frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

```
[Out] 2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/
2)/(e*x+d)^(1/2))/g^(1/2)/(-a*e*g+c*d*f)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {888, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x
]
```

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f
 - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*Sqrt[c*d*f - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = (2e^2)\,\mathrm{Subst}\left(\int \frac{1}{-e\,(cd^2+ae^2)\,g+cde(ef+dg)+e^2gx^2}\,dx\right.$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{cdf-aeg}\,\sqrt{d+ex}}\right)}{\sqrt{g}\,\sqrt{cdf-aeg}}$$

Mathematica [A]
time = 0.07, size = 93, normalized size = 1.16

$$\frac{2\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\,\sqrt{cdf-aeg}\,\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[
c*d*f - a*e*g]])/(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

Maple [A]
time = 0.15, size = 77, normalized size = 0.96

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)\,(ex+d)}\;\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)\,g}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)), x)
```

Fricas [A]
time = 2.71, size = 259, normalized size = 3.24

$$\left[-\frac{\sqrt{-cdfg+ag^2e}\,\log\left(-\frac{cd^2gx-cd^2f+2\,agxe^2+(cdgx^2-cdfx+2\,adg)e-2\,\sqrt{-cdfg+ag^2e}\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{dgx+df+(gx^2+fx)e}\right)}{cdfg-ag^2e},-\frac{2\,\arctan\left(\frac{\sqrt{cdfg-ag^2e}\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{cd^2gx+agxe^2+(cdgx^2+adg)e}\right)}{\sqrt{cdfg-ag^2e}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [-sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g
*x^2 - c*d*f*x + 2*a*d*g)*e - 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^2 + f*x)*e))/(
c*d*f*g - a*g^2*e), -2*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^
2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 +
a*d*g)*e))/sqrt(c*d*f*g - a*g^2*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,\,(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

Giac [A]
time = 1.49, size = 120, normalized size = 1.50

$$\frac{2\,\arctan\left(\frac{\sqrt{(xe+d)cde-cd^2e+ae^3}\,ge^{(-1)}}{\sqrt{cdfg-ag^2e}}\right)}{\sqrt{cdfg-ag^2e}}-\frac{2\,\arctan\left(\frac{\sqrt{-cd^2e+ae^3}\,ge^{(-1)}}{\sqrt{cdfg-ag^2e}}\right)}{\sqrt{cdfg-ag^2e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] 2*arctan(sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*g*e^(-1)/sqrt(c*d*f*g - a*
g^2*e))/sqrt(c*d*f*g - a*g^2*e) - 2*arctan(sqrt(-c*d^2*e + a*e^3)*g*e^(-1)/
sqrt(c*d*f*g - a*g^2*e))/sqrt(c*d*f*g - a*g^2*e)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{\left(f+g\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)), x)
```

3.662 $\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{\sqrt{g}\,(cdf-aeg)^{3/2}}$$

```
[Out] c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(
1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(3/2)/g^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {886, 888, 211}

$$\frac{cd\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}\,(cdf-aeg)^{3/2}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/((c*d*f - a*e*g)*Sqrt[d + e*x]*
(f + g*x)) + (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(Sqrt[g]*(c*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
```

```
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{(cd)\int \frac{\sqrt{\cdots}}{(f+gx)\sqrt{ade+(c\cdots}}}{2(cdf\cdots}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{(cde^2)\,\text{Subst}\left(\int \frac{\cdots}{-e(cd^2+\cdots}\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ade+\cdots}}{\sqrt{cdf\cdots}}\right.}{\sqrt{g}\,(cdf\cdots}$$

Mathematica [A]
time = 0.27, size = 136, normalized size = 0.97

$$\frac{\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{cdf-aeg}\,(ae+cdx)+cd\sqrt{ae+cdx}\,(f+gx)\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{\sqrt{g}\,(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*Sqrt[a*e +
c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/
(Sqrt[g]*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))
```

Maple [A]
time = 0.15, size = 158, normalized size = 1.13

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdgx+\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdf-\sqrt{cdx+ae}\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,(aeg-cdf)(gx+f)\sqrt{(aeg-cdf)g}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g*x+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(131) = 262$.
time = 3.94, size = 727, normalized size = 5.19

$$\left[\frac{(cd^2gx+cd^2f+(cdgx^2+cdfx)e)\sqrt{-cdfg+ag^2e}\log\left(-\frac{cd^2gx-cd^2f+2agxe^2+(cdgx^2-cdfx+2adg)e+2\sqrt{-cdfg+ag^2e}\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{dgx+df+(gx^2+fx)e}\right)+2(cdfg-ag^2e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{2(c^2d^3f^2g^2x+c^2d^3f^3g+(a^2g^4x^2+a^2fg^3x)e^3-(2acdfg^3x^2-a^2dfg^3+(2acdf^2g^2-a^2dg^4)x)e^2+(c^2d^2f^2g^2x^2-2acd^2f^2g^2+(c^2d^2f^3g-2acd^2fg^3)x)e)},-\frac{(cd^2gx+cd^2f+(cdgx^2+cdfx)e)\sqrt{cdfg-ag^2e}\arctan\left(\frac{\sqrt{cdfg-ag^2e}\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{cd^2gx+agxe^2+(cdgx^2+adg)e}\right)-(cdfg-ag^2e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{c^2d^3f^2g^2x+c^2d^3f^3g+(a^2g^4x^2+a^2fg^3x)e^3-(2acdfg^3x^2-a^2dfg^3+(2acdf^2g^2-a^2dg^4)x)e^2+(c^2d^2f^2g^2x^2-2acd^2f^2g^2+(c^2d^2f^3g-2acd^2fg^3)x)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*((c*d^2*g*x + c*d^2*f + (c*d*g*x^2 + c*d*f*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e + 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f^2*g^2*x + c^2*d^3*f^3*g + (a^2*g^4*x^2 + a^2*f*g^3*x)*e^3 - (2*a*c*d*f*g^3*x^2 - a^2*d*f*g^3 + (2*a*c*d*f^2*g^2 - a^2*d*g^4)*x)*e^2 + (c^2*d^2*f^2*g^2*x^2

```
- 2*a*c*d^2*f^2*g^2 + (c^2*d^2*f^3*g - 2*a*c*d^2*f*g^3)*x)*e), -((c*d^2*g*x
 + c*d^2*f + (c*d*g*x^2 + c*d*f*x)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c
*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)
/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (c*d*f*g - a*g^2*e)*sqr
t(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f^2*g^2*x
+ c^2*d^3*f^3*g + (a^2*g^4*x^2 + a^2*f*g^3*x)*e^3 - (2*a*c*d*f*g^3*x^2 - a^
2*d*f*g^3 + (2*a*c*d*f^2*g^2 - a^2*d*g^4)*x)*e^2 + (c^2*d^2*f^2*g^2*x^2 - 2
*a*c*d^2*f^2*g^2 + (c^2*d^2*f^3*g - 2*a*c*d^2*f*g^3)*x)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}\,(f+gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.663 $\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)}+\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{4\sqrt{g}\,(cdf-aeg)^{5/2}}$$

```
[Out] 3/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)/g^(1/2)+1/2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)+3/4*c*d*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2
)
```

Rubi [A]
time = 0.20, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {886, 888, 211}

$$\frac{3c^2d^2\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4\sqrt{g}\,(cdf-aeg)^{5/2}}+\frac{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^2) + (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*
d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{(3cd)\int \frac{\sqrt{}}{(f+gx)^2\sqrt{ade+}}}{4(cd}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+(cd^2+ae^2}}{4(cdf-aeg)^2\sqrt{d+e}}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+(cd^2+ae^2}}{4(cdf-aeg)^2\sqrt{d+e}}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{3cd\sqrt{ade+(cd^2+ae^2}}{4(cdf-aeg)^2\sqrt{d+e}}$$

Mathematica [A]
time = 0.34, size = 163, normalized size = 0.77

$$\frac{\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{cdf-aeg}\,(ae+cdx)(-2aeg+cd(5f+3gx))+3c^2d^2\sqrt{ae+cdx}\,(f+gx)^2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{4\sqrt{g}\,(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x)*(-2*a*e*g + c*d*(
5*f + 3*g*x)) + 3*c^2*d^2*Sqrt[a*e + c*d*x]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqr
```

```
t[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*Sqrt[g]*(c*d*f - a*e*g)^(5/2)*Sqr
t[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

Maple [A]
time = 0.17, size = 275, normalized size = 1.29

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^2x^2+6\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx+\right.}{4\sqrt{ex}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/4/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*g^2*x^2+6*arctanh(g*(c*d*x+a*e)^(1/2)/(
(a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g
-c*d*f)*g)^(1/2))*c^2*d^2*f^2-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c
*d*g*x+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g-5*((a*e*g-c*d*f)*g
)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^2/(g*x+f)^
2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(197) = 394$.
time = 3.65, size = 1325, normalized size = 6.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^3*g^2*x^2 + 2*c^2*d^3*f*g*x + c^2*d^3*f^2 + (c^2*d^2*g^2*x^
3 + 2*c^2*d^2*f*g*x^2 + c^2*d^2*f^2*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*
d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e - 2*sqr
t(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e
 + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) - 2*(3*c^2*d^2*f*g^2*x + 5*c^2*d^2*f
^2*g + 2*a^2*g^3*e^2 - (3*a*c*d*g^3*x + 7*a*c*d*f*g^2)*e)*sqrt(c*d^2*x + a*
x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^3*d^4*f^3*g^3*x^2 + 2*c^3*d^4*
f^4*g^2*x + c^3*d^4*f^5*g - (a^3*g^6*x^3 + 2*a^3*f*g^5*x^2 + a^3*f^2*g^4*x)
*e^4 + (3*a^2*c*d*f*g^5*x^3 - a^3*d*f^2*g^4 + (6*a^2*c*d*f^2*g^4 - a^3*d*g^
6)*x^2 + (3*a^2*c*d*f^3*g^3 - 2*a^3*d*f*g^5)*x)*e^3 - 3*(a*c^2*d^2*f^2*g^4*
x^3 - a^2*c*d^2*f^3*g^3 + (2*a*c^2*d^2*f^3*g^3 - a^2*c*d^2*f*g^5)*x^2 + (a*
c^2*d^2*f^4*g^2 - 2*a^2*c*d^2*f^2*g^4)*x)*e^2 + (c^3*d^3*f^3*g^3*x^3 - 3*a*
c^2*d^3*f^4*g^2 + (2*c^3*d^3*f^4*g^2 - 3*a*c^2*d^3*f^2*g^4)*x^2 + (c^3*d^3*
f^5*g - 6*a*c^2*d^3*f^3*g^3)*x)*e), -1/4*(3*(c^2*d^3*g^2*x^2 + 2*c^2*d^3*f*
g*x + c^2*d^3*f^2 + (c^2*d^2*g^2*x^3 + 2*c^2*d^2*f*g*x^2 + c^2*d^2*f^2*x)*e
)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2
 + a*d*g)*e)) - (3*c^2*d^2*f*g^2*x + 5*c^2*d^2*f^2*g + 2*a^2*g^3*e^2 - (3*a
*c*d*g^3*x + 7*a*c*d*f*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*
sqrt(x*e + d))/(c^3*d^4*f^3*g^3*x^2 + 2*c^3*d^4*f^4*g^2*x + c^3*d^4*f^5*g -
 (a^3*g^6*x^3 + 2*a^3*f*g^5*x^2 + a^3*f^2*g^4*x)*e^4 + (3*a^2*c*d*f*g^5*x^3
 - a^3*d*f^2*g^4 + (6*a^2*c*d*f^2*g^4 - a^3*d*g^6)*x^2 + (3*a^2*c*d*f^3*g^3
 - 2*a^3*d*f*g^5)*x)*e^3 - 3*(a*c^2*d^2*f^2*g^4*x^3 - a^2*c*d^2*f^3*g^3 + (
2*a*c^2*d^2*f^3*g^3 - a^2*c*d^2*f*g^5)*x^2 + (a*c^2*d^2*f^4*g^2 - 2*a^2*c*d
^2*f^2*g^4)*x)*e^2 + (c^3*d^3*f^3*g^3*x^3 - 3*a*c^2*d^3*f^4*g^2 + (2*c^3*d^
3*f^4*g^2 - 3*a*c^2*d^3*f^2*g^4)*x^2 + (c^3*d^3*f^5*g - 6*a*c^2*d^3*f^3*g^3
)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.664 $\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=280

$$\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^2}+\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8(cdf-aeg)^3\sqrt{d+ex}\,(f+gx)}$$

```
[Out] 5/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)/g^(1/2)+1/3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)+5/12*c*d*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(
1/2)+5/8*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(
g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {886, 888, 211}

$$\frac{5c^3d^3\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8\sqrt{g}\,(cdf-aeg)^{7/2}}+\frac{5c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*(c*d*f - a*e*g)*Sqrt[d + e*x
]*(f + g*x)^3) + (5*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(8*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (
5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqr
t[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*Sqrt[g]*(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
```

```
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{(5cd)\int \frac{}{(f+gx)^3\sqrt{ade+ \cdots}}}{6(c \cdots}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae \cdots}}{12(cdf-aeg)^2\sqrt{d+ \cdots}}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae \cdots}}{12(cdf-aeg)^2\sqrt{d+ \cdots}}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae \cdots}}{12(cdf-aeg)^2\sqrt{d+ \cdots}}$$

$$= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{5cd\sqrt{ade+(cd^2+ae \cdots}}{12(cdf-aeg)^2\sqrt{d+ \cdots}}$$

Mathematica [A]
time = 0.51, size = 191, normalized size = 0.68

$$\frac{c^3d^3\sqrt{d+ex}\left(\frac{(ae+cdx)\left(8a^2e^2g^2-2acdeg(13f+5gx)+c^2d^2\left(33f^2+40fgx+15g^2x^2\right)\right)}{c^3d^3(cdf-aeg)^3(f+gx)^3}+\frac{15\sqrt{ae+cdx}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{g}\,(cdf-aeg)^{7/2}}\right)}{24\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (c^3*d^3*Sqrt[d + e*x]*(((a*e + c*d*x)*(8*a^2*e^2*g^2 - 2*a*c*d*e*g*(13*f +
 5*g*x) + c^2*d^2*(33*f^2 + 40*f*g*x + 15*g^2*x^2)))/(c^3*d^3*(c*d*f - a*e*
g)^3*(f + g*x)^3) + (15*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]])/(Sqrt[g]*(c*d*f - a*e*g)^(7/2))))/(24*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Maple [A]
time = 0.16, size = 440, normalized size = 1.57

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^3d^3g^3x^3+45\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^3d^3f\,g^2x^2+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^3*d^3*f^3-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*g^
2*x^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x-40*((a*e*g
-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x-8*((a*e*g-c*d*f)*g)^(1/2)*
(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*
a*c*d*e*f*g-33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*x+
d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(261) = 522$.

time = 3.32, size = 2119, normalized size = 7.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/48*(15*(c^3*d^4*g^3*x^3 + 3*c^3*d^4*f*g^2*x^2 + 3*c^3*d^4*f^2*g*x + c^3*
d^4*f^3 + (c^3*d^3*g^3*x^4 + 3*c^3*d^3*f*g^2*x^3 + 3*c^3*d^3*f^2*g*x^2 + c^
3*d^3*f^3*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*
x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e + 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt
(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^
2 + f*x)*e)) + 2*(15*c^3*d^3*f*g^3*x^2 + 40*c^3*d^3*f^2*g^2*x + 33*c^3*d^3*
f^3*g - 8*a^3*g^4*e^3 + 2*(5*a^2*c*d*g^4*x + 17*a^2*c*d*f*g^3)*e^2 - (15*a*
c^2*d^2*g^4*x^2 + 50*a*c^2*d^2*f*g^3*x + 59*a*c^2*d^2*f^2*g^2)*e)*sqrt(c*d^
2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^4*d^5*f^4*g^4*x^3 + 3*
c^4*d^5*f^5*g^3*x^2 + 3*c^4*d^5*f^6*g^2*x + c^4*d^5*f^7*g + (a^4*g^8*x^4 +
3*a^4*f*g^7*x^3 + 3*a^4*f^2*g^6*x^2 + a^4*f^3*g^5*x)*e^5 - (4*a^3*c*d*f*g^7
*x^4 - a^4*d*f^3*g^5 + (12*a^3*c*d*f^2*g^6 - a^4*d*g^8)*x^3 + 3*(4*a^3*c*d*
f^3*g^5 - a^4*d*f*g^7)*x^2 + (4*a^3*c*d*f^4*g^4 - 3*a^4*d*f^2*g^6)*x)*e^4 +
 2*(3*a^2*c^2*d^2*f^2*g^6*x^4 - 2*a^3*c*d^2*f^4*g^4 + (9*a^2*c^2*d^2*f^3*g^
5 - 2*a^3*c*d^2*f*g^7)*x^3 + 3*(3*a^2*c^2*d^2*f^4*g^4 - 2*a^3*c*d^2*f^2*g^6
)*x^2 + 3*(a^2*c^2*d^2*f^5*g^3 - 2*a^3*c*d^2*f^3*g^5)*x)*e^3 - 2*(2*a*c^3*d
^3*f^3*g^5*x^4 - 3*a^2*c^2*d^3*f^5*g^3 + 3*(2*a*c^3*d^3*f^4*g^4 - a^2*c^2*d
^3*f^2*g^6)*x^3 + 3*(2*a*c^3*d^3*f^5*g^3 - 3*a^2*c^2*d^3*f^3*g^5)*x^2 + (2*
a*c^3*d^3*f^6*g^2 - 9*a^2*c^2*d^3*f^4*g^4)*x)*e^2 + (c^4*d^4*f^4*g^4*x^4 -
4*a*c^3*d^4*f^6*g^2 + (3*c^4*d^4*f^5*g^3 - 4*a*c^3*d^4*f^3*g^5)*x^3 + 3*(c^
4*d^4*f^6*g^2 - 4*a*c^3*d^4*f^4*g^4)*x^2 + (c^4*d^4*f^7*g - 12*a*c^3*d^4*f^
5*g^3)*x)*e), -1/24*(15*(c^3*d^4*g^3*x^3 + 3*c^3*d^4*f*g^2*x^2 + 3*c^3*d^4*
f^2*g*x + c^3*d^4*f^3 + (c^3*d^3*g^3*x^4 + 3*c^3*d^3*f*g^2*x^3 + 3*c^3*d^3*
f^2*g*x^2 + c^3*d^3*f^3*x)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g -
 a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*
g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (15*c^3*d^3*f*g^3*x^2 + 40*c^3*
d^3*f^2*g^2*x + 33*c^3*d^3*f^3*g - 8*a^3*g^4*e^3 + 2*(5*a^2*c*d*g^4*x + 17*
a^2*c*d*f*g^3)*e^2 - (15*a*c^2*d^2*g^4*x^2 + 50*a*c^2*d^2*f*g^3*x + 59*a*c^
2*d^2*f^2*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)
)/(c^4*d^5*f^4*g^4*x^3 + 3*c^4*d^5*f^5*g^3*x^2 + 3*c^4*d^5*f^6*g^2*x + c^4*
d^5*f^7*g + (a^4*g^8*x^4 + 3*a^4*f*g^7*x^3 + 3*a^4*f^2*g^6*x^2 + a^4*f^3*g^
5*x)*e^5 - (4*a^3*c*d*f*g^7*x^4 - a^4*d*f^3*g^5 + (12*a^3*c*d*f^2*g^6 - a^4
*d*g^8)*x^3 + 3*(4*a^3*c*d*f^3*g^5 - a^4*d*f*g^7)*x^2 + (4*a^3*c*d*f^4*g^4
- 3*a^4*d*f^2*g^6)*x)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^6*x^4 - 2*a^3*c*d^2*f^4*
g^4 + (9*a^2*c^2*d^2*f^3*g^5 - 2*a^3*c*d^2*f*g^7)*x^3 + 3*(3*a^2*c^2*d^2*f^
```

```
4*g^4 - 2*a^3*c*d^2*f^2*g^6)*x^2 + 3*(a^2*c^2*d^2*f^5*g^3 - 2*a^3*c*d^2*f^3
*g^5)*x)*e^3 - 2*(2*a*c^3*d^3*f^3*g^5*x^4 - 3*a^2*c^2*d^3*f^5*g^3 + 3*(2*a*
c^3*d^3*f^4*g^4 - a^2*c^2*d^3*f^2*g^6)*x^3 + 3*(2*a*c^3*d^3*f^5*g^3 - 3*a^2
*c^2*d^3*f^3*g^5)*x^2 + (2*a*c^3*d^3*f^6*g^2 - 9*a^2*c^2*d^3*f^4*g^4)*x)*e^
2 + (c^4*d^4*f^4*g^4*x^4 - 4*a*c^3*d^4*f^6*g^2 + (3*c^4*d^4*f^5*g^3 - 4*a*c
^3*d^4*f^3*g^5)*x^3 + 3*(c^4*d^4*f^6*g^2 - 4*a*c^3*d^4*f^4*g^4)*x^2 + (c^4*
d^4*f^7*g - 12*a*c^3*d^4*f^5*g^3)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{{\left(f+g\,x\right)}^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.665 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}-\frac{16g(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5c^4d^4e\sqrt{d+ex}}$$

```
[Out] -2*(g*x+f)^3*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-16/5
*g*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/c^4/d^4/e/(e*x+d)^(1/2)+12/5*g*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+16/5*g^2*(-a*e*g+c*d*f)*(e*x+d)^(1/
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A]

time = 0.21, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {880, 884, 808, 662}

$$-\frac{16g\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)\left(2ae^2g-cd(3ef-dg)\right)}{5c^4d^4e\sqrt{d+ex}}+\frac{16g^2\sqrt{d+ex}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)}{5c^3d^3e}+\frac{12g(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5c^2d^2\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^3)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (16*g*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^2*(c*d*
f - a*e*g)*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^
3*d^3*e) + (12*g*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
5*c^2*d^2*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
```

```
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{(6g)\int \frac{\sqrt{d+ex}\,(f+g}{\sqrt{ade+(cd^2+ae^2)}}}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{12g(f+gx)^2\sqrt{ade+(cd^2+}}{5c^2d^2\sqrt{d+e}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{16g^2(cdf-aeg)\sqrt{d+ex}\,\sqrt{}}{5c}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^3}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{16g(cdf-aeg)\,(2ae^2g-cd(}{5}$$

Mathematica [A]
time = 0.10, size = 134, normalized size = 0.52

$$\frac{2\sqrt{d+ex}\,(16a^3e^3g^3+8a^2cde^2g^2(-5f+gx)-2ac^2d^2eg(-15f^2+10fgx+g^2x^2)+c^3d^3(-5f^3+15f^2gx+5fg^2x^2+g^3x^3))}{5c^4d^4\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(-5*f + g*x) - 2*a*c^2
*d^2*e*g*(-15*f^2 + 10*f*g*x + g^2*x^2) + c^3*d^3*(-5*f^3 + 15*f^2*g*x + 5*
f*g^2*x^2 + g^3*x^3)))/(5*c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.13, size = 179, normalized size = 0.70

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(g^3x^3c^3d^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-4}{5\sqrt{ex+d}\,(cdx+ae)c^4d^4}$
gosper	$\frac{2(cdx+ae)(g^3x^3c^3d^3-2a\,c^2d^2e\,g^3x^2+5c^3d^3f\,g^2x^2+8a^2cd\,e^2g^3x-20a\,c^2d^2ef\,g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd\,e^2f\,g^2+30a\,c^2d^2}{5c^4d^4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/5/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(c^3*d^3*g^3*x^3-2*a*c^2*d^2*
e*g^3*x^2+5*c^3*d^3*f*g^2*x^2+8*a^2*c*d*e^2*g^3*x-20*a*c^2*d^2*e*f*g^2*x+15
*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-40*a^2*c*d*e^2*f*g^2+30*a*c^2*d^2*e*f^2*g-5
*c^3*d^3*f^3)/(c*d*x+a*e)/c^4/d^4
```

Maxima [A]
time = 0.33, size = 169, normalized size = 0.66

$$-\frac{2\,f^3}{\sqrt{cdx+ae}\,cd}+\frac{6\,(cdx+2\,ae)f^2g}{\sqrt{cdx+ae}\,c^2d^2}+\frac{2\,(c^2d^2x^2-4\,acdxe-8\,a^2e^2)fg^2}{\sqrt{cdx+ae}\,c^3d^3}+\frac{2\,(c^3d^3x^3-2\,ac^2d^2x^2e+8\,a^2cdxe^2+16\,a^3e^3)g^3}{5\,\sqrt{cdx+ae}\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^3/(sqrt(c*d*x + a*e)*c*d) + 6*(c*d*x + 2*a*e)*f^2*g/(sqrt(c*d*x + a*e)
*c^2*d^2) + 2*(c^2*d^2*x^2 - 4*a*c*d*x*e - 8*a^2*e^2)*f*g^2/(sqrt(c*d*x + a
*e)*c^3*d^3) + 2/5*(c^3*d^3*x^3 - 2*a*c^2*d^2*x^2*e + 8*a^2*c*d*x*e^2 + 16*
a^3*e^3)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)
```

Fricas [A]
time = 2.78, size = 216, normalized size = 0.84

$$\frac{2\,(c^3d^3g^3x^3+5\,c^3d^3fg^2x^2+15\,c^3d^3f^2gx-5\,c^3d^3f^3+16\,a^3g^3e^3+8\,(a^2cdg^3x-5\,a^2cdfg^2)e^2-2\,(ac^2d^2g^3x^2+10\,ac^2d^2fg^2x-15\,ac^2d^2f^2g)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{5\,(c^5d^6x+ac^4d^4xe^2+(c^5d^5x^2+ac^4d^5)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/5*(c^3*d^3*g^3*x^3 + 5*c^3*d^3*f*g^2*x^2 + 15*c^3*d^3*f^2*g*x - 5*c^3*d^3
*f^3 + 16*a^3*g^3*e^3 + 8*(a^2*c*d*g^3*x - 5*a^2*c*d*f*g^2)*e^2 - 2*(a*c^2*
d^2*g^3*x^2 + 10*a*c^2*d^2*f*g^2*x - 15*a*c^2*d^2*f^2*g)*e)*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^5*d^6*x + a*c^4*d^4*x*e^2 + (
c^5*d^5*x^2 + a*c^4*d^5)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}(f+gx)^3}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**3/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(241) = 482.
time = 2.36, size = 496, normalized size = 1.93

$$\frac{2\,(c^3d^6g^3-5\,c^3d^5fg^2e+15\,c^3d^4f^2ge^2+2\,ac^2d^4g^3e^2+5\,c^3d^3f^3e^3-20\,ac^2d^3fg^2e^3-30\,ac^2d^2f^2ge^4+8\,a^2cd^2g^3e^4+40\,a^2cdfg^2e^5-16\,a^3g^3e^6)e^{(-2)}}{5\,\sqrt{-cd^2e+ae^3}\,c^4d^4}-\frac{2\,(c^3d^3f^3e-3\,ac^2d^2f^2ge^2+3\,a^2cdfg^2e^3-a^3g^3e^4)}{\sqrt{(xe+d)cde-cd^2e+ae^3}\,c^4d^4}+\frac{2\left(15\,\sqrt{(xe+d)cde-cd^2e+ae^3}\,c^{18}d^{18}f^2ge^{24}-30\,\sqrt{(xe+d)cde-cd^2e+ae^3}\,ac^{17}d^{17}fg^2e^{25}+5\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^{17}d^{17}fg^2e^{22}+15\,\sqrt{(xe+d)cde-cd^2e+ae^3}\,a^2c^{16}d^{16}g^3e^{26}-5\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}ac^{16}d^{16}g^3e^{23}+((xe+d)cde-cd^2e+ae^3)^{\frac{5}{2}}c^{16}d^{16}g^3e^{20}\right)e^{(-25)}}{5\,c^{20}d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] 2/5*(c^3*d^6*g^3 - 5*c^3*d^5*f*g^2*e + 15*c^3*d^4*f^2*g*e^2 + 2*a*c^2*d^4*g
^3*e^2 + 5*c^3*d^3*f^3*e^3 - 20*a*c^2*d^3*f*g^2*e^3 - 30*a*c^2*d^2*f^2*g*e^
4 + 8*a^2*c*d^2*g^3*e^4 + 40*a^2*c*d*f*g^2*e^5 - 16*a^3*g^3*e^6)*e^(-2)/(sq
rt(-c*d^2*e + a*e^3)*c^4*d^4) - 2*(c^3*d^3*f^3*e - 3*a*c^2*d^2*f^2*g*e^2 +
3*a^2*c*d*f*g^2*e^3 - a^3*g^3*e^4)/(sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)
*c^4*d^4) + 2/5*(15*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*c^18*d^18*f^2*g
*e^24 - 30*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*a*c^17*d^17*f*g^2*e^25 +
 5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^17*d^17*f*g^2*e^22 + 15*sqrt
((x*e + d)*c*d*e - c*d^2*e + a*e^3)*a^2*c^16*d^16*g^3*e^26 - 5*((x*e + d)*c
*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^16*d^16*g^3*e^23 + ((x*e + d)*c*d*e - c*d
^2*e + a*e^3)^(5/2)*c^16*d^16*g^3*e^20)*e^(-25)/(c^20*d^20)
```

Mupad [B]
time = 3.61, size = 252, normalized size = 0.98

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(32\,a^3\,e^3\,g^3-80\,a^2\,c\,d\,e^2\,f\,g^2+60\,a\,c^2\,d^2\,e\,f^2\,g-10\,c^3\,d^3\,f^3\right)}{5\,c^5\,d^5\,e}+\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{5\,c^2\,d^2\,e}-\frac{2\,g^2\,x^2\,\left(2\,a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{5\,c^3\,d^3\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}\,\left(8\,a^2\,e^2\,g^2-20\,a\,c\,d\,e\,f\,g+15\,c^2\,d^2\,f^2\right)}{5\,c^4\,d^4\,e}\right)}{\frac{a}{c}+x^2+\frac{x\,\left(5\,c^5\,d^6+5\,a\,c^4\,d^4\,e^2\right)}{5\,c^5\,d^5\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(32*a^3*e^
3*g^3 - 10*c^3*d^3*f^3 + 60*a*c^2*d^2*e*f^2*g - 80*a^2*c*d*e^2*f*g^2))/(5*c
^5*d^5*e) + (2*g^3*x^3*(d + e*x)^(1/2))/(5*c^2*d^2*e) - (2*g^2*x^2*(2*a*e*g
 - 5*c*d*f)*(d + e*x)^(1/2))/(5*c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2)*(8*a^2*
e^2*g^2 + 15*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(5*c^4*d^4*e)))/(a/c + x^2 + (x
*(5*c^5*d^6 + 5*a*c^4*d^4*e^2))/(5*c^5*d^5*e))
```

3.666 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}-\frac{8g(2ae^2g-cd(3ef-dg))\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^3d^3e\sqrt{d+ex}}+\frac{8g^2\sqrt{d+e}}{}$$

```
[Out] -2*(g*x+f)^2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*
g*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/
d^3/e/(e*x+d)^(1/2)+8/3*g^2*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/c^2/d^2/e
```

Rubi [A]
time = 0.12, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {880, 808, 662}

$$-\frac{8g\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(2ae^2g-cd(3ef-dg))}{3c^3d^3e\sqrt{d+ex}}+\frac{8g^2\sqrt{d+ex}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2e}-\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^2)/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (8*g*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(3*c^3*d^3*e*Sqrt[d + e*x]) + (8*g^2*Sqrt[d + e*x]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{(4g)\int \frac{\sqrt{d+ex}\,(f\cdot}{\sqrt{ade+(cd^2+ae^2}}}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{8g^2\sqrt{d+ex}\,\sqrt{ade+(cd^2}}{3c^2d^2e}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^2}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}-\frac{8g(2ae^2g-cd(3ef-dg))}{3c^3d^3}$$

Mathematica [A]
time = 0.07, size = 88, normalized size = 0.49

$$\frac{2\sqrt{d+ex}\,(-8a^2e^2g^2-4acdeg(-3f+gx)+c^2d^2(-3f^2+6fgx+g^2x^2))}{3c^3d^3\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(-8*a^2*e^2*g^2 - 4*a*c*d*e*g*(-3*f + g*x) + c^2*d^2*(-3*f
^2 + 6*f*g*x + g^2*x^2)))/(3*c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 108, normalized size = 0.60

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,\left(-g^2x^2c^2d^2+4acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-12acdefg+3f^2c^2d^2\right)}{3\sqrt{ex+d}\,(cdx+ae)c^3d^3}$	108

gosper	$-\frac{2(cdx+ae)\left(-g^{2}x^{2}c^{2}d^{2}+4acde\,g^{2}x-6c^{2}d^{2}fgx+8a^{2}e^{2}g^{2}-12acdefg+3f^{2}c^{2}d^{2}\right)(ex+d)^{\frac{3}{2}}}{3c^{3}d^{3}\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c^2*d^2*g^2*x^2+4*a*c*d*e*
g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^2*g^2-12*a*c*d*e*f*g+3*c^2*d^2*f^2)/(c*d*x+a*
e)/c^3/d^3
```

Maxima [A]
time = 0.33, size = 102, normalized size = 0.56

$$-\frac{2\,f^{2}}{\sqrt{cdx+ae}\,cd}+\frac{4\,(cdx+2\,ae)fg}{\sqrt{cdx+ae}\,c^{2}d^{2}}+\frac{2\,(c^{2}d^{2}x^{2}-4\,acdxe-8\,a^{2}e^{2})g^{2}}{3\,\sqrt{cdx+ae}\,c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] -2*f^2/(sqrt(c*d*x + a*e)*c*d) + 4*(c*d*x + 2*a*e)*f*g/(sqrt(c*d*x + a*e)*c
^2*d^2) + 2/3*(c^2*d^2*x^2 - 4*a*c*d*x*e - 8*a^2*e^2)*g^2/(sqrt(c*d*x + a*e
)*c^3*d^3)
```

Fricas [A]
time = 2.32, size = 147, normalized size = 0.81

$$\frac{2\,(c^{2}d^{2}g^{2}x^{2}+6\,c^{2}d^{2}fgx-3\,c^{2}d^{2}f^{2}-8\,a^{2}g^{2}e^{2}-4\,(acdg^{2}x-3\,acdfg)e)\,\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,\sqrt{xe+d}}{3\,(c^{4}d^{5}x+ac^{3}d^{3}xe^{2}+(c^{4}d^{4}x^{2}+ac^{3}d^{4})e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^2*d^2*g^2*x^2 + 6*c^2*d^2*f*g*x - 3*c^2*d^2*f^2 - 8*a^2*g^2*e^2 - 4*
(a*c*d*g^2*x - 3*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*
sqrt(x*e + d)/(c^4*d^5*x + a*c^3*d^3*x*e^2 + (c^4*d^4*x^2 + a*c^3*d^4)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^{2}}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [A]
time = 1.98, size = 285, normalized size = 1.57

$$-\frac{2\,(c^2d^4g^2-6\,c^2d^3fge-3\,c^2d^2f^2e^2+4\,acd^2g^2e^2+12\,acdfge^3-8\,a^2g^2e^4)e^{(-1)}}{3\,\sqrt{-cd^2e+ae^3}\,c^3d^3}-\frac{2\,(c^2d^2f^2e-2\,acdfge^2+a^2g^2e^3)}{\sqrt{(xe+d)cde-cd^2e+ae^3}\,c^3d^3}+\frac{2\left(6\,\sqrt{(xe+d)cde-cd^2e+ae^3}\,c^7d^7fge^8-6\,\sqrt{(xe+d)cde-cd^2e+ae^3}\,ac^6d^6g^2e^9+((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^6d^6g^2e^6\right)e^{(-9)}}{3\,c^9d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] -2/3*(c^2*d^4*g^2 - 6*c^2*d^3*f*g*e - 3*c^2*d^2*f^2*e^2 + 4*a*c*d^2*g^2*e^2
 + 12*a*c*d*f*g*e^3 - 8*a^2*g^2*e^4)*e^(-1)/(sqrt(-c*d^2*e + a*e^3)*c^3*d^3
) - 2*(c^2*d^2*f^2*e - 2*a*c*d*f*g*e^2 + a^2*g^2*e^3)/(sqrt((x*e + d)*c*d*e
 - c*d^2*e + a*e^3)*c^3*d^3) + 2/3*(6*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^
3)*c^7*d^7*f*g*e^8 - 6*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*a*c^6*d^6*g^
2*e^9 + ((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^6*d^6*g^2*e^6)*e^(-9)/(
c^9*d^9)
```

Mupad [B]
time = 3.43, size = 178, normalized size = 0.98

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,a^2\,e^2\,g^2-24\,a\,c\,d\,e\,f\,g+6\,c^2\,d^2\,f^2\right)}{3\,c^4\,d^4\,e}-\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}+\frac{4\,g\,x\,\left(2\,a\,e\,g-3\,c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,c^3\,d^3\,e}\right)}{\frac{a}{c}+x^2+\frac{x\,\left(3\,c^4\,d^5+3\,a\,c^3\,d^3\,e^2\right)}{3\,c^4\,d^4\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e
^2*g^2 + 6*c^2*d^2*f^2 - 24*a*c*d*e*f*g))/(3*c^4*d^4*e) - (2*g^2*x^2*(d + e
*x)^(1/2))/(3*c^2*d^2*e) + (4*g*x*(2*a*e*g - 3*c*d*f)*(d + e*x)^(1/2))/(3*c
^3*d^3*e)))/(a/c + x^2 + (x*(3*c^4*d^5 + 3*a*c^3*d^3*e^2))/(3*c^4*d^4*e))
```

3.667 $\int \frac{(d+ex)^{3/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2(2ae^2g-cd(ef+dg))\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2\left(cd^2-ae^2\right)\sqrt{d+ex}}$$

```
[Out] -2*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)-2*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/c^2/d^2/(-a*e^2+c*d^2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {802, 662}

$$-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(2ae^2g-cd(dg+ef))}{c^2d^2\sqrt{d+ex}\,\left(cd^2-ae^2\right)}-\frac{2(d+ex)^{3/2}(cdf-aeg)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\,(cd^2-ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{(2\big(-\frac{1}{2}e(2cdef}{}$$
$$= -\frac{2(cdf-aeg)(d+ex)^{3/2}}{cd\,(cd^2-ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{2(2ae^2g-cd(}{}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.34

$$\frac{2\sqrt{d+ex}\,(2aeg+cd(-f+gx))}{c^2d^2\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(2*a*e*g + c*d*(-f + g*x)))/(c^2*d^2*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

Maple [A]
time = 0.14, size = 58, normalized size = 0.39

method	result	size
default	$\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,(cdgx+2aeg-cdf)}{\sqrt{ex+d}\,(cdx+ae)c^2d^2}$	58
gosper	$\frac{2(cdx+ae)(cdgx+2aeg-cdf)(ex+d)^{\frac{3}{2}}}{c^2d^2(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=
_RETURNVERBOSE)
```

```
[Out] 2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(c*d*g*x+2*a*e*g-c*d*f)/(c*d*x+
a*e)/c^2/d^2
```

Maxima [A]
time = 0.31, size = 51, normalized size = 0.34

$$-\frac{2\,f}{\sqrt{cdx+ae}\,cd} + \frac{2\,(cdx+2\,ae)g}{\sqrt{cdx+ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] -2*f/(sqrt(c*d*x + a*e)*c*d) + 2*(c*d*x + 2*a*e)*g/(sqrt(c*d*x + a*e)*c^2*d
^2)
```

Fricas [A]
time = 1.88, size = 98, normalized size = 0.65

$$\frac{2\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,(cdgx - cdf + 2\,age)\sqrt{xe + d}}{c^3d^4x + ac^2d^2xe^2 + (c^3d^3x^2 + ac^2d^3)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*g*x - c*d*f + 2*a*g*e)*s
qrt(x*e + d)/(c^3*d^4*x + a*c^2*d^2*x*e^2 + (c^3*d^3*x^2 + a*c^2*d^3)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^{\frac{3}{2}}\,(f + gx)}{((d + ex)\,(ae + cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [A]
time = 1.53, size = 127, normalized size = 0.85

$$\frac{2\sqrt{(xe + d)cde - cd^2e + ae^3}\,ge^{(-1)}}{c^2d^2} + \frac{2\,(cd^2g + cdfe - 2\,age^2)}{\sqrt{-cd^2e + ae^3}\,c^2d^2} - \frac{2\,(cdfe - age^2)}{\sqrt{(xe + d)cde - cd^2e + ae^3}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] 2*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*g*e^(-1)/(c^2*d^2) + 2*(c*d^2*g +
 c*d*f*e - 2*a*g*e^2)/(sqrt(-c*d^2*e + a*e^3)*c^2*d^2) - 2*(c*d*f*e - a*g*e
^2)/(sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*c^2*d^2)
```

Mupad [B]
time = 3.37, size = 118, normalized size = 0.79

$$\frac{\left(\frac{(4aeg-2cdf)\sqrt{d+ex}}{c^3d^3e}+\frac{2gx\sqrt{d+ex}}{c^2d^2e}\right)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{\frac{a}{c}+x^2+\frac{x(c^3d^4+ac^2d^2e^2)}{c^3d^3e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/
2),x)
```

```
[Out] ((((4*a*e*g - 2*c*d*f)*(d + e*x)^(1/2))/(c^3*d^3*e) + (2*g*x*(d + e*x)^(1/2
))/(c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(a/c + x^2 +
 (x*(c^3*d^4 + a*c^2*d^2*e^2))/(c^3*d^3*e))
```

3.668 $\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {662}

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.76

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
[Out] (-2*Sqrt[d + e*x])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.15, size = 42, normalized size = 0.91

method	result	size
default	$-\frac{2\sqrt{\left(c d x+a e\right)\left(e x+d\right)}}{\sqrt{e x+d}\,\left(c d x+a e\right) c d}$	42
gosper	$-\frac{2 \left(c d x+a e\right) \left(e x+d\right)^{\frac{3}{2}}}{c d \left(c d e\,x^{2}+a\,e^{2} x+c\,d^{2} x+a d e\right)^{\frac{3}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=_RETURNV
ERBOSE)
[Out] -2/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)/c/d
```

Maxima [A]
time = 0.31, size = 19, normalized size = 0.41

$$-\frac{2}{\sqrt{c d x+a e}\, c d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="maxima")
[Out] -2/(sqrt(c*d*x + a*e)*c*d)
```

Fricas [A]
time = 2.26, size = 75, normalized size = 1.63

$$-\frac{2 \sqrt{c d^2 x + a x e^2 + (c d x^2 + a d) e}\,\sqrt{x e + d}}{c^2 d^3 x + a c d x e^2 + (c^2 d^2 x^2 + a c d^2) e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="fricas")
[Out] -2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^2*d^3*x + a
*c*d*x*e^2 + (c^2*d^2*x^2 + a*c*d^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + e x)^{\frac{3}{2}}}{((d + e x)(a e + c d x))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Giac [A]
time = 2.00, size = 62, normalized size = 1.35

$$-\frac{2\,e}{\sqrt{(xe+d)cde-cd^2e+ae^3}\,cd}+\frac{2\,e}{\sqrt{-cd^2e+ae^3}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorith
m="giac")
```

```
[Out] -2*e/(sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*c*d) + 2*e/(sqrt(-c*d^2*e + a
*e^3)*c*d)
```

Mupad [B]
time = 3.27, size = 82, normalized size = 1.78

$$-\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c^2\,d^2\,e\,\left(\frac{a}{c}+x^2+\frac{x\,\left(c^2\,d^3+a\,c\,d\,e^2\right)}{c^2\,d^2\,e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(c^2*d^2
*e*(a/c + x^2 + (x*(c^2*d^3 + a*c*d*e^2))/(c^2*d^2*e)))
```

3.669 $\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{(cdf-aeg)^{3/2}}$$

```
[Out] -2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1
/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(3/2)-2*(e*x+d)^(1/2)/(-a*e*g+c*d
*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {882, 888, 211}

$$-\frac{2\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}-\frac{2\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2
)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{g\int \frac{}{(f+gx)}}{}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(2e^2g\right)\text{Su}}{}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\sqrt{g}\,\tan}{}$$

Mathematica [A]
time = 0.11, size = 109, normalized size = 0.82

$$-\frac{2\sqrt{d+ex}\left(\sqrt{cdf-aeg}+\sqrt{g}\,\sqrt{ae+cdx}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g] + Sqrt[g]*Sqrt[a*e + c*d*x]*ArcTan[(
Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/((c*d*f - a*e*g)^(3/2)*Sq
rt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 118, normalized size = 0.89

method	result	size

default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(g\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}-\sqrt{(aeg-cdf)g}\right)}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)\sqrt{(aeg-cdf)g}}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*(c*d*x+a*e)^(1/2)-((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x
+a*e)/(a*e*g-c*d*f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(124) = 248$.
time = 2.74, size = 555, normalized size = 4.17

$$\left[-\frac{(cd^2x+axe^2+(cdx^2+ad)e)\sqrt{-\frac{g}{cdf-age}}\log\left(-\frac{cd^2gx-cd^2f+2agxe^2+2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(cdf-age)\sqrt{xe+d}\sqrt{-\frac{g}{cdf-age}}+(cdgx^2-cdfx+2adg)e}{dgx+df+(gx^2+fx)e}\right)+2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{c^2d^3fx-a^2gxe^3-(acdgx^2-acdfx+a^2dg)e^2+(c^2d^2fx^2-acd^2gx+acd^2f)e}, -\frac{2\left((cd^2x+axe^2+(cdx^2+ad)e)\sqrt{\frac{g}{cdf-age}}\arctan\left(-\frac{\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(cdf-age)\sqrt{xe+d}\sqrt{\frac{g}{cdf-age}}}{cd^2gx+agxe^2+(cdgx^2+adg)e}\right)+\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}\right)}{c^2d^3fx-a^2gxe^3-(acdgx^2-acdfx+a^2dg)e^2+(c^2d^2fx^2-acd^2gx+acd^2f)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] [-((c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-g/(c*d*f - a*g*e))*log(-(c
*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*
d)*e)*(c*d*f - a*g*e)*sqrt(x*e + d)*sqrt(-g/(c*d*f - a*g*e)) + (c*d*g*x^2 -
 c*d*f*x + 2*a*d*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f*x - a^2*g*x*e^3 - (a
*c*d*g*x^2 - a*c*d*f*x + a^2*d*g)*e^2 + (c^2*d^2*f*x^2 - a*c*d^2*g*x + a*c*
d^2*f)*e), -2*((c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g/(c*d*f - a*g*
e))*arctan(-sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*f - a*g*e)*sqr
t(x*e + d)*sqrt(g/(c*d*f - a*g*e))/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*
```

```
d*g)*e)) + sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*
d^3*f*x - a^2*g*x*e^3 - (a*c*d*g*x^2 - a*c*d*f*x + a^2*d*g)*e^2 + (c^2*d^2*
f*x^2 - a*c*d^2*g*x + a*c*d^2*f)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2
),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(124) = 248.
time = 2.42, size = 276, normalized size = 2.08

$$-2\left(\frac{g\arctan\left(\frac{\sqrt{(xe+d)cde-cd^2e+ae^3}\,ge^{(-1)}}{\sqrt{cdfg-ag^2e}}\right)e^{(-1)}}{\sqrt{cdfg-ag^2e}\,(cdf-age)}+\frac{1}{\sqrt{(xe+d)cde-cd^2e+ae^3}\,(cdf-age)}\right)e+\frac{2\left(\sqrt{-cd^2e+ae^3}\,g\arctan\left(\frac{\sqrt{-cd^2e+ae^3}\,ge^{(-1)}}{\sqrt{cdfg-ag^2e}}\right)+\sqrt{cdfg-ag^2e}\,e\right)}{\sqrt{cdfg-ag^2e}\,\sqrt{-cd^2e+ae^3}\,cdf-\sqrt{cdfg-ag^2e}\,\sqrt{-cd^2e+ae^3}\,age}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,
algorithm="giac")
```

```
[Out] -2*(g*arctan(sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*g*e^(-1)/sqrt(c*d*f*g
- a*g^2*e))*e^(-1)/(sqrt(c*d*f*g - a*g^2*e)*(c*d*f - a*g*e)) + 1/(sqrt((x*e
 + d)*c*d*e - c*d^2*e + a*e^3)*(c*d*f - a*g*e)))*e + 2*(sqrt(-c*d^2*e + a*e
^3)*g*arctan(sqrt(-c*d^2*e + a*e^3)*g*e^(-1)/sqrt(c*d*f*g - a*g^2*e)) + sqr
t(c*d*f*g - a*g^2*e)*e)/(sqrt(c*d*f*g - a*g^2*e)*sqrt(-c*d^2*e + a*e^3)*c*d
*f - sqrt(c*d*f*g - a*g^2*e)*sqrt(-c*d^2*e + a*e^3)*a*g*e)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2
)), x)
```

3.670 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{3g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)}-\frac{3cd\sqrt{g}\,\tan^{-1}\left(\right.}{}$$

[Out] -3*c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(5/2)-2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {882, 886, 888, 211}

$$-\frac{3cd\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}-\frac{3g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]

[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) - (3*c*d*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(5/2)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Dist[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
```

```
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} -$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} -$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} -$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} -$$

Mathematica [A]
time = 0.39, size = 141, normalized size = 0.70

$$-\frac{\sqrt{d+ex}\left(\sqrt{cdf-aeg}\,(aeg+cd(2f+3gx))+3cd\sqrt{g}\,\sqrt{ae+cdx}\,(f+gx)\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] -((Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g]*(a*e*g + c*d*(2*f + 3*g*x)) + 3*c*d*S
qrt[g]*Sqrt[a*e + c*d*x]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[
c*d*f - a*e*g]]))/((c*d*f - a*e*g)^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f +
 g*x)))
```

Maple [A]
time = 0.16, size = 215, normalized size = 1.06

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cd\,g^2x+3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{c}}{\sqrt{(ae}}\right.\right.}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)^2(gx+f)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*(c*d*x+a*e)^(1/2)*arctanh(g*
(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g^2*x+3*(c*d*x+a*e)^(1/2)*ar
ctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f*g-3*((a*e*g-c*d*f)
*g)^(1/2)*c*d*g*x-((a*e*g-c*d*f)*g)^(1/2)*a*e*g-2*((a*e*g-c*d*f)*g)^(1/2)*c
*d*f)/(c*d*x+a*e)/(a*e*g-c*d*f)^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(192) = 384.
time = 1.82, size = 1097, normalized size = 5.43

$$\left[\frac{3\,(c^2d^3gx^2+c^2d^3fx+(acdgx^2+acdfx)e^2+(c^2d^2gx^3+c^2d^2fx^2+acd^2gx+acd^2f)e)\sqrt{-\frac{g}{cdf-age}}\log\left(-\frac{cd^2gx-cd^2f+2agxe^2-2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(cdf-age)\sqrt{xe+d}\sqrt{-\frac{g}{cdf-age}}+(cdgx^2-cdfx+2adg)e}{dgx+df+(gx^2+fx)e}\right)-2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(3\,cdgx+2\,cdf+age)\sqrt{xe+d}}{2\,(c^3d^4f^2gx^2+c^3d^4f^3x+(a^3g^3x^2+a^3fg^2x)e^4+(a^2cdg^3x^3-a^2cdfg^2x^2+a^3dfg^2-(2\,a^2cdf^2g-a^3dg^3)x)e^3-(2\,ac^2d^2fg^2x^3+2\,a^2cd^2f^2g+(ac^2d^2f^2g-a^2cd^2g^3)x^2-(ac^2d^2f^3-a^2cd^2fg^2)x)e^2+(c^3d^3f^2gx^3-ac^2d^3f^2gx+ac^2d^3f^3+(c^3d^3f^3-2\,ac^2d^3fg^2)x^2)e)},\right.$$
$$\left.-\frac{3\,(c^2d^3gx^2+c^2d^3fx+(acdgx^2+acdfx)e^2+(c^2d^2gx^3+c^2d^2fx^2+acd^2gx+acd^2f)e)\sqrt{\frac{g}{cdf-age}}\arctan\left(-\frac{\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(cdf-age)\sqrt{xe+d}\sqrt{\frac{g}{cdf-age}}}{cd^2gx+agxe^2+(cdgx^2+adg)e}\right)+\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(3\,cdgx+2\,cdf+age)\sqrt{xe+d}}{c^3d^4f^2gx^2+c^3d^4f^3x+(a^3g^3x^2+a^3fg^2x)e^4+(a^2cdg^3x^3-a^2cdfg^2x^2+a^3dfg^2-(2\,a^2cdf^2g-a^3dg^3)x)e^3-(2\,ac^2d^2fg^2x^3+2\,a^2cd^2f^2g+(ac^2d^2f^2g-a^2cd^2g^3)x^2-(ac^2d^2f^3-a^2cd^2fg^2)x)e^2+(c^3d^3f^2gx^3-ac^2d^3f^2gx+ac^2d^3f^3+(c^3d^3f^3-2\,ac^2d^3fg^2)x^2)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c^2*d^3*g*x^2 + c^2*d^3*f*x + (a*c*d*g*x^2 + a*c*d*f*x)*e^2 + (c^2
*d^2*g*x^3 + c^2*d^2*f*x^2 + a*c*d^2*g*x + a*c*d^2*f)*e)*sqrt(-g/(c*d*f - a
*g*e))*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 - 2*sqrt(c*d^2*x + a*x*e^2 +
 (c*d*x^2 + a*d)*e)*(c*d*f - a*g*e)*sqrt(x*e + d)*sqrt(-g/(c*d*f - a*g*e))
+ (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) - 2*s
qrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(3*c*d*g*x + 2*c*d*f + a*g*e)*sq
rt(x*e + d))/(c^3*d^4*f^2*g*x^2 + c^3*d^4*f^3*x + (a^3*g^3*x^2 + a^3*f*g^2*
x)*e^4 + (a^2*c*d*g^3*x^3 - a^2*c*d*f*g^2*x^2 + a^3*d*f*g^2 - (2*a^2*c*d*f^
2*g - a^3*d*g^3)*x)*e^3 - (2*a*c^2*d^2*f*g^2*x^3 + 2*a^2*c*d^2*f^2*g + (a*c
^2*d^2*f^2*g - a^2*c*d^2*g^3)*x^2 - (a*c^2*d^2*f^3 - a^2*c*d^2*f*g^2)*x)*e^
2 + (c^3*d^3*f^2*g*x^3 - a*c^2*d^3*f^2*g*x + a*c^2*d^3*f^3 + (c^3*d^3*f^3 -
 2*a*c^2*d^3*f*g^2)*x^2)*e), -(3*(c^2*d^3*g*x^2 + c^2*d^3*f*x + (a*c*d*g*x^
2 + a*c*d*f*x)*e^2 + (c^2*d^2*g*x^3 + c^2*d^2*f*x^2 + a*c*d^2*g*x + a*c*d^2
*f)*e)*sqrt(g/(c*d*f - a*g*e))*arctan(-sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
a*d)*e)*(c*d*f - a*g*e)*sqrt(x*e + d)*sqrt(g/(c*d*f - a*g*e))/(c*d^2*g*x +
a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) + sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*(3*c*d*g*x + 2*c*d*f + a*g*e)*sqrt(x*e + d))/(c^3*d^4*f^2*g*x^2 + c^
3*d^4*f^3*x + (a^3*g^3*x^2 + a^3*f*g^2*x)*e^4 + (a^2*c*d*g^3*x^3 - a^2*c*d*
f*g^2*x^2 + a^3*d*f*g^2 - (2*a^2*c*d*f^2*g - a^3*d*g^3)*x)*e^3 - (2*a*c^2*d
^2*f*g^2*x^3 + 2*a^2*c*d^2*f^2*g + (a*c^2*d^2*f^2*g - a^2*c*d^2*g^3)*x^2 -
(a*c^2*d^2*f^3 - a^2*c*d^2*f*g^2)*x)*e^2 + (c^3*d^3*f^2*g*x^3 - a*c^2*d^3*f
^2*g*x + a*c^2*d^3*f^3 + (c^3*d^3*f^3 - 2*a*c^2*d^3*f*g^2)*x^2)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

[Out] Timed out

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^2\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)), x)
```

3.671 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{5g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^2}-\frac{15cdg\sqrt{ade+(}}{4(cdf-aeg)^{}}$$

```
[Out] -15/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))*g^(1/2)/(-a*e*g+c*d*f)^(7/2)-2*(e*x+d)^(1/2)/
(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/2*g*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2
)-15/4*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+
f)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {882, 886, 888, 211}

$$-\frac{15c^2d^2\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}}-\frac{15cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}-\frac{5g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) - (5*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*(c
*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) - (15*c*d*g*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (1
5*c^2*d^2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
```

```
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^{3/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx &= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\
&= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}
\end{aligned}$$

Mathematica [A]
time = 0.57, size = 185, normalized size = 0.68

$$-\frac{\sqrt{d+ex}\left(\sqrt{cdf-aeg}\,(-2a^2e^2g^2+acdeg(9f+5gx)+c^2d^2(8f^2+25fgx+15g^2x^2))+15c^2d^2\sqrt{g}\,\sqrt{ae+cdx}\,(f+gx)^2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{4(cdf-aeg)^{7/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)),x]
```

```
[Out] -1/4*(Sqrt[d + e*x]*(Sqrt[c*d*f - a*e*g]*(-2*a^2*e^2*g^2 + a*c*d*e*g*(9*f +
 5*g*x) + c^2*d^2*(8*f^2 + 25*f*g*x + 15*g^2*x^2)) + 15*c^2*d^2*Sqrt[g]*Sqr
t[a*e + c*d*x]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f -
a*e*g]]))/((c*d*f - a*e*g)^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

Maple [A]
time = 0.14, size = 369, normalized size = 1.35

method	result
default	$-\frac{\sqrt{(cdx+ae)\,(ex+d)}\left(15\sqrt{cdx+ae}\ \operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^2d^2g^3x^2+30\sqrt{cdx+ae}\ \operatorname{arctanh}\left(\frac{}{\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/4/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*(c*d*x+a*e)^(1/2)*arctan
h(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*g^3*x^2+30*(c*d*x+a*
e)^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g^2
*x+15*(c*d*x+a*e)^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^2*d^2*f^2*g-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-5*((a*e*g-c*d*f)
*g)^(1/2)*a*c*d*e*g^2*x-25*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x+2*((a*e*g-
c*d*f)*g)^(1/2)*a^2*e^2*g^2-9*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g-8*((a*e*g
-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3/(g*x+f)^2/((a*e*g
-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

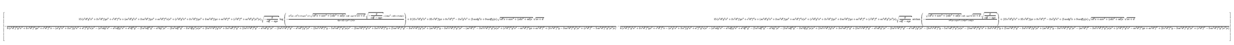
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 950 vs. $2(257) = 514$.
time = 3.00, size = 1941, normalized size = 7.08



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] [-1/8*(15*(c^3*d^4*g^2*x^3 + 2*c^3*d^4*f*g*x^2 + c^3*d^4*f^2*x + (a*c^2*d^2
*g^2*x^3 + 2*a*c^2*d^2*f*g*x^2 + a*c^2*d^2*f^2*x)*e^2 + (c^3*d^3*g^2*x^4 +
2*c^3*d^3*f*g*x^3 + 2*a*c^2*d^3*f*g*x + a*c^2*d^3*f^2 + (c^3*d^3*f^2 + a*c^
2*d^3*g^2)*x^2)*e)*sqrt(-g/(c*d*f - a*g*e))*log(-(c*d^2*g*x - c*d^2*f + 2*a
*g*x*e^2 + 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*f - a*g*e)*sq
rt(x*e + d)*sqrt(-g/(c*d*f - a*g*e)) + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e)/(
d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(15*c^2*d^2*g^2*x^2 + 25*c^2*d^2*f*g*x
+ 8*c^2*d^2*f^2 - 2*a^2*g^2*e^2 + (5*a*c*d*g^2*x + 9*a*c*d*f*g)*e)*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^4*d^5*f^3*g^2*x^3 + 2
*c^4*d^5*f^4*g*x^2 + c^4*d^5*f^5*x - (a^4*g^5*x^3 + 2*a^4*f*g^4*x^2 + a^4*f
^2*g^3*x)*e^5 - (a^3*c*d*g^5*x^4 - a^3*c*d*f*g^4*x^3 + a^4*d*f^2*g^3 - (5*a
^3*c*d*f^2*g^3 - a^4*d*g^5)*x^2 - (3*a^3*c*d*f^3*g^2 - 2*a^4*d*f*g^4)*x)*e^
4 + (3*a^2*c^2*d^2*f*g^4*x^4 + 3*a^3*c*d^2*f^3*g^2 + (3*a^2*c^2*d^2*f^2*g^3
 - a^3*c*d^2*g^5)*x^3 - (3*a^2*c^2*d^2*f^3*g^2 - a^3*c*d^2*f*g^4)*x^2 - (3*
a^2*c^2*d^2*f^4*g - 5*a^3*c*d^2*f^2*g^3)*x)*e^3 - (3*a*c^3*d^3*f^2*g^3*x^4
+ 3*a^2*c^2*d^3*f^4*g + (5*a*c^3*d^3*f^3*g^2 - 3*a^2*c^2*d^3*f*g^4)*x^3 + (
a*c^3*d^3*f^4*g - 3*a^2*c^2*d^3*f^2*g^3)*x^2 - (a*c^3*d^3*f^5 - 3*a^2*c^2*d
^3*f^3*g^2)*x)*e^2 + (c^4*d^4*f^3*g^2*x^4 - a*c^3*d^4*f^4*g*x + a*c^3*d^4*f
^5 + (2*c^4*d^4*f^4*g - 3*a*c^3*d^4*f^2*g^3)*x^3 + (c^4*d^4*f^5 - 5*a*c^3*d
^4*f^3*g^2)*x^2)*e), -1/4*(15*(c^3*d^4*g^2*x^3 + 2*c^3*d^4*f*g*x^2 + c^3*d^
4*f^2*x + (a*c^2*d^2*g^2*x^3 + 2*a*c^2*d^2*f*g*x^2 + a*c^2*d^2*f^2*x)*e^2 +
 (c^3*d^3*g^2*x^4 + 2*c^3*d^3*f*g*x^3 + 2*a*c^2*d^3*f*g*x + a*c^2*d^3*f^2 +
 (c^3*d^3*f^2 + a*c^2*d^3*g^2)*x^2)*e)*sqrt(g/(c*d*f - a*g*e))*arctan(-sqrt
(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*f - a*g*e)*sqrt(x*e + d)*sqrt(
g/(c*d*f - a*g*e))/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) + (15*c
^2*d^2*g^2*x^2 + 25*c^2*d^2*f*g*x + 8*c^2*d^2*f^2 - 2*a^2*g^2*e^2 + (5*a*c*
d*g^2*x + 9*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(
x*e + d))/(c^4*d^5*f^3*g^2*x^3 + 2*c^4*d^5*f^4*g*x^2 + c^4*d^5*f^5*x - (a^4
*g^5*x^3 + 2*a^4*f*g^4*x^2 + a^4*f^2*g^3*x)*e^5 - (a^3*c*d*g^5*x^4 - a^3*c*
d*f*g^4*x^3 + a^4*d*f^2*g^3 - (5*a^3*c*d*f^2*g^3 - a^4*d*g^5)*x^2 - (3*a^3*
c*d*f^3*g^2 - 2*a^4*d*f*g^4)*x)*e^4 + (3*a^2*c^2*d^2*f*g^4*x^4 + 3*a^3*c*d^
2*f^3*g^2 + (3*a^2*c^2*d^2*f^2*g^3 - a^3*c*d^2*g^5)*x^3 - (3*a^2*c^2*d^2*f^
```

```
3*g^2 - a^3*c*d^2*f*g^4)*x^2 - (3*a^2*c^2*d^2*f^4*g - 5*a^3*c*d^2*f^2*g^3)*
x)*e^3 - (3*a*c^3*d^3*f^2*g^3*x^4 + 3*a^2*c^2*d^3*f^4*g + (5*a*c^3*d^3*f^3*
g^2 - 3*a^2*c^2*d^3*f*g^4)*x^3 + (a*c^3*d^3*f^4*g - 3*a^2*c^2*d^3*f^2*g^3)*
x^2 - (a*c^3*d^3*f^5 - 3*a^2*c^2*d^3*f^3*g^2)*x)*e^2 + (c^4*d^4*f^3*g^2*x^4
 - a*c^3*d^4*f^4*g*x + a*c^3*d^4*f^5 + (2*c^4*d^4*f^4*g - 3*a*c^3*d^4*f^2*g
^3)*x^3 + (c^4*d^4*f^5 - 5*a*c^3*d^4*f^3*g^2)*x^2)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3
/2)), x)
```

3.672 $\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{16g^2(2ae^2g-cd(3ef-dg))}{3c^4d^4e}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^3/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-4*
g*(g*x+f)^2*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1
6/3*g^2*(2*a*e^2*g-c*d*(-d*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c^4/d^4/e/(e*x+d)^(1/2)+16/3*g^3*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A]
time = 0.18, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {880, 808, 662}

$$-\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(2ae^2g-cd(3ef-dg))}{3c^4d^4e\sqrt{d+ex}}+\frac{16g^3\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3c^3d^3e}-\frac{4g\sqrt{d+ex}\,(f+gx)^2}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (4*g*Sqrt[d + e*x]*(f + g*x)^2)/(c^2*d^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2]) - (16*g^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^4*d^4*e*Sqrt[d + e*x]) + (16*g^3*
Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
```

```
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(2g)\int \frac{(d+ex)^{3/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}}{cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{4g\sqrt{d+ex}\,(f+g}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{4g\sqrt{d+ex}\,(f+g}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^3}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{4g\sqrt{d+ex}\,(f+g}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)}}$$

Mathematica [A]
time = 0.11, size = 131, normalized size = 0.55

$$\frac{2(d+ex)^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-6ac^2d^2eg\left(f^2-6fgx+g^2x^2\right)+c^3d^3\left(-f^3-9f^2gx+9fg^2x^2+g^3x^3\right)\right)}{3c^4d^4((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(f - g*x) - 6*a*c^
2*d^2*e*g*(f^2 - 6*f*g*x + g^2*x^2) + c^3*d^3*(-f^3 - 9*f^2*g*x + 9*f*g^2*x
^2 + g^3*x^3)))/(3*c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A]
time = 0.15, size = 179, normalized size = 0.75

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g}{3\sqrt{ex+d}\,(cdx+ae)^2c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-g^3x^3c^3d^3+6a\,c^2d^2e\,g^3x^2-9c^3d^3f\,g^2x^2+24a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x+9c^3d^3f^2gx+16a^3e^3g^3-24a^2cd\,e^2f\,g^2+6a\,c}{3c^4d^4(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c^3*d^3*g^3*x^3+6*a*c^2*d^
2*e*g^3*x^2-9*c^3*d^3*f*g^2*x^2+24*a^2*c*d*e^2*g^3*x-36*a*c^2*d^2*e*f*g^2*x
+9*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-24*a^2*c*d*e^2*f*g^2+6*a*c^2*d^2*e*f^2*g+
c^3*d^3*f^3)/(c*d*x+a*e)^2/c^4/d^4
```

Maxima [A]
time = 0.36, size = 227, normalized size = 0.95

$$-\frac{2\,(3\,cdx+2\,ae)f^2g}{(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}}+\frac{2\,(3\,c^2d^2x^2+12\,acdxe+8\,a^2e^2)fg^2}{(c^4d^4x+ac^3d^3e)\sqrt{cdx+ae}}+\frac{2\,(c^3d^3x^3-6\,ac^2d^2x^2e-24\,a^2cdxe^2-16\,a^3e^3)g^3}{3\,(c^5d^5x+ac^4d^4e)\sqrt{cdx+ae}}-\frac{2\,f^3}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -2*(3*c*d*x + 2*a*e)*f^2*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2*(3*c^2*d^2*x^2 + 12*a*c*d*x*e + 8*a^2*e^2)*f*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) + 2/3*(c^3*d^3*x^3 - 6*a*c^2*d^2*x^2*e - 24*a^2*c*d*x
*e^2 - 16*a^3*e^3)*g^3/((c^5*d^5*x + a*c^4*d^4*e)*sqrt(c*d*x + a*e)) - 2/3*
f^3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [A]
time = 1.85, size = 247, normalized size = 1.03

$$\frac{2\,(c^3d^3g^3x^3+9\,c^3d^3fg^2x^2-9\,c^3d^3f^2gx-c^3d^3f^3-16\,a^3g^3e^3-24\,(a^2cdg^3x-a^2cdfg^2)e^2-6\,(ac^2d^2g^3x^2-6\,ac^2d^2fg^2x+ac^2d^2f^2g)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{3\,(c^6d^7x^2+a^2c^4d^4xe^3+(2\,ac^5d^5x^2+a^2c^4d^5)e^2+(c^6d^6x^3+2\,ac^5d^6x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(c^3*d^3*g^3*x^3 + 9*c^3*d^3*f*g^2*x^2 - 9*c^3*d^3*f^2*g*x - c^3*d^3*f^
3 - 16*a^3*g^3*e^3 - 24*(a^2*c*d*g^3*x - a^2*c*d*f*g^2)*e^2 - 6*(a*c^2*d^2*
g^3*x^2 - 6*a*c^2*d^2*f*g^2*x + a*c^2*d^2*f^2*g)*e)*sqrt(c*d^2*x + a*x*e^2
+ (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^6*d^7*x^2 + a^2*c^4*d^4*x*e^3 + (2*a*
c^5*d^5*x^2 + a^2*c^4*d^5)*e^2 + (c^6*d^6*x^3 + 2*a*c^5*d^6*x)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(221) = 442.
time = 1.37, size = 509, normalized size = 2.13

$$-\frac{2\left(c^3d^6g^3-9c^3d^5fg^2e-9c^3d^4f^2ge^2+6ac^2d^4g^3e^2+c^3d^3f^3e^3+36ac^2d^3fg^2e^3+6ac^2d^2f^2ge^4-24a^2cd^2g^3e^4-24a^2cdfg^2e^5+16a^3g^3e^6\right)}{3\left(\sqrt{-cd^2e+ae^3}\,c^5d^6e-\sqrt{-cd^2e+ae^3}\,ac^4d^4e^3\right)}-\frac{2\left(c^3d^3f^3e^3-3ac^2d^2f^2ge^4+9\left((xe+d)cde-cd^2e+ae^3\right)c^2d^2f^2ge+3a^2cdfg^2e^5-18\left((xe+d)cde-cd^2e+ae^3\right)acdfg^2e^2-a^3g^3e^6+9\left((xe+d)cde-cd^2e+ae^3\right)a^2g^3e^3\right)}{3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}c^4d^4}+\frac{2\left(9\sqrt{(xe+d)cde-cd^2e+ae^3}\,c^9d^9fg^2e^8-9\sqrt{(xe+d)cde-cd^2e+ae^3}\,ac^8d^8g^3e^9+\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}c^8d^8g^3e^6\right)e^{(-9)}}{3c^{12}d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] -2/3*(c^3*d^6*g^3 - 9*c^3*d^5*f*g^2*e - 9*c^3*d^4*f^2*g*e^2 + 6*a*c^2*d^4*g
^3*e^2 + c^3*d^3*f^3*e^3 + 36*a*c^2*d^3*f*g^2*e^3 + 6*a*c^2*d^2*f^2*g*e^4 -
 24*a^2*c*d^2*g^3*e^4 - 24*a^2*c*d*f*g^2*e^5 + 16*a^3*g^3*e^6)/(sqrt(-c*d^2
*e + a*e^3)*c^5*d^6*e - sqrt(-c*d^2*e + a*e^3)*a*c^4*d^4*e^3) - 2/3*(c^3*d^
3*f^3*e^3 - 3*a*c^2*d^2*f^2*g*e^4 + 9*((x*e + d)*c*d*e - c*d^2*e + a*e^3)*c
^2*d^2*f^2*g*e + 3*a^2*c*d*f*g^2*e^5 - 18*((x*e + d)*c*d*e - c*d^2*e + a*e^
3)*a*c*d*f*g^2*e^2 - a^3*g^3*e^6 + 9*((x*e + d)*c*d*e - c*d^2*e + a*e^3)*a^
2*g^3*e^3)/(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4) + 2/3*(9*sqr
t((x*e + d)*c*d*e - c*d^2*e + a*e^3)*c^9*d^9*f*g^2*e^8 - 9*sqrt((x*e + d)*c
*d*e - c*d^2*e + a*e^3)*a*c^8*d^8*g^3*e^9 + ((x*e + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*c^8*d^8*g^3*e^6)*e^(-9)/(c^12*d^12)
```

Mupad [B]
time = 3.77, size = 278, normalized size = 1.16

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(\frac{32\,a^3\,e^3\,g^3}{3}-16\,a^2\,c\,d\,e^2\,f\,g^2+4\,a\,c^2\,d^2\,e\,f^2\,g+\frac{2\,c^3\,d^3\,f^3}{3}\right)}{c^6\,d^6\,e}-\frac{2\,g^3\,x^3\,\sqrt{d+e\,x}}{3\,c^3\,d^3\,e}+\frac{g^2\,x^2\,\left(4\,a\,e\,g-6\,c\,d\,f\right)\,\sqrt{d+e\,x}}{c^4\,d^4\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}\,\left(8\,a^2\,e^2\,g^2-12\,a\,c\,d\,e\,f\,g+3\,c^2\,d^2\,f^2\right)}{c^5\,d^5\,e}\right)}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^6\,d^7+2\,a\,c^5\,d^5\,e^2\right)}{c^6\,d^6\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*((32*a^3*
e^3*g^3)/3 + (2*c^3*d^3*f^3)/3 + 4*a*c^2*d^2*e*f^2*g - 16*a^2*c*d*e^2*f*g^2
```

```
))/(c^6*d^6*e) - (2*g^3*x^3*(d + e*x)^(1/2))/(3*c^3*d^3*e) + (g^2*x^2*(4*a*
e*g - 6*c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d + e*x)^(1/2)*(8*a^2
*e^2*g^2 + 3*c^2*d^2*f^2 - 12*a*c*d*e*f*g))/(c^5*d^5*e)))/(x^3 + (a^2*e)/(c
^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^6*d^7 + 2*a*c^5*d^5*e^2
))/(c^6*d^6*e))
```

3.673 $\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{8g(cdf-aeg)(d+ex)^{3/2}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{8g(2ae^2g-cd(ef}{3c}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^2/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-8/
3*g*(-a*e*g+c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)-8/3*g*(2*a*e^2*g-c*d*(d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^3/d^3/(-a*e^2+c*d^2)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {880, 802, 662}

$$-\frac{8g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(2ae^2g-cd(dg+ef)\right)}{3c^3d^3\sqrt{d+ex}\left(cd^2-ae^2\right)}-\frac{8g(d+ex)^{3/2}(cdf-aeg)}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^2)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) - (8*g*(c*d*f - a*e*g)*(d + e*x)^(3/2))/(3*c^2*d^2*(c*d^2 - a*e
^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*(2*a*e^2*g - c*d*(e
*f + d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*(c*d^2 -
 a*e^2)*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
```

```
[p, -1] && GtQ[m, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{(4g)\int\frac{(d+ex)^{3/2}(f+gx}{(ade+(cd^2+ae^2)x+cde}}{3cd}$$

$$=-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{8g(cdf-ae}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade}}$$

$$=-\frac{2(d+ex)^{3/2}(f+gx)^2}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{8g(cdf-ae}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade}}$$

Mathematica [A]
time = 0.07, size = 87, normalized size = 0.41

$$\frac{2(d+ex)^{3/2}\left(8a^2e^2g^2-4acdeg(f-3gx)-c^2d^2\left(f^2+6fgx-3g^2x^2\right)\right)}{3c^3d^3((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(f - 3*g*x) - c^2*d^2*(f^2
+ 6*f*g*x - 3*g^2*x^2)))/(3*c^3*d^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A]
time = 0.13, size = 108, normalized size = 0.51

method	result	size

default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3g^2x^2c^2d^2+12acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-4acdefg-f^2c^2d^2)}{3\sqrt{ex+d}\,(cdx+ae)^2c^3d^3}$	108
gosper	$\frac{2(cdx+ae)(3g^2x^2c^2d^2+12acde\,g^2x-6c^2d^2fgx+8a^2e^2g^2-4acdefg-f^2c^2d^2)(ex+d)^{\frac{5}{2}}}{3c^3d^3(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*g^2*x^2+12*a*c*d*e
*g^2*x-6*c^2*d^2*f*g*x+8*a^2*e^2*g^2-4*a*c*d*e*f*g-c^2*d^2*f^2)/(c*d*x+a*e)
^2/c^3/d^3
```

Maxima [A]
time = 0.35, size = 145, normalized size = 0.69

$$-\frac{4\,(3\,cdx+2\,ae)fg}{3\,(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}}+\frac{2\,(3\,c^2d^2x^2+12\,acdxe+8\,a^2e^2)g^2}{3\,(c^4d^4x+ac^3d^3e)\sqrt{cdx+ae}}-\frac{2\,f^2}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] -4/3*(3*c*d*x + 2*a*e)*f*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) +
2/3*(3*c^2*d^2*x^2 + 12*a*c*d*x*e + 8*a^2*e^2)*g^2/((c^4*d^4*x + a*c^3*d^3*
e)*sqrt(c*d*x + a*e)) - 2/3*f^2/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [A]
time = 2.80, size = 181, normalized size = 0.86

$$\frac{2\,(3\,c^2d^2g^2x^2-6\,c^2d^2fgx-c^2d^2f^2+8\,a^2g^2e^2+4\,(3\,acdg^2x-acdfg)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{3\,(c^5d^6x^2+a^2c^3d^3xe^3+(2\,ac^4d^4x^2+a^2c^3d^4)e^2+(c^5d^5x^3+2\,ac^4d^5x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3*(3*c^2*d^2*g^2*x^2 - 6*c^2*d^2*f*g*x - c^2*d^2*f^2 + 8*a^2*g^2*e^2 + 4*
(3*a*c*d*g^2*x - a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*
sqrt(x*e + d)/(c^5*d^6*x^2 + a^2*c^3*d^3*x*e^3 + (2*a*c^4*d^4*x^2 + a^2*c^3
*d^4)*e^2 + (c^5*d^5*x^3 + 2*a*c^4*d^5*x)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.02, size = 290, normalized size = 1.37

$$\frac{2\left(3\,c^2d^4g^2+6\,c^2d^3fge-c^2d^2f^2e^2-12\,acd^2g^2e^2-4\,acdfge^3+8\,a^2g^2e^4\right)}{3\left(\sqrt{-cd^2e+ae^3}\,c^4d^5-\sqrt{-cd^2e+ae^3}\,ac^3d^3e^2\right)}+\frac{2\,\sqrt{(xe+d)cde-cd^2e+ae^3}\,g^2e^{(-1)}}{c^3d^3}-\frac{2\left(c^2d^2f^2e^3-2\,acdfge^4+6\left((xe+d)cde-cd^2e+ae^3\right)cdfge+a^2g^2e^5-6\left((xe+d)cde-cd^2e+ae^3\right)ag^2e^2\right)}{3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] 2/3*(3*c^2*d^4*g^2 + 6*c^2*d^3*f*g*e - c^2*d^2*f^2*e^2 - 12*a*c*d^2*g^2*e^2
 - 4*a*c*d*f*g*e^3 + 8*a^2*g^2*e^4)/(sqrt(-c*d^2*e + a*e^3)*c^4*d^5 - sqrt(
-c*d^2*e + a*e^3)*a*c^3*d^3*e^2) + 2*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3
)*g^2*e^(-1)/(c^3*d^3) - 2/3*(c^2*d^2*f^2*e^3 - 2*a*c*d*f*g*e^4 + 6*((x*e +
 d)*c*d*e - c*d^2*e + a*e^3)*c*d*f*g*e + a^2*g^2*e^5 - 6*((x*e + d)*c*d*e -
 c*d^2*e + a*e^3)*a*g^2*e^2)/(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3
*d^3)
```

Mupad [B]
time = 3.61, size = 206, normalized size = 0.98

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^2\,\sqrt{d+e\,x}}{c^3\,d^3\,e}-\frac{\sqrt{d+e\,x}\,\left(-16\,a^2\,e^2\,g^2+8\,a\,c\,d\,e\,f\,g+2\,c^2\,d^2\,f^2\right)}{3\,c^5\,d^5\,e}+\frac{4\,g\,x\,\left(2\,a\,e\,g-c\,d\,f\right)\,\sqrt{d+e\,x}}{c^4\,d^4\,e}\right)}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(3\,c^5\,d^6+6\,a\,c^4\,d^4\,e^2\right)}{3\,c^5\,d^5\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^2*(d + e*x)^(1/2))
/(c^3*d^3*e) - ((d + e*x)^(1/2)*(2*c^2*d^2*f^2 - 16*a^2*e^2*g^2 + 8*a*c*d*e
*f*g))/(3*c^5*d^5*e) + (4*g*x*(2*a*e*g - c*d*f)*(d + e*x)^(1/2))/(c^4*d^4*e
)))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(3*c^
5*d^6 + 6*a*c^4*d^4*e^2))/(3*c^5*d^5*e))
```

3.674 $\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2(2ae^2g+cd(ef-3dg))\sqrt{d+ex}}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2/3*(-a*e*g+c*d*f)*(e*x+d)^(5/2)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)+2/3*(2*a*e^2*g+c*d*(-3*d*g+e*f))*(e*x+d)^(1/2)/c^2/d^2/(-
a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {802, 662}

$$\frac{2\sqrt{d+ex}\,(2ae^2g+cd(ef-3dg))}{3c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/
2),x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(d + e*x)^(5/2))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*Sqrt[d
 + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(2ae^2g+cd\cdots}{\cdots}$$

$$= -\frac{2(cdf-aeg)(d+ex)^{5/2}}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{2(2ae\cdots}{3c^2d^2\left(cd^2-\cdots\right.}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.34

$$-\frac{2(d+ex)^{3/2}(2aeg+cd(f+3gx))}{3c^2d^2((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(2*a*e*g + c*d*(f + 3*g*x)))/(3*c^2*d^2*((a*e + c*d*x)*
(d + e*x))^(3/2))
```

Maple [A]
time = 0.14, size = 58, normalized size = 0.38

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(3cdgx+2aeg+cdf)}{3\sqrt{ex+d}\,(cdx+ae)^2c^2d^2}$	58
gosper	$-\frac{2(cdx+ae)(3cdgx+2aeg+cdf)(ex+d)^{\frac{5}{2}}}{3c^2d^2(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c*d*g*x+2*a*e*g+c*d*f)/(c
*d*x+a*e)^2/c^2/d^2
```

Maxima [A]
time = 0.32, size = 78, normalized size = 0.51

$$-\frac{2\,(3\,cdx+2\,ae)g}{3\,(c^3d^3x+ac^2d^2e)\sqrt{cdx+ae}} - \frac{2\,f}{3\,(c^2d^2x+acde)\sqrt{cdx+ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] -2/3*(3*c*d*x + 2*a*e)*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) - 2/
3*f/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [**A**]
time = 2.40, size = 130, normalized size = 0.84

$$-\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(3\,cdgx+cdf+2\,age)\sqrt{xe+d}}{3\,(c^4d^5x^2+a^2c^2d^2xe^3+(2\,ac^3d^3x^2+a^2c^2d^3)e^2+(c^4d^4x^3+2\,ac^3d^4x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(3*c*d*g*x + c*d*f + 2*a*g
*e)*sqrt(x*e + d)/(c^4*d^5*x^2 + a^2*c^2*d^2*x*e^3 + (2*a*c^3*d^3*x^2 + a^2
*c^2*d^3)*e^2 + (c^4*d^4*x^3 + 2*a*c^3*d^4*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.38, size = 153, normalized size = 0.99

$$\frac{2\,(3\,cd^2ge-cdfe^2-2\,age^3)}{3\left(\sqrt{-cd^2e+ae^3}\,c^3d^4-\sqrt{-cd^2e+ae^3}\,ac^2d^2e^2\right)}-\frac{2\,(cdfe^3-age^4+3\,((xe+d)cde-cd^2e+ae^3)ge)}{3\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] 2/3*(3*c*d^2*g*e - c*d*f*e^2 - 2*a*g*e^3)/(sqrt(-c*d^2*e + a*e^3)*c^3*d^4 -
 sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*e^2) - 2/3*(c*d*f*e^3 - a*g*e^4 + 3*((x*e
 + d)*c*d*e - c*d^2*e + a*e^3)*g*e)/(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3
/2)*c^2*d^2)
```

Mupad [B]
time = 3.50, size = 149, normalized size = 0.97

$$-\frac{\left(\frac{\left(\frac{4\,a\,e\,g}{3}+\frac{2\,c\,d\,f}{3}\right)\,\sqrt{d+e\,x}}{c^4\,d^4\,e}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c^3\,d^3\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c^4\,d^5+2\,a\,c^3\,d^3\,e^2\right)}{c^4\,d^4\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/
2),x)
```

```
[Out] -(((((4*a*e*g)/3 + (2*c*d*f)/3)*(d + e*x)^(1/2))/(c^4*d^4*e) + (2*g*x*(d +
e*x)^(1/2))/(c^3*d^3*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^
3 + (a^2*e)/(c^2*d) + (a*x*(a*e^2 + 2*c*d^2))/(c^2*d^2) + (x^2*(c^4*d^5 + 2
*a*c^3*d^3*e^2))/(c^4*d^4*e))
```

3.675 $\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {662}

$$-\frac{2(d+ex)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.77

$$-\frac{2(d+ex)^{3/2}}{3cd((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A]
time = 0.15, size = 42, normalized size = 0.88

method	result	size
default	$-\frac{2\sqrt{\left(c d x +a e \right) \left(e x +d \right)}}{3\sqrt{e x +d}\, \left(c d x +a e \right)^{2} c d}$	42
gosper	$-\frac{2 \left(c d x +a e \right) \left(e x +d \right)^{\frac{5}{2}}}{3 c d \left(c d e \,x^{2}+a \,e^{2} x +c \,d^{2} x +a d e \right)^{\frac{5}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^2/c/d
```

Maxima [A]
time = 0.30, size = 30, normalized size = 0.62

$$-\frac{2}{3\,{\left(c^{2} d^{2} x + a c d e\right)} \sqrt{c d x + a e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="maxima")
```

```
[Out] -2/3/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(44) = 88$.
time = 3.51, size = 107, normalized size = 2.23

$$-\frac{2\,\sqrt{c d^{2} x + a x e^{2} + {\left(c d x^{2} + a d\right)} e}\,\sqrt{x e + d}}{3\,{\left(c^{3} d^{4} x^{2} + a^{2} c d x e^{3} + {\left(2\,a c^{2} d^{2} x^{2} + a^{2} c d^{2}\right)} e^{2} + {\left(c^{3} d^{3} x^{3} + 2\,a c^{2} d^{3} x\right)} e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="fricas")
```

```
[Out] -2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^3*d^4*x^2
 + a^2*c*d*x*e^3 + (2*a*c^2*d^2*x^2 + a^2*c*d^2)*e^2 + (c^3*d^3*x^3 + 2*a*c
^2*d^3*x)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.40, size = 88, normalized size = 1.83

$$-\frac{2\,e^2}{3\left(\sqrt{-cd^2e+ae^3}\,c^2d^3-\sqrt{-cd^2e+ae^3}\,acde^2\right)}-\frac{2\,e^3}{3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorith
m="giac")
```

```
[Out] -2/3*e^2/(sqrt(-c*d^2*e + a*e^3)*c^2*d^3 - sqrt(-c*d^2*e + a*e^3)*a*c*d*e^2
) - 2/3*e^3/(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d)
```

Mupad [B]
time = 3.32, size = 110, normalized size = 2.29

$$-\frac{2\,\sqrt{d+e\,x}\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^2\,c\,d^2\,e^2+a^2\,c\,d\,e^3\,x+2\,a\,c^2\,d^3\,e\,x+2\,a\,c^2\,d^2\,e^2\,x^2+c^3\,d^4\,x^2+c^3\,d^3\,e\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(c^3*
d^4*x^2 + a^2*c*d^2*e^2 + c^3*d^3*e*x^3 + 2*a*c^2*d^3*e*x + a^2*c*d*e^3*x +
 2*a*c^2*d^2*e^2*x^2))
```

3.676 $\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2g\sqrt{d+ex}}{(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2g^{3/2}\tan^{-1}}{}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+2
*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d
*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(5/2)+2*g*(e*x+d)^(1/2)/(-a*e*g+c*d
*f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {882, 888, 211}

$$\frac{2g^{3/2}\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}+\frac{2g\sqrt{d+ex}}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2
)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) + (2*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)
^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
```

```
Q[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{g\int \frac{}{(f+}}{}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf-}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf-}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{(cdf-}$$

Mathematica [A]
time = 0.22, size = 129, normalized size = 0.69

$$\frac{2(d+ex)^{3/2}\left(\sqrt{cdf-aeg}\,(4aeg-cd(f-3gx))+3g^{3/2}(ae+cdx)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{3(cdf-aeg)^{5/2}((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(Sqrt[c*d*f - a*e*g]*(4*a*e*g - c*d*(f - 3*g*x)) + 3*g^(
3/2)*(a*e + c*d*x)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*
e*g]]))/(3*(c*d*f - a*e*g)^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A]
time = 0.15, size = 209, normalized size = 1.11

method	result
default	$-\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,\left(3\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)cd\,g^2x+3\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right.\right.}{3\sqrt{ex+d}\,(cdx+ae)^2(aeg-cdf)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*(c*d*x+a*e)^(1/2)*arctanh(g*(c*d*x+a*e)
^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g^2*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a
*e*g-c*d*f)*g)^(1/2))*a*e*g^2*(c*d*x+a*e)^(1/2)-3*((a*e*g-c*d*f)*g)^(1/2)*c
*d*g*x-4*((a*e*g-c*d*f)*g)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*c*d*f)/(e*x+
d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(5/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(176) = 352$.
time = 4.66, size = 1029, normalized size = 5.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="fricas")
```

```
[Out] [1/3*(3*(c^2*d^3*g*x^2 + a^2*g*x*e^3 + (2*a*c*d*g*x^2 + a^2*d*g)*e^2 + (c^2
*d^2*g*x^3 + 2*a*c*d^2*g*x)*e)*sqrt(-g/(c*d*f - a*g*e))*log(-(c*d^2*g*x - c
*d^2*f + 2*a*g*x*e^2 + 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*f
 - a*g*e)*sqrt(x*e + d)*sqrt(-g/(c*d*f - a*g*e)) + (c*d*g*x^2 - c*d*f*x + 2
*a*d*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*(3*c*d*g*x - c*d*f + 4*a*g*e)*sqrt(x*e + d))/(c^4*d^5*f^2*
```

```
x^2 + a^4*g^2*x*e^5 + (2*a^3*c*d*g^2*x^2 - 2*a^3*c*d*f*g*x + a^4*d*g^2)*e^4
 + (a^2*c^2*d^2*g^2*x^3 - 4*a^2*c^2*d^2*f*g*x^2 - 2*a^3*c*d^2*f*g + (a^2*c^
2*d^2*f^2 + 2*a^3*c*d^2*g^2)*x)*e^3 - (2*a*c^3*d^3*f*g*x^3 + 4*a^2*c^2*d^3*
f*g*x - a^2*c^2*d^3*f^2 - (2*a*c^3*d^3*f^2 + a^2*c^2*d^3*g^2)*x^2)*e^2 + (c
^4*d^4*f^2*x^3 - 2*a*c^3*d^4*f*g*x^2 + 2*a*c^3*d^4*f^2*x)*e), 2/3*(3*(c^2*d
^3*g*x^2 + a^2*g*x*e^3 + (2*a*c*d*g*x^2 + a^2*d*g)*e^2 + (c^2*d^2*g*x^3 + 2
*a*c*d^2*g*x)*e)*sqrt(g/(c*d*f - a*g*e))*arctan(-sqrt(c*d^2*x + a*x*e^2 + (
c*d*x^2 + a*d)*e)*(c*d*f - a*g*e)*sqrt(x*e + d)*sqrt(g/(c*d*f - a*g*e))/(c*
d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) + sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*(3*c*d*g*x - c*d*f + 4*a*g*e)*sqrt(x*e + d))/(c^4*d^5*f^2*
x^2 + a^4*g^2*x*e^5 + (2*a^3*c*d*g^2*x^2 - 2*a^3*c*d*f*g*x + a^4*d*g^2)*e^4
 + (a^2*c^2*d^2*g^2*x^3 - 4*a^2*c^2*d^2*f*g*x^2 - 2*a^3*c*d^2*f*g + (a^2*c^
2*d^2*f^2 + 2*a^3*c*d^2*g^2)*x)*e^3 - (2*a*c^3*d^3*f*g*x^3 + 4*a^2*c^2*d^3*
f*g*x - a^2*c^2*d^3*f^2 - (2*a*c^3*d^3*f^2 + a^2*c^2*d^3*g^2)*x^2)*e^2 + (c
^4*d^4*f^2*x^3 - 2*a*c^3*d^4*f*g*x^2 + 2*a*c^3*d^4*f^2*x)*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2
),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(176) = 352.
time = 1.91, size = 661, normalized size = 3.52

$$\frac{2}{3}\left(\frac{3\,g^{2}\arctan\left(\frac{\sqrt{(xe+d)cde-cd^{2}e+ae^{3}}\,ge^{(-1)}}{\sqrt{cdfg-ag^{2}e}}\right)e^{(-1)}}{\left(c^{2}d^{2}f^{2}e-2\,acdfge^{2}+a^{2}g^{2}e^{3}\right)\sqrt{cdfg-ag^{2}e}}-\frac{cdfe^{2}-age^{3}-3\left(\left(xe+d\right)cde-cd^{2}e+ae^{3}\right)g}{\left(c^{2}d^{2}f^{2}e-2\,acdfge^{2}+a^{2}g^{2}e^{3}\right)\left(\left(xe+d\right)cde-cd^{2}e+ae^{3}\right)^{\frac{3}{2}}}\right)e^{2}-\frac{2\left(3\sqrt{-cd^{2}e+ae^{3}}\,cd^{2}g^{2}\arctan\left(\frac{\sqrt{-cd^{2}e+ae^{3}}\,ge^{(-1)}}{\sqrt{cdfg-ag^{2}e}}\right)-3\sqrt{-cd^{2}e+ae^{3}}\,ag^{2}\arctan\left(\frac{\sqrt{-cd^{2}e+ae^{3}}\,ge^{(-1)}}{\sqrt{cdfg-ag^{2}e}}\right)e^{2}+3\sqrt{cdfg-ag^{2}e}\,cd^{2}ge+\sqrt{cdfg-ag^{2}e}\,cdfe^{2}-4\sqrt{cdfg-ag^{2}e}\,age^{3}\right)}{3\left(\sqrt{cdfg-ag^{2}e}\sqrt{-cd^{2}e+ae^{3}}\,c^{3}d^{4}f^{2}-2\sqrt{cdfg-ag^{2}e}\sqrt{-cd^{2}e+ae^{3}}\,ac^{2}d^{3}fge-\sqrt{cdfg-ag^{2}e}\sqrt{-cd^{2}e+ae^{3}}\,ac^{2}d^{2}f^{2}e^{2}+\sqrt{cdfg-ag^{2}e}\sqrt{-cd^{2}e+ae^{3}}\,a^{2}cd^{2}g^{2}e^{2}+2\sqrt{cdfg-ag^{2}e}\sqrt{-cd^{2}e+ae^{3}}\,a^{2}cdfge^{3}-\sqrt{cdfg-ag^{2}e}\sqrt{-cd^{2}e+ae^{3}}\,a^{3}g^{2}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,
algorithm="giac")
```

```
[Out] 2/3*(3*g^2*arctan(sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*g*e^(-1)/sqrt(c*d
*f*g - a*g^2*e))*e^(-1)/((c^2*d^2*f^2*e - 2*a*c*d*f*g*e^2 + a^2*g^2*e^3)*sq
rt(c*d*f*g - a*g^2*e)) - (c*d*f*e^2 - a*g*e^3 - 3*((x*e + d)*c*d*e - c*d^2*
e + a*e^3)*g)/((c^2*d^2*f^2*e - 2*a*c*d*f*g*e^2 + a^2*g^2*e^3)*((x*e + d)*c
*d*e - c*d^2*e + a*e^3)^(3/2)))*e^2 - 2/3*(3*sqrt(-c*d^2*e + a*e^3)*c*d^2*g
^2*arctan(sqrt(-c*d^2*e + a*e^3)*g*e^(-1)/sqrt(c*d*f*g - a*g^2*e)) - 3*sqrt
(-c*d^2*e + a*e^3)*a*g^2*arctan(sqrt(-c*d^2*e + a*e^3)*g*e^(-1)/sqrt(c*d*f*
g - a*g^2*e))*e^2 + 3*sqrt(c*d*f*g - a*g^2*e)*c*d^2*g*e + sqrt(c*d*f*g - a*
```

```
g^2*e)*c*d*f*e^2 - 4*sqrt(c*d*f*g - a*g^2*e)*a*g*e^3)/(sqrt(c*d*f*g - a*g^2
*e)*sqrt(-c*d^2*e + a*e^3)*c^3*d^4*f^2 - 2*sqrt(c*d*f*g - a*g^2*e)*sqrt(-c*
d^2*e + a*e^3)*a*c^2*d^3*f*g*e - sqrt(c*d*f*g - a*g^2*e)*sqrt(-c*d^2*e + a*
e^3)*a*c^2*d^2*f^2*e^2 + sqrt(c*d*f*g - a*g^2*e)*sqrt(-c*d^2*e + a*e^3)*a^2
*c*d^2*g^2*e^2 + 2*sqrt(c*d*f*g - a*g^2*e)*sqrt(-c*d^2*e + a*e^3)*a^2*c*d*f
*g*e^3 - sqrt(c*d*f*g - a*g^2*e)*sqrt(-c*d^2*e + a*e^3)*a^3*g^2*e^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2
)), x)
```

3.677 $\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{10g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)+5*c*d*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(7/2)+10/3*g*(e*x+d)^(1
/2)/(-a*e*g+c*d*f)^2/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*g^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1
/2)
```

Rubi [A]
time = 0.24, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {882, 886, 888, 211}

$$\frac{5cdg^{3/2}\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}}+\frac{5g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3(f+gx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2)) + (10*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*x)
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) + (5*c
*d*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sq
rt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(c*d*f - a*e*g)^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
```

```
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}$$

Mathematica [A]
time = 0.44, size = 180, normalized size = 0.67

$$\frac{(d+ex)^{3/2}\left(\sqrt{cdf-aeg}\,(3a^2e^2g^2+2acdeg(7f+10gx)+c^2d^2(-2f^2+10fgx+15g^2x^2))+15cdg^{3/2}(ae+cdx)^{3/2}(f+gx)\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{3(cdf-aeg)^{7/2}((ae+cdx)(d+ex))^{3/2}(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] ((d + e*x)^(3/2)*(Sqrt[c*d*f - a*e*g]*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(7*f + 1
0*g*x) + c^2*d^2*(-2*f^2 + 10*f*g*x + 15*g^2*x^2)) + 15*c*d*g^(3/2)*(a*e +
c*d*x)^(3/2)*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*
g]]))/(3*(c*d*f - a*e*g)^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x))
```

Maple [A]
time = 0.14, size = 414, normalized size = 1.54

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\sqrt{cdx+ae}\,\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^3x^2+15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*(c*d*x+a*e)^(1/2)*arctanh(g*(c*d*x+a*e)
^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*g^3*x^2+15*arctanh(g*(c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*g^3*x*(c*d*x+a*e)^(1/2)+15*(c*d*x+a*e)
^(1/2)*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g^2*x
+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g^2*(c*d
*x+a*e)^(1/2)-15*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*g^2*x^2-20*((a*e*g-c*d*f)*
g)^(1/2)*a*c*d*e*g^2*x-10*((a*e*g-c*d*f)*g)^(1/2)*c^2*d^2*f*g*x-3*((a*e*g-c
*d*f)*g)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*a*c*d*e*f*g+2*((a*e*g
-c*d*f)*g)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^3/(
g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(5/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(253) = 506$.
time = 3.62, size = 1969, normalized size = 7.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^3*d^4*g^2*x^3 + c^3*d^4*f*g*x^2 + (a^2*c*d*g^2*x^2 + a^2*c*d*f
*g*x)*e^3 + (2*a*c^2*d^2*g^2*x^3 + 2*a*c^2*d^2*f*g*x^2 + a^2*c*d^2*g^2*x +
a^2*c*d^2*f*g)*e^2 + (c^3*d^3*g^2*x^4 + c^3*d^3*f*g*x^3 + 2*a*c^2*d^3*g^2*x
^2 + 2*a*c^2*d^3*f*g*x)*e)*sqrt(-g/(c*d*f - a*g*e))*log(-(c*d^2*g*x - c*d^2
*f + 2*a*g*x*e^2 - 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*f - a
*g*e)*sqrt(x*e + d)*sqrt(-g/(c*d*f - a*g*e)) + (c*d*g*x^2 - c*d*f*x + 2*a*d
*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) - 2*(15*c^2*d^2*g^2*x^2 + 10*c^2*d^
2*f*g*x - 2*c^2*d^2*f^2 + 3*a^2*g^2*e^2 + 2*(10*a*c*d*g^2*x + 7*a*c*d*f*g)*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^5*d^6*f^3*
g*x^3 + c^5*d^6*f^4*x^2 - (a^5*g^4*x^2 + a^5*f*g^3*x)*e^6 - (2*a^4*c*d*g^4*
x^3 - a^4*c*d*f*g^3*x^2 + a^5*d*f*g^3 - (3*a^4*c*d*f^2*g^2 - a^5*d*g^4)*x)*
e^5 - (a^3*c^2*d^2*g^4*x^4 - 5*a^3*c^2*d^2*f*g^3*x^3 - 3*a^4*c*d^2*f^2*g^2
- (3*a^3*c^2*d^2*f^2*g^2 - 2*a^4*c*d^2*g^4)*x^2 + (3*a^3*c^2*d^2*f^3*g - a^
4*c*d^2*f*g^3)*x)*e^4 + (3*a^2*c^3*d^3*f*g^3*x^4 - 3*a^3*c^2*d^3*f^3*g - (3
*a^2*c^3*d^3*f^2*g^2 + a^3*c^2*d^3*g^4)*x^3 - 5*(a^2*c^3*d^3*f^3*g - a^3*c^
2*d^3*f*g^3)*x^2 + (a^2*c^3*d^3*f^4 + 3*a^3*c^2*d^3*f^2*g^2)*x)*e^3 - (3*a*
c^4*d^4*f^2*g^2*x^4 + 5*a^2*c^3*d^4*f^3*g*x - a^2*c^3*d^4*f^4 + (a*c^4*d^4*
f^3*g - 3*a^2*c^3*d^4*f*g^3)*x^3 - (2*a*c^4*d^4*f^4 - 3*a^2*c^3*d^4*f^2*g^2
)*x^2)*e^2 + (c^5*d^5*f^3*g*x^4 - a*c^4*d^5*f^3*g*x^2 + 2*a*c^4*d^5*f^4*x +
 (c^5*d^5*f^4 - 3*a*c^4*d^5*f^2*g^2)*x^3)*e), 1/3*(15*(c^3*d^4*g^2*x^3 + c^
3*d^4*f*g*x^2 + (a^2*c*d*g^2*x^2 + a^2*c*d*f*g*x)*e^3 + (2*a*c^2*d^2*g^2*x^
3 + 2*a*c^2*d^2*f*g*x^2 + a^2*c*d^2*g^2*x + a^2*c*d^2*f*g)*e^2 + (c^3*d^3*g
^2*x^4 + c^3*d^3*f*g*x^3 + 2*a*c^2*d^3*g^2*x^2 + 2*a*c^2*d^3*f*g*x)*e)*sqrt
(g/(c*d*f - a*g*e))*arctan(-sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*
d*f - a*g*e)*sqrt(x*e + d)*sqrt(g/(c*d*f - a*g*e))/(c*d^2*g*x + a*g*x*e^2 +
 (c*d*g*x^2 + a*d*g)*e)) + (15*c^2*d^2*g^2*x^2 + 10*c^2*d^2*f*g*x - 2*c^2*d
^2*f^2 + 3*a^2*g^2*e^2 + 2*(10*a*c*d*g^2*x + 7*a*c*d*f*g)*e)*sqrt(c*d^2*x +
 a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^5*d^6*f^3*g*x^3 + c^5*d^6*f
^4*x^2 - (a^5*g^4*x^2 + a^5*f*g^3*x)*e^6 - (2*a^4*c*d*g^4*x^3 - a^4*c*d*f*g
^3*x^2 + a^5*d*f*g^3 - (3*a^4*c*d*f^2*g^2 - a^5*d*g^4)*x)*e^5 - (a^3*c^2*d^
```

```
2*g^4*x^4 - 5*a^3*c^2*d^2*f*g^3*x^3 - 3*a^4*c*d^2*f^2*g^2 - (3*a^3*c^2*d^2*
f^2*g^2 - 2*a^4*c*d^2*g^4)*x^2 + (3*a^3*c^2*d^2*f^3*g - a^4*c*d^2*f*g^3)*x)
*e^4 + (3*a^2*c^3*d^3*f*g^3*x^4 - 3*a^3*c^2*d^3*f^3*g - (3*a^2*c^3*d^3*f^2*
g^2 + a^3*c^2*d^3*g^4)*x^3 - 5*(a^2*c^3*d^3*f^3*g - a^3*c^2*d^3*f*g^3)*x^2
+ (a^2*c^3*d^3*f^4 + 3*a^3*c^2*d^3*f^2*g^2)*x)*e^3 - (3*a*c^4*d^4*f^2*g^2*x
^4 + 5*a^2*c^3*d^4*f^3*g*x - a^2*c^3*d^4*f^4 + (a*c^4*d^4*f^3*g - 3*a^2*c^3
*d^4*f*g^3)*x^3 - (2*a*c^4*d^4*f^4 - 3*a^2*c^3*d^4*f^2*g^2)*x^2)*e^2 + (c^5
*d^5*f^3*g*x^4 - a*c^4*d^5*f^3*g*x^2 + 2*a*c^4*d^5*f^4*x + (c^5*d^5*f^4 - 3
*a*c^4*d^5*f^2*g^2)*x^3)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)), x)
```

3.678 $\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=342

$$-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{14g\sqrt{d+ex}}{3(cdf-aeg)^2(f+gx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)+35/4*c^2*d^2*g^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/(-a*e*g+c*d*f)^(9/2)+14/3*g*(
e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)+35/6*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x
+f)^2/(e*x+d)^(1/2)+35/4*c*d*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-
a*e*g+c*d*f)^4/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {882, 886, 888, 211}

$$\frac{35c^2d^2g^{3/2}\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}}+\frac{35cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^4}+\frac{35g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{6\sqrt{d+ex}(f+gx)^2(cdf-aeg)^3}+\frac{14g\sqrt{d+ex}}{3(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2)) + (14*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*
x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (35*g^2*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(6*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)^
2) + (35*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a
*e*g)^4*Sqrt[d + e*x]*(f + g*x)) + (35*c^2*d^2*g^(3/2)*ArcTan[(Sqrt[g]*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x]
)])/(4*(c*d*f - a*e*g)^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
```

```
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}$$

$$= -\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}$$

Mathematica [**A**]
time = 0.80, size = 240, normalized size = 0.70

$$\frac{c^2d^2\sqrt{d+ex}\left(\frac{-6a^3e^3g^3+3a^2cde^2g^2(13f+7gx)+2ac^2d^2eg(40f^2+119fgx+70g^2x^2)+c^3d^3(-8f^3+56f^2gx+175fg^2x^2+105g^3x^3)}{c^2d^2(cdf-aeg)^4(ae+cdx)(f+gx)^2}+\frac{105g^{3/2}\sqrt{ae+cdx}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{9/2}}\right)}{12\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(5/2)),x]
```

```
[Out] (c^2*d^2*Sqrt[d + e*x]*((-6*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(13*f + 7*g*x)
+ 2*a*c^2*d^2*e*g*(40*f^2 + 119*f*g*x + 70*g^2*x^2) + c^3*d^3*(-8*f^3 + 56*
f^2*g*x + 175*f*g^2*x^2 + 105*g^3*x^3))/(c^2*d^2*(c*d*f - a*e*g)^4*(a*e + c
*d*x)*(f + g*x)^2) + (105*g^(3/2)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*
e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(9/2)))/(12*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(304) = 608$.
time = 0.14, size = 660, normalized size = 1.93

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(105\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^3d^3g^4x^3\sqrt{cdx+ae}+105\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+a}}{\sqrt{(aeg-cd}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/12*((c*d*x+a*e)*(e*x+d))^(1/2)*(105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^3*d^3*g^4*x^3*(c*d*x+a*e)^(1/2)+105*arctanh(g*(c*d*x+a*e
)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e*g^4*x^2*(c*d*x+a*e)^(1/2)+210*
arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*f*g^3*x^2*(c*d
*x+a*e)^(1/2)+210*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^
2*d^2*e*f*g^3*x*(c*d*x+a*e)^(1/2)+105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*f^2*g^2*x*(c*d*x+a*e)^(1/2)-105*((a*e*g-c*d*f)*g)^(
1/2)*c^3*d^3*g^3*x^3+105*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
))*a*c^2*d^2*e*f^2*g^2*(c*d*x+a*e)^(1/2)-140*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*
d^2*e*g^3*x^2-175*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f*g^2*x^2-21*((a*e*g-c*d*
f)*g)^(1/2)*a^2*c*d*e^2*g^3*x-238*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f*g^2
*x-56*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^2*g*x+6*((a*e*g-c*d*f)*g)^(1/2)*a^3
*e^3*g^3-39*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-80*((a*e*g-c*d*f)*g)^
(1/2)*a*c^2*d^2*e*f^2*g+8*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2
)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(5/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(320) = 640$.
time = 6.27, size = 3043, normalized size = 8.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")

[Out] [1/24*(105*(c^4*d^5*g^3*x^4 + 2*c^4*d^5*f*g^2*x^3 + c^4*d^5*f^2*g*x^2 + (a^
2*c^2*d^2*g^3*x^3 + 2*a^2*c^2*d^2*f*g^2*x^2 + a^2*c^2*d^2*f^2*g*x)*e^3 + (2
*a*c^3*d^3*g^3*x^4 + 4*a*c^3*d^3*f*g^2*x^3 + 2*a^2*c^2*d^3*f*g^2*x + a^2*c^
2*d^3*f^2*g + (2*a*c^3*d^3*f^2*g + a^2*c^2*d^3*g^3)*x^2)*e^2 + (c^4*d^4*g^3
*x^5 + 2*c^4*d^4*f*g^2*x^4 + 4*a*c^3*d^4*f*g^2*x^2 + 2*a*c^3*d^4*f^2*g*x +
(c^4*d^4*f^2*g + 2*a*c^3*d^4*g^3)*x^3)*e)*sqrt(-g/(c*d*f - a*g*e))*log(-(c*
d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d
)*e)*(c*d*f - a*g*e)*sqrt(x*e + d)*sqrt(-g/(c*d*f - a*g*e)) + (c*d*g*x^2 -
c*d*f*x + 2*a*d*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(105*c^3*d^3*g^3
*x^3 + 175*c^3*d^3*f*g^2*x^2 + 56*c^3*d^3*f^2*g*x - 8*c^3*d^3*f^3 - 6*a^3*g
^3*e^3 + 3*(7*a^2*c*d*g^3*x + 13*a^2*c*d*f*g^2)*e^2 + 2*(70*a*c^2*d^2*g^3*x
^2 + 119*a*c^2*d^2*f*g^2*x + 40*a*c^2*d^2*f^2*g)*e)*sqrt(c*d^2*x + a*x*e^2
+ (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^6*d^7*f^4*g^2*x^4 + 2*c^6*d^7*f^5*g*
x^3 + c^6*d^7*f^6*x^2 + (a^6*g^6*x^3 + 2*a^6*f*g^5*x^2 + a^6*f^2*g^4*x)*e^7
 + (2*a^5*c*d*g^6*x^4 + a^6*d*f^2*g^4 - (6*a^5*c*d*f^2*g^4 - a^6*d*g^6)*x^2
 - 2*(2*a^5*c*d*f^3*g^3 - a^6*d*f*g^5)*x)*e^6 + (a^4*c^2*d^2*g^6*x^5 - 6*a^
4*c^2*d^2*f*g^5*x^4 + 4*a^4*c^2*d^2*f^3*g^3*x^2 - 4*a^5*c*d^2*f^3*g^3 - (9*
a^4*c^2*d^2*f^2*g^4 - 2*a^5*c*d^2*g^6)*x^3 + 6*(a^4*c^2*d^2*f^4*g^2 - a^5*c
*d^2*f^2*g^4)*x)*e^5 - (4*a^3*c^3*d^3*f*g^5*x^5 - 6*a^4*c^2*d^3*f^4*g^2 - (
4*a^3*c^3*d^3*f^2*g^4 + a^4*c^2*d^3*g^6)*x^4 - 2*(8*a^3*c^3*d^3*f^3*g^3 - 3
*a^4*c^2*d^3*f*g^5)*x^3 - (4*a^3*c^3*d^3*f^4*g^2 - 9*a^4*c^2*d^3*f^2*g^4)*x
^2 + 4*(a^3*c^3*d^3*f^5*g - a^4*c^2*d^3*f^3*g^3)*x)*e^4 + (6*a^2*c^4*d^4*f^
2*g^4*x^5 - 4*a^3*c^3*d^4*f^5*g + 4*(a^2*c^4*d^4*f^3*g^3 - a^3*c^3*d^4*f*g^
5)*x^4 - (9*a^2*c^4*d^4*f^4*g^2 - 4*a^3*c^3*d^4*f^2*g^4)*x^3 - 2*(3*a^2*c^4
*d^4*f^5*g - 8*a^3*c^3*d^4*f^3*g^3)*x^2 + (a^2*c^4*d^4*f^6 + 4*a^3*c^3*d^4*
f^4*g^2)*x)*e^3 - (4*a*c^5*d^5*f^3*g^3*x^5 - 4*a^2*c^4*d^5*f^3*g^3*x^3 + 6*
a^2*c^4*d^5*f^5*g*x - a^2*c^4*d^5*f^6 + 6*(a*c^5*d^5*f^4*g^2 - a^2*c^4*d^5*
f^2*g^4)*x^4 - (2*a*c^5*d^5*f^6 - 9*a^2*c^4*d^5*f^4*g^2)*x^2)*e^2 + (c^6*d^
6*f^4*g^2*x^5 + 2*a*c^5*d^6*f^6*x + 2*(c^6*d^6*f^5*g - 2*a*c^5*d^6*f^3*g^3)
*x^4 + (c^6*d^6*f^6 - 6*a*c^5*d^6*f^4*g^2)*x^3)*e), 1/12*(105*(c^4*d^5*g^3*
x^4 + 2*c^4*d^5*f*g^2*x^3 + c^4*d^5*f^2*g*x^2 + (a^2*c^2*d^2*g^3*x^3 + 2*a^
2*c^2*d^2*f*g^2*x^2 + a^2*c^2*d^2*f^2*g*x)*e^3 + (2*a*c^3*d^3*g^3*x^4 + 4*a
*c^3*d^3*f*g^2*x^3 + 2*a^2*c^2*d^3*f*g^2*x + a^2*c^2*d^3*f^2*g + (2*a*c^3*d
^3*f^2*g + a^2*c^2*d^3*g^3)*x^2)*e^2 + (c^4*d^4*g^3*x^5 + 2*c^4*d^4*f*g^2*x
^4 + 4*a*c^3*d^4*f*g^2*x^2 + 2*a*c^3*d^4*f^2*g*x + (c^4*d^4*f^2*g + 2*a*c^3
*d^4*g^3)*x^3)*e)*sqrt(g/(c*d*f - a*g*e))*arctan(-sqrt(c*d^2*x + a*x*e^2 +
(c*d*x^2 + a*d)*e)*(c*d*f - a*g*e)*sqrt(x*e + d)*sqrt(g/(c*d*f - a*g*e))/(c
*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) + (105*c^3*d^3*g^3*x^3 + 175
*c^3*d^3*f*g^2*x^2 + 56*c^3*d^3*f^2*g*x - 8*c^3*d^3*f^3 - 6*a^3*g^3*e^3 + 3
*(7*a^2*c*d*g^3*x + 13*a^2*c*d*f*g^2)*e^2 + 2*(70*a*c^2*d^2*g^3*x^2 + 119*a
*c^2*d^2*f*g^2*x + 40*a*c^2*d^2*f^2*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*sqrt(x*e + d))/(c^6*d^7*f^4*g^2*x^4 + 2*c^6*d^7*f^5*g*x^3 + c^6*
```

```
d^7*f^6*x^2 + (a^6*g^6*x^3 + 2*a^6*f*g^5*x^2 + a^6*f^2*g^4*x)*e^7 + (2*a^5*
c*d*g^6*x^4 + a^6*d*f^2*g^4 - (6*a^5*c*d*f^2*g^4 - a^6*d*g^6)*x^2 - 2*(2*a^
5*c*d*f^3*g^3 - a^6*d*f*g^5)*x)*e^6 + (a^4*c^2*d^2*g^6*x^5 - 6*a^4*c^2*d^2*
f*g^5*x^4 + 4*a^4*c^2*d^2*f^3*g^3*x^2 - 4*a^5*c*d^2*f^3*g^3 - (9*a^4*c^2*d^
2*f^2*g^4 - 2*a^5*c*d^2*g^6)*x^3 + 6*(a^4*c^2*d^2*f^4*g^2 - a^5*c*d^2*f^2*g
^4)*x)*e^5 - (4*a^3*c^3*d^3*f*g^5*x^5 - 6*a^4*c^2*d^3*f^4*g^2 - (4*a^3*c^3*
d^3*f^2*g^4 + a^4*c^2*d^3*g^6)*x^4 - 2*(8*a^3*c^3*d^3*f^3*g^3 - 3*a^4*c^2*d
^3*f*g^5)*x^3 - (4*a^3*c^3*d^3*f^4*g^2 - 9*a^4*c^2*d^3*f^2*g^4)*x^2 + 4*(a^
3*c^3*d^3*f^5*g - a^4*c^2*d^3*f^3*g^3)*x)*e^4 + (6*a^2*c^4*d^4*f^2*g^4*x^5
- 4*a^3*c^3*d^4*f^5*g + 4*(a^2*c^4*d^4*f^3*g^3 - a^3*c^3*d^4*f*g^5)*x^4 - (
9*a^2*c^4*d^4*f^4*g^2 - 4*a^3*c^3*d^4*f^2*g^4)*x^3 - 2*(3*a^2*c^4*d^4*f^5*g
 - 8*a^3*c^3*d^4*f^3*g^3)*x^2 + (a^2*c^4*d^4*f^6 + 4*a^3*c^3*d^4*f^4*g^2)*x
)*e^3 - (4*a*c^5*d^5*f^3*g^3*x^5 - 4*a^2*c^4*d^5*f^3*g^3*x^3 + 6*a^2*c^4*d^
5*f^5*g*x - a^2*c^4*d^5*f^6 + 6*(a*c^5*d^5*f^4*g^2 - a^2*c^4*d^5*f^2*g^4)*x
^4 - (2*a*c^5*d^5*f^6 - 9*a^2*c^4*d^5*f^4*g^2)*x^2)*e^2 + (c^6*d^6*f^4*g^2*
x^5 + 2*a*c^5*d^6*f^6*x + 2*(c^6*d^6*f^5*g - 2*a*c^5*d^6*f^3*g^3)*x^4 + (c^
6*d^6*f^6 - 6*a*c^5*d^6*f^4*g^2)*x^3)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{5/2}}{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)),x)
```

```
[Out] int((d + e*x)^(5/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5
/2)), x)
```

3.679 $\int \frac{(f+gx)^4 \sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} \, dx$

Optimal. Leaf size=336

$$-\frac{128(cdf-aeg)^3\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3465c^5d^5e(d+ex)^{3/2}}+\frac{128g(cdf-aeg)^3\left(ade+\left(cd^2\right.\right.}{1155c^4d^4e\sqrt{d+}}$$

```
[Out] -128/3465*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)/c^5/d^5/e/(e*x+d)^(3/2)+32/231*(-a*e*g+c*d*f)^2*(g*x
+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/(e*x+d)^(3/2)+16/99*(
-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*
x+d)^(3/2)+2/11*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+
d)^(3/2)+128/1155*g*(-a*e*g+c*d*f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/c^4/d^4/e/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^3\left(2ae^2g-cd(5ef-3dg)\right)}{3465c^5d^5e(d+ex)^{3/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^3}{1155c^4d^4e\sqrt{d+ex}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{231c^3d^3(d+ex)^{3/2}}+\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{99c^2d^2(d+ex)^{3/2}}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(3465*c^5*d^5*e*(d + e*x)^(3/2)) + (128*g*(c*
d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(1155*c^4*d^4
*e*Sqrt[d + e*x]) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(231*c^3*d^3*(d + e*x)^(3/2)) + (16*(c*d*f - a*e*
g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(99*c^2*d^2*(
d + e*x)^(3/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3
/2))/(11*c*d*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{11cd(d+ex)^{3/2}} + \frac{(8(cdf-\ldots}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{99c^2d^2(d+ex)^{3/2}}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/\ldots}}{231c^3d^3(d+ex)^{3/2}}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{1155c^4d^4e\sqrt{d+ex}} + \frac{32(\ldots}{}$$

$$= \frac{128(cdf-aeg)^3\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+(cd^2+ae^2)x+\ldots\right.}{3465c^4d^4(d+ex)^{3/2}}$$

Mathematica [A]
time = 0.15, size = 195, normalized size = 0.58

$$\frac{2((ae+cdx)(d+ex))^{3/2}\left(128a^4e^4g^4-64a^3cde^3g^3(11f+3gx)+48a^2c^2d^2e^2g^2(33f^2+22fgx+5g^2x^2)-8ac^3d^3eg(231f^3+297f^2gx+165fg^2x^2+35g^3x^3)+c^4d^4(1155f^4+2772f^3gx+2970f^2g^2x^2+1540fg^3x^3+315g^4x^4)\right)}{3465c^5d^5(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
1*f + 3*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(33*f^2 + 22*f*g*x + 5*g^2*x^2) - 8*a
*c^3*d^3*e*g*(231*f^3 + 297*f^2*g*x + 165*f*g^2*x^2 + 35*g^3*x^3) + c^4*d^4
*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 1540*f*g^3*x^3 + 315*g^4*x^4
)))/(3465*c^5*d^5*(d + e*x)^(3/2))
```

Maple [A]
time = 0.14, size = 273, normalized size = 0.81

method	result
default	$\frac{2(cdx+ae)(315g^4x^4c^4d^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e^3g^4\cdots}{}$
gosper	$\frac{2(cdx+ae)(315g^4x^4c^4d^4-280a\,c^3d^3e\,g^4x^3+1540c^4d^4f\,g^3x^3+240a^2c^2d^2e^2g^4x^2-1320a\,c^3d^3ef\,g^3x^2+2970c^4d^4f^2g^2x^2-192a^3cd\,e^3g^4\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/3465*(c*d*x+a*e)*(315*c^4*d^4*g^4*x^4-280*a*c^3*d^3*e*g^4*x^3+1540*c^4*d^
4*f*g^3*x^3+240*a^2*c^2*d^2*e^2*g^4*x^2-1320*a*c^3*d^3*e*f*g^3*x^2+2970*c^4
*d^4*f^2*g^2*x^2-192*a^3*c*d*e^3*g^4*x+1056*a^2*c^2*d^2*e^2*f*g^3*x-2376*a*
c^3*d^3*e*f^2*g^2*x+2772*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-704*a^3*c*d*e^3*f*
g^3+1584*a^2*c^2*d^2*e^2*f^2*g^2-1848*a*c^3*d^3*e*f^3*g+1155*c^4*d^4*f^4)*(
(c*d*x+a*e)*(e*x+d))^(1/2)/c^5/d^5/(e*x+d)^(1/2)
```

Maxima [A]
time = 0.36, size = 319, normalized size = 0.95

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f^4}{3\,cd}+\frac{8\,(3\,c^2d^2x^2+acdxe-2\,a^2e^2)\sqrt{cdx+ae}\,f^3g}{15\,c^2d^2}+\frac{4\,(15\,c^3d^3x^3+3\,ac^2d^2x^2e-4\,a^2cdxe^2+8\,a^3e^3)\sqrt{cdx+ae}\,f^2g^2}{35\,c^3d^3}+\frac{8\,(35\,c^4d^4x^4+5\,ac^3d^3x^3e-6\,a^2c^2d^2x^2e^2+8\,a^3cdxe^3-16\,a^4e^4)\sqrt{cdx+ae}\,fg^3}{315\,c^4d^4}+\frac{2\,(315\,c^5d^5x^5+35\,ac^4d^4x^4e-40\,a^2c^3d^3x^3e^2+48\,a^3c^2d^2x^2e^3-64\,a^4cdxe^4+128\,a^5e^5)\sqrt{cdx+ae}\,g^4}{3465\,c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^4/(c*d) + 8/15*(3*c^2*d^2*x^2 + a*c*d*x*e - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2) + 4/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^
2*x^2*e - 4*a^2*c*d*x*e^2 + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f^2*g^2/(c^3*d^3)
+ 8/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*x^3*e - 6*a^2*c^2*d^2*x^2*e^2 + 8*a^3
*c*d*x*e^3 - 16*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/3465*(315*c^
5*d^5*x^5 + 35*a*c^4*d^4*x^4*e - 40*a^2*c^3*d^3*x^3*e^2 + 48*a^3*c^2*d^2*x^
2*e^3 - 64*a^4*c*d*x*e^4 + 128*a^5*e^5)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Fricas [**A**]
time = 2.75, size = 374, normalized size = 1.11

$$\frac{2\,(315\,c^5d^5g^4x^5+1540\,c^5d^5fg^3x^4+2970\,c^5d^5f^2g^2x^3+2772\,c^5d^5f^3gx^2+1155\,c^5d^5f^4x+128\,a^5g^4e^5-64\,(a^4cdg^4x+11\,a^4cdfg^3)e^4+16\,(3\,a^3c^2d^2g^4x^2+22\,a^3c^2d^2fg^3x+99\,a^3c^2d^2f^2g^2)e^3-8\,(5\,a^2c^3d^3g^4x^3+33\,a^2c^3d^3fg^3x^2+99\,a^2c^3d^3f^2g^2x+231\,a^2c^3d^3f^3g)e^2+(35\,ac^4d^4g^4x^4+220\,ac^4d^4fg^3x^3+594\,ac^4d^4f^2g^2x^2+924\,ac^4d^4f^3gx+1155\,ac^4d^4f^4)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{3465\,(c^5d^5xe+c^5d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(315*c^5*d^5*g^4*x^5 + 1540*c^5*d^5*f*g^3*x^4 + 2970*c^5*d^5*f^2*g^2
*x^3 + 2772*c^5*d^5*f^3*g*x^2 + 1155*c^5*d^5*f^4*x + 128*a^5*g^4*e^5 - 64*(
a^4*c*d*g^4*x + 11*a^4*c*d*f*g^3)*e^4 + 16*(3*a^3*c^2*d^2*g^4*x^2 + 22*a^3*
c^2*d^2*f*g^3*x + 99*a^3*c^2*d^2*f^2*g^2)*e^3 - 8*(5*a^2*c^3*d^3*g^4*x^3 +
33*a^2*c^3*d^3*f*g^3*x^2 + 99*a^2*c^3*d^3*f^2*g^2*x + 231*a^2*c^3*d^3*f^3*g
)*e^2 + (35*a*c^4*d^4*g^4*x^4 + 220*a*c^4*d^4*f*g^3*x^3 + 594*a*c^4*d^4*f^2
*g^2*x^2 + 924*a*c^4*d^4*f^3*g*x + 1155*a*c^4*d^4*f^4)*e)*sqrt(c*d^2*x + a*
x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^5*d^5*x*e + c^5*d^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^4}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**4/sqrt(d + e*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(317) = 634$.
time = 1.01, size = 1088, normalized size = 3.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/3465*(1155*f^4*(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) +
(sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d))*e^(-1)
 + 198*f^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^
3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
 + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^
```

```
3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*
((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - 44*f*
g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^
6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)
*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105
*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e -
 c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(
7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4)
)*e^(-1) + g^4*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a
*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-
c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8
- 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-4)/(c^5*d^5) + (1155*((x*e + d)*
c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((x*e + d)*c*d*e - c*d^2*e +
 a*e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*
e^6 - 1540*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((x*e + d)
*c*d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c^5*d^5))*e^(-1) - 924*f^3*g*((5*
((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^
2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 -
sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2
*d^2))*e^(-2))*e^(-1)
```

Mupad [B]
time = 3.60, size = 347, normalized size = 1.03

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5}{11}+\frac{256\,a^5\,e^5\,g^4-1408\,a^4\,c\,d\,e^4\,f\,g^3+3168\,a^3\,c^2\,d^2\,e^3\,f^2\,g^2-3696\,a^2\,c^3\,d^3\,e^2\,f^3\,g+2310\,a\,c^4\,d^4\,e\,f^4}{3465\,c^5\,d^5}+\frac{x\,\left(-128\,a^4\,c\,d\,e^4\,g^4+704\,a^3\,c^2\,d^2\,e^3\,f\,g^3-1584\,a^2\,c^3\,d^3\,e^2\,f^2\,g^2+1848\,a\,c^4\,d^4\,e\,f^3\,g+2310\,c^5\,d^5\,f^4\right)}{3465\,c^5\,d^5}+\frac{4\,g\,x^2\,\left(8\,a^3\,e^3\,g^3-44\,a^2\,c\,d\,e^2\,f\,g^2+99\,a\,c^2\,d^2\,e\,f^2\,g+462\,c^3\,d^3\,f^3\right)}{1155\,c^3\,d^3}+\frac{4\,g^2\,x^3\,\left(-4\,a^2\,e^2\,g^2+22\,a\,c\,d\,e\,f\,g+297\,c^2\,d^2\,f^2\right)}{693\,c^2\,d^2}+\frac{2\,g^3\,x^4\,\left(a\,e\,g+44\,c\,d\,f\right)}{99\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5)/11 + (256*a^5*e
^5*g^4 + 2310*a*c^4*d^4*e*f^4 - 3696*a^2*c^3*d^3*e^2*f^3*g - 1408*a^4*c*d*e
^4*f*g^3 + 3168*a^3*c^2*d^2*e^3*f^2*g^2)/(3465*c^5*d^5) + (x*(2310*c^5*d^5*
f^4 - 128*a^4*c*d*e^4*g^4 + 704*a^3*c^2*d^2*e^3*f*g^3 + 1848*a*c^4*d^4*e*f^
3*g - 1584*a^2*c^3*d^3*e^2*f^2*g^2))/(3465*c^5*d^5) + (4*g*x^2*(8*a^3*e^3*g
^3 + 462*c^3*d^3*f^3 + 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*
c^3*d^3) + (4*g^2*x^3*(297*c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(
693*c^2*d^2) + (2*g^3*x^4*(a*e*g + 44*c*d*f))/(99*c*d)))/(d + e*x)^(1/2)
```

3.680 $\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=269

$$-\frac{16(cdf-aeg)^2\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{315c^4d^4e(d+ex)^{3/2}}+\frac{16g(cdf-aeg)^2\left(ade+(cd^2+\right.}{105c^3d^3e\sqrt{d+}}$$

```
[Out] -16/315*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)/c^4/d^4/e/(e*x+d)^(3/2)+4/21*(-a*e*g+c*d*f)*(g*x+f)^2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/(e*x+d)^(3/2)+2/9*(g*x+f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)+16/105*g*(-a*e*g
+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {884, 808, 662}

$$-\frac{16\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2\left(2ae^2g-cd(5ef-3dg)\right)}{315c^4d^4e(d+ex)^{3/2}}+\frac{16g\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)^2}{105c^3d^3e\sqrt{d+ex}}+\frac{4(f+gx)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{21c^2d^2(d+ex)^{3/2}}+\frac{2(f+gx)^3\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(3/2))/(315*c^4*d^4*e*(d + e*x)^(3/2)) + (16*g*(c*d*f
 - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*S
qrt[d + e*x]) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2))/(21*c^2*d^2*(d + e*x)^(3/2)) + (2*(f + g*x)^3*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*c*d*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{9cd(d+ex)^{3/2}} + \frac{(2(cde^2f+$$

$$= \frac{4(cdf-aeg)(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{21c^2d^2(d+ex)^{3/2}} +$$

$$= \frac{16g(cdf-aeg)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{105c^3d^3e\sqrt{d+ex}} + \frac{4(cdf$$

$$= \frac{16(cdf-aeg)^2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)\left(ade+(cd^2+ae^2)\,x+c\right.}{315c^3d^3(d+ex)^{3/2}}$$

Mathematica [A]
time = 0.11, size = 136, normalized size = 0.51

$$\frac{2((ae+cdx)(d+ex))^{3/2}\left(-16a^3e^3g^3+24a^2cde^2g^2(3f+gx)-6ac^2d^2eg(21f^2+18fgx+5g^2x^2)+c^3d^3(105f^3+189f^2gx+135fg^2x^2+35g^3x^3)\right)}{315c^4d^4(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-16*a^3*e^3*g^3 + 24*a^2*c*d*e^2*g^2*(3
*f + g*x) - 6*a*c^2*d^2*e*g*(21*f^2 + 18*f*g*x + 5*g^2*x^2) + c^3*d^3*(105*
f^3 + 189*f^2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3)))/(315*c^4*d^4*(d + e*x)^(3
/2))
```

Maple [A]
time = 0.13, size = 178, normalized size = 0.66

method	result
default	$-\frac{2(cdx+ae)\left(-35g^3x^3c^3d^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\,e^2\right.}{315c^4d^4\sqrt{ex+d}}$
gosper	$-\frac{2(cdx+ae)\left(-35g^3x^3c^3d^3+30a\,c^2d^2e\,g^3x^2-135c^3d^3f\,g^2x^2-24a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-189c^3d^3f^2gx+16a^3e^3g^3-72a^2cd\,e^2\right.}{315c^4d^4\sqrt{ex+d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/315*(c*d*x+a*e)*(-35*c^3*d^3*g^3*x^3+30*a*c^2*d^2*e*g^3*x^2-135*c^3*d^3*
f*g^2*x^2-24*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-189*c^3*d^3*f^2*g*x+
16*a^3*e^3*g^3-72*a^2*c*d*e^2*f*g^2+126*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*
((c*d*x+a*e)*(e*x+d))^(1/2)/c^4/d^4/(e*x+d)^(1/2)
```

Maxima [A]
time = 0.34, size = 219, normalized size = 0.81

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f^3}{3\,cd}+\frac{2\,(3\,c^2d^2x^2+acdxe-2\,a^2e^2)\sqrt{cdx+ae}\,f^2g}{5\,c^2d^2}+\frac{2\,(15\,c^3d^3x^3+3\,ac^2d^2x^2e-4\,a^2cdxe^2+8\,a^3e^3)\sqrt{cdx+ae}\,fg^2}{35\,c^3d^3}+\frac{2\,(35\,c^4d^4x^4+5\,ac^3d^3x^3e-6\,a^2c^2d^2x^2e^2+8\,a^3cdxe^3-16\,a^4e^4)\sqrt{cdx+ae}\,g^3}{315\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^3/(c*d) + 2/5*(3*c^2*d^2*x^2 + a*c*d*x*e - 2*a^2*
e^2)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2) + 2/35*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*x^2*e - 4*a^2*c*d*x*e^2 + 8*a^3*e^3)*sqrt(c*d*x + a*e)*f*g^2/(c^3*d^3) + 2
/315*(35*c^4*d^4*x^4 + 5*a*c^3*d^3*x^3*e - 6*a^2*c^2*d^2*x^2*e^2 + 8*a^3*c*
d*x*e^3 - 16*a^4*e^4)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

Fricas [A]
time = 1.30, size = 263, normalized size = 0.98

$$\frac{2\,(35\,c^4d^4g^3x^4+135\,c^4d^4fg^2x^3+189\,c^4d^4f^2gx^2+105\,c^4d^4f^3x-16\,a^4g^3e^4+8\,(a^3cdg^3x+9\,a^3cdfg^2)e^3-6\,(a^2c^2d^2g^3x^2+6\,a^2c^2d^2fg^2x+21\,a^2c^2d^2f^2g)e^2+(5\,ac^3d^3g^3x^3+27\,ac^3d^3fg^2x^2+63\,ac^3d^3f^2gx+105\,ac^3d^3f^3)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{315\,(c^4d^4xe+c^4d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^3*x^4 + 135*c^4*d^4*f*g^2*x^3 + 189*c^4*d^4*f^2*g*x^2 +
 105*c^4*d^4*f^3*x - 16*a^4*g^3*e^4 + 8*(a^3*c*d*g^3*x + 9*a^3*c*d*f*g^2)*e
```

```
^3 - 6*(a^2*c^2*d^2*g^3*x^2 + 6*a^2*c^2*d^2*f*g^2*x + 21*a^2*c^2*d^2*f^2*g)
*e^2 + (5*a*c^3*d^3*g^3*x^3 + 27*a*c^3*d^3*f*g^2*x^2 + 63*a*c^3*d^3*f^2*g*x
 + 105*a*c^3*d^3*f^3)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x
*e + d)/(c^4*d^4*x*e + c^4*d^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^3}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**3/sqrt(d + e*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. 2(253) = 506.
time = 3.89, size = 747, normalized size = 2.78

$$\frac{2}{315}\left(105\,f^3\left(\frac{\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}e^{(-1)}}{cd}+\frac{\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2}{cd}\right)e^{(-1)}+9\,fg^2\left(\frac{\left(15\,\sqrt{-cd^2e+ae^3}\,c^3d^6-3\,\sqrt{-cd^2e+ae^3}\,ac^2d^4e^2-4\,\sqrt{-cd^2e+ae^3}\,a^2cd^2e^4-8\,\sqrt{-cd^2e+ae^3}\,a^3e^6\right)e^{(-2)}}{c^3d^3}+\frac{\left(35\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a^2e^6-42\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}ae^3+15\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{7}{2}}\right)e^{(-5)}}{c^3d^3}\right)e^{(-1)}-g^3\left(\frac{\left(35\,\sqrt{-cd^2e+ae^3}\,c^4d^8-5\,\sqrt{-cd^2e+ae^3}\,ac^3d^6e^2-6\,\sqrt{-cd^2e+ae^3}\,a^2c^2d^4e^4-8\,\sqrt{-cd^2e+ae^3}\,a^3cd^2e^6-16\,\sqrt{-cd^2e+ae^3}\,a^4e^8\right)e^{(-3)}}{c^4d^4}+\frac{\left(105\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a^3e^9-189\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}a^2e^6+135\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{7}{2}}ae^3-35\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{9}{2}}\right)e^{(-7)}}{c^4d^4}\right)e^{(-1)}-63\,f^2g\left(\frac{\left(5\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}ae^3-3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}\right)e^{(-2)}}{c^2d^2}+\frac{3\,\sqrt{-cd^2e+ae^3}\,c^2d^4-\sqrt{-cd^2e+ae^3}\,acd^2e^2-2\,\sqrt{-cd^2e+ae^3}\,a^2e^4}{c^2d^2}\right)e^{(-2)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/315*(105*f^3*(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (s
qrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d))*e^(-1) +
 9*f*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c
^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e
^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/
2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e
+ d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - g^3*((35*sq
rt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*s
qrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*
e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*((x*e + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e - c*d^2*e +
a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3
- 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4))*e^(-1) -
63*f^2*g*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)
*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^
3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*
a^2*e^4)/(c^2*d^2))*e^(-2))*e^(-1)
```

Mupad [B]
time = 3.37, size = 242, normalized size = 0.90

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^3\,x^4}{9}-\frac{32\,a^4\,e^4\,g^3-144\,a^3\,c\,d\,e^3\,f\,g^2+252\,a^2\,c^2\,d^2\,e^2\,f^2\,g-210\,a\,c^3\,d^3\,e\,f^3}{315\,c^4\,d^4}+\frac{x\,\left(16\,a^3\,c\,d\,e^3\,g^3-72\,a^2\,c^2\,d^2\,e^2\,f\,g^2+126\,a\,c^3\,d^3\,e\,f^2\,g+210\,c^4\,d^4\,f^3\right)}{315\,c^4\,d^4}+\frac{2\,g\,x^2\,\left(-2\,a^2\,e^2\,g^2+9\,a\,c\,d\,e\,f\,g+63\,c^2\,d^2\,f^2\right)}{105\,c^2\,d^2}+\frac{2\,g^2\,x^3\,\left(a\,e\,g+27\,c\,d\,f\right)}{63\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^3*x^4)/9 - (32*a^4*e^4
*g^3 - 210*a*c^3*d^3*e*f^3 + 252*a^2*c^2*d^2*e^2*f^2*g - 144*a^3*c*d*e^3*f*
g^2)/(315*c^4*d^4) + (x*(210*c^4*d^4*f^3 + 16*a^3*c*d*e^3*g^3 - 72*a^2*c^2*
d^2*e^2*f*g^2 + 126*a*c^3*d^3*e*f^2*g))/(315*c^4*d^4) + (2*g*x^2*(63*c^2*d^
2*f^2 - 2*a^2*e^2*g^2 + 9*a*c*d*e*f*g))/(105*c^2*d^2) + (2*g^2*x^3*(a*e*g +
 27*c*d*f))/(63*c*d)))/(d + e*x)^(1/2)
```

3.681 $\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=200

$$-\frac{8(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{105c^3d^3e(d+ex)^{3/2}}+\frac{8g(cdf-aeg)\left(ade+(cd^2+ae^2)\right.}{35c^2d^2e\sqrt{d+ex}}$$

```
[Out] -8/105*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)/c^3/d^3/e/(e*x+d)^(3/2)+2/7*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)+8/35*g*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{8\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)}{105c^3d^3e(d+ex)^{3/2}}+\frac{8g\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{35c^2d^2e\sqrt{d+ex}}+\frac{2(f+gx)^2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(105*c^3*d^3*e*(d + e*x)^(3/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(35*c^2*d^2*e*Sqrt[d +
 e*x]) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*c
*d*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2(f+gx)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{7cd(d+ex)^{3/2}} + \frac{(4(cde^2f\ \cdots}{\cdots}$$

$$= \frac{8g(cdf-aeg)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{35c^2d^2e\sqrt{d+ex}} + \frac{2(f+\cdots}{\cdots}$$

$$= -\frac{8(cdf-aeg)\left(2ae^2g-cd(5ef-3dg)\right)\left(ade+(cd^2+ae\cdots\right.}{105c^3d^3e(d+ex)^{3/2}}$$

Mathematica [A]
time = 0.07, size = 90, normalized size = 0.45

$$\frac{2((ae+cdx)(d+ex))^{3/2}\left(8a^2e^2g^2-4acdeg(7f+3gx)+c^2d^2(35f^2+42fgx+15g^2x^2)\right)}{105c^3d^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(7*f + 3*g*
x) + c^2*d^2*(35*f^2 + 42*f*g*x + 15*g^2*x^2)))/(105*c^3*d^3*(d + e*x)^(3/2
))
```

Maple [A]
time = 0.14, size = 106, normalized size = 0.53

method	result	si

default	$\frac{2(cdx+ae)\left(15g^2x^2c^2d^2-12acde\,g^2x+42c^2d^2fgx+8a^2e^2g^2-28acdefg+35f^2c^2d^2\right)\sqrt{(cdx+ae)\,(ex+d)}}{105c^3d^3\sqrt{ex+d}}$	106
gosper	$\frac{2(cdx+ae)\left(15g^2x^2c^2d^2-12acde\,g^2x+42c^2d^2fgx+8a^2e^2g^2-28acdefg+35f^2c^2d^2\right)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{105c^3d^3\sqrt{ex+d}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/105*(c*d*x+a*e)*(15*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+42*c^2*d^2*f*g*x+8*a
^2*e^2*g^2-28*a*c*d*e*f*g+35*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)/c^3/d
^3/(e*x+d)^(1/2)
```

Maxima [A]
time = 0.33, size = 135, normalized size = 0.68

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f^2}{3\,cd}+\frac{4\,(3\,c^2d^2x^2+acdxe-2\,a^2e^2)\sqrt{cdx+ae}\,fg}{15\,c^2d^2}+\frac{2\,(15\,c^3d^3x^3+3\,ac^2d^2x^2e-4\,a^2cdxe^2+8\,a^3e^3)\sqrt{cdx+ae}\,g^2}{105\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f^2/(c*d) + 4/15*(3*c^2*d^2*x^2 + a*c*d*x*e - 2*a^2
*e^2)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) + 2/105*(15*c^3*d^3*x^3 + 3*a*c^2*d^2
*x^2*e - 4*a^2*c*d*x*e^2 + 8*a^3*e^3)*sqrt(c*d*x + a*e)*g^2/(c^3*d^3)
```

Fricas [A]
time = 1.06, size = 174, normalized size = 0.87

$$\frac{2\,(15\,c^3d^3g^2x^3+42\,c^3d^3fgx^2+35\,c^3d^3f^2x+8\,a^3g^2e^3-4\,(a^2cdg^2x+7\,a^2cdfg)e^2+(3\,ac^2d^2g^2x^2+14\,ac^2d^2fgx+35\,ac^2d^2f^2)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{105\,(c^3d^3xe+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(15*c^3*d^3*g^2*x^3 + 42*c^3*d^3*f*g*x^2 + 35*c^3*d^3*f^2*x + 8*a^3*g
^2*e^3 - 4*(a^2*c*d*g^2*x + 7*a^2*c*d*f*g)*e^2 + (3*a*c^2*d^2*g^2*x^2 + 14*
a*c^2*d^2*f*g*x + 35*a*c^2*d^2*f^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
a*d)*e)*sqrt(x*e + d)/(c^3*d^3*x*e + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^2}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2/sqrt(d + e*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(187) = 374$.
time = 2.41, size = 465, normalized size = 2.32

$$\frac{2}{105}\left(35\,f^2\left(\frac{\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}e^{(-1)}}{cd}+\frac{\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2}{cd}\right)e^{(-1)}+g^2\left(\frac{\left(15\sqrt{-cd^2e+ae^3}\,c^3d^6-3\sqrt{-cd^2e+ae^3}\,ac^2d^4e^2-4\sqrt{-cd^2e+ae^3}\,a^2cd^2e^4-8\sqrt{-cd^2e+ae^3}\,a^3e^6\right)e^{(-2)}}{c^3d^3}+\frac{\left(35\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a^2e^6-42\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}ae^3+15\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{7}{2}}\right)e^{(-5)}}{c^3d^3}\right)e^{(-1)}-14\,fg\left(\frac{\left(5\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}ae^3-3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}\right)e^{(-2)}}{c^2d^2}+\frac{3\sqrt{-cd^2e+ae^3}\,c^2d^4-\sqrt{-cd^2e+ae^3}\,acd^2e^2-2\sqrt{-cd^2e+ae^3}\,a^2e^4}{c^2d^2}\right)e^{(-2)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] 2/105*(35*f^2*(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sq
rt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d))*e^(-1) +
g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^
4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a
^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^
2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*
c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - 14*f*g*((5*((x*e
 + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e +
 a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(
-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2)
)*e^(-2))*e^(-1)
```

Mupad [B]
time = 3.25, size = 157, normalized size = 0.78

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^3}{7}+\frac{16\,a^3\,e^3\,g^2-56\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,c^3\,d^3}+\frac{x\,\left(-8\,a^2\,c\,d\,e^2\,g^2+28\,a\,c^2\,d^2\,e\,f\,g+70\,c^3\,d^3\,f^2\right)}{105\,c^3\,d^3}+\frac{2\,g\,x^2\,\left(a\,e\,g+14\,c\,d\,f\right)}{35\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3)/7 + (16*a^3*e^3
*g^2 + 70*a*c^2*d^2*e*f^2 - 56*a^2*c*d*e^2*f*g)/(105*c^3*d^3) + (x*(70*c^3*
d^3*f^2 - 8*a^2*c*d*e^2*g^2 + 28*a*c^2*d^2*e*f*g))/(105*c^3*d^3) + (2*g*x^2
*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

3.682 $\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=125

$$-\frac{2(2ae^2g-cd(5ef-3dg))\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{15c^2d^2e(d+ex)^{3/2}}+\frac{2g\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}$$

```
[Out] -2/15*(2*a*e^2*g-c*d*(-3*d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/c^2/d^2/e/(e*x+d)^(3/2)+2/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d
/e/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {808, 662}

$$\frac{2g\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{5cde\sqrt{d+ex}}-\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(2ae^2g-cd(5ef-3dg))}{15c^2d^2e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],x
]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2))/(15*c^2*d^2*e*(d + e*x)^(3/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(3/2))/(5*c*d*e*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{2g(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5cde\sqrt{d+ex}} + \frac{1}{5}\bigg(5f-\frac{3dg}{e}-\frac{2ae}{cd}$$
$$= \frac{2\left(5f-\frac{3dg}{e}-\frac{2aeg}{cd}\right)(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{15cd(d+ex)^{3/2}} + \frac{2g}{}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.43

$$\frac{2((ae+cdx)(d+ex))^{3/2}(-2aeg+cd(5f+3gx))}{15c^2d^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*a*e*g + c*d*(5*f + 3*g*x)))/(15*c^2*d^2*(d + e*x)^(3/2))
```

Maple [A]
time = 0.15, size = 57, normalized size = 0.46

method	result	size
default	$-\frac{2(cdx+ae)(-3cdgx+2aeg-5cdf)\sqrt{(cdx+ae)\,(ex+d)}}{15c^2d^2\sqrt{ex+d}}$	57
gosper	$-\frac{2(cdx+ae)(-3cdgx+2aeg-5cdf)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{15c^2d^2\sqrt{ex+d}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(c*d*x+a*e)*(-3*c*d*g*x+2*a*e*g-5*c*d*f)*((c*d*x+a*e)*(e*x+d))^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Maxima [A]
time = 0.32, size = 67, normalized size = 0.54

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}f}{3\,cd} + \frac{2\,(3\,c^2d^2x^2+acdxe-2\,a^2e^2)\sqrt{cdx+ae}\;g}{15\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)*f/(c*d) + 2/15*(3*c^2*d^2*x^2 + a*c*d*x*e - 2*a^2*e
^2)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

Fricas [A]
time = 1.42, size = 104, normalized size = 0.83

$$\frac{2\left(3\,c^2d^2gx^2+5\,c^2d^2fx-2\,a^2ge^2+(acdgx+5\,acdf)e\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{15\left(c^2d^2xe+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(3*c^2*d^2*g*x^2 + 5*c^2*d^2*f*x - 2*a^2*g*e^2 + (a*c*d*g*x + 5*a*c*d*
f)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^2*d^2*x*
e + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)/sqrt(d + e*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(114) = 228.
time = 1.74, size = 246, normalized size = 1.97

$$\frac{2}{15}\left(5\,f\left(\frac{((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}e^{(-1)}}{cd}+\frac{\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2}{cd}\right)e^{(-1)}-g\left(\frac{\left(5\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}ae^3-3\,((xe+d)cde-cd^2e+ae^3)^{\frac{5}{2}}\right)e^{(-2)}}{c^2d^2}+\frac{3\,\sqrt{-cd^2e+ae^3}\,c^2d^4-\sqrt{-cd^2e+ae^3}\,acd^2e^2-2\,\sqrt{-cd^2e+ae^3}\,a^2e^4}{c^2d^2}\right)e^{(-2)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] 2/15*(5*f*(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sqrt(-
c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d))*e^(-1) - g*((
5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*
d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4
```

```
- sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c
^2*d^2))*e^(-2))*e^(-1)
```

Mupad [B]
time = 3.13, size = 93, normalized size = 0.74

$$\frac{\left(\frac{2\,g\,x^2}{5}-\frac{4\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,c^2\,d^2}+\frac{x\,\left(10\,f\,c^2\,d^2+2\,a\,e\,g\,c\,d\right)}{15\,c^2\,d^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/
2),x)
```

```
[Out] (((2*g*x^2)/5 - (4*a^2*e^2*g - 10*a*c*d*e*f)/(15*c^2*d^2) + (x*(10*c^2*d^2*
f + 2*a*c*d*e*g))/(15*c^2*d^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2))/(d + e*x)^(1/2)
```

3.683 $\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/(e*x+d)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {662}

$$\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3cd(d+ex)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.77

$$\frac{2((ae+cdx)(d+ex))^{3/2}}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*c*d*(d + e*x)^(3/2))
```

Maple [A]
time = 0.14, size = 40, normalized size = 0.83

method	result	size
default	$\frac{2(cdx+ae)\sqrt{(cdx+ae)(ex+d)}}{3cd\sqrt{ex+d}}$	40
gosper	$\frac{2(cdx+ae)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{3cd\sqrt{ex+d}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2/3*(c*d*x+a*e)*((c*d*x+a*e)*(e*x+d))^(1/2)/c/d/(e*x+d)^(1/2)
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.40

$$\frac{2\,(cdx+ae)^{\frac{3}{2}}}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c*d*x + a*e)^(3/2)/(c*d)
```

Fricas [A]
time = 1.17, size = 60, normalized size = 1.25

$$\frac{2\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(cdx+ae)\sqrt{xe+d}}{3\,(cdxe+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*x + a*e)*sqrt(x*e + d)
/(c*d*x*e + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)
```

Giac [A]
time = 2.45, size = 88, normalized size = 1.83

$$\frac{2}{3}\left(\frac{\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}e^{(-1)}}{cd}+\frac{\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2}{cd}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorith
m="giac")
```

```
[Out] 2/3*(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sqrt(-c*d^2*
e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d))*e^(-2)
```

Mupad [B]
time = 3.05, size = 49, normalized size = 1.02

$$\frac{\left(\frac{2\,x}{3}+\frac{2\,a\,e}{3\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

```
[Out] (((2*x)/3 + (2*a*e)/(3*c*d))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))
/(d + e*x)^(1/2)
```

3.684 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)}\,dx$

Optimal. Leaf size=124

$$\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\,\sqrt{d+ex}}\right)}{g^{3/2}}$$

```
[Out] -2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1
/2)/(e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(3/2)+2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {878, 888, 211}

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{2\sqrt{cdf-aeg}\,\text{ArcTan}\left(\frac{\sqrt{g}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)),x
]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]) - (2*Sqrt
[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cde^2f+cd^2eg-e(cd^2+ae^2)\,g}{}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{(2e^2(cdf-aeg))\,\mathrm{Subst}\left(\int \frac{}{-e(cd^2}\right.}{}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}} - \frac{2\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ad}}{\sqrt{}}\right.}{g^{3/2}}$$

Mathematica [A]
time = 0.12, size = 114, normalized size = 0.92

$$\frac{2\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{ae+cdx}-\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x] - Sqrt[c*d*f
- a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(g^(3/2)
*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.17, size = 143, normalized size = 1.15

method	result

default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)aeg-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdf-\sqrt{cdx+a}\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g\sqrt{(aeg-cdf)g}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*a*e*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-
(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/
g/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)*sqrt(x*e +
 d)), x)
```

Fricas [A]
time = 1.32, size = 329, normalized size = 2.65

$$\left[\frac{(xe+d)\sqrt{-\frac{cdf-age}{g}}\log\left(-\frac{cd^2gx-cd^2f+2agxe^2-2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}\,g\sqrt{-\frac{cdf-age}{g}}+(cdgx^2-cdfx+2adg)e}{dgx+df+(gx^2+fx)e}\right)+2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{gxe+dg},\ \frac{2\left((xe+d)\sqrt{\frac{cdf-age}{g}}\arctan\left(\frac{\sqrt{xe+d}\sqrt{\frac{cdf-age}{g}}}{\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}\right)+\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}\right)}{gxe+dg}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((x*e + d)*sqrt(-(c*d*f - a*g*e)/g)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^
2 - 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)*g*sqrt(-(c*
d*f - a*g*e)/g) + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e)/(d*g*x + d*f + (g*x^2
+ f*x)*e)) + 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(
g*x*e + d*g), 2*((x*e + d)*sqrt((c*d*f - a*g*e)/g)*arctan(sqrt(x*e + d)*sqr
t((c*d*f - a*g*e)/g)/sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)) + sqrt(c*
d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(g*x*e + d*g)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)/(e*x+d)**(1/2
),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)/(e*x+d)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(f+g\,x\right)\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)*(d + e*x)^(1/2
)), x)
```

3.685 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^2}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\,\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

```
[Out] c*d*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(
1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)-(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/g/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {876, 888, 211}

$$\frac{cd\text{ArcTan}\left(\frac{\sqrt{g}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^2)
,x]
```

```
[Out] -(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)))
+ (c*d*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c
*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^2}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2}}}{2g}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{(cde^2)\,\text{Subst}\left(\int \frac{1}{-e(cd^2+ae^2)g+cde(}\right.}{}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\,(f+gx)} + \frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+}}{\sqrt{cdf-aeg}\sqrt{}}\right.}{g^{3/2}\sqrt{cdf-aeg}}$$

Mathematica [**A**]
time = 0.26, size = 110, normalized size = 0.83

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}}{f+gx}+\frac{cd\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^2),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[g]/(f + g*x)) + (c*d*ArcTan[(Sqrt[g]
*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c
*d*x])))/(g^(3/2)*Sqrt[d + e*x])
```

Maple [**A**]
time = 0.14, size = 151, normalized size = 1.14

method	result
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)cdgx-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)cdf-\sqrt{cdx+ae}\,\sqrt{(aeg-cdf)\,g}\right)\sqrt{}}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,g(gx+f)\sqrt{(aeg-cdf)\,g}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] (-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*g*x-arctanh(g*(c
*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*f-(c*d*x+a*e)^(1/2)*((a*e*g-c*
d*f)*g)^(1/2))*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/
g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^2*sqrt(x*e
 + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(122) = 244.
time = 1.17, size = 578, normalized size = 4.38

$$\left[-\frac{(cd^2gx+cd^2f+(cdgx^2+cdfx)e)\sqrt{-cdfg+ag^2e}\,\log\left(-\frac{cd^2gx-cd^2f+2\,agxe^2+(cdgx^2-cdfx+2\,adg)e-2\,\sqrt{-cdfg+ag^2e}\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{dgx+df+(gx^2+fx)e}\right)+2\,(cdfg-ag^2e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{2\,(cd^2fg^3x+cd^2f^2g^2-(ag^4x^2+afg^3x)e^2+(cdfg^3x^2-adfg^3+(cdf^2g^2-adg^4)x)e)},-\frac{(cd^2gx+cd^2f+(cdgx^2+cdfx)e)\sqrt{cdfg-ag^2e}\,\arctan\left(\frac{\sqrt{cdfg-ag^2e}\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{cd^2gx+agxe^2+(cdgx^2+adg)e}\right)+(cdfg-ag^2e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{cd^2fg^3x+cd^2f^2g^2-(ag^4x^2+afg^3x)e^2+(cdfg^3x^2-adfg^3+(cdf^2g^2-adg^4)x)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2*g*x + c*d^2*f + (c*d*g*x^2 + c*d*f*x)*e)*sqrt(-c*d*f*g + a*g^
2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d
*g)*e - 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(c*d*f*g - a*g^2*e)
*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c*d^2*f*g^3*x
+ c*d^2*f^2*g^2 - (a*g^4*x^2 + a*f*g^3*x)*e^2 + (c*d*f*g^3*x^2 - a*d*f*g^3
+ (c*d*f^2*g^2 - a*d*g^4)*x)*e), -((c*d^2*g*x + c*d^2*f + (c*d*g*x^2 + c*d*
```

```
f*x)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x
 + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d
*g*x^2 + a*d*g)*e)) + (c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*sqrt(x*e + d))/(c*d^2*f*g^3*x + c*d^2*f^2*g^2 - (a*g^4*x^2 + a*f
*g^3*x)*e^2 + (c*d*f*g^3*x^2 - a*d*f*g^3 + (c*d*f^2*g^2 - a*d*g^4)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**2/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^2/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^2\,\sqrt{d+e\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^2*(d + e*x)^(1
/2)), x)
```

3.686 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^3}\,dx$

Optimal. Leaf size=207

$$-\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)\sqrt{d+ex}\,(f+gx)}+\frac{c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2}}{\sqrt{cdf-aeg}\sqrt{d}}\right.}{4g^{3/2}(cdf-aeg)^{3/2}}$$

```
[Out] 1/4*c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)-1/2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^2/(e*x+d)^(1/2)+1/4*c*d*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{c^2d^2\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}}+\frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^3)
,x]
```

```
[Out] -1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)
^2) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g
)*Sqrt[d + e*x]*(f + g*x)) + (c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(3/2)*(c
*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
```

```
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^3}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^2\sqrt{ade+(cd^2+ae}}}{4g}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex}}{4g(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex}}{4g(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex}}{4g(cdf-aeg)\sqrt{d+ex}\,(f+gx}$$

Mathematica [A]
time = 0.54, size = 165, normalized size = 0.80

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{cdf-aeg}\,\sqrt{ae+cdx}\,(2aeg+cd(-f+gx))+c^2d^2(f+gx)^2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{4g^{3/2}(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^3),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*
d*x]*(2*a*e*g + c*d*(-f + g*x)) + c^2*d^2*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[
a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(4*g^(3/2)*(c*d*f - a*e*g)^(3/2)*Sqrt[
(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2)
```

Maple [A]
time = 0.14, size = 275, normalized size = 1.33

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2g^2x^2+2\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^2d^2fgx+\operatorname{arct}\right.}{4\sqrt{ex+d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^2*d^2*g^2*x^2+2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2))*c^2*d^2*f*g*x+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c
^2*d^2*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*g*x-2*((a*e*g-c*d*
f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/
2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+f)^2/((a*e*g
-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^3*sqrt(x*e
 + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(190) = 380.
time = 1.61, size = 1095, normalized size = 5.29

$$\left[\frac{(c^2d^3g^2x^2+2c^2d^3fgx+c^2d^3f^2+(c^2d^2g^2x^3+2c^2d^2fgx^2+c^2d^2f^2x)e)\sqrt{-cdfg+ag^2e}\log\left(-\frac{cd^2gx-cd^2f+2agxe^2+(cdgx^2-cdfx+2adg)e+2\sqrt{-cdfg+ag^2e}\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{dgx+df+(gx^2+fx)e}\right)+2(c^2d^2fg^2x-c^2d^2f^2g-2a^2g^3e^2-(acdg^3x-3acdfg^2)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{8(c^2d^3f^2g^4x^2+2c^2d^3f^3g^3x+c^2d^3f^4g^2+(a^2g^6x^3+2a^2fg^5x^2+a^2f^2g^4x)e^3-(2acdfg^5x^3-a^2df^2g^4+(4acdf^2g^4-a^2dg^6)x^2+2(acdf^3g^3-a^2dfg^5)x)e^2+(c^2d^2f^2g^4x^3-2acd^2f^3g^3+2(c^2d^2f^3g^3-acd^2fg^5)x^2+(c^2d^2f^4g^2-4acd^2f^2g^4)x)e)},-\frac{(c^2d^3g^2x^2+2c^2d^3fgx+c^2d^3f^2+(c^2d^2g^2x^3+2c^2d^2fgx^2+c^2d^2f^2x)e)\sqrt{cdfg-ag^2e}\arctan\left(\frac{\sqrt{cdfg-ag^2e}\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{cd^2gx+agxe^2+(cdgx^2+adg)e}\right)-(c^2d^2fg^2x-c^2d^2f^2g-2a^2g^3e^2-(acdg^3x-3acdfg^2)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{4(c^2d^3f^2g^4x^2+2c^2d^3f^3g^3x+c^2d^3f^4g^2+(a^2g^6x^3+2a^2fg^5x^2+a^2f^2g^4x)e^3-(2acdfg^5x^3-a^2df^2g^4+(4acdf^2g^4-a^2dg^6)x^2+2(acdf^3g^3-a^2dfg^5)x)e^2+(c^2d^2f^2g^4x^3-2acd^2f^3g^3+2(c^2d^2f^3g^3-acd^2fg^5)x^2+(c^2d^2f^4g^2-4acd^2f^2g^4)x)e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((c^2*d^3*g^2*x^2 + 2*c^2*d^3*f*g*x + c^2*d^3*f^2 + (c^2*d^2*g^2*x^3 +
 2*c^2*d^2*f*g*x^2 + c^2*d^2*f^2*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2
*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e + 2*sqrt(-
c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d
))/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(c^2*d^2*f*g^2*x - c^2*d^2*f^2*g -
2*a^2*g^3*e^2 - (a*c*d*g^3*x - 3*a*c*d*f*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (
c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f^2*g^4*x^2 + 2*c^2*d^3*f^3*g^3*x
 + c^2*d^3*f^4*g^2 + (a^2*g^6*x^3 + 2*a^2*f*g^5*x^2 + a^2*f^2*g^4*x)*e^3 -
(2*a*c*d*f*g^5*x^3 - a^2*d*f^2*g^4 + (4*a*c*d*f^2*g^4 - a^2*d*g^6)*x^2 + 2*
(a*c*d*f^3*g^3 - a^2*d*f*g^5)*x)*e^2 + (c^2*d^2*f^2*g^4*x^3 - 2*a*c*d^2*f^3
*g^3 + 2*(c^2*d^2*f^3*g^3 - a*c*d^2*f*g^5)*x^2 + (c^2*d^2*f^4*g^2 - 4*a*c*d
^2*f^2*g^4)*x)*e), -1/4*((c^2*d^3*g^2*x^2 + 2*c^2*d^3*f*g*x + c^2*d^3*f^2 +
 (c^2*d^2*g^2*x^3 + 2*c^2*d^2*f*g*x^2 + c^2*d^2*f^2*x)*e)*sqrt(c*d*f*g - a*
g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (c^
2*d^2*f*g^2*x - c^2*d^2*f^2*g - 2*a^2*g^3*e^2 - (a*c*d*g^3*x - 3*a*c*d*f*g^
2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f
^2*g^4*x^2 + 2*c^2*d^3*f^3*g^3*x + c^2*d^3*f^4*g^2 + (a^2*g^6*x^3 + 2*a^2*f
*g^5*x^2 + a^2*f^2*g^4*x)*e^3 - (2*a*c*d*f*g^5*x^3 - a^2*d*f^2*g^4 + (4*a*c
*d*f^2*g^4 - a^2*d*g^6)*x^2 + 2*(a*c*d*f^3*g^3 - a^2*d*f*g^5)*x)*e^2 + (c^2
*d^2*f^2*g^4*x^3 - 2*a*c*d^2*f^3*g^3 + 2*(c^2*d^2*f^3*g^3 - a*c*d^2*f*g^5)*
x^2 + (c^2*d^2*f^4*g^2 - 4*a*c*d^2*f^2*g^4)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**3/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^3/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^3\,\sqrt{d+e\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^3*(d + e*x)^(1
/2)), x)
```

3.687 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^4}\,dx$

Optimal. Leaf size=277

$$-\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8g(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)}+$$

```
[Out] 1/8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(5/2)-1/3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^3/(e*x+d)^(1/2)+1/12*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)+1/8*c^2*
d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x
+d)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{c^3d^3\,\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}}+\frac{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^4)
,x]
```

```
[Out] -1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)
^3) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*
g)*Sqrt[d + e*x]*(f + g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(8*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)) + (c^3*d^3*ArcTa
n[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g
]*Sqrt[d + e*x])])/(8*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
```

```
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^4}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^3\sqrt{ade+(cd^2+a}}}{6g}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{12g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

Mathematica [A]
time = 0.81, size = 202, normalized size = 0.73

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{cdf-aeg}\,\sqrt{ae+cdx}\,(-8a^2e^2g^2-2acdeg(-7f+gx)+c^2d^2(-3f^2+8fgx+3g^2x^2))+3c^3d^3(f+gx)^3\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{24g^{3/2}(cdf-aeg)^{5/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^4),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*
d*x]*(-8*a^2*e^2*g^2 - 2*a*c*d*e*g*(-7*f + g*x) + c^2*d^2*(-3*f^2 + 8*f*g*x
 + 3*g^2*x^2)) + 3*c^3*d^3*(f + g*x)^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/S
qrt[c*d*f - a*e*g]]))/(24*g^(3/2)*(c*d*f - a*e*g)^(5/2)*Sqrt[(a*e + c*d*x)*
(d + e*x)]*(f + g*x)^3)
```

Maple [A]
time = 0.14, size = 443, normalized size = 1.60

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+9\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3fg^2x^2+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^3*d^3*f*g^2*x^2+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^3*d^3*f^2*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*g^2*x^
2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x-8*((a*e*g-c*d*f
)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x
+a*e)^(1/2)*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*
e*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2
)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="maxima")

[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^4*sqrt(x*e
 + d)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(257) = 514$.
time = 1.32, size = 1801, normalized size = 6.50

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="fricas")

[Out] [-1/48*(3*(c^3*d^4*g^3*x^3 + 3*c^3*d^4*f*g^2*x^2 + 3*c^3*d^4*f^2*g*x + c^3*
d^4*f^3 + (c^3*d^3*g^3*x^4 + 3*c^3*d^3*f*g^2*x^3 + 3*c^3*d^3*f^2*g*x^2 + c^
3*d^3*f^3*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*
x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e - 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt
(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^
2 + f*x)*e)) - 2*(3*c^3*d^3*f*g^3*x^2 + 8*c^3*d^3*f^2*g^2*x - 3*c^3*d^3*f^3
*g + 8*a^3*g^4*e^3 + 2*(a^2*c*d*g^4*x - 11*a^2*c*d*f*g^3)*e^2 - (3*a*c^2*d^
2*g^4*x^2 + 10*a*c^2*d^2*f*g^3*x - 17*a*c^2*d^2*f^2*g^2)*e)*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^3*d^4*f^3*g^5*x^3 + 3*c^3*d^
4*f^4*g^4*x^2 + 3*c^3*d^4*f^5*g^3*x + c^3*d^4*f^6*g^2 - (a^3*g^8*x^4 + 3*a^
3*f*g^7*x^3 + 3*a^3*f^2*g^6*x^2 + a^3*f^3*g^5*x)*e^4 + (3*a^2*c*d*f*g^7*x^4
 - a^3*d*f^3*g^5 + (9*a^2*c*d*f^2*g^6 - a^3*d*g^8)*x^3 + 3*(3*a^2*c*d*f^3*g
^5 - a^3*d*f*g^7)*x^2 + 3*(a^2*c*d*f^4*g^4 - a^3*d*f^2*g^6)*x)*e^3 - 3*(a*c
^2*d^2*f^2*g^6*x^4 - a^2*c*d^2*f^4*g^4 + (3*a*c^2*d^2*f^3*g^5 - a^2*c*d^2*f
*g^7)*x^3 + 3*(a*c^2*d^2*f^4*g^4 - a^2*c*d^2*f^2*g^6)*x^2 + (a*c^2*d^2*f^5*
g^3 - 3*a^2*c*d^2*f^3*g^5)*x)*e^2 + (c^3*d^3*f^3*g^5*x^4 - 3*a*c^2*d^3*f^5*
g^3 + 3*(c^3*d^3*f^4*g^4 - a*c^2*d^3*f^2*g^6)*x^3 + 3*(c^3*d^3*f^5*g^3 - 3*
a*c^2*d^3*f^3*g^5)*x^2 + (c^3*d^3*f^6*g^2 - 9*a*c^2*d^3*f^4*g^4)*x)*e), -1/
24*(3*(c^3*d^4*g^3*x^3 + 3*c^3*d^4*f*g^2*x^2 + 3*c^3*d^4*f^2*g*x + c^3*d^4*
f^3 + (c^3*d^3*g^3*x^4 + 3*c^3*d^3*f*g^2*x^3 + 3*c^3*d^3*f^2*g*x^2 + c^3*d^
3*f^3*x)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 +
(c*d*g*x^2 + a*d*g)*e)) - (3*c^3*d^3*f*g^3*x^2 + 8*c^3*d^3*f^2*g^2*x - 3*c^
3*d^3*f^3*g + 8*a^3*g^4*e^3 + 2*(a^2*c*d*g^4*x - 11*a^2*c*d*f*g^3)*e^2 - (3
*a*c^2*d^2*g^4*x^2 + 10*a*c^2*d^2*f*g^3*x - 17*a*c^2*d^2*f^2*g^2)*e)*sqrt(c
*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^3*d^4*f^3*g^5*x^3 +
 3*c^3*d^4*f^4*g^4*x^2 + 3*c^3*d^4*f^5*g^3*x + c^3*d^4*f^6*g^2 - (a^3*g^8*x
^4 + 3*a^3*f*g^7*x^3 + 3*a^3*f^2*g^6*x^2 + a^3*f^3*g^5*x)*e^4 + (3*a^2*c*d*
f*g^7*x^4 - a^3*d*f^3*g^5 + (9*a^2*c*d*f^2*g^6 - a^3*d*g^8)*x^3 + 3*(3*a^2*
c*d*f^3*g^5 - a^3*d*f*g^7)*x^2 + 3*(a^2*c*d*f^4*g^4 - a^3*d*f^2*g^6)*x)*e^3

```
 - 3*(a*c^2*d^2*f^2*g^6*x^4 - a^2*c*d^2*f^4*g^4 + (3*a*c^2*d^2*f^3*g^5 - a^
2*c*d^2*f*g^7)*x^3 + 3*(a*c^2*d^2*f^4*g^4 - a^2*c*d^2*f^2*g^6)*x^2 + (a*c^2
*d^2*f^5*g^3 - 3*a^2*c*d^2*f^3*g^5)*x)*e^2 + (c^3*d^3*f^3*g^5*x^4 - 3*a*c^2
*d^3*f^5*g^3 + 3*(c^3*d^3*f^4*g^4 - a*c^2*d^3*f^2*g^6)*x^3 + 3*(c^3*d^3*f^5
*g^3 - 3*a*c^2*d^3*f^3*g^5)*x^2 + (c^3*d^3*f^6*g^2 - 9*a*c^2*d^3*f^4*g^4)*x
)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**4/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^4/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^4\,\sqrt{d+e\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^4*(d + e*x)^(1
/2)), x)
```

3.688 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^5}\,dx$

Optimal. Leaf size=347

$$-\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4}+\frac{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex}}{96g(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)}$$

```
[Out] 5/64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g
+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(7/2)-1/4*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(g*x+f)^4/(e*x+d)^(1/2)+1/24*c*d*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)+5/96*c^
2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)^2/
(e*x+d)^(1/2)+5/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*
g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{5c^4d^4\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}}+\frac{5c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g\sqrt{d+ex}(f+gx)(cdf-aeg)^3}+\frac{5c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{96g\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2}+\frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24g\sqrt{d+ex}(f+gx)^3(cdf-aeg)}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^5)
,x]
```

```
[Out] -1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*Sqrt[d + e*x]*(f + g*x)
^4) + (c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*g*(c*d*f - a*e*
g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(96*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^2) + (5*c^3*d^
3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g*(c*d*f - a*e*g)^3*Sqrt
[d + e*x]*(f + g*x)) + (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(3/2)*(c*d*
f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
```

```
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^5}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4} + \frac{(cd)\int \frac{\sqrt{d+ex}}{(f+gx)^4\sqrt{ade+(cd^2+a}}}{8g}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

$$= -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}\,(f+gx)^4} + \frac{cd\sqrt{ade+(cd^2+ae^2)x+cd}}{24g(cdf-aeg)\sqrt{d+ex}\,(f+}$$

Mathematica [A]
time = 1.20, size = 234, normalized size = 0.67

$$\frac{c^4d^4\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{g}\,(48a^3e^3g^3+8a^2cde^2g^2(-17f+gx)-2ac^2d^2eg(-59f^2+18fgx+5g^2x^2)+c^3d^3(-15f^3+73f^2gx+55fg^2x^2+15g^3x^3))}{c^4d^4(cdf-aeg)^3(f+gx)^4}+\frac{15\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}\sqrt{ae+cdx}}\right)}{192g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^5),x]
```

```
[Out] (c^4*d^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[g]*(48*a^3*e^3*g^3 + 8*a^2*c*
d*e^2*g^2*(-17*f + g*x) - 2*a*c^2*d^2*e*g*(-59*f^2 + 18*f*g*x + 5*g^2*x^2)
+ c^3*d^3*(-15*f^3 + 73*f^2*g*x + 55*f*g^2*x^2 + 15*g^3*x^3)))/(c^4*d^4*(c*
d*f - a*e*g)^3*(f + g*x)^4) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c
*d*f - a*e*g]])/((c*d*f - a*e*g)^(7/2)*Sqrt[a*e + c*d*x])))/(192*g^(3/2)*Sq
rt[d + e*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(309) = 618$.
time = 0.14, size = 686, normalized size = 1.98

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4g^4x^4+60\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4fg^3x^3+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/192*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^4*d^4*f^3*g*x-15*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^4*
d^4*f^4+10*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-55
*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*a^2*c*d*e^2*
g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*a*c^2*d^2*e*f*g^2*x*(c*d
*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-73*c^3*d^3*f^2*g*x*(c*d*x+a*e)^(1/2)*
((a*e*g-c*d*f)*g)^(1/2)-48*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*e^
3*g^3+136*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2-118*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+15*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2)/(g*x+f)^4/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d
*x+a*e)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^5*sqrt(x*e
 + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. $2(324) = 648$.
time = 3.52, size = 2715, normalized size = 7.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^5*g^4*x^4 + 4*c^4*d^5*f*g^3*x^3 + 6*c^4*d^5*f^2*g^2*x^2 +
 4*c^4*d^5*f^3*g*x + c^4*d^5*f^4 + (c^4*d^4*g^4*x^5 + 4*c^4*d^4*f*g^3*x^4 +
 6*c^4*d^4*f^2*g^2*x^3 + 4*c^4*d^4*f^3*g*x^2 + c^4*d^4*f^4*x)*e)*sqrt(-c*d*
f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f
*x + 2*a*d*g)*e + 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(15*c^4*d
^4*f*g^4*x^3 + 55*c^4*d^4*f^2*g^3*x^2 + 73*c^4*d^4*f^3*g^2*x - 15*c^4*d^4*f
^4*g - 48*a^4*g^5*e^4 - 8*(a^3*c*d*g^5*x - 23*a^3*c*d*f*g^4)*e^3 + 2*(5*a^2
*c^2*d^2*g^5*x^2 + 22*a^2*c^2*d^2*f*g^4*x - 127*a^2*c^2*d^2*f^2*g^3)*e^2 -
(15*a*c^3*d^3*g^5*x^3 + 65*a*c^3*d^3*f*g^4*x^2 + 109*a*c^3*d^3*f^2*g^3*x -
133*a*c^3*d^3*f^3*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(
x*e + d))/(c^4*d^5*f^4*g^6*x^4 + 4*c^4*d^5*f^5*g^5*x^3 + 6*c^4*d^5*f^6*g^4*
x^2 + 4*c^4*d^5*f^7*g^3*x + c^4*d^5*f^8*g^2 + (a^4*g^10*x^5 + 4*a^4*f*g^9*x
^4 + 6*a^4*f^2*g^8*x^3 + 4*a^4*f^3*g^7*x^2 + a^4*f^4*g^6*x)*e^5 - (4*a^3*c*
d*f*g^9*x^5 - a^4*d*f^4*g^6 + (16*a^3*c*d*f^2*g^8 - a^4*d*g^10)*x^4 + 4*(6*
a^3*c*d*f^3*g^7 - a^4*d*f*g^9)*x^3 + 2*(8*a^3*c*d*f^4*g^6 - 3*a^4*d*f^2*g^8
)*x^2 + 4*(a^3*c*d*f^5*g^5 - a^4*d*f^3*g^7)*x)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g
^8*x^5 - 2*a^3*c*d^2*f^5*g^5 + 2*(6*a^2*c^2*d^2*f^3*g^7 - a^3*c*d^2*f*g^9)*
x^4 + 2*(9*a^2*c^2*d^2*f^4*g^6 - 4*a^3*c*d^2*f^2*g^8)*x^3 + 12*(a^2*c^2*d^2
*f^5*g^5 - a^3*c*d^2*f^3*g^7)*x^2 + (3*a^2*c^2*d^2*f^6*g^4 - 8*a^3*c*d^2*f^
4*g^6)*x)*e^3 - 2*(2*a*c^3*d^3*f^3*g^7*x^5 - 3*a^2*c^2*d^3*f^6*g^4 + (8*a*c
^3*d^3*f^4*g^6 - 3*a^2*c^2*d^3*f^2*g^8)*x^4 + 12*(a*c^3*d^3*f^5*g^5 - a^2*c
^2*d^3*f^3*g^7)*x^3 + 2*(4*a*c^3*d^3*f^6*g^4 - 9*a^2*c^2*d^3*f^4*g^6)*x^2 +
 2*(a*c^3*d^3*f^7*g^3 - 6*a^2*c^2*d^3*f^5*g^5)*x)*e^2 + (c^4*d^4*f^4*g^6*x^
5 - 4*a*c^3*d^4*f^7*g^3 + 4*(c^4*d^4*f^5*g^5 - a*c^3*d^4*f^3*g^7)*x^4 + 2*(
3*c^4*d^4*f^6*g^4 - 8*a*c^3*d^4*f^4*g^6)*x^3 + 4*(c^4*d^4*f^7*g^3 - 6*a*c^3
*d^4*f^5*g^5)*x^2 + (c^4*d^4*f^8*g^2 - 16*a*c^3*d^4*f^6*g^4)*x)*e), -1/192*
(15*(c^4*d^5*g^4*x^4 + 4*c^4*d^5*f*g^3*x^3 + 6*c^4*d^5*f^2*g^2*x^2 + 4*c^4*
d^5*f^3*g*x + c^4*d^5*f^4 + (c^4*d^4*g^4*x^5 + 4*c^4*d^4*f*g^3*x^4 + 6*c^4*
d^4*f^2*g^2*x^3 + 4*c^4*d^4*f^3*g*x^2 + c^4*d^4*f^4*x)*e)*sqrt(c*d*f*g - a*
g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (15
*c^4*d^4*f*g^4*x^3 + 55*c^4*d^4*f^2*g^3*x^2 + 73*c^4*d^4*f^3*g^2*x - 15*c^4
*d^4*f^4*g - 48*a^4*g^5*e^4 - 8*(a^3*c*d*g^5*x - 23*a^3*c*d*f*g^4)*e^3 + 2*
(5*a^2*c^2*d^2*g^5*x^2 + 22*a^2*c^2*d^2*f*g^4*x - 127*a^2*c^2*d^2*f^2*g^3)*
e^2 - (15*a*c^3*d^3*g^5*x^3 + 65*a*c^3*d^3*f*g^4*x^2 + 109*a*c^3*d^3*f^2*g^
3*x - 133*a*c^3*d^3*f^3*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
*sqrt(x*e + d))/(c^4*d^5*f^4*g^6*x^4 + 4*c^4*d^5*f^5*g^5*x^3 + 6*c^4*d^5*f^
6*g^4*x^2 + 4*c^4*d^5*f^7*g^3*x + c^4*d^5*f^8*g^2 + (a^4*g^10*x^5 + 4*a^4*f
*g^9*x^4 + 6*a^4*f^2*g^8*x^3 + 4*a^4*f^3*g^7*x^2 + a^4*f^4*g^6*x)*e^5 - (4*
```

```
a^3*c*d*f*g^9*x^5 - a^4*d*f^4*g^6 + (16*a^3*c*d*f^2*g^8 - a^4*d*g^10)*x^4 +
 4*(6*a^3*c*d*f^3*g^7 - a^4*d*f*g^9)*x^3 + 2*(8*a^3*c*d*f^4*g^6 - 3*a^4*d*f
^2*g^8)*x^2 + 4*(a^3*c*d*f^5*g^5 - a^4*d*f^3*g^7)*x)*e^4 + 2*(3*a^2*c^2*d^2
*f^2*g^8*x^5 - 2*a^3*c*d^2*f^5*g^5 + 2*(6*a^2*c^2*d^2*f^3*g^7 - a^3*c*d^2*f
*g^9)*x^4 + 2*(9*a^2*c^2*d^2*f^4*g^6 - 4*a^3*c*d^2*f^2*g^8)*x^3 + 12*(a^2*c
^2*d^2*f^5*g^5 - a^3*c*d^2*f^3*g^7)*x^2 + (3*a^2*c^2*d^2*f^6*g^4 - 8*a^3*c*
d^2*f^4*g^6)*x)*e^3 - 2*(2*a*c^3*d^3*f^3*g^7*x^5 - 3*a^2*c^2*d^3*f^6*g^4 +
(8*a*c^3*d^3*f^4*g^6 - 3*a^2*c^2*d^3*f^2*g^8)*x^4 + 12*(a*c^3*d^3*f^5*g^5 -
 a^2*c^2*d^3*f^3*g^7)*x^3 + 2*(4*a*c^3*d^3*f^6*g^4 - 9*a^2*c^2*d^3*f^4*g^6)
*x^2 + 2*(a*c^3*d^3*f^7*g^3 - 6*a^2*c^2*d^3*f^5*g^5)*x)*e^2 + (c^4*d^4*f^4*
g^6*x^5 - 4*a*c^3*d^4*f^7*g^3 + 4*(c^4*d^4*f^5*g^5 - a*c^3*d^4*f^3*g^7)*x^4
 + 2*(3*c^4*d^4*f^6*g^4 - 8*a*c^3*d^4*f^4*g^6)*x^3 + 4*(c^4*d^4*f^7*g^3 - 6
*a*c^3*d^4*f^5*g^5)*x^2 + (c^4*d^4*f^8*g^2 - 16*a*c^3*d^4*f^6*g^4)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**5/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^5\,\sqrt{d+e\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(1
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^5*(d + e*x)^(1
/2)), x)
```

3.689 $\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{128(cdf-aeg)^3\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{15015c^5d^5e(d+ex)^{5/2}}+\frac{128g(cdf-aeg)^3\left(ade+(cd^2\right.}{3003c^4d^4e(d+}$$

```
[Out] -128/15015*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(5/2)/c^5/d^5/e/(e*x+d)^(5/2)+128/3003*g*(-a*e*g+c*d*f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/e/(e*x+d)^(3/2)+32/429*(-a
*e*g+c*d*f)^2*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/(e*
x+d)^(5/2)+16/143*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)/c^2/d^2/(e*x+d)^(5/2)+2/13*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A]

time = 0.39, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^3\left(2ae^2g-cd(7ef-5dg)\right)}{15015c^5d^5e(d+ex)^{5/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^3}{3003c^4d^4e(d+ex)^{3/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{429c^3d^3(d+ex)^{5/2}}+\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{143c^2d^2(d+ex)^{5/2}}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(5/2))/(15015*c^5*d^5*e*(d + e*x)^(5/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3003*c^4*d^
4*e*(d + e*x)^(3/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(429*c^3*d^3*(d + e*x)^(5/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(143*c^2*d
^2*(d + e*x)^(5/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2))/(13*c*d*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
```

```
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(f+gx)^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{13cd(d+ex)^{5/2}} + \frac{(8(cdf-\ldots}{}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{143c^2d^2(d+ex)^{5/2}}$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/\ldots}}{429c^3d^3(d+ex)^{5/2}}$$

$$= \frac{128g(cdf-aeg)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3003c^4d^4e(d+ex)^{3/2}} + 32(\ldots$$

$$= \frac{128(cdf-aeg)^3\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+\ldots\right.}{15015c^4d^4(d+ex)^{5/2}}$$

Mathematica [A]
time = 0.20, size = 195, normalized size = 0.58

$$\frac{2((ae+cdx)(d+ex))^{5/2}\left(128a^4e^4g^4-64a^3cde^3g^3(13f+5gx)+16a^2c^2d^2e^2g^2(143f^2+130fgx+35g^2x^2)-8ac^3d^3eg(429f^3+715f^2gx+455fg^2x^2+105g^3x^3)+c^4d^4(3003f^4+8580f^3gx+10010f^2g^2x^2+5460fg^3x^3+1155g^4x^4)\right)}{15015c^5d^5(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(1
3*f + 5*g*x) + 16*a^2*c^2*d^2*e^2*g^2*(143*f^2 + 130*f*g*x + 35*g^2*x^2) -
8*a*c^3*d^3*e*g*(429*f^3 + 715*f^2*g*x + 455*f*g^2*x^2 + 105*g^3*x^3) + c^4
*d^4*(3003*f^4 + 8580*f^3*g*x + 10010*f^2*g^2*x^2 + 5460*f*g^3*x^3 + 1155*g
^4*x^4)))/(15015*c^5*d^5*(d + e*x)^(5/2))
```

Maple [A]
time = 0.14, size = 275, normalized size = 0.82

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2(1155g^4x^4c^4d^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3}{}$
gosper	$\frac{2(cdx+ae)(1155g^4x^4c^4d^4-840a\,c^3d^3e\,g^4x^3+5460c^4d^4f\,g^3x^3+560a^2c^2d^2e^2g^4x^2-3640a\,c^3d^3ef\,g^3x^2+10010c^4d^4f^2g^2x^2-320a^3cd\,e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/15015*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(1155*c^4*d
^4*g^4*x^4-840*a*c^3*d^3*e*g^4*x^3+5460*c^4*d^4*f*g^3*x^3+560*a^2*c^2*d^2*e
^2*g^4*x^2-3640*a*c^3*d^3*e*f*g^3*x^2+10010*c^4*d^4*f^2*g^2*x^2-320*a^3*c*d
*e^3*g^4*x+2080*a^2*c^2*d^2*e^2*f*g^3*x-5720*a*c^3*d^3*e*f^2*g^2*x+8580*c^4
*d^4*f^3*g*x+128*a^4*e^4*g^4-832*a^3*c*d*e^3*f*g^3+2288*a^2*c^2*d^2*e^2*f^2
*g^2-3432*a*c^3*d^3*e*f^3*g+3003*c^4*d^4*f^4)/c^5/d^5
```

Maxima [A]
time = 0.36, size = 408, normalized size = 1.21

$$\frac{2\,(c^2d^2x^2+2\,acdxe+a^2e^2)\sqrt{cdx+ae}\,f^4}{5\,cd}+\frac{8\,(5\,c^3d^3x^3+8\,ac^2d^2x^2e+a^2cdxe^2-2\,a^3e^3)\sqrt{cdx+ae}\,f^3g}{35\,c^2d^2}+\frac{4\,(35\,c^4d^4x^4+50\,ac^3d^3x^3e+3\,a^2c^2d^2x^2e^2-4\,a^3cdxe^3+8\,a^4e^4)\sqrt{cdx+ae}\,f^2g^2}{105\,c^3d^3}+\frac{8\,(105\,c^5d^5x^5+140\,ac^4d^4x^4e+5\,a^2c^3d^3x^3e^2-6\,a^3c^2d^2x^2e^3+8\,a^4cdxe^4-16\,a^5e^5)\sqrt{cdx+ae}\,fg^3}{1155\,c^4d^4}+\frac{2\,(1155\,c^6d^6x^6+1470\,ac^5d^5x^5e+35\,a^2c^4d^4x^4e^2-40\,a^3c^3d^3x^3e^3+48\,a^4c^2d^2x^2e^4-64\,a^5cdxe^5+128\,a^6e^6)\sqrt{cdx+ae}\,g^4}{15015\,c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*x*e + a^2*e^2)*sqrt(c*d*x + a*e)*f^4/(c*d) + 8/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*x^2*e + a^2*c*d*x*e^2 - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^3*g/(c^2*d^2) + 4/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*x^3*e + 3*a
^2*c^2*d^2*x^2*e^2 - 4*a^3*c*d*x*e^3 + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g^2
/(c^3*d^3) + 8/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*x^4*e + 5*a^2*c^3*d^3*
x^3*e^2 - 6*a^3*c^2*d^2*x^2*e^3 + 8*a^4*c*d*x*e^4 - 16*a^5*e^5)*sqrt(c*d*x
+ a*e)*f*g^3/(c^4*d^4) + 2/15015*(1155*c^6*d^6*x^6 + 1470*a*c^5*d^5*x^5*e +
 35*a^2*c^4*d^4*x^4*e^2 - 40*a^3*c^3*d^3*x^3*e^3 + 48*a^4*c^2*d^2*x^2*e^4 -
 64*a^5*c*d*x*e^5 + 128*a^6*e^6)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Fricas [A]
time = 1.82, size = 469, normalized size = 1.40

$$\frac{2\,\left(1155\,c^6d^6g^4x^6+5460\,c^6d^6fg^3x^5+10010\,c^6d^6f^2g^2x^4+8580\,c^6d^6f^3gx^3+3003\,c^6d^6f^4x^2+128\,a^6g^4e^6-64\,\left(a^5cdg^4x+13\,a^5cdfg^3\right)e^5+16\,\left(3\,a^4c^2d^2g^4x^2+26\,a^4c^2d^2fg^3x+143\,a^4c^2d^2f^2g^2\right)e^4-8\,\left(5\,a^3c^3d^3g^4x^3+39\,a^3c^3d^3fg^3x^2+143\,a^3c^3d^3f^2g^2x+429\,a^3c^3d^3f^3g\right)e^3+\left(35\,a^2c^4d^4g^4x^4+260\,a^2c^4d^4fg^3x^3+858\,a^2c^4d^4f^2g^2x^2+1716\,a^2c^4d^4f^3gx+3003\,a^2c^4d^4f^4\right)e^2+2\,\left(735\,ac^5d^5g^4x^5+3640\,ac^5d^5fg^3x^4+7150\,ac^5d^5f^2g^2x^3+6864\,ac^5d^5f^3gx^2+3003\,ac^5d^5f^4x\right)e\right)\sqrt{cd^2x+axe^2+\left(cdx^2+ad\right)e}\,\sqrt{xe+d}}{15015\,\left(c^5d^5xe+c^5d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/15015*(1155*c^6*d^6*g^4*x^6 + 5460*c^6*d^6*f*g^3*x^5 + 10010*c^6*d^6*f^2*
g^2*x^4 + 8580*c^6*d^6*f^3*g*x^3 + 3003*c^6*d^6*f^4*x^2 + 128*a^6*g^4*e^6 -
 64*(a^5*c*d*g^4*x + 13*a^5*c*d*f*g^3)*e^5 + 16*(3*a^4*c^2*d^2*g^4*x^2 + 26
*a^4*c^2*d^2*f*g^3*x + 143*a^4*c^2*d^2*f^2*g^2)*e^4 - 8*(5*a^3*c^3*d^3*g^4*
x^3 + 39*a^3*c^3*d^3*f*g^3*x^2 + 143*a^3*c^3*d^3*f^2*g^2*x + 429*a^3*c^3*d^
3*f^3*g)*e^3 + (35*a^2*c^4*d^4*g^4*x^4 + 260*a^2*c^4*d^4*f*g^3*x^3 + 858*a^
2*c^4*d^4*f^2*g^2*x^2 + 1716*a^2*c^4*d^4*f^3*g*x + 3003*a^2*c^4*d^4*f^4)*e^
2 + 2*(735*a*c^5*d^5*g^4*x^5 + 3640*a*c^5*d^5*f*g^3*x^4 + 7150*a*c^5*d^5*f^
2*g^2*x^3 + 6864*a*c^5*d^5*f^3*g*x^2 + 3003*a*c^5*d^5*f^4*x)*e)*sqrt(c*d^2*
x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^5*d^5*x*e + c^5*d^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(f+gx\right)^{4}}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**4/(d + e*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2489 vs. $2(317) = 634$.
time = 3.68, size = 2489, normalized size = 7.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] 2/45045*(1716*c*d*f^3*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2
*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt
(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2
```

*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - 858*c*d*f^2*g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4))*e^(-1) + 52*c*d*f*g^3*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-4)/(c^5*d^5) + (1155*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c^5*d^5))*e^(-1) - 5*c*d*g^4*((693*sqrt(-c*d^2*e + a*e^3)*c^6*d^12 - 63*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^10*e^2 - 70*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^8*e^4 - 80*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^6*e^6 - 96*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*c*d^2*e^10 - 256*sqrt(-c*d^2*e + a*e^3)*a^6*e^12)*e^(-5)/(c^6*d^6) + (3003*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^5*e^15 - 9009*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^4*e^12 + 12870*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^3*e^9 - 10010*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a^2*e^6 + 4095*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(11/2)*a*e^3 - 693*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(13/2))*e^(-11)/(c^6*d^6))*e^(-1) - 3003*c*d*f^4*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-2) - 12012*a*f^3*g*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-1) + 15015*a*f^4*((((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d)) + 2574*a*f^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3)) - 572*a*f*g^3*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4

```
*d^4)) + 13*a*g^4*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e
+ a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqr
t(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e
^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-4)/(c^5*d^5) + (1155*((x*e +
d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((x*e + d)*c*d*e - c*d^2*
e + a*e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a
^2*e^6 - 1540*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((x*e +
 d)*c*d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c^5*d^5)))*e^(-1)
```

Mupad [B]
time = 3.80, size = 445, normalized size = 1.32

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g^3\,x^5\,\left(7\,a\,e\,g+26\,c\,d\,f\right)}{143}+\frac{256\,a^6\,e^6\,g^4-1664\,a^5\,c\,d\,e^5\,f\,g^3+4576\,a^4\,c^2\,d^2\,e^4\,f^2\,g^2-6864\,a^3\,c^3\,d^3\,e^3\,f^3\,g+6006\,a^2\,c^4\,d^4\,e^2\,f^4}{15015\,c^5\,d^5}+\frac{x^2\,\left(96\,a^4\,c^2\,d^2\,e^4\,g^4-624\,a^3\,c^3\,d^3\,e^3\,f\,g^3+1716\,a^2\,c^4\,d^4\,e^2\,f^2\,g^2+27456\,a\,c^5\,d^5\,e\,f^3\,g+6006\,c^6\,d^6\,f^4\right)}{15015\,c^5\,d^5}+\frac{x\,\left(-128\,a^5\,c\,d\,e^5\,g^4+832\,a^4\,c^2\,d^2\,e^4\,f\,g^3-2288\,a^3\,c^3\,d^3\,e^3\,f^2\,g^2+3432\,a^2\,c^4\,d^4\,e^2\,f^3\,g+12012\,a\,c^5\,d^5\,e\,f^4\right)}{15015\,c^5\,d^5}+\frac{2\,c\,d\,g^4\,x^6}{13}+\frac{8\,g\,x^3\,\left(-2\,a^3\,e^3\,g^3+13\,a^2\,c\,d\,e^2\,f\,g^2+715\,a\,c^2\,d^2\,e\,f^2\,g+429\,c^3\,d^3\,f^3\right)}{3003\,c^2\,d^2}+\frac{2\,g^2\,x^4\,\left(a^2\,e^2\,g^2+208\,a\,c\,d\,e\,f\,g+286\,c^2\,d^2\,f^2\right)}{429\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^3*x^5*(7*a*e*g + 26*c*
d*f))/143 + (256*a^6*e^6*g^4 + 6006*a^2*c^4*d^4*e^2*f^4 - 6864*a^3*c^3*d^3*
e^3*f^3*g - 1664*a^5*c*d*e^5*f*g^3 + 4576*a^4*c^2*d^2*e^4*f^2*g^2)/(15015*c
^5*d^5) + (x^2*(6006*c^6*d^6*f^4 + 96*a^4*c^2*d^2*e^4*g^4 - 624*a^3*c^3*d^3
*e^3*f*g^3 + 27456*a*c^5*d^5*e*f^3*g + 1716*a^2*c^4*d^4*e^2*f^2*g^2))/(1501
5*c^5*d^5) + (x*(12012*a*c^5*d^5*e*f^4 - 128*a^5*c*d*e^5*g^4 + 3432*a^2*c^4
*d^4*e^2*f^3*g + 832*a^4*c^2*d^2*e^4*f*g^3 - 2288*a^3*c^3*d^3*e^3*f^2*g^2))
/(15015*c^5*d^5) + (2*c*d*g^4*x^6)/13 + (8*g*x^3*(429*c^3*d^3*f^3 - 2*a^3*e
^3*g^3 + 715*a*c^2*d^2*e*f^2*g + 13*a^2*c*d*e^2*f*g^2))/(3003*c^2*d^2) + (2
*g^2*x^4*(a^2*e^2*g^2 + 286*c^2*d^2*f^2 + 208*a*c*d*e*f*g))/(429*c*d)))/(d
+ e*x)^(1/2)
```

3.690 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16(cdf-aeg)^2\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{1155c^4d^4e(d+ex)^{5/2}}+\frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+\right.\right.}{231c^3d^3e(d+e}$$

```
[Out] -16/1155*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(5/2)/c^4/d^4/e/(e*x+d)^(5/2)+16/231*g*(-a*e*g+c*d*f)^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^3/d^3/e/(e*x+d)^(3/2)+4/33*(-a*e*g+c
*d*f)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/(e*x+d)^(5/
2)+2/11*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A]

time = 0.26, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2\left(2ae^2g-cd(7ef-5dg)\right)}{1155c^4d^4e(d+ex)^{5/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)^2}{231c^3d^3e(d+ex)^{3/2}}+\frac{4(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{33c^2d^2(d+ex)^{5/2}}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(5/2))/(1155*c^4*d^4*e*(d + e*x)^(5/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*c^3*d^3*e*
(d + e*x)^(3/2)) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2))/(33*c^2*d^2*(d + e*x)^(5/2)) + (2*(f + g*x)^3*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*c*d*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
```

```
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11cd(d+ex)^{5/2}} + \frac{(6(cdf-\ldots}{\ldots}$$

$$= \frac{4(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{33c^2d^2(d+ex)^{5/2}} \ldots$$

$$= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{231c^3d^3e(d+ex)^{3/2}} + \frac{4(cd\ldots}{\ldots}$$

$$= \frac{16(cdf-aeg)^2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+\ldots\right.}{1155c^3d^3(d+ex)^{5/2}}$$

Mathematica [A]
time = 0.14, size = 137, normalized size = 0.51

$$\frac{2((ae+cdx)(d+ex))^{5/2}\left(-16a^3e^3g^3+8a^2cde^2g^2(11f+5gx)-2ac^2d^2eg\left(99f^2+110fgx+35g^2x^2\right)+c^3d^3\left(231f^3+495f^2gx+385fg^2x^2+105g^3x^3\right)\right)}{1155c^4d^4(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(11
*f + 5*g*x) - 2*a*c^2*d^2*e*g*(99*f^2 + 110*f*g*x + 35*g^2*x^2) + c^3*d^3*(
231*f^3 + 495*f^2*g*x + 385*f*g^2*x^2 + 105*g^3*x^3)))/(1155*c^4*d^4*(d + e
*x)^(5/2))
```

Maple [A]
time = 0.16, size = 180, normalized size = 0.67

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2(-105g^3x^3c^3d^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-49}{1155\sqrt{ex+d}\,c^4d^4}$
gosper	$-\frac{2(cdx+ae)(-105g^3x^3c^3d^3+70a\,c^2d^2e\,g^3x^2-385c^3d^3f\,g^2x^2-40a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-495c^3d^3f^2gx+16a^3e^3g^3-88a^2cd\,e}{1155c^4d^4(ex+d)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/1155*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-105*c^3*d
^3*g^3*x^3+70*a*c^2*d^2*e*g^3*x^2-385*c^3*d^3*f*g^2*x^2-40*a^2*c*d*e^2*g^3*
x+220*a*c^2*d^2*e*f*g^2*x-495*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-88*a^2*c*d*e^2
*f*g^2+198*a*c^2*d^2*e*f^2*g-231*c^3*d^3*f^3)/c^4/d^4
```

Maxima [A]
time = 0.35, size = 292, normalized size = 1.09

$$\frac{2\,(c^2d^2x^2+2\,acdxe+a^2e^2)\sqrt{cdx+ae}\,f^3}{5\,cd}+\frac{6\,(5\,c^3d^3x^3+8\,ac^2d^2x^2e+a^2cdxe^2-2\,a^3e^3)\sqrt{cdx+ae}\,f^2g}{35\,c^2d^2}+\frac{2\,(35\,c^4d^4x^4+50\,ac^3d^3x^3e+3\,a^2c^2d^2x^2e^2-4\,a^3cdxe^3+8\,a^4e^4)\sqrt{cdx+ae}\,fg^2}{105\,c^3d^3}+\frac{2\,(105\,c^5d^5x^5+140\,ac^4d^4x^4e+5\,a^2c^3d^3x^3e^2-6\,a^3c^2d^2x^2e^3+8\,a^4cdxe^4-16\,a^5e^5)\sqrt{cdx+ae}\,g^3}{1155\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*x*e + a^2*e^2)*sqrt(c*d*x + a*e)*f^3/(c*d) + 6/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*x^2*e + a^2*c*d*x*e^2 - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f^2*g/(c^2*d^2) + 2/105*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*x^3*e + 3*a
^2*c^2*d^2*x^2*e^2 - 4*a^3*c*d*x*e^3 + 8*a^4*e^4)*sqrt(c*d*x + a*e)*f*g^2/(
c^3*d^3) + 2/1155*(105*c^5*d^5*x^5 + 140*a*c^4*d^4*x^4*e + 5*a^2*c^3*d^3*x^
3*e^2 - 6*a^3*c^2*d^2*x^2*e^3 + 8*a^4*c*d*x*e^4 - 16*a^5*e^5)*sqrt(c*d*x +
a*e)*g^3/(c^4*d^4)
```

Fricas [A]
time = 1.62, size = 339, normalized size = 1.26

$$\frac{2\,(105\,c^5d^5g^3x^5+385\,c^5d^5fg^2x^4+495\,c^5d^5f^2gx^3+231\,c^5d^5f^3x^2-16\,a^5g^3e^5+8\,(a^4cdg^3x+11\,a^4cdfg^2)e^4-2\,(3\,a^3c^2d^2g^3x^2+22\,a^3c^2d^2fg^2x+99\,a^3c^2d^2f^2g)e^3+(5\,a^2c^3d^3g^3x^3+33\,a^2c^3d^3fg^2x^2+99\,a^2c^3d^3f^2gx+231\,a^2c^3d^3f^3)e^2+2\,(70\,ac^4d^4g^3x^4+275\,ac^4d^4fg^2x^3+396\,ac^4d^4f^2gx^2+231\,ac^4d^4f^3x)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{1155\,(c^4d^4xe+c^4d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^5*d^5*g^3*x^5 + 385*c^5*d^5*f*g^2*x^4 + 495*c^5*d^5*f^2*g*x^3
 + 231*c^5*d^5*f^3*x^2 - 16*a^5*g^3*e^5 + 8*(a^4*c*d*g^3*x + 11*a^4*c*d*f*g
```

```
^2)*e^4 - 2*(3*a^3*c^2*d^2*g^3*x^2 + 22*a^3*c^2*d^2*f*g^2*x + 99*a^3*c^2*d^
2*f^2*g)*e^3 + (5*a^2*c^3*d^3*g^3*x^3 + 33*a^2*c^3*d^3*f*g^2*x^2 + 99*a^2*c
^3*d^3*f^2*g*x + 231*a^2*c^3*d^3*f^3)*e^2 + 2*(70*a*c^4*d^4*g^3*x^4 + 275*a
*c^4*d^4*f*g^2*x^3 + 396*a*c^4*d^4*f^2*g*x^2 + 231*a*c^4*d^4*f^3*x)*e)*sqrt
(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^4*d^4*x*e + c^4*d^
5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)(ae+cdx))^{\frac{3}{2}}(f+gx)^3}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**3/(d + e*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(253) = 506.
time = 2.61, size = 1742, normalized size = 6.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] 2/3465*(99*c*d*f^2*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e
+ a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c
*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e
+ a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3
 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) -
 33*c*d*f*g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3
)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*
e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d
^4) + (105*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e +
d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e
+ a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)
/(c^4*d^4))*e^(-1) + c*d*g^3*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqr
t(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e
^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*
a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-4)/(c^5*d^5) + (11
55*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((x*e + d)*c*d
```

```
*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d*e - c*d^2*e + a*e
^3)^(7/2)*a^2*e^6 - 1540*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 +
315*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c^5*d^5))*e^(-1) -
231*c*d*f^3*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e +
 d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a
*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^
3)*a^2*e^4)/(c^2*d^2))*e^(-2) - 693*a*f^2*g*((5*((x*e + d)*c*d*e - c*d^2*e
+ a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/
(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*
d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-1) + 1155*a*f^3*
(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sqrt(-c*d^2*e +
a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d)) + 99*a*f*g^2*((15*sqrt(
-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt
(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)
/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x
*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*d^2*
e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3)) - 11*a*g^3*((35*sqrt(-c*d^2*e + a*e^3)*
c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)
*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*
e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*((x*e + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6
+ 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e
 - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4)))*e^(-1)
```

Mupad [B]
time = 3.65, size = 310, normalized size = 1.15

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^4\,\left(4\,a\,e\,g+11\,c\,d\,f\right)}{33}-\frac{32\,a^5\,e^5\,g^3-176\,a^4\,c\,d\,e^4\,f\,g^2+396\,a^3\,c^2\,d^2\,e^3\,f^2\,g-462\,a^2\,c^3\,d^3\,e^2\,f^3}{1155\,c^4\,d^4}+\frac{x^2\,\left(-12\,a^3\,c^2\,d^2\,e^3\,g^3+66\,a^2\,c^3\,d^3\,e^2\,f\,g^2+1584\,a\,c^4\,d^4\,e\,f^2\,g+462\,c^5\,d^5\,f^3\right)}{1155\,c^4\,d^4}+\frac{2\,c\,d\,g^3\,x^5}{11}+\frac{2\,g\,x^3\,\left(a^2\,e^2\,g^2+110\,a\,c\,d\,e\,f\,g+99\,c^2\,d^2\,f^2\right)}{231\,c\,d}+\frac{2\,a\,e\,x\,\left(8\,a^3\,e^3\,g^3-44\,a^2\,c\,d\,e^2\,f\,g^2+99\,a\,c^2\,d^2\,e\,f^2\,g+462\,c^3\,d^3\,f^3\right)}{1155\,c^3\,d^3}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^4*(4*a*e*g + 11*c*
d*f))/33 - (32*a^5*e^5*g^3 - 462*a^2*c^3*d^3*e^2*f^3 + 396*a^3*c^2*d^2*e^3*
f^2*g - 176*a^4*c*d*e^4*f*g^2)/(1155*c^4*d^4) + (x^2*(462*c^5*d^5*f^3 - 12*
a^3*c^2*d^2*e^3*g^3 + 66*a^2*c^3*d^3*e^2*f*g^2 + 1584*a*c^4*d^4*e*f^2*g))/(
1155*c^4*d^4) + (2*c*d*g^3*x^5)/11 + (2*g*x^3*(a^2*e^2*g^2 + 99*c^2*d^2*f^2
 + 110*a*c*d*e*f*g))/(231*c*d) + (2*a*e*x*(8*a^3*e^3*g^3 + 462*c^3*d^3*f^3
+ 99*a*c^2*d^2*e*f^2*g - 44*a^2*c*d*e^2*f*g^2))/(1155*c^3*d^3)))/(d + e*x)^
(1/2)
```

3.691 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{315c^3d^3e(d+ex)^{5/2}}+\frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)\right.}{63c^2d^2e(d+ex)^{3/2}}$$

```
[Out] -8/315*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(5/2)/c^3/d^3/e/(e*x+d)^(5/2)+8/63*g*(-a*e*g+c*d*f)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/e/(e*x+d)^(3/2)+2/9*(g*x+f)^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A]

time = 0.15, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)}{315c^3d^3e(d+ex)^{5/2}}+\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}(cdf-aeg)}{63c^2d^2e(d+ex)^{3/2}}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(5/2))/(315*c^3*d^3*e*(d + e*x)^(5/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(63*c^2*d^2*e*(d + e*x
)^(3/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9
*c*d*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{9cd(d+ex)^{5/2}}+\frac{(4(cde^2 f\ldots}{\ldots}$$

$$=\frac{8g(cdf-aeg)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{63c^2d^2e(d+ex)^{3/2}}+\frac{2(f+\ldots}{\ldots}$$

$$=-\frac{8(cdf-aeg)\left(2ae^2g-cd(7ef-5dg)\right)\left(ade+(cd^2+\ldots\right.}{315c^3d^3e(d+ex)^{5/2}}$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 0.45

$$\frac{2((ae+cdx)(d+ex))^{5/2}\left(8a^2e^2g^2-4acdeg(9f+5gx)+c^2d^2(63f^2+90fgx+35g^2x^2)\right)}{315c^3d^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(8*a^2*e^2*g^2 - 4*a*c*d*e*g*(9*f + 5*g*
x) + c^2*d^2*(63*f^2 + 90*f*g*x + 35*g^2*x^2)))/(315*c^3*d^3*(d + e*x)^(5/2
))
```

Maple [A]
time = 0.15, size = 108, normalized size = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2(35g^2x^2c^2d^2-20acde\,g^2x+90c^2d^2fgx+8a^2e^2g^2-36acdefg+63f^2c^2d^2)}{315\sqrt{ex+d}\,c^3d^3}$	108

gosper	$\frac{2(cdx+ae)(35g^2x^2c^2d^2-20acde\,g^2x+90c^2d^2fgx+8a^2e^2g^2-36acdefg+63f^2c^2d^2)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}{315c^3d^3(ex+d)^{\frac{3}{2}}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/315*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(35*c^2*d^2*g
^2*x^2-20*a*c*d*e*g^2*x+90*c^2*d^2*f*g*x+8*a^2*e^2*g^2-36*a*c*d*e*f*g+63*c^
2*d^2*f^2)/c^3/d^3
```

Maxima [A]
time = 0.32, size = 192, normalized size = 0.96

$$\frac{2\left(c^2d^2x^2+2\,acdxe+a^2e^2\right)\sqrt{cdx+ae}\,f^2}{5\,cd}+\frac{4\left(5\,c^3d^3x^3+8\,ac^2d^2x^2e+a^2cdxe^2-2\,a^3e^3\right)\sqrt{cdx+ae}\,fg}{35\,c^2d^2}+\frac{2\left(35\,c^4d^4x^4+50\,ac^3d^3x^3e+3\,a^2c^2d^2x^2e^2-4\,a^3cdxe^3+8\,a^4e^4\right)\sqrt{cdx+ae}\,g^2}{315\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*x*e + a^2*e^2)*sqrt(c*d*x + a*e)*f^2/(c*d) + 4/3
5*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*x^2*e + a^2*c*d*x*e^2 - 2*a^3*e^3)*sqrt(c*d*
x + a*e)*f*g/(c^2*d^2) + 2/315*(35*c^4*d^4*x^4 + 50*a*c^3*d^3*x^3*e + 3*a^2
*c^2*d^2*x^2*e^2 - 4*a^3*c*d*x*e^3 + 8*a^4*e^4)*sqrt(c*d*x + a*e)*g^2/(c^3*
d^3)
```

Fricas [A]
time = 2.05, size = 229, normalized size = 1.14

$$\frac{2\left(35\,c^4d^4g^2x^4+90\,c^4d^4fgx^3+63\,c^4d^4f^2x^2+8\,a^4g^2e^4-4\left(a^3cdg^2x+9\,a^3cdfg\right)e^3+3\left(a^2c^2d^2g^2x^2+6\,a^2c^2d^2fgx+21\,a^2c^2d^2f^2\right)e^2+2\left(25\,ac^3d^3g^2x^3+72\,ac^3d^3fgx^2+63\,ac^3d^3f^2x\right)e\right)\sqrt{cd^2x+axe^2+\left(cdx^2+ad\right)e}\,\sqrt{xe+d}}{315\left(c^3d^3xe+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] 2/315*(35*c^4*d^4*g^2*x^4 + 90*c^4*d^4*f*g*x^3 + 63*c^4*d^4*f^2*x^2 + 8*a^4
*g^2*e^4 - 4*(a^3*c*d*g^2*x + 9*a^3*c*d*f*g)*e^3 + 3*(a^2*c^2*d^2*g^2*x^2 +
 6*a^2*c^2*d^2*f*g*x + 21*a^2*c^2*d^2*f^2)*e^2 + 2*(25*a*c^3*d^3*g^2*x^3 +
72*a*c^3*d^3*f*g*x^2 + 63*a*c^3*d^3*f^2*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d
*x^2 + a*d)*e)*sqrt(x*e + d)/(c^3*d^3*x*e + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(f+gx)^2}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)**2/(d + e*x)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1119 vs. $2(187) = 374$.
time = 3.11, size = 1119, normalized size = 5.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] 2/315*(6*c*d*f*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*
e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2
*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*
e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 1
5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - c*d
*g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d
^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3
)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (10
5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e
- c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^
(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4
))*e^(-1) - 21*c*d*f^2*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3
- 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-
c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d
^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-2) - 42*a*f*g*((5*((x*e + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))
*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e
^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-1) + 105
*a*f^2*(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sqrt(-c*d
^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d)) + 3*a*g^2*((15*s
qrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*
sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)*e^
(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42
*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*
d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3)))*e^(-1)
```

Mupad [**B**]
time = 3.43, size = 206, normalized size = 1.03

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g\,x^3\,\left(5\,a\,e\,g+9\,c\,d\,f\right)}{63}+\frac{16\,a^4\,e^4\,g^2-72\,a^3\,c\,d\,e^3\,f\,g+126\,a^2\,c^2\,d^2\,e^2\,f^2}{315\,c^3\,d^3}+\frac{x^2\,\left(6\,a^2\,c^2\,d^2\,e^2\,g^2+288\,a\,c^3\,d^3\,e\,f\,g+126\,c^4\,d^4\,f^2\right)}{315\,c^3\,d^3}+\frac{2\,c\,d\,g^2\,x^4}{9}+\frac{4\,a\,e\,x\,\left(-2\,a^2\,e^2\,g^2+9\,a\,c\,d\,e\,f\,g+63\,c^2\,d^2\,f^2\right)}{315\,c^2\,d^2}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g*x^3*(5*a*e*g + 9*c*d*f
))/63 + (16*a^4*e^4*g^2 + 126*a^2*c^2*d^2*e^2*f^2 - 72*a^3*c*d*e^3*f*g)/(31
5*c^3*d^3) + (x^2*(126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 + 288*a*c^3*d^3*
e*f*g))/(315*c^3*d^3) + (2*c*d*g^2*x^4)/9 + (4*a*e*x*(63*c^2*d^2*f^2 - 2*a^
2*e^2*g^2 + 9*a*c*d*e*f*g))/(315*c^2*d^2)))/(d + e*x)^(1/2)
```

3.692 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2(2ae^2g-cd(7ef-5dg))\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{35c^2d^2e(d+ex)^{5/2}}+\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}$$

```
[Out] -2/35*(2*a*e^2*g-c*d*(-5*d*g+7*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2
)/c^2/d^2/e/(e*x+d)^(5/2)+2/7*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d
/e/(e*x+d)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {808, 662}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}\left(2ae^2g-cd(7ef-5dg)\right)}{35c^2d^2e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(35*c^2*d^2*e*(d + e*x)^(5/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(5/2))/(7*c*d*e*(d + e*x)^(3/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2g\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{7cde(d+ex)^{3/2}} + \frac{1}{7}\left(7f-\frac{5dg}{e}-\frac{2ae}{cd}\right.$$

$$= \frac{2\left(7f-\frac{5dg}{e}-\frac{2aeg}{cd}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{35cd(d+ex)^{5/2}} + \frac{2g}{}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.43

$$\frac{2((ae+cdx)(d+ex))^{5/2}(-2aeg+cd(7f+5gx))}{35c^2d^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-2*a*e*g + c*d*(7*f + 5*g*x)))/(35*c^2*
d^2*(d + e*x)^(5/2))
```

Maple [A]
time = 0.12, size = 59, normalized size = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^2(-5cdgx+2aeg-7cdf)}{35\sqrt{ex+d}\,c^2d^2}$	59
gosper	$-\frac{2(cdx+ae)(-5cdgx+2aeg-7cdf)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{35c^2d^2(ex+d)^{\frac{3}{2}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/35*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2*(-5*c*d*g*x+2
*a*e*g-7*c*d*f)/c^2/d^2
```

Maxima [A]
time = 0.31, size = 108, normalized size = 0.86

$$\frac{2\left(c^2d^2x^2+2\,acdxe+a^2e^2\right)\sqrt{cdx+ae}\,f}{5\,cd}+\frac{2\left(5\,c^3d^3x^3+8\,ac^2d^2x^2e+a^2cdxe^2-2\,a^3e^3\right)\sqrt{cdx+ae}\,g}{35\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*x*e + a^2*e^2)*sqrt(c*d*x + a*e)*f/(c*d) + 2/35*
(5*c^3*d^3*x^3 + 8*a*c^2*d^2*x^2*e + a^2*c*d*x*e^2 - 2*a^3*e^3)*sqrt(c*d*x
+ a*e)*g/(c^2*d^2)
```

Fricas [A]
time = 1.62, size = 139, normalized size = 1.11

$$\frac{2\left(5\,c^3d^3gx^3+7\,c^3d^3fx^2-2\,a^3ge^3+\left(a^2cdgx+7\,a^2cdf\right)e^2+2\left(4\,ac^2d^2gx^2+7\,ac^2d^2fx\right)e\right)\sqrt{cd^2x+axe^2+\left(cdx^2+ad\right)e}\,\sqrt{xe+d}}{35\left(c^2d^2xe+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="fricas")
```

```
[Out] 2/35*(5*c^3*d^3*g*x^3 + 7*c^3*d^3*f*x^2 - 2*a^3*g*e^3 + (a^2*c*d*g*x + 7*a^
2*c*d*f)*e^2 + 2*(4*a*c^2*d^2*g*x^2 + 7*a*c^2*d^2*f*x)*e)*sqrt(c*d^2*x + a*
x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^2*d^2*x*e + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(f+gx\right)}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2
),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*(f + g*x)/(d + e*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(114) = 228.
time = 1.37, size = 616, normalized size = 4.93

$$\frac{2}{105}\left(cdg\left(\frac{\left(15\sqrt{-cd^2e+ae^3}\,c^3d^6-3\sqrt{-cd^2e+ae^3}\,ac^2d^4e^2-4\sqrt{-cd^2e+ae^3}\,a^2cd^2e^4-8\sqrt{-cd^2e+ae^3}\,a^3e^6\right)e^{(-2)}}{c^3d^3}+\frac{\left(35\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a^2e^6-42\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}ae^3+15\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{7}{2}}\right)e^{(-5)}}{c^3d^3}\right)e^{(-1)}-7\,cdf\left(\frac{\left(5\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}ae^3-3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}\right)e^{(-2)}}{c^2d^2}+\frac{3\sqrt{-cd^2e+ae^3}\,c^2d^4-\sqrt{-cd^2e+ae^3}\,acd^2e^2-2\sqrt{-cd^2e+ae^3}\,a^2e^4}{c^2d^2}\right)e^{(-2)}-7\,ag\left(\frac{\left(5\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}ae^3-3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}\right)e^{(-2)}}{c^2d^2}+\frac{3\sqrt{-cd^2e+ae^3}\,c^2d^4-\sqrt{-cd^2e+ae^3}\,acd^2e^2-2\sqrt{-cd^2e+ae^3}\,a^2e^4}{c^2d^2}\right)e^{(-1)}+35\,af\left(\frac{\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}e^{(-1)}}{cd}+\frac{\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2}{cd}\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,
algorithm="giac")
```

```
[Out] 2/105*(c*d*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)
*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e +
 a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)
^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((
```

```
x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - 7*c*d*f
*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e -
 c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d
^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)
/(c^2*d^2))*e^(-2) - 7*a*g*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*
e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sq
rt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(
-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-1) + 35*a*f*(((x*e + d)*c*d*e - c
*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-
c*d^2*e + a*e^3)*a*e^2)/(c*d)))*e^(-1)
```

Mupad [B]
time = 3.25, size = 109, normalized size = 0.87

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(x^2\,\left(\frac{16\,a\,e\,g}{35}+\frac{2\,c\,d\,f}{5}\right)-\frac{4\,a^3\,e^3\,g-14\,a^2\,c\,d\,e^2\,f}{35\,c^2\,d^2}+\frac{2\,c\,d\,g\,x^3}{7}+\frac{2\,a\,e\,x\,\left(a\,e\,g+14\,c\,d\,f\right)}{35\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(3/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x^2*((16*a*e*g)/35 + (2*c*d
*f)/5) - (4*a^3*e^3*g - 14*a^2*c*d*e^2*f)/(35*c^2*d^2) + (2*c*d*g*x^3)/7 +
(2*a*e*x*(a*e*g + 14*c*d*f))/(35*c*d)))/(d + e*x)^(1/2)
```

3.693 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/(e*x+d)^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{662\}$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5cd(d+ex)^{5/2}}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.77

$$\frac{2((ae+cdx)(d+ex))^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*c*d*(d + e*x)^(5/2))
```

Maple [A]
time = 0.14, size = 42, normalized size = 0.88

method	result	size
default	$\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,(cdx+ae)^{2}}{5\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{3}{2}}}{5cd(ex+d)^{\frac{3}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2/5*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^2/c/d
```

Maxima [A]
time = 0.31, size = 44, normalized size = 0.92

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdxe+a^{2}e^{2}\right)\sqrt{cdx+ae}}{5\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="maxima")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*x*e + a^2*e^2)*sqrt(c*d*x + a*e)/(c*d)
```

Fricas [A]
time = 1.85, size = 76, normalized size = 1.58

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdxe+a^{2}e^{2}\right)\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,\sqrt{xe+d}}{5\left(cdxe+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*x*e + a^2*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*sqrt(x*e + d)/(c*d*x*e + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(44) = 88$.
time = 3.12, size = 244, normalized size = 5.08

$$-\frac{2}{15}\left(cd\left(\frac{\left(5\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}ae^3-3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}\right)e^{(-2)}}{c^2d^2}+\frac{3\sqrt{-cd^2e+ae^3}\,c^2d^4-\sqrt{-cd^2e+ae^3}\,acd^2e^2-2\sqrt{-cd^2e+ae^3}\,a^2e^4}{c^2d^2}\right)e^{(-2)}-5a\left(\frac{\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}e^{(-1)}}{cd}+\frac{\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2}{cd}\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorith
m="giac")
```

```
[Out] -2/15*(c*d*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e +
d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*
e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3
)*a^2*e^4)/(c^2*d^2))*e^(-2) - 5*a*(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/
2)*e^(-1)/(c*d) + (sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*
e^2)/(c*d)))*e^(-1)
```

Mupad [B]
time = 3.08, size = 62, normalized size = 1.29

$$\frac{\left(\frac{4\,a\,e\,x}{5}+\frac{2\,c\,d\,x^2}{5}+\frac{2\,a^2\,e^2}{5\,c\,d}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^(3/2),x)
```

```
[Out] (((4*a*e*x)/5 + (2*c*d*x^2)/5 + (2*a^2*e^2)/(5*c*d))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(1/2)
```

3.694 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx$

Optimal. Leaf size=179

$$-\frac{2(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}+\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}+\frac{2(cdf-aeg)^{3/2}\tan^{-1}\left(\right.}{}$$

[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+2*(-a*e*g+c*d*f
)^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)-2*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {878, 888, 211}

$$\frac{2(cdf-aeg)^{3/2}\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{g^2\sqrt{d+ex}}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)),x]

[Out] (-2*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[
d + e*x]) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x
)^(3/2)) + (2*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
```

```
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)}\,dx = \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3g(d+ex)^{3/2}} - \frac{(cde^2f + cd^2eg - e(cd^2 + ae$$

$$= -\frac{2(cdf - aeg)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^2\sqrt{d+ex}} + \frac{2(ade + (cd^2 + }{3g(d - }$$

$$= -\frac{2(cdf - aeg)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^2\sqrt{d+ex}} + \frac{2(ade + (cd^2 + }{3g(d - }$$

$$= -\frac{2(cdf - aeg)\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^2\sqrt{d+ex}} + \frac{2(ade + (cd^2 + }{3g(d - }$$

Mathematica [A]
time = 0.21, size = 132, normalized size = 0.74

$$\frac{2\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{ae+cdx}\,(4aeg+cd(-3f+gx))+3(cdf-aeg)^{3/2}\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{3g^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(4*a*e*g + c*
d*(-3*f + g*x)) + 3*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x]
)/Sqrt[c*d*f - a*e*g]]))/(3*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.15, size = 253, normalized size = 1.41

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)a^{2}e^{2}g^{2}-6\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)acdefg+3\,\mathrm{arc}\right.}{3\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*a^2*e^2*g^2-6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2))*a*c*d*e*f*g+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^
2*d^2*f^2-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*g*x-4*((a*e*g-c*d*f
)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1
/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)*(x*e + d
)^(3/2)), x)
```

Fricas [A]
time = 1.71, size = 419, normalized size = 2.34

$$\left[-\frac{3\left(cd^{2}f-agxe^{2}+(cdfx-adg)e\right)\sqrt{-\frac{cdf-age}{g}}\log\left(-\frac{cd^{2}gx-cd^{2}f+2\,agxe^{2}-2\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\sqrt{xe+d}\,g\sqrt{-\frac{cdf-age}{g}}+(cdgx^{2}-cdfx+2\,adg)e}{dgx+df+(gx^{2}+fx)e}\right)-2\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,(cdgx-3\,cdf+4\,age)\sqrt{xe+d}}{3\left(g^{2}xe+dg^{2}\right)},-\frac{2\left(3\left(cd^{2}f-agxe^{2}+(cdfx-adg)e\right)\sqrt{\frac{cdf-age}{g}}\arctan\left(\frac{\sqrt{xe+d}\sqrt{\frac{cdf-age}{g}}}{\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}}\right)-\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,(cdgx-3\,cdf+4\,age)\sqrt{xe+d}\right)}{3\left(g^{2}xe+dg^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [-1/3*(3*(c*d^2*f - a*g*x*e^2 + (c*d*f*x - a*d*g)*e)*sqrt(-(c*d*f - a*g*e)/
g)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 - 2*sqrt(c*d^2*x + a*x*e^2 + (c*
d*x^2 + a*d)*e)*sqrt(x*e + d)*g*sqrt(-(c*d*f - a*g*e)/g) + (c*d*g*x^2 - c*d
*f*x + 2*a*d*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) - 2*sqrt(c*d^2*x + a*x*
e^2 + (c*d*x^2 + a*d)*e)*(c*d*g*x - 3*c*d*f + 4*a*g*e)*sqrt(x*e + d))/(g^2*
```

```
x*e + d*g^2), -2/3*(3*(c*d^2*f - a*g*x*e^2 + (c*d*f*x - a*d*g)*e)*sqrt((c*d
*f - a*g*e)/g)*arctan(sqrt(x*e + d)*sqrt((c*d*f - a*g*e)/g)/sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)) - sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
*(c*d*g*x - 3*c*d*f + 4*a*g*e)*sqrt(x*e + d))/(g^2*x*e + d*g^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{\left(f+g\,x\right)\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)*(d + e*x)^(3/2
)), x)
```

3.695 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx$

Optimal. Leaf size=178

$$\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}-\frac{3cd\sqrt{cdf-aeg}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ade-}}{\sqrt{cdj}}\right.}{g^{5/2}}$$

```
[Out] -(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)-3*c*d*arct
an(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*
x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(5/2)+3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 878, 888, 211}

$$-\frac{3cd\sqrt{cdf-aeg}\,\text{ArcTan}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{5/2}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)}+\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^2),x]
```

```
[Out] (3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]) - (
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(g*(d + e*x)^(3/2)*(f + g*x))
- (3*c*d*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/g^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^2}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}(f+gx)} + \frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right.}}{\sqrt{d+ex}\,(f\cdots}}{2g}$$

$$= \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cde\cdots\right.}{g(d+ex)^{3/2}(f+gx)}$$

$$= \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cde\cdots\right.}{g(d+ex)^{3/2}(f+gx)}$$

$$= \frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cde\cdots\right.}{g(d+ex)^{3/2}(f+gx)}$$

Mathematica [A]
time = 0.37, size = 144, normalized size = 0.81

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{ae+cdx}\,(-aeg+cd(3f+2gx))-3cd\sqrt{cdf-aeg}\,(f+gx)\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{g^{5/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^2),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-(a*e*g) + c*d
*(3*f + 2*g*x)) - 3*c*d*Sqrt[c*d*f - a*e*g]*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[
a*e + c*d*x])/Sqrt[c*d*f - a*e*g]]))/(g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]
*(f + g*x))
```

Maple [A]
time = 0.16, size = 296, normalized size = 1.66

method	result
default	$\frac{\left(-3\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)acde\,g^{2}x+3\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^{2}d^{2}fgx-3\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x,metho
d=_RETURNVERBOSE)
```

```
[Out] (-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*g^2*x+3*ar
ctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f*g*x-3*arctanh(
g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c*d*e*f*g+3*arctanh(g*(c*d*x
+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*f^2+2*((a*e*g-c*d*f)*g)^(1/2)*
(c*d*x+a*e)^(1/2)*c*d*g*x-((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3
*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*f)*((c*d*x+a*e)*(e*x+d))^(1/
2)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^2*(x*e +
 d)^(3/2)), x)
```

Fricas [A]
time = 2.08, size = 457, normalized size = 2.57

$$\left[\frac{3\,(cd^{2}gx+cd^{2}f+(cdgx^{2}+cdfx)e)\sqrt{-\frac{cdf-age}{g}}\,\log\left(-\frac{cd^{2}gx-cd^{2}f+2\,agxe^{2}-2\,\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,\sqrt{xe+d}\,g\sqrt{-\frac{cdf-age}{g}}+(cdgx^{2}-cdfx+2\,adg)e}{dgx+df+(gx^{2}+fx)e}\right)+2\,\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,(2\,cdgx+3\,cdf-age)\sqrt{xe+d}}{2\,(dg^{3}x+dfg^{2}+(g^{3}x^{2}+fg^{2}x)e)},\ \frac{3\,(cd^{2}gx+cd^{2}f+(cdgx^{2}+cdfx)e)\sqrt{\frac{cdf-age}{g}}\,\arctan\left(\frac{\sqrt{xe+d}\,\sqrt{\frac{cdf-age}{g}}}{\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}}\right)+\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,(2\,cdgx+3\,cdf-age)\sqrt{xe+d}}{dg^{3}x+dfg^{2}+(g^{3}x^{2}+fg^{2}x)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [1/2*(3*(c*d^2*g*x + c*d^2*f + (c*d*g*x^2 + c*d*f*x)*e)*sqrt(-(c*d*f - a*g*
e)/g)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 - 2*sqrt(c*d^2*x + a*x*e^2 +
(c*d*x^2 + a*d)*e)*sqrt(x*e + d)*g*sqrt(-(c*d*f - a*g*e)/g) + (c*d*g*x^2 -
c*d*f*x + 2*a*d*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*sqrt(c*d^2*x + a
*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + 3*c*d*f - a*g*e)*sqrt(x*e + d))/(d
*g^3*x + d*f*g^2 + (g^3*x^2 + f*g^2*x)*e), (3*(c*d^2*g*x + c*d^2*f + (c*d*g
*x^2 + c*d*f*x)*e)*sqrt((c*d*f - a*g*e)/g)*arctan(sqrt(x*e + d)*sqrt((c*d*f
 - a*g*e)/g)/sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)) + sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + 3*c*d*f - a*g*e)*sqrt(x*e + d))/(
d*g^3*x + d*f*g^2 + (g^3*x^2 + f*g^2*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^2*(d + e*x)^(3
/2)), x)
```

3.696 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx$

Optimal. Leaf size=195

$$-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}+\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ade+\left(cd^2+a\right.}}{\sqrt{cdf-aeg}\,\sqrt{}}\right.}{4g^{5/2}\sqrt{cdf-aeg}}$$

```
[Out] -1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^2+3/4*
c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(1/2)-3/4*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {876, 888, 211}

$$\frac{3c^2d^2\text{ArcTan}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{4g^{5/2}\sqrt{cdf-aeg}}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^3),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]*(
f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(2*g*(d + e*x)^(3
/2)*(f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(4*g^(5/2)*Sqrt[c*d*f
 - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
```

```
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3}\,dx = -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2} + \frac{(3cd)\int \frac{\sqrt{ade+(cd^2+ae^2)}}{\sqrt{d+ex}\,(f+}}{4g}$$

$$= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{(ade+(cd^2+ae^2)x+cd}{2g(d+ex)^{3/2}(f+gx}$$

$$= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{(ade+(cd^2+ae^2)x+cd}{2g(d+ex)^{3/2}(f+gx}$$

$$= -\frac{3cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)} - \frac{(ade+(cd^2+ae^2)x+cd}{2g(d+ex)^{3/2}(f+gx}$$

Mathematica [A]
time = 0.51, size = 135, normalized size = 0.69

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}\,(2aeg+cd(3f+5gx))}{(f+gx)^2}+\frac{3c^2d^2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}\right)}{4g^{5/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^3),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(2*a*e*g + c*d*(3*f + 5*g*x)))/(
f + g*x)^2) + (3*c^2*d^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*
e*g]])/(Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*d*x])))/(4*g^(5/2)*Sqrt[d + e*x])
```

Maple [A]
time = 0.14, size = 266, normalized size = 1.36

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^2d^2g^2x^2+6\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^2d^2fgx+3\,\mathrm{a}\right.}{4\sqrt{ex}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^2*d^2*g^2*x^2+6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c^2*d^2*f*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*c^2*d^2*f^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c*d*g*x+2*((a*e
*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*e*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x
+a*e)^(1/2)*c*d*f)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^2/(g*x+f)^2/((a*e*g-c*
d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^3*(x*e +
 d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(177) = 354.
time = 2.12, size = 864, normalized size = 4.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [-1/8*(3*(c^2*d^3*g^2*x^2 + 2*c^2*d^3*f*g*x + c^2*d^3*f^2 + (c^2*d^2*g^2*x^
3 + 2*c^2*d^2*f*g*x^2 + c^2*d^2*f^2*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*
```

```
d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e - 2*sqr
t(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e
+ d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(5*c^2*d^2*f*g^2*x + 3*c^2*d^2*f
^2*g - 2*a^2*g^3*e^2 - (5*a*c*d*g^3*x + a*c*d*f*g^2)*e)*sqrt(c*d^2*x + a*x*
e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c*d^2*f*g^5*x^2 + 2*c*d^2*f^2*g^4*
x + c*d^2*f^3*g^3 - (a*g^6*x^3 + 2*a*f*g^5*x^2 + a*f^2*g^4*x)*e^2 + (c*d*f*
g^5*x^3 - a*d*f^2*g^4 + (2*c*d*f^2*g^4 - a*d*g^6)*x^2 + (c*d*f^3*g^3 - 2*a*
d*f*g^5)*x)*e), -1/4*(3*(c^2*d^3*g^2*x^2 + 2*c^2*d^3*f*g*x + c^2*d^3*f^2 +
(c^2*d^2*g^2*x^3 + 2*c^2*d^2*f*g*x^2 + c^2*d^2*f^2*x)*e)*sqrt(c*d*f*g - a*g
^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*
d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) + (5*c
^2*d^2*f*g^2*x + 3*c^2*d^2*f^2*g - 2*a^2*g^3*e^2 - (5*a*c*d*g^3*x + a*c*d*f
*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c*d^2*
f*g^5*x^2 + 2*c*d^2*f^2*g^4*x + c*d^2*f^3*g^3 - (a*g^6*x^3 + 2*a*f*g^5*x^2
+ a*f^2*g^4*x)*e^2 + (c*d*f*g^5*x^3 - a*d*f^2*g^4 + (2*c*d*f^2*g^4 - a*d*g^
6)*x^2 + (c*d*f^3*g^3 - 2*a*d*f*g^5)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^3,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(f+g\,x\right)}^3\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^3*(d + e*x)^(3
/2)), x)
```

3.697 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx$

Optimal. Leaf size=265

$$-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

```
[Out] -1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^3+1/8*
c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(3/2)-1/4*c*d*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^2/(e*x+d)^(1/2)+1/8*c^2*d^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{c^3d^3\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{5/2}(cdf-aeg)^{3/2}}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^4),x]
```

```
[Out] -1/4*(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*(
f + g*x)^2) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*
(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)/(3*g*(d + e*x)^(3/2)*(f + g*x)^3) + (c^3*d^3*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
e*x])])/(8*g^(5/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
```

```
2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] && !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^4}\,dx=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^3}+\frac{(cd)\int\frac{\sqrt{ade+\left(cd^2+ae^2\right)x}}{\sqrt{d+ex}\,(f+gx)}}{2g}$$

$$=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{3g(d+ex)^{3/2}(f+gx)^3}$$

$$=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f}$$

$$=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f}$$

$$=-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}\,(f+gx)^2}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{8g^2(cdf-aeg)\sqrt{d+ex}\,(f}$$

Mathematica [A]
time = 0.83, size = 201, normalized size = 0.76

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{cdf-aeg}\,\sqrt{ae+cdx}\,(8a^2e^2g^2-2acdeg(f-7gx)+c^2d^2(-3f^2-8fgx+3g^2x^2))+3c^3d^3(f+gx)^3\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{24g^{5/2}(cdf-aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^4),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[c*d*f - a*e*g]*Sqrt[a*e + c*
d*x]*(8*a^2*e^2*g^2 - 2*a*c*d*e*g*(f - 7*g*x) + c^2*d^2*(-3*f^2 - 8*f*g*x +
 3*g^2*x^2)) + 3*c^3*d^3*(f + g*x)^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqr
t[c*d*f - a*e*g]]))/(24*g^(5/2)*(c*d*f - a*e*g)^(3/2)*Sqrt[(a*e + c*d*x)*(d
 + e*x)]*(f + g*x)^3)
```

Maple [A]
time = 0.14, size = 443, normalized size = 1.67

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^3d^3g^3x^3+9\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^3d^3f\,g^2x^2+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^3*d^3*g^3*x^3+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c^3*d^3*f*g^2*x^2+9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^3*d^3*f^2*g*x+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*g^2*x^2
-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x+8*((a*e*g-c*d*f
)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x-8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x
+a*e)^(1/2)*a^2*e^2*g^2+2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e
*f*g+3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*x+d)^(1/2)
/(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g^2/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^4*(x*e +
 d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(244) = 488$.
time = 2.05, size = 1493, normalized size = 5.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [1/48*(3*(c^3*d^4*g^3*x^3 + 3*c^3*d^4*f*g^2*x^2 + 3*c^3*d^4*f^2*g*x + c^3*d
^4*f^3 + (c^3*d^3*g^3*x^4 + 3*c^3*d^3*f*g^2*x^3 + 3*c^3*d^3*f^2*g*x^2 + c^3
*d^3*f^3*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x
*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e + 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt(
c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^2
 + f*x)*e)) + 2*(3*c^3*d^3*f*g^3*x^2 - 8*c^3*d^3*f^2*g^2*x - 3*c^3*d^3*f^3*
g - 8*a^3*g^4*e^3 - 2*(7*a^2*c*d*g^4*x - 5*a^2*c*d*f*g^3)*e^2 - (3*a*c^2*d^
2*g^4*x^2 - 22*a*c^2*d^2*f*g^3*x - a*c^2*d^2*f^2*g^2)*e)*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f^2*g^6*x^3 + 3*c^2*d^3*f
^3*g^5*x^2 + 3*c^2*d^3*f^4*g^4*x + c^2*d^3*f^5*g^3 + (a^2*g^8*x^4 + 3*a^2*f
*g^7*x^3 + 3*a^2*f^2*g^6*x^2 + a^2*f^3*g^5*x)*e^3 - (2*a*c*d*f*g^7*x^4 - a^
2*d*f^3*g^5 + (6*a*c*d*f^2*g^6 - a^2*d*g^8)*x^3 + 3*(2*a*c*d*f^3*g^5 - a^2*
d*f*g^7)*x^2 + (2*a*c*d*f^4*g^4 - 3*a^2*d*f^2*g^6)*x)*e^2 + (c^2*d^2*f^2*g^
6*x^4 - 2*a*c*d^2*f^4*g^4 + (3*c^2*d^2*f^3*g^5 - 2*a*c*d^2*f*g^7)*x^3 + 3*(
c^2*d^2*f^4*g^4 - 2*a*c*d^2*f^2*g^6)*x^2 + (c^2*d^2*f^5*g^3 - 6*a*c*d^2*f^3
*g^5)*x)*e), -1/24*(3*(c^3*d^4*g^3*x^3 + 3*c^3*d^4*f*g^2*x^2 + 3*c^3*d^4*f^
2*g*x + c^3*d^4*f^3 + (c^3*d^3*g^3*x^4 + 3*c^3*d^3*f*g^2*x^3 + 3*c^3*d^3*f^
2*g*x^2 + c^3*d^3*f^3*x)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a
*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*
x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (3*c^3*d^3*f*g^3*x^2 - 8*c^3*d^3*
f^2*g^2*x - 3*c^3*d^3*f^3*g - 8*a^3*g^4*e^3 - 2*(7*a^2*c*d*g^4*x - 5*a^2*c*
d*f*g^3)*e^2 - (3*a*c^2*d^2*g^4*x^2 - 22*a*c^2*d^2*f*g^3*x - a*c^2*d^2*f^2*
g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3
*f^2*g^6*x^3 + 3*c^2*d^3*f^3*g^5*x^2 + 3*c^2*d^3*f^4*g^4*x + c^2*d^3*f^5*g^
3 + (a^2*g^8*x^4 + 3*a^2*f*g^7*x^3 + 3*a^2*f^2*g^6*x^2 + a^2*f^3*g^5*x)*e^3
 - (2*a*c*d*f*g^7*x^4 - a^2*d*f^3*g^5 + (6*a*c*d*f^2*g^6 - a^2*d*g^8)*x^3 +
 3*(2*a*c*d*f^3*g^5 - a^2*d*f*g^7)*x^2 + (2*a*c*d*f^4*g^4 - 3*a^2*d*f^2*g^6
)*x)*e^2 + (c^2*d^2*f^2*g^6*x^4 - 2*a*c*d^2*f^4*g^4 + (3*c^2*d^2*f^3*g^5 -
2*a*c*d^2*f*g^7)*x^3 + 3*(c^2*d^2*f^4*g^4 - 2*a*c*d^2*f^2*g^6)*x^2 + (c^2*d
^2*f^5*g^3 - 6*a*c*d^2*f^3*g^5)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**4,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^4\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^4*(d + e*x)^(3
/2)), x)
```

3.698 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx$

Optimal. Leaf size=335

$$-\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\frac{3c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{64g^2(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

```
[Out] -1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^4+3/64
*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d
*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(5/2)-1/8*c*d*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^3/(e*x+d)^(1/2)+1/32*c^2*d^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d)^(1/2)
+3/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)^2/
(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{3c^4d^4\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{5/2}(cdf-aeg)^{5/2}}+\frac{3c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^5),x]
```

```
[Out] -1/8*(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*(
f + g*x)^3) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^2
*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^3*d^3*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x
)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(4*g*(d + e*x)^(3/2)*(f
+ g*x)^4) + (3*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(5/2)*(c*d*f - a*e*g)^
(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
```

```
b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^5}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x}}{\sqrt{d+ex}\,(f+g}}{8g}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{4g(d+ex)^{3/2}(f+gx)^4}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f}$$

$$= -\frac{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^2\sqrt{d+ex}\,(f+gx)^3}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+}}{32g^2(cdf-aeg)\sqrt{d+ex}\,(f}$$

Mathematica [A]
time = 1.25, size = 240, normalized size = 0.72

$$\frac{c^4d^4((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{g}\left(-16a^3e^3g^3+24a^2cde^2g^2(f-gx)-2ac^2d^2eg\left(f^2-22fgx+g^2x^2\right)+c^3d^3\left(-3f^3-11f^2gx+11fg^2x^2+3g^3x^3\right)\right)}{c^4d^4(cdf-aeg)^2(ae+cdx)(f+gx)^4}+\frac{3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}(ae+cdx)^{3/2}}\right)}{64g^{5/2}(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^5),x]
```

```
[Out] (c^4*d^4*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[g]*(-16*a^3*e^3*g^3 + 24*a^
2*c*d*e^2*g^2*(f - g*x) - 2*a*c^2*d^2*e*g*(f^2 - 22*f*g*x + g^2*x^2) + c^3*
d^3*(-3*f^3 - 11*f^2*g*x + 11*f*g^2*x^2 + 3*g^3*x^3)))/(c^4*d^4*(c*d*f - a*
e*g)^2*(a*e + c*d*x)*(f + g*x)^4) + (3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/S
qrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(5/2)*(a*e + c*d*x)^(3/2))))/(64*g^(5
/2)*(d + e*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(297) = 594.
time = 0.14, size = 655, normalized size = 1.96

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4g^4x^4+12\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^4d^4fg^3x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/64*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+12*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^4*d^4*f*g^3*x^3+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+12*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^4*d^4*f^3*g*x-3*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d
*f)*g)^(1/2)+3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^4*d^4
*f^4+2*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-11*c^3
*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+24*a^2*c*d*e^2*g^3
*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-44*a*c^2*d^2*e*f*g^2*x*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+11*c^3*d^3*f^2*g*x*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)+16*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*e^3*g
^3-24*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2+2*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+3*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g
*x+f)^4/g^2/(a*e*g-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^5*(x*e +
 d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(311) = 622$.
time = 3.28, size = 2317, normalized size = 6.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="fricas")

[Out] [-1/128*(3*(c^4*d^5*g^4*x^4 + 4*c^4*d^5*f*g^3*x^3 + 6*c^4*d^5*f^2*g^2*x^2 +
 4*c^4*d^5*f^3*g*x + c^4*d^5*f^4 + (c^4*d^4*g^4*x^5 + 4*c^4*d^4*f*g^3*x^4 +
 6*c^4*d^4*f^2*g^2*x^3 + 4*c^4*d^4*f^3*g*x^2 + c^4*d^4*f^4*x)*e)*sqrt(-c*d*
f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f
*x + 2*a*d*g)*e - 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) - 2*(3*c^4*d^
4*f*g^4*x^3 + 11*c^4*d^4*f^2*g^3*x^2 - 11*c^4*d^4*f^3*g^2*x - 3*c^4*d^4*f^4
*g + 16*a^4*g^5*e^4 + 8*(3*a^3*c*d*g^5*x - 5*a^3*c*d*f*g^4)*e^3 + 2*(a^2*c^
2*d^2*g^5*x^2 - 34*a^2*c^2*d^2*f*g^4*x + 13*a^2*c^2*d^2*f^2*g^3)*e^2 - (3*a
*c^3*d^3*g^5*x^3 + 13*a*c^3*d^3*f*g^4*x^2 - 55*a*c^3*d^3*f^2*g^3*x - a*c^3*
d^3*f^3*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/
(c^3*d^4*f^3*g^7*x^4 + 4*c^3*d^4*f^4*g^6*x^3 + 6*c^3*d^4*f^5*g^5*x^2 + 4*c^
3*d^4*f^6*g^4*x + c^3*d^4*f^7*g^3 - (a^3*g^10*x^5 + 4*a^3*f*g^9*x^4 + 6*a^3
*f^2*g^8*x^3 + 4*a^3*f^3*g^7*x^2 + a^3*f^4*g^6*x)*e^4 + (3*a^2*c*d*f*g^9*x^
5 - a^3*d*f^4*g^6 + (12*a^2*c*d*f^2*g^8 - a^3*d*g^10)*x^4 + 2*(9*a^2*c*d*f^
3*g^7 - 2*a^3*d*f*g^9)*x^3 + 6*(2*a^2*c*d*f^4*g^6 - a^3*d*f^2*g^8)*x^2 + (3
*a^2*c*d*f^5*g^5 - 4*a^3*d*f^3*g^7)*x)*e^3 - 3*(a*c^2*d^2*f^2*g^8*x^5 - a^2
*c*d^2*f^5*g^5 + (4*a*c^2*d^2*f^3*g^7 - a^2*c*d^2*f*g^9)*x^4 + 2*(3*a*c^2*d
^2*f^4*g^6 - 2*a^2*c*d^2*f^2*g^8)*x^3 + 2*(2*a*c^2*d^2*f^5*g^5 - 3*a^2*c*d^
2*f^3*g^7)*x^2 + (a*c^2*d^2*f^6*g^4 - 4*a^2*c*d^2*f^4*g^6)*x)*e^2 + (c^3*d^
3*f^3*g^7*x^5 - 3*a*c^2*d^3*f^6*g^4 + (4*c^3*d^3*f^4*g^6 - 3*a*c^2*d^3*f^2*
g^8)*x^4 + 6*(c^3*d^3*f^5*g^5 - 2*a*c^2*d^3*f^3*g^7)*x^3 + 2*(2*c^3*d^3*f^6
*g^4 - 9*a*c^2*d^3*f^4*g^6)*x^2 + (c^3*d^3*f^7*g^3 - 12*a*c^2*d^3*f^5*g^5)*
x)*e), -1/64*(3*(c^4*d^5*g^4*x^4 + 4*c^4*d^5*f*g^3*x^3 + 6*c^4*d^5*f^2*g^2*
x^2 + 4*c^4*d^5*f^3*g*x + c^4*d^5*f^4 + (c^4*d^4*g^4*x^5 + 4*c^4*d^4*f*g^3*
x^4 + 6*c^4*d^4*f^2*g^2*x^3 + 4*c^4*d^4*f^3*g*x^2 + c^4*d^4*f^4*x)*e)*sqrt(
c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 +
(c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*
g)*e)) - (3*c^4*d^4*f*g^4*x^3 + 11*c^4*d^4*f^2*g^3*x^2 - 11*c^4*d^4*f^3*g^2
*x - 3*c^4*d^4*f^4*g + 16*a^4*g^5*e^4 + 8*(3*a^3*c*d*g^5*x - 5*a^3*c*d*f*g^
4)*e^3 + 2*(a^2*c^2*d^2*g^5*x^2 - 34*a^2*c^2*d^2*f*g^4*x + 13*a^2*c^2*d^2*f
^2*g^3)*e^2 - (3*a*c^3*d^3*g^5*x^3 + 13*a*c^3*d^3*f*g^4*x^2 - 55*a*c^3*d^3*
f^2*g^3*x - a*c^3*d^3*f^3*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*
e)*sqrt(x*e + d))/(c^3*d^4*f^3*g^7*x^4 + 4*c^3*d^4*f^4*g^6*x^3 + 6*c^3*d^4*
f^5*g^5*x^2 + 4*c^3*d^4*f^6*g^4*x + c^3*d^4*f^7*g^3 - (a^3*g^10*x^5 + 4*a^3
*f*g^9*x^4 + 6*a^3*f^2*g^8*x^3 + 4*a^3*f^3*g^7*x^2 + a^3*f^4*g^6*x)*e^4 + (
3*a^2*c*d*f*g^9*x^5 - a^3*d*f^4*g^6 + (12*a^2*c*d*f^2*g^8 - a^3*d*g^10)*x^4
 + 2*(9*a^2*c*d*f^3*g^7 - 2*a^3*d*f*g^9)*x^3 + 6*(2*a^2*c*d*f^4*g^6 - a^3*d
*f^2*g^8)*x^2 + (3*a^2*c*d*f^5*g^5 - 4*a^3*d*f^3*g^7)*x)*e^3 - 3*(a*c^2*d^2
*f^2*g^8*x^5 - a^2*c*d^2*f^5*g^5 + (4*a*c^2*d^2*f^3*g^7 - a^2*c*d^2*f*g^9)*
x^4 + 2*(3*a*c^2*d^2*f^4*g^6 - 2*a^2*c*d^2*f^2*g^8)*x^3 + 2*(2*a*c^2*d^2*f^
5*g^5 - 3*a^2*c*d^2*f^3*g^7)*x^2 + (a*c^2*d^2*f^6*g^4 - 4*a^2*c*d^2*f^4*g^6
```

```
)*x)*e^2 + (c^3*d^3*f^3*g^7*x^5 - 3*a*c^2*d^3*f^6*g^4 + (4*c^3*d^3*f^4*g^6
- 3*a*c^2*d^3*f^2*g^8)*x^4 + 6*(c^3*d^3*f^5*g^5 - 2*a*c^2*d^3*f^3*g^7)*x^3
+ 2*(2*c^3*d^3*f^6*g^4 - 9*a*c^2*d^3*f^4*g^6)*x^2 + (c^3*d^3*f^7*g^3 - 12*a
*c^2*d^3*f^5*g^5)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**5,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^5\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^5*(d + e*x)^(3
/2)), x)
```

3.699 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx$

Optimal. Leaf size=405

$$-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{64g^2(cdf-aeg)^2\sqrt{d+ex}\,(f+}$$

```
[Out] -1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^5+3/12
8*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(5/2)/(-a*e*g+c*d*f)^(7/2)-3/40*c*d*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(g*x+f)^4/(e*x+d)^(1/2)+1/80*c^2*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x+d)^(1/
2)+1/64*c^3*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(-a*e*g+c*d*f)^
2/(g*x+f)^2/(e*x+d)^(1/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/g^2/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{3c^5d^5\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}}+\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^2\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^2\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{80g^2\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}-\frac{3cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^6),x]
```

```
[Out] (-3*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(40*g^2*Sqrt[d + e*x]*
(f + g*x)^4) + (c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(80*g^
2*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)
^2) + (3*c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^2*(c*d
*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)/(5*g*(d + e*x)^(3/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
e*x])])/(128*g^(5/2)*(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^6}\,dx=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5}+\frac{(3cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x}}{\sqrt{d+ex}\,(f+g}}{10g}$$

$$=-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{5g(d+ex)^{3/2}(f+gx)}$$

$$=-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(}$$

$$=-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(}$$

$$=-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(}$$

$$=-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(}$$

$$=-\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40g^2\sqrt{d+ex}\,(f+gx)^4}+\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x}}{80g^2(cdf-aeg)\sqrt{d+ex}\,(}$$

<u>Mathematica</u> [A]
time = 2.39, size = 302, normalized size = 0.75

$$\frac{c^5d^5((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{g}\,(128a^4e^4g^4+16a^3cde^3g^3(-21f+11gx)+8a^2c^2d^2e^2g^2(31f^2-64fgx+g^2x^2)-2ac^3d^3eg(5f^3-233f^2gx+23fg^2x^2+5g^3x^3)+c^4d^4(-15f^4-70f^3gx+128f^2g^2x^2+70fg^3x^3+15g^4x^4))}{c^5d^5(cdf-aeg)^3(ae+cdx)(f+gx)^5}+\frac{15\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}(ae+cdx)^{3/2}}\right)}{640g^{5/2}(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^6),x]
```

```
[Out] (c^5*d^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[g]*(128*a^4*e^4*g^4 + 16*a^
3*c*d*e^3*g^3*(-21*f + 11*g*x) + 8*a^2*c^2*d^2*e^2*g^2*(31*f^2 - 64*f*g*x +
 g^2*x^2) - 2*a*c^3*d^3*e*g*(5*f^3 - 233*f^2*g*x + 23*f*g^2*x^2 + 5*g^3*x^3
) + c^4*d^4*(-15*f^4 - 70*f^3*g*x + 128*f^2*g^2*x^2 + 70*f*g^3*x^3 + 15*g^4
*x^4)))/(c^5*d^5*(c*d*f - a*e*g)^3*(a*e + c*d*x)*(f + g*x)^5) + (15*ArcTan[
```

(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(7/2)*(a*e + c*d*x)^(3/2))))/(640*g^(5/2)*(d + e*x)^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(361) = 722$.
time = 0.13, size = 945, normalized size = 2.33

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5g^5x^5+75\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5fg^4x^4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/640*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^5*d^5*g^5*x^5+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^5*d^5*f*g^4*x^4+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^5*d^5*f^2*g^3*x^3+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^5*d^5*f^3*g^2*x^2-15*c^4*d^4*g^4*x^4*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)
)*c^5*d^5*f^4*g*x+10*a*c^3*d^3*e*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)-70*c^4*d^4*f*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*a
rctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5-8*a^2*c^2*d
^2*e^2*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+46*a*c^3*d^3*e*f*g
^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-128*c^4*d^4*f^2*g^2*x^2*(c
*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-176*a^3*c*d*e^3*g^4*x*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)+512*a^2*c^2*d^2*e^2*f*g^3*x*(c*d*x+a*e)^(1/2)*
((a*e*g-c*d*f)*g)^(1/2)-466*a*c^3*d^3*e*f^2*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g
-c*d*f)*g)^(1/2)+70*c^4*d^4*f^3*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/
2)-128*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*e^4*g^4+336*(c*d*x+a*e
)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c*d*e^3*f*g^3-248*(c*d*x+a*e)^(1/2)*((a
*e*g-c*d*f)*g)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)*a*c^3*d^3*e*f^3*g+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/
2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^5/g^2/(a*e*g-
c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^6*(x*e +
 d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1653 vs. $2(378) = 756$.
time = 7.49, size = 3345, normalized size = 8.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [1/1280*(15*(c^5*d^6*g^5*x^5 + 5*c^5*d^6*f*g^4*x^4 + 10*c^5*d^6*f^2*g^3*x^3
 + 10*c^5*d^6*f^3*g^2*x^2 + 5*c^5*d^6*f^4*g*x + c^5*d^6*f^5 + (c^5*d^5*g^5*
x^6 + 5*c^5*d^5*f*g^4*x^5 + 10*c^5*d^5*f^2*g^3*x^4 + 10*c^5*d^5*f^3*g^2*x^3
 + 5*c^5*d^5*f^4*g*x^2 + c^5*d^5*f^5*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c
*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e + 2*sq
rt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e
 + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(15*c^5*d^5*f*g^5*x^4 + 70*c^5*
d^5*f^2*g^4*x^3 + 128*c^5*d^5*f^3*g^3*x^2 - 70*c^5*d^5*f^4*g^2*x - 15*c^5*d
^5*f^5*g - 128*a^5*g^6*e^5 - 16*(11*a^4*c*d*g^6*x - 29*a^4*c*d*f*g^5)*e^4 -
 8*(a^3*c^2*d^2*g^6*x^2 - 86*a^3*c^2*d^2*f*g^5*x + 73*a^3*c^2*d^2*f^2*g^4)*
e^3 + 2*(5*a^2*c^3*d^3*g^6*x^3 + 27*a^2*c^3*d^3*f*g^5*x^2 - 489*a^2*c^3*d^3
*f^2*g^4*x + 129*a^2*c^3*d^3*f^3*g^3)*e^2 - (15*a*c^4*d^4*g^6*x^4 + 80*a*c^
4*d^4*f*g^5*x^3 + 174*a*c^4*d^4*f^2*g^4*x^2 - 536*a*c^4*d^4*f^3*g^3*x - 5*a
*c^4*d^4*f^4*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e +
 d))/(c^4*d^5*f^4*g^8*x^5 + 5*c^4*d^5*f^5*g^7*x^4 + 10*c^4*d^5*f^6*g^6*x^3
+ 10*c^4*d^5*f^7*g^5*x^2 + 5*c^4*d^5*f^8*g^4*x + c^4*d^5*f^9*g^3 + (a^4*g^1
2*x^6 + 5*a^4*f*g^11*x^5 + 10*a^4*f^2*g^10*x^4 + 10*a^4*f^3*g^9*x^3 + 5*a^4
*f^4*g^8*x^2 + a^4*f^5*g^7*x)*e^5 - (4*a^3*c*d*f*g^11*x^6 - a^4*d*f^5*g^7 +
 (20*a^3*c*d*f^2*g^10 - a^4*d*g^12)*x^5 + 5*(8*a^3*c*d*f^3*g^9 - a^4*d*f*g^
11)*x^4 + 10*(4*a^3*c*d*f^4*g^8 - a^4*d*f^2*g^10)*x^3 + 10*(2*a^3*c*d*f^5*g
^7 - a^4*d*f^3*g^9)*x^2 + (4*a^3*c*d*f^6*g^6 - 5*a^4*d*f^4*g^8)*x)*e^4 + 2*
(3*a^2*c^2*d^2*f^2*g^10*x^6 - 2*a^3*c*d^2*f^6*g^6 + (15*a^2*c^2*d^2*f^3*g^9
 - 2*a^3*c*d^2*f*g^11)*x^5 + 10*(3*a^2*c^2*d^2*f^4*g^8 - a^3*c*d^2*f^2*g^10
)*x^4 + 10*(3*a^2*c^2*d^2*f^5*g^7 - 2*a^3*c*d^2*f^3*g^9)*x^3 + 5*(3*a^2*c^2
*d^2*f^6*g^6 - 4*a^3*c*d^2*f^4*g^8)*x^2 + (3*a^2*c^2*d^2*f^7*g^5 - 10*a^3*c
*d^2*f^5*g^7)*x)*e^3 - 2*(2*a*c^3*d^3*f^3*g^9*x^6 - 3*a^2*c^2*d^3*f^7*g^5 +
 (10*a*c^3*d^3*f^4*g^8 - 3*a^2*c^2*d^3*f^2*g^10)*x^5 + 5*(4*a*c^3*d^3*f^5*g
^7 - 3*a^2*c^2*d^3*f^3*g^9)*x^4 + 10*(2*a*c^3*d^3*f^6*g^6 - 3*a^2*c^2*d^3*f
^4*g^8)*x^3 + 10*(a*c^3*d^3*f^7*g^5 - 3*a^2*c^2*d^3*f^5*g^7)*x^2 + (2*a*c^3
*d^3*f^8*g^4 - 15*a^2*c^2*d^3*f^6*g^6)*x)*e^2 + (c^4*d^4*f^4*g^8*x^6 - 4*a*
```

```
c^3*d^4*f^8*g^4 + (5*c^4*d^4*f^5*g^7 - 4*a*c^3*d^4*f^3*g^9)*x^5 + 10*(c^4*d
^4*f^6*g^6 - 2*a*c^3*d^4*f^4*g^8)*x^4 + 10*(c^4*d^4*f^7*g^5 - 4*a*c^3*d^4*f
^5*g^7)*x^3 + 5*(c^4*d^4*f^8*g^4 - 8*a*c^3*d^4*f^6*g^6)*x^2 + (c^4*d^4*f^9*
g^3 - 20*a*c^3*d^4*f^7*g^5)*x)*e), -1/640*(15*(c^5*d^6*g^5*x^5 + 5*c^5*d^6*
f*g^4*x^4 + 10*c^5*d^6*f^2*g^3*x^3 + 10*c^5*d^6*f^3*g^2*x^2 + 5*c^5*d^6*f^4
*g*x + c^5*d^6*f^5 + (c^5*d^5*g^5*x^6 + 5*c^5*d^5*f*g^4*x^5 + 10*c^5*d^5*f^
2*g^3*x^4 + 10*c^5*d^5*f^3*g^2*x^3 + 5*c^5*d^5*f^4*g*x^2 + c^5*d^5*f^5*x)*e
)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2
 + a*d*g)*e)) - (15*c^5*d^5*f*g^5*x^4 + 70*c^5*d^5*f^2*g^4*x^3 + 128*c^5*d^
5*f^3*g^3*x^2 - 70*c^5*d^5*f^4*g^2*x - 15*c^5*d^5*f^5*g - 128*a^5*g^6*e^5 -
 16*(11*a^4*c*d*g^6*x - 29*a^4*c*d*f*g^5)*e^4 - 8*(a^3*c^2*d^2*g^6*x^2 - 86
*a^3*c^2*d^2*f*g^5*x + 73*a^3*c^2*d^2*f^2*g^4)*e^3 + 2*(5*a^2*c^3*d^3*g^6*x
^3 + 27*a^2*c^3*d^3*f*g^5*x^2 - 489*a^2*c^3*d^3*f^2*g^4*x + 129*a^2*c^3*d^3
*f^3*g^3)*e^2 - (15*a*c^4*d^4*g^6*x^4 + 80*a*c^4*d^4*f*g^5*x^3 + 174*a*c^4*
d^4*f^2*g^4*x^2 - 536*a*c^4*d^4*f^3*g^3*x - 5*a*c^4*d^4*f^4*g^2)*e)*sqrt(c*
d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^4*d^5*f^4*g^8*x^5 +
5*c^4*d^5*f^5*g^7*x^4 + 10*c^4*d^5*f^6*g^6*x^3 + 10*c^4*d^5*f^7*g^5*x^2 + 5
*c^4*d^5*f^8*g^4*x + c^4*d^5*f^9*g^3 + (a^4*g^12*x^6 + 5*a^4*f*g^11*x^5 + 1
0*a^4*f^2*g^10*x^4 + 10*a^4*f^3*g^9*x^3 + 5*a^4*f^4*g^8*x^2 + a^4*f^5*g^7*x
)*e^5 - (4*a^3*c*d*f*g^11*x^6 - a^4*d*f^5*g^7 + (20*a^3*c*d*f^2*g^10 - a^4*
d*g^12)*x^5 + 5*(8*a^3*c*d*f^3*g^9 - a^4*d*f*g^11)*x^4 + 10*(4*a^3*c*d*f^4*
g^8 - a^4*d*f^2*g^10)*x^3 + 10*(2*a^3*c*d*f^5*g^7 - a^4*d*f^3*g^9)*x^2 + (4
*a^3*c*d*f^6*g^6 - 5*a^4*d*f^4*g^8)*x)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^10*x^6
- 2*a^3*c*d^2*f^6*g^6 + (15*a^2*c^2*d^2*f^3*g^9 - 2*a^3*c*d^2*f*g^11)*x^5 +
 10*(3*a^2*c^2*d^2*f^4*g^8 - a^3*c*d^2*f^2*g^10)*x^4 + 10*(3*a^2*c^2*d^2*f^
5*g^7 - 2*a^3*c*d^2*f^3*g^9)*x^3 + 5*(3*a^2*c^2*d^2*f^6*g^6 - 4*a^3*c*d^2*f
^4*g^8)*x^2 + (3*a^2*c^2*d^2*f^7*g^5 - 10*a^3*c*d^2*f^5*g^7)*x)*e^3 - 2*(2*
a*c^3*d^3*f^3*g^9*x^6 - 3*a^2*c^2*d^3*f^7*g^5 + (10*a*c^3*d^3*f^4*g^8 - 3*a
^2*c^2*d^3*f^2*g^10)*x^5 + 5*(4*a*c^3*d^3*f^5*g^7 - 3*a^2*c^2*d^3*f^3*g^9)*
x^4 + 10*(2*a*c^3*d^3*f^6*g^6 - 3*a^2*c^2*d^3*f^4*g^8)*x^3 + 10*(a*c^3*d^3*
f^7*g^5 - 3*a^2*c^2*d^3*f^5*g^7)*x^2 + (2*a*c^3*d^3*f^8*g^4 - 15*a^2*c^2*d^
3*f^6*g^6)*x)*e^2 + (c^4*d^4*f^4*g^8*x^6 - 4*a*c^3*d^4*f^8*g^4 + (5*c^4*d^4
*f^5*g^7 - 4*a*c^3*d^4*f^3*g^9)*x^5 + 10*(c^4*d^4*f^6*g^6 - 2*a*c^3*d^4*f^4
*g^8)*x^4 + 10*(c^4*d^4*f^7*g^5 - 4*a*c^3*d^4*f^5*g^7)*x^3 + 5*(c^4*d^4*f^8
*g^4 - 8*a*c^3*d^4*f^6*g^6)*x^2 + (c^4*d^4*f^9*g^3 - 20*a*c^3*d^4*f^7*g^5)*
x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**6,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^6,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^6\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(3
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^6*(d + e*x)^(3
/2)), x)
```

3.700 $\int \frac{(f+gx)^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}} dx$

Optimal. Leaf size=336

$$-\frac{128(cdf-aeg)^3\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{45045c^5d^5e(d+ex)^{7/2}}+\frac{128g(cdf-aeg)^3\left(ade+\left(cd^2\right.\right.}{6435c^4d^4e(d+}$$

```
[Out] -128/45045*(-a*e*g+c*d*f)^3*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(7/2)/c^5/d^5/e/(e*x+d)^(7/2)+128/6435*g*(-a*e*g+c*d*f)^3
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/e/(e*x+d)^(5/2)+32/715*(-a
*e*g+c*d*f)^2*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/(e*
x+d)^(7/2)+16/195*(-a*e*g+c*d*f)*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(7/2)/c^2/d^2/(e*x+d)^(7/2)+2/15*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A]
time = 0.39, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{128\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^3\left(2ae^2g-cd(9ef-7dg)\right)}{45045c^5d^5e(d+ex)^{7/2}}+\frac{128g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^3}{6435c^4d^4e(d+ex)^{5/2}}+\frac{32(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{715c^3d^3(d+ex)^{7/2}}+\frac{16(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{195c^2d^2(d+ex)^{7/2}}+\frac{2(f+gx)^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(7/2))/(45045*c^5*d^5*e*(d + e*x)^(7/2)) + (128*g*(c
*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(6435*c^4*d^
4*e*(d + e*x)^(5/2)) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(715*c^3*d^3*(d + e*x)^(7/2)) + (16*(c*d*f - a
*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(195*c^2*d
^2*(d + e*x)^(7/2)) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(7/2))/(15*c*d*(d + e*x)^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
```

```
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{15cd(d+ex)^{7/2}} + \frac{(8(cdf-\cdots}{\cdots}$$

$$= \frac{16(cdf-aeg)(f+gx)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{195c^2d^2(d+ex)^{7/2}} \cdots$$

$$= \frac{32(cdf-aeg)^2(f+gx)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/\cdots}}{715c^3d^3(d+ex)^{7/2}} \cdots$$

$$= \frac{128g(cdf-aeg)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{6435c^4d^4e(d+ex)^{5/2}} + \frac{32(\cdots}{\cdots}$$

$$= \frac{128(cdf-aeg)^3\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+(cd^2+ae^2)x+\cdots\right.}{45045c^4d^4(d+ex)^{7/2}}$$

Mathematica [A]
time = 0.18, size = 205, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\,\left(128a^4e^4g^4-64a^3cde^3g^3(15f+7gx)+48a^2c^2d^2e^2g^2(65f^2+70fgx+21g^2x^2)-8ac^3d^3eg(715f^3+1365f^2gx+945fg^2x^2+231g^3x^3)+c^4d^4(6435f^4+20020f^3gx+24570f^2g^2x^2+13860fg^3x^3+3003g^4x^4)\right)}{45045c^5d^5\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^4*g^4 - 64*a^3*
c*d*e^3*g^3*(15*f + 7*g*x) + 48*a^2*c^2*d^2*e^2*g^2*(65*f^2 + 70*f*g*x + 21
*g^2*x^2) - 8*a*c^3*d^3*e*g*(715*f^3 + 1365*f^2*g*x + 945*f*g^2*x^2 + 231*g
^3*x^3) + c^4*d^4*(6435*f^4 + 20020*f^3*g*x + 24570*f^2*g^2*x^2 + 13860*f*g
^3*x^3 + 3003*g^4*x^4)))/(45045*c^5*d^5*Sqrt[d + e*x])
```

Maple [A]
time = 0.14, size = 275, normalized size = 0.82

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3(3003g^4x^4c^4d^4-1848a\,c^3d^3e\,g^4x^3+13860c^4d^4f\,g^3x^3+1008a^2c^2d^2e^2g^4x^2-7560a\,c^3d^3e f}{}$
gosper	$\frac{2(cdx+ae)(3003g^4x^4c^4d^4-1848a\,c^3d^3e\,g^4x^3+13860c^4d^4f\,g^3x^3+1008a^2c^2d^2e^2g^4x^2-7560a\,c^3d^3ef\,g^3x^2+24570c^4d^4f^2g^2x^2-448a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/45045*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(3003*c^4*d
^4*g^4*x^4-1848*a*c^3*d^3*e*g^4*x^3+13860*c^4*d^4*f*g^3*x^3+1008*a^2*c^2*d^
2*e^2*g^4*x^2-7560*a*c^3*d^3*e*f*g^3*x^2+24570*c^4*d^4*f^2*g^2*x^2-448*a^3*
c*d*e^3*g^4*x+3360*a^2*c^2*d^2*e^2*f*g^3*x-10920*a*c^3*d^3*e*f^2*g^2*x+2002
0*c^4*d^4*f^3*g*x+128*a^4*e^4*g^4-960*a^3*c*d*e^3*f*g^3+3120*a^2*c^2*d^2*e^
2*f^2*g^2-5720*a*c^3*d^3*e*f^3*g+6435*c^4*d^4*f^4)/c^5/d^5
```

Maxima [A]
time = 0.38, size = 488, normalized size = 1.45

$$\frac{2\,(c^3d^3x^3+3\,ac^2d^2x^2e+3\,a^2cdxe^2+a^3e^3)\sqrt{cdx+ae}\,f^4}{7\,cd}+\frac{8\,(7\,c^4d^4x^4+19\,ac^3d^3x^3e+15\,a^2c^2d^2x^2e^2+a^3cdxe^3-2\,a^4e^4)\sqrt{cdx+ae}\,f^3g}{63\,c^2d^2}+\frac{4\,(63\,c^5d^5x^5+161\,ac^4d^4x^4e+113\,a^2c^3d^3x^3e^2+3\,a^3c^2d^2x^2e^3-4\,a^4cdxe^4+8\,a^5e^5)\sqrt{cdx+ae}\,f^2g^2}{231\,c^3d^3}+\frac{8\,(231\,c^6d^6x^6+567\,ac^5d^5x^5e+371\,a^2c^4d^4x^4e^2+5\,a^3c^3d^3x^3e^3-6\,a^4c^2d^2x^2e^4+8\,a^5cdxe^5-16\,a^6e^6)\sqrt{cdx+ae}\,fg^3}{3003\,c^4d^4}+\frac{2\,(3003\,c^7d^7x^7+7161\,ac^6d^6x^6e+4473\,a^2c^5d^5x^5e^2+35\,a^3c^4d^4x^4e^3-40\,a^4c^3d^3x^3e^4+48\,a^5c^2d^2x^2e^5-64\,a^6cdxe^6+128\,a^7e^7)\sqrt{cdx+ae}\,g^4}{45045\,c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*x^2*e + 3*a^2*c*d*x*e^2 + a^3*e^3)*sqrt(c*d*
x + a*e)*f^4/(c*d) + 8/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*x^3*e + 15*a^2*c^2*
d^2*x^2*e^2 + a^3*c*d*x*e^3 - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^3*g/(c^2*d^2)
+ 4/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*x^4*e + 113*a^2*c^3*d^3*x^3*e^2 + 3
*a^3*c^2*d^2*x^2*e^3 - 4*a^4*c*d*x*e^4 + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f^2*g
^2/(c^3*d^3) + 8/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*x^5*e + 371*a^2*c^4*
d^4*x^4*e^2 + 5*a^3*c^3*d^3*x^3*e^3 - 6*a^4*c^2*d^2*x^2*e^4 + 8*a^5*c*d*x*e
^5 - 16*a^6*e^6)*sqrt(c*d*x + a*e)*f*g^3/(c^4*d^4) + 2/45045*(3003*c^7*d^7*
x^7 + 7161*a*c^6*d^6*x^6*e + 4473*a^2*c^5*d^5*x^5*e^2 + 35*a^3*c^4*d^4*x^4*
```

```
e^3 - 40*a^4*c^3*d^3*x^3*e^4 + 48*a^5*c^2*d^2*x^2*e^5 - 64*a^6*c*d*x*e^6 +
128*a^7*e^7)*sqrt(c*d*x + a*e)*g^4/(c^5*d^5)
```

Fricas [A]
time = 1.80, size = 562, normalized size = 1.67

$$\frac{2\left(3003\,c^{7}d^{7}g^{4}x^{7}+13860\,c^{7}d^{7}fg^{3}x^{6}+24570\,c^{7}d^{7}f^{2}g^{2}x^{5}+20020\,c^{7}d^{7}f^{3}gx^{4}+6435\,c^{7}d^{7}f^{4}x^{3}+128\,a^{7}g^{4}e^{7}-64\left(a^{6}cdg^{4}x+15\,a^{6}cdfg^{3}\right)e^{6}+48\left(a^{5}c^{2}d^{2}g^{4}x^{2}+10\,a^{5}c^{2}d^{2}fg^{3}x+65\,a^{5}c^{2}d^{2}f^{2}g^{2}\right)e^{5}-40\left(a^{4}c^{3}d^{3}g^{4}x^{3}+9\,a^{4}c^{3}d^{3}fg^{3}x^{2}+39\,a^{4}c^{3}d^{3}f^{2}g^{2}x+143\,a^{4}c^{3}d^{3}f^{3}g\right)e^{4}+5\left(7\,a^{3}c^{4}d^{4}g^{4}x^{4}+60\,a^{3}c^{4}d^{4}fg^{3}x^{3}+234\,a^{3}c^{4}d^{4}f^{2}g^{2}x^{2}+572\,a^{3}c^{4}d^{4}f^{3}gx+1287\,a^{3}c^{4}d^{4}f^{4}\right)e^{3}+3\left(1491\,a^{2}c^{5}d^{5}g^{4}x^{5}+7420\,a^{2}c^{5}d^{5}fg^{3}x^{4}+14690\,a^{2}c^{5}d^{5}f^{2}g^{2}x^{3}+14300\,a^{2}c^{5}d^{5}f^{3}gx^{2}+6435\,a^{2}c^{5}d^{5}f^{4}x\right)e^{2}+\left(7161\,ac^{6}d^{6}g^{4}x^{6}+34020\,ac^{6}d^{6}fg^{3}x^{5}+62790\,ac^{6}d^{6}f^{2}g^{2}x^{4}+54340\,ac^{6}d^{6}f^{3}gx^{3}+19305\,ac^{6}d^{6}f^{4}x^{2}\right)e\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\,\sqrt{xe+d}}{45045\left(c^{5}d^{5}xe+c^{5}d^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/45045*(3003*c^7*d^7*g^4*x^7 + 13860*c^7*d^7*f*g^3*x^6 + 24570*c^7*d^7*f^2
*g^2*x^5 + 20020*c^7*d^7*f^3*g*x^4 + 6435*c^7*d^7*f^4*x^3 + 128*a^7*g^4*e^7
 - 64*(a^6*c*d*g^4*x + 15*a^6*c*d*f*g^3)*e^6 + 48*(a^5*c^2*d^2*g^4*x^2 + 10
*a^5*c^2*d^2*f*g^3*x + 65*a^5*c^2*d^2*f^2*g^2)*e^5 - 40*(a^4*c^3*d^3*g^4*x^
3 + 9*a^4*c^3*d^3*f*g^3*x^2 + 39*a^4*c^3*d^3*f^2*g^2*x + 143*a^4*c^3*d^3*f^
3*g)*e^4 + 5*(7*a^3*c^4*d^4*g^4*x^4 + 60*a^3*c^4*d^4*f*g^3*x^3 + 234*a^3*c^
4*d^4*f^2*g^2*x^2 + 572*a^3*c^4*d^4*f^3*g*x + 1287*a^3*c^4*d^4*f^4)*e^3 + 3
*(1491*a^2*c^5*d^5*g^4*x^5 + 7420*a^2*c^5*d^5*f*g^3*x^4 + 14690*a^2*c^5*d^5
*f^2*g^2*x^3 + 14300*a^2*c^5*d^5*f^3*g*x^2 + 6435*a^2*c^5*d^5*f^4*x)*e^2 +
(7161*a*c^6*d^6*g^4*x^6 + 34020*a*c^6*d^6*f*g^3*x^5 + 62790*a*c^6*d^6*f^2*g
^2*x^4 + 54340*a*c^6*d^6*f^3*g*x^3 + 19305*a*c^6*d^6*f^4*x^2)*e)*sqrt(c*d^2
*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^5*d^5*x*e + c^5*d^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4231 vs. $2(317) = 634$.
time = 6.42, size = 4231, normalized size = 12.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

[Out] 2/45045*(429*c^2*d^2*f^4*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - 572*c^2*d^2*f^3*g*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4))*e^(-1) + 78*c^2*d^2*f^2*g^2*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-4)/(c^5*d^5) + (1155*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c^5*d^5))*e^(-1) - 20*c^2*d^2*f*g^3*((693*sqrt(-c*d^2*e + a*e^3)*c^6*d^12 - 63*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^10*e^2 - 70*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^8*e^4 - 80*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^6*e^6 - 96*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*c*d^2*e^10 - 256*sqrt(-c*d^2*e + a*e^3)*a^6*e^12)*e^(-5)/(c^6*d^6) + (3003*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^5*e^15 - 9009*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^4*e^12 + 12870*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^3*e^9 - 10010*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a^2*e^6 + 4095*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(11/2)*a*e^3 - 693*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(13/2))*e^(-11)/(c^6*d^6))*e^(-1) + c^2*d^2*g^4*((3003*sqrt(-c*d^2*e + a*e^3)*c^7*d^14 - 231*sqrt(-c*d^2*e + a*e^3)*a*c^6*d^12*e^2 - 252*sqrt(-c*d^2*e + a*e^3)*a^2*c^5*d^10*e^4 - 280*sqrt(-c*d^2*e + a*e^3)*a^3*c^4*d^8*e^6 - 320*sqrt(-c*d^2*e + a*e^3)*a^4*c^3*d^6*e^8 - 384*sqrt(-c*d^2*e + a*e^3)*a^5*c^2*d^4*e^10 - 512*sqrt(-c*d^2*e + a*e^3)*a^6*c*d^2*e^12 - 1024*sqrt(-c*d^2*e + a*e^3)*a^7*e^14)*e^(-6)/(c^7*d^7) + (15015*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^6*e^18 - 54054*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^5*e^15 + 96525*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^4*e^12 - 100100*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a^3*e^9 + 61425*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(11/2)*a^2*e^6 - 20790*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(13/2)*a*e^3 + 3003*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(15/2))*e^(-13)/(c^7*d^7))*e^(-1) - 6006*a*c*d*f^4*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-1) + 3432*a*c*d*f^3*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a

```
^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^
2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*
c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3)) - 1716*a*c*d*f^2*g^2*((35
*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 -
6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d
^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*((x*e +
 d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e - c*d^2*e
 + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e
^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4)) + 104*
a*c*d*f*g^3*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^
3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d
^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 1
28*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-4)/(c^5*d^5) + (1155*((x*e + d)*c*d
*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((x*e + d)*c*d*e - c*d^2*e + a*
e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6
 - 1540*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((x*e + d)*c*
d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c^5*d^5)) - 10*a*c*d*g^4*((693*sqrt(
-c*d^2*e + a*e^3)*c^6*d^12 - 63*sqrt(-c*d^2*e + a*e^3)*a*c^5*d^10*e^2 - 70*
sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d^8*e^4 - 80*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*
d^6*e^6 - 96*sqrt(-c*d^2*e + a*e^3)*a^4*c^2*d^4*e^8 - 128*sqrt(-c*d^2*e + a
*e^3)*a^5*c*d^2*e^10 - 256*sqrt(-c*d^2*e + a*e^3)*a^6*e^12)*e^(-5)/(c^6*d^6
) + (3003*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(...
```

Mupad [B]
time = 4.09, size = 523, normalized size = 1.56

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^5\,\left(71\,a^2\,e^2\,g^2+540\,a\,c\,d\,e\,f\,g+390\,c^2\,d^2\,f^2\right)}{715}+\frac{256\,a^7\,e^7\,g^4-1920\,a^6\,c\,d\,e^6\,f\,g^3+6240\,a^5\,c^2\,d^2\,e^5\,f^2\,g^2-11440\,a^4\,c^3\,d^3\,e^4\,f^3\,g+12870\,a^3\,c^4\,d^4\,e^3\,f^4}{45045\,c^5\,d^5}+\frac{x^3\,\left(-80\,a^4\,c^3\,d^3\,e^4\,g^4+600\,a^3\,c^4\,d^4\,e^3\,f\,g^3+88140\,a^2\,c^5\,d^5\,e^2\,f^2\,g^2+108680\,a\,c^6\,d^6\,e\,f^3\,g+12870\,c^7\,d^7\,f^4\right)}{45045\,c^5\,d^5}+\frac{2\,c^2\,d^2\,g^4\,x^7}{15}+\frac{2\,c\,d\,g^3\,x^6\,\left(31\,a\,e\,g+60\,c\,d\,f\right)}{195}+\frac{2\,g\,x^4\,\left(a^3\,e^3\,g^3+636\,a^2\,c\,d\,e^2\,f\,g^2+1794\,a\,c^2\,d^2\,e\,f^2\,g+572\,c^3\,d^3\,f^3\right)}{1287\,c\,d}+\frac{2\,a^2\,e^2\,x\,\left(-64\,a^4\,e^4\,g^4+480\,a^3\,c\,d\,e^3\,f\,g^3-1560\,a^2\,c^2\,d^2\,e^2\,f^2\,g^2+2860\,a\,c^3\,d^3\,e\,f^3\,g+19305\,c^4\,d^4\,f^4\right)}{45045\,c^4\,d^4}+\frac{2\,a\,e\,x^2\,\left(16\,a^4\,e^4\,g^4-120\,a^3\,c\,d\,e^3\,f\,g^3+390\,a^2\,c^2\,d^2\,e^2\,f^2\,g^2+14300\,a\,c^3\,d^3\,e\,f^3\,g+6435\,c^4\,d^4\,f^4\right)}{15015\,c^3\,d^3}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^5*(71*a^2*e^2*g^2
+ 390*c^2*d^2*f^2 + 540*a*c*d*e*f*g))/715 + (256*a^7*e^7*g^4 + 12870*a^3*c^
4*d^4*e^3*f^4 - 11440*a^4*c^3*d^3*e^4*f^3*g - 1920*a^6*c*d*e^6*f*g^3 + 6240
*a^5*c^2*d^2*e^5*f^2*g^2)/(45045*c^5*d^5) + (x^3*(12870*c^7*d^7*f^4 - 80*a^
4*c^3*d^3*e^4*g^4 + 600*a^3*c^4*d^4*e^3*f*g^3 + 108680*a*c^6*d^6*e*f^3*g +
88140*a^2*c^5*d^5*e^2*f^2*g^2))/(45045*c^5*d^5) + (2*c^2*d^2*g^4*x^7)/15 +
(2*c*d*g^3*x^6*(31*a*e*g + 60*c*d*f))/195 + (2*g*x^4*(a^3*e^3*g^3 + 572*c^3
*d^3*f^3 + 1794*a*c^2*d^2*e*f^2*g + 636*a^2*c*d*e^2*f*g^2))/(1287*c*d) + (2
*a^2*e^2*x*(19305*c^4*d^4*f^4 - 64*a^4*e^4*g^4 + 2860*a*c^3*d^3*e*f^3*g + 4
80*a^3*c*d*e^3*f*g^3 - 1560*a^2*c^2*d^2*e^2*f^2*g^2))/(45045*c^4*d^4) + (2*
a*e*x^2*(16*a^4*e^4*g^4 + 6435*c^4*d^4*f^4 + 14300*a*c^3*d^3*e*f^3*g - 120*
a^3*c*d*e^3*f*g^3 + 390*a^2*c^2*d^2*e^2*f^2*g^2))/(15015*c^3*d^3)))/(d + e*
x)^(1/2)
```

3.701 $\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{16(cdf-aeg)^2\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{3003c^4d^4e(d+ex)^{7/2}}+\frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+\right.\right.}{429c^3d^3e(d+e}$$

```
[Out] -16/3003*(-a*e*g+c*d*f)^2*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(7/2)/c^4/d^4/e/(e*x+d)^(7/2)+16/429*g*(-a*e*g+c*d*f)^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^(5/2)+12/143*(-a*e*g
+c*d*f)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/(e*x+d)^(
7/2)+2/13*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/
2)
```

Rubi [A]
time = 0.26, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{16\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2\left(2ae^2g-cd(9ef-7dg)\right)}{3003c^4d^4e(d+ex)^{7/2}}+\frac{16g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)^2}{429c^3d^3e(d+ex)^{5/2}}+\frac{12(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{143c^2d^2(d+ex)^{7/2}}+\frac{2(f+gx)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^(7/2))/(3003*c^4*d^4*e*(d + e*x)^(7/2)) + (16*g*(c*d*
f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*c^3*d^3*e*
(d + e*x)^(5/2)) + (12*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(7/2))/(143*c^2*d^2*(d + e*x)^(7/2)) + (2*(f + g*x)^3*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*c*d*(d + e*x)^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13cd(d+ex)^{7/2}}+\frac{(6(cdf-\ldots}{\ldots}$$

$$= \frac{12(cdf-aeg)(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143c^2d^2(d+ex)^{7/2}} \ldots$$

$$= \frac{16g(cdf-aeg)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{429c^3d^3e(d+ex)^{5/2}}+\frac{12(c\ldots}{\ldots}$$

$$= \frac{16(cdf-aeg)^2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+\ldots\right.}{3003c^3d^3(d+ex)^{7/2}}$$

Mathematica [A]
time = 0.14, size = 147, normalized size = 0.55

$$\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\left(-16a^3e^3g^3+8a^2cde^2g^2(13f+7gx)-2ac^2d^2eg\left(143f^2+182fgx+63g^2x^2\right)+c^3d^3\left(429f^3+1001f^2gx+819fg^2x^2+231g^3x^3\right)\right)}{3003c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3 + 8*a^2*c
*d*e^2*g^2*(13*f + 7*g*x) - 2*a*c^2*d^2*e*g*(143*f^2 + 182*f*g*x + 63*g^2*x
^2) + c^3*d^3*(429*f^3 + 1001*f^2*g*x + 819*f*g^2*x^2 + 231*g^3*x^3)))/(300
3*c^4*d^4*Sqrt[d + e*x])
```

Maple [A]
time = 0.16, size = 180, normalized size = 0.67

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{3}(-231g^{3}x^{3}c^{3}d^{3}+126a\,c^{2}d^{2}e\,g^{3}x^{2}-819c^{3}d^{3}f\,g^{2}x^{2}-56a^{2}cd\,e^{2}g^{3}x+364a\,c^{2}d^{2}ef\,g^{2}x-}{3003\sqrt{ex+d}\,c^{4}d^{4}}$
gosper	$-\frac{2(cdx+ae)(-231g^{3}x^{3}c^{3}d^{3}+126a\,c^{2}d^{2}e\,g^{3}x^{2}-819c^{3}d^{3}f\,g^{2}x^{2}-56a^{2}cd\,e^{2}g^{3}x+364a\,c^{2}d^{2}ef\,g^{2}x-1001c^{3}d^{3}f^{2}gx+16a^{3}e^{3}g^{3}-104a^{2}}{3003c^{4}d^{4}(ex+d)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3003*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-231*c^3*d
^3*g^3*x^3+126*a*c^2*d^2*e*g^3*x^2-819*c^3*d^3*f*g^2*x^2-56*a^2*c*d*e^2*g^3
*x+364*a*c^2*d^2*e*f*g^2*x-1001*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-104*a^2*c*d*
e^2*f*g^2+286*a*c^2*d^2*e*f^2*g-429*c^3*d^3*f^3)/c^4/d^4
```

Maxima [A]
time = 0.34, size = 356, normalized size = 1.32

$$\frac{2\,(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}x^{2}e+3\,a^{2}cdxe^{2}+a^{3}e^{3})\sqrt{cdx+ae}\,f^{3}}{7\,cd}+\frac{2\,(7\,c^{4}d^{4}x^{4}+19\,ac^{3}d^{3}x^{3}e+15\,a^{2}c^{2}d^{2}x^{2}e^{2}+a^{3}cdxe^{3}-2\,a^{4}e^{4})\sqrt{cdx+ae}\,f^{2}g}{21\,c^{2}d^{2}}+\frac{2\,(63\,c^{5}d^{5}x^{5}+161\,ac^{4}d^{4}x^{4}e+113\,a^{2}c^{3}d^{3}x^{3}e^{2}+3\,a^{3}c^{2}d^{2}x^{2}e^{3}-4\,a^{4}cdxe^{4}+8\,a^{5}e^{5})\sqrt{cdx+ae}\,fg^{2}}{231\,c^{3}d^{3}}+\frac{2\,(231\,c^{6}d^{6}x^{6}+567\,ac^{5}d^{5}x^{5}e+371\,a^{2}c^{4}d^{4}x^{4}e^{2}+5\,a^{3}c^{3}d^{3}x^{3}e^{3}-6\,a^{4}c^{2}d^{2}x^{2}e^{4}+8\,a^{5}cdxe^{5}-16\,a^{6}e^{6})\sqrt{cdx+ae}\,g^{3}}{3003\,c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*x^2*e + 3*a^2*c*d*x*e^2 + a^3*e^3)*sqrt(c*d*
x + a*e)*f^3/(c*d) + 2/21*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*x^3*e + 15*a^2*c^2*
d^2*x^2*e^2 + a^3*c*d*x*e^3 - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f^2*g/(c^2*d^2)
+ 2/231*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*x^4*e + 113*a^2*c^3*d^3*x^3*e^2 + 3
*a^3*c^2*d^2*x^2*e^3 - 4*a^4*c*d*x*e^4 + 8*a^5*e^5)*sqrt(c*d*x + a*e)*f*g^2
/(c^3*d^3) + 2/3003*(231*c^6*d^6*x^6 + 567*a*c^5*d^5*x^5*e + 371*a^2*c^4*d^
4*x^4*e^2 + 5*a^3*c^3*d^3*x^3*e^3 - 6*a^4*c^2*d^2*x^2*e^4 + 8*a^5*c*d*x*e^5
 - 16*a^6*e^6)*sqrt(c*d*x + a*e)*g^3/(c^4*d^4)
```

Fricas [A]
time = 1.77, size = 412, normalized size = 1.53

$$\frac{2\,(231\,c^{6}d^{6}g^{3}x^{6}+819\,c^{6}d^{6}fg^{2}x^{5}+1001\,c^{6}d^{6}f^{2}gx^{4}+429\,c^{6}d^{6}f^{3}x^{3}-16\,a^{6}g^{3}e^{6}+8\,(a^{5}cdg^{3}x+13\,a^{5}cdfg^{2})e^{5}-2\,(3\,a^{4}c^{2}d^{2}g^{3}x^{2}+26\,a^{4}c^{2}d^{2}fg^{2}x+143\,a^{4}c^{2}d^{2}f^{2}g)e^{4}+(5\,a^{3}c^{3}d^{3}g^{3}x^{3}+39\,a^{3}c^{3}d^{3}fg^{2}x^{2}+143\,a^{3}c^{3}d^{3}f^{2}gx+429\,a^{3}c^{3}d^{3}f^{3})e^{3}+(371\,a^{2}c^{4}d^{4}g^{3}x^{4}+1469\,a^{2}c^{4}d^{4}fg^{2}x^{3}+2145\,a^{2}c^{4}d^{4}f^{2}gx^{2}+1287\,a^{2}c^{4}d^{4}f^{3}x)e^{2}+(567\,ac^{5}d^{5}g^{3}x^{5}+2093\,ac^{5}d^{5}fg^{2}x^{4}+2717\,ac^{5}d^{5}f^{2}gx^{3}+1287\,ac^{5}d^{5}f^{3}x^{2})e)\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,\sqrt{xe+d}}{3003\,(c^{4}d^{4}xe+c^{4}d^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/3003*(231*c^6*d^6*g^3*x^6 + 819*c^6*d^6*f*g^2*x^5 + 1001*c^6*d^6*f^2*g*x^
4 + 429*c^6*d^6*f^3*x^3 - 16*a^6*g^3*e^6 + 8*(a^5*c*d*g^3*x + 13*a^5*c*d*f*
g^2)*e^5 - 2*(3*a^4*c^2*d^2*g^3*x^2 + 26*a^4*c^2*d^2*f*g^2*x + 143*a^4*c^2*
d^2*f^2*g)*e^4 + (5*a^3*c^3*d^3*g^3*x^3 + 39*a^3*c^3*d^3*f*g^2*x^2 + 143*a^
3*c^3*d^3*f^2*g*x + 429*a^3*c^3*d^3*f^3)*e^3 + (371*a^2*c^4*d^4*g^3*x^4 + 1
469*a^2*c^4*d^4*f*g^2*x^3 + 2145*a^2*c^4*d^4*f^2*g*x^2 + 1287*a^2*c^4*d^4*f
^3*x)*e^2 + (567*a*c^5*d^5*g^3*x^5 + 2093*a*c^5*d^5*f*g^2*x^4 + 2717*a*c^5*
d^5*f^2*g*x^3 + 1287*a*c^5*d^5*f^3*x^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^
2 + a*d)*e)*sqrt(x*e + d)/(c^4*d^4*x*e + c^4*d^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3013 vs. 2(253) = 506.
time = 6.10, size = 3013, normalized size = 11.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] 2/45045*(429*c^2*d^2*f^3*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^
2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqr
t(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^
2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a
*e^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-
1) - 429*c^2*d^2*f^2*g*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*
e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqr
t(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-
3)/(c^4*d^4) + (105*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189
*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e -
 c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2
))*e^(-7)/(c^4*d^4))*e^(-1) + 39*c^2*d^2*f*g^2*((315*sqrt(-c*d^2*e + a*e^3)
*c^5*d^10 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*
e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(
-c*d^2*e + a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-
```

4)/(c^5*d^5) + (1155*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2
772*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d
*e - c*d^2*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((x*e + d)*c*d*e - c*d^2*e + a*e
^3)^(9/2)*a*e^3 + 315*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c
^5*d^5))*e^(-1) - 5*c^2*d^2*g^3*((693*sqrt(-c*d^2*e + a*e^3)*c^6*d^12 - 63*
sqrt(-c*d^2*e + a*e^3)*a*c^5*d^10*e^2 - 70*sqrt(-c*d^2*e + a*e^3)*a^2*c^4*d
^8*e^4 - 80*sqrt(-c*d^2*e + a*e^3)*a^3*c^3*d^6*e^6 - 96*sqrt(-c*d^2*e + a*e
^3)*a^4*c^2*d^4*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*c*d^2*e^10 - 256*sqrt(
-c*d^2*e + a*e^3)*a^6*e^12)*e^(-5)/(c^6*d^6) + (3003*((x*e + d)*c*d*e - c*d
^2*e + a*e^3)^(3/2)*a^5*e^15 - 9009*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/
2)*a^4*e^12 + 12870*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^3*e^9 - 100
10*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a^2*e^6 + 4095*((x*e + d)*c*d*
e - c*d^2*e + a*e^3)^(11/2)*a*e^3 - 693*((x*e + d)*c*d*e - c*d^2*e + a*e^3)
^(13/2))*e^(-11)/(c^6*d^6))*e^(-1) - 6006*a*c*d*f^3*((5*((x*e + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))
*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e
^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-1) + 257
4*a*c*d*f^2*g*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3
)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
+ a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3
)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*(
(x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3)) - 858*a*c*d*f*g
^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6
*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*
a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*
((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e -
c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7
/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4))
 + 26*a*c*d*g^3*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10 - 35*sqrt(-c*d^2*e +
a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^6*e^4 - 48*sqrt(
-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*e^8
 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-4)/(c^5*d^5) + (1155*((x*e + d)
*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((x*e + d)*c*d*e - c*d^2*e
+ a*e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2
e^6 - 1540((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*a*e^3 + 315*((x*e + d
)*c*d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c^5*d^5)) + 15015*a^2*f^3*(((x*e
 + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sqrt(-c*d^2*e + a*e^3)
*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)/(c*d))*e + 1287*a^2*f*g^2*((15*sqrt(
-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt
(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)
/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x
*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*d^2*
e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e - 143*a^2*g^3*((35*sqrt(-c*d^2*e + a*
e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a
*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c

```
*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*((x*e + d)*c*d*e - c*d^2*e
 + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2
*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*
c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4))*e - 9009*a^2*f^2*g*((5*((
x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*
e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sq
rt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^...
```

Mupad [B]

time = 3.81, size = 379, normalized size = 1.41

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g\,x^4\,\left(53\,a^2\,e^2\,g^2+299\,a\,c\,d\,e\,f\,g+143\,c^2\,d^2\,f^2\right)}{429}-\frac{32\,a^6\,e^6\,g^3-208\,a^5\,c\,d\,e^5\,f\,g^2+572\,a^4\,c^2\,d^2\,e^4\,f^2\,g-858\,a^3\,c^3\,d^3\,e^3\,f^3}{3003\,c^4\,d^4}+\frac{x^3\,\left(10\,a^3\,c^3\,d^3\,e^3\,g^3+2938\,a^2\,c^4\,d^4\,e^2\,f\,g^2+5434\,a\,c^5\,d^5\,e\,f^2\,g+858\,c^6\,d^6\,f^3\right)}{3003\,c^4\,d^4}+\frac{2\,c^2\,d^2\,g^3\,x^6}{13}+\frac{6\,c\,d\,g^2\,x^5\,\left(9\,a\,e\,g+13\,c\,d\,f\right)}{143}+\frac{2\,a^2\,e^2\,x\,\left(8\,a^3\,e^3\,g^3-52\,a^2\,c\,d\,e^2\,f\,g^2+143\,a\,c^2\,d^2\,e\,f^2\,g+1287\,c^3\,d^3\,f^3\right)}{3003\,c^3\,d^3}+\frac{2\,a\,e\,x^2\,\left(-2\,a^3\,e^3\,g^3+13\,a^2\,c\,d\,e^2\,f\,g^2+715\,a\,c^2\,d^2\,e\,f^2\,g+429\,c^3\,d^3\,f^3\right)}{1001\,c^2\,d^2}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g*x^4*(53*a^2*e^2*g^2 +
143*c^2*d^2*f^2 + 299*a*c*d*e*f*g))/429 - (32*a^6*e^6*g^3 - 858*a^3*c^3*d^3
*e^3*f^3 + 572*a^4*c^2*d^2*e^4*f^2*g - 208*a^5*c*d*e^5*f*g^2)/(3003*c^4*d^4
) + (x^3*(858*c^6*d^6*f^3 + 10*a^3*c^3*d^3*e^3*g^3 + 2938*a^2*c^4*d^4*e^2*f
*g^2 + 5434*a*c^5*d^5*e*f^2*g))/(3003*c^4*d^4) + (2*c^2*d^2*g^3*x^6)/13 + (
6*c*d*g^2*x^5*(9*a*e*g + 13*c*d*f))/143 + (2*a^2*e^2*x*(8*a^3*e^3*g^3 + 128
7*c^3*d^3*f^3 + 143*a*c^2*d^2*e*f^2*g - 52*a^2*c*d*e^2*f*g^2))/(3003*c^3*d^
3) + (2*a*e*x^2*(429*c^3*d^3*f^3 - 2*a^3*e^3*g^3 + 715*a*c^2*d^2*e*f^2*g +
13*a^2*c*d*e^2*f*g^2))/(1001*c^2*d^2)))/(d + e*x)^(1/2)
```

3.702 $\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{8(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{693c^3d^3e(d+ex)^{7/2}}+\frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right.\right.}{99c^2d^2e(d+ex)^{5/2}}$$

```
[Out] -8/693*(-a*e*g+c*d*f)*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(7/2)/c^3/d^3/e/(e*x+d)^(7/2)+8/99*g*(-a*e*g+c*d*f)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c^2/d^2/e/(e*x+d)^(5/2)+2/11*(g*x+f)^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A]

time = 0.16, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {884, 808, 662}

$$-\frac{8\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)\left(2ae^2g-cd(9ef-7dg)\right)}{693c^3d^3e(d+ex)^{7/2}}+\frac{8g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(cdf-aeg)}{99c^2d^2e(d+ex)^{5/2}}+\frac{2(f+gx)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(7/2))/(693*c^3*d^3*e*(d + e*x)^(7/2)) + (8*g*(c*d*f - a
*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(99*c^2*d^2*e*(d + e*x
)^(5/2)) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(1
1*c*d*(d + e*x)^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int \frac{(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{11cd(d+ex)^{7/2}} + \frac{(4(cdf-\cdots}{\cdots}$$

$$= \frac{8g(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{99c^2d^2e(d+ex)^{5/2}} + \frac{2(f+\cdots}{\cdots}$$

$$= \frac{8(cdf-aeg)\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+\left(cd^2+ae^2\right)x+cd\cdots\right.}{693c^2d^2(d+ex)^{7/2}}$$

Mathematica [A]
time = 0.10, size = 100, normalized size = 0.50

$$\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\left(8a^2e^2g^2-4acdeg(11f+7gx)+c^2d^2\left(99f^2+154fgx+63g^2x^2\right)\right)}{693c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^2*g^2 - 4*a*c*d*e
*g*(11*f + 7*g*x) + c^2*d^2*(99*f^2 + 154*f*g*x + 63*g^2*x^2)))/(693*c^3*d^
3*Sqrt[d + e*x])
```

Maple [A]
time = 0.13, size = 108, normalized size = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3\left(63g^2x^2c^2d^2-28acde\,g^2x+154c^2d^2fgx+8a^2e^2g^2-44acdefg+99f^2c^2d^2\right)}{693\sqrt{ex+d}\,c^3d^3}$	108

gosper	$\frac{2(cdx+ae)\left(63g^2x^2c^2d^2-28acde\,g^2x+154c^2d^2fgx+8a^2e^2g^2-44acdefg+99f^2c^2d^2\right)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{693c^3d^3(ex+d)^{\frac{5}{2}}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/693*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(63*c^2*d^2*g
^2*x^2-28*a*c*d*e*g^2*x+154*c^2*d^2*f*g*x+8*a^2*e^2*g^2-44*a*c*d*e*f*g+99*c
^2*d^2*f^2)/c^3/d^3
```

Maxima [A]
time = 0.35, size = 240, normalized size = 1.20

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2x^2e+3\,a^2cdxe^2+a^3e^3\right)\sqrt{cdx+ae}\,f^2}{7\,cd}+\frac{4\left(7\,c^4d^4x^4+19\,ac^3d^3x^3e+15\,a^2c^2d^2x^2e^2+a^3cdxe^3-2\,a^4e^4\right)\sqrt{cdx+ae}\,fg}{63\,c^2d^2}+\frac{2\left(63\,c^5d^5x^5+161\,ac^4d^4x^4e+113\,a^2c^3d^3x^3e^2+3\,a^3c^2d^2x^2e^3-4\,a^4cdxe^4+8\,a^5e^5\right)\sqrt{cdx+ae}\,g^2}{693\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*x^2*e + 3*a^2*c*d*x*e^2 + a^3*e^3)*sqrt(c*d*
x + a*e)*f^2/(c*d) + 4/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*x^3*e + 15*a^2*c^2*
d^2*x^2*e^2 + a^3*c*d*x*e^3 - 2*a^4*e^4)*sqrt(c*d*x + a*e)*f*g/(c^2*d^2) +
2/693*(63*c^5*d^5*x^5 + 161*a*c^4*d^4*x^4*e + 113*a^2*c^3*d^3*x^3*e^2 + 3*a
^3*c^2*d^2*x^2*e^3 - 4*a^4*c*d*x*e^4 + 8*a^5*e^5)*sqrt(c*d*x + a*e)*g^2/(c^
3*d^3)
```

Fricas [A]
time = 1.28, size = 282, normalized size = 1.41

$$\frac{2\left(63\,c^5d^5g^2x^5+154\,c^5d^5fgx^4+99\,c^5d^5f^2x^3+8\,a^5g^2e^5-4\left(a^4cdg^2x+11\,a^4cdfg\right)e^4+\left(3\,a^3c^2d^2g^2x^2+22\,a^3c^2d^2fgx+99\,a^3c^2d^2f^2\right)e^3+\left(113\,a^2c^3d^3g^2x^3+330\,a^2c^3d^3fgx^2+297\,a^2c^3d^3f^2x\right)e^2+\left(161\,ac^4d^4g^2x^4+418\,ac^4d^4fgx^3+297\,ac^4d^4f^2x^2\right)e\right)\sqrt{cd^2x+axe^2+\left(cdx^2+ad\right)e}\,\sqrt{xe+d}}{693\left(c^3d^3xe+c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] 2/693*(63*c^5*d^5*g^2*x^5 + 154*c^5*d^5*f*g*x^4 + 99*c^5*d^5*f^2*x^3 + 8*a^
5*g^2*e^5 - 4*(a^4*c*d*g^2*x + 11*a^4*c*d*f*g)*e^4 + (3*a^3*c^2*d^2*g^2*x^2
 + 22*a^3*c^2*d^2*f*g*x + 99*a^3*c^2*d^2*f^2)*e^3 + (113*a^2*c^3*d^3*g^2*x^
3 + 330*a^2*c^3*d^3*f*g*x^2 + 297*a^2*c^3*d^3*f^2*x)*e^2 + (161*a*c^4*d^4*g
^2*x^4 + 418*a*c^4*d^4*f*g*x^3 + 297*a*c^4*d^4*f^2*x^2)*e)*sqrt(c*d^2*x + a
*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^3*d^3*x*e + c^3*d^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. $2(187) = 374$.
time = 4.92, size = 1976, normalized size = 9.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] 2/3465*(33*c^2*d^2*f^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*
e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(
-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*
e + a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e
^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1)
 - 22*c^2*d^2*f*g*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a
*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*
d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c
^4*d^4) + (105*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*
e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^
2*e + a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^
(-7)/(c^4*d^4))*e^(-1) + c^2*d^2*g^2*((315*sqrt(-c*d^2*e + a*e^3)*c^5*d^10
- 35*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^8*e^2 - 40*sqrt(-c*d^2*e + a*e^3)*a^2*c
^3*d^6*e^4 - 48*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*e^6 - 64*sqrt(-c*d^2*e +
 a*e^3)*a^4*c*d^2*e^8 - 128*sqrt(-c*d^2*e + a*e^3)*a^5*e^10)*e^(-4)/(c^5*d^
5) + (1155*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*e^12 - 2772*((x*e
+ d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*e^9 + 2970*((x*e + d)*c*d*e - c*d^2
*e + a*e^3)^(7/2)*a^2*e^6 - 1540*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*
a*e^3 + 315*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(11/2))*e^(-9)/(c^5*d^5))*e
^(-1) - 462*a*c*d*f^2*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 -
 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c
*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^
2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-1) + 132*a*c*d*f*g*((15*sqrt(-c*d^2*e
+ a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e
 + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3
) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c
*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3
)^(7/2))*e^(-5)/(c^3*d^3)) - 22*a*c*d*g^2*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d
^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*
```

```
c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a
*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) + (105*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^3*e^9 - 189*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135
*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*
d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4*d^4)) + 1155*a^2*f^2*(((x*e + d)*c*d*e -
c*d^2*e + a*e^3)^(3/2)*e^(-1)/(c*d) + (sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(
-c*d^2*e + a*e^3)*a*e^2)/(c*d))*e + 33*a^2*g^2*((15*sqrt(-c*d^2*e + a*e^3)*
c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)
*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*(
(x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*
d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*
e^(-5)/(c^3*d^3))*e - 462*a^2*f*g*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(
3/2)*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2)
+ (3*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 -
2*sqrt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2)))*e^(-1)
```

Mupad [B]

time = 3.56, size = 259, normalized size = 1.30

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,a^5\,e^5\,g^2-88\,a^4\,c\,d\,e^4\,f\,g+198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,c^3\,d^3}+\frac{x^3\,\left(226\,a^2\,c^3\,d^3\,e^2\,g^2+836\,a\,c^4\,d^4\,e\,f\,g+198\,c^5\,d^5\,f^2\right)}{693\,c^3\,d^3}+\frac{2\,c^2\,d^2\,g^2\,x^5}{11}+\frac{2\,c\,d\,g\,x^4\,\left(23\,a\,e\,g+22\,c\,d\,f\right)}{99}+\frac{2\,a^2\,e^2\,x\,\left(-4\,a^2\,e^2\,g^2+22\,a\,c\,d\,e\,f\,g+297\,c^2\,d^2\,f^2\right)}{693\,c^2\,d^2}+\frac{2\,a\,e\,x^2\,\left(a^2\,e^2\,g^2+110\,a\,c\,d\,e\,f\,g+99\,c^2\,d^2\,f^2\right)}{231\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*a^5*e^5*g^2 + 198*a^3*c
^2*d^2*e^3*f^2 - 88*a^4*c*d*e^4*f*g)/(693*c^3*d^3) + (x^3*(198*c^5*d^5*f^2
+ 226*a^2*c^3*d^3*e^2*g^2 + 836*a*c^4*d^4*e*f*g))/(693*c^3*d^3) + (2*c^2*d^
2*g^2*x^5)/11 + (2*c*d*g*x^4*(23*a*e*g + 22*c*d*f))/99 + (2*a^2*e^2*x*(297*
c^2*d^2*f^2 - 4*a^2*e^2*g^2 + 22*a*c*d*e*f*g))/(693*c^2*d^2) + (2*a*e*x^2*(
a^2*e^2*g^2 + 99*c^2*d^2*f^2 + 110*a*c*d*e*f*g))/(231*c*d)))/(d + e*x)^(1/2
)
```

3.703 $\int \frac{(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2(2ae^2g-cd(9ef-7dg))\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63c^2d^2e(d+ex)^{7/2}}+\frac{2g\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}$$

```
[Out] -2/63*(2*a*e^2*g-c*d*(-7*d*g+9*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2
)/c^2/d^2/e/(e*x+d)^(7/2)+2/9*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d
/e/(e*x+d)^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {808, 662}

$$\frac{2g\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}(2ae^2g-cd(9ef-7dg))}{63c^2d^2e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x]
```

```
[Out] (-2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(63*c^2*d^2*e*(d + e*x)^(7/2)) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2)^(7/2))/(9*c*d*e*(d + e*x)^(5/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2g\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{9cde(d+ex)^{5/2}} + \frac{1}{9}\left(9f - \frac{7dg}{e} - \frac{2a\ldots}{c\ldots}\right.$$
$$= \frac{2\left(9f-\frac{7dg}{e}-\frac{2aeg}{cd}\right)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{7/2}}{63cd(d+ex)^{7/2}} + \frac{2\ldots}{\ldots}$$

Mathematica [A]
time = 0.07, size = 64, normalized size = 0.51

$$\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\,(-2aeg+cd(9f+7gx))}{63c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(9*f + 7*g
*x)))/(63*c^2*d^2*Sqrt[d + e*x])
```

Maple [A]
time = 0.13, size = 59, normalized size = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3(-7cdgx+2aeg-9cdf)}{63\sqrt{ex+d}\,c^2d^2}$	59
gosper	$-\frac{2(cdx+ae)(-7cdgx+2aeg-9cdf)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{5}{2}}}{63c^2d^2(ex+d)^{\frac{5}{2}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2/63*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3*(-7*c*d*g*x+2
*a*e*g-9*c*d*f)/c^2/d^2
```

Maxima [A]
time = 0.30, size = 140, normalized size = 1.12

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2x^2e+3\,a^2cdxe^2+a^3e^3\right)\sqrt{cdx+ae}\,f}{7\,cd}+\frac{2\left(7\,c^4d^4x^4+19\,ac^3d^3x^3e+15\,a^2c^2d^2x^2e^2+a^3cdxe^3-2\,a^4e^4\right)\sqrt{cdx+ae}\,g}{63\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*x^2*e + 3*a^2*c*d*x*e^2 + a^3*e^3)*sqrt(c*d*
x + a*e)*f/(c*d) + 2/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*x^3*e + 15*a^2*c^2*d^
2*x^2*e^2 + a^3*c*d*x*e^3 - 2*a^4*e^4)*sqrt(c*d*x + a*e)*g/(c^2*d^2)
```

Fricas [A]
time = 1.51, size = 173, normalized size = 1.38

$$\frac{2\,(7\,c^4d^4gx^4 + 9\,c^4d^4fx^3 - 2\,a^4ge^4 + (a^3cdgx + 9\,a^3cdf)e^3 + 3\,(5\,a^2c^2d^2gx^2 + 9\,a^2c^2d^2fx)e^2 + (19\,ac^3d^3gx^3 + 27\,ac^3d^3fx^2)e)\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,\sqrt{xe + d}}{63\,(c^2d^2xe + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="fricas")
```

```
[Out] 2/63*(7*c^4*d^4*g*x^4 + 9*c^4*d^4*f*x^3 - 2*a^4*g*e^4 + (a^3*c*d*g*x + 9*a^
3*c*d*f)*e^3 + 3*(5*a^2*c^2*d^2*g*x^2 + 9*a^2*c^2*d^2*f*x)*e^2 + (19*a*c^3*
d^3*g*x^3 + 27*a*c^3*d^3*f*x^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e)*sqrt(x*e + d)/(c^2*d^2*x*e + c^2*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2
),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. 2(114) = 228.
time = 6.08, size = 1124, normalized size = 8.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,
algorithm="giac")
```

```
[Out] 2/315*(3*c^2*d^2*f*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e +
a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d
^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 +
```

```
15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - c
^2*d^2*g*((35*sqrt(-c*d^2*e + a*e^3)*c^4*d^8 - 5*sqrt(-c*d^2*e + a*e^3)*a*c
^3*d^6*e^2 - 6*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*e^4 - 8*sqrt(-c*d^2*e + a
*e^3)*a^3*c*d^2*e^6 - 16*sqrt(-c*d^2*e + a*e^3)*a^4*e^8)*e^(-3)/(c^4*d^4) +
 (105*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^3*e^9 - 189*((x*e + d)*c*
d*e - c*d^2*e + a*e^3)^(5/2)*a^2*e^6 + 135*((x*e + d)*c*d*e - c*d^2*e + a*e
^3)^(7/2)*a*e^3 - 35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2))*e^(-7)/(c^4
*d^4))*e^(-1) - 42*a*c*d*f*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*
e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sq
rt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(
-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2))*e^(-1) + 6*a*c*d*g*((15*sqrt(-c*d^2*e
 + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*
e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^
3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*e^6 - 42*((x*e + d)*
c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^
3)^(7/2))*e^(-5)/(c^3*d^3)) + 105*a^2*f*(((x*e + d)*c*d*e - c*d^2*e + a*e^3
)^(3/2)*e^(-1)/(c*d) + (sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^
3)*a*e^2)/(c*d))*e - 21*a^2*g*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)
*a*e^3 - 3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3
*sqrt(-c*d^2*e + a*e^3)*c^2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sq
rt(-c*d^2*e + a*e^3)*a^2*e^4)/(c^2*d^2)))*e^(-1)
```

Mupad [B]
time = 3.37, size = 134, normalized size = 1.07

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,c^2\,d^2\,g\,x^4}{9}+\frac{2\,a\,e\,x^2\,\left(5\,a\,e\,g+9\,c\,d\,f\right)}{21}+\frac{2\,c\,d\,x^3\,\left(19\,a\,e\,g+9\,c\,d\,f\right)}{63}-\frac{2\,a^3\,e^3\,\left(2\,a\,e\,g-9\,c\,d\,f\right)}{63\,c^2\,d^2}+\frac{2\,a^2\,e^2\,x\,\left(a\,e\,g+27\,c\,d\,f\right)}{63\,c\,d}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(5/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*c^2*d^2*g*x^4)/9 + (2*a*
e*x^2*(5*a*e*g + 9*c*d*f))/21 + (2*c*d*x^3*(19*a*e*g + 9*c*d*f))/63 - (2*a^
3*e^3*(2*a*e*g - 9*c*d*f))/(63*c^2*d^2) + (2*a^2*e^2*x*(a*e*g + 27*c*d*f))/
(63*c*d))/(d + e*x)^(1/2)
```

3.704 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/(e*x+d)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{662\}$

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7cd(d+ex)^{7/2}}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 0.77

$$\frac{2((ae+cdx)(d+ex))^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*c*d*(d + e*x)^(7/2))
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.88

method	result	size
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^3}{7\sqrt{ex+d}\,cd}$	42
gosper	$\frac{2(cdx+ae)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}{7cd(ex+d)^{\frac{5}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2/7*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(c*d*x+a*e)^3/c/d
```

Maxima [A]
time = 0.30, size = 60, normalized size = 1.25

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2x^2e+3\,a^2cdxe^2+a^3e^3\right)\sqrt{cdx+ae}}{7\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="maxima")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*x^2*e + 3*a^2*c*d*x*e^2 + a^3*e^3)*sqrt(c*d*
x + a*e)/(c*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.
time = 1.64, size = 92, normalized size = 1.92

$$\frac{2\left(c^3d^3x^3+3\,ac^2d^2x^2e+3\,a^2cdxe^2+a^3e^3\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{7\left(cdxe+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*x^2*e + 3*a^2*c*d*x*e^2 + a^3*e^3)*sqrt(c*d^
2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d*x*e + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((d+ex)\,(ae+cdx)\right)^{\frac{5}{2}}}{(d+ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(d + e*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(44) = 88.
time = 6.04, size = 469, normalized size = 9.77

$$\frac{2}{105}\left(c^2d^2\left(\frac{\left(15\sqrt{-cd^2e+ae^3}\,c^3d^6-3\sqrt{-cd^2e+ae^3}\,ac^2d^4e^2-4\sqrt{-cd^2e+ae^3}\,a^2cd^2e^4-8\sqrt{-cd^2e+ae^3}\,a^3e^6\right)e^{(-2)}}{c^3d^3}+\frac{\left(35\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}a^2e^6-42\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}ae^3+15\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{7}{2}}\right)e^{(-5)}}{c^3d^3}\right)e^{(-1)}-14acd\left(\frac{\left(5\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}ae^3-3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{5}{2}}\right)e^{(-2)}}{c^2d^2}+\frac{3\sqrt{-cd^2e+ae^3}\,c^2d^4-\sqrt{-cd^2e+ae^3}\,acd^2e^2-2\sqrt{-cd^2e+ae^3}\,a^2e^4}{c^2d^2}\right)e^{(-1)}+35a^2\left(\frac{\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}e^{(-1)}}{cd}+\frac{\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2}{cd}\right)e\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorith
m="giac")
```

```
[Out] 2/105*(c^2*d^2*((15*sqrt(-c*d^2*e + a*e^3)*c^3*d^6 - 3*sqrt(-c*d^2*e + a*e^
3)*a*c^2*d^4*e^2 - 4*sqrt(-c*d^2*e + a*e^3)*a^2*c*d^2*e^4 - 8*sqrt(-c*d^2*e
 + a*e^3)*a^3*e^6)*e^(-2)/(c^3*d^3) + (35*((x*e + d)*c*d*e - c*d^2*e + a*e^
3)^(3/2)*a^2*e^6 - 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*e^3 + 15*
((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2))*e^(-5)/(c^3*d^3))*e^(-1) - 14*a*
c*d*((5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*e^3 - 3*((x*e + d)*c*d*
e - c*d^2*e + a*e^3)^(5/2))*e^(-2)/(c^2*d^2) + (3*sqrt(-c*d^2*e + a*e^3)*c^
2*d^4 - sqrt(-c*d^2*e + a*e^3)*a*c*d^2*e^2 - 2*sqrt(-c*d^2*e + a*e^3)*a^2*e
^4)/(c^2*d^2))*e^(-1) + 35*a^2*(((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e
^(-1)/(c*d) + (sqrt(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)
/(c*d))*e)*e^(-1)
```

Mupad [B]
time = 3.16, size = 79, normalized size = 1.65

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{6\,a^2\,e^2\,x}{7}+\frac{2\,c^2\,d^2\,x^3}{7}+\frac{2\,a^3\,e^3}{7\,c\,d}+\frac{6\,a\,c\,d\,e\,x^2}{7}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^(5/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((6*a^2*e^2*x)/7 + (2*c^2*d^
2*x^3)/7 + (2*a^3*e^3)/(7*c*d) + (6*a*c*d*e*x^2)/7))/(d + e*x)^(1/2)
```

3.705 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx$

Optimal. Leaf size=236

$$\frac{2(cdf-aeg)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{2(cdf-aeg)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}+\frac{2(ade+(cd}{5g}$$

```
[Out] -2/3*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/
2)+2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)-2*(-a*e*g+c*
d*f)^(5/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)+2*(-a*e*g+c*d*f)^2*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {878, 888, 211}

$$-\frac{2(cdf-aeg)^{5/2}\text{ArcTan}\left(\frac{\sqrt{g}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{7/2}}+\frac{2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^2}{g^3\sqrt{d+ex}}-\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}\,(cdf-aeg)}{3g^2(d+ex)^{3/2}}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)),x]
```

```
[Out] (2*(c*d*f - a*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt
[d + e*x]) - (2*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/
2))/(3*g^2*(d + e*x)^(3/2)) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5
/2))/(5*g*(d + e*x)^(5/2)) - (2*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])
])/g^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
```

```
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)}\,dx = \frac{2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{5g(d+ex)^{5/2}} - \frac{(cde^2f + cd^2eg - e(cd^2 + ae^2)}{e}$$

$$= -\frac{2(cdf - aeg)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}} + \frac{2(ade + (cd^2 +}{5g(d}$$

$$= \frac{2(cdf - aeg)^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf - aeg)(ade}{3g}$$

$$= \frac{2(cdf - aeg)^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf - aeg)(ade}{3g}$$

$$= \frac{2(cdf - aeg)^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{g^3\sqrt{d+ex}} - \frac{2(cdf - aeg)(ade}{3g}$$

Mathematica [A]

time = 0.23, size = 168, normalized size = 0.71

$$\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}\,(23a^2e^2g^2 + acdeg(-35f+11gx) + c^2d^2(15f^2 - 5fgx + 3g^2x^2)) - 15(cdf-aeg)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{15g^{7/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)),x]
```

[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(23*a^2*e^2*g
^2 + a*c*d*e*g*(-35*f + 11*g*x) + c^2*d^2*(15*f^2 - 5*f*g*x + 3*g^2*x^2)) -
 15*(c*d*f - a*e*g)^(5/2)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]]))/(15*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(208) = 416$.
time = 0.15, size = 421, normalized size = 1.78

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)a^3e^3g^3-45\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)a^2cd\,e^2f}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,method=
_RETURNVERBOSE)

[Out] -2/15*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*a^3*e^3*g^3-45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*a^2*c*d*e^2*f*g^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^
(1/2))*c^3*d^3*f^3-3*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*g^2*
x^2-11*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x+5*((a*e*g-c*
d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x-23*((a*e*g-c*d*f)*g)^(1/2)*(c
*d*x+a*e)^(1/2)*a^2*e^2*g^2+35*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*
c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*x+d)
^(1/2)/(c*d*x+a*e)^(1/2)/g^3/((a*e*g-c*d*f)*g)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="maxima")

[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)*(x*e + d
)^(5/2)), x)

Fricas [A]
time = 1.68, size = 596, normalized size = 2.53

$$\left[\frac{15\left(c^2d^3f^2+a^2g^2xe^3-(2\,acdfgx-a^2dg^2)e^2+(c^2d^2f^2x-2\,acd^2fg)e\right)\sqrt{-\frac{cdf-age}{g}}\log\left(-\frac{cd^2gx-cd^2f+2\,agxe^2-2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}\,g\sqrt{-\frac{cdf-age}{g}}+(cdgx^2-cdfx+2\,adg)e}{dgx+df+(gx^2+fx)e}\right)+2\left(3\,c^2d^2g^2x^2-5\,c^2d^2fgx+15\,c^2d^2f^2+23\,a^2g^2e^2+(11\,acdg^2x-35\,acdfg)e\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{15\,(g^3xe+dg^3)},\ \frac{2\left(15\left(c^2d^3f^2+a^2g^2xe^3-(2\,acdfgx-a^2dg^2)e^2+(c^2d^2f^2x-2\,acd^2fg)e\right)\sqrt{\frac{cdf-age}{g}}\arctan\left(\frac{\sqrt{xe+d}\sqrt{\frac{cdf-age}{g}}}{\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}\right)+\left(3\,c^2d^2g^2x^2-5\,c^2d^2fgx+15\,c^2d^2f^2+23\,a^2g^2e^2+(11\,acdg^2x-35\,acdfg)e\right)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}\right)}{15\,(g^3xe+dg^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="fricas")
```

```
[Out] [1/15*(15*(c^2*d^3*f^2 + a^2*g^2*x*e^3 - (2*a*c*d*f*g*x - a^2*d*g^2)*e^2 +
(c^2*d^2*f^2*x - 2*a*c*d^2*f*g)*e)*sqrt(-(c*d*f - a*g*e)/g)*log(-(c*d^2*g*x
 - c*d^2*f + 2*a*g*x*e^2 - 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sq
rt(x*e + d)*g*sqrt(-(c*d*f - a*g*e)/g) + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e)
/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(3*c^2*d^2*g^2*x^2 - 5*c^2*d^2*f*g*x
+ 15*c^2*d^2*f^2 + 23*a^2*g^2*e^2 + (11*a*c*d*g^2*x - 35*a*c*d*f*g)*e)*sqrt
(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(g^3*x*e + d*g^3), 2
/15*(15*(c^2*d^3*f^2 + a^2*g^2*x*e^3 - (2*a*c*d*f*g*x - a^2*d*g^2)*e^2 + (c
^2*d^2*f^2*x - 2*a*c*d^2*f*g)*e)*sqrt((c*d*f - a*g*e)/g)*arctan(sqrt(x*e +
d)*sqrt((c*d*f - a*g*e)/g)/sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)) + (
3*c^2*d^2*g^2*x^2 - 5*c^2*d^2*f*g*x + 15*c^2*d^2*f^2 + 23*a^2*g^2*e^2 + (11
*a*c*d*g^2*x - 35*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
*sqrt(x*e + d))/(g^3*x*e + d*g^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f),x,
algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/2
)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)*(d + e*x)^(5/2
)), x)
```

3.706 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx$

Optimal. Leaf size=235

$$-\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}+\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x\right.}{g(d+ex)^{5/2}(f}$$

```
[Out] 5/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)-(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)+5*c*d*(-a*e*g+c*d*f)
^(3/2)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^(1/2)/(e*x+d)^(1/2))/g^(7/2)-5*c*d*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 878, 888, 211}

$$\frac{5cd(cdf-aeg)^{3/2}\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}}-\frac{5cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)}{g^3\sqrt{d+ex}}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}+\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^2),x]
```

```
[Out] (-5*c*d*(c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*S
qrt[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g^
2*(d + e*x)^(3/2)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(g*(d +
e*x)^(5/2)*(f + g*x)) + (5*c*d*(c*d*f - a*e*g)^(3/2)*ArcTan[(Sqrt[g]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])]
)/g^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
```

```
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^2}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}(f+gx)}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{(d+ex)^{3/2}(f+gx)}}{2g}$$

$$=\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{g(d+ex)^{5/2}(f+gx}$$

$$=-\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}+\frac{5cd(ade+(ca}{3g^2}$$

$$=-\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}+\frac{5cd(ade+(ca}{3g^2}$$

$$=-\frac{5cd(cdf-aeg)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}+\frac{5cd(ade+(ca}{3g^2}$$

Mathematica [A]

time = 0.50, size = 183, normalized size = 0.78

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{ae+cdx}\,(-3a^2e^2g^2+2acdeg(10f+7gx)+c^2d^2(-15f^2-10fgx+2g^2x^2))+15cd(cdf-aeg)^{3/2}(f+gx)\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{3g^{7/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^2),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-3*a^2*e^2*g^2
 + 2*a*c*d*e*g*(10*f + 7*g*x) + c^2*d^2*(-15*f^2 - 10*f*g*x + 2*g^2*x^2)) +
 15*c*d*(c*d*f - a*e*g)^(3/2)*(f + g*x)*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/
Sqrt[c*d*f - a*e*g]]))/(3*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(209) = 418.
time = 0.14, size = 513, normalized size = 2.18

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^2cde^2g^3x-30\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)ac^2d^2ef\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*a^2*c*d*e^2*g^3*x-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*a*c^2*d^2*e*f*g^2*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*a^2*c*d*e^2*f*g^2-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*a*c^2*d^2*e*f^2*g+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c^3*d^3*f^3-2*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*g
^2*x^2-14*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x+10*((a*e*
g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+3*((a*e*g-c*d*f)*g)^(1/2)
*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2-20*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)
*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*x
+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^2*(x*e +
 d)^(5/2)), x)
```

Fricas [A]
time = 2.01, size = 681, normalized size = 2.90

$$\left[-\frac{15\left(c^{2}d^{3}fgx+c^{2}d^{3}f^{2}-\left(acdg^{2}x^{2}+acdfgx\right)e^{2}+\left(c^{2}d^{2}fgx^{2}-acd^{2}fg+\left(c^{2}d^{2}f^{2}-acd^{2}g^{2}\right)x\right)e\right)\sqrt{-\frac{cdf-age}{g}}\log\left(-\frac{cd^{2}gx-cd^{2}f+2agxe^{2}-2\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{xe+d}\,g\sqrt{-\frac{cdf-age}{g}}+\left(cdgx^{2}-cdfx+2adg\right)e}{dgx+df+\left(gx^{2}+fx\right)e}\right)-2\left(2c^{2}d^{2}g^{2}x^{2}-10c^{2}d^{2}fgx-15c^{2}d^{2}f^{2}-3a^{2}g^{2}e^{2}+2\left(7acdg^{2}x+10acdfg\right)e\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{xe+d}}{6\left(dg^{4}x+dfg^{3}+\left(g^{4}x^{2}+fg^{3}x\right)e\right)},\ -\frac{15\left(c^{2}d^{3}fgx+c^{2}d^{3}f^{2}-\left(acdg^{2}x^{2}+acdfgx\right)e^{2}+\left(c^{2}d^{2}fgx^{2}-acd^{2}fg+\left(c^{2}d^{2}f^{2}-acd^{2}g^{2}\right)x\right)e\right)\sqrt{\frac{cdf-age}{g}}\arctan\left(\frac{\sqrt{xe+d}\sqrt{\frac{cdf-age}{g}}}{\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}}\right)-\left(2c^{2}d^{2}g^{2}x^{2}-10c^{2}d^{2}fgx-15c^{2}d^{2}f^{2}-3a^{2}g^{2}e^{2}+2\left(7acdg^{2}x+10acdfg\right)e\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{xe+d}}{3\left(dg^{4}x+dfg^{3}+\left(g^{4}x^{2}+fg^{3}x\right)e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="fricas")
```

```
[Out] [-1/6*(15*(c^2*d^3*f*g*x + c^2*d^3*f^2 - (a*c*d*g^2*x^2 + a*c*d*f*g*x)*e^2
+ (c^2*d^2*f*g*x^2 - a*c*d^2*f*g + (c^2*d^2*f^2 - a*c*d^2*g^2)*x)*e)*sqrt(-
(c*d*f - a*g*e)/g)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 - 2*sqrt(c*d^2*x
 + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)*g*sqrt(-(c*d*f - a*g*e)/g) +
(c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) - 2*(2*
c^2*d^2*g^2*x^2 - 10*c^2*d^2*f*g*x - 15*c^2*d^2*f^2 - 3*a^2*g^2*e^2 + 2*(7*
a*c*d*g^2*x + 10*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*
sqrt(x*e + d))/(d*g^4*x + d*f*g^3 + (g^4*x^2 + f*g^3*x)*e), -1/3*(15*(c^2*d
^3*f*g*x + c^2*d^3*f^2 - (a*c*d*g^2*x^2 + a*c*d*f*g*x)*e^2 + (c^2*d^2*f*g*x
^2 - a*c*d^2*f*g + (c^2*d^2*f^2 - a*c*d^2*g^2)*x)*e)*sqrt((c*d*f - a*g*e)/g
)*arctan(sqrt(x*e + d)*sqrt((c*d*f - a*g*e)/g)/sqrt(c*d^2*x + a*x*e^2 + (c*
d*x^2 + a*d)*e)) - (2*c^2*d^2*g^2*x^2 - 10*c^2*d^2*f*g*x - 15*c^2*d^2*f^2 -
 3*a^2*g^2*e^2 + 2*(7*a*c*d*g^2*x + 10*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2
 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g^4*x + d*f*g^3 + (g^4*x^2 + f*g^3*
x)*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^2,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^2\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^2*(d + e*x)^(5
/2)), x)
```

3.707 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx$

Optimal. Leaf size=246

$$\frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}{2g(d+ex)^{5/2}(f+gx)^2}$$

```
[Out] -5/4*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)-
1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^2-15/4*
c^2*d^2*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*
f)^(1/2)/(e*x+d)^(1/2))*(-a*e*g+c*d*f)^(1/2)/g^(7/2)+15/4*c^2*d^2*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 878, 888, 211}

$$-\frac{15c^2d^2\sqrt{cdf-aeg}\,\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{7/2}}+\frac{15c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^3),x]
```

```
[Out] (15*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*Sqrt[d + e*
x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g^2*(d + e*x
)^(3/2)*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*g*(d
+ e*x)^(5/2)*(f + g*x)^2) - (15*c^2*d^2*Sqrt[c*d*f - a*e*g]*ArcTan[(Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d +
 e*x])])/(4*g^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
```

```
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^3}\,dx=-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}+\frac{(5cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{(d+ex)^{3/2}(f+gx)^2}}{4g}$$

$$=-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{2g(d+ex)^{5/2}(f+gx}$$

$$=\frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+\right.}{4g^2(d+ex)^{3/2}(f+}$$

$$=\frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+\right.}{4g^2(d+ex)^{3/2}(f+}$$

$$=\frac{15c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+\right.}{4g^2(d+ex)^{3/2}(f+}$$

Mathematica [A]

time = 0.56, size = 189, normalized size = 0.77

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{g}\,\sqrt{ae+cdx}\,(-2a^2e^2g^2-acdeg(5f+9gx)+c^2d^2(15f^2+25fgx+8g^2x^2))-15c^2d^2\sqrt{cdf-aeg}\,(f+gx)^2\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{4g^{7/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^3),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-2*a^2*e^2*g^2
 - a*c*d*e*g*(5*f + 9*g*x) + c^2*d^2*(15*f^2 + 25*f*g*x + 8*g^2*x^2)) - 15*
c^2*d^2*Sqrt[c*d*f - a*e*g]*(f + g*x)^2*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/
Sqrt[c*d*f - a*e*g]]))/(4*g^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^2
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(214) = 428$.
time = 0.14, size = 516, normalized size = 2.10

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a\,c^2d^2e\,g^3x^2-15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*
d*f)*g)^(1/2))*a*c^2*d^2*e*f*g^2*x-30*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*a*c^2*d^2*e*f^2*g-15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f^3-8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2
*g^2*x^2+9*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x-25*((a*e
*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+2*((a*e*g-c*d*f)*g)^(1/2
)*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+5*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)
*a*c*d*e*f*g-15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*x
+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^2/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^3*(x*e +
 d)^(5/2)), x)
```

Fricas [A]
time = 2.08, size = 696, normalized size = 2.83

$$\left[\frac{15\,(c^2d^3g^2x^2+2\,c^2d^3fgx+c^2d^3f^2+(c^2d^2g^2x^3+2\,c^2d^2fgx^2+c^2d^2f^2x)e)\sqrt{-\frac{cdf-age}{g}}\log\left(-\frac{cd^2gx-cd^2f+2\,agxe^2-2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}\,g\sqrt{-\frac{cdf-age}{g}}+(cdgx^2-cdfx+2\,adg)e}{dgx+df+(gx^2+fx)e}\right)+2\,(8\,c^2d^2g^2x^2+25\,c^2d^2fgx+15\,c^2d^2f^2-2\,a^2g^2e^2-(9\,acdg^2x+5\,acdfg)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{8\,(dg^5x^2+2\,dfg^4x+df^2g^3+(g^5x^3+2\,fg^4x^2+f^2g^3x)e)},\ \frac{15\,(c^2d^3g^2x^2+2\,c^2d^3fgx+c^2d^3f^2+(c^2d^2g^2x^3+2\,c^2d^2fgx^2+c^2d^2f^2x)e)\sqrt{\frac{cdf-age}{g}}\arctan\left(\frac{\sqrt{xe+d}\sqrt{\frac{cdf-age}{g}}}{\sqrt{cd^2x+axe^2+(cdx^2+ad)e}}\right)+(8\,c^2d^2g^2x^2+25\,c^2d^2fgx+15\,c^2d^2f^2-2\,a^2g^2e^2-(9\,acdg^2x+5\,acdfg)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{xe+d}}{4\,(dg^5x^2+2\,dfg^4x+df^2g^3+(g^5x^3+2\,fg^4x^2+f^2g^3x)e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="fricas")
```

```
[Out] [1/8*(15*(c^2*d^3*g^2*x^2 + 2*c^2*d^3*f*g*x + c^2*d^3*f^2 + (c^2*d^2*g^2*x^
3 + 2*c^2*d^2*f*g*x^2 + c^2*d^2*f^2*x)*e)*sqrt(-(c*d*f - a*g*e)/g)*log(-(c*
d^2*g*x - c*d^2*f + 2*a*g*x*e^2 - 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d
)*e)*sqrt(x*e + d)*g*sqrt(-(c*d*f - a*g*e)/g) + (c*d*g*x^2 - c*d*f*x + 2*a*
d*g)*e)/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(8*c^2*d^2*g^2*x^2 + 25*c^2*d^
2*f*g*x + 15*c^2*d^2*f^2 - 2*a^2*g^2*e^2 - (9*a*c*d*g^2*x + 5*a*c*d*f*g)*e)
*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g^5*x^2 + 2*
d*f*g^4*x + d*f^2*g^3 + (g^5*x^3 + 2*f*g^4*x^2 + f^2*g^3*x)*e), 1/4*(15*(c^
2*d^3*g^2*x^2 + 2*c^2*d^3*f*g*x + c^2*d^3*f^2 + (c^2*d^2*g^2*x^3 + 2*c^2*d^
2*f*g*x^2 + c^2*d^2*f^2*x)*e)*sqrt((c*d*f - a*g*e)/g)*arctan(sqrt(x*e + d)*
sqrt((c*d*f - a*g*e)/g)/sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)) + (8*c
^2*d^2*g^2*x^2 + 25*c^2*d^2*f*g*x + 15*c^2*d^2*f^2 - 2*a^2*g^2*e^2 - (9*a*c
*d*g^2*x + 5*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt
(x*e + d))/(d*g^5*x^2 + 2*d*f*g^4*x + d*f^2*g^3 + (g^5*x^3 + 2*f*g^4*x^2 +
f^2*g^3*x)*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^3,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^3\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^3*(d + e*x)^(5
/2)), x)
```

3.708 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx$

Optimal. Leaf size=253

$$-\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{3g(d+ex)^{5/2}(f+gx)^3}$$

```
[Out] -5/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^2-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^3+5/
8*c^3*d^3*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*
d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(1/2)-5/8*c^2*d^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {876, 888, 211}

$$\frac{5c^3d^3\text{ArcTan}\left(\frac{\sqrt{g}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{8g^{7/2}\sqrt{cdf-aeg}}-\frac{5c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^4),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*Sqrt[d + e*
x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g
^2*(d + e*x)^(3/2)*(f + g*x)^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2)/(3*g*(d + e*x)^(5/2)*(f + g*x)^3) + (5*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])
/(8*g^(7/2)*Sqrt[c*d*f - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
```

```
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^4}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^3}+\frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{(d+ex)^{3/2}(f+gx)^3}}{6g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^2}-\frac{\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g(d+ex)^{5/2}(f+g}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x\right.}{12g^2(d+ex)^{3/2}(f}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x\right.}{12g^2(d+ex)^{3/2}(f}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}\,(f+gx)}-\frac{5cd\left(ade+\left(cd^2+ae^2\right)x\right.}{12g^2(d+ex)^{3/2}(f}$$

Mathematica [A]
time = 0.82, size = 171, normalized size = 0.68

$$\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{g}\left(8a^2e^2g^2+2acdeg(5f+13gx)+c^2d^2\left(15f^2+40fgx+33g^2x^2\right)\right)}{(f+gx)^3}+\frac{15c^3d^3\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{\sqrt{cdf-aeg}\sqrt{ae+cdx}}\right)}{24g^{7/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^4),x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[g]*(8*a^2*e^2*g^2 + 2*a*c*d*e*g*(5*
f + 13*g*x) + c^2*d^2*(15*f^2 + 40*f*g*x + 33*g^2*x^2)))/(f + g*x)^3) + (15
*c^3*d^3*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(Sqrt[c*d
*f - a*e*g]*Sqrt[a*e + c*d*x])))/(24*g^(7/2)*Sqrt[d + e*x])
```

Maple [A]
time = 0.15, size = 431, normalized size = 1.70

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3g^3x^3+45\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^3fg^2x \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^3*d^3*g^3*x^3+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^3*d^3*f*g^2*x^2+45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^3*d^3*f^2*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^3*d^3*f^3+33*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*g
^2*x^2+26*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c*d*e*g^2*x+40*((a*e*
g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f*g*x+8*((a*e*g-c*d*f)*g)^(1/2)
*(c*d*x+a*e)^(1/2)*a^2*e^2*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)
*a*c*d*e*f*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^2*d^2*f^2)/(e*x
+d)^(1/2)/(c*d*x+a*e)^(1/2)/g^3/(g*x+f)^3/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^4*(x*e +
 d)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(231) = 462$.
time = 1.60, size = 1176, normalized size = 4.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="fricas")
```

```
[Out] [-1/48*(15*(c^3*d^4*g^3*x^3 + 3*c^3*d^4*f*g^2*x^2 + 3*c^3*d^4*f^2*g*x + c^3
*d^4*f^3 + (c^3*d^3*g^3*x^4 + 3*c^3*d^3*f*g^2*x^3 + 3*c^3*d^3*f^2*g*x^2 + c
^3*d^3*f^3*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g
*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e - 2*sqrt(-c*d*f*g + a*g^2*e)*sqr
t(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x
^2 + f*x)*e)) + 2*(33*c^3*d^3*f*g^3*x^2 + 40*c^3*d^3*f^2*g^2*x + 15*c^3*d^3
*f^3*g - 8*a^3*g^4*e^3 - 2*(13*a^2*c*d*g^4*x + a^2*c*d*f*g^3)*e^2 - (33*a*c
^2*d^2*g^4*x^2 + 14*a*c^2*d^2*f*g^3*x + 5*a*c^2*d^2*f^2*g^2)*e)*sqrt(c*d^2*
x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c*d^2*f*g^7*x^3 + 3*c*d^2*
f^2*g^6*x^2 + 3*c*d^2*f^3*g^5*x + c*d^2*f^4*g^4 - (a*g^8*x^4 + 3*a*f*g^7*x^
3 + 3*a*f^2*g^6*x^2 + a*f^3*g^5*x)*e^2 + (c*d*f*g^7*x^4 - a*d*f^3*g^5 + (3*
c*d*f^2*g^6 - a*d*g^8)*x^3 + 3*(c*d*f^3*g^5 - a*d*f*g^7)*x^2 + (c*d*f^4*g^4
 - 3*a*d*f^2*g^6)*x)*e), -1/24*(15*(c^3*d^4*g^3*x^3 + 3*c^3*d^4*f*g^2*x^2 +
 3*c^3*d^4*f^2*g*x + c^3*d^4*f^3 + (c^3*d^3*g^3*x^4 + 3*c^3*d^3*f*g^2*x^3 +
 3*c^3*d^3*f^2*g*x^2 + c^3*d^3*f^3*x)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqr
t(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e +
 d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) + (33*c^3*d^3*f*g^3*x^
2 + 40*c^3*d^3*f^2*g^2*x + 15*c^3*d^3*f^3*g - 8*a^3*g^4*e^3 - 2*(13*a^2*c*d
*g^4*x + a^2*c*d*f*g^3)*e^2 - (33*a*c^2*d^2*g^4*x^2 + 14*a*c^2*d^2*f*g^3*x
+ 5*a*c^2*d^2*f^2*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(
x*e + d))/(c*d^2*f*g^7*x^3 + 3*c*d^2*f^2*g^6*x^2 + 3*c*d^2*f^3*g^5*x + c*d^
2*f^4*g^4 - (a*g^8*x^4 + 3*a*f*g^7*x^3 + 3*a*f^2*g^6*x^2 + a*f^3*g^5*x)*e^2
 + (c*d*f*g^7*x^4 - a*d*f^3*g^5 + (3*c*d*f^2*g^6 - a*d*g^8)*x^3 + 3*(c*d*f^
3*g^5 - a*d*f*g^7)*x^2 + (c*d*f^4*g^4 - 3*a*d*f^2*g^6)*x)*e)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^4,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^4\,\left(d+e\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^4*(d + e*x)^(5
/2)), x)
```

3.709 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx$

Optimal. Leaf size=323

$$-\frac{5c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}+\frac{5c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx)}-\frac{5cd(ade+(cd^2+ae^2)\,x+c}{24g^2(d+ex)^{3/2}(f+g}$$

```
[Out] -5/24*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^3-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^4+5/
64*c^4*d^4*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(3/2)-5/32*c^2*d^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^2/(e*x+d)^(1/2)+5/64*c^3*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)/(e*x+d)^
(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{5c^4d^4\,\text{ArcTan}\left(\frac{\sqrt{g}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{64g^{7/2}(cdf-aeg)^{3/2}}+\frac{5c^3d^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)}-\frac{5c^2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2}-\frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^5),x]
```

```
[Out] (-5*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e
*x]*(f + g*x)^2) + (5*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(64*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(24*g^2*(d + e*x)^(3/2)*(f + g*x)^3) - (a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(4*g*(d + e*x)^(5/2)*(f + g*x)^4) +
 (5*c^4*d^4*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(S
qrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(64*g^(7/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
```

```
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^5}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}(f+gx)^4} + \frac{(5cd)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)}{(d+ex)^{3/2}(f+gx)^4}}{8g}$$

$$= -\frac{5cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24g^2(d+ex)^{3/2}(f+gx)^3} - \frac{\left(ade+\left(cd^2+ae^2\right)x+c\right.}{4g(d+ex)^{5/2}(f+g}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2} - \frac{5cd\left(ade+\left(cd^2+ae^2\right)x\right.}{24g^2(d+ex)^{3/2}(f}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae\right.}}{64g^3(cdf-aeg)\sqrt{d+}}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae\right.}}{64g^3(cdf-aeg)\sqrt{d+}}$$

$$= -\frac{5c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^2} + \frac{5c^3d^3\sqrt{ade+\left(cd^2+ae\right.}}{64g^3(cdf-aeg)\sqrt{d+}}$$

Mathematica [A]
time = 1.36, size = 244, normalized size = 0.76

$$\frac{c^4d^4((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}\,(48a^3e^3g^3-8a^2cde^2g^2(f-17gx)+2ac^2d^2eg(-5f^2-18fgx+59g^2x^2)-c^3d^3(15f^3+55f^2gx+73fg^2x^2-15g^3x^3))}{c^4d^4(cdf-aeg)(ae+cdx)^2(f+gx)^4}+\frac{15\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}(ae+cdx)^{5/2}}\right)}{192g^{7/2}(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^5),x]
```

```
[Out] (c^4*d^4*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(48*a^3*e^3*g^3 - 8*a^2*
c*d*e^2*g^2*(f - 17*g*x) + 2*a*c^2*d^2*e*g*(-5*f^2 - 18*f*g*x + 59*g^2*x^2)
 - c^3*d^3*(15*f^3 + 55*f^2*g*x + 73*f*g^2*x^2 - 15*g^3*x^3)))/(c^4*d^4*(c*
d*f - a*e*g)*(a*e + c*d*x)^2*(f + g*x)^4) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e +
c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(3/2)*(a*e + c*d*x)^(5/2))))
/(192*g^(7/2)*(d + e*x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(285) = 570$.
time = 0.16, size = 655, normalized size = 2.03

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^4d^4g^4x^4+60\,\text{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^4d^4f\,g^3x^3+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/192*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^4*d^4*g^4*x^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^4*d^4*f*g^3*x^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f
)*g)^(1/2))*c^4*d^4*f^2*g^2*x^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^4*d^4*f^3*g*x-15*c^3*d^3*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^4*
d^4*f^4-118*a*c^2*d^2*e*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+7
3*c^3*d^3*f*g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-136*a^2*c*d*e
^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*a*c^2*d^2*e*f*g^2*x*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+55*c^3*d^3*f^2*g*x*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-48*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3
*e^3*g^3+8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c*d*e^2*f*g^2+10*(
c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^2*d^2*e*f^2*g+15*(c*d*x+a*e)^(
1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/(
a*e*g-c*d*f)/g^3/(g*x+f)^4/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^5*(x*e +
 d)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(298) = 596$.
time = 2.65, size = 1929, normalized size = 5.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="fricas")
```

```
[Out] [1/384*(15*(c^4*d^5*g^4*x^4 + 4*c^4*d^5*f*g^3*x^3 + 6*c^4*d^5*f^2*g^2*x^2 +
 4*c^4*d^5*f^3*g*x + c^4*d^5*f^4 + (c^4*d^4*g^4*x^5 + 4*c^4*d^4*f*g^3*x^4 +
 6*c^4*d^4*f^2*g^2*x^3 + 4*c^4*d^4*f^3*g*x^2 + c^4*d^4*f^4*x)*e)*sqrt(-c*d*
f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f
*x + 2*a*d*g)*e + 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(15*c^4*d
^4*f*g^4*x^3 - 73*c^4*d^4*f^2*g^3*x^2 - 55*c^4*d^4*f^3*g^2*x - 15*c^4*d^4*f
^4*g - 48*a^4*g^5*e^4 - 8*(17*a^3*c*d*g^5*x - 7*a^3*c*d*f*g^4)*e^3 - 2*(59*
a^2*c^2*d^2*g^5*x^2 - 86*a^2*c^2*d^2*f*g^4*x - a^2*c^2*d^2*f^2*g^3)*e^2 - (
15*a*c^3*d^3*g^5*x^3 - 191*a*c^3*d^3*f*g^4*x^2 - 19*a*c^3*d^3*f^2*g^3*x - 5
*a*c^3*d^3*f^3*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e
 + d))/(c^2*d^3*f^2*g^8*x^4 + 4*c^2*d^3*f^3*g^7*x^3 + 6*c^2*d^3*f^4*g^6*x^2
 + 4*c^2*d^3*f^5*g^5*x + c^2*d^3*f^6*g^4 + (a^2*g^10*x^5 + 4*a^2*f*g^9*x^4
+ 6*a^2*f^2*g^8*x^3 + 4*a^2*f^3*g^7*x^2 + a^2*f^4*g^6*x)*e^3 - (2*a*c*d*f*g
^9*x^5 - a^2*d*f^4*g^6 + (8*a*c*d*f^2*g^8 - a^2*d*g^10)*x^4 + 4*(3*a*c*d*f^
3*g^7 - a^2*d*f*g^9)*x^3 + 2*(4*a*c*d*f^4*g^6 - 3*a^2*d*f^2*g^8)*x^2 + 2*(a
*c*d*f^5*g^5 - 2*a^2*d*f^3*g^7)*x)*e^2 + (c^2*d^2*f^2*g^8*x^5 - 2*a*c*d^2*f
^5*g^5 + 2*(2*c^2*d^2*f^3*g^7 - a*c*d^2*f*g^9)*x^4 + 2*(3*c^2*d^2*f^4*g^6 -
 4*a*c*d^2*f^2*g^8)*x^3 + 4*(c^2*d^2*f^5*g^5 - 3*a*c*d^2*f^3*g^7)*x^2 + (c^
2*d^2*f^6*g^4 - 8*a*c*d^2*f^4*g^6)*x)*e), -1/192*(15*(c^4*d^5*g^4*x^4 + 4*c
^4*d^5*f*g^3*x^3 + 6*c^4*d^5*f^2*g^2*x^2 + 4*c^4*d^5*f^3*g*x + c^4*d^5*f^4
+ (c^4*d^4*g^4*x^5 + 4*c^4*d^4*f*g^3*x^4 + 6*c^4*d^4*f^2*g^2*x^3 + 4*c^4*d^
4*f^3*g*x^2 + c^4*d^4*f^4*x)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g
 - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^
2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (15*c^4*d^4*f*g^4*x^3 - 73*c^
4*d^4*f^2*g^3*x^2 - 55*c^4*d^4*f^3*g^2*x - 15*c^4*d^4*f^4*g - 48*a^4*g^5*e^
4 - 8*(17*a^3*c*d*g^5*x - 7*a^3*c*d*f*g^4)*e^3 - 2*(59*a^2*c^2*d^2*g^5*x^2
- 86*a^2*c^2*d^2*f*g^4*x - a^2*c^2*d^2*f^2*g^3)*e^2 - (15*a*c^3*d^3*g^5*x^3
 - 191*a*c^3*d^3*f*g^4*x^2 - 19*a*c^3*d^3*f^2*g^3*x - 5*a*c^3*d^3*f^3*g^2)*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f^2*
g^8*x^4 + 4*c^2*d^3*f^3*g^7*x^3 + 6*c^2*d^3*f^4*g^6*x^2 + 4*c^2*d^3*f^5*g^5
*x + c^2*d^3*f^6*g^4 + (a^2*g^10*x^5 + 4*a^2*f*g^9*x^4 + 6*a^2*f^2*g^8*x^3
+ 4*a^2*f^3*g^7*x^2 + a^2*f^4*g^6*x)*e^3 - (2*a*c*d*f*g^9*x^5 - a^2*d*f^4*g
^6 + (8*a*c*d*f^2*g^8 - a^2*d*g^10)*x^4 + 4*(3*a*c*d*f^3*g^7 - a^2*d*f*g^9)
*x^3 + 2*(4*a*c*d*f^4*g^6 - 3*a^2*d*f^2*g^8)*x^2 + 2*(a*c*d*f^5*g^5 - 2*a^2
*d*f^3*g^7)*x)*e^2 + (c^2*d^2*f^2*g^8*x^5 - 2*a*c*d^2*f^5*g^5 + 2*(2*c^2*d^
2*f^3*g^7 - a*c*d^2*f*g^9)*x^4 + 2*(3*c^2*d^2*f^4*g^6 - 4*a*c*d^2*f^2*g^8)*
x^3 + 4*(c^2*d^2*f^5*g^5 - 3*a*c*d^2*f^3*g^7)*x^2 + (c^2*d^2*f^6*g^4 - 8*a*
c*d^2*f^4*g^6)*x)*e)]
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^5\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^5*(d + e*x)^(5
/2)), x)
```

3.710 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx$

Optimal. Leaf size=393

$$-\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}+\frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}+\frac{3c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+}}{128g^3(cdf-aeg)^2\sqrt{d+ex}\,(}$$

```
[Out] -1/8*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^5+3/1
28*c^5*d^5*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c
*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(5/2)-1/16*c^2*d^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^3/(e*x+d)^(1/2)+1/64*c^3*d^3*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)^2/(e*x+d
)^(1/2)+3/128*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c
*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{3c^5d^5\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}}+\frac{3c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{128g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{64g^3\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3}-\frac{cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^6),x]
```

```
[Out] -1/16*(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e
*x]*(f + g*x)^3) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(6
4*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) + (3*c^4*d^4*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f
 + g*x)) - (c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d +
e*x)^(3/2)*(f + g*x)^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*
g*(d + e*x)^(5/2)*(f + g*x)^5) + (3*c^5*d^5*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(128*g^
(7/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^6}\,dx = -\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5} + \frac{(cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/}}{(d+ex)^{3/2}(f+gx)^5}}{2g}$$

$$= -\frac{cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cd\right.}{5g(d+ex)^{5/2}(f+gx}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3} - \frac{cd\left(ade+\left(cd^2+ae^2\right)x+\right.}{8g^2(d+ex)^{3/2}(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)}}{64g^3(cdf-aeg)\sqrt{d+ex}}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)}}{64g^3(cdf-aeg)\sqrt{d+ex}}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)}}{64g^3(cdf-aeg)\sqrt{d+ex}}$$

$$= -\frac{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}\,(f+gx)^3} + \frac{c^3d^3\sqrt{ade+\left(cd^2+ae^2\right)}}{64g^3(cdf-aeg)\sqrt{d+ex}}$$

<u>Mathematica</u> [A]
time = 1.90, size = 301, normalized size = 0.77

$$\frac{c^5d^5((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}\left(-128a^4e^4g^4+16a^3cde^3g^3(11f-21gx)-8a^2c^2d^2e^2g^2\left(f^2-64fgx+31g^2x^2\right)-2ac^3d^3eg\left(5f^3+23f^2gx-233fg^2x^2+5g^3x^3\right)+c^4d^4\left(-15f^4-70f^3gx-128f^2g^2x^2+70fg^3x^3+15g^4x^4\right)\right)}{c^5d^5(cdf-aeg)^2(ae+cdx)^2(f+gx)^5}+\frac{15\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}(ae+cdx)^{5/2}}\right)}{640g^{7/2}(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^6),x]
```

```
[Out] (c^5*d^5*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(-128*a^4*e^4*g^4 + 16*a
^3*c*d*e^3*g^3*(11*f - 21*g*x) - 8*a^2*c^2*d^2*e^2*g^2*(f^2 - 64*f*g*x + 31
*g^2*x^2) - 2*a*c^3*d^3*e*g*(5*f^3 + 23*f^2*g*x - 233*f*g^2*x^2 + 5*g^3*x^3
) + c^4*d^4*(-15*f^4 - 70*f^3*g*x - 128*f^2*g^2*x^2 + 70*f*g^3*x^3 + 15*g^4
*x^4)))/(c^5*d^5*(c*d*f - a*e*g)^2*(a*e + c*d*x)^2*(f + g*x)^5) + (15*ArcTa
n[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/((c*d*f - a*e*g)^(5/2)*
(a*e + c*d*x)^(5/2))))/(640*g^(7/2)*(d + e*x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(349) = 698$.
time = 0.14, size = 914, normalized size = 2.33

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5g^5x^5+75\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^5d^5fg^4x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/640*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c^5*d^5*g^5*x^5+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^5*d^5*f*g^4*x^4+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d
*f)*g)^(1/2))*c^5*d^5*f^2*g^3*x^3+150*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c
*d*f)*g)^(1/2))*c^5*d^5*f^3*g^2*x^2-15*c^4*d^4*g^4*x^4*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)+75*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
))*c^5*d^5*f^4*g*x+10*a*c^3*d^3*e*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*
g)^(1/2)-70*c^4*d^4*f*g^3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*
arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^5*d^5*f^5+248*a^2*c^
2*d^2*e^2*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-466*a*c^3*d^3*e
*f*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+128*c^4*d^4*f^2*g^2*x^
2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+336*a^3*c*d*e^3*g^4*x*(c*d*x+a*
e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-512*a^2*c^2*d^2*e^2*f*g^3*x*(c*d*x+a*e)^(1
/2)*((a*e*g-c*d*f)*g)^(1/2)+46*a*c^3*d^3*e*f^2*g^2*x*(c*d*x+a*e)^(1/2)*((a*
e*g-c*d*f)*g)^(1/2)+70*c^4*d^4*f^3*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^
(1/2)+128*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*e^4*g^4-176*(c*d*x+
a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c*d*e^3*f*g^3+8*(c*d*x+a*e)^(1/2)*((
a*e*g-c*d*f)*g)^(1/2)*a^2*c^2*d^2*e^2*f^2*g^2+10*(c*d*x+a*e)^(1/2)*((a*e*g-
c*d*f)*g)^(1/2)*a*c^3*d^3*e*f^3*g+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1
/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^5/g^3/(a*e*g
-c*d*f)^2/(c*d*x+a*e)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^6*(x*e +
 d)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1413 vs. $2(365) = 730$.
time = 5.70, size = 2865, normalized size = 7.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(c^5*d^6*g^5*x^5 + 5*c^5*d^6*f*g^4*x^4 + 10*c^5*d^6*f^2*g^3*x^
3 + 10*c^5*d^6*f^3*g^2*x^2 + 5*c^5*d^6*f^4*g*x + c^5*d^6*f^5 + (c^5*d^5*g^5
*x^6 + 5*c^5*d^5*f*g^4*x^5 + 10*c^5*d^5*f^2*g^3*x^4 + 10*c^5*d^5*f^3*g^2*x^
3 + 5*c^5*d^5*f^4*g*x^2 + c^5*d^5*f^5*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(
c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e - 2*s
qrt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*
e + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) - 2*(15*c^5*d^5*f*g^5*x^4 + 70*c^5
*d^5*f^2*g^4*x^3 - 128*c^5*d^5*f^3*g^3*x^2 - 70*c^5*d^5*f^4*g^2*x - 15*c^5*
d^5*f^5*g + 128*a^5*g^6*e^5 + 16*(21*a^4*c*d*g^6*x - 19*a^4*c*d*f*g^5)*e^4
+ 8*(31*a^3*c^2*d^2*g^6*x^2 - 106*a^3*c^2*d^2*f*g^5*x + 23*a^3*c^2*d^2*f^2*
g^4)*e^3 + 2*(5*a^2*c^3*d^3*g^6*x^3 - 357*a^2*c^3*d^3*f*g^5*x^2 + 279*a^2*c
^3*d^3*f^2*g^4*x + a^2*c^3*d^3*f^3*g^3)*e^2 - (15*a*c^4*d^4*g^6*x^4 + 80*a*
c^4*d^4*f*g^5*x^3 - 594*a*c^4*d^4*f^2*g^4*x^2 - 24*a*c^4*d^4*f^3*g^3*x - 5*
a*c^4*d^4*f^4*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e
+ d))/(c^3*d^4*f^3*g^9*x^5 + 5*c^3*d^4*f^4*g^8*x^4 + 10*c^3*d^4*f^5*g^7*x^3
 + 10*c^3*d^4*f^6*g^6*x^2 + 5*c^3*d^4*f^7*g^5*x + c^3*d^4*f^8*g^4 - (a^3*g^
12*x^6 + 5*a^3*f*g^11*x^5 + 10*a^3*f^2*g^10*x^4 + 10*a^3*f^3*g^9*x^3 + 5*a^
3*f^4*g^8*x^2 + a^3*f^5*g^7*x)*e^4 + (3*a^2*c*d*f*g^11*x^6 - a^3*d*f^5*g^7
+ (15*a^2*c*d*f^2*g^10 - a^3*d*g^12)*x^5 + 5*(6*a^2*c*d*f^3*g^9 - a^3*d*f*g
^11)*x^4 + 10*(3*a^2*c*d*f^4*g^8 - a^3*d*f^2*g^10)*x^3 + 5*(3*a^2*c*d*f^5*g
^7 - 2*a^3*d*f^3*g^9)*x^2 + (3*a^2*c*d*f^6*g^6 - 5*a^3*d*f^4*g^8)*x)*e^3 -
3*(a*c^2*d^2*f^2*g^10*x^6 - a^2*c*d^2*f^6*g^6 + (5*a*c^2*d^2*f^3*g^9 - a^2*
c*d^2*f*g^11)*x^5 + 5*(2*a*c^2*d^2*f^4*g^8 - a^2*c*d^2*f^2*g^10)*x^4 + 10*(
a*c^2*d^2*f^5*g^7 - a^2*c*d^2*f^3*g^9)*x^3 + 5*(a*c^2*d^2*f^6*g^6 - 2*a^2*c
*d^2*f^4*g^8)*x^2 + (a*c^2*d^2*f^7*g^5 - 5*a^2*c*d^2*f^5*g^7)*x)*e^2 + (c^3
*d^3*f^3*g^9*x^6 - 3*a*c^2*d^3*f^7*g^5 + (5*c^3*d^3*f^4*g^8 - 3*a*c^2*d^3*f
^2*g^10)*x^5 + 5*(2*c^3*d^3*f^5*g^7 - 3*a*c^2*d^3*f^3*g^9)*x^4 + 10*(c^3*d^
3*f^6*g^6 - 3*a*c^2*d^3*f^4*g^8)*x^3 + 5*(c^3*d^3*f^7*g^5 - 6*a*c^2*d^3*f^5
*g^7)*x^2 + (c^3*d^3*f^8*g^4 - 15*a*c^2*d^3*f^6*g^6)*x)*e), -1/640*(15*(c^5
*d^6*g^5*x^5 + 5*c^5*d^6*f*g^4*x^4 + 10*c^5*d^6*f^2*g^3*x^3 + 10*c^5*d^6*f^
3*g^2*x^2 + 5*c^5*d^6*f^4*g*x + c^5*d^6*f^5 + (c^5*d^5*g^5*x^6 + 5*c^5*d^5*
f*g^4*x^5 + 10*c^5*d^5*f^2*g^3*x^4 + 10*c^5*d^5*f^3*g^2*x^3 + 5*c^5*d^5*f^4
```

```
*g*x^2 + c^5*d^5*f^5*x)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*
g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x
 + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (15*c^5*d^5*f*g^5*x^4 + 70*c^5*d^5
*f^2*g^4*x^3 - 128*c^5*d^5*f^3*g^3*x^2 - 70*c^5*d^5*f^4*g^2*x - 15*c^5*d^5*
f^5*g + 128*a^5*g^6*e^5 + 16*(21*a^4*c*d*g^6*x - 19*a^4*c*d*f*g^5)*e^4 + 8*
(31*a^3*c^2*d^2*g^6*x^2 - 106*a^3*c^2*d^2*f*g^5*x + 23*a^3*c^2*d^2*f^2*g^4)
*e^3 + 2*(5*a^2*c^3*d^3*g^6*x^3 - 357*a^2*c^3*d^3*f*g^5*x^2 + 279*a^2*c^3*d
^3*f^2*g^4*x + a^2*c^3*d^3*f^3*g^3)*e^2 - (15*a*c^4*d^4*g^6*x^4 + 80*a*c^4*
d^4*f*g^5*x^3 - 594*a*c^4*d^4*f^2*g^4*x^2 - 24*a*c^4*d^4*f^3*g^3*x - 5*a*c^
4*d^4*f^4*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)
)/(c^3*d^4*f^3*g^9*x^5 + 5*c^3*d^4*f^4*g^8*x^4 + 10*c^3*d^4*f^5*g^7*x^3 + 1
0*c^3*d^4*f^6*g^6*x^2 + 5*c^3*d^4*f^7*g^5*x + c^3*d^4*f^8*g^4 - (a^3*g^12*x
^6 + 5*a^3*f*g^11*x^5 + 10*a^3*f^2*g^10*x^4 + 10*a^3*f^3*g^9*x^3 + 5*a^3*f^
4*g^8*x^2 + a^3*f^5*g^7*x)*e^4 + (3*a^2*c*d*f*g^11*x^6 - a^3*d*f^5*g^7 + (1
5*a^2*c*d*f^2*g^10 - a^3*d*g^12)*x^5 + 5*(6*a^2*c*d*f^3*g^9 - a^3*d*f*g^11)
*x^4 + 10*(3*a^2*c*d*f^4*g^8 - a^3*d*f^2*g^10)*x^3 + 5*(3*a^2*c*d*f^5*g^7 -
 2*a^3*d*f^3*g^9)*x^2 + (3*a^2*c*d*f^6*g^6 - 5*a^3*d*f^4*g^8)*x)*e^3 - 3*(a
*c^2*d^2*f^2*g^10*x^6 - a^2*c*d^2*f^6*g^6 + (5*a*c^2*d^2*f^3*g^9 - a^2*c*d^
2*f*g^11)*x^5 + 5*(2*a*c^2*d^2*f^4*g^8 - a^2*c*d^2*f^2*g^10)*x^4 + 10*(a*c^
2*d^2*f^5*g^7 - a^2*c*d^2*f^3*g^9)*x^3 + 5*(a*c^2*d^2*f^6*g^6 - 2*a^2*c*d^2
*f^4*g^8)*x^2 + (a*c^2*d^2*f^7*g^5 - 5*a^2*c*d^2*f^5*g^7)*x)*e^2 + (c^3*d^3
*f^3*g^9*x^6 - 3*a*c^2*d^3*f^7*g^5 + (5*c^3*d^3*f^4*g^8 - 3*a*c^2*d^3*f^2*g
^10)*x^5 + 5*(2*c^3*d^3*f^5*g^7 - 3*a*c^2*d^3*f^3*g^9)*x^4 + 10*(c^3*d^3*f^
6*g^6 - 3*a*c^2*d^3*f^4*g^8)*x^3 + 5*(c^3*d^3*f^7*g^5 - 6*a*c^2*d^3*f^5*g^7
)*x^2 + (c^3*d^3*f^8*g^4 - 15*a*c^2*d^3*f^6*g^6)*x)*e)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^6,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^6\,\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^6*(d + e*x)^(5
/2)), x)
```

3.711 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx$

Optimal. Leaf size=463

$$-\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}+\frac{c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{192g^3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}+\frac{5c^4d^4\sqrt{ade+(cd^2+ae^2)\,x+}}{768g^3(cdf-aeg)^2\sqrt{d+ex}\,(f}$$

```
[Out] -1/12*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)
^5-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^6+5/
512*c^6*d^6*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+
c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(7/2)/(-a*e*g+c*d*f)^(7/2)-1/32*c^2*d^2*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(g*x+f)^4/(e*x+d)^(1/2)+1/192*c^3*d^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)/(g*x+f)^3/(e*x
+d)^(1/2)+5/768*c^4*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(-a*e*g
+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2)+5/512*c^5*d^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/g^3/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {876, 886, 888, 211}

$$\frac{5c^6d^6\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{512g^{7/2}(cdf-aeg)^{7/2}}+\frac{5c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{512g^3\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}+\frac{5c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{768g^3\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}+\frac{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{192g^3\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}-\frac{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{32g^3\sqrt{d+ex}\,(f+gx)^4}-\frac{cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^7),x]
```

```
[Out] -1/32*(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e
*x]*(f + g*x)^4) + (c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1
92*g^3*(c*d*f - a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) + (5*c^4*d^4*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(768*g^3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(
f + g*x)^2) + (5*c^5*d^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(512*
g^3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (c*d*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)*(f + g*x)^5) - (a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(6*g*(d + e*x)^(5/2)*(f + g*x)^6) + (5*
c^6*d^6*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[
c*d*f - a*e*g]*Sqrt[d + e*x])])/(512*g^(7/2)*(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^7}\,dx = -\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} + \frac{(5cd)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/}}{(d+ex)^{3/2}(f+gx)^6}}{12g}$$

$$= -\frac{cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{(ade+(cd^2+ae^2)x+cde}{6g(d+ex)^{5/2}(f+gx)}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} - \frac{cd(ade+(cd^2+ae^2)x+}{12g^2(d+ex)^{3/2}(f+}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x}}{192g^3(cdf-aeg)\sqrt{d+ex}}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x}}{192g^3(cdf-aeg)\sqrt{d+ex}}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x}}{192g^3(cdf-aeg)\sqrt{d+ex}}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x}}{192g^3(cdf-aeg)\sqrt{d+ex}}$$

$$= -\frac{c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32g^3\sqrt{d+ex}(f+gx)^4} + \frac{c^3d^3\sqrt{ade+(cd^2+ae^2)x}}{192g^3(cdf-aeg)\sqrt{d+ex}}$$

Mathematica [A]
time = 2.99, size = 370, normalized size = 0.80

$$\frac{c^6d^6((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{g}\,(256a^5e^5g^5+640a^4cde^4g^4(-f+gx)+16a^3c^2d^2e^3g^3(27f^2-106fgx+27g^2x^2)+8a^2c^3d^3e^2g^2(-f^3+159f^2gx-159fg^2x^2+g^3x^3)-2ac^4d^4eg(5f^4+28f^3gx-594f^2g^2x^2+28fg^3x^3+5g^4x^4)+c^5d^5(-15f^5-85f^4gx-198f^3g^2x^2+198f^2g^3x^3+85fg^4x^4+15g^5x^5))}{c^6d^6(cdf-aeg)^3(ae+cdx)^2(f+gx)^6}+\frac{15\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}(ae+cdx)^{5/2}}\right)}{1536g^{7/2}(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^7),x]
```

```
[Out] (c^6*d^6*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[g]*(256*a^5*e^5*g^5 + 640*a
^4*c*d*e^4*g^4*(-f + g*x) + 16*a^3*c^2*d^2*e^3*g^3*(27*f^2 - 106*f*g*x + 27
*g^2*x^2) + 8*a^2*c^3*d^3*e^2*g^2*(-f^3 + 159*f^2*g*x - 159*f*g^2*x^2 + g^3
```

```
*x^3) - 2*a*c^4*d^4*e*g*(5*f^4 + 28*f^3*g*x - 594*f^2*g^2*x^2 + 28*f*g^3*x^
3 + 5*g^4*x^4) + c^5*d^5*(-15*f^5 - 85*f^4*g*x - 198*f^3*g^2*x^2 + 198*f^2*
g^3*x^3 + 85*f*g^4*x^4 + 15*g^5*x^5)))/(c^6*d^6*(c*d*f - a*e*g)^3*(a*e + c*
d*x)^2*(f + g*x)^6) + (15*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]])/((c*d*f - a*e*g)^(7/2)*(a*e + c*d*x)^(5/2))))/(1536*g^(7/2)*(d + e*
x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(413) = 826$.

time = 0.14, size = 1251, normalized size = 2.70

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(-15c^5d^5g^5x^5\sqrt{cdx+ae}\sqrt{(aeg-cdf)g}+15\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^6d^6 \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/1536*((c*d*x+a*e)*(e*x+d))^(1/2)*(-15*c^5*d^5*g^5*x^5*(c*d*x+a*e)^(1/2)*(
(a*e*g-c*d*f)*g)^(1/2)+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*c^6*d^6*f*g^5*x^5+225*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/
2))*c^6*d^6*f^2*g^4*x^4+300*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2))*c^6*d^6*f^3*g^3*x^3+225*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)
^(1/2))*c^6*d^6*f^4*g^2*x^2+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g
)^(1/2))*c^6*d^6*f^5*g*x+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(
1/2))*c^6*d^6*f^6+10*a*c^4*d^4*e*g^5*x^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g
)^(1/2)+56*a*c^4*d^4*e*f*g^4*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+
1272*a^2*c^3*d^3*e^2*f*g^4*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-11
88*a*c^4*d^4*e*f^2*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1696*a
^3*c^2*d^2*e^3*f*g^4*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1272*a^2*c
^3*d^3*e^2*f^2*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+56*a*c^4*d^4
*e*f^3*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*a^2*c^3*d^3*e^2*g^
5*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-432*a^3*c^2*d^2*e^3*g^5*x^2
*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-640*a^4*c*d*e^4*g^5*x*(c*d*x+a*e
)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*c^6*d^6*g^6*x^6-256*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*
a^5*e^5*g^5+15*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*c^5*d^5*f^5-85*c^5
*d^5*f*g^4*x^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-198*c^5*d^5*f^2*g^
3*x^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+198*c^5*d^5*f^3*g^2*x^2*(c*
d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+85*c^5*d^5*f^4*g*x*(c*d*x+a*e)^(1/2)
*((a*e*g-c*d*f)*g)^(1/2)+640*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^4*
c*d*e^4*f*g^4-432*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^3*c^2*d^2*e^3
```

```
*f^2*g^3+8*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a^2*c^3*d^3*e^2*f^3*g^
2+10*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*a*c^4*d^4*e*f^4*g)/(e*x+d)^(
1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^6/g^3/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c
*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^7*(x*e +
 d)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1994 vs. $2(432) = 864$.
time = 23.52, size = 4027, normalized size = 8.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="fricas")
```

```
[Out] [1/3072*(15*(c^6*d^7*g^6*x^6 + 6*c^6*d^7*f*g^5*x^5 + 15*c^6*d^7*f^2*g^4*x^4
 + 20*c^6*d^7*f^3*g^3*x^3 + 15*c^6*d^7*f^4*g^2*x^2 + 6*c^6*d^7*f^5*g*x + c^
6*d^7*f^6 + (c^6*d^6*g^6*x^7 + 6*c^6*d^6*f*g^5*x^6 + 15*c^6*d^6*f^2*g^4*x^5
 + 20*c^6*d^6*f^3*g^3*x^4 + 15*c^6*d^6*f^4*g^2*x^3 + 6*c^6*d^6*f^5*g*x^2 +
c^6*d^6*f^6*x)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a*
g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e + 2*sqrt(-c*d*f*g + a*g^2*e)*sq
rt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g*
x^2 + f*x)*e)) + 2*(15*c^6*d^6*f*g^6*x^5 + 85*c^6*d^6*f^2*g^5*x^4 + 198*c^6
*d^6*f^3*g^4*x^3 - 198*c^6*d^6*f^4*g^3*x^2 - 85*c^6*d^6*f^5*g^2*x - 15*c^6*
d^6*f^6*g - 256*a^6*g^7*e^6 - 128*(5*a^5*c*d*g^7*x - 7*a^5*c*d*f*g^6)*e^5 -
 16*(27*a^4*c^2*d^2*g^7*x^2 - 146*a^4*c^2*d^2*f*g^6*x + 67*a^4*c^2*d^2*f^2*
g^5)*e^4 - 8*(a^3*c^3*d^3*g^7*x^3 - 213*a^3*c^3*d^3*f*g^6*x^2 + 371*a^3*c^3
*d^3*f^2*g^5*x - 55*a^3*c^3*d^3*f^3*g^4)*e^3 + 2*(5*a^2*c^4*d^4*g^7*x^4 + 3
2*a^2*c^4*d^4*f*g^6*x^3 - 1230*a^2*c^4*d^4*f^2*g^5*x^2 + 664*a^2*c^4*d^4*f^
3*g^4*x + a^2*c^4*d^4*f^4*g^3)*e^2 - (15*a*c^5*d^5*g^7*x^5 + 95*a*c^5*d^5*f
*g^6*x^4 + 254*a*c^5*d^5*f^2*g^5*x^3 - 1386*a*c^5*d^5*f^3*g^4*x^2 - 29*a*c^
5*d^5*f^4*g^3*x - 5*a*c^5*d^5*f^5*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*sqrt(x*e + d))/(c^4*d^5*f^4*g^10*x^6 + 6*c^4*d^5*f^5*g^9*x^5 + 1
5*c^4*d^5*f^6*g^8*x^4 + 20*c^4*d^5*f^7*g^7*x^3 + 15*c^4*d^5*f^8*g^6*x^2 + 6
```

*c^4*d^5*f^9*g^5*x + c^4*d^5*f^10*g^4 + (a^4*g^14*x^7 + 6*a^4*f*g^13*x^6 +
15*a^4*f^2*g^12*x^5 + 20*a^4*f^3*g^11*x^4 + 15*a^4*f^4*g^10*x^3 + 6*a^4*f^5
*g^9*x^2 + a^4*f^6*g^8*x)*e^5 - (4*a^3*c*d*f*g^13*x^7 - a^4*d*f^6*g^8 + (24
*a^3*c*d*f^2*g^12 - a^4*d*g^14)*x^6 + 6*(10*a^3*c*d*f^3*g^11 - a^4*d*f*g^13
)*x^5 + 5*(16*a^3*c*d*f^4*g^10 - 3*a^4*d*f^2*g^12)*x^4 + 20*(3*a^3*c*d*f^5*
g^9 - a^4*d*f^3*g^11)*x^3 + 3*(8*a^3*c*d*f^6*g^8 - 5*a^4*d*f^4*g^10)*x^2 +
2*(2*a^3*c*d*f^7*g^7 - 3*a^4*d*f^5*g^9)*x)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^12*
x^7 - 2*a^3*c*d^2*f^7*g^7 + 2*(9*a^2*c^2*d^2*f^3*g^11 - a^3*c*d^2*f*g^13)*x
^6 + 3*(15*a^2*c^2*d^2*f^4*g^10 - 4*a^3*c*d^2*f^2*g^12)*x^5 + 30*(2*a^2*c^2
*d^2*f^5*g^9 - a^3*c*d^2*f^3*g^11)*x^4 + 5*(9*a^2*c^2*d^2*f^6*g^8 - 8*a^3*c
*d^2*f^4*g^10)*x^3 + 6*(3*a^2*c^2*d^2*f^7*g^7 - 5*a^3*c*d^2*f^5*g^9)*x^2 +
3*(a^2*c^2*d^2*f^8*g^6 - 4*a^3*c*d^2*f^6*g^8)*x)*e^3 - 2*(2*a*c^3*d^3*f^3*g
^11*x^7 - 3*a^2*c^2*d^3*f^8*g^6 + 3*(4*a*c^3*d^3*f^4*g^10 - a^2*c^2*d^3*f^2
*g^12)*x^6 + 6*(5*a*c^3*d^3*f^5*g^9 - 3*a^2*c^2*d^3*f^3*g^11)*x^5 + 5*(8*a*
c^3*d^3*f^6*g^8 - 9*a^2*c^2*d^3*f^4*g^10)*x^4 + 30*(a*c^3*d^3*f^7*g^7 - 2*a
^2*c^2*d^3*f^5*g^9)*x^3 + 3*(4*a*c^3*d^3*f^8*g^6 - 15*a^2*c^2*d^3*f^6*g^8)*
x^2 + 2*(a*c^3*d^3*f^9*g^5 - 9*a^2*c^2*d^3*f^7*g^7)*x)*e^2 + (c^4*d^4*f^4*g
^10*x^7 - 4*a*c^3*d^4*f^9*g^5 + 2*(3*c^4*d^4*f^5*g^9 - 2*a*c^3*d^4*f^3*g^11
)*x^6 + 3*(5*c^4*d^4*f^6*g^8 - 8*a*c^3*d^4*f^4*g^10)*x^5 + 20*(c^4*d^4*f^7*
g^7 - 3*a*c^3*d^4*f^5*g^9)*x^4 + 5*(3*c^4*d^4*f^8*g^6 - 16*a*c^3*d^4*f^6*g^
8)*x^3 + 6*(c^4*d^4*f^9*g^5 - 10*a*c^3*d^4*f^7*g^7)*x^2 + (c^4*d^4*f^10*g^4
 - 24*a*c^3*d^4*f^8*g^6)*x)*e), -1/1536*(15*(c^6*d^7*g^6*x^6 + 6*c^6*d^7*f*
g^5*x^5 + 15*c^6*d^7*f^2*g^4*x^4 + 20*c^6*d^7*f^3*g^3*x^3 + 15*c^6*d^7*f^4*
g^2*x^2 + 6*c^6*d^7*f^5*g*x + c^6*d^7*f^6 + (c^6*d^6*g^6*x^7 + 6*c^6*d^6*f*
g^5*x^6 + 15*c^6*d^6*f^2*g^4*x^5 + 20*c^6*d^6*f^3*g^3*x^4 + 15*c^6*d^6*f^4*
g^2*x^3 + 6*c^6*d^6*f^5*g*x^2 + c^6*d^6*f^6*x)*e)*sqrt(c*d*f*g - a*g^2*e)*a
rctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*s
qrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (15*c^6*d^6
*f*g^6*x^5 + 85*c^6*d^6*f^2*g^5*x^4 + 198*c^6*d^6*f^3*g^4*x^3 - 198*c^6*d^6
*f^4*g^3*x^2 - 85*c^6*d^6*f^5*g^2*x - 15*c^6*d^6*f^6*g - 256*a^6*g^7*e^6 -
128*(5*a^5*c*d*g^7*x - 7*a^5*c*d*f*g^6)*e^5 - 16*(27*a^4*c^2*d^2*g^7*x^2 -
146*a^4*c^2*d^2*f*g^6*x + 67*a^4*c^2*d^2*f^2*g^5)*e^4 - 8*(a^3*c^3*d^3*g^7*
x^3 - 213*a^3*c^3*d^3*f*g^6*x^2 + 371*a^3*c^3*d^3*f^2*g^5*x - 55*a^3*c^3*d^
3*f^3*g^4)*e^3 + 2*(5*a^2*c^4*d^4*g^7*x^4 + 32*a^2*c^4*d^4*f*g^6*x^3 - 1230
*a^2*c^4*d^4*f^2*g^5*x^2 + 664*a^2*c^4*d^4*f^3*g^4*x + a^2*c^4*d^4*f^4*g^3)
*e^2 - (15*a*c^5*d^5*g^7*x^5 + 95*a*c^5*d^5*f*g^6*x^4 + 254*a*c^5*d^5*f^2*g
^5*x^3 - 1386*a*c^5*d^5*f^3*g^4*x^2 - 29*a*c^5*d^5*f^4*g^3*x - 5*a*c^5*d^5*
f^5*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^4
*d^5*f^4*g^10*x^6 + 6*c^4*d^5*f^5*g^9*x^5 + 15*c^4*d^5*f^6*g^8*x^4 + 20*c^4
*d^5*f^7*g^7*x^3 + 15*c^4*d^5*f^8*g^6*x^2 + 6*c^4*d^5*f^9*g^5*x + c^4*d^5*f
^10*g^4 + (a^4*g^14*x^7 + 6*a^4*f*g^13*x^6 + 15*a^4*f^2*g^12*x^5 + 20*a^4*f
^3*g^11*x^4 + 15*a^4*f^4*g^10*x^3 + 6*a^4*f^5*g^9*x^2 + a^4*f^6*g^8*x)*e^5
- (4*a^3*c*d*f*g^13*x^7 - a^4*d*f^6*g^8 + (24*a^3*c*d*f^2*g^12 - a^4*d*g^14
)*x^6 + 6*(10*a^3*c*d*f^3*g^11 - a^4*d*f*g^13)*x^5 + 5*(16*a^3*c*d*f^4*g^10
 - 3*a^4*d*f^2*g^12)*x^4 + 20*(3*a^3*c*d*f^5*g^9 - a^4*d*f^3*g^11)*x^3 + 3*

```
(8*a^3*c*d*f^6*g^8 - 5*a^4*d*f^4*g^10)*x^2 + 2*(2*a^3*c*d*f^7*g^7 - 3*a^4*d
*f^5*g^9)*x)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^12*x^7 - 2*a^3*c*d^2*f^7*g^7 + 2*
(9*a^2*c^2*d^2*f^3*g^11 - a^3*c*d^2*f*g^13)*x^6...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**7,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^7\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(5
/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^7*(d + e*x)^(5
/2)), x)
```

3.712 $\int \frac{\sqrt{d+ex}\,(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=313

$$\frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}}+\frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+c}}{12c^2d^2\sqrt{d+ex}}$$

```
[Out] 5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/g^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/12*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+1/3*(g*x+f)^(5/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+5/8*(-a*e*g+c*d*f)^2*
(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {884, 905, 65, 223, 212}

$$\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}+\frac{5\sqrt{f+gx}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}}+\frac{5(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)}{12c^2d^2\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(8*c^3*d^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^2*d^2*Sqrt[d + e*x]) + ((f + g*
x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*Sqrt[d + e*x])
 + (5*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(7/2)*d^(7/2)*Sqrt[
g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd\sqrt{d+ex}} + \frac{(5(cde^2f+cd^2eg-\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-\cdots}{\cdots}$$

$$= \frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8c^3d^3\sqrt{d+ex}} + \frac{5(cdf-\cdots}{\cdots}$$

Mathematica [A]
time = 0.33, size = 207, normalized size = 0.66

$$\frac{(cdf-aeg)^3\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{ae+cdx}\,\sqrt{f+gx}\,(15a^2e^2g^2-10acdeg(4f+gx)+c^2d^2(33f^2+26fgx+8g^2x^2))}{(cdf-aeg)^3}+\frac{15\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{\sqrt{g}}\right)}{24c^{7/2}d^{7/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(5/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] ((c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((Sqrt[c]*Sqrt[d]*Sqrt[a
*e + c*d*x]*Sqrt[f + g*x]*(15*a^2*e^2*g^2 - 10*a*c*d*e*g*(4*f + g*x) + c^2*
d^2*(33*f^2 + 26*f*g*x + 8*g^2*x^2)))/(c*d*f - a*e*g)^3 + (15*ArcTanh[(Sqrt
```

[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/Sqrt[g]))/(24*c^(7
/2)*d^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])

Maple [A]
time = 0.15, size = 501, normalized size = 1.60

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a^{3}e^{3}g^{3}-45\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)

[Out] -1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^3*e^3*
g^3-45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*
((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f^2*g
-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/
2))/(d*g*c)^(1/2))*c^3*d^3*f^3-16*c^2*d^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/
2)*(d*g*c)^(1/2)+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x
-52*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*
d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2+80*((g*x+f)*(c*d*x+a*e))^(1/2)*(d
*g*c)^(1/2)*a*c*d*e*f*g-66*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^
2*f^2)/(e*x+d)^(1/2)/c^3/d^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(d*g*c)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")

[Out] integrate((g*x + f)^(5/2)*sqrt(x*e + d)/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a
*e^2)*x), x)

Fricas [A]
time = 3.37, size = 845, normalized size = 2.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 26*c^3*d^3*f*g^2*x + 33*c^3*d^3*f^2*g + 15*a^
2*c*d*g^3*e^2 - 10*(a*c^2*d^2*g^3*x + 4*a*c^2*d^2*f*g^2)*e)*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^3*d^4*f^3
- a^3*g^3*x*e^4 + (3*a^2*c*d*f*g^2*x - a^3*d*g^3)*e^3 - 3*(a*c^2*d^2*f^2*g*
x - a^2*c*d^2*f*g^2)*e^2 + (c^3*d^3*f^3*x - 3*a*c^2*d^3*f^2*g)*e)*sqrt(c*d*
g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3
- 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c*d*f + a*g*e)
*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x
 + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g
+ (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^4*d^4*g*x*e + c^4*d^5*
g), 1/48*(2*(8*c^3*d^3*g^3*x^2 + 26*c^3*d^3*f*g^2*x + 33*c^3*d^3*f^2*g + 15
*a^2*c*d*g^3*e^2 - 10*(a*c^2*d^2*g^3*x + 4*a*c^2*d^2*f*g^2)*e)*sqrt(c*d^2*x
 + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^3*d^4*f
^3 - a^3*g^3*x*e^4 + (3*a^2*c*d*f*g^2*x - a^3*d*g^3)*e^3 - 3*(a*c^2*d^2*f^2
*g*x - a^2*c*d^2*f*g^2)*e^2 + (c^3*d^3*f^3*x - 3*a*c^2*d^3*f^2*g)*e)*sqrt(-
c*d*g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sq
rt(g*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2
 + c*d*f*x + a*d*g)*e)))/(c^4*d^4*g*x*e + c^4*d^5*g)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(5/2)*sqrt(x*e + d)/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{5/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

3.713 $\int \frac{\sqrt{d+ex}\,(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=244

$$\frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}}+\frac{3(cdf}{}$$

```
[Out] 3/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/c/d/(e*x+d)^(1/2)+3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {884, 905, 65, 223, 212}

$$\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}+\frac{3\sqrt{f+gx}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)}{4c^2d^2\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2],x]
```

```
[Out] (3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(4*c^2*d^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(2*c*d*Sqrt[d + e*x]) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sq
rt[f + g*x])])/(4*c^(5/2)*d^(5/2)*Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cd\sqrt{d+ex}} + \frac{(3(cde^2f+cd^2eg-}{}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)}{}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)}{}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)}{}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)}{}$$

$$= \frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)}{}$$

Mathematica [A]
time = 0.28, size = 179, normalized size = 0.73

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}\,(f+gx)(-3aeg+cd(5f+2gx))+3(cdf-aeg)^2\sqrt{f+gx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)\right)}{4c^{5/2}d^{5/2}\sqrt{g}\,\sqrt{(ae+cdx)(d+ex)}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^(3/2))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]
*(f + g*x)*(-3*a*e*g + c*d*(5*f + 2*g*x)) + 3*(c*d*f - a*e*g)^2*Sqrt[f + g*
x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])]))/(
4*c^(5/2)*d^(5/2)*Sqrt[g]*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A]
time = 0.15, size = 318, normalized size = 1.30

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a^2e^2g^2-6\ln\left(\frac{2cdgx}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*(g*x+f)^(1/2)/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*
d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2
))*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2
)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*f^2+4*
((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c*d*g*x-6*(d*g*c)^(1/2)*((g*x+f)*
(c*d*x+a*e))^(1/2)*a*e*g+10*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f
)/((g*x+f)*(c*d*x+a*e))^(1/2)/c^2/d^2/(d*g*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)*sqrt(x*e + d)/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

Fricas [A]
time = 3.40, size = 669, normalized size = 2.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*g^2*e)*sqrt(c*d^2*x + a
*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 3*(c^2*d^3*f^2 +
a^2*g^2*x*e^3 - (2*a*c*d*f*g*x - a^2*d*g^2)*e^2 + (c^2*d^2*f^2*x - 2*a*c*d^
2*f*g)*e)*sqrt(c*d*g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f
```

```
^2 + a^2*g^2*x*e^3 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g
*x + c*d*f + a*g*e)*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*
x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x
^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^3*d
^3*g*x*e + c^3*d^4*g), 1/8*(2*(2*c^2*d^2*g^2*x + 5*c^2*d^2*f*g - 3*a*c*d*g^
2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d
) - 3*(c^2*d^3*f^2 + a^2*g^2*x*e^3 - (2*a*c*d*f*g*x - a^2*d*g^2)*e^2 + (c^2
*d^2*f^2*x - 2*a*c*d^2*f*g)*e)*sqrt(-c*d*g)*arctan(2*sqrt(c*d^2*x + a*x*e^2
 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x
 + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c^3*d^3*g*x*
e + c^3*d^4*g)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(f + g*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*sqrt(x*e + d)/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a
*e^2)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^{3/2}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

3.714 $\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}}+\frac{(cdf-aeg)\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] (-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1
/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(1/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c/d/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {884, 905, 65, 223, 212}

$$\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}+\frac{\sqrt{f+gx}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*Sqrt[d + e
*x]) + ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[g]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} + \frac{(cde^2f+cd^2eg-e(cd^2 \cdots}{\cdots}$$

$$= \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left((cde^2f+cd^2eg-e(c \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left((cde^2f+cd^2eg-e(c \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} + \frac{\left((cde^2f+cd^2eg-e(c \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{cd\sqrt{d+ex}} + \frac{(cdf-aeg)\sqrt{ae+cdx} \cdots}{c^{3/2}d^{3/2}\sqrt{g}\,\cdots}$$

Mathematica [A]
time = 0.34, size = 129, normalized size = 0.76

$$\frac{\sqrt{d+ex}\left(g(ae+cdx)\sqrt{f+gx}+\sqrt{\frac{g}{cd}}\,(-cdf+aeg)\sqrt{ae+cdx}\,\log\left(-\sqrt{\frac{g}{cd}}\,\sqrt{ae+cdx}+\sqrt{f+gx}\right)\right)}{cdg\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*(g*(a*e + c*d*x)*Sqrt[f + g*x] + Sqrt[g/(c*d)]*(-(c*d*f) + a
*e*g)*Sqrt[a*e + c*d*x]*Log[-(Sqrt[g/(c*d)]*Sqrt[a*e + c*d*x]) + Sqrt[f + g
*x]]))/(c*d*g*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 191, normalized size = 1.13

method	result

default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)aeg-\ln\left(\frac{2cdgx+c}{}\right.\right.}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/2*(g*x+f)^(1/2)/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d
*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2)
)*a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)
^(1/2))/(d*g*c)^(1/2))*c*d*f-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(
(g*x+f)*(c*d*x+a*e))^(1/2)/c/d/(d*g*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*sqrt(x*e + d)/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Fricas [A]
time = 2.04, size = 531, normalized size = 3.14

$$\left[\frac{4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}\,cdg-(cd^2f-agxe^2+(cdfx-adg)e)\sqrt{cdg}\,\log\left(-\frac{8c^2d^3g^2x^2+8c^2d^3fgx+c^2d^3f^2+a^2g^2xe^3-4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2cdgx+cdf+age)\sqrt{cdg}\,\sqrt{gx+f}\,\sqrt{xe+d}+(8acdg^2x^2+6acdfgx+a^2dg^2)e^2+(8c^2d^2g^2x^3+8c^2d^2fgx^2+6acd^2fg+(c^2d^2f^2+8acd^2g^2)x)e}{xe+d}\right)}{4(c^2d^2gxe+c^2d^3g)},\ \frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}\,cdg-(cd^2f-agxe^2+(cdfx-adg)e)\sqrt{-cdg}\,\arctan\left(\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{-cdg}\,\sqrt{gx+f}\,\sqrt{xe+d}}{2cd^2gx+cd^2f+agxe^2+(2cdgx^2+cdfx+adg)e}\right)}{2(c^2d^2gxe+c^2d^3g)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e
+ d)*c*d*g - (c*d^2*f - a*g*x*e^2 + (c*d*f*x - a*d*g)*e)*sqrt(c*d*g)*log(-(
8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 - 4*sqrt(
c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c*d*f + a*g*e)*sqrt(c*d
*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d*
g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d^
2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^2*d^2*g*x*e + c^2*d^3*g), 1/2*(
2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*c
*d*g - (c*d^2*f - a*g*x*e^2 + (c*d*f*x - a*d*g)*e)*sqrt(-c*d*g)*arctan(2*sq
```

```
rt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x
*e + d)/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g
)*e)))/(c^2*d^2*g*x*e + c^2*d^3*g)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\sqrt{f+gx}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*sqrt(x*e + d)/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2), x)
```

3.715 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=105

$$\frac{2\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a
*e)^(1/2)*(e*x+d)^(1/2)/c^(1/2)/d^(1/2)/g^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {905, 65, 223, 212}

$$\frac{2\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{1}{\sqrt{ae+cdx}\,\sqrt{f+gx}}\,dx}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$= \frac{\left(2\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{\sqrt{f-\frac{aeg}{cd}+\frac{gx^2}{cd}}}\,dx\right.}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$= \frac{\left(2\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\text{Subst}\left(\int \frac{1}{1-\frac{gx^2}{cd}}\,dx,x,\frac{\sqrt{ae+cd}}{\sqrt{f+g}}\right.}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$= \frac{2\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

Mathematica [A]
time = 0.10, size = 94, normalized size = 0.90

$$\frac{2\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqr
t[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[(a*e + c*d*x)*(
d + e*x)])
```

Maple [A]
time = 0.14, size = 102, normalized size = 0.97

method	result	size
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)\,(ex+d)}\,\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)\,(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)}{\sqrt{ex+d}\,\sqrt{dgc}\,\sqrt{(gx+f)\,(cdx+ae)}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*ln(1/2*(2*c*d*g*x
+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))/(d
*g*c)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g
*x + f)), x)
```

Fricas [A]
time = 2.62, size = 349, normalized size = 3.32

$$\left[\frac{\sqrt{cdg}\,\log\left(-\frac{8\,c^2d^3g^2x^2+8\,c^2d^3fgx+c^2d^3f^2+a^2g^2xe^3+4\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2\,cdgx+cdf+age)\sqrt{cdg}\,\sqrt{gx+f}\,\sqrt{xe+d}+(8\,acdg^2x^2+6\,acdfgx+a^2dg^2)e^2+(8\,c^2d^2g^2x^3+8\,c^2d^2fgx^2+6\,acd^2fg+(c^2d^2f^2+8\,acd^2g^2)x)e}{xe+d}\right)}{2\,cdg}, -\frac{\sqrt{-cdg}\,\arctan\left(\frac{2\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{-cdg}\,\sqrt{gx+f}\,\sqrt{xe+d}}{2\,cd^2gx+cd^2f+agxe^2+(2\,cdgx^2+cdfx+adg)e}\right)}{cdg}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c*d*g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 +
a^2*g^2*x*e^3 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x +
c*d*f + a*g*e)*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 +
 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 +
```

```
6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d))/(c*d*g), -sq
rt(-c*d*g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g
)*sqrt(g*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g
*x^2 + c*d*f*x + a*d*g)*e))/(c*d*g)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)), x)
```

3.716 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,\sqrt{f+gx}}$$

```
[Out] 2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(e*x+d)^(1/2)/(g*x
+f)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {874}

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)*Sqrt[d + e
*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)\sqrt{d+ex}\,\sqrt{f+gx}}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.82

$$\frac{2\sqrt{(ae+cdx)(d+ex)}}{(cdf-aeg)\sqrt{d+ex}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)])/((c*d*f - a*e*g)*Sqrt[d + e*x]*Sqrt[f + g
*x])
```

Maple [A]
time = 0.14, size = 45, normalized size = 0.74

method	result	size
default	$-\frac{2\sqrt{\left(c d x +a e \right) \left(e x +d \right)}}{\sqrt{e x +d}\, \sqrt{g x +f}\, \left(a e g -c d f \right)}$	45
gosper	$-\frac{2 \left(c d x +a e \right) \sqrt{e x +d}}{\sqrt{g x +f}\, \left(a e g -c d f \right) \sqrt{c d e \,x^{2}+a \,e^{2} x +c \,d^{2} x +a d e}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(a*e*g-c*d*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(3/2)), x)
```

Fricas [A]
time = 3.03, size = 115, normalized size = 1.89

$$\frac{2 \sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\, \sqrt{gx + f}\, \sqrt{xe + d}}{cd^2fgx + cd^2f^2 - (ag^2x^2 + afgx)e^2 + (cdfgx^2 - adfg + (cdf^2 - adg^2)x)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(
c*d^2*f*g*x + c*d^2*f^2 - (a*g^2*x^2 + a*f*g*x)*e^2 + (c*d*f*g*x^2 - a*d*f*
g + (c*d*f^2 - a*d*g^2)*x)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,\,(f+gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.64, size = 100, normalized size = 1.64

$$-\frac{2\,\sqrt{d+e\,x}\,\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(x\,\sqrt{f+g\,x}-\frac{\sqrt{f+g\,x}\,\left(c\,d^2\,f-a\,d\,e\,g\right)}{a\,e^2\,g-c\,d\,e\,f}\right)\,\left(a\,e^2\,g-c\,d\,e\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -(2*(d + e*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((x*(f +
 g*x)^(1/2) - ((f + g*x)^(1/2)*(c*d^2*f - a*d*e*g))/(a*e^2*g - c*d*e*f))*(a
*e^2*g - c*d*e*f))
```

3.717 $\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=129

$$\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{3/2}}+\frac{4cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}\,\sqrt{f+gx}}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(e
*x+d)^(1/2)+4/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^
2/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(3/2)) + (4*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
```

```
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{3/2}} + \frac{(2cd)\int \frac{}{(f+gx)^{3/2}\sqrt{}}}{}$$
$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{3/2}} + \frac{4cd\sqrt{ade+(cd^2}}{3(cdf-aeg)^2\sqrt{d}}$$

Mathematica [A]
time = 0.12, size = 69, normalized size = 0.53

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(-aeg+cd(3f+2gx))}{3(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(a*e*g) + c*d*(3*f + 2*g*x)))/(3*(c*d*f
- a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2))
```

Maple [A]
time = 0.16, size = 61, normalized size = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-2cdgx+aeg-3cdf)}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}(aeg-cdf)^2}$	61
gosper	$-\frac{2(cdx+ae)(-2cdgx+aeg-3cdf)\sqrt{ex+d}}{3(gx+f)^{\frac{3}{2}}(a^2e^2g^2-2acdefg+f^2c^2d^2)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-2*c*d*g*x+a*
e*g-3*c*d*f)/(a*e*g-c*d*f)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(119) = 238.
time = 3.21, size = 302, normalized size = 2.34

$$\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2\,cdgx+3\,cdf-age)\sqrt{gx+f}\,\sqrt{xe+d}}{3\,(c^2d^3f^2g^2x^2+2\,c^2d^3f^3gx+c^2d^3f^4+(a^2g^4x^3+2\,a^2fg^3x^2+a^2f^2g^2x)e^3-(2\,acdfg^3x^3-a^2df^2g^2+(4\,acdf^2g^2-a^2dg^4)x^2+2\,(acdf^3g-a^2dfg^3)x)e^2+(c^2d^2f^2g^2x^3-2\,acd^2f^3g+2\,(c^2d^2f^3g-acd^2fg^3)x^2+(c^2d^2f^4-4\,acd^2f^2g^2)x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + 3*c*d*f - a*g*
e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^2*d^3*f^2*g^2*x^2 + 2*c^2*d^3*f^3*g*x + c
^2*d^3*f^4 + (a^2*g^4*x^3 + 2*a^2*f*g^3*x^2 + a^2*f^2*g^2*x)*e^3 - (2*a*c*d
*f*g^3*x^3 - a^2*d*f^2*g^2 + (4*a*c*d*f^2*g^2 - a^2*d*g^4)*x^2 + 2*(a*c*d*f
^3*g - a^2*d*f*g^3)*x)*e^2 + (c^2*d^2*f^2*g^2*x^3 - 2*a*c*d^2*f^3*g + 2*(c^
2*d^2*f^3*g - a*c*d^2*f*g^3)*x^2 + (c^2*d^2*f^4 - 4*a*c*d^2*f^2*g^2)*x)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(5/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.90, size = 147, normalized size = 1.14

$$-\frac{\left(\frac{(2aeg-6cdf)\sqrt{d+ex}}{3eg(aeg-cdf)^2}-\frac{4cdx\sqrt{d+ex}}{3e(aeg-cdf)^2}\right)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^2\sqrt{f+gx}+\frac{df\sqrt{f+gx}}{eg}+\frac{x\sqrt{f+gx}\,(dg+ef)}{eg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((((2*a*e*g - 6*c*d*f)*(d + e*x)^(1/2))/(3*e*g*(a*e*g - c*d*f)^2) - (4*c*d
*x*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c
*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (d*f*(f + g*x)^(1/2))/(e*g) + (x*(f
 + g*x)^(1/2)*(d*g + e*f))/(e*g))
```

3.718 $\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=198

$$\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{5/2}}+\frac{8cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^{3/2}}+\frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)x+cde}}{15(cdf-aeg)^3\sqrt{d+ex}\,\sqrt{f+g}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(e
*x+d)^(1/2)+8/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)
^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)+16/15*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{16c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{15\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{15\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(5/2)) + (8*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(15*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) + (16*c^2*d^2*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*
Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
```

```
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{5/2}} + \frac{(4cd)\int \frac{}{(f+gx)^{5/2}\sqrt{}}}{}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+(cd^2}}{15(cdf-aeg)^2\sqrt{a}}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{5(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{5/2}} + \frac{8cd\sqrt{ade+(cd^2}}{15(cdf-aeg)^2\sqrt{a}}$$

Mathematica [A]
time = 0.14, size = 105, normalized size = 0.53

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(3a^2e^2g^2-2acdeg(5f+2gx)+c^2d^2(15f^2+20fgx+8g^2x^2))}{15(cdf-aeg)^3\sqrt{d+ex}\,(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(3*a^2*e^2*g^2 - 2*a*c*d*e*g*(5*f + 2*g*x)
 + c^2*d^2*(15*f^2 + 20*f*g*x + 8*g^2*x^2)))/(15*(c*d*f - a*e*g)^3*Sqrt[d +
 e*x]*(f + g*x)^(5/2))
```

Maple [A]
time = 0.14, size = 111, normalized size = 0.56

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(8g^2x^2c^2d^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15f^2c^2d^2)}{15\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(aeg-cdf)^3}$	111
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2-4acde\,g^2x+20c^2d^2fgx+3a^2e^2g^2-10acdefg+15f^2c^2d^2)\sqrt{ex+d}}{15(gx+f)^{\frac{5}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	169

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/15/(e*x+d)^(1/2)/(g*x+f)^(5/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(8*c^2*d^2*g^
2*x^2-4*a*c*d*e*g^2*x+20*c^2*d^2*f*g*x+3*a^2*e^2*g^2-10*a*c*d*e*f*g+15*c^2*
d^2*f^2)/(a*e*g-c*d*f)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(7/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(183) = 366.
time = 5.08, size = 599, normalized size = 3.03

$$\frac{2\,(8\,c^2d^2g^2x^2+20\,c^2d^2fgx+15\,c^2d^2f^2+3\,a^2g^2e^2-2\,(2\,acdg^2x+5\,acdfg)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{15\,(c^3d^4f^3g^3x^3+3\,c^3d^4f^4g^2x^2+3\,c^3d^4f^5gx+c^3d^4f^6-(a^3g^6x^4+3\,a^3fg^5x^3+3\,a^3f^2g^4x^2+a^3f^3g^3x)e^4+(3\,a^2cdfg^5x^4-a^3df^3g^3+(9\,a^2cdf^2g^4-a^3dg^6)x^3+3\,(3\,a^2cdf^3g^3-a^3dfg^5)x^2+3\,(a^2cdf^4g^2-a^3df^2g^4)x)e^3-3\,(ac^2d^2f^2g^4x^4-a^2cd^2f^4g^2+(3\,ac^2d^2f^3g^3-a^2cd^2fg^5)x^3+3\,(ac^2d^2f^4g^2-a^2cd^2f^2g^4)x^2+(ac^2d^2f^5g-3\,a^2cd^2f^3g^3)x)e^2+(c^3d^3f^3g^3x^4-3\,ac^2d^3f^5g+3\,(c^3d^3f^4g^2-ac^2d^3f^2g^4)x^3+3\,(c^3d^3f^5g-3\,ac^2d^3f^3g^3)x^2+(c^3d^3f^6-9\,ac^2d^3f^4g^2)x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*c^2*d^2*g^2*x^2 + 20*c^2*d^2*f*g*x + 15*c^2*d^2*f^2 + 3*a^2*g^2*e^2
 - 2*(2*a*c*d*g^2*x + 5*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^3*d^4*f^3*g^3*x^3 + 3*c^3*d^4*f^4*g^2
*x^2 + 3*c^3*d^4*f^5*g*x + c^3*d^4*f^6 - (a^3*g^6*x^4 + 3*a^3*f*g^5*x^3 + 3
*a^3*f^2*g^4*x^2 + a^3*f^3*g^3*x)*e^4 + (3*a^2*c*d*f*g^5*x^4 - a^3*d*f^3*g^
3 + (9*a^2*c*d*f^2*g^4 - a^3*d*g^6)*x^3 + 3*(3*a^2*c*d*f^3*g^3 - a^3*d*f*g^
5)*x^2 + 3*(a^2*c*d*f^4*g^2 - a^3*d*f^2*g^4)*x)*e^3 - 3*(a*c^2*d^2*f^2*g^4*
x^4 - a^2*c*d^2*f^4*g^2 + (3*a*c^2*d^2*f^3*g^3 - a^2*c*d^2*f*g^5)*x^3 + 3*(
a*c^2*d^2*f^4*g^2 - a^2*c*d^2*f^2*g^4)*x^2 + (a*c^2*d^2*f^5*g - 3*a^2*c*d^2
*f^3*g^3)*x)*e^2 + (c^3*d^3*f^3*g^3*x^4 - 3*a*c^2*d^3*f^5*g + 3*(c^3*d^3*f^
4*g^2 - a*c^2*d^3*f^2*g^4)*x^3 + 3*(c^3*d^3*f^5*g - 3*a*c^2*d^3*f^3*g^3)*x^
2 + (c^3*d^3*f^6 - 9*a*c^2*d^3*f^4*g^2)*x)*e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.17, size = 242, normalized size = 1.22

$$-\frac{\left(\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2-20\,a\,c\,d\,e\,f\,g+30\,c^2\,d^2\,f^2\right)}{15\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^2\,d^2\,x^2\,\sqrt{d+e\,x}}{15\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c\,d\,x\,\left(a\,e\,g-5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{15\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3\,\sqrt{f+g\,x}+\frac{d\,f^2\,\sqrt{f+g\,x}}{e\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(d\,g+2\,e\,f\right)}{e\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,\left(2\,d\,g+e\,f\right)}{e\,g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((((d + e*x)^(1/2)*(6*a^2*e^2*g^2 + 30*c^2*d^2*f^2 - 20*a*c*d*e*f*g))/(15*
e*g^2*(a*e*g - c*d*f)^3) + (16*c^2*d^2*x^2*(d + e*x)^(1/2))/(15*e*(a*e*g -
c*d*f)^3) - (8*c*d*x*(a*e*g - 5*c*d*f)*(d + e*x)^(1/2))/(15*e*g*(a*e*g - c*
d*f)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2
) + (d*f^2*(f + g*x)^(1/2))/(e*g^2) + (x^2*(f + g*x)^(1/2)*(d*g + 2*e*f))/(
e*g) + (f*x*(f + g*x)^(1/2)*(2*d*g + e*f))/(e*g^2))
```

3.719 $\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=267

$$\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}}+\frac{12cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{35(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^{5/2}}+\frac{16c^2d^2\sqrt{ade+(cd^2+ae^2)x+cde}}{35(cdf-aeg)^3\sqrt{d+ex}\,(f+gx)}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(7/2)/(e
*x+d)^(1/2)+12/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f
)^2/(g*x+f)^(5/2)/(e*x+d)^(1/2)+16/35*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)+32/35*c^3*d^3*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(
1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{32c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^4}+\frac{16c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}+\frac{12cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7\sqrt{d+ex}\,(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]),x]
```

```
[Out] (2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*(c*d*f - a*e*g)*Sqrt[d +
 e*x]*(f + g*x)^(7/2)) + (12*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(35*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) + (16*c^2*d^2*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*(c*d*f - a*e*g)^3*Sqrt[d + e*x]
*(f + g*x)^(3/2)) + (32*c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(35*(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
```

```
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{9/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{(6cd)\int \frac{}{(f+gx)^{7/2}\sqrt{}}}{}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+(cd^2}}{35(cdf-aeg)^2\sqrt{d}}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+(cd^2}}{35(cdf-aeg)^2\sqrt{d}}$$

$$= \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{7(cdf-aeg)\sqrt{d+ex}\,(f+gx)^{7/2}} + \frac{12cd\sqrt{ade+(cd^2}}{35(cdf-aeg)^2\sqrt{d}}$$

Mathematica [A]
time = 0.17, size = 152, normalized size = 0.57

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,\left(-5a^3e^3g^3+3a^2cde^2g^2(7f+2gx)-ac^2d^2eg(35f^2+28fgx+8g^2x^2)+c^3d^3(35f^3+70f^2gx+56fg^2x^2+16g^3x^3)\right)}{35(cdf-aeg)^4\sqrt{d+ex}\,(f+gx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/((f + g*x)^(9/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]),x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-5*a^3*e^3*g^3 + 3*a^2*c*d*e^2*g^2*(7*f +
 2*g*x) - a*c^2*d^2*e*g*(35*f^2 + 28*f*g*x + 8*g^2*x^2) + c^3*d^3*(35*f^3 +
 70*f^2*g*x + 56*f*g^2*x^2 + 16*g^3*x^3)))/(35*(c*d*f - a*e*g)^4*Sqrt[d + e
*x]*(f + g*x)^(7/2))
```

Maple [A]
time = 0.14, size = 183, normalized size = 0.69

method	result

default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g}{35\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(-16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2-56c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x+28a\,c^2d^2ef\,g^2x-70c^3d^3f^2gx+5a^3e^3g^3-21a^2cd\,e^2f\,g^2+35a}{35(gx+f)^{\frac{7}{2}}(g^4e^4a^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4)\sqrt{cde\,x^2+a\,e^2x+c\,d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/35/(e*x+d)^(1/2)/(g*x+f)^(7/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-16*c^3*d^3*
g^3*x^3+8*a*c^2*d^2*e*g^3*x^2-56*c^3*d^3*f*g^2*x^2-6*a^2*c*d*e^2*g^3*x+28*a
*c^2*d^2*e*f*g^2*x-70*c^3*d^3*f^2*g*x+5*a^3*e^3*g^3-21*a^2*c*d*e^2*f*g^2+35
*a*c^2*d^2*e*f^2*g-35*c^3*d^3*f^3)/(a*e*g-c*d*f)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x +
 f)^(9/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(247) = 494$.
time = 4.10, size = 1002, normalized size = 3.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*c^3*d^3*g^3*x^3 + 56*c^3*d^3*f*g^2*x^2 + 70*c^3*d^3*f^2*g*x + 35*c
^3*d^3*f^3 - 5*a^3*g^3*e^3 + 3*(2*a^2*c*d*g^3*x + 7*a^2*c*d*f*g^2)*e^2 - (8
*a*c^2*d^2*g^3*x^2 + 28*a*c^2*d^2*f*g^2*x + 35*a*c^2*d^2*f^2*g)*e)*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^4*d^5*f^
4*g^4*x^4 + 4*c^4*d^5*f^5*g^3*x^3 + 6*c^4*d^5*f^6*g^2*x^2 + 4*c^4*d^5*f^7*g
*x + c^4*d^5*f^8 + (a^4*g^8*x^5 + 4*a^4*f*g^7*x^4 + 6*a^4*f^2*g^6*x^3 + 4*a
^4*f^3*g^5*x^2 + a^4*f^4*g^4*x)*e^5 - (4*a^3*c*d*f*g^7*x^5 - a^4*d*f^4*g^4
```

```
+ (16*a^3*c*d*f^2*g^6 - a^4*d*g^8)*x^4 + 4*(6*a^3*c*d*f^3*g^5 - a^4*d*f*g^7
)*x^3 + 2*(8*a^3*c*d*f^4*g^4 - 3*a^4*d*f^2*g^6)*x^2 + 4*(a^3*c*d*f^5*g^3 -
a^4*d*f^3*g^5)*x)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^6*x^5 - 2*a^3*c*d^2*f^5*g^3
+ 2*(6*a^2*c^2*d^2*f^3*g^5 - a^3*c*d^2*f*g^7)*x^4 + 2*(9*a^2*c^2*d^2*f^4*g^
4 - 4*a^3*c*d^2*f^2*g^6)*x^3 + 12*(a^2*c^2*d^2*f^5*g^3 - a^3*c*d^2*f^3*g^5)
*x^2 + (3*a^2*c^2*d^2*f^6*g^2 - 8*a^3*c*d^2*f^4*g^4)*x)*e^3 - 2*(2*a*c^3*d^
3*f^3*g^5*x^5 - 3*a^2*c^2*d^3*f^6*g^2 + (8*a*c^3*d^3*f^4*g^4 - 3*a^2*c^2*d^
3*f^2*g^6)*x^4 + 12*(a*c^3*d^3*f^5*g^3 - a^2*c^2*d^3*f^3*g^5)*x^3 + 2*(4*a*
c^3*d^3*f^6*g^2 - 9*a^2*c^2*d^3*f^4*g^4)*x^2 + 2*(a*c^3*d^3*f^7*g - 6*a^2*c
^2*d^3*f^5*g^3)*x)*e^2 + (c^4*d^4*f^4*g^4*x^5 - 4*a*c^3*d^4*f^7*g + 4*(c^4*
d^4*f^5*g^3 - a*c^3*d^4*f^3*g^5)*x^4 + 2*(3*c^4*d^4*f^6*g^2 - 8*a*c^3*d^4*f
^4*g^4)*x^3 + 4*(c^4*d^4*f^7*g - 6*a*c^3*d^4*f^5*g^3)*x^2 + (c^4*d^4*f^8 -
16*a*c^3*d^4*f^6*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.51, size = 357, normalized size = 1.34

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(10\,a^3\,e^3\,g^3-42\,a^2\,c\,d\,e^2\,f\,g^2+70\,a\,c^2\,d^2\,e\,f^2\,g-70\,c^3\,d^3\,f^3\right)}{35\,e\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{32\,c^3\,d^3\,x^3\,\sqrt{d+e\,x}}{35\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{4\,c\,d\,x\,\sqrt{d+e\,x}\,\left(3\,a^2\,e^2\,g^2-14\,a\,c\,d\,e\,f\,g+35\,c^2\,d^2\,f^2\right)}{35\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{16\,c^2\,d^2\,x^2\,\left(a\,e\,g-7\,c\,d\,f\right)\,\sqrt{d+e\,x}}{35\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^4\,\sqrt{f+g\,x}+\frac{d\,f^3\,\sqrt{f+g\,x}}{e\,g^3}+\frac{x^3\,\sqrt{f+g\,x}\,\left(d\,g+3\,e\,f\right)}{e\,g}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\left(d\,g+e\,f\right)}{e\,g^2}+\frac{f^2\,x\,\sqrt{f+g\,x}\,\left(3\,d\,g+e\,f\right)}{e\,g^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(9/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(10*a^3*e
^3*g^3 - 70*c^3*d^3*f^3 + 70*a*c^2*d^2*e*f^2*g - 42*a^2*c*d*e^2*f*g^2))/(35
*e*g^3*(a*e*g - c*d*f)^4) - (32*c^3*d^3*x^3*(d + e*x)^(1/2))/(35*e*(a*e*g -
 c*d*f)^4) - (4*c*d*x*(d + e*x)^(1/2)*(3*a^2*e^2*g^2 + 35*c^2*d^2*f^2 - 14*
a*c*d*e*f*g))/(35*e*g^2*(a*e*g - c*d*f)^4) + (16*c^2*d^2*x^2*(a*e*g - 7*c*d
*f)*(d + e*x)^(1/2))/(35*e*g*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (d
*f^3*(f + g*x)^(1/2))/(e*g^3) + (x^3*(f + g*x)^(1/2)*(d*g + 3*e*f))/(e*g) +
 (3*f*x^2*(f + g*x)^(1/2)*(d*g + e*f))/(e*g^2) + (f^2*x*(f + g*x)^(1/2)*(3*
d*g + e*f))/(e*g^3))
```

3.720 $\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{15g(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^3d^3\sqrt{d+ex}}+\frac{5g(f+gx}{}$$

```
[Out] -2*(g*x+f)^(5/2)*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
15/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*g^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/2*g*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)+15/4*g*(-a*e*g+c*d*f)*(g*x+f)^(1/2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {880, 884, 905, 65, 223, 212}

$$\frac{15\sqrt{g}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}+\frac{15g\sqrt{f+gx}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)}{4c^3d^3\sqrt{d+ex}}+\frac{5g(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2c^2d^2\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(5/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (15*g*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(4*c^3*d^3*Sqrt[d + e*x]) + (5*g*(f + g*x)^(3/2)*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2*Sqrt[d + e*x]) + (15*Sqrt[g
]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a
*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*c^(7/2)*d^(7/2)*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{(5g)\int \frac{\sqrt{d+ex}\,(f+}{\sqrt{ade+(cd^2+ae^2)}}}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+(cd^2}}{2c^2d^2\sqrt{d+}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}}{4c^3d^3}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}}{4c^3d^3}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}}{4c^3d^3}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}}{4c^3d^3}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{5/2}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{15g(cdf-aeg)\sqrt{f+gx}}{4c^3d^3}$$

Mathematica [A]
time = 0.38, size = 183, normalized size = 0.61

$$\frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{f+gx}\left(-15a^2e^2g^2-5acdeg(-5f+gx)+c^2d^2(-8f^2+9fgx+2g^2x^2)\right)+15\sqrt{g}\,(cdf-aeg)^2\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{4c^{7/2}d^{7/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(-15*a^2*e^2*g^2 - 5*a*c*d*e*
g*(-5*f + g*x) + c^2*d^2*(-8*f^2 + 9*f*g*x + 2*g^2*x^2)) + 15*Sqrt[g]*(c*d*
f - a*e*g)^2*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqr
t[g]*Sqrt[a*e + c*d*x])]))/(4*c^(7/2)*d^(7/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(255) = 510$.

time = 0.15, size = 638, normalized size = 2.12

method	result
default	$\frac{\left(15 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{2\sqrt{dgc}}\right) a^{2} c d e^{2} g^{3} x-30 \ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+a}}{2\sqrt{dgc}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)
^(1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f*g^
2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2))/(d*g*c)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((
g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^3*e^3*g^3-30*ln(1
/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g
*c)^(1/2))*a^2*c*d*e^2*f*g^2+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c
*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f^2*g+4*c^2*d^2*
g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)-10*((g*x+f)*(c*d*x+a*e))^
(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x+18*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1
/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2+
50*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*f*g-16*((g*x+f)*(c*d*x
+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*x+f)
^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(d*g*c)^(1/2)/(c*d*x+a*e)/d^3/c^3/(e*x+d
)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(5/2)*(x*e + d)^(3/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

Fricas [A]

time = 9.02, size = 979, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 + 9*c^2*d^2*f*g*x - 8*c^2*d^2*f^2 - 15*a^2*g^2*
e^2 - 5*(a*c*d*g^2*x - 5*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 15*(c^3*d^4*f^2*x + a^3*g^2*x*e^4 + (
a^2*c*d*g^2*x^2 - 2*a^2*c*d*f*g*x + a^3*d*g^2)*e^3 - (2*a*c^2*d^2*f*g*x^2 +
 2*a^2*c*d^2*f*g - (a*c^2*d^2*f^2 + a^2*c*d^2*g^2)*x)*e^2 + (c^3*d^3*f^2*x^
2 - 2*a*c^2*d^3*f*g*x + a*c^2*d^3*f^2)*e)*sqrt(g/(c*d))*log(-(8*c^2*d^3*g^2
*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*(2*c^2*d^2*g*x + c
^2*d^2*f + a*c*d*g*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x
+ f)*sqrt(x*e + d)*sqrt(g/(c*d)) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d
*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d
^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^4*d^5*x + a*c^3*d^3*x*e^2 + (c
^4*d^4*x^2 + a*c^3*d^4)*e), 1/8*(2*(2*c^2*d^2*g^2*x^2 + 9*c^2*d^2*f*g*x - 8
*c^2*d^2*f^2 - 15*a^2*g^2*e^2 - 5*(a*c*d*g^2*x - 5*a*c*d*f*g)*e)*sqrt(c*d^2
*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^3*d^4
*f^2*x + a^3*g^2*x*e^4 + (a^2*c*d*g^2*x^2 - 2*a^2*c*d*f*g*x + a^3*d*g^2)*e^
3 - (2*a*c^2*d^2*f*g*x^2 + 2*a^2*c*d^2*f*g - (a*c^2*d^2*f^2 + a^2*c*d^2*g^2
)*x)*e^2 + (c^3*d^3*f^2*x^2 - 2*a*c^2*d^3*f*g*x + a*c^2*d^3*f^2)*e)*sqrt(-g
/(c*d))*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*
sqrt(x*e + d)*c*d*sqrt(-g/(c*d))/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*
d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c^4*d^5*x + a*c^3*d^3*x*e^2 + (c^4*d^4*x^2
 + a*c^3*d^4)*e)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(5/2)*(x*e + d)^(3/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{5/2}\,(d+e\,x)^{3/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

3.721 $\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{3g\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{c^2d^2\sqrt{d+ex}}+\frac{3\sqrt{g}\,(cdf-aeg)\sqrt{ae+}}{c^{5/2}d^{5/2}}$$

```
[Out] -2*(g*x+f)^(3/2)*(e*x+d)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+
3*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^
(1/2))*g^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)+3*g*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/c^2/d^2/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {880, 884, 905, 65, 223, 212}

$$\frac{3\sqrt{g}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}+\frac{3g\sqrt{f+gx}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^(3/2))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + (3*g*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(c^2*d^2*Sqrt[d + e*x]) + (3*Sqrt[g]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqr
t[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*
x])])/(c^(5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{(3g)\int \frac{\sqrt{d+ex}\,\sqrt{f}}{\sqrt{ade+(cd^2+ae^2}}}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+(cd^2}}{c^2d^2\sqrt{d+e}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+(cd^2}}{c^2d^2\sqrt{d+e}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+(cd^2}}{c^2d^2\sqrt{d+e}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+(cd^2}}{c^2d^2\sqrt{d+e}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^{3/2}}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{3g\sqrt{f+gx}\,\sqrt{ade+(cd^2}}{c^2d^2\sqrt{d+e}}$$

Mathematica [A]
time = 0.58, size = 133, normalized size = 0.59

$$\frac{\sqrt{d+ex}\left(cd\sqrt{f+gx}\,(-2cdf+3aeg+cdgx)+3\sqrt{\frac{cd}{g}}\,g(-cdf+aeg)\sqrt{ae+cdx}\,\log\left(\sqrt{ae+cdx}-\sqrt{\frac{cd}{g}}\,\sqrt{f+gx}\right)\right)}{c^3d^3\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*(c*d*Sqrt[f + g*x]*(-2*c*d*f + 3*a*e*g + c*d*g*x) + 3*Sqrt[(
c*d)/g]*g*(-(c*d*f) + a*e*g)*Sqrt[a*e + c*d*x]*Log[Sqrt[a*e + c*d*x] - Sqrt
[(c*d)/g]*Sqrt[f + g*x]]))/(c^3*d^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 386, normalized size = 1.70

method	result
default	$-\frac{\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)acde\,g^2x-3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae}}{2\sqrt{dgc}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)
^(1/2))/(d*g*c)^(1/2))*a*c*d*e*g^2*x-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*
x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*f*g*x+3*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*
c)^(1/2))*a^2*e^2*g^2-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e
))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c*d*e*f*g-2*((g*x+f)*(c*d*x+a*e))^
(1/2)*(d*g*c)^(1/2)*c*d*g*x-6*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e
*g+4*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*((c*d*x+a*e)*(e*x+d))
^(1/2)*(g*x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)/(d*g*c)^(1/2)/
c^2/d^2/(e*x+d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)*(x*e + d)^(3/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

Fricas [A]
time = 5.82, size = 727, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*g*x - 2*c*d*f + 3*
a*g*e)*sqrt(g*x + f)*sqrt(x*e + d) - 3*(c^2*d^3*f*x - a^2*g*x*e^3 - (a*c*d*
```

```
g*x^2 - a*c*d*f*x + a^2*d*g)*e^2 + (c^2*d^2*f*x^2 - a*c*d^2*g*x + a*c*d^2*f
)*e)*sqrt(g/(c*d))*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2
+ a^2*g^2*x*e^3 - 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*g*e)*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(g/(c*d)) + (8
*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^
2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e +
d)))/(c^3*d^4*x + a*c^2*d^2*x*e^2 + (c^3*d^3*x^2 + a*c^2*d^3)*e), 1/2*(2*sq
rt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*g*x - 2*c*d*f + 3*a*g*e)*sqr
t(g*x + f)*sqrt(x*e + d) - 3*(c^2*d^3*f*x - a^2*g*x*e^3 - (a*c*d*g*x^2 - a*
c*d*f*x + a^2*d*g)*e^2 + (c^2*d^2*f*x^2 - a*c*d^2*g*x + a*c*d^2*f)*e)*sqrt(
-g/(c*d))*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f
)*sqrt(x*e + d)*c*d*sqrt(-g/(c*d))/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*
c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c^3*d^4*x + a*c^2*d^2*x*e^2 + (c^3*d^3*x
^2 + a*c^2*d^3)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*(x*e + d)^(3/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}\,(d+e\,x)^{3/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

3.722 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{g}\,\sqrt{ae+cdx}\,\sqrt{d+ex}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*g^(1/2)*
(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {880, 905, 65, 223, 212}

$$\frac{2\sqrt{g}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + (2*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[
a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(3/2)*d^(3/2)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{g\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{ade+(cd^2}}}{cd}$$

$$= -\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\left(g\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int}{cd\sqrt{ade+(cd^2+}}$$

$$= -\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\left(2g\sqrt{ae+cdx}\,\sqrt{d+ex}\right)}{c^2d^2\sqrt{}}$$

$$= -\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\left(2g\sqrt{ae+cdx}\,\sqrt{d+ex}\right)}{c^2d^2\sqrt{ade+(}}$$

$$= -\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{2\sqrt{g}\,\sqrt{ae+cdx}\,\sqrt{d+ex}}{c^{3/2}d^{3/2}\sqrt{ade+(c}}$$

Mathematica [A]
time = 0.12, size = 117, normalized size = 0.73

$$-\frac{2\sqrt{d+ex}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}-\sqrt{g}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{ae+cdx}}\right)\right)}{c^{3/2}d^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x] - Sqrt[g]*Sqrt[a*e + c*d*x
]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(c
^(3/2)*d^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 200, normalized size = 1.24

method	result

default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)cdgx+\ln\left(\frac{2cdgx+ae}{}\right.\right.}{\sqrt{dgc}\,(cdx+ae)\sqrt{(gx+f)(cdx+ae}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*
((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c*d*g*x+ln(1/2*(2
*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(
1/2))*a*e*g-2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2)/(c*d
*x+a*e)/((g*x+f)*(c*d*x+a*e))^(1/2)/d/c/(e*x+d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^(3/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2
)*x)^(3/2), x)
```

Fricas [A]
time = 4.83, size = 577, normalized size = 3.58

$$\left[\frac{(cd^2x+axe^2+(cdx^2+ad)e)\sqrt{\frac{g}{cd}}\log\left(-\frac{8c^2d^3g^2x^2+8c^2d^3fgx+c^2d^3f^2+a^2g^2xe^3+4(2c^2d^2gx+c^2d^2f+acdge)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}\,\sqrt{\frac{g}{cd}}+(8acdg^2x^2+6acdfgx+a^2dg^2)e^2+(8c^2d^2g^2x^3+8c^2d^2fgx^2+6acd^2fg+(c^2d^2f^2+8acd^2g^2)x)e}{xe+d}\right)-4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{2(c^2d^3x+acdxe^2+(c^2d^2x^2+acd^2)e)},\ -\frac{(cd^2x+axe^2+(cdx^2+ad)e)\sqrt{-\frac{g}{cd}}\arctan\left(\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}\,cd\sqrt{-\frac{g}{cd}}}{2cd^2gx+cd^2f+agxe^2+(2cdgx^2+cdfx+adg)e}\right)+2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{c^2d^3x+acdxe^2+(c^2d^2x^2+acd^2)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g/(c*d))*log(-(8*c^2*d^3
*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*(2*c^2*d^2*g*x
 + c^2*d^2*f + a*c*d*g*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(
g*x + f)*sqrt(x*e + d)*sqrt(g/(c*d)) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a
^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c
^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)) - 4*sqrt(c*d^2*x + a*x*e^2 + (
c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d))/(c^2*d^3*x + a*c*d*x*e^2 + (
c^2*d^2*x^2 + a*c*d^2)*e), -((c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-
```

```
g/(c*d))*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)
*sqrt(x*e + d)*c*d*sqrt(-g/(c*d))/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c
*d*g*x^2 + c*d*f*x + a*d*g)*e)) + 2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d
)*e)*sqrt(g*x + f)*sqrt(x*e + d))/(c^2*d^3*x + a*c*d*x*e^2 + (c^2*d^2*x^2 +
 a*c*d^2)*e)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*sqrt(f + g*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(3/2), x)
```

3.723 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] -2*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = $\{874\}$

$$-\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{(cdf-aeg)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.82

$$-\frac{2\sqrt{d+ex}\,\sqrt{f+gx}}{(cdf-aeg)\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*Sqrt[f + g*x])/((c*d*f - a*e*g)*Sqrt[(a*e + c*d*x)*(d + e
*x)])
```

Maple [A]
time = 0.14, size = 55, normalized size = 0.90

method	result	size
default	$\frac{2\sqrt{gx+f}\,\sqrt{(cdx+ae)\,(ex+d)}}{\sqrt{ex+d}\,(cdx+ae)(aeg-cdf)}$	55
gosper	$\frac{2\sqrt{gx+f}\,(cdx+ae)(ex+d)^{\frac{3}{2}}}{(aeg-cdf)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 2/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)/(a*e*
g-c*d*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sq
rt(g*x + f)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(58) = 116$.
time = 4.93, size = 122, normalized size = 2.00

$$-\frac{2\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{c^2d^3fx-a^2gxe^3-(acdgx^2-acdfx+a^2dg)e^2+(c^2d^2fx^2-acd^2gx+acd^2f)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/
(c^2*d^3*f*x - a^2*g*x*e^3 - (a*c*d*g*x^2 - a*c*d*f*x + a^2*d*g)*e^2 + (c^2
*d^2*f*x^2 - a*c*d^2*g*x + a*c*d^2*f)*e)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{((d+ex)(ae+cdx))^{\frac{3}{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.68, size = 147, normalized size = 2.41

$$\frac{\left(\frac{2\,f\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)}+\frac{2\,g\,x\,\sqrt{d+e\,x}}{c\,d\,e\,(a\,e\,g-c\,d\,f)}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)}{c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] (((2*f*(d + e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)) + (2*g*x*(d + e*x)^(1/2))/(
c*d*e*(a*e*g - c*d*f)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2
*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1/2)*(a*e^2 + c*d^
2))/(c*d*e))
```

3.724 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}-\frac{4g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(cdf-aeg)^2\sqrt{d+ex}\sqrt{f+gx}}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-4*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(e*
x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {882, 874}

$$-\frac{4g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]) - (4*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c
*d*f - a*e*g)^2*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
```

```
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$=-\frac{2\sqrt{d+ex}}{(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [A]
time = 0.13, size = 64, normalized size = 0.52

$$-\frac{2\sqrt{d+ex}\,(aeg+cd(f+2gx))}{(cdf-aeg)^2\sqrt{(ae+cdx)(d+ex)}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
[Out] (-2*Sqrt[d + e*x]*(a*e*g + c*d*(f + 2*g*x)))/((c*d*f - a*e*g)^2*Sqrt[(a*e +
 c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A]
time = 0.14, size = 70, normalized size = 0.56

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(2cdgx+aeg+cdf)}{\sqrt{ex+d}\,\sqrt{gx+f}\,(cdx+ae)(aeg-cdf)^2}$	70
gosper	$-\frac{2(cdx+ae)(2cdgx+aeg+cdf)(ex+d)^{\frac{3}{2}}}{\sqrt{gx+f}\,(a^2e^2g^2-2acdefg+f^2c^2d^2)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
[Out] -2/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(2*c*d*g*x+a*e*g
+c*d*f)/(c*d*x+a*e)/(a*e*g-c*d*f)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(3/2)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(118) = 236$.
time = 2.06, size = 336, normalized size = 2.71

$$-\frac{2\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,(2\,cdgx + cdf + age)\sqrt{gx + f}\,\sqrt{xe + d}}{c^3d^4f^2gx^2 + c^3d^4f^3x + (a^3g^3x^2 + a^3fg^2x)e^4 + (a^2cdg^3x^3 - a^2cdfg^2x^2 + a^3dfg^2 - (2\,a^2cdf^2g - a^3dg^3)x)e^3 - (2\,ac^2d^2fg^2x^3 + 2\,a^2cd^2f^2g + (ac^2d^2f^2g - a^2cd^2g^3)x^2 - (ac^2d^2f^3 - a^2cd^2fg^2)x)e^2 + (c^3d^3f^2gx^3 - ac^2d^3f^2gx + ac^2d^3f^3 + (c^3d^3f^3 - 2\,ac^2d^3fg^2)x^2)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c*d*f + a*g*e)*
sqrt(g*x + f)*sqrt(x*e + d)/(c^3*d^4*f^2*g*x^2 + c^3*d^4*f^3*x + (a^3*g^3*x
^2 + a^3*f*g^2*x)*e^4 + (a^2*c*d*g^3*x^3 - a^2*c*d*f*g^2*x^2 + a^3*d*f*g^2
- (2*a^2*c*d*f^2*g - a^3*d*g^3)*x)*e^3 - (2*a*c^2*d^2*f*g^2*x^3 + 2*a^2*c*d
^2*f^2*g + (a*c^2*d^2*f^2*g - a^2*c*d^2*g^3)*x^2 - (a*c^2*d^2*f^3 - a^2*c*d
^2*f*g^2)*x)*e^2 + (c^3*d^3*f^2*g*x^3 - a*c^2*d^3*f^2*g*x + a*c^2*d^3*f^3 +
 (c^3*d^3*f^3 - 2*a*c^2*d^3*f*g^2)*x^2)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.98, size = 151, normalized size = 1.22

$$-\frac{\left(\frac{4\,g\,x\,\sqrt{d+e\,x}}{e\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{\left(2\,a\,e\,g+2\,c\,d\,f\right)\,\sqrt{d+e\,x}}{c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}+\frac{a\,\sqrt{f+g\,x}}{c}+\frac{x\,\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)}{c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] -((((4*g*x*(d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^2) + ((2*a*e*g + 2*c*d*f)*(d
+ e*x)^(1/2))/(c*d*e*(a*e*g - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^(1/2))/(x^2*(f + g*x)^(1/2) + (a*(f + g*x)^(1/2))/c + (x*(f + g*x)^(1
/2)*(a*e^2 + c*d^2))/(c*d*e))
```

3.725 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}-\frac{8g\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^{3/2}}-\frac{16cdg\sqrt{ade+\cdots}}{3(cdf-aeg\cdots}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-8/3*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/(
g*x+f)^(3/2)/(e*x+d)^(1/2)-16/3*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {882, 886, 874}

$$-\frac{16cdg\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^3}-\frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (8*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
3*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(3/2)) - (16*c*d*g*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
```

```
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[2*p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

Mathematica [A]
time = 0.15, size = 105, normalized size = 0.55

$$-\frac{2\sqrt{d+ex}\left(-a^2e^2g^2+2acdeg(3f+2gx)+c^2d^2\left(3f^2+12fgx+8g^2x^2\right)\right)}{3(cdf-aeg)^3\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(-(a^2*e^2*g^2) + 2*a*c*d*e*g*(3*f + 2*g*x) + c^2*d^2*(3*
f^2 + 12*f*g*x + 8*g^2*x^2)))/(3*(c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d +
e*x)]*(f + g*x)^(3/2))
```

Maple [A]
time = 0.14, size = 120, normalized size = 0.62

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-8g^2x^2c^2d^2-4acde\,g^2x-12c^2d^2fgx+a^2e^2g^2-6acdefg-3f^2c^2d^2)}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}(cdx+ae)(aeg-cdf)^3}$	120
gosper	$-\frac{2(cdx+ae)(-8g^2x^2c^2d^2-4acde\,g^2x-12c^2d^2fgx+a^2e^2g^2-6acdefg-3f^2c^2d^2)(ex+d)^{\frac{3}{2}}}{3(gx+f)^{\frac{3}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$	168

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-8*c^2*d^2*g^
2*x^2-4*a*c*d*e*g^2*x-12*c^2*d^2*f*g*x+a^2*e^2*g^2-6*a*c*d*e*f*g-3*c^2*d^2*
f^2)/(c*d*x+a*e)/(a*e*g-c*d*f)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g
*x + f)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(179) = 358.
time = 3.23, size = 679, normalized size = 3.54

$$-\frac{2\,(8\,c^2d^2g^2x^2+12\,c^2d^2fgx+3\,c^2d^2f^2-a^2g^2e^2+2\,(2\,acdg^2x+3\,acdfg)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{3\,(c^4d^5f^3g^2x^3+2\,c^4d^5f^4gx^2+c^4d^5f^5x-(a^4g^5x^3+2\,a^4fg^4x^2+a^4f^2g^3x)e^5-(a^3cdg^5x^4-a^3cdfg^4x^3+a^4df^2g^3-(5\,a^3cdf^2g^3-a^4dg^5)x^2-(3\,a^3cdf^3g^2-2\,a^4dfg^4)x)e^4+(3\,a^2c^2d^2fg^4x^4+3\,a^3cd^2f^3g^2+(3\,a^2c^2d^2f^2g^3-a^3cd^2g^5)x^3-(3\,a^2c^2d^2f^3g^2-a^3cd^2fg^4)x^2-(3\,a^2c^2d^2f^4g-5\,a^3cd^2f^2g^3)x)e^3-(3\,ac^3d^3f^2g^3x^4+3\,a^2c^2d^3f^4g+(5\,ac^3d^3f^3g^2-3\,a^2c^2d^3fg^4)x^3+(ac^3d^3f^4g-3\,a^2c^2d^3f^2g^3)x^2-(ac^3d^3f^5-3\,a^2c^2d^3f^3g^2)x)e^2+(c^4d^4f^3g^2x^4-ac^3d^4f^4gx+ac^3d^4f^5+(2\,c^4d^4f^4g-3\,ac^3d^4f^2g^3)x^3+(c^4d^4f^5-5\,ac^3d^4f^3g^2)x^2)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*c^2*d^2*g^2*x^2 + 12*c^2*d^2*f*g*x + 3*c^2*d^2*f^2 - a^2*g^2*e^2 +
2*(2*a*c*d*g^2*x + 3*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^4*d^5*f^3*g^2*x^3 + 2*c^4*d^5*f^4*g*x^2
+ c^4*d^5*f^5*x - (a^4*g^5*x^3 + 2*a^4*f*g^4*x^2 + a^4*f^2*g^3*x)*e^5 - (a^
3*c*d*g^5*x^4 - a^3*c*d*f*g^4*x^3 + a^4*d*f^2*g^3 - (5*a^3*c*d*f^2*g^3 - a^
```

```
4*d*g^5)*x^2 - (3*a^3*c*d*f^3*g^2 - 2*a^4*d*f*g^4)*x)*e^4 + (3*a^2*c^2*d^2*
f*g^4*x^4 + 3*a^3*c*d^2*f^3*g^2 + (3*a^2*c^2*d^2*f^2*g^3 - a^3*c*d^2*g^5)*x
^3 - (3*a^2*c^2*d^2*f^3*g^2 - a^3*c*d^2*f*g^4)*x^2 - (3*a^2*c^2*d^2*f^4*g -
 5*a^3*c*d^2*f^2*g^3)*x)*e^3 - (3*a*c^3*d^3*f^2*g^3*x^4 + 3*a^2*c^2*d^3*f^4
*g + (5*a*c^3*d^3*f^3*g^2 - 3*a^2*c^2*d^3*f*g^4)*x^3 + (a*c^3*d^3*f^4*g - 3
*a^2*c^2*d^3*f^2*g^3)*x^2 - (a*c^3*d^3*f^5 - 3*a^2*c^2*d^3*f^3*g^2)*x)*e^2
+ (c^4*d^4*f^3*g^2*x^4 - a*c^3*d^4*f^4*g*x + a*c^3*d^4*f^5 + (2*c^4*d^4*f^4
*g - 3*a*c^3*d^4*f^2*g^3)*x^3 + (c^4*d^4*f^5 - 5*a*c^3*d^4*f^3*g^2)*x^2)*e)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.33, size = 268, normalized size = 1.40

$$\frac{\left(\frac{8x(aeg+3cdf)\sqrt{d+ex}}{3e(aeg-cdf)^3}+\frac{\sqrt{d+ex}\,(-2a^2e^2g^2+12acdefg+6c^2d^2f^2)}{3cdeg(aeg-cdf)^3}+\frac{16cdgx^2\sqrt{d+ex}}{3e(aeg-cdf)^3}\right)\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}{x^3\sqrt{f+gx}+\frac{af\sqrt{f+gx}}{cg}+\frac{x\sqrt{f+gx}\,(cfd^2+agde+afe^2)}{cdeg}+\frac{x^2\sqrt{f+gx}\,(cgd^2+cfde+age^2)}{cdeg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] (((8*x*(a*e*g + 3*c*d*f)*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3) + ((d + e
*x)^(1/2)*(6*c^2*d^2*f^2 - 2*a^2*e^2*g^2 + 12*a*c*d*e*f*g))/(3*c*d*e*g*(a*e
*g - c*d*f)^3) + (16*c*d*g*x^2*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)^3))*(x
*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3*(f + g*x)^(1/2) + (a*f*(f
 + g*x)^(1/2))/(c*g) + (x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + a*d*e*g))/(c
*d*e*g) + (x^2*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + c*d*e*f))/(c*d*e*g))
```

3.726 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{12g\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^{5/2}}-\frac{16cdg\sqrt{ade+}}{5(cdf-aeg}$$

```
[Out] -2*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)-12/5*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^2/
(g*x+f)^(5/2)/(e*x+d)^(1/2)-16/5*c*d*g*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)-32/5*c^2*d^2*g*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {882, 886, 874}

$$-\frac{32c^2d^2g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}\,(cdf-aeg)^4}-\frac{16cdg\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{3/2}(cdf-aeg)^3}-\frac{12g\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{5\sqrt{d+ex}\,(f+gx)^{5/2}(cdf-aeg)^2}-\frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(3/2)),x]
```

```
[Out] (-2*Sqrt[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2]) - (12*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(5*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(5/2)) - (16*c*d*g*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g
*x)^(3/2)) - (32*c^2*d^2*g*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*
(c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
```

```
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

$$= -\frac{2\sqrt{d+ex}}{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}$$

Mathematica [A]
time = 0.19, size = 139, normalized size = 0.53

$$-\frac{2(ae+cdx)^4(d+ex)^{3/2}\left(g^3-\frac{5cdg^2(f+gx)}{ae+cdx}+\frac{15c^2d^2g(f+gx)^2}{(ae+cdx)^2}+\frac{5c^3d^3(f+gx)^3}{(ae+cdx)^3}\right)}{5(cdf-aeg)^4((ae+cdx)(d+ex))^{3/2}(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^(7/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)),x]
```

[Out] (-2*(a*e + c*d*x)^4*(d + e*x)^(3/2)*(g^3 - (5*c*d*g^2*(f + g*x))/(a*e + c*d *x) + (15*c^2*d^2*g*(f + g*x)^2)/(a*e + c*d*x)^2 + (5*c^3*d^3*(f + g*x)^3)/ (a*e + c*d*x)^3))/(5*(c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x)^(5/2))

Maple [A]
time = 0.14, size = 192, normalized size = 0.73

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2+40c^3d^3f\,g^2x^2-2a^2cd\,e^2g^3x+20a\,c^2d^2ef\,g^2x+30c^3d^3f^2gx+a^3e^3g^3-}{5\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(cdx+ae)(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(16g^3x^3c^3d^3+8a\,c^2d^2e\,g^3x^2+40c^3d^3f\,g^2x^2-2a^2cd\,e^2g^3x+20a\,c^2d^2ef\,g^2x+30c^3d^3f^2gx+a^3e^3g^3-5a^2cd\,e^2f\,g^2+15a\,c^2d}{5(gx+f)^{\frac{5}{2}}(g^4e^4a^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x,m ethod=_RETURNVERBOSE)

[Out] -2/5/(e*x+d)^(1/2)/(g*x+f)^(5/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(16*c^3*d^3*g^ 3*x^3+8*a*c^2*d^2*e*g^3*x^2+40*c^3*d^3*f*g^2*x^2-2*a^2*c*d*e^2*g^3*x+20*a*c ^2*d^2*e*f*g^2*x+30*c^3*d^3*f^2*g*x+a^3*e^3*g^3-5*a^2*c*d*e^2*f*g^2+15*a*c^ 2*d^2*e*f^2*g+5*c^3*d^3*f^3)/(c*d*x+a*e)/(a*e*g-c*d*f)^4

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/ 2),x, algorithm="maxima")

[Out] integrate((x*e + d)^(3/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g *x + f)^(7/2)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(244) = 488.
time = 3.76, size = 1114, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/ 2),x, algorithm="fricas")

```
[Out] -2/5*(16*c^3*d^3*g^3*x^3 + 40*c^3*d^3*f*g^2*x^2 + 30*c^3*d^3*f^2*g*x + 5*c^
3*d^3*f^3 + a^3*g^3*e^3 - (2*a^2*c*d*g^3*x + 5*a^2*c*d*f*g^2)*e^2 + (8*a*c^
2*d^2*g^3*x^2 + 20*a*c^2*d^2*f*g^2*x + 15*a*c^2*d^2*f^2*g)*e)*sqrt(c*d^2*x
+ a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^5*d^6*f^4*g^3
*x^4 + 3*c^5*d^6*f^5*g^2*x^3 + 3*c^5*d^6*f^6*g*x^2 + c^5*d^6*f^7*x + (a^5*g
^7*x^4 + 3*a^5*f*g^6*x^3 + 3*a^5*f^2*g^5*x^2 + a^5*f^3*g^4*x)*e^6 + (a^4*c*
d*g^7*x^5 - a^4*c*d*f*g^6*x^4 + a^5*d*f^3*g^4 - (9*a^4*c*d*f^2*g^5 - a^5*d*
g^7)*x^3 - (11*a^4*c*d*f^3*g^4 - 3*a^5*d*f*g^6)*x^2 - (4*a^4*c*d*f^4*g^3 -
3*a^5*d*f^2*g^5)*x)*e^5 - (4*a^3*c^2*d^2*f*g^6*x^5 + 4*a^4*c*d^2*f^4*g^3 +
(6*a^3*c^2*d^2*f^2*g^5 - a^4*c*d^2*g^7)*x^4 - (6*a^3*c^2*d^2*f^3*g^4 - a^4*
c*d^2*f*g^6)*x^3 - (14*a^3*c^2*d^2*f^4*g^3 - 9*a^4*c*d^2*f^2*g^5)*x^2 - (6*
a^3*c^2*d^2*f^5*g^2 - 11*a^4*c*d^2*f^3*g^4)*x)*e^4 + 2*(3*a^2*c^3*d^3*f^2*g
^5*x^5 + 3*a^3*c^2*d^3*f^5*g^2 + (7*a^2*c^3*d^3*f^3*g^4 - 2*a^3*c^2*d^3*f*g
^6)*x^4 + 3*(a^2*c^3*d^3*f^4*g^3 - a^3*c^2*d^3*f^2*g^5)*x^3 - 3*(a^2*c^3*d^
3*f^5*g^2 - a^3*c^2*d^3*f^3*g^4)*x^2 - (2*a^2*c^3*d^3*f^6*g - 7*a^3*c^2*d^3
*f^4*g^3)*x)*e^3 - (4*a*c^4*d^4*f^3*g^4*x^5 + 4*a^2*c^3*d^4*f^6*g + (11*a*c
^4*d^4*f^4*g^3 - 6*a^2*c^3*d^4*f^2*g^5)*x^4 + (9*a*c^4*d^4*f^5*g^2 - 14*a^2
*c^3*d^4*f^3*g^4)*x^3 + (a*c^4*d^4*f^6*g - 6*a^2*c^3*d^4*f^4*g^3)*x^2 - (a*
c^4*d^4*f^7 - 6*a^2*c^3*d^4*f^5*g^2)*x)*e^2 + (c^5*d^5*f^4*g^3*x^5 - a*c^4*
d^5*f^6*g*x + a*c^4*d^5*f^7 + (3*c^5*d^5*f^5*g^2 - 4*a*c^4*d^5*f^3*g^4)*x^4
 + (3*c^5*d^5*f^6*g - 11*a*c^4*d^5*f^4*g^3)*x^3 + (c^5*d^5*f^7 - 9*a*c^4*d^
5*f^5*g^2)*x^2)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.70, size = 414, normalized size = 1.58

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,x\,\sqrt{d+e\,x}\,\left(-a^2\,e^2\,g^2+10\,a\,c\,d\,e\,f\,g+15\,c^2\,d^2\,f^2\right)}{5\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{\sqrt{d+e\,x}\,\left(\frac{2\,a^3\,e^3\,g^3}{5}-2\,a^2\,c\,d\,e^2\,f\,g^2+6\,a\,c^2\,d^2\,e\,f^2\,g+2\,c^3\,d^3\,f^3\right)}{c\,d\,e\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{32\,c^2\,d^2\,g\,x^3\,\sqrt{d+e\,x}}{5\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{16\,c\,d\,x^2\,\left(a\,e\,g+5\,c\,d\,f\right)\,\sqrt{d+e\,x}}{5\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^4\,\sqrt{f+g\,x}+\frac{a\,f^2\,\sqrt{f+g\,x}}{c\,g^2}+\frac{x^2\,\sqrt{f+g\,x}\,\left(2\,c\,d^2\,f\,g+c\,d\,e\,f^2+a\,d\,e\,g^2+2\,a\,e^2\,f\,g\right)}{c\,d\,e\,g^2}+\frac{x^3\,\sqrt{f+g\,x}\,\left(c\,g\,d^2+2\,c\,f\,d\,e+a\,g\,e^2\right)}{c\,d\,e\,g}+\frac{f\,x\,\sqrt{f+g\,x}\,\left(c\,f\,d^2+2\,a\,g\,d\,e+a\,f\,e^2\right)}{c\,d\,e\,g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(7/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*x*(d + e*x)^(1/2)*(15*c
^2*d^2*f^2 - a^2*e^2*g^2 + 10*a*c*d*e*f*g))/(5*e*g*(a*e*g - c*d*f)^4) + ((d
 + e*x)^(1/2)*((2*a^3*e^3*g^3)/5 + 2*c^3*d^3*f^3 + 6*a*c^2*d^2*e*f^2*g - 2*
a^2*c*d*e^2*f*g^2))/(c*d*e*g^2*(a*e*g - c*d*f)^4) + (32*c^2*d^2*g*x^3*(d +
e*x)^(1/2))/(5*e*(a*e*g - c*d*f)^4) + (16*c*d*x^2*(a*e*g + 5*c*d*f)*(d + e*
x)^(1/2))/(5*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (a*f^2*(f + g*x)
^(1/2))/(c*g^2) + (x^2*(f + g*x)^(1/2)*(a*d*e*g^2 + c*d*e*f^2 + 2*a*e^2*f*g
 + 2*c*d^2*f*g))/(c*d*e*g^2) + (x^3*(f + g*x)^(1/2)*(a*e^2*g + c*d^2*g + 2*
c*d*e*f))/(c*d*e*g) + (f*x*(f + g*x)^(1/2)*(a*e^2*f + c*d^2*f + 2*a*d*e*g))
/(c*d*e*g^2))
```

3.727 $\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=289

$$-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{5g^2\sqrt{f+gx}\,\sqrt{ade+(cd^2}}{c^3d^3\sqrt{d+e}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(5/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)-10/3*g*(g*x+f)^(3/2)*(e*x+d)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)+5*g^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2
)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*g^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {880, 884, 905, 65, 223, 212}

$$\frac{5g^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5g^2\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}}-\frac{10g\sqrt{d+ex}\,(f+gx)^{3/2}}{3c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(5/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (10*g*Sqrt[d + e*x]*(f + g*x)^(3/2))/(3*c^2*d^2*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(c^3*d^3*Sqrt[d + e*x]) + (5*g^(3/2)*(c*d*f -
a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/
(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(7/2)*d^(7/2)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} + \frac{(5g)\int \frac{(d+ex)^{3/2}(f+gx)^{3}}{(ade+(cd^2+ae^2)x+cde}}{3cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+}{3c^2d^2\sqrt{ade+(cd^2+ae}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+}{3c^2d^2\sqrt{ade+(cd^2+ae}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+}{3c^2d^2\sqrt{ade+(cd^2+ae}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+}{3c^2d^2\sqrt{ade+(cd^2+ae}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+}{3c^2d^2\sqrt{ade+(cd^2+ae}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}} - \frac{10g\sqrt{d+ex}\,(f+}{3c^2d^2\sqrt{ade+(cd^2+ae}}$$

Mathematica [A]
time = 1.08, size = 179, normalized size = 0.62

$$\frac{(d+ex)^{3/2}\left(\sqrt{\frac{cd}{g}}\sqrt{f+gx}\,(15a^2e^2g^2-10acdeg(f-2gx)+c^2d^2(-2f^2-14fgx+3g^2x^2))-15g(cdf-aeg)(ae+cdx)^{3/2}\log\left(\sqrt{ae+cdx}-\sqrt{\frac{cd}{g}}\sqrt{f+gx}\right)\right)}{3c^3d^3\sqrt{\frac{cd}{g}}\,((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] ((d + e*x)^(3/2)*(Sqrt[(c*d)/g]*Sqrt[f + g*x]*(15*a^2*e^2*g^2 - 10*a*c*d*e*
g*(f - 2*g*x) + c^2*d^2*(-2*f^2 - 14*f*g*x + 3*g^2*x^2)) - 15*g*(c*d*f - a*
e*g)*(a*e + c*d*x)^(3/2)*Log[Sqrt[a*e + c*d*x] - Sqrt[(c*d)/g]*Sqrt[f + g*x
]]))/(3*c^3*d^3*Sqrt[(c*d)/g]*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(245) = 490$.

time = 0.15, size = 642, normalized size = 2.22

method	result
default	$-\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a\,c^{2}d^{2}e\,g^{3}x^{2}-15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+}}{2\sqrt{dgc}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c
)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*
f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*f*g^2
*x^2+30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)
^(1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f*g^
2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2))/(d*g*c)^(1/2))*a^3*e^3*g^3-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+
f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*f*g^2-6*c^2
*d^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)-40*((g*x+f)*(c*d*x+a
*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x+28*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*
c)^(1/2)*c^2*d^2*f*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2
*g^2+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*f*g+4*((g*x+f)*(c
*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*
x+f)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(c*d*x+a*e)^2/(d*g*c)^(1/2)/c^3/d^3/
(e*x+d)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(5/2)*(x*e + d)^(5/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

Fricas [A]

time = 3.95, size = 1061, normalized size = 3.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 - 14*c^2*d^2*f*g*x - 2*c^2*d^2*f^2 + 15*a^2*g^2
*e^2 + 10*(2*a*c*d*g^2*x - a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
+ a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^3*d^4*f*g*x^2 - a^3*g^2*x*e^4
 - (2*a^2*c*d*g^2*x^2 - a^2*c*d*f*g*x + a^3*d*g^2)*e^3 - (a*c^2*d^2*g^2*x^3
 - 2*a*c^2*d^2*f*g*x^2 + 2*a^2*c*d^2*g^2*x - a^2*c*d^2*f*g)*e^2 + (c^3*d^3*
f*g*x^3 - a*c^2*d^3*g^2*x^2 + 2*a*c^2*d^3*f*g*x)*e)*sqrt(g/(c*d))*log(-(8*c
^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 - 4*(2*c^2*d
^2*g*x + c^2*d^2*f + a*c*d*g*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(g/(c*d)) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g
*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*
g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^5*d^6*x^2 + a^2*c^3*
d^3*x*e^3 + (2*a*c^4*d^4*x^2 + a^2*c^3*d^4)*e^2 + (c^5*d^5*x^3 + 2*a*c^4*d^
5*x)*e), 1/6*(2*(3*c^2*d^2*g^2*x^2 - 14*c^2*d^2*f*g*x - 2*c^2*d^2*f^2 + 15*
a^2*g^2*e^2 + 10*(2*a*c*d*g^2*x - a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^3*d^4*f*g*x^2 - a^3*g^
2*x*e^4 - (2*a^2*c*d*g^2*x^2 - a^2*c*d*f*g*x + a^3*d*g^2)*e^3 - (a*c^2*d^2*
g^2*x^3 - 2*a*c^2*d^2*f*g*x^2 + 2*a^2*c*d^2*g^2*x - a^2*c*d^2*f*g)*e^2 + (c
^3*d^3*f*g*x^3 - a*c^2*d^3*g^2*x^2 + 2*a*c^2*d^3*f*g*x)*e)*sqrt(-g/(c*d))*a
rctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e
+ d)*c*d*sqrt(-g/(c*d))/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 +
 c*d*f*x + a*d*g)*e)))/(c^5*d^6*x^2 + a^2*c^3*d^3*x*e^3 + (2*a*c^4*d^4*x^2
+ a^2*c^3*d^4)*e^2 + (c^5*d^5*x^3 + 2*a*c^4*d^5*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(5/2)*(x*e + d)^(5/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{5/2}\,(d+e\,x)^{5/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2), x)
```

3.728 $\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2g^{3/2}\sqrt{ae+cdx}\,\sqrt{d+ex}\ \mathrm{ta}}{c^{5/2}d^{5/2}\sqrt{ade+(cd}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)+2*g^(3/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2)
)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)-2*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {880, 905, 65, 223, 212}

$$\frac{2g^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2g\sqrt{d+ex}\,\sqrt{f+gx}}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) - (2*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(c^2*d^2*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*g^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(c^(
5/2)*d^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 880

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m - 1)*(f + g*x)^n*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e*g*(n/(c*(p + 1))), Int[(d
 + e*x)^(m - 1)*(f + g*x)^(n - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -
1] && GtQ[n, 0]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} + \frac{g\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{(ade+(cd^2+ae^2)x+cdex^2)^{3}}}{cd}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{2g\sqrt{d+ex}\,\sqrt{f-}}{c^2d^2\sqrt{ade+(cd^2+ae^2)}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{2g\sqrt{d+ex}\,\sqrt{f-}}{c^2d^2\sqrt{ade+(cd^2+ae^2)}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{2g\sqrt{d+ex}\,\sqrt{f-}}{c^2d^2\sqrt{ade+(cd^2+ae^2)}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{2g\sqrt{d+ex}\,\sqrt{f-}}{c^2d^2\sqrt{ade+(cd^2+ae^2)}}$$

$$= -\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}} - \frac{2g\sqrt{d+ex}\,\sqrt{f-}}{c^2d^2\sqrt{ade+(cd^2+ae^2)}}$$

Mathematica [A]
time = 0.21, size = 134, normalized size = 0.61

$$-\frac{2(d+ex)^{3/2}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}\,(3aeg+cd(f+4gx))-3g^{3/2}(ae+cdx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)\right)}{3c^{5/2}d^{5/2}((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x]*(3*a*e*g + c*d*(f + 4*g*
x)) - 3*g^(3/2)*(a*e + c*d*x)^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sq
rt[c]*Sqrt[d]*Sqrt[f + g*x])]))/(3*c^(5/2)*d^(5/2)*((a*e + c*d*x)*(d + e*x)
)^(3/2))
```

Maple [A]
time = 0.15, size = 333, normalized size = 1.52

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)c^2d^2g^2x^2+6\ln\left(\frac{2cd}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/3*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*
d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*g^2
*x^2+6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2))/(d*g*c)^(1/2))*a*c*d*e*g^2*x+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x
+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*e^2*g^2-8*((g*x+f)
*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c*d*g*x-6*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a
*e))^(1/2)*a*e*g-2*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(d*g*c)
^(1/2)/(c*d*x+a*e)^2/((g*x+f)*(c*d*x+a*e))^(1/2)/d^2/c^2/(e*x+d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)*(x*e + d)^(5/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

Fricas [A]
time = 4.55, size = 765, normalized size = 3.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(4*c*d*g*x + c*d*f + 3
*a*g*e)*sqrt(g*x + f)*sqrt(x*e + d) - 3*(c^2*d^3*g*x^2 + a^2*g*x*e^3 + (2*a
*c*d*g*x^2 + a^2*d*g)*e^2 + (c^2*d^2*g*x^3 + 2*a*c*d^2*g*x)*e)*sqrt(g/(c*d)
)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 +
```

```
 4*(2*c^2*d^2*g*x + c^2*d^2*f + a*c*d*g*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^
2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(g/(c*d)) + (8*a*c*d*g^2*x^2 +
6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6
*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^4*d^5*x^2
 + a^2*c^2*d^2*x*e^3 + (2*a*c^3*d^3*x^2 + a^2*c^2*d^3)*e^2 + (c^4*d^4*x^3 +
 2*a*c^3*d^4*x)*e), -1/3*(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(4*
c*d*g*x + c*d*f + 3*a*g*e)*sqrt(g*x + f)*sqrt(x*e + d) + 3*(c^2*d^3*g*x^2 +
 a^2*g*x*e^3 + (2*a*c*d*g*x^2 + a^2*d*g)*e^2 + (c^2*d^2*g*x^3 + 2*a*c*d^2*g
*x)*e)*sqrt(-g/(c*d))*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*
sqrt(g*x + f)*sqrt(x*e + d)*c*d*sqrt(-g/(c*d))/(2*c*d^2*g*x + c*d^2*f + a*g
*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c^4*d^5*x^2 + a^2*c^2*d^2*x*
e^3 + (2*a*c^3*d^3*x^2 + a^2*c^2*d^3)*e^2 + (c^4*d^4*x^3 + 2*a*c^3*d^4*x)*e
)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*(x*e + d)^(5/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}\,(d+e\,x)^{5/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2), x)
```

3.729 $\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {874}

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}\sqrt{f+gx}}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.83

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(cdf-aeg)((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2))/(3*(c*d*f - a*e*g)*((a*e + c*d*x)*(d +
 e*x))^(3/2))
```

Maple [A]
time = 0.15, size = 55, normalized size = 0.87

method	result	size
default	$\frac{2(gx+f)^{\frac{3}{2}}\sqrt{(cdx+ae)(ex+d)}}{3\sqrt{ex+d}\,(cdx+ae)^{2}(aeg-cdf)}$	55
gosper	$\frac{2(gx+f)^{\frac{3}{2}}(cdx+ae)(ex+d)^{\frac{5}{2}}}{3(aeg-cdf)(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 2/3/(e*x+d)^(1/2)*(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/(c*d*x+a*e)^2/(
a*e*g-c*d*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^(5/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2
)*x)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(58) = 116.
time = 4.63, size = 188, normalized size = 2.98

$$-\frac{2\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,(gx+f)^{\frac{3}{2}}\sqrt{xe+d}}{3\,(c^{3}d^{4}fx^{2}-a^{3}gxe^{4}-(2\,a^{2}cdgx^{2}-a^{2}cdfx+a^{3}dg)e^{3}-(ac^{2}d^{2}gx^{3}-2\,ac^{2}d^{2}fx^{2}+2\,a^{2}cd^{2}gx-a^{2}cd^{2}f)e^{2}+(c^{3}d^{3}fx^{3}-ac^{2}d^{3}gx^{2}+2\,ac^{2}d^{3}fx)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(g*x + f)^(3/2)*sqrt(x*e +
 d)/(c^3*d^4*f*x^2 - a^3*g*x*e^4 - (2*a^2*c*d*g*x^2 - a^2*c*d*f*x + a^3*d*g
)*e^3 - (a*c^2*d^2*g*x^3 - 2*a*c^2*d^2*f*x^2 + 2*a^2*c*d^2*g*x - a^2*c*d^2*
f)*e^2 + (c^3*d^3*f*x^3 - a*c^2*d^3*g*x^2 + 2*a*c^2*d^3*f*x)*e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^(5/2)/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2
)*x)^(5/2), x)
```

Mupad [B]
time = 4.32, size = 169, normalized size = 2.68

$$\frac{\left(\frac{2\,f\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,\left(a\,e\,g-c\,d\,f\right)}+\frac{2\,g\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,\left(a\,e\,g-c\,d\,f\right)}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^3+\frac{a^2\,e}{c^2\,d}+\frac{a\,x\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}+\frac{x^2\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(5/2),x)
```

```
[Out] (((2*f*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)) + (2*
g*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/(3*c^2*d^2*e*(a*e*g - c*d*f)))*(x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^3 + (a^2*e)/(c^2*d) + (a*x*(a*e^
2 + 2*c*d^2))/(c^2*d^2) + (x^2*(2*a*e^2 + c*d^2))/(c*d*e))
```

3.730 $\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{4g\sqrt{d+ex}\,\sqrt{f+gx}}{3(cdf-aeg)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

```
[Out] -2/3*(e*x+d)^(3/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+4/3*g*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(-a*e*g+c*d*f)^2/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {882, 874}

$$\frac{4g\sqrt{d+ex}\,\sqrt{f+gx}}{3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*Sqrt[f + g*x])/(3*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x]*Sqrt[f + g*x])/(3*(c*d*f - a
*e*g)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
```

```
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{(2g)}{}$$
$$= -\frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(cdf-aeg)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{}{3(cdf}$$

Mathematica [A]
time = 0.12, size = 68, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}\sqrt{f+gx}\,(3aeg-cd(f-2gx))}{3(cdf-aeg)^2((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2)),x]
[Out] (2*(d + e*x)^(3/2)*Sqrt[f + g*x]*(3*a*e*g - c*d*(f - 2*g*x)))/(3*(c*d*f - a
*e*g)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Maple [A]
time = 0.14, size = 72, normalized size = 0.56

method	result	size
default	$\frac{2\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,(2cdgx+3aeg-cdf)}{3\sqrt{ex+d}\,(cdx+ae)^2(aeg-cdf)^2}$	72
gosper	$\frac{2\sqrt{gx+f}\,(cdx+ae)(2cdgx+3aeg-cdf)(ex+d)^{\frac{5}{2}}}{3(a^2e^2g^2-2acdefg+f^2c^2d^2)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
[Out] 2/3/(e*x+d)^(1/2)*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(2*c*d*g*x+3*a*
e*g-c*d*f)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(5/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sq
rt(g*x + f)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(118) = 236$.
time = 8.18, size = 320, normalized size = 2.50

$$\frac{2\sqrt{cd^2x + axe^2 + (cdx^2 + ad)e}\,(2\,cdgx - cdf + 3\,age)\sqrt{gx + f}\,\sqrt{xe + d}}{3\,(c^4d^5f^2x^2 + a^4g^2xe^5 + (2\,a^3cdg^2x^2 - 2\,a^3cdfgx + a^4dg^2)e^4 + (a^2c^2d^2g^2x^3 - 4\,a^2c^2d^2fgx^2 - 2\,a^3cd^2fg + (a^2c^2d^2f^2 + 2\,a^3cd^2g^2)x)e^3 - (2\,ac^3d^3fgx^3 + 4\,a^2c^2d^3fgx - a^2c^2d^3f^2 - (2\,ac^3d^3f^2 + a^2c^2d^3g^2)x^2)e^2 + (c^4d^4f^2x^3 - 2\,ac^3d^4fgx^2 + 2\,ac^3d^4f^2x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x - c*d*f + 3*a*g*
e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^4*d^5*f^2*x^2 + a^4*g^2*x*e^5 + (2*a^3*c*
d*g^2*x^2 - 2*a^3*c*d*f*g*x + a^4*d*g^2)*e^4 + (a^2*c^2*d^2*g^2*x^3 - 4*a^2
*c^2*d^2*f*g*x^2 - 2*a^3*c*d^2*f*g + (a^2*c^2*d^2*f^2 + 2*a^3*c*d^2*g^2)*x)
*e^3 - (2*a*c^3*d^3*f*g*x^3 + 4*a^2*c^2*d^3*f*g*x - a^2*c^2*d^3*f^2 - (2*a*
c^3*d^3*f^2 + a^2*c^2*d^3*g^2)*x^2)*e^2 + (c^4*d^4*f^2*x^3 - 2*a*c^3*d^4*f*
g*x^2 + 2*a*c^3*d^4*f^2*x)*e)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.06, size = 246, normalized size = 1.92

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{4\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{\left(2\,c\,d\,f^2-6\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{x\,\left(6\,a\,e\,g^2+2\,c\,d\,f\,g\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((4*g^2*x^2*(d + e*x)^(1/2))
/(3*c*d*e*(a*e*g - c*d*f)^2) - ((2*c*d*f^2 - 6*a*e*f*g)*(d + e*x)^(1/2))/(3
*c^2*d^2*e*(a*e*g - c*d*f)^2) + (x*(6*a*e*g^2 + 2*c*d*f*g)*(d + e*x)^(1/2))
/(3*c^2*d^2*e*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2) + (a^2*e*(f + g*x)^
(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(c*d*e) + (a*x*(f
+ g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

3.731 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=194

$$-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{8g\sqrt{d+ex}}{3(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(
g*x+f)^(1/2)+8/3*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(1/2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)/(-a*e*g+c*d*f)^3/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {882, 874}

$$\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\,\sqrt{f+gx}\,(cdf-aeg)^3}+\frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}\,\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2)) + (8*g*Sqrt[d + e*x])/(3*(c*d*f - a*e*g)^2*Sqrt[f
 + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*Sqrt[f
+ g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
```

```
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}$$

Mathematica [A]
time = 0.16, size = 102, normalized size = 0.53

$$-\frac{2(d+ex)^{3/2}(f+gx)^{3/2}\left(c^2d^2-\frac{3g^2(ae+cdx)^2}{(f+gx)^2}-\frac{6cdg(ae+cdx)}{f+gx}\right)}{3(cdf-aeg)^3((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^(3/2)*(c^2*d^2 - (3*g^2*(a*e + c*d*x)^2)/(f +
 g*x)^2 - (6*c*d*g*(a*e + c*d*x))/(f + g*x)))/(3*(c*d*f - a*e*g)^3*((a*e +
c*d*x)*(d + e*x))^(3/2))
```

Maple [A]
time = 0.14, size = 121, normalized size = 0.62

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(8g^2x^2c^2d^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-f^2c^2d^2)}{3\sqrt{ex+d}\,\sqrt{gx+f}\,(cdx+ae)^2(aeg-cdf)^3}$	121
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2+12acde\,g^2x+4c^2d^2fgx+3a^2e^2g^2+6acdefg-f^2c^2d^2)(ex+d)^{\frac{5}{2}}}{3\sqrt{gx+f}\,(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}$	169

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)/(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(8*c^2*d^2*g^2
*x^2+12*a*c*d*e*g^2*x+4*c^2*d^2*f*g*x+3*a^2*e^2*g^2+6*a*c*d*e*f*g-c^2*d^2*f
^2)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(5/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(179) = 358$.
time = 6.90, size = 690, normalized size = 3.56

$$\frac{2\,(8\,c^2d^2g^2x^2+4\,c^2d^2fgx-c^2d^2f^2+3\,a^2g^2e^2+6\,(2\,acdg^2x+acdfg)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{3\,(c^5d^6f^3gx^3+c^5d^6f^4x^2-(a^5g^4x^2+a^5fg^3x)e^6-(2\,a^4cdg^4x^3-a^4cdfg^3x^2+a^5dfg^3-(3\,a^4cdf^2g^2-a^5dg^4)x)e^5-(a^3c^2d^2g^4x^4-5\,a^3c^2d^2fg^3x^3-3\,a^4cd^2f^2g^2-(3\,a^3c^2d^2f^2g^2-2\,a^4cd^2g^4)x^2+(3\,a^3c^2d^2f^3g-a^4cd^2fg^3)x)e^4+(3\,a^2c^3d^3fg^3x^4-3\,a^3c^2d^3f^3g-(3\,a^2c^3d^3f^2g^2+a^3c^2d^3g^4)x^3-5\,(a^2c^3d^3f^3g-a^3c^2d^3fg^3)x^2+(a^2c^3d^3f^4+3\,a^3c^2d^3f^2g^2)x)e^3-(3\,ac^4d^4f^2g^2x^4+5\,a^2c^3d^4f^3gx-a^2c^3d^4f^4+(ac^4d^4f^3g-3\,a^2c^3d^4fg^3)x^3-(2\,ac^4d^4f^4-3\,a^2c^3d^4f^2g^2)x^2)e^2+(c^5d^5f^3gx^4-ac^4d^5f^3gx^2+2\,ac^4d^5f^4x+(c^5d^5f^4-3\,ac^4d^5f^2g^2)x^3)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*c^2*d^2*g^2*x^2 + 4*c^2*d^2*f*g*x - c^2*d^2*f^2 + 3*a^2*g^2*e^2 + 6*
(2*a*c*d*g^2*x + a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*
sqrt(g*x + f)*sqrt(x*e + d)/(c^5*d^6*f^3*g*x^3 + c^5*d^6*f^4*x^2 - (a^5*g^4
*x^2 + a^5*f*g^3*x)*e^6 - (2*a^4*c*d*g^4*x^3 - a^4*c*d*f*g^3*x^2 + a^5*d*f*
g^3 - (3*a^4*c*d*f^2*g^2 - a^5*d*g^4)*x)*e^5 - (a^3*c^2*d^2*g^4*x^4 - 5*a^3
*c^2*d^2*f*g^3*x^3 - 3*a^4*c*d^2*f^2*g^2 - (3*a^3*c^2*d^2*f^2*g^2 - 2*a^4*c
*d^2*g^4)*x^2 + (3*a^3*c^2*d^2*f^3*g - a^4*c*d^2*f*g^3)*x)*e^4 + (3*a^2*c^3
*d^3*f*g^3*x^4 - 3*a^3*c^2*d^3*f^3*g - (3*a^2*c^3*d^3*f^2*g^2 + a^3*c^2*d^3
*g^4)*x^3 - 5*(a^2*c^3*d^3*f^3*g - a^3*c^2*d^3*f*g^3)*x^2 + (a^2*c^3*d^3*f^
4 + 3*a^3*c^2*d^3*f^2*g^2)*x)*e^3 - (3*a*c^4*d^4*f^2*g^2*x^4 + 5*a^2*c^3*d^
4*f^3*g*x - a^2*c^3*d^4*f^4 + (a*c^4*d^4*f^3*g - 3*a^2*c^3*d^4*f*g^3)*x^3 -
 (2*a*c^4*d^4*f^4 - 3*a^2*c^3*d^4*f^2*g^2)*x^2)*e^2 + (c^5*d^5*f^3*g*x^4 -
a*c^4*d^5*f^3*g*x^2 + 2*a*c^4*d^5*f^4*x + (c^5*d^5*f^4 - 3*a*c^4*d^5*f^2*g^
2)*x^3)*e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.28, size = 255, normalized size = 1.31

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g^2\,x^2\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{\sqrt{d+e\,x}\,\left(6\,a^2\,e^2\,g^2+12\,a\,c\,d\,e\,f\,g-2\,c^2\,d^2\,f^2\right)}{3\,c^2\,d^2\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{8\,g\,x\,\left(3\,a\,e\,g+c\,d\,f\right)\,\sqrt{d+e\,x}}{3\,c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^3\,\sqrt{f+g\,x}+\frac{a^2\,e\,\sqrt{f+g\,x}}{c^2\,d}+\frac{x^2\,\sqrt{f+g\,x}\,\left(c\,d^2+2\,a\,e^2\right)}{c\,d\,e}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,d^2+a\,e^2\right)}{c^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g^2*x^2*(d + e*x)^(1/2
))/(3*e*(a*e*g - c*d*f)^3) + ((d + e*x)^(1/2)*(6*a^2*e^2*g^2 - 2*c^2*d^2*f^
2 + 12*a*c*d*e*f*g))/(3*c^2*d^2*e*(a*e*g - c*d*f)^3) + (8*g*x*(3*a*e*g + c*
d*f)*(d + e*x)^(1/2))/(3*c*d*e*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2) +
(a^2*e*(f + g*x)^(1/2))/(c^2*d) + (x^2*(f + g*x)^(1/2)*(2*a*e^2 + c*d^2))/(
c*d*e) + (a*x*(f + g*x)^(1/2)*(a*e^2 + 2*c*d^2))/(c^2*d^2))
```

3.732 $\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{4g\sqrt{d+ex}}{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x}}$$

```
[Out] -2/3*(e*x+d)^(3/2)/(-a*e*g+c*d*f)/(g*x+f)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)+4*g*(e*x+d)^(1/2)/(-a*e*g+c*d*f)^2/(g*x+f)^(3/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+16/3*g^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/(-a*e*g+c*d*f)^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)+32/3*c*d*g^2*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {882, 886, 874}

$$\frac{32cdg^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4}+\frac{16g^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(cdf-aeg)^2}-\frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^(5/2)),x]
```

```
[Out] (-2*(d + e*x)^(3/2))/(3*(c*d*f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2)) + (4*g*Sqrt[d + e*x])/((c*d*f - a*e*g)^2*(f + g
*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*g^2*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f +
g*x)^(3/2)) + (32*c*d*g^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(
c*d*f - a*e*g)^4*Sqrt[d + e*x]*Sqrt[f + g*x])
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 882

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(c*e*f + c*d*g - b*e*g))), x] + Di
```

```
st[e^2*g*((m - n - 2)/((p + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m
- 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g, n}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*
d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && Rational
Q[n]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}$$

$$=-\frac{2(d+ex)^{3/2}}{3(cdf-aeg)(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right.}$$

Mathematica [**A**]
time = 0.20, size = 138, normalized size = 0.53

$$-\frac{2(ae+cdx)^4(d+ex)^{5/2}\left(g^3-\frac{9cdg^2(f+gx)}{ae+cdx}-\frac{9c^2d^2g(f+gx)^2}{(ae+cdx)^2}+\frac{c^3d^3(f+gx)^3}{(ae+cdx)^3}\right)}{3(cdf-aeg)^4((ae+cdx)(d+ex))^{5/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(5/2)/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2)),x]
```

```
[Out] (-2*(a*e + c*d*x)^4*(d + e*x)^(5/2)*(g^3 - (9*c*d*g^2*(f + g*x))/(a*e + c*d
*x) - (9*c^2*d^2*g*(f + g*x)^2)/(a*e + c*d*x)^2 + (c^3*d^3*(f + g*x)^3)/(a*
e + c*d*x)^3))/(3*(c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x))^(5/2)*(f + g*
x)^(3/2))
```

Maple [A]
time = 0.13, size = 191, normalized size = 0.73

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-16g^3x^3c^3d^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3}{3\sqrt{ex+d}\,(gx+f)^{\frac{3}{2}}(cdx+ae)^2(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(-16g^3x^3c^3d^3-24a\,c^2d^2e\,g^3x^2-24c^3d^3f\,g^2x^2-6a^2cd\,e^2g^3x-36a\,c^2d^2ef\,g^2x-6c^3d^3f^2gx+a^3e^3g^3-9a^2cd\,e^2f\,g^2-9a}{3(gx+f)^{\frac{3}{2}}(g^4e^4a^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4)(cde\,x^2+a\,e^2x+c\,d^2x+ade)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)/(g*x+f)^(3/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-16*c^3*d^3*g
^3*x^3-24*a*c^2*d^2*e*g^3*x^2-24*c^3*d^3*f*g^2*x^2-6*a^2*c*d*e^2*g^3*x-36*a
*c^2*d^2*e*f*g^2*x-6*c^3*d^3*f^2*g*x+a^3*e^3*g^3-9*a^2*c*d*e^2*f*g^2-9*a*c^
2*d^2*e*f^2*g+c^3*d^3*f^3)/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(5/2)/((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g
*x + f)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. 2(242) = 484.
time = 6.49, size = 1108, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*c^3*d^3*g^3*x^3 + 24*c^3*d^3*f*g^2*x^2 + 6*c^3*d^3*f^2*g*x - c^3*d^
3*f^3 - a^3*g^3*e^3 + 3*(2*a^2*c*d*g^3*x + 3*a^2*c*d*f*g^2)*e^2 + 3*(8*a*c^
2*d^2*g^3*x^2 + 12*a*c^2*d^2*f*g^2*x + 3*a*c^2*d^2*f^2*g)*e)*sqrt(c*d^2*x +
 a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^6*d^7*f^4*g^2*
x^4 + 2*c^6*d^7*f^5*g*x^3 + c^6*d^7*f^6*x^2 + (a^6*g^6*x^3 + 2*a^6*f*g^5*x^
2 + a^6*f^2*g^4*x)*e^7 + (2*a^5*c*d*g^6*x^4 + a^6*d*f^2*g^4 - (6*a^5*c*d*f^
2*g^4 - a^6*d*g^6)*x^2 - 2*(2*a^5*c*d*f^3*g^3 - a^6*d*f*g^5)*x)*e^6 + (a^4*
c^2*d^2*g^6*x^5 - 6*a^4*c^2*d^2*f*g^5*x^4 + 4*a^4*c^2*d^2*f^3*g^3*x^2 - 4*a
^5*c*d^2*f^3*g^3 - (9*a^4*c^2*d^2*f^2*g^4 - 2*a^5*c*d^2*g^6)*x^3 + 6*(a^4*c
^2*d^2*f^4*g^2 - a^5*c*d^2*f^2*g^4)*x)*e^5 - (4*a^3*c^3*d^3*f*g^5*x^5 - 6*a
^4*c^2*d^3*f^4*g^2 - (4*a^3*c^3*d^3*f^2*g^4 + a^4*c^2*d^3*g^6)*x^4 - 2*(8*a
^3*c^3*d^3*f^3*g^3 - 3*a^4*c^2*d^3*f*g^5)*x^3 - (4*a^3*c^3*d^3*f^4*g^2 - 9*
a^4*c^2*d^3*f^2*g^4)*x^2 + 4*(a^3*c^3*d^3*f^5*g - a^4*c^2*d^3*f^3*g^3)*x)*e
^4 + (6*a^2*c^4*d^4*f^2*g^4*x^5 - 4*a^3*c^3*d^4*f^5*g + 4*(a^2*c^4*d^4*f^3*
g^3 - a^3*c^3*d^4*f*g^5)*x^4 - (9*a^2*c^4*d^4*f^4*g^2 - 4*a^3*c^3*d^4*f^2*g
^4)*x^3 - 2*(3*a^2*c^4*d^4*f^5*g - 8*a^3*c^3*d^4*f^3*g^3)*x^2 + (a^2*c^4*d^
4*f^6 + 4*a^3*c^3*d^4*f^4*g^2)*x)*e^3 - (4*a*c^5*d^5*f^3*g^3*x^5 - 4*a^2*c^
4*d^5*f^3*g^3*x^3 + 6*a^2*c^4*d^5*f^5*g*x - a^2*c^4*d^5*f^6 + 6*(a*c^5*d^5*
f^4*g^2 - a^2*c^4*d^5*f^2*g^4)*x^4 - (2*a*c^5*d^5*f^6 - 9*a^2*c^4*d^5*f^4*g
^2)*x^2)*e^2 + (c^6*d^6*f^4*g^2*x^5 + 2*a*c^5*d^6*f^6*x + 2*(c^6*d^6*f^5*g
- 2*a*c^5*d^6*f^3*g^3)*x^4 + (c^6*d^6*f^6 - 6*a*c^5*d^6*f^4*g^2)*x^3)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)/(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.86, size = 416, normalized size = 1.60

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{16\,g\,x^2\,\left(a\,e\,g+c\,d\,f\right)\,\sqrt{d+e\,x}}{e\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{\sqrt{d+e\,x}\,\left(2\,a^3\,e^3\,g^3-18\,a^2\,c\,d\,e^2\,f\,g^2-18\,a\,c^2\,d^2\,e\,f^2\,g+2\,c^3\,d^3\,f^3\right)}{3\,c^2\,d^2\,e\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{32\,c\,d\,g^2\,x^3\,\sqrt{d+e\,x}}{3\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{4\,x\,\sqrt{d+e\,x}\,\left(a^2\,e^2\,g^2+6\,a\,c\,d\,e\,f\,g+c^2\,d^2\,f^2\right)}{c\,d\,e\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^4\,\sqrt{f+g\,x}+\frac{x^2\,\sqrt{f+g\,x}\,\left(g\,a^2\,e^3+2\,g\,a\,c\,d^2\,e+2\,f\,a\,c\,d\,e^2+f\,c^2\,d^3\right)}{c^2\,d^2\,e\,g}+\frac{a\,x\,\sqrt{f+g\,x}\,\left(2\,c\,f\,d^2+a\,g\,d\,e+a\,f\,e^2\right)}{c^2\,d^2\,g}+\frac{a^2\,e\,f\,\sqrt{f+g\,x}}{c^2\,d\,g}+\frac{x^3\,\sqrt{f+g\,x}\,\left(c\,g\,d^2+c\,f\,d\,e+2\,a\,g\,e^2\right)}{c\,d\,e\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(5/2)/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(5/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((16*g*x^2*(a*e*g + c*d*f)*(
d + e*x)^(1/2))/(e*(a*e*g - c*d*f)^4) - ((d + e*x)^(1/2)*(2*a^3*e^3*g^3 + 2
*c^3*d^3*f^3 - 18*a*c^2*d^2*e*f^2*g - 18*a^2*c*d*e^2*f*g^2))/(3*c^2*d^2*e*g
*(a*e*g - c*d*f)^4) + (32*c*d*g^2*x^3*(d + e*x)^(1/2))/(3*e*(a*e*g - c*d*f)
^4) + (4*x*(d + e*x)^(1/2)*(a^2*e^2*g^2 + c^2*d^2*f^2 + 6*a*c*d*e*f*g))/(c*
d*e*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2) + (x^2*(f + g*x)^(1/2)*(a^2*e
^3*g + c^2*d^3*f + 2*a*c*d*e^2*f + 2*a*c*d^2*e*g))/(c^2*d^2*e*g) + (a*x*(f
+ g*x)^(1/2)*(a*e^2*f + 2*c*d^2*f + a*d*e*g))/(c^2*d^2*g) + (a^2*e*f*(f + g
*x)^(1/2))/(c^2*d*g) + (x^3*(f + g*x)^(1/2)*(2*a*e^2*g + c*d^2*g + c*d*e*f)
)/(c*d*e*g))
```

3.733 $\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=385

$$-\frac{5(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64c^3d^3g\sqrt{d+ex}}-\frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+}}{96c^2d^2g\sqrt{d+ex}}$$

```
[Out] -5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(7/2)/d^(7/2)/g^(3/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/96*(-a*e*g+c*d*f)^2*(g*x+f)^(3/2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)+1/24*(a*e/c/d-f/
g)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)+1/4*
(g*x+f)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-5/64*
(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/
d^3/g/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$-\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}}-\frac{5(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^2}{96c^2d^2g\sqrt{d+ex}}+\frac{(f+gx)^{7/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{24\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c^3*d^3*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*c^2*d^2*g*Sqrt[d + e*x]) +
(((a*e)/(c*d) - f/g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(24*Sqrt[d + e*x]) + ((f + g*x)^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(4*g*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}dx = \frac{(f+gx)^{7/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g\sqrt{d+ex}} - \frac{(cdf-aeg}{}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24\sqrt{d+ex}} +$$

$$= -\frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex}}{96c^2d^2g\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64c^3d^3g\sqrt{d+ex}}$$

<u>Mathematica</u> [A]
time = 0.48, size = 227, normalized size = 0.59

$$\frac{(cdf-aeg)^4\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}(f+gx)^{7/2}\left(15c^3d^3+\frac{15g^3(ae+cdx)^3}{(f+gx)^3}-\frac{55cdg^2(ae+cdx)^2}{(f+gx)^2}+\frac{73c^2d^2g(ae+cdx)}{f+gx}\right)}{(cdf-aeg)^4}-\frac{15\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{ae+cdx}}\right)}{192c^{7/2}d^{7/2}g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] ((c*d*f - a*e*g)^4*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[c]*Sqrt[d]*Sqrt[g]*
(f + g*x)^(7/2)*(15*c^3*d^3 + (15*g^3*(a*e + c*d*x)^3)/(f + g*x)^3 - (55*c*
d*g^2*(a*e + c*d*x)^2)/(f + g*x)^2 + (73*c^2*d^2*g*(a*e + c*d*x))/(f + g*x)
))/(c*d*f - a*e*g)^4 - (15*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqr
t[d]*Sqrt[f + g*x])])/Sqrt[a*e + c*d*x]))/(192*c^(7/2)*d^(7/2)*g^(3/2)*Sqrt
[d + e*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(329) = 658$.

time = 0.16, size = 732, normalized size = 1.90

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(-96c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}+15\ln\left(\frac{2cdgx+aeg+cdf+2\cdots}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/384*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-96*c^3*d^3*g^3*x^3*((g*x
+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^4*e^4*g^4-60*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*
c)^(1/2))*a^3*c*d*e^3*f*g^3+90*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*
d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-60*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d
*g*c)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*
(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^4*d^4*f^4-16*a*c^2*d^2*e
*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)-272*c^3*d^3*f*g^2*x^2*((
g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g
*c)^(1/2)*a^2*c*d*e^2*g^3*x-72*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*
c^2*d^2*e*f*g^2*x-236*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^3*d^3*f^2
*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^3*e^3*g^3+110*((g*x+f)*
(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*c*d*e^2*f*g^2-146*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(d*g*c)^(1/2)*a*c^2*d^2*e*f^2*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g
*c)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/g/((g*x+f)*(c*d*x+a*e))^(1/2)/d^3/c^3/
(d*g*c)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(5/2)/sqrt(
x*e + d), x)
```

Fricas [A]
time = 6.20, size = 1075, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 136*c^4*d^4*f*g^3*x^2 + 118*c^4*d^4*f^2*g^2
*x + 15*c^4*d^4*f^3*g + 15*a^3*c*d*g^4*e^3 - 5*(2*a^2*c^2*d^2*g^4*x + 11*a^
2*c^2*d^2*f*g^3)*e^2 + (8*a*c^3*d^3*g^4*x^2 + 36*a*c^3*d^3*f*g^3*x + 73*a*c
^3*d^3*f^2*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f
)*sqrt(x*e + d) + 15*(c^4*d^5*f^4 + a^4*g^4*x*e^5 - (4*a^3*c*d*f*g^3*x - a^
4*d*g^4)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^2*x - 2*a^3*c*d^2*f*g^3)*e^3 - 2*(2*a
*c^3*d^3*f^3*g*x - 3*a^2*c^2*d^3*f^2*g^2)*e^2 + (c^4*d^4*f^4*x - 4*a*c^3*d^
4*f^3*g)*e)*sqrt(c*d*g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3
*f^2 + a^2*g^2*x*e^3 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d
*g*x + c*d*f + a*g*e)*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^
2*x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g
*x^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^4
*d^4*g^2*x*e + c^4*d^5*g^2), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 136*c^4*d^4*f*g
^3*x^2 + 118*c^4*d^4*f^2*g^2*x + 15*c^4*d^4*f^3*g + 15*a^3*c*d*g^4*e^3 - 5*
(2*a^2*c^2*d^2*g^4*x + 11*a^2*c^2*d^2*f*g^3)*e^2 + (8*a*c^3*d^3*g^4*x^2 + 3
6*a*c^3*d^3*f*g^3*x + 73*a*c^3*d^3*f^2*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*
d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 15*(c^4*d^5*f^4 + a^4*g^4*x*e
^5 - (4*a^3*c*d*f*g^3*x - a^4*d*g^4)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^2*x - 2*a
^3*c*d^2*f*g^3)*e^3 - 2*(2*a*c^3*d^3*f^3*g*x - 3*a^2*c^2*d^3*f^2*g^2)*e^2 +
 (c^4*d^4*f^4*x - 4*a*c^3*d^4*f^3*g)*e)*sqrt(-c*d*g)*arctan(2*sqrt(c*d^2*x
+ a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x*e + d)/(2*
c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c^4
*d^4*g^2*x*e + c^4*d^5*g^2)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{5/2}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

3.734 $\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=313

$$-\frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}+\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12\sqrt{d+ex}}$$

```
[Out] -1/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(3/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/12*(a*e/c/d-f/g)*(g*x+f)^(3/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)+1/3*(g*x+f)^(5/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)-1/8*(-a*e*g+c*d*f)^2*(g*x+f)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8c^{5/2}d^{5/2}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{\sqrt{f+gx}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^2}{8c^2d^2g\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{12\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +
e*x],x]
```

```
[Out] -1/8*((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(c^2*d^2*g*Sqrt[d + e*x]) + (((a*e)/(c*d) - f/g)*(f + g*x)^(3/2)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*Sqrt[d + e*x]) + ((f + g*x)^
(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*g*Sqrt[d + e*x]) - ((
c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
+ c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(5/2)*d^(5/2)*g^(3/2)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{(f+gx)^{5/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g\sqrt{d+ex}} - \frac{(cdf-aeg}{}$$

$$= \frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12\sqrt{d+ex}} + $$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}$$

$$= -\frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2g\sqrt{d+ex}}$$

Mathematica [A]
time = 0.37, size = 200, normalized size = 0.64

$$\frac{(cdf-aeg)^3\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\left(-3a^2e^2g^2+2acdeg(4f+gx)+c^2d^2(3f^2+14fgx+8g^2x^2)\right)}{(cdf-aeg)^3}-\frac{3\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{ae+cdx}}\right)}{24c^{5/2}d^{5/2}g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqr
t[d + e*x],x]
```

```
[Out] ((c*d*f - a*e*g)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[c]*Sqrt[d]*Sqrt[g]*
Sqrt[f + g*x]*(-3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + g*x) + c^2*d^2*(3*f^2 +
```

```
14*f*g*x + 8*g^2*x^2)))/(c*d*f - a*e*g)^3 - (3*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/Sqrt[a*e + c*d*x]))/(24*c^(5/2)*d
^(5/2)*g^(3/2)*Sqrt[d + e*x])
```

Maple [A]
time = 0.14, size = 504, normalized size = 1.61

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a^3e^3g^3-9\ln\left(\frac{2cd \cdots}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+c
*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^3*e^3*g^
3-9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/
2))/(d*g*c)^(1/2))*a^2*c*d*e^2*f*g^2+9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*
x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f^2*g-3*l
n(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(
d*g*c)^(1/2))*c^3*d^3*f^3+16*c^2*d^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d
*g*c)^(1/2)+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x+28*((
g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*x+a*e
))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2+16*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(
1/2)*a*c*d*e*f*g+6*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f^2)/(
e*x+d)^(1/2)/g/((g*x+f)*(c*d*x+a*e))^(1/2)/d^2/c^2/(d*g*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(
x*e + d), x)
```

Fricas [A]
time = 5.20, size = 853, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 14*c^3*d^3*f*g^2*x + 3*c^3*d^3*f^2*g - 3*a^2*
c*d*g^3*e^2 + 2*(a*c^2*d^2*g^3*x + 4*a*c^2*d^2*f*g^2)*e)*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 3*(c^3*d^4*f^3 - a^
3*g^3*x*e^4 + (3*a^2*c*d*f*g^2*x - a^3*d*g^3)*e^3 - 3*(a*c^2*d^2*f^2*g*x -
a^2*c*d^2*f*g^2)*e^2 + (c^3*d^3*f^3*x - 3*a*c^2*d^3*f^2*g)*e)*sqrt(c*d*g)*l
og(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*
sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c*d*f + a*g*e)*sqr
t(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a
^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c
^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^3*d^3*g^2*x*e + c^3*d^4*g^
2), 1/48*(2*(8*c^3*d^3*g^3*x^2 + 14*c^3*d^3*f*g^2*x + 3*c^3*d^3*f^2*g - 3*a
^2*c*d*g^3*e^2 + 2*(a*c^2*d^2*g^3*x + 4*a*c^2*d^2*f*g^2)*e)*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 3*(c^3*d^4*f^3 -
 a^3*g^3*x*e^4 + (3*a^2*c*d*f*g^2*x - a^3*d*g^3)*e^3 - 3*(a*c^2*d^2*f^2*g*x
 - a^2*c*d^2*f*g^2)*e^2 + (c^3*d^3*f^3*x - 3*a*c^2*d^3*f^2*g)*e)*sqrt(-c*d*
g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g
*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c
*d*f*x + a*d*g)*e)))/(c^3*d^3*g^2*x*e + c^3*d^4*g^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**(3/2)/sqrt(d + e*x), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

3.735 $\int \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=241

$$\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-ae\cdots}{\cdots}$$

```
[Out] -1/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*
x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(3/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/2*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)+1/4*(a*e/c/d-f/g)*(g*x+f)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$-\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}+\frac{(f+gx)^{3/2}\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}}+\frac{\sqrt{f+gx}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*
x],x]
```

```
[Out] (((a*e)/(c*d) - f/g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(4*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(2*g*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt
[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
])])/(4*c^(3/2)*d^(3/2)*g^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx=\frac{(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2g\sqrt{d+ex}}-\frac{(cdf-aeg)}{}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f}{}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f}{}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f}{}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f}{}$$

$$=\frac{\left(\frac{ae}{cd}-\frac{f}{g}\right)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4\sqrt{d+ex}}+\frac{(f}{}$$

Mathematica [A]
time = 0.40, size = 162, normalized size = 0.67

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}\,\sqrt{f+gx}\,(aeg+cd(f+2gx))-(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[
d + e*x],x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]
*Sqrt[f + g*x]*(a*e*g + c*d*(f + 2*g*x)) - (c*d*f - a*e*g)^2*ArcTanh[(Sqrt[
g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])]))/(4*c^(3/2)*d^(3/2)
*g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 319, normalized size = 1.32

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a^2e^2g^2-2\ln\left(\frac{2c\cdots}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/8*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*e^2*g^2-2*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c*d*e*f*g+ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*f^2-4*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c*d*g*x-2*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e*g-2*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(e*x+d)^(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/d/g/c/(d*g*c)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(x*e + d), x)

Fricas [A]
time = 4.75, size = 671, normalized size = 2.78

$$\left[\frac{4\,(2\,c^2d^2g^2x+c^2d^2fg+acdg^2e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}+(c^2d^3f^2+a^2g^2xe^3-(2\,acdfgx-a^2dg^2)e^2+(c^2d^2f^2x-2\,acd^2fg)e)\sqrt{cdg}\,\log\left(\cdots\right)}{16\,(c^2d^2g^2xe+c^2d^3g^2)},\ \cdots\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + (c^2*d^3*f^2 + a^2*g^2*x*e^3 - (2*a*c*d*f*g*x - a^2*d*g^2)*e^2 + (c^2*d^2*f^2*x - 2*a*c*d^2*f*g)*e)*sqrt(c*d*g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a

```
^2*g^2*x*e^3 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c
*d*f + a*g*e)*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 +
6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6
*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^2*d^2*g^2
*x*e + c^2*d^3*g^2), 1/8*(2*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*g^2*e)*s
qrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + (c
^2*d^3*f^2 + a^2*g^2*x*e^3 - (2*a*c*d*f*g*x - a^2*d*g^2)*e^2 + (c^2*d^2*f^2
*x - 2*a*c*d^2*f*g)*e)*sqrt(-c*d*g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x + c*d^2
*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c^2*d^2*g^2*x*e + c^
2*d^3*g^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}\,\sqrt{f+gx}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))*sqrt(f + g*x)/sqrt(d + e*x), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*
x)^(1/2), x)
```

3.736 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=167

$$\frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}}-\frac{(cdf-aeg)\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\,g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

```
[Out] -(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(
1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(3/2)/c^(1/2)/d^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)+(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/g/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {878, 905, 65, 223, 212}

$$\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}}-\frac{\sqrt{d+ex}\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\,g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f + g*x
]),x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x
]) - ((c*d*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(Sqrt[c]*Sqrt[d]*g^(3/2)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\int \frac{}{\sqrt{f+}}}{}$$

$$= \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} - \frac{\Big((cdf-aeg)\sqrt{ae+c}}{2g\sqrt{a}}$$

$$= \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} - \frac{\Big((cdf-aeg)\sqrt{ae+c}}{}$$

$$= \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} - \frac{\Big((cdf-aeg)\sqrt{ae+c}}{cdg}$$

$$= \frac{\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{g\sqrt{d+ex}} - \frac{(cdf-aeg)\sqrt{ae+cd}}{\sqrt{c}\,\sqrt{d}\,g^{3/}}$$

Mathematica [A]
time = 0.31, size = 127, normalized size = 0.76

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(g\sqrt{ae+cdx}\,\sqrt{f+gx}+\sqrt{\frac{g}{cd}}\,(cdf-aeg)\log\left(-\sqrt{\frac{g}{cd}}\,\sqrt{ae+cdx}+\sqrt{f+gx}\right)\right)}{g^2\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*Sqrt[f
 + g*x]),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(g*Sqrt[a*e + c*d*x]*Sqrt[f + g*x] + Sqrt[
g/(c*d)]*(c*d*f - a*e*g)*Log[-(Sqrt[g/(c*d)]*Sqrt[a*e + c*d*x]) + Sqrt[f +
g*x]]))/(g^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 188, normalized size = 1.13

method	result

default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\,\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)aeg-\ln\left(\frac{2cdgx+aeg+\cdots}{\cdots}\right)\cdots\right)}{2\sqrt{ex+d}\,\sqrt{(gx+f)(cdx+ae)}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/2*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*x+f)^(1/2)/(e*x+d)^(1/2)*(ln(1/2*(2*c*d*
g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))
*a*e*g-ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2))/(d*g*c)^(1/2))*c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/((
g*x+f)*(c*d*x+a*e))^(1/2)/g/(d*g*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(g*x + f)*sqrt(x
*e + d)), x)
```

Fricas [A]
time = 4.52, size = 526, normalized size = 3.15

$$\left[\frac{4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}\,cdg-(cd^2f-agxe^2+(cdfx-adg)e)\sqrt{cdg}\log\left(-\frac{8c^2d^3g^2x^2+8c^2d^3fgx+c^2d^3f^2+a^2g^2xe^3+4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(2cdgx+cdf+age)\sqrt{cdg}\,\sqrt{gx+f}\,\sqrt{xe+d}+(8acdg^2x^2+6acdfgx+a^2dg^2)e^2+(8c^2d^2g^2x^3+8c^2d^2fgx^2+6acd^2fg+(c^2d^2f^2+8acd^2g^2)x)e}{xe+d}\right)}{4(cdg^2xe+cd^2g^2)},\ \frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}\,cdg+(cd^2f-agxe^2+(cdfx-adg)e)\sqrt{-cdg}\arctan\left(\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{-cdg}\,\sqrt{gx+f}\,\sqrt{xe+d}}{2cd^2gx+cd^2f+agxe^2+(2cdgx^2+cdfx+adg)e}\right)}{2(cdg^2xe+cd^2g^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e
+ d)*c*d*g - (c*d^2*f - a*g*x*e^2 + (c*d*f*x - a*d*g)*e)*sqrt(c*d*g)*log(-(
8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*sqrt(
c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c*d*f + a*g*e)*sqrt(c*d
*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d*
g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d^
2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c*d*g^2*x*e + c*d^2*g^2), 1/2*(2*
sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*c*d
*g + (c*d^2*f - a*g*x*e^2 + (c*d*f*x - a*d*g)*e)*sqrt(-c*d*g)*arctan(2*sqrt
```

```
(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x*e
 + d)/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*
e)))/(c*d*g^2*x*e + c*d^2*g^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(1/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x
)^(1/2)), x)
```

3.737 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$

```
[Out] 2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*c^(1/2)*
d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(e*x+d)^(1/2)/(g*x
+f)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {876, 905, 65, 223, 212}

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(3
/2)),x]
```

```
[Out] (-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*Sqrt[d + e*x]*Sqrt[f +
g*x]) + (2*Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]
*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(3/2)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}}dx = -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{(cd)\int \frac{\sqrt{d+e}}{\sqrt{f+gx}\sqrt{ade+(cd^2}}}{g}$$

$$= -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{\left(cd\sqrt{ae+cdx}\sqrt{d+ex}\right)\int}{g\sqrt{ade+(cd^2+ae}}$$

$$= -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subs}}{g\sqrt{ade-}}$$

$$= -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{\left(2\sqrt{ae+cdx}\sqrt{d+ex}\right)\text{Subs}}{g\sqrt{ade+(cd^2}}$$

$$= -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}} + \frac{2\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}}{g^{3/2}\sqrt{ade+(cd^2}}$$

Mathematica [A]
time = 0.14, size = 133, normalized size = 0.84

$$\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{g}\sqrt{ae+cdx}+\sqrt{c}\sqrt{d}\sqrt{f+gx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)\right)}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]) + Sqrt[c]*
Sqrt[d]*Sqrt[f + g*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*
Sqrt[f + g*x])]))/(g^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f + g*x])
```

Maple [A]
time = 0.14, size = 187, normalized size = 1.18

method	result

default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{2\sqrt{dgc}}\right)cdgx+\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(g}{}\right.\right.}{\sqrt{dgc}\sqrt{(gx+f)(cdx+ae)}\,g\sqrt{ex+d}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] ((c*d*x+a*e)*(e*x+d))^(1/2)*(ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*
x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c*d*g*x+ln(1/2*(2*c*d*g*x+a*e*g
+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c*d*f-2*
((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2)/((g*x+f)*(c*d*x+a*
e))^(1/2)/g/(e*x+d)^(1/2)/(g*x+f)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^(3/2)*sqrt
(x*e + d)), x)
```

Fricas [A]
time = 3.81, size = 533, normalized size = 3.37

$$\left[\frac{(dgx+df+(gx^2+fx)e)\sqrt{\frac{cd}{g}}\log\left(-\frac{8c^2d^3g^2x^2+8c^2d^3fgx+c^2d^3f^2+a^2g^2xe^3+4(2cdg^2x+cdfg+ag^2e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{gx+f}\sqrt{xe+d}\sqrt{\frac{cd}{g}}+(8acdg^2x^2+6acdfgx+a^2dg^2)e^2+(8c^2d^2g^2x^3+8c^2d^2fgx^2+6acd^2fg+(c^2d^2f^2+8acd^2g^2)x)e}{xe+d}\right)-4\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{gx+f}\sqrt{xe+d}}{2(dg^2x+dfg+(g^2x^2+fgx)e)}, -\frac{(dgx+df+(gx^2+fx)e)\sqrt{-\frac{cd}{g}}\arctan\left(\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{gx+f}\sqrt{xe+d}\sqrt{-\frac{cd}{g}}\,g}{2cd^2gx+cd^2f+agxe^2+(2cdgx^2+cdfx+adg)e}\right)+2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\sqrt{gx+f}\sqrt{xe+d}}{dg^2x+dfg+(g^2x^2+fgx)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*((d*g*x + d*f + (g*x^2 + f*x)*e)*sqrt(c*d/g)*log(-(8*c^2*d^3*g^2*x^2 +
 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*(2*c*d*g^2*x + c*d*f*g +
 a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*
e + d)*sqrt(c*d/g) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8
*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c
*d^2*g^2)*x)*e)/(x*e + d)) - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*
sqrt(g*x + f)*sqrt(x*e + d))/(d*g^2*x + d*f*g + (g^2*x^2 + f*g*x)*e), -((d*
g*x + d*f + (g*x^2 + f*x)*e)*sqrt(-c*d/g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 +
```

```
(c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-c*d/g)*g/(2*c*d^2*g*x
 + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)) + 2*sqrt(c*d^2
*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d))/(d*g^2*x + d
*f*g + (g^2*x^2 + f*g*x)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(3/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(3/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(f+g\,x\right)}^{3/2}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(3/2)*(d + e*x
)^(1/2)), x)
```

3.738 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[Out] 2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {874}

$$\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(5
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{5/2}}\,dx=\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 0.83

$$\frac{2((ae+cdx)(d+ex))^{3/2}}{3(cdf-aeg)(d+ex)^{3/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(5/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2))/(3*(c*d*f - a*e*g)*(d + e*x)^(3/2)*(f +
 g*x)^(3/2))
```

Maple [A]
time = 0.14, size = 53, normalized size = 0.84

method	result	size
default	$-\frac{2(cdx+ae)\sqrt{(cdx+ae)(ex+d)}}{3(gx+f)^{\frac{3}{2}}(aeg-cdf)\sqrt{ex+d}}$	53
gosper	$-\frac{2(cdx+ae)\sqrt{cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade}}{3(gx+f)^{\frac{3}{2}}(aeg-cdf)\sqrt{ex+d}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3/(g*x+f)^(3/2)*(c*d*x+a*e)/(a*e*g-c*d*f)*((c*d*x+a*e)*(e*x+d))^(1/2)/(e
*x+d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^(5/2)*sqrt
(x*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(58) = 116$.
time = 2.00, size = 176, normalized size = 2.79

$$\frac{2\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(cdx+ae)\sqrt{gx+f}\,\sqrt{xe+d}}{3\,(cd^2fg^2x^2+2\,cd^2f^2gx+cd^2f^3-(ag^3x^3+2\,afg^2x^2+af^2gx)e^2+(cdfg^2x^3-adf^2g+(2\,cdf^2g-adg^3)x^2+(cdf^3-2\,adfg^2)x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*x + a*e)*sqrt(g*x + f)
*sqrt(x*e + d)/(c*d^2*f*g^2*x^2 + 2*c*d^2*f^2*g*x + c*d^2*f^3 - (a*g^3*x^3
+ 2*a*f*g^2*x^2 + a*f^2*g*x)*e^2 + (c*d*f*g^2*x^3 - a*d*f^2*g + (2*c*d*f^2*
g - a*d*g^3)*x^2 + (c*d*f^3 - 2*a*d*f*g^2)*x)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{\sqrt{d+ex}\,\,(f+gx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(5/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(sqrt(d + e*x)*(f + g*x)**(5/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.92, size = 136, normalized size = 2.16

$$-\frac{\left(\frac{2\,a\,e}{3\,a\,e\,g^2-3\,c\,d\,f\,g}+\frac{2\,c\,d\,x}{3\,a\,e\,g^2-3\,c\,d\,f\,g}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(3\,c\,d\,f^2-3\,a\,e\,f\,g\right)\,\sqrt{d+e\,x}}{3\,a\,e\,g^2-3\,c\,d\,f\,g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(5/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] -(((2*a*e)/(3*a*e*g^2 - 3*c*d*f*g) + (2*c*d*x)/(3*a*e*g^2 - 3*c*d*f*g))*(x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x*(f + g*x)^(1/2)*(d + e*x)^(1
/2) - ((f + g*x)^(1/2)*(3*c*d*f^2 - 3*a*e*f*g)*(d + e*x)^(1/2))/(3*a*e*g^2
- 3*c*d*f*g))
```

3.739 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=129

$$\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{4cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{3/2}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(5/2)+4/15*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(7
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(5/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(15*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
```

```
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{7/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{(2cd)\int \frac{\sqrt{ade+(cd^2+ae^2)}}{\sqrt{d+ex}\,(f+g}}{5(cdf-aeg)}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5(cdf-aeg)(d+ex)^{3/2}(f+gx)^{5/2}} + \frac{4cd(ade+(cd^2+ae^2)x+}{15(cdf-aeg)^2(d+ex)^{3/2}(f}$$

Mathematica [A]
time = 0.12, size = 69, normalized size = 0.53

$$\frac{2((ae+cdx)(d+ex))^{3/2}(-3aeg+cd(5f+2gx))}{15(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-3*a*e*g + c*d*(5*f + 2*g*x)))/(15*(c*d
*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2))
```

Maple [A]
time = 0.15, size = 70, normalized size = 0.54

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)(-2cdgx+3aeg-5cdf)}{15(gx+f)^{\frac{5}{2}}\sqrt{ex+d}\,(aeg-cdf)^2}$	70
gosper	$-\frac{2(cdx+ae)(-2cdgx+3aeg-5cdf)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{15(gx+f)^{\frac{5}{2}}(a^2e^2g^2-2acdefg+f^2c^2d^2)\sqrt{ex+d}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/15*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(5/2)/(e*x+d)^(1/2)*(c*d*x+a*e)*(
-2*c*d*g*x+3*a*e*g-5*c*d*f)/(a*e*g-c*d*f)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^(7/2)*sqrt
(x*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(119) = 238$.
time = 1.27, size = 424, normalized size = 3.29

$$\frac{2\left(2\,c^2d^2gx^2+5\,c^2d^2fx-3\,a^2ge^2-\left(acdgx-5\,acdf\right)e\right)\sqrt{cd^2x+axe^2+\left(cdx^2+ad\right)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{15\left(c^2d^3f^2g^3x^3+3\,c^2d^3f^3g^2x^2+3\,c^2d^3f^4gx+c^2d^3f^5+\left(a^2g^5x^4+3\,a^2fg^4x^3+3\,a^2f^2g^3x^2+a^2f^3g^2x\right)e^3-\left(2\,acdfg^4x^4-a^2df^3g^2+\left(6\,acdf^2g^3-a^2dg^5\right)x^3+3\left(2\,acdf^3g^2-a^2dfg^4\right)x^2+\left(2\,acdf^4g-3\,a^2df^2g^3\right)x\right)e^2+\left(c^2d^2f^2g^3x^4-2\,acd^2f^4g+\left(3\,c^2d^2f^3g^2-2\,acd^2fg^4\right)x^3+3\left(c^2d^2f^4g-2\,acd^2f^2g^3\right)x^2+\left(c^2d^2f^5-6\,acd^2f^3g^2\right)x\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*c^2*d^2*g*x^2 + 5*c^2*d^2*f*x - 3*a^2*g*e^2 - (a*c*d*g*x - 5*a*c*d*
f)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e +
d)/(c^2*d^3*f^2*g^3*x^3 + 3*c^2*d^3*f^3*g^2*x^2 + 3*c^2*d^3*f^4*g*x + c^2*d
^3*f^5 + (a^2*g^5*x^4 + 3*a^2*f*g^4*x^3 + 3*a^2*f^2*g^3*x^2 + a^2*f^3*g^2*x
)*e^3 - (2*a*c*d*f*g^4*x^4 - a^2*d*f^3*g^2 + (6*a*c*d*f^2*g^3 - a^2*d*g^5)*
x^3 + 3*(2*a*c*d*f^3*g^2 - a^2*d*f*g^4)*x^2 + (2*a*c*d*f^4*g - 3*a^2*d*f^2*
g^3)*x)*e^2 + (c^2*d^2*f^2*g^3*x^4 - 2*a*c*d^2*f^4*g + (3*c^2*d^2*f^3*g^2 -
 2*a*c*d^2*f*g^4)*x^3 + 3*(c^2*d^2*f^4*g - 2*a*c*d^2*f^2*g^3)*x^2 + (c^2*d^
2*f^5 - 6*a*c*d^2*f^3*g^2)*x)*e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(7/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.08, size = 187, normalized size = 1.45

$$\frac{\left(\frac{x\,\left(10\,c^2\,d^2\,f-2\,a\,c\,d\,e\,g\right)}{15\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{6\,a^2\,e^2\,g-10\,a\,c\,d\,e\,f}{15\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{4\,c^2\,d^2\,x^2}{15\,g\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{2\,f\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(7/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] (((x*(10*c^2*d^2*f - 2*a*c*d*e*g))/(15*g^2*(a*e*g - c*d*f)^2) - (6*a^2*e^2*
g - 10*a*c*d*e*f)/(15*g^2*(a*e*g - c*d*f)^2) + (4*c^2*d^2*x^2)/(15*g*(a*e*g
 - c*d*f)^2))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)
^(1/2)*(d + e*x)^(1/2) + (f^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (2*f*x
*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g)
```

3.740 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=198

$$\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{8cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{35(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{5/2}}+\frac{16c^2d^2(ade+(cd^2+ae^2)x+cd}{105(cdf-aeg)^3(d+ex)^{3/2}(f+}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(7/2)+8/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(5/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{16c^2d^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3}+\frac{8cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(9
/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(7/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(5/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
```

```
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{9/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{(4cd)\int \frac{\sqrt{ade+(cd^2+ae^2)}}{\sqrt{d+ex}\,(f+g}}{7(cdf-aeg)}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{8cd(ade+(cd^2+ae^2)x+}{35(cdf-aeg)^2(d+ex)^{3/2}(f}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{7(cdf-aeg)(d+ex)^{3/2}(f+gx)^{7/2}} + \frac{8cd(ade+(cd^2+ae^2)x+}{35(cdf-aeg)^2(d+ex)^{3/2}(f}$$

Mathematica [A]
time = 0.14, size = 105, normalized size = 0.53

$$\frac{2((ae+cdx)(d+ex))^{3/2}\,(15a^2e^2g^2-6acdeg(7f+2gx)+c^2d^2(35f^2+28fgx+8g^2x^2))}{105(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(9/2)),x]
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(15*a^2*e^2*g^2 - 6*a*c*d*e*g*(7*f + 2*g
*x) + c^2*d^2*(35*f^2 + 28*f*g*x + 8*g^2*x^2)))/(105*(c*d*f - a*e*g)^3*(d +
 e*x)^(3/2)*(f + g*x)^(7/2))
```

Maple [A]
time = 0.14, size = 119, normalized size = 0.60

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)(8g^2x^2c^2d^2-12acde\,g^2x+28c^2d^2fgx+15a^2e^2g^2-42acdefg+35f^2c^2d^2)}{105(gx+f)^{\frac{7}{2}}\sqrt{ex+d}\,(aeg-cdf)^3}$
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2-12acde\,g^2x+28c^2d^2fgx+15a^2e^2g^2-42acdefg+35f^2c^2d^2)\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}{105(gx+f)^{\frac{7}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)\sqrt{ex+d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/105*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(7/2)/(e*x+d)^(1/2)*(c*d*x+a*e)*
(8*c^2*d^2*g^2*x^2-12*a*c*d*e*g^2*x+28*c^2*d^2*f*g*x+15*a^2*e^2*g^2-42*a*c*
d*e*f*g+35*c^2*d^2*f^2)/(a*e*g-c*d*f)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^(9/2)*sqrt
(x*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 783 vs. 2(183) = 366.
time = 1.51, size = 783, normalized size = 3.95

$$\frac{2\left(8c^3d^3g^2x^3+28c^3d^3fgx^2+35c^3d^3f^2x+15a^3g^2e^3+3\left(a^2cdg^2x-14a^2cdfg\right)e^2-\left(4ac^2d^2g^2x^2+14ac^2d^2fgx-35ac^2d^2f^2\right)e\right)\sqrt{cd^2x+axe^2+\left(cdx^2+ad\right)e}\sqrt{gx+f}\sqrt{xe+d}}{105\left(c^3d^4f^3g^4x^4+4c^3d^4f^4g^3x^3+6c^3d^4f^5g^2x^2+4c^3d^4f^6gx+c^3d^4f^7-\left(a^3g^7x^5+4a^3fg^6x^4+6a^3f^2g^5x^3+4a^3f^3g^4x^2+a^3f^4g^3x\right)e^4+\left(3a^2cdfg^6x^5-a^3df^4g^3+\left(12a^2cdf^2g^5-a^3dg^7\right)x^4+2\left(9a^2cdf^3g^4-2a^3dfg^6\right)x^3+6\left(2a^2cdf^4g^3-a^3df^2g^5\right)x^2+\left(3a^2cdf^5g^2-4a^3df^3g^4\right)x\right)e^3-3\left(ac^2d^2f^2g^5x^5-a^2cd^2f^5g^2+\left(4ac^2d^2f^3g^4-a^2cd^2fg^6\right)x^4+2\left(3ac^2d^2f^4g^3-2a^2cd^2f^2g^5\right)x^3+2\left(2ac^2d^2f^5g^2-3a^2cd^2f^3g^4\right)x^2+\left(ac^2d^2f^6g-4a^2cd^2f^4g^3\right)x\right)e^2+\left(c^3d^3f^3g^4x^5-3ac^2d^3f^6g+\left(4c^3d^3f^4g^3-3ac^2d^3f^2g^5\right)x^4+6\left(c^3d^3f^5g^2-2ac^2d^3f^3g^4\right)x^3+2\left(2c^3d^3f^6g-9ac^2d^3f^4g^3\right)x^2+\left(c^3d^3f^7-12ac^2d^3f^5g^2\right)x\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*c^3*d^3*g^2*x^3 + 28*c^3*d^3*f*g*x^2 + 35*c^3*d^3*f^2*x + 15*a^3*g
^2*e^3 + 3*(a^2*c*d*g^2*x - 14*a^2*c*d*f*g)*e^2 - (4*a*c^2*d^2*g^2*x^2 + 14
*a*c^2*d^2*f*g*x - 35*a*c^2*d^2*f^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
 a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^3*d^4*f^3*g^4*x^4 + 4*c^3*d^4*f^4*g
^3*x^3 + 6*c^3*d^4*f^5*g^2*x^2 + 4*c^3*d^4*f^6*g*x + c^3*d^4*f^7 - (a^3*g^7
*x^5 + 4*a^3*f*g^6*x^4 + 6*a^3*f^2*g^5*x^3 + 4*a^3*f^3*g^4*x^2 + a^3*f^4*g^
3*x)*e^4 + (3*a^2*c*d*f*g^6*x^5 - a^3*d*f^4*g^3 + (12*a^2*c*d*f^2*g^5 - a^3
*d*g^7)*x^4 + 2*(9*a^2*c*d*f^3*g^4 - 2*a^3*d*f*g^6)*x^3 + 6*(2*a^2*c*d*f^4*
g^3 - a^3*d*f^2*g^5)*x^2 + (3*a^2*c*d*f^5*g^2 - 4*a^3*d*f^3*g^4)*x)*e^3 - 3
*(a*c^2*d^2*f^2*g^5*x^5 - a^2*c*d^2*f^5*g^2 + (4*a*c^2*d^2*f^3*g^4 - a^2*c*
d^2*f*g^6)*x^4 + 2*(3*a*c^2*d^2*f^4*g^3 - 2*a^2*c*d^2*f^2*g^5)*x^3 + 2*(2*a
*c^2*d^2*f^5*g^2 - 3*a^2*c*d^2*f^3*g^4)*x^2 + (a*c^2*d^2*f^6*g - 4*a^2*c*d^
2*f^4*g^3)*x)*e^2 + (c^3*d^3*f^3*g^4*x^5 - 3*a*c^2*d^3*f^6*g + (4*c^3*d^3*f
^4*g^3 - 3*a*c^2*d^3*f^2*g^5)*x^4 + 6*(c^3*d^3*f^5*g^2 - 2*a*c^2*d^3*f^3*g^
4)*x^3 + 2*(2*c^3*d^3*f^6*g - 9*a*c^2*d^3*f^4*g^3)*x^2 + (c^3*d^3*f^7 - 12*
a*c^2*d^3*f^5*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(9/2)/(e*x+d
)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.29, size = 289, normalized size = 1.46

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{30\,a^3\,e^3\,g^2-84\,a^2\,c\,d\,e^2\,f\,g+70\,a\,c^2\,d^2\,e\,f^2}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x\,\left(6\,a^2\,c\,d\,e^2\,g^2-28\,a\,c^2\,d^2\,e\,f\,g+70\,c^3\,d^3\,f^2\right)}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^3\,d^3\,x^3}{105\,g\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c^2\,d^2\,x^2\,\left(a\,e\,g-7\,c\,d\,f\right)}{105\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(9/2)*(d + e*x
)^(1/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((30*a^3*e^3*g^2 + 70*a*c^2
*d^2*e*f^2 - 84*a^2*c*d*e^2*f*g)/(105*g^3*(a*e*g - c*d*f)^3) + (x*(70*c^3*d
^3*f^2 + 6*a^2*c*d*e^2*g^2 - 28*a*c^2*d^2*e*f*g))/(105*g^3*(a*e*g - c*d*f)^
3) + (16*c^3*d^3*x^3)/(105*g*(a*e*g - c*d*f)^3) - (8*c^2*d^2*x^2*(a*e*g - 7
*c*d*f))/(105*g^2*(a*e*g - c*d*f)^3)))/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2)
 + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f + g*x)^(1/2)*(d
+ e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.741 $\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=267

$$\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{21(cdf-aeg)^2(d+ex)^{3/2}(f+gx)^{7/2}}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+cc\right.}{105(cdf-aeg)^3(d+ex)^{3/2}(f+}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)/(e*x+d)^(3/2)/(g
*x+f)^(9/2)+4/21*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(3/2)/(g*x+f)^(7/2)+16/105*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(3/2)/(g*x+f)^(5/2)+32/315*c^3*d^3*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(3/2)/(g*x+f)^
(3/2)
```

Rubi [A]
time = 0.21, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^(1
1/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(3/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(3/2))/(21*(c*d*f - a*e*g)^2*(d + e*x)^(3/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(105*(c*d*f - a*e*g)^3*(d
+ e*x)^(3/2)*(f + g*x)^(5/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2))/(315*(c*d*f - a*e*g)^4*(d + e*x)^(3/2)*(f + g*x)^(3/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
```

```
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}\,(f+gx)^{11/2}}\,dx = \frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{(2cd)\int \frac{\sqrt{ade+(cd^2+ae^2)}}{\sqrt{d+ex}\,(f+g}}{3(cdf-aeg)}$$

$$= \frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd(ade+(cd^2+ae^2)\,x+}{21(cdf-aeg)^2(d+ex)^{3/2}(f}$$

$$= \frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd(ade+(cd^2+ae^2)\,x+}{21(cdf-aeg)^2(d+ex)^{3/2}(f}$$

$$= \frac{2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{9(cdf-aeg)(d+ex)^{3/2}(f+gx)^{9/2}} + \frac{4cd(ade+(cd^2+ae^2)\,x+}{21(cdf-aeg)^2(d+ex)^{3/2}(f}$$

Mathematica [A]
time = 0.17, size = 152, normalized size = 0.57

$$\frac{2((ae+cdx)(d+ex))^{3/2}\left(-35a^3e^3g^3+15a^2cde^2g^2(9f+2gx)-3ac^2d^2eg(63f^2+36fgx+8g^2x^2)+c^3d^3(105f^3+126f^2gx+72fg^2x^2+16g^3x^3)\right)}{315(cdf-aeg)^4(d+ex)^{3/2}(f+gx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g
*x)^(11/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(-35*a^3*e^3*g^3 + 15*a^2*c*d*e^2*g^2*(9
*f + 2*g*x) - 3*a*c^2*d^2*e*g*(63*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(10
5*f^3 + 126*f^2*g*x + 72*f*g^2*x^2 + 16*g^3*x^3)))/(315*(c*d*f - a*e*g)^4*(
d + e*x)^(3/2)*(f + g*x)^(9/2))
```

Maple [A]
time = 0.14, size = 191, normalized size = 0.72

method	result

default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)(-16g^3x^3c^3d^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d}{315(gx+f)^{\frac{9}{2}}\sqrt{ex+d}\,(aeg-cdf)^4}$
gosper	$-\frac{2(cdx+ae)(-16g^3x^3c^3d^3+24a\,c^2d^2e\,g^3x^2-72c^3d^3f\,g^2x^2-30a^2cd\,e^2g^3x+108a\,c^2d^2ef\,g^2x-126c^3d^3f^2gx+35a^3e^3g^3-135a^2cd\,e^2f}{315(gx+f)^{\frac{9}{2}}(g^4e^4a^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/315*((c*d*x+a*e)*(e*x+d))^(1/2)/(g*x+f)^(9/2)/(e*x+d)^(1/2)*(c*d*x+a*e)*
(-16*c^3*d^3*g^3*x^3+24*a*c^2*d^2*e*g^3*x^2-72*c^3*d^3*f*g^2*x^2-30*a^2*c*d
*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-126*c^3*d^3*f^2*g*x+35*a^3*e^3*g^3-135*a
^2*c*d*e^2*f*g^2+189*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)/(a*e*g-c*d*f)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)/((g*x + f)^(11/2)*sqr
t(x*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. 2(247) = 494.
time = 5.17, size = 1243, normalized size = 4.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*c^4*d^4*g^3*x^4 + 72*c^4*d^4*f*g^2*x^3 + 126*c^4*d^4*f^2*g*x^2 +
105*c^4*d^4*f^3*x - 35*a^4*g^3*e^4 - 5*(a^3*c*d*g^3*x - 27*a^3*c*d*f*g^2)*e
^3 + 3*(2*a^2*c^2*d^2*g^3*x^2 + 9*a^2*c^2*d^2*f*g^2*x - 63*a^2*c^2*d^2*f^2*
g)*e^2 - (8*a*c^3*d^3*g^3*x^3 + 36*a*c^3*d^3*f*g^2*x^2 + 63*a*c^3*d^3*f^2*g
*x - 105*a*c^3*d^3*f^3)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt
(g*x + f)*sqrt(x*e + d)/(c^4*d^5*f^4*g^5*x^5 + 5*c^4*d^5*f^5*g^4*x^4 + 10*c
^4*d^5*f^6*g^3*x^3 + 10*c^4*d^5*f^7*g^2*x^2 + 5*c^4*d^5*f^8*g*x + c^4*d^5*f
```

```
^9 + (a^4*g^9*x^6 + 5*a^4*f*g^8*x^5 + 10*a^4*f^2*g^7*x^4 + 10*a^4*f^3*g^6*x
^3 + 5*a^4*f^4*g^5*x^2 + a^4*f^5*g^4*x)*e^5 - (4*a^3*c*d*f*g^8*x^6 - a^4*d*
f^5*g^4 + (20*a^3*c*d*f^2*g^7 - a^4*d*g^9)*x^5 + 5*(8*a^3*c*d*f^3*g^6 - a^4
*d*f*g^8)*x^4 + 10*(4*a^3*c*d*f^4*g^5 - a^4*d*f^2*g^7)*x^3 + 10*(2*a^3*c*d*
f^5*g^4 - a^4*d*f^3*g^6)*x^2 + (4*a^3*c*d*f^6*g^3 - 5*a^4*d*f^4*g^5)*x)*e^4
 + 2*(3*a^2*c^2*d^2*f^2*g^7*x^6 - 2*a^3*c*d^2*f^6*g^3 + (15*a^2*c^2*d^2*f^3
*g^6 - 2*a^3*c*d^2*f*g^8)*x^5 + 10*(3*a^2*c^2*d^2*f^4*g^5 - a^3*c*d^2*f^2*g
^7)*x^4 + 10*(3*a^2*c^2*d^2*f^5*g^4 - 2*a^3*c*d^2*f^3*g^6)*x^3 + 5*(3*a^2*c
^2*d^2*f^6*g^3 - 4*a^3*c*d^2*f^4*g^5)*x^2 + (3*a^2*c^2*d^2*f^7*g^2 - 10*a^3
*c*d^2*f^5*g^4)*x)*e^3 - 2*(2*a*c^3*d^3*f^3*g^6*x^6 - 3*a^2*c^2*d^3*f^7*g^2
 + (10*a*c^3*d^3*f^4*g^5 - 3*a^2*c^2*d^3*f^2*g^7)*x^5 + 5*(4*a*c^3*d^3*f^5*
g^4 - 3*a^2*c^2*d^3*f^3*g^6)*x^4 + 10*(2*a*c^3*d^3*f^6*g^3 - 3*a^2*c^2*d^3*
f^4*g^5)*x^3 + 10*(a*c^3*d^3*f^7*g^2 - 3*a^2*c^2*d^3*f^5*g^4)*x^2 + (2*a*c^
3*d^3*f^8*g - 15*a^2*c^2*d^3*f^6*g^3)*x)*e^2 + (c^4*d^4*f^4*g^5*x^6 - 4*a*c
^3*d^4*f^8*g + (5*c^4*d^4*f^5*g^4 - 4*a*c^3*d^4*f^3*g^6)*x^5 + 10*(c^4*d^4*
f^6*g^3 - 2*a*c^3*d^4*f^4*g^5)*x^4 + 10*(c^4*d^4*f^7*g^2 - 4*a*c^3*d^4*f^5*
g^4)*x^3 + 5*(c^4*d^4*f^8*g - 8*a*c^3*d^4*f^6*g^3)*x^2 + (c^4*d^4*f^9 - 20*
a*c^3*d^4*f^7*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(11/2)/(e*x+
d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^(11/2)/(e*x+d)^(1
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.50, size = 409, normalized size = 1.53

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{x\,\left(-10\,a^3\,c\,d\,e^3\,g^3+54\,a^2\,c^2\,d^2\,e^2\,f\,g^2-126\,a\,c^3\,d^3\,e\,f^2\,g+210\,c^4\,d^4\,f^3\right)}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{70\,a^4\,e^4\,g^3-270\,a^3\,c\,d\,e^3\,f\,g^2+378\,a^2\,c^2\,d^2\,e^2\,f^2\,g-210\,a\,c^3\,d^3\,e\,f^3}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{32\,c^4\,d^4\,x^4}{315\,g\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{4\,c^2\,d^2\,x^2\,\left(a^2\,e^2\,g^2-6\,a\,c\,d\,e\,f\,g+21\,c^2\,d^2\,f^2\right)}{105\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{16\,c^3\,d^3\,x^3\,\left(a\,e\,g-9\,c\,d\,f\right)}{315\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/((f + g*x)^(11/2)*(d + e*
x)^(1/2)),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((x*(210*c^4*d^4*f^3 - 10*a^
3*c*d*e^3*g^3 + 54*a^2*c^2*d^2*e^2*f*g^2 - 126*a*c^3*d^3*e*f^2*g))/(315*g^4
*(a*e*g - c*d*f)^4) - (70*a^4*e^4*g^3 - 210*a*c^3*d^3*e*f^3 + 378*a^2*c^2*d
^2*e^2*f^2*g - 270*a^3*c*d*e^3*f*g^2)/(315*g^4*(a*e*g - c*d*f)^4) + (32*c^4
*d^4*x^4)/(315*g*(a*e*g - c*d*f)^4) + (4*c^2*d^2*x^2*(a^2*e^2*g^2 + 21*c^2*
d^2*f^2 - 6*a*c*d*e*f*g))/(105*g^3*(a*e*g - c*d*f)^4) - (16*c^3*d^3*x^3*(a*
e*g - 9*c*d*f))/(315*g^2*(a*e*g - c*d*f)^4)))/(x^4*(f + g*x)^(1/2)*(d + e*x
)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (6
*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.742 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=382

$$\frac{3(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2g^2\sqrt{d+ex}}+\frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+c}$$

```
[Out] 1/4*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+3
/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x
+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2)/g^(5/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/32*(-a*e*g+c*d*f)^2*(g*x+f)^(3/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^2/(e*x+d)^(1/2)-1/8*(-a*e*g+c*d*f)*(
g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)+3/64
*(-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2
/d^2/g^2/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{32cdg^2\sqrt{d+ex}}-\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{8g^2\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x]
```

```
[Out] (3*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(64*c^2*d^2*g^2*Sqrt[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*c*d*g^2*Sqrt[d + e*x]) - ((c
*d*f - a*e*g)*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(8*g^2*Sqrt[d + e*x]) + ((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2))/(4*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*x]
*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])])/(64*c^(5/2)*d^(5/2)*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4g(d+ex)^{3/2}} - \frac{(3(cdf}{}$$

$$= -\frac{(cdf-aeg)(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{8g^2\sqrt{d+ex}}$$

$$= \frac{(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{32cdg^2\sqrt{d+ex}}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64c^2d^2g^2\sqrt{d+ex}}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64c^2d^2g^2\sqrt{d+ex}}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64c^2d^2g^2\sqrt{d+ex}}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64c^2d^2g^2\sqrt{d+ex}}$$

$$= \frac{3(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64c^2d^2g^2\sqrt{d+ex}}$$

Mathematica [A]
time = 0.63, size = 238, normalized size = 0.62

$$\frac{(cdf-aeg)^4((ae+cdx)(d+ex))^{3/2}\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{g}(ae+cdx)^2\sqrt{f+gx}\left(3g^3-\frac{11cdg^2(f+gx)}{ae+cdx}-\frac{11c^2d^2g(f+gx)^2}{(ae+cdx)^2}+\frac{3c^3d^3(f+gx)^3}{(ae+cdx)^3}\right)}{(cdf-aeg)^4}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{(ae+cdx)^{3/2}}\right)}{64c^{5/2}d^{5/2}g^{5/2}(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
d + e*x)^(3/2),x]
```

```
[Out] ((c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x))^(3/2)*(-((Sqrt[c]*Sqrt[d]*Sqrt
[g]*(a*e + c*d*x)^2*Sqrt[f + g*x]*(3*g^3 - (11*c*d*g^2*(f + g*x))/(a*e + c*
d*x) - (11*c^2*d^2*g*(f + g*x)^2)/(a*e + c*d*x)^2 + (3*c^3*d^3*(f + g*x)^3)
/(a*e + c*d*x)^3))/(c*d*f - a*e*g)^4) + (3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f
+ g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(3/2)))/(64*c^(5/2)*d^(
5/2)*g^(5/2)*(d + e*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(326) = 652$.
time = 0.15, size = 732, normalized size = 1.92

method	result
default	$\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(32c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}+3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx}{2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/128*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(32*c^3*d^3*g^3*x^3*((g*x+f
)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+
f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^4*e^4*g^4-12*ln(1/2*(
2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^
(1/2))*a^3*c*d*e^3*f*g^3+18*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x
+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-12*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*
c)^(1/2))*a*c^3*d^3*e*f^3*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d
*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^4*d^4*f^4+48*a*c^2*d^2*e*g^3
*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+48*c^3*d^3*f*g^2*x^2*((g*x+f
)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1
/2)*a^2*c*d*e^2*g^3*x+88*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c^2*d^
2*e*f*g^2*x+4*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^3*d^3*f^2*g*x-6*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^3*e^3*g^3+22*((g*x+f)*(c*d*x+a*e
))^(1/2)*(d*g*c)^(1/2)*a^2*c*d*e^2*f*g^2+22*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*
g*c)^(1/2)*a*c^2*d^2*e*f^2*g-6*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^
3*d^3*f^3)/(e*x+d)^(1/2)/d^2/g^2/c^2/((g*x+f)*(c*d*x+a*e))^(1/2)/(d*g*c)^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(x*
e + d)^(3/2), x)
```

Fricas [**A**]
time = 6.10, size = 1073, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/256*(4*(16*c^4*d^4*g^4*x^3 + 24*c^4*d^4*f*g^3*x^2 + 2*c^4*d^4*f^2*g^2*x
- 3*c^4*d^4*f^3*g - 3*a^3*c*d*g^4*e^3 + (2*a^2*c^2*d^2*g^4*x + 11*a^2*c^2*d
^2*f*g^3)*e^2 + (24*a*c^3*d^3*g^4*x^2 + 44*a*c^3*d^3*f*g^3*x + 11*a*c^3*d^3
*f^2*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt
(x*e + d) + 3*(c^4*d^5*f^4 + a^4*g^4*x*e^5 - (4*a^3*c*d*f*g^3*x - a^4*d*g^4
)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^2*x - 2*a^3*c*d^2*f*g^3)*e^3 - 2*(2*a*c^3*d^
3*f^3*g*x - 3*a^2*c^2*d^3*f^2*g^2)*e^2 + (c^4*d^4*f^4*x - 4*a*c^3*d^4*f^3*g
)*e)*sqrt(c*d*g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 +
a^2*g^2*x*e^3 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x +
c*d*f + a*g*e)*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 +
 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 +
6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^3*d^3*g^
3*x*e + c^3*d^4*g^3), 1/128*(2*(16*c^4*d^4*g^4*x^3 + 24*c^4*d^4*f*g^3*x^2 +
 2*c^4*d^4*f^2*g^2*x - 3*c^4*d^4*f^3*g - 3*a^3*c*d*g^4*e^3 + (2*a^2*c^2*d^2
*g^4*x + 11*a^2*c^2*d^2*f*g^3)*e^2 + (24*a*c^3*d^3*g^4*x^2 + 44*a*c^3*d^3*f
*g^3*x + 11*a*c^3*d^3*f^2*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*
e)*sqrt(g*x + f)*sqrt(x*e + d) - 3*(c^4*d^5*f^4 + a^4*g^4*x*e^5 - (4*a^3*c*
d*f*g^3*x - a^4*d*g^4)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^2*x - 2*a^3*c*d^2*f*g^3
)*e^3 - 2*(2*a*c^3*d^3*f^3*g*x - 3*a^2*c^2*d^3*f^2*g^2)*e^2 + (c^4*d^4*f^4*
x - 4*a*c^3*d^4*f^3*g)*e)*sqrt(-c*d*g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x + c*
d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c^3*d^3*g^3*x*e +
 c^3*d^4*g^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2), x)
```

3.743 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=310

$$\frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8cdg^2\sqrt{d+ex}}-\frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{4g^2\sqrt{d+ex}}$$

```
[Out] 1/3*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)+1
/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+
f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2)/g^(5/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/4*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)+1/8*(-a*e*g+c*d*f)^2*(g*x+f)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^2/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$\frac{\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}}-\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^2\sqrt{d+ex}}+\frac{(f+gx)^{3/2}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)
^(3/2),x]
```

```
[Out] ((c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(8*c*d*g^2*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(3/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*Sqrt[d + e*x]) + ((f + g*x)^(3/2)*
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e*x)^(3/2)) + ((c*
d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*c^(3/2)*d^(3/2)*g^(5/2)*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{3g(d+ex)^{3/2}} - \frac{(cdf-\ldots}{}$$

$$= -\frac{(cdf-aeg)(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\ldots}{4g^2\sqrt{d+ex}}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8cdg^2\sqrt{d+ex}}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8cdg^2\sqrt{d+ex}}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8cdg^2\sqrt{d+ex}}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8cdg^2\sqrt{d+ex}}$$

$$= \frac{(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{8cdg^2\sqrt{d+ex}}$$

Mathematica [A]
time = 0.38, size = 201, normalized size = 0.65

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,\sqrt{ae+cdx}\,\sqrt{f+gx}\,(3a^2e^2g^2+2acdeg(4f+7gx)+c^2d^2(-3f^2+2fgx+8g^2x^2))+3(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{ae+cdx}}\right)\right)}{24c^{3/2}d^{3/2}g^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d
+ e*x)^(3/2),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]
*Sqrt[f + g*x]*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + 7*g*x) + c^2*d^2*(-3*f^2
 + 2*f*g*x + 8*g^2*x^2)) + 3*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqr
t[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(24*c^(3/2)*d^(3/2)*g^(5/2)*Sqrt
[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.16, size = 504, normalized size = 1.63

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a^{3}e^{3}g^{3}-9\ln\left(\frac{2cc}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/48*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+
c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^3*e^3*g
^3-9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1
/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*f*g^2+9*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f^2*g-3*
ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/
(d*g*c)^(1/2))*c^3*d^3*f^3-16*c^2*d^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(
d*g*c)^(1/2)-28*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x-4*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*x+a*
e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2-16*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2)*a*c*d*e*f*g+6*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f^2)/
(e*x+d)^(1/2)/c/d/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(d*g*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*sqrt(g*x + f)/(x*e
+ d)^(3/2), x)
```

Fricas [A]
time = 4.35, size = 855, normalized size = 2.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 + 2*c^3*d^3*f*g^2*x - 3*c^3*d^3*f^2*g + 3*a^2*c
*d*g^3*e^2 + 2*(7*a*c^2*d^2*g^3*x + 4*a*c^2*d^2*f*g^2)*e)*sqrt(c*d^2*x + a*
x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 3*(c^3*d^4*f^3 - a
^3*g^3*x*e^4 + (3*a^2*c*d*f*g^2*x - a^3*d*g^3)*e^3 - 3*(a*c^2*d^2*f^2*g*x -
 a^2*c*d^2*f*g^2)*e^2 + (c^3*d^3*f^3*x - 3*a*c^2*d^3*f^2*g)*e)*sqrt(c*d*g)*
log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 - 4
*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c*d*f + a*g*e)*sq
rt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x +
a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (
c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^2*d^2*g^3*x*e + c^2*d^3*g
^3), 1/48*(2*(8*c^3*d^3*g^3*x^2 + 2*c^3*d^3*f*g^2*x - 3*c^3*d^3*f^2*g + 3*a
^2*c*d*g^3*e^2 + 2*(7*a*c^2*d^2*g^3*x + 4*a*c^2*d^2*f*g^2)*e)*sqrt(c*d^2*x
+ a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 3*(c^3*d^4*f^3
 - a^3*g^3*x*e^4 + (3*a^2*c*d*f*g^2*x - a^3*d*g^3)*e^3 - 3*(a*c^2*d^2*f^2*g
*x - a^2*c*d^2*f*g^2)*e^2 + (c^3*d^3*f^3*x - 3*a*c^2*d^3*f^2*g)*e)*sqrt(-c*
d*g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt
(g*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 +
 c*d*f*x + a*d*g)*e)))/(c^2*d^2*g^3*x*e + c^2*d^3*g^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\sqrt{f+gx}}{\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d
)**(3/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)*sqrt(f + g*x)/(d + e*x)**(3/2), x
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*
x)^(3/2), x)
```

3.744 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=238

$$-\frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^2\sqrt{d+ex}}+\frac{\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}+\frac{3(c\cdots}{\cdots}$$

```
[Out] 1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g/(e*x+d)^(3/2)+3
/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+
f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/c^(1/2)/d^(1/2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {878, 905, 65, 223, 212}

$$\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4\sqrt{c}\,\sqrt{d}\,g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^2\sqrt{d+ex}}+\frac{\sqrt{f+gx}\,\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sqrt[f +
 g*x]),x]
```

```
[Out] (-3*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(4*g^2*Sqrt[d + e*x]) + (Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2))/(2*g*(d + e*x)^(3/2)) + (3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(4*Sqrt[c]*Sqrt[d]*g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = \frac{\sqrt{f+gx}\,(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2g(d+ex)^{3/2}} - \frac{(3(cdf-aeg))\int \sqrt{}}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+}}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+}}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+}}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+}}{}$$

$$= -\frac{3(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+}}{}$$

Mathematica [A]
time = 0.28, size = 161, normalized size = 0.68

$$\frac{\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{g}\,(ae+cdx)\sqrt{f+gx}\,(5aeg+cd(-3f+2gx))+3(cdf-aeg)^2\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)\right)}{4\sqrt{c}\sqrt{d}\,g^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[g]*(a*e + c*d*x)*Sqrt[f + g*x]*(5*a*e*
g + c*d*(-3*f + 2*g*x)) + 3*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*ArcTanh[(Sq
rt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[c]*Sqrt
[d]*g^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 315, normalized size = 1.32

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\,\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a^{2}e^{2}g^{2}-6\ln\left(\frac{2cdgx}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)*(g*x+f)^(1/2)*(3*ln(1/2*(2*c*
d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2
))*a^2*e^2*g^2-6*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2
)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c*d*e*f*g+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*f^2+4*
((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c*d*g*x+10*(d*g*c)^(1/2)*((g*x+f)
*(c*d*x+a*e))^(1/2)*a*e*g-6*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f
)/((g*x+f)*(c*d*x+a*e))^(1/2)/g^2/(d*g*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(sqrt(g*x + f)*(x*e
 + d)^(3/2)), x)
```

Fricas [A]
time = 4.07, size = 665, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*g^2*e)*sqrt(c*d^2*x + a
*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 3*(c^2*d^3*f^2 +
a^2*g^2*x*e^3 - (2*a*c*d*f*g*x - a^2*d*g^2)*e^2 + (c^2*d^2*f^2*x - 2*a*c*d^
2*f*g)*e)*sqrt(c*d*g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f
```

```
^2 + a^2*g^2*x*e^3 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g
*x + c*d*f + a*g*e)*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*
x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x
^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c*d*g
^3*x*e + c*d^2*g^3), 1/8*(2*(2*c^2*d^2*g^2*x - 3*c^2*d^2*f*g + 5*a*c*d*g^2*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)
- 3*(c^2*d^3*f^2 + a^2*g^2*x*e^3 - (2*a*c*d*f*g*x - a^2*d*g^2)*e^2 + (c^2*d
^2*f^2*x - 2*a*c*d^2*f*g)*e)*sqrt(-c*d*g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 +
 (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x +
 c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c*d*g^3*x*e +
c*d^2*g^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(1/2),x)
```

```
[Out] Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/((d + e*x)**(3/2)*sqrt(f + g*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(1/2)*(d + e*x
)^(3/2)), x)
```

3.745 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=222

$$\frac{3cd\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}-\frac{3\sqrt{c}\,\sqrt{d}\,(cdf-aeg)\sqrt{ae}}{g^{5/2}\sqrt{}}$$

```
[Out] -2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(1/2)-3*
(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1
/2))*c^(1/2)*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+3*c*d*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/g^2/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {876, 878, 905, 65, 223, 212}

$$-\frac{3\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3cd\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (3*c*d*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt
[d + e*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(g*(d + e*x)
^(3/2)*Sqrt[f + g*x]) - (3*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f
 + g*x])])/(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}} + \frac{(3cd)\int \frac{\sqrt{ade+(cd^2+ae^2)}}{\sqrt{d+ex}\,\sqrt{f}}}{g}$$

$$= \frac{3cd\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(ade+(cd^2+ae}{g(d+ex)^{3/2}}$$

$$= \frac{3cd\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(ade+(cd^2+ae}{g(d+ex)^{3/2}}$$

$$= \frac{3cd\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(ade+(cd^2+ae}{g(d+ex)^{3/2}}$$

$$= \frac{3cd\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(ade+(cd^2+ae}{g(d+ex)^{3/2}}$$

$$= \frac{3cd\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(ade+(cd^2+ae}{g(d+ex)^{3/2}}$$

Mathematica [A]
time = 0.56, size = 148, normalized size = 0.67

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{ae+cdx}\,(-2aeg+cd(3f+gx))+3\sqrt{\frac{cd}{g}}\,(cdf-aeg)\sqrt{f+gx}\,\log\left(\sqrt{ae+cdx}-\sqrt{\frac{cd}{g}}\,\sqrt{f+gx}\right)\right)}{g^2\sqrt{(ae+cdx)(d+ex)}\,\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a*e + c*d*x]*(-2*a*e*g + c*d*(3*f +
g*x)) + 3*Sqrt[(c*d)/g]*(c*d*f - a*e*g)*Sqrt[f + g*x]*Log[Sqrt[a*e + c*d*x]
 - Sqrt[(c*d)/g]*Sqrt[f + g*x]]))/(g^2*Sqrt[(a*e + c*d*x)*(d + e*x)]*Sqrt[f
 + g*x])
```

Maple [A]
time = 0.16, size = 373, normalized size = 1.68

method	result
default	$\frac{\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)acde\,g^{2}x-3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}}{2\sqrt{dgc}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/2*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2))/(d*g*c)^(1/2))*a*c*d*e*g^2*x-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x
+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*f*g*x+3*ln(1/2
*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c
)^(1/2))*a*c*d*e*f*g-3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*f^2+2*((g*x+f)*(c*d*x+a*e))^(
1/2)*(d*g*c)^(1/2)*c*d*g*x-4*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e*
g+6*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)*((c*d*x+a*e)*(e*x+d))^
(1/2)/((g*x+f)*(c*d*x+a*e))^(1/2)/(d*g*c)^(1/2)/g^2/(g*x+f)^(1/2)/(e*x+d)^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^(3/2)*(x
*e + d)^(3/2)), x)
```

Fricas [A]
time = 2.19, size = 675, normalized size = 3.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*g*x + 3*c*d*f - 2*
a*g*e)*sqrt(g*x + f)*sqrt(x*e + d) - 3*(c*d^2*f*g*x + c*d^2*f^2 - (a*g^2*x^
```

```
2 + a*f*g*x)*e^2 + (c*d*f*g*x^2 - a*d*f*g + (c*d*f^2 - a*d*g^2)*x)*e)*sqrt(
c*d/g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*
e^3 + 4*(2*c*d*g^2*x + c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(c*d/g) + (8*a*c*d*g^2*x^2 + 6*a
*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*
c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(d*g^3*x + d*f*
g^2 + (g^3*x^2 + f*g^2*x)*e), 1/2*(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*
d)*e)*(c*d*g*x + 3*c*d*f - 2*a*g*e)*sqrt(g*x + f)*sqrt(x*e + d) + 3*(c*d^2*
f*g*x + c*d^2*f^2 - (a*g^2*x^2 + a*f*g*x)*e^2 + (c*d*f*g*x^2 - a*d*f*g + (c
*d*f^2 - a*d*g^2)*x)*e)*sqrt(-c*d/g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d
*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-c*d/g)*g/(2*c*d^2*g*x + c*
d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(d*g^3*x + d*f*g^2
 + (g^3*x^2 + f*g^2*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(f+g\,x\right)^{3/2}\,\left(d+e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(3/2)*(d + e*x
)^(3/2)), x)
```

3.746 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{2c^{3/2}d^{3/2}\sqrt{ae+cdx}\,\sqrt{d+ex}\ \mathrm{ta}}{g^{5/2}\sqrt{ade+\left(cd^2+\right.}}$$

```
[Out] -2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g/(e*x+d)^(3/2)/(g*x+f)^(3/2)+
2*c^(3/2)*d^(3/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)
^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)-2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^2/(e*x+d)^(1/2
)/(g*x+f)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {876, 905, 65, 223, 212}

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (-2*c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*Sqrt[d + e*x]*Sqr
t[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*g*(d + e
*x)^(3/2)*(f + g*x)^(3/2)) + (2*c^(3/2)*d^(3/2)*Sqrt[a*e + c*d*x]*Sqrt[d +
e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/
(g^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}} + \frac{(cd)\int \frac{\sqrt{ade+\left(cd^2+ae^2\right.}}{\sqrt{d+ex}\,(f+}}{g}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g(d+ex)^{3/2}(f+gx}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g(d+ex)^{3/2}(f+gx}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g(d+ex)^{3/2}(f+gx}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g(d+ex)^{3/2}(f+gx}$$

$$= -\frac{2cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^2\sqrt{d+ex}\,\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+c\right.}{3g(d+ex)^{3/2}(f+gx}$$

Mathematica [A]
time = 0.27, size = 152, normalized size = 0.71

$$\frac{2\sqrt{ae+cdx}\,\sqrt{d+ex}\left(-\sqrt{g}\,\sqrt{ae+cdx}\,(aeg+cd(3f+4gx))+3c^{3/2}d^{3/2}(f+gx)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{ae+cdx}}\right)\right)}{3g^{5/2}\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[g]*Sqrt[a*e + c*d*x]*(a*e*g + c*
d*(3*f + 4*g*x))) + 3*c^(3/2)*d^(3/2)*(f + g*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt
[d]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])]))/(3*g^(5/2)*Sqrt[(a*e + c*
d*x)*(d + e*x)]*(f + g*x)^(3/2))
```

Maple [A]
time = 0.15, size = 321, normalized size = 1.50

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(3\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{2\sqrt{dgc}}\right)c^2d^2g^2x^2+6\ln\left(\frac{2cdgx+aeg+cdf+2\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/3*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)
*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*g^2*x^2+6*ln(1/2*
(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)
^(1/2))*c^2*d^2*f*g*x+3*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e
))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^2*d^2*f^2-8*((g*x+f)*(c*d*x+a*e))^
(1/2)*(d*g*c)^(1/2)*c*d*g*x-2*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*a*e
*g-6*(d*g*c)^(1/2)*((g*x+f)*(c*d*x+a*e))^(1/2)*c*d*f)/(d*g*c)^(1/2)/((g*x+f
)*(c*d*x+a*e))^(1/2)/g^2/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^(5/2)*(x
*e + d)^(3/2)), x)
```

Fricas [A]
time = 1.48, size = 699, normalized size = 3.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(4*c*d*g*x + 3*c*d*f +
 a*g*e)*sqrt(g*x + f)*sqrt(x*e + d) - 3*(c*d^2*g^2*x^2 + 2*c*d^2*f*g*x + c*
d^2*f^2 + (c*d*g^2*x^3 + 2*c*d*f*g*x^2 + c*d*f^2*x)*e)*sqrt(c*d/g)*log(-(8*
c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*(2*c*d*
```

```
g^2*x + c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt
(g*x + f)*sqrt(x*e + d)*sqrt(c*d/g) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^
2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^
2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(d*g^4*x^2 + 2*d*f*g^3*x + d*f
^2*g^2 + (g^4*x^3 + 2*f*g^3*x^2 + f^2*g^2*x)*e), -1/3*(2*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*(4*c*d*g*x + 3*c*d*f + a*g*e)*sqrt(g*x + f)*sqrt(
x*e + d) + 3*(c*d^2*g^2*x^2 + 2*c*d^2*f*g*x + c*d^2*f^2 + (c*d*g^2*x^3 + 2*
c*d*f*g*x^2 + c*d*f^2*x)*e)*sqrt(-c*d/g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 +
(c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-c*d/g)*g/(2*c*d^2*g*x
+ c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(d*g^4*x^2 + 2
*d*f*g^3*x + d*f^2*g^2 + (g^4*x^3 + 2*f*g^3*x^2 + f^2*g^2*x)*e)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{{\left(f+g\,x\right)}^{5/2}\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(5/2)*(d + e*x
)^(3/2)), x)
```

3.747 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

```
[Out] 2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {874}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.83

$$\frac{2((ae+cdx)(d+ex))^{5/2}}{5(cdf-aeg)(d+ex)^{5/2}(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2))/(5*(c*d*f - a*e*g)*(d + e*x)^(5/2)*(f +
 g*x)^(5/2))
```

Maple [A]
time = 0.16, size = 55, normalized size = 0.87

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(cdx+ae)^{2}}{5\sqrt{ex+d}\,(gx+f)^{\frac{5}{2}}(aeg-cdf)}$	55
gosper	$-\frac{2(cdx+ae)(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}{5(gx+f)^{\frac{5}{2}}(aeg-cdf)(ex+d)^{\frac{3}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/5*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(5/2)*(c*d*x+a*e)^2/
(a*e*g-c*d*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^(7/2)*(x
*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(58) = 116$.
time = 0.86, size = 244, normalized size = 3.87

$$\frac{2\left(c^{2}d^{2}x^{2}+2\,acdxe+a^{2}e^{2}\right)\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{5\left(cd^{2}fg^{3}x^{3}+3\,cd^{2}f^{2}g^{2}x^{2}+3\,cd^{2}f^{3}gx+cd^{2}f^{4}-(ag^{4}x^{4}+3\,afg^{3}x^{3}+3\,af^{2}g^{2}x^{2}+af^{3}gx)e^{2}+(cdfg^{3}x^{4}-adf^{3}g+(3\,cdf^{2}g^{2}-adg^{4})x^{3}+3\,(cdf^{3}g-adfg^{3})x^{2}+(cdf^{4}-3\,adf^{2}g^{2})x)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] 2/5*(c^2*d^2*x^2 + 2*a*c*d*x*e + a^2*e^2)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c*d^2*f*g^3*x^3 + 3*c*d^2*f^2*g^2*x
^2 + 3*c*d^2*f^3*g*x + c*d^2*f^4 - (a*g^4*x^4 + 3*a*f*g^3*x^3 + 3*a*f^2*g^2
*x^2 + a*f^3*g*x)*e^2 + (c*d*f*g^3*x^4 - a*d*f^3*g + (3*c*d*f^2*g^2 - a*d*g
^4)*x^3 + 3*(c*d*f^3*g - a*d*f*g^3)*x^2 + (c*d*f^4 - 3*a*d*f^2*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.07, size = 232, normalized size = 3.68

$$-\frac{\left(\frac{2\,a^2\,e^2}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{2\,c^2\,d^2\,x^2}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{4\,a\,c\,d\,e\,x}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(5\,c\,d\,f^3-5\,a\,e\,f^2\,g\right)\,\sqrt{d+e\,x}}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}+\frac{x\,\sqrt{f+g\,x}\,\left(10\,a\,e\,f\,g^2-10\,c\,d\,f^2\,g\right)\,\sqrt{d+e\,x}}{5\,a\,e\,g^3-5\,c\,d\,f\,g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(7/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] -(((2*a^2*e^2)/(5*a*e*g^3 - 5*c*d*f*g^2) + (2*c^2*d^2*x^2)/(5*a*e*g^3 - 5*c
*d*f*g^2) + (4*a*c*d*e*x)/(5*a*e*g^3 - 5*c*d*f*g^2))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^
(1/2)*(5*c*d*f^3 - 5*a*e*f^2*g)*(d + e*x)^(1/2))/(5*a*e*g^3 - 5*c*d*f*g^2)
+ (x*(f + g*x)^(1/2)*(10*a*e*f*g^2 - 10*c*d*f^2*g)*(d + e*x)^(1/2))/(5*a*e*
g^3 - 5*c*d*f*g^2))
```

3.748 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=129

$$\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{4cd(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{5/2}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(7/2)+4/35*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^2}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(7/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(35*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
```

*p]

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{(2cd)\int\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3}}{(d+ex)^{3/2}(f+gx)^{7/2}}}{7(cdf-aeg)}$$

$$=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7(cdf-aeg)(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+c\right.}{35(cdf-aeg)^2(d+ex)^{5/2}(f}$$

Mathematica [A]
time = 0.16, size = 69, normalized size = 0.53

$$\frac{2((ae+cdx)(d+ex))^{5/2}(-5aeg+cd(7f+2gx))}{35(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-5*a*e*g + c*d*(7*f + 2*g*x)))/(35*(c*d
*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2))
```

Maple [A]
time = 0.14, size = 100, normalized size = 0.78

method	result	size
gosper	$-\frac{2(cdx+ae)(-2cdgx+5aeg-7cdf)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{\frac{3}{2}}}{35(gx+f)^{\frac{7}{2}}\left(a^2e^2g^2-2acdefg+f^2c^2d^2\right)(ex+d)^{\frac{3}{2}}}$	99
default	$-\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,\left(-2g\,x^2c^2d^2+3acdegx-7c^2d^2fx+5a^2e^2g-7acdef\right)(cdx+ae)}{35\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}(aeg-cdf)^2}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/35*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(7/2)*(-2*c^2*d^2*g
*x^2+3*a*c*d*e*g*x-7*c^2*d^2*f*x+5*a^2*e^2*g-7*a*c*d*e*f)*(c*d*x+a*e)/(a*e*
g-c*d*f)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^(9/2)*(x
*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(119) = 238$.
time = 0.72, size = 553, normalized size = 4.29

$$\frac{2\left(2c^{3}d^{3}gx^{3}+7c^{3}d^{3}fx^{2}-5a^{3}ge^{3}-\left(8a^{2}cdgx-7a^{2}cdf\right)e^{2}-\left(ac^{2}d^{2}gx^{2}-14ac^{2}d^{2}fx\right)e\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{gx+f}\sqrt{xe+d}}{35\left(c^{2}d^{3}f^{2}g^{4}x^{4}+4c^{2}d^{3}f^{3}g^{3}x^{3}+6c^{2}d^{3}f^{4}g^{2}x^{2}+4c^{2}d^{3}f^{5}gx+c^{2}d^{3}f^{6}+\left(a^{2}g^{6}x^{5}+4a^{2}fg^{5}x^{4}+6a^{2}f^{2}g^{4}x^{3}+4a^{2}f^{3}g^{3}x^{2}+a^{2}f^{4}g^{2}x\right)e^{3}-\left(2acdfg^{5}x^{5}-a^{2}df^{4}g^{2}+\left(8acdf^{2}g^{4}-a^{2}dg^{6}\right)x^{4}+4\left(3acdf^{3}g^{3}-a^{2}dfg^{5}\right)x^{3}+2\left(4acdf^{4}g^{2}-3a^{2}df^{2}g^{4}\right)x^{2}+2\left(acdf^{5}g-2a^{2}df^{3}g^{3}\right)x\right)e^{2}+\left(c^{2}d^{2}f^{2}g^{4}x^{5}-2acd^{2}f^{5}g+2\left(2c^{2}d^{2}f^{3}g^{3}-acd^{2}fg^{5}\right)x^{4}+2\left(3c^{2}d^{2}f^{4}g^{2}-4acd^{2}f^{2}g^{4}\right)x^{3}+4\left(c^{2}d^{2}f^{5}g-3acd^{2}f^{3}g^{3}\right)x^{2}+\left(c^{2}d^{2}f^{6}-8acd^{2}f^{4}g^{2}\right)x\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*c^3*d^3*g*x^3 + 7*c^3*d^3*f*x^2 - 5*a^3*g*e^3 - (8*a^2*c*d*g*x - 7*
a^2*c*d*f)*e^2 - (a*c^2*d^2*g*x^2 - 14*a*c^2*d^2*f*x)*e)*sqrt(c*d^2*x + a*x
*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^2*d^3*f^2*g^4*x^4
+ 4*c^2*d^3*f^3*g^3*x^3 + 6*c^2*d^3*f^4*g^2*x^2 + 4*c^2*d^3*f^5*g*x + c^2*d
^3*f^6 + (a^2*g^6*x^5 + 4*a^2*f*g^5*x^4 + 6*a^2*f^2*g^4*x^3 + 4*a^2*f^3*g^3
*x^2 + a^2*f^4*g^2*x)*e^3 - (2*a*c*d*f*g^5*x^5 - a^2*d*f^4*g^2 + (8*a*c*d*f
^2*g^4 - a^2*d*g^6)*x^4 + 4*(3*a*c*d*f^3*g^3 - a^2*d*f*g^5)*x^3 + 2*(4*a*c*
d*f^4*g^2 - 3*a^2*d*f^2*g^4)*x^2 + 2*(a*c*d*f^5*g - 2*a^2*d*f^3*g^3)*x)*e^2
 + (c^2*d^2*f^2*g^4*x^5 - 2*a*c*d^2*f^5*g + 2*(2*c^2*d^2*f^3*g^3 - a*c*d^2*
f*g^5)*x^4 + 2*(3*c^2*d^2*f^4*g^2 - 4*a*c*d^2*f^2*g^4)*x^3 + 4*(c^2*d^2*f^5
*g - 3*a*c*d^2*f^3*g^3)*x^2 + (c^2*d^2*f^6 - 8*a*c*d^2*f^4*g^2)*x)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.31, size = 247, normalized size = 1.91

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^2\,e^2\,\left(5\,a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c^3\,d^3\,x^3}{35\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,c^2\,d^2\,x^2\,\left(a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{4\,a\,c\,d\,e\,x\,\left(4\,a\,e\,g-7\,c\,d\,f\right)}{35\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{3\,f\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{3\,f^2\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(9/2)*(d + e*x
)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^2*e^2*(5*a*e*g - 7*c*
d*f))/(35*g^3*(a*e*g - c*d*f)^2) - (4*c^3*d^3*x^3)/(35*g^2*(a*e*g - c*d*f)^
2) + (2*c^2*d^2*x^2*(a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2) + (4*a*c*
d*e*x*(4*a*e*g - 7*c*d*f))/(35*g^3*(a*e*g - c*d*f)^2)))/(x^3*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3 + (3*f*x^2*(f
 + g*x)^(1/2)*(d + e*x)^(1/2))/g + (3*f^2*x*(f + g*x)^(1/2)*(d + e*x)^(1/2)
)/g^2)
```

3.749 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=198

$$\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{8cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{63(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{7/2}}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x+\right.}{315(cdf-aeg)^3(d+ex)^{5/2}(f}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(g
*x+f)^(9/2)+8/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(5/2)/(g*x+f)^(7/2)+16/315*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(5/2)
```

Rubi [A]
time = 0.15, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^3}+\frac{8cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(5/2)*(f + g*x)^(9/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(5/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(7/2)) + (16*c^2*d^
2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(315*(c*d*f - a*e*g)^3*(d
+ e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
```

```
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{(4cd)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3}}{(d+ex)^{3/2}(f+gx)^{9/2}}}{9(cdf-aeg)}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{8cd(ade+(cd^2+ae^2)x+c}{63(cdf-aeg)^2(d+ex)^{5/2}(f}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{9(cdf-aeg)(d+ex)^{5/2}(f+gx)^{9/2}} + \frac{8cd(ade+(cd^2+ae^2)x+c}{63(cdf-aeg)^2(d+ex)^{5/2}(f}$$

Mathematica [A]
time = 0.19, size = 113, normalized size = 0.57

$$\frac{2(ae+cdx)^3((ae+cdx)(d+ex))^{3/2}\left(35g^2-\frac{90cdg(f+gx)}{ae+cdx}+\frac{63c^2d^2(f+gx)^2}{(ae+cdx)^2}\right)}{315(cdf-aeg)^3(d+ex)^{3/2}(f+gx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*((a*e + c*d*x)*(d + e*x))^(3/2)*(35*g^2 - (90*c*d*g*(f +
 g*x))/(a*e + c*d*x) + (63*c^2*d^2*(f + g*x)^2)/(a*e + c*d*x)^2))/(315*(c*d
*f - a*e*g)^3*(d + e*x)^(3/2)*(f + g*x)^(9/2))
```

Maple [A]
time = 0.14, size = 172, normalized size = 0.87

method	result
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2-20acde\,g^2x+36c^2d^2fgx+35a^2e^2g^2-90acdefg+63f^2c^2d^2)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}{315(gx+f)^{\frac{9}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(ex+d)^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(8g^2x^3c^3d^3-12a\,c^2d^2e\,g^2x^2+36c^3d^3fg\,x^2+15a^2cd\,e^2g^2x-54a\,c^2d^2efgx+63c^3d^3f^2x+35a^3e^3g^2-}{315\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}(aeg-cdf)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/315*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(9/2)*(8*c^3*d^3*g
^2*x^3-12*a*c^2*d^2*e*g^2*x^2+36*c^3*d^3*f*g*x^2+15*a^2*c*d*e^2*g^2*x-54*a*
c^2*d^2*e*f*g*x+63*c^3*d^3*f^2*x+35*a^3*e^3*g^2-90*a^2*c*d*e^2*f*g+63*a*c^2
*d^2*e*f^2)*(c*d*x+a*e)/(a*e*g-c*d*f)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^(11/2)*(
x*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 968 vs. $2(183) = 366$.
time = 1.05, size = 968, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/315*(8*c^4*d^4*g^2*x^4 + 36*c^4*d^4*f*g*x^3 + 63*c^4*d^4*f^2*x^2 + 35*a^4
*g^2*e^4 + 10*(5*a^3*c*d*g^2*x - 9*a^3*c*d*f*g)*e^3 + 3*(a^2*c^2*d^2*g^2*x^
2 - 48*a^2*c^2*d^2*f*g*x + 21*a^2*c^2*d^2*f^2)*e^2 - 2*(2*a*c^3*d^3*g^2*x^3
 + 9*a*c^3*d^3*f*g*x^2 - 63*a*c^3*d^3*f^2*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c
*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^3*d^4*f^3*g^5*x^5 + 5*c^3*d
^4*f^4*g^4*x^4 + 10*c^3*d^4*f^5*g^3*x^3 + 10*c^3*d^4*f^6*g^2*x^2 + 5*c^3*d^
4*f^7*g*x + c^3*d^4*f^8 - (a^3*g^8*x^6 + 5*a^3*f*g^7*x^5 + 10*a^3*f^2*g^6*x
^4 + 10*a^3*f^3*g^5*x^3 + 5*a^3*f^4*g^4*x^2 + a^3*f^5*g^3*x)*e^4 + (3*a^2*c
*d*f*g^7*x^6 - a^3*d*f^5*g^3 + (15*a^2*c*d*f^2*g^6 - a^3*d*g^8)*x^5 + 5*(6*
a^2*c*d*f^3*g^5 - a^3*d*f*g^7)*x^4 + 10*(3*a^2*c*d*f^4*g^4 - a^3*d*f^2*g^6)
*x^3 + 5*(3*a^2*c*d*f^5*g^3 - 2*a^3*d*f^3*g^5)*x^2 + (3*a^2*c*d*f^6*g^2 - 5
*a^3*d*f^4*g^4)*x)*e^3 - 3*(a*c^2*d^2*f^2*g^6*x^6 - a^2*c*d^2*f^6*g^2 + (5*
a*c^2*d^2*f^3*g^5 - a^2*c*d^2*f*g^7)*x^5 + 5*(2*a*c^2*d^2*f^4*g^4 - a^2*c*d
^2*f^2*g^6)*x^4 + 10*(a*c^2*d^2*f^5*g^3 - a^2*c*d^2*f^3*g^5)*x^3 + 5*(a*c^2
*d^2*f^6*g^2 - 2*a^2*c*d^2*f^4*g^4)*x^2 + (a*c^2*d^2*f^7*g - 5*a^2*c*d^2*f^
5*g^3)*x)*e^2 + (c^3*d^3*f^3*g^5*x^6 - 3*a*c^2*d^3*f^7*g + (5*c^3*d^3*f^4*g
```

```
^4 - 3*a*c^2*d^3*f^2*g^6)*x^5 + 5*(2*c^3*d^3*f^5*g^3 - 3*a*c^2*d^3*f^3*g^5)
*x^4 + 10*(c^3*d^3*f^6*g^2 - 3*a*c^2*d^3*f^4*g^4)*x^3 + 5*(c^3*d^3*f^7*g -
6*a*c^2*d^3*f^5*g^3)*x^2 + (c^3*d^3*f^8 - 15*a*c^2*d^3*f^6*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.48, size = 377, normalized size = 1.90

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{70\,a^4\,e^4\,g^2-180\,a^3\,c\,d\,e^3\,f\,g+126\,a^2\,c^2\,d^2\,e^2\,f^2}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x^2\,\left(6\,a^2\,c^2\,d^2\,e^2\,g^2-36\,a\,c^3\,d^3\,e\,f\,g+126\,c^4\,d^4\,f^2\right)}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^4\,d^4\,x^4}{315\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c^3\,d^3\,x^3\,\left(a\,e\,g-9\,c\,d\,f\right)}{315\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{4\,a\,c\,d\,e\,x\,\left(25\,a^2\,e^2\,g^2-72\,a\,c\,d\,e\,f\,g+63\,c^2\,d^2\,f^2\right)}{315\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(11/2)*(d + e*
x)^(3/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((70*a^4*e^4*g^2 + 126*a^2*
c^2*d^2*e^2*f^2 - 180*a^3*c*d*e^3*f*g)/(315*g^4*(a*e*g - c*d*f)^3) + (x^2*(
126*c^4*d^4*f^2 + 6*a^2*c^2*d^2*e^2*g^2 - 36*a*c^3*d^3*e*f*g))/(315*g^4*(a*
e*g - c*d*f)^3) + (16*c^4*d^4*x^4)/(315*g^2*(a*e*g - c*d*f)^3) - (8*c^3*d^3
*x^3*(a*e*g - 9*c*d*f))/(315*g^3*(a*e*g - c*d*f)^3) + (4*a*c*d*e*x*(25*a^2*
e^2*g^2 + 63*c^2*d^2*f^2 - 72*a*c*d*e*f*g))/(315*g^4*(a*e*g - c*d*f)^3)))/(
x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))
/g^4 + (4*f*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/
2)*(d + e*x)^(1/2))/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.750 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx$

Optimal. Leaf size=267

$$\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}}+\frac{4cd(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{33(cdf-aeg)^2(d+ex)^{5/2}(f+gx)^{9/2}}+\frac{16c^2d^2(ade+(cd^2+ae^2)\,x\ +}{231(cdf-aeg)^3(d+ex)^{5/2}(}$$

```
[Out] 2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)/(e*x+d)^(5/2)/(
g*x+f)^(11/2)+4/33*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(5/2)/(g*x+f)^(9/2)+16/231*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(5/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(5/2)/(g*x+f)^(7/2)+32/1155*c^3*d^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(5/2)/(g*x+
f)^(5/2)
```

Rubi [A]

time = 0.22, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{32c^3d^3(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf-aeg)^4}+\frac{16c^2d^2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{33(d+ex)^{5/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(5/2)*(f + g*x)^(11/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(5/2))/(33*(c*d*f - a*e*g)^2*(d + e*x)^(5/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(231*(c*d*f - a*e*g)^3*(
d + e*x)^(5/2)*(f + g*x)^(7/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(5/2))/(1155*(c*d*f - a*e*g)^4*(d + e*x)^(5/2)*(f + g*x)^(5/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
```

```
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}}\,dx = \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{(6cd)\int \frac{(ade+(cd^2+ae^2)x+cdex}{(d+ex)^{3/2}(f+gx)^{11/}}}{11(cdf-aeg)}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd(ade+\left(cd^2+ae^2\right)x+}{33(cdf-aeg)^2(d+ex)^{5/2}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd(ade+\left(cd^2+ae^2\right)x+}{33(cdf-aeg)^2(d+ex)^{5/2}}$$

$$= \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{11(cdf-aeg)(d+ex)^{5/2}(f+gx)^{11/2}} + \frac{4cd(ade+\left(cd^2+ae^2\right)x+}{33(cdf-aeg)^2(d+ex)^{5/2}}$$

Mathematica [A]
time = 0.24, size = 141, normalized size = 0.53

$$\frac{2(ae+cdx)^4((ae+cdx)(d+ex))^{3/2}\left(-105g^3+\frac{385cdg^2(f+gx)}{ae+cdx}-\frac{495c^2d^2g(f+gx)^2}{(ae+cdx)^2}+\frac{231c^3d^3(f+gx)^3}{(ae+cdx)^3}\right)}{1155(cdf-aeg)^4(d+ex)^{3/2}(f+gx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^4*((a*e + c*d*x)*(d + e*x))^(3/2)*(-105*g^3 + (385*c*d*g^2
*(f + g*x))/(a*e + c*d*x) - (495*c^2*d^2*g*(f + g*x)^2)/(a*e + c*d*x)^2 + (
231*c^3*d^3*(f + g*x)^3)/(a*e + c*d*x)^3))/(1155*(c*d*f - a*e*g)^4*(d + e*x
)^(3/2)*(f + g*x)^(11/2))
```

Maple [A]
time = 0.14, size = 267, normalized size = 1.00

method	result
gosper	$-\frac{2(cdx+ae)\left(-16g^3x^3c^3d^3+40a\,c^2d^2e\,g^3x^2-88c^3d^3f\,g^2x^2-70a^2cd\,e^2g^3x+220a\,c^2d^2ef\,g^2x-198c^3d^3f^2gx+105a^3e^3g^3-385a^2cd\,e^2\right.}{1155(gx+f)^{\frac{11}{2}}\left(g^4e^4a^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c^4d^4\right.}$

default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-16g^3x^4c^4d^4+24a\,c^3d^3e\,g^3x^3-88c^4d^4f\,g^2x^3-30a^2c^2d^2e^2g^3x^2+132a\,c^3d^3ef\,g^2x^2-198c^4d^4f^2}{1155\sqrt{ex}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/1155*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(11/2)*(-16*c^4*d
^4*g^3*x^4+24*a*c^3*d^3*e*g^3*x^3-88*c^4*d^4*f*g^2*x^3-30*a^2*c^2*d^2*e^2*g
^3*x^2+132*a*c^3*d^3*e*f*g^2*x^2-198*c^4*d^4*f^2*g*x^2+35*a^3*c*d*e^3*g^3*x
-165*a^2*c^2*d^2*e^2*f*g^2*x+297*a*c^3*d^3*e*f^2*g*x-231*c^4*d^4*f^3*x+105*
a^4*e^4*g^3-385*a^3*c*d*e^3*f*g^2+495*a^2*c^2*d^2*e^2*f^2*g-231*a*c^3*d^3*e
*f^3)*(c*d*x+a*e)/(a*e*g-c*d*f)^4
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((g*x + f)^(13/2)*(
x*e + d)^(3/2)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1496 vs. $2(247) = 494$.
time = 1.01, size = 1496, normalized size = 5.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*c^5*d^5*g^3*x^5 + 88*c^5*d^5*f*g^2*x^4 + 198*c^5*d^5*f^2*g*x^3 +
 231*c^5*d^5*f^3*x^2 - 105*a^5*g^3*e^5 - 35*(4*a^4*c*d*g^3*x - 11*a^4*c*d*f
*g^2)*e^4 - 5*(a^3*c^2*d^2*g^3*x^2 - 110*a^3*c^2*d^2*f*g^2*x + 99*a^3*c^2*d
^2*f^2*g)*e^3 + 3*(2*a^2*c^3*d^3*g^3*x^3 + 11*a^2*c^3*d^3*f*g^2*x^2 - 264*a
^2*c^3*d^3*f^2*g*x + 77*a^2*c^3*d^3*f^3)*e^2 - (8*a*c^4*d^4*g^3*x^4 + 44*a*
c^4*d^4*f*g^2*x^3 + 99*a*c^4*d^4*f^2*g*x^2 - 462*a*c^4*d^4*f^3*x)*e)*sqrt(c
*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^4*d^5*
f^4*g^6*x^6 + 6*c^4*d^5*f^5*g^5*x^5 + 15*c^4*d^5*f^6*g^4*x^4 + 20*c^4*d^5*f
^7*g^3*x^3 + 15*c^4*d^5*f^8*g^2*x^2 + 6*c^4*d^5*f^9*g*x + c^4*d^5*f^10 + (a
```

```
^4*g^10*x^7 + 6*a^4*f*g^9*x^6 + 15*a^4*f^2*g^8*x^5 + 20*a^4*f^3*g^7*x^4 + 1
5*a^4*f^4*g^6*x^3 + 6*a^4*f^5*g^5*x^2 + a^4*f^6*g^4*x)*e^5 - (4*a^3*c*d*f*g
^9*x^7 - a^4*d*f^6*g^4 + (24*a^3*c*d*f^2*g^8 - a^4*d*g^10)*x^6 + 6*(10*a^3*
c*d*f^3*g^7 - a^4*d*f*g^9)*x^5 + 5*(16*a^3*c*d*f^4*g^6 - 3*a^4*d*f^2*g^8)*x
^4 + 20*(3*a^3*c*d*f^5*g^5 - a^4*d*f^3*g^7)*x^3 + 3*(8*a^3*c*d*f^6*g^4 - 5*
a^4*d*f^4*g^6)*x^2 + 2*(2*a^3*c*d*f^7*g^3 - 3*a^4*d*f^5*g^5)*x)*e^4 + 2*(3*
a^2*c^2*d^2*f^2*g^8*x^7 - 2*a^3*c*d^2*f^7*g^3 + 2*(9*a^2*c^2*d^2*f^3*g^7 -
a^3*c*d^2*f*g^9)*x^6 + 3*(15*a^2*c^2*d^2*f^4*g^6 - 4*a^3*c*d^2*f^2*g^8)*x^5
 + 30*(2*a^2*c^2*d^2*f^5*g^5 - a^3*c*d^2*f^3*g^7)*x^4 + 5*(9*a^2*c^2*d^2*f^
6*g^4 - 8*a^3*c*d^2*f^4*g^6)*x^3 + 6*(3*a^2*c^2*d^2*f^7*g^3 - 5*a^3*c*d^2*f
^5*g^5)*x^2 + 3*(a^2*c^2*d^2*f^8*g^2 - 4*a^3*c*d^2*f^6*g^4)*x)*e^3 - 2*(2*a
*c^3*d^3*f^3*g^7*x^7 - 3*a^2*c^2*d^3*f^8*g^2 + 3*(4*a*c^3*d^3*f^4*g^6 - a^2
*c^2*d^3*f^2*g^8)*x^6 + 6*(5*a*c^3*d^3*f^5*g^5 - 3*a^2*c^2*d^3*f^3*g^7)*x^5
 + 5*(8*a*c^3*d^3*f^6*g^4 - 9*a^2*c^2*d^3*f^4*g^6)*x^4 + 30*(a*c^3*d^3*f^7*
g^3 - 2*a^2*c^2*d^3*f^5*g^5)*x^3 + 3*(4*a*c^3*d^3*f^8*g^2 - 15*a^2*c^2*d^3*
f^6*g^4)*x^2 + 2*(a*c^3*d^3*f^9*g - 9*a^2*c^2*d^3*f^7*g^3)*x)*e^2 + (c^4*d^
4*f^4*g^6*x^7 - 4*a*c^3*d^4*f^9*g + 2*(3*c^4*d^4*f^5*g^5 - 2*a*c^3*d^4*f^3*
g^7)*x^6 + 3*(5*c^4*d^4*f^6*g^4 - 8*a*c^3*d^4*f^4*g^6)*x^5 + 20*(c^4*d^4*f^
7*g^3 - 3*a*c^3*d^4*f^5*g^5)*x^4 + 5*(3*c^4*d^4*f^8*g^2 - 16*a*c^3*d^4*f^6*
g^4)*x^3 + 6*(c^4*d^4*f^9*g - 10*a*c^3*d^4*f^7*g^3)*x^2 + (c^4*d^4*f^10 - 2
4*a*c^3*d^4*f^8*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.83, size = 519, normalized size = 1.94

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{210\,a^5\,e^5\,g^3-770\,a^4\,c\,d\,e^4\,f\,g^2+990\,a^3\,c^2\,d^2\,e^3\,f^2\,g-462\,a^2\,c^3\,d^3\,e^2\,f^3}{1155\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{x^2\,\left(-10\,a^3\,c^2\,d^2\,e^3\,g^3+66\,a^2\,c^3\,d^3\,e^2\,f\,g^2-198\,a\,c^4\,d^4\,e\,f^2\,g+462\,c^5\,d^5\,f^3\right)}{1155\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{32\,c^5\,d^5\,x^5}{1155\,g^2\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{4\,c^3\,d^3\,x^3\,\left(3\,a^2\,e^2\,g^2-22\,a\,c\,d\,e\,f\,g+99\,c^2\,d^2\,f^2\right)}{1155\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{16\,c^4\,d^4\,x^4\,\left(a\,e\,g-11\,c\,d\,f\right)}{1155\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{4\,a\,c\,d\,e\,x\,\left(70\,a^3\,e^3\,g^3-275\,a^2\,c\,d\,e^2\,f\,g^2+396\,a\,c^2\,d^2\,e\,f^2\,g-231\,c^3\,d^3\,f^3\right)}{1155\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{5\,f\,x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{5\,f^4\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{10\,f^2\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{10\,f^3\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/((f + g*x)^(13/2)*(d + e*x)^(3/2)),x)

[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((210*a^5*e^5*g^3 - 462*a^2*c^3*d^3*e^2*f^3 + 990*a^3*c^2*d^2*e^3*f^2*g - 770*a^4*c*d*e^4*f*g^2)/(1155*g^5*(a*e*g - c*d*f)^4) - (x^2*(462*c^5*d^5*f^3 - 10*a^3*c^2*d^2*e^3*g^3 + 66*a^2*c^3*d^3*e^2*f*g^2 - 198*a*c^4*d^4*e*f^2*g))/(1155*g^5*(a*e*g - c*d*f)^4) - (32*c^5*d^5*x^5)/(1155*g^2*(a*e*g - c*d*f)^4) - (4*c^3*d^3*x^3*(3*a^2*e^2*g^2 + 99*c^2*d^2*f^2 - 22*a*c*d*e*f*g))/(1155*g^4*(a*e*g - c*d*f)^4) + (16*c^4*d^4*x^4*(a*e*g - 11*c*d*f))/(1155*g^3*(a*e*g - c*d*f)^4) + (4*a*c*d*e*x*(70*a^3*e^3*g^3 - 231*c^3*d^3*f^3 + 396*a*c^2*d^2*e*f^2*g - 275*a^2*c*d*e^2*f*g^2))/(1155*g^5*(a*e*g - c*d*f)^4)))/(x^5*(f + g*x)^(1/2)*(d + e*x)^(1/2) + (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (10*f^2*x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^3)

3.751 $\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=448

$$-\frac{3(cdf-aeg)^4\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2g^3\sqrt{d+ex}}-\frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+\cdots}}{64cdg^3\sqrt{d+ex}}$$

```
[Out] -1/8*(-a*e*g+c*d*f)*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g
^2/(e*x+d)^(3/2)+1/5*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/
g/(e*x+d)^(5/2)-3/128*(-a*e*g+c*d*f)^5*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(5/2)/d^(5/2
)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/64*(-a*e*g+c*d*f)^3*(g*
x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g^3/(e*x+d)^(1/2)+1/
16*(-a*e*g+c*d*f)^2*(g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g
^3/(e*x+d)^(1/2)-3/128*(-a*e*g+c*d*f)^4*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/c^2/d^2/g^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$-\frac{3\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^5\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{3\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^4}{128c^2d^2g^3\sqrt{d+ex}}-\frac{(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}}+\frac{(f+gx)^{5/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{16g^3\sqrt{d+ex}}-\frac{(f+gx)^{5/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{8g^2(d+ex)^{3/2}}+\frac{(f+gx)^{5/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x]
```

```
[Out] (-3*(c*d*f - a*e*g)^4*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(128*c^2*d^2*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)^3*(f + g*x)^(3/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (
(c*d*f - a*e*g)^2*(f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(16*g^3*Sqrt[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^(5/2)*(a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(5/2
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)) - (3
*(c*d*f - a*e*g)^5*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*
e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(128*c^(5/2)*d^(5/2)*g^(7/2)*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}} - \frac{(cdf-}{}$$

$$= -\frac{(cdf-aeg)(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{8g^2(d+ex)^{3/2}}$$

$$= \frac{(cdf-aeg)^2(f+gx)^{5/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{16g^3\sqrt{d+ex}}$$

$$= -\frac{(cdf-aeg)^3(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{64cdg^3\sqrt{d+ex}}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{128c^2d^2g^3\sqrt{d+ex}}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{128c^2d^2g^3\sqrt{d+ex}}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{128c^2d^2g^3\sqrt{d+ex}}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{128c^2d^2g^3\sqrt{d+ex}}$$

$$= -\frac{3(cdf-aeg)^4\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}{128c^2d^2g^3\sqrt{d+ex}}$$

Mathematica [A]
time = 0.92, size = 265, normalized size = 0.59

$$\frac{(cdf-aeg)^5((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{g}(f+gx)^{9/2}\left(15c^4d^4-\frac{15g^4(ae+cdx)^4}{(f+gx)^4}+\frac{70cdg^3(ae+cdx)^3}{(f+gx)^3}+\frac{128c^2d^2g^2(ae+cdx)^2}{(f+gx)^2}-\frac{70c^3d^3g(ae+cdx)}{f+gx}\right)}{(cdf-aeg)^5(ae+cdx)^2}-\frac{15\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{(ae+cdx)^{5/2}}\right)}{640c^{5/2}d^{5/2}g^{7/2}(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/2),x]

[Out] ((c*d*f - a*e*g)^5*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g]*(f + g*x)^(9/2)*(15*c^4*d^4 - (15*g^4*(a*e + c*d*x)^4)/(f + g*x)^4 + (70*c*d*g^3*(a*e + c*d*x)^3)/(f + g*x)^3 + (128*c^2*d^2*g^2*(a*e + c*d*x)^2)/(f + g*x)^2 - (70*c^3*d^3*g*(a*e + c*d*x))/(f + g*x)))/((c*d*f - a*e*g)^5*(a*e + c*d*x)^2) - (15*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(a*e + c*d*x)^(5/2)))/(640*c^(5/2)*d^(5/2)*g^(7/2)*(d + e*x)^(5/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(384) = 768$.

time = 0.14, size = 1005, normalized size = 2.24

method	result
default	$\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(256c^4d^4g^4x^4\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}+672a\,c^3d^3e\,g^4x^3\sqrt{(gx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/1280*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(256*c^4*d^4*g^4*x^4*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+672*a*c^3*d^3*e*g^4*x^3*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+352*c^4*d^4*f*g^3*x^3*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^5*e^5*g^5-75*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^4*c*d*e^4*f*g^4+150*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^3*c^2*d^2*e^3*f^2*g^3-150*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*c^3*d^3*e^2*f^3*g^2+75*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^4*d^4*e*f^4*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^5*d^5*f^5+496*a^2*c^2*d^2*e^2*g^4*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+1024*a*c^3*d^3*e*f*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+16*c^4*d^4*f^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^3*c*d*e^3*g^4*x+932*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*c^2*d^2*e^2*f*g^3*x+92*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c^3*d^3*e*f^2*g^2*x-20*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^4*d^4*f^3*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^4*e^4*g^4+140*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^3*c*d*e^3*f*g^3+256*a^2*c^2*d^2*e^2*f^2*g^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)-140*((g*x

```
+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c^3*d^3*e*f^3*g+30*((g*x+f)*(c*d*x+a
*e))^(1/2)*(d*g*c)^(1/2)*c^4*d^4*f^4)/(e*x+d)^(1/2)/c^2/d^2/g^3/((g*x+f)*(c
*d*x+a*e))^(1/2)/(d*g*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^(3/2)/(x*
e + d)^(5/2), x)
```

Fricas [A]
time = 6.33, size = 1331, normalized size = 2.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/2560*(4*(128*c^5*d^5*g^5*x^4 + 176*c^5*d^5*f*g^4*x^3 + 8*c^5*d^5*f^2*g^3
*x^2 - 10*c^5*d^5*f^3*g^2*x + 15*c^5*d^5*f^4*g - 15*a^4*c*d*g^5*e^4 + 10*(a
^3*c^2*d^2*g^5*x + 7*a^3*c^2*d^2*f*g^4)*e^3 + 2*(124*a^2*c^3*d^3*g^5*x^2 +
233*a^2*c^3*d^3*f*g^4*x + 64*a^2*c^3*d^3*f^2*g^3)*e^2 + 2*(168*a*c^4*d^4*g^
5*x^3 + 256*a*c^4*d^4*f*g^4*x^2 + 23*a*c^4*d^4*f^2*g^3*x - 35*a*c^4*d^4*f^3
*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e
 + d) - 15*(c^5*d^6*f^5 - a^5*g^5*x*e^6 + (5*a^4*c*d*f*g^4*x - a^5*d*g^5)*e
^5 - 5*(2*a^3*c^2*d^2*f^2*g^3*x - a^4*c*d^2*f*g^4)*e^4 + 10*(a^2*c^3*d^3*f^
3*g^2*x - a^3*c^2*d^3*f^2*g^3)*e^3 - 5*(a*c^4*d^4*f^4*g*x - 2*a^2*c^3*d^4*f
^3*g^2)*e^2 + (c^5*d^5*f^5*x - 5*a*c^4*d^5*f^4*g)*e)*sqrt(c*d*g)*log(-(8*c^
2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*sqrt(c*d^
2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c*d*f + a*g*e)*sqrt(c*d*g)*
sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)
*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^
2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^3*d^3*g^4*x*e + c^3*d^4*g^4), 1/128
0*(2*(128*c^5*d^5*g^5*x^4 + 176*c^5*d^5*f*g^4*x^3 + 8*c^5*d^5*f^2*g^3*x^2 -
 10*c^5*d^5*f^3*g^2*x + 15*c^5*d^5*f^4*g - 15*a^4*c*d*g^5*e^4 + 10*(a^3*c^2
*d^2*g^5*x + 7*a^3*c^2*d^2*f*g^4)*e^3 + 2*(124*a^2*c^3*d^3*g^5*x^2 + 233*a^
2*c^3*d^3*f*g^4*x + 64*a^2*c^3*d^3*f^2*g^3)*e^2 + 2*(168*a*c^4*d^4*g^5*x^3
+ 256*a*c^4*d^4*f*g^4*x^2 + 23*a*c^4*d^4*f^2*g^3*x - 35*a*c^4*d^4*f^3*g^2)*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)
```

```
+ 15*(c^5*d^6*f^5 - a^5*g^5*x*e^6 + (5*a^4*c*d*f*g^4*x - a^5*d*g^5)*e^5 - 5
*(2*a^3*c^2*d^2*f^2*g^3*x - a^4*c*d^2*f*g^4)*e^4 + 10*(a^2*c^3*d^3*f^3*g^2*
x - a^3*c^2*d^3*f^2*g^3)*e^3 - 5*(a*c^4*d^4*f^4*g*x - 2*a^2*c^3*d^4*f^3*g^2
)*e^2 + (c^5*d^5*f^5*x - 5*a*c^4*d^5*f^4*g)*e)*sqrt(-c*d*g)*arctan(2*sqrt(c
*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x*e +
 d)/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)
))/(c^3*d^3*g^4*x*e + c^3*d^4*g^4)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{(d+e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x)
```

```
[Out] int(((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2), x)
```

3.752 $\int \frac{\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{5(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64cdg^3\sqrt{d+ex}}+\frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+\left(cd^2+ae^2\right)x+}}{32g^3\sqrt{d+ex}}$$

```
[Out] -5/24*(-a*e*g+c*d*f)*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/
g^2/(e*x+d)^(3/2)+1/4*(g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
/g/(e*x+d)^(5/2)-5/64*(-a*e*g+c*d*f)^4*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/c^(3/2)/d^(3/2
)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/32*(-a*e*g+c*d*f)^2*(g*
x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)-5/64*(
-a*e*g+c*d*f)^3*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g
^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {878, 884, 905, 65, 223, 212}

$$-\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^4\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}}+\frac{5(f+gx)^{3/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{32g^3\sqrt{d+ex}}-\frac{5(f+gx)^{3/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{24g^2(d+ex)^{3/2}}+\frac{(f+gx)^{3/2}\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{4g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)
^(5/2),x]
```

```
[Out] (-5*(c*d*f - a*e*g)^3*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(64*c*d*g^3*Sqrt[d + e*x]) + (5*(c*d*f - a*e*g)^2*(f + g*x)^(3/2)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*Sqrt[d + e*x]) - (5*(c*d*
f - a*e*g)*(f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(
24*g^2*(d + e*x)^(3/2)) + ((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(5/2))/(4*g*(d + e*x)^(5/2)) - (5*(c*d*f - a*e*g)^4*Sqrt[a*e + c*d*
x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
f + g*x])])/(64*c^(3/2)*d^(3/2)*g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{4g(d+ex)^{5/2}} - \frac{(5(cdf -}{}$$

$$= -\frac{5(cdf-aeg)(f+gx)^{3/2}\,(ade+(cd^2+ae^2)\,x+cdex^2)}{24g^2(d+ex)^{3/2}}$$

$$= \frac{5(cdf-aeg)^2(f+gx)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{32g^3\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64cdg^3\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64cdg^3\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64cdg^3\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64cdg^3\sqrt{d+ex}}$$

$$= -\frac{5(cdf-aeg)^3\sqrt{f+gx}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64cdg^3\sqrt{d+ex}}$$

Mathematica [A]
time = 0.61, size = 235, normalized size = 0.62

$$\frac{(cdf-aeg)^4((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{g}\,(ae+cdx)\sqrt{f+gx}\,\left(15g^3+\frac{73cdg^2(f+gx)}{ae+cdx}-\frac{55c^2d^2g(f+gx)^2}{(ae+cdx)^2}+\frac{15c^3d^3(f+gx)^3}{(ae+cdx)^3}\right)}{(cdf-aeg)^4}-\frac{15\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{ae+cdx}}\right)}{(ae+cdx)^{5/2}}\right)}{192c^{3/2}d^{3/2}g^{7/2}(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d
+ e*x)^(5/2),x]
```

```
[Out] ((c*d*f - a*e*g)^4*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[g
]*(a*e + c*d*x)*Sqrt[f + g*x]*(15*g^3 + (73*c*d*g^2*(f + g*x))/(a*e + c*d*x
) - (55*c^2*d^2*g*(f + g*x)^2)/(a*e + c*d*x)^2 + (15*c^3*d^3*(f + g*x)^3)/(
a*e + c*d*x)^3))/(c*d*f - a*e*g)^4 - (15*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f +
g*x])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(a*e + c*d*x)^(5/2)))/(192*c^(3/2)*d^(3
/2)*g^(7/2)*(d + e*x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(320) = 640$.

time = 0.14, size = 732, normalized size = 1.95

method	result
default	$-\frac{\sqrt{gx+f}\,\sqrt{(cdx+ae)(ex+d)}\,\left(-96c^3d^3g^3x^3\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}+15\ln\left(\frac{2cdgx+aeg+cdf+2\cdots}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/384*(g*x+f)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-96*c^3*d^3*g^3*x^3*((g*x
+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^4*e^4*g^4-60*ln(1/
2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*
c)^(1/2))*a^3*c*d*e^3*f*g^3+90*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*
d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*c^2*d^2*e^2*f^2*g^2-60*ln
(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d
*g*c)^(1/2))*a*c^3*d^3*e*f^3*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*
(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^4*d^4*f^4-272*a*c^2*d^2*
e*g^3*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)-16*c^3*d^3*f*g^2*x^2*((
g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)-236*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*
g*c)^(1/2)*a^2*c*d*e^2*g^3*x-72*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a
*c^2*d^2*e*f*g^2*x+20*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^3*d^3*f^2
*g*x-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^3*e^3*g^3-146*((g*x+f)*
(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*c*d*e^2*f*g^2+110*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(d*g*c)^(1/2)*a*c^2*d^2*e*f^2*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g
*c)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/c/d/((g*x+f)*(c*d*x+a*e))^(1/2)/g^3/(d
*g*c)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)/(x*e
+ d)^(5/2), x)
```

Fricas [A]
time = 2.43, size = 1073, normalized size = 2.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(48*c^4*d^4*g^4*x^3 + 8*c^4*d^4*f*g^3*x^2 - 10*c^4*d^4*f^2*g^2*x
+ 15*c^4*d^4*f^3*g + 15*a^3*c*d*g^4*e^3 + (118*a^2*c^2*d^2*g^4*x + 73*a^2*c
^2*d^2*f*g^3)*e^2 + (136*a*c^3*d^3*g^4*x^2 + 36*a*c^3*d^3*f*g^3*x - 55*a*c^
3*d^3*f^2*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)
*sqrt(x*e + d) + 15*(c^4*d^5*f^4 + a^4*g^4*x*e^5 - (4*a^3*c*d*f*g^3*x - a^4
*d*g^4)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^2*x - 2*a^3*c*d^2*f*g^3)*e^3 - 2*(2*a*
c^3*d^3*f^3*g*x - 3*a^2*c^2*d^3*f^2*g^2)*e^2 + (c^4*d^4*f^4*x - 4*a*c^3*d^4
*f^3*g)*e)*sqrt(c*d*g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*
f^2 + a^2*g^2*x*e^3 - 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*
g*x + c*d*f + a*g*e)*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2
*x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*
x^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c^2*
d^2*g^4*x*e + c^2*d^3*g^4), 1/384*(2*(48*c^4*d^4*g^4*x^3 + 8*c^4*d^4*f*g^3*
x^2 - 10*c^4*d^4*f^2*g^2*x + 15*c^4*d^4*f^3*g + 15*a^3*c*d*g^4*e^3 + (118*a
^2*c^2*d^2*g^4*x + 73*a^2*c^2*d^2*f*g^3)*e^2 + (136*a*c^3*d^3*g^4*x^2 + 36*
a*c^3*d^3*f*g^3*x - 55*a*c^3*d^3*f^2*g^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*
x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 15*(c^4*d^5*f^4 + a^4*g^4*x*e^5
 - (4*a^3*c*d*f*g^3*x - a^4*d*g^4)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^2*x - 2*a^3
*c*d^2*f*g^3)*e^3 - 2*(2*a*c^3*d^3*f^3*g*x - 3*a^2*c^2*d^3*f^2*g^2)*e^2 + (
c^4*d^4*f^4*x - 4*a*c^3*d^4*f^3*g)*e)*sqrt(-c*d*g)*arctan(2*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g)*sqrt(g*x + f)*sqrt(x*e + d)/(2*c*
d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(c^2*d
^2*g^4*x*e + c^2*d^3*g^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d
)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2),x)
```

```
[Out] int(((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*
x)^(5/2), x)
```

3.753 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=304

$$\frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}}-\frac{5(cdf-aeg)\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{12g^2(d+ex)^{3/2}}$$

[Out] -5/12*(-a*e*g+c*d*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g^2/(e*x+d)^(3/2)+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*(g*x+f)^(1/2)/g/(e*x+d)^(5/2)-5/8*(-a*e*g+c*d*f)^3*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/c^(1/2)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/8*(-a*e*g+c*d*f)^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)

Rubi [A]
time = 0.31, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {878, 905, 65, 223, 212}

$$-\frac{5\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{8\sqrt{c}\,\sqrt{d}\,g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2}{8g^3\sqrt{d+ex}}-\frac{5\sqrt{f+gx}\,\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}(cdf-aeg)}{12g^2(d+ex)^{3/2}}+\frac{\sqrt{f+gx}\,\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]

[Out] (5*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*Sqrt[d + e*x]) - (5*(c*d*f - a*e*g)*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(12*g^2*(d + e*x)^(3/2)) + (Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(3*g*(d + e*x)^(5/2)) - (5*(c*d*f - a*e*g)^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(8*Sqrt[c]*Sqrt[d]*g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx=\frac{\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}}-\frac{(5(cdf-aeg))\int\frac{(ade}{(}}{6}$$

$$=-\frac{5(cdf-aeg)\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12g^2(d+ex)^{3/2}}+\frac{\sqrt{f+}}{}$$

$$=\frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}}-\frac{5(cdf-}{}$$

$$=\frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}}-\frac{5(cdf-}{}$$

$$=\frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}}-\frac{5(cdf-}{}$$

$$=\frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}}-\frac{5(cdf-}{}$$

$$=\frac{5(cdf-aeg)^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8g^3\sqrt{d+ex}}-\frac{5(cdf-}{}$$

Mathematica [A]
time = 0.35, size = 191, normalized size = 0.63

$$\frac{((ae+cdx)(d+ex))^{5/2}\left(\sqrt{g}\,\sqrt{ae+cdx}\,\sqrt{f+gx}\,\left(33a^2e^2g^2+2acdeg(-20f+13gx)+c^2d^2\left(15f^2-10fgx+8g^2x^2\right)\right)-\frac{15(cdf-aeg)^3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{ae+cdx}}\right)}{\sqrt{c}\,\sqrt{d}}\right)}{24g^{7/2}(ae+cdx)^{5/2}(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sq
rt[f + g*x]),x]
```

```
[Out] (((a*e + c*d*x)*(d + e*x))^(5/2)*(Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[f + g*x]*(
33*a^2*e^2*g^2 + 2*a*c*d*e*g*(-20*f + 13*g*x) + c^2*d^2*(15*f^2 - 10*f*g*x
 + 8*g^2*x^2)) - (15*(c*d*f - a*e*g)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x
```

```
])/(Sqrt[g]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d])))/(24*g^(7/2)*(a*e + c*d
*x)^(5/2)*(d + e*x)^(5/2))
```

Maple [A]
time = 0.14, size = 498, normalized size = 1.64

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\,\sqrt{gx+f}\,\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a^3e^3g^3-45\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/48*((c*d*x+a*e)*(e*x+d))^(1/2)*(g*x+f)^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+
c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^3*e^3*g
^3-45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(
1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*f*g^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*(
(g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f^2*g-
15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2
))/(d*g*c)^(1/2))*c^3*d^3*f^3+16*c^2*d^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2
)*(d*g*c)^(1/2)+52*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x-
20*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f*g*x+66*((g*x+f)*(c*d
*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2-80*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*
g*c)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2
*f^2)/(e*x+d)^(1/2)/g^3/((g*x+f)*(c*d*x+a*e))^(1/2)/(d*g*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(sqrt(g*x + f)*(x*e
 + d)^(5/2)), x)
```

Fricas [A]
time = 1.96, size = 843, normalized size = 2.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(8*c^3*d^3*g^3*x^2 - 10*c^3*d^3*f*g^2*x + 15*c^3*d^3*f^2*g + 33*a^
2*c*d*g^3*e^2 + 2*(13*a*c^2*d^2*g^3*x - 20*a*c^2*d^2*f*g^2)*e)*sqrt(c*d^2*x
 + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^3*d^4*f
^3 - a^3*g^3*x*e^4 + (3*a^2*c*d*f*g^2*x - a^3*d*g^3)*e^3 - 3*(a*c^2*d^2*f^2
*g*x - a^2*c*d^2*f*g^2)*e^2 + (c^3*d^3*f^3*x - 3*a*c^2*d^3*f^2*g)*e)*sqrt(c
*d*g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e
^3 + 4*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(2*c*d*g*x + c*d*f + a*g
*e)*sqrt(c*d*g)*sqrt(g*x + f)*sqrt(x*e + d) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*
g*x + a^2*d*g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f
*g + (c^2*d^2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(c*d*g^4*x*e + c*d^2*g
^4), 1/48*(2*(8*c^3*d^3*g^3*x^2 - 10*c^3*d^3*f*g^2*x + 15*c^3*d^3*f^2*g + 3
3*a^2*c*d*g^3*e^2 + 2*(13*a*c^2*d^2*g^3*x - 20*a*c^2*d^2*f*g^2)*e)*sqrt(c*d
^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 15*(c^3*d
^4*f^3 - a^3*g^3*x*e^4 + (3*a^2*c*d*f*g^2*x - a^3*d*g^3)*e^3 - 3*(a*c^2*d^2
*f^2*g*x - a^2*c*d^2*f*g^2)*e^2 + (c^3*d^3*f^3*x - 3*a*c^2*d^3*f^2*g)*e)*sq
rt(-c*d*g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(-c*d*g
)*sqrt(g*x + f)*sqrt(x*e + d)/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g
*x^2 + c*d*f*x + a*d*g)*e)))/(c*d*g^4*x*e + c*d^2*g^4)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*x
)^(5/2)),x)

[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(1/2)*(d + e*x
)^(5/2)), x)

3.754 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=294

$$-\frac{15cd(cdf-aeg)\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+gx}\,\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}}$$

```
[Out] -2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(1/2)+5/
2*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*(g*x+f)^(1/2)/g^2/(e*x+d)^(3/
2)+15/4*(-a*e*g+c*d*f)^2*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/
(g*x+f)^(1/2))*c^(1/2)*d^(1/2)*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-15/4*c*d*(-a*e*g+c*d*f)*(g*x+f)^(1/2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {876, 878, 905, 65, 223, 212}

$$\frac{15\sqrt{c}\,\sqrt{d}\,\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{15cd\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)}{4g^3\sqrt{d+ex}}+\frac{5cd\sqrt{f+gx}\,\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}}-\frac{2\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(3/2)),x]
```

```
[Out] (-15*c*d*(c*d*f - a*e*g)*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(4*g^3*Sqrt[d + e*x]) + (5*c*d*Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2)^(3/2))/(2*g^2*(d + e*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2)^(5/2))/(g*(d + e*x)^(5/2)*Sqrt[f + g*x]) + (15*Sqrt[c]*
Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*
Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(4*g^(7/2)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{(d+ex)^{3/2}\sqrt{f+gx}}}{g}$$

$$= \frac{5cd\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2\left(ade+\left(cd^2+a\right.\right.}{g(d+ex)^{5/}}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd}{}$$

$$= -\frac{15cd(cdf-aeg)\sqrt{f+gx}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4g^3\sqrt{d+ex}} + \frac{5cd}{}$$

Mathematica [A]
time = 0.38, size = 199, normalized size = 0.68

$$\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}\left(-8a^2e^2g^2+acdeg(25f+9gx)+c^2d^2\left(-15f^2-5fgx+2g^2x^2\right)\right)+15\sqrt{c}\sqrt{d}\,(cdf-aeg)^2\sqrt{f+gx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)\right)}{4g^{7/2}\sqrt{(ae+cdx)(d+ex)}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(3/2)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[g]*Sqrt[a*e + c*d*x]*(-8*a^2*e^2*g^2
 + a*c*d*e*g*(25*f + 9*g*x) + c^2*d^2*(-15*f^2 - 5*f*g*x + 2*g^2*x^2)) + 15
*Sqrt[c]*Sqrt[d]*(c*d*f - a*e*g)^2*Sqrt[f + g*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
+ c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])]))/(4*g^(7/2)*Sqrt[(a*e + c*d*x)*
(d + e*x)]*Sqrt[f + g*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(248) = 496$.

time = 0.16, size = 625, normalized size = 2.13

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a^{2}cd\,e^{2}g^{3}x-30\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+}}{2\sqrt{dgc}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/8*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)
^(1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*g^3*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2
*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f*g^
2*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2))/(d*g*c)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((
g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*c*d*e^2*f*g^2-3
0*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)
)/(d*g*c)^(1/2))*a*c^2*d^2*e*f^2*g+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x
+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*f^3+4*c^2*d^2*
g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)+18*((g*x+f)*(c*d*x+a*e))^
(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x-10*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1
/2)*c^2*d^2*f*g*x-16*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2+
50*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*f*g-30*((g*x+f)*(c*d*x
+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)/((g*x+f
)*(c*d*x+a*e))^(1/2)/(d*g*c)^(1/2)/g^3/(g*x+f)^(1/2)/(e*x+d)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^(3/2)*(x
*e + d)^(5/2)), x)
```

Fricas [A]

time = 1.88, size = 931, normalized size = 3.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(2*c^2*d^2*g^2*x^2 - 5*c^2*d^2*f*g*x - 15*c^2*d^2*f^2 - 8*a^2*g^2*
e^2 + (9*a*c*d*g^2*x + 25*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 +
 a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 15*(c^2*d^3*f^2*g*x + c^2*d^3*f^3 +
(a^2*g^3*x^2 + a^2*f*g^2*x)*e^3 - (2*a*c*d*f*g^2*x^2 - a^2*d*f*g^2 + (2*a*c
*d*f^2*g - a^2*d*g^3)*x)*e^2 + (c^2*d^2*f^2*g*x^2 - 2*a*c*d^2*f^2*g + (c^2*
d^2*f^3 - 2*a*c*d^2*f*g^2)*x)*e)*sqrt(c*d/g)*log(-(8*c^2*d^3*g^2*x^2 + 8*c^
2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*(2*c*d*g^2*x + c*d*f*g + a*g^
2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d
)*sqrt(c*d/g) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)*e^2 + (8*c^2*
d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^2 + 8*a*c*d^2*
g^2)*x)*e)/(x*e + d)))/(d*g^4*x + d*f*g^3 + (g^4*x^2 + f*g^3*x)*e), 1/8*(2*
(2*c^2*d^2*g^2*x^2 - 5*c^2*d^2*f*g*x - 15*c^2*d^2*f^2 - 8*a^2*g^2*e^2 + (9*
a*c*d*g^2*x + 25*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*
sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^2*d^3*f^2*g*x + c^2*d^3*f^3 + (a^2*g^3*
x^2 + a^2*f*g^2*x)*e^3 - (2*a*c*d*f*g^2*x^2 - a^2*d*f*g^2 + (2*a*c*d*f^2*g
- a^2*d*g^3)*x)*e^2 + (c^2*d^2*f^2*g*x^2 - 2*a*c*d^2*f^2*g + (c^2*d^2*f^3 -
 2*a*c*d^2*f*g^2)*x)*e)*sqrt(-c*d/g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d
*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(-c*d/g)*g/(2*c*d^2*g*x + c*
d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(d*g^4*x + d*f*g^3
 + (g^4*x^2 + f*g^3*x)*e)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{\left(f+g\,x\right)^{3/2}\,\left(d+e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(3/2)*(d + e*x
)^(5/2)), x)
```

3.755 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx$

Optimal. Leaf size=284

$$\frac{5c^2d^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}-\frac{2\left(ade+\left(cd^2+ae^2\right)\right.}{3g(d+ex)^{5/2}(f}$$

```
[Out] -2/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(3/2)-
10/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
(1/2)-5*c^(3/2)*d^(3/2)*(-a*e*g+c*d*f)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^
(1/2)/d^(1/2)/(g*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5*c^2*d^2*(g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {876, 878, 905, 65, 223, 212}

$$-\frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{5c^2d^2\sqrt{f+gx}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}}-\frac{10cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(5/2)),x]
```

```
[Out] (5*c^2*d^2*Sqrt[f + g*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*
Sqrt[d + e*x]) - (10*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*Sqrt[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2))/(3*g*(d + e*x)^(5/2)*(f + g*x)^(3/2)) - (5*c^(3/2)*d^(3/2)*(c*d
*f - a*e*g)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d
*x])/(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 878

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((
a + b*x + c*x^2)^p/(g*(m - n - 1))), x] - Dist[m*((c*e*f + c*d*g - b*e*g)/(
e^2*g*(m - n - 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &&
 NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && E
qQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(Intege
rQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}}\,dx = -\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} + \frac{(5cd)\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)}{(d+ex)^{3/2}(f+gx)^{3/2}}}{3g}$$

$$= -\frac{10cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+\right.}{3g(d+ex)^{5/2}(f+g}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2\right.\right.}{3g^2(d+ex}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2\right.\right.}{3g^2(d+ex}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2\right.\right.}{3g^2(d+ex}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2\right.\right.}{3g^2(d+ex}$$

$$= \frac{5c^2d^2\sqrt{f+gx}\,\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}} - \frac{10cd\left(ade+\left(cd^2\right.\right.}{3g^2(d+ex}$$

Mathematica [A]
time = 0.96, size = 189, normalized size = 0.67

$$\frac{\sqrt{ae+cdx}\,\sqrt{d+ex}\left(\sqrt{ae+cdx}\left(-2a^2e^2g^2-2acdeg(5f+7gx)+c^2d^2\left(15f^2+20fgx+3g^2x^2\right)\right)+15\left(\frac{cd}{g}\right)^{3/2}g(cdf-aeg)(f+gx)^{3/2}\log\left(\sqrt{ae+cdx}-\sqrt{\frac{cd}{g}}\,\sqrt{f+gx}\right)\right)}{3g^3\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(5/2)),x]
```

```
[Out] (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a*e + c*d*x]*(-2*a^2*e^2*g^2 - 2*a*c
*d*e*g*(5*f + 7*g*x) + c^2*d^2*(15*f^2 + 20*f*g*x + 3*g^2*x^2)) + 15*((c*d)
/g)^(3/2)*g*(c*d*f - a*e*g)*(f + g*x)^(3/2)*Log[Sqrt[a*e + c*d*x] - Sqrt[(c
*d)/g]*Sqrt[f + g*x]]))/(3*g^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^(3/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(240) = 480$.
time = 0.15, size = 628, normalized size = 2.21

method	result
default	$\frac{\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\,\sqrt{dgc}}{2\sqrt{dgc}}\right)a\,c^2d^2e\,g^3x^2-15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx}}{2\sqrt{dgc}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/6*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)
^(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*g^3*x^2-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f
+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*f*g^2*
x^2+30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f*g^2*x-30*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+
2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*f^2*g*x
+15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/
2))/(d*g*c)^(1/2))*a*c^2*d^2*e*f^2*g-15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g
*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*f^3+6*c^2*d^
2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)-28*((g*x+f)*(c*d*x+a*e)
)^(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x+40*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^
(1/2)*c^2*d^2*f*g*x-4*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a^2*e^2*g^2
-20*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*f*g+30*((g*x+f)*(c*d*
x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f^2)*((c*d*x+a*e)*(e*x+d))^(1/2)/((g*x+
f)*(c*d*x+a*e))^(1/2)/(d*g*c)^(1/2)/g^3/(g*x+f)^(3/2)/(e*x+d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^(5/2)*(x
*e + d)^(5/2)), x)
```

Fricas [A]
time = 1.78, size = 993, normalized size = 3.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*c^2*d^2*g^2*x^2 + 20*c^2*d^2*f*g*x + 15*c^2*d^2*f^2 - 2*a^2*g^2
*e^2 - 2*(7*a*c*d*g^2*x + 5*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^2*d^3*f*g^2*x^2 + 2*c^2*d^3*
f^2*g*x + c^2*d^3*f^3 - (a*c*d*g^3*x^3 + 2*a*c*d*f*g^2*x^2 + a*c*d*f^2*g*x)
*e^2 + (c^2*d^2*f*g^2*x^3 - a*c*d^2*f^2*g + (2*c^2*d^2*f^2*g - a*c*d^2*g^3)
*x^2 + (c^2*d^2*f^3 - 2*a*c*d^2*f*g^2)*x)*e)*sqrt(c*d/g)*log(-(8*c^2*d^3*g^
2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*(2*c*d*g^2*x + c*
d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*
sqrt(x*e + d)*sqrt(c*d/g) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d*g^2)*e
^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d^2*f^2
+ 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(d*g^5*x^2 + 2*d*f*g^4*x + d*f^2*g^3 + (
g^5*x^3 + 2*f*g^4*x^2 + f^2*g^3*x)*e), 1/6*(2*(3*c^2*d^2*g^2*x^2 + 20*c^2*d
^2*f*g*x + 15*c^2*d^2*f^2 - 2*a^2*g^2*e^2 - 2*(7*a*c*d*g^2*x + 5*a*c*d*f*g)
*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)
 + 15*(c^2*d^3*f*g^2*x^2 + 2*c^2*d^3*f^2*g*x + c^2*d^3*f^3 - (a*c*d*g^3*x^3
 + 2*a*c*d*f*g^2*x^2 + a*c*d*f^2*g*x)*e^2 + (c^2*d^2*f*g^2*x^3 - a*c*d^2*f^
2*g + (2*c^2*d^2*f^2*g - a*c*d^2*g^3)*x^2 + (c^2*d^2*f^3 - 2*a*c*d^2*f*g^2)
*x)*e)*sqrt(-c*d/g)*arctan(2*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sq
rt(g*x + f)*sqrt(x*e + d)*sqrt(-c*d/g)*g/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2
 + (2*c*d*g*x^2 + c*d*f*x + a*d*g)*e)))/(d*g^5*x^2 + 2*d*f*g^4*x + d*f^2*g^
3 + (g^5*x^3 + 2*f*g^4*x^2 + f^2*g^3*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^{5/2}\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(5/2)*(d + e*x
)^(5/2)), x)
```

3.756 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{2c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

```
[Out] -2/3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/g^2/(e*x+d)^(3/2)/(g*x+f)^
(3/2)-2/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/g/(e*x+d)^(5/2)/(g*x+f)^(
5/2)+2*c^(5/2)*d^(5/2)*arctanh(g^(1/2)*(c*d*x+a*e)^(1/2)/c^(1/2)/d^(1/2)/(g
*x+f)^(1/2))*(c*d*x+a*e)^(1/2)*(e*x+d)^(1/2)/g^(7/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)-2*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g^3/(e*
x+d)^(1/2)/(g*x+f)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {876, 905, 65, 223, 212}

$$\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\,\sqrt{ae+cdx}\,\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{c}\,\sqrt{d}\,\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{g^3\sqrt{d+ex}\,\sqrt{f+gx}}-\frac{2cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}}-\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(7/2)),x]
```

```
[Out] (-2*c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*Sqrt[d + e*x]
*Sqrt[f + g*x]) - (2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(3*
g^2*(d + e*x)^(3/2)*(f + g*x)^(3/2)) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(5/2))/(5*g*(d + e*x)^(5/2)*(f + g*x)^(5/2)) + (2*c^(5/2)*d^(5/2)*Sq
rt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e + c*d*x])/(Sqrt[c]*
Sqrt[d]*Sqrt[f + g*x])])/(g^(7/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 876

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a +
 b*x + c*x^2)^p/(g*(n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^
(m + 1)*(f + g*x)^(n + 1)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && GtQ[p, 0] && LtQ
[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}}\,dx = -\frac{2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}} + \frac{(cd)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/}}{(d+ex)^{3/2}(f+gx)^{5/2}}}{g}$$

$$= -\frac{2cd(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}} - \frac{2(ade+(cd^2+ae^2)x+c}{5g(d+ex)^{5/2}(f+gx}$$

$$= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(ade+(cd^2+ae^2)x}{3g^2(d+ex)^{3/2}(f+}$$

$$= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(ade+(cd^2+ae^2)x}{3g^2(d+ex)^{3/2}(f+}$$

$$= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(ade+(cd^2+ae^2)x}{3g^2(d+ex)^{3/2}(f+}$$

$$= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(ade+(cd^2+ae^2)x}{3g^2(d+ex)^{3/2}(f+}$$

$$= -\frac{2c^2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(ade+(cd^2+ae^2)x}{3g^2(d+ex)^{3/2}(f+}$$

Mathematica [A]
time = 0.27, size = 166, normalized size = 0.61

$$\frac{2((ae+cdx)(d+ex))^{5/2}\left(-\frac{\sqrt{g}\left(3g^2+\frac{5cdg(f+gx)}{ae+cdx}+\frac{15c^2d^2(f+gx)^2}{(ae+cdx)^2}\right)}{(f+gx)^{5/2}}+\frac{15c^{5/2}d^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{(ae+cdx)^{5/2}}\right)}{15g^{7/2}(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(7/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(-((Sqrt[g]*(3*g^2 + (5*c*d*g*(f + g*x))
/(a*e + c*d*x) + (15*c^2*d^2*(f + g*x)^2)/(a*e + c*d*x)^2))/(f + g*x)^(5/2)
```

```
) + (15*c^(5/2)*d^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[f + g*x])/(Sqrt[g]*Sq
rt[a*e + c*d*x])])/(a*e + c*d*x)^(5/2)))/(15*g^(7/2)*(d + e*x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(230) = 460$.

time = 0.14, size = 501, normalized size = 1.83

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(15\ln\left(\frac{2cdgx+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{2\sqrt{dgc}}\right)c^3d^3g^3x^3+45\ln\left(\frac{2cdgx+aeg+c}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/15*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+
f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*g^3*x^3+45*ln(1
/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g
*c)^(1/2))*c^3*d^3*f*g^2*x^2+45*ln(1/2*(2*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c
*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c^3*d^3*f^2*g*x+15*ln(1/2*(2
*c*d*g*x+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(
1/2))*c^3*d^3*f^3-46*c^2*d^2*g^2*x^2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1
/2)-22*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*c*d*e*g^2*x-70*((g*x+f)*
(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f*g*x-6*((g*x+f)*(c*d*x+a*e))^(1/2
)*(d*g*c)^(1/2)*a^2*e^2*g^2-10*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*a*
c*d*e*f*g-30*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)*c^2*d^2*f^2)/((g*x+f
)*(c*d*x+a*e))^(1/2)/(d*g*c)^(1/2)/g^3/(g*x+f)^(5/2)/(e*x+d)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^(7/2)*(x
*e + d)^(5/2)), x)
```

Fricas [A]

time = 1.40, size = 949, normalized size = 3.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="fricas")
```

```
[Out] [-1/30*(4*(23*c^2*d^2*g^2*x^2 + 35*c^2*d^2*f*g*x + 15*c^2*d^2*f^2 + 3*a^2*g
^2*e^2 + (11*a*c*d*g^2*x + 5*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^
2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) - 15*(c^2*d^3*g^3*x^3 + 3*c^2*d^3*f
*g^2*x^2 + 3*c^2*d^3*f^2*g*x + c^2*d^3*f^3 + (c^2*d^2*g^3*x^4 + 3*c^2*d^2*f
*g^2*x^3 + 3*c^2*d^2*f^2*g*x^2 + c^2*d^2*f^3*x)*e)*sqrt(c*d/g)*log(-(8*c^2*
d^3*g^2*x^2 + 8*c^2*d^3*f*g*x + c^2*d^3*f^2 + a^2*g^2*x*e^3 + 4*(2*c*d*g^2*
x + c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x
 + f)*sqrt(x*e + d)*sqrt(c*d/g) + (8*a*c*d*g^2*x^2 + 6*a*c*d*f*g*x + a^2*d*
g^2)*e^2 + (8*c^2*d^2*g^2*x^3 + 8*c^2*d^2*f*g*x^2 + 6*a*c*d^2*f*g + (c^2*d^
2*f^2 + 8*a*c*d^2*g^2)*x)*e)/(x*e + d)))/(d*g^6*x^3 + 3*d*f*g^5*x^2 + 3*d*f
^2*g^4*x + d*f^3*g^3 + (g^6*x^4 + 3*f*g^5*x^3 + 3*f^2*g^4*x^2 + f^3*g^3*x)*
e), -1/15*(2*(23*c^2*d^2*g^2*x^2 + 35*c^2*d^2*f*g*x + 15*c^2*d^2*f^2 + 3*a^
2*g^2*e^2 + (11*a*c*d*g^2*x + 5*a*c*d*f*g)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d
*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d) + 15*(c^2*d^3*g^3*x^3 + 3*c^2*d^
3*f*g^2*x^2 + 3*c^2*d^3*f^2*g*x + c^2*d^3*f^3 + (c^2*d^2*g^3*x^4 + 3*c^2*d^
2*f*g^2*x^3 + 3*c^2*d^2*f^2*g*x^2 + c^2*d^2*f^3*x)*e)*sqrt(-c*d/g)*arctan(2
*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sq
rt(-c*d/g)*g/(2*c*d^2*g*x + c*d^2*f + a*g*x*e^2 + (2*c*d*g*x^2 + c*d*f*x +
a*d*g)*e)))/(d*g^6*x^3 + 3*d*f*g^5*x^2 + 3*d*f^2*g^4*x + d*f^3*g^3 + (g^6*x
^4 + 3*f*g^5*x^3 + 3*f^2*g^4*x^2 + f^3*g^3*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(7/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(f+g\,x\right)}^{7/2}\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*x
)^(5/2)),x)
[Out] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(7/2)*(d + e*x
)^(5/2)), x)
```

3.757 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

```
[Out] 2/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(g
*x+f)^(7/2)
```

Rubi [A]

time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {874}

$$\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(9/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rubi steps

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}}\,dx=\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

Mathematica [A]

time = 0.10, size = 52, normalized size = 0.83

$$\frac{2((ae+cdx)(d+ex))^{7/2}}{7(cdf-aeg)(d+ex)^{7/2}(f+gx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(9/2)),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f +
 g*x)^(7/2))
```

Maple [A]
time = 0.15, size = 78, normalized size = 1.24

method	result	size
gosper	$-\frac{2(cdx+ae)\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}{7(gx+f)^{\frac{7}{2}}(aeg-cdf)(ex+d)^{\frac{5}{2}}}$	63
default	$-\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,(c^{2}d^{2}x^{2}+2acdex+a^{2}e^{2})(cdx+ae)}{7\sqrt{ex+d}\,(gx+f)^{\frac{7}{2}}(aeg-cdf)}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/7*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(7/2)*(c^2*d^2*x^2+2
*a*c*d*e*x+a^2*e^2)*(c*d*x+a*e)/(a*e*g-c*d*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^(9/2)*(x
*e + d)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(58) = 116$.
time = 0.78, size = 314, normalized size = 4.98

$$\frac{2\left(c^{3}d^{3}x^{3}+3\,ac^{2}d^{2}x^{2}e+3\,a^{2}cdxe^{2}+a^{3}e^{3}\right)\sqrt{cd^{2}x+axe^{2}+(cdx^{2}+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{7\left(cd^{2}fg^{4}x^{4}+4\,cd^{2}f^{2}g^{3}x^{3}+6\,cd^{2}f^{3}g^{2}x^{2}+4\,cd^{2}f^{4}gx+cd^{2}f^{5}-(ag^{5}x^{5}+4\,afg^{4}x^{4}+6\,af^{2}g^{3}x^{3}+4\,af^{3}g^{2}x^{2}+af^{4}gx)e^{2}+(cdfg^{4}x^{5}-adf^{4}g+(4\,cdf^{2}g^{3}-adg^{5})x^{4}+2\,(3\,cdf^{3}g^{2}-2\,adfg^{4})x^{3}+2\,(2\,cdf^{4}g-3\,adf^{2}g^{3})x^{2}+(cdf^{5}-4\,adf^{3}g^{2})x)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="fricas")
```

```
[Out] 2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*x^2*e + 3*a^2*c*d*x*e^2 + a^3*e^3)*sqrt(c*d^
2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c*d^2*f*g^4
*x^4 + 4*c*d^2*f^2*g^3*x^3 + 6*c*d^2*f^3*g^2*x^2 + 4*c*d^2*f^4*g*x + c*d^2*
f^5 - (a*g^5*x^5 + 4*a*f*g^4*x^4 + 6*a*f^2*g^3*x^3 + 4*a*f^3*g^2*x^2 + a*f^
4*g*x)*e^2 + (c*d*f*g^4*x^5 - a*d*f^4*g + (4*c*d*f^2*g^3 - a*d*g^5)*x^4 + 2
*(3*c*d*f^3*g^2 - 2*a*d*f*g^4)*x^3 + 2*(2*c*d*f^4*g - 3*a*d*f^2*g^3)*x^2 +
(c*d*f^5 - 4*a*d*f^3*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(9/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.34, size = 325, normalized size = 5.16

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{2\,c^3\,d^3\,x^3}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a^2\,c\,d\,e^2\,x}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{6\,a\,c^2\,d^2\,e\,x^2}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}\right)}{x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}-\frac{\sqrt{f+g\,x}\,\left(7\,c\,d\,f^4-7\,a\,e\,f^3\,g\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}+\frac{x^2\,\sqrt{f+g\,x}\,\left(21\,a\,e\,f\,g^3-21\,c\,d\,f^2\,g^2\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}-\frac{x\,\sqrt{f+g\,x}\,\left(21\,c\,d\,f^3\,g-21\,a\,e\,f^2\,g^2\right)\,\sqrt{d+e\,x}}{7\,a\,e\,g^4-7\,c\,d\,f\,g^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(9/2)*(d + e*x
)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3)/(7*a*e*g^4 - 7
*c*d*f*g^3) + (2*c^3*d^3*x^3)/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a^2*c*d*e^2*x)
/(7*a*e*g^4 - 7*c*d*f*g^3) + (6*a*c^2*d^2*e*x^2)/(7*a*e*g^4 - 7*c*d*f*g^3))
)/(x^3*(f + g*x)^(1/2)*(d + e*x)^(1/2) - ((f + g*x)^(1/2)*(7*c*d*f^4 - 7*a*
e*f^3*g)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3) + (x^2*(f + g*x)^(1/2)*
(21*a*e*f*g^3 - 21*c*d*f^2*g^2)*(d + e*x)^(1/2))/(7*a*e*g^4 - 7*c*d*f*g^3)
- (x*(f + g*x)^(1/2)*(21*c*d*f^3*g - 21*a*e*f^2*g^2)*(d + e*x)^(1/2))/(7*a*
e*g^4 - 7*c*d*f*g^3))
```

3.758 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx$

Optimal. Leaf size=129

$$\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{4cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{63(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{7/2}}$$

```
[Out] 2/9*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(g
*x+f)^(9/2)+4/63*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)
^2/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {886, 874}

$$\frac{4cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(11/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(9*(c*d*f - a*e*g)*(d + e
*x)^(7/2)*(f + g*x)^(9/2)) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(7/2))/(63*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
```

*p]

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{(2cd)\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5}}{(d+ex)^{5/2}(f+gx)^{9/2}}}{9(cdf-aeg)}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{9(cdf-aeg)(d+ex)^{7/2}(f+gx)^{9/2}} + \frac{4cd(ade+(cd^2+ae^2)x+c}{63(cdf-aeg)^2(d+ex)^{7/2}(f}$$

Mathematica [A]
time = 0.17, size = 79, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\,(-7aeg+cd(9f+2gx))}{63(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(11/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-7*a*e*g + c*d*(9*f + 2*g
*x)))/(63*(c*d*f - a*e*g)^2*Sqrt[d + e*x]*(f + g*x)^(9/2))
```

Maple [A]
time = 0.14, size = 136, normalized size = 1.05

method	result
gosper	$-\frac{2(cdx+ae)(-2cdgx+7aeg-9cdf)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}{63(gx+f)^{\frac{9}{2}}(a^2e^2g^2-2acdefg+f^2c^2d^2)(ex+d)^{\frac{5}{2}}}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-2c^3d^3g\,x^3+3a\,c^2d^2eg\,x^2-9c^3d^3f\,x^2+12a^2cd\,e^2gx-18a\,c^2d^2efx+7a^3e^3g-9a^2cd\,e^2f)(cdx+ae)}{63\sqrt{ex+d}\,(gx+f)^{\frac{9}{2}}(aeg-cdf)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/63*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(9/2)*(-2*c^3*d^3*g
*x^3+3*a*c^2*d^2*e*g*x^2-9*c^3*d^3*f*x^2+12*a^2*c*d*e^2*g*x-18*a*c^2*d^2*e*
f*x+7*a^3*e^3*g-9*a^2*c*d*e^2*f)*(c*d*x+a*e)/(a*e*g-c*d*f)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^(11/2)*(
x*e + d)^(5/2)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(119) = 238$.
time = 0.81, size = 677, normalized size = 5.25

$$\frac{2\,(2\,c^4d^4gx^4+9\,c^4d^4fx^3-7\,a^4ge^4-(19\,a^3cdgx-9\,a^3cdf)e^3-3\,(5\,a^2c^2d^2gx^2-9\,a^2c^2d^2fx)e^2-(ac^3d^3gx^3-27\,ac^3d^3fx^2)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{gx+f}\,\sqrt{xe+d}}{63\,(c^2d^3f^2g^5x^5+5\,c^2d^3f^3g^4x^4+10\,c^2d^3f^4g^3x^3+10\,c^2d^3f^5g^2x^2+5\,c^2d^3f^6gx+c^2d^3f^7+(a^2g^7x^6+5\,a^2fg^6x^5+10\,a^2f^2g^5x^4+10\,a^2f^3g^4x^3+5\,a^2f^4g^3x^2+a^2f^5g^2x)e^3-(2\,acdfg^6x^6-a^2df^5g^2+(10\,acdf^2g^5-a^2dg^7)x^5+5\,(4\,acdf^3g^4-a^2dfg^6)x^4+10\,(2\,acdf^4g^3-a^2df^2g^5)x^3+10\,(acdf^5g^2-a^2df^3g^4)x^2+(2\,acdf^6g-5\,a^2df^4g^3)x)e^2+(c^2d^2f^2g^5x^6-2\,acd^2f^6g+(5\,c^2d^2f^3g^4-2\,acd^2fg^6)x^5+10\,(c^2d^2f^4g^3-acd^2f^2g^5)x^4+10\,(c^2d^2f^5g^2-2\,acd^2f^3g^4)x^3+5\,(c^2d^2f^6g-4\,acd^2f^4g^3)x^2+(c^2d^2f^7-10\,acd^2f^5g^2)x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*c^4*d^4*g*x^4 + 9*c^4*d^4*f*x^3 - 7*a^4*g*e^4 - (19*a^3*c*d*g*x - 9
*a^3*c*d*f)*e^3 - 3*(5*a^2*c^2*d^2*g*x^2 - 9*a^2*c^2*d^2*f*x)*e^2 - (a*c^3*
d^3*g*x^3 - 27*a*c^3*d^3*f*x^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^2*d^3*f^2*g^5*x^5 + 5*c^2*d^3*f^3*g^4*x^
4 + 10*c^2*d^3*f^4*g^3*x^3 + 10*c^2*d^3*f^5*g^2*x^2 + 5*c^2*d^3*f^6*g*x + c
^2*d^3*f^7 + (a^2*g^7*x^6 + 5*a^2*f*g^6*x^5 + 10*a^2*f^2*g^5*x^4 + 10*a^2*f
^3*g^4*x^3 + 5*a^2*f^4*g^3*x^2 + a^2*f^5*g^2*x)*e^3 - (2*a*c*d*f*g^6*x^6 -
a^2*d*f^5*g^2 + (10*a*c*d*f^2*g^5 - a^2*d*g^7)*x^5 + 5*(4*a*c*d*f^3*g^4 - a
^2*d*f*g^6)*x^4 + 10*(2*a*c*d*f^4*g^3 - a^2*d*f^2*g^5)*x^3 + 10*(a*c*d*f^5*
g^2 - a^2*d*f^3*g^4)*x^2 + (2*a*c*d*f^6*g - 5*a^2*d*f^4*g^3)*x)*e^2 + (c^2*
d^2*f^2*g^5*x^6 - 2*a*c*d^2*f^6*g + (5*c^2*d^2*f^3*g^4 - 2*a*c*d^2*f*g^6)*x
^5 + 10*(c^2*d^2*f^4*g^3 - a*c*d^2*f^2*g^5)*x^4 + 10*(c^2*d^2*f^5*g^2 - 2*a
*c*d^2*f^3*g^4)*x^3 + 5*(c^2*d^2*f^6*g - 4*a*c*d^2*f^4*g^3)*x^2 + (c^2*d^2*
f^7 - 10*a*c*d^2*f^5*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(11
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.54, size = 315, normalized size = 2.44

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,a^3\,e^3\,\left(7\,a\,e\,g-9\,c\,d\,f\right)}{63\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}-\frac{4\,c^4\,d^4\,x^4}{63\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,c^3\,d^3\,x^3\,\left(a\,e\,g-9\,c\,d\,f\right)}{63\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,a^2\,c\,d\,e^2\,x\,\left(19\,a\,e\,g-27\,c\,d\,f\right)}{63\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}+\frac{2\,a\,c^2\,d^2\,e\,x^2\,\left(5\,a\,e\,g-9\,c\,d\,f\right)}{21\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^2}\right)}{x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{4\,f\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{4\,f^3\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{6\,f^2\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(11/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*a^3*e^3*(7*a*e*g - 9*c*
d*f))/(63*g^4*(a*e*g - c*d*f)^2) - (4*c^4*d^4*x^4)/(63*g^3*(a*e*g - c*d*f)^
2) + (2*c^3*d^3*x^3*(a*e*g - 9*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a^2*
c*d*e^2*x*(19*a*e*g - 27*c*d*f))/(63*g^4*(a*e*g - c*d*f)^2) + (2*a*c^2*d^2*
e*x^2*(5*a*e*g - 9*c*d*f))/(21*g^4*(a*e*g - c*d*f)^2)))/(x^4*(f + g*x)^(1/2
)*(d + e*x)^(1/2) + (f^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (4*f*x^3*(f
 + g*x)^(1/2)*(d + e*x)^(1/2))/g + (4*f^3*x*(f + g*x)^(1/2)*(d + e*x)^(1/2)
)/g^3 + (6*f^2*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2)
```

3.759 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx$

Optimal. Leaf size=198

$$\frac{2(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{8cd(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{99(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{9/2}}+\frac{16c^2d^2(ade+(cd^2+ae^2)\,x\,\cdots}{693(cdf-aeg)^3(d+ex)^{7/2}(\cdots}$$

```
[Out] 2/11*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(11/2)+8/99*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*
f)^2/(e*x+d)^(7/2)/(g*x+f)^(9/2)+16/693*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(7/2)
```

Rubi [A]

time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{16c^2d^2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^3}+\frac{8cd(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^2}+\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(13/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(11*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(11/2)) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^(7/2))/(99*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (16*c^2*
d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(693*(c*d*f - a*e*g)^3*(
d + e*x)^(7/2)*(f + g*x)^(7/2))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
```

```
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{(ade + (cd^2+ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}}\,dx = \frac{2(ade + (cd^2+ae^2)x + cdex^2)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{(4cd)\int \frac{(ade+(cd^2+ae^2)x+cdex}{(d+ex)^{5/2}(f+gx)^{11/}}}{11(cdf-aeg)}$$

$$= \frac{2(ade + (cd^2+ae^2)x + cdex^2)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{8cd(ade + (cd^2+ae^2)x +}{99(cdf-aeg)^2(d+ex)^{7/2}}$$

$$= \frac{2(ade + (cd^2+ae^2)x + cdex^2)^{7/2}}{11(cdf-aeg)(d+ex)^{7/2}(f+gx)^{11/2}} + \frac{8cd(ade + (cd^2+ae^2)x +}{99(cdf-aeg)^2(d+ex)^{7/2}}$$

Mathematica [A]
time = 0.22, size = 113, normalized size = 0.57

$$\frac{2(ae+cdx)^3((ae+cdx)(d+ex))^{5/2}\left(63g^2 - \frac{154cdg(f+gx)}{ae+cdx} + \frac{99c^2d^2(f+gx)^2}{(ae+cdx)^2}\right)}{693(cdf-aeg)^3(d+ex)^{5/2}(f+gx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(13/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^3*((a*e + c*d*x)*(d + e*x))^(5/2)*(63*g^2 - (154*c*d*g*(f
+ g*x))/(a*e + c*d*x) + (99*c^2*d^2*(f + g*x)^2)/(a*e + c*d*x)^2))/(693*(c*
d*f - a*e*g)^3*(d + e*x)^(5/2)*(f + g*x)^(11/2))
```

Maple [A]
time = 0.15, size = 231, normalized size = 1.17

method	result
gosper	$-\frac{2(cdx+ae)(8g^2x^2c^2d^2-28acde\,g^2x+44c^2d^2fgx+63a^2e^2g^2-154acdefg+99f^2c^2d^2)(cde\,x^2+a\,e^2x+c\,d^2x+ade)^{\frac{5}{2}}}{693(gx+f)^{\frac{11}{2}}(a^3e^3g^3-3a^2cd\,e^2f\,g^2+3a\,c^2d^2e\,f^2g-f^3c^3d^3)(ex+d)^{\frac{5}{2}}}$
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(8c^4d^4g^2x^4-12a\,c^3d^3e\,g^2x^3+44c^4d^4fg\,x^3+15a^2c^2d^2e^2g^2x^2-66a\,c^3d^3efg\,x^2+99c^4d^4f^2x^2+98a^3}{693\sqrt{ex+d}\,(gx+f)^{\frac{11}{2}}(aeg-cdf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/693*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(11/2)*(8*c^4*d^4*
g^2*x^4-12*a*c^3*d^3*e*g^2*x^3+44*c^4*d^4*f*g*x^3+15*a^2*c^2*d^2*e^2*g^2*x^
2-66*a*c^3*d^3*e*f*g*x^2+99*c^4*d^4*f^2*x^2+98*a^3*c*d*e^3*g^2*x-264*a^2*c^
2*d^2*e^2*f*g*x+198*a*c^3*d^3*e*f^2*x+63*a^4*e^4*g^2-154*a^3*c*d*e^3*f*g+99
*a^2*c^2*d^2*e^2*f^2)*(c*d*x+a*e)/(a*e*g-c*d*f)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^(13/2)*(
x*e + d)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. 2(183) = 366.
time = 0.93, size = 1159, normalized size = 5.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="fricas")
```

```
[Out] 2/693*(8*c^5*d^5*g^2*x^5 + 44*c^5*d^5*f*g*x^4 + 99*c^5*d^5*f^2*x^3 + 63*a^5
*g^2*e^5 + 7*(23*a^4*c*d*g^2*x - 22*a^4*c*d*f*g)*e^4 + (113*a^3*c^2*d^2*g^2
*x^2 - 418*a^3*c^2*d^2*f*g*x + 99*a^3*c^2*d^2*f^2)*e^3 + 3*(a^2*c^3*d^3*g^2
*x^3 - 110*a^2*c^3*d^3*f*g*x^2 + 99*a^2*c^3*d^3*f^2*x)*e^2 - (4*a*c^4*d^4*g
^2*x^4 + 22*a*c^4*d^4*f*g*x^3 - 297*a*c^4*d^4*f^2*x^2)*e)*sqrt(c*d^2*x + a*
x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^3*d^4*f^3*g^6*x^6
 + 6*c^3*d^4*f^4*g^5*x^5 + 15*c^3*d^4*f^5*g^4*x^4 + 20*c^3*d^4*f^6*g^3*x^3
+ 15*c^3*d^4*f^7*g^2*x^2 + 6*c^3*d^4*f^8*g*x + c^3*d^4*f^9 - (a^3*g^9*x^7 +
 6*a^3*f*g^8*x^6 + 15*a^3*f^2*g^7*x^5 + 20*a^3*f^3*g^6*x^4 + 15*a^3*f^4*g^5
*x^3 + 6*a^3*f^5*g^4*x^2 + a^3*f^6*g^3*x)*e^4 + (3*a^2*c*d*f*g^8*x^7 - a^3*
d*f^6*g^3 + (18*a^2*c*d*f^2*g^7 - a^3*d*g^9)*x^6 + 3*(15*a^2*c*d*f^3*g^6 -
2*a^3*d*f*g^8)*x^5 + 15*(4*a^2*c*d*f^4*g^5 - a^3*d*f^2*g^7)*x^4 + 5*(9*a^2*
c*d*f^5*g^4 - 4*a^3*d*f^3*g^6)*x^3 + 3*(6*a^2*c*d*f^6*g^3 - 5*a^3*d*f^4*g^5
)*x^2 + 3*(a^2*c*d*f^7*g^2 - 2*a^3*d*f^5*g^4)*x)*e^3 - 3*(a*c^2*d^2*f^2*g^7
*x^7 - a^2*c*d^2*f^7*g^2 + (6*a*c^2*d^2*f^3*g^6 - a^2*c*d^2*f*g^8)*x^6 + 3*
```

```
(5*a*c^2*d^2*f^4*g^5 - 2*a^2*c*d^2*f^2*g^7)*x^5 + 5*(4*a*c^2*d^2*f^5*g^4 -
3*a^2*c*d^2*f^3*g^6)*x^4 + 5*(3*a*c^2*d^2*f^6*g^3 - 4*a^2*c*d^2*f^4*g^5)*x^
3 + 3*(2*a*c^2*d^2*f^7*g^2 - 5*a^2*c*d^2*f^5*g^4)*x^2 + (a*c^2*d^2*f^8*g -
6*a^2*c*d^2*f^6*g^3)*x)*e^2 + (c^3*d^3*f^3*g^6*x^7 - 3*a*c^2*d^3*f^8*g + 3*
(2*c^3*d^3*f^4*g^5 - a*c^2*d^3*f^2*g^7)*x^6 + 3*(5*c^3*d^3*f^5*g^4 - 6*a*c^
2*d^3*f^3*g^6)*x^5 + 5*(4*c^3*d^3*f^6*g^3 - 9*a*c^2*d^3*f^4*g^5)*x^4 + 15*(
c^3*d^3*f^7*g^2 - 4*a*c^2*d^3*f^5*g^4)*x^3 + 3*(2*c^3*d^3*f^8*g - 15*a*c^2*
d^3*f^6*g^3)*x^2 + (c^3*d^3*f^9 - 18*a*c^2*d^3*f^7*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(13
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 4.82, size = 465, normalized size = 2.35

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{126\,a^5\,e^5\,g^2-308\,a^4\,c\,d\,e^4\,f\,g+198\,a^3\,c^2\,d^2\,e^3\,f^2}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{x^3\,\left(6\,a^2\,c^3\,d^3\,e^2\,g^2-44\,a\,c^4\,d^4\,e\,f\,g+198\,c^5\,d^5\,f^2\right)}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{16\,c^5\,d^5\,x^5}{693\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^3}-\frac{8\,c^4\,d^4\,x^4\,\left(a\,e\,g-11\,c\,d\,f\right)}{693\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{2\,a^2\,c\,d\,e^2\,x\,\left(161\,a^2\,e^2\,g^2-418\,a\,c\,d\,e\,f\,g+297\,c^2\,d^2\,f^2\right)}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}+\frac{2\,a\,c^2\,d^2\,e\,x^2\,\left(113\,a^2\,e^2\,g^2-330\,a\,c\,d\,e\,f\,g+297\,c^2\,d^2\,f^2\right)}{693\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^3}\right)}{x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{5\,f\,x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{5\,f^4\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}+\frac{10\,f^2\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{10\,f^3\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(13/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((126*a^5*e^5*g^2 + 198*a^3
*c^2*d^2*e^3*f^2 - 308*a^4*c*d*e^4*f*g)/(693*g^5*(a*e*g - c*d*f)^3) + (x^3*
(198*c^5*d^5*f^2 + 6*a^2*c^3*d^3*e^2*g^2 - 44*a*c^4*d^4*e*f*g))/(693*g^5*(a
*e*g - c*d*f)^3) + (16*c^5*d^5*x^5)/(693*g^3*(a*e*g - c*d*f)^3) - (8*c^4*d^
4*x^4*(a*e*g - 11*c*d*f))/(693*g^4*(a*e*g - c*d*f)^3) + (2*a^2*c*d*e^2*x*(1
61*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 418*a*c*d*e*f*g))/(693*g^5*(a*e*g - c*d*
```

```
f)^3) + (2*a*c^2*d^2*e*x^2*(113*a^2*e^2*g^2 + 297*c^2*d^2*f^2 - 330*a*c*d*e
*f*g))/(693*g^5*(a*e*g - c*d*f)^3)))/(x^5*(f + g*x)^(1/2)*(d + e*x)^(1/2) +
 (f^5*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (5*f*x^4*(f + g*x)^(1/2)*(d +
e*x)^(1/2))/g + (5*f^4*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4 + (10*f^2*x^3
*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (10*f^3*x^2*(f + g*x)^(1/2)*(d + e*
x)^(1/2))/g^3)
```

3.760 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx$

Optimal. Leaf size=267

$$\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}}+\frac{12cd\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{143(cdf-aeg)^2(d+ex)^{7/2}(f+gx)^{11/2}}+\frac{16c^2d^2\left(ade+\left(cd^2+ae^2\right)x\right.}{429(cdf-aeg)^3(d+ex)^{7/2}}$$

```
[Out] 2/13*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)/(e*x+d)^(7/2)/(
g*x+f)^(13/2)+12/143*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*
d*f)^2/(e*x+d)^(7/2)/(g*x+f)^(11/2)+16/429*c^2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^3/(e*x+d)^(7/2)/(g*x+f)^(9/2)+32/3003*c^3*d^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/(-a*e*g+c*d*f)^4/(e*x+d)^(7/2)/(g
*x+f)^(7/2)
```

Rubi [A]
time = 0.21, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {886, 874}

$$\frac{32c^3d^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf-aeg)^4}+\frac{16c^2d^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf-aeg)^3}+\frac{12cd\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{143(d+ex)^{7/2}(f+gx)^{11/2}(cdf-aeg)^2}+\frac{2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x
)^(15/2)),x]
```

```
[Out] (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(13*(c*d*f - a*e*g)*(d +
e*x)^(7/2)*(f + g*x)^(13/2)) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(7/2))/(143*(c*d*f - a*e*g)^2*(d + e*x)^(7/2)*(f + g*x)^(11/2)) + (16*c
^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(429*(c*d*f - a*e*g)^
3*(d + e*x)^(7/2)*(f + g*x)^(9/2)) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(7/2))/(3003*(c*d*f - a*e*g)^4*(d + e*x)^(7/2)*(f + g*x)^(7/2
))
```

Rule 874

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2
- 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p,
 0] && EqQ[m - n - 2, 0]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
```

```
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rubi steps

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}}\,dx = \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{(6cd)\int \frac{(ade+(cd^2+ae^2)x+cd}{(d+ex)^{5/2}(f+gx)^1}}{13(cdf-aeg)}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd(ade+(cd^2+ae^2)}{143(cdf-aeg)^2(d+ex)}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd(ade+(cd^2+ae^2)}{143(cdf-aeg)^2(d+ex)}$$

$$= \frac{2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{13(cdf-aeg)(d+ex)^{7/2}(f+gx)^{13/2}} + \frac{12cd(ade+(cd^2+ae^2)}{143(cdf-aeg)^2(d+ex)}$$

Mathematica [A]
time = 0.28, size = 141, normalized size = 0.53

$$\frac{2(ae+cdx)^4((ae+cdx)(d+ex))^{5/2}\left(-231g^3+\frac{819cdg^2(f+gx)}{ae+cdx}-\frac{1001c^2d^2g(f+gx)^2}{(ae+cdx)^2}+\frac{429c^3d^3(f+gx)^3}{(ae+cdx)^3}\right)}{3003(cdf-aeg)^4(d+ex)^{5/2}(f+gx)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f
 + g*x)^(15/2)),x]
```

```
[Out] (2*(a*e + c*d*x)^4*((a*e + c*d*x)*(d + e*x))^(5/2)*(-231*g^3 + (819*c*d*g^2
*(f + g*x))/(a*e + c*d*x) - (1001*c^2*d^2*g*(f + g*x)^2)/(a*e + c*d*x)^2 +
(429*c^3*d^3*(f + g*x)^3)/(a*e + c*d*x)^3))/(3003*(c*d*f - a*e*g)^4*(d + e*
x)^(5/2)*(f + g*x)^(13/2))
```

Maple [A]
time = 0.15, size = 349, normalized size = 1.31

method	result
gosper	$-\frac{2(cdx+ae)(-16g^3x^3c^3d^3+56a\,c^2d^2e\,g^3x^2-104c^3d^3f\,g^2x^2-126a^2cd\,e^2g^3x+364a\,c^2d^2ef\,g^2x-286c^3d^3f^2gx+231a^3e^3g^3-819a^2c}{3003(gx+f)^{\frac{13}{2}}(g^4e^4a^4-4a^3cd\,e^3f\,g^3+6a^2c^2d^2e^2f^2g^2-4a\,c^3d^3e\,f^3g+f^4c}$

default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-16c^5d^5g^3x^5+24a\,c^4d^4e\,g^3x^4-104c^5d^5f\,g^2x^4-30a^2c^3d^3e^2g^3x^3+156a\,c^4d^4ef\,g^2x^3-286c^5d^5f^2g$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15/2),x,
method=_RETURNVERBOSE)
```

```
[Out] -2/3003*((c*d*x+a*e)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(13/2)*(-16*c^5*d
^5*g^3*x^5+24*a*c^4*d^4*e*g^3*x^4-104*c^5*d^5*f*g^2*x^4-30*a^2*c^3*d^3*e^2*
g^3*x^3+156*a*c^4*d^4*e*f*g^2*x^3-286*c^5*d^5*f^2*g*x^3+35*a^3*c^2*d^2*e^3*
g^3*x^2-195*a^2*c^3*d^3*e^2*f*g^2*x^2+429*a*c^4*d^4*e*f^2*g*x^2-429*c^5*d^5
*f^3*x^2+336*a^4*c*d*e^4*g^3*x-1274*a^3*c^2*d^2*e^3*f*g^2*x+1716*a^2*c^3*d^
3*e^2*f^2*g*x-858*a*c^4*d^4*e*f^3*x+231*a^5*e^5*g^3-819*a^4*c*d*e^4*f*g^2+1
001*a^3*c^2*d^2*e^3*f^2*g-429*a^2*c^3*d^3*e^2*f^3)*(c*d*x+a*e)/(a*e*g-c*d*f
)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((g*x + f)^(15/2)*(
x*e + d)^(5/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(247) = 494$.
time = 0.95, size = 1741, normalized size = 6.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*c^6*d^6*g^3*x^6 + 104*c^6*d^6*f*g^2*x^5 + 286*c^6*d^6*f^2*g*x^4
+ 429*c^6*d^6*f^3*x^3 - 231*a^6*g^3*e^6 - 63*(9*a^5*c*d*g^3*x - 13*a^5*c*d*
f*g^2)*e^5 - 7*(53*a^4*c^2*d^2*g^3*x^2 - 299*a^4*c^2*d^2*f*g^2*x + 143*a^4*
c^2*d^2*f^2*g)*e^4 - (5*a^3*c^3*d^3*g^3*x^3 - 1469*a^3*c^3*d^3*f*g^2*x^2 +
2717*a^3*c^3*d^3*f^2*g*x - 429*a^3*c^3*d^3*f^3)*e^3 + 3*(2*a^2*c^4*d^4*g^3*
x^4 + 13*a^2*c^4*d^4*f*g^2*x^3 - 715*a^2*c^4*d^4*f^2*g*x^2 + 429*a^2*c^4*d^
4*f^3*x)*e^2 - (8*a*c^5*d^5*g^3*x^5 + 52*a*c^5*d^5*f*g^2*x^4 + 143*a*c^5*d^
```

```
5*f^2*g*x^3 - 1287*a*c^5*d^5*f^3*x^2)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2
+ a*d)*e)*sqrt(g*x + f)*sqrt(x*e + d)/(c^4*d^5*f^4*g^7*x^7 + 7*c^4*d^5*f^5*
g^6*x^6 + 21*c^4*d^5*f^6*g^5*x^5 + 35*c^4*d^5*f^7*g^4*x^4 + 35*c^4*d^5*f^8*
g^3*x^3 + 21*c^4*d^5*f^9*g^2*x^2 + 7*c^4*d^5*f^10*g*x + c^4*d^5*f^11 + (a^4
*g^11*x^8 + 7*a^4*f*g^10*x^7 + 21*a^4*f^2*g^9*x^6 + 35*a^4*f^3*g^8*x^5 + 35
*a^4*f^4*g^7*x^4 + 21*a^4*f^5*g^6*x^3 + 7*a^4*f^6*g^5*x^2 + a^4*f^7*g^4*x)*
e^5 - (4*a^3*c*d*f*g^10*x^8 - a^4*d*f^7*g^4 + (28*a^3*c*d*f^2*g^9 - a^4*d*g
^11)*x^7 + 7*(12*a^3*c*d*f^3*g^8 - a^4*d*f*g^10)*x^6 + 7*(20*a^3*c*d*f^4*g^
7 - 3*a^4*d*f^2*g^9)*x^5 + 35*(4*a^3*c*d*f^5*g^6 - a^4*d*f^3*g^8)*x^4 + 7*(
12*a^3*c*d*f^6*g^5 - 5*a^4*d*f^4*g^7)*x^3 + 7*(4*a^3*c*d*f^7*g^4 - 3*a^4*d*
f^5*g^6)*x^2 + (4*a^3*c*d*f^8*g^3 - 7*a^4*d*f^6*g^5)*x)*e^4 + 2*(3*a^2*c^2*
d^2*f^2*g^9*x^8 - 2*a^3*c*d^2*f^8*g^3 + (21*a^2*c^2*d^2*f^3*g^8 - 2*a^3*c*d
^2*f*g^10)*x^7 + 7*(9*a^2*c^2*d^2*f^4*g^7 - 2*a^3*c*d^2*f^2*g^9)*x^6 + 21*(
5*a^2*c^2*d^2*f^5*g^6 - 2*a^3*c*d^2*f^3*g^8)*x^5 + 35*(3*a^2*c^2*d^2*f^6*g^
5 - 2*a^3*c*d^2*f^4*g^7)*x^4 + 7*(9*a^2*c^2*d^2*f^7*g^4 - 10*a^3*c*d^2*f^5*
g^6)*x^3 + 21*(a^2*c^2*d^2*f^8*g^3 - 2*a^3*c*d^2*f^6*g^5)*x^2 + (3*a^2*c^2*
d^2*f^9*g^2 - 14*a^3*c*d^2*f^7*g^4)*x)*e^3 - 2*(2*a*c^3*d^3*f^3*g^8*x^8 - 3
*a^2*c^2*d^3*f^9*g^2 + (14*a*c^3*d^3*f^4*g^7 - 3*a^2*c^2*d^3*f^2*g^9)*x^7 +
 21*(2*a*c^3*d^3*f^5*g^6 - a^2*c^2*d^3*f^3*g^8)*x^6 + 7*(10*a*c^3*d^3*f^6*g
^5 - 9*a^2*c^2*d^3*f^4*g^7)*x^5 + 35*(2*a*c^3*d^3*f^7*g^4 - 3*a^2*c^2*d^3*f
^5*g^6)*x^4 + 21*(2*a*c^3*d^3*f^8*g^3 - 5*a^2*c^2*d^3*f^6*g^5)*x^3 + 7*(2*a
*c^3*d^3*f^9*g^2 - 9*a^2*c^2*d^3*f^7*g^4)*x^2 + (2*a*c^3*d^3*f^10*g - 21*a^
2*c^2*d^3*f^8*g^3)*x)*e^2 + (c^4*d^4*f^4*g^7*x^8 - 4*a*c^3*d^4*f^10*g + (7*
c^4*d^4*f^5*g^6 - 4*a*c^3*d^4*f^3*g^8)*x^7 + 7*(3*c^4*d^4*f^6*g^5 - 4*a*c^3
*d^4*f^4*g^7)*x^6 + 7*(5*c^4*d^4*f^7*g^4 - 12*a*c^3*d^4*f^5*g^6)*x^5 + 35*(
c^4*d^4*f^8*g^3 - 4*a*c^3*d^4*f^6*g^5)*x^4 + 7*(3*c^4*d^4*f^9*g^2 - 20*a*c^
3*d^4*f^7*g^4)*x^3 + 7*(c^4*d^4*f^10*g - 12*a*c^3*d^4*f^8*g^3)*x^2 + (c^4*d
^4*f^11 - 28*a*c^3*d^4*f^9*g^2)*x)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f
)**(15/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(15
/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.12, size = 627, normalized size = 2.35

$$-\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{462\,a^6\,e^6\,g^3-1638\,a^5\,c\,d\,e^5\,f\,g^2+2002\,a^4\,c^2\,d^2\,e^4\,f^2\,g-858\,a^3\,c^3\,d^3\,e^3\,f^3}{3003\,g^6\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{x^3\,\left(-10\,a^3\,c^3\,d^3\,e^3\,g^3+78\,a^2\,c^4\,d^4\,e^2\,f\,g^2-286\,a\,c^5\,d^5\,e\,f^2\,g+858\,c^6\,d^6\,f^3\right)}{3003\,g^6\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{32\,c^6\,d^6\,x^6}{3003\,g^3\,{\left(a\,e\,g-c\,d\,f\right)}^4}-\frac{4\,c^4\,d^4\,x^4\,\left(3\,a^2\,e^2\,g^2-26\,a\,c\,d\,e\,f\,g+143\,c^2\,d^2\,f^2\right)}{3003\,g^5\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{16\,c^5\,d^5\,x^5\,\left(a\,e\,g-13\,c\,d\,f\right)}{3003\,g^4\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{2\,a^2\,c\,d\,e^2\,x\,\left(567\,a^3\,e^3\,g^3-2093\,a^2\,c\,d\,e^2\,f\,g^2+2717\,a\,c^2\,d^2\,e\,f^2\,g-1287\,c^3\,d^3\,f^3\right)}{3003\,g^6\,{\left(a\,e\,g-c\,d\,f\right)}^4}+\frac{2\,a\,c^2\,d^2\,e\,x^2\,\left(371\,a^3\,e^3\,g^3-1469\,a^2\,c\,d\,e^2\,f\,g^2+2145\,a\,c^2\,d^2\,e\,f^2\,g-1287\,c^3\,d^3\,f^3\right)}{3003\,g^6\,{\left(a\,e\,g-c\,d\,f\right)}^4}\right)}{x^6\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}+\frac{f^6\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^6}+\frac{6\,f\,x^5\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g}+\frac{6\,f^5\,x\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^5}+\frac{15\,f^2\,x^4\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^2}+\frac{20\,f^3\,x^3\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^3}+\frac{15\,f^4\,x^2\,\sqrt{f+g\,x}\,\sqrt{d+e\,x}}{g^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/((f + g*x)^(15/2)*(d + e*
x)^(5/2)),x)
```

```
[Out] -((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((462*a^6*e^6*g^3 - 858*a^3
*c^3*d^3*e^3*f^3 + 2002*a^4*c^2*d^2*e^4*f^2*g - 1638*a^5*c*d*e^5*f*g^2)/(30
03*g^6*(a*e*g - c*d*f)^4) - (x^3*(858*c^6*d^6*f^3 - 10*a^3*c^3*d^3*e^3*g^3
+ 78*a^2*c^4*d^4*e^2*f*g^2 - 286*a*c^5*d^5*e*f^2*g))/(3003*g^6*(a*e*g - c*d
*f)^4) - (32*c^6*d^6*x^6)/(3003*g^3*(a*e*g - c*d*f)^4) - (4*c^4*d^4*x^4*(3*
a^2*e^2*g^2 + 143*c^2*d^2*f^2 - 26*a*c*d*e*f*g))/(3003*g^5*(a*e*g - c*d*f)^
4) + (16*c^5*d^5*x^5*(a*e*g - 13*c*d*f))/(3003*g^4*(a*e*g - c*d*f)^4) + (2*
a^2*c*d*e^2*x*(567*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2717*a*c^2*d^2*e*f^2*g
- 2093*a^2*c*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4) + (2*a*c^2*d^2*e*x^
2*(371*a^3*e^3*g^3 - 1287*c^3*d^3*f^3 + 2145*a*c^2*d^2*e*f^2*g - 1469*a^2*c
*d*e^2*f*g^2))/(3003*g^6*(a*e*g - c*d*f)^4)))/(x^6*(f + g*x)^(1/2)*(d + e*x
)^(1/2) + (f^6*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^6 + (6*f*x^5*(f + g*x)^(1
/2)*(d + e*x)^(1/2))/g + (6*f^5*x*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^5 + (1
5*f^2*x^4*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^2 + (20*f^3*x^3*(f + g*x)^(1/2
)*(d + e*x)^(1/2))/g^3 + (15*f^4*x^2*(f + g*x)^(1/2)*(d + e*x)^(1/2))/g^4)
```

3.761 $\int \frac{(d+ex)^{5/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(ae+cdx)(d+ex)^{5/2}(f+gx)^{1+n}\,{}_2F_1\left(1,-\frac{1}{2}+n;2+n;\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^(5/2)*(g*x+f)^(1+n)*hypergeom([1, -1/2+n],[2+n],c*d*(g
*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(5/2)
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$-\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
5/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((g*(a*e +
 c*d*x))/(c*d*f - a*e*g))])/(3*c*d*(a*e + c*d*x)*((c*d*(f + g*x))/(c*d*f -
a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx &= \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{(ae+cdx)^{5/2}}\,dx}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} \\
&= \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{5/2}}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} \\
&= -\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd(ae+cdx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 100, normalized size = 0.96

$$-\frac{2(d+ex)^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};\frac{g(ae+cdx)}{-cdf+aeg}\right)}{3cd((ae+cdx)(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(5/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(5/2),x]
```

```
[Out] (-2*(d + e*x)^(3/2)*(f + g*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, (g*(a*e +
 c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*((a*e + c*d*x)*(d + e*x))^(3/2)*((c*d*
(f + g*x))/(c*d*f - a*e*g))^n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{5}{2}}\,(gx+f)^n}{(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

```
[Out] int((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(5/2)*(g*x + f)^n/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*
x)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(x^2*e^2 + 2*d*x*e + d
^2)*sqrt(x*e + d)*(g*x + f)^n/(c^3*d^6*x^3 + a^3*x^3*e^6 + 3*(a^2*c*d^2*x^3
 + (a^2*c*d*x^4 + a^3*d*x^2)*e)*e^4 + (c^3*d^3*x^6 + 3*a*c^2*d^3*x^4 + 3*a^
2*c*d^3*x^2 + a^3*d^3)*e^3 + 3*(c^3*d^4*x^5 + 2*a*c^2*d^4*x^3 + a^2*c*d^4*x
)*e^2 + 3*(a*c^2*d^4*x^3 + (a*c^2*d^2*x^5 + 2*a^2*c*d^2*x^3 + a^3*d^2*x)*e^
2 + 2*(a*c^2*d^3*x^4 + a^2*c*d^3*x^2)*e)*e^2 + 3*(c^3*d^5*x^4 + a*c^2*d^5*x
^2)*e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(5/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(5/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((x*e + d)^(5/2)*(g*x + f)^n/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*
x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,(d+e\,x)^{5/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(5/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
5/2), x)
```

3.762 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(ae+cdx)(d+ex)^{3/2}(f+gx)^{1+n}\,{}_2F_1\left(1,\frac{1}{2}+n;2+n;\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^(3/2)*(g*x+f)^(1+n)*hypergeom([1, 1/2+n],[2+n],c*d*(g*
x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$-\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((g*(a*e +
c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g))^n*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{(ae+cdx)^{3/2}}\,dx}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{(ae+cdx)^{3/2}}}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$= -\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

Mathematica [**A**]
time = 0.21, size = 98, normalized size = 0.94

$$-\frac{2\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};\frac{g(ae+cdx)}{-cdf+aeg}\right)}{cd\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, (g*(a*e + c*
d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[(a*e + c*d*x)*(d + e*x)]*((c*d*(f + g*
x))/(c*d*f - a*e*g))^n)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}}\,(gx+f)^n}{\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

```
[Out] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)*(g*x + f)^n/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*
x)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(x*e + d)^(3/2)*(g*x +
 f)^n/(c^2*d^4*x^2 + a^2*x^2*e^4 + (c^2*d^2*x^4 + 2*a*c*d^2*x^2 + a^2*d^2)*
e^2 + 2*(a*c*d^2*x^2 + (a*c*d*x^3 + a^2*d*x)*e)*e^2 + 2*(c^2*d^3*x^3 + a*c*
d^3*x)*e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((x*e + d)^(3/2)*(g*x + f)^n/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*
x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,(d+e\,x)^{3/2}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
3/2), x)
```

3.763 $\int \frac{\sqrt{d+ex}\,(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(ae+cdx)\sqrt{d+ex}\,(f+gx)^{1+n}\,{}_2F_1\left(1,\frac{3}{2}+n;2+n;\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom([1, 3/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c
*d*f))*(e*x+d)^(1/2)/(-a*e*g+c*d*f)/(1+n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$\frac{2\sqrt{d+ex}\,(f+gx)^n(ae+cdx)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (2*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
-((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c*d*((c*d*(f + g*x))/(c*d*f - a*e*g
))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\right)\int \frac{(f+gx)^n}{\sqrt{ae+cdx}}\,dx}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$= \frac{\left(\sqrt{ae+cdx}\,\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\right)\int \frac{\left(\frac{cdf}{cdf-aeg}+\frac{cdgx}{cdf-aeg}\right)^n}{\sqrt{ae+cdx}}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

$$= \frac{2(ae+cdx)\sqrt{d+ex}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{g(ae+cd}{cdf-ae}\right.}{cd\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}$$

Mathematica [A]
time = 0.14, size = 98, normalized size = 0.94

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{g(ae+cdx)}{-cdf+aeg}\right)}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2
, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(c*d*Sqrt[d + e*x]*((c*d*(f + g*x)
)/(c*d*f - a*e*g))^n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d}\,\left(gx+f\right)^{n}}{\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] int((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*(g*x + f)^n/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(x*e + d)*(g*x + f)^n/sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(x*e + d)*(g*x + f)^n/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2
)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,\sqrt{d+e\,x}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(1/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2), x)
```

3.764 $\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=104

$$-\frac{(ae+cdx)(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\;{}_2F_1\left(1,\frac{5}{2}+n;2+n;\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)\sqrt{d+ex}}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom([1, 5/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c
*d*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$\frac{2(f+gx)^n(ae+cdx)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d + e*x]
,x]
```

```
[Out] (2*(a*e + c*d*x)*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*Hy
pergeometric2F1[3/2, -n, 5/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(3*c*d
*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^n\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\int\sqrt{ae+cdx}\,(f+gx)^n\,dx}{\sqrt{ae+cdx}\,\sqrt{d+ex}}$$

$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\right)}{\sqrt{ae+cdx}\,\sqrt{d+ex}}$$

$$= \frac{2(ae+cdx)(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)\,x}}{3cd\sqrt{d+ex}}$$

Mathematica [A]
time = 0.14, size = 100, normalized size = 0.96

$$\frac{2((ae+cdx)(d+ex))^{3/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};\frac{g(ae+cdx)}{-cdf+aeg}\right)}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d
+ e*x],x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(3/2)*(f + g*x)^n*Hypergeometric2F1[3/2, -n, 5
/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(3*c*d*(d + e*x)^(3/2)*((c*d*(f
+ g*x))/(c*d*f - a*e*g))^n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^n\sqrt{ade+(a\,e^2+c\,d^2)\,x+cde\,x^2}}{\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(x*e
+ d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(g*x + f)^n/sqrt(x*e +
 d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^n/sqrt(x*e
+ d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2),x)
```

```
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^(
1/2), x)
```

3.765 $\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}} \, dx$

Optimal. Leaf size=104

$$-\frac{(ae+cdx)(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2} {}_2F_1\left(1,\frac{7}{2}+n;2+n;\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)(d+ex)^{3/2}}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*hypergeo
m([1, 7/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$\frac{2(f+gx)^n(ae+cdx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x]
```

```
[Out] (2*(a*e + c*d*x)^2*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*
Hypergeometric2F1[5/2, -n, 7/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(5*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^n \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\int (ae+cdx)^{3/2}(f+gx)^n}{\sqrt{ae+cdx}\,\sqrt{d+ex}}$$

$$= \frac{\left((f+gx)^n \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\right.}{\sqrt{ae+cdx}\,\sqrt{d+}}$$

$$= \frac{2(ae+cdx)^2(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+\left(cd^2+ae^2\right)}}{5cd\sqrt{d+ex}}$$

Mathematica [A]
time = 0.20, size = 100, normalized size = 0.96

$$\frac{2((ae+cdx)(d+ex))^{5/2}(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\,{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};\frac{g(ae+cdx)}{-cdf+aeg}\right)}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d +
e*x)^(3/2),x]
```

```
[Out] (2*((a*e + c*d*x)*(d + e*x))^(5/2)*(f + g*x)^n*Hypergeometric2F1[5/2, -n, 7
/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(5*c*d*(d + e*x)^(5/2)*((c*d*(f
+ g*x))/(c*d*f - a*e*g))^n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^n \left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

```
[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(x*e +
d)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral((c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)^(3/2)*sqrt(x*e + d)*(g*x +
 f)^n/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^n/(x*e +
d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{(d+e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2),x)
```

```
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^(
3/2), x)
```

3.766 $\int \frac{(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{(ae+cdx)(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}\,{}_2F_1\left(1,\frac{9}{2}+n;2+n;\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)(d+ex)^{5/2}}$$

```
[Out] -(c*d*x+a*e)*(g*x+f)^(1+n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*hypergeo
m([1, 9/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/(e*x+d)
^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$\frac{2(f+gx)^n(ae+cdx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{7}{2},-n;\frac{9}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*(f + g*x)^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]*
Hypergeometric2F1[7/2, -n, 9/2, -((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(7*c
*d*Sqrt[d + e*x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(f+gx)^n \left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}\int (ae+cdx)^{5/2}(f+gx)^n$$

$$= \frac{\left((f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}\right.}{\sqrt{ae+cdx}\,\sqrt{d+}$$

$$= \frac{2(ae+cdx)^3(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}\sqrt{ade+(cd^2+ae^2)}}{7cd\sqrt{d+ex}}$$

Mathematica [A]
time = 0.19, size = 110, normalized size = 1.06

$$\frac{2(ae+cdx)^3\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^n\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{7}{2},-n;\frac{9}{2};\frac{g(ae+cdx)}{-cdf+aeg}\right)}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d +
e*x)^(5/2),x]
```

```
[Out] (2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*Hypergeometric
2F1[7/2, -n, 9/2, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/(7*c*d*Sqrt[d + e*
x]*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^n\left(ade+(a\,e^2+c\,d^2)x+cde\,x^2\right)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

```
[Out] int((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(x*e +
d)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((c^2*d^4*x^2 + a^2*x^2*e^4 + (c^2*d^2*x^4 + 2*a*c*d^2*x^2 + a^2*d^
2)*e^2 + 2*(a*c*d^2*x^2 + (a*c*d*x^3 + a^2*d*x)*e)*e^2 + 2*(c^2*d^3*x^3 + a
*c*d^3*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)*(g*x
 + f)^n/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(
5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(x*e +
d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{(d+e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2),x)
```

```
[Out] int(((f + g*x)^n*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^(
5/2), x)
```

3.767 $\int (d+ex)^m(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal. Leaf size=103

$$-\frac{(ae+cdx)(d+ex)^m(f+gx)^{1+n}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(1,2-m+n;2+n;\frac{cd(f+gx)}{cdf-aeg}\right)}{(cdf-aeg)(1+n)}$$

```
[Out] -(c*d*x+a*e)*(e*x+d)^m*(g*x+f)^(1+n)*hypergeom([1, 2-m+n],[2+n],c*d*(g*x+f)
/(-a*e*g+c*d*f))/(-a*e*g+c*d*f)/(1+n)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {905, 72, 71}

$$\frac{(d+ex)^m(f+gx)^{n+1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\,{}_2F_1\left(m,n+1;n+2;\frac{cd(f+gx)}{cdf-aeg}\right)}{g(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hyp
ergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 + n
)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
```

```
e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m(f+gx)^n\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x\right.\right.$$
$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+a\right.\right.\right.$$
$$= \frac{\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{1+n}\left(ade+\left(\right.\right.}{}$$

Mathematica [A]
time = 0.07, size = 95, normalized size = 0.92

$$\frac{\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{1+n}\,{}_2F_1\left(m,1+n;2+n;\frac{cd(f+gx)}{cdf-aeg}\right)}{g(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^n)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] (((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(1 + n)*Hyp
ergeometric2F1[m, 1 + n, 2 + n, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(1 + n
)*((a*e + c*d*x)*(d + e*x))^m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^m\,(gx+f)^n\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((g*x + f)^n*(x*e + d)^m/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((g*x + f)^n*(x*e + d)^m/(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)^m,
 x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((g*x + f)^n*(x*e + d)^m/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^m
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,(d+e\,x)^m}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

3.768 $\int (d+ex)^m (f+gx)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal. Leaf size=343

$$-\frac{6(cdf-aeg)^2\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}+\frac{6g(cd$$

```
[Out] -6*(-a*e*g+c*d*f)^2*(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^4/d^4/e/(m^2-7*m+12)/(m^2-3*m+2)+6*g*
(-a*e*g+c*d*f)^2*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^3/d^3/
e/(2-m)/(3-m)/(4-m)+3*(-a*e*g+c*d*f)*(e*x+d)^(-1+m)*(g*x+f)^2*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/d^2/(3-m)/(4-m)+(e*x+d)^(-1+m)*(g*x+f)^3*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(4-m)
```

Rubi [A]

time = 0.43, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {884, 808, 662}

$$-\frac{6(d+ex)^{m-1}(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^4d^4e(1-m)(2-m)(3-m)(4-m)}+\frac{6g(d+ex)^m(cdf-aeg)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{c^3d^3e(2-m)(3-m)(4-m)}+\frac{3(f+gx)^2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{c^2d^2(3-m)(4-m)}+\frac{(f+gx)^3(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(4-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-6*(c*d*f - a*e*g)^2*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)
^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^4*d^4*e*(1 -
m)*(2 - m)*(3 - m)*(4 - m)) + (6*g*(c*d*f - a*e*g)^2*(d + e*x)^m*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(2 - m)*(3 - m)*(4 - m))
+ (3*(c*d*f - a*e*g)*(d + e*x)^(-1 + m)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*(3 - m)*(4 - m)) + ((d + e*x)^(-1 + m)*(
f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(4 - m))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
```

```
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int (d+ex)^m(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{(d+ex)^{-1+m}(f+gx)^3\left(ade+\left(cd^2+ae^2\right)x\right.}{cd(4-m)}$$

$$=\frac{3(cdf-aeg)(d+ex)^{-1+m}(f+gx)^2\left(ade+\right.}{c^2d^2(3-m)(4-m)}$$

$$=\frac{6g(cdf-aeg)^2(d+ex)^m\left(ade+\left(cd^2+ae^2\right)\right.}{c^3d^3e(2-m)(3-m)(4-m)}$$

$$=-\frac{6(cdf-aeg)^2\left(ae^2g+cd(dg(1-m)-ef(\right.}{c^4d^4e(1-m}$$

Mathematica [A]
time = 0.27, size = 134, normalized size = 0.39

$$\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\left(-\frac{(cdf-aeg)^3}{-1+m}-\frac{3g(cdf-aeg)^2(ae+cdx)}{-2+m}+\frac{3g^2(-cdf+aeg)(ae+cdx)^2}{-3+m}-\frac{g^3(ae+cdx)^3}{-4+m}\right)}{c^4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(-((c*d*f - a*e*g)^3/
(-1 + m)) - (3*g*(c*d*f - a*e*g)^2*(a*e + c*d*x))/(-2 + m) + (3*g^2*(-(c*d*
f) + a*e*g)*(a*e + c*d*x)^2)/(-3 + m) - (g^3*(a*e + c*d*x)^3)/(-4 + m)))/(c
^4*d^4)
```

Maple [A]
time = 0.17, size = 527, normalized size = 1.54

method	result
gosper	$-\frac{(ex+d)^m (c^3d^3g^3m^3x^3+3c^3d^3fg^2m^3x^2-6c^3d^3g^3m^2x^3+3ac^2d^2eg^3m^2x^2+3c^3d^3f^2gm^3x-21c^3d^3fg^2m^2x^2+11c^3d^3g^3mx^3+\dots}{}$
risch	$-\frac{(c^4d^4g^3m^3x^4+ac^3d^3eg^3m^3x^3+3c^4d^4fg^2m^3x^3-6c^4d^4g^3m^2x^4+3ac^3d^3efg^2m^3x^2-3ac^3d^3eg^3m^2x^3+3c^4d^4f^2gm^3x^2-\dots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RET
URNVERBOSE)
```

```
[Out] -(e*x+d)^m*(c^3*d^3*g^3*m^3*x^3+3*c^3*d^3*f*g^2*m^3*x^2-6*c^3*d^3*g^3*m^2*x
^3+3*a*c^2*d^2*e*g^3*m^2*x^2+3*c^3*d^3*f^2*g*m^3*x-21*c^3*d^3*f*g^2*m^2*x^2
+11*c^3*d^3*g^3*m*x^3+6*a*c^2*d^2*e*f*g^2*m^2*x-9*a*c^2*d^2*e*g^3*m*x^2+c^3
*d^3*f^3*m^3-24*c^3*d^3*f^2*g*m^2*x+42*c^3*d^3*f*g^2*m*x^2-6*c^3*d^3*g^3*x^
3+6*a^2*c*d*e^2*g^3*m*x+3*a*c^2*d^2*e*f^2*g*m^2-30*a*c^2*d^2*e*f*g^2*m*x+6*
a*c^2*d^2*e*g^3*x^2-9*c^3*d^3*f^3*m^2+57*c^3*d^3*f^2*g*m*x-24*c^3*d^3*f*g^2
*x^2+6*a^2*c*d*e^2*f*g^2*m-6*a^2*c*d*e^2*g^3*x-21*a*c^2*d^2*e*f^2*g*m+24*a*
c^2*d^2*e*f*g^2*x+26*c^3*d^3*f^3*m-36*c^3*d^3*f^2*g*x+6*a^3*e^3*g^3-24*a^2*
c*d*e^2*f*g^2+36*a*c^2*d^2*e*f^2*g-24*c^3*d^3*f^3)*(c*d*x+a*e)/((c*d*e*x^2+
a*e^2*x+c*d^2*x+a*d*e)^m)/c^4/d^4/(m^4-10*m^3+35*m^2-50*m+24)
```

Maxima [A]
time = 0.32, size = 333, normalized size = 0.97

$$-\frac{(cdx+ae)f^3}{(cdx+ae)^m cd(m-1)} - \frac{3\,(c^2d^2(m-1)x^2+acdmxe+a^2e^2)f^2g}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2} - \frac{3\,((m^2-3\,m+2)c^3d^3x^3+(m^2-m)ac^2d^2x^2e+2\,a^2cdmxe^2+2\,a^3e^3)fg^2}{(m^3-6\,m^2+11\,m-6)(cdx+ae)^m c^3d^3} - \frac{((m^3-6\,m^2+11\,m-6)c^4d^4x^4+(m^3-3\,m^2+2\,m)ac^3d^3x^3e+3\,(m^2-m)a^2c^2d^2x^2e^2+6\,a^3cdmxe^3+6\,a^4e^4)g^3}{(m^4-10\,m^3+35\,m^2-50\,m+24)(cdx+ae)^m c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^3/((c*d*x + a*e)^m*c*d*(m - 1)) - 3*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*m*x*e + a^2*e^2)*f^2*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - 3
*((m^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*x^2*e + 2*a^2*c*d*m*x*e
^2 + 2*a^3*e^3)*f*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3) -
((m^3 - 6*m^2 + 11*m - 6)*c^4*d^4*x^4 + (m^3 - 3*m^2 + 2*m)*a*c^3*d^3*x^3*e
 + 3*(m^2 - m)*a^2*c^2*d^2*x^2*e^2 + 6*a^3*c*d*m*x*e^3 + 6*a^4*e^4)*g^3/((m
^4 - 10*m^3 + 35*m^2 - 50*m + 24)*(c*d*x + a*e)^m*c^4*d^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(330) = 660$.
time = 0.82, size = 699, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(6*a^4*g^3*e^4 + (c^4*d^4*g^3*m^3 - 6*c^4*d^4*g^3*m^2 + 11*c^4*d^4*g^3*m -
 6*c^4*d^4*g^3)*x^4 + 3*(c^4*d^4*f*g^2*m^3 - 7*c^4*d^4*f*g^2*m^2 + 14*c^4*d
^4*f*g^2*m - 8*c^4*d^4*f*g^2)*x^3 + 3*(c^4*d^4*f^2*g*m^3 - 8*c^4*d^4*f^2*g*
m^2 + 19*c^4*d^4*f^2*g*m - 12*c^4*d^4*f^2*g)*x^2 + (c^4*d^4*f^3*m^3 - 9*c^4
*d^4*f^3*m^2 + 26*c^4*d^4*f^3*m - 24*c^4*d^4*f^3)*x + 6*(a^3*c*d*g^3*m*x +
a^3*c*d*f*g^2*m - 4*a^3*c*d*f*g^2)*e^3 + 3*(a^2*c^2*d^2*f^2*g*m^2 - 7*a^2*c
^2*d^2*f^2*g*m + 12*a^2*c^2*d^2*f^2*g + (a^2*c^2*d^2*g^3*m^2 - a^2*c^2*d^2*
g^3*m)*x^2 + 2*(a^2*c^2*d^2*f*g^2*m^2 - 4*a^2*c^2*d^2*f*g^2*m)*x)*e^2 + (a*
c^3*d^3*f^3*m^3 - 9*a*c^3*d^3*f^3*m^2 + 26*a*c^3*d^3*f^3*m - 24*a*c^3*d^3*f
^3 + (a*c^3*d^3*g^3*m^3 - 3*a*c^3*d^3*g^3*m^2 + 2*a*c^3*d^3*g^3*m)*x^3 + 3*
(a*c^3*d^3*f*g^2*m^3 - 5*a*c^3*d^3*f*g^2*m^2 + 4*a*c^3*d^3*f*g^2*m)*x^2 + 3
*(a*c^3*d^3*f^2*g*m^3 - 7*a*c^3*d^3*f^2*g*m^2 + 12*a*c^3*d^3*f^2*g*m)*x)*e)
*(x*e + d)^m/((c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m
 + 24*c^4*d^4)*(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)^m)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2024 vs. $2(330) = 660$.
time = 3.34, size = 2024, normalized size = 5.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((x*e + d)^m*c^4*d^4*g^3*m^3*x^4*e^(-m*log(c*d*x + a*e) - m*log(x*e + d))
+ 3*(x*e + d)^m*c^4*d^4*f*g^2*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d)) - 6*(x*e + d)^m*c^4*d^4*g^3*m^2*x^4*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) + (x*e + d)^m*a*c^3*d^3*g^3*m^3*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*
e + d) + 1) + 3*(x*e + d)^m*c^4*d^4*f^2*g*m^3*x^2*e^(-m*log(c*d*x + a*e) -
m*log(x*e + d)) - 21*(x*e + d)^m*c^4*d^4*f*g^2*m^2*x^3*e^(-m*log(c*d*x + a*
e) - m*log(x*e + d)) + 11*(x*e + d)^m*c^4*d^4*g^3*m*x^4*e^(-m*log(c*d*x + a
```

```
*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a*c^3*d^3*f*g^2*m^3*x^2*e^(-m*log(c*d
*x + a*e) - m*log(x*e + d) + 1) - 3*(x*e + d)^m*a*c^3*d^3*g^3*m^2*x^3*e^(-m
*log(c*d*x + a*e) - m*log(x*e + d) + 1) + (x*e + d)^m*c^4*d^4*f^3*m^3*x*e^(
-m*log(c*d*x + a*e) - m*log(x*e + d)) - 24*(x*e + d)^m*c^4*d^4*f^2*g*m^2*x^
2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 42*(x*e + d)^m*c^4*d^4*f*g^2*m
*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - 6*(x*e + d)^m*c^4*d^4*g^3*x
^4*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a*c^3*d^3*f^2*g
*m^3*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 15*(x*e + d)^m*a*c^3*
d^3*f*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) + 2*(x*e + d
)^m*a*c^3*d^3*g^3*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 9*(x
*e + d)^m*c^4*d^4*f^3*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 57*(
x*e + d)^m*c^4*d^4*f^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - 2
4*(x*e + d)^m*c^4*d^4*f*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) +
3*(x*e + d)^m*a^2*c^2*d^2*g^3*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 2) + (x*e + d)^m*a*c^3*d^3*f^3*m^3*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d) + 1) - 21*(x*e + d)^m*a*c^3*d^3*f^2*g*m^2*x*e^(-m*log(c*d*x + a*e) - m
*log(x*e + d) + 1) + 12*(x*e + d)^m*a*c^3*d^3*f*g^2*m*x^2*e^(-m*log(c*d*x +
 a*e) - m*log(x*e + d) + 1) + 26*(x*e + d)^m*c^4*d^4*f^3*m*x*e^(-m*log(c*d*
x + a*e) - m*log(x*e + d)) - 36*(x*e + d)^m*c^4*d^4*f^2*g*x^2*e^(-m*log(c*d
*x + a*e) - m*log(x*e + d)) + 6*(x*e + d)^m*a^2*c^2*d^2*f*g^2*m^2*x*e^(-m*l
og(c*d*x + a*e) - m*log(x*e + d) + 2) - 3*(x*e + d)^m*a^2*c^2*d^2*g^3*m*x^2
*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) - 9*(x*e + d)^m*a*c^3*d^3*f^3
*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) + 36*(x*e + d)^m*a*c^3*d^
3*f^2*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 24*(x*e + d)^m*c
^4*d^4*f^3*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + 3*(x*e + d)^m*a^2*c
^2*d^2*f^2*g*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) - 24*(x*e + d
)^m*a^2*c^2*d^2*f*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 2) + 26
*(x*e + d)^m*a*c^3*d^3*f^3*m*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) +
 6*(x*e + d)^m*a^3*c*d*g^3*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 3)
 - 21*(x*e + d)^m*a^2*c^2*d^2*f^2*g*m*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 2) - 24*(x*e + d)^m*a*c^3*d^3*f^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
 d) + 1) + 6*(x*e + d)^m*a^3*c*d*f*g^2*m*e^(-m*log(c*d*x + a*e) - m*log(x*e
 + d) + 3) + 36*(x*e + d)^m*a^2*c^2*d^2*f^2*g*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 2) - 24*(x*e + d)^m*a^3*c*d*f*g^2*e^(-m*log(c*d*x + a*e) - m*l
og(x*e + d) + 3) + 6*(x*e + d)^m*a^4*g^3*e^(-m*log(c*d*x + a*e) - m*log(x*e
 + d) + 4))/(c^4*d^4*m^4 - 10*c^4*d^4*m^3 + 35*c^4*d^4*m^2 - 50*c^4*d^4*m +
 24*c^4*d^4)
```

Mupad [B]

time = 3.75, size = 615, normalized size = 1.79

$$-\frac{\frac{g^3\,x^4\,{\left(d+e\,x\right)}^m\,\left(m^3-6\,m^2+11\,m-6\right)}{m^4-10\,m^3+35\,m^2-50\,m+24}+\frac{x\,{\left(d+e\,x\right)}^m\,\left(6\,a^3\,c\,d\,e^3\,g^3\,m+6\,a^2\,c^2\,d^2\,e^2\,f\,g^2\,m^2-24\,a^2\,c^2\,d^2\,e^2\,f\,g^2\,m+3\,a\,c^3\,d^3\,e\,f^2\,g\,m^3-21\,a\,c^3\,d^3\,e\,f^2\,g\,m^2+36\,a\,c^3\,d^3\,e\,f^2\,g\,m+c^4\,d^4\,f^3\,m^3-9\,c^4\,d^4\,f^3\,m^2+26\,c^4\,d^4\,f^3\,m-24\,c^4\,d^4\,f^3\right)}{c^4\,d^4\,\left(m^4-10\,m^3+35\,m^2-50\,m+24\right)}+\frac{a\,e\,{\left(d+e\,x\right)}^m\,\left(6\,a^3\,e^3\,g^3+6\,a^2\,c\,d\,e^2\,f\,g^2\,m-24\,a^2\,c\,d\,e^2\,f\,g^2+3\,a\,c^2\,d^2\,e\,f^2\,g\,m^2-21\,a\,c^2\,d^2\,e\,f^2\,g\,m+36\,a\,c^2\,d^2\,e\,f^2\,g+c^3\,d^3\,f^3\,m^3-9\,c^3\,d^3\,f^3\,m^2+26\,c^3\,d^3\,f^3\,m-24\,c^3\,d^3\,f^3\right)}{c^4\,d^4\,\left(m^4-10\,m^3+35\,m^2-50\,m+24\right)}+\frac{3\,g\,x^2\,\left(m-1\right)\,{\left(d+e\,x\right)}^m\,\left(a^2\,e^2\,g^2\,m+a\,c\,d\,e\,f\,g\,m^2-4\,a\,c\,d\,e\,f\,g\,m+c^2\,d^2\,f^2\,m^2-7\,c^2\,d^2\,f^2\,m+12\,c^2\,d^2\,f^2\right)}{c^2\,d^2\,\left(m^4-10\,m^3+35\,m^2-50\,m+24\right)}+\frac{g^2\,x^3\,{\left(d+e\,x\right)}^m\,\left(a\,e\,g\,m-12\,c\,d\,f+3\,c\,d\,f\,m\right)\,\left(m^2-3\,m+2\right)}{c\,d\,\left(m^4-10\,m^3+35\,m^2-50\,m+24\right)}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g^3*x^4*(d + e*x)^m*(11*m - 6*m^2 + m^3 - 6))/(35*m^2 - 50*m - 10*m^3 +
m^4 + 24) + (x*(d + e*x)^m*(26*c^4*d^4*f^3*m - 24*c^4*d^4*f^3 - 9*c^4*d^4*f
^3*m^2 + c^4*d^4*f^3*m^3 + 6*a^3*c*d*e^3*g^3*m - 24*a^2*c^2*d^2*e^2*f*g^2*m
 + 36*a*c^3*d^3*e*f^2*g*m + 6*a^2*c^2*d^2*e^2*f*g^2*m^2 - 21*a*c^3*d^3*e*f^
2*g*m^2 + 3*a*c^3*d^3*e*f^2*g*m^3))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4
+ 24)) + (a*e*(d + e*x)^m*(6*a^3*e^3*g^3 - 24*c^3*d^3*f^3 + 26*c^3*d^3*f^3*
m - 9*c^3*d^3*f^3*m^2 + c^3*d^3*f^3*m^3 + 36*a*c^2*d^2*e*f^2*g - 24*a^2*c*d
*e^2*f*g^2 - 21*a*c^2*d^2*e*f^2*g*m + 6*a^2*c*d*e^2*f*g^2*m + 3*a*c^2*d^2*e
*f^2*g*m^2))/(c^4*d^4*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)) + (3*g*x^2*(m -
1)*(d + e*x)^m*(12*c^2*d^2*f^2 + a^2*e^2*g^2*m - 7*c^2*d^2*f^2*m + c^2*d^2*
f^2*m^2 - 4*a*c*d*e*f*g*m + a*c*d*e*f*g*m^2))/(c^2*d^2*(35*m^2 - 50*m - 10*
m^3 + m^4 + 24)) + (g^2*x^3*(d + e*x)^m*(a*e*g*m - 12*c*d*f + 3*c*d*f*m)*(m
^2 - 3*m + 2))/(c*d*(35*m^2 - 50*m - 10*m^3 + m^4 + 24)))/(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^m
```

3.769 $\int (d+ex)^m(f+gx)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal. Leaf size=246

$$-\frac{2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^3d^3e(1-m)(2-m)(3-m)}+\frac{2g(cdf$$

```
[Out] -2*(-a*e*g+c*d*f)*(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^3/d^3/e/(1-m)/(2-m)/(3-m)+2*g*(-a*e*g+c
*d*f)*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c^2/d^2/e/(2-m)/(3-
m)+(e*x+d)^(-1+m)*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(3-
m)
```

Rubi [A]

time = 0.26, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {884, 808, 662}

$$-\frac{2(d+ex)^{m-1}(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^3d^3e(1-m)(2-m)(3-m)}+\frac{2g(d+ex)^m(cdf-aeg)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{c^2d^2e(2-m)(3-m)}+\frac{(f+gx)^2(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] (-2*(c*d*f - a*e*g)*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(
-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^3*d^3*e*(1 - m)
*(2 - m)*(3 - m)) + (2*g*(c*d*f - a*e*g)*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(2 - m)*(3 - m)) + ((d + e*x)^(-1 + m
)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(3 - m)
)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \frac{(d+ex)^{-1+m}(f+gx)^2\left(ade+\left(cd^2+ae^2\right)x\right.}{cd(3-m)}$$

$$= \frac{2g(cdf-aeg)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x\right.}{c^2d^2e(2-m)(3-m)}$$

$$= -\frac{2(cdf-aeg)\left(ae^2g+cd(dg(1-m)-ef(2\right.}{c^3d^3e(1}$$

Mathematica [A]
time = 0.19, size = 131, normalized size = 0.53

$$-\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\left(2a^2e^2g^2+2acdeg(f(-3+m)+g(-1+m)x)+c^2d^2\left(f^2\left(6-5m+m^2\right)+2fg\left(3-4m+m^2\right)x+g^2\left(2-3m+m^2\right)x^2\right)\right)}{c^3d^3(-3+m)(-2+m)(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(2*a^2*e^2*g^2 + 2*
a*c*d*e*g*(f*(-3 + m) + g*(-1 + m)*x) + c^2*d^2*(f^2*(6 - 5*m + m^2) + 2*f*
g*(3 - 4*m + m^2)*x + g^2*(2 - 3*m + m^2)*x^2)))/(c^3*d^3*(-3 + m)*(-2 + m)
*(-1 + m)))
```

Maple [A]
time = 0.15, size = 235, normalized size = 0.96

method	result
gosper	$-\frac{(cdx+ae)\left(c^2d^2g^2m^2x^2+2c^2d^2fg\,m^2x-3c^2d^2g^2m\,x^2+2acde\,g^2mx+c^2d^2f^2m^2-8c^2d^2fgmx+2g^2x^2c^2d^2+2acdefgm-2acde\,g^2x\right.}{c^3d^3(m^3-6m^2+11m-6)}$

risch	$-\frac{(ex+d)^m (c^3d^3g^2m^2x^3+a\,c^2d^2e\,g^2m^2x^2+2c^3d^3\,fg\,m^2x^2-3c^3d^3g^2m\,x^3+2a\,c^2d^2efg\,m^2x-a\,c^2d^2e\,g^2m\,x^2+c^3d^3f^2m^2x-8c^3d^3\,fgm}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RET
URNVERBOSE)
```

```
[Out] -(c*d*x+a*e)*(c^2*d^2*g^2*m^2*x^2+2*c^2*d^2*f*g*m^2*x-3*c^2*d^2*g^2*m*x^2+2
*a*c*d*e*g^2*m*x+c^2*d^2*f^2*m^2-8*c^2*d^2*f*g*m*x+2*c^2*d^2*g^2*x^2+2*a*c*
d*e*f*g*m-2*a*c*d*e*g^2*x-5*c^2*d^2*f^2*m+6*c^2*d^2*f*g*x+2*a^2*e^2*g^2-6*a
*c*d*e*f*g+6*c^2*d^2*f^2)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)/c
^3/d^3/(m^3-6*m^2+11*m-6)
```

Maxima [A]
time = 0.31, size = 196, normalized size = 0.80

$$-\frac{(cdx+ae)f^2}{(cdx+ae)^m cd(m-1)}-\frac{2\,(c^2d^2(m-1)x^2+acdmxe+a^2e^2)fg}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}-\frac{((m^2-3\,m+2)c^3d^3x^3+(m^2-m)ac^2d^2x^2e+2\,a^2cdmxe^2+2\,a^3e^3)g^2}{(m^3-6\,m^2+11\,m-6)(cdx+ae)^m c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] -(c*d*x + a*e)*f^2/((c*d*x + a*e)^m*c*d*(m - 1)) - 2*(c^2*d^2*(m - 1)*x^2 +
 a*c*d*m*x*e + a^2*e^2)*f*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2) - ((m
^2 - 3*m + 2)*c^3*d^3*x^3 + (m^2 - m)*a*c^2*d^2*x^2*e + 2*a^2*c*d*m*x*e^2 +
 2*a^3*e^3)*g^2/((m^3 - 6*m^2 + 11*m - 6)*(c*d*x + a*e)^m*c^3*d^3)
```

Fricas [A]
time = 0.99, size = 348, normalized size = 1.41

$$-\frac{(2\,a^3g^2e^3+(c^3d^3g^2m^2-3\,c^3d^3g^2m+2\,c^3d^3g^2)x^3+2\,(c^3d^3fgm^2-4\,c^3d^3fgm+3\,c^3d^3fg)x^2+(c^3d^3f^2m^2-5\,c^3d^3f^2m+6\,c^3d^3f^2)x+2\,(a^2cdg^2mx+a^2cdfgm-3\,a^2cdfg)e^2+(ac^2d^2f^2m^2-5\,ac^2d^2f^2m+6\,ac^2d^2f^2+(ac^2d^2g^2m^2-ac^2d^2g^2m)x^2+2\,(ac^2d^2fgm^2-3\,ac^2d^2fgm)x)e)(xe+d)^m}{(c^3d^3m^3-6\,c^3d^3m^2+11\,c^3d^3m-6\,c^3d^3)(cd^2x+axe^2+(cdx^2+ad)e)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] -(2*a^3*g^2*e^3 + (c^3*d^3*g^2*m^2 - 3*c^3*d^3*g^2*m + 2*c^3*d^3*g^2)*x^3 +
 2*(c^3*d^3*f*g*m^2 - 4*c^3*d^3*f*g*m + 3*c^3*d^3*f*g)*x^2 + (c^3*d^3*f^2*m
^2 - 5*c^3*d^3*f^2*m + 6*c^3*d^3*f^2)*x + 2*(a^2*c*d*g^2*m*x + a^2*c*d*f*g*
m - 3*a^2*c*d*f*g)*e^2 + (a*c^2*d^2*f^2*m^2 - 5*a*c^2*d^2*f^2*m + 6*a*c^2*d
^2*f^2 + (a*c^2*d^2*g^2*m^2 - a*c^2*d^2*g^2*m)*x^2 + 2*(a*c^2*d^2*f*g*m^2 -
 3*a*c^2*d^2*f*g*m)*x)*e)*(x*e + d)^m/((c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^
3*d^3*m - 6*c^3*d^3)*(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)^m)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. 2(238) = 476.
time = 6.87, size = 981, normalized size = 3.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] -((x*e + d)^m*c^3*d^3*g^2*m^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d))
+ 2*(x*e + d)^m*c^3*d^3*f*g*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) - 3*(x*e + d)^m*c^3*d^3*g^2*m*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) + (x*e + d)^m*a*c^2*d^2*g^2*m^2*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 1) + (x*e + d)^m*c^3*d^3*f^2*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) - 8*(x*e + d)^m*c^3*d^3*f*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d)) + 2*(x*e + d)^m*c^3*d^3*g^2*x^3*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d)) + 2*(x*e + d)^m*a*c^2*d^2*f*g*m^2*x*e^(-m*log(c*d*x + a*e) - m*log(x*e
+ d) + 1) - (x*e + d)^m*a*c^2*d^2*g^2*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 1) - 5*(x*e + d)^m*c^3*d^3*f^2*m*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d)) + 6*(x*e + d)^m*c^3*d^3*f*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d)) + (x*e + d)^m*a*c^2*d^2*f^2*m^2*e^(-m*log(c*d*x + a*e) - m*log(x*
e + d) + 1) - 6*(x*e + d)^m*a*c^2*d^2*f*g*m*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) + 6*(x*e + d)^m*c^3*d^3*f^2*x*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d)) + 2*(x*e + d)^m*a^2*c*d*g^2*m*x*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 2) - 5*(x*e + d)^m*a*c^2*d^2*f^2*m*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) + 2*(x*e + d)^m*a^2*c*d*f*g*m*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 2) + 6*(x*e + d)^m*a*c^2*d^2*f^2*e^(-m*log(c*d*x + a*e) - m*lo
g(x*e + d) + 1) - 6*(x*e + d)^m*a^2*c*d*f*g*e^(-m*log(c*d*x + a*e) - m*log(
x*e + d) + 2) + 2*(x*e + d)^m*a^3*g^2*e^(-m*log(c*d*x + a*e) - m*log(x*e +
d) + 3))/(c^3*d^3*m^3 - 6*c^3*d^3*m^2 + 11*c^3*d^3*m - 6*c^3*d^3)
```

Mupad [**B**]
time = 3.52, size = 327, normalized size = 1.33

$$-\frac{\frac{g^2\,x^3\,{\left(d+e\,x\right)}^m\,\left(m^2-3\,m+2\right)}{m^3-6\,m^2+11\,m-6}+\frac{x\,{\left(d+e\,x\right)}^m\,\left(2\,a^2\,c\,d\,e^2\,g^2\,m+2\,a\,c^2\,d^2\,e\,f\,g\,m^2-6\,a\,c^2\,d^2\,e\,f\,g\,m+c^3\,d^3\,f^2\,m^2-5\,c^3\,d^3\,f^2\,m+6\,c^3\,d^3\,f^2\right)}{c^3\,d^3\,\left(m^3-6\,m^2+11\,m-6\right)}+\frac{a\,e\,{\left(d+e\,x\right)}^m\,\left(2\,a^2\,e^2\,g^2+2\,a\,c\,d\,e\,f\,g\,m-6\,a\,c\,d\,e\,f\,g+c^2\,d^2\,f^2\,m^2-5\,c^2\,d^2\,f^2\,m+6\,c^2\,d^2\,f^2\right)}{c^3\,d^3\,\left(m^3-6\,m^2+11\,m-6\right)}+\frac{g\,x^2\,\left(m-1\right)\,{\left(d+e\,x\right)}^m\,\left(a\,e\,g\,m-6\,c\,d\,f+2\,c\,d\,f\,m\right)}{c\,d\,\left(m^3-6\,m^2+11\,m-6\right)}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g^2*x^3*(d + e*x)^m*(m^2 - 3*m + 2))/(11*m - 6*m^2 + m^3 - 6) + (x*(d +
e*x)^m*(6*c^3*d^3*f^2 - 5*c^3*d^3*f^2*m + c^3*d^3*f^2*m^2 + 2*a^2*c*d*e^2*g
^2*m + 2*a*c^2*d^2*e*f*g*m^2 - 6*a*c^2*d^2*e*f*g*m))/(c^3*d^3*(11*m - 6*m^2
 + m^3 - 6)) + (a*e*(d + e*x)^m*(2*a^2*e^2*g^2 + 6*c^2*d^2*f^2 - 5*c^2*d^2*
f^2*m + c^2*d^2*f^2*m^2 - 6*a*c*d*e*f*g + 2*a*c*d*e*f*g*m))/(c^3*d^3*(11*m
- 6*m^2 + m^3 - 6)) + (g*x^2*(m - 1)*(d + e*x)^m*(a*e*g*m - 6*c*d*f + 2*c*d
*f*m))/(c*d*(11*m - 6*m^2 + m^3 - 6)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x
^2)^m
```

3.770 $\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal. Leaf size=150

$$-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{c^2d^2e(1-m)(2-m)}+\frac{g(d+ex)^m\left(ade+\right.}{c}$$

```
[Out] -(a*e^2*g+c*d*(d*g*(1-m)-e*f*(2-m)))*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1-m)/c^2/d^2/e/(1-m)/(2-m)+g*(e*x+d)^m*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1-m)/c/d/e/(2-m)
```

Rubi [A]
time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {808, 662}

$$\frac{g(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cde(2-m)}-\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)}{c^2d^2e(1-m)(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^(-1 + m)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c^2*d^2*e*(1 - m)*(2 - m))) + (g*(
d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*e*(2 - m))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{g(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cde(2-m)} -$$

$$=-\frac{\left(ae^2g+cd(dg(1-m)-ef(2-m))\right)(d+ex}{c^2d^2e(1-m)}$$

Mathematica [A]
time = 0.11, size = 67, normalized size = 0.45

$$-\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}(aeg+cd(f(-2+m)+g(-1+m)x))}{c^2d^2(-2+m)(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*(a*e*g + c*d*(f*(-2
 + m) + g*(-1 + m)*x)))/(c^2*d^2*(-2 + m)*(-1 + m)))
```

Maple [A]
time = 0.15, size = 89, normalized size = 0.59

method	result
gosper	$-\frac{(ex+d)^m(cdgmx+cdfm-cdgx+aeg-2cdf)(cdx+ae)\left(cde\,x^2+a\,e^2x+c\,d^2x+ade\right)^{-m}}{c^2d^2(m^2-3m+2)}$
risch	$-\frac{\left(g\,x^2c^2d^2m+acdegmx+c^2d^2fmx-g\,x^2c^2d^2+acdefm-2c^2d^2fx+a^2e^2g-2acdef\right)(ex+d)^m\mathrm{e}^{\frac{m\left(i\pi\mathrm{csgn}(i(ex+d)(cdx+ae))^3-i\pi\mathrm{csgn}(i(ex}{}}}{c^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RETUR
NVERBOSE)
```

```
[Out] -(e*x+d)^m*(c*d*g*m*x+c*d*f*m-c*d*g*x+a*e*g-2*c*d*f)*(c*d*x+a*e)/((c*d*e*x^
2+a*e^2*x+c*d^2*x+a*d*e)^m)/c^2/d^2/(m^2-3*m+2)
```

Maxima [A]
time = 0.33, size = 97, normalized size = 0.65

$$-\frac{(cdx+ae)f}{(cdx+ae)^m cd(m-1)}-\frac{(c^2d^2(m-1)x^2+acdmxe+a^2e^2)g}{(m^2-3\,m+2)(cdx+ae)^m c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
```

```
[Out] -(c*d*x + a*e)*f/((c*d*x + a*e)^m*c*d*(m - 1)) - (c^2*d^2*(m - 1)*x^2 + a*c
*d*m*x*e + a^2*e^2)*g/((m^2 - 3*m + 2)*(c*d*x + a*e)^m*c^2*d^2)
```

Fricas [A]
time = 0.80, size = 144, normalized size = 0.96

$$-\frac{(a^2ge^2 + (c^2d^2gm - c^2d^2g)x^2 + (c^2d^2fm - 2\,c^2d^2f)x + (acdgmx + acdfm - 2\,acdf)e)(xe + d)^m}{(c^2d^2m^2 - 3\,c^2d^2m + 2\,c^2d^2)(cd^2x + axe^2 + (cdx^2 + ad)e)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
```

```
[Out] -(a^2*g*e^2 + (c^2*d^2*g*m - c^2*d^2*g)*x^2 + (c^2*d^2*f*m - 2*c^2*d^2*f)*x
 + (a*c*d*g*m*x + a*c*d*f*m - 2*a*c*d*f)*e)*(x*e + d)^m/((c^2*d^2*m^2 - 3*c
^2*d^2*m + 2*c^2*d^2)*(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)^m)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(143) = 286.
time = 3.12, size = 369, normalized size = 2.46

$$-\frac{(xe+d)^m c^2d^2gmx^2e^{(-m\log(cdx+ae)-m\log(xe+d))} + (xe+d)^m c^2d^2fmxe^{(-m\log(cdx+ae)-m\log(xe+d))} - (xe+d)^m c^2d^2gx^2e^{(-m\log(cdx+ae)-m\log(xe+d))} + (xe+d)^m acdgmxe^{(-m\log(cdx+ae)-m\log(xe+d)+1)} - 2\,(xe+d)^m c^2d^2fxe^{(-m\log(cdx+ae)-m\log(xe+d))} + (xe+d)^m acdfme^{(-m\log(cdx+ae)-m\log(xe+d)+1)} - 2\,(xe+d)^m acdfe^{(-m\log(cdx+ae)-m\log(xe+d)+1)} + (xe+d)^m a^2ge^{(-m\log(cdx+ae)-m\log(xe+d)+2)}}{c^2d^2m^2 - 3\,c^2d^2m + 2\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] -((x*e + d)^m*c^2*d^2*g*m*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x
*e + d)^m*c^2*d^2*f*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) - (x*e + d
)^m*c^2*d^2*g*x^2*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*a*
c*d*g*m*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 2*(x*e + d)^m*c^2*
d^2*f*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*a*c*d*f*m*e^
(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1) - 2*(x*e + d)^m*a*c*d*f*e^(-m*lo
```

```
g(c*d*x + a*e) - m*log(x*e + d) + 1) + (x*e + d)^m*a^2*g*e^(-m*log(c*d*x +
a*e) - m*log(x*e + d) + 2))/(c^2*d^2*m^2 - 3*c^2*d^2*m + 2*c^2*d^2)
```

Mupad [B]
time = 3.36, size = 139, normalized size = 0.93

$$-\frac{\frac{g\,x^2\,(m-1)\,(d+e\,x)^m}{m^2-3\,m+2}+\frac{x\,(d+e\,x)^m\,(a\,e\,g\,m-2\,c\,d\,f+c\,d\,f\,m)}{c\,d\,(m^2-3\,m+2)}+\frac{a\,e\,(d+e\,x)^m\,(a\,e\,g-2\,c\,d\,f+c\,d\,f\,m)}{c^2\,d^2\,(m^2-3\,m+2)}}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((g*x^2*(m - 1)*(d + e*x)^m)/(m^2 - 3*m + 2) + (x*(d + e*x)^m*(a*e*g*m - 2
*c*d*f + c*d*f*m))/(c*d*(m^2 - 3*m + 2)) + (a*e*(d + e*x)^m*(a*e*g - 2*c*d*
f + c*d*f*m))/(c^2*d^2*(m^2 - 3*m + 2)))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^m
```

3.771 $\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx$

Optimal. Leaf size=54

$$\frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

```
[Out] (e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1-m)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {662}

$$\frac{(d+ex)^{m-1}\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m}}{cd(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] ((d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(1 - m))/(c*d*(
1 - m))
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rubi steps

$$\int (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\frac{(d+ex)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m)}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.78

$$-\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}}{cd(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m))/(c*d*(-1 + m)))
```

Maple [A]
time = 0.14, size = 57, normalized size = 1.06

method	result
gosper	$-\frac{(cdx+ae)(ex+d)^{m}\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{-m}}{cd(-1+m)}$
norman	$\left(-\frac{x\,\mathrm{e}^{m\ln(ex+d)}}{-1+m}-\frac{ae\,\mathrm{e}^{m\ln(ex+d)}}{cd(-1+m)}\right)\mathrm{e}^{-m\ln\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)}$
risch	$-\frac{(ex+d)^{m}(cdx+ae)\mathrm{e}^{\frac{m\left(i\pi\mathrm{csgn}(i(ex+d)(cdx+ae))^{3}-i\pi\mathrm{csgn}(i(ex+d)(cdx+ae))^{2}\mathrm{csgn}(i(ex+d))-i\pi\mathrm{csgn}(i(ex+d)(cdx+ae))^{2}\mathrm{csgn}(i(cdx+ae))+i\pi\,\mathrm{cs}\right.}{2}}}{cd(-1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=_RETURNVERBOSE
)
```

```
[Out] -(c*d*x+a*e)/c/d/(-1+m)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)
```

Maxima [A]
time = 0.32, size = 35, normalized size = 0.65

$$-\frac{cdx+ae}{(cdx+ae)^{m}cd(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="max
ima")
```

```
[Out] -(c*d*x + a*e)/((c*d*x + a*e)^m*c*d*(m - 1))
```

Fricas [A]
time = 1.33, size = 59, normalized size = 1.09

$$-\frac{(cdx+ae)(xe+d)^{m}}{(cdm-cd)(cd^{2}x+axe^{2}+(cdx^{2}+ad)e)^{m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fri
cas")
```

```
[Out] -(c*d*x + a*e)*(x*e + d)^m/((c*d*m - c*d)*(c*d^2*x + a*x*e^2 + (c*d*x^2 + a
*d)*e)^m)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Giac [A]
time = 3.98, size = 87, normalized size = 1.61

$$-\frac{(xe+d)^m cdxe^{(-m\log(cdx+ae)-m\log(xe+d))}+(xe+d)^m ae^{(-m\log(cdx+ae)-m\log(xe+d)+1)}}{cdm-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="gia
c")
```

```
[Out] -((x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)) + (x*e + d)^m*
a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) + 1))/(c*d*m - c*d)
```

Mupad [B]
time = 3.25, size = 57, normalized size = 1.06

$$-\frac{(a\,e+c\,d\,x)\,\left(d+e\,x\right)^m}{c\,d\,\left(m-1\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] -((a*e + c*d*x)*(d + e*x)^m)/(c*d*(m - 1)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*
e*x^2)^m)
```

3.772 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx$

Optimal. Leaf size=99

$$\frac{(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(1,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)(1-m)}$$

```
[Out] (c*d*x+a*e)*(e*x+d)^m*hypergeom([1, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c*d*f
))/(-a*e*g+c*d*f)/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A]
time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {905, 70}

$$\frac{(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(1,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] ((a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[1, 1 - m, 2 - m, -((g*(a*e + c
*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)*(1 - m)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{f+gx}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)\right.$$

$$=\frac{(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(cdf-aeg)(1-m)}$$

Mathematica [A]
time = 0.06, size = 82, normalized size = 0.83

$$-\frac{(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\,{}_2F_1\left(1,1-m;2-m;\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] -(((d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2F1[1
, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)*(-1
 + m)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{-m}}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="maxima")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="fricas")
```

```
[Out] integral((x*e + d)^m/((g*x + f)*(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)^m),
 x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algori
thm="giac")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^m)
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{(f+g\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

```
[Out] int((d + e*x)^m/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

3.773 $\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^2}\,dx$

Optimal. Leaf size=101

$$\frac{cd(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}{}_2F_1\left(2,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)^2(1-m)}$$

```
[Out] c*d*(c*d*x+a*e)*(e*x+d)^m*hypergeom([2, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e*g+c
*d*f))/(-a*e*g+c*d*f)^2/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {905, 70}

$$\frac{cd(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}{}_2F_1\left(2,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] (c*d*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[2, 1 - m, 2 - m, -((g*(a*e
 + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^2*(1 - m)*(a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2)^m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^2}\,dx = \left((ae+cdx)^m(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-\right.$$

$$= \frac{cd(ae+cdx)(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(cdf-aeg)^2(1-m)}$$

Mathematica [**A**]
time = 0.07, size = 84, normalized size = 0.83

$$-\frac{cd(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\,{}_2F_1\left(2,1-m;2-m;\frac{g(ae+cdx)}{-cdf+aeg}\right)}{(cdf-aeg)^2(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m),x]
```

```
[Out] -((c*d*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeometric2
F1[2, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*e*g)
^2*(-1 + m)))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m (ade+(a\,e^2+c\,d^2)x+cde\,x^2)^{-m}}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)^2*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((x*e + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d^2*x + a*x*e^2 + (c*d*x
^2 + a*d)*e)^m), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^2/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)^2*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{(f+g\,x)^2\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

3.774 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^3}\,dx$

Optimal. Leaf size=105

$$\frac{c^2d^2(ae+cdx)(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(3,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(cdf-aeg)^3(1-m)}$$

```
[Out] c^2*d^2*(c*d*x+a*e)*(e*x+d)^m*hypergeom([3, 1-m],[2-m],-g*(c*d*x+a*e)/(-a*e
*g+c*d*f))/(-a*e*g+c*d*f)^3/(1-m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {905, 70}

$$\frac{c^2d^2(d+ex)^m(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\,{}_2F_1\left(3,1-m;2-m;-\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x]
```

```
[Out] (c^2*d^2*(a*e + c*d*x)*(d + e*x)^m*Hypergeometric2F1[3, 1 - m, 2 - m, -((g*
(a*e + c*d*x))/(c*d*f - a*e*g))])/((c*d*f - a*e*g)^3*(1 - m)*(a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m\,(ade+(cd^2+ae^2)\,x+cdex^2)^{-m}}{(f+gx)^3}\,dx = \Big((ae+cdx)^m(d+ex)^m\,\big(ade+\big(cd^2+ae^2\big)\,x+cdex^2\big)$$

$$= \frac{c^2d^2(ae+cdx)(d+ex)^m\,(ade+(cd^2+ae^2)\,x+cdex^2)}{(cdf-aeg)^3(1-m}$$

Mathematica [A]
time = 0.07, size = 88, normalized size = 0.84

$$-\frac{c^2d^2(d+ex)^{-1+m}((ae+cdx)(d+ex))^{1-m}\,{}_2F_1\Big(3,1-m;2-m;\frac{g(ae+cdx)}{-cdf+aeg}\Big)}{(cdf-aeg)^3(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
m),x]
```

```
[Out] -((c^2*d^2*(d + e*x)^(-1 + m)*((a*e + c*d*x)*(d + e*x))^(1 - m)*Hypergeomet
ric2F1[3, 1 - m, 2 - m, (g*(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*f - a*
e*g)^3*(-1 + m)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\,(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2)^{-m}}{(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="maxima")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)^3*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="fricas")
```

```
[Out] integral((x*e + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)^m), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x
)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^3/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algo
rithm="giac")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)^3*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^
m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{(f+g\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m),x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m), x)
```

3.775 $\int (d+ex)^m(f+gx)^{3/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-}$

Optimal. Leaf size=105

$$\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{5/2}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,{}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

```
[Out] 2/5*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(5/2)*hypergeom([5/
2, m],[7/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^m)
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$\frac{2(f+gx)^{5/2}(d+ex)^m\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m\,{}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \Big(\left((ae+cdx)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x\right.\right.$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+\right.\right.\right.$$

$$= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m (f+gx)^{5/2} \left(ade+\right.}{}$$

Mathematica [A]
time = 0.17, size = 93, normalized size = 0.89

$$\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m ((ae+cdx)(d+ex))^{-m} (f+gx)^{5/2} {}_2F_1\left(\frac{5}{2},m;\frac{7}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{5g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^(3/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(5/2)*Hyp
ergeometric2F1[5/2, m, 7/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(5*g*((a*e +
c*d*x)*(d + e*x))^m)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^m (gx+f)^{\frac{3}{2}} \left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(3/2)*(x*e + d)^m/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral((g*x + f)^(3/2)*(x*e + d)^m/(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e
)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)*(x*e + d)^m/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*
x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^{3/2}\,(d+e\,x)^m}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
```

```
[Out] int(((f + g*x)^(3/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
, x)
```

3.776 $\int (d+ex)^m \sqrt{f+gx} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal. Leaf size=105

$$\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m (f+gx)^{3/2} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} {}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

```
[Out] 2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*(g*x+f)^(3/2)*hypergeom([3/
2, m],[5/2],c*d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^m)
```

Rubi [A]

time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$\frac{2(f+gx)^{3/2}(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x
]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx=\left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x\right.\right.$$

$$=\left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+a\right.\right.\right.$$

$$=\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m(f+gx)^{3/2}\left(ade+\right.}{3}$$

Mathematica [A]
time = 0.12, size = 93, normalized size = 0.89

$$\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}(f+gx)^{3/2}\,{}_2F_1\left(\frac{3}{2},m;\frac{5}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*Sqrt[f + g*x])/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2)^m,x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*(f + g*x)^(3/2)*Hyp
ergeometric2F1[3/2, m, 5/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e +
c*d*x)*(d + e*x))^m)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^m\sqrt{gx+f}\,\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^m/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(x*e + d)^m/(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)^
m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^m/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)
^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}\,\left(d+e\,x\right)^m}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
```

```
[Out] int(((f + g*x)^(1/2)*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
, x)
```

3.777 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=103

$$\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\sqrt{f+gx}\,(ade+(cd^2+ae^2)\,x+cdex^2)^{-m}\,{}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

```
[Out] 2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([1/2, m],[3/2],c*d*
(g*x+f)/(-a*e*g+c*d*f))*(g*x+f)^(1/2)/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
m)
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$\frac{2\sqrt{f+gx}\,(d+ex)^m\,(x(ae^2+cd^2)+ade+cdex^2)^{-m}\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m{}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m),x
]
```

```
[Out] (2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*(a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{\sqrt{f+gx}}\,dx = \left((ae+cdx)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-\cdots}\right.$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cd\cdots\right.\right.$$

$$= \frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m(d+ex)^m\sqrt{f+gx}\left(ade+\left(cd^2+ae^2\right)\cdots\right.}{g}$$

Mathematica [A]
time = 0.13, size = 91, normalized size = 0.88

$$\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m(d+ex)^m((ae+cdx)(d+ex))^{-m}\sqrt{f+gx}\;{}_2F_1\left(\frac{1}{2},m;\frac{3}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
)^m),x]
```

```
[Out] (2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Sqrt[f + g*x]*Hyper
geometric2F1[1/2, m, 3/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*
x)*(d + e*x))^m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{-m}}{\sqrt{gx+f}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((x*e + d)^m/(sqrt(g*x + f)*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral((x*e + d)^m/(sqrt(g*x + f)*(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
^m), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(1/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(1/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((x*e + d)^m/(sqrt(g*x + f)*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x
)^m), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{\sqrt{f+g\,x}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

3.778 $\int \frac{(d+ex)^m\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} {}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

```
[Out] -2*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-1/2, m],[1/2],c*
d*(g*x+f)/(-a*e*g+c*d*f))/g/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m)/(g*x+f)^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$-\frac{2(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*Sqrt[f + g*x]*(a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{3/2}}\,dx = \left((ae+cdx)^m(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-\text{}}\right.$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cd\right.\right.$$

$$= -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex\right.}{g\sqrt{f+gx}}$$

Mathematica [A]
time = 0.19, size = 91, normalized size = 0.88

$$-\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m((ae+cdx)(d+ex))^{-m}\,{}_2F_1\left(-\frac{1}{2},m;\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m),x]
```

```
[Out] (-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1[
-1/2, m, 1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(g*((a*e + c*d*x)*(d + e*x)
)^m*Sqrt[f + g*x])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m \left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{-m}}{(gx+f)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)^(3/2)*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(x*e + d)^m/((g^2*x^2 + 2*f*g*x + f^2)*(c*d^2*x + a*
x*e^2 + (c*d*x^2 + a*d)*e)^m), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(3/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)^(3/2)*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{(f+g\,x)^{3/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^(3/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

3.779 $\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}} \, dx$

Optimal. Leaf size=105

$$-\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} {}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

```
[Out] -2/3*(-g*(c*d*x+a*e)/(-a*e*g+c*d*f))^m*(e*x+d)^m*hypergeom([-3/2, m],[-1/2]
,c*d*(g*x+f)/(-a*e*g+c*d*f))/g/(g*x+f)^(3/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {905, 72, 71}

$$-\frac{2(d+ex)^m \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
,x]
```

```
[Out] (-2*(-((g*(a*e + c*d*x))/(c*d*f - a*e*g)))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*(f + g*x)^(3/2)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}}{(f+gx)^{5/2}}\,dx = \left((ae+cdx)^m(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-\cdots}\right.$$

$$= \left(\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cd\cdots\right.\right.$$

$$= -\frac{2\left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m (d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex\cdots\right.}{3g(f+gx)^{3/2}}$$

Mathematica [A]
time = 0.25, size = 93, normalized size = 0.89

$$-\frac{2\left(\frac{g(ae+cdx)}{-cdf+aeg}\right)^m (d+ex)^m((ae+cdx)(d+ex))^{-m}\,{}_2F_1\left(-\frac{3}{2},m;-\frac{1}{2};\frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/((f + g*x)^(5/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^m),x]
```

```
[Out] (-2*((g*(a*e + c*d*x))/(-(c*d*f) + a*e*g))^m*(d + e*x)^m*Hypergeometric2F1[
-3/2, m, -1/2, (c*d*(f + g*x))/(c*d*f - a*e*g)])/(3*g*((a*e + c*d*x)*(d + e
*x))^m*(f + g*x)^(3/2))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m \left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{-m}}{(gx+f)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)^(5/2)*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] integral(sqrt(g*x + f)*(x*e + d)^m/((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^
3)*(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)^m), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)**(5/2)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)^(5/2)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] integrate((x*e + d)^m/((g*x + f)^(5/2)*(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)
*x)^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{(f+g\,x)^{5/2}\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
,x)
```

```
[Out] int((d + e*x)^m/((f + g*x)^(5/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m)
, x)
```

3.780 $\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}$

Optimal. Leaf size=65

$$\frac{(ae+cdx)^n(d+ex)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m+n)}$$

```
[Out] (c*d*x+a*e)^n*(e*x+d)^(-1+m)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1-m)/c/d/(1
-m+n)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {872}

$$\frac{(d+ex)^{m-1} \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{1-m} (ae+cdx)^n}{cd(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m
,x]
```

```
[Out] ((a*e + c*d*x)^n*(d + e*x)^(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)
^(1 - m))/(c*d*(1 - m + n))
```

Rule 872

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] /; FreeQ[{a, b, c, d, e, f,
 g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && EqQ[c*e*f + c*d*g -
 b*e*g, 0] && NeQ[m - n - 1, 0]
```

Rubi steps

$$\int (ae+cdx)^n(d+ex)^m \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m} dx = \frac{(ae+cdx)^n(d+ex)^{-1+m} \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{1-m}}{cd(1-m+n)}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.82

$$\frac{(ae+cdx)^{1+n}(d+ex)^m((ae+cdx)(d+ex))^{-m}}{cd-cdm+cdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*e + c*d*x)^n*(d + e*x)^m)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^m,x]
```

```
[Out] ((a*e + c*d*x)^(1 + n)*(d + e*x)^m)/((c*d - c*d*m + c*d*n)*((a*e + c*d*x)*(
d + e*x))^m)
```

Maple [A]
time = 0.17, size = 64, normalized size = 0.98

method	result
gosper	$-\frac{(cdx+ae)^{1+n}(ex+d)^{m}\left(cde\,x^{2}+a\,e^{2}x+c\,d^{2}x+ade\right)^{-m}}{cd(-1+m-n)}$
risch	$-\frac{(ex+d)^{m}(cdx+ae)^{n}(cdx+ae)\mathrm{e}^{\frac{m\left(i\pi\mathrm{csgn}(i(ex+d)(cdx+ae))^{3}-i\pi\mathrm{csgn}(i(ex+d)(cdx+ae))^{2}\mathrm{csgn}(i(ex+d))-i\pi\mathrm{csgn}(i(ex+d)(cdx+ae))^{2}\mathrm{csgn}(i(cdx}{2}}}{cd(-1+m-n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,method=
_RETURNVERBOSE)
```

```
[Out] -(c*d*x+a*e)^(1+n)/c/d/(-1+m-n)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e
)^m)
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.80

$$-\frac{(cdx+ae)e^{(-m\log(cdx+ae)+n\log(cdx+ae))}}{cd(m-n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="maxima")
```

```
[Out] -(c*d*x + a*e)*e^(-m*log(c*d*x + a*e) + n*log(c*d*x + a*e))/(c*d*(m - n - 1
))
```

Fricas [A]
time = 4.09, size = 71, normalized size = 1.09

$$-\frac{(cdx+ae)(cdx+ae)^{n}(xe+d)^{m}e^{(-m\log(cdx+ae)-m\log(xe+d))}}{cdm-cdn-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="fricas")
```

```
[Out] -(c*d*x + a*e)*(c*d*x + a*e)^n*(x*e + d)^m*e^(-m*log(c*d*x + a*e) - m*log(x
*e + d))/(c*d*m - c*d*n - c*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
m),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.29, size = 114, normalized size = 1.75

$$-\frac{(cdx+ae)^n(xe+d)^m cdxe^{(-m\log(cdx+ae)-m\log(xe+d))}+(cdx+ae)^n(xe+d)^m ae^{(-m\log(cdx+ae)-m\log(xe+d)+1)}}{cdm-cdn-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x,
algorithm="giac")
```

```
[Out] -((c*d*x + a*e)^n*(x*e + d)^m*c*d*x*e^(-m*log(c*d*x + a*e) - m*log(x*e + d)
) + (c*d*x + a*e)^n*(x*e + d)^m*a*e^(-m*log(c*d*x + a*e) - m*log(x*e + d) +
 1))/(c*d*m - c*d*n - c*d)
```

Mupad [**B**]
time = 3.54, size = 63, normalized size = 0.97

$$\frac{(a\,e+c\,d\,x)^{n+1}\,(d+e\,x)^m}{c\,d\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^m\,(n-m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*e + c*d*x)^n*(d + e*x)^m)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m
,x)
```

```
[Out] ((a*e + c*d*x)^(n + 1)*(d + e*x)^m)/(c*d*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e
*x^2)^m*(n - m + 1))
```

3.781 $\int (d+ex)^m \left(cd^2eg - e(cd^2+ae^2)\, g - cde^2gx\right)^{-1+m} (a$

Optimal. Leaf size=78

$$-\frac{(d+ex)^m\left(-ae^3g-cde^2gx\right)^m\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{-m}\log(ae+cdx)}{cde^2g}$$

```
[Out] -(e*x+d)^m*(-c*d*e^2*g*x-a*e^3*g)^m*ln(c*d*x+a*e)/c/d/e^2/g/((a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}} = 0.041$, Rules used = {905, 23, 31}

$$-\frac{(d+ex)^m\left(x(ae^2+cd^2)+ade+cdex^2\right)^{-m}\log(ae+cdx)\left(-ae^3g-cde^2gx\right)^m}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1 + m))/
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((d + e*x)^m*(-(a*e^3*g) - c*d*e^2*g*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*(a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && !IntegerQ[p] && !IGtQ[m, 0] && !IGtQ[n, 0]
```

Rubi steps

$$\int (d+ex)^m \left(cd^2eg-e\left(cd^2+ae^2\right)g-cde^2gx\right)^{-1+m}\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{-m}\,dx = \Big((ae+cdx)^m($$

$$= \Big((d+ex)^m\,(c$$

$$= -\frac{(d+ex)^m\,(-}{}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.82

$$-\frac{(-e^2g(ae+cdx))^m\,(d+ex)^m((ae+cdx)(d+ex))^{-m}\log(ae+cdx)}{cde^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(c*d^2*e*g - e*(c*d^2 + a*e^2)*g - c*d*e^2*g*x)^(-1
+ m))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m,x]
```

```
[Out] -(((-(e^2*g*(a*e + c*d*x)))^m*(d + e*x)^m*Log[a*e + c*d*x])/(c*d*e^2*g*((a*
e + c*d*x)*(d + e*x))^m))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (ex+d)^m\left(c\,d^2eg-e\left(a\,e^2+c\,d^2\right)g-cd\,e^2gx\right)^{-1+m}\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

```
[Out] int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^m),x)
```

Maxima [A]
time = 0.30, size = 32, normalized size = 0.41

$$-\frac{(-g)^m\,e^{(2\,m-2)}\log\left(cdx+ae\right)}{cdg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="maxima")
```

```
[Out] -(-g)^m*e^(2*m - 2)*log(c*d*x + a*e)/(c*d*g)
```

Fricas [A]
time = 2.90, size = 34, normalized size = 0.44

$$-\frac{e^{(-2)}\log\left(cdx+ae\right)}{cdg\left(-\frac{e^{(-2)}}{g}\right)^{m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="fricas")
```

```
[Out] -e^(-2)*log(c*d*x + a*e)/(c*d*g*(-e^(-2)/g)^m)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*d**2*e*g-e*(a*e**2+c*d**2)*g-c*d*e**2*g*x)**(-1+m)/
((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x, algorithm="giac")
```

```
[Out] integrate((-c*d*g*x*e^2 + c*d^2*g*e - (c*d^2 + a*e^2)*g*e)^(m - 1)*(x*e + d
)^m/(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^m\,{\left(c\,d^2\,e\,g-e\,g\,\left(c\,d^2+a\,e^2\right)-c\,d\,e^2\,g\,x\right)}^{m-1}}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^m} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m,x)
```

```
[Out] int(((d + e*x)^m*(c*d^2*e*g - e*g*(a*e^2 + c*d^2) - c*d*e^2*g*x)^(m - 1))/(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^m, x)
```

3.782 $\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=213

$$\frac{2e(f+gx)^{1+n}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}}+\frac{(2ae^2g(1+n)+cd(ef-dg(3+2n)))\,(ae+cdx)\sqrt{d+ex}}{cdg(cdf-aeg)(1+n)(3+2n)\sqrt{ade+}}$$

```
[Out] (2*a*e^2*g*(1+n)+c*d*(e*f-d*g*(3+2*n)))*(c*d*x+a*e)*(g*x+f)^(1+n)*hypergeom
([1, 3/2+n],[2+n],c*d*(g*x+f)/(-a*e*g+c*d*f))*(e*x+d)^(1/2)/c/d/g/(-a*e*g+c
*d*f)/(1+n)/(3+2*n)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*e*(g*x+f)^(1+
n)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(3+2*n)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {894, 905, 72, 71}

$$\frac{2e(f+gx)^{n+1}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\,(f+gx)^n(ae+cdx)\left(2ae^2g(n+1)+cd(ef-dg(2n+3))\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{g(ae+cdx)}{cdf-aeg}\right)}{c^2d^2g(2n+3)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (2*e*(f + g*x)^(1 + n)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*
(3 + 2*n)*Sqrt[d + e*x]) - (2*(2*a*e^2*g*(1 + n) + c*d*(e*f - d*g*(3 + 2*n)
))*(a*e + c*d*x)*Sqrt[d + e*x]*(f + g*x)^n*Hypergeometric2F1[1/2, -n, 3/2,
-((g*(a*e + c*d*x))/(c*d*f - a*e*g))])/(c^2*d^2*g*(3 + 2*n)*((c*d*(f + g*x)
)/(c*d*f - a*e*g))^n*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rule 905

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((d +
 e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f +
g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &
& NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx = \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{(2ae^2g(1+n)+cd}{}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left((2ae^2g(1+n)+\right.}{}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{\left((2ae^2g(1+n)+\right.}{}$$

$$= \frac{2e(f+gx)^{1+n}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{cdg(3+2n)\sqrt{d+ex}} - \frac{2(2ae^2g(1+n)+c}{}$$

Mathematica [A]
time = 0.25, size = 145, normalized size = 0.68

$$\frac{\sqrt{(ae+cdx)(d+ex)}\,(f+gx)^n\left(cde(f+gx)+\left(-2ae^2g(1+n)+cd(-ef+dg(3+2n))\right)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{g(ae+cdx)}{-cdf+aeg}\right)\right)}{c^2d^2g\left(\frac{3}{2}+n\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^n)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (Sqrt[(a*e + c*d*x)*(d + e*x)]*(f + g*x)^n*(c*d*e*(f + g*x) + ((-2*a*e^2*g*
(1 + n) + c*d*(-(e*f) + d*g*(3 + 2*n)))*Hypergeometric2F1[1/2, -n, 3/2, (g*
(a*e + c*d*x))/(-(c*d*f) + a*e*g)])/((c*d*(f + g*x))/(c*d*f - a*e*g))^n))/(
c^2*d^2*g*(3/2 + n)*Sqrt[d + e*x])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{\frac{3}{2}} (gx+f)^{n}}{\sqrt{ade+(a\,e^{2}+c\,d^{2})\,x+cde\,x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

```
[Out] int((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)*(g*x + f)^n/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((x*e + d)^(3/2)*(g*x + f)^n/sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*
d)*e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**n/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^n/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((x*e + d)^(3/2)*(g*x + f)^n/sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e
^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^n\,(d+e\,x)^{3/2}}{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] int(((f + g*x)^n*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2), x)
```

3.783 $\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=501

$$\frac{128(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3465c^6d^6eg\sqrt{d+ex}}-\frac{128(cd\cdots}{}$$

```
[Out] 128/3465*(-a*e*g+c*d*f)^3*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(2*a*e^2*g-c*d*(-d
*g+3*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^6/d^6/e/g/(e*x+d)^(1/2
)-32/1155*(-a*e*g+c*d*f)^2*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+f)^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/g/(e*x+d)^(1/2)-16/693*(-a*e*g+c*d
*f)*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/c^3/d^3/g/(e*x+d)^(1/2)-2/99*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(g*x+
f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(1/2)+2/11*e
*(g*x+f)^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)-128/
3465*(-a*e*g+c*d*f)^3*(10*a*e^2*g+c*d*(-11*d*g+e*f))*(e*x+d)^(1/2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e
```

Rubi [A]
time = 0.55, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {894, 884, 808, 662}

$$\frac{128\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{3465c^6d^6eg\sqrt{d+ex}}-\frac{128\sqrt{d+ex}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^3\left(10ae^2g+cd(ef-11dg)\right)}{3465c^5d^5e}-\frac{32(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)^2\left(10ae^2g+cd(ef-11dg)\right)}{1155c^4d^4g\sqrt{d+ex}}-\frac{16(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)\left(10ae^2g+cd(ef-11dg)\right)}{693c^3d^3g\sqrt{d+ex}}-\frac{2(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(10ae^2g+cd(ef-11dg)\right)}{99c^2d^2g\sqrt{d+ex}}+\frac{2e(f+gx)^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{11cdg\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(2*a*e^2*g - c*d*(
3*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^6*d^6*e*
g*Sqrt[d + e*x]) - (128*(c*d*f - a*e*g)^3*(10*a*e^2*g + c*d*(e*f - 11*d*g))
*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3465*c^5*d^5*e
) - (32*(c*d*f - a*e*g)^2*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^2*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1155*c^4*d^4*g*Sqrt[d + e*x]) -
(16*(c*d*f - a*e*g)*(10*a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^3*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(693*c^3*d^3*g*Sqrt[d + e*x]) - (2*(10*
a*e^2*g + c*d*(e*f - 11*d*g))*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(99*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^5*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(11*c*d*g*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
```

```
x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2e(f+gx)^5\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{11cdg\sqrt{d+ex}} - \frac{1}{11}\bigg(-11d+\frac{10ae^2}{cd}$$

$$= -\frac{2(10ae^2g+cd(ef-11dg))\,(f+gx)^4\sqrt{ade+(cd^2+ae^2)\,x+cd}}{99c^2d^2g\sqrt{d+ex}}$$

$$= -\frac{16(cdf-aeg)\,(10ae^2g+cd(ef-11dg))\,(f+gx)^3\sqrt{ade+(cd^2}}{693c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{32(cdf-aeg)^2\,(10ae^2g+cd(ef-11dg))\,(f+gx)^2\sqrt{ade+(cd^2}}{1155c^4d^4g\sqrt{d+ex}}$$

$$= -\frac{128(cdf-aeg)^3\,(10ae^2g+cd(ef-11dg))\sqrt{d+ex}\sqrt{ade+(cd}}{3465c^5d^5e}$$

$$= \frac{128(cdf-aeg)^3\,(10ae^2g+cd(ef-11dg))\,(2ae^2g-cd(3ef-dg))}{3465c^6d^6eg\sqrt{d+ex}}$$

Mathematica [A]
time = 0.30, size = 380, normalized size = 0.76

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(-1280a^5e^6g^4+128a^4cde^4g^3(44ef+11dg+5egx)-32a^3c^2d^2e^3g^2(22dg(9f+gx)+e(297f^2+88fgx+15g^2x^2))+16a^2c^3d^3e^2g(33dg(21f^2+6fgx+g^2x^2)+e(462f^3+297f^2gx+132fg^2x^2+25g^3x^3))-2ac^4d^4e(44dg(105f^3+63f^2gx+27fg^2x^2+5g^3x^3)+e(1155f^4+1848f^3gx+1782f^2g^2x^2+880fg^3x^3+175g^4x^4))+c^5d^5(11d(315f^4+420f^3gx+378f^2g^2x^2+180fg^3x^3+35g^4x^4)+ex(1155f^4+2772f^3gx+2970f^2g^2x^2+1540fg^3x^3+315g^4x^4)))}{3465c^6d^6\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-1280*a^5*e^6*g^4 + 128*a^4*c*d*e^4*g^3*(
44*e*f + 11*d*g + 5*e*g*x) - 32*a^3*c^2*d^2*e^3*g^2*(22*d*g*(9*f + g*x) + e
*(297*f^2 + 88*f*g*x + 15*g^2*x^2)) + 16*a^2*c^3*d^3*e^2*g*(33*d*g*(21*f^2
+ 6*f*g*x + g^2*x^2) + e*(462*f^3 + 297*f^2*g*x + 132*f*g^2*x^2 + 25*g^3*x^
3)) - 2*a*c^4*d^4*e*(44*d*g*(105*f^3 + 63*f^2*g*x + 27*f*g^2*x^2 + 5*g^3*x^
3) + e*(1155*f^4 + 1848*f^3*g*x + 1782*f^2*g^2*x^2 + 880*f*g^3*x^3 + 175*g^
4*x^4)) + c^5*d^5*(11*d*(315*f^4 + 420*f^3*g*x + 378*f^2*g^2*x^2 + 180*f*g^
3*x^3 + 35*g^4*x^4) + e*x*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 154
0*f*g^3*x^3 + 315*g^4*x^4))))/(3465*c^6*d^6*Sqrt[d + e*x])
```

Maple [A]
time = 0.12, size = 623, normalized size = 1.24

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4}{}$
gosper	$-\frac{2(cdx+ae)(-315e\,g^4x^5c^5d^5+350a\,c^4d^4e^2g^4x^4-385c^5d^6g^4x^4-1540c^5d^5ef\,g^3x^4-400a^2c^3d^3e^3g^4x^3+440a\,c^4d^5e\,g^4x^3+1760a\,c^4d^4e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3465/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-315*c^5*d^5*e*g^4*x^5+3
50*a*c^4*d^4*e^2*g^4*x^4-385*c^5*d^6*g^4*x^4-1540*c^5*d^5*e*f*g^3*x^4-400*a
^2*c^3*d^3*e^3*g^4*x^3+440*a*c^4*d^5*e*g^4*x^3+1760*a*c^4*d^4*e^2*f*g^3*x^3
-1980*c^5*d^6*f*g^3*x^3-2970*c^5*d^5*e*f^2*g^2*x^3+480*a^3*c^2*d^2*e^4*g^4*
x^2-528*a^2*c^3*d^4*e^2*g^4*x^2-2112*a^2*c^3*d^3*e^3*f*g^3*x^2+2376*a*c^4*d
^5*e*f*g^3*x^2+3564*a*c^4*d^4*e^2*f^2*g^2*x^2-4158*c^5*d^6*f^2*g^2*x^2-2772
*c^5*d^5*e*f^3*g*x^2-640*a^4*c*d*e^5*g^4*x+704*a^3*c^2*d^3*e^3*g^4*x+2816*a
^3*c^2*d^2*e^4*f*g^3*x-3168*a^2*c^3*d^4*e^2*f*g^3*x-4752*a^2*c^3*d^3*e^3*f^
2*g^2*x+5544*a*c^4*d^5*e*f^2*g^2*x+3696*a*c^4*d^4*e^2*f^3*g*x-4620*c^5*d^6*
f^3*g*x-1155*c^5*d^5*e*f^4*x+1280*a^5*e^6*g^4-1408*a^4*c*d^2*e^4*g^4-5632*a
^4*c*d*e^5*f*g^3+6336*a^3*c^2*d^3*e^3*f*g^3+9504*a^3*c^2*d^2*e^4*f^2*g^2-11
088*a^2*c^3*d^4*e^2*f^2*g^2-7392*a^2*c^3*d^3*e^3*f^3*g+9240*a*c^4*d^5*e*f^3
*g+2310*a*c^4*d^4*e^2*f^4-3465*c^5*d^6*f^4)/c^6/d^6
```

Maxima [A]
time = 0.37, size = 678, normalized size = 1.35

$$\frac{2\,(c^2d^2x^2e+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f^4}{3\,\sqrt{cdx+ae}\,c^2d^2}+\frac{8\,(3\,c^3d^3x^3e-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)f^3g}{15\,\sqrt{cdx+ae}\,c^3d^3}+\frac{4\,(15\,c^4d^4x^4e+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2-6\,a^3cde^4)x)f^2g^2}{35\,\sqrt{cdx+ae}\,c^4d^4}+\frac{8\,(35\,c^5d^5x^5e-144\,a^4cd^2e^4+128\,a^5e^6+5\,(9\,c^5d^6-ac^4d^4e^2)x^4-(9\,ac^4d^5e-8\,a^2c^3d^3e^3)x^3+2\,(9\,a^2c^3d^4e^2-8\,a^3c^2d^2e^4)x^2-8\,(9\,a^3c^2d^3e^3-8\,a^4cde^5)x)fg^3}{315\,\sqrt{cdx+ae}\,c^5d^5}+\frac{2\,(315\,c^6d^6x^6e+1408\,a^5cd^2e^5-1280\,a^6e^7+35\,(11\,c^6d^7-ac^5d^5e^2)x^5-5\,(11\,ac^5d^6e-10\,a^2c^4d^4e^3)x^4+8\,(11\,a^2c^4d^5e^2-10\,a^3c^3d^3e^4)x^3-16\,(11\,a^3c^3d^4e^3-10\,a^4c^2d^2e^5)x^2+64\,(11\,a^4c^2d^3e^4-10\,a^5cde^6)x)g^4}{3465\,\sqrt{cdx+ae}\,c^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*x^2*e + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^4/(sqrt(c*d*x + a*e)*c^2*d^2) + 8/15*(3*c^3*d^3*x^3*e - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
^3)*x)*f^3*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 4/35*(15*c^4*d^4*x^4*e + 56*a^3*
c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
 - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f^2*g^
2/(sqrt(c*d*x + a*e)*c^4*d^4) + 8/315*(35*c^5*d^5*x^5*e - 144*a^4*c*d^2*e^4
 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2
*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^
3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*f*g^3/(sqrt(c*d*x + a*e)*c^5*d^5) + 2/346
```

```
5*(315*c^6*d^6*x^6*e + 1408*a^5*c*d^2*e^5 - 1280*a^6*e^7 + 35*(11*c^6*d^7 -
 a*c^5*d^5*e^2)*x^5 - 5*(11*a*c^5*d^6*e - 10*a^2*c^4*d^4*e^3)*x^4 + 8*(11*a
^2*c^4*d^5*e^2 - 10*a^3*c^3*d^3*e^4)*x^3 - 16*(11*a^3*c^3*d^4*e^3 - 10*a^4*
c^2*d^2*e^5)*x^2 + 64*(11*a^4*c^2*d^3*e^4 - 10*a^5*c*d*e^6)*x)*g^4/(sqrt(c*
d*x + a*e)*c^6*d^6)
```

Fricas [A]
time = 3.85, size = 608, normalized size = 1.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/3465*(385*c^5*d^6*g^4*x^4 + 1980*c^5*d^6*f*g^3*x^3 + 4158*c^5*d^6*f^2*g^2
*x^2 + 4620*c^5*d^6*f^3*g*x + 3465*c^5*d^6*f^4 - 1280*a^5*g^4*e^6 + 128*(5*
a^4*c*d*g^4*x + 44*a^4*c*d*f*g^3)*e^5 - 32*(15*a^3*c^2*d^2*g^4*x^2 + 88*a^3
*c^2*d^2*f*g^3*x + 297*a^3*c^2*d^2*f^2*g^2 - 44*a^4*c*d^2*g^4)*e^4 + 16*(25
*a^2*c^3*d^3*g^4*x^3 + 132*a^2*c^3*d^3*f*g^3*x^2 + 462*a^2*c^3*d^3*f^3*g -
396*a^3*c^2*d^3*f*g^3 + 11*(27*a^2*c^3*d^3*f^2*g^2 - 4*a^3*c^2*d^3*g^4)*x)*
e^3 - 2*(175*a*c^4*d^4*g^4*x^4 + 880*a*c^4*d^4*f*g^3*x^3 + 1155*a*c^4*d^4*f
^4 - 5544*a^2*c^3*d^4*f^2*g^2 + 66*(27*a*c^4*d^4*f^2*g^2 - 4*a^2*c^3*d^4*g^
4)*x^2 + 264*(7*a*c^4*d^4*f^3*g - 6*a^2*c^3*d^4*f*g^3)*x)*e^2 + (315*c^5*d^
5*g^4*x^5 + 1540*c^5*d^5*f*g^3*x^4 - 9240*a*c^4*d^5*f^3*g + 110*(27*c^5*d^5
*f^2*g^2 - 4*a*c^4*d^5*g^4)*x^3 + 396*(7*c^5*d^5*f^3*g - 6*a*c^4*d^5*f*g^3)
*x^2 + 231*(5*c^5*d^5*f^4 - 24*a*c^4*d^5*f^2*g^2)*x)*e)*sqrt(c*d^2*x + a*x*
e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^6*d^6*x*e + c^6*d^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^4}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**4/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. 2(483) = 966.
time = 3.95, size = 1786, normalized size = 3.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2*(c^5*d^6*f^4 - 4*a*c^4*d^5*f^3*g*e - a*c^4*d^4*f^4*e^2 + 6*a^2*c^3*d^4*f^
2*g^2*e^2 + 4*a^2*c^3*d^3*f^3*g*e^3 - 4*a^3*c^2*d^3*f*g^3*e^3 - 6*a^3*c^2*d
^2*f^2*g^2*e^4 + a^4*c*d^2*g^4*e^4 + 4*a^4*c*d*f*g^3*e^5 - a^5*g^4*e^6)*sqr
t((x*e + d)*c*d*e - c*d^2*e + a*e^3)*e^(-1)/(c^6*d^6) - 4/3465*(35*sqrt(-c*
d^2*e + a*e^3)*c^5*d^10*g^4 - 220*sqrt(-c*d^2*e + a*e^3)*c^5*d^9*f*g^3*e +
594*sqrt(-c*d^2*e + a*e^3)*c^5*d^8*f^2*g^2*e^2 + 45*sqrt(-c*d^2*e + a*e^3)*
a*c^4*d^8*g^4*e^2 - 924*sqrt(-c*d^2*e + a*e^3)*c^5*d^7*f^3*g*e^3 - 308*sqrt
(-c*d^2*e + a*e^3)*a*c^4*d^7*f*g^3*e^3 + 1155*sqrt(-c*d^2*e + a*e^3)*c^5*d^
6*f^4*e^4 + 990*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^6*f^2*g^2*e^4 + 64*sqrt(-c*d
^2*e + a*e^3)*a^2*c^3*d^6*g^4*e^4 - 2772*sqrt(-c*d^2*e + a*e^3)*a*c^4*d^5*f
^3*g*e^5 - 528*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^5*f*g^3*e^5 - 1155*sqrt(-c*
d^2*e + a*e^3)*a*c^4*d^4*f^4*e^6 + 3168*sqrt(-c*d^2*e + a*e^3)*a^2*c^3*d^4*
f^2*g^2*e^6 + 112*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^4*g^4*e^6 + 3696*sqrt(-c
*d^2*e + a*e^3)*a^2*c^3*d^3*f^3*g*e^7 - 1760*sqrt(-c*d^2*e + a*e^3)*a^3*c^2
*d^3*f*g^3*e^7 - 4752*sqrt(-c*d^2*e + a*e^3)*a^3*c^2*d^2*f^2*g^2*e^8 + 384*
sqrt(-c*d^2*e + a*e^3)*a^4*c*d^2*g^4*e^8 + 2816*sqrt(-c*d^2*e + a*e^3)*a^4*
c*d*f*g^3*e^9 - 640*sqrt(-c*d^2*e + a*e^3)*a^5*g^4*e^10)*e^(-5)/(c^6*d^6) +
 2/3465*(4620*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^5*f^3*g*e^7 +
 1155*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^4*d^4*f^4*e^8 - 13860*((x
*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c^3*d^4*f^2*g^2*e^8 + 4158*((x*e +
 d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^3*d^4*f^2*g^2*e^5 - 9240*((x*e + d)*c*
d*e - c*d^2*e + a*e^3)^(3/2)*a*c^3*d^3*f^3*g*e^9 + 13860*((x*e + d)*c*d*e -
 c*d^2*e + a*e^3)^(3/2)*a^2*c^2*d^3*f*g^3*e^9 + 2772*((x*e + d)*c*d*e - c*d
^2*e + a*e^3)^(5/2)*c^3*d^3*f^3*g*e^6 - 8316*((x*e + d)*c*d*e - c*d^2*e + a
*e^3)^(5/2)*a*c^2*d^3*f*g^3*e^6 + 1980*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^
(7/2)*c^2*d^3*f*g^3*e^3 + 20790*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a
^2*c^2*d^2*f^2*g^2*e^10 - 4620*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^
3*c*d^2*g^4*e^10 - 12474*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c^2*d^
2*f^2*g^2*e^7 + 4158*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*c*d^2*g^
4*e^7 + 2970*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c^2*d^2*f^2*g^2*e^4
- 1980*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*c*d^2*g^4*e^4 + 385*((x*
e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)*c*d^2*g^4*e - 18480*((x*e + d)*c*d*e
- c*d^2*e + a*e^3)^(3/2)*a^3*c*d*f*g^3*e^11 + 16632*((x*e + d)*c*d*e - c*d^
2*e + a*e^3)^(5/2)*a^2*c*d*f*g^3*e^8 - 7920*((x*e + d)*c*d*e - c*d^2*e + a*
e^3)^(7/2)*a*c*d*f*g^3*e^5 + 1540*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(9/2)
*c*d*f*g^3*e^2 + 5775*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^4*g^4*e^1
2 - 6930*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^3*g^4*e^9 + 4950*((x*e
 + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a^2*g^4*e^6 - 1925*((x*e + d)*c*d*e -
c*d^2*e + a*e^3)^(9/2)*a*g^4*e^3 + 315*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^
(11/2)*g^4)*e^(-10)/(c^6*d^6)
```

Mupad [B]
time = 4.09, size = 653, normalized size = 1.30

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^4\,x^5\,\sqrt{d+e\,x}}{11\,c\,d}-\frac{\sqrt{d+e\,x}\,\left(2560\,a^5\,e^6\,g^4-2816\,a^4\,c\,d^2\,e^4\,g^4-11264\,a^4\,c\,d\,e^5\,f\,g^3+12672\,a^3\,c^2\,d^3\,e^3\,f\,g^3+19008\,a^3\,c^2\,d^2\,e^4\,f^2\,g^2-22176\,a^2\,c^3\,d^4\,e^2\,f^2\,g^2-14784\,a^2\,c^3\,d^3\,e^3\,f^3\,g+18480\,a\,c^4\,d^5\,e\,f^3\,g+4620\,a\,c^4\,d^4\,e^2\,f^4-6930\,c^5\,d^6\,f^4\right)}{3465\,c^6\,d^6\,e}+\frac{x\,\sqrt{d+e\,x}\,\left(1280\,a^4\,c\,d\,e^5\,g^4-1408\,a^3\,c^2\,d^3\,e^3\,g^4-5632\,a^3\,c^2\,d^2\,e^4\,f\,g^3+6336\,a^2\,c^3\,d^4\,e^2\,f\,g^3+9504\,a^2\,c^3\,d^3\,e^3\,f^2\,g^2-11088\,a\,c^4\,d^5\,e\,f^2\,g^2-7392\,a\,c^4\,d^4\,e^2\,f^3\,g+9240\,c^5\,d^6\,f^3\,g+2310\,c^5\,d^5\,e\,f^4\right)}{3465\,c^6\,d^6\,e}+\frac{x^2\,\sqrt{d+e\,x}\,\left(-960\,a^3\,c^2\,d^2\,e^4\,g^4+1056\,a^2\,c^3\,d^4\,e^2\,g^4+4224\,a^2\,c^3\,d^3\,e^3\,f\,g^3-4752\,a\,c^4\,d^5\,e\,f\,g^3-7128\,a\,c^4\,d^4\,e^2\,f^2\,g^2+8316\,c^5\,d^6\,f^2\,g^2+5544\,c^5\,d^5\,e\,f^3\,g\right)}{3465\,c^6\,d^6\,e}+\frac{4\,g^2\,x^3\,\sqrt{d+e\,x}\,\left(40\,a^2\,e^3\,g^2-44\,a\,c\,d^2\,e\,g^2-176\,a\,c\,d\,e^2\,f\,g+198\,c^2\,d^3\,f\,g+297\,c^2\,d^2\,e\,f^2\right)}{693\,c^3\,d^3\,e}+\frac{2\,g^3\,x^4\,\sqrt{d+e\,x}\,\left(11\,c\,g\,d^2+44\,c\,f\,d\,e-10\,a\,g\,e^2\right)}{99\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^4*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^4*x^5*(d + e*x)^(1/2))
/(11*c*d) - ((d + e*x)^(1/2)*(2560*a^5*e^6*g^4 - 6930*c^5*d^6*f^4 + 4620*a*
c^4*d^4*e^2*f^4 - 2816*a^4*c*d^2*e^4*g^4 - 14784*a^2*c^3*d^3*e^3*f^3*g + 12
672*a^3*c^2*d^3*e^3*f*g^3 + 18480*a*c^4*d^5*e*f^3*g - 11264*a^4*c*d*e^5*f*g
^3 - 22176*a^2*c^3*d^4*e^2*f^2*g^2 + 19008*a^3*c^2*d^2*e^4*f^2*g^2))/(3465*
c^6*d^6*e) + (x*(d + e*x)^(1/2)*(2310*c^5*d^5*e*f^4 + 9240*c^5*d^6*f^3*g -
1408*a^3*c^2*d^3*e^3*g^4 + 1280*a^4*c*d*e^5*g^4 - 7392*a*c^4*d^4*e^2*f^3*g
- 11088*a*c^4*d^5*e*f^2*g^2 + 6336*a^2*c^3*d^4*e^2*f*g^3 - 5632*a^3*c^2*d^2
*e^4*f*g^3 + 9504*a^2*c^3*d^3*e^3*f^2*g^2))/(3465*c^6*d^6*e) + (x^2*(d + e*
x)^(1/2)*(8316*c^5*d^6*f^2*g^2 + 1056*a^2*c^3*d^4*e^2*g^4 - 960*a^3*c^2*d^2
*e^4*g^4 + 5544*c^5*d^5*e*f^3*g - 7128*a*c^4*d^4*e^2*f^2*g^2 + 4224*a^2*c^3
*d^3*e^3*f*g^3 - 4752*a*c^4*d^5*e*f*g^3))/(3465*c^6*d^6*e) + (4*g^2*x^3*(d
+ e*x)^(1/2)*(40*a^2*e^3*g^2 + 297*c^2*d^2*e*f^2 + 198*c^2*d^3*f*g - 44*a*c
*d^2*e*g^2 - 176*a*c*d*e^2*f*g))/(693*c^3*d^3*e) + (2*g^3*x^4*(d + e*x)^(1/
2)*(11*c*d^2*g - 10*a*e^2*g + 44*c*d*e*f))/(99*c^2*d^2*e)))/(x + d/e)
```

3.784 $\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\,dx$

Optimal. Leaf size=412

$$\frac{16(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{315c^5d^5eg\sqrt{d+ex}}-\frac{16(cdf-ae$$

```
[Out] 16/315*(-a*e*g+c*d*f)^2*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(2*a*e^2*g-c*d*(-d*g+3
*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e/g/(e*x+d)^(1/2)-4/
105*(-a*e*g+c*d*f)*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(g*x+f)^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/g/(e*x+d)^(1/2)-2/63*(8*a*e^2*g+c*d*(-9*d*g+
e*f))*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/g/(e*x+d)^(
1/2)+2/9*e*(g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^
(1/2)-16/315*(-a*e*g+c*d*f)^2*(8*a*e^2*g+c*d*(-9*d*g+e*f))*(e*x+d)^(1/2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e
```

Rubi [A]
time = 0.40, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {894, 884, 808, 662}

$$\frac{16\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{315c^5d^5eg\sqrt{d+ex}}-\frac{16\sqrt{d+ex}\,\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)^2\left(8ae^2g+cd(ef-9dg)\right)}{315c^4d^4e}-\frac{4(f+gx)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(cdf-aeg)\left(8ae^2g+cd(ef-9dg)\right)}{105c^3d^3g\sqrt{d+ex}}-\frac{2(f+gx)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(8ae^2g+cd(ef-9dg)\right)}{63c^2d^2g\sqrt{d+ex}}+\frac{2e(f+gx)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{9cdg\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(2*a*e^2*g - c*d*(3*e
*f - d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^5*d^5*e*g*Sq
rt[d + e*x]) - (16*(c*d*f - a*e*g)^2*(8*a*e^2*g + c*d*(e*f - 9*d*g))*Sqrt[d
 + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(315*c^4*d^4*e) - (4*(
c*d*f - a*e*g)*(8*a*e^2*g + c*d*(e*f - 9*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*g*Sqrt[d + e*x]) - (2*(8*a*e^2*g
+ c*d*(e*f - 9*d*g))*(f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/(63*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(9*c*d*g*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{9cdg\sqrt{d+ex}} - \frac{1}{9}\bigg(-9d+\frac{8ae^2}{cd}+\frac{ef}{g}$$

$$= -\frac{2(8ae^2g+cd(ef-9dg))(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{63c^2d^2g\sqrt{d+ex}}$$

$$= -\frac{4(cdf-aeg)(8ae^2g+cd(ef-9dg))(f+gx)^2\sqrt{ade+(cd^2+ae^2}}{105c^3d^3g\sqrt{d+ex}}$$

$$= -\frac{16(cdf-aeg)^2(8ae^2g+cd(ef-9dg))\sqrt{d+ex}\sqrt{ade+(cd^2+a}}{315c^4d^4e}$$

$$= \frac{16(cdf-aeg)^2(8ae^2g+cd(ef-9dg))(2ae^2g-cd(3ef-dg))\sqrt{ad}}{315c^5d^5eg\sqrt{d+ex}}$$

Mathematica [A]

time = 0.20, size = 264, normalized size = 0.64

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(128a^4e^5g^3-16a^3cde^3g^2(27ef+9dg+4egx)+24a^2c^2d^2e^2g(3dg(7f+gx)+e(21f^2+9fgx+2g^2x^2))-2ac^3d^3e(9dg(35f^2+14fgx+3g^2x^2)+e(105f^3+126f^2gx+81fg^2x^2+20g^3x^3))+c^4d^4(9d(35f^3+35f^2gx+21fg^2x^2+5g^3x^3)+ex(105f^3+189f^2gx+135fg^2x^2+35g^3x^3)))}{315c^5d^5\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^3)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(128*a^4*e^5*g^3 - 16*a^3*c*d*e^3*g^2*(27*
e*f + 9*d*g + 4*e*g*x) + 24*a^2*c^2*d^2*e^2*g*(3*d*g*(7*f + g*x) + e*(21*f^
2 + 9*f*g*x + 2*g^2*x^2)) - 2*a*c^3*d^3*e*(9*d*g*(35*f^2 + 14*f*g*x + 3*g^2
*x^2) + e*(105*f^3 + 126*f^2*g*x + 81*f*g^2*x^2 + 20*g^3*x^3)) + c^4*d^4*(9
*d*(35*f^3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3) + e*x*(105*f^3 + 189*f^
2*g*x + 135*f*g^2*x^2 + 35*g^3*x^3))))/(315*c^5*d^5*Sqrt[d + e*x])
```

Maple [A]

time = 0.13, size = 407, normalized size = 0.99

method	result
default	$\frac{2\sqrt{(cdx+ae)(ex+d)}\,(35e\,g^3x^4c^4d^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-1}{}$
gosper	$\frac{2(cdx+ae)(35e\,g^3x^4c^4d^4-40a\,c^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4ef\,g^2x^3+48a^2c^2d^2e^3g^3x^2-54a\,c^3d^4e\,g^3x^2-162a\,c^3d^3e^2f\,g^2x^2+189}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/315/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(35*c^4*d^4*e*g^3*x^4-40*a*
c^3*d^3*e^2*g^3*x^3+45*c^4*d^5*g^3*x^3+135*c^4*d^4*e*f*g^2*x^3+48*a^2*c^2*d
^2*e^3*g^3*x^2-54*a*c^3*d^4*e*g^3*x^2-162*a*c^3*d^3*e^2*f*g^2*x^2+189*c^4*d
^5*f*g^2*x^2+189*c^4*d^4*e*f^2*g*x^2-64*a^3*c*d*e^4*g^3*x+72*a^2*c^2*d^3*e^
2*g^3*x+216*a^2*c^2*d^2*e^3*f*g^2*x-252*a*c^3*d^4*e*f*g^2*x-252*a*c^3*d^3*e
^2*f^2*g*x+315*c^4*d^5*f^2*g*x+105*c^4*d^4*e*f^3*x+128*a^4*e^5*g^3-144*a^3*
c*d^2*e^3*g^3-432*a^3*c*d*e^4*f*g^2+504*a^2*c^2*d^3*e^2*f*g^2+504*a^2*c^2*d
^2*e^3*f^2*g-630*a*c^3*d^4*e*f^2*g-210*a*c^3*d^3*e^2*f^3+315*c^4*d^5*f^3)/c
^5/d^5
```

Maxima [A]
time = 0.34, size = 476, normalized size = 1.16

$$\frac{2\,(c^2d^2x^2e+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f^3}{3\,\sqrt{cdx+ae}\,c^2d^2}+\frac{2\,(3\,c^3d^3x^3e-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)f^2g}{5\,\sqrt{cdx+ae}\,c^3d^3}+\frac{2\,(15\,c^4d^4x^4e+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2-6\,a^3cde^4)x)fg^2}{35\,\sqrt{cdx+ae}\,c^4d^4}+\frac{2\,(35\,c^5d^5x^5e-144\,a^4cd^2e^4+128\,a^5e^6+5\,(9\,c^5d^6-ac^4d^4e^2)x^4-(9\,ac^4d^5e-8\,a^2c^3d^3e^3)x^3+2\,(9\,a^2c^3d^4e^2-8\,a^3c^2d^2e^4)x^2-8\,(9\,a^3c^2d^3e^3-8\,a^4cde^5)x)g^3}{315\,\sqrt{cdx+ae}\,c^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*x^2*e + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^3/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3*e - 10*a^2*c*d^2*e^2 +
8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^
3)*x)*f^2*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(15*c^4*d^4*x^4*e + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*f*g^2/(
sqrt(c*d*x + a*e)*c^4*d^4) + 2/315*(35*c^5*d^5*x^5*e - 144*a^4*c*d^2*e^4 +
128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2*c^
3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^3*c
^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*g^3/(sqrt(c*d*x + a*e)*c^5*d^5)
```

Fricas [A]
time = 4.91, size = 411, normalized size = 1.00

$$\frac{2\,(45\,c^4d^5g^3x^3+189\,c^4d^5fg^2x^2+315\,c^4d^5f^2gx+315\,c^4d^5f^3+128\,a^4g^3e^5-16\,(4\,a^3cdg^3x+27\,a^3cdfg^2)e^4+24\,(2\,a^2c^2d^2g^3x^2+9\,a^2c^2d^2fg^2x+21\,a^2c^2d^2f^2g-6\,a^3cd^2g^3)e^3-2\,(20\,ac^3d^3g^3x^3+81\,ac^3d^3fg^2x^2+105\,ac^3d^3f^3-252\,a^2c^2d^3fg^2+18\,(7\,ac^3d^3f^2g-2\,a^2c^2d^3g^3)x)e^2+(35\,c^4d^4g^3x^4+135\,c^4d^4fg^2x^3-630\,ac^3d^4f^2g+27\,(7\,c^4d^4f^2g-2\,ac^3d^4g^3)x^2+21\,(5\,c^4d^4f^3-12\,ac^3d^4fg^2)x)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{315\,(c^5d^5xe+c^5d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

[Out] 2/315*(45*c^4*d^5*g^3*x^3 + 189*c^4*d^5*f*g^2*x^2 + 315*c^4*d^5*f^2*g*x + 3
15*c^4*d^5*f^3 + 128*a^4*g^3*e^5 - 16*(4*a^3*c*d*g^3*x + 27*a^3*c*d*f*g^2)*
e^4 + 24*(2*a^2*c^2*d^2*g^3*x^2 + 9*a^2*c^2*d^2*f*g^2*x + 21*a^2*c^2*d^2*f^
2*g - 6*a^3*c*d^2*g^3)*e^3 - 2*(20*a*c^3*d^3*g^3*x^3 + 81*a*c^3*d^3*f*g^2*x
^2 + 105*a*c^3*d^3*f^3 - 252*a^2*c^2*d^3*f*g^2 + 18*(7*a*c^3*d^3*f^2*g - 2*
a^2*c^2*d^3*g^3)*x)*e^2 + (35*c^4*d^4*g^3*x^4 + 135*c^4*d^4*f*g^2*x^3 - 630
*a*c^3*d^4*f^2*g + 27*(7*c^4*d^4*f^2*g - 2*a*c^3*d^4*g^3)*x^2 + 21*(5*c^4*d
^4*f^3 - 12*a*c^3*d^4*f*g^2)*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)
*e)*sqrt(x*e + d)/(c^5*d^5*x*e + c^5*d^6)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^3}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)

[Out] Integral((d + e*x)**(3/2)*(f + g*x)**3/sqrt((d + e*x)*(a*e + c*d*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. 2(396) = 792.
time = 3.18, size = 1181, normalized size = 2.87

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")

[Out] 2*(c^4*d^5*f^3 - 3*a*c^3*d^4*f^2*g*e - a*c^3*d^3*f^3*e^2 + 3*a^2*c^2*d^3*f*
g^2*e^2 + 3*a^2*c^2*d^2*f^2*g*e^3 - a^3*c*d^2*g^3*e^3 - 3*a^3*c*d*f*g^2*e^4
 + a^4*g^3*e^5)*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*e^(-1)/(c^5*d^5) +
4/315*(5*sqrt(-c*d^2*e + a*e^3)*c^4*d^8*g^3 - 27*sqrt(-c*d^2*e + a*e^3)*c^4
*d^7*f*g^2*e + 63*sqrt(-c*d^2*e + a*e^3)*c^4*d^6*f^2*g*e^2 + 7*sqrt(-c*d^2*
e + a*e^3)*a*c^3*d^6*g^3*e^2 - 105*sqrt(-c*d^2*e + a*e^3)*c^4*d^5*f^3*e^3 -
 45*sqrt(-c*d^2*e + a*e^3)*a*c^3*d^5*f*g^2*e^3 + 189*sqrt(-c*d^2*e + a*e^3)
*a*c^3*d^4*f^2*g*e^4 + 12*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^4*g^3*e^4 + 105*
sqrt(-c*d^2*e + a*e^3)*a*c^3*d^3*f^3*e^5 - 144*sqrt(-c*d^2*e + a*e^3)*a^2*c
^2*d^3*f*g^2*e^5 - 252*sqrt(-c*d^2*e + a*e^3)*a^2*c^2*d^2*f^2*g*e^6 + 40*sq
rt(-c*d^2*e + a*e^3)*a^3*c*d^2*g^3*e^6 + 216*sqrt(-c*d^2*e + a*e^3)*a^3*c*d
*f*g^2*e^7 - 64*sqrt(-c*d^2*e + a*e^3)*a^4*g^3*e^8)*e^(-4)/(c^5*d^5) + 2/31
5*(315*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^4*f^2*g*e^5 + 105*((

```
x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^3*d^3*f^3*e^6 - 630*((x*e + d)*c*
d*e - c*d^2*e + a*e^3)^(3/2)*a*c^2*d^3*f*g^2*e^6 + 189*((x*e + d)*c*d*e - c
*d^2*e + a*e^3)^(5/2)*c^2*d^3*f*g^2*e^3 - 630*((x*e + d)*c*d*e - c*d^2*e +
a*e^3)^(3/2)*a*c^2*d^2*f^2*g*e^7 + 315*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^
(3/2)*a^2*c*d^2*g^3*e^7 + 189*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c^2
*d^2*f^2*g*e^4 - 189*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a*c*d^2*g^3*
e^4 + 45*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*c*d^2*g^3*e + 945*((x*e
+ d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*c*d*f*g^2*e^8 - 567*((x*e + d)*c*d*
e - c*d^2*e + a*e^3)^(5/2)*a*c*d*f*g^2*e^5 + 135*((x*e + d)*c*d*e - c*d^2*e
 + a*e^3)^(7/2)*c*d*f*g^2*e^2 - 420*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/
2)*a^3*g^3*e^9 + 378*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*a^2*g^3*e^6
- 180*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*a*g^3*e^3 + 35*((x*e + d)*c
*d*e - c*d^2*e + a*e^3)^(9/2)*g^3)*e^(-8)/(c^5*d^5)
```

Mupad [B]
time = 3.86, size = 438, normalized size = 1.06

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(256\,a^4\,e^5\,g^3-288\,a^3\,c\,d^2\,e^3\,g^3-864\,a^3\,c\,d\,e^4\,f\,g^2+1008\,a^2\,c^2\,d^3\,e^2\,f\,g^2+1008\,a^2\,c^2\,d^2\,e^3\,f^2\,g-1260\,a\,c^3\,d^4\,e\,f^2\,g-420\,a\,c^3\,d^3\,e^2\,f^3+630\,c^4\,d^5\,f^3\right)}{315\,c^5\,d^5\,e}+\frac{2\,g^3\,x^4\,\sqrt{d+e\,x}}{9\,c\,d}+\frac{x\,\sqrt{d+e\,x}\,\left(-128\,a^3\,c\,d\,e^4\,g^3+144\,a^2\,c^2\,d^3\,e^2\,g^3+432\,a^2\,c^2\,d^2\,e^3\,f\,g^2-504\,a\,c^3\,d^4\,e\,f\,g^2-504\,a\,c^3\,d^3\,e^2\,f^2\,g+630\,c^4\,d^5\,f^2\,g+210\,c^4\,d^4\,e\,f^3\right)}{315\,c^5\,d^5\,e}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}\,\left(16\,a^2\,e^3\,g^2-18\,a\,c\,d^2\,e\,g^2-54\,a\,c\,d\,e^2\,f\,g+63\,c^2\,d^3\,f\,g+63\,c^2\,d^2\,e\,f^2\right)}{105\,c^3\,d^3\,e}+\frac{2\,g^2\,x^3\,\sqrt{d+e\,x}\,\left(9\,c\,g\,d^2+27\,c\,f\,d\,e-8\,a\,g\,e^2\right)}{63\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(256*a^4*e
^5*g^3 + 630*c^4*d^5*f^3 - 420*a*c^3*d^3*e^2*f^3 - 288*a^3*c*d^2*e^3*g^3 +
1008*a^2*c^2*d^2*e^3*f^2*g + 1008*a^2*c^2*d^3*e^2*f*g^2 - 1260*a*c^3*d^4*e*
f^2*g - 864*a^3*c*d*e^4*f*g^2))/(315*c^5*d^5*e) + (2*g^3*x^4*(d + e*x)^(1/2
))/(9*c*d) + (x*(d + e*x)^(1/2)*(210*c^4*d^4*e*f^3 + 630*c^4*d^5*f^2*g + 14
4*a^2*c^2*d^3*e^2*g^3 - 128*a^3*c*d*e^4*g^3 - 504*a*c^3*d^3*e^2*f^2*g + 432
*a^2*c^2*d^2*e^3*f*g^2 - 504*a*c^3*d^4*e*f*g^2))/(315*c^5*d^5*e) + (2*g*x^2
*(d + e*x)^(1/2)*(16*a^2*e^3*g^2 + 63*c^2*d^2*e*f^2 + 63*c^2*d^3*f*g - 18*a
*c*d^2*e*g^2 - 54*a*c*d*e^2*f*g))/(105*c^3*d^3*e) + (2*g^2*x^3*(d + e*x)^(1
/2)*(9*c*d^2*g - 8*a*e^2*g + 27*c*d*e*f))/(63*c^2*d^2*e)))/(x + d/e)
```

3.785 $\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=321

$$\frac{8(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{105c^4d^4eg\sqrt{d+ex}}-\frac{8(cdf-aeg)\,(}{}$$

```
[Out] 8/105*(-a*e*g+c*d*f)*(6*a*e^2*g+c*d*(-7*d*g+e*f))*(2*a*e^2*g-c*d*(-d*g+3*e*
f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e/g/(e*x+d)^(1/2)-2/35*
(6*a*e^2*g+c*d*(-7*d*g+e*f))*(g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/c^2/d^2/g/(e*x+d)^(1/2)+2/7*e*(g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/c/d/g/(e*x+d)^(1/2)-8/105*(-a*e*g+c*d*f)*(6*a*e^2*g+c*d*(-7*d*g+e*
f))*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e
```

Rubi [A]

time = 0.27, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {894, 884, 808, 662}

$$\frac{8\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)\left(2ae^2g-cd(3ef-dg)\right)}{105c^4d^4eg\sqrt{d+ex}}-\frac{8\sqrt{d+ex}\,\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(cdf-aeg)\left(6ae^2g+cd(ef-7dg)\right)}{105c^3d^3e}-\frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(6ae^2g+cd(ef-7dg)\right)}{35c^2d^2g\sqrt{d+ex}}+\frac{2e(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cdg\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2],x]
```

```
[Out] (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(2*a*e^2*g - c*d*(3*e*f
- d*g))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^4*d^4*e*g*Sqrt[
d + e*x]) - (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*Sqrt[d + e*x
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*e) - (2*(6*a*e^
2*g + c*d*(e*f - 7*d*g))*(f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(35*c^2*d^2*g*Sqrt[d + e*x]) + (2*e*(f + g*x)^3*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])/(7*c*d*g*Sqrt[d + e*x])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
```

```
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 884

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*
((a + b*x + c*x^2)^(p + 1)/(c*(m - n - 1))), x] - Dist[n*((c*e*f + c*d*g -
b*e*g)/(c*e*(m - n - 1))), Int[(d + e*x)^m*(f + g*x)^(n - 1)*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0]
 && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &
& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Inte
gerQ[n])
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2e(f+gx)^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{7cdg\sqrt{d+ex}} - \frac{1}{7}\left(-7d+\frac{6ae^2}{cd}+\frac{e}{g}\right.$$

$$= -\frac{2(6ae^2g+cd(ef-7dg))\,(f+gx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex}}{35c^2d^2g\sqrt{d+ex}}$$

$$= -\frac{8(cdf-aeg)\,(6ae^2g+cd(ef-7dg))\sqrt{d+ex}\,\sqrt{ade+(cd^2+ae}}{105c^3d^3e}$$

$$= \frac{8(cdf-aeg)\,(6ae^2g+cd(ef-7dg))\,(2ae^2g-cd(3ef-dg))\,\sqrt{ad}}{105c^4d^4eg\sqrt{d+ex}}$$

Mathematica [A]

time = 0.14, size = 169, normalized size = 0.53

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(-48a^3e^4g^2+8a^2cde^2g(14ef+7dg+3egx)-2ac^2d^2e(14dg(5f+gx)+e(35f^2+28fgx+9g^2x^2))+c^3d^3(7d(15f^2+10fgx+3g^2x^2)+ex(35f^2+42fgx+15g^2x^2)))}{105c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x)^2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-48*a^3*e^4*g^2 + 8*a^2*c*d*e^2*g*(14*e*f
 + 7*d*g + 3*e*g*x) - 2*a*c^2*d^2*e*(14*d*g*(5*f + g*x) + e*(35*f^2 + 28*f*
g*x + 9*g^2*x^2)) + c^3*d^3*(7*d*(15*f^2 + 10*f*g*x + 3*g^2*x^2) + e*x*(35*
f^2 + 42*f*g*x + 15*g^2*x^2))))/(105*c^4*d^4*Sqrt[d + e*x])
```

Maple [A]
time = 0.14, size = 237, normalized size = 0.74

method	result
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-15g^2ex^3c^3d^3+18a\,c^2d^2e^2g^2x^2-21c^3d^4g^2x^2-42c^3d^3efg\,x^2-24a^2cd\,e^3g^2x+28a\,c^2d^3e\,g^2x+56a}{105\sqrt{ex+d}}$
gosper	$-\frac{2(cdx+ae)(-15g^2ex^3c^3d^3+18a\,c^2d^2e^2g^2x^2-21c^3d^4g^2x^2-42c^3d^3efg\,x^2-24a^2cd\,e^3g^2x+28a\,c^2d^3e\,g^2x+56a\,c^2d^2e^2fgx-70c^3d^4fg}{105c^4d^4\sqrt{cde\,x^2+a\,e^2x+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/105/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-15*c^3*d^3*e*g^2*x^3+18*
a*c^2*d^2*e^2*g^2*x^2-21*c^3*d^4*g^2*x^2-42*c^3*d^3*e*f*g*x^2-24*a^2*c*d*e^
3*g^2*x+28*a*c^2*d^3*e*g^2*x+56*a*c^2*d^2*e^2*f*g*x-70*c^3*d^4*f*g*x-35*c^3
*d^3*e*f^2*x+48*a^3*e^4*g^2-56*a^2*c*d^2*e^2*g^2-112*a^2*c*d*e^3*f*g+140*a*
c^2*d^3*e*f*g+70*a*c^2*d^2*e^2*f^2-105*c^3*d^4*f^2)/c^4/d^4
```

Maxima [A]
time = 0.36, size = 306, normalized size = 0.95

$$\frac{2\,(c^2d^2x^2e+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f^2}{3\,\sqrt{cdx+ae}\,c^2d^2}+\frac{4\,(3\,c^3d^3x^3e-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)fg}{15\,\sqrt{cdx+ae}\,c^3d^3}+\frac{2\,(15\,c^4d^4x^4e+56\,a^3cd^2e^3-48\,a^4e^5+3\,(7\,c^4d^5-ac^3d^3e^2)x^3-(7\,ac^3d^4e-6\,a^2c^2d^2e^3)x^2+4\,(7\,a^2c^2d^3e^2-6\,a^3cde^4)x)g^2}{105\,\sqrt{cdx+ae}\,c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*x^2*e + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
^2/(sqrt(c*d*x + a*e)*c^2*d^2) + 4/15*(3*c^3*d^3*x^3*e - 10*a^2*c*d^2*e^2 +
 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e
```

```
^3)*x)*f*g/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/105*(15*c^4*d^4*x^4*e + 56*a^3*c
*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e
- 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*c*d*e^4)*x)*g^2/(sq
rt(c*d*x + a*e)*c^4*d^4)
```

Fricas [A]
time = 1.53, size = 254, normalized size = 0.79

$$\frac{2\left(21\,c^3d^4g^2x^2+70\,c^3d^4fgx+105\,c^3d^4f^2-48\,a^3g^2e^4+8\left(3\,a^2cdg^2x+14\,a^2cdfg\right)e^3-2\left(9\,ac^2d^2g^2x^2+28\,ac^2d^2fgx+35\,ac^2d^2f^2-28\,a^2cd^2g^2\right)e^2+\left(15\,c^3d^3g^2x^3+42\,c^3d^3fgx^2-140\,ac^2d^3fg+7\left(5\,c^3d^3f^2-4\,ac^2d^3g^2\right)x\right)e\right)\sqrt{cd^2x+axe^2+\left(cdx^2+ad\right)e}\,\sqrt{xe+d}}{105\left(c^4d^4xe+c^4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] 2/105*(21*c^3*d^4*g^2*x^2 + 70*c^3*d^4*f*g*x + 105*c^3*d^4*f^2 - 48*a^3*g^2
*e^4 + 8*(3*a^2*c*d*g^2*x + 14*a^2*c*d*f*g)*e^3 - 2*(9*a*c^2*d^2*g^2*x^2 +
28*a*c^2*d^2*f*g*x + 35*a*c^2*d^2*f^2 - 28*a^2*c*d^2*g^2)*e^2 + (15*c^3*d^3
*g^2*x^3 + 42*c^3*d^3*f*g*x^2 - 140*a*c^2*d^3*f*g + 7*(5*c^3*d^3*f^2 - 4*a*
c^2*d^3*g^2)*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d
)/(c^4*d^4*x*e + c^4*d^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)^2}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)**2/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. 2(307) = 614.
time = 3.54, size = 700, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] 2*(c^3*d^4*f^2 - 2*a*c^2*d^3*f*g*e - a*c^2*d^2*f^2*e^2 + a^2*c*d^2*g^2*e^2
+ 2*a^2*c*d*f*g*e^3 - a^3*g^2*e^4)*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*
e^(-1)/(c^4*d^4) - 4/105*(3*sqrt(-c*d^2*e + a*e^3)*c^3*d^6*g^2 - 14*sqrt(-c
```

```
*d^2*e + a*e^3)*c^3*d^5*f*g*e + 35*sqrt(-c*d^2*e + a*e^3)*c^3*d^4*f^2*e^2 +
 5*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^4*g^2*e^2 - 42*sqrt(-c*d^2*e + a*e^3)*a*c
^2*d^3*f*g*e^3 - 35*sqrt(-c*d^2*e + a*e^3)*a*c^2*d^2*f^2*e^4 + 16*sqrt(-c*d
^2*e + a*e^3)*a^2*c*d^2*g^2*e^4 + 56*sqrt(-c*d^2*e + a*e^3)*a^2*c*d*f*g*e^5
 - 24*sqrt(-c*d^2*e + a*e^3)*a^3*g^2*e^6)*e^(-3)/(c^4*d^4) + 2/105*(70*((x*
e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c^2*d^3*f*g*e^3 + 35*((x*e + d)*c*d*e
 - c*d^2*e + a*e^3)^(3/2)*c^2*d^2*f^2*e^4 - 70*((x*e + d)*c*d*e - c*d^2*e +
 a*e^3)^(3/2)*a*c*d^2*g^2*e^4 + 21*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2
)*c*d^2*g^2*e - 140*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*c*d*f*g*e^5
 + 42*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(5/2)*c*d*f*g*e^2 + 105*((x*e + d
)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a^2*g^2*e^6 - 63*((x*e + d)*c*d*e - c*d^2*
e + a*e^3)^(5/2)*a*g^2*e^3 + 15*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(7/2)*g
^2)*e^(-6)/(c^4*d^4)
```

Mupad [B]
time = 3.71, size = 279, normalized size = 0.87

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{2\,g^2\,x^3\,\sqrt{d+e\,x}}{7\,c\,d}-\frac{\sqrt{d+e\,x}\,\left(96\,a^3\,e^4\,g^2-112\,a^2\,c\,d^2\,e^2\,g^2-224\,a^2\,c\,d\,e^3\,f\,g+280\,a\,c^2\,d^3\,e\,f\,g+140\,a\,c^2\,d^2\,e^2\,f^2-210\,c^3\,d^4\,f^2\right)}{105\,c^4\,d^4\,e}+\frac{x\,\sqrt{d+e\,x}\,\left(48\,a^2\,c\,d\,e^3\,g^2-56\,a\,c^2\,d^3\,e\,g^2-112\,a\,c^2\,d^2\,e^2\,f\,g+140\,c^3\,d^4\,f\,g+70\,c^3\,d^3\,e\,f^2\right)}{105\,c^4\,d^4\,e}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}\,\left(7\,c\,g\,d^2+14\,c\,f\,d\,e-6\,a\,g\,e^2\right)}{35\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(
1/2),x)

[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((2*g^2*x^3*(d + e*x)^(1/2))
/(7*c*d) - ((d + e*x)^(1/2)*(96*a^3*e^4*g^2 - 210*c^3*d^4*f^2 + 140*a*c^2*d
^2*e^2*f^2 - 112*a^2*c*d^2*e^2*g^2 + 280*a*c^2*d^3*e*f*g - 224*a^2*c*d*e^3*
f*g))/(105*c^4*d^4*e) + (x*(d + e*x)^(1/2)*(70*c^3*d^3*e*f^2 + 140*c^3*d^4*
f*g - 56*a*c^2*d^3*e*g^2 + 48*a^2*c*d*e^3*g^2 - 112*a*c^2*d^2*e^2*f*g))/(10
5*c^4*d^4*e) + (2*g*x^2*(d + e*x)^(1/2)*(7*c*d^2*g - 6*a*e^2*g + 14*c*d*e*f
))/(35*c^2*d^2*e)))/(x + d/e)
```

3.786 $\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=209

$$-\frac{4(cd^2-ae^2)(4ae^2g-cd(5ef-dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{15c^3d^3e\sqrt{d+ex}}-\frac{2(4ae^2g-cd(5ef-dg))\sqrt{d+ex}}{15c^2d}$$

```
[Out] 2/5*g*(e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e-4/15*(-a*
e^2+c*d^2)*(4*a*e^2*g-c*d*(-d*g+5*e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/c^3/d^3/e/(e*x+d)^(1/2)-2/15*(4*a*e^2*g-c*d*(-d*g+5*e*f))*(e*x+d)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
```

Rubi [A]
time = 0.13, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {808, 670, 662}

$$-\frac{4(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(4ae^2g-cd(5ef-dg))}{15c^3d^3e\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(4ae^2g-cd(5ef-dg))}{15c^2d^2e}+\frac{2g(d+ex)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cde}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
,x]
```

```
[Out] (-4*(c*d^2 - a*e^2)*(4*a*e^2*g - c*d*(5*e*f - d*g))*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*Sqrt[d + e*x]) - (2*(4*a*e^2*g - c*d*(5
*e*f - d*g))*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15
*c^2*d^2*e) + (2*g*(d + e*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(5*c*d*e)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2g(d+ex)^{3/2}\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{5cde} + \frac{1}{5}\left(5f-\frac{dg}{e}-\frac{4aeg}{cd}\right.$$

$$= -\frac{2(4ae^2g-cd(5ef-dg))\sqrt{d+ex}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{15c^2d^2e}$$

$$= -\frac{4(cd^2-ae^2)\,(4ae^2g-cd(5ef-dg))\sqrt{ade+(cd^2+ae^2)\,x+cdex}}{15c^3d^3e\sqrt{d+ex}}$$

Mathematica [A]
time = 0.08, size = 96, normalized size = 0.46

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(8a^2e^3g-2acde(5ef+5dg+2egx)+c^2d^2(5d(3f+gx)+ex(5f+3gx)))}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(f + g*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(8*a^2*e^3*g - 2*a*c*d*e*(5*e*f + 5*d*g +
2*e*g*x) + c^2*d^2*(5*d*(3*f + g*x) + e*x*(5*f + 3*g*x))))/(15*c^3*d^3*Sqrt
[d + e*x])
```

Maple [A]
time = 0.13, size = 113, normalized size = 0.54

method	result	size
default	$\frac{2\sqrt{(cdx+ae)\,(ex+d)}\,(3eg\,x^2c^2d^2-4acd\,e^2gx+5c^2d^3gx+5c^2d^2efx+8a^2e^3g-10ac\,d^2eg-10acd\,e^2f+15d^3f\,c^2)}{15\sqrt{ex+d}\,c^3d^3}$	113
gosper	$\frac{2(cdx+ae)(3eg\,x^2c^2d^2-4acd\,e^2gx+5c^2d^3gx+5c^2d^2efx+8a^2e^3g-10ac\,d^2eg-10acd\,e^2f+15d^3f\,c^2)\sqrt{ex+d}}{15c^3d^3\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] 2/15/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(3*c^2*d^2*e*g*x^2-4*a*c*d*e
^2*g*x+5*c^2*d^3*g*x+5*c^2*d^2*e*f*x+8*a^2*e^3*g-10*a*c*d^2*e*g-10*a*c*d*e^
2*f+15*c^2*d^3*f)/c^3/d^3
```

Maxima [A]
time = 0.31, size = 168, normalized size = 0.80

$$\frac{2\,(c^2d^2x^2e+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)f}{3\,\sqrt{cdx+ae}\,c^2d^2}+\frac{2\,(3\,c^3d^3x^3e-10\,a^2cd^2e^2+8\,a^3e^4+(5\,c^3d^4-ac^2d^2e^2)x^2-(5\,ac^2d^3e-4\,a^2cde^3)x)g}{15\,\sqrt{cdx+ae}\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] 2/3*(c^2*d^2*x^2*e + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)*f
/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/15*(3*c^3*d^3*x^3*e - 10*a^2*c*d^2*e^2 + 8
*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^3
)*x)*g/(sqrt(c*d*x + a*e)*c^3*d^3)
```

Fricas [A]
time = 2.93, size = 137, normalized size = 0.66

$$\frac{2\,(5\,c^2d^3gx+15\,c^2d^3f+8\,a^2ge^3-2\,(2\,acdgx+5\,acdf)e^2+(3\,c^2d^2gx^2+5\,c^2d^2fx-10\,acd^2g)e)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{15\,(c^3d^3xe+c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] 2/15*(5*c^2*d^3*g*x + 15*c^2*d^3*f + 8*a^2*g*e^3 - 2*(2*a*c*d*g*x + 5*a*c*d
*f)*e^2 + (3*c^2*d^2*g*x^2 + 5*c^2*d^2*f*x - 10*a*c*d^2*g)*e)*sqrt(c*d^2*x
+ a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c^3*d^3*x*e + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\,(f+gx)}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(f + g*x)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [A]
time = 6.25, size = 342, normalized size = 1.64

$$\frac{2\left(c^{2}d^{3}f-acd^{2}ge-acdfe^{2}+a^{2}ge^{3}\right)\sqrt{(xe+d)cde-cd^{2}e+ae^{3}}\,e^{(-1)}}{c^{3}d^{3}}+\frac{4\left(\sqrt{-cd^{2}e+ae^{3}}\,c^{2}d^{4}g-5\sqrt{-cd^{2}e+ae^{3}}\,c^{2}d^{3}fe+3\sqrt{-cd^{2}e+ae^{3}}\,acd^{2}ge^{2}+5\sqrt{-cd^{2}e+ae^{3}}\,acdfe^{3}-4\sqrt{-cd^{2}e+ae^{3}}\,a^{2}ge^{4}\right)e^{(-2)}}{15\,c^{3}d^{3}}+\frac{2\left(5\left((xe+d)cde-cd^{2}e+ae^{3}\right)^{\frac{3}{2}}cd^{2}ge+5\left((xe+d)cde-cd^{2}e+ae^{3}\right)^{\frac{3}{2}}cdfe^{2}-10\left((xe+d)cde-cd^{2}e+ae^{3}\right)^{\frac{3}{2}}age^{3}+3\left((xe+d)cde-cd^{2}e+ae^{3}\right)^{\frac{5}{2}}g\right)e^{(-4)}}{15\,c^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] 2*(c^2*d^3*f - a*c*d^2*g*e - a*c*d*f*e^2 + a^2*g*e^3)*sqrt((x*e + d)*c*d*e
- c*d^2*e + a*e^3)*e^(-1)/(c^3*d^3) + 4/15*(sqrt(-c*d^2*e + a*e^3)*c^2*d^4*
g - 5*sqrt(-c*d^2*e + a*e^3)*c^2*d^3*f*e + 3*sqrt(-c*d^2*e + a*e^3)*a*c*d^2
*g*e^2 + 5*sqrt(-c*d^2*e + a*e^3)*a*c*d*f*e^3 - 4*sqrt(-c*d^2*e + a*e^3)*a^
2*g*e^4)*e^(-2)/(c^3*d^3) + 2/15*(5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/
2)*c*d^2*g*e + 5*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*c*d*f*e^2 - 10*(
(x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*a*g*e^3 + 3*((x*e + d)*c*d*e - c*d
^2*e + a*e^3)^(5/2)*g)*e^(-4)/(c^3*d^3)
```

Mupad [B]
time = 3.48, size = 152, normalized size = 0.73

$$\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(\frac{\sqrt{d+e\,x}\,\left(16\,g\,a^2\,e^3-20\,g\,a\,c\,d^2\,e-20\,f\,a\,c\,d\,e^2+30\,f\,c^2\,d^3\right)}{15\,c^3\,d^3\,e}+\frac{2\,g\,x^2\,\sqrt{d+e\,x}}{5\,c\,d}+\frac{2\,x\,\sqrt{d+e\,x}\,\left(5\,c\,g\,d^2+5\,c\,f\,d\,e-4\,a\,g\,e^2\right)}{15\,c^2\,d^2\,e}\right)}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^(3/2))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/
2),x)
```

```
[Out] ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(((d + e*x)^(1/2)*(16*a^2*e^
3*g + 30*c^2*d^3*f - 20*a*c*d*e^2*f - 20*a*c*d^2*e*g))/(15*c^3*d^3*e) + (2*
g*x^2*(d + e*x)^(1/2))/(5*c*d) + (2*x*(d + e*x)^(1/2)*(5*c*d^2*g - 4*a*e^2*
g + 5*c*d*e*f))/(15*c^2*d^2*e)))/(x + d/e)
```

3.787 $\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$

Optimal. Leaf size=109

$$\frac{4(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd}$$

```
[Out] 4/3*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(e*x+d)^
(1/2)+2/3*(e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {670, 662}

$$\frac{4(cd^2-ae^2)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (4*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*
Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/(3*c*d)
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
 && IGtQ[Simplify[m + p], 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2\sqrt{d+ex}\,\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cd} + \frac{\left(2\left(d^2-\frac{ae^2}{c}\right)\right)\int \frac{}{\sqrt{ad}}}{}$$

$$= \frac{4(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\sqrt{ade+(}}{3}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.50

$$\frac{2\sqrt{(ae+cdx)(d+ex)}\,(-2ae^2+cd(3d+ex))}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

```
[Out] (2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*S
qrt[d + e*x])
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.47

method	result	size
default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\,(-cdex+2a\,e^2-3c\,d^2)}{3\sqrt{ex+d}\,c^2d^2}$	51
gosper	$-\frac{2(cdx+ae)(-cdex+2a\,e^2-3c\,d^2)\sqrt{ex+d}}{3c^2d^2\sqrt{cde\,x^2+a\,e^2x+c\,d^2x+ade}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -2/3/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)*(-c*d*e*x+2*a*e^2-3*c*d^2)/c
^2/d^2
```

Maxima [A]
time = 0.32, size = 66, normalized size = 0.61

$$\frac{2\,(c^2d^2x^2e+3\,acd^2e-2\,a^2e^3+(3\,c^2d^3-acde^2)x)}{3\,\sqrt{cdx+ae}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="maxima")
```

```
[Out] 2/3*(c^2*d^2*x^2*e + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d*e^2)*x)/(
sqrt(c*d*x + a*e)*c^2*d^2)
```

Fricas [A]
time = 3.02, size = 75, normalized size = 0.69

$$\frac{2\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,(cdxe+3\,cd^2-2\,ae^2)\sqrt{xe+d}}{3\,(c^2d^2xe+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="fricas")
```

```
[Out] 2/3*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*(c*d*x*e + 3*c*d^2 - 2*a*e^
2)*sqrt(x*e + d)/(c^2*d^2*x*e + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/sqrt((d + e*x)*(a*e + c*d*x)), x)
```

Giac [A]
time = 5.06, size = 135, normalized size = 1.24

$$\frac{2\,\sqrt{(xe+d)cde-cd^2e+ae^3}\,(cd^2-ae^2)e^{(-1)}}{c^2d^2}+\frac{2\,((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}e^{(-2)}}{3\,c^2d^2}-\frac{4\left(\sqrt{-cd^2e+ae^3}\,cd^2-\sqrt{-cd^2e+ae^3}\,ae^2\right)e^{(-1)}}{3\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorith
m="giac")
```

```
[Out] 2*sqrt((x*e + d)*c*d*e - c*d^2*e + a*e^3)*(c*d^2 - a*e^2)*e^(-1)/(c^2*d^2)
+ 2/3*((x*e + d)*c*d*e - c*d^2*e + a*e^3)^(3/2)*e^(-2)/(c^2*d^2) - 4/3*(sqr
t(-c*d^2*e + a*e^3)*c*d^2 - sqrt(-c*d^2*e + a*e^3)*a*e^2)*e^(-1)/(c^2*d^2)
```

Mupad [B]
time = 3.36, size = 85, normalized size = 0.78

$$\frac{\left(\frac{2\,x\,\sqrt{d+e\,x}}{3\,c\,d}-\frac{\left(4\,a\,e^2-6\,c\,d^2\right)\,\sqrt{d+e\,x}}{3\,c^2\,d^2\,e}\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x+\frac{d}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

```
[Out] (((2*x*(d + e*x)^(1/2))/(3*c*d) - ((4*a*e^2 - 6*c*d^2)*(d + e*x)^(1/2))/(3*
c^2*d^2*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(x + d/e)
```

3.788 $\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=139

$$\frac{2e\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{cdf-aeg}\sqrt{d+ex}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

```
[Out] -2*(-d*g+e*f)*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e*
g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(1/2)+2*e*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/g/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {894, 888, 211}

$$\frac{2e\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg\sqrt{d+ex}}-\frac{2(ef-dg)\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
,x]
```

```
[Out] (2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*g*Sqrt[d + e*x]) - (
2*(e*f - d*g)*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(g^(3/2)*Sqrt[c*d*f - a*e*g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 894

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n
+ 1)*((a + b*x + c*x^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(b*e*g*(n + 1
) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(c*g*(n + p + 2)), Int[(d + e*x)^(
m - 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 -
b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] &&
IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{2e\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{\left(2\left(\frac{1}{2}cde^2f-\frac{1}{2}cd^2eg\right)\right)}{}$$

$$= \frac{2e\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{(2e^2(ef-dg))\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{2e\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cdg\sqrt{d+ex}} - \frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}}{}\right.}{g^3}$$

Mathematica [A]
time = 0.16, size = 140, normalized size = 1.01

$$\frac{2\sqrt{d+ex}\left(e\sqrt{g}\,\sqrt{cdf-aeg}\,(ae+cdx)+cd(-ef+dg)\sqrt{ae+cdx}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)\right)}{cdg^{3/2}\sqrt{cdf-aeg}\,\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2]),x]
```

```
[Out] (2*Sqrt[d + e*x]*(e*Sqrt[g]*Sqrt[c*d*f - a*e*g]*(a*e + c*d*x) + c*d*(-(e*f)
 + d*g)*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a
*e*g]]))/(c*d*g^(3/2)*Sqrt[c*d*f - a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [A]
time = 0.14, size = 153, normalized size = 1.10

method	result

default	$-\frac{2\sqrt{(cdx+ae)(ex+d)}\left(\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c\,d^{2}g-\operatorname{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)cdef-e\sqrt{cdx}\right.}{\sqrt{ex+d}\,\sqrt{cdx+ae}\,cdg\sqrt{(aeg-cdf)g}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -2*((c*d*x+a*e)*(e*x+d))^(1/2)*(arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*
g)^(1/2))*c*d^2*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*
e*f-e*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^
(1/2)/c/d/g/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)), x)
```

Fricas [A]
time = 2.89, size = 520, normalized size = 3.74

$$\left[\frac{(cd^3g-cdfxe^2+(cd^2gx-cd^2f)e)\sqrt{-cdfg+ag^2e}\,\log\left(-\frac{cd^2gx-cd^2f+2agxe^2+(cdgx^2-cdfx+2adg)e+2\sqrt{-cdfg+ag^2e}\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{dgx+df+(gx^2+fx)e}\right)+2\,(cdfge-ag^2e^2)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{c^2d^3fg^2-acdg^3xe^2+(c^2d^2fg^2x-acd^2g^3)e},\ -\frac{2\left((cd^3g-cdfxe^2+(cd^2gx-cd^2f)e)\sqrt{cdfg-ag^2e}\,\arctan\left(\frac{\sqrt{cdfg-ag^2e}\,\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}}{cd^2gx+agxe^2+(cdgx^2+adg)e}\right)-(cdfge-ag^2e^2)\sqrt{cd^2x+axe^2+(cdx^2+ad)e}\,\sqrt{xe+d}\right)}{c^2d^3fg^2-acdg^3xe^2+(c^2d^2fg^2x-acd^2g^3)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] [((c*d^3*g - c*d*f*x*e^2 + (c*d^2*g*x - c*d^2*f)*e)*sqrt(-c*d*f*g + a*g^2*e
)*log(-(c*d^2*g*x - c*d^2*f + 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)
*e + 2*sqrt(-c*d*f*g + a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)
*sqrt(x*e + d))/(d*g*x + d*f + (g*x^2 + f*x)*e)) + 2*(c*d*f*g*e - a*g^2*e^2
)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f*g^2
 - a*c*d*g^3*x*e^2 + (c^2*d^2*f*g^2*x - a*c*d^2*g^3)*e), -2*((c*d^3*g - c*d
*f*x*e^2 + (c*d^2*g*x - c*d^2*f)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d
*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(
c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e)) - (c*d*f*g*e - a*g^2*e^2)*s
```

```
qrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f*g^2 -
a*c*d*g^3*x*e^2 + (c^2*d^2*f*g^2*x - a*c*d^2*g^3)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2
),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
)), x)
```

3.789 $\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=170

$$-\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)}-\frac{(2ae^2g-cd(ef+dg))\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ade+(cd^2+ae^2)x+cd}}{\sqrt{cdf-aeg}\,\sqrt{d+ex}}\right.}{g^{3/2}(cdf-aeg)^{3/2}}$$

```
[Out] -(2*a*e^2*g-c*d*(d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)^(3/2)-(-d
*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)/(g*x+f)/(e
*x+d)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {892, 888, 211}

$$-\frac{(2ae^2g-cd(dg+ef))\,\text{ArcTan}\left(\frac{\sqrt{g}\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\,\sqrt{cdf-aeg}}\right)}{g^{3/2}(cdf-aeg)^{3/2}}-\frac{(ef-dg)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\,(f+gx)(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -(((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f - a*e
*g)*Sqrt[d + e*x]*(f + g*x))) - ((2*a*e^2*g - c*d*(e*f + d*g))*ArcTan[(Sqrt
[g]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[
d + e*x])])/(g^(3/2)*(c*d*f - a*e*g)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Dist[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} + \frac{\left(e\left(\frac{1}{2}cde^2f\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} - \frac{\left(e^2(2ae^2g-\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{g(cdf-aeg)\sqrt{d+ex}\,(f+gx)} - \frac{(2ae^2g-cd}{}$$

Mathematica [A]
time = 0.49, size = 154, normalized size = 0.91

$$\frac{\sqrt{d+ex}\left(-\frac{\sqrt{g}(-ef+dg)(ae+cdx)}{(-cdf+aeg)(f+gx)}+\frac{(-2ae^2g+cd(ef+dg))\sqrt{ae+cdx}\,\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}\right)}{g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(-((Sqrt[g]*(-(e*f) + d*g)*(a*e + c*d*x))/((-(c*d*f) + a*e*g
)*(f + g*x))) + ((-2*a*e^2*g + c*d*(e*f + d*g))*Sqrt[a*e + c*d*x]*ArcTan[(S
qrt[g]*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(3/2)))/(g^
(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(154) = 308.
time = 0.14, size = 337, normalized size = 1.98

method	result
default	$\frac{\left(-2\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)a\,e^{2}g^{2}x+\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c\,d^{2}g^{2}x+\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] (-2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*g^2*x+arctan
h(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d^2*g^2*x+arctanh(g*(c*d*x
+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*e*f*g*x-2*arctanh(g*(c*d*x+a*e)^(1
/2)/((a*e*g-c*d*f)*g)^(1/2))*a*e^2*f*g+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-
c*d*f)*g)^(1/2))*c*d^2*f*g+arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*c*d*e*f^2-(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)*d*g+(c*d*x+a*e)^(1
/2)*((a*e*g-c*d*f)*g)^(1/2)*e*f)/(e*x+d)^(1/2)*((c*d*x+a*e)*(e*x+d))^(1/2)/
(c*d*x+a*e)^(1/2)/(a*e*g-c*d*f)/g/(g*x+f)/((a*e*g-c*d*f)*g)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(159) = 318.
time = 2.65, size = 921, normalized size = 5.42

$$\left[-\frac{\left(cd^{3}g^{2}x+cd^{3}fg-2\left(ag^{2}x^{2}+afgx\right)e^{3}+\left(cdfgx^{2}-2\,adfg+\left(cdf^{2}-2\,adg^{2}\right)x\right)e^{2}+\left(cd^{2}g^{2}x^{2}+2\,cd^{2}fgx+cd^{2}f^{2}\right)e\right)\sqrt{-cdfg+ag^{2}e}\,\log\left(-\frac{cd^{2}gx-cd^{2}f+2\,agxe^{2}+\left(cdgx^{2}-cdfx+2\,adg\right)e-2\sqrt{-cdfg+ag^{2}e}\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{xe+d}}{dgx+df+\left(gx^{2}+fx\right)e}\right)-2\left(cd^{2}fg^{2}+afg^{2}e^{2}-\left(cdf^{2}g+adg^{3}\right)e\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{xe+d}}{2\left(c^{2}d^{3}f^{2}g^{3}x+c^{2}d^{3}f^{3}g^{2}+\left(a^{2}g^{5}x^{2}+a^{2}fg^{4}x\right)e^{3}-\left(2\,acdfg^{4}x^{2}-a^{2}dfg^{4}+\left(2\,acdf^{2}g^{3}-a^{2}dg^{5}\right)x\right)e^{2}+\left(c^{2}d^{2}f^{2}g^{3}x^{2}-2\,acd^{2}f^{2}g^{3}+\left(c^{2}d^{2}f^{3}g^{2}-2\,acd^{2}fg^{4}\right)x\right)e\right)},\ -\frac{\left(cd^{3}g^{2}x+cd^{3}fg-2\left(ag^{2}x^{2}+afgx\right)e^{3}+\left(cdfgx^{2}-2\,adfg+\left(cdf^{2}-2\,adg^{2}\right)x\right)e^{2}+\left(cd^{2}g^{2}x^{2}+2\,cd^{2}fgx+cd^{2}f^{2}\right)e\right)\sqrt{cdfg-ag^{2}e}\,\arctan\left(\frac{\sqrt{cdfg-ag^{2}e}\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{xe+d}}{cd^{2}gx+agxe^{2}+\left(cdgx^{2}+adg\right)e}\right)-\left(cd^{2}fg^{2}+afg^{2}e^{2}-\left(cdf^{2}g+adg^{3}\right)e\right)\sqrt{cd^{2}x+axe^{2}+\left(cdx^{2}+ad\right)e}\sqrt{xe+d}}{c^{2}d^{3}f^{2}g^{3}x+c^{2}d^{3}f^{3}g^{2}+\left(a^{2}g^{5}x^{2}+a^{2}fg^{4}x\right)e^{3}-\left(2\,acdfg^{4}x^{2}-a^{2}dfg^{4}+\left(2\,acdf^{2}g^{3}-a^{2}dg^{5}\right)x\right)e^{2}+\left(c^{2}d^{2}f^{2}g^{3}x^{2}-2\,acd^{2}f^{2}g^{3}+\left(c^{2}d^{2}f^{3}g^{2}-2\,acd^{2}fg^{4}\right)x\right)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^3*g^2*x + c*d^3*f*g - 2*(a*g^2*x^2 + a*f*g*x)*e^3 + (c*d*f*g*x^
2 - 2*a*d*f*g + (c*d*f^2 - 2*a*d*g^2)*x)*e^2 + (c*d^2*g^2*x^2 + 2*c*d^2*f*g
*x + c*d^2*f^2)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f + 2*a
```

```
*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e - 2*sqrt(-c*d*f*g + a*g^2*e)*s
qrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f + (g
*x^2 + f*x)*e)) - 2*(c*d^2*f*g^2 + a*f*g^2*e^2 - (c*d*f^2*g + a*d*g^3)*e)*s
qrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f^2*g^3*
x + c^2*d^3*f^3*g^2 + (a^2*g^5*x^2 + a^2*f*g^4*x)*e^3 - (2*a*c*d*f*g^4*x^2
- a^2*d*f*g^4 + (2*a*c*d*f^2*g^3 - a^2*d*g^5)*x)*e^2 + (c^2*d^2*f^2*g^3*x^2
 - 2*a*c*d^2*f^2*g^3 + (c^2*d^2*f^3*g^2 - 2*a*c*d^2*f*g^4)*x)*e), -((c*d^3*
g^2*x + c*d^3*f*g - 2*(a*g^2*x^2 + a*f*g*x)*e^3 + (c*d*f*g*x^2 - 2*a*d*f*g
+ (c*d*f^2 - 2*a*d*g^2)*x)*e^2 + (c*d^2*g^2*x^2 + 2*c*d^2*f*g*x + c*d^2*f^2
)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x +
a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*
x^2 + a*d*g)*e)) - (c*d^2*f*g^2 + a*f*g^2*e^2 - (c*d*f^2*g + a*d*g^3)*e)*sq
rt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^2*d^3*f^2*g^3*x
 + c^2*d^3*f^3*g^2 + (a^2*g^5*x^2 + a^2*f*g^4*x)*e^3 - (2*a*c*d*f*g^4*x^2 -
 a^2*d*f*g^4 + (2*a*c*d*f^2*g^3 - a^2*d*g^5)*x)*e^2 + (c^2*d^2*f^2*g^3*x^2
- 2*a*c*d^2*f^2*g^3 + (c^2*d^2*f^3*g^2 - 2*a*c*d^2*f*g^4)*x)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{(d+ex)\,(ae+cdx)}\,\,(f+gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(sqrt((d + e*x)*(a*e + c*d*x))*(f + g*x)**2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^{3/2}}{{\left(f+g\,x\right)}^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.790 $\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal. Leaf size=261

$$-\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2}-\frac{(4ae^2g-cd(ef+3dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4g(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)}-\frac{cd(4a$$

```
[Out] -1/4*c*d*(4*a*e^2*g-c*d*(3*d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*d*f)
^(5/2)-1/2*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d
*f)/(g*x+f)^2/(e*x+d)^(1/2)-1/4*(4*a*e^2*g-c*d*(3*d*g+e*f))*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {892, 886, 888, 211}

$$-\frac{cd(4ae^2g-cd(3dg+ef))\,\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{5/2}}-\frac{(ef-dg)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(4ae^2g-cd(3dg+ef))}{4g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -1/2*((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f -
a*e*g)*Sqrt[d + e*x]*(f + g*x)^2) - ((4*a*e^2*g - c*d*(e*f + 3*d*g))*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x]
*(f + g*x)) - (c*d*(4*a*e^2*g - c*d*(e*f + 3*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/
(4*g^(3/2)*(c*d*f - a*e*g)^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Dist[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} + \frac{\left(e\left(\tfrac{1}{2}cde^2f\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} - \frac{(4ae^2g-c}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} - \frac{(4ae^2g-c}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^2} - \frac{(4ae^2g-c}{}$$

Mathematica [A]

time = 0.92, size = 200, normalized size = 0.77

$$\frac{cd\sqrt{d+ex}\left(\frac{\sqrt{g}\,(ae+cdx)(-2aeg(dg+e(f+2gx))+cd(ef(-f+gx)+dg(5f+3gx)))}{cd(cdf-aeg)^2(f+gx)^2}+\frac{(-4ae^2g+cd(ef+3dg))\sqrt{ae+cdx}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}\right)}{4g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (c*d*Sqrt[d + e*x]*((Sqrt[g]*(a*e + c*d*x)*(-2*a*e*g*(d*g + e*(f + 2*g*x))
+ c*d*(e*f*(-f + g*x) + d*g*(5*f + 3*g*x))))/(c*d*(c*d*f - a*e*g)^2*(f + g*
x)^2) + ((-4*a*e^2*g + c*d*(e*f + 3*d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]
*Sqrt[a*e + c*d*x])/Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(5/2)))/(4*g^(3/2
)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(235) = 470$.
time = 0.14, size = 663, normalized size = 2.54

method	result
default	$\frac{\sqrt{(cdx+ae)(ex+d)}\left(4\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)acd\,e^2g^3x^2-3\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)\,g}}\right)c^2d^3g^3x^2-\mathrm{ar}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/4*((c*d*x+a*e)*(e*x+d))^(1/2)*(4*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*
f)*g)^(1/2))*a*c*d*e^2*g^3*x^2-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)
*g)^(1/2))*c^2*d^3*g^3*x^2-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*c^2*d^2*e*f*g^2*x^2+8*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1
/2))*a*c*d*e^2*f*g^2*x-6*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2
))*c^2*d^3*f*g^2*x-2*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c
^2*d^2*e*f^2*g*x+4*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c
*d*e^2*f^2*g-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^3
*f^2*g-arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^2*d^2*e*f^3-4
*a*e^2*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*c*d^2*g^2*x*(c*d*x
+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+c*d*e*f*g*x*(c*d*x+a*e)^(1/2)*((a*e*g-c
*d*f)*g)^(1/2)-2*a*d*e*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-2*a*e^
2*f*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+5*c*d^2*f*g*(c*d*x+a*e)^(1/
2)*((a*e*g-c*d*f)*g)^(1/2)-c*d*e*f^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1
```

```
/2))/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^2/g/(a*e*g-c*d*f)^2/(c*d
*x+a*e)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

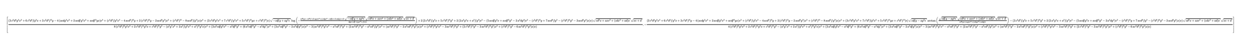
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(244) = 488$.
time = 2.28, size = 1761, normalized size = 6.75



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [1/8*((3*c^2*d^4*g^3*x^2 + 6*c^2*d^4*f*g^2*x + 3*c^2*d^4*f^2*g - 4*(a*c*d*g
^3*x^3 + 2*a*c*d*f*g^2*x^2 + a*c*d*f^2*g*x)*e^3 + (c^2*d^2*f*g^2*x^3 - 4*a*
c*d^2*f^2*g + 2*(c^2*d^2*f^2*g - 2*a*c*d^2*g^3)*x^2 + (c^2*d^2*f^3 - 8*a*c*
d^2*f*g^2)*x)*e^2 + (3*c^2*d^3*g^3*x^3 + 7*c^2*d^3*f*g^2*x^2 + 5*c^2*d^3*f^
2*g*x + c^2*d^3*f^3)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f
+ 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e + 2*sqrt(-c*d*f*g + a*g^2
*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f
 + (g*x^2 + f*x)*e)) + 2*(3*c^2*d^3*f*g^3*x + 5*c^2*d^3*f^2*g^2 + 2*(2*a^2*
g^4*x + a^2*f*g^3)*e^3 - (5*a*c*d*f*g^3*x + a*c*d*f^2*g^2 - 2*a^2*d*g^4)*e^
2 - (c^2*d^2*f^3*g + 7*a*c*d^2*f*g^3 - (c^2*d^2*f^2*g^2 - 3*a*c*d^2*g^4)*x)
*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^3*d^4*f^3
*g^4*x^2 + 2*c^3*d^4*f^4*g^3*x + c^3*d^4*f^5*g^2 - (a^3*g^7*x^3 + 2*a^3*f*g
^6*x^2 + a^3*f^2*g^5*x)*e^4 + (3*a^2*c*d*f*g^6*x^3 - a^3*d*f^2*g^5 + (6*a^2
*c*d*f^2*g^5 - a^3*d*g^7)*x^2 + (3*a^2*c*d*f^3*g^4 - 2*a^3*d*f*g^6)*x)*e^3
- 3*(a*c^2*d^2*f^2*g^5*x^3 - a^2*c*d^2*f^3*g^4 + (2*a*c^2*d^2*f^3*g^4 - a^2
*c*d^2*f*g^6)*x^2 + (a*c^2*d^2*f^4*g^3 - 2*a^2*c*d^2*f^2*g^5)*x)*e^2 + (c^3
*d^3*f^3*g^4*x^3 - 3*a*c^2*d^3*f^4*g^3 + (2*c^3*d^3*f^4*g^3 - 3*a*c^2*d^3*f
^2*g^5)*x^2 + (c^3*d^3*f^5*g^2 - 6*a*c^2*d^3*f^3*g^4)*x)*e), -1/4*((3*c^2*d
^4*g^3*x^2 + 6*c^2*d^4*f*g^2*x + 3*c^2*d^4*f^2*g - 4*(a*c*d*g^3*x^3 + 2*a*c
*d*f*g^2*x^2 + a*c*d*f^2*g*x)*e^3 + (c^2*d^2*f*g^2*x^3 - 4*a*c*d^2*f^2*g +
2*(c^2*d^2*f^2*g - 2*a*c*d^2*g^3)*x^2 + (c^2*d^2*f^3 - 8*a*c*d^2*f*g^2)*x)*
```

```
e^2 + (3*c^2*d^3*g^3*x^3 + 7*c^2*d^3*f*g^2*x^2 + 5*c^2*d^3*f^2*g*x + c^2*d^
3*f^3)*e)*sqrt(c*d*f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2
*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c
*d*g*x^2 + a*d*g)*e)) - (3*c^2*d^3*f*g^3*x + 5*c^2*d^3*f^2*g^2 + 2*(2*a^2*g
^4*x + a^2*f*g^3)*e^3 - (5*a*c*d*f*g^3*x + a*c*d*f^2*g^2 - 2*a^2*d*g^4)*e^2
 - (c^2*d^2*f^3*g + 7*a*c*d^2*f*g^3 - (c^2*d^2*f^2*g^2 - 3*a*c*d^2*g^4)*x)*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^3*d^4*f^3*
g^4*x^2 + 2*c^3*d^4*f^4*g^3*x + c^3*d^4*f^5*g^2 - (a^3*g^7*x^3 + 2*a^3*f*g^
6*x^2 + a^3*f^2*g^5*x)*e^4 + (3*a^2*c*d*f*g^6*x^3 - a^3*d*f^2*g^5 + (6*a^2*
c*d*f^2*g^5 - a^3*d*g^7)*x^2 + (3*a^2*c*d*f^3*g^4 - 2*a^3*d*f*g^6)*x)*e^3 -
 3*(a*c^2*d^2*f^2*g^5*x^3 - a^2*c*d^2*f^3*g^4 + (2*a*c^2*d^2*f^3*g^4 - a^2*
c*d^2*f*g^6)*x^2 + (a*c^2*d^2*f^4*g^3 - 2*a^2*c*d^2*f^2*g^5)*x)*e^2 + (c^3*
d^3*f^3*g^4*x^3 - 3*a*c^2*d^3*f^4*g^3 + (2*c^3*d^3*f^4*g^3 - 3*a*c^2*d^3*f^
2*g^5)*x^2 + (c^3*d^3*f^5*g^2 - 6*a*c^2*d^3*f^3*g^4)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.791 $\int \frac{(d+ex)^{3/2}}{(f+gx)^4 \sqrt{ade+(cd^2+ae^2)x+cdex^2}} \, dx$

Optimal. Leaf size=351

$$-\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3}-\frac{(6ae^2g-cd(ef+5dg))\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12g(cdf-aeg)^2\sqrt{d+ex}\,(f+gx)^2}-cd(6$$

```
[Out] -1/8*c^2*d^2*(6*a*e^2*g-c*d*(5*d*g+e*f))*arctan(g^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/(-a*e*g+c*d*f)^(1/2)/(e*x+d)^(1/2))/g^(3/2)/(-a*e*g+c*
d*f)^(7/2)-1/3*(-d*g+e*f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g
+c*d*f)/(g*x+f)^3/(e*x+d)^(1/2)-1/12*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g/(-a*e*g+c*d*f)^2/(g*x+f)^2/(e*x+d)^(1/2)-1/
8*c*d*(6*a*e^2*g-c*d*(5*d*g+e*f))*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/g
/(-a*e*g+c*d*f)^3/(g*x+f)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {892, 886, 888, 211}

$$-\frac{c^2d^2(6ae^2g-cd(5dg+ef))\,\text{ArcTan}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{7/2}}-\frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(6ae^2g-cd(5dg+ef))}{8g\sqrt{d+ex}\,(f+gx)(cdf-aeg)^3}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\,(6ae^2g-cd(5dg+ef))}{12g\sqrt{d+ex}\,(f+gx)^2(cdf-aeg)^2}-\frac{(ef-dg)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}\,(f+gx)^3(cdf-aeg)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]),x]
```

```
[Out] -1/3*((e*f - d*g)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*(c*d*f -
a*e*g)*Sqrt[d + e*x]*(f + g*x)^3) - ((6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*g*(c*d*f - a*e*g)^2*Sqrt[d + e*x
]*(f + g*x)^2) - (c*d*(6*a*e^2*g - c*d*(e*f + 5*d*g))*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])/(8*g*(c*d*f - a*e*g)^3*Sqrt[d + e*x]*(f + g*x)) - (
c^2*d^2*(6*a*e^2*g - c*d*(e*f + 5*d*g))*ArcTan[(Sqrt[g]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(Sqrt[c*d*f - a*e*g]*Sqrt[d + e*x])])/(8*g^(3/2)*
(c*d*f - a*e*g)^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 886

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^
```

```
(n + 1)*((a + b*x + c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g - b*e*g))), x] -
 Dist[c*e*((m - n - 2)/((n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^m*
(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*
e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2
*p]
```

Rule 888

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2]), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) - b*e*
g + e^2*g*x^2), x], x, Sqrt[a + b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a
, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(
f + g*x)^(n + 1)*((a + b*x + c*x^2)^(p + 1)/(g*(n + 1)*(c*e*f + c*d*g - b*e
*g))), x] - Dist[e*((b*e*g*(n + 1) + c*e*f*(p + 1) - c*d*g*(2*n + p + 3))/(
g*(n + 1)*(c*e*f + c*d*g - b*e*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^(n + 1
)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && N
eQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
  !IntegerQ[p] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} + \frac{\left(e\left(\tfrac{1}{2}cde^2f\right.\right.}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} - \frac{(6ae^2g-c}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} - \frac{(6ae^2g-c}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} - \frac{(6ae^2g-c}{}$$

$$= -\frac{(ef-dg)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3g(cdf-aeg)\sqrt{d+ex}\,(f+gx)^3} - \frac{(6ae^2g-c}{}$$

Mathematica [A]
time = 1.19, size = 279, normalized size = 0.79

$$\frac{c^2d^2\sqrt{d+ex}\left(\frac{\sqrt{g}\,(ae+cdx)\left(4a^2e^2g^2(2dg+e(f+3gx))-2acdeg\left(dg(13f+5gx)+e\left(8f^2+25fgx+9g^2x^2\right)\right)+c^2d^2\left(ef\left(-3f^2+8fgx+3g^2x^2\right)+dg\left(33f^2+40fgx+15g^2x^2\right)\right)\right)}{c^2d^2(cdf-aeg)^3(f+gx)^3}+\frac{3\left(-6ae^2g+cd(ef+5dg)\right)\sqrt{ae+cdx}\,\tan^{-1}\left(\frac{\sqrt{g}\,\sqrt{ae+cdx}}{\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{7/2}}\right)}{24g^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]),x]
```

```
[Out] (c^2*d^2*Sqrt[d + e*x]*((Sqrt[g]*(a*e + c*d*x)*(4*a^2*e^2*g^2*(2*d*g + e*(f
 + 3*g*x)) - 2*a*c*d*e*g*(d*g*(13*f + 5*g*x) + e*(8*f^2 + 25*f*g*x + 9*g^2*
x^2)) + c^2*d^2*(e*f*(-3*f^2 + 8*f*g*x + 3*g^2*x^2) + d*g*(33*f^2 + 40*f*g*
x + 15*g^2*x^2))))/(c^2*d^2*(c*d*f - a*e*g)^3*(f + g*x)^3) + (3*(-6*a*e^2*g
 + c*d*(e*f + 5*d*g))*Sqrt[a*e + c*d*x]*ArcTan[(Sqrt[g]*Sqrt[a*e + c*d*x])/
Sqrt[c*d*f - a*e*g]])/(c*d*f - a*e*g)^(7/2)))/(24*g^(3/2)*Sqrt[(a*e + c*d*x
)*(d + e*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(319) = 638$.
time = 0.14, size = 1132, normalized size = 3.23

method	result
default	$-\frac{\sqrt{(cdx+ae)(ex+d)}\left(-45\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^4fg^3x^2-45\,\mathrm{arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)c^3d^4f^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x,metho d=_RETURNVERBOSE)

[Out] -1/24*((c*d*x+a*e)*(e*x+d))^(1/2)*(15*c^2*d^3*g^3*x^2*(c*d*x+a*e)^(1/2)*((a *e*g-c*d*f)*g)^(1/2)-3*c^2*d^2*e*f^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1 /2)-45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^4*f*g^3*x ^2-45*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^4*f^2*g^2* x+8*a^2*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+4*a^2*e^3*f*g^2 *(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+33*c^2*d^3*f^2*g*(c*d*x+a*e)^(1/ 2)*((a*e*g-c*d*f)*g)^(1/2)+12*a^2*e^3*g^3*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-3*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*e *f*g^3*x^3-9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*e *f^2*g^2*x^2-9*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3 *e*f^3*g*x+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^ 2*e^2*f^3*g+40*c^2*d^3*f*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-50 *a*c*d*e^2*f*g^2*x*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+18*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e^2*g^4*x^3+54*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e^2*f*g^3*x^2+54*arc tanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*a*c^2*d^2*e^2*f^2*g^2*x-1 8*a*c*d*e^2*g^3*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+3*c^2*d^2*e*f *g^2*x^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-10*a*c*d^2*e*g^3*x*(c*d* x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+8*c^2*d^2*e*f^2*g*x*(c*d*x+a*e)^(1/2)* ((a*e*g-c*d*f)*g)^(1/2)-26*a*c*d^2*e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f) *g)^(1/2)-16*a*c*d*e^2*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-15*a rctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^4*g^4*x^3-15*arct anh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^4*f^3*g-3*arctanh(g* (c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^3*d^3*e*f^4)/(e*x+d)^(1/2)/((a *e*g-c*d*f)*g)^(1/2)/(g*x+f)^3/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c ^2*d^2*f^2)/(c*d*x+a*e)^(1/2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x , algorithm="maxima")

```
[Out] integrate((x*e + d)^(3/2)/(sqrt(c*d*x^2*e + a*d*e + (c*d^2 + a*e^2)*x)*(g*x
 + f)^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(330) = 660$.
time = 3.74, size = 2867, normalized size = 8.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="fricas")
```

```
[Out] [-1/48*(3*(5*c^3*d^5*g^4*x^3 + 15*c^3*d^5*f*g^3*x^2 + 15*c^3*d^5*f^2*g^2*x
+ 5*c^3*d^5*f^3*g - 6*(a*c^2*d^2*g^4*x^4 + 3*a*c^2*d^2*f*g^3*x^3 + 3*a*c^2*
d^2*f^2*g^2*x^2 + a*c^2*d^2*f^3*g*x)*e^3 + (c^3*d^3*f*g^3*x^4 - 6*a*c^2*d^3
*f^3*g + 3*(c^3*d^3*f^2*g^2 - 2*a*c^2*d^3*g^4)*x^3 + 3*(c^3*d^3*f^3*g - 6*a
*c^2*d^3*f*g^3)*x^2 + (c^3*d^3*f^4 - 18*a*c^2*d^3*f^2*g^2)*x)*e^2 + (5*c^3*
d^4*g^4*x^4 + 16*c^3*d^4*f*g^3*x^3 + 18*c^3*d^4*f^2*g^2*x^2 + 8*c^3*d^4*f^3
*g*x + c^3*d^4*f^4)*e)*sqrt(-c*d*f*g + a*g^2*e)*log(-(c*d^2*g*x - c*d^2*f +
 2*a*g*x*e^2 + (c*d*g*x^2 - c*d*f*x + 2*a*d*g)*e - 2*sqrt(-c*d*f*g + a*g^2*
e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(d*g*x + d*f
+ (g*x^2 + f*x)*e)) - 2*(15*c^3*d^4*f*g^4*x^2 + 40*c^3*d^4*f^2*g^3*x + 33*c
^3*d^4*f^3*g^2 - 4*(3*a^3*g^5*x + a^3*f*g^4)*e^4 + 2*(9*a^2*c*d*g^5*x^2 + 3
1*a^2*c*d*f*g^4*x + 10*a^2*c*d*f^2*g^3 - 4*a^3*d*g^5)*e^3 - (21*a*c^2*d^2*f
*g^4*x^2 + 13*a*c^2*d^2*f^3*g^2 - 34*a^2*c*d^2*f*g^4 + 2*(29*a*c^2*d^2*f^2*
g^3 - 5*a^2*c*d^2*g^5)*x)*e^2 - (3*c^3*d^3*f^4*g + 59*a*c^2*d^3*f^2*g^3 - 3
*(c^3*d^3*f^2*g^3 - 5*a*c^2*d^3*g^5)*x^2 - 2*(4*c^3*d^3*f^3*g^2 - 25*a*c^2*
d^3*f*g^4)*x)*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))
/(c^4*d^5*f^4*g^5*x^3 + 3*c^4*d^5*f^5*g^4*x^2 + 3*c^4*d^5*f^6*g^3*x + c^4*d
^5*f^7*g^2 + (a^4*g^9*x^4 + 3*a^4*f*g^8*x^3 + 3*a^4*f^2*g^7*x^2 + a^4*f^3*g
^6*x)*e^5 - (4*a^3*c*d*f*g^8*x^4 - a^4*d*f^3*g^6 + (12*a^3*c*d*f^2*g^7 - a^
4*d*g^9)*x^3 + 3*(4*a^3*c*d*f^3*g^6 - a^4*d*f*g^8)*x^2 + (4*a^3*c*d*f^4*g^5
 - 3*a^4*d*f^2*g^7)*x)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^7*x^4 - 2*a^3*c*d^2*f^4
*g^5 + (9*a^2*c^2*d^2*f^3*g^6 - 2*a^3*c*d^2*f*g^8)*x^3 + 3*(3*a^2*c^2*d^2*f
^4*g^5 - 2*a^3*c*d^2*f^2*g^7)*x^2 + 3*(a^2*c^2*d^2*f^5*g^4 - 2*a^3*c*d^2*f^
3*g^6)*x)*e^3 - 2*(2*a*c^3*d^3*f^3*g^6*x^4 - 3*a^2*c^2*d^3*f^5*g^4 + 3*(2*a
*c^3*d^3*f^4*g^5 - a^2*c^2*d^3*f^2*g^7)*x^3 + 3*(2*a*c^3*d^3*f^5*g^4 - 3*a^
2*c^2*d^3*f^3*g^6)*x^2 + (2*a*c^3*d^3*f^6*g^3 - 9*a^2*c^2*d^3*f^4*g^5)*x)*e
^2 + (c^4*d^4*f^4*g^5*x^4 - 4*a*c^3*d^4*f^6*g^3 + (3*c^4*d^4*f^5*g^4 - 4*a*
c^3*d^4*f^3*g^6)*x^3 + 3*(c^4*d^4*f^6*g^3 - 4*a*c^3*d^4*f^4*g^5)*x^2 + (c^4
*d^4*f^7*g^2 - 12*a*c^3*d^4*f^5*g^4)*x)*e), -1/24*(3*(5*c^3*d^5*g^4*x^3 + 1
5*c^3*d^5*f*g^3*x^2 + 15*c^3*d^5*f^2*g^2*x + 5*c^3*d^5*f^3*g - 6*(a*c^2*d^2
*g^4*x^4 + 3*a*c^2*d^2*f*g^3*x^3 + 3*a*c^2*d^2*f^2*g^2*x^2 + a*c^2*d^2*f^3*
g*x)*e^3 + (c^3*d^3*f*g^3*x^4 - 6*a*c^2*d^3*f^3*g + 3*(c^3*d^3*f^2*g^2 - 2*
```

```
a*c^2*d^3*g^4)*x^3 + 3*(c^3*d^3*f^3*g - 6*a*c^2*d^3*f*g^3)*x^2 + (c^3*d^3*f
^4 - 18*a*c^2*d^3*f^2*g^2)*x)*e^2 + (5*c^3*d^4*g^4*x^4 + 16*c^3*d^4*f*g^3*x
^3 + 18*c^3*d^4*f^2*g^2*x^2 + 8*c^3*d^4*f^3*g*x + c^3*d^4*f^4)*e)*sqrt(c*d*
f*g - a*g^2*e)*arctan(sqrt(c*d*f*g - a*g^2*e)*sqrt(c*d^2*x + a*x*e^2 + (c*d
*x^2 + a*d)*e)*sqrt(x*e + d)/(c*d^2*g*x + a*g*x*e^2 + (c*d*g*x^2 + a*d*g)*e
)) - (15*c^3*d^4*f*g^4*x^2 + 40*c^3*d^4*f^2*g^3*x + 33*c^3*d^4*f^3*g^2 - 4*
(3*a^3*g^5*x + a^3*f*g^4)*e^4 + 2*(9*a^2*c*d*g^5*x^2 + 31*a^2*c*d*f*g^4*x +
 10*a^2*c*d*f^2*g^3 - 4*a^3*d*g^5)*e^3 - (21*a*c^2*d^2*f*g^4*x^2 + 13*a*c^2
*d^2*f^3*g^2 - 34*a^2*c*d^2*f*g^4 + 2*(29*a*c^2*d^2*f^2*g^3 - 5*a^2*c*d^2*g
^5)*x)*e^2 - (3*c^3*d^3*f^4*g + 59*a*c^2*d^3*f^2*g^3 - 3*(c^3*d^3*f^2*g^3 -
 5*a*c^2*d^3*g^5)*x^2 - 2*(4*c^3*d^3*f^3*g^2 - 25*a*c^2*d^3*f*g^4)*x)*e)*sq
rt(c*d^2*x + a*x*e^2 + (c*d*x^2 + a*d)*e)*sqrt(x*e + d))/(c^4*d^5*f^4*g^5*x
^3 + 3*c^4*d^5*f^5*g^4*x^2 + 3*c^4*d^5*f^6*g^3*x + c^4*d^5*f^7*g^2 + (a^4*g
^9*x^4 + 3*a^4*f*g^8*x^3 + 3*a^4*f^2*g^7*x^2 + a^4*f^3*g^6*x)*e^5 - (4*a^3*
c*d*f*g^8*x^4 - a^4*d*f^3*g^6 + (12*a^3*c*d*f^2*g^7 - a^4*d*g^9)*x^3 + 3*(4
*a^3*c*d*f^3*g^6 - a^4*d*f*g^8)*x^2 + (4*a^3*c*d*f^4*g^5 - 3*a^4*d*f^2*g^7)
*x)*e^4 + 2*(3*a^2*c^2*d^2*f^2*g^7*x^4 - 2*a^3*c*d^2*f^4*g^5 + (9*a^2*c^2*d
^2*f^3*g^6 - 2*a^3*c*d^2*f*g^8)*x^3 + 3*(3*a^2*c^2*d^2*f^4*g^5 - 2*a^3*c*d^
2*f^2*g^7)*x^2 + 3*(a^2*c^2*d^2*f^5*g^4 - 2*a^3*c*d^2*f^3*g^6)*x)*e^3 - 2*(
2*a*c^3*d^3*f^3*g^6*x^4 - 3*a^2*c^2*d^3*f^5*g^4 + 3*(2*a*c^3*d^3*f^4*g^5 -
a^2*c^2*d^3*f^2*g^7)*x^3 + 3*(2*a*c^3*d^3*f^5*g^4 - 3*a^2*c^2*d^3*f^3*g^6)*
x^2 + (2*a*c^3*d^3*f^6*g^3 - 9*a^2*c^2*d^3*f^4*g^5)*x)*e^2 + (c^4*d^4*f^4*g
^5*x^4 - 4*a*c^3*d^4*f^6*g^3 + (3*c^4*d^4*f^5*g^4 - 4*a*c^3*d^4*f^3*g^6)*x^
3 + 3*(c^4*d^4*f^6*g^3 - 4*a*c^3*d^4*f^4*g^5)*x^2 + (c^4*d^4*f^7*g^2 - 12*a
*c^3*d^4*f^5*g^4)*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)), x)
```

3.792 $\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=324

$$-\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\sqrt{1-d^2x^2}}{15d^6}-\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)\,x\sqrt{1-d}}{16d^6}$$

```
[Out] 1/16*(16*a^3*d^6+24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c^3)
*arcsin(d*x)/d^7-1/15*b*(45*a^2*d^4+60*a*c*d^2+10*b^2*d^2+24*c^2)*(-d^2*x^2
+1)^(1/2)/d^6-1/16*(24*a^2*c*d^4+24*a*b^2*d^4+18*a*c^2*d^2+18*b^2*c*d^2+5*c
^3)*x*(-d^2*x^2+1)^(1/2)/d^6-1/15*b*(30*a*c*d^2+5*b^2*d^2+12*c^2)*x^2*(-d^2
*x^2+1)^(1/2)/d^4-1/24*c*(18*a*c*d^2+18*b^2*d^2+5*c^2)*x^3*(-d^2*x^2+1)^(1/
2)/d^4-3/5*b*c^2*x^4*(-d^2*x^2+1)^(1/2)/d^2-1/6*c^3*x^5*(-d^2*x^2+1)^(1/2)/
d^2
```

Rubi [A]
time = 0.58, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {913, 1829, 655, 222}

$$-\frac{b\sqrt{1-d^2x^2}\,(45a^2d^4+60acd^2+10b^2d^2+24c^2)}{15d^6}-\frac{x\sqrt{1-d^2x^2}\,(24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3)}{16d^6}+\frac{\text{ArcSin}(dx)\,(16a^3d^6+24a^2cd^4+24ab^2d^4+18ac^2d^2+18b^2cd^2+5c^3)}{16d^7}-\frac{bx^2\sqrt{1-d^2x^2}\,(30acd^2+5b^2d^2+12c^2)}{15d^4}-\frac{cx^3\sqrt{1-d^2x^2}\,(18acd^2+18b^2d^2+5c^2)}{24d^4}-\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2}-\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/15*(b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2 + 45*a^2*d^4)*Sqrt[1 - d^2*x^2])
/d^6 - ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4)
*x*Sqrt[1 - d^2*x^2])/(16*d^6) - (b*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*S
qrt[1 - d^2*x^2])/(15*d^4) - (c*(5*c^2 + 18*b^2*d^2 + 18*a*c*d^2)*x^3*Sqrt[
1 - d^2*x^2])/(24*d^4) - (3*b*c^2*x^4*Sqrt[1 - d^2*x^2])/(5*d^2) - (c^3*x^5
*Sqrt[1 - d^2*x^2])/(6*d^2) + ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*
b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x])/(16*d^7)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx = \int \frac{(a+bx+cx^2)^3}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} - \frac{\int \frac{-6a^3d^2-18a^2bd^2x-18a(b^2+ac)d^2x^2-6b(b^2+6ac)d^2x^3-c(5c^2+18b^2d^2+18\cdots}{\sqrt{1-d^2x^2}}}{6d^2}$$

$$= -\frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} + \frac{\int \frac{30a^3d^4+90a^2bd^4x+90a(b^2+ac)d^4x^2+6bd^2(12\cdots}{\sqrt{1\cdots}}}{\cdots}$$

$$= -\frac{c(5c^2+18b^2d^2+18acd^2)\,x^3\sqrt{1-d^2x^2}}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2\cdots}}{6d^2}$$

$$= -\frac{b(12c^2+5b^2d^2+30acd^2)\,x^2\sqrt{1-d^2x^2}}{15d^4} - \frac{c(5c^2+18b^2d^2+18acd^2)\,x^3\sqrt{1-\cdots}}{24d^4}$$

$$= -\frac{(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4)\,x\sqrt{1-d^2x^2}}{16d^6} - \frac{b(12c^2+5b^2\cdots}{\cdots}$$

$$= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\,\sqrt{1-d^2x^2}}{15d^6} - \frac{(5c^3+18b^2cd^2+18ac^2d^2\cdots}{\cdots}$$

$$= -\frac{b(24c^2+10b^2d^2+60acd^2+45a^2d^4)\,\sqrt{1-d^2x^2}}{15d^6} - \frac{(5c^3+18b^2cd^2+18ac^2d^2\cdots}{\cdots}$$

Mathematica [A]
time = 0.73, size = 264, normalized size = 0.81

$$\frac{-d^2\sqrt{1-d^2x^2}\,(80b^3d^2(2+d^2x^2)+90b^2d^2x(4ad^2+c(3+2d^2x^2))+48b(15a^2d^4+10acd^2(2+d^2x^2)+c^2(8+4d^2x^2+3d^4x^4))+5cx(72a^2d^4+18acd^2(3+2d^2x^2)+c^2(15+10d^2x^2+8d^4x^4)))+15\sqrt{-d^2}\,(5c^3+18b^2cd^2+18ac^2d^2+24ab^2d^4+24a^2cd^4+16a^3d^6)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{240d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d^2*Sqrt[1 - d^2*x^2]*(80*b^3*d^2*(2 + d^2*x^2) + 90*b^2*d^2*x*(4*a*d^2
+ c*(3 + 2*d^2*x^2)) + 48*b*(15*a^2*d^4 + 10*a*c*d^2*(2 + d^2*x^2) + c^2*(8
 + 4*d^2*x^2 + 3*d^4*x^4)) + 5*c*x*(72*a^2*d^4 + 18*a*c*d^2*(3 + 2*d^2*x^2)
 + c^2*(15 + 10*d^2*x^2 + 8*d^4*x^4)))) + 15*Sqrt[-d^2]*(5*c^3 + 18*b^2*c*d
^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*Log[-(Sqrt[-d
^2]*x) + Sqrt[1 - d^2*x^2]])/(240*d^8)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 602, normalized size = 1.86

method	result
risch	$\frac{(40c^3x^5d^4+144b\,c^2x^4d^4+180a\,c^2d^4x^3+180b^2c\,d^4x^3+480abc\,d^4x^2+80b^3d^4x^2+360a^2c\,d^4x+360a\,b^2d^4x+50c^3d^2x^3+720a^2b\,d^4+192b\,c^2\cdots}{240d^6\sqrt{-(dx-1)(dx+\cdots}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(40\,\mathrm{csgn}(d)c^3d^5x^5\sqrt{-d^2x^2+1}+144\,\mathrm{csgn}(d)b\,c^2d^5x^4\sqrt{-d^2x^2+1}+180\,\mathrm{csgn}(d)a\,c^2d^5x^3\sqrt{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(40*csgn(d)*c^3*d^5*x^5*(-d^2*x^2+1)^(1
/2)+144*csgn(d)*b*c^2*d^5*x^4*(-d^2*x^2+1)^(1/2)+180*csgn(d)*a*c^2*d^5*x^3*
(-d^2*x^2+1)^(1/2)+180*csgn(d)*b^2*c*d^5*x^3*(-d^2*x^2+1)^(1/2)+480*csgn(d)
*a*b*c*d^5*x^2*(-d^2*x^2+1)^(1/2)+80*csgn(d)*b^3*d^5*x^2*(-d^2*x^2+1)^(1/2)
+360*(-d^2*x^2+1)^(1/2)*csgn(d)*d^5*a^2*c*x+360*(-d^2*x^2+1)^(1/2)*csgn(d)*
d^5*a*b^2*x+50*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*c^3*x^3+720*csgn(d)*d^5*(-d^2
*x^2+1)^(1/2)*a^2*b+192*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*b*c^2*x^2-240*arctan
(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^3*d^6+270*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3
*a*c^2*x+270*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*b^2*c*x+960*csgn(d)*d^3*(-d^2*x
^2+1)^(1/2)*a*b*c+160*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*b^3-360*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*a^2*c*d^4-360*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2
))*a*b^2*d^4+75*(-d^2*x^2+1)^(1/2)*csgn(d)*d*c^3*x+384*csgn(d)*d*(-d^2*x^2+
1)^(1/2)*b*c^2-270*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*c^2*d^2-270*arc
tan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b^2*c*d^2-75*arctan(csgn(d)*d*x/(-d^2*x
^2+1)^(1/2))*c^3)*csgn(d)/d^7/(-d^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.50, size = 365, normalized size = 1.13

$$-\frac{\sqrt{-d^2x^2+1}\,c^3x^5}{6\,d^2}-\frac{3\,\sqrt{-d^2x^2+1}\,bc^2x^4}{5\,d^2}+\frac{a^3\arcsin(dx)}{d}-\frac{5\,\sqrt{-d^2x^2+1}\,c^3x^3}{24\,d^4}-\frac{3\,\sqrt{-d^2x^2+1}\,(b^2c+ac^2)x^3}{4\,d^2}-\frac{3\,\sqrt{-d^2x^2+1}\,a^2b}{d^2}-\frac{4\,\sqrt{-d^2x^2+1}\,bc^2x^2}{5\,d^4}-\frac{\sqrt{-d^2x^2+1}\,(b^3+6\,abc)x^2}{3\,d^2}-\frac{3\,\sqrt{-d^2x^2+1}\,(ab^2+a^2c)x}{2\,d^2}+\frac{3\,(ab^2+a^2c)\arcsin(dx)}{2\,d^3}-\frac{5\,\sqrt{-d^2x^2+1}\,c^3x}{16\,d^6}-\frac{9\,\sqrt{-d^2x^2+1}\,(b^2c+ac^2)x}{8\,d^4}-\frac{8\,\sqrt{-d^2x^2+1}\,bc^2}{5\,d^6}-\frac{2\,\sqrt{-d^2x^2+1}\,(b^3+6\,abc)}{3\,d^4}+\frac{5\,c^3\arcsin(dx)}{16\,d^7}+\frac{9\,(b^2c+ac^2)\arcsin(dx)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*sqrt(-d^2*x^2 + 1)*c^3*x^5/d^2 - 3/5*sqrt(-d^2*x^2 + 1)*b*c^2*x^4/d^2
+ a^3*arcsin(d*x)/d - 5/24*sqrt(-d^2*x^2 + 1)*c^3*x^3/d^4 - 3/4*sqrt(-d^2*x
^2 + 1)*(b^2*c + a*c^2)*x^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*a^2*b/d^2 - 4/5*sqrt
(-d^2*x^2 + 1)*b*c^2*x^2/d^4 - 1/3*sqrt(-d^2*x^2 + 1)*(b^3 + 6*a*b*c)*x^2/d
^2 - 3/2*sqrt(-d^2*x^2 + 1)*(a*b^2 + a^2*c)*x/d^2 + 3/2*(a*b^2 + a^2*c)*arc
sin(d*x)/d^3 - 5/16*sqrt(-d^2*x^2 + 1)*c^3*x/d^6 - 9/8*sqrt(-d^2*x^2 + 1)*(
b^2*c + a*c^2)*x/d^4 - 8/5*sqrt(-d^2*x^2 + 1)*b*c^2/d^6 - 2/3*sqrt(-d^2*x^2
 + 1)*(b^3 + 6*a*b*c)/d^4 + 5/16*c^3*arcsin(d*x)/d^7 + 9/8*(b^2*c + a*c^2)*
arcsin(d*x)/d^5
```

Fricas [**A**]
time = 1.72, size = 251, normalized size = 0.77

$$-\frac{(40\,c^3d^5x^5 + 144\,bc^2d^5x^4 + 720\,a^2bd^5 + 384\,bc^2d + 160\,(b^3 + 6\,abc)d^3 + 10\,(5\,c^3d^3 + 18\,(b^2c + ac^2)d^5)x^3 + 16\,(12\,bc^2d^3 + 5\,(b^3 + 6\,abc)d^5)x^2 + 15\,(24\,(ab^2 + a^2c)d^5 + 5\,c^3d + 18\,(b^2c + ac^2)d^3)x)\sqrt{dx + 1}\,\sqrt{-dx + 1} + 30\,(16\,a^3d^6 + 24\,(ab^2 + a^2c)d^4 + 5\,c^3 + 18\,(b^2c + ac^2)d^2)\arctan\left(\frac{\sqrt{dx + 1}\,\sqrt{-dx + 1} - 1}{dx}\right)}{240\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*((40*c^3*d^5*x^5 + 144*b*c^2*d^5*x^4 + 720*a^2*b*d^5 + 384*b*c^2*d +
 160*(b^3 + 6*a*b*c)*d^3 + 10*(5*c^3*d^3 + 18*(b^2*c + a*c^2)*d^5)*x^3 + 16
*(12*b*c^2*d^3 + 5*(b^3 + 6*a*b*c)*d^5)*x^2 + 15*(24*(a*b^2 + a^2*c)*d^5 +
5*c^3*d + 18*(b^2*c + a*c^2)*d^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(16*
a^3*d^6 + 24*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 18*(b^2*c + a*c^2)*d^2)*arctan((
sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^7
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.79, size = 353, normalized size = 1.09

$$-\frac{(720\,a^2bd^5 - 360\,ab^2d^4 - 360\,a^2cd^4 + 240\,b^3d^3 + 1440\,abcd^3 - 450\,b^2cd^2 - 450\,ac^2d^2 + 720\,bc^2d - 165\,c^3 + (360\,ab^2d^4 + 360\,a^2cd^4 - 160\,b^3d^3 - 960\,abcd^3 + 810\,b^2cd^2 + 810\,ac^2d^2 - 960\,bc^2d + 425\,c^3 + 2\,(40\,b^3d^3 + 240\,abcd^3 - 270\,b^2cd^2 - 270\,ac^2d^2 + 528\,bc^2d - 275\,c^3 + (90\,b^2cd^2 + 90\,ac^2d^2 - 288\,bc^2d + 225\,c^3 + 4\,(5\,(dx + 1)c^3 + 18\,bc^2d - 25\,c^3)(dx + 1))(dx + 1))(dx + 1))(dx + 1))\sqrt{dx + 1}\,\sqrt{-dx + 1} - 30\,(16\,a^3d^6 + 24\,ab^2d^4 + 24\,a^2cd^4 + 18\,b^2cd^2 + 18\,ac^2d^2 + 5\,c^3)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx + 1}\right)}{240\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/240*((720*a^2*b*d^5 - 360*a*b^2*d^4 - 360*a^2*c*d^4 + 240*b^3*d^3 + 1440
*a*b*c*d^3 - 450*b^2*c*d^2 - 450*a*c^2*d^2 + 720*b*c^2*d - 165*c^3 + (360*a
*b^2*d^4 + 360*a^2*c*d^4 - 160*b^3*d^3 - 960*a*b*c*d^3 + 810*b^2*c*d^2 + 81
0*a*c^2*d^2 - 960*b*c^2*d + 425*c^3 + 2*(40*b^3*d^3 + 240*a*b*c*d^3 - 270*b
^2*c*d^2 - 270*a*c^2*d^2 + 528*b*c^2*d - 275*c^3 + (90*b^2*c*d^2 + 90*a*c^2
*d^2 - 288*b*c^2*d + 225*c^3 + 4*(5*(d*x + 1)*c^3 + 18*b*c^2*d - 25*c^3)*(d
*x + 1))*(d*x + 1))*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30
*(16*a^3*d^6 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 18*b^2*c*d^2 + 18*a*c^2*d^2 +
5*c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^7
```

Mupad [B]
time = 31.33, size = 1768, normalized size = 5.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^23*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6
*a^2*c*d^4 + (9*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2)
 - 1)*((5*c^3)/4 + 6*a*b^2*d^4 + (9*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (9*b^2*c*d
^2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^3*((175*c^3)/12 + 6*
a*b^2*d^4 + (105*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1
)^(1/2) - 1)^3 + (((1 - d*x)^(1/2) - 1)^21*((175*c^3)/12 + 6*a*b^2*d^4 + (1
05*a*c^2*d^2)/2 + 6*a^2*c*d^4 + (105*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^2
1 + (((1 - d*x)^(1/2) - 1)^5*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)
/2 + 126*a^2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d
*x)^(1/2) - 1)^19*(126*a*b^2*d^4 - (311*c^3)/4 + (669*a*c^2*d^2)/2 + 126*a^
2*c*d^4 + (669*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^19 + (((1 - d*x)^(1/2)
- 1)^7*((8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c*d^4 +
 (1533*b^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^17*(
(8361*c^3)/4 + 510*a*b^2*d^4 + (1533*a*c^2*d^2)/2 + 510*a^2*c*d^4 + (1533*b
^2*c*d^2)/2))/((d*x + 1)^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^11*((25295*
c^3)/2 + 420*a*b^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/((
d*x + 1)^(1/2) - 1)^11 - (((1 - d*x)^(1/2) - 1)^13*((25295*c^3)/2 + 420*a*b
^2*d^4 - 549*a*c^2*d^2 + 420*a^2*c*d^4 - 549*b^2*c*d^2))/((d*x + 1)^(1/2) -
 1)^13 - (((1 - d*x)^(1/2) - 1)^9*((42259*c^3)/6 - 804*a*b^2*d^4 + 165*a*c^
2*d^2 - 804*a^2*c*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*
x)^(1/2) - 1)^15*((42259*c^3)/6 - 804*a*b^2*d^4 + 165*a*c^2*d^2 - 804*a^2*c
*d^4 + 165*b^2*c*d^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^6*
((1024*b^3*d^3)/3 + 1080*a^2*b*d^5 + 2048*b*c^2*d + 2048*a*b*c*d^3))/((d*x
+ 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^18*((1024*b^3*d^3)/3 + 1080*a^2*
b*d^5 + 2048*b*c^2*d + 2048*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^18 + (((1 - d
*x)^(1/2) - 1)^10*(1024*b^3*d^3 + 5040*a^2*b*d^5 + (6144*b*c^2*d)/5 + 6144*
a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)^14*(1024*b^3*
```

d^3 + 5040*a^2*b*d^5 + (6144*b*c^2*d)/5 + 6144*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)^(1/2) - 1)^12*((3200*b^3*d^3)/3 + 6048*a^2*b*d^5 + (32768*b*c^2*d)/5 + 6400*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^4*(64*b^3*d^3 + 240*a^2*b*d^5 + 384*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^20*(64*b^3*d^3 + 240*a^2*b*d^5 + 384*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^20 + (((1 - d*x)^(1/2) - 1)^8*(768*b^3*d^3 + 2880*a^2*b*d^5 + 4608*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2) - 1)^16*(768*b^3*d^3 + 2880*a^2*b*d^5 + 4608*a*b*c*d^3))/((d*x + 1)^(1/2) - 1)^16 + (24*a^2*b*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (24*a^2*b*d^5*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22)/(d^7 + (12*d^7*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (66*d^7*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (220*d^7*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (495*d^7*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (792*d^7*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (924*d^7*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (792*d^7*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (495*d^7*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (220*d^7*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (66*d^7*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (12*d^7*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22 + (d^7*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24) - (atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(5*c^3 + 16*a^3*d^6 + 24*a*b^2*d^4 + 18*a*c^2*d^2 + 24*a^2*c*d^4 + 18*b^2*c*d^2))/(4*d^7)

3.793 $\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=166

$$-\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4}-\frac{\left(4b^2+c\left(8a+\frac{3c}{d^2}\right)\right)x\sqrt{1-d^2x^2}}{8d^2}-\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}+\frac{(3c^2+$$

```
[Out] 1/8*(8*a^2*d^4+8*a*c*d^2+4*b^2*d^2+3*c^2)*arcsin(d*x)/d^5-2/3*b*(3*a*d^2+2*
c)*(-d^2*x^2+1)^(1/2)/d^4-1/8*(4*b^2+c*(8*a+3*c/d^2))*x*(-d^2*x^2+1)^(1/2)/
d^2-2/3*b*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/4*c^2*x^3*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A]
time = 0.21, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {913, 1829, 655, 222}

$$\frac{\text{ArcSin}(dx)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{8d^5}-\frac{x\sqrt{1-d^2x^2}\left(c\left(8a+\frac{3c}{d^2}\right)+4b^2\right)}{8d^2}-\frac{2b\sqrt{1-d^2x^2}\left(3ad^2+2c\right)}{3d^4}-\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2}-\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-2*b*(2*c + 3*a*d^2)*Sqrt[1 - d^2*x^2])/(3*d^4) - ((4*b^2 + c*(8*a + (3*c)
/d^2))*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (2*b*c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2)
 - (c^2*x^3*Sqrt[1 - d^2*x^2])/(4*d^2) + ((3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 +
8*a^2*d^4)*ArcSin[d*x])/(8*d^5)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{(a+bx+cx^2)^2}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} - \frac{\int \frac{-4a^2d^2-8abd^2x-(3c^2+4b^2d^2+8acd^2)x^2-8bcd^2x^3}{\sqrt{1-d^2x^2}}\,dx}{4d^2}\\
&= -\frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} + \frac{\int \frac{12a^2d^4+8bd^2(2c+3ad^2)x+3d^2(3c^2+4b^2d^2+8a}{\sqrt{1-d^2x^2}}}{12d^4}\\
&= -\frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2} -\\
&= -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1}}{3a}\\
&= -\frac{2b(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{(3c^2+4b^2d^2+8acd^2)x\sqrt{1-d^2x^2}}{8d^4} - \frac{2bcx^2\sqrt{1}}{3a}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 151, normalized size = 0.91

$$\frac{\sqrt{1-d^2x^2}\,(-32bc-48abd^2-9c^2x-12b^2d^2x-24acd^2x-16bcd^2x^2-6c^2d^2x^3)}{24d^4}+\frac{\sqrt{-d^2}\,(3c^2+4b^2d^2+8acd^2+8a^2d^4)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{8d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(-32*b*c - 48*a*b*d^2 - 9*c^2*x - 12*b^2*d^2*x - 24*a*c*
d^2*x - 16*b*c*d^2*x^2 - 6*c^2*d^2*x^3))/(24*d^4) + (Sqrt[-d^2]*(3*c^2 + 4*
b^2*d^2 + 8*a*c*d^2 + 8*a^2*d^4)*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/
(8*d^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 291, normalized size = 1.75

method	result

risch	$\frac{(6c^2x^3d^2+16bc\,x^2d^2+24ac\,d^2x+12b^2d^2x+48ab\,d^2+9c^2x+32bc)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)\,(dx+1)}}{24d^4\sqrt{-(dx-1)\,(dx+1)}\,\sqrt{-dx+1}}+\frac{\Bigg(\frac{\arctan\Big(\frac{}{\sqrt{}}}{}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\Big(6\,\mathrm{csgn}(d)c^2d^3x^3\sqrt{-d^2x^2+1}+16\,\mathrm{csgn}(d)bc\,d^3x^2\sqrt{-d^2x^2+1}+24\sqrt{-d^2x^2+1}\,\mathrm{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*csgn(d)*c^2*d^3*x^3*(-d^2*x^2+1)^(1/2
)+16*csgn(d)*b*c*d^3*x^2*(-d^2*x^2+1)^(1/2)+24*(-d^2*x^2+1)^(1/2)*csgn(d)*d
^3*a*c*x+12*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*b^2*x+48*csgn(d)*d^3*(-d^2*x^2+1
)^(1/2)*a*b-24*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^2*d^4+9*(-d^2*x^2+1
)^(1/2)*csgn(d)*d*c^2*x+32*csgn(d)*d*(-d^2*x^2+1)^(1/2)*b*c-24*arctan(csgn(
d)*d*x/(-d^2*x^2+1)^(1/2))*a*c*d^2-12*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)
)*b^2*d^2-9*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c^2)*csgn(d)/d^5/(-d^2*x
^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 171, normalized size = 1.03

$$-\frac{\sqrt{-d^2x^2+1}\,c^2x^3}{4\,d^2}-\frac{2\,\sqrt{-d^2x^2+1}\,bcx^2}{3\,d^2}+\frac{a^2\arcsin(dx)}{d}-\frac{2\,\sqrt{-d^2x^2+1}\,ab}{d^2}-\frac{\sqrt{-d^2x^2+1}\,(b^2+2\,ac)x}{2\,d^2}-\frac{3\,\sqrt{-d^2x^2+1}\,c^2x}{8\,d^4}+\frac{(b^2+2\,ac)\arcsin(dx)}{2\,d^3}-\frac{4\,\sqrt{-d^2x^2+1}\,bc}{3\,d^4}+\frac{3\,c^2\arcsin(dx)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-d^2*x^2 + 1)*c^2*x^3/d^2 - 2/3*sqrt(-d^2*x^2 + 1)*b*c*x^2/d^2 +
a^2*arcsin(d*x)/d - 2*sqrt(-d^2*x^2 + 1)*a*b/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(
b^2 + 2*a*c)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*c^2*x/d^4 + 1/2*(b^2 + 2*a*c)*a
rcsin(d*x)/d^3 - 4/3*sqrt(-d^2*x^2 + 1)*b*c/d^4 + 3/8*c^2*arcsin(d*x)/d^5
```

Fricas [A]
time = 2.40, size = 134, normalized size = 0.81

$$-\frac{(6\,c^2d^3x^3+16\,bcd^3x^2+48\,abd^3+32\,bcd+3\,(4\,(b^2+2\,ac)d^3+3\,c^2d)x)\sqrt{dx+1}\,\sqrt{-dx+1}+6\,(8\,a^2d^4+4\,(b^2+2\,ac)d^2+3\,c^2)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{24\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

[Out] -1/24*((6*c^2*d^3*x^3 + 16*b*c*d^3*x^2 + 48*a*b*d^3 + 32*b*c*d + 3*(4*(b^2 + 2*a*c)*d^3 + 3*c^2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*a^2*d^4 + 4*(b^2 + 2*a*c)*d^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^5

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)

[Out] Timed out

Giac [**A**]
time = 6.26, size = 159, normalized size = 0.96

$$-\frac{\left(48\,abd^{3}-12\,b^{2}d^{2}-24\,acd^{2}+48\,bcd+\left(12\,b^{2}d^{2}+24\,acd^{2}-32\,bcd+2\left(3\left(dx+1\right)c^{2}+8\,bcd-9\,c^{2}\right)\left(dx+1\right)+27\,c^{2}\right)\left(dx+1\right)-15\,c^{2}\right)\sqrt{dx+1}\,\sqrt{-dx+1}\,-6\left(8\,a^{2}d^{4}+4\,b^{2}d^{2}+8\,acd^{2}+3\,c^{2}\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{24\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")

[Out] -1/24*((48*a*b*d^3 - 12*b^2*d^2 - 24*a*c*d^2 + 48*b*c*d + (12*b^2*d^2 + 24*a*c*d^2 - 32*b*c*d + 2*(3*(d*x + 1)*c^2 + 8*b*c*d - 9*c^2)*(d*x + 1) + 27*c^2)*(d*x + 1) - 15*c^2)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(8*a^2*d^4 + 4*b^2*d^2 + 8*a*c*d^2 + 3*c^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^5

Mupad [**B**]
time = 13.85, size = 897, normalized size = 5.40

$$-\frac{\frac{\left(\sqrt{1-dx}-1\right)^{15}\left(2b^2d^2+\frac{3c^2}{2}+4acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{15}}+\frac{\left(\sqrt{1-dx}-1\right)^{3}\left(6b^2d^2-\frac{23c^2}{2}+12acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{3}}-\frac{\left(\sqrt{1-dx}-1\right)^{13}\left(6b^2d^2-\frac{23c^2}{2}+12acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{13}}+\frac{\left(\sqrt{1-dx}-1\right)^{5}\left(30b^2d^2+\frac{333c^2}{2}+60acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{5}}-\frac{\left(\sqrt{1-dx}-1\right)^{11}\left(30b^2d^2+\frac{333c^2}{2}+60acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{11}}+\frac{\left(\sqrt{1-dx}-1\right)^{7}\left(22b^2d^2-\frac{671c^2}{2}+44acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{7}}-\frac{\left(\sqrt{1-dx}-1\right)^{9}\left(22b^2d^2-\frac{671c^2}{2}+44acd^2\right)}{\left(\sqrt{dx+1}-1\right)^{9}}+\frac{\left(\sqrt{1-dx}-1\right)^{4}\left(96abd^3+128bcd\right)}{\left(\sqrt{dx+1}-1\right)^{4}}+\frac{\left(\sqrt{1-dx}-1\right)^{12}\left(96abd^3+128bcd\right)}{\left(\sqrt{dx+1}-1\right)^{12}}+\frac{\left(\sqrt{1-dx}-1\right)^{8}\left(320abd^3+\frac{256bcd}{3}\right)}{\left(\sqrt{dx+1}-1\right)^{8}}+\frac{\left(\sqrt{1-dx}-1\right)^{6}\left(240abd^3+\frac{512bcd}{3}\right)}{\left(\sqrt{dx+1}-1\right)^{6}}+\frac{\left(\sqrt{1-dx}-1\right)^{10}\left(240abd^3+\frac{512bcd}{3}\right)}{\left(\sqrt{dx+1}-1\right)^{10}}-\frac{\left(\sqrt{1-dx}-1\right)\left(2b^2d^2+\frac{3c^2}{2}+4acd^2\right)}{\sqrt{dx+1}-1}+\frac{16abd^3\left(\sqrt{1-dx}-1\right)^{2}}{\left(\sqrt{dx+1}-1\right)^{2}}+\frac{16abd^3\left(\sqrt{1-dx}-1\right)^{14}}{\left(\sqrt{dx+1}-1\right)^{14}}}{d^5+\frac{8d^5\left(\sqrt{1-dx}-1\right)^{2}}{\left(\sqrt{dx+1}-1\right)^{2}}+\frac{28d^5\left(\sqrt{1-dx}-1\right)^{4}}{\left(\sqrt{dx+1}-1\right)^{4}}+\frac{56d^5\left(\sqrt{1-dx}-1\right)^{6}}{\left(\sqrt{dx+1}-1\right)^{6}}+\frac{70d^5\left(\sqrt{1-dx}-1\right)^{8}}{\left(\sqrt{dx+1}-1\right)^{8}}+\frac{56d^5\left(\sqrt{1-dx}-1\right)^{10}}{\left(\sqrt{dx+1}-1\right)^{10}}+\frac{28d^5\left(\sqrt{1-dx}-1\right)^{12}}{\left(\sqrt{dx+1}-1\right)^{12}}+\frac{8d^5\left(\sqrt{1-dx}-1\right)^{14}}{\left(\sqrt{dx+1}-1\right)^{14}}+\frac{d^5\left(\sqrt{1-dx}-1\right)^{16}}{\left(\sqrt{dx+1}-1\right)^{16}}}-\frac{\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)\left(8a^2d^4+8acd^2+4b^2d^2+3c^2\right)}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^2/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)

[Out] - ((((1 - d*x)^(1/2) - 1)^15*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^3*(6*b^2*d^2 - (23*c^2)/2 + 12*a*c*d^2))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^13*(6*b^2*d^2 - (23*c^2)/2 + 12*a*c*d^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d*x)^(1/2) - 1)^11*((333*c^2)/2 + 30*b^2*d^2 + 60*a*c*d^2))/((d*x + 1)^(1/2) - 1)^11 + (((1 - d*x)^(1/2) - 1)^7*(22*b^2*d^2 - (671*c^2)/2 + 44*a*c*d^2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^9*(22*b^2*d^2 - (671*c^2)/2 + 44*a*c*d^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1)^4*(128*b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^12*(1

28*b*c*d + 96*a*b*d^3))/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^8 *((256*b*c*d)/3 + 320*a*b*d^3))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2) - 1)^6*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^10*((512*b*c*d)/3 + 240*a*b*d^3))/((d*x + 1)^(1/2) - 1)^10 - (((1 - d*x)^(1/2) - 1)*((3*c^2)/2 + 2*b^2*d^2 + 4*a*c*d^2))/((d*x + 1)^(1/2) - 1) + (16*a*b*d^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (16* a*b*d^3*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^5 + (8*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (70*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (56*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1) ^(1/2) - 1)^16) - (atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(3*c^2 + 8*a^2*d^4 + 4*b^2*d^2 + 8*a*c*d^2))/(2*d^5)

3.794 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {913, 1829, 655, 222}

$$\frac{(2ad^2+c)\,\text{ArcSin}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
```

```
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{cx\sqrt{1-d^2x^2}}{2d^2}-\frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\ &= -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}-\frac{(-c-2ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\ &= -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 82, normalized size = 1.30

$$\frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2}+\frac{\sqrt{-d^2}\,(c+2ad^2)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + (Sqrt[-d^2]*(c + 2*a*d^2)*Log[-(
Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(2*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 117, normalized size = 1.86

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(\mathrm{csgn}(d)d\sqrt{-d^2x^2+1}\,cx-2\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)a\,d^2+2\,\mathrm{csgn}(d)d\sqrt{-d^2x^2+1}\,b-\right.}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(cx+2b)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)a}{\sqrt{d^2}}+\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)c}{2d^2\sqrt{d^2}}\right)\sqrt{}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(csgn(d)*d*(-d^2*x^2+1)^(1/2)*c*x-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Maxima [A]
time = 0.49, size = 57, normalized size = 0.90

$$\frac{a\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}\,cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}\,b}{d^2} + \frac{c\arcsin(dx)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

Fricas [A]
time = 2.20, size = 67, normalized size = 1.06

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.60, size = 60, normalized size = 0.95

$$-\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\,\sqrt{-dx+1}-2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2 +
 c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B]
time = 7.76, size = 232, normalized size = 3.68

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

3.795 $\int \frac{1}{\sqrt{1-dx}\,\sqrt{1+dx}\,(a+bx+cx^2)}\,dx$

Optimal. Leaf size=282

$$-\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\,\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\,\sqrt{1-d^2x^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}}+\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{}{\sqrt{2}\,\sqrt{2c^2+2a}}\right.}{\sqrt{b^2-4ac}\,\sqrt{2c^2+}}$$

```
[Out] -c*arctanh(1/2*(2*c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2
)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(-4*a*c+b^2
)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+c*arctanh(1/2*
(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*
d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+
2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {913, 999, 739, 212}

$$\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\,\sqrt{1-d^2x^2}\,\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}-\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\,\sqrt{1-d^2x^2}\,\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] -((Sqrt[2]*c*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*
c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2])) + (
Sqrt[2]*c*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2
 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[b^
2 - 4*a*c]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 999

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[2*(c/q), Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{1-dx}\,\sqrt{1+dx}\,(a+bx+cx^2)}\,dx &= \int \frac{1}{(a+bx+cx^2)\sqrt{1-d^2x^2}}\,dx \\
&= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{1-d^2x^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}\right.}}{\sqrt{b^2-}} \\
&= -\frac{(2c)\text{Subst}\left(\int \frac{1}{4c^2-\left(b-\sqrt{b^2-4ac}\right)^2d^2-x^2}\,dx, x, \frac{2c+\left(b-\sqrt{b^2-4ac}\right.}{\sqrt{1-d^2x^2}}\right.}{\sqrt{b^2-4ac}} \\
&= -\frac{\sqrt{2}\,c\tanh^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)d^2x}{\sqrt{2}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\,\sqrt{1}}\right.}{\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}}
\end{aligned}
$$

Mathematica [A]

time = 10.42, size = 455, normalized size = 1.61

$$
\frac{\sqrt{2}\,c\left(\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\,\log\left(-b+\sqrt{b^2-4ac}-2cx\right)-\sqrt{2c^2+2acd^2+b\left(-b+\sqrt{b^2-4ac}\right)d^2}\,\log\left(b+\sqrt{b^2-4ac}+2cx\right)-\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}\,\log\left(-2c-bd^2x+\sqrt{b^2-4ac}\,d^2x-\sqrt{4c^2+4acd^2+2b\left(-b+\sqrt{b^2-4ac}\right)d^2}\,\sqrt{1-d^2x^2}\right)+\sqrt{2c^2+2acd^2+b\left(-b+\sqrt{b^2-4ac}\right)d^2}\,\log\left(2c+bd^2x+\sqrt{b^2-4ac}\,d^2x+\sqrt{4c^2+4acd^2-2b\left(b+\sqrt{b^2-4ac}\right)d^2}\,\sqrt{1-d^2x^2}\right)\right)}{\sqrt{b^2-4ac}\,\sqrt{2c^2+2acd^2+b\left(-b+\sqrt{b^2-4ac}\right)d^2}\,\sqrt{2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (Sqrt[2]*c*(Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x] - Sqrt[2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 -
4*a*c])*d^2]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x] - Sqrt[2*c^2 + 2*a*c*d^2 -
b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Log[-2*c - b*d^2*x + Sqrt[b^2 - 4*a*c]*d^2*x
 - Sqrt[4*c^2 + 4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*
x^2]] + Sqrt[2*c^2 + 2*a*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Log[2*c +
b*d^2*x + Sqrt[b^2 - 4*a*c]*d^2*x + Sqrt[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[
b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2]]))/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 + 2*a
*c*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sq
rt[b^2 - 4*a*c])*d^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 1759, normalized size = 6.24

method	result	size
default	Expression too large to display	1759

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 32*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*c^2*(ln(2*((-4*a*c+b^2)^(1/2)*d^2
*x+b*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2
+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2
)^(1/2)))*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+
b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)-ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*d^2*x
+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*
c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))
*a^2*d^4*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)
+2*a*c-b^2)/a^2/c^2)^(1/2)+2*ln(2*((-4*a*c+b^2)^(1/2)*d^2*x+b*d^2*x+(-d^2*x
^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1
/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*a*c*d^2*
(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b
^2)/a^2/c^2)^(1/2)-ln(2*((-4*a*c+b^2)^(1/2)*d^2*x+b*d^2*x+(-d^2*x^2+1)^(1/2
)*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-
b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))*b^2*d^2*(-(b*(-4*a
*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^
2)^(1/2)-2*ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(
-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^
2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))*a*c*d^2*(-(b*(-4*a*c+b^2)
^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)
+ln(2*(b*d^2*x-(-4*a*c+b^2)^(1/2)*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2
)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/
```

```
2)*c+2*c)/(b+2*c*x-(-4*a*c+b^2)^(1/2)))*b^2*d^2*(-(b*(-4*a*c+b^2)^(1/2)-2*a
*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)+ln(2*((-4
*a*c+b^2)^(1/2)*d^2*x+b*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)-2*
a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2)*c+2*c)/(
b+2*c*x+(-4*a*c+b^2)^(1/2)))*c^2*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2
*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)-ln(2*(b*d^2*x-(-4*a*c+b
^2)^(1/2)*d^2*x+(-d^2*x^2+1)^(1/2)*(-(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)*(-2*a
^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2)^(1/2)*c+2*c)/(b+2*c*x-(-4*a
*c+b^2)^(1/2)))*c^2*(-(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c
+b^2)^(1/2)+2*a*c-b^2)/a^2/c^2)^(1/2))/(-d^2*x^2+1)^(1/2)/(b*d-d*(-4*a*c+b^
2)^(1/2)-2*c)/(b*d-d*(-4*a*c+b^2)^(1/2)+2*c)/(-4*a*c+b^2)^(1/2)/(-(b*(-4*a*
c+b^2)^(1/2)+2*a*c-b^2)*(-2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a^2/c^2
)^(1/2)/(d*(-4*a*c+b^2)^(1/2)+b*d-2*c)/(d*(-4*a*c+b^2)^(1/2)+b*d+2*c)/(-(b*
(-4*a*c+b^2)^(1/2)-2*a*c+b^2)*(2*a^2*d^2+b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/a^
2/c^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4313 vs. $2(248) = 496$.
time = 2.60, size = 4313, normalized size = 15.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b
^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^
2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*
a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 -
6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^
3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)
*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*
c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((
a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^
```

4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^
6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c
- 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2
*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b
^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*
c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(
-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3
- (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8
- 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*
a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 -
((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^
2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c +
8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24
*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*
a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*log((4
*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 + 2*(
b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*
a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^
2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^
2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - sqrt(
2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c
+ 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*sqrt(b^2*d^4
/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2
*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b
^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3
*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^
3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*
d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 +
b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2
*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*
c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)))/x) - 1/2*sqrt(2)*sqrt(
-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3
- (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8
- 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*
a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d^2)))/((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d^2))*log((4*sqrt(d*x + 1)*sqrt(-d*x + 1)*a*b*c*d^2 - 2*b^
2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^
4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c
)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^
6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3
+ 8*a^2*c^4)*d^2))*x + sqrt(2)*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a

```
*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4
*a^2*b*c^3)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3
*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2
*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2))*x - ((
a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*sqrt(-((b^2 - 2*a*c)*d
^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*
c + 8*a^2*c^2)*d^2)*sqrt(b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*
a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*
b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)))/((
a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*
d^2)))/x) + 1/2*sqrt(2)*sqrt(-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^
3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*sqrt(b^2*
d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-dx+1}\,\sqrt{dx+1}\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(248) = 496.
time = 5.46, size = 681, normalized size = 2.41

$$-\left(\frac{(ad^2-bd+c)\left(\frac{ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}-1\right)\sqrt{\frac{ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}}\arctan\left(-\frac{\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}}{2\sqrt{\frac{ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}}}\right)}{\left(ad^2-c+\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}\right)\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}-\frac{(ad^2-bd+c)\left(\frac{ad^2-c-\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}-1\right)\sqrt{\frac{ad^2-c-\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}}\arctan\left(-\frac{\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}}{2\sqrt{\frac{ad^2-c-\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}{ad^2-bd+c}}}\right)}{\left(ad^2-c-\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}\right)\sqrt{-(ad^2+bd+c)(ad^2-bd+c)+(ad^2-c)^2}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -((a*d^2 - b*d + c)*((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c)
 + (a*d^2 - c)^2))/(a*d^2 - b*d + c) - 1)*sqrt((a*d^2 - c + sqrt(-(a*d^2 +
b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c))*arctan(-1/2
*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(
-d*x + 1)))/sqrt((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (
a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c + sqrt(-(a*d^2 + b*d + c)*(a
*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c)
 + (a*d^2 - c)^2)) - (a*d^2 - b*d + c)*((a*d^2 - c - sqrt(-(a*d^2 + b*d + c
)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c) - 1)*sqrt((a*d^2 -
c - sqrt(-(a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*
```

```
d + c))*arctan(-1/2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x +
1)/(sqrt(2) - sqrt(-d*x + 1)))/sqrt((a*d^2 - c - sqrt(-(a*d^2 + b*d + c)*(a
*d^2 - b*d + c) + (a*d^2 - c)^2))/(a*d^2 - b*d + c)))/((a*d^2 - c - sqrt(-(
a*d^2 + b*d + c)*(a*d^2 - b*d + c) + (a*d^2 - c)^2))*sqrt(-(a*d^2 + b*d + c
)*(a*d^2 - b*d + c) + (a*d^2 - c)^2)))*d
```

Mupad [B]
time = 82.37, size = 2500, normalized size = 8.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] - atan(((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^
2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 3
2*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d
^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16
*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 +
 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^
(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) +
8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2
*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32
*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^
2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*
a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 +
16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(
1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8
*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*
c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 16*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*
a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1/2)*((-(8*a*c^3 - 2*b^2*c^2 + b^4*d^2
 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*c^2*d^2 - 6*a*b^2*c*d^2)/(2*(16*a
^2*c^4 + b^4*c^2 - b^6*d^2 - 8*a*b^2*c^3 + a^2*b^4*d^4 + 32*a^3*c^3*d^2 + 1
6*a^4*c^2*d^4 - 8*a^3*b^2*c*d^4 - 32*a^2*b^2*c^2*d^2 + 10*a*b^4*c*d^2)))^(1
/2)*((((1 - d*x)^(1/2) - 1)^2*(1073741824*a*b^10*d^12 - 2147483648*a^3*b^8*
d^14 + 1073741824*a^5*b^6*d^16 - 36283883716608*a^3*c^8*d^6 + 3628388371660
8*a^4*c^7*d^8 + 210900074102784*a^5*c^6*d^10 + 167812962189312*a^6*c^5*d^12
 + 29480655519744*a^7*c^4*d^14 - 2267742732288*a*b^4*c^6*d^6 + 760209211392
*a*b^6*c^4*d^8 + 1504312295424*a*b^8*c^2*d^10 + 75161927680*a^2*b^8*c*d^12
- 66571993088*a^4*b^6*c*d^14 - 8589934592*a^6*b^4*c*d^16 + 18141941858304*a
^2*b^2*c^7*d^6 - 3813930958848*a^2*b^4*c^5*d^8 - 5978594476032*a^3*b^2*c^6*
d^8 - 21930103013376*a^2*b^6*c^3*d^10 + 116415088558080*a^3*b^4*c^4*d^10 -
263779711451136*a^4*b^2*c^5*d^10 - 4173634469888*a^3*b^6*c^2*d^12 + 3999473
5460352*a^4*b^4*c^3*d^12 - 140239272148992*a^5*b^2*c^4*d^12 + 2478196129792
```

*a^5*b^4*c^2*d^14 - 16080357556224*a^6*b^2*c^3*d^14 + 17179869184*a^7*b^2*c
^2*d^16))/((d*x + 1)^(1/2) - 1)^2 + 1073741824*a*b^10*d^12 + (((1 - d*x)^(1
/2) - 1)*(1176821039104*a*b^7*c^3*d^9 - 21440476741632*a^3*b*c^7*d^7 - 1340
029796352*a*b^5*c^5*d^7 - 11544872091648*a^4*b*c^6*d^9 + 42193758715904*a^5
*b*c^5*d^11 - 210453397504*a^3*b^7*c*d^13 + 32985348833280*a^6*b*c^4*d^13 +
42949672960*a^5*b^5*c*d^15 + 687194767360*a^7*b*c^3*d^15 + 10720238370816*
a^2*b^3*c^6*d^7 - 10136122818560*a^2*b^5*c^4*d^9 + 24601572671488*a^3*b^3*c
^5*d^9 - 3646427234304*a^2*b^7*c^2*d^11 + 23768349016064*a^3*b^5*c^3*d^11 -
57999238365184*a^4*b^3*c^4*d^11 + 3745211482112*a^4*b^5*c^2*d^13 - 1985992
8776704*a^5*b^3*c^3*d^13 - 343597383680*a^6*b^3*c^2*d^15 + 167503724544*a*b
^9*c*d^11))/((d*x + 1)^(1/2) - 1) - 2147483648*a^3*b^8*d^14 + 1073741824*a^
5*b^6*d^16 + 1099511627776*a^3*c^8*d^6 - 4947802324992*a^4*c^7*d^8 - 158054
7964928*a^5*c^6*d^10 + 16080357556224*a^6*c^5*d^12 + 11613591568384*a^7*c^4
*d^14 + 68719476736*a*b^4*c^6*d^6 - 115964116992*a*b^6*c^4*d^8 + 4831838208
0*a*b^8*c^2*d^10 + 23622320128*a^2*b^8*c*d^12 - 15032385536*a^4*b^6*c*d^14
- 8589934592*a^6*b^4*c*d^16 - 549755813888*a^2*b^2*c^7*d^6 + 618475290624*a
^2*b^4*c^5*d^8 + 618475290624*a^3*b^2*c^6*d^8 - 77309411328*a^2*b^6*c^3*d^1
0 - 1799591297024*a^3*b^4*c^4*d^10 + 5738076307456*a^4*b^2*c^5*d^10 - 10812
58016768*a^3*b^6*c^2*d^12 + 8246337208320*a^4*b^4*c^3*d^12 - 21492016349184
*a^5*b^2*c^4*d^12 + 949187772416*a^5*b^4*c^2*d^14 - 6322191859712*a^6*b^2*c
^3*d^14 + 17179869184*a^7*b^2*c^2*d^16) + (((1 - d*x)^(1/2) - 1)^2*(1778116
460544*a*b^5*c^4*d^8 + 28449863368704*a^3*b*c^6*d^8 - 1767379042304*a*b^7*c
^2*d^10 + 57312043597824*a^4*b*c^5*d^10 - 47244640256*a^2*b^7*c*d^12 + 2961
8094473216*a^5*b*c^4*d^12 + 47244640256*a^4*b^5*c*d^14 + 755914244096*a^6*b
*c^3*d^14 - 14224931684352*a^2*b^3*c^5*d^8 + 17721035063296*a^2*b^5*c^3*d^1
0 - 56934086475776*a^3*b^3*c^4*d^10 + 2229088026624*a^3*b^5*c^2*d^12 - 1556
4961480704*a^4*b^3*c^3*d^12 - 377957122048*a^5*b^3*c^2*d^14))/((d*x + 1)^(1
/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)*(30236569763840*a^3*c^7*d^7 + 574494825
51296*a^4*c^6*d^9 + 24189255811072*a^5*c^5*d^11 - 3023656976384*a^6*c^4*d^1
3 + 1889785610240*a*b^4*c^5*d^7 - 1778116460544*a*b^6*c^3*d^9 + 12884901888
0*a^3*b^6*c*d^13 - 15118284881920*a^2*b^2*c^6*d^7 + 17815524343808*a^2*b^4*
c^4*d^9 - 57174604644352*a^3*b^2*c^5*d^9 + 1494648619008*a^2*b^6*c^2*d^11 -
4260607557632*a^3*b^4*c^3*d^11 - 4672924418048*a^4*b^2*c^4*d^11 - 12197707
12064*a^4*b^4*c^2*d^13 + 3573412790272*a^5*b^2*c^3*d^13 - 128849018880*a*b^
8*c*d^11))/((d*x + 1)^(1/2) - 1) + 77309411328*a*b^5*c^4*d^8 + 123695058124
8*a^3*b*c^6*d^8 - 88046829568*a*b^7*c^2*d^10 + ...

3.796 $\int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=571

$$-\frac{\left(b\left(b^2d^2-c\left(c+3ad^2\right)\right)-c\left(2c^2-b^2d^2+2acd^2\right)x\right)\sqrt{1-d^2x^2}}{\left(b^2-4ac\right)\left(b^2d^2-\left(c+ad^2\right)^2\right)\left(a+bx+cx^2\right)}-\frac{c\left(4c^3+12ac^2d^2-ab\left(b+\sqrt{b^2-4ac}\right)d^{\cdots}\right.}{\sqrt{2}\,(b\cdots}$$

```
[Out] -(b*(b^2*d^2-c*(3*a*d^2+c))-c*(2*a*c*d^2-b^2*d^2+2*c^2)*x)*(-d^2*x^2+1)^(1/
2)/(-4*a*c+b^2)/(b^2*d^2-(a*d^2+c)^2)/(c*x^2+b*x+a)-1/2*c*arctanh(1/2*(2*c+
d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b
*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(4*c^3+12*a*c^2*d^2-a*b*d^4*(b+(-4*a*c+
b^2)^(1/2))-c*d^2*(5*b^2-8*a^2*d^2-b*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2
)/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2
)))^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^
2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(4*c^3
+12*a*c^2*d^2-2*a*b^2*d^4-4*c*d^2*(-2*a^2*d^2+b^2)-b*d^2*(-a*d^2+c)*(b+(-4*
a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 4.04, antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {913, 989, 1048, 739, 212}

$$-\frac{c\left(-cd^2\left(-8a^2d^2-b\sqrt{b^2-4ac}+5b^2\right)-abd^4\left(\sqrt{b^2-4ac}+b\right)+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}+\frac{c\left(-4cd^2\left(b^2-2a^2d^2\right)-bd^2\left(\sqrt{b^2-4ac}+b\right)\left(c-ad^2\right)-2ab^2d^4+12ac^2d^2+4c^3\right)\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{\sqrt{1-d^2x^2}\left(b\left(b^2d^2-c\left(3ad^2+c\right)\right)-cx\left(2acd^2-b^2d^2+2c^2\right)\right)}{\left(b^2-4ac\right)\left(b^2d^2-\left(ad^2+c\right)^2\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] -(((b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d^2 + 2*a*c*d^2)*x)*Sqrt
[1 - d^2*x^2])/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^2)))
 - (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 - c*d^2*(5*b^
2 - b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2))*ArcTanh[(2*c + (b - Sqrt[b^2 - 4*a*c]
)*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*S
qrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b
*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) + (c*(4*c^3 + 12*a
*c^2*d^2 - 2*a*b^2*d^4 - b*(b + Sqrt[b^2 - 4*a*c])*d^2*(c - a*d^2) - 4*c*d^
2*(b^2 - 2*a^2*d^2))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]
*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])
```

```
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-dx}\,\sqrt{1+dx}\,(a+bx+cx^2)^2}\,dx = \int \frac{1}{(a+bx+cx^2)^2\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{(b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)}$$

$$= -\frac{(b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)}$$

$$= -\frac{(b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)}$$

$$= -\frac{(b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x)\sqrt{1-d^2x^2}}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)}$$

<u>Mathematica</u> **[A]**
time = 11.06, size = 800, normalized size = 1.40

$$-\frac{-\frac{2(b^3d^2-bc(c+3ad^2)+b^2cd^2x-2c^2(c+ad^2)x)\sqrt{1-d^2x^2}}{(b^2-4ac)(a+x(b+cx))}+\frac{c\left(4c^3+12ac^2d^2-ab\left(b+\sqrt{b^2-4ac}\right)d^4+cd^2\left(-5b^2+b\sqrt{b^2-4ac}+8a^2d^2\right)\right)\log\left(-b+\sqrt{b^2-4ac}-2cx\right)}{(b^2-4ac)^{3/2}\sqrt{c^2+acd^2+\frac{1}{2}b\left(-b+\sqrt{b^2-4ac}\right)d^2}}+\frac{c\left(-4c^3-12ac^2d^2+ab\left(b-\sqrt{b^2-4ac}\right)d^4+cd^2\left(5b^2+b\sqrt{b^2-4ac}-8a^2d^2\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)}{(b^2-4ac)^{3/2}\sqrt{c^2+acd^2-\frac{1}{2}b\left(b+\sqrt{b^2-4ac}\right)d^2}}-\frac{c\left(4c^3+12ac^2d^2-ab\left(b+\sqrt{b^2-4ac}\right)d^4+cd^2\left(-5b^2+b\sqrt{b^2-4ac}+8a^2d^2\right)\right)\log\left(-2c-bd^2x+\sqrt{b^2-4ac}\,d^2x-\sqrt{4c^2+4acd^2+2b\left(-b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}\right)}{(b^2-4ac)^{3/2}\sqrt{c^2+acd^2+\frac{1}{2}b\left(-b+\sqrt{b^2-4ac}\right)d^2}}+\frac{c\left(4c^3+12ac^2d^2+ab\left(-b+\sqrt{b^2-4ac}\right)d^4+cd^2\left(-5b^2-b\sqrt{b^2-4ac}+8a^2d^2\right)\right)\log\left(2c+bd^2x+\sqrt{b^2-4ac}\,d^2x+\sqrt{4c^2+4acd^2-2b\left(b+\sqrt{b^2-4ac}\right)d^2}\sqrt{1-d^2x^2}\right)}{(b^2-4ac)^{3/2}\sqrt{c^2+acd^2-\frac{1}{2}b\left(b+\sqrt{b^2-4ac}\right)d^2}}}{2\left(c^2-b^2d^2+2acd^2+a^2d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(a + b*x + c*x^2)^2),x]
```

```
[Out] -1/2*((-2*(b^3*d^2 - b*c*(c + 3*a*d^2) + b^2*c*d^2*x - 2*c^2*(c + a*d^2)*x)
*Sqrt[1 - d^2*x^2])/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (c*(4*c^3 + 12*a*c^
2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 + c*d^2*(-5*b^2 + b*Sqrt[b^2 - 4*a*
c] + 8*a^2*d^2))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x])/((b^2 - 4*a*c)^(3/2)*
Sqrt[c^2 + a*c*d^2 + (b*(-b + Sqrt[b^2 - 4*a*c])*d^2)/2]) + (c*(-4*c^3 - 12
*a*c^2*d^2 + a*b*(b - Sqrt[b^2 - 4*a*c])*d^4 + c*d^2*(5*b^2 + b*Sqrt[b^2 -
4*a*c] - 8*a^2*d^2))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/((b^2 - 4*a*c)^(3/
2)*Sqrt[c^2 + a*c*d^2 - (b*(b + Sqrt[b^2 - 4*a*c])*d^2)/2]) - (c*(4*c^3 + 1
2*a*c^2*d^2 - a*b*(b + Sqrt[b^2 - 4*a*c])*d^4 + c*d^2*(-5*b^2 + b*Sqrt[b^2
- 4*a*c] + 8*a^2*d^2))*Log[-2*c - b*d^2*x + Sqrt[b^2 - 4*a*c]*d^2*x - Sqrt[
4*c^2 + 4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[c^2 + a*c*d^2 + (b*(-b + Sqrt[b^2 - 4*a*c])*d^2)/2
```

```
]) + (c*(4*c^3 + 12*a*c^2*d^2 + a*b*(-b + Sqrt[b^2 - 4*a*c])*d^4 + c*d^2*(-
5*b^2 - b*Sqrt[b^2 - 4*a*c] + 8*a^2*d^2))*Log[2*c + b*d^2*x + Sqrt[b^2 - 4*
a*c]*d^2*x + Sqrt[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt
[1 - d^2*x^2]])/((b^2 - 4*a*c)^(3/2)*Sqrt[c^2 + a*c*d^2 - (b*(b + Sqrt[b^2
- 4*a*c])*d^2)/2]))/(c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.60, size = 41834, normalized size = 73.26

method	result	size
default	Expression too large to display	41834

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*sqrt(d*x + 1)*sqrt(-d*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35403 vs. 2(529) = 1058.
time = 81.34, size = 35403, normalized size = 62.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/2*(2*a*b*c^2 - 2*(a*b^3 - 3*a^2*b*c)*d^2 + 2*(b*c^3 - (b^3*c - 3*a*b*c^2
)*d^2)*x^2 + sqrt(2)*(a^2*b^2*c^2 - 4*a^3*c^3 + (a^4*b^2 - 4*a^5*c)*d^4 - (
a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^2 + (a*b^2*c^3 - 4*a^2*c^4 + (a^3*b^2*
c - 4*a^4*c^2)*d^4 - (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d^2)*x^2 + (a*b^
3*c^2 - 4*a^2*b*c^3 + (a^3*b^3 - 4*a^4*b*c)*d^4 - (a*b^5 - 6*a^2*b^3*c + 8*
a^3*b*c^2)*d^2)*x)*sqrt(-((a^2*b^6 - 12*a^3*b^4*c + 42*a^4*b^2*c^2 - 32*a^5
*c^3)*d^10 + (4*a*b^6*c - 39*a^2*b^4*c^2 + 114*a^3*b^2*c^3 - 128*a^4*c^4)*d
^8 - 8*c^8 + 2*(2*b^6*c^2 - 18*a*b^4*c^3 + 75*a^2*b^2*c^4 - 100*a^3*c^5)*d^
```

6 - (21*b^4*c^4 - 102*a*b^2*c^5 + 152*a^2*c^6)*d^4 + 8*(3*b^2*c^6 - 7*a*c^7)*d^2 + ((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^12 + b^6* c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9 - 3*(a^4*b^8 - 14*a^5*b^6* c + 72*a^6*b^4*c^2 - 160*a^7*b^2*c^3 + 128*a^8*c^4)*d^10 + 3*(a^2*b^10 - 16 *a^3*b^8*c + 101*a^4*b^6*c^2 - 316*a^5*b^4*c^3 + 496*a^6*b^2*c^4 - 320*a^7* c^5)*d^8 - (b^12 - 18*a*b^10*c + 138*a^2*b^8*c^2 - 588*a^3*b^6*c^3 + 1488*a ^4*b^4*c^4 - 2112*a^5*b^2*c^5 + 1280*a^6*c^6)*d^6 + 3*(b^10*c^2 - 16*a*b^8* c^3 + 101*a^2*b^6*c^4 - 316*a^3*b^4*c^5 + 496*a^4*b^2*c^6 - 320*a^5*c^7)*d^ 4 - 3*(b^8*c^4 - 14*a*b^6*c^5 + 72*a^2*b^4*c^6 - 160*a^3*b^2*c^7 + 128*a^4* c^8)*d^2)*sqrt(((a^4*b^6 - 12*a^5*b^4*c + 36*a^6*b^2*c^2)*d^20 + 2*(4*a^3*b ^6*c - 39*a^4*b^4*c^2 + 90*a^5*b^2*c^3)*d^18 + 9*b^2*c^8*d^8 + 3*(8*a^2*b^6 *c^2 - 64*a^3*b^4*c^3 + 123*a^4*b^2*c^4)*d^16 + 2*(16*a*b^6*c^3 - 111*a^2*b ^4*c^4 + 198*a^3*b^2*c^5)*d^14 + 2*(8*b^6*c^4 - 60*a*b^4*c^5 + 117*a^2*b^2* c^6)*d^12 - 24*(b^4*c^6 - 3*a*b^2*c^7)*d^10)/((a^12*b^6 - 12*a^13*b^4*c + 4 8*a^14*b^2*c^2 - 64*a^15*c^3)*d^24 - 6*(a^10*b^8 - 14*a^11*b^6*c + 72*a^12* b^4*c^2 - 160*a^13*b^2*c^3 + 128*a^14*c^4)*d^22 + 3*(5*a^8*b^10 - 80*a^9*b^ 8*c + 502*a^10*b^6*c^2 - 1544*a^11*b^4*c^3 + 2336*a^12*b^2*c^4 - 1408*a^13* c^5)*d^20 - 10*(2*a^6*b^12 - 36*a^7*b^10*c + 267*a^8*b^8*c^2 - 1050*a^9*b^6 *c^3 + 2328*a^10*b^4*c^4 - 2784*a^11*b^2*c^5 + 1408*a^12*c^6)*d^18 + b^6*c^ 12 - 12*a*b^4*c^13 + 48*a^2*b^2*c^14 - 64*a^3*c^15 + 15*(a^4*b^14 - 20*a^5* b^12*c + 172*a^6*b^10*c^2 - 832*a^7*b^8*c^3 + 2465*a^8*b^6*c^4 - 4492*a^9*b ^4*c^5 + 4656*a^10*b^2*c^6 - 2112*a^11*c^7)*d^16 - 6*(a^2*b^16 - 22*a^3*b^1 4*c + 218*a^4*b^12*c^2 - 1284*a^5*b^10*c^3 + 4930*a^6*b^8*c^4 - 12572*a^7*b ^6*c^5 + 20624*a^8*b^4*c^6 - 19776*a^9*b^2*c^7 + 8448*a^10*c^8)*d^14 + (b^1 8 - 24*a*b^16*c + 282*a^2*b^14*c^2 - 2120*a^3*b^12*c^3 + 10938*a^4*b^10*c^4 - 39072*a^5*b^8*c^5 + 95068*a^6*b^6*c^6 - 150864*a^7*b^4*c^7 + 141120*a^8* b^2*c^8 - 59136*a^9*c^9)*d^12 - 6*(b^16*c^2 - 22*a*b^14*c^3 + 218*a^2*b^12* c^4 - 1284*a^3*b^10*c^5 + 4930*a^4*b^8*c^6 - 12572*a^5*b^6*c^7 + 20624*a^6* b^4*c^8 - 19776*a^7*b^2*c^9 + 8448*a^8*c^10)*d^10 + 15*(b^14*c^4 - 20*a*b^1 2*c^5 + 172*a^2*b^10*c^6 - 832*a^3*b^8*c^7 + 2465*a^4*b^6*c^8 - 4492*a^5*b^ 4*c^9 + 4656*a^6*b^2*c^10 - 2112*a^7*c^11)*d^8 - 10*(2*b^12*c^6 - 36*a*b^10 *c^7 + 267*a^2*b^8*c^8 - 1050*a^3*b^6*c^9 + 2328*a^4*b^4*c^10 - 2784*a^5*b^ 2*c^11 + 1408*a^6*c^12)*d^6 + 3*(5*b^10*c^8 - 80*a*b^8*c^9 + 502*a^2*b^6*c^ 10 - 1544*a^3*b^4*c^11 + 2336*a^4*b^2*c^12 - 1408*a^5*c^13)*d^4 - 6*(b^8*c^ 10 - 14*a*b^6*c^11 + 72*a^2*b^4*c^12 - 160*a^3*b^2*c^13 + 128*a^4*c^14)*d^2)))/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*d^12 + b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9 - 3*(a^4*b^8 - 14*a^5*b^6*c + 72*a^6*b^4*c^2 - 160*a^7*b^2*c^3 + 128*a^8*c^4)*d^10 + 3*(a^2*b^10 - 16*a^3 *b^8*c + 101*a^4*b^6*c^2 - 316*a^5*b^4*c^3 + 496*a^6*b^2*c^4 - 320*a^7*c^5) *d^8 - (b^12 - 18*a*b^10*c + 138*a^2*b^8*c^2 - 588*a^3*b^6*c^3 + 1488*a^4*b ^4*c^4 - 2112*a^5*b^2*c^5 + 1280*a^6*c^6)*d^6 + 3*(b^10*c^2 - 16*a*b^8*c^3 + 101*a^2*b^6*c^4 - 316*a^3*b^4*c^5 + 496*a^4*b^2*c^6 - 320*a^5*c^7)*d^4 - 3*(b^8*c^4 - 14*a*b^6*c^5 + 72*a^2*b^4*c^6 - 160*a^3*b^2*c^7 + 128*a^4*c^8) *d^2))*log(-(48*a*b*c^9*d^4 + 4*(3*a^4*b^5*c^2 - 34*a^5*b^3*c^3 + 96*a^6*b* c^4)*d^14 + 4*(18*a^3*b^5*c^3 - 161*a^4*b^3*c^4 + 336*a^5*b*c^5)*d^12 + 24*

```
(6*a^2*b^5*c^4 - 43*a^3*b^3*c^5 + 76*a^4*b*c^6)*d^10 + 4*(24*a*b^5*c^5 - 16
1*a^2*b^3*c^6 + 300*a^3*b*c^7)*d^8 - 8*(17*a*b^3*c^7 - 48*a^2*b*c^8)*d^6 +
2*((3*a^7*b^6*c^2 - 40*a^8*b^4*c^3 + 176*a^9*b^2*c^4 - 256*a^10*c^5)*d^16 -
 4*b^4*c^11 + 32*a*b^2*c^12 - 64*a^2*c^13 - (9*a^5*b^8*c^2 - 144*a^6*b^6*c^
3 + 832*a^7*b^4*c^4 - 2048*a^8*b^2*c^5 + 1792*a^9*c^6)*d^14 + (9*a^3*b^10*c
^2 - 174*a^4*b^8*c^3 + 1281*a^5*b^6*c^4 - 4540*a^6*b^4*c^5 + 7856*a^7*b^2*c
^6 - 5440*a^8*c^7)*d^12 - (3*a*b^12*c^2 - 76*a^2*b^10*c^3 + 734*a^3*b^8*c^4
 - 3634*a^4*b^6*c^5 + 10040*a^5*b^4*c^6 - 14944*a^6*b^2*c^7 + 9344*a^7*c^8)
*d^10 - (6*b^12*c^3 - 109*a*b^10*c^4 + 884*a^2*b^8*c^5 - 4069*a^3*b^6*c^6 +
 10964*a^4*b^4*c^7 - 16048*a^5*b^2*c^8 + 9920*a^6*c^9)*d^8 + (22*b^10*c^5 -
 329*a*b^8*c^6 + 1996*a^2*b^6*c^7 - 6224*a^3*b^4*c^8 + 10048*a^4*b^2*c^9 -
6656*a^5*c^10)*d^6 - (30*b^8*c^7 - 375*a*b^6*c^8 + 1732*a^2*b^4*c^9 - 3536*
a^3*b^2*c^10 + 2752*a^4*c^11)*d^4 + 2*(9*b^6*c^9 - 92*a*b^4*c^10 + 304*a^2*
b^2*c^11 - 320*a^3*c^12)*d^2)*x*sqrt(((a^4*b^6 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(a+bx+cx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*x + 1)*sqrt(d*x + 1)*(a + b*x + c*x**2)**2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] \text{Hanged}
```

3.797 $\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)\left(c^2+3b^2d^2+2acd^2+a^2d^4\right)x}{d^6\sqrt{1-d^2x^2}}+\frac{b(5c^2+b^2d^2+6acd^2)\sqrt{1-d^2x^2}}{d^6}+$$

```
[Out] -3/8*(8*a^2*c*d^4+8*a*b^2*d^4+12*a*c^2*d^2+12*b^2*c*d^2+5*c^3)*arcsin(d*x)/
d^7+(b*(3*a^2+3*c^2/d^4+b^2/d^2+6*a*c/d^2)*d^4+(a*d^2+c)*(a^2*d^4+2*a*c*d^2
+3*b^2*d^2+c^2)*x)/d^6/(-d^2*x^2+1)^(1/2)+b*(6*a*c*d^2+b^2*d^2+5*c^2)*(-d^2
*x^2+1)^(1/2)/d^6+1/8*c*(12*a*c*d^2+12*b^2*d^2+7*c^2)*x*(-d^2*x^2+1)^(1/2)/
d^6+b*c^2*x^2*(-d^2*x^2+1)^(1/2)/d^4+1/4*c^3*x^3*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A]
time = 0.37, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {913, 1828, 1829, 655, 222}

$$-\frac{3\text{ArcSin}(dx)\left(8a^2cd^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3\right)}{8d^7}+\frac{x(ad^2+c)\left(a^2d^4+2acd^2+3b^2d^2+c^2\right)+bd^4\left(3a^2+\frac{6ac}{d^2}+\frac{b^2}{d^2}+\frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}}+\frac{cx\sqrt{1-d^2x^2}\left(12acd^2+12b^2d^2+7c^2\right)}{8d^6}+\frac{b\sqrt{1-d^2x^2}\left(6acd^2+b^2d^2+5c^2\right)}{d^6}+\frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4}+\frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b*(3*a^2 + (3*c^2)/d^4 + b^2/d^2 + (6*a*c)/d^2)*d^4 + (c + a*d^2)*(c^2 + 3
*b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^6*Sqrt[1 - d^2*x^2]) + (b*(5*c^2 + b^
2*d^2 + 6*a*c*d^2)*Sqrt[1 - d^2*x^2])/d^6 + (c*(7*c^2 + 12*b^2*d^2 + 12*a*c
*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^6) + (b*c^2*x^2*Sqrt[1 - d^2*x^2])/d^4 + (c
^3*x^3*Sqrt[1 - d^2*x^2])/(4*d^4) - (3*(5*c^3 + 12*b^2*c*d^2 + 12*a*c^2*d^2
 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*ArcSin[d*x])/(8*d^7)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
```

```
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{(a+bx+cx^2)^3}{(1-d^2x^2)^{3/2}}\,dx$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)(c^2+3b^2d^2+2acd^2+a^2d^4)x}{d^6\sqrt{1-d^2x^2}} - \int \frac{c^3}{}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)(c^2+3b^2d^2+2acd^2+a^2d^4)x}{d^6\sqrt{1-d^2x^2}} + \frac{c^3x^3}{}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)(c^2+3b^2d^2+2acd^2+a^2d^4)x}{d^6\sqrt{1-d^2x^2}} + \frac{bc^2x^2}{}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)(c^2+3b^2d^2+2acd^2+a^2d^4)x}{d^6\sqrt{1-d^2x^2}} + \frac{c(7c^2}{}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)(c^2+3b^2d^2+2acd^2+a^2d^4)x}{d^6\sqrt{1-d^2x^2}} + \frac{b(5c^2}{}$$

$$= \frac{b\left(3a^2+\frac{3c^2}{d^4}+\frac{b^2}{d^2}+\frac{6ac}{d^2}\right)d^4+(c+ad^2)(c^2+3b^2d^2+2acd^2+a^2d^4)x}{d^6\sqrt{1-d^2x^2}} + \frac{b(5c^2}{}$$

Mathematica [A]
time = 0.91, size = 261, normalized size = 0.95

$$-\frac{\frac{d^2(8b^3d^2(-2+d^2x^2)+12b^2d^2x(-2ad^2+c(-3+d^2x^2))+8b(-3a^2d^4+6acd^2(-2+d^2x^2)+c^2(-8+4d^2x^2+d^4x^4))+x(-24a^2cd^4-8a^3d^6+12ac^2d^2(-3+d^2x^2)+c^3(-15+5d^2x^2+2d^4x^4)))}{\sqrt{1-d^2x^2}}+3\sqrt{-d^2}\,(5c^3+12b^2cd^2+12ac^2d^2+8ab^2d^4+8a^2cd^4)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{8d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] -1/8*((d^2*(8*b^3*d^2*(-2 + d^2*x^2) + 12*b^2*d^2*x*(-2*a*d^2 + c*(-3 + d^2
*x^2)) + 8*b*(-3*a^2*d^4 + 6*a*c*d^2*(-2 + d^2*x^2) + c^2*(-8 + 4*d^2*x^2 +
 d^4*x^4)) + x*(-24*a^2*c*d^4 - 8*a^3*d^6 + 12*a*c^2*d^2*(-3 + d^2*x^2) + c
^3*(-15 + 5*d^2*x^2 + 2*d^4*x^4))))/Sqrt[1 - d^2*x^2] + 3*Sqrt[-d^2]*(5*c^3
 + 12*b^2*c*d^2 + 12*a*c^2*d^2 + 8*a*b^2*d^4 + 8*a^2*c*d^4)*Log[-(Sqrt[-d^2
]*x) + Sqrt[1 - d^2*x^2]])/d^8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 755, normalized size = 2.74

method	result
default	$-\frac{\sqrt{-dx+1}\,\left(96\,\mathrm{csgn}(d)d^3\sqrt{-d^2x^2+1}\,abc+24\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^5a^2cx+24\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^5a\,b^2x+\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-d*x+1)^(1/2)*(96*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*a*b*c+24*(-d^2*x^2+1
)^(1/2)*csgn(d)*d^5*a^2*c*x+24*(-d^2*x^2+1)^(1/2)*csgn(d)*d^5*a*b^2*x+36*(-
d^2*x^2+1)^(1/2)*csgn(d)*d^3*a*c^2*x+36*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*b^2*
c*x-15*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c^3+16*csgn(d)*d^3*(-d^2*x^2+
1)^(1/2)*b^3-36*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b^2*c*d^2-24*arctan(
csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^2*c*d^4-24*arctan(csgn(d)*d*x/(-d^2*x^2+1
)^(1/2))*a*b^2*d^4-36*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*c^2*d^2+15*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c^3*d^2*x^2-8*csgn(d)*b*c^2*d^5*x^4*(
-d^2*x^2+1)^(1/2)-12*csgn(d)*a*c^2*d^5*x^3*(-d^2*x^2+1)^(1/2)-12*csgn(d)*b^
2*c*d^5*x^3*(-d^2*x^2+1)^(1/2)-32*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*b*c^2*x^2-
48*csgn(d)*a*b*c*d^5*x^2*(-d^2*x^2+1)^(1/2)-8*csgn(d)*b^3*d^5*x^2*(-d^2*x^2
+1)^(1/2)-5*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*c^3*x^3+8*csgn(d)*d^7*(-d^2*x^2+
1)^(1/2)*a^3*x-2*csgn(d)*c^3*d^5*x^5*(-d^2*x^2+1)^(1/2)+64*csgn(d)*d*(-d^2*
x^2+1)^(1/2)*b*c^2+24*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*a^2*b+36*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*a*c^2*d^4*x^2+36*arctan(csgn(d)*d*x/(-d^2*x^2+1)^
(1/2))*b^2*c*d^4*x^2+24*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^2*c*d^6*x^
2+24*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*b^2*d^6*x^2+15*(-d^2*x^2+1)^(
1/2)*csgn(d)*d*c^3*x)*csgn(d)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^7/(d*x+1)^(1/2)
```

Maxima [A]
time = 0.51, size = 371, normalized size = 1.34

$$-\frac{c^3x^5}{4\sqrt{-d^2x^2+1}\,d^2} - \frac{bc^2x^4}{\sqrt{-d^2x^2+1}\,d^2} + \frac{a^3x}{\sqrt{-d^2x^2+1}} - \frac{5c^3x^3}{8\sqrt{-d^2x^2+1}\,d^4} - \frac{3(b^2c+ac^2)x^3}{2\sqrt{-d^2x^2+1}\,d^2} + \frac{3a^2b}{\sqrt{-d^2x^2+1}\,d^2} - \frac{4bc^2x^2}{\sqrt{-d^2x^2+1}\,d^4} - \frac{(b^3+6abc)x^2}{\sqrt{-d^2x^2+1}\,d^2} + \frac{3(ab^2+a^2c)x}{\sqrt{-d^2x^2+1}\,d^2} - \frac{3(ab^2+a^2c)\arcsin(dx)}{d^3} + \frac{15c^3x}{8\sqrt{-d^2x^2+1}\,d^6} + \frac{9(b^2c+ac^2)x}{2\sqrt{-d^2x^2+1}\,d^4} - \frac{15c^3\arcsin(dx)}{8d^7} - \frac{9(b^2c+ac^2)\arcsin(dx)}{2d^5} + \frac{8bc^2}{\sqrt{-d^2x^2+1}\,d^6} + \frac{2(b^3+6abc)}{\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/4*c^3*x^5/(sqrt(-d^2*x^2 + 1)*d^2) - b*c^2*x^4/(sqrt(-d^2*x^2 + 1)*d^2)
+ a^3*x/sqrt(-d^2*x^2 + 1) - 5/8*c^3*x^3/(sqrt(-d^2*x^2 + 1)*d^4) - 3/2*(b^
2*c + a*c^2)*x^3/(sqrt(-d^2*x^2 + 1)*d^2) + 3*a^2*b/(sqrt(-d^2*x^2 + 1)*d^2
) - 4*b*c^2*x^2/(sqrt(-d^2*x^2 + 1)*d^4) - (b^3 + 6*a*b*c)*x^2/(sqrt(-d^2*x
^2 + 1)*d^2) + 3*(a*b^2 + a^2*c)*x/(sqrt(-d^2*x^2 + 1)*d^2) - 3*(a*b^2 + a^
2*c)*arcsin(d*x)/d^3 + 15/8*c^3*x/(sqrt(-d^2*x^2 + 1)*d^6) + 9/2*(b^2*c + a
*c^2)*x/(sqrt(-d^2*x^2 + 1)*d^4) - 15/8*c^3*arcsin(d*x)/d^7 - 9/2*(b^2*c +
a*c^2)*arcsin(d*x)/d^5 + 8*b*c^2/(sqrt(-d^2*x^2 + 1)*d^6) + 2*(b^3 + 6*a*b*
c)/(sqrt(-d^2*x^2 + 1)*d^4)
```

Fricas [A]
time = 1.89, size = 376, normalized size = 1.36

$$-\frac{24a^2bd^5+64bc^2d+16(b^3+6abc)d^3-8(3a^2bd^7+8bc^2d^3+2(b^3+6abc)d^5)x^2-\left(2c^3d^5x^5+8bc^2d^5x^4-24a^2bd^5-64bc^2d-16(b^3+6abc)d^3+(5c^3d^3+12(b^2c+ac^2)d^5)x^3+8(4bc^2d^3+(b^3+6abc)d^5)x^2-(8a^3d^7+24(ab^2+a^2c)d^5+15c^3d+36(b^2c+ac^2)d^3)x\right)\sqrt{dx+1}\sqrt{-dx+1}+6\left(8(ab^2+a^2c)d^4+5c^3+12(b^2c+ac^2)d^2-(8(ab^2+a^2c)d^6+5c^3d^2+12(b^2c+ac^2)d^4)x^2\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{8(d^9x^2-d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/8*(24*a^2*b*d^5 + 64*b*c^2*d + 16*(b^3 + 6*a*b*c)*d^3 - 8*(3*a^2*b*d^7 +
 8*b*c^2*d^3 + 2*(b^3 + 6*a*b*c)*d^5)*x^2 - (2*c^3*d^5*x^5 + 8*b*c^2*d^5*x^
4 - 24*a^2*b*d^5 - 64*b*c^2*d - 16*(b^3 + 6*a*b*c)*d^3 + (5*c^3*d^3 + 12*(b
^2*c + a*c^2)*d^5)*x^3 + 8*(4*b*c^2*d^3 + (b^3 + 6*a*b*c)*d^5)*x^2 - (8*a^3
*d^7 + 24*(a*b^2 + a^2*c)*d^5 + 15*c^3*d + 36*(b^2*c + a*c^2)*d^3)*x)*sqrt(
d*x + 1)*sqrt(-d*x + 1) + 6*(8*(a*b^2 + a^2*c)*d^4 + 5*c^3 + 12*(b^2*c + a*
c^2)*d^2 - (8*(a*b^2 + a^2*c)*d^6 + 5*c^3*d^2 + 12*(b^2*c + a*c^2)*d^4)*x^2
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^9*x^2 - d^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(260) = 520.
time = 5.44, size = 736, normalized size = 2.67

$$\frac{\frac{\left(\left((dx+1)\left(2(dx+1)\left(\frac{(dx+1)c^3}{d^6}+\frac{4bc^2d^{31}-5c^3d^{30}}{d^{36}}\right)+\frac{12b^2cd^{32}+12ac^2d^{32}-32bc^2d^{31}+25c^3d^{30}}{d^{36}}\right)+\frac{8b^3d^{33}+48abcd^{33}-36b^2cd^{32}-36ac^2d^{32}+80bc^2d^{31}-35c^3d^{30}}{d^{36}}\right)(dx+1)-\frac{2\left(2a^3d^{36}+6a^2bd^{35}+6ab^2d^{34}+6a^2cd^{34}+10b^3d^{33}+60abcd^{33}-6b^2cd^{32}-6ac^2d^{32}+54bc^2d^{31}-7c^3d^{30}\right)}{d^{36}}\right)\sqrt{dx+1}\sqrt{-dx+1}}{dx-1}-\frac{6\left(8ab^2d^4+8a^2cd^4+12b^2cd^2+12ac^2d^2+5c^3\right)\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^6}+\frac{2\left(\frac{a^3d^6\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{3a^2bd^5\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{3ab^2d^4\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{3a^2cd^4\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{b^3d^3\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{6abcd^3\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{3b^2cd^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{3ac^2d^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{3bc^2d\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{c^3\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}\right)}{d^6}-\frac{2\left(a^3d^6-3a^2bd^5+3ab^2d^4+3a^2cd^4-b^3d^3-6abcd^3+3b^2cd^2+3ac^2d^2-3bc^2d+c^3\right)\sqrt{dx+1}}{d^6\left(\sqrt{2}-\sqrt{-dx+1}\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((((d*x + 1)*(2*(d*x + 1)*((d*x + 1)*c^3/d^6 + (4*b*c^2*d^31 - 5*c^3*d^
30)/d^36) + (12*b^2*c*d^32 + 12*a*c^2*d^32 - 32*b*c^2*d^31 + 25*c^3*d^30)/d
^36) + (8*b^3*d^33 + 48*a*b*c*d^33 - 36*b^2*c*d^32 - 36*a*c^2*d^32 + 80*b*c
^2*d^31 - 35*c^3*d^30)/d^36)*(d*x + 1) - 2*(2*a^3*d^36 + 6*a^2*b*d^35 + 6*a
*b^2*d^34 + 6*a^2*c*d^34 + 10*b^3*d^33 + 60*a*b*c*d^33 - 6*b^2*c*d^32 - 6*a
*c^2*d^32 + 54*b*c^2*d^31 - 7*c^3*d^30)/d^36)*sqrt(d*x + 1)*sqrt(-d*x + 1)/
(d*x - 1) - 6*(8*a*b^2*d^4 + 8*a^2*c*d^4 + 12*b^2*c*d^2 + 12*a*c^2*d^2 + 5*
c^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^6 + 2*(a^3*d^6*(sqrt(2) - sqrt(-d*
x + 1))/sqrt(d*x + 1) - 3*a^2*b*d^5*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1
) + 3*a*b^2*d^4*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a^2*c*d^4*(sqr
t(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - b^3*d^3*(sqrt(2) - sqrt(-d*x + 1))/s
qrt(d*x + 1) - 6*a*b*c*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*b^2
*c*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 3*a*c^2*d^2*(sqrt(2) - sq
rt(-d*x + 1))/sqrt(d*x + 1) - 3*b*c^2*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
 + 1) + c^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^6 - 2*(a^3*d^6 - 3*
a^2*b*d^5 + 3*a*b^2*d^4 + 3*a^2*c*d^4 - b^3*d^3 - 6*a*b*c*d^3 + 3*b^2*c*d^2
 + 3*a*c^2*d^2 - 3*b*c^2*d + c^3)*sqrt(d*x + 1)/(d^6*(sqrt(2) - sqrt(-d*x +
 1))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^3}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)^3/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.798 $\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}}+\frac{2bc\sqrt{1-d^2x^2}}{d^4}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}-\frac{\left(2b^2+c\left(4a+\frac{3c}{d^2}\right)\right)\sin^{-1}(dx)}{2d^3}$$

```
[Out] -1/2*(2*b^2+c*(4*a+3*c/d^2))*arcsin(d*x)/d^3+(2*b*(a+c/d^2)*d^2+(a^2*d^4+2*
a*c*d^2+b^2*d^2+c^2)*x)/d^4/(-d^2*x^2+1)^(1/2)+2*b*c*(-d^2*x^2+1)^(1/2)/d^4
+1/2*c^2*x*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A]
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {913, 1828, 1829, 655, 222}

$$\frac{x(a^2d^4+2acd^2+b^2d^2+c^2)+2bd^2\left(a+\frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}}-\frac{\text{ArcSin}(dx)\left(c\left(4a+\frac{3c}{d^2}\right)+2b^2\right)}{2d^3}+\frac{2bc\sqrt{1-d^2x^2}}{d^4}+\frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (2*b*(a + c/d^2)*d^2 + (c^2 + b^2*d^2 + 2*a*c*d^2 + a^2*d^4)*x)/(d^4*Sqrt[1
 - d^2*x^2]) + (2*b*c*Sqrt[1 - d^2*x^2])/d^4 + (c^2*x*Sqrt[1 - d^2*x^2])/(2
*d^4) - ((2*b^2 + c*(4*a + (3*c)/d^2))*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx = \int \frac{(a+bx+cx^2)^2}{(1-d^2x^2)^{3/2}}\,dx$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} - \int \frac{\frac{c^2+b^2d^2+2acd^2}{d^4}+\frac{2bcx}{d^2}+\frac{c^2x}{d^2}\cdots}{\sqrt{1-d^2x^2}}\cdots$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4} + \frac{\int \frac{-2b^2-c(\cdots}{\sqrt{1\cdots}}}{\cdots}$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-\cdots}}{2d^4}\cdots$$

$$= \frac{2b\left(a+\frac{c}{d^2}\right)d^2+\left(c^2+b^2d^2+2acd^2+a^2d^4\right)x}{d^4\sqrt{1-d^2x^2}} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-\cdots}}{2d^4}\cdots$$

Mathematica [A]
time = 0.58, size = 148, normalized size = 1.10

$$\frac{\frac{d^4\left(2b^2d^2x+4b\left(ad^2+c\left(2-d^2x^2\right)\right)+x\left(4acd^2+2a^2d^4+c^2\left(3-d^2x^2\right)\right)\right)}{\sqrt{1-d^2x^2}}+\left(-d^2\right)^{3/2}\left(3c^2+2b^2d^2+4acd^2\right)\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] ((d^4*(2*b^2*d^2*x + 4*b*(a*d^2 + c*(2 - d^2*x^2)) + x*(4*a*c*d^2 + 2*a^2*d
^4 + c^2*(3 - d^2*x^2))))/Sqrt[1 - d^2*x^2] + (-d^2)^(3/2)*(3*c^2 + 2*b^2*d
^2 + 4*a*c*d^2)*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2]])/(2*d^8)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 381, normalized size = 2.82

method	result
default	$-\frac{\sqrt{-dx+1}\,\left(2\,\mathrm{csgn}(d)d^5\sqrt{-d^2x^2+1}\,a^2x-\mathrm{csgn}(d)c^2d^3x^3\sqrt{-d^2x^2+1}-4\,\mathrm{csgn}(d)bc\,d^3x^2\sqrt{-d^2x^2+1}+4\,\mathrm{arcta}\right.}{}$
risch	$-\frac{c(cx+4b)(dx-1)\sqrt{dx+1}\,\sqrt{(-dx+1)(dx+1)}}{2d^4\sqrt{-(dx-1)(dx+1)}\,\sqrt{-dx+1}}-\frac{\left(\frac{2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)ac}{d^2\sqrt{d^2}}+\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)b}{d^2\sqrt{d^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(2*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*a^2*x-csgn(d)*c^2*d^3
*x^3*(-d^2*x^2+1)^(1/2)-4*csgn(d)*b*c*d^3*x^2*(-d^2*x^2+1)^(1/2)+4*arctan(c
sgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*c*d^4*x^2+2*arctan(csgn(d)*d*x/(-d^2*x^2+1
)^(1/2))*b^2*d^4*x^2+4*(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*a*c*x+2*(-d^2*x^2+1)^
(1/2)*csgn(d)*d^3*b^2*x+4*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*a*b+3*arctan(csgn(
d)*d*x/(-d^2*x^2+1)^(1/2))*c^2*d^2*x^2+3*(-d^2*x^2+1)^(1/2)*csgn(d)*d*c^2*x
+8*csgn(d)*d*(-d^2*x^2+1)^(1/2)*b*c-4*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)
)*a*c*d^2-2*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b^2*d^2-3*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*c^2)*csgn(d)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^5/(d*x+1
)^(1/2)
```

Maxima [A]
time = 0.50, size = 176, normalized size = 1.30

$$\frac{a^2x}{\sqrt{-d^2x^2+1}}-\frac{c^2x^3}{2\sqrt{-d^2x^2+1}\,d^2}-\frac{2\,bcx^2}{\sqrt{-d^2x^2+1}\,d^2}+\frac{2\,ab}{\sqrt{-d^2x^2+1}\,d^2}+\frac{(b^2+2\,ac)x}{\sqrt{-d^2x^2+1}\,d^2}-\frac{(b^2+2\,ac)\arcsin(dx)}{d^3}+\frac{3\,c^2x}{2\sqrt{-d^2x^2+1}\,d^4}-\frac{3\,c^2\arcsin(dx)}{2\,d^5}+\frac{4\,bc}{\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] a^2*x/sqrt(-d^2*x^2 + 1) - 1/2*c^2*x^3/(sqrt(-d^2*x^2 + 1)*d^2) - 2*b*c*x^2
/(sqrt(-d^2*x^2 + 1)*d^2) + 2*a*b/(sqrt(-d^2*x^2 + 1)*d^2) + (b^2 + 2*a*c)*
x/(sqrt(-d^2*x^2 + 1)*d^2) - (b^2 + 2*a*c)*arcsin(d*x)/d^3 + 3/2*c^2*x/(sqr
t(-d^2*x^2 + 1)*d^4) - 3/2*c^2*arcsin(d*x)/d^5 + 4*b*c/(sqrt(-d^2*x^2 + 1)*
d^4)
```

Fricas [A]
time = 1.64, size = 204, normalized size = 1.51

$$-\frac{4\,abd^3+8\,bcd-4\,(abd^5+2\,bcd^3)x^2-(c^2d^3x^3+4\,bcd^3x^2-4\,abd^3-8\,bcd-(2\,a^2d^5+2\,(b^2+2\,ac)d^3+3\,c^2d)x)\sqrt{dx+1}\,\sqrt{-dx+1}+2\,(2\,(b^2+2\,ac)d^2-(2\,(b^2+2\,ac)d^4+3\,c^2d^2)x^2+3\,c^2)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,(d^7x^2-d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*(4*a*b*d^3 + 8*b*c*d - 4*(a*b*d^5 + 2*b*c*d^3)*x^2 - (c^2*d^3*x^3 + 4*
b*c*d^3*x^2 - 4*a*b*d^3 - 8*b*c*d - (2*a^2*d^5 + 2*(b^2 + 2*a*c)*d^3 + 3*c^
2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*(b^2 + 2*a*c)*d^2 - (2*(b^2 + 2
*a*c)*d^4 + 3*c^2*d^2)*x^2 + 3*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) -
1)/(d*x)))/(d^7*x^2 - d^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^2\right)^2}{\left(-dx + 1\right)^{\frac{3}{2}} \left(dx + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)**2/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(125) = 250.
time = 3.52, size = 391, normalized size = 2.90

$$\frac{\frac{2\sqrt{dx+1}\sqrt{-dx+1}\left((dx+1)\left(\frac{(dx+1)c^2}{d^4}+\frac{4bcd^{13}-3c^2d^{12}}{d^{16}}\right)-\frac{a^2d^{16}+2abd^{15}+b^2d^{14}+2acd^{14}+10bcd^{13}-c^2d^{12}}{d^{16}}\right)}{dx-1}-\frac{4\left(2b^2d^2+4acd^2+3c^2\right)\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^4}+\frac{\frac{a^2d^4\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{2abd^3\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{b^2d^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{2acd^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{2bcd\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{c^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}}{d^4}-\frac{\left(a^2d^4-2abd^3+b^2d^2+2acd^2-2bcd+c^2\right)\sqrt{dx+1}}{d^4\left(\sqrt{2}-\sqrt{-dx+1}\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*((d*x + 1)*c^2/d^4 + (4*b*c*
d^13 - 3*c^2*d^12)/d^16) - (a^2*d^16 + 2*a*b*d^15 + b^2*d^14 + 2*a*c*d^14 +
 10*b*c*d^13 - c^2*d^12)/d^16)/(d*x - 1) - 4*(2*b^2*d^2 + 4*a*c*d^2 + 3*c^2
)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4 + (a^2*d^4*(sqrt(2) - sqrt(-d*x + 1
))/sqrt(d*x + 1) - 2*a*b*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + b^2
*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 2*a*c*d^2*(sqrt(2) - sqrt(-
d*x + 1))/sqrt(d*x + 1) - 2*b*c*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1)
+ c^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^4 - (a^2*d^4 - 2*a*b*d^3
+ b^2*d^2 + 2*a*c*d^2 - 2*b*c*d + c^2)*sqrt(d*x + 1)/(d^4*(sqrt(2) - sqrt(-
d*x + 1))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^2}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
[Out] int((a + b*x + c*x^2)^2/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.799 $\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}}-\frac{c\sin^{-1}(dx)}{d^3}$$

[Out] -c*arcsin(d*x)/d^3+(b+(a*d^2+c)*x)/d^2/(-d^2*x^2+1)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {913, 1828, 12, 222}

$$\frac{x(ad^2+c)+b}{d^2\sqrt{1-d^2x^2}}-\frac{c\text{ArcSin}(dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]

[Out] (b + (c + a*d^2)*x)/(d^2*Sqrt[1 - d^2*x^2]) - (c*ArcSin[d*x])/d^3

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}}\,dx &= \int \frac{a+bx+cx^2}{(1-d^2x^2)^{3/2}}\,dx \\
&= \frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}} - \int \frac{c}{d^2\sqrt{1-d^2x^2}}\,dx \\
&= \frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}} - \frac{c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{d^2} \\
&= \frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}} - \frac{c\sin^{-1}(dx)}{d^3}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 70, normalized size = 1.75

$$\frac{b+(c+ad^2)\,x}{d^2\sqrt{1-d^2x^2}} - \frac{c\log\left(-\sqrt{-d^2}\,x+\sqrt{1-d^2x^2}\right)}{(-d^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)),x]
```

```
[Out] (b + (c + a*d^2)*x)/(d^2*Sqrt[1 - d^2*x^2]) - (c*Log[-(Sqrt[-d^2]*x) + Sqrt
[1 - d^2*x^2]])/(-d^2)^(3/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 151, normalized size = 3.78

method	result
default	$\frac{\left(-\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^3ax-\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-(dx-1)(dx+1)}}\right)c\,d^2x^2-\mathrm{csgn}(d)d\sqrt{-d^2x^2+1}\,cx-\mathrm{csgn}(d)d\sqrt{-\right.}{(dx-1)\sqrt{-d^2x^2+1}\,d^3\sqrt{dx+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d^2*x^2+1)^(1/2)*csgn(d)*d^3*a*x-arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^
(1/2))*c*d^2*x^2-csgn(d)*d*(-d^2*x^2+1)^(1/2)*c*x-csgn(d)*d*(-d^2*x^2+1)^(1
/2)*b+arctan(csgn(d)*d*x/(-(d*x-1)*(d*x+1))^(1/2))*c)*(-d*x+1)^(1/2)*csgn(d
)/(d*x-1)/(-d^2*x^2+1)^(1/2)/d^3/(d*x+1)^(1/2)
```

Maxima [A]
time = 0.48, size = 61, normalized size = 1.52

$$\frac{ax}{\sqrt{-d^2x^2+1}}+\frac{cx}{\sqrt{-d^2x^2+1}\,d^2}-\frac{c\arcsin\left(dx\right)}{d^3}+\frac{b}{\sqrt{-d^2x^2+1}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] a*x/sqrt(-d^2*x^2 + 1) + c*x/(sqrt(-d^2*x^2 + 1)*d^2) - c*arcsin(d*x)/d^3 +
 b/(sqrt(-d^2*x^2 + 1)*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(38) = 76.
time = 5.95, size = 101, normalized size = 2.52

$$\frac{bd^3x^2-(bd+(ad^3+cd)x)\sqrt{dx+1}\,\sqrt{-dx+1}-bd+2\left(cd^2x^2-c\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{d^5x^2-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d^3*x^2 - (b*d + (a*d^3 + c*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - b*d + 2
*(c*d^2*x^2 - c)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*x^2
 - d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(38) = 76.
time = 2.73, size = 186, normalized size = 4.65

$$-\frac{\frac{8\,c\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}-\frac{\frac{ad^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}-\frac{bd\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}+\frac{c\left(\sqrt{2}-\sqrt{-dx+1}\right)}{\sqrt{dx+1}}}{d^2}+\frac{(ad^2-bd+c)\sqrt{dx+1}}{d^2\left(\sqrt{2}-\sqrt{-dx+1}\right)}+\frac{2\,(ad^4+bd^3+cd^2)\sqrt{dx+1}\,\sqrt{-dx+1}}{(dx-1)d^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(8*c*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2 - (a*d^2*(sqrt(2) - sqrt(-d
*x + 1))/sqrt(d*x + 1) - b*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + c*(
sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^2 + (a*d^2 - b*d + c)*sqrt(d*x +
 1)/(d^2*(sqrt(2) - sqrt(-d*x + 1))) + 2*(a*d^4 + b*d^3 + c*d^2)*sqrt(d*x +
 1)*sqrt(-d*x + 1)/((d*x - 1)*d^4))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c\,x^2+b\,x+a}{{\left(1-d\,x\right)}^{3/2}\,{\left(d\,x+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)), x)
```

3.800 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)}\,dx$

Optimal. Leaf size=443

$$\frac{d^2(b-(c+ad^2)\,x)}{(b^2d^2-(c+ad^2)^2)\sqrt{1-d^2x^2}}+\frac{c\left(2c^2+2acd^2-b\left(b+\sqrt{b^2-4ac}\right)d^2\right)\tanh^{-1}\left(\frac{2c+\left(b-\right.}{\sqrt{2}\sqrt{2c^2+2acd^2-b}}\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2+2acd^2-b\left(b-\sqrt{b^2-4ac}\right)d^2}\;(}$$

```
[Out] d^2*(b-(a*d^2+c)*x)/(b^2*d^2-(a*d^2+c)^2)/(-d^2*x^2+1)^(1/2)+1/2*c*arctanh(
1/2*(2*c+d^2*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*
a*c+b^2)^(1/2)))/(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+2*
a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*c*arctanh(1/2*(2*c+d^2*x*(b
+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))/
(b^2*d^2-(a*d^2+c)^2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+
(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.97, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {913, 990, 1048, 739, 212}

$$\frac{c\left(-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2}\,\left(b^2d^2-\left(ad^2+c\right)^2\right)}-\frac{c\left(-bd^2\left(b-\sqrt{b^2-4ac}\right)+2acd^2+2c^2\right)\tanh^{-1}\left(\frac{d^2x\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bd^2\left(\sqrt{b^2-4ac}+b\right)+2acd^2+2c^2}\,\left(b^2d^2-\left(ad^2+c\right)^2\right)}+\frac{d^2\left(b-x\left(ad^2+c\right)\right)}{\sqrt{1-d^2x^2}\,\left(b^2d^2-\left(ad^2+c\right)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (d^2*(b - (c + a*d^2)*x))/((b^2*d^2 - (c + a*d^2)^2)*Sqrt[1 - d^2*x^2]) + (
c*(2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b - S
qrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2
- 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2)) - (c
*(2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2)*ArcTanh[(2*c + (b + Sq
rt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 -
 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2 +
2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(b^2*d^2 - (c + a*d^2)^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 990

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p + 1
)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*((-c)*e*(2*
p + q + 4)))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} - \frac{\int \frac{2d^2\left(c^2-b^2d^2+acd^2\right)-2bcd^4x}{\left(a+bx+cx^2\right)\sqrt{1-d^2x^2}}\,d}{2d^2\left(b^2d^2-\left(c+ad^2\right)^2\right)}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{\left(c\left(2c^2+2acd^2-b\left(b-\right.\right.\right.}{\sqrt{}}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} - \frac{\left(c\left(2c^2+2acd^2-b\left(b-\right.\right.\right.}{}$$

$$= \frac{d^2\left(b-\left(c+ad^2\right)x\right)}{\left(b^2d^2-\left(c+ad^2\right)^2\right)\sqrt{1-d^2x^2}} + \frac{c\left(2c^2+2acd^2-b\left(b+\sqrt{}\right.\right.}{\sqrt{2}\sqrt{b^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.68, size = 1546, normalized size = 3.49

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]
```

```
[Out] (4*d^2*(-b + (c + a*d^2)*x)*Sqrt[1 - d^2*x^2] + 2*c^2*Sqrt[-d^2]*(c + a*d^2
)*(-1 + d*x)*(1 + d*x)*RootSum[c^2 - 4*c^2*#1 + 4*b^2*d^2*#1 - 8*a*c*d^2*#1
 + 6*c^2*#1^2 - 8*b^2*d^2*#1^2 + 16*a*c*d^2*#1^2 + 16*a^2*d^4*#1^2 - 4*c^2*
#1^3 + 4*b^2*d^2*#1^3 - 8*a*c*d^2*#1^3 + c^2*#1^4 & , Log[-1 + 2*d^2*x^2 +
2*Sqrt[-d^2]*x*Sqrt[1 - d^2*x^2] + #1]/(-c^2 + b^2*d^2 - 2*a*c*d^2 + 3*c^2*
#1 - 4*b^2*d^2*#1 + 8*a*c*d^2*#1 + 8*a^2*d^4*#1 - 3*c^2*#1^2 + 3*b^2*d^2*#1
^2 - 6*a*c*d^2*#1^2 + c^2*#1^3) & ] - 4*Sqrt[-d^2]*(c^3 + 3*a*c^2*d^2 - 2*a
*b^2*d^4 + 2*a^2*c*d^4)*(-1 + d*x)*(1 + d*x)*RootSum[c^2 - 4*c^2*#1 + 4*b^2
*d^2*#1 - 8*a*c*d^2*#1 + 6*c^2*#1^2 - 8*b^2*d^2*#1^2 + 16*a*c*d^2*#1^2 + 16
*a^2*d^4*#1^2 - 4*c^2*#1^3 + 4*b^2*d^2*#1^3 - 8*a*c*d^2*#1^3 + c^2*#1^4 & ,
 (Log[-1 + 2*d^2*x^2 + 2*Sqrt[-d^2]*x*Sqrt[1 - d^2*x^2] + #1]*#1)/(-c^2 + b
```

```
^2*d^2 - 2*a*c*d^2 + 3*c^2*#1 - 4*b^2*d^2*#1 + 8*a*c*d^2*#1 + 8*a^2*d^4*#1
 - 3*c^2*#1^2 + 3*b^2*d^2*#1^2 - 6*a*c*d^2*#1^2 + c^2*#1^3) & ] + 2*c^2*Sqrt
[-d^2]*(c + a*d^2)*(-1 + d*x)*(1 + d*x)*RootSum[c^2 - 4*c^2*#1 + 4*b^2*d^2*
#1 - 8*a*c*d^2*#1 + 6*c^2*#1^2 - 8*b^2*d^2*#1^2 + 16*a*c*d^2*#1^2 + 16*a^2*
d^4*#1^2 - 4*c^2*#1^3 + 4*b^2*d^2*#1^3 - 8*a*c*d^2*#1^3 + c^2*#1^4 & , (Log
[-1 + 2*d^2*x^2 + 2*Sqrt[-d^2]*x*Sqrt[1 - d^2*x^2] + #1]*#1^2)/(-c^2 + b^2*
d^2 - 2*a*c*d^2 + 3*c^2*#1 - 4*b^2*d^2*#1 + 8*a*c*d^2*#1 + 8*a^2*d^4*#1 - 3
*c^2*#1^2 + 3*b^2*d^2*#1^2 - 6*a*c*d^2*#1^2 + c^2*#1^3) & ] + b*d^2*(-1 + d
*x)*(1 + d*x)*RootSum[c^2 - 4*c^2*#1^2 + 4*b^2*d^2*#1^2 - 8*a*c*d^2*#1^2 +
6*c^2*#1^4 - 8*b^2*d^2*#1^4 + 16*a*c*d^2*#1^4 + 16*a^2*d^4*#1^4 - 4*c^2*#1^
6 + 4*b^2*d^2*#1^6 - 8*a*c*d^2*#1^6 + c^2*#1^8 & , (-(c^2*Log[-(Sqrt[-d^2]*
x) + Sqrt[1 - d^2*x^2] - #1]) + 7*c^2*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^
2] - #1]*#1^2 - 4*b^2*d^2*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2] - #1]*#1^
2 + 8*a*c*d^2*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2] - #1]*#1^2 - 7*c^2*Lo
g[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2] - #1]*#1^4 + 4*b^2*d^2*Log[-(Sqrt[-d^
2]*x) + Sqrt[1 - d^2*x^2] - #1]*#1^4 - 8*a*c*d^2*Log[-(Sqrt[-d^2]*x) + Sqrt
[1 - d^2*x^2] - #1]*#1^4 + c^2*Log[-(Sqrt[-d^2]*x) + Sqrt[1 - d^2*x^2] - #1
]*#1^6)/(-(c^2*#1) + b^2*d^2*#1 - 2*a*c*d^2*#1 + 3*c^2*#1^3 - 4*b^2*d^2*#1^
3 + 8*a*c*d^2*#1^3 + 8*a^2*d^4*#1^3 - 3*c^2*#1^5 + 3*b^2*d^2*#1^5 - 6*a*c*d
^2*#1^5 + c^2*#1^7) & ])/(4*(c + d*(-b + a*d))*(c + d*(b + a*d))*(1 - d*x)*
(1 + d*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 11142, normalized size = 25.15

method	result	size
default	Expression too large to display	11142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21628 vs. $2(404) = 808$.

time = 28.51, size = 21628, normalized size = 48.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*b*d^4*x^2 - 2*b*d^2 - sqrt(2)*(a^2*d^4 - (b^2 - 2*a*c)*d^2 - (a^2*d^
6 - (b^2 - 2*a*c)*d^4 + c^2*d^2)*x^2 + c^2)*sqrt(-((b^6 - 6*a*b^4*c + 9*a^2
*b^2*c^2 - 2*a^3*c^3)*d^6 - 2*c^6 - 3*(b^4*c^2 - 4*a*b^2*c^3 + 2*a^2*c^4)*d
^4 + 3*(b^2*c^4 - 2*a*c^5)*d^2 + ((a^6*b^2 - 4*a^7*c)*d^12 - 3*(a^4*b^4 - 6
*a^5*b^2*c + 8*a^6*c^2)*d^10 + 3*(a^2*b^6 - 8*a^3*b^4*c + 21*a^4*b^2*c^2 -
20*a^5*c^3)*d^8 + b^2*c^6 - 4*a*c^7 - (b^8 - 10*a*b^6*c + 42*a^2*b^4*c^2 -
92*a^3*b^2*c^3 + 80*a^4*c^4)*d^6 + 3*(b^6*c^2 - 8*a*b^4*c^3 + 21*a^2*b^2*c^
4 - 20*a^3*c^5)*d^4 - 3*(b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*d^2)*sqrt((9*b^
2*c^8*d^4 + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2
*c^4)*d^12 - 6*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5)*d^1
0 + 3*(5*b^6*c^4 - 20*a*b^4*c^5 + 18*a^2*b^2*c^6)*d^8 - 18*(b^4*c^6 - 2*a*b
^2*c^7)*d^6)/((a^12*b^2 - 4*a^13*c)*d^24 - 6*(a^10*b^4 - 6*a^11*b^2*c + 8*a
^12*c^2)*d^22 + 3*(5*a^8*b^6 - 40*a^9*b^4*c + 102*a^10*b^2*c^2 - 88*a^11*c^
3)*d^20 - 10*(2*a^6*b^8 - 20*a^7*b^6*c + 75*a^8*b^4*c^2 - 130*a^9*b^2*c^3 +
 88*a^10*c^4)*d^18 + 15*(a^4*b^10 - 12*a^5*b^8*c + 60*a^6*b^6*c^2 - 160*a^7
*b^4*c^3 + 225*a^8*b^2*c^4 - 132*a^9*c^5)*d^16 - 6*(a^2*b^12 - 14*a^3*b^10*
c + 90*a^4*b^8*c^2 - 340*a^5*b^6*c^3 + 770*a^6*b^4*c^4 - 972*a^7*b^2*c^5 +
528*a^8*c^6)*d^14 + b^2*c^12 - 4*a*c^13 + (b^14 - 16*a*b^12*c + 138*a^2*b^1
0*c^2 - 760*a^3*b^8*c^3 + 2650*a^4*b^6*c^4 - 5712*a^5*b^4*c^5 + 6972*a^6*b^
2*c^6 - 3696*a^7*c^7)*d^12 - 6*(b^12*c^2 - 14*a*b^10*c^3 + 90*a^2*b^8*c^4 -
 340*a^3*b^6*c^5 + 770*a^4*b^4*c^6 - 972*a^5*b^2*c^7 + 528*a^6*c^8)*d^10 +
15*(b^10*c^4 - 12*a*b^8*c^5 + 60*a^2*b^6*c^6 - 160*a^3*b^4*c^7 + 225*a^4*b^
2*c^8 - 132*a^5*c^9)*d^8 - 10*(2*b^8*c^6 - 20*a*b^6*c^7 + 75*a^2*b^4*c^8 -
130*a^3*b^2*c^9 + 88*a^4*c^10)*d^6 + 3*(5*b^6*c^8 - 40*a*b^4*c^9 + 102*a^2*
b^2*c^10 - 88*a^3*c^11)*d^4 - 6*(b^4*c^10 - 6*a*b^2*c^11 + 8*a^2*c^12)*d^2)
))/((a^6*b^2 - 4*a^7*c)*d^12 - 3*(a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d^10 +
 3*(a^2*b^6 - 8*a^3*b^4*c + 21*a^4*b^2*c^2 - 20*a^5*c^3)*d^8 + b^2*c^6 - 4*
a*c^7 - (b^8 - 10*a*b^6*c + 42*a^2*b^4*c^2 - 92*a^3*b^2*c^3 + 80*a^4*c^4)*d
^6 + 3*(b^6*c^2 - 8*a*b^4*c^3 + 21*a^2*b^2*c^4 - 20*a^3*c^5)*d^4 - 3*(b^4*c
^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*d^2))*log(-(12*a*b*c^7*d^2 + 4*(a*b^5*c^3 - 4
*a^2*b^3*c^4 + 3*a^3*b*c^5)*d^6 - 12*(a*b^3*c^5 - 2*a^2*b*c^6)*d^4 + 2*((a^
6*b^2*c^3 - 4*a^7*c^4)*d^12 + b^2*c^9 - 4*a*c^10 - 3*(a^4*b^4*c^3 - 6*a^5*b
^2*c^4 + 8*a^6*c^5)*d^10 + 3*(a^2*b^6*c^3 - 8*a^3*b^4*c^4 + 21*a^4*b^2*c^5
- 20*a^5*c^6)*d^8 - (b^8*c^3 - 10*a*b^6*c^4 + 42*a^2*b^4*c^5 - 92*a^3*b^2*c
^6 + 80*a^4*c^7)*d^6 + 3*(b^6*c^5 - 8*a*b^4*c^6 + 21*a^2*b^2*c^7 - 20*a^3*c
^8)*d^4 - 3*(b^4*c^7 - 6*a*b^2*c^8 + 8*a^2*c^9)*d^2)*x*sqrt((9*b^2*c^8*d^4
+ (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^12
```

```
 - 6*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5)*d^10 + 3*(5*b
^6*c^4 - 20*a*b^4*c^5 + 18*a^2*b^2*c^6)*d^8 - 18*(b^4*c^6 - 2*a*b^2*c^7)*d^
6)/((a^12*b^2 - 4*a^13*c)*d^24 - 6*(a^10*b^4 - 6*a^11*b^2*c + 8*a^12*c^2)*d
^22 + 3*(5*a^8*b^6 - 40*a^9*b^4*c + 102*a^10*b^2*c^2 - 88*a^11*c^3)*d^20 -
10*(2*a^6*b^8 - 20*a^7*b^6*c + 75*a^8*b^4*c^2 - 130*a^9*b^2*c^3 + 88*a^10*c
^4)*d^18 + 15*(a^4*b^10 - 12*a^5*b^8*c + 60*a^6*b^6*c^2 - 160*a^7*b^4*c^3 +
 225*a^8*b^2*c^4 - 132*a^9*c^5)*d^16 - 6*(a^2*b^12 - 14*a^3*b^10*c + 90*a^4
*b^8*c^2 - 340*a^5*b^6*c^3 + 770*a^6*b^4*c^4 - 972*a^7*b^2*c^5 + 528*a^8*c^
6)*d^14 + b^2*c^12 - 4*a*c^13 + (b^14 - 16*a*b^12*c + 138*a^2*b^10*c^2 - 76
0*a^3*b^8*c^3 + 2650*a^4*b^6*c^4 - 5712*a^5*b^4*c^5 + 6972*a^6*b^2*c^6 - 36
96*a^7*c^7)*d^12 - 6*(b^12*c^2 - 14*a*b^10*c^3 + 90*a^2*b^8*c^4 - 340*a^3*b
^6*c^5 + 770*a^4*b^4*c^6 - 972*a^5*b^2*c^7 + 528*a^6*c^8)*d^10 + 15*(b^10*c
^4 - 12*a*b^8*c^5 + 60*a^2*b^6*c^6 - 160*a^3*b^4*c^7 + 225*a^4*b^2*c^8 - 13
2*a^5*c^9)*d^8 - 10*(2*b^8*c^6 - 20*a*b^6*c^7 + 75*a^2*b^4*c^8 - 130*a^3*b^
2*c^9 + 88*a^4*c^10)*d^6 + 3*(5*b^6*c^8 - 40*a*b^4*c^9 + 102*a^2*b^2*c^10 -
 88*a^3*c^11)*d^4 - 6*(b^4*c^10 - 6*a*b^2*c^11 + 8*a^2*c^12)*d^2)) - 4*(3*a
*b*c^7*d^2 + (a*b^5*c^3 - 4*a^2*b^3*c^4 + 3*a^3*b*c^5)*d^6 - 3*(a*b^3*c^5 -
 2*a^2*b*c^6)*d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(3*b^2*c^7*d^2 + (b^6*c
^3 - 4*a*b^4*c^4 + 3*a^2*b^2*c^5)*d^6 - 3*(b^4*c^5 - 2*a*b^2*c^6)*d^4)*x +
sqrt(2)*(((a^7*b^5 - 7*a^8*b^3*c + 12*a^9*b*c^2)*d^16 - (3*a^5*b^7 - 27*a^6
*b^5*c + 80*a^7*b^3*c^2 - 80*a^8*b*c^3)*d^14 + (3*a^3*b^9 - 33*a^4*b^7*c +
141*a^5*b^5*c^2 - 284*a^6*b^3*c^3 + 224*a^7*b*c^4)*d^12 + b^3*c^9 - 4*a*b*c
^10 - (a*b^11 - 13*a^2*b^9*c + 78*a^3*b^7*c^2 - 263*a^4*b^5*c^3 + 464*a^5*b
^3*c^4 - 336*a^6*b*c^5)*d^10 + 5*(a*b^9*c^2 - 10*a^2*b^7*c^3 + 39*a^3*b^5*c
^4 - 74*a^4*b^3*c^5 + 56*a^5*b*c^6)*d^8 - (b^9*c^3 - a*b^7*c^4 - 33*a^2*b^5
*c^5 + 112*a^3*b^3*c^6 - 112*a^4*b*c^7)*d^6 + (3*b^7*c^5 - 17*a*b^5*c^6 + 2
0*a^2*b^3*c^7)*d^4 - (3*b^5*c^7 - 16*a*b^3*c^8 + 16*a^2*b*c^9)*d^2)*x*sqrt(
(9*b^2*c^8*d^4 + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^
4*b^2*c^4)*d^12 - 6*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5
)*d^10 + 3*(5*b^6*c^4 - 20*a*b^4*c^5 + 18*a^2*b...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-dx+1)^{\frac{3}{2}}\,(dx+1)^{\frac{3}{2}}\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(1/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)*(a + b*x + c*x**2)), x)
```

Giac [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(1-d\,x\right)^{3/2}\,\left(d\,x+1\right)^{3/2}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)),x)
```

```
[Out] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)), x)
```

3.801 $\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=939

$$-\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+b^2d^4(2b^2+a^2d^2)-c^2d^2(b^2+6a^2d^2)-c(6ab^2d^4+}{(b^2-4ac)\,(c-bd+ad^2)^2\,(c+bd+ad^2)^2\,\sqrt{1-d^2x^2}}$$

```
[Out] -d^2*(b*(-11*a^2*c*d^4+3*a*b^2*d^4-10*a*c^2*d^2+2*b^2*c*d^2+c^3)-(2*c^4+b^2
*d^4*(a^2*d^2+2*b^2)-c^2*d^2*(6*a^2*d^2+b^2)-c*(4*a^3*d^6+6*a*b^2*d^4))*x)/
(-4*a*c+b^2)/(a*d^2-b*d+c)^2/(a*d^2+b*d+c)^2/(-d^2*x^2+1)^(1/2)+(-b*(b^2*d^
2-c*(3*a*d^2+c))+c*(2*a*c*d^2-b^2*d^2+2*c^2)*x)/(-4*a*c+b^2)/(b^2*d^2-(a*d^
2+c)^2)/(c*x^2+b*x+a)/(-d^2*x^2+1)^(1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b-(-
4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4
*a*c+b^2)^(1/2)))^(1/2))*(4*c^5+24*a*c^4*d^2+3*a*b^3*d^6*(b+(-4*a*c+b^2)^(1
/2))-c^3*d^2*(9*b^2-36*a^2*d^2-b*(-4*a*c+b^2)^(1/2))-2*a*c^2*d^4*(7*b^2-8*a
^2*d^2+5*b*(-4*a*c+b^2)^(1/2))+b*c*d^4*(2*b^3-17*a^2*b*d^2+2*b^2*(-4*a*c+b^
2)^(1/2)-11*a^2*d^2*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/(a^2*d^4+2*a*c*
d^2-b^2*d^2+c^2)^2*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b-(-4*a*c+b^2)^(1/2)))^(
1/2)+1/2*c*arctanh(1/2*(2*c+d^2*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-d^2*x^2
+1)^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*c^5*d^2
-24*a*c^4*d^4-6*a*b^4*d^8-4*b^2*c*d^6*(-7*a^2*d^2+b^2)+2*c^3*(-18*a^2*d^6+4
*b^2*d^4)+8*c^2*(-2*a^3*d^8+3*a*b^2*d^6)+b*d^4*(-11*a^2*c*d^4+3*a*b^2*d^4-1
0*a*c^2*d^2+2*b^2*c*d^2+c^3)*(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(3/2)/d^2
/(a^2*d^4+2*a*c*d^2-b^2*d^2+c^2)^2*2^(1/2)/(2*c^2+2*a*c*d^2-b*d^2*(b+(-4*a*
c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 11.23, antiderivative size = 938, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {913, 989, 1076, 1048, 739, 212}

Antiderivative was successfully verified.

```
[In] Int[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((d^2*(b*(c^3 + 2*b^2*c*d^2 - 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) -
 (2*c^4 + b^2*d^4*(2*b^2 + a^2*d^2) - c^2*d^2*(b^2 + 6*a^2*d^2) - c*(6*a*b^
2*d^4 + 4*a^3*d^6))*x))/((b^2 - 4*a*c)*(c - b*d + a*d^2)^2*(c + b*d + a*d^2
)^2*Sqrt[1 - d^2*x^2])) - (b*(b^2*d^2 - c*(c + 3*a*d^2)) - c*(2*c^2 - b^2*d
```

```
^2 + 2*a*c*d^2)*x)/((b^2 - 4*a*c)*(b^2*d^2 - (c + a*d^2)^2)*(a + b*x + c*x^
2)*Sqrt[1 - d^2*x^2]) + (c*(4*c^5 + 24*a*c^4*d^2 + 3*a*b^3*(b + Sqrt[b^2 -
4*a*c])*d^6 - c^3*d^2*(9*b^2 - b*Sqrt[b^2 - 4*a*c] - 36*a^2*d^2) - 2*a*c^2*
d^4*(7*b^2 + 5*b*Sqrt[b^2 - 4*a*c] - 8*a^2*d^2) + b*c*d^4*(2*b^3 + 2*b^2*Sq
rt[b^2 - 4*a*c] - 17*a^2*b*d^2 - 11*a^2*Sqrt[b^2 - 4*a*c]*d^2))*ArcTanh[(2*
c + (b - Sqrt[b^2 - 4*a*c])*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b -
 Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*
Sqrt[2*c^2 + 2*a*c*d^2 - b*(b - Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*
a*c*d^2 + a^2*d^4)^2) - (c*(4*c^5*d^2 + 24*a*c^4*d^4 + 6*a*b^4*d^8 + 4*b^2*
c*d^6*(b^2 - 7*a^2*d^2) - b*(b + Sqrt[b^2 - 4*a*c])*d^4*(c^3 + 2*b^2*c*d^2
- 10*a*c^2*d^2 + 3*a*b^2*d^4 - 11*a^2*c*d^4) - 4*c^3*(2*b^2*d^4 - 9*a^2*d^6
) - 8*c^2*(3*a*b^2*d^6 - 2*a^3*d^8))*ArcTanh[(2*c + (b + Sqrt[b^2 - 4*a*c])
*d^2*x)/(Sqrt[2]*Sqrt[2*c^2 + 2*a*c*d^2 - b*(b + Sqrt[b^2 - 4*a*c])*d^2]*Sq
rt[1 - d^2*x^2])])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*d^2*Sqrt[2*c^2 + 2*a*c*d^2
- b*(b + Sqrt[b^2 - 4*a*c])*d^2]*(c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
```

```
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d
+ e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*((A*c
- a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*d - c*(2*a*f)) + c*(A*(2*c^2*d - c*(2*a*
f)) - B*(-2*a*c*e) + C*(-2*a*(c*d - a*f)))*x), x] + Dist[1/((-4*a*c)*(a*c*e
^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*S
imp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p + 1) + (2*(A*c*(c*
d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*
c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2) -
(2*f*((A*c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p
 + q + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*((-c)*e*(2*p
+ q + 4)))*x - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p +
 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e
^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !Int
egerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}\left(a+bx+cx^2\right)^2}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2\left(1-d^2x^2\right)^{3/2}}\,dx$$

$$= -\frac{b(b^2d^2-c(c+3ad^2))-c(2c^2-b^2d^2+2acd^2)\,x}{(b^2-4ac)\left(b^2d^2-(c+ad^2)^2\right)(a+bx+cx^2)\sqrt{1-d^2x^2}}$$

$$= -\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+ \cdots}{(b^2-4ac)\left(b^2d^2-(\cdots\right.}$$

$$= -\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+ \cdots}{(b^2-4ac)\left(b^2d^2-(\cdots\right.}$$

$$= -\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+ \cdots}{(b^2-4ac)\left(b^2d^2-(\cdots\right.}$$

$$= -\frac{d^2(b(c^3+2b^2cd^2-10ac^2d^2+3ab^2d^4-11a^2cd^4)-(2c^4+ \cdots}{(b^2-4ac)\left(b^2d^2-(\cdots\right.}$$

Mathematica [A]
time = 14.69, size = 1302, normalized size = 1.39

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-2*Sqrt[1 - d^2*x^2]*((d^4*(-2*b*(c + a*d^2) + b^2*d^2*x + (c + a*d^2)^2*x
))/(-1 + d^2*x^2) + (b^5*d^4 - b^3*c*d^2*(2*c + 5*a*d^2) + b*c^2*(c^2 + 6*a
*c*d^2 + 5*a^2*d^4) + b^4*c*d^4*x + 2*c^3*(c + a*d^2)^2*x - 2*b^2*c^2*d^2*(
c + 2*a*d^2)*x)/((b^2 - 4*a*c)*(a + x*(b + c*x)))) - (c*(4*c^5 + 24*a*c^4*d
^2 + 3*a*b^3*(b + Sqrt[b^2 - 4*a*c])*d^6 + 2*a*c^2*d^4*(-7*b^2 - 5*b*Sqrt[b
^2 - 4*a*c] + 8*a^2*d^2) + c^3*d^2*(-9*b^2 + b*Sqrt[b^2 - 4*a*c] + 36*a^2*d
^2) + b*c*d^4*(2*b^3 + 2*b^2*Sqrt[b^2 - 4*a*c] - 17*a^2*b*d^2 - 11*a^2*Sqrt
[b^2 - 4*a*c]*d^2))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x])/((b^2 - 4*a*c)^(3/
2)*Sqrt[c^2 + a*c*d^2 + (b*(-b + Sqrt[b^2 - 4*a*c])*d^2)/2]) + (c*(4*c^5 +
```

```
24*a*c^4*d^2 + 3*a*b^3*(b - Sqrt[b^2 - 4*a*c])*d^6 + 2*a*c^2*d^4*(-7*b^2 +
5*b*Sqrt[b^2 - 4*a*c] + 8*a^2*d^2) + c^3*d^2*(-9*b^2 - b*Sqrt[b^2 - 4*a*c]
+ 36*a^2*d^2) + b*c*d^4*(2*b^3 - 2*b^2*Sqrt[b^2 - 4*a*c] - 17*a^2*b*d^2 + 1
1*a^2*Sqrt[b^2 - 4*a*c]*d^2))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/((b^2 - 4
*a*c)^(3/2)*Sqrt[c^2 + a*c*d^2 - (b*(b + Sqrt[b^2 - 4*a*c])*d^2)/2]) + (c*(
4*c^5 + 24*a*c^4*d^2 + 3*a*b^3*(b + Sqrt[b^2 - 4*a*c])*d^6 + 2*a*c^2*d^4*(-
7*b^2 - 5*b*Sqrt[b^2 - 4*a*c] + 8*a^2*d^2) + c^3*d^2*(-9*b^2 + b*Sqrt[b^2 -
 4*a*c] + 36*a^2*d^2) + b*c*d^4*(2*b^3 + 2*b^2*Sqrt[b^2 - 4*a*c] - 17*a^2*b
*d^2 - 11*a^2*Sqrt[b^2 - 4*a*c]*d^2))*Log[-2*c - b*d^2*x + Sqrt[b^2 - 4*a*c
]*d^2*x - Sqrt[4*c^2 + 4*a*c*d^2 + 2*b*(-b + Sqrt[b^2 - 4*a*c])*d^2]*Sqrt[1
 - d^2*x^2]])/((b^2 - 4*a*c)^(3/2)*Sqrt[c^2 + a*c*d^2 + (b*(-b + Sqrt[b^2 -
 4*a*c])*d^2)/2]) + (c*(-4*c^5 - 24*a*c^4*d^2 + 3*a*b^3*(-b + Sqrt[b^2 - 4*
a*c])*d^6 + c^3*d^2*(9*b^2 + b*Sqrt[b^2 - 4*a*c] - 36*a^2*d^2) - 2*a*c^2*d^
4*(-7*b^2 + 5*b*Sqrt[b^2 - 4*a*c] + 8*a^2*d^2) + b*c*d^4*(-2*b^3 + 2*b^2*Sq
rt[b^2 - 4*a*c] + 17*a^2*b*d^2 - 11*a^2*Sqrt[b^2 - 4*a*c]*d^2))*Log[2*c + b
*d^2*x + Sqrt[b^2 - 4*a*c]*d^2*x + Sqrt[4*c^2 + 4*a*c*d^2 - 2*b*(b + Sqrt[b
^2 - 4*a*c])*d^2]*Sqrt[1 - d^2*x^2]])/((b^2 - 4*a*c)^(3/2)*Sqrt[c^2 + a*c*d
^2 - (b*(b + Sqrt[b^2 - 4*a*c])*d^2)/2]))/(2*(c^2 - b^2*d^2 + 2*a*c*d^2 + a
^2*d^4)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.56, size = 108974, normalized size = 116.05

method	result	size
default	Expression too large to display	108974

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(1-d\,x)^{3/2}\,(d\,x+1)^{3/2}\,(c\,x^2+b\,x+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] int(1/((1 - d*x)^(3/2)*(d*x + 1)^(3/2)*(a + b*x + c*x^2)^2), x)
```

3.802 $\int (1-ex)^m (1+ex)^m \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=54

$$x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)$$

```
[Out] x*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2,-c*x^2/a)/((c*x^2/a+1)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {531, 441, 440}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (c
*x^2)/a)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rubi steps

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p dx = \int \left(a+cx^2\right)^p \left(1-e^2x^2\right)^m dx$$
$$= \left(\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right) \int \left(1+\frac{cx^2}{a}\right)^p \left(1-e^2x^2\right)^m dx$$
$$= x\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(54) = 108.
time = 0.23, size = 167, normalized size = 3.09

$$\frac{3ax(a+cx^2)^p(1-e^2x^2)^m F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)}{3aF_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},e^2x^2\right)+2x^2\left(cpF_1\left(\frac{3}{2};1-p,-m;\frac{5}{2};-\frac{cx^2}{a},e^2x^2\right)-ae^2mF_1\left(\frac{3}{2};-p,1-m;\frac{5}{2};-\frac{cx^2}{a},e^2x^2\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
[Out] (3*a*x*(a + c*x^2)^p*(1 - e^2*x^2)^m*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a
), e^2*x^2])/(3*a*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2] + 2*x^2
*(c*p*AppellF1[3/2, 1 - p, -m, 5/2, -((c*x^2)/a), e^2*x^2] - a*e^2*m*Appell
F1[3/2, -p, 1 - m, 5/2, -((c*x^2)/a), e^2*x^2]))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(-ex+1\right)^m \left(ex+1\right)^m \left(c\,x^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
[Out] int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(x*e + 1)^m*(-x*e + 1)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(x*e + 1)^m*(-x*e + 1)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)**m*(e*x+1)**m*(c*x**2+a)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(x*e + 1)^m*(-x*e + 1)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x^2+a)^p\,(1-e\,x)^m\,(e\,x+1)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m,x)
[Out] int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m, x)
```

3.803 $\int (d-ex)^m (d+ex)^m (a+cx^2)^p \, dx$

Optimal. Leaf size=89

$$x(d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[Out] x*(-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,-c*x^
2/a)/((c*x^2/a+1)^p)/((1-e^2*x^2/d^2)^m)
```

Rubi [A]

time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {533, 441, 440}

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d-ex)^m(d+ex)^m\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^
2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx &= \left((d-ex)^m(d+ex)^m\left(d^2-e^2x^2\right)^{-m}\right)\int \left(a+cx^2\right)^p\left(d^2-e^2x^2\right)^m\,dx \\
&= \left((d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d^2-e^2x^2\right)^{-m}\right)\int \Bigg(\\
&= \left((d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\right)\int \Bigg(\\
&= x(d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}F_1\Bigg(\frac{1}{2}
\end{aligned}
$$

Mathematica [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
[Out] Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p, x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(-ex+d\right)^{m}\left(ex+d\right)^{m}\left(c\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
[Out] int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(x*e + d)^m*(-x*e + d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^2 + a)^p*(x*e + d)^m*(-x*e + d)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)**m*(e*x+d)**m*(c*x**2+a)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^2 + a)^p*(x*e + d)^m*(-x*e + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+a\right)^p\,{\left(d+e\,x\right)}^m\,{\left(d-e\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m,x)
[Out] int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m, x)
```

3.804 $\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \, dx$

Optimal. Leaf size=92

$$x(d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(1-\frac{e^2x^2}{d^2}\right)^{-m} F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

```
[Out] x*(e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,-
c*x^2/a)/((c*x^2/a+1)^p)/((1-e^2*x^2/d^2)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {533, 441, 440}

$$x\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} (df-efx)^m F_1\left(\frac{1}{2};-p,-m;\frac{3}{2};-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

```
[Out] (x*(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((
c*x^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \, dx = \left((d+ex)^m(df-efx)^m \left(d^2f-e^2fx^2\right)^{-m}\right) \int \left(a+cx^2\right)^p \left(d^2f-\right.$$

$$= \left((d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d^2f-e^2fx^2\right)\right.$$

$$= \left((d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(1-\frac{e^2x^2}{d^2}\right)^{-m}\right.$$

$$= x(d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(1-\frac{e^2x^2}{d^2}\right)^{-m}$$

Mathematica [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p, x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (ex+d)^m \left(-efx+df\right)^m \left(c\,x^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
[Out] int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^p*(-f*x*e + d*f)^m*(x*e + d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + a)^p*(-f*x*e + d*f)^m*(x*e + d)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(-e*f*x+d*f)**m*(c*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(-f*x*e + d*f)^m*(x*e + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,f-e\,f\,x)^m\,\left(c\,x^2+a\right)^p\,(d+e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m,x)
```

```
[Out] int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m, x)
```

3.805 $\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=275

$$-\frac{(ef-dg)^3\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^6(1+n)}+\frac{(ef-dg)^2\left(3aeg^2+c(5e^2f^2-10defg+2d^2g^2)\right)(f+gx)}{g^6(2+n)}$$

```
[Out] -(-d*g+e*f)^3*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^6/(1+n)+(-d*g+e*f)^2
*(3*a*e*g^2+c*(2*d^2*g^2-10*d*e*f*g+5*e^2*f^2))*(g*x+f)^(2+n)/g^6/(2+n)-e*(
-d*g+e*f)*(3*a*e*g^2+c*(7*d^2*g^2-20*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3+n)/g^6
/(3+n)+e^2*(a*e*g^2+c*(9*d^2*g^2-20*d*e*f*g+10*e^2*f^2))*(g*x+f)^(4+n)/g^6/
(4+n)-5*c*e^3*(-d*g+e*f)*(g*x+f)^(5+n)/g^6/(5+n)+c*e^4*(g*x+f)^(6+n)/g^6/(6
+n)
```

Rubi [A]

time = 0.17, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{961\}$

$$\frac{(ef-dg)^2(f+gx)^{n+2}\left(3aeg^2+c(2d^2g^2-10defg+5e^2f^2)\right)}{g^6(n+2)}-\frac{e(ef-dg)(f+gx)^{n+3}\left(3aeg^2+c(7d^2g^2-20defg+10e^2f^2)\right)}{g^6(n+3)}+\frac{e^2(f+gx)^{n+4}\left(aeg^2+c(9d^2g^2-20defg+10e^2f^2)\right)}{g^6(n+4)}-\frac{(ef-dg)^3(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^6(n+1)}-\frac{5ce^3(ef-dg)(f+gx)^{n+5}}{g^6(n+5)}+\frac{ce^4(f+gx)^{n+6}}{g^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)^3*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^6*(1 + n
))) + ((e*f - d*g)^2*(3*a*e*g^2 + c*(5*e^2*f^2 - 10*d*e*f*g + 2*d^2*g^2))*(
f + g*x)^(2 + n))/(g^6*(2 + n)) - (e*(e*f - d*g)*(3*a*e*g^2 + c*(10*e^2*f^2
 - 20*d*e*f*g + 7*d^2*g^2))*(f + g*x)^(3 + n))/(g^6*(3 + n)) + (e^2*(a*e*g^
2 + c*(10*e^2*f^2 - 20*d*e*f*g + 9*d^2*g^2))*(f + g*x)^(4 + n))/(g^6*(4 + n
)) - (5*c*e^3*(e*f - d*g)*(f + g*x)^(5 + n))/(g^6*(5 + n)) + (c*e^4*(f + g*
x)^(6 + n))/(g^6*(6 + n))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^3(f+gx)^n \left(a+2cdx+cex^2\right)\, dx = \int \left(\frac{(ef-dg)^3 \left(-ag^2-cf(ef-2dg)\right) (f+gx)^n}{g^5} + \frac{(ef-dg}{}\right.$$
$$= -\frac{(ef-dg)^3 \left(ag^2+cf(ef-2dg)\right) (f+gx)^{1+n}}{g^6(1+n)} + \frac{(ef-dg)^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 577 vs. 2(275) = 550.
time = 0.58, size = 577, normalized size = 2.10

$$\frac{(f+gx)^{1+n}\left(ag^2(30+11n+n^2)\left(d^3g^3(24+26n+9n^2+n^3)+3d^2eg^2(12+7n+n^2)(-f+g(1+n)x)+3de^2g(4+n)\left(2f^2-2fg(1+n)x+g^2(2+3n+n^2)x^2\right)+e^3\left(-6f^3+6f^2g(1+n)x-3fg^2(2+3n+n^2)x^2+g^3(6+11n+6n^2+n^3)x^3\right)\right)+c\left(2d^4g^4(360+342n+119n^2+18n^3+n^4)(-f+g(1+n)x)+7d^3eg^3(120+74n+15n^2+n^3)\left(2f^2-2fg(1+n)x+g^2(2+3n+n^2)x^2\right)+9d^2e^2g^2(30+11n+n^2)\left(-6f^3+6f^2g(1+n)x-3fg^2(2+3n+n^2)x^2+g^3(6+11n+6n^2+n^3)x^3\right)+5de^3g(6+n)\left(24f^4-24f^3g(1+n)x+12f^2g^2(2+3n+n^2)x^2-4fg^3(6+11n+6n^2+n^3)x^3+g^4(24+50n+35n^2+10n^3+n^4)x^4\right)-e^4\left(120f^5-120f^4g(1+n)x+60f^3g^2(2+3n+n^2)x^2-20f^2g^3(6+11n+6n^2+n^3)x^3+5fg^4(24+50n+35n^2+10n^3+n^4)x^4-g^5(120+274n+225n^2+85n^3+15n^4+n^5)x^5\right)\right)\right)}{g^6(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(a*g^2*(30 + 11*n + n^2)*(d^3*g^3*(24 + 26*n + 9*n^2 + n
^3) + 3*d^2*e*g^2*(12 + 7*n + n^2)*(-f + g*(1 + n)*x) + 3*d*e^2*g*(4 + n)*(
2*f^2 - 2*f*g*(1 + n)*x + g^2*(2 + 3*n + n^2)*x^2) + e^3*(-6*f^3 + 6*f^2*g*
(1 + n)*x - 3*f*g^2*(2 + 3*n + n^2)*x^2 + g^3*(6 + 11*n + 6*n^2 + n^3)*x^3)
) + c*(2*d^4*g^4*(360 + 342*n + 119*n^2 + 18*n^3 + n^4)*(-f + g*(1 + n)*x)
+ 7*d^3*e*g^3*(120 + 74*n + 15*n^2 + n^3)*(2*f^2 - 2*f*g*(1 + n)*x + g^2*(2
 + 3*n + n^2)*x^2) + 9*d^2*e^2*g^2*(30 + 11*n + n^2)*(-6*f^3 + 6*f^2*g*(1 +
 n)*x - 3*f*g^2*(2 + 3*n + n^2)*x^2 + g^3*(6 + 11*n + 6*n^2 + n^3)*x^3) + 5
*d*e^3*g*(6 + n)*(24*f^4 - 24*f^3*g*(1 + n)*x + 12*f^2*g^2*(2 + 3*n + n^2)*
x^2 - 4*f*g^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + g^4*(24 + 50*n + 35*n^2 + 10*n
^3 + n^4)*x^4) - e^4*(120*f^5 - 120*f^4*g*(1 + n)*x + 60*f^3*g^2*(2 + 3*n +
 n^2)*x^2 - 20*f^2*g^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + 5*f*g^4*(24 + 50*n +
35*n^2 + 10*n^3 + n^4)*x^4 - g^5*(120 + 274*n + 225*n^2 + 85*n^3 + 15*n^4 +
 n^5)*x^5))))/(g^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. 2(275) = 550.
time = 0.14, size = 1786, normalized size = 6.49

method	result	size
norman	Expression too large to display	1786
gosper	Expression too large to display	2017
risch	Expression too large to display	2642

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*e^4/(6+n)*x^6*exp(n*ln(g*x+f))+f*(a*d^3*g^5*n^5-2*c*d^4*f*g^4*n^4+20*a*d^
3*g^5*n^4-3*a*d^2*e*f*g^4*n^4-36*c*d^4*f*g^4*n^3+14*c*d^3*e*f^2*g^3*n^3+155
*a*d^3*g^5*n^3-54*a*d^2*e*f*g^4*n^3+6*a*d*e^2*f^2*g^3*n^3-238*c*d^4*f*g^4*n
^2+210*c*d^3*e*f^2*g^3*n^2-54*c*d^2*e^2*f^3*g^2*n^2+580*a*d^3*g^5*n^2-357*a
*d^2*e*f*g^4*n^2+90*a*d*e^2*f^2*g^3*n^2-6*a*e^3*f^3*g^2*n^2-684*c*d^4*f*g^4
*n+1036*c*d^3*e*f^2*g^3*n-594*c*d^2*e^2*f^3*g^2*n+120*c*d*e^3*f^4*g*n+1044*
a*d^3*g^5*n-1026*a*d^2*e*f*g^4*n+444*a*d*e^2*f^2*g^3*n-66*a*e^3*f^3*g^2*n-7
20*c*d^4*f*g^4+1680*c*d^3*e*f^2*g^3-1620*c*d^2*e^2*f^3*g^2+720*c*d*e^3*f^4*
g-120*c*e^4*f^5+720*a*d^3*g^5-1080*a*d^2*e*f*g^4+720*a*d*e^2*f^2*g^3-180*a*
e^3*f^3*g^2)/g^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*exp(n*ln(
g*x+f))+(2*c*d^4*g^4*n^4+7*c*d^3*e*f*g^3*n^4+3*a*d^2*e*g^4*n^4+3*a*d*e^2*f*
g^3*n^4+36*c*d^4*g^4*n^3+105*c*d^3*e*f*g^3*n^3-27*c*d^2*e^2*f^2*g^2*n^3+54*
a*d^2*e*g^4*n^3+45*a*d*e^2*f*g^3*n^3-3*a*e^3*f^2*g^2*n^3+238*c*d^4*g^4*n^2+
518*c*d^3*e*f*g^3*n^2-297*c*d^2*e^2*f^2*g^2*n^2+60*c*d*e^3*f^3*g*n^2+357*a*
d^2*e*g^4*n^2+222*a*d*e^2*f*g^3*n^2-33*a*e^3*f^2*g^2*n^2+684*c*d^4*g^4*n+84
0*c*d^3*e*f*g^3*n-810*c*d^2*e^2*f^2*g^2*n+360*c*d*e^3*f^3*g*n-60*c*e^4*f^4*
n+1026*a*d^2*e*g^4*n+360*a*d*e^2*f*g^3*n-90*a*e^3*f^2*g^2*n+720*c*d^4*g^4+1
080*a*d^2*e*g^4)/g^4/(n^5+20*n^4+155*n^3+580*n^2+1044*n+720)*x^2*exp(n*ln(g
*x+f))+(2*c*d^4*f*g^4*n^5+a*d^3*g^5*n^5+3*a*d^2*e*f*g^4*n^5+36*c*d^4*f*g^4*
n^4-14*c*d^3*e*f^2*g^3*n^4+20*a*d^3*g^5*n^4+54*a*d^2*e*f*g^4*n^4-6*a*d*e^2*
f^2*g^3*n^4+238*c*d^4*f*g^4*n^3-210*c*d^3*e*f^2*g^3*n^3+54*c*d^2*e^2*f^3*g^
2*n^3+155*a*d^3*g^5*n^3+357*a*d^2*e*f*g^4*n^3-90*a*d*e^2*f^2*g^3*n^3+6*a*e^
3*f^3*g^2*n^3+684*c*d^4*f*g^4*n^2-1036*c*d^3*e*f^2*g^3*n^2+594*c*d^2*e^2*f^
3*g^2*n^2-120*c*d*e^3*f^4*g*n^2+580*a*d^3*g^5*n^2+1026*a*d^2*e*f*g^4*n^2-44
4*a*d*e^2*f^2*g^3*n^2+66*a*e^3*f^3*g^2*n^2+720*c*d^4*f*g^4*n-1680*c*d^3*e*f
^2*g^3*n+1620*c*d^2*e^2*f^3*g^2*n-720*c*d*e^3*f^4*g*n+120*c*e^4*f^5*n+1044*
a*d^3*g^5*n+1080*a*d^2*e*f*g^4*n-720*a*d*e^2*f^2*g^3*n+180*a*e^3*f^3*g^2*n+
720*a*d^3*g^5)/g^5/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*x*exp(n
*ln(g*x+f))+(9*c*d^2*g^2*n^2+5*c*d*e*f*g*n^2+a*e*g^2*n^2+99*c*d^2*g^2*n+30*
c*d*e*f*g*n-5*c*e^2*f^2*n+11*a*e*g^2*n+270*c*d^2*g^2+30*a*e*g^2)*e^2/g^2/(n
^3+15*n^2+74*n+120)*x^4*exp(n*ln(g*x+f))+(7*c*d^3*g^3*n^3+9*c*d^2*e*f*g^2*n
^3+3*a*d*e*g^3*n^3+a*e^2*f*g^2*n^3+105*c*d^3*g^3*n^2+99*c*d^2*e*f*g^2*n^2-2
0*c*d*e^2*f^2*g*n^2+45*a*d*e*g^3*n^2+11*a*e^2*f*g^2*n^2+518*c*d^3*g^3*n+270
*c*d^2*e*f*g^2*n-120*c*d*e^2*f^2*g*n+20*c*e^3*f^3*n+222*a*d*e*g^3*n+30*a*e^
2*f*g^2*n+840*c*d^3*g^3+360*a*d*e*g^3)*e/g^3/(n^4+18*n^3+119*n^2+342*n+360)
*x^3*exp(n*ln(g*x+f))+(5*d*g*n+e*f*n+30*d*g)*c/g*e^3/(n^2+11*n+30)*x^5*exp(
n*ln(g*x+f))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(275) = 550$.
time = 0.33, size = 811, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^4/((n^2 + 3*n + 2)*g^2)
 + 7*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d^3*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 3*(g^2*(n + 1)*x^2 + f
*g*n*x - f^2)*(g*x + f)^n*a*d^2*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)
*a*d^3/(g*(n + 1)) + 9*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2
*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*
c*d^2*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + 3*((n^2 + 3*n + 2)*g^
3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*d*e^2/((n^
3 + 6*n^2 + 11*n + 6)*g^3) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5
 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x
^3 + 12*(n^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*d*e^3/
((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5) + ((n^3 + 6*n^2 + 11*
n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 +
6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*a*e^3/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)
*g^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^6*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*f*g^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*
f^2*g^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*f^3*g^3*x^3 - 60*(n^2 + n)*f^4*g^2*x^2
 + 120*f^5*g*n*x - 120*f^6)*(g*x + f)^n*c*e^4/((n^6 + 21*n^5 + 175*n^4 + 73
5*n^3 + 1624*n^2 + 1764*n + 720)*g^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2095 vs. $2(282) = 564$.

time = 2.92, size = 2095, normalized size = 7.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d^3*f*g^5*n^5 - 720*c*d^4*f^2*g^4 + 720*a*d^3*f*g^5 - 2*(c*d^4*f^2*g^4 -
 10*a*d^3*f*g^5)*n^4 - (36*c*d^4*f^2*g^4 - 155*a*d^3*f*g^5)*n^3 - 2*(119*c*
d^4*f^2*g^4 - 290*a*d^3*f*g^5)*n^2 + 2*(c*d^4*g^6*n^5 + 19*c*d^4*g^6*n^4 +
137*c*d^4*g^6*n^3 + 461*c*d^4*g^6*n^2 + 702*c*d^4*g^6*n + 360*c*d^4*g^6)*x^
2 - 36*(19*c*d^4*f^2*g^4 - 29*a*d^3*f*g^5)*n + (720*a*d^3*g^6 + (2*c*d^4*f*
g^5 + a*d^3*g^6)*n^5 + 4*(9*c*d^4*f*g^5 + 5*a*d^3*g^6)*n^4 + (238*c*d^4*f*g
^5 + 155*a*d^3*g^6)*n^3 + 4*(171*c*d^4*f*g^5 + 145*a*d^3*g^6)*n^2 + 36*(20*
c*d^4*f*g^5 + 29*a*d^3*g^6)*n)*x + (120*c*f^5*g*n*x - 120*c*f^6 + (c*g^6*n^
5 + 15*c*g^6*n^4 + 85*c*g^6*n^3 + 225*c*g^6*n^2 + 274*c*g^6*n + 120*c*g^6)*
x^6 + (c*f*g^5*n^5 + 10*c*f*g^5*n^4 + 35*c*f*g^5*n^3 + 50*c*f*g^5*n^2 + 24*
c*f*g^5*n)*x^5 - 5*(c*f^2*g^4*n^4 + 6*c*f^2*g^4*n^3 + 11*c*f^2*g^4*n^2 + 6*
c*f^2*g^4*n)*x^4 + 20*(c*f^3*g^3*n^3 + 3*c*f^3*g^3*n^2 + 2*c*f^3*g^3*n)*x^3
 - 60*(c*f^4*g^2*n^2 + c*f^4*g^2*n)*x^2)*e^4 - (6*a*f^4*g^2*n^2 - 720*c*d*f
^5*g + 180*a*f^4*g^2 - 5*(c*d*g^6*n^5 + 16*c*d*g^6*n^4 + 95*c*d*g^6*n^3 + 2
60*c*d*g^6*n^2 + 324*c*d*g^6*n + 144*c*d*g^6)*x^5 - (180*a*g^6 + (5*c*d*f*g
```

```
^5 + a*g^6)*n^5 + (60*c*d*f*g^5 + 17*a*g^6)*n^4 + (235*c*d*f*g^5 + 107*a*g^
6)*n^3 + (360*c*d*f*g^5 + 307*a*g^6)*n^2 + 36*(5*c*d*f*g^5 + 11*a*g^6)*n)*x
^4 - (a*f*g^5*n^5 - 2*(10*c*d*f^2*g^4 - 7*a*f*g^5)*n^4 - 5*(36*c*d*f^2*g^4
- 13*a*f*g^5)*n^3 - 16*(25*c*d*f^2*g^4 - 7*a*f*g^5)*n^2 - 60*(4*c*d*f^2*g^4
 - a*f*g^5)*n)*x^3 + 3*(a*f^2*g^4*n^4 - 4*(5*c*d*f^3*g^3 - 3*a*f^2*g^4)*n^3
 - (140*c*d*f^3*g^3 - 41*a*f^2*g^4)*n^2 - 30*(4*c*d*f^3*g^3 - a*f^2*g^4)*n)
*x^2 - 6*(20*c*d*f^5*g - 11*a*f^4*g^2)*n - 6*(a*f^3*g^3*n^3 - (20*c*d*f^4*g
^2 - 11*a*f^3*g^3)*n^2 - 30*(4*c*d*f^4*g^2 - a*f^3*g^3)*n)*x)*e^3 + 3*(2*a*
d*f^3*g^3*n^3 - 540*c*d^2*f^4*g^2 + 240*a*d*f^3*g^3 + 3*(c*d^2*g^6*n^5 + 17
*c*d^2*g^6*n^4 + 107*c*d^2*g^6*n^3 + 307*c*d^2*g^6*n^2 + 396*c*d^2*g^6*n +
180*c*d^2*g^6)*x^4 + (240*a*d*g^6 + (3*c*d^2*f*g^5 + a*d*g^6)*n^5 + 6*(7*c*
d^2*f*g^5 + 3*a*d*g^6)*n^4 + (195*c*d^2*f*g^5 + 121*a*d*g^6)*n^3 + 12*(28*c
*d^2*f*g^5 + 31*a*d*g^6)*n^2 + 4*(45*c*d^2*f*g^5 + 127*a*d*g^6)*n)*x^3 - 6*
(3*c*d^2*f^4*g^2 - 5*a*d*f^3*g^3)*n^2 + (a*d*f*g^5*n^5 - (9*c*d^2*f^2*g^4 -
 16*a*d*f*g^5)*n^4 - (108*c*d^2*f^2*g^4 - 89*a*d*f*g^5)*n^3 - (369*c*d^2*f^
2*g^4 - 194*a*d*f*g^5)*n^2 - 30*(9*c*d^2*f^2*g^4 - 4*a*d*f*g^5)*n)*x^2 - 2*
(99*c*d^2*f^4*g^2 - 74*a*d*f^3*g^3)*n - 2*(a*d*f^2*g^4*n^4 - 3*(3*c*d^2*f^3
*g^3 - 5*a*d*f^2*g^4)*n^3 - (99*c*d^2*f^3*g^3 - 74*a*d*f^2*g^4)*n^2 - 30*(9
*c*d^2*f^3*g^3 - 4*a*d*f^2*g^4)*n)*x)*e^2 - (3*a*d^2*f^2*g^4*n^4 - 1680*c*d
^3*f^3*g^3 + 1080*a*d^2*f^2*g^4 - 2*(7*c*d^3*f^3*g^3 - 27*a*d^2*f^2*g^4)*n^
3 - 7*(c*d^3*g^6*n^5 + 18*c*d^3*g^6*n^4 + 121*c*d^3*g^6*n^3 + 372*c*d^3*g^6
*n^2 + 508*c*d^3*g^6*n + 240*c*d^3*g^6)*x^3 - 21*(10*c*d^3*f^3*g^3 - 17*a*d
^2*f^2*g^4)*n^2 - (1080*a*d^2*g^6 + (7*c*d^3*f*g^5 + 3*a*d^2*g^6)*n^5 + (11
2*c*d^3*f*g^5 + 57*a*d^2*g^6)*n^4 + (623*c*d^3*f*g^5 + 411*a*d^2*g^6)*n^3 +
 (1358*c*d^3*f*g^5 + 1383*a*d^2*g^6)*n^2 + 6*(140*c*d^3*f*g^5 + 351*a*d^2*g
^6)*n)*x^2 - 2*(518*c*d^3*f^3*g^3 - 513*a*d^2*f^2*g^4)*n - (3*a*d^2*f*g^5*n
^5 - 2*(7*c*d^3*f^2*g^4 - 27*a*d^2*f*g^5)*n^4 - 21*(10*c*d^3*f^2*g^4 - 17*a
*d^2*f*g^5)*n^3 - 2*(518*c*d^3*f^2*g^4 - 513*a*d^2*f*g^5)*n^2 - 120*(14*c*d
^3*f^2*g^4 - 9*a*d^2*f*g^5)*n)*x)*e)*(g*x + f)^n/(g^6*n^6 + 21*g^6*n^5 + 17
5*g^6*n^4 + 735*g^6*n^3 + 1624*g^6*n^2 + 1764*g^6*n + 720*g^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 24206 vs. $2(260) = 520$.
time = 4.97, size = 24206, normalized size = 88.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4
/4 + c*d**4*x**2 + 7*c*d**3*e*x**3/3 + 9*c*d**2*e**2*x**4/4 + c*d*e**3*x**5
 + c*e**4*x**6/6), Eq(g, 0)), (-12*a*d**3*g**5/(60*f**5*g**6 + 300*f**4*g**
7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11
*x**5) - 9*a*d**2*e*f*g**4/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*
```

x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 45*a*d**2*e
*g**5*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**
9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 6*a*d*e**2*f**2*g**3/(60*f**5*
g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g*
*10*x**4 + 60*g**11*x**5) - 30*a*d*e**2*f*g**4*x/(60*f**5*g**6 + 300*f**4*g
7*x + 600*f3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**
11*x**5) - 60*a*d*e**2*g**5*x**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3
*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 3*a*e
3*f3*g**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f*
*2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 15*a*e**3*f**2*g**3*x/(6
0*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 3
00*f*g**10*x**4 + 60*g**11*x**5) - 30*a*e**3*f*g**4*x**2/(60*f**5*g**6 + 30
0*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4
+ 60*g**11*x**5) - 30*a*e**3*g**5*x**3/(60*f**5*g**6 + 300*f**4*g**7*x + 60
0*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) -
6*c*d**4*f*g**4/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600
*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 30*c*d**4*g**5*x/(60*
f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300
*f*g**10*x**4 + 60*g**11*x**5) - 14*c*d**3*e*f**2*g**3/(60*f**5*g**6 + 300*
f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 +
60*g**11*x**5) - 70*c*d**3*e*f*g**4*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600
*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) -
140*c*d**3*e*g**5*x**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2
+ 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 27*c*d**2*e**2*
f**3*g**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g
9*x3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 135*c*d**2*e**2*f**2*g**3*x/
(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 +
300*f*g**10*x**4 + 60*g**11*x**5) - 270*c*d**2*e**2*f*g**4*x**2/(60*f**5*g
6 + 300*f4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**
10*x**4 + 60*g**11*x**5) - 270*c*d**2*e**2*g**5*x**3/(60*f**5*g**6 + 300*f*
*4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60
*g**11*x**5) - 60*c*d*e**3*f**4*g/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**
3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 300*
c*d*e**3*f**3*g**2*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 +
600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 600*c*d*e**3*f**2
*g**3*x**2/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*
g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) - 600*c*d*e**3*f*g**4*x**3/(6
0*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 3
00*f*g**10*x**4 + 60*g**11*x**5) - 300*c*d*e**3*g**5*x**4/(60*f**5*g**6 + 3
00*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4
+ 60*g**11*x**5) + 60*c*e**4*f**5*log(f/g + x)/(60*f**5*g**6 + 300*f**4*g*
*7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**1
1*x**5) + 137*c*e**4*f**5/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x
2 + 600*f2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) + 300*c*e**4*f
4*g*x*log(f/g + x)/(60*f5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 +

```
 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5) + 625*c*e**4*f**4*g
*x/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**
3 + 300*f*g**10*x**4 + 60*g**11*x**5) + 600*c*e**4*f**3*g**2*x**2*log(f/g +
 x)/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x*
*3 + 300*f*g**10*x**4 + 60*g**11*x**5) + 1100*c*e**4*f**3*g**2*x**2/(60*f**
5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*
g**10*x**4 + 60*g**11*x**5) + 600*c*e**4*f**2*g**3*x**3*log(f/g + x)/(60*f*
*5*g**6 + 300*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f
*g**10*x**4 + 60*g**11*x**5) + 900*c*e**4*f**2*g**3*x**3/(60*f**5*g**6 + 30
0*f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4
+ 60*g**11*x**5) + 300*c*e**4*f*g**4*x**4*log(f/g + x)/(60*f**5*g**6 + 300*
f**4*g**7*x + 600*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 +
60*g**11*x**5) + 300*c*e**4*f*g**4*x**4/(60*f**5*g**6 + 300*f**4*g**7*x + 6
00*f**3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5)
+ 60*c*e**4*g**5*x**5*log(f/g + x)/(60*f**5*g**6 + 300*f**4*g**7*x + 600*f*
*3*g**8*x**2 + 600*f**2*g**9*x**3 + 300*f*g**10*x**4 + 60*g**11*x**5), Eq(n
, -6)), (-3*a*d**3*g**5/(12*f**4*g**6 + 48*f**3...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3760 vs. $2(282) = 564$.
time = 4.41, size = 3760, normalized size = 13.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^6*n^5*x^6*e^4 + 5*(g*x + f)^n*c*d*g^6*n^5*x^5*e^3 + 9*(g*x
 + f)^n*c*d^2*g^6*n^5*x^4*e^2 + 7*(g*x + f)^n*c*d^3*g^6*n^5*x^3*e + 2*(g*x
+ f)^n*c*d^4*g^6*n^5*x^2 + (g*x + f)^n*c*f*g^5*n^5*x^5*e^4 + 15*(g*x + f)^n
*c*g^6*n^4*x^6*e^4 + 5*(g*x + f)^n*c*d*f*g^5*n^5*x^4*e^3 + 80*(g*x + f)^n*c
*d*g^6*n^4*x^5*e^3 + 9*(g*x + f)^n*c*d^2*f*g^5*n^5*x^3*e^2 + 153*(g*x + f)^
n*c*d^2*g^6*n^4*x^4*e^2 + 7*(g*x + f)^n*c*d^3*f*g^5*n^5*x^2*e + 126*(g*x +
f)^n*c*d^3*g^6*n^4*x^3*e + 2*(g*x + f)^n*c*d^4*f*g^5*n^5*x + 38*(g*x + f)^n
*c*d^4*g^6*n^4*x^2 + 10*(g*x + f)^n*c*f*g^5*n^4*x^5*e^4 + 85*(g*x + f)^n*c*
g^6*n^3*x^6*e^4 + 60*(g*x + f)^n*c*d*f*g^5*n^4*x^4*e^3 + (g*x + f)^n*a*g^6*
n^5*x^4*e^3 + 475*(g*x + f)^n*c*d*g^6*n^3*x^5*e^3 + 126*(g*x + f)^n*c*d^2*f
*g^5*n^4*x^3*e^2 + 3*(g*x + f)^n*a*d*g^6*n^5*x^3*e^2 + 963*(g*x + f)^n*c*d^
2*g^6*n^3*x^4*e^2 + 112*(g*x + f)^n*c*d^3*f*g^5*n^4*x^2*e + 3*(g*x + f)^n*a
*d^2*g^6*n^5*x^2*e + 847*(g*x + f)^n*c*d^3*g^6*n^3*x^3*e + 36*(g*x + f)^n*c
*d^4*f*g^5*n^4*x + (g*x + f)^n*a*d^3*g^6*n^5*x + 274*(g*x + f)^n*c*d^4*g^6*
n^3*x^2 - 5*(g*x + f)^n*c*f^2*g^4*n^4*x^4*e^4 + 35*(g*x + f)^n*c*f*g^5*n^3*
x^5*e^4 + 225*(g*x + f)^n*c*g^6*n^2*x^6*e^4 - 20*(g*x + f)^n*c*d*f^2*g^4*n^
4*x^3*e^3 + (g*x + f)^n*a*f*g^5*n^5*x^3*e^3 + 235*(g*x + f)^n*c*d*f*g^5*n^3
*x^4*e^3 + 17*(g*x + f)^n*a*g^6*n^4*x^4*e^3 + 1300*(g*x + f)^n*c*d*g^6*n^2*
```

x^5*e^3 - 27*(g*x + f)^n*c*d^2*f^2*g^4*n^4*x^2*e^2 + 3*(g*x + f)^n*a*d*f*g^
5*n^5*x^2*e^2 + 585*(g*x + f)^n*c*d^2*f*g^5*n^3*x^3*e^2 + 54*(g*x + f)^n*a*
d*g^6*n^4*x^3*e^2 + 2763*(g*x + f)^n*c*d^2*g^6*n^2*x^4*e^2 - 14*(g*x + f)^n
*c*d^3*f^2*g^4*n^4*x*e + 3*(g*x + f)^n*a*d^2*f*g^5*n^5*x*e + 623*(g*x + f)^
n*c*d^3*f*g^5*n^3*x^2*e + 57*(g*x + f)^n*a*d^2*g^6*n^4*x^2*e + 2604*(g*x +
f)^n*c*d^3*g^6*n^2*x^3*e - 2*(g*x + f)^n*c*d^4*f^2*g^4*n^4 + (g*x + f)^n*a*
d^3*f*g^5*n^5 + 238*(g*x + f)^n*c*d^4*f*g^5*n^3*x + 20*(g*x + f)^n*a*d^3*g^
6*n^4*x + 922*(g*x + f)^n*c*d^4*g^6*n^2*x^2 - 30*(g*x + f)^n*c*f^2*g^4*n^3*
x^4*e^4 + 50*(g*x + f)^n*c*f*g^5*n^2*x^5*e^4 + 274*(g*x + f)^n*c*g^6*n*x^6*
e^4 - 180*(g*x + f)^n*c*d*f^2*g^4*n^3*x^3*e^3 + 14*(g*x + f)^n*a*f*g^5*n^4*
x^3*e^3 + 360*(g*x + f)^n*c*d*f*g^5*n^2*x^4*e^3 + 107*(g*x + f)^n*a*g^6*n^3
*x^4*e^3 + 1620*(g*x + f)^n*c*d*g^6*n*x^5*e^3 - 324*(g*x + f)^n*c*d^2*f^2*g
^4*n^3*x^2*e^2 + 48*(g*x + f)^n*a*d*f*g^5*n^4*x^2*e^2 + 1008*(g*x + f)^n*c*
d^2*f*g^5*n^2*x^3*e^2 + 363*(g*x + f)^n*a*d*g^6*n^3*x^3*e^2 + 3564*(g*x + f
)^n*c*d^2*g^6*n*x^4*e^2 - 210*(g*x + f)^n*c*d^3*f^2*g^4*n^3*x*e + 54*(g*x +
f)^n*a*d^2*f*g^5*n^4*x*e + 1358*(g*x + f)^n*c*d^3*f*g^5*n^2*x^2*e + 411*(g
*x + f)^n*a*d^2*g^6*n^3*x^2*e + 3556*(g*x + f)^n*c*d^3*g^6*n*x^3*e - 36*(g*
x + f)^n*c*d^4*f^2*g^4*n^3 + 20*(g*x + f)^n*a*d^3*f*g^5*n^4 + 684*(g*x + f)
^n*c*d^4*f*g^5*n^2*x + 155*(g*x + f)^n*a*d^3*g^6*n^3*x + 1404*(g*x + f)^n*c
*d^4*g^6*n*x^2 + 20*(g*x + f)^n*c*f^3*g^3*n^3*x^3*e^4 - 55*(g*x + f)^n*c*f^
2*g^4*n^2*x^4*e^4 + 24*(g*x + f)^n*c*f*g^5*n*x^5*e^4 + 120*(g*x + f)^n*c*g^
6*x^6*e^4 + 60*(g*x + f)^n*c*d*f^3*g^3*n^3*x^2*e^3 - 3*(g*x + f)^n*a*f^2*g^
4*n^4*x^2*e^3 - 400*(g*x + f)^n*c*d*f^2*g^4*n^2*x^3*e^3 + 65*(g*x + f)^n*a*
f*g^5*n^3*x^3*e^3 + 180*(g*x + f)^n*c*d*f*g^5*n*x^4*e^3 + 307*(g*x + f)^n*a
*g^6*n^2*x^4*e^3 + 720*(g*x + f)^n*c*d*g^6*x^5*e^3 + 54*(g*x + f)^n*c*d^2*f
^3*g^3*n^3*x*e^2 - 6*(g*x + f)^n*a*d*f^2*g^4*n^4*x*e^2 - 1107*(g*x + f)^n*c
*d^2*f^2*g^4*n^2*x^2*e^2 + 267*(g*x + f)^n*a*d*f*g^5*n^3*x^2*e^2 + 540*(g*x
+ f)^n*c*d^2*f*g^5*n*x^3*e^2 + 1116*(g*x + f)^n*a*d*g^6*n^2*x^3*e^2 + 1620
*(g*x + f)^n*c*d^2*g^6*x^4*e^2 + 14*(g*x + f)^n*c*d^3*f^3*g^3*n^3*e - 3*(g*
x + f)^n*a*d^2*f^2*g^4*n^4*e - 1036*(g*x + f)^n*c*d^3*f^2*g^4*n^2*x*e + 357
*(g*x + f)^n*a*d^2*f*g^5*n^3*x*e + 840*(g*x + f)^n*c*d^3*f*g^5*n*x^2*e + 13
83*(g*x + f)^n*a*d^2*g^6*n^2*x^2*e + 1680*(g*x + f)^n*c*d^3*g^6*x^3*e - 238
*(g*x + f)^n*c*d^4*f^2*g^4*n^2 + 155*(g*x + f)^n*a*d^3*f*g^5*n^3 + 720*(g*x
+ f)^n*c*d^4*f*g^5*n*x + 580*(g*x + f)^n*a*d^3*g^6*n^2*x + 720*(g*x + f)^n
*c*d^4*g^6*x^2 + 60*(g*x + f)^n*c*f^3*g^3*n^2*x^3*e^4 - 30*(g*x + f)^n*c*f^
2*g^4*n*x^4*e^4 + 420*(g*x + f)^n*c*d*f^3*g^3*n^2*x^2*e^3 - 36*(g*x + f)^n*
a*f^2*g^4*n^3*x^2*e^3 - 240*(g*x + f)^n*c*d*f^2*g^4*n*x^3*e^3 + 112*(g*x +
f)^n*a*f*g^5*n^2*x^3*e^3 + 396*(g*x + f)^n*a*g^6*n*x^4*e^3 + 594*(g*x + f)^
n*c*d^2*f^3*g^3*n^2*x*e^2 - 90*(g*x + f)^n*a*d*f^2*g^4*n^3*x*e^2 - 810*(g*x
+ f)^n*c*d^2*f^2*g^4*n*x^2*e^2 + 582*(g*x + f)^n*a*d*f*g^5*n^2*x^2*e^2 + 1
524*(g*x + f)^n*a*d*g^6*n*x^3*e^2 + 210*(g*x + f)^n*c*d^3*f^3*g^3*n^2*e - 5
4*(g*x + f)^n*a*d^2*f^2*g^4*n^3*e - 1680*(g*x + f)^n*c*d^3*f^2*g^4*n*x*e +
1026*(g*x + f)^n*a*d^2*f*g^5*n^2*x*e + 2106*(g*x + f)^n*a*d^2*g^6*n*x^2*e -
684*(g*x + f)^n*c*d^4*f^2*g^4*n + 580*(g*x + f)^n*a*d^3*f*g^5*n^2 + 1044*(
g*x + f)^n*a*d^3*g^6*n*x - 60*(g*x + f)^n*c*f^4*g^2*n^2*x^2*e^4 + 40*(g*x +

```
f)^n*c*f^3*g^3*n*x^3*e^4 - 120*(g*x + f)^n*c*d*f^4*g^2*n^2*x*e^3 + 6*(g*x
+ f)^n*a*f^3*g^3*n^3*x*e^3 + 360*(g*x + f)^n*c*d*f^3*g^3*n*x^2*e^3 - 123*(g
*x + f)^n*a*f^2*g^4*n^2*x^2*e^3 + 60*(g*x + f)^n*a*f*g^5*n*x^3*e^3 + 180*(g
*x + f)^n*a*g^6*x^4*e^3 - 54*(g*x + f)^n*c*d^2*...
```

Mupad [B]
time = 3.90, size = 1943, normalized size = 7.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^3*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (x*(f + g*x)^n*(720*a*d^3*g^6 + 580*a*d^3*g^6*n^2 + 155*a*d^3*g^6*n^3 + 20*
a*d^3*g^6*n^4 + a*d^3*g^6*n^5 + 1044*a*d^3*g^6*n + 720*c*d^4*f*g^5*n + 120*
c*e^4*f^5*g*n + 180*a*e^3*f^3*g^3*n + 684*c*d^4*f*g^5*n^2 + 238*c*d^4*f*g^5
*n^3 + 36*c*d^4*f*g^5*n^4 + 2*c*d^4*f*g^5*n^5 + 66*a*e^3*f^3*g^3*n^2 + 6*a*
e^3*f^3*g^3*n^3 - 444*a*d*e^2*f^2*g^4*n^2 - 90*a*d*e^2*f^2*g^4*n^3 - 6*a*d*
e^2*f^2*g^4*n^4 + 1620*c*d^2*e^2*f^3*g^3*n - 120*c*d*e^3*f^4*g^2*n^2 - 1036
*c*d^3*e*f^2*g^4*n^2 - 210*c*d^3*e*f^2*g^4*n^3 - 14*c*d^3*e*f^2*g^4*n^4 + 1
080*a*d^2*e*f*g^5*n + 594*c*d^2*e^2*f^3*g^3*n^2 + 54*c*d^2*e^2*f^3*g^3*n^3
- 720*a*d*e^2*f^2*g^4*n + 1026*a*d^2*e*f*g^5*n^2 + 357*a*d^2*e*f*g^5*n^3 +
54*a*d^2*e*f*g^5*n^4 + 3*a*d^2*e*f*g^5*n^5 - 720*c*d*e^3*f^4*g^2*n - 1680*c
*d^3*e*f^2*g^4*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n
^6 + 720)) - ((f + g*x)^n*(120*c*e^4*f^6 + 180*a*e^3*f^4*g^2 + 720*c*d^4*f^
2*g^4 - 720*a*d^3*f*g^5 - 720*c*d*e^3*f^5*g - 1044*a*d^3*f*g^5*n - 720*a*d*
e^2*f^3*g^3 + 1080*a*d^2*e*f^2*g^4 - 1680*c*d^3*e*f^3*g^3 - 580*a*d^3*f*g^5
*n^2 - 155*a*d^3*f*g^5*n^3 - 20*a*d^3*f*g^5*n^4 - a*d^3*f*g^5*n^5 + 66*a*e^
3*f^4*g^2*n + 684*c*d^4*f^2*g^4*n + 1620*c*d^2*e^2*f^4*g^2 + 6*a*e^3*f^4*g^
2*n^2 + 238*c*d^4*f^2*g^4*n^2 + 36*c*d^4*f^2*g^4*n^3 + 2*c*d^4*f^2*g^4*n^4
- 90*a*d*e^2*f^3*g^3*n^2 + 357*a*d^2*e*f^2*g^4*n^2 - 6*a*d*e^2*f^3*g^3*n^3
+ 54*a*d^2*e*f^2*g^4*n^3 + 3*a*d^2*e*f^2*g^4*n^4 + 594*c*d^2*e^2*f^4*g^2*n
- 210*c*d^3*e*f^3*g^3*n^2 - 14*c*d^3*e*f^3*g^3*n^3 - 120*c*d*e^3*f^5*g*n +
54*c*d^2*e^2*f^4*g^2*n^2 - 444*a*d*e^2*f^3*g^3*n + 1026*a*d^2*e*f^2*g^4*n -
 1036*c*d^3*e*f^3*g^3*n))/(g^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*
n^5 + n^6 + 720)) + (c*e^4*x^6*(f + g*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n
^4 + n^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 72
0) + (x^2*(f + g*x)^n*(n + 1)*(720*c*d^4*g^4 + 238*c*d^4*g^4*n^2 + 36*c*d^4
*g^4*n^3 + 2*c*d^4*g^4*n^4 + 1080*a*d^2*e*g^4 + 684*c*d^4*g^4*n - 60*c*e^4*
f^4*n + 1026*a*d^2*e*g^4*n + 357*a*d^2*e*g^4*n^2 + 54*a*d^2*e*g^4*n^3 + 3*a
*d^2*e*g^4*n^4 - 90*a*e^3*f^2*g^2*n - 33*a*e^3*f^2*g^2*n^2 - 3*a*e^3*f^2*g^
2*n^3 - 810*c*d^2*e^2*f^2*g^2*n + 360*a*d*e^2*f*g^3*n + 360*c*d*e^3*f^3*g*n
 + 840*c*d^3*e*f*g^3*n - 297*c*d^2*e^2*f^2*g^2*n^2 - 27*c*d^2*e^2*f^2*g^2*n
^3 + 222*a*d*e^2*f*g^3*n^2 + 45*a*d*e^2*f*g^3*n^3 + 3*a*d*e^2*f*g^3*n^4 + 6
0*c*d*e^3*f^3*g*n^2 + 518*c*d^3*e*f*g^3*n^2 + 105*c*d^3*e*f*g^3*n^3 + 7*c*d
```

```
^3*e*f*g^3*n^4))/(g^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6
 + 720)) + (e*x^3*(f + g*x)^n*(3*n + n^2 + 2)*(840*c*d^3*g^3 + 105*c*d^3*g^
3*n^2 + 7*c*d^3*g^3*n^3 + 360*a*d*e*g^3 + 518*c*d^3*g^3*n + 20*c*e^3*f^3*n
+ 45*a*d*e*g^3*n^2 + 3*a*d*e*g^3*n^3 + 30*a*e^2*f*g^2*n + 11*a*e^2*f*g^2*n^
2 + a*e^2*f*g^2*n^3 + 222*a*d*e*g^3*n - 120*c*d*e^2*f^2*g*n + 270*c*d^2*e*f
*g^2*n - 20*c*d*e^2*f^2*g*n^2 + 99*c*d^2*e*f*g^2*n^2 + 9*c*d^2*e*f*g^2*n^3)
)/(g^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (e^2
*x^4*(f + g*x)^n*(11*n + 6*n^2 + n^3 + 6)*(270*c*d^2*g^2 + 30*a*e*g^2 + 9*c
*d^2*g^2*n^2 + 11*a*e*g^2*n + a*e*g^2*n^2 + 99*c*d^2*g^2*n - 5*c*e^2*f^2*n
+ 5*c*d*e*f*g*n^2 + 30*c*d*e*f*g*n))/(g^2*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720)) + (c*e^3*x^5*(f + g*x)^n*(30*d*g + 5*d*g*n + e
*f*n)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(g*(1764*n + 1624*n^2 + 735*n^3
+ 175*n^4 + 21*n^5 + n^6 + 720))
```

3.806 $\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=208

$$\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^5(1+n)}-\frac{2(ef-dg)\left(aeg^2+c(2e^2f^2-4defg+d^2g^2)\right)(f+gx)^{2+n}}{g^5(2+n)}+$$

```
[Out] (-d*g+e*f)^2*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^5/(1+n)-2*(-d*g+e*f)*
(a*e*g^2+c*(d^2*g^2-4*d*e*f*g+2*e^2*f^2))*(g*x+f)^(2+n)/g^5/(2+n)+e*(a*e*g^
2+c*(5*d^2*g^2-12*d*e*f*g+6*e^2*f^2))*(g*x+f)^(3+n)/g^5/(3+n)-4*c*e^2*(-d*g
+e*f)*(g*x+f)^(4+n)/g^5/(4+n)+c*e^3*(g*x+f)^(5+n)/g^5/(5+n)
```

Rubi [A]
time = 0.11, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {961}

$$-\frac{2(ef-dg)(f+gx)^{n+2}\left(aeg^2+c(d^2g^2-4defg+2e^2f^2)\right)}{g^5(n+2)}+\frac{e(f+gx)^{n+3}\left(aeg^2+c(5d^2g^2-12defg+6e^2f^2)\right)}{g^5(n+3)}+\frac{(ef-dg)^2(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^5(n+1)}-\frac{4ce^2(ef-dg)(f+gx)^{n+4}}{g^5(n+4)}+\frac{ce^3(f+gx)^{n+5}}{g^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n))
 - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x)
^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g
^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^(4 +
 n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^2(f+gx)^n \left(a+2cdx+cex^2\right) dx = \int \left(\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^4}+\frac{2(ef-dg}{}\right.$$

$$=\frac{(ef-dg)^2\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^5(1+n)}-\frac{2(ef-dg)(}{}$$

Mathematica [**A**]
time = 0.31, size = 348, normalized size = 1.67

$$\frac{(f+gx)^{1+n}\left(ag^2(20+9n+n^2)\left(d^2g^2(6+5n+n^2)+2deg(3+n)(-f+g(1+n)x)+e^2(2f^2-2fg(1+n)x+g^2(2+3n+n^2)x^2)\right)+c\left(2d^3g^3(60+47n+12n^2+n^3)(-f+g(1+n)x)+5d^2eg^2(20+9n+n^2)(2f^2-2fg(1+n)x+g^2(2+3n+n^2)x^2)+4de^2g(5+n)(-6f^3+6f^2g(1+n)x-3fg^2(2+3n+n^2)x^2+g^3(6+11n+6n^2+n^3)x^3)+e^3(24f^4-24f^3g(1+n)x+12f^2g^2(2+3n+n^2)x^2-4fg^3(6+11n+6n^2+n^3)x^3+g^4(24+50n+35n^2+10n^3+n^4)x^4)\right)\right)}{g^5(1+n)(2+n)(3+n)(4+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(a*g^2*(20 + 9*n + n^2)*(d^2*g^2*(6 + 5*n + n^2) + 2*d*e
*g*(3 + n)*(-f + g*(1 + n)*x) + e^2*(2*f^2 - 2*f*g*(1 + n)*x + g^2*(2 + 3*n
 + n^2)*x^2)) + c*(2*d^3*g^3*(60 + 47*n + 12*n^2 + n^3)*(-f + g*(1 + n)*x)
+ 5*d^2*e*g^2*(20 + 9*n + n^2)*(2*f^2 - 2*f*g*(1 + n)*x + g^2*(2 + 3*n + n^
2)*x^2) + 4*d*e^2*g*(5 + n)*(-6*f^3 + 6*f^2*g*(1 + n)*x - 3*f*g^2*(2 + 3*n
+ n^2)*x^2 + g^3*(6 + 11*n + 6*n^2 + n^3)*x^3) + e^3*(24*f^4 - 24*f^3*g*(1
+ n)*x + 12*f^2*g^2*(2 + 3*n + n^2)*x^2 - 4*f*g^3*(6 + 11*n + 6*n^2 + n^3)*
x^3 + g^4*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^4))))/(g^5*(1 + n)*(2 + n)*
(3 + n)*(4 + n)*(5 + n))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. 2(208) = 416.
time = 0.11, size = 1029, normalized size = 4.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*e^3/(5+n)*x^5*exp(n*ln(g*x+f))+f*(a*d^2*g^4*n^4-2*c*d^3*f*g^3*n^3+14*a*d^
2*g^4*n^3-2*a*d*e*f*g^3*n^3-24*c*d^3*f*g^3*n^2+10*c*d^2*e*f^2*g^2*n^2+71*a*
d^2*g^4*n^2-24*a*d*e*f*g^3*n^2+2*a*e^2*f^2*g^2*n^2-94*c*d^3*f*g^3*n+90*c*d^
2*e*f^2*g^2*n-24*c*d*e^2*f^3*g*n+154*a*d^2*g^4*n-94*a*d*e*f*g^3*n+18*a*e^2*
f^2*g^2*n-120*c*d^3*f*g^3+200*c*d^2*e*f^2*g^2-120*c*d*e^2*f^3*g+24*c*e^3*f^
4+120*a*d^2*g^4-120*a*d*e*f*g^3+40*a*e^2*f^2*g^2)/g^5/(n^5+15*n^4+85*n^3+22
5*n^2+274*n+120)*exp(n*ln(g*x+f))+(2*c*d^3*g^3*n^3+5*c*d^2*e*f*g^2*n^3+2*a*
d*e*g^3*n^3+a*e^2*f*g^2*n^3+24*c*d^3*g^3*n^2+45*c*d^2*e*f*g^2*n^2-12*c*d*e^
2*f^2*g*n^2+24*a*d*e*g^3*n^2+9*a*e^2*f*g^2*n^2+94*c*d^3*g^3*n+100*c*d^2*e*f
*g^2*n-60*c*d*e^2*f^2*g*n+12*c*e^3*f^3*n+94*a*d*e*g^3*n+20*a*e^2*f*g^2*n+12
0*c*d^3*g^3+120*a*d*e*g^3)/g^3/(n^4+14*n^3+71*n^2+154*n+120)*x^2*exp(n*ln(g
*x+f))+(2*c*d^3*f*g^3*n^4+a*d^2*g^4*n^4+2*a*d*e*f*g^3*n^4+24*c*d^3*f*g^3*n^
3-10*c*d^2*e*f^2*g^2*n^3+14*a*d^2*g^4*n^3+24*a*d*e*f*g^3*n^3-2*a*e^2*f^2*g^
2*n^3+94*c*d^3*f*g^3*n^2-90*c*d^2*e*f^2*g^2*n^2+24*c*d*e^2*f^3*g*n^2+71*a*d
^2*g^4*n^2+94*a*d*e*f*g^3*n^2-18*a*e^2*f^2*g^2*n^2+120*c*d^3*f*g^3*n-200*c*
d^2*e*f^2*g^2*n+120*c*d*e^2*f^3*g*n-24*c*e^3*f^4*n+154*a*d^2*g^4*n+120*a*d*
e*f*g^3*n-40*a*e^2*f^2*g^2*n+120*a*d^2*g^4)/g^4/(n^5+15*n^4+85*n^3+225*n^2+
274*n+120)*x*exp(n*ln(g*x+f))+(5*c*d^2*g^2*n^2+4*c*d*e*f*g*n^2+a*e*g^2*n^2+
45*c*d^2*g^2*n+20*c*d*e*f*g*n-4*c*e^2*f^2*n+9*a*e*g^2*n+100*c*d^2*g^2+20*a*
e*g^2)*e/g^2/(n^3+12*n^2+47*n+60)*x^3*exp(n*ln(g*x+f))+(4*d*g*n+e*f*n+20*d*
g)*c*e^2/g/(n^2+9*n+20)*x^4*exp(n*ln(g*x+f))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(208) = 416$.
time = 0.31, size = 512, normalized size = 2.46

$$\frac{2\,(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n cd^3}{(n^2+3\,n+2)g^2}+\frac{5\,((n^2+3\,n+2)g^3x^3+(n^2+n)fg^2x^2-2\,f^2gnx+2\,f^3)(gx+f)^n cd^2e}{(n^3+6\,n^2+11\,n+6)g^3}+\frac{2\,(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n ade}{(n^2+3\,n+2)g^2}+\frac{(gx+f)^{n+1}ad^2}{g(n+1)}+\frac{4\,((n^3+6\,n^2+11\,n+6)g^4x^4+(n^3+3\,n^2+2\,n)fg^3x^3-3\,(n^2+n)f^2g^2x^2+6\,f^3gnx-6\,f^4)(gx+f)^n cde^2}{(n^4+10\,n^3+35\,n^2+50\,n+24)g^4}+\frac{((n^2+3\,n+2)g^3x^3+(n^2+n)fg^2x^2-2\,f^2gnx+2\,f^3)(gx+f)^n ae^2}{(n^3+6\,n^2+11\,n+6)g^3}+\frac{((n^4+10\,n^3+35\,n^2+50\,n+24)g^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)fg^4x^4-4\,(n^3+3\,n^2+2\,n)f^2g^3x^3+12\,(n^2+n)f^3g^2x^2-24\,f^4gnx+24\,f^5)(gx+f)^n ce^3}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^3/((n^2 + 3*n + 2)*g^2)
 + 5*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d^2*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + 2*(g^2*(n + 1)*x^2 + f
*g*n*x - f^2)*(g*x + f)^n*a*d*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a
*d^2/(g*(n + 1)) + 4*((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n
)*f*g^3*x^3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*
d*e^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^4) + ((n^2 + 3*n + 2)*g^3*x^3
+ (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x + f)^n*a*e^2/((n^3 + 6*n^
2 + 11*n + 6)*g^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*g^5*x^5 + (n^4 +
6*n^3 + 11*n^2 + 6*n)*f*g^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*f^2*g^3*x^3 + 12*(n
^2 + n)*f^3*g^2*x^2 - 24*f^4*g*n*x + 24*f^5)*(g*x + f)^n*c*e^3/((n^5 + 15*n
^4 + 85*n^3 + 225*n^2 + 274*n + 120)*g^5)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(216) = 432$.
time = 1.89, size = 1188, normalized size = 5.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d^2*f*g^4*n^4 - 120*c*d^3*f^2*g^3 + 120*a*d^2*f*g^4 - 2*(c*d^3*f^2*g^3 -
 7*a*d^2*f*g^4)*n^3 - (24*c*d^3*f^2*g^3 - 71*a*d^2*f*g^4)*n^2 + 2*(c*d^3*g^
5*n^4 + 13*c*d^3*g^5*n^3 + 59*c*d^3*g^5*n^2 + 107*c*d^3*g^5*n + 60*c*d^3*g^
5)*x^2 - 2*(47*c*d^3*f^2*g^3 - 77*a*d^2*f*g^4)*n + (120*a*d^2*g^5 + (2*c*d^
3*f*g^4 + a*d^2*g^5)*n^4 + 2*(12*c*d^3*f*g^4 + 7*a*d^2*g^5)*n^3 + (94*c*d^3
*f*g^4 + 71*a*d^2*g^5)*n^2 + 2*(60*c*d^3*f*g^4 + 77*a*d^2*g^5)*n)*x - (24*c
*f^4*g*n*x - 24*c*f^5 - (c*g^5*n^4 + 10*c*g^5*n^3 + 35*c*g^5*n^2 + 50*c*g^5
*n + 24*c*g^5)*x^5 - (c*f*g^4*n^4 + 6*c*f*g^4*n^3 + 11*c*f*g^4*n^2 + 6*c*f*
g^4*n)*x^4 + 4*(c*f^2*g^3*n^3 + 3*c*f^2*g^3*n^2 + 2*c*f^2*g^3*n)*x^3 - 12*(
c*f^3*g^2*n^2 + c*f^3*g^2*n)*x^2)*e^3 + (2*a*f^3*g^2*n^2 - 120*c*d*f^4*g +
40*a*f^3*g^2 + 4*(c*d*g^5*n^4 + 11*c*d*g^5*n^3 + 41*c*d*g^5*n^2 + 61*c*d*g^
5*n + 30*c*d*g^5)*x^4 + (40*a*g^5 + (4*c*d*f*g^4 + a*g^5)*n^4 + 4*(8*c*d*f*
g^4 + 3*a*g^5)*n^3 + (68*c*d*f*g^4 + 49*a*g^5)*n^2 + 2*(20*c*d*f*g^4 + 39*a
*g^5)*n)*x^3 + (a*f*g^4*n^4 - 2*(6*c*d*f^2*g^3 - 5*a*f*g^4)*n^3 - (72*c*d*f
```

```
^2*g^3 - 29*a*f*g^4)*n^2 - 20*(3*c*d*f^2*g^3 - a*f*g^4)*n)*x^2 - 6*(4*c*d*f
^4*g - 3*a*f^3*g^2)*n - 2*(a*f^2*g^3*n^3 - 3*(4*c*d*f^3*g^2 - 3*a*f^2*g^3)*
n^2 - 20*(3*c*d*f^3*g^2 - a*f^2*g^3)*n)*x)*e^2 - (2*a*d*f^2*g^3*n^3 - 200*c
*d^2*f^3*g^2 + 120*a*d*f^2*g^3 - 5*(c*d^2*g^5*n^4 + 12*c*d^2*g^5*n^3 + 49*c
*d^2*g^5*n^2 + 78*c*d^2*g^5*n + 40*c*d^2*g^5)*x^3 - 2*(5*c*d^2*f^3*g^2 - 12
*a*d*f^2*g^3)*n^2 - (120*a*d*g^5 + (5*c*d^2*f*g^4 + 2*a*d*g^5)*n^4 + 2*(25*
c*d^2*f*g^4 + 13*a*d*g^5)*n^3 + (145*c*d^2*f*g^4 + 118*a*d*g^5)*n^2 + 2*(50
*c*d^2*f*g^4 + 107*a*d*g^5)*n)*x^2 - 2*(45*c*d^2*f^3*g^2 - 47*a*d*f^2*g^3)*
n - 2*(a*d*f*g^4*n^4 - (5*c*d^2*f^2*g^3 - 12*a*d*f*g^4)*n^3 - (45*c*d^2*f^2
*g^3 - 47*a*d*f*g^4)*n^2 - 20*(5*c*d^2*f^2*g^3 - 3*a*d*f*g^4)*n)*x)*e)*(g*x
 + f)^n/(g^5*n^5 + 15*g^5*n^4 + 85*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*
g^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 11946 vs. $2(197) = 394$.

time = 2.50, size = 11946, normalized size = 57.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + c*d**3*x**2 + 5*c*
d**2*e*x**3/3 + c*d*e**2*x**4 + c*e**3*x**5/5), Eq(g, 0)), (-3*a*d**2*g**4/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g*
*9*x**4) - 2*a*d*e*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**
2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 8*a*d*e*g**4*x/(12*f**4*g**5 + 48*f**3
*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - a*e**2*f**2*
g**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 +
12*g**9*x**4) - 4*a*e**2*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*
g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 6*a*e**2*g**4*x**2/(12*f**4*g*
*5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) -
2*c*d**3*f*g**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g
**8*x**3 + 12*g**9*x**4) - 8*c*d**3*g**4*x/(12*f**4*g**5 + 48*f**3*g**6*x +
 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 5*c*d**2*e*f**2*g**2/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g*
*9*x**4) - 20*c*d**2*e*f*g**3*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g*
*7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 30*c*d**2*e*g**4*x**2/(12*f**4*g
**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) -
 12*c*d*e**2*f**3*g/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48
*f*g**8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*f**2*g**2*x/(12*f**4*g**5 + 48*f
**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 72*c*d*e*
*2*f*g**3*x**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g*
*8*x**3 + 12*g**9*x**4) - 48*c*d*e**2*g**4*x**3/(12*f**4*g**5 + 48*f**3*g**
6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 12*c*e**3*f**4*l
```

og(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 25*c*e**3*f**4/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f**3*g*x*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 88*c*e**3*f**3*g*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 72*c*e**3*f**2*g**2*x**2*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 108*c*e**3*f**2*g**2*x**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f*g**3*x**3*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 48*c*e**3*f*g**3*x**3/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) + 12*c*e**3*g**4*x**4*log(f/g + x)/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4), Eq(n, -5)), (-a*d**2*g**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - a*d*e*f*g**3/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*d*e*g**4*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - a*e**2*f**2*g**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*e**2*f*g**3*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*a*e**2*g**4*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - c*d**3*f*g**3/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 3*c*d**3*g**4*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 5*c*d**2*e*f**2*g**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 15*c*d**2*e*f*g**3*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 15*c*d**2*e*g**4*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 12*c*d*e**2*f**3*g*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 22*c*d*e**2*f**3*g/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f**2*g**2*x*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 54*c*d*e**2*f**2*g**2*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f*g**3*x**2*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 36*c*d*e**2*f*g**3*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 12*c*d*e**2*g**4*x**3*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 12*c*e**3*f**4*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 22*c*e**3*f**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36*c*e**3*f**3*g*x*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 54*c*e**3*f**3*g*x/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36*c*e**3*f**2*g**2*x**2*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 36*c*e**3*f**2*g**2*x**2/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) - 12*c*e**3*f*g**3*x**3*log(f/g + x)/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3) + 3*c*e**3*g**4*x**4/(3*f**3*g**5 + 9*f**2*g**6*x + 9*f*g**7*x**2 + 3*g**8*x**3), Eq(n, -4)), (-a*d**2*g**4/(2*f**2*g...

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. $2(216) = 432$.

time = 3.91, size = 2114, normalized size = 10.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^5*n^4*x^5*e^3 + 4*(g*x + f)^n*c*d*g^5*n^4*x^4*e^2 + 5*(g*x
 + f)^n*c*d^2*g^5*n^4*x^3*e + 2*(g*x + f)^n*c*d^3*g^5*n^4*x^2 + (g*x + f)^n
*c*f*g^4*n^4*x^4*e^3 + 10*(g*x + f)^n*c*g^5*n^3*x^5*e^3 + 4*(g*x + f)^n*c*d
*f*g^4*n^4*x^3*e^2 + 44*(g*x + f)^n*c*d*g^5*n^3*x^4*e^2 + 5*(g*x + f)^n*c*d
^2*f*g^4*n^4*x^2*e + 60*(g*x + f)^n*c*d^2*g^5*n^3*x^3*e + 2*(g*x + f)^n*c*d
^3*f*g^4*n^4*x + 26*(g*x + f)^n*c*d^3*g^5*n^3*x^2 + 6*(g*x + f)^n*c*f*g^4*n
^3*x^4*e^3 + 35*(g*x + f)^n*c*g^5*n^2*x^5*e^3 + 32*(g*x + f)^n*c*d*f*g^4*n^
3*x^3*e^2 + (g*x + f)^n*a*g^5*n^4*x^3*e^2 + 164*(g*x + f)^n*c*d*g^5*n^2*x^4
*e^2 + 50*(g*x + f)^n*c*d^2*f*g^4*n^3*x^2*e + 2*(g*x + f)^n*a*d*g^5*n^4*x^2
*e + 245*(g*x + f)^n*c*d^2*g^5*n^2*x^3*e + 24*(g*x + f)^n*c*d^3*f*g^4*n^3*x
 + (g*x + f)^n*a*d^2*g^5*n^4*x + 118*(g*x + f)^n*c*d^3*g^5*n^2*x^2 - 4*(g*x
 + f)^n*c*f^2*g^3*n^3*x^3*e^3 + 11*(g*x + f)^n*c*f*g^4*n^2*x^4*e^3 + 50*(g*
x + f)^n*c*g^5*n*x^5*e^3 - 12*(g*x + f)^n*c*d*f^2*g^3*n^3*x^2*e^2 + (g*x +
f)^n*a*f*g^4*n^4*x^2*e^2 + 68*(g*x + f)^n*c*d*f*g^4*n^2*x^3*e^2 + 12*(g*x +
 f)^n*a*g^5*n^3*x^3*e^2 + 244*(g*x + f)^n*c*d*g^5*n*x^4*e^2 - 10*(g*x + f)^
n*c*d^2*f^2*g^3*n^3*x*e + 2*(g*x + f)^n*a*d*f*g^4*n^4*x*e + 145*(g*x + f)^n
*c*d^2*f*g^4*n^2*x^2*e + 26*(g*x + f)^n*a*d*g^5*n^3*x^2*e + 390*(g*x + f)^n
*c*d^2*g^5*n*x^3*e - 2*(g*x + f)^n*c*d^3*f^2*g^3*n^3 + (g*x + f)^n*a*d^2*f*
g^4*n^4 + 94*(g*x + f)^n*c*d^3*f*g^4*n^2*x + 14*(g*x + f)^n*a*d^2*g^5*n^3*x
 + 214*(g*x + f)^n*c*d^3*g^5*n*x^2 - 12*(g*x + f)^n*c*f^2*g^3*n^2*x^3*e^3 +
 6*(g*x + f)^n*c*f*g^4*n*x^4*e^3 + 24*(g*x + f)^n*c*g^5*x^5*e^3 - 72*(g*x +
 f)^n*c*d*f^2*g^3*n^2*x^2*e^2 + 10*(g*x + f)^n*a*f*g^4*n^3*x^2*e^2 + 40*(g*
x + f)^n*c*d*f*g^4*n*x^3*e^2 + 49*(g*x + f)^n*a*g^5*n^2*x^3*e^2 + 120*(g*x
+ f)^n*c*d*g^5*x^4*e^2 - 90*(g*x + f)^n*c*d^2*f^2*g^3*n^2*x*e + 24*(g*x + f
)^n*a*d*f*g^4*n^3*x*e + 100*(g*x + f)^n*c*d^2*f*g^4*n*x^2*e + 118*(g*x + f)
^n*a*d*g^5*n^2*x^2*e + 200*(g*x + f)^n*c*d^2*g^5*x^3*e - 24*(g*x + f)^n*c*d
^3*f^2*g^3*n^2 + 14*(g*x + f)^n*a*d^2*f*g^4*n^3 + 120*(g*x + f)^n*c*d^3*f*g
^4*n*x + 71*(g*x + f)^n*a*d^2*g^5*n^2*x + 120*(g*x + f)^n*c*d^3*g^5*x^2 + 1
2*(g*x + f)^n*c*f^3*g^2*n^2*x^2*e^3 - 8*(g*x + f)^n*c*f^2*g^3*n*x^3*e^3 + 2
4*(g*x + f)^n*c*d*f^3*g^2*n^2*x*e^2 - 2*(g*x + f)^n*a*f^2*g^3*n^3*x*e^2 - 6
0*(g*x + f)^n*c*d*f^2*g^3*n*x^2*e^2 + 29*(g*x + f)^n*a*f*g^4*n^2*x^2*e^2 +
78*(g*x + f)^n*a*g^5*n*x^3*e^2 + 10*(g*x + f)^n*c*d^2*f^3*g^2*n^2*e - 2*(g*
x + f)^n*a*d*f^2*g^3*n^3*e - 200*(g*x + f)^n*c*d^2*f^2*g^3*n*x*e + 94*(g*x
+ f)^n*a*d*f*g^4*n^2*x*e + 214*(g*x + f)^n*a*d*g^5*n*x^2*e - 94*(g*x + f)^n
*c*d^3*f^2*g^3*n + 71*(g*x + f)^n*a*d^2*f*g^4*n^2 + 154*(g*x + f)^n*a*d^2*g
^5*n*x + 12*(g*x + f)^n*c*f^3*g^2*n*x^2*e^3 + 120*(g*x + f)^n*c*d*f^3*g^2*n
```

```
*x*e^2 - 18*(g*x + f)^n*a*f^2*g^3*n^2*x*e^2 + 20*(g*x + f)^n*a*f*g^4*n*x^2*
e^2 + 40*(g*x + f)^n*a*g^5*x^3*e^2 + 90*(g*x + f)^n*c*d^2*f^3*g^2*n*e - 24*
(g*x + f)^n*a*d*f^2*g^3*n^2*e + 120*(g*x + f)^n*a*d*f*g^4*n*x*e + 120*(g*x
+ f)^n*a*d*g^5*x^2*e - 120*(g*x + f)^n*c*d^3*f^2*g^3 + 154*(g*x + f)^n*a*d^
2*f*g^4*n + 120*(g*x + f)^n*a*d^2*g^5*x - 24*(g*x + f)^n*c*f^4*g*n*x*e^3 -
24*(g*x + f)^n*c*d*f^4*g*n*e^2 + 2*(g*x + f)^n*a*f^3*g^2*n^2*e^2 - 40*(g*x
+ f)^n*a*f^2*g^3*n*x*e^2 + 200*(g*x + f)^n*c*d^2*f^3*g^2*e - 94*(g*x + f)^n
*a*d*f^2*g^3*n*e + 120*(g*x + f)^n*a*d^2*f*g^4 - 120*(g*x + f)^n*c*d*f^4*g*
e^2 + 18*(g*x + f)^n*a*f^3*g^2*n*e^2 - 120*(g*x + f)^n*a*d*f^2*g^3*e + 24*(
g*x + f)^n*c*f^5*e^3 + 40*(g*x + f)^n*a*f^3*g^2*e^2)/(g^5*n^5 + 15*g^5*n^4
+ 85*g^5*n^3 + 225*g^5*n^2 + 274*g^5*n + 120*g^5)
```

Mupad [B]
time = 3.52, size = 1133, normalized size = 5.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^2*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] ((f + g*x)^n*(24*c*e^3*f^5 + 40*a*e^2*f^3*g^2 - 120*c*d^3*f^2*g^3 + 120*a*d
^2*f*g^4 - 120*a*d*e*f^2*g^3 - 120*c*d*e^2*f^4*g + 154*a*d^2*f*g^4*n + 200*
c*d^2*e*f^3*g^2 + 71*a*d^2*f*g^4*n^2 + 14*a*d^2*f*g^4*n^3 + a*d^2*f*g^4*n^4
 + 18*a*e^2*f^3*g^2*n - 94*c*d^3*f^2*g^3*n + 2*a*e^2*f^3*g^2*n^2 - 24*c*d^3
*f^2*g^3*n^2 - 2*c*d^3*f^2*g^3*n^3 + 10*c*d^2*e*f^3*g^2*n^2 - 94*a*d*e*f^2*
g^3*n - 24*c*d*e^2*f^4*g*n - 24*a*d*e*f^2*g^3*n^2 - 2*a*d*e*f^2*g^3*n^3 + 9
0*c*d^2*e*f^3*g^2*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
 + (x*(f + g*x)^n*(120*a*d^2*g^5 + 71*a*d^2*g^5*n^2 + 14*a*d^2*g^5*n^3 + a*
d^2*g^5*n^4 + 154*a*d^2*g^5*n + 120*c*d^3*f*g^4*n - 24*c*e^3*f^4*g*n - 40*a
*e^2*f^2*g^3*n + 94*c*d^3*f*g^4*n^2 + 24*c*d^3*f*g^4*n^3 + 2*c*d^3*f*g^4*n^
4 - 18*a*e^2*f^2*g^3*n^2 - 2*a*e^2*f^2*g^3*n^3 + 120*a*d*e*f*g^4*n + 24*c*d
*e^2*f^3*g^2*n^2 - 90*c*d^2*e*f^2*g^3*n^2 - 10*c*d^2*e*f^2*g^3*n^3 + 94*a*d
*e*f*g^4*n^2 + 24*a*d*e*f*g^4*n^3 + 2*a*d*e*f*g^4*n^4 + 120*c*d*e^2*f^3*g^2
*n - 200*c*d^2*e*f^2*g^3*n))/(g^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5
+ 120)) + (c*e^3*x^5*(f + g*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*
n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (x^2*(f + g*x)^n*(n + 1)*(120*
c*d^3*g^3 + 24*c*d^3*g^3*n^2 + 2*c*d^3*g^3*n^3 + 120*a*d*e*g^3 + 94*c*d^3*g
^3*n + 12*c*e^3*f^3*n + 24*a*d*e*g^3*n^2 + 2*a*d*e*g^3*n^3 + 20*a*e^2*f*g^2
*n + 9*a*e^2*f*g^2*n^2 + a*e^2*f*g^2*n^3 + 94*a*d*e*g^3*n - 60*c*d*e^2*f^2*
g*n + 100*c*d^2*e*f*g^2*n - 12*c*d*e^2*f^2*g*n^2 + 45*c*d^2*e*f*g^2*n^2 + 5
*c*d^2*e*f*g^2*n^3))/(g^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
+ (e*x^3*(f + g*x)^n*(3*n + n^2 + 2)*(100*c*d^2*g^2 + 20*a*e*g^2 + 5*c*d^2*
g^2*n^2 + 9*a*e*g^2*n + a*e*g^2*n^2 + 45*c*d^2*g^2*n - 4*c*e^2*f^2*n + 4*c*
d*e*f*g*n^2 + 20*c*d*e*f*g*n))/(g^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^
5 + 120)) + (c*e^2*x^4*(f + g*x)^n*(20*d*g + 4*d*g*n + e*f*n)*(11*n + 6*n^2
 + n^3 + 6))/(g*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
```

3.807 $\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=146

$$-\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^4(1+n)}+\frac{\left(aeg^2+c\left(3e^2f^2-6defg+2d^2g^2\right)\right)(f+gx)^{2+n}}{g^4(2+n)}-\frac{3ce(ef-d}{g^4(}$$

```
[Out] -(-d*g+e*f)*(a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^4/(1+n)+(a*e*g^2+c*(2*
d^2*g^2-6*d*e*f*g+3*e^2*f^2))*(g*x+f)^(2+n)/g^4/(2+n)-3*c*e*(-d*g+e*f)*(g*x
+f)^(3+n)/g^4/(3+n)+c*e^2*(g*x+f)^(4+n)/g^4/(4+n)
```

Rubi [A]

time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {785}

$$\frac{(f+gx)^{n+2}\left(aeg^2+c\left(2d^2g^2-6defg+3e^2f^2\right)\right)}{g^4(n+2)}-\frac{(ef-dg)(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^4(n+1)}-\frac{3ce(ef-dg)(f+gx)^{n+3}}{g^4(n+3)}+\frac{ce^2(f+gx)^{n+4}}{g^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -(((e*f - d*g)*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^4*(1 + n))
) + ((a*e*g^2 + c*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(2 + n))/(
g^4*(2 + n)) - (3*c*e*(e*f - d*g)*(f + g*x)^(3 + n))/(g^4*(3 + n)) + (c*e^2
*(f + g*x)^(4 + n))/(g^4*(4 + n))
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)(f+gx)^n \left(a+2cdx+cex^2\right) dx = \int \left(\frac{(ef-dg)\left(-ag^2-cf(ef-2dg)\right)(f+gx)^n}{g^3}+\frac{(aeg^2+c(3}{}\right.$$

$$= -\frac{(ef-dg)\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^4(1+n)}+\frac{(aeg^2+c(3e^2}{}$$

Mathematica [A]

time = 0.20, size = 190, normalized size = 1.30

$$\frac{(f+gx)^{1+n}\left(ag^2(12+7n+n^2)\left(-ef+dg(2+n)+eg(1+n)x\right)+c\left(2d^2g^2(12+7n+n^2)(-f+g(1+n)x)+3deg(4+n)\left(2f^2-2fg(1+n)x+g^2(2+3n+n^2)x^2\right)-e^2\left(6f^3-6f^2g(1+n)x+3fg^2(2+3n+n^2)x^2-g^3(6+11n+6n^2+n^3)x^3\right)\right)\right)}{g^4(1+n)(2+n)(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(a*g^2*(12 + 7*n + n^2)*(-(e*f) + d*g*(2 + n) + e*g*(1 +
 n)*x) + c*(2*d^2*g^2*(12 + 7*n + n^2)*(-f + g*(1 + n)*x) + 3*d*e*g*(4 + n)
*(2*f^2 - 2*f*g*(1 + n)*x + g^2*(2 + 3*n + n^2)*x^2) - e^2*(6*f^3 - 6*f^2*g
*(1 + n)*x + 3*f*g^2*(2 + 3*n + n^2)*x^2 - g^3*(6 + 11*n + 6*n^2 + n^3)*x^3
))))/(g^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(146) = 292$.
time = 0.11, size = 449, normalized size = 3.08

method	result
gosper	$\frac{(gx+f)^{1+n}\left(c e^2 g^3 n^3 x^3+3cde\, g^3 n^3 x^2+6c e^2 g^3 n^2 x^3+2c d^2 g^3 n^3 x+21cde\, g^3 n^2 x^2-3c e^2 f\, g^2 n^2 x^2+11c e^2 g^3 n\, x^3+ae\, g^3 n^3 x+16c d^2 g^3\right.}{}$
norman	$\frac{c e^2 x^4 \mathrm{e}^{n \ln(gx+f)}}{4+n}+\frac{f\left(ad\, g^3 n^3-2c\, d^2 f\, g^2 n^2+9ad\, g^3 n^2-aef\, g^2 n^2-14c\, d^2 f\, g^2 n+6cde\, f^2 gn+26ad\, g^3 n-7aef\, g^2 n-24c\, d^2 f\, g^2+24c\right.}{g^4(n^4+10n^3+35n^2+50n+24)}$
risch	$\frac{\left(c e^2 g^4 n^3 x^4+3cde\, g^4 n^3 x^3+c e^2 f\, g^3 n^3 x^3+6c e^2 g^4 n^2 x^4+2c\, d^2 g^4 n^3 x^2+3cdef\, g^3 n^3 x^2+21cde\, g^4 n^2 x^3+3c e^2 f\, g^3 n^2 x^3+11c e^2 g^4 n\, x^4+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (g*x+f)^(1+n)*(c*e^2*g^3*n^3*x^3+3*c*d*e*g^3*n^3*x^2+6*c*e^2*g^3*n^2*x^3+2*
c*d^2*g^3*n^3*x+21*c*d*e*g^3*n^2*x^2-3*c*e^2*f*g^2*n^2*x^2+11*c*e^2*g^3*n*x
^3+a*e*g^3*n^3*x+16*c*d^2*g^3*n^2*x-6*c*d*e*f*g^2*n^2*x+42*c*d*e*g^3*n*x^2-
9*c*e^2*f*g^2*n*x^2+6*c*e^2*g^3*x^3+a*d*g^3*n^3+8*a*e*g^3*n^2*x-2*c*d^2*f*g
^2*n^2+38*c*d^2*g^3*n*x-30*c*d*e*f*g^2*n*x+24*c*d*e*g^3*x^2+6*c*e^2*f^2*g*n
*x-6*c*e^2*f*g^2*x^2+9*a*d*g^3*n^2-a*e*f*g^2*n^2+19*a*e*g^3*n*x-14*c*d^2*f*
g^2*n+24*c*d^2*g^3*x+6*c*d*e*f^2*g*n-24*c*d*e*f*g^2*x+6*c*e^2*f^2*g*x+26*a*
d*g^3*n-7*a*e*f*g^2*n+12*a*e*g^3*x-24*c*d^2*f*g^2+24*c*d*e*f^2*g-6*c*e^2*f^
3+24*a*d*g^3-12*a*e*f*g^2)/g^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.31, size = 289, normalized size = 1.98

$$\frac{2\,(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n cd^2}{(n^2+3n+2)g^2}+\frac{3\,((n^2+3n+2)g^3x^3+(n^2+n)fg^2x^2-2f^2gnx+2f^3)(gx+f)^n cde}{(n^3+6n^2+11n+6)g^3}+\frac{(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n ae}{(n^2+3n+2)g^2}+\frac{(gx+f)^{n+1}ad}{g(n+1)}+\frac{((n^3+6n^2+11n+6)g^4x^4+(n^3+3n^2+2n)fg^3x^3-3\,(n^2+n)f^2g^2x^2+6f^3gnx-6f^4)(gx+f)^n ce^2}{(n^4+10n^3+35n^2+50n+24)g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d^2/((n^2 + 3*n + 2)*g^2)
 + 3*((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*
(g*x + f)^n*c*d*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g^2*(n + 1)*x^2 + f*g*n
*x - f^2)*(g*x + f)^n*a*e/((n^2 + 3*n + 2)*g^2) + (g*x + f)^(n + 1)*a*d/(g*
```

```
(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*g^4*x^4 + (n^3 + 3*n^2 + 2*n)*f*g^3*x^
3 - 3*(n^2 + n)*f^2*g^2*x^2 + 6*f^3*g*n*x - 6*f^4)*(g*x + f)^n*c*e^2/((n^4
+ 10*n^3 + 35*n^2 + 50*n + 24)*g^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(152) = 304$.
time = 4.02, size = 577, normalized size = 3.95

$$\frac{\left(adfg^3n^3 - 24\,cd^2f^2g^2 + 24\,adfg^3 - \left(2\,cd^2f^2g^2 - 9\,adfg^3\right)n^2 + 2\left(cd^2g^4n^3 + 8\,cd^2g^4n^2 + 19\,cd^2g^4n + 12\,cd^2g^4\right)x^2 - 2\left(7\,cd^2f^2g^2 - 13\,adfg^3\right)n + \left(24\,adg^4 + \left(2\,cd^2fg^3 + adg^4\right)n^3 + \left(14\,cd^2fg^3 + 9\,adg^4\right)n^2 + 2\left(12\,cd^2fg^3 + 13\,adg^4\right)n\right)x + \left(6\,cf^3gnx - 6\,cf^4 + \left(cg^4n^3 + 6\,cg^4n^2 + 11\,cg^4n + 6\,cg^4\right)x^4 + \left(cfg^3n^3 + 3\,cfg^3n^2 + 2\,cfg^3n\right)x^3 - 3\left(cf^2g^2n^2 + cf^2g^2n\right)x^2\right)e^2 - \left(af^2g^2n^2 - 24\,cdf^3g + 12\,af^2g^2 - 3\left(cdg^4n^3 + 7\,cdg^4n^2 + 14\,cdg^4n + 8\,cdg^4\right)x^3 - \left(12\,ag^4 + \left(3\,cdfg^3 + ag^4\right)n^3 + \left(15\,cdfg^3 + 8\,ag^4\right)n^2 + \left(12\,cdfg^3 + 19\,ag^4\right)n\right)x^2 - \left(6\,cdf^3g - 7\,af^2g^2\right)n - \left(afg^3n^3 - \left(6\,cdf^2g^2 - 7\,afg^3\right)n^2 - 12\left(2\,cdf^2g^2 - afg^3\right)n\right)x\right)e\right)(gx + f)^n}{g^4n^4 + 10\,g^4n^3 + 35\,g^4n^2 + 50\,g^4n + 24\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*d*f*g^3*n^3 - 24*c*d^2*f^2*g^2 + 24*a*d*f*g^3 - (2*c*d^2*f^2*g^2 - 9*a*d
*f*g^3)*n^2 + 2*(c*d^2*g^4*n^3 + 8*c*d^2*g^4*n^2 + 19*c*d^2*g^4*n + 12*c*d^
2*g^4)*x^2 - 2*(7*c*d^2*f^2*g^2 - 13*a*d*f*g^3)*n + (24*a*d*g^4 + (2*c*d^2*
f*g^3 + a*d*g^4)*n^3 + (14*c*d^2*f*g^3 + 9*a*d*g^4)*n^2 + 2*(12*c*d^2*f*g^3
 + 13*a*d*g^4)*n)*x + (6*c*f^3*g*n*x - 6*c*f^4 + (c*g^4*n^3 + 6*c*g^4*n^2 +
 11*c*g^4*n + 6*c*g^4)*x^4 + (c*f*g^3*n^3 + 3*c*f*g^3*n^2 + 2*c*f*g^3*n)*x^
3 - 3*(c*f^2*g^2*n^2 + c*f^2*g^2*n)*x^2)*e^2 - (a*f^2*g^2*n^2 - 24*c*d*f^3*
g + 12*a*f^2*g^2 - 3*(c*d*g^4*n^3 + 7*c*d*g^4*n^2 + 14*c*d*g^4*n + 8*c*d*g^
4)*x^3 - (12*a*g^4 + (3*c*d*f*g^3 + a*g^4)*n^3 + (15*c*d*f*g^3 + 8*a*g^4)*n
^2 + (12*c*d*f*g^3 + 19*a*g^4)*n)*x^2 - (6*c*d*f^3*g - 7*a*f^2*g^2)*n - (a*
f*g^3*n^3 - (6*c*d*f^2*g^2 - 7*a*f*g^3)*n^2 - 12*(2*c*d*f^2*g^2 - a*f*g^3)*
n)*x)*e)*(g*x + f)^n/(g^4*n^4 + 10*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 24*g^4
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4952 vs. $2(134) = 268$.
time = 1.22, size = 4952, normalized size = 33.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d*x + a*e*x**2/2 + c*d**2*x**2 + c*d*e*x**3 + c*e**2*x**
4/4), Eq(g, 0)), (-2*a*d*g**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**
2 + 6*g**7*x**3) - a*e*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**
2 + 6*g**7*x**3) - 3*a*e*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x
**2 + 6*g**7*x**3) - 2*c*d**2*f*g**2/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g
**6*x**2 + 6*g**7*x**3) - 6*c*d**2*g**3*x/(6*f**3*g**4 + 18*f**2*g**5*x + 1
8*f*g**6*x**2 + 6*g**7*x**3) - 6*c*d*e*f**2*g/(6*f**3*g**4 + 18*f**2*g**5*x
 + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*f*g**2*x/(6*f**3*g**4 + 18*f**2
*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) - 18*c*d*e*g**3*x**2/(6*f**3*g**4 +
 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*f**3*log(f/g + x
```

)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 11*c*e**2
*f**3/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*
e**2*f**2*g*x*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 +
 6*g**7*x**3) + 27*c*e**2*f**2*g*x/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**
6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2*log(f/g + x)/(6*f**3*g**4 + 1
8*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 18*c*e**2*f*g**2*x**2/(6*f*
*3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**3) + 6*c*e**2*g**3*x*
*3*log(f/g + x)/(6*f**3*g**4 + 18*f**2*g**5*x + 18*f*g**6*x**2 + 6*g**7*x**
3), Eq(n, -4)), (-a*d*g**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - a*e*f
*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 2*a*e*g**3*x/(2*f**2*g**4
+ 4*f*g**5*x + 2*g**6*x**2) - 2*c*d**2*f*g**2/(2*f**2*g**4 + 4*f*g**5*x + 2
*g**6*x**2) - 4*c*d**2*g**3*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*
c*d*e*f**2*g*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 9*c*d*
e*f**2*g/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x*log(f
/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 12*c*d*e*f*g**2*x/(2*f**
2*g**4 + 4*f*g**5*x + 2*g**6*x**2) + 6*c*d*e*g**3*x**2*log(f/g + x)/(2*f**2
*g**4 + 4*f*g**5*x + 2*g**6*x**2) - 6*c*e**2*f**3*log(f/g + x)/(2*f**2*g**4
 + 4*f*g**5*x + 2*g**6*x**2) - 9*c*e**2*f**3/(2*f**2*g**4 + 4*f*g**5*x + 2*
g**6*x**2) - 12*c*e**2*f**2*g*x*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*
g**6*x**2) - 12*c*e**2*f**2*g*x/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2) -
6*c*e**2*f*g**2*x**2*log(f/g + x)/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2)
+ 2*c*e**2*g**3*x**3/(2*f**2*g**4 + 4*f*g**5*x + 2*g**6*x**2), Eq(n, -3)),
(-2*a*d*g**3/(2*f*g**4 + 2*g**5*x) + 2*a*e*f*g**2*log(f/g + x)/(2*f*g**4 +
2*g**5*x) + 2*a*e*f*g**2/(2*f*g**4 + 2*g**5*x) + 2*a*e*g**3*x*log(f/g + x)/
(2*f*g**4 + 2*g**5*x) + 4*c*d**2*f*g**2*log(f/g + x)/(2*f*g**4 + 2*g**5*x)
+ 4*c*d**2*f*g**2/(2*f*g**4 + 2*g**5*x) + 4*c*d**2*g**3*x*log(f/g + x)/(2*f
*g**4 + 2*g**5*x) - 12*c*d*e*f**2*g*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 12
*c*d*e*f**2*g/(2*f*g**4 + 2*g**5*x) - 12*c*d*e*f*g**2*x*log(f/g + x)/(2*f*g
4 + 2*g5*x) + 6*c*d*e*g**3*x**2/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3*l
og(f/g + x)/(2*f*g**4 + 2*g**5*x) + 6*c*e**2*f**3/(2*f*g**4 + 2*g**5*x) + 6
*c*e**2*f**2*g*x*log(f/g + x)/(2*f*g**4 + 2*g**5*x) - 3*c*e**2*f*g**2*x**2/
(2*f*g**4 + 2*g**5*x) + c*e**2*g**3*x**3/(2*f*g**4 + 2*g**5*x), Eq(n, -2)),
 (a*d*log(f/g + x)/g - a*e*f*log(f/g + x)/g**2 + a*e*x/g - 2*c*d**2*f*log(f
/g + x)/g**2 + 2*c*d**2*x/g + 3*c*d*e*f**2*log(f/g + x)/g**3 - 3*c*d*e*f*x/
g**2 + 3*c*d*e*x**2/(2*g) - c*e**2*f**3*log(f/g + x)/g**4 + c*e**2*f**2*x/g
3 - c*e2*f*x**2/(2*g**2) + c*e**2*x**3/(3*g), Eq(n, -1)), (a*d*f*g**3*n
3*(f + g*x)n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*
g**4) + 9*a*d*f*g**3*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 26*a*d*f*g**3*n*(f + g*x)**n/(g**4*n**4 + 10*
g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*f*g**3*(f + g*x)**
n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*d*g**
4*n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n
+ 24*g**4) + 9*a*d*g**4*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 26*a*d*g**4*n*x*(f + g*x)**n/(g**4*n**4
+ 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 24*a*d*g**4*x*(f + g

```
*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - a*
e*f**2*g**2*n**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50
*g**4*n + 24*g**4) - 7*a*e*f**2*g**2*n*(f + g*x)**n/(g**4*n**4 + 10*g**4*n*
*3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) - 12*a*e*f**2*g**2*(f + g*x)**n/(g
**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*f*g**3*
n**3*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n +
24*g**4) + 7*a*e*f*g**3*n**2*x*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*
g**4*n**2 + 50*g**4*n + 24*g**4) + 12*a*e*f*g**3*n*x*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + a*e*g**4*n**3*x**2
*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**
4) + 8*a*e*g**4*n**2*x**2*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*
n**2 + 50*g**4*n + 24*g**4) + 19*a*e*g**4*n*x**...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(152) = 304$.

time = 6.46, size = 1018, normalized size = 6.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^4*n^3*x^4*e^2 + 3*(g*x + f)^n*c*d*g^4*n^3*x^3*e + 2*(g*x +
 f)^n*c*d^2*g^4*n^3*x^2 + (g*x + f)^n*c*f*g^3*n^3*x^3*e^2 + 6*(g*x + f)^n*c
*g^4*n^2*x^4*e^2 + 3*(g*x + f)^n*c*d*f*g^3*n^3*x^2*e + 21*(g*x + f)^n*c*d*g
^4*n^2*x^3*e + 2*(g*x + f)^n*c*d^2*f*g^3*n^3*x + 16*(g*x + f)^n*c*d^2*g^4*n
^2*x^2 + 3*(g*x + f)^n*c*f*g^3*n^2*x^3*e^2 + 11*(g*x + f)^n*c*g^4*n*x^4*e^2
 + 15*(g*x + f)^n*c*d*f*g^3*n^2*x^2*e + (g*x + f)^n*a*g^4*n^3*x^2*e + 42*(g
*x + f)^n*c*d*g^4*n*x^3*e + 14*(g*x + f)^n*c*d^2*f*g^3*n^2*x + (g*x + f)^n*
a*d*g^4*n^3*x + 38*(g*x + f)^n*c*d^2*g^4*n*x^2 - 3*(g*x + f)^n*c*f^2*g^2*n^
2*x^2*e^2 + 2*(g*x + f)^n*c*f*g^3*n*x^3*e^2 + 6*(g*x + f)^n*c*g^4*x^4*e^2 -
 6*(g*x + f)^n*c*d*f^2*g^2*n^2*x*e + (g*x + f)^n*a*f*g^3*n^3*x*e + 12*(g*x
+ f)^n*c*d*f*g^3*n*x^2*e + 8*(g*x + f)^n*a*g^4*n^2*x^2*e + 24*(g*x + f)^n*c
*d*g^4*x^3*e - 2*(g*x + f)^n*c*d^2*f^2*g^2*n^2 + (g*x + f)^n*a*d*f*g^3*n^3
+ 24*(g*x + f)^n*c*d^2*f*g^3*n*x + 9*(g*x + f)^n*a*d*g^4*n^2*x + 24*(g*x +
f)^n*c*d^2*g^4*x^2 - 3*(g*x + f)^n*c*f^2*g^2*n*x^2*e^2 - 24*(g*x + f)^n*c*d
*f^2*g^2*n*x*e + 7*(g*x + f)^n*a*f*g^3*n^2*x*e + 19*(g*x + f)^n*a*g^4*n*x^2
*e - 14*(g*x + f)^n*c*d^2*f^2*g^2*n + 9*(g*x + f)^n*a*d*f*g^3*n^2 + 26*(g*x
 + f)^n*a*d*g^4*n*x + 6*(g*x + f)^n*c*f^3*g*n*x*e^2 + 6*(g*x + f)^n*c*d*f^3
*g*n*e - (g*x + f)^n*a*f^2*g^2*n^2*e + 12*(g*x + f)^n*a*f*g^3*n*x*e + 12*(g
*x + f)^n*a*g^4*x^2*e - 24*(g*x + f)^n*c*d^2*f^2*g^2 + 26*(g*x + f)^n*a*d*f
*g^3*n + 24*(g*x + f)^n*a*d*g^4*x + 24*(g*x + f)^n*c*d*f^3*g*e - 7*(g*x + f
)^n*a*f^2*g^2*n*e + 24*(g*x + f)^n*a*d*f*g^3 - 6*(g*x + f)^n*c*f^4*e^2 - 12
*(g*x + f)^n*a*f^2*g^2*e)/(g^4*n^4 + 10*g^4*n^3 + 35*g^4*n^2 + 50*g^4*n + 2
4*g^4)
```

Mupad [B]
time = 3.29, size = 572, normalized size = 3.92

$$\frac{x\,{\left(f+g\,x\right)}^n\,\left(2\,c\,d^2\,f\,g^3\,n^3+14\,c\,d^2\,f\,g^3\,n^2+24\,c\,d^2\,f\,g^3\,n-6\,c\,d\,e\,f^2\,g^2\,n^2-24\,c\,d\,e\,f^2\,g^2\,n+a\,d\,g^4\,n^3+9\,a\,d\,g^4\,n^2+26\,a\,d\,g^4\,n+24\,a\,d\,g^4+6\,c\,e^2\,f^3\,g\,n+a\,e\,f\,g^3\,n^3+7\,a\,e\,f\,g^3\,n^2+12\,a\,e\,f\,g^3\,n\right)}{g^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{{\left(f+g\,x\right)}^n\,\left(2\,c\,d^2\,f^2\,g^2\,n^2+14\,c\,d^2\,f^2\,g^2\,n+24\,c\,d^2\,f^2\,g^2-6\,c\,d\,e\,f^3\,g\,n-24\,c\,d\,e\,f^3\,g-a\,d\,f\,g^3\,n^3-9\,a\,d\,f\,g^3\,n^2-26\,a\,d\,f\,g^3\,n-24\,a\,d\,f\,g^3+6\,c\,e^2\,f^4+a\,e\,f^2\,g^2\,n^2+7\,a\,e\,f^2\,g^2\,n+12\,a\,e\,f^2\,g^2\right)}{g^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{c\,e^2\,x^4\,{\left(f+g\,x\right)}^n\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{x^2\,{\left(f+g\,x\right)}^n\,\left(n+1\right)\,\left(2\,c\,d^2\,g^2\,n^2+14\,c\,d^2\,g^2\,n+24\,c\,d^2\,g^2+3\,c\,d\,e\,f\,g\,n^2+12\,c\,d\,e\,f\,g\,n-3\,c\,e^2\,f^2\,n+a\,e\,g^2\,n^2+7\,a\,e\,g^2\,n+12\,a\,e\,g^2\right)}{g^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{c\,e\,x^3\,{\left(f+g\,x\right)}^n\,\left(12\,d\,g+3\,d\,g\,n+e\,f\,n\right)\,\left(n^2+3\,n+2\right)}{g\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (x*(f + g*x)^n*(24*a*d*g^4 + 26*a*d*g^4*n + 9*a*d*g^4*n^2 + a*d*g^4*n^3 + 7
*a*e*f*g^3*n^2 + a*e*f*g^3*n^3 + 24*c*d^2*f*g^3*n + 6*c*e^2*f^3*g*n + 14*c*
d^2*f*g^3*n^2 + 2*c*d^2*f*g^3*n^3 + 12*a*e*f*g^3*n - 24*c*d*e*f^2*g^2*n - 6
*c*d*e*f^2*g^2*n^2))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - ((f + g*x)
^n*(6*c*e^2*f^4 + 24*c*d^2*f^2*g^2 - 24*a*d*f*g^3 + 12*a*e*f^2*g^2 - 9*a*d*
f*g^3*n^2 - a*d*f*g^3*n^3 + 7*a*e*f^2*g^2*n + a*e*f^2*g^2*n^2 + 14*c*d^2*f^
2*g^2*n - 24*c*d*e*f^3*g - 26*a*d*f*g^3*n + 2*c*d^2*f^2*g^2*n^2 - 6*c*d*e*f
^3*g*n))/(g^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e^2*x^4*(f + g*x)^n
*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24) + (x^2*(f +
g*x)^n*(n + 1)*(24*c*d^2*g^2 + 12*a*e*g^2 + 2*c*d^2*g^2*n^2 + 7*a*e*g^2*n +
 a*e*g^2*n^2 + 14*c*d^2*g^2*n - 3*c*e^2*f^2*n + 3*c*d*e*f*g*n^2 + 12*c*d*e*
f*g*n))/(g^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (c*e*x^3*(f + g*x)^n*(1
2*d*g + 3*d*g*n + e*f*n)*(3*n + n^2 + 2))/(g*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24))
```

3.808 $\int (f+gx)^n \left(a+2cdx+cex^2\right) \, dx$

Optimal. Leaf size=84

$$\frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^3(1+n)}-\frac{2c(ef-dg)(f+gx)^{2+n}}{g^3(2+n)}+\frac{ce(f+gx)^{3+n}}{g^3(3+n)}$$

```
[Out] (a*g^2+c*f*(-2*d*g+e*f))*(g*x+f)^(1+n)/g^3/(1+n)-2*c*(-d*g+e*f)*(g*x+f)^(2+
n)/g^3/(2+n)+c*e*(g*x+f)^(3+n)/g^3/(3+n)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {712}

$$\frac{(f+gx)^{n+1}\left(ag^2+cf(ef-2dg)\right)}{g^3(n+1)}-\frac{2c(ef-dg)(f+gx)^{n+2}}{g^3(n+2)}+\frac{ce(f+gx)^{n+3}}{g^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^3*(1 + n)) - (2*c*(e*f -
 d*g)*(f + g*x)^(2 + n))/(g^3*(2 + n)) + (c*e*(f + g*x)^(3 + n))/(g^3*(3 +
n))
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int (f+gx)^n \left(a+2cdx+cex^2\right) dx &= \int \left(\frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^n}{g^2}+\frac{2c(-ef+dg)(f+gx)^{1+n}}{g^2}+\frac{ce\ldots}{}\right)\\ &= \frac{\left(ag^2+cf(ef-2dg)\right)(f+gx)^{1+n}}{g^3(1+n)}-\frac{2c(ef-dg)(f+gx)^{2+n}}{g^3(2+n)}+\frac{ce(f+\ldots}{g^3(3\ldots}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 92, normalized size = 1.10

$$\frac{(f+gx)^{1+n}\left(ag^2(6+5n+n^2)+2cdg(3+n)(-f+g(1+n)x)+ce(2f^2-2fg(1+n)x+g^2(2+3n+n^2)x^2)\right)}{g^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((f + g*x)^(1 + n)*(a*g^2*(6 + 5*n + n^2) + 2*c*d*g*(3 + n)*(-f + g*(1 + n)
*x) + c*e*(2*f^2 - 2*f*g*(1 + n)*x + g^2*(2 + 3*n + n^2)*x^2)))/(g^3*(1 + n
)*(2 + n)*(3 + n))
```

Maple [A]
time = 0.10, size = 147, normalized size = 1.75

method	result
gosper	$\frac{(gx+f)^{1+n}(ce\,g^2n^2x^2+2cd\,g^2n^2x+3ce\,g^2n\,x^2+8cd\,g^2nx-2cefgnx+2ce\,x^2g^2+a\,g^2n^2-2cdfgn+6cd\,g^2x-2cefgx+5a\,g^2n-6cdfg+}{g^3(n^3+6n^2+11n+6)}$
norman	$\frac{ce\,x^3e^{n\ln(gx+f)}}{3+n}+\frac{f\left(a\,g^2n^2-2cdfgn+5a\,g^2n-6cdfg+2ce\,f^2+6a\,g^2\right)e^{n\ln(gx+f)}}{g^3(n^3+6n^2+11n+6)}+\frac{(2cdfg\,n^2+a\,g^2n^2+6cdfgn-2ce\,f^2n+5a\,g^2n}{g^2(n^3+6n^2+11n+6)}$
risch	$\frac{(ce\,g^3n^2x^3+2cd\,g^3n^2x^2+cef\,g^2n^2x^2+3ce\,g^3n\,x^3+2cdf\,g^2n^2x+8cd\,g^3n\,x^2+cef\,g^2n\,x^2+2ce\,x^3g^3+a\,g^3n^2x+6cdf\,g^2nx+6cd\,g^3x^2-}{(2+n)(3+n)(1+n)g^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (g*x+f)^(1+n)*(c*e*g^2*n^2*x^2+2*c*d*g^2*n^2*x+3*c*e*g^2*n*x^2+8*c*d*g^2*n*
x-2*c*e*f*g*n*x+2*c*e*g^2*x^2+a*g^2*n^2-2*c*d*f*g*n+6*c*d*g^2*x-2*c*e*f*g*x
+5*a*g^2*n-6*c*d*f*g+2*c*e*f^2+6*a*g^2)/g^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.31, size = 135, normalized size = 1.61

$$\frac{2\,(g^2(n+1)x^2+fgnx-f^2)(gx+f)^n cd}{(n^2+3\,n+2)g^2}+\frac{((n^2+3\,n+2)g^3x^3+(n^2+n)fg^2x^2-2\,f^2gnx+2\,f^3)(gx+f)^n ce}{(n^3+6\,n^2+11\,n+6)g^3}+\frac{(gx+f)^{n+1}a}{g(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
```

```
[Out] 2*(g^2*(n + 1)*x^2 + f*g*n*x - f^2)*(g*x + f)^n*c*d/((n^2 + 3*n + 2)*g^2) +
 ((n^2 + 3*n + 2)*g^3*x^3 + (n^2 + n)*f*g^2*x^2 - 2*f^2*g*n*x + 2*f^3)*(g*x
 + f)^n*c*e/((n^3 + 6*n^2 + 11*n + 6)*g^3) + (g*x + f)^(n + 1)*a/(g*(n + 1)
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(90) = 180.
time = 4.93, size = 224, normalized size = 2.67

$$\frac{(afg^2n^2-6\,cdf^2g+6\,afg^2+2\,(cdg^3n^2+4\,cdg^3n+3\,cdg^3)x^2-(2\,cdf^2g-5\,afg^2)n+(6\,ag^3+(2\,cdfg^2+ag^3)n^2+(6\,cdfg^2+5\,ag^3)n)x-(2\,cf^2gnx-2\,cf^3-(cg^3n^2+3\,cg^3n+2\,cg^3)x^3-(cfg^2n^2+cfg^2n)x^2)e)(gx+f)^n}{g^3n^3+6\,g^3n^2+11\,g^3n+6\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
```

```
[Out] (a*f*g^2*n^2 - 6*c*d*f^2*g + 6*a*f*g^2 + 2*(c*d*g^3*n^2 + 4*c*d*g^3*n + 3*c
*d*g^3)*x^2 - (2*c*d*f^2*g - 5*a*f*g^2)*n + (6*a*g^3 + (2*c*d*f*g^2 + a*g^3
)*n^2 + (6*c*d*f*g^2 + 5*a*g^3)*n)*x - (2*c*f^2*g*n*x - 2*c*f^3 - (c*g^3*n^
2 + 3*c*g^3*n + 2*c*g^3)*x^3 - (c*f*g^2*n^2 + c*f*g^2*n)*x^2)*e)*(g*x + f)^
n/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. $2(75) = 150$.

time = 0.57, size = 1489, normalized size = 17.73

$$\begin{cases} f^{n}\left(ax + cdx^{2} + \frac{cex^{3}}{3}\right) & \text{for}\ g = 0 \\ -\frac{ag^{2}}{2f^{2}g^{3}+4fg^{4}x+2g^{5}x^{2}} - \frac{2cdfg}{2f^{2}g^{3}+4fg^{4}x+2g^{5}x^{2}} - \frac{4cdg^{2}x}{2f^{2}g^{3}+4fg^{4}x+2g^{5}x^{2}} + \frac{2cef^{2}\log\left(\frac{f}{g}+x\right)}{2f^{2}g^{3}+4fg^{4}x+2g^{5}x^{2}} + \frac{3cef^{2}}{2f^{2}g^{3}+4fg^{4}x+2g^{5}x^{2}} + \frac{4cefgx\log\left(\frac{f}{g}+x\right)}{2f^{2}g^{3}+4fg^{4}x+2g^{5}x^{2}} + \frac{4cefgx}{2f^{2}g^{3}+4fg^{4}x+2g^{5}x^{2}} + \frac{2ceg^{2}x^{2}\log\left(\frac{f}{g}+x\right)}{2f^{2}g^{3}+4fg^{4}x+2g^{5}x^{2}} & \text{for}\ n = -3 \\ -\frac{ag^{2}}{fg^{3}+g^{4}x} + \frac{2cdfg\log\left(\frac{f}{g}+x\right)}{fg^{3}+g^{4}x} + \frac{2cdfg}{fg^{3}+g^{4}x} + \frac{2cdg^{2}x\log\left(\frac{f}{g}+x\right)}{fg^{3}+g^{4}x} - \frac{2cef^{2}\log\left(\frac{f}{g}+x\right)}{fg^{3}+g^{4}x} - \frac{2cef^{2}}{fg^{3}+g^{4}x} - \frac{2cefgx\log\left(\frac{f}{g}+x\right)}{fg^{3}+g^{4}x} + \frac{ceg^{2}x^{2}}{fg^{3}+g^{4}x} & \text{for}\ n = -2 \\ \frac{a\log\left(\frac{f}{g}+x\right)}{g} - \frac{2cdf\log\left(\frac{f}{g}+x\right)}{g^{2}} + \frac{2cdx}{g} + \frac{cef^{2}\log\left(\frac{f}{g}+x\right)}{g^{3}} - \frac{cefx}{g^{2}} + \frac{cex^{2}}{2g} & \text{for}\ n = -1 \\ \frac{afg^{2}n^{2}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{5afg^{2}n(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{6afg^{2}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{ag^{3}n^{2}x(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{5ag^{3}nx(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{6ag^{3}x(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} - \frac{2cdf^{2}gn(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} - \frac{6cdf^{2}g(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{2cdfg^{2}n^{2}x(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{6cdfg^{2}nx(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{2cdg^{3}n^{2}x^{2}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{8cdg^{3}nx^{2}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{6cdg^{3}x^{2}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{2cef^{3}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} - \frac{2cef^{2}gnx(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{cefg^{2}n^{2}x^{2}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{cefg^{2}nx^{2}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{ceg^{3}n^{2}x^{3}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{3ceg^{3}nx^{3}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} + \frac{2ceg^{3}x^{3}(f+gx)^{n}}{g^{3}n^{3}+6g^{3}n^{2}+11g^{3}n+6g^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*x + c*d*x**2 + c*e*x**3/3), Eq(g, 0)), (-a*g**2/(2*f**2*
g**3 + 4*f*g**4*x + 2*g**5*x**2) - 2*c*d*f*g/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) - 4*c*d*g**2*x/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 2*c*e*
f**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 3*c*e*f**2/(2*
f**2*g**3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x*log(f/g + x)/(2*f**2*g*
*3 + 4*f*g**4*x + 2*g**5*x**2) + 4*c*e*f*g*x/(2*f**2*g**3 + 4*f*g**4*x + 2*
g**5*x**2) + 2*c*e*g**2*x**2*log(f/g + x)/(2*f**2*g**3 + 4*f*g**4*x + 2*g**
5*x**2), Eq(n, -3)), (-a*g**2/(f*g**3 + g**4*x) + 2*c*d*f*g*log(f/g + x)/(f
*g**3 + g**4*x) + 2*c*d*f*g/(f*g**3 + g**4*x) + 2*c*d*g**2*x*log(f/g + x)/(
f*g**3 + g**4*x) - 2*c*e*f**2*log(f/g + x)/(f*g**3 + g**4*x) - 2*c*e*f**2/(
f*g**3 + g**4*x) - 2*c*e*f*g*x*log(f/g + x)/(f*g**3 + g**4*x) + c*e*g**2*x*
*2/(f*g**3 + g**4*x), Eq(n, -2)), (a*log(f/g + x)/g - 2*c*d*f*log(f/g + x)/
g**2 + 2*c*d*x/g + c*e*f**2*log(f/g + x)/g**3 - c*e*f*x/g**2 + c*e*x**2/(2*
g), Eq(n, -1)), (a*f*g**2*n**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g
**3*n + 6*g**3) + 5*a*f*g**2*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g
**3*n + 6*g**3) + 6*a*f*g**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**
3*n + 6*g**3) + a*g**3*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 5*a*g**3*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 6*a*g**3*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3
*n + 6*g**3) - 2*c*d*f**2*g*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) - 6*c*d*f**2*g*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g*
*3*n + 6*g**3) + 2*c*d*f*g**2*n**2*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2
+ 11*g**3*n + 6*g**3) + 6*c*d*f*g**2*n*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n
**2 + 11*g**3*n + 6*g**3) + 2*c*d*g**3*n**2*x**2*(f + g*x)**n/(g**3*n**3 +
6*g**3*n**2 + 11*g**3*n + 6*g**3) + 8*c*d*g**3*n*x**2*(f + g*x)**n/(g**3*n*
*3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 6*c*d*g**3*x**2*(f + g*x)**n/(g**3
*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e*f**3*(f + g*x)**n/(g**3*n
**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) - 2*c*e*f**2*g*n*x*(f + g*x)**n/(g*
*3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n**2*x**2*(f + g*x
)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g**2*n*x**2*(f
```

```
+ g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*g**3*n**2*x*
*3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 3*c*e*g**3
*n*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e
*g**3*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3), Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(90) = 180.
time = 3.81, size = 373, normalized size = 4.44

$$\frac{(gx+f)^n cg^3n^2x^3e + 2\,(gx+f)^n cdg^3n^2x^2 + (gx+f)^n cfg^2n^2x^2e + 3\,(gx+f)^n cg^3nx^3e + 2\,(gx+f)^n cdfg^2n^2x + 8\,(gx+f)^n cdg^3nx^2 + (gx+f)^n cfg^2nx^2e + 2\,(gx+f)^n cg^3x^3e + 6\,(gx+f)^n cdfg^2nx + (gx+f)^n ag^3n^2x + 6\,(gx+f)^n cdg^3x^2 - 2\,(gx+f)^n cf^2gnxe - 2\,(gx+f)^n cdf^2gn + (gx+f)^n afg^2n^2 + 5\,(gx+f)^n ag^3nx - 6\,(gx+f)^n cdf^2g + 5\,(gx+f)^n afg^2n + 6\,(gx+f)^n ag^3x + 2\,(gx+f)^n cf^3e + 6\,(gx+f)^n afg^2}{g^3n^3 + 6\,g^3n^2 + 11\,g^3n + 6\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
```

```
[Out] ((g*x + f)^n*c*g^3*n^2*x^3*e + 2*(g*x + f)^n*c*d*g^3*n^2*x^2 + (g*x + f)^n*
c*f*g^2*n^2*x^2*e + 3*(g*x + f)^n*c*g^3*n*x^3*e + 2*(g*x + f)^n*c*d*f*g^2*n
^2*x + 8*(g*x + f)^n*c*d*g^3*n*x^2 + (g*x + f)^n*c*f*g^2*n*x^2*e + 2*(g*x +
 f)^n*c*g^3*x^3*e + 6*(g*x + f)^n*c*d*f*g^2*n*x + (g*x + f)^n*a*g^3*n^2*x +
 6*(g*x + f)^n*c*d*g^3*x^2 - 2*(g*x + f)^n*c*f^2*g*n*x*e - 2*(g*x + f)^n*c*
d*f^2*g*n + (g*x + f)^n*a*f*g^2*n^2 + 5*(g*x + f)^n*a*g^3*n*x - 6*(g*x + f)
^n*c*d*f^2*g + 5*(g*x + f)^n*a*f*g^2*n + 6*(g*x + f)^n*a*g^3*x + 2*(g*x + f
)^n*c*f^3*e + 6*(g*x + f)^n*a*f*g^2)/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^
3)
```

Mupad [B]
time = 3.07, size = 211, normalized size = 2.51

$$(f+g\,x)^n\left(\frac{f\,\left(2\,c\,e\,f^2-2\,c\,d\,f\,g\,n-6\,c\,d\,f\,g+a\,g^2\,n^2+5\,a\,g^2\,n+6\,a\,g^2\right)}{g^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{x\,\left(-2\,c\,e\,f^2\,g\,n+2\,c\,d\,f\,g^2\,n^2+6\,c\,d\,f\,g^2\,n+a\,g^3\,n^2+5\,a\,g^3\,n+6\,a\,g^3\right)}{g^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{c\,e\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{c\,x^2\,\left(n+1\right)\,\left(6\,d\,g+2\,d\,g\,n+e\,f\,n\right)}{g\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x)
```

```
[Out] (f + g*x)^n*((f*(6*a*g^2 + a*g^2*n^2 + 2*c*e*f^2 + 5*a*g^2*n - 6*c*d*f*g -
2*c*d*f*g*n))/(g^3*(11*n + 6*n^2 + n^3 + 6)) + (x*(6*a*g^3 + a*g^3*n^2 + 5*
a*g^3*n + 2*c*d*f*g^2*n^2 + 6*c*d*f*g^2*n - 2*c*e*f^2*g*n))/(g^3*(11*n + 6*
n^2 + n^3 + 6)) + (c*e*x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (c*x
^2*(n + 1)*(6*d*g + 2*d*g*n + e*f*n))/(g*(11*n + 6*n^2 + n^3 + 6)))
```

3.809 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{d+ex}\,dx$

Optimal. Leaf size=114

$$-\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)}+\frac{c(f+gx)^{2+n}}{g^2(2+n)}+\frac{(cd^2-ae)\,(f+gx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)(1+n)}$$

```
[Out] -c*(-d*g+e*f)*(g*x+f)^(1+n)/e/g^2/(1+n)+c*(g*x+f)^(2+n)/g^2/(2+n)+(c*d^2-a*
e)*(g*x+f)^(1+n)*hypergeom([1, 1+n],[2+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f
)/(1+n)
```

Rubi [A]

time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {965, 81, 70}

$$\frac{(cd^2-ae)\,(f+gx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)}-\frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)}+\frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

```
[Out] -((c*(e*f - d*g)*(f + g*x)^(1 + n))/(e*g^2*(1 + n))) + (c*(f + g*x)^(2 + n)
)/(g^2*(2 + n)) + ((c*d^2 - a*e)*(f + g*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
```

```
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{d+ex}\,dx = \frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{\int \frac{(f+gx)^n(-eg(cdf-ag)(2+n)-ceg(ef-dg)(2+n)x)}{d+ex}\,dx}{eg^2(2+n)}$$
$$= -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)} + \frac{c(f+gx)^{2+n}}{g^2(2+n)} - \frac{\left(cd^2-ae\right)\int \frac{(f+gx)^n}{d+ex}\,dx}{e}$$
$$= -\frac{c(ef-dg)(f+gx)^{1+n}}{eg^2(1+n)} + \frac{c(f+gx)^{2+n}}{g^2(2+n)} + \frac{\left(cd^2-ae\right)(f+gx)^{1+n}\,{}_2F$$

Mathematica [A]
time = 0.34, size = 125, normalized size = 1.10

$$\frac{(f+gx)^n\left(\frac{c\left(\frac{dg(f+gx)}{e}+\frac{fgnx+g^2(1+n)x^2+f^2\left(-1+\left(1+\frac{gx}{f}\right)^{-n}\right)}{2+n}\right)}{g^2}+\frac{\left(cd^2-ae\right)(f+gx)\,{}_2F_1\left(1,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)}\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x]
```

```
[Out] ((f + g*x)^n*((c*((d*g*(f + g*x))/e + (f*g*n*x + g^2*(1 + n)*x^2 + f^2*(-1
+ (1 + (g*x)/f)^(-n)))/(2 + n)))/g^2 + ((c*d^2 - a*e)*(f + g*x)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g))))/(1 +
n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^n\left(ce\,x^2+2cdx+a\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)
```

[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="maxima")

[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d), x)

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="fricas")

[Out] integral((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d), x)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d),x)

[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x), x)

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d),x, algorithm="giac")

[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,(c\,e\,x^2+2\,c\,d\,x+a)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x),x)

[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x), x)

3.810 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^2}\,dx$

Optimal. Leaf size=88

$$\frac{c(f+gx)^{1+n}}{eg(1+n)}-\frac{(cd^2-ae)\,g(f+gx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)^2(1+n)}$$

```
[Out] c*(g*x+f)^(1+n)/e/g/(1+n)-(c*d^2-a*e)*g*(g*x+f)^(1+n)*hypergeom([2, 1+n],[2
+n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f)^2/(1+n)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {961, 70}

$$\frac{c(f+gx)^{n+1}}{eg(n+1)}-\frac{g(cd^2-ae)\,(f+gx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

```
[Out] (c*(f + g*x)^(1 + n))/(e*g*(1 + n)) - ((c*d^2 - a*e)*g*(f + g*x)^(1 + n)*Hy
pergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(e*(e*f - d*g)
^2*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx = \int \left(\frac{c(f+gx)^n}{e}+\frac{(-cd^2+ae)\,(f+gx)^n}{e(d+ex)^2}\right)dx$$
$$= \frac{c(f+gx)^{1+n}}{eg(1+n)}+\frac{(-cd^2+ae)\int \frac{(f+gx)^n}{(d+ex)^2}\,dx}{e}$$
$$= \frac{c(f+gx)^{1+n}}{eg(1+n)}-\frac{(cd^2-ae)\,g(f+gx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)^2(1+n)}$$

Mathematica [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
[Out] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2, x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^n\,(ce\,x^2+2cdx+a)}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^n\,\left(c\,e\,x^2+2\,c\,d\,x+a\right)}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x)
```

```
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2, x)
```

3.811 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^3}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2}-\frac{(cd^2-ae)\,g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)}+\frac{(aeg^2(1-n)n-c(2e^2f^2-4defg+d^2g^2(2+n-\cdots}{2e(ef-d\cdots}$$

```
[Out] -1/2*(a-c*d^2/e)*(g*x+f)^(1+n)/(-d*g+e*f)/(e*x+d)^2-1/2*(c*d^2-a*e)*g*(1-n)
*(g*x+f)^(1+n)/e/(-d*g+e*f)^2/(e*x+d)+1/2*(a*e*g^2*(1-n)*n-c*(2*e^2*f^2-4*d
*e*f*g+d^2*g^2*(-n^2+n+2)))*(g*x+f)^(1+n)*hypergeom([1, 1+n],[2+n],e*(g*x+f
)/(-d*g+e*f))/e/(-d*g+e*f)^3/(1+n)
```

Rubi [A]
time = 0.14, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {963, 79, 70}

$$\frac{(f+gx)^{n+1}\left(aeg^2(1-n)n-c(d^2g^2(-n^2+n+2)-4defg+2e^2f^2)\right)\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{2e(n+1)(ef-dg)^3}-\frac{g(1-n)\left(cd^2-ae\right)(f+gx)^{n+1}}{2e(d+ex)(ef-dg)^2}-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{2(d+ex)^2(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

```
[Out] -1/2*((a - (c*d^2)/e)*(f + g*x)^(1 + n))/((e*f - d*g)*(d + e*x)^2) - ((c*d^
2 - a*e)*g*(1 - n)*(f + g*x)^(1 + n))/(2*e*(e*f - d*g)^2*(d + e*x)) + ((a*e
*g^2*(1 - n)*n - c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2*(2 + n - n^2)))*(f + g*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(
2*e*(e*f - d*g)^3*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2} - \frac{\int \frac{(f+gx)^n\left(ag(1-n)-\frac{cd(2ef-dg(1+n))}{e}-2c(ef-dg)x\right)}{(d+ex)^2}\,d}{2(ef-dg)}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2} - \frac{\left(cd^2-ae\right)g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)} - \frac{(aeg^2(1}{}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{2(ef-dg)(d+ex)^2} - \frac{\left(cd^2-ae\right)g(1-n)(f+gx)^{1+n}}{2e(ef-dg)^2(d+ex)} + \frac{(aeg^2(1}{}$$

Mathematica [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
[Out] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3, x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^n\left(ce\,x^2+2cdx+a\right)}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="maxima")
[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="fricas")
[Out] integral((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2
*x*e + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**3,x)
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(f+g\,x\right)}^n\,\left(c\,e\,x^2+2\,c\,d\,x+a\right)}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3, x)
```

3.812 $\int \frac{(f+gx)^n(a+2cdx+cex^2)}{(d+ex)^4}\,dx$

Optimal. Leaf size=197

$$-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3}-\frac{(cd^2-ae)\,g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2}+\frac{g(aeg^2(2-3n+n^2)+c(6e^2f^2-12defg+d^2g}{6e(e}$$

```
[Out] -1/3*(a-c*d^2/e)*(g*x+f)^(1+n)/(-d*g+e*f)/(e*x+d)^3-1/6*(c*d^2-a*e)*g*(2-n)
*(g*x+f)^(1+n)/e/(-d*g+e*f)^2/(e*x+d)^2+1/6*g*(a*e*g^2*(n^2-3*n+2)+c*(6*e^2
*f^2-12*d*e*f*g+d^2*g^2*(-n^2+3*n+4)))*(g*x+f)^(1+n)*hypergeom([2, 1+n],[2+
n],e*(g*x+f)/(-d*g+e*f))/e/(-d*g+e*f)^4/(1+n)
```

Rubi [A]

time = 0.15, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {963, 79, 70}

$$\frac{g(f+gx)^{n+1}\left(aeg^2(n^2-3n+2)+c(d^2g^2(-n^2+3n+4)-12defg+6e^2f^2)\right)\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)}{6e(n+1)(ef-dg)^4}-\frac{g(2-n)\,(cd^2-ae)\,(f+gx)^{n+1}}{6e(d+ex)^2(ef-dg)^2}-\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{n+1}}{3(d+ex)^3(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

```
[Out] -1/3*((a - (c*d^2)/e)*(f + g*x)^(1 + n))/((e*f - d*g)*(d + e*x)^3) - ((c*d^
2 - a*e)*g*(2 - n)*(f + g*x)^(1 + n))/(6*e*(e*f - d*g)^2*(d + e*x)^2) + (g*
(a*e*g^2*(2 - 3*n + n^2) + c*(6*e^2*f^2 - 12*d*e*f*g + d^2*g^2*(4 + 3*n - n
^2)))*(f + g*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (e*(f + g*x))/(e
*f - d*g)])/(6*e*(e*f - d*g)^4*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx = -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3} - \frac{\int \frac{(f+gx)^n\left(ag(2-n)-\frac{cd(3ef-dg(1+n))}{e}-3c(ef-dg)x\right)}{(d+ex)^3}\,dx}{3(ef-dg)}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3} - \frac{\left(cd^2-ae\right)g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2} + \frac{(aeg^2(2}{}$$

$$= -\frac{\left(a-\frac{cd^2}{e}\right)(f+gx)^{1+n}}{3(ef-dg)(d+ex)^3} - \frac{\left(cd^2-ae\right)g(2-n)(f+gx)^{1+n}}{6e(ef-dg)^2(d+ex)^2} + \frac{g(aeg^2(2}{}$$

Mathematica [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
[Out] Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4, x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^n\left(ce\,x^2+2cdx+a\right)}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
[Out] int((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="maxima")
[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="fricas")
[Out] integral((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2
*x^2*e^2 + 4*d^3*x*e + d^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a)/(e*x+d)**4,x)
[Out] Integral((f + g*x)**n*(a + 2*c*d*x + c*e*x**2)/(d + e*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^n*(c*e*x^2+2*c*d*x+a)/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n/(x*e + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(f+g\,x\right)}^n\,\left(c\,e\,x^2+2\,c\,d\,x+a\right)}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x)
[Out] int(((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4, x)
```

3.813 $\int (d+ex)^m (f+gx)^n \left(a+2cdx+cex^2\right) dx$

Optimal. Leaf size=231

$$-\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)}+\frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)}+\frac{(c(ef-dg)(2+m)(ef(1+m)+$$

```
[Out] -c*(-d*g+e*f)*(2+m)*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e/g^2/(2+m+n)/(3+m+n)+c*(e*
x+d)^(2+m)*(g*x+f)^(1+n)/e/g/(3+m+n)+(c*(-d*g+e*f)*(2+m)*(e*f*(1+m)+d*g*(1+
n))+g*(2+m+n)*(a*e*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*(1+n))))*(e*x+d)^(1+m)*(g*x
+f)^n*hypergeom([-n, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/e^2/g^2/(1+m)/(2+m+n
)/(3+m+n)/((e*(g*x+f)/(-d*g+e*f))^n)
```

Rubi [A]

time = 0.17, antiderivative size = 227, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {965, 81, 72, 71}

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(aeg(m+n+3)+\frac{c(m+2)(ef-dg)(dg(n+1)+ef(m+1))}{g(m+n+2)}-cd(dg(n+1)+ef(m+2))\right){}_2F_1\left(m+1,-n;m+2;-\frac{g(d+ex)}{ef-dg}\right)}{e^2g(m+1)(m+n+3)}-\frac{c(m+2)(ef-dg)(d+ex)^{m+1}(f+gx)^{n+1}}{eg^2(m+n+2)(m+n+3)}+\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{eg(m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] -((c*(e*f - d*g)*(2 + m)*(d + e*x)^(1 + m)*(f + g*x)^(1 + n))/(e*g^2*(2 + m
 + n)*(3 + m + n))) + (c*(d + e*x)^(2 + m)*(f + g*x)^(1 + n))/(e*g*(3 + m +
 n)) + ((a*e*g*(3 + m + n) + (c*(e*f - d*g)*(2 + m)*(e*f*(1 + m) + d*g*(1 +
 n)))/(g*(2 + m + n)) - c*d*(e*f*(2 + m) + d*g*(1 + n)))*(d + e*x)^(1 + m)*
(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g)
)])/(e^2*g*(1 + m)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int (d+ex)^m(f+gx)^n\left(a+2cdx+cex^2\right)\,dx = \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{eg(3+m+n)} + \frac{\int (d+ex)^m(f+gx)^n\,(e(aeg(3$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+}}{eg(3+}$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+}}{eg(3+}$$

$$= -\frac{c(ef-dg)(2+m)(d+ex)^{1+m}(f+gx)^{1+n}}{eg^2(2+m+n)(3+m+n)} + \frac{c(d+ex)^{2+}}{eg(3+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.32, size = 190, normalized size = 0.82

$$\frac{1}{3}(d+ex)^m(f+gx)^n\left(3cdx^2\left(1+\frac{ex}{d}\right)^{-m}\left(1+\frac{gx}{f}\right)^{-n}F_1\left(2;-m,-n;3;-\frac{ex}{d},-\frac{gx}{f}\right)+cex^3\left(1+\frac{ex}{d}\right)^{-m}\left(1+\frac{gx}{f}\right)^{-n}F_1\left(3;-m,-n;4;-\frac{ex}{d},-\frac{gx}{f}\right)+\frac{3a\left(\frac{g(d+ex)}{-ef+dg}\right)^{-m}(f+gx)\,{}_2F_1\left(-m,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{g(1+n)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]
```

```
[Out] ((d + e*x)^m*(f + g*x)^n*((3*c*d*x^2*AppellF1[2, -m, -n, 3, -((e*x)/d), -((
g*x)/f)])/((1 + (e*x)/d)^m*(1 + (g*x)/f)^n) + (c*e*x^3*AppellF1[3, -m, -n,
4, -((e*x)/d), -((g*x)/f)])/((1 + (e*x)/d)^m*(1 + (g*x)/f)^n) + (3*a*(f + g
*x)*Hypergeometric2F1[-m, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(g*(1 +
 n)*((g*(d + e*x))/(-(e*f) + d*g))^m)))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex+d\right)^{m} \left(gx+f\right)^{n} \left(ce\,x^{2}+2cdx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
[Out] int((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="maxima")
[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n*(x*e + d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="fricas")
[Out] integral((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n*(x*e + d)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x, algorithm="giac")
[Out] integrate((c*x^2*e + 2*c*d*x + a)*(g*x + f)^n*(x*e + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^n\,(d+e\,x)^m\,\left(c\,e\,x^2+2\,c\,d\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^m*(a + 2*c*d*x + c*e*x^2),x)
[Out] int((f + g*x)^n*(d + e*x)^m*(a + 2*c*d*x + c*e*x^2), x)
```

3.814 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=83

$$\frac{cx}{eg}+\frac{(cd^2-bde+ae^2)\log(d+ex)}{e^2(ef-dg)}-\frac{(cf^2-bfg+ag^2)\log(f+gx)}{g^2(ef-dg)}$$

```
[Out] c*x/e/g+(a*e^2-b*d*e+c*d^2)*ln(e*x+d)/e^2/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)*ln
(g*x+f)/g^2/(-d*g+e*f)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {907}

$$\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)}{e^2(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)}{g^2(ef-dg)}+\frac{cx}{eg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) + ((c*d^2 - b*d*e + a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) - ((
c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(d+ex)(f+gx)}\,dx&=\int \left(\frac{c}{eg}+\frac{cd^2-bde+ae^2}{e(ef-dg)(d+ex)}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)(f+gx)}\right)dx\\&=\frac{cx}{eg}+\frac{(cd^2-bde+ae^2)\log(d+ex)}{e^2(ef-dg)}-\frac{(cf^2-bfg+ag^2)\log(f+gx)}{g^2(ef-dg)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 85, normalized size = 1.02

$$\frac{cx}{eg}-\frac{(-cd^2+bde-ae^2)\log(d+ex)}{e^2(ef-dg)}-\frac{(cf^2-bfg+ag^2)\log(f+gx)}{g^2(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (c*x)/(e*g) - ((-(c*d^2) + b*d*e - a*e^2)*Log[d + e*x])/(e^2*(e*f - d*g)) -
 ((c*f^2 - b*f*g + a*g^2)*Log[f + g*x])/(g^2*(e*f - d*g))
```

Maple [A]
time = 0.10, size = 84, normalized size = 1.01

method	result	size
default	$\frac{cx}{eg}+\frac{\left(-a\,e^{2}+bde-c\,d^{2}\right)\ln \left(ex+d\right)}{\left(dg-ef\right)e^{2}}+\frac{\left(a\,g^{2}-bfg+c\,f^{2}\right)\ln \left(gx+f\right)}{g^{2}\left(dg-ef\right)}$	84
norman	$\frac{cx}{eg}+\frac{\left(a\,g^{2}-bfg+c\,f^{2}\right)\ln \left(gx+f\right)}{g^{2}\left(dg-ef\right)}-\frac{\left(a\,e^{2}-bde+c\,d^{2}\right)\ln \left(ex+d\right)}{\left(dg-ef\right)e^{2}}$	84
risch	$\frac{cx}{eg}+\frac{\ln \left(-gx-f\right)a}{dg-ef}-\frac{\ln \left(-gx-f\right)bf}{g\left(dg-ef\right)}+\frac{\ln \left(-gx-f\right)c\,f^{2}}{g^{2}\left(dg-ef\right)}-\frac{\ln \left(ex+d\right)a}{dg-ef}+\frac{\ln \left(ex+d\right)bd}{\left(dg-ef\right)e}-\frac{\ln \left(ex+d\right)c\,d^{2}}{\left(dg-ef\right)e^{2}}$	151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e/g+(-a*e^2+b*d*e-c*d^2)/(d*g-e*f)/e^2*ln(e*x+d)+1/g^2*(a*g^2-b*f*g+c*f
^2)/(d*g-e*f)*ln(g*x+f)
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.04

$$\frac{cxe^{(-1)}}{g}+\frac{(cf^{2}-bfg+ag^{2})\log(gx+f)}{dg^{3}-fg^{2}e}-\frac{(cd^{2}-bde+ae^{2})\log(xe+d)}{dge^{2}-fe^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] c*x*e^(-1)/g + (c*f^2 - b*f*g + a*g^2)*log(g*x + f)/(d*g^3 - f*g^2*e) - (c*
d^2 - b*d*e + a*e^2)*log(x*e + d)/(d*g*e^2 - f*e^3)
```

Fricas [A]
time = 1.22, size = 96, normalized size = 1.16

$$\frac{cdg^{2}xe-cfgxe^{2}+(cf^{2}-bfg+ag^{2})e^{2}\log(gx+f)-(cd^{2}g^{2}-bdg^{2}e+ag^{2}e^{2})\log(xe+d)}{dg^{3}e^{2}-fg^{2}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] (c*d*g^2*x*e - c*f*g*x*e^2 + (c*f^2 - b*f*g + a*g^2)*e^2*log(g*x + f) - (c*
d^2*g^2 - b*d*g^2*e + a*g^2*e^2)*log(x*e + d))/(d*g^3*e^2 - f*g^2*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(70) = 140$.
time = 74.15, size = 420, normalized size = 5.06

$$\frac{cx}{eg} + \frac{(ag^2 - bfg + cf^2)\log\left(x + \frac{adeg^2 + ae^2fg - 2bdefg + cd^2fg + cdef^2 - \frac{d^2eg(ag^2 - bfg + cf^2)}{dg - ef} + \frac{2de^2f(ag^2 - bfg + cf^2)}{dg - ef} - \frac{e^3f^2(ag^2 - bfg + cf^2)}{g(dg - ef)}}{2ae^2g^2 - bdeg^2 - be^2fg + cd^2g^2 + ce^2f^2}\right)}{g^2(dg - ef)} - \frac{(ae^2 - bde + cd^2)\log\left(x + \frac{adeg^2 + ae^2fg - 2bdefg + cd^2fg + cdef^2 + \frac{d^2g^3(ae^2 - bde + cd^2)}{e(dg - ef)} - \frac{2dfg^2(ae^2 - bde + cd^2)}{dg - ef} + \frac{ef^2g(ae^2 - bde + cd^2)}{dg - ef}}{2ae^2g^2 - bdeg^2 - be^2fg + cd^2g^2 + ce^2f^2}\right)}{e^2(dg - ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f),x)
```

```
[Out] c*x/(e*g) + (a*g**2 - b*f*g + c*f**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*
b*d*e*f*g + c*d**2*f*g + c*d*e*f**2 - d**2*e*g*(a*g**2 - b*f*g + c*f**2)/(d
*g - e*f) + 2*d*e**2*f*(a*g**2 - b*f*g + c*f**2)/(d*g - e*f) - e**3*f**2*(a
*g**2 - b*f*g + c*f**2)/(g*(d*g - e*f)))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e*
*2*f*g + c*d**2*g**2 + c*e**2*f**2))/(g**2*(d*g - e*f)) - (a*e**2 - b*d*e +
 c*d**2)*log(x + (a*d*e*g**2 + a*e**2*f*g - 2*b*d*e*f*g + c*d**2*f*g + c*d*
e*f**2 + d**2*g**3*(a*e**2 - b*d*e + c*d**2)/(e*(d*g - e*f)) - 2*d*f*g**2*(
a*e**2 - b*d*e + c*d**2)/(d*g - e*f) + e*f**2*g*(a*e**2 - b*d*e + c*d**2)/(
d*g - e*f))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e**2*f*g + c*d**2*g**2 + c*e**2
*f**2))/(e**2*(d*g - e*f))
```

Giac [A]
time = 4.68, size = 88, normalized size = 1.06

$$\frac{cxe^{(-1)}}{g} + \frac{(cf^2 - bfg + ag^2)\log(|gx + f|)}{dg^3 - fg^2e} - \frac{(cd^2 - bde + ae^2)\log(|xe + d|)}{dge^2 - fe^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] c*x*e^(-1)/g + (c*f^2 - b*f*g + a*g^2)*log(abs(g*x + f))/(d*g^3 - f*g^2*e)
- (c*d^2 - b*d*e + a*e^2)*log(abs(x*e + d))/(d*g*e^2 - f*e^3)
```

Mupad [B]
time = 3.42, size = 84, normalized size = 1.01

$$\frac{\ln(d + e\,x)\,(c\,d^2 - b\,d\,e + a\,e^2)}{e^3\,f - d\,e^2\,g} + \frac{\ln(f + g\,x)\,(c\,f^2 - b\,f\,g + a\,g^2)}{g^2\,(d\,g - e\,f)} + \frac{c\,x}{e\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(a*e^2 + c*d^2 - b*d*e))/(e^3*f - d*e^2*g) + (log(f + g*x)*(a
*g^2 + c*f^2 - b*f*g))/(g^2*(d*g - e*f)) + (c*x)/(e*g)
```

3.815 $\int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)} dx$

Optimal. Leaf size=184

$$\frac{(b^2e^2g^2 - 2ceg(bef + bdg - aeg) + c^2(e^2f^2 + defg + d^2g^2))\,x}{e^3g^3} - \frac{c(cef + cdg - 2beg)x^2}{2e^2g^2} + \frac{c^2x^3}{3eg} + \frac{(cd^2 - bde -}{e^4(}$$

```
[Out] (b^2*e^2*g^2-2*c*e*g*(-a*e*g+b*d*g+b*e*f)+c^2*(d^2*g^2+d*e*f*g+e^2*f^2))*x/
e^3/g^3-1/2*c*(-2*b*e*g+c*d*g+c*e*f)*x^2/e^2/g^2+1/3*c^2*x^3/e/g+(a*e^2-b*d
*e+c*d^2)^2*ln(e*x+d)/e^4/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^2*ln(g*x+f)/g^4/(-
d*g+e*f)
```

Rubi [A]

time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {907}

$$\frac{x(-2ceg(-aeg + bdg + bef) + b^2e^2g^2 + c^2(d^2g^2 + defg + e^2f^2))}{e^3g^3} + \frac{\log(d + ex)\,(ae^2 - bde + cd^2)^2}{e^4(ef - dg)} - \frac{\log(f + gx)\,(ag^2 - bfg + cf^2)^2}{g^4(ef - dg)} - \frac{cx^2(-2beg + cdg + cef)}{2e^2g^2} + \frac{c^2x^3}{3eg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((b^2*e^2*g^2 - 2*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g +
d^2*g^2))*x)/(e^3*g^3) - (c*(c*e*f + c*d*g - 2*b*e*g)*x^2)/(2*e^2*g^2) + (c
^2*x^3)/(3*e*g) + ((c*d^2 - b*d*e + a*e^2)^2*Log[d + e*x])/(e^4*(e*f - d*g)
) - ((c*f^2 - b*f*g + a*g^2)^2*Log[f + g*x])/(g^4*(e*f - d*g))
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{(a + bx + cx^2)^2}{(d + ex)(f + gx)}\,dx = \int \left(\frac{b^2e^2g^2 - 2ceg(bef + bdg - aeg) + c^2(e^2f^2 + defg + d^2g^2)}{e^3g^3} - \frac{c(cef + cdg -}{e^2g^2}\right.$$

$$= \frac{(b^2e^2g^2 - 2ceg(bef + bdg - aeg) + c^2(e^2f^2 + defg + d^2g^2))\,x}{e^3g^3} - \frac{c(cef + cdg - 2}{2e^2g^2}$$

Mathematica [A]

time = 0.10, size = 177, normalized size = 0.96

$$-\frac{eg(-ef+dg)x(6b^2e^2g^2+6ceg(2aeg+b(-2ef-2dg+egx))+c^2(6d^2g^2-3deg(-2f+gx)+e^2(6f^2-3fgx+2g^2x^2)))-6(cd^2+e(-bd+ae))^2g^4\log(d+ex)+6e^4(cf^2+g(-bf+ag))^2\log(f+gx)}{6e^4g^4(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2/((d + e*x)*(f + g*x)),x]
```

```
[Out] -1/6*(e*g*(-(e*f) + d*g)*x*(6*b^2*e^2*g^2 + 6*c*e*g*(2*a*e*g + b*(-2*e*f -
2*d*g + e*g*x)) + c^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*
g*x + 2*g^2*x^2))) - 6*(c*d^2 + e*(-(b*d) + a*e))^2*g^4*Log[d + e*x] + 6*e^
4*(c*f^2 + g*(-(b*f) + a*g))^2*Log[f + g*x])/(e^4*g^4*(e*f - d*g))
```

Maple [A]

time = 0.15, size = 280, normalized size = 1.52

method	result
norman	$\frac{(2ac\,e^2g^2+b^2e^2g^2-2bcde\,g^2-2bc\,e^2fg+c^2d^2g^2+c^2defg+c^2e^2f^2)x}{e^3g^3}+\frac{c^2x^3}{3eg}+\frac{c(2beg-dgc-cef)x^2}{2e^2g^2}+\frac{(a^2g^4-2abf\,g^3+2ac\,f^2g^2+}{g^4}$
default	$\frac{\frac{1}{3}c^2x^3e^2g^2+bc\,e^2g^2x^2-\frac{1}{2}c^2de\,g^2x^2-\frac{1}{2}c^2e^2fg\,x^2+2ac\,e^2g^2x+b^2e^2g^2x-2bcde\,g^2x-2bc\,e^2fgx+c^2d^2g^2x+c^2defgx+c^2e^2f^2x}{e^3g^3}+\frac{(-a^2}{}$
risch	$\frac{c^2x^3}{3eg}+\frac{bc\,x^2}{eg}-\frac{c^2d\,x^2}{2e^2g}-\frac{c^2f\,x^2}{2e\,g^2}+\frac{2acx}{eg}+\frac{b^2x}{eg}-\frac{2bcdx}{e^2g}-\frac{2bcfx}{e\,g^2}+\frac{c^2d^2x}{e^3g}+\frac{c^2dfx}{e^2g^2}+\frac{c^2f^2x}{e\,g^3}+\frac{\ln(-gx-f)a^2}{dg-ef}-\frac{2\ln(}{g}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^3/g^3*(1/3*c^2*x^3*e^2*g^2+b*c*e^2*g^2*x^2-1/2*c^2*d*e*g^2*x^2-1/2*c^2*
e^2*f*g*x^2+2*a*c*e^2*g^2*x+b^2*e^2*g^2*x-2*b*c*d*e*g^2*x-2*b*c*e^2*f*g*x+c
^2*d^2*g^2*x+c^2*d*e*f*g*x+c^2*e^2*f^2*x)+(-a^2*e^4+2*a*b*d*e^3-2*a*c*d^2*e
^2-b^2*d^2*e^2+2*b*c*d^3*e-c^2*d^4)/e^4/(d*g-e*f)*ln(e*x+d)+1/g^4*(a^2*g^4-
2*a*b*f*g^3+2*a*c*f^2*g^2+b^2*f^2*g^2-2*b*c*f^3*g+c^2*f^4)/(d*g-e*f)*ln(g*x
+f)
```

Maxima [A]

time = 0.27, size = 253, normalized size = 1.38

$$\frac{(c^2f^4-2\,bcf^3g-2\,abfg^3+a^2g^4+(b^2+2\,ac)f^2g^2)\log(gx+f)}{dg^5-fg^4e}-\frac{(c^2d^4-2\,bcd^3e-2\,abde^3+(b^2e^2+2\,ace^2)d^2+a^2e^4)\log(xe+d)}{dge^4-fe^5}+\frac{(2\,c^2g^2x^3e^2-3\,(c^2fge^2+(c^2de-2\,bce^2)g^2)x^2+6\,(c^2f^2e^2+(c^2de-2\,bce^2)fg+(c^2d^2-2\,bcde+b^2e^2+2\,ace^2)g^2)x)e^{(-3)}}{6\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^2*f^4 - 2*b*c*f^3*g - 2*a*b*f*g^3 + a^2*g^4 + (b^2 + 2*a*c)*f^2*g^2)*log
(g*x + f)/(d*g^5 - f*g^4*e) - (c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (b^2*e
^2 + 2*a*c*e^2)*d^2 + a^2*e^4)*log(x*e + d)/(d*g*e^4 - f*e^5) + 1/6*(2*c^2*
g^2*x^3*e^2 - 3*(c^2*f*g*e^2 + (c^2*d*e - 2*b*c*e^2)*g^2)*x^2 + 6*(c^2*f^2*
```

```
e^2 + (c^2*d*e - 2*b*c*e^2)*f*g + (c^2*d^2 - 2*b*c*d*e + b^2*e^2 + 2*a*c*e^
2)*g^2)*x)*e^(-3)/g^3
```

Fricas [A]
time = 1.38, size = 306, normalized size = 1.66

$$\frac{6\,c^2d^3g^4xe + 6\,(c^2f^4 - 2\,bcf^3g - 2\,abfg^3 + a^2g^4 + (b^2 + 2\,ac)f^2g^2)e^4\log\left(gx + f\right) - (2\,c^2fg^3x^3 - 3\,(c^2f^2g^2 - 2\,bcfg^3)x^2 + 6\,(c^2f^3g - 2\,bcf^2g^2 + (b^2 + 2\,ac)fg^3)x)e^4 + 2\,(c^2dg^4x^3 + 3\,bcdg^4x^2 + 3\,(b^2 + 2\,ac)dg^4x)e^3 - 3\,(c^2d^2g^4x^2 + 4\,bcd^2g^4x)e^2 - 6\,(c^2d^4g^4 - 2\,bcd^3g^4e - 2\,abdg^4e^3 + (b^2 + 2\,ac)d^2g^4e^2 + a^2g^4e^4)\log\left(xe + d\right)}{6\,(dg^5e^4 - fg^4e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/6*(6*c^2*d^3*g^4*x*e + 6*(c^2*f^4 - 2*b*c*f^3*g - 2*a*b*f*g^3 + a^2*g^4 +
 (b^2 + 2*a*c)*f^2*g^2)*e^4*log(g*x + f) - (2*c^2*f*g^3*x^3 - 3*(c^2*f^2*g^
2 - 2*b*c*f*g^3)*x^2 + 6*(c^2*f^3*g - 2*b*c*f^2*g^2 + (b^2 + 2*a*c)*f*g^3)*
x)*e^4 + 2*(c^2*d*g^4*x^3 + 3*b*c*d*g^4*x^2 + 3*(b^2 + 2*a*c)*d*g^4*x)*e^3
- 3*(c^2*d^2*g^4*x^2 + 4*b*c*d^2*g^4*x)*e^2 - 6*(c^2*d^4*g^4 - 2*b*c*d^3*g^
4*e - 2*a*b*d*g^4*e^3 + (b^2 + 2*a*c)*d^2*g^4*e^2 + a^2*g^4*e^4)*log(x*e +
d))/(d*g^5*e^4 - f*g^4*e^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.77, size = 281, normalized size = 1.53

$$\frac{(c^2f^4 - 2\,bcf^3g + b^2f^2g^2 + 2\,acf^2g^2 - 2\,abfg^3 + a^2g^4)\log\left(|gx + f|\right)}{dg^5 - fg^4e} - \frac{(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4)\log\left(|xe + d|\right)}{dge^4 - fe^5} + \frac{(2\,c^2g^2x^3e^2 - 3\,c^2dg^2x^2e + 6\,c^2d^2g^2x - 3\,c^2fgx^2e^2 + 6\,bcg^2x^2e^2 + 6\,c^2dfgxe - 12\,bcdg^2xe + 6\,c^2f^2xe^2 - 12\,bcfgxe^2 + 6\,b^2g^2xe^2 + 12\,acg^2xe^2)e^{(-3)}}{6\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] (c^2*f^4 - 2*b*c*f^3*g + b^2*f^2*g^2 + 2*a*c*f^2*g^2 - 2*a*b*f*g^3 + a^2*g^
4)*log(abs(g*x + f))/(d*g^5 - f*g^4*e) - (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e
^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*log(abs(x*e + d))/(d*g*e^4 - f*
e^5) + 1/6*(2*c^2*g^2*x^3*e^2 - 3*c^2*d*g^2*x^2*e + 6*c^2*d^2*g^2*x - 3*c^2
*f*g*x^2*e^2 + 6*b*c*g^2*x^2*e^2 + 6*c^2*d*f*g*x*e - 12*b*c*d*g^2*x*e + 6*c
^2*f^2*x*e^2 - 12*b*c*f*g*x*e^2 + 6*b^2*g^2*x*e^2 + 12*a*c*g^2*x*e^2)*e^(-3
)/g^3
```

Mupad [B]

time = 3.51, size = 266, normalized size = 1.45

$$x\left(\frac{b^2+2\,a\,c}{e\,g}+\frac{\left(\frac{c^2\,(d\,g+e\,f)}{e^2\,g^2}-\frac{2\,b\,c}{e\,g}\right)\,(d\,g+e\,f)}{e\,g}-\frac{c^2\,d\,f}{e^2\,g^2}\right)-x^2\left(\frac{c^2\,(d\,g+e\,f)}{2\,e^2\,g^2}-\frac{b\,c}{e\,g}\right)+\frac{\ln(d+e\,x)\,\left(e^2\,(b^2\,d^2+2\,a\,c\,d^2)+a^2\,e^4+c^2\,d^4-2\,a\,b\,d\,e^3-2\,b\,c\,d^3\,e\right)}{e^5\,f-d\,e^4\,g}+\frac{\ln(f+g\,x)\,\left(g^2\,(b^2\,f^2+2\,a\,c\,f^2)+a^2\,g^4+c^2\,f^4-2\,a\,b\,f\,g^3-2\,b\,c\,f^3\,g\right)}{d\,g^5-e\,f\,g^4}+\frac{c^2\,x^3}{3\,e\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^2/((f + g*x)*(d + e*x)),x)
```

```
[Out] x*((2*a*c + b^2)/(e*g) + (((c^2*(d*g + e*f))/(e^2*g^2) - (2*b*c)/(e*g))*(d*
g + e*f))/(e*g) - (c^2*d*f)/(e^2*g^2)) - x^2*((c^2*(d*g + e*f))/(2*e^2*g^2)
 - (b*c)/(e*g)) + (log(d + e*x)*(e^2*(b^2*d^2 + 2*a*c*d^2) + a^2*e^4 + c^2*
d^4 - 2*a*b*d*e^3 - 2*b*c*d^3*e))/(e^5*f - d*e^4*g) + (log(f + g*x)*(g^2*(b
^2*f^2 + 2*a*c*f^2) + a^2*g^4 + c^2*f^4 - 2*a*b*f*g^3 - 2*b*c*f^3*g))/(d*g^
5 - e*f*g^4) + (c^2*x^3)/(3*e*g)
```

3.816 $\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=531

$$-\frac{(b^2e^3g^3(bef+bdg-3aeg)-c^3(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4)-3ce^2g^2(a^2e^2g^2-2abeg(ef}{e}$$

```
[Out] -(b^2*e^3*g^3*(-3*a*e*g+b*d*g+b*e*f)-c^3*(d^4*g^4+d^3*e*f*g^3+d^2*e^2*f^2*g
^2+d*e^3*f^3*g+e^4*f^4)-3*c*e^2*g^2*(a^2*e^2*g^2-2*a*b*e*g*(d*g+e*f)+b^2*(d
^2*g^2+d*e*f*g+e^2*f^2))-3*c^2*e*g*(a*e*g*(d^2*g^2+d*e*f*g+e^2*f^2)-b*(d^3*
g^3+d^2*e*f*g^2+d*e^2*f^2*g+e^3*f^3)))*x/e^5/g^5+1/2*(b^3*e^3*g^3-3*b*c*e^2
*g^2*(-2*a*e*g+b*d*g+b*e*f)-c^3*(d^3*g^3+d^2*e*f*g^2+d*e^2*f^2*g+e^3*f^3)-3
*c^2*e*g*(a*e*g*(d*g+e*f)-b*(d^2*g^2+d*e*f*g+e^2*f^2)))*x^2/e^4/g^4+1/3*c*(
3*b^2*e^2*g^2-3*c*e*g*(-a*e*g+b*d*g+b*e*f)+c^2*(d^2*g^2+d*e*f*g+e^2*f^2))*x
^3/e^3/g^3-1/4*c^2*(-3*b*e*g+c*d*g+c*e*f)*x^4/e^2/g^2+1/5*c^3*x^5/e/g+(a*e^
2-b*d*e+c*d^2)^3*ln(e*x+d)/e^6/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^3*ln(g*x+f)/g
^6/(-d*g+e*f)
```

Rubi [A]

time = 0.62, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{907\}$

$$-\frac{x(-3ce^2g^2(a^2e^2g^2-2abeg(dg+ef)+b^2(d^2g^2+defg+e^2f^2))+b^2e^3g^3(-3aeg+bdg+bef)-3c^2eg(aeg(d^2g^2+defg+e^2f^2)-b(d^3g^3+d^2efg^2+de^2f^2g+e^3f^3))-(c^3(d^4g^4+d^3efg^3+d^2e^2f^2g^2+de^3f^3g+e^4f^4)))}{e^5g^5}+\frac{x^2(-3c^2eg(aeg(dg+ef)-b(d^2g^2+defg+e^2f^2))-3bce^2g^2(-2aeg+bdg+bef)+b^3e^3g^3-(c^3(d^3g^3+d^2efg^2+de^2f^2g+e^3f^3)))}{2e^4g^4}+\frac{cx^3(-3ceg(-aeg+bdg+bef)+3b^2e^2g^2+c^2(d^2g^2+defg+e^2f^2))}{3e^3g^3}+\frac{\log(d+ex)\left(ae^2-bde+cd^2\right)^3}{e^6(ef-dg)}-\frac{\log(f+gx)\left(ag^2-bfg+cf^2\right)^3}{g^6(ef-dg)}-\frac{c^2x^4(-3beg+cdg+cef)}{4e^2g^2}+\frac{c^3x^5}{5eg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -(((b^2*e^3*g^3*(b*e*f + b*d*g - 3*a*e*g) - c^3*(e^4*f^4 + d*e^3*f^3*g + d^
2*e^2*f^2*g^2 + d^3*e*f*g^3 + d^4*g^4) - 3*c*e^2*g^2*(a^2*e^2*g^2 - 2*a*b*e
*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) - 3*c^2*e*g*(a*e*g*(e^2
*f^2 + d*e*f*g + d^2*g^2) - b*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^
3)))*x)/(e^5*g^5)) + ((b^3*e^3*g^3 - 3*b*c*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g
) - c^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3) - 3*c^2*e*g*(a*e*g*
(e*f + d*g) - b*(e^2*f^2 + d*e*f*g + d^2*g^2)))*x^2)/(2*e^4*g^4) + (c*(3*b^
2*e^2*g^2 - 3*c*e*g*(b*e*f + b*d*g - a*e*g) + c^2*(e^2*f^2 + d*e*f*g + d^2*
g^2))*x^3)/(3*e^3*g^3) - (c^2*(c*e*f + c*d*g - 3*b*e*g)*x^4)/(4*e^2*g^2) +
(c^3*x^5)/(5*e*g) + ((c*d^2 - b*d*e + a*e^2)^3*Log[d + e*x])/(e^6*(e*f - d*
g)) - ((c*f^2 - b*f*g + a*g^2)^3*Log[f + g*x])/(g^6*(e*f - d*g))
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)}\,dx = \int \Bigg(\frac{-b^2e^3g^3(bef+bdg-3aeg)+c^3(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4)}{}$$

$$= -\frac{(b^2e^3g^3(bef+bdg-3aeg)-c^3(e^4f^4+de^3f^3g+d^2e^2f^2g^2+d^3efg^3+d^4g^4)-3}{}$$

Mathematica [**A**]
time = 0.29, size = 476, normalized size = 0.90

$$-\frac{egx(-30b^2e^3g^3(ef-dg)(6aeg+b(-2ef-2dg+egx))+c^3(60d^5g^5-30d^4eg^5x+20d^3e^2g^5x^2-15d^2e^3g^5x^3+12de^4g^5x^4+e^5f(-60f^4+30f^3gx-20f^2g^2x^2+15fg^3x^3-12g^4x^4))-30ce^2g^2(ef-dg)(6a^2e^2g^2+6abeg(-2ef-2dg+egx)+b^2(6d^2g^2-3deg(-2f+gx)+e^2(6f^2-3fgx+2g^2x^2)))+15c^2eg(-2aeg(ef-dg)(6d^2g^2-3deg(-2f+gx)+e^2(6f^2-3fgx+2g^2x^2))+b(-12d^4g^4+6d^3eg^4x-4d^2e^2g^4x^2+3de^3g^4x^3+e^4f(12f^3-6f^2gx+4fg^2x^2-3g^3x^3))))-60(cd^2+e(-bd+ae))^3g^6\log(d+ex)+60e^6(cf^2+g(-bf+ag))^3\log(f+gx)}{60e^6g^6(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]
```

```
[Out] -1/60*(e*g*x*(-30*b^2*e^3*g^3*(e*f - d*g)*(6*a*e*g + b*(-2*e*f - 2*d*g + e*
g*x)) + c^3*(60*d^5*g^5 - 30*d^4*e*g^5*x + 20*d^3*e^2*g^5*x^2 - 15*d^2*e^3*
g^5*x^3 + 12*d*e^4*g^5*x^4 + e^5*f*(-60*f^4 + 30*f^3*g*x - 20*f^2*g^2*x^2 +
 15*f*g^3*x^3 - 12*g^4*x^4)) - 30*c*e^2*g^2*(e*f - d*g)*(6*a^2*e^2*g^2 + 6*
a*b*e*g*(-2*e*f - 2*d*g + e*g*x) + b^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) +
e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2))) + 15*c^2*e*g*(-2*a*e*g*(e*f - d*g)*(6*d
^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)) + b*(-12
*d^4*g^4 + 6*d^3*e*g^4*x - 4*d^2*e^2*g^4*x^2 + 3*d*e^3*g^4*x^3 + e^4*f*(12*
f^3 - 6*f^2*g*x + 4*f*g^2*x^2 - 3*g^3*x^3)))) - 60*(c*d^2 + e*(-(b*d) + a*e
))^3*g^6*Log[d + e*x] + 60*e^6*(c*f^2 + g*(-(b*f) + a*g))^3*Log[f + g*x])/(
e^6*g^6*(e*f - d*g))
```

Maple [**A**]
time = 0.14, size = 940, normalized size = 1.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] (-a^3*e^6+3*a^2*b*d*e^5-3*a^2*c*d^2*e^4-3*a*b^2*d^2*e^4+6*a*b*c*d^3*e^3-3*a
*c^2*d^4*e^2+b^3*d^3*e^3-3*b^2*c*d^4*e^2+3*b*c^2*d^5*e-c^3*d^6)/e^6/(d*g-e*
f)*ln(e*x+d)+1/e^5/g^5*(1/2*b^3*e^4*g^4*x^2+c^3*d^4*g^4*x+c^3*e^4*f^4*x+1/5
*c^3*x^5*e^4*g^4-b*c^2*d*e^3*g^4*x^3-b*c^2*e^4*f*g^3*x^3+1/3*c^3*d*e^3*f*g^
3*x^3+3*a*b*c*e^4*g^4*x^2-3/2*a*c^2*d*e^3*g^4*x^2-3/2*a*c^2*e^4*f*g^3*x^2-3
```

```
/2*b^2*c*d*e^3*g^4*x^2-3/2*b^2*c*e^4*f*g^3*x^2+3/2*b*c^2*d^2*e^2*g^4*x^2+3/
2*b*c^2*e^4*f^2*g^2*x^2-1/2*c^3*d^2*e^2*f*g^3*x^2-1/2*c^3*d*e^3*f^2*g^2*x^2
+a*c^2*e^4*g^4*x^3+b^2*c*e^4*g^4*x^3+3/2*b*c^2*d*e^3*f*g^3*x^2-6*a*b*c*d*e^
3*g^4*x-6*a*b*c*e^4*f*g^3*x+3*a^2*c*e^4*g^4*x+3*a*b^2*e^4*g^4*x-b^3*d*e^3*g
^4*x-b^3*e^4*f*g^3*x+3/4*b*c^2*e^4*g^4*x^4-1/4*c^3*d*e^3*g^4*x^4-1/4*c^3*e^
4*f*g^3*x^4+1/3*c^3*d^2*e^2*g^4*x^3+1/3*c^3*e^4*f^2*g^2*x^3-1/2*c^3*d^3*e*g
^4*x^2-1/2*c^3*e^4*f^3*g*x^2+3*a*c^2*d^2*e^2*g^4*x+3*a*c^2*e^4*f^2*g^2*x+3*
b^2*c*d^2*e^2*g^4*x+3*b^2*c*e^4*f^2*g^2*x-3*b*c^2*d^3*e*g^4*x-3*b*c^2*e^4*f
^3*g*x+c^3*d^3*e*f*g^3*x+c^3*d^2*e^2*f^2*g^2*x+c^3*d*e^3*f^3*g*x+3*a*c^2*d*
e^3*f*g^3*x+3*b^2*c*d*e^3*f*g^3*x-3*b*c^2*d^2*e^2*f*g^3*x-3*b*c^2*d*e^3*f^2
*g^2*x)+1/g^6*(a^3*g^6-3*a^2*b*f*g^5+3*a^2*c*f^2*g^4+3*a*b^2*f^2*g^4-6*a*b*
c*f^3*g^3+3*a*c^2*f^4*g^2-b^3*f^3*g^3+3*b^2*c*f^4*g^2-3*b*c^2*f^5*g+c^3*f^6
)/(d*g-e*f)*ln(g*x+f)
```

Maxima [**A**]
time = 0.30, size = 710, normalized size = 1.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] (c^3*f^6 - 3*b*c^2*f^5*g - 3*a^2*b*f*g^5 + a^3*g^6 + 3*(b^2*c + a*c^2)*f^4*
g^2 - (b^3 + 6*a*b*c)*f^3*g^3 + 3*(a*b^2 + a^2*c)*f^2*g^4)*log(g*x + f)/(d*
g^7 - f*g^6*e) - (c^3*d^6 - 3*b*c^2*d^5*e + 3*(b^2*c*e^2 + a*c^2*e^2)*d^4 -
 3*a^2*b*d*e^5 - (b^3*e^3 + 6*a*b*c*e^3)*d^3 + a^3*e^6 + 3*(a*b^2*e^4 + a^2
*c*e^4)*d^2)*log(x*e + d)/(d*g*e^6 - f*e^7) + 1/60*(12*c^3*g^4*x^5*e^4 - 15
*(c^3*f*g^3*e^4 + (c^3*d*e^3 - 3*b*c^2*e^4)*g^4)*x^4 + 20*(c^3*f^2*g^2*e^4
+ (c^3*d*e^3 - 3*b*c^2*e^4)*f*g^3 + (c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*b^2*c*
e^4 + 3*a*c^2*e^4)*g^4)*x^3 - 30*(c^3*f^3*g*e^4 + (c^3*d*e^3 - 3*b*c^2*e^4)
*f^2*g^2 + (c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*b^2*c*e^4 + 3*a*c^2*e^4)*f*g^3
+ (c^3*d^3*e - 3*b*c^2*d^2*e^2 - b^3*e^4 - 6*a*b*c*e^4 + 3*(b^2*c*e^3 + a*c
^2*e^3)*d)*g^4)*x^2 + 60*(c^3*f^4*e^4 + (c^3*d*e^3 - 3*b*c^2*e^4)*f^3*g + (
c^3*d^2*e^2 - 3*b*c^2*d*e^3 + 3*b^2*c*e^4 + 3*a*c^2*e^4)*f^2*g^2 + (c^3*d^3
*e - 3*b*c^2*d^2*e^2 - b^3*e^4 - 6*a*b*c*e^4 + 3*(b^2*c*e^3 + a*c^2*e^3)*d)
*f*g^3 + (c^3*d^4 - 3*b*c^2*d^3*e + 3*a*b^2*e^4 + 3*a^2*c*e^4 + 3*(b^2*c*e^
2 + a*c^2*e^2)*d^2 - (b^3*e^3 + 6*a*b*c*e^3)*d)*g^4)*x)*e^(-5)/g^5
```

Fricas [**A**]
time = 5.02, size = 722, normalized size = 1.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] 1/60*(60*c^3*d^5*g^6*x*e + 60*(c^3*f^6 - 3*b*c^2*f^5*g - 3*a^2*b*f*g^5 + a^
3*g^6 + 3*(b^2*c + a*c^2)*f^4*g^2 - (b^3 + 6*a*b*c)*f^3*g^3 + 3*(a*b^2 + a^
2*c)*f^2*g^4)*e^6*log(g*x + f) - (12*c^3*f*g^5*x^5 - 15*(c^3*f^2*g^4 - 3*b*
c^2*f*g^5)*x^4 + 20*(c^3*f^3*g^3 - 3*b*c^2*f^2*g^4 + 3*(b^2*c + a*c^2)*f*g^
5)*x^3 - 30*(c^3*f^4*g^2 - 3*b*c^2*f^3*g^3 + 3*(b^2*c + a*c^2)*f^2*g^4 - (b
^3 + 6*a*b*c)*f*g^5)*x^2 + 60*(c^3*f^5*g - 3*b*c^2*f^4*g^2 + 3*(b^2*c + a*c
^2)*f^3*g^3 - (b^3 + 6*a*b*c)*f^2*g^4 + 3*(a*b^2 + a^2*c)*f*g^5)*x)*e^6 + 3
*(4*c^3*d*g^6*x^5 + 15*b*c^2*d*g^6*x^4 + 20*(b^2*c + a*c^2)*d*g^6*x^3 + 10*
(b^3 + 6*a*b*c)*d*g^6*x^2 + 60*(a*b^2 + a^2*c)*d*g^6*x)*e^5 - 15*(c^3*d^2*g
^6*x^4 + 4*b*c^2*d^2*g^6*x^3 + 6*(b^2*c + a*c^2)*d^2*g^6*x^2 + 4*(b^3 + 6*a
*b*c)*d^2*g^6*x)*e^4 + 10*(2*c^3*d^3*g^6*x^3 + 9*b*c^2*d^3*g^6*x^2 + 18*(b^
2*c + a*c^2)*d^3*g^6*x)*e^3 - 30*(c^3*d^4*g^6*x^2 + 6*b*c^2*d^4*g^6*x)*e^2
- 60*(c^3*d^6*g^6 - 3*b*c^2*d^5*g^6*e + 3*(b^2*c + a*c^2)*d^4*g^6*e^2 - 3*a
^2*b*d*g^6*e^5 - (b^3 + 6*a*b*c)*d^3*g^6*e^3 + a^3*g^6*e^6 + 3*(a*b^2 + a^2
*c)*d^2*g^6*e^4)*log(x*e + d))/(d*g^7*e^6 - f*g^6*e^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.53, size = 907, normalized size = 1.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] (c^3*f^6 - 3*b*c^2*f^5*g + 3*b^2*c*f^4*g^2 + 3*a*c^2*f^4*g^2 - b^3*f^3*g^3
- 6*a*b*c*f^3*g^3 + 3*a*b^2*f^2*g^4 + 3*a^2*c*f^2*g^4 - 3*a^2*b*f*g^5 + a^3
*g^6)*log(abs(g*x + f))/(d*g^7 - f*g^6*e) - (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^
2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2
*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*log(abs(x*e + d))/(d*g*e^
6 - f*e^7) + 1/60*(12*c^3*g^4*x^5*e^4 - 15*c^3*d*g^4*x^4*e^3 + 20*c^3*d^2*g
^4*x^3*e^2 - 30*c^3*d^3*g^4*x^2*e + 60*c^3*d^4*g^4*x - 15*c^3*f*g^3*x^4*e^4
 + 45*b*c^2*g^4*x^4*e^4 + 20*c^3*d*f*g^3*x^3*e^3 - 60*b*c^2*d*g^4*x^3*e^3 -
 30*c^3*d^2*f*g^3*x^2*e^2 + 90*b*c^2*d^2*g^4*x^2*e^2 + 60*c^3*d^3*f*g^3*x*e
 - 180*b*c^2*d^3*g^4*x*e + 20*c^3*f^2*g^2*x^3*e^4 - 60*b*c^2*f*g^3*x^3*e^4
+ 60*b^2*c*g^4*x^3*e^4 + 60*a*c^2*g^4*x^3*e^4 - 30*c^3*d*f^2*g^2*x^2*e^3 +
90*b*c^2*d*f*g^3*x^2*e^3 - 90*b^2*c*d*g^4*x^2*e^3 - 90*a*c^2*d*g^4*x^2*e^3
```

```
+ 60*c^3*d^2*f^2*g^2*x*e^2 - 180*b*c^2*d^2*f*g^3*x*e^2 + 180*b^2*c*d^2*g^4*
x*e^2 + 180*a*c^2*d^2*g^4*x*e^2 - 30*c^3*f^3*g*x^2*e^4 + 90*b*c^2*f^2*g^2*x
^2*e^4 - 90*b^2*c*f*g^3*x^2*e^4 - 90*a*c^2*f*g^3*x^2*e^4 + 30*b^3*g^4*x^2*e
^4 + 180*a*b*c*g^4*x^2*e^4 + 60*c^3*d*f^3*g*x*e^3 - 180*b*c^2*d*f^2*g^2*x*e
^3 + 180*b^2*c*d*f*g^3*x*e^3 + 180*a*c^2*d*f*g^3*x*e^3 - 60*b^3*d*g^4*x*e^3
 - 360*a*b*c*d*g^4*x*e^3 + 60*c^3*f^4*x*e^4 - 180*b*c^2*f^3*g*x*e^4 + 180*b
^2*c*f^2*g^2*x*e^4 + 180*a*c^2*f^2*g^2*x*e^4 - 60*b^3*f*g^3*x*e^4 - 360*a*b
*c*f*g^3*x*e^4 + 180*a*b^2*g^4*x*e^4 + 180*a^2*c*g^4*x*e^4)*e^(-5)/g^5
```

Mupad [B]
time = 4.20, size = 794, normalized size = 1.50

$$x^4\left(\frac{3bc^2}{4eg}-\frac{c^3(dg+ef)}{4e^2g^2}\right)-x^3\left(\frac{(dg+ef)\left(\frac{3bc^2}{eg}-\frac{c^3(dg+ef)}{e^2g^2}\right)}{3eg}-\frac{c(b^2+ac)}{eg}+\frac{c^3df}{3e^2g^2}\right)+x^2\left(\frac{b^3+6acb}{2eg}+\frac{(dg+ef)\left(\frac{(dg+ef)\left(\frac{3bc^2}{eg}-\frac{c^3(dg+ef)}{e^2g^2}\right)}{eg}-\frac{3c(b^2+ac)}{eg}+\frac{c^3df}{e^2g^2}\right)}{2eg}-\frac{df\left(\frac{3bc^2}{eg}-\frac{c^3(dg+ef)}{e^2g^2}\right)}{2eg}\right)+x\left(\frac{3a(b^2+ac)}{eg}-\frac{(dg+ef)\left(\frac{b^3+6acb}{eg}+\frac{(dg+ef)\left(\frac{(dg+ef)\left(\frac{3bc^2}{eg}-\frac{c^3(dg+ef)}{e^2g^2}\right)}{eg}-\frac{3c(b^2+ac)}{eg}+\frac{c^3df}{e^2g^2}\right)}{eg}-\frac{df\left(\frac{3bc^2}{eg}-\frac{c^3(dg+ef)}{e^2g^2}\right)}{eg}\right)}{eg}+\frac{df\left(\frac{(dg+ef)\left(\frac{3bc^2}{eg}-\frac{c^3(dg+ef)}{e^2g^2}\right)}{eg}-\frac{3c(b^2+ac)}{eg}+\frac{c^3df}{e^2g^2}\right)}{eg}\right)+\frac{\ln(d+ex)\left(e^4\left(3ca^2d^2+3ab^2d^2\right)+e^2\left(3b^2cd^4+3ac^2d^4\right)-e^3\left(b^3d^3+6acbd^3\right)+a^3e^6+c^3d^6-3a^2bde^5-3bc^2d^5e\right)}{e^7f-de^6g}+\frac{\ln(f+gx)\left(g^4\left(3ca^2f^2+3ab^2f^2\right)+g^2\left(3b^2cf^4+3ac^2f^4\right)-g^3\left(b^3f^3+6acbf^3\right)+a^3g^6+c^3f^6-3a^2bfg^5-3bc^2f^5g\right)}{dg^7-efg^6}+\frac{c^3x^5}{5eg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^3/((f + g*x)*(d + e*x)),x)
```

```
[Out] x^4*((3*b*c^2)/(4*e*g) - (c^3*(d*g + e*f))/(4*e^2*g^2)) - x^3*(((d*g + e*f)
*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(3*e*g) - (c*(a*c + b^2))
/(e*g) + (c^3*d*f)/(3*e^2*g^2)) + x^2*((b^3 + 6*a*b*c)/(2*e*g) + ((d*g + e*
f)*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (
3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(2*e*g) - (d*f*((3*b*c^2)/(e
*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(2*e*g)) + x*((3*a*(a*c + b^2))/(e*g) -
 ((d*g + e*f)*((b^3 + 6*a*b*c)/(e*g) + ((d*g + e*f)*(((d*g + e*f)*((3*b*c^2
)/(e*g) - (c^3*(d*g + e*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (
c^3*d*f)/(e^2*g^2)))/(e*g) - (d*f*((3*b*c^2)/(e*g) - (c^3*(d*g + e*f))/(e^2
*g^2)))/(e*g)))/(e*g) + (d*f*(((d*g + e*f)*((3*b*c^2)/(e*g) - (c^3*(d*g + e
*f))/(e^2*g^2)))/(e*g) - (3*c*(a*c + b^2))/(e*g) + (c^3*d*f)/(e^2*g^2)))/(e
*g)) + (log(d + e*x)*(e^4*(3*a*b^2*d^2 + 3*a^2*c*d^2) + e^2*(3*a*c^2*d^4 +
3*b^2*c*d^4) - e^3*(b^3*d^3 + 6*a*b*c*d^3) + a^3*e^6 + c^3*d^6 - 3*a^2*b*d*
e^5 - 3*b*c^2*d^5*e))/(e^7*f - d*e^6*g) + (log(f + g*x)*(g^4*(3*a*b^2*f^2 +
 3*a^2*c*f^2) + g^2*(3*a*c^2*f^4 + 3*b^2*c*f^4) - g^3*(b^3*f^3 + 6*a*b*c*f^
3) + a^3*g^6 + c^3*f^6 - 3*a^2*b*f*g^5 - 3*b*c^2*f^5*g))/(d*g^7 - e*f*g^6)
+ (c^3*x^5)/(5*e*g)
```

3.817 $\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)}\,dx$

Optimal. Leaf size=246

$$-\frac{\left(2c^2df+b^2eg-c(bef+bdg+2aeg)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2\log(d+ex)}{\left(cd^2-bde+ae^2\right)(ef-dg)}-\frac{g^2\log(}{(ef-dg)\,(cf}$$

```
[Out] e^2*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)-g^2*ln(g*x+f)/(-d*g+e*f)/(a*g^
2-b*f*g+c*f^2)-1/2*(-b*e*g+c*d*g+c*e*f)*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)
/(c*f^2-g*(-a*g+b*f))-(2*c^2*d*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e*f))*arctanh((
2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(c*f^2-g*(-a*g+b*f))/(-4*a
*c+b^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {907, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\sqrt{b^2-4ac}\,\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)}{2\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}+\frac{e^2\log(d+ex)}{(ef-dg)\left(ae^2-bde+cd^2\right)}-\frac{g^2\log(f+gx)}{(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g)))) + (e^2*Log[d + e*x])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)) -
(g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)) - ((c*e*f + c*d*g
- b*e*g)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f -
 a*g)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)}\,dx = \int \left(-\frac{e^3}{(cd^2-bde+ae^2)(-ef+dg)(d+ex)} - \frac{g}{(ef-dg)(cf^2-b\ldots}\right.$$

$$= \frac{e^2\log(d+ex)}{(cd^2-bde+ae^2)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)} + \frac{\int \ldots}{(\ldots}$$

$$= \frac{e^2\log(d+ex)}{(cd^2-bde+ae^2)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)} + \frac{(\ldots}{2(\ldots}$$

$$= \frac{e^2\log(d+ex)}{(cd^2-bde+ae^2)(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)(cf^2-bfg+ag^2)} - \frac{(c\ldots}{2(\ldots}$$

$$= -\frac{(2c^2df+b^2eg-c(bef+bdg+2aeg))\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)(cf^2-g(bf-ag))} + \ldots$$

Mathematica [A]
time = 0.22, size = 246, normalized size = 1.00

$$\frac{(2c^2df+b^2eg-c(bef+bdg+2aeg))\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,(cd^2+e(-bd+ae))(cf^2+g(-bf+ag))} + \frac{e^2\log(d+ex)}{(cd^2+e(-bd+ae))(ef-dg)} - \frac{g^2\log(f+gx)}{(ef-dg)(cf^2+g(-bf+ag))} - \frac{(cef+cdg-beg)\log(a+x(b+cx))}{2(cd^2+e(-bd+ae))(cf^2+g(-bf+ag))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)),x]
```

```
[Out] ((2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(c*f^2 + g
*(-(b*f) + a*g))) + (e^2*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))*(e*f - d
*g)) - (g^2*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))) - ((c*e*
f + c*d*g - b*e*g)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))*(c*f
^2 + g*(-(b*f) + a*g)))
```

Maple [A]
time = 0.28, size = 244, normalized size = 0.99

method	result
default	$-\frac{e^{2} \ln(ex+d)}{(a\,e^{2}-bde+c\,d^{2})(dg-ef)}+\frac{g^{2} \ln(gx+f)}{(dg-ef)(a\,g^{2}-bfg+c\,f^{2})}+\frac{\frac{\left(bceg-c^{2}dg-c^{2}ef\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-aceg+b^{2}eg-bcdg-bcef+c^{2}df-\frac{(bce}{}\right.}{\sqrt{4}}}{(a\,e^{2}-bde+c\,d^{2})(a\,g^{2}-bfg+c\,f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/(a*e^2-b*d*e+c*d^2)/(d*g-e*f)*ln(e*x+d)+g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f
^2)*ln(g*x+f)+1/(a*e^2-b*d*e+c*d^2)/(a*g^2-b*f*g+c*f^2)*(1/2*(b*c*e*g-c^2*d
*g-c^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-a*c*e*g+b^2*e*g-b*c*d*g-b*c*e*f+c^2*d*f-1
/2*(b*c*e*g-c^2*d*g-c^2*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.43, size = 392, normalized size = 1.59

$$\frac{g^3 \log\left(|gx+f|\right)}{cdf^2g^2 - bdfg^3 + adg^4 - cf^3ge + bf^2g^2e - afg^3e} - \frac{(cdg+cfe-bge)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2f^2 - bcd^2fg + acd^2g^2 - bcdf^2e + b^2dfge - abdg^2e + acf^2e^2 - abfge^2 + a^2g^2e^2\right)} - \frac{e^3 \log\left(|xe+d|\right)}{cd^3ge - cd^2fe^2 - bd^2ge^2 + bdfe^3 + adge^3 - afe^4} + \frac{\left(2\,c^2df - bcdg - bcfe + b^2ge - 2\,acge\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(c^2d^2f^2 - bcd^2fg + acd^2g^2 - bcdf^2e + b^2dfge - abdg^2e + acf^2e^2 - abfge^2 + a^2g^2e^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] g^3*log(abs(g*x + f))/(c*d*f^2*g^2 - b*d*f*g^3 + a*d*g^4 - c*f^3*g*e + b*f^
2*g^2*e - a*f*g^3*e) - 1/2*(c*d*g + c*f*e - b*g*e)*log(c*x^2 + b*x + a)/(c^
2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b^2*d*f*g*e - a*b*d*g
^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2) - e^3*log(abs(x*e + d))/(c*
d^3*g*e - c*d^2*f*e^2 - b*d^2*g*e^2 + b*d*f*e^3 + a*d*g*e^3 - a*f*e^4) + (2
*c^2*d*f - b*c*d*g - b*c*f*e + b^2*g*e - 2*a*c*g*e)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((c^2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b
^2*d*f*g*e - a*b*d*g^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2)*sqrt(-b
^2 + 4*a*c))
```

Mupad [**B**]
time = 19.25, size = 2500, normalized size = 10.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(6*a^2*c^4*d^5*g^5 + 6*a^2*c^4*e^5*f^5 - a^3*b^3*e^5*g^5 - a^3*b^2*e^5*
g^5*(b^2 - 4*a*c)^(1/2) - c^5*d^3*e^2*f^5*(b^2 - 4*a*c)^(1/2) - c^5*d^5*f^3
*g^2*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3*d^3*e^2*g^5 + b^2*c^4*d^2*e^3*f^5 - 1
8*a^3*c^3*e^5*f^3*g^2 + b^2*c^4*d^5*f^2*g^3 + 4*a^4*b*c*e^5*g^5 + 4*a^4*c*e
^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*c^3*d^5*g^5 - 2*a*b^2*c^3*e^5*f^5 + 2*
a*b^5*d^2*e^3*g^5 - 10*a*c^5*d^2*e^3*f^5 + a^2*b^4*d*e^4*g^5 + b*c^5*d^3*e^
2*f^5 - 8*a^4*c^2*d*e^4*g^5 + 2*a*b^5*e^5*f^2*g^3 - 10*a*c^5*d^5*f^2*g^3 +
```

a^2*b^4*e^5*f*g^4 + b*c^5*d^5*f^3*g^2 - 8*a^4*c^2*e^5*f*g^4 - a^2*b^4*e^5*g
^5*x - 8*a^4*c^2*e^5*g^5*x - 2*b^3*c^3*d^5*g^5*x - 2*b^3*c^3*e^5*f^5*x + 2*
b^6*d^2*e^3*g^5*x + 2*c^6*d^3*e^2*f^5*x + 2*b^6*e^5*f^2*g^3*x + 2*c^6*d^5*f
^3*g^2*x - 2*a*b*c^3*d^5*g^5*(b^2 - 4*a*c)^(1/2) - 2*a*b*c^3*e^5*f^5*(b^2 -
 4*a*c)^(1/2) + 7*a*c^4*d*e^4*f^5*(b^2 - 4*a*c)^(1/2) + 7*a*c^4*d^5*f*g^4*(
b^2 - 4*a*c)^(1/2) + 2*c^5*d^4*e*f^4*g*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*d^5*g^
5*x*(b^2 - 4*a*c)^(1/2) + 3*a*c^4*e^5*f^5*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^3*c
^2*d^4*e*g^5 - 6*a*b^4*c*d^3*e^2*g^5 - 21*a^2*b*c^3*d^4*e*g^5 - 2*a^3*b^2*c
*d*e^4*g^5 + 6*a*b^3*c^2*e^5*f^4*g - 6*a*b^4*c*e^5*f^3*g^2 - 21*a^2*b*c^3*e
^5*f^4*g - 2*a^3*b^2*c*e^5*f*g^4 + 10*a*c^5*d^3*e^2*f^4*g + 10*a*c^5*d^4*e*
f^3*g^2 + 26*a^2*c^4*d*e^4*f^4*g + 26*a^2*c^4*d^4*e*f*g^4 + 6*a^3*b^2*c*e^5
*g^5*x - 3*b*c^5*d^2*e^3*f^5*x + 14*a^2*c^4*d^4*e*g^5*x + 5*b^2*c^4*d*e^4*f
^5*x + 6*b^4*c^2*d^4*e*g^5*x - 6*b^5*c*d^3*e^2*g^5*x - 3*b*c^5*d^5*f^2*g^3*
x + 14*a^2*c^4*e^5*f^4*g*x + 5*b^2*c^4*d^5*f*g^4*x + 6*b^4*c^2*e^5*f^4*g*x
- 6*b^5*c*e^5*f^3*g^2*x + 2*a*b^4*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + a^2*b^3
*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) - b*c^4*d^2*e^3*f^5*(b^2 - 4*a*c)^(1/2) - 7*
a^2*c^3*d^4*e*g^5*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*e^5*f^2*g^3*(b^2 - 4*a*c)^(
1/2) + a^2*b^3*e^5*f*g^4*(b^2 - 4*a*c)^(1/2) - b*c^4*d^5*f^2*g^3*(b^2 - 4*a
*c)^(1/2) - 7*a^2*c^3*e^5*f^4*g*(b^2 - 4*a*c)^(1/2) - a^2*b^3*e^5*g^5*x*(b^
2 - 4*a*c)^(1/2) - 2*b^2*c^3*d^5*g^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^3*e^5*
f^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^5*d^2*e^3*g^5*x*(b^2 - 4*a*c)^(1/2) - 5*c^5
*d^2*e^3*f^5*x*(b^2 - 4*a*c)^(1/2) + 2*b^5*e^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2
) - 5*c^5*d^5*f^2*g^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c*d^2*e^3*g^5 + 21
*a^3*b*c^2*d^2*e^3*g^5 - 13*a^2*b^3*c*e^5*f^2*g^3 + 21*a^3*b*c^2*e^5*f^2*g^
3 + 2*a^3*c^3*d*e^4*f^2*g^3 + 2*a^3*c^3*d^2*e^3*f*g^4 - b^2*c^4*d^3*e^2*f^4
*g - b^2*c^4*d^4*e*f^3*g^2 - b^3*c^3*d^2*e^3*f^4*g - b^3*c^3*d^4*e*f^2*g^3
- b^5*c*d^2*e^3*f^2*g^3 - 10*a^3*c^3*d^2*e^3*g^5*x - 10*a^3*c^3*e^5*f^2*g^3
*x + 3*a*b*c^4*d*e^4*f^5 + 5*a^3*c^2*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 3*a*
b*c^4*d^5*f*g^4 + 5*a^3*c^2*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 5*a*b^5*d*e^4
*f*g^4 - 2*b*c^5*d^4*e*f^4*g + 7*a*b*c^4*d^5*g^5*x + 7*a*b*c^4*e^5*f^5*x +
a*b^5*d*e^4*g^5*x - 14*a*c^5*d*e^4*f^5*x + a*b^5*e^5*f*g^4*x - 14*a*c^5*d^5
*f*g^4*x - 5*b^6*d*e^4*f*g^4*x - 4*c^6*d^4*e*f^4*g*x + 27*a^2*b^2*c^2*d^3*e
^2*g^5 + 27*a^2*b^2*c^2*e^5*f^3*g^2 - 40*a^2*c^4*d^2*e^3*f^3*g^2 - 40*a^2*c
^4*d^3*e^2*f^2*g^3 + b^3*c^3*d^3*e^2*f^3*g^2 + b^4*c^2*d^2*e^3*f^3*g^2 + b^
4*c^2*d^3*e^2*f^2*g^3 + 32*a*b^3*c^2*d^3*e^2*g^5*x - 35*a^2*b*c^3*d^3*e^2*g
^5*x + 32*a*b^3*c^2*e^5*f^3*g^2*x - 35*a^2*b*c^3*e^5*f^3*g^2*x + 48*a*c^5*d
^3*e^2*f^3*g^2*x + 14*a^2*c^4*d*e^4*f^3*g^2*x + 14*a^2*c^4*d^3*e^2*f*g^4*x
+ 3*b^2*c^4*d^2*e^3*f^4*g*x + 3*b^2*c^4*d^4*e*f^2*g^3*x + 4*b^4*c^2*d*e^4*f
^3*g^2*x + 4*b^4*c^2*d^3*e^2*f*g^4*x + 13*a^2*b*c^2*d^3*e^2*g^5*(b^2 - 4*a*
c)^(1/2) - 7*a^2*b^2*c*d^2*e^3*g^5*(b^2 - 4*a*c)^(1/2) + 13*a^2*b*c^2*e^5*f
^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*b^2*c*e^5*f^2*g^3*(b^2 - 4*a*c)^(1/2) -
24*a*c^4*d^3*e^2*f^3*g^2*(b^2 - 4*a*c)^(1/2) - 7*a^2*c^3*d*e^4*f^3*g^2*(b^2
 - 4*a*c)^(1/2) - 7*a^2*c^3*d^3*e^2*f*g^4*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^2
*e^3*f^4*g*(b^2 - 4*a*c)^(1/2) + b^2*c^3*d^4*e*f^2*g^3*(b^2 - 4*a*c)^(1/2)
+ b^4*c*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/2) - 9*a^2*c^3*d^3*e^2*g^5*x*(b^2

- 4*a*c)^(1/2) - 9*a^2*c^3*e^5*f^3*g^2*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^3
*d^2*e^3*f^3*g^2 + 10*a*b^2*c^3*d^3*e^2*f^2*g^3 - 23*a*b^3*c^2*d^2*e^3*f^2*
g^3 + 96*a^2*b*c^3*d^2*e^3*f^2*g^3 - 39*a^2*b^2*c^2*d*e^4*f^2*g^3 - 39*a^2*
b^2*c^2*d^2*e^3*f*g^4 + 27*a^2*b^2*c^2*d^2*e^3*g^5*x + 27*a^2*b^2*c^2*e^5*f
^2*g^3*x - 48*a^2*c^4*d^2*e^3*f^2*g^3*x - 18*b^2*c^4*d^3*e^2*f^3*g^2*x + 17
*b^3*c^3*d^2*e^3*f^3*g^2*x + 17*b^3*c^3*d^3*e^2*f^2*g^3*x - 27*b^4*c^2*d^2*
e^3*f^2*g^3*x - 4*a^3*b*c*d*e^4*g^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*b*c*e^5*f*g
^4*(b^2 - 4*a*c)^(1/2) - 5*a*b^4*d*e^4*f*g^4*(b^2 - 4*a*c)^(1/2) + 4*a^3*b*
c*e^5*g^5*x*(b^2 - 4*a*c)^(1/2) + a*b^4*d*e^4*g^5*x*(b^2 - 4*a*c)^(1/2) + 5
*b*c^4*d*e^4*f^5*x*(b^2 - 4*a*c)^(1/2) + a*b^4*e^5*f*g^4*x*(b^2 - 4*a*c)^(1
/2) + 5*b*c^4*d^5*f*g^4*x*(b^2 - 4*a*c)^(1/2) - 5*b^5*d*e^4*f*g^4*x*(b^2 -
4*a*c)^(1/2) + 7*a*b*c^4*d^2*e^3*f^4*g + 7*a*b*c^4*d^4*e*f^2*g^3 - 10*a*b^2
*c^3*d*e^4*f^4*g - 10*a*b^2*c^3*d^4*e*f*g^4 + 10*a*b^4*c*d*e^4*f^2*g^3 + 10
*a*b^4*c*d^2*e^3*f*g^4 + 19*a^2*b^3*c*d*e^4*f*g^4 + 2*a^3*b*c^2*d*e^4*f*g^4
 + 24*a^2*c^3*d^2*e^3*f^2*g^3*(b^2 - 4*a*c)^(1/...

3.818 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=644

$$-\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c(2c^2df+b^2eg-c(bef+bdg+2aeg))\,x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right)\left(a+bx+cx^2\right)}+\frac{2c(2c^2\ldots}{\ldots}$$

```
[Out] (-b^3*e*g+b^2*c*(d*g+e*f)-2*a*c^2*(d*g+e*f)-b*c*(-3*a*e*g+c*d*f)-c*(2*c^2*d
*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*
f^2-g*(-a*g+b*f))/(c*x^2+b*x+a)+2*c*(2*c^2*d*f+b^2*e*g-c*(2*a*e*g+b*d*g+b*e
*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+
c*d^2)/(c*f^2-g*(-a*g+b*f))+e^4*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)-
g^4*ln(g*x+f)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2-1/2*(-b*e*g+c*d*g+c*e*f)*(c*
(d^2*g^2+e^2*f^2)+e*g*(2*a*e*g-b*(d*g+e*f)))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c
*d^2)^2/(c*f^2-g*(-a*g+b*f))^2+(b^2*e^2*g^2*(-2*a*e*g+b*d*g+b*e*f)-2*c^3*d*
f*(d^2*g^2+d*e*f*g+e^2*f^2)+2*c*e*g*(a^2*e^2*g^2+a*b*e*g*(d*g+e*f)-b^2*(d*g
+e*f)^2)-c^2*(4*a*d*e^2*f*g^2-b*(d^3*g^3+5*d^2*e*f*g^2+5*d*e^2*f^2*g+e^3*f^
3)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^2/(c*f^2-g*(
-a*g+b*f))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 1.26, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {907, 652, 632, 212, 648, 642}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ceg\left(a^2e^2g^2+abeg(dg+ef)-b^2(dg+ef)^2\right)+b^2e^2g^2(-2aeg+bdg+bef)-c^2\left(4ade^2fg^2-b\left(d^3g^3+5d^2efg^2+5de^2f^2g+e^3f^3\right)\right)-2c^3df\left(d^2g^2+defg+e^2f^2\right)\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag)\right)^2}+\frac{2c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{cx\left(-c(2aeg+bdg+bef)+b^2eg+2c^2df\right)+bc(cdf-3aeg)+2ac^2(dg+ef)+b^3eg-b^2c(dg+ef)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)\left(cf^2-g(bf-ag)\right)}-\frac{\log\left(a+bx+cx^2\right)(-beg+cdg+cef)\left(eg(2aeg-b(dg+ef))+c\left(d^2g^2+e^2f^2\right)\right)}{2\left(ae^2-bde+cd^2\right)^2\left(cf^2-g(bf-ag)\right)^2}+\frac{e^4\log(d+ex)}{(ef-dg)\left(ae^2-bde+cd^2\right)^2}-\frac{g^4\log(f+gx)}{(ef-dg)\left(ag^2-bfg+cf^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((b^3*e*g - b^2*c*(e*f + d*g) + 2*a*c^2*(e*f + d*g) + b*c*(c*d*f - 3*a*e*g
) + c*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*x)/((b^2 - 4*a*c)
*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a*g))*(a + b*x + c*x^2))) + (2*c
*(2*c^2*d*f + b^2*e*g - c*(b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(
b*f - a*g))) + ((b^2*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*c^3*d*f*(e^2*f^2
 + d*e*f*g + d^2*g^2) + 2*c*e*g*(a^2*e^2*g^2 + a*b*e*g*(e*f + d*g) - b^2*(e
*f + d*g)^2) - c^2*(4*a*d*e^2*f*g^2 - b*(e^3*f^3 + 5*d*e^2*f^2*g + 5*d^2*e*
f*g^2 + d^3*g^3)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2) + (e^4*Log[d + e*x]
)/((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)
*(c*f^2 - b*f*g + a*g^2)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^
2) + e*g*(2*a*e*g - b*(e*f + d*g)))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e
 + a*e^2)^2*(c*f^2 - g*(b*f - a*g))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^2}\,dx = \int \left(-\frac{e^5}{\left(cd^2-bde+ae^2\right)^2(-ef+dg)(d+ex)} - \frac{}{(ef-dg)\left(cf^2-\right.}\right.$$

$$= \frac{e^4\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2(ef-dg)} - \frac{g^4\log(f+gx)}{(ef-dg)\left(cf^2-bfg+ag^2\right)^2} + \int$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c(2c^2df}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right.}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c(2c^2df}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right.}$$

$$= -\frac{b^3eg-b^2c(ef+dg)+2ac^2(ef+dg)+bc(cdf-3aeg)+c(2c^2df}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(cf^2-g(bf-ag)\right.}$$

Mathematica [**A**]
time = 1.66, size = 710, normalized size = 1.10

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*(f -
 g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x))))/((b^2 - 4*a*c)*(-(c*d^
2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*(a + x*(b + c*x))) + ((4*c^5
*d^3*f^3 + b^4*e^2*g^2*(b*e*f + b*d*g - 2*a*e*g) - 2*b^2*c*e*g*(-6*a^2*e^2*
g^2 + 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)) + 2*c^4*d*
f*(-3*b*d*f*(e*f + d*g) + 2*a*(3*e^2*f^2 + d*e*f*g + 3*d^2*g^2)) + c^2*(-12
*a^3*e^3*g^3 - 6*a^2*b*e^2*g^2*(e*f + d*g) + 12*a*b^2*e*g*(e^2*f^2 + d*e*f*
g + d^2*g^2) + b^3*(e^3*f^3 + d*e^2*f^2*g + d^2*e*f*g^2 + d^3*g^3)) - 2*c^3
*(-4*b^2*d^2*e*f^2*g + 2*a^2*e*g*(e^2*f^2 - 5*d*e*f*g + d^2*g^2) + a*b*(3*e
^3*f^3 + 11*d*e^2*f^2*g + 11*d^2*e*f*g^2 + 3*d^3*g^3)))*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2*(c*
f^2 + g*(-(b*f) + a*g))^2) + (e^4*Log[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))
^2*(e*f - d*g)) - (g^4*Log[f + g*x])/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a*g)
)^2) - ((c*e*f + c*d*g - b*e*g)*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(
e*f + d*g)))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^2*(c*f^2 +
 g*(-(b*f) + a*g))^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2227 vs. 2(635) = 1270.
time = 0.94, size = 2228, normalized size = 3.46

method	result	size
default	Expression too large to display	2228
risch	Expression too large to display	29824

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e^4/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)+g^4/(d*g-e*f)/(a*g^2-b*f*g+c
*f^2)^2*ln(g*x+f)-1/(a*e^2-b*d*e+c*d^2)^2/(a*g^2-b*f*g+c*f^2)^2*((c*(2*a^3*
c*e^3*g^3-a^2*b^2*e^3*g^3-a^2*b*c*d*e^2*g^3-a^2*b*c*e^3*f*g^2+2*a^2*c^2*d^2
*e*g^3-2*a^2*c^2*d*e^2*f*g^2+2*a^2*c^2*e^3*f^2*g+a*b^3*d*e^2*g^3+a*b^3*e^3*
f*g^2-2*a*b^2*c*d^2*e*g^3-2*a*b^2*c*e^3*f^2*g+a*b*c^2*d^3*g^3+a*b*c^2*d^2*e
*f*g^2+a*b*c^2*d*e^2*f^2*g+a*b*c^2*e^3*f^3-2*a*c^3*d^3*f*g^2+2*a*c^3*d^2*e*
f^2*g-2*a*c^3*d*e^2*f^3-b^4*d*e^2*f*g^2+2*b^3*c*d^2*e*f*g^2+2*b^3*c*d*e^2*f
^2*g-b^2*c^2*d^3*f*g^2-5*b^2*c^2*d^2*e*f^2*g-b^2*c^2*d*e^2*f^3+3*b*c^3*d^3*
f^2*g+3*b*c^3*d^2*e*f^3-2*c^4*d^3*f^3)/(4*a*c-b^2)*x+(3*a^3*b*c*e^3*g^3-2*a
^3*c^2*d*e^2*g^3-2*a^3*c^2*e^3*f*g^2-a^2*b^3*e^3*g^3-2*a^2*b^2*c*d*e^2*g^3-
2*a^2*b^2*c*e^3*f*g^2+5*a^2*b*c^2*d^2*e*g^3+3*a^2*b*c^2*d*e^2*f*g^2+5*a^2*b
*c^2*e^3*f^2*g-2*a^2*c^3*d^3*g^3-2*a^2*c^3*d^2*e*f*g^2-2*a^2*c^3*d*e^2*f^2*
g-2*a^2*c^3*e^3*f^3+a*b^4*d*e^2*g^3+a*b^4*e^3*f*g^2-2*a*b^3*c*d^2*e*g^3+a*b
^3*c*d*e^2*f*g^2-2*a*b^3*c*e^3*f^2*g+a*b^2*c^2*d^3*g^3-3*a*b^2*c^2*d^2*e*f*
g^2-3*a*b^2*c^2*d*e^2*f^2*g+a*b^2*c^2*e^3*f^3+a*b*c^3*d^3*f*g^2+7*a*b*c^3*d
^2*e*f^2*g+a*b*c^3*d*e^2*f^3-2*a*c^4*d^3*f^2*g-2*a*c^4*d^2*e*f^3-b^5*d*e^2*
f*g^2+2*b^4*c*d^2*e*f*g^2+2*b^4*c*d*e^2*f^2*g-b^3*c^2*d^3*f*g^2-4*b^3*c^2*d
^2*e*f^2*g-b^3*c^2*d*e^2*f^3+2*b^2*c^3*d^3*f^2*g+2*b^2*c^3*d^2*e*f^3-b*c^4*
d^3*f^3)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(-8*a^2*b*c^2*e^3*g^
3+8*a^2*c^3*d*e^2*g^3+8*a^2*c^3*e^3*f*g^2+2*a*b^3*c*e^3*g^3+2*a*b^2*c^2*d*e
^2*g^3+2*a*b^2*c^2*e^3*f*g^2-8*a*b*c^3*d^2*e*g^3-8*a*b*c^3*d*e^2*f*g^2-8*a*
b*c^3*e^3*f^2*g+4*a*c^4*d^3*g^3+4*a*c^4*d^2*e*f*g^2+4*a*c^4*d*e^2*f^2*g+4*a
*c^4*e^3*f^3-b^4*c*d*e^2*g^3-b^4*c*e^3*f*g^2+2*b^3*c^2*d^2*e*g^3+2*b^3*c^2*
d*e^2*f*g^2+2*b^3*c^2*e^3*f^2*g-b^2*c^3*d^3*g^3-b^2*c^3*d^2*e*f*g^2-b^2*c^3
*d*e^2*f^2*g-b^2*c^3*e^3*f^3)/c*ln(c*x^2+b*x+a)+2*(13*a*b*c^3*d*e^2*f^2*g+2
*a*b^4*e^3*g^3-b^5*d*e^2*g^3-b^5*e^3*f*g^2-b^3*c^2*d^3*g^3-b^3*c^2*e^3*f^3+
6*a^3*c^2*e^3*g^3-2*c^5*d^3*f^3-10*a^2*c^3*d*e^2*f*g^2+3*a*b^3*c*d*e^2*g^3+
3*a*b^3*c*e^3*f*g^2-10*a*b^2*c^2*d^2*e*g^3-10*a*b^2*c^2*e^3*f^2*g-2*a*c^4*d
^2*e*f^2*g+2*b^4*c*d*e^2*f*g^2-b^3*c^2*d^2*e*f*g^2-b^3*c^2*d*e^2*f^2*g-4*b^
2*c^3*d^2*e*f^2*g-1/2*(-8*a^2*b*c^2*e^3*g^3+8*a^2*c^3*d*e^2*g^3+8*a^2*c^3*e
^3*f*g^2+2*a*b^3*c*e^3*g^3+2*a*b^2*c^2*d*e^2*g^3+2*a*b^2*c^2*e^3*f*g^2-8*a*
b*c^3*d^2*e*g^3-8*a*b*c^3*d*e^2*f*g^2-8*a*b*c^3*e^3*f^2*g+4*a*c^4*d^3*g^3+4
*a*c^4*d^2*e*f*g^2+4*a*c^4*d*e^2*f^2*g+4*a*c^4*e^3*f^3-b^4*c*d*e^2*g^3-b^4*
c*e^3*f*g^2+2*b^3*c^2*d^2*e*g^3+2*b^3*c^2*d*e^2*f*g^2+2*b^3*c^2*e^3*f^2*g-b
^2*c^3*d^3*g^3-b^2*c^3*d^2*e*f*g^2-b^2*c^3*d*e^2*f^2*g-b^2*c^3*e^3*f^3)*b/c
+7*a^2*b*c^2*d*e^2*g^3+7*a^2*b*c^2*e^3*f*g^2+2*b^4*c*d^2*e*g^3+2*b^4*c*e^3*
```

```
f^2*g-10*a*b^2*c^2*d*e^2*f*g^2+13*a*b*c^3*d^2*e*f*g^2-10*a^2*b^2*c*e^3*g^3+
2*a^2*c^3*d^2*e*g^3+2*a^2*c^3*e^3*f^2*g+5*a*b*c^3*d^3*g^3+5*a*b*c^3*e^3*f^3
-6*a*c^4*d^3*f*g^2-6*a*c^4*d*e^2*f^3+3*b*c^4*d^3*f^2*g+3*b*c^4*d^2*e*f^3)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3315 vs. $2(653) = 1306$.
time = 4.82, size = 3315, normalized size = 5.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

[Out] g^5*log(abs(g*x + f))/(c^2*d*f^4*g^2 - 2*b*c*d*f^3*g^3 + b^2*d*f^2*g^4 + 2*
a*c*d*f^2*g^4 - 2*a*b*d*f*g^5 + a^2*d*g^6 - c^2*f^5*g*e + 2*b*c*f^4*g^2*e -
b^2*f^3*g^3*e - 2*a*c*f^3*g^3*e + 2*a*b*f^2*g^4*e - a^2*f*g^5*e) - 1/2*(c^
2*d^3*g^3 + c^2*d^2*f*g^2*e - 2*b*c*d^2*g^3*e + c^2*d*f^2*g*e^2 - 2*b*c*d*f
*g^2*e^2 + b^2*d*g^3*e^2 + 2*a*c*d*g^3*e^2 + c^2*f^3*e^3 - 2*b*c*f^2*g*e^3
+ b^2*f*g^2*e^3 + 2*a*c*f*g^2*e^3 - 2*a*b*g^3*e^3)*log(c*x^2 + b*x + a)/(c^
4*d^4*f^4 - 2*b*c^3*d^4*f^3*g + b^2*c^2*d^4*f^2*g^2 + 2*a*c^3*d^4*f^2*g^2 -
2*a*b*c^2*d^4*f*g^3 + a^2*c^2*d^4*g^4 - 2*b*c^3*d^3*f^4*e + 4*b^2*c^2*d^3*
f^3*g*e - 2*b^3*c*d^3*f^2*g^2*e - 4*a*b*c^2*d^3*f^2*g^2*e + 4*a*b^2*c*d^3*f
*g^3*e - 2*a^2*b*c*d^3*g^4*e + b^2*c^2*d^2*f^4*e^2 + 2*a*c^3*d^2*f^4*e^2 -
2*b^3*c*d^2*f^3*g*e^2 - 4*a*b*c^2*d^2*f^3*g*e^2 + b^4*d^2*f^2*g^2*e^2 + 4*a
*b^2*c*d^2*f^2*g^2*e^2 + 4*a^2*c^2*d^2*f^2*g^2*e^2 - 2*a*b^3*d^2*f*g^3*e^2
- 4*a^2*b*c*d^2*f*g^3*e^2 + a^2*b^2*d^2*g^4*e^2 + 2*a^3*c*d^2*g^4*e^2 - 2*a
*b*c^2*d*f^4*e^3 + 4*a*b^2*c*d*f^3*g*e^3 - 2*a*b^3*d*f^2*g^2*e^3 - 4*a^2*b*
c*d*f^2*g^2*e^3 + 4*a^2*b^2*d*f*g^3*e^3 - 2*a^3*b*d*g^4*e^3 + a^2*c^2*f^4*e
^4 - 2*a^2*b*c*f^3*g*e^4 + a^2*b^2*f^2*g^2*e^4 + 2*a^3*c*f^2*g^2*e^4 - 2*a^
3*b*f*g^3*e^4 + a^4*g^4*e^4) - e^5*log(abs(x*e + d))/(c^2*d^5*g*e - c^2*d^4
*f*e^2 - 2*b*c*d^4*g*e^2 + 2*b*c*d^3*f*e^3 + b^2*d^3*g*e^3 + 2*a*c*d^3*g*e^
3 - b^2*d^2*f*e^4 - 2*a*c*d^2*f*e^4 - 2*a*b*d^2*g*e^4 + 2*a*b*d*f*e^5 + a^2
*d*g*e^5 - a^2*f*e^6) - (4*c^5*d^3*f^3 - 6*b*c^4*d^3*f^2*g + 12*a*c^4*d^3*f
*g^2 + b^3*c^2*d^3*g^3 - 6*a*b*c^3*d^3*g^3 - 6*b*c^4*d^2*f^3*e + 8*b^2*c^3*
d^2*f^2*g*e + 4*a*c^4*d^2*f^2*g*e + b^3*c^2*d^2*f*g^2*e - 22*a*b*c^3*d^2*f*
g^2*e - 2*b^4*c*d^2*g^3*e + 12*a*b^2*c^2*d^2*g^3*e - 4*a^2*c^3*d^2*g^3*e +
12*a*c^4*d*f^3*e^2 + b^3*c^2*d*f^2*g*e^2 - 22*a*b*c^3*d*f^2*g*e^2 - 2*b^4*c
*d*f*g^2*e^2 + 12*a*b^2*c^2*d*f*g^2*e^2 + 20*a^2*c^3*d*f*g^2*e^2 + b^5*d*g^
3*e^2 - 4*a*b^3*c*d*g^3*e^2 - 6*a^2*b*c^2*d*g^3*e^2 + b^3*c^2*f^3*e^3 - 6*a
*b*c^3*f^3*e^3 - 2*b^4*c*f^2*g*e^3 + 12*a*b^2*c^2*f^2*g*e^3 - 4*a^2*c^3*f^2
*g*e^3 + b^5*f*g^2*e^3 - 4*a*b^3*c*f*g^2*e^3 - 6*a^2*b*c^2*f*g^2*e^3 - 2*a*
b^4*g^3*e^3 + 12*a^2*b^2*c*g^3*e^3 - 12*a^3*c^2*g^3*e^3)*arctan((2*c*x + b)
/sqrt(-b^2 + 4*a*c))/((b^2*c^4*d^4*f^4 - 4*a*c^5*d^4*f^4 - 2*b^3*c^3*d^4*f^
3*g + 8*a*b*c^4*d^4*f^3*g + b^4*c^2*d^4*f^2*g^2 - 2*a*b^2*c^3*d^4*f^2*g^2 -
8*a^2*c^4*d^4*f^2*g^2 - 2*a*b^3*c^2*d^4*f*g^3 + 8*a^2*b*c^3*d^4*f*g^3 + a^
2*b^2*c^2*d^4*g^4 - 4*a^3*c^3*d^4*g^4 - 2*b^3*c^3*d^3*f^4*e + 8*a*b*c^4*d^3
*f^4*e + 4*b^4*c^2*d^3*f^3*g*e - 16*a*b^2*c^3*d^3*f^3*g*e - 2*b^5*c*d^3*f^2
*g^2*e + 4*a*b^3*c^2*d^3*f^2*g^2*e + 16*a^2*b*c^3*d^3*f^2*g^2*e + 4*a*b^4*c
*d^3*f*g^3*e - 16*a^2*b^2*c^2*d^3*f*g^3*e - 2*a^2*b^3*c*d^3*g^4*e + 8*a^3*b
*c^2*d^3*g^4*e + b^4*c^2*d^2*f^4*e^2 - 2*a*b^2*c^3*d^2*f^4*e^2 - 8*a^2*c^4*
d^2*f^4*e^2 - 2*b^5*c*d^2*f^3*g*e^2 + 4*a*b^3*c^2*d^2*f^3*g*e^2 + 16*a^2*b*
c^3*d^2*f^3*g*e^2 + b^6*d^2*f^2*g^2*e^2 - 12*a^2*b^2*c^2*d^2*f^2*g^2*e^2 -
16*a^3*c^3*d^2*f^2*g^2*e^2 - 2*a*b^5*d^2*f*g^3*e^2 + 4*a^2*b^3*c*d^2*f*g^3*
e^2 + 16*a^3*b*c^2*d^2*f*g^3*e^2 + a^2*b^4*d^2*g^4*e^2 - 2*a^3*b^2*c*d^2*g^
4*e^2 - 8*a^4*c^2*d^2*g^4*e^2 - 2*a*b^3*c^2*d*f^4*e^3 + 8*a^2*b*c^3*d*f^4*e
^3 + 4*a*b^4*c*d*f^3*g*e^3 - 16*a^2*b^2*c^2*d*f^3*g*e^3 - 2*a*b^5*d*f^2*g^2
*e^3 + 4*a^2*b^3*c*d*f^2*g^2*e^3 + 16*a^3*b*c^2*d*f^2*g^2*e^3 + 4*a^2*b^4*d
*f*g^3*e^3 - 16*a^3*b^2*c*d*f*g^3*e^3 - 2*a^3*b^3*d*g^4*e^3 + 8*a^4*b*c*d*g

```
^4*e^3 + a^2*b^2*c^2*f^4*e^4 - 4*a^3*c^3*f^4*e^4 - 2*a^2*b^3*c*f^3*g*e^4 +
8*a^3*b*c^2*f^3*g*e^4 + a^2*b^4*f^2*g^2*e^4 - 2*a^3*b^2*c*f^2*g^2*e^4 - 8*a
^4*c^2*f^2*g^2*e^4 - 2*a^3*b^3*f*g^3*e^4 + 8*a^4*b*c*f*g^3*e^4 + a^4*b^2*g^
4*e^4 - 4*a^5*c*g^4*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^4*d^3*f^3 - 2*b^2*c^3*d
^3*f^2*g + 2*a*c^4*d^3*f^2*g + b^3*c^2*d^3*f*g^2 - a*b*c^3*d^3*f*g^2 - a*b^
2*c^2*d^3*g^3 + 2*a^2*c^3*d^3*g^3 - 2*b^2*c^3*d^2*f^3*e + 2*a*c^4*d^2*f^3*e
 + 4*b^3*c^2*d^2*f^2*g*e - 7*a*b*c^3*d^2*f^2*g*e - 2*b^4*c*d^2*f*g^2*e + 3*
a*b^2*c^2*d^2*f*g^2*e + 2*a^2*c^3*d^2*f*g^2*e + 2*a*b^3*c*d^2*g^3*e - 5*a^2
*b*c^2*d^2*g^3*e + b^3*c^2*d*f^3*e^2 - a*b*c^3*d*f^3*e^2 - 2*b^4*c*d*f^2*g*
e^2 + 3*a*b^2*c^2*d*f^2*g*e^2 + 2*a^2*c^3*d*f^2*g*e^2 + b^5*d*f*g^2*e^2 - a
*b^3*c*d*f*g^2*e^2 - 3*a^2*b*c^2*d*f*g^2*e^2 - a*b^4*d*g^3*e^2 + 2*a^2*b^2*
c*d*g^3*e^2 + 2*a^3*c^2*d*g^3*e^2 - a*b^2*c^2*f^3*e^3 + 2*a^2*c^3*f^3*e^3 +
 2*a*b^3*c*f^2*g*e^3 - 5*a^2*b*c^2*f^2*g*e^3 - a*b^4*f*g^2*e^3 + 2*a^2*b^2*
c*f*g^2*e^3 + 2*a^3*c^2*f*g^2*e^3 + a^2*b^3*g^3*e^3 - 3*a^3*b*c*g^3*e^3 + (
2*c^5*d^3*f^3 - 3*b*c^4*d^3*f^2*g + b^2*c^3*d^3*f*g^2 + 2*a*c^4*d^3*f*g^2 -
 a*b*c^3*d^3*g^3 - 3*b*c^4*d^2*f^3*e + 5*b^2*c^3*d^2*f^2*g*e - 2*a*c^4*d^2*
f^2*g*e - 2*b^3*c^2*d^2*f*g^2*e - a*b*c^3*d^2*f*g^2*e + 2*a*b^2*c^2*d^2*g^3
*e - 2*a^2*c^3*d^2*g^3*e + b^2*c^3*d*f^3*e^2 + 2*a*c^4*d*f^3*e^2 - 2*b^3*c^
2*d*f^2*g*e^2 - a*b*c^3*d*f^2*g*e^2 + b^4*c*d*f*g^2*e^2 + 2*a^2*c^3*d*f*g^2
*e^2 - a*b^3*c*d*g^3*e^2 + a^2*b*c^2*d*g^3*e^2 - a*b*c^3*f^3*e^3 + 2*a*b^2*
c^2*f^2*g*e^3 - 2*a^2*c^3*f^2*g*e^3 - a*b^3*c*f*g^2*e^3 + a^2*b*c^2*f*g^2*e
^3 + a^2*b^2*c*g^3*e^3 - 2*a^3*c^2*g^3*e^3)*x)/...
```

Mupad [B]
time = 32.63, size = 2500, normalized size = 3.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((b^3*e*g + 2*a*c^2*d*g + 2*a*c^2*e*f + b*c^2*d*f - b^2*c*d*g - b^2*c*e*f -
 3*a*b*c*e*g)/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b^2*e^2*g^2 + 4*a^2*
c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^3*d*e*g^2 + b^3*c*d
*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2 - a*b^2*c*e^2*f^2
- b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 - 4*a*b*c^2*d^2*f*g -
 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g) - (x*(2*a*c^2*e*g - 2*c^3*d*f + b*c
^2*d*g + b*c^2*e*f - b^2*c*e*g))/(4*a*c^3*d^2*f^2 + 4*a^3*c*e^2*g^2 - a^2*b
^2*e^2*g^2 + 4*a^2*c^2*d^2*g^2 + 4*a^2*c^2*e^2*f^2 - b^2*c^2*d^2*f^2 + a*b^
3*d*e*g^2 + b^3*c*d*e*f^2 + a*b^3*e^2*f*g + b^3*c*d^2*f*g - a*b^2*c*d^2*g^2
 - a*b^2*c*e^2*f^2 - b^4*d*e*f*g - 4*a*b*c^2*d*e*f^2 - 4*a^2*b*c*d*e*g^2 -
4*a*b*c^2*d^2*f*g - 4*a^2*b*c*e^2*f*g + 4*a*b^2*c*d*e*f*g))/(a + b*x + c*x^
2) + symsum(log((12*a^2*c^5*e^6*g^6 - 3*b^2*c^5*d^2*e^4*g^6 - 3*b^2*c^5*e^6
*f^2*g^4 + 4*c^7*d^2*e^4*f^2*g^4 - 2*a*b^2*c^4*e^6*g^6 + 16*a*c^6*d^2*e^4*g
^6 + 3*b^3*c^4*d*e^5*g^6 + 16*a*c^6*e^6*f^2*g^4 + 3*b^3*c^4*e^6*f*g^5 - 4*b
```

*c^6*d*e^5*f^2*g^4 - 4*b*c^6*d^2*e^4*f*g^5 - 16*a*b*c^5*d*e^5*g^6 - 16*a*b*c^5*e^6*f*g^5 + 16*a*c^6*d*e^5*f*g^5)/(16*a^2*c^6*d^4*f^4 + a^4*b^4*e^4*g^4 + 16*a^4*c^4*d^4*g^4 + 16*a^4*c^4*e^4*f^4 + b^4*c^4*d^4*f^4 + 16*a^6*c^2*e^4*g^4 + a^2*b^4*c^2*d^4*g^4 + a^2*b^4*c^2*e^4*f^4 - 8*a^3*b^2*c^3*d^4*g^4 - 8*a^3*b^2*c^3*e^4*f^4 + a^2*b^6*d^2*e^2*g^4 + 32*a^3*c^5*d^2*e^2*f^4 + 32*a^5*c^3*d^2*e^2*g^4 + b^6*c^2*d^2*e^2*f^4 + a^2*b^6*e^4*f^2*g^2 + 32*a^3*c^5*d^4*f^2*g^2 + 32*a^5*c^3*e^4*f^2*g^2 + b^6*c^2*d^4*f^2*g^2 + b^8*d^2*e^2*f^2*g^2 - 8*a*b^2*c^5*d^4*f^4 - 8*a^5*b^2*c*e^4*g^4 - 2*a^3*b^5*d*e^3*g^4 - 2*b^5*c^3*d^3*e*f^4 - 2*a^3*b^5*e^4*f*g^3 - 2*b^5*c^3*d^4*f^3*g + 16*a*b^3*c^4*d^3*e*f^4 - 2*a*b^5*c^2*d*e^3*f^4 - 32*a^2*b*c^5*d^3*e*f^4 - 32*a^3*b*c^4*d*e^3*f^4 - 2*a^2*b^5*c*d^3*e*g^4 - 32*a^4*b*c^3*d^3*e*g^4 + 16*a^4*b^3*c*d*e^3*g^4 - 32*a^5*b*c^2*d*e^3*g^4 + 16*a*b^3*c^4*d^4*f^3*g - 2*a*b^5*c^2*d^4*f*g^3 - 32*a^2*b*c^5*d^4*f^3*g - 32*a^3*b*c^4*d^4*f*g^3 - 2*a^2*b^5*c*e^4*f^3*g - 32*a^4*b*c^3*e^4*f^3*g + 16*a^4*b^3*c*e^4*f*g^3 - 32*a^5*b*c^2*e^4*f*g^3 - 2*a*b^7*d*e^3*f^2*g^2 - 2*a*b^7*d^2*e^2*f*g^3 + 4*a^2*b^6*d*e^3*f*g^3 + 4*b^6*c^2*d^3*e*f^3*g - 2*b^7*c*d^2*e^2*f^3*g - 2*b^7*c*d^3*e*f^2*g^2 - 6*a*b^4*c^3*d^2*e^2*f^4 + 16*a^2*b^3*c^3*d*e^3*f^4 + 16*a^3*b^3*c^2*d^3*e*g^4 - 6*a^3*b^4*c*d^2*e^2*g^4 - 6*a*b^4*c^3*d^4*f^2*g^2 + 16*a^2*b^3*c^3*d^4*f*g^3 + 16*a^3*b^3*c^2*e^4*f^3*g - 6*a^3*b^4*c*e^4*f^2*g^2 + 64*a^4*c^4*d^2*e^2*f^2*g^2 + 4*a*b^6*c*d*e^3*f^3*g + 4*a*b^6*c*d^3*e*f*g^3 - 32*a*b^4*c^3*d^3*e*f^3*g - 32*a^3*b^4*c*d*e^3*f*g^3 - 12*a^2*b^4*c^2*d^2*e^2*f^2*g^2 + 32*a^3*b^2*c^3*d^2*e^2*f^2*g^2 + 12*a*b^5*c^2*d^2*e^2*f^3*g + 12*a*b^5*c^2*d^3*e*f^2*g^2 - 4*a*b^6*c*d^2*e^2*f^2*g^2 + 64*a^2*b^2*c^4*d^3*e*f^3*g - 32*a^2*b^4*c^2*d*e^3*f^3*g - 32*a^2*b^4*c^2*d^3*e*f*g^3 + 12*a^2*b^5*c*d*e^3*f^2*g^2 + 12*a^2*b^5*c*d^2*e^2*f*g^3 - 64*a^3*b*c^4*d^2*e^2*f^3*g - 64*a^3*b*c^4*d^3*e*f^2*g^2 + 64*a^3*b^2*c^3*d*e^3*f^3*g + 64*a^3*b^2*c^3*d^3*e*f*g^3 - 64*a^4*b*c^3*d*e^3*f^2*g^2 - 64*a^4*b*c^3*d^2*e^2*f*g^3 + 64*a^4*b^2*c^2*d*e^3*f*g^3) - root(1120*a^6*b^2*c^6*d^9*e*f*g^9*z^4 + 1120*a^6*b^2*c^6*d*e^9*f^9*g*z^4 - 792*a^5*b^4*c^5*d^9*e*f*g^9*z^4 - 792*a^5*b^4*c^5*d*e^9*f^9*g*z^4 + 512*a^9*b*c^4*d^4*e^6*f*g^9*z^4 + 512*a^9*b*c^4*d*e^9*f^4*g^6*z^4 - 512*a^7*b*c^6*d^8*e^2*f*g^9*z^4 - 512*a^7*b*c^6*d*e^9*f^8*g^2*z^4 - 512*a^6*b*c^7*d^9*e*f^2*g^8*z^4 - 512*a^6*b*c^7*d^2*e^8*f^9*g*z^4 + 512*a^4*b*c^9*d^9*e*f^6*g^4*z^4 + 512*a^4*b*c^9*d^6*e^4*f^9*g*z^4 + 256*a^10*b*c^3*d^2*e^8*f*g^9*z^4 + 256*a^10*b*c^3*d*e^9*f^2*g^8*z^4 + 256*a^3*b*c^10*d^9*e*f^8*g^2*z^4 + 256*a^3*b*c^10*d^8*e^2*f^9*g*z^4 - 200*a^6*b^7*c*d^4*e^6*f*g^9*z^4 - 200*a^6*b^7*c*d*e^9*f^4*g^6*z^4 - 200*a*b^7*c^6*d^9*e*f^6*g^4*z^4 - 200*a*b^7*c^6*d^6*e^4*f^9*g*z^4 + 194*a^4*b^6*c^4*d^9*e*f*g^9*z^4 + 194*a^4*b^6*c^4*d*e^9*f^9*g*z^4 + 144*a^5*b^8*c*d^5*e^5*f*g^9*z^4 + 144*a^5*b^8*c*d*e^9*f^5*g^5*z^4 + 144*a*b^8*c^5*d^9*e*f^5*g^5*z^4 + 144*a*b^8*c^5*d^5*e^5*f^9*g*z^4 + 96*a^10*b^2*c^2*d*e^9*f*g^9*z^4 + 96*a^2*b^2*c^10*d^9*e*f^9*g*z^4 + 56*a^7*b^6*c*d^3*e^7*f*g^9*z^4 + 56*a^7*b^6*c*d*e^9*f^3*g^7*z^4 + 56*a*b^6*c^7*d^9*e*f^7*g^3*z^4 + 56*a*b^6*c^7*d^7*e^3*f^9*g*z^4 + 48*a^8*b^5*c*d^2*e^8*f*g^9*z^4 + 48*a^8*b^5*c*d*e^9*f^2*g^8*z^4 + 48*a*b^5*c^8*d^9*e*f^8*g^2*z^4 + 48*a*b^5*c^8*d^8*e^2*f^9*g*z^4 + 20*a*b^12*c*d^6*e^4*f^4*g^6*z^4 + 20*a*b^12*c*d^4*e^6*f^6*g^4*z^4 - 16*a^3*b^10*c*d^7*e^3*f*g^9*

z^4 - 16*a^3*b^10*c*d*e^9*f^7*g^3*z^4 - 16*a^3*b^8*c^3*d^9*e*f*g^9*z^4 - 16
*a^3*b^8*c^3*d*e^9*f^9*g*z^4 - 16*a*b^12*c*d^7*e^3*f^3*g^7*z^4 - 16*a*b^12*
c*d^3*e^7*f^7*g^3*z^4 - 16*a*b^10*c^3*d^9*e*f^3*g^7*z^4 - 16*a*b^10*c^3*d^3
*e^7*f^9*g*z^4 - 8*a^4*b^9*c*d^6*e^4*f*g^9*z^4 - 8*a^4*b^9*c*d*e^9*f^6*g^4*
z^4 - 8*a*b^12*c*d^5*e^5*f^5*g^5*z^4 - 8*a*b^9*c^4*d^9*e*f^4*g^6*z^4 - 8*a*
b^9*c^4*d^4*e^6*f^9*g*z^4 - 9984*a^7*b^2*c^5*d^4*e^6*f^4*g^6*z^4 - 9984*a^5
*b^2*c^7*d^6*e^4*f^6*g^4*z^4 - 8640*a^6*b^2*c^6*d^6*e^4*f^4*g^6*z^4 - 8640*
a^6*b^2*c^6*d^4*e^6*f^6*g^4*z^4 - 8544*a^5*b^4*...

3.819 $\int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=287

$$-\frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))(f+gx)^{\cdots}}{3g^6}$$

```
[Out] 2/3*(-d*g+e*f)^2*(c*f*(-2*d*g+5*e*f)-g*(-3*a*e*g-b*d*g+4*b*e*f))*(g*x+f)^(3
/2)/g^6+2/5*(-d*g+e*f)*(3*e*g*(-a*e*g-b*d*g+2*b*e*f)-c*(d^2*g^2-8*d*e*f*g+1
0*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*e*(e*g*(-a*e*g-3*b*d*g+4*b*e*f)-c*(3*d^2*
g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*e^2*(-b*e*g-3*c*d*g+5*c*e
*f)*(g*x+f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6-2*(-d*g+e*f)^3*(a*g^2-b
*f*g+c*f^2)*(g*x+f)^(1/2)/g^6
```

Rubi [A]

time = 0.32, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {911, 1167}

$$-\frac{2e(f+gx)^{7/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}+\frac{2(f+gx)^{5/2}(ef-dg)\left(3eg(-aeg-bdg+2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}-\frac{2\sqrt{f+gx}\,(ef-dg)^3\left(ag^2-bfg+cf^2\right)}{g^6}+\frac{2(f+gx)^{3/2}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{3g^6}-\frac{2e^2(f+gx)^{9/2}(-beg-3cdg+5cef)}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*
g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^(3/2))
/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2
- 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^6) - (2*e*(e*g*(4*b*e*f - 3*b
*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(7/2))/(
7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6)
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\mathrm{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx,x,\sqrt{f+g\ldots}\right.}{g}$$

$$= \frac{2\mathrm{Subst}\left(\int\left(\frac{(-ef+dg)^3(cf^2-bfg+ag^2)}{g^5}+\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))x^2}{g^5}+\ldots\right.\right.}{}$$

$$= -\frac{2(ef-dg)^3\,(cf^2-bfg+ag^2)\,\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)^2(cf(5ef-2dg)\ldots}{}$$

Mathematica [A]
time = 0.32, size = 412, normalized size = 1.44

$$\frac{2\sqrt{f+gx}\,(c(231d^3g^3(8f^2-4fgx+3g^2x^2)+297d^2eg^2(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3)+33de^2g(128f^4-64f^3gx+48f^2g^2x^2-40fg^3x^3+35g^4x^4)-5e^3(256f^5-128f^4gx+96f^3g^2x^2-80f^2g^3x^3+70fg^4x^4-63g^5x^5))+11g(9ag(35d^3g^3+35d^2eg^2(-2f+gx)+7de^2g(8f^2-4fgx+3g^2x^2)+e^3(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3))+b(105d^3g^3(-2f+gx)+63d^2eg^2(8f^2-4fgx+3g^2x^2)+27de^2g(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3)+e^3(128f^4-64f^3gx+48f^2g^2x^2-40fg^3x^3+35g^4x^4))))}{3465g^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 297*d^2*e*
g^2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 33*d*e^2*g*(128*f^4 -
 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) - 5*e^3*(256*f^5
- 128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5*x^5
)) + 11*g*(9*a*g*(35*d^3*g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e^2*g*(8*f^2
 - 4*f*g*x + 3*g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^
3)) + b*(105*d^3*g^3*(-2*f + g*x) + 63*d^2*e*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x
^2) + 27*d*e^2*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^3*(128
*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4)))))/(3465*g
^6)
```

Maple [A]
time = 0.08, size = 285, normalized size = 0.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^6*(1/11*c*e^3*(g*x+f)^(11/2)+1/9*(3*(d*g-e*f)*e^2*c+e^3*(b*g-2*c*f))*(g
*x+f)^(9/2)+1/7*(3*(d*g-e*f)^2*e*c+3*(d*g-e*f)*e^2*(b*g-2*c*f)+e^3*(a*g^2-b
*f*g+c*f^2))*(g*x+f)^(7/2)+1/5*((d*g-e*f)^3*c+3*(d*g-e*f)^2*e*(b*g-2*c*f)+3
```

```
*(d*g-e*f)*e^2*(a*g^2-b*f*g+c*f^2))*(g*x+f)^(5/2)+1/3*((d*g-e*f)^3*(b*g-2*c
*f)+3*(d*g-e*f)^2*e*(a*g^2-b*f*g+c*f^2))*(g*x+f)^(3/2)+(d*g-e*f)^3*(a*g^2-b
*f*g+c*f^2)*(g*x+f)^(1/2))
```

Maxima [A]
time = 0.32, size = 411, normalized size = 1.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*f*e^3 - (3*c*d*e^2 + b*e^3)*g
)*(g*x + f)^(9/2) + 495*(10*c*f^2*e^3 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^
2*e + 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(7/2) - 693*(10*c*f^3*e^3 - 6*(3*c*
d*e^2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3
*b*d^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*f^4*e^3 - 4*(3*c*d*e
^2 + b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*b*d^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*(g*x + f)^(3/2) + 3
465*(a*d^3*g^5 - c*f^5*e^3 + (3*c*d*e^2 + b*e^3)*f^4*g - (3*c*d^2*e + 3*b*d
*e^2 + a*e^3)*f^3*g^2 + (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 - (b*d^3 +
3*a*d^2*e)*f*g^4)*sqrt(g*x + f))/g^6
```

Fricas [A]
time = 2.19, size = 458, normalized size = 1.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(693*c*d^3*g^5*x^2 + 1848*c*d^3*f^2*g^3 - 2310*b*d^3*f*g^4 + 3465*a*
d^3*g^5 - 231*(4*c*d^3*f*g^4 - 5*b*d^3*g^5)*x + (315*c*g^5*x^5 - 1280*c*f^5
 + 1408*b*f^4*g - 1584*a*f^3*g^2 - 35*(10*c*f*g^4 - 11*b*g^5)*x^4 + 5*(80*c
*f^2*g^3 - 88*b*f*g^4 + 99*a*g^5)*x^3 - 6*(80*c*f^3*g^2 - 88*b*f^2*g^3 + 99
*a*f*g^4)*x^2 + 8*(80*c*f^4*g - 88*b*f^3*g^2 + 99*a*f^2*g^3)*x)*e^3 + 33*(3
5*c*d*g^5*x^4 + 128*c*d*f^4*g - 144*b*d*f^3*g^2 + 168*a*d*f^2*g^3 - 5*(8*c*
d*f*g^4 - 9*b*d*g^5)*x^3 + 3*(16*c*d*f^2*g^3 - 18*b*d*f*g^4 + 21*a*d*g^5)*x
^2 - 4*(16*c*d*f^3*g^2 - 18*b*d*f^2*g^3 + 21*a*d*f*g^4)*x)*e^2 + 99*(15*c*d
^2*g^5*x^3 - 48*c*d^2*f^3*g^2 + 56*b*d^2*f^2*g^3 - 70*a*d^2*f*g^4 - 3*(6*c*
d^2*f*g^4 - 7*b*d^2*g^5)*x^2 + (24*c*d^2*f^2*g^3 - 28*b*d^2*f*g^4 + 35*a*d^
2*g^5)*x)*e)*sqrt(g*x + f)/g^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(291) = 582$.
time = 73.95, size = 1544, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Piecewise(((-2*a*d**3*f/sqrt(f + g*x) - 2*a*d**3*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 6*a*d**2*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 6*a*d**2*e*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 6*a*d*e**2
*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 6*a
*d*e**2*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) -
(f + g*x)**(5/2)/5)/g**2 - 2*a*e**3*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f
+ g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 2*a*e**3*(f**4/sqr
t(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)
**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*b*d**3*f*(-f/sqrt(f + g*x) - sqrt(
f + g*x))/g - 2*b*d**3*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)*
*(3/2)/3)/g - 6*b*d**2*e*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g
*x)**(3/2)/3)/g**2 - 6*b*d**2*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x)
 + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 6*b*d*e**2*f*(-f**3/sqrt
(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)
/g**3 - 6*b*d*e**2*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f +
 g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*b*e**3
*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4
*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*b*e**3*(-f**5/sqrt(f +
 g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x
)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 - 2*c*d**3*f*(
f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d**
3*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g
*x)**(5/2)/5)/g**2 - 6*c*d**2*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*
x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 6*c*d**2*e*(f**4/sqrt(
f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**
(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 6*c*d*e**2*f*(f**4/sqrt(f + g*x) + 4*f
**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f +
 g*x)**(7/2)/7)/g**4 - 6*c*d*e**2*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*
x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**
(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 - 2*c*e**3*f*(-f**5/sqrt(f + g*x) - 5*f*
*4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5
*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**5 - 2*c*e**3*(f**6/sqrt(f +
g*x) + 6*f**5*sqrt(f + g*x) - 5*f**4*(f + g*x)**(3/2) + 4*f**3*(f + g*x)**(
5/2) - 15*f**2*(f + g*x)**(7/2)/7 + 2*f*(f + g*x)**(9/2)/3 - (f + g*x)**(11
/2)/11)/g**5)/g, Ne(g, 0)), ((a*d**3*x + c*e**3*x**6/6 + x**5*(b*e**3 + 3*c
*d*e**2)/5 + x**4*(a*e**3 + 3*b*d*e**2 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 +
 3*b*d**2*e + c*d**3)/3 + x**2*(3*a*d**2*e + b*d**3)/2)/sqrt(f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(271) = 542.

time = 4.32, size = 565, normalized size = 1.97

$$\frac{2\left(3465\sqrt{gx+f}\,ad^3+\frac{1155\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)bd^3}{g}+\frac{3465\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ad^2e}{g}+\frac{231\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)cd^3}{g^2}+\frac{693\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)bd^2e}{g^2}+\frac{693\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)ade^2}{g^2}+\frac{297\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)cd^2e}{g^3}+\frac{297\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)bde^2}{g^3}+\frac{99\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^2-35\sqrt{gx+f}\,f^3\right)ae^3}{g^3}+\frac{33\left(35(gx+f)^{\frac{9}{2}}-180(gx+f)^{\frac{7}{2}}f+378(gx+f)^{\frac{5}{2}}f^2-420(gx+f)^{\frac{3}{2}}f^3+315\sqrt{gx+f}\,f^4\right)cde^2}{g^4}+\frac{11\left(35(gx+f)^{\frac{9}{2}}-180(gx+f)^{\frac{7}{2}}f+378(gx+f)^{\frac{5}{2}}f^2-420(gx+f)^{\frac{3}{2}}f^3+315\sqrt{gx+f}\,f^4\right)be^3}{g^4}+\frac{5\left(63(gx+f)^{\frac{11}{2}}-385(gx+f)^{\frac{9}{2}}f+990(gx+f)^{\frac{7}{2}}f^2-1386(gx+f)^{\frac{5}{2}}f^3+1155(gx+f)^{\frac{3}{2}}f^4-693\sqrt{gx+f}\,f^5\right)ce^3}{g^5}\right)}{3465\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(g*x + f)*a*d^3 + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*
f)*b*d^3/g + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d^2*e/g + 231*(3*
(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^3/g^2 +
693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d^2
*e/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x +
 f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 297*(5*(g*x + f)^(7/2)
- 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*d
*e^2/g^3 + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2
)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x
 + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqr
t(g*x + f)*f^4)*c*d*e^2/g^4 + 11*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*
f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f
^4)*b*e^3/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x +
 f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*s
qrt(g*x + f)*f^5)*c*e^3/g^5)/g
```

Mupad [B]
time = 0.15, size = 283, normalized size = 0.99

$$\frac{(f+gx)^{9/2}\,(2be^3g-10ce^3f+6cde^2g)}{9g^6}+\frac{(f+gx)^{7/2}\,(6cd^2eg^2-24cde^2fg+6bde^2g^2+20ce^3f^2-8be^3fg+2ae^3g^2)}{7g^6}+\frac{2\,(f+gx)^{5/2}\,(dg-ef)\,(cd^2g^2-8cdefg+3bdeg^2+10ce^2f^2-6be^2fg+3ae^2g^2)}{5g^6}+\frac{2\,\sqrt{f+gx}\,(dg-ef)^3\,(cf^2-bfg+ag^2)}{g^6}+\frac{2\,(f+gx)^{3/2}\,(dg-ef)^2\,(3aeg^2+bdg^2+5cef^2-4befg-2cdfg)}{3g^6}+\frac{2ce^3\,(f+gx)^{11/2}}{11g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(9/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(9*g^6) + ((f + g*
x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g^2 + 6*c*d^
2*e*g^2 - 24*c*d*e^2*f*g))/(7*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(3*a*e^
2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8*c*d*e*f*g)
)/(5*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^3*(a*g^2 + c*f^2 - b*f*g))/g^6 +
 (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^2 + 5*c*e*f^2 - 4*b*e*
f*g - 2*c*d*f*g))/(3*g^6) + (2*c*e^3*(f + g*x)^(11/2))/(11*g^6)
```

3.820 $\int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=212

$$\frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))(f+gx)^{3/2}}{3g^5}-$$

```
[Out] -2/3*(-d*g+e*f)*(2*c*f*(-d*g+2*e*f)-g*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(3/
2)/g^5-2/5*(e*g*(-a*e*g-2*b*d*g+3*b*e*f)-c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(
g*x+f)^(5/2)/g^5-2/7*e*(-b*e*g-2*c*d*g+4*c*e*f)*(g*x+f)^(7/2)/g^5+2/9*c*e^2
*(g*x+f)^(9/2)/g^5+2*(-d*g+e*f)^2*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^5
```

Rubi [A]
time = 0.21, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {911, 1167}

$$-\frac{2(f+gx)^{5/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}\,(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5}-\frac{2(f+gx)^{3/2}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{3g^5}-\frac{2e(f+gx)^{7/2}(-beg-2cdg+4cef)}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^5 - (2*(e*f - d*g
)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^(3/2))/(3
*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^
2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*
x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\text{Subst}\left(\int \left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)dx, x, \sqrt{f+g}\right.}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(-ef+dg)^2\left(cf^2-bfg+ag^2\right)}{g^4}+\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))x^2}{g^4}+\right.\right.}{}$$

$$= \frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^5}-\frac{2(ef-dg)(2cf(2ef-dg)-}{}$$

Mathematica [A]

time = 0.20, size = 256, normalized size = 1.21

$$\frac{2\sqrt{f+gx}\left(c\left(21d^2g^2\left(8f^2-4fgx+3g^2x^2\right)+18deg\left(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3\right)+e^2\left(128f^4-64f^3gx+48f^2g^2x^2-40fg^3x^3+35g^4x^4\right)\right)+3g\left(7ag\left(15d^2g^2+10deg(-2f+gx)+e^2\left(8f^2-4fgx+3g^2x^2\right)\right)+b\left(35d^2g^2(-2f+gx)+14deg\left(8f^2-4fgx+3g^2x^2\right)-3e^2\left(16f^3-8f^2gx+6fg^2x^2-5g^3x^3\right)\right)\right)\right)}{315g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*(21*d^2*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 18*d*e*g*(-
16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^2*(128*f^4 - 64*f^3*g*x +
 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4)) + 3*g*(7*a*g*(15*d^2*g^2 + 10
*d*e*g*(-2*f + g*x) + e^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + b*(35*d^2*g^2*(-
2*f + g*x) + 14*d*e*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) - 3*e^2*(16*f^3 - 8*f^2
*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))))/(315*g^5)
```

Maple [A]

time = 0.10, size = 205, normalized size = 0.97

method	result
derivativedivides	$\frac{\frac{2ce^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec+e^2(bg-2cf)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c+2(dg-ef)e(bg-2cf)+e^2\left(ag^2-bfg+cf^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((d\right.}{}}{g^5}$
default	$\frac{\frac{2ce^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2(dg-ef)ec+e^2(bg-2cf)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c+2(dg-ef)e(bg-2cf)+e^2\left(ag^2-bfg+cf^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((d\right.}{}}{g^5}$
gosper	$\frac{2\sqrt{gx+f}\left(35ce^2x^4g^4+45be^2g^4x^3+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+126bdeg^4x^2-54be^2fg^3x^2+63cd^2g^4x\right.}{}$
trager	$\frac{2\sqrt{gx+f}\left(35ce^2x^4g^4+45be^2g^4x^3+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+126bdeg^4x^2-54be^2fg^3x^2+63cd^2g^4x\right.}{}$
risch	$\frac{2\sqrt{gx+f}\left(35ce^2x^4g^4+45be^2g^4x^3+90cdeg^4x^3-40ce^2fg^3x^3+63ae^2g^4x^2+126bdeg^4x^2-54be^2fg^3x^2+63cd^2g^4x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^5*(1/9*c*e^2*(g*x+f)^(9/2)+1/7*(2*(d*g-e*f)*e*c+e^2*(b*g-2*c*f))*(g*x+f
)^(7/2)+1/5*((d*g-e*f)^2*c+2*(d*g-e*f)*e*(b*g-2*c*f)+e^2*(a*g^2-b*f*g+c*f^2
))*(g*x+f)^(5/2)+1/3*((d*g-e*f)^2*(b*g-2*c*f)+2*(d*g-e*f)*e*(a*g^2-b*f*g+c*
f^2))*(g*x+f)^(3/2)+(d*g-e*f)^2*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2))
```

Maxima [A]
time = 0.31, size = 258, normalized size = 1.22

$$\frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^2-45\,(4\,cfe^2-(2\,cde+be^2)g)(gx+f)^{\frac{7}{2}}+63\,(6\,cf^2e^2-3\,(2\,cde+be^2)fg+(cd^2+2\,bde+ae^2)g^2)(gx+f)^{\frac{5}{2}}-105\,(4\,cf^3e^2-3\,(2\,cde+be^2)f^2g+2\,(cd^2+2\,bde+ae^2)fg^2-(bd^2+2\,ade)g^3)(gx+f)^{\frac{3}{2}}+315\,(ad^2g^4+cf^4e^2-(2\,cde+be^2)f^3g+(cd^2+2\,bde+ae^2)f^2g^2-(bd^2+2\,ade)fg^3)\sqrt{gx+f}\right)}{315\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(g*x + f)^(9/2)*c*e^2 - 45*(4*c*f*e^2 - (2*c*d*e + b*e^2)*g)*(g*x
 + f)^(7/2) + 63*(6*c*f^2*e^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
+ a*e^2)*g^2)*(g*x + f)^(5/2) - 105*(4*c*f^3*e^2 - 3*(2*c*d*e + b*e^2)*f^2*
g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*(g*x + f)^(3
/2) + 315*(a*d^2*g^4 + c*f^4*e^2 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2*b*d
*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)*sqrt(g*x + f))/g^5
```

Fricas [A]
time = 2.78, size = 278, normalized size = 1.31

$$\frac{2\,(63\,cd^2g^4x^2+168\,cd^2f^2g^2-210\,bd^2fg^3+315\,ad^2g^4-21\,(4\,cd^2fg^3-5\,bd^2g^4)x+(35\,cg^4x^4+128\,cf^4-144\,bf^3g+168\,af^2g^2-5\,(8\,cfg^3-9\,bg^4)x^3+3\,(16\,cf^2g^2-18\,bfg^3+21\,ag^4)x^2-4\,(16\,cf^3g-18\,bf^2g^2+21\,afg^3)x)e^2+6\,(15\,cdg^4x^3-48\,cdf^3g+56\,bdf^2g^2-70\,adfg^3-3\,(6\,cdfg^3-7\,bdg^4)x^2+(24\,cdf^2g^2-28\,bdfg^3+35\,adg^4)x)e)\sqrt{gx+f}}{315\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(63*c*d^2*g^4*x^2 + 168*c*d^2*f^2*g^2 - 210*b*d^2*f*g^3 + 315*a*d^2*g
^4 - 21*(4*c*d^2*f*g^3 - 5*b*d^2*g^4)*x + (35*c*g^4*x^4 + 128*c*f^4 - 144*b
*f^3*g + 168*a*f^2*g^2 - 5*(8*c*f*g^3 - 9*b*g^4)*x^3 + 3*(16*c*f^2*g^2 - 18
*b*f*g^3 + 21*a*g^4)*x^2 - 4*(16*c*f^3*g - 18*b*f^2*g^2 + 21*a*f*g^3)*x)*e^
2 + 6*(15*c*d*g^4*x^3 - 48*c*d*f^3*g + 56*b*d*f^2*g^2 - 70*a*d*f*g^3 - 3*(6
*c*d*f*g^3 - 7*b*d*g^4)*x^2 + (24*c*d*f^2*g^2 - 28*b*d*f*g^3 + 35*a*d*g^4)*
x)*e)*sqrt(g*x + f)/g^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(218) = 436$.
time = 46.65, size = 1001, normalized size = 4.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d**2*f/sqrt(f + g*x) - 2*a*d**2*(-f/sqrt(f + g*x) - sqrt(f
 + g*x)) - 4*a*d*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 4*a*d*e*(f**2/s
qrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*a*e**2*f*(f**2
/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*a*e**2*(-
f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)*
*(5/2)/5)/g**2 - 2*b*d**2*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*b*d**2
*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 4*b*d*e*
f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 4*b*
d*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
 g*x)**(5/2)/5)/g**2 - 2*b*e**2*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*
x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 - 2*b*e**2*(f**4/sqrt(f
+ g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5
/2)/5 - (f + g*x)**(7/2)/7)/g**3 - 2*c*d**2*f*(f**2/sqrt(f + g*x) + 2*f*sqr
t(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d**2*(-f**3/sqrt(f + g*x) - 3*f
**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 4*c*d*e
*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
g*x)**(5/2)/5)/g**3 - 4*c*d*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) -
2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3
 - 2*c*e**2*f*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)
**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**4 - 2*c*e**2*(-f*
*5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f*
*2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4)/g,
 Ne(g, 0)), ((a*d**2*x + c*e**2*x**5/5 + x**4*(b*e**2 + 2*c*d*e)/4 + x**3*(
a*e**2 + 2*b*d*e + c*d**2)/3 + x**2*(2*a*d*e + b*d**2)/2)/sqrt(f), True))
```

Giac [A]
time = 3.04, size = 363, normalized size = 1.71

$$\frac{2\left(315\sqrt{gx+f}\,ad^{2}+\frac{105\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)bd^{2}}{g}+\frac{210\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)ade}{g}+\frac{21\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^{2}\right)cd^{2}}{g^{2}}+\frac{42\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^{2}\right)bde}{g^{2}}+\frac{21\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^{2}\right)ae^{2}}{g^{2}}+\frac{18\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^{2}-35\sqrt{gx+f}\,f^{3}\right)cde}{g^{3}}+\frac{9\left(5(gx+f)^{\frac{7}{2}}-21(gx+f)^{\frac{5}{2}}f+35(gx+f)^{\frac{3}{2}}f^{2}-35\sqrt{gx+f}\,f^{3}\right)be^{2}}{g^{3}}+\frac{\left(35(gx+f)^{\frac{9}{2}}-180(gx+f)^{\frac{7}{2}}f+378(gx+f)^{\frac{5}{2}}f^{2}-420(gx+f)^{\frac{3}{2}}f^{3}+315\sqrt{gx+f}\,f^{4}\right)ce^{2}}{g^{4}}\right)}{315\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(g*x + f)*a*d^2 + 105*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*
b*d^2/g + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d*e/g + 21*(3*(g*x +
f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^2/g^2 + 42*(3*(
g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*d*e/g^2 + 2
1*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*e^2/g
^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2
- 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + 9*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/
2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*b*e^2/g^3 + (35*(g*x
+ f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f
)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

Mupad [B]
time = 3.17, size = 204, normalized size = 0.96

$$\frac{(f+g\,x)^{7/2}\,(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g)}{7\,g^5}+\frac{(f+g\,x)^{5/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+2\,a\,e^2\,g^2)}{5\,g^5}+\frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,(2\,a\,e\,g^2+b\,d\,g^2+4\,c\,e\,f^2-3\,b\,e\,f\,g-2\,c\,d\,f\,g)}{3\,g^5}+\frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^2\,(c\,f^2-b\,f\,g+a\,g^2)}{g^5}+\frac{2\,c\,e^2\,(f+g\,x)^{9/2}}{9\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(7*g^5) + ((f + g*x)^
(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d*e*g^2 - 6*b*e^2*f*g
 - 12*c*d*e*f*g))/(5*g^5) + (2*(f + g*x)^(3/2)*(d*g - e*f)*(2*a*e*g^2 + b*d
*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*
g - e*f)^2*(a*g^2 + c*f^2 - b*f*g))/g^5 + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

3.821 $\int \frac{(d+ex)(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=137

$$-\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))(f+gx)^{3/2}}{3g^4}-\frac{2(3cef-$$

```
[Out] 2/3*(c*f*(-2*d*g+3*e*f)-g*(-a*e*g-b*d*g+2*b*e*f))*(g*x+f)^(3/2)/g^4-2/5*(-b
*e*g-c*d*g+3*c*e*f)*(g*x+f)^(5/2)/g^4+2/7*c*e*(g*x+f)^(7/2)/g^4-2*(-d*g+e*f
)*(a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2)/g^4
```

Rubi [A]
time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$,
Rules used = {785}

$$-\frac{2\sqrt{f+gx}\,(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4}+\frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4}-\frac{2(f+gx)^{5/2}(-beg-cdg+3cef)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(c*f*(3*e*f
 - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^(3/2))/(3*g^4) - (2*(3*c
*e*f - c*d*g - b*e*g)*(f + g*x)^(5/2))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7
*g^4)
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)(a+bx+cx^2)}{\sqrt{f+gx}}\,dx &= \int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3\sqrt{f+gx}}+\frac{(cf(3ef-2dg)-g(2bef-bdg-a\right.\\ &= -\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}{g^4}+\frac{2(cf(3ef-2dg)-g(2bef-}{3g^4}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 131, normalized size = 0.96

$$\frac{2\sqrt{f+gx}\,(7g(5bdg(-2f+gx)+5ag(-2ef+3dg+egx)+be(8f^2-4fgx+3g^2x^2))+c(7dg(8f^2-4fgx+3g^2x^2)-3e(16f^3-8f^2gx+6fg^2x^2-5g^3x^3)))}{105g^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(7*g*(5*b*d*g*(-2*f + g*x) + 5*a*g*(-2*e*f + 3*d*g + e*g*x
) + b*e*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + c*(7*d*g*(8*f^2 - 4*f*g*x + 3*g^2*
x^2) - 3*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3))))/(105*g^4)
```

Maple [A]
time = 0.08, size = 125, normalized size = 0.91

method	result
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c+e(bg-2cf))(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((dg-ef)(bg-2cf)+e\left(a\,g^2-bfg+c\,f^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^2-bfg+c\,f^2\right.}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c+e(bg-2cf))(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left((dg-ef)(bg-2cf)+e\left(a\,g^2-bfg+c\,f^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(a\,g^2-bfg+c\,f^2\right.}{g^4}$
gosper	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21be\,g^3x^2+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x+35bd\,g^3x-28bef\,g^2x-28cdf\,g^2x+24ce\,f^2gx+1}{105g^4}$
trager	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21be\,g^3x^2+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x+35bd\,g^3x-28bef\,g^2x-28cdf\,g^2x+24ce\,f^2gx+1}{105g^4}$
risch	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21be\,g^3x^2+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x+35bd\,g^3x-28bef\,g^2x-28cdf\,g^2x+24ce\,f^2gx+1}{105g^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^4*(1/7*c*e*(g*x+f)^(7/2)+1/5*((d*g-e*f)*c+e*(b*g-2*c*f))*(g*x+f)^(5/2)+
1/3*((d*g-e*f)*(b*g-2*c*f)+e*(a*g^2-b*f*g+c*f^2))*(g*x+f)^(3/2)+(d*g-e*f)*(
a*g^2-b*f*g+c*f^2)*(g*x+f)^(1/2))
```

Maxima [A]
time = 0.28, size = 138, normalized size = 1.01

$$\frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce-21\,(3\,cfe-(cd+be)g)(gx+f)^{\frac{5}{2}}+35\,(3\,cf^2e-2\,(cd+be)fg+(bd+ae)g^2)(gx+f)^{\frac{3}{2}}+105\,(adg^3-cf^3e+(cd+be)f^2g-(bd+ae)fg^2)\sqrt{gx+f}\right)}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*f*e - (c*d + b*e)*g)*(g*x + f)^(5/2
) + 35*(3*c*f^2*e - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*(g*x + f)^(3/2) +
105*(a*d*g^3 - c*f^3*e + (c*d + b*e)*f^2*g - (b*d + a*e)*f*g^2)*sqrt(g*x +
f))/g^4
```

Fricas [A]
time = 8.78, size = 140, normalized size = 1.02

$$\frac{2\,(21\,cdg^3x^2+56\,cdf^2g-70\,bdfg^2+105\,adg^3-7\,(4\,cdfg^2-5\,bdg^3)x+(15\,cg^3x^3-48\,cf^3+56\,bf^2g-70\,afg^2-3\,(6\,cfg^2-7\,bg^3)x^2+(24\,cf^2g-28\,bfg^2+35\,ag^3)x)e)\sqrt{gx+f}}{105\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(21*c*d*g^3*x^2 + 56*c*d*f^2*g - 70*b*d*f*g^2 + 105*a*d*g^3 - 7*(4*c*
d*f*g^2 - 5*b*d*g^3)*x + (15*c*g^3*x^3 - 48*c*f^3 + 56*b*f^2*g - 70*a*f*g^2
 - 3*(6*c*f*g^2 - 7*b*g^3)*x^2 + (24*c*f^2*g - 28*b*f*g^2 + 35*a*g^3)*x)*e)
*sqrt(g*x + f)/g^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(143) = 286$.
time = 25.05, size = 549, normalized size = 4.01

$$\begin{cases} \frac{-\frac{2adf}{\sqrt{f+gx}} - 2ad\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2aef\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2ae\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2bdf\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2bd\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2bef\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2be\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2} - \frac{2cdf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2cd\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2} - \frac{2cef\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^3} - \frac{2ce\left(\frac{f^4}{\sqrt{f+gx}} + 4f^3\sqrt{f+gx} - 2f^2(f+gx)^{\frac{3}{2}} + \frac{4f(f+gx)^{\frac{5}{2}}}{5} - \frac{(f+gx)^{\frac{7}{2}}}{7}\right)}{g^3}}{g} & \text{for}\ g \neq 0 \\ \frac{adx + \frac{cex^4}{4} + \frac{x^3(be+cd)}{3} + \frac{x^2(ae+bd)}{2}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*d*f/sqrt(f + g*x) - 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x
)) - 2*a*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*a*e*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*b*d*f*(-f/sqrt(f + g*x)
- sqrt(f + g*x))/g - 2*b*d*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g
*x)**(3/2)/3)/g - 2*b*e*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*
x)**(3/2)/3)/g**2 - 2*b*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(
f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 2*c*d*f*(f**2/sqrt(f + g*x) +
2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*d*(-f**3/sqrt(f + g*x) -
 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 - 2*c
*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f
+ g*x)**(5/2)/5)/g**3 - 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) -
2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3
)/g, Ne(g, 0)), ((a*d*x + c*e*x**4/4 + x**3*(b*e + c*d)/3 + x**2*(a*e + b*d
)/2)/sqrt(f), True))
```

Giac [A]
time = 4.35, size = 199, normalized size = 1.45

$$\frac{2\left(105\sqrt{gx+f}\,ad + \frac{35\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}\,f\right)bd}{g} + \frac{35\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}\,f\right)ae}{g} + \frac{7\left(3(gx+f)^{\frac{5}{2}} - 10(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}\,f^2\right)cd}{g^2} + \frac{7\left(3(gx+f)^{\frac{5}{2}} - 10(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}\,f^2\right)be}{g^2} + \frac{3\left(5(gx+f)^{\frac{7}{2}} - 21(gx+f)^{\frac{5}{2}}f + 35(gx+f)^{\frac{3}{2}}f^2 - 35\sqrt{gx+f}\,f^3\right)ce}{g^3}\right)}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b*d
/g + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e/g + 7*(3*(g*x + f)^(5/2)
- 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d/g^2 + 7*(3*(g*x + f)^(5/
2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*b*e/g^2 + 3*(5*(g*x + f)^
```

```
(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^
3)*c*e/g^3)/g
```

Mupad [B]
time = 0.08, size = 125, normalized size = 0.91

$$\frac{(f+gx)^{5/2}(2beg+2cdg-6cef)}{5g^4}+\frac{(f+gx)^{3/2}(2aeg^2+2bdg^2+6cef^2-4befg-4cdfg)}{3g^4}+\frac{2\sqrt{f+gx}\,(dg-ef)\,(cf^2-bfg+ag^2)}{g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] ((f + g*x)^(5/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(5*g^4) + ((f + g*x)^(3/2)*
(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g - 4*c*d*f*g))/(3*g^4) + (2*(
f + g*x)^(1/2)*(d*g - e*f)*(a*g^2 + c*f^2 - b*f*g))/g^4 + (2*c*e*(f + g*x)^
(7/2))/(7*g^4)
```

3.822 $\int \frac{a+bx+cx^2}{\sqrt{f+gx}} \, dx$

Optimal. Leaf size=73

$$\frac{2(cf^2-bfg+ag^2)\sqrt{f+gx}}{g^3}-\frac{2(2cf-bg)(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

```
[Out] -2/3*(-b*g+2*c*f)*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^(5/2)/g^3+2*(a*g^2-b*f*g+
c*f^2)*(g*x+f)^(1/2)/g^3
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {712}

$$\frac{2\sqrt{f+gx}\,(ag^2-bfg+cf^2)}{g^3}-\frac{2(f+gx)^{3/2}(2cf-bg)}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)^
(3/2))/(3*g^3) + (2*c*(f + g*x)^(5/2))/(5*g^3)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{f+gx}}\,dx &= \int \left(\frac{cf^2-bfg+ag^2}{g^2\sqrt{f+gx}}+\frac{(-2cf+bg)\sqrt{f+gx}}{g^2}+\frac{c(f+gx)^{3/2}}{g^2}\right)dx\\ &= \frac{2(cf^2-bfg+ag^2)\sqrt{f+gx}}{g^3}-\frac{2(2cf-bg)(f+gx)^{3/2}}{3g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 0.74

$$\frac{2\sqrt{f+gx}\,(5g(-2bf+3ag+bgx)+c(8f^2-4fgx+3g^2x^2))}{15g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(5*g*(-2*b*f + 3*a*g + b*g*x) + c*(8*f^2 - 4*f*g*x + 3*g^2
*x^2)))/(15*g^3)
```

Maple [A]
time = 0.08, size = 75, normalized size = 1.03

method	result	size
gosper	$\frac{2\sqrt{gx+f}\,(3c\,x^2g^2+5b\,g^2x-4cfgx+15a\,g^2-10bfg+8c\,f^2)}{15g^3}$	53
trager	$\frac{2\sqrt{gx+f}\,(3c\,x^2g^2+5b\,g^2x-4cfgx+15a\,g^2-10bfg+8c\,f^2)}{15g^3}$	53
risch	$\frac{2\sqrt{gx+f}\,(3c\,x^2g^2+5b\,g^2x-4cfgx+15a\,g^2-10bfg+8c\,f^2)}{15g^3}$	53
derivativedivides	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}+\frac{2bg(gx+f)^{\frac{3}{2}}}{3}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^2\sqrt{gx+f}-2bfg\sqrt{gx+f}+2c\,f^2\sqrt{gx+f}}{g^3}$	75
default	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}+\frac{2bg(gx+f)^{\frac{3}{2}}}{3}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^2\sqrt{gx+f}-2bfg\sqrt{gx+f}+2c\,f^2\sqrt{gx+f}}{g^3}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^3*(1/5*c*(g*x+f)^(5/2)+1/3*b*g*(g*x+f)^(3/2)-2/3*c*f*(g*x+f)^(3/2)+a*g^
2*(g*x+f)^(1/2)-b*f*g*(g*x+f)^(1/2)+c*f^2*(g*x+f)^(1/2))
```

Maxima [A]
time = 0.28, size = 77, normalized size = 1.05

$$\frac{2\left(15\sqrt{gx+f}\,a+\frac{5\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}\,f\right)b}{g}+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

Fricas [A]
time = 2.45, size = 54, normalized size = 0.74

$$\frac{2\,(3\,cg^2x^2+8\,cf^2-10\,bfg+15\,ag^2-(4\,cfg-5\,bg^2)x)\sqrt{gx+f}}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*g^2*x^2 + 8*c*f^2 - 10*b*f*g + 15*a*g^2 - (4*c*f*g - 5*b*g^2)*x)*
sqrt(g*x + f)/g^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(73) = 146$.

time = 4.76, size = 223, normalized size = 3.05

$$\begin{cases} \frac{-\frac{2af}{\sqrt{f+gx}} - 2a\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) - \frac{2bf\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} - \frac{2b\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} - \frac{2cf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} - \frac{2c\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g \neq 0 \\ \frac{ax + \frac{bx^2}{2} + \frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*f/sqrt(f + g*x) - 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) -
 2*b*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g - 2*b*(f**2/sqrt(f + g*x) + 2*f
*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g - 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sq
rt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 - 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*
sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0))
, ((a*x + b*x**2/2 + c*x**3/3)/sqrt(f), True))
```

Giac [A]

time = 3.96, size = 77, normalized size = 1.05

$$\frac{2\left(15\sqrt{gx+f}\,a + \frac{5\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}\,f\right)b}{g} + \frac{\left(3(gx+f)^{\frac{5}{2}} - 10(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}\,f^2\right)c}{g^2}\right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + 5*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*b/g + (3
*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c/g^2)/g
```

Mupad [B]

time = 3.12, size = 58, normalized size = 0.79

$$\frac{2\sqrt{f+g\,x}\,\left(3\,c\,{(f+g\,x)}^2+15\,a\,g^2+15\,c\,f^2+5\,b\,g\,(f+g\,x)-10\,c\,f\,(f+g\,x)-15\,b\,f\,g\right)}{15\,g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(f + g*x)^(1/2),x)
```

```
[Out] (2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 + 5*b*g*(f + g*x)
 - 10*c*f*(f + g*x) - 15*b*f*g))/(15*g^3)
```

3.823 $\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=116

$$\frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2(cd^2-bde+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2-b*d*e+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/
2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1/2)+2*(b*e*g-c*(d*g+e*f))*(g*x+f)
^(1/2)/e^2/g^2
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {911, 1167, 214}

$$-\frac{2(ae^2-bde+cd^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2\sqrt{f+gx}\,(beg-c(dg+ef))}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*(b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))
/(3*e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqr
t[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\mathrm{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2\mathrm{Subst}\left(\int \left(\frac{beg-c(ef+dg)}{e^2g}+\frac{cx^2}{eg}+\frac{cd^2-bde+ae^2}{e^2\left(d-\frac{ef}{g}+\frac{ex^2}{g}\right)}\right)dx,x,\sqrt{f+gx}\right)}{g}$$

$$= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}+\frac{(2(cd^2-bde+ae^2))\,\mathrm{Subst}\big(}{e}$$

$$= \frac{2(beg-c(ef+dg))\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2}-\frac{2(cd^2-bde+ae^2)\tanh^{-1}\big(}{e^{5/2}\sqrt{ef-dg}}$$

Mathematica [A]
time = 0.16, size = 104, normalized size = 0.90

$$\frac{2\sqrt{f+gx}\,(3beg+c(-2ef-3dg+egx))}{3e^2g^2}+\frac{2(cd^2+e(-bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}\sqrt{-ef+dg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(3*b*e*g + c*(-2*e*f - 3*d*g + e*g*x)))/(3*e^2*g^2) + (2*(
c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]
])/(e^(5/2)*Sqrt[-(e*f) + d*g])
```

Maple [A]
time = 0.11, size = 115, normalized size = 0.99

method	result

derivativedivides	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{3}{2}}e}{3}+beg\sqrt{gx+f}-cdg\sqrt{gx+f}-cef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(a\,e^2-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{e^2\sqrt{(dg-ef)\,e}}}{g^2}$
default	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{3}{2}}e}{3}+beg\sqrt{gx+f}-cdg\sqrt{gx+f}-cef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(a\,e^2-bde+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{e^2\sqrt{(dg-ef)\,e}}}{g^2}$
risch	$\frac{2(cegx+3beg-3dgc-2cef)\sqrt{gx+f}}{3g^2e^2}+\frac{2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)a}{\sqrt{(dg-ef)\,e}}-\frac{2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)bd}{e\sqrt{(dg-ef)\,e}}$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^2*(1/e^2*(1/3*c*(g*x+f)^(3/2)*e+b*e*g*(g*x+f)^(1/2)-c*d*g*(g*x+f)^(1/2)
-c*e*f*(g*x+f)^(1/2))+g^2*(a*e^2-b*d*e+c*d^2)/e^2/((d*g-e*f)*e)^(1/2)*arcta
n(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [A]
time = 1.42, size = 334, normalized size = 2.88

$$\left[-\frac{3\,(cd^2g^2-bdg^2e+ag^2e^2)\sqrt{-dge+fe^2}\log\left(-\frac{dg-(gx+2\,f)e+2\,\sqrt{-dge+fe^2}\,\sqrt{gx+f}}{xe+d}\right)+2\,(3\,cd^2g^2e+(cfgx-2\,cf^2+3\,bfg)e^3-(cdg^2x+cdfg+3\,bdg^2)e^2)\sqrt{gx+f}}{3\,(dg^3e^3-fg^2e^4)},-\frac{2\left(3\,(cd^2g^2-bdg^2e+ag^2e^2)\sqrt{dge-fe^2}\arctan\left(-\frac{\sqrt{dge-fe^2}\,\sqrt{gx+f}}{dg-fe}\right)+(3\,cd^2g^2e+(cfgx-2\,cf^2+3\,bfg)e^3-(cdg^2x+cdfg+3\,bdg^2)e^2)\sqrt{gx+f}\right)}{3\,(dg^3e^3-fg^2e^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(c*d^2*g^2 - b*d*g^2*e + a*g^2*e^2)*sqrt(-d*g*e + f*e^2)*log(-(d*g
 - (g*x + 2*f)*e + 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e + d)) + 2*(3*
c*d^2*g^2*e + (c*f*g*x - 2*c*f^2 + 3*b*f*g)*e^3 - (c*d*g^2*x + c*d*f*g + 3*
```

```
b*d*g^2)*e^2)*sqrt(g*x + f))/(d*g^3*e^3 - f*g^2*e^4), -2/3*(3*(c*d^2*g^2 -
b*d*g^2*e + a*g^2*e^2)*sqrt(d*g*e - f*e^2)*arctan(-sqrt(d*g*e - f*e^2)*sqrt
(g*x + f)/(d*g - f*e)) + (3*c*d^2*g^2*e + (c*f*g*x - 2*c*f^2 + 3*b*f*g)*e^3
 - (c*d*g^2*x + c*d*f*g + 3*b*d*g^2)*e^2)*sqrt(g*x + f))/(d*g^3*e^3 - f*g^2
*e^4)]
```

Sympy [A]
time = 14.42, size = 112, normalized size = 0.97

$$\frac{2c(f+gx)^{\frac{3}{2}}}{3eg^2}-\frac{2(ae^2-bde+cd^2)\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{e^2\sqrt{\frac{e}{dg-ef}}\,(dg-ef)}+\frac{2\sqrt{f+gx}\,(beg-cdg-cef)}{e^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*e*g**2) - 2*(a*e**2 - b*d*e + c*d**2)*atan(1/(sqrt(
e/(d*g - e*f))*sqrt(f + g*x)))/(e**2*sqrt(e/(d*g - e*f))*(d*g - e*f)) + 2*s
qrt(f + g*x)*(b*e*g - c*d*g - c*e*f)/(e**2*g**2)
```

Giac [A]
time = 5.44, size = 128, normalized size = 1.10

$$\frac{2\,(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)e^{(-2)}}{\sqrt{dge-fe^2}}-\frac{2\left(3\,\sqrt{gx+f}\,cdg^5e-(gx+f)^{\frac{3}{2}}cg^4e^2+3\,\sqrt{gx+f}\,cfg^4e^2-3\,\sqrt{gx+f}\,bg^5e^2\right)e^{(-3)}}{3\,g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))*e^(-2
)/sqrt(d*g*e - f*e^2) - 2/3*(3*sqrt(g*x + f)*c*d*g^5*e - (g*x + f)^(3/2)*c*
g^4*e^2 + 3*sqrt(g*x + f)*c*f*g^4*e^2 - 3*sqrt(g*x + f)*b*g^5*e^2)*e^(-3)/g
^6
```

Mupad [B]
time = 0.14, size = 117, normalized size = 1.01

$$\sqrt{f+g\,x}\,\left(\frac{2\,b\,g-4\,c\,f}{e\,g^2}-\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}\right)+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}}+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

```
[Out] (f + g*x)^(1/2)*((2*b*g - 4*c*f)/(e*g^2) - (2*c*(d*g^3 - e*f*g^2))/(e^2*g^4
)) + (2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2 -
b*d*e))/(e^(5/2)*(d*g - e*f)^(1/2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

3.824 $\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=140

$$\frac{2c\sqrt{f+gx}}{e^2g}-\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)}+\frac{(cd(4ef-3dg)-e(2bef-bdg-aeg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

```
[Out] (c*d*(-3*d*g+4*e*f)-e*(-a*e*g-b*d*g+2*b*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)
/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(3/2)+2*c*(g*x+f)^(1/2)/e^2/g-(a+d*(-
b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)
```

Rubi [A]
time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {911, 1171, 396, 214}

$$-\frac{\sqrt{f+gx}\,(ae^2-bde+cd^2)}{e^2(d+ex)(ef-dg)}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}}+\frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (2*c*Sqrt[f + g*x])/(e^2*g) - ((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x])/(e^2*
(e*f - d*g)*(d + e*x)) + ((c*d*(4*e*f - 3*d*g) - e*(2*b*e*f - b*d*g - a*e*g
))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*(e*f - d*g)^(
3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx,x,\sqrt{f+gx}\right)}{g}$$

$$= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{\text{Subst}\left(\int \frac{-a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{2cf^2}{g^2}+\frac{2bf}{g}+\frac{2c(ef-dg)x^2}{eg^2}}{\frac{-ef+dg}{g}+\frac{ex^2}{g}}\,dx,x,\sqrt{\right.}}{ef-dg}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} - \frac{(cd(4ef-3dg)-e(2bef-bdg-}{e}$$

$$= \frac{2c\sqrt{f+gx}}{e^2g} - \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)(d+ex)} + \frac{(cd(4ef-3dg)-e(2bef-bdg-}{e^{5/2}(ef-}$$

Mathematica [A]
time = 0.48, size = 150, normalized size = 1.07

$$\frac{\sqrt{f+gx}\,(e(bd-ae)g+c(-3d^2g+2e^2fx+2de(f-gx)))}{e^2g(ef-dg)(d+ex)} - \frac{(cd(-4ef+3dg)+e(2bef-bdg-aeg))\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}(-ef+dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

[Out] (Sqrt[f + g*x]*(e*(b*d - a*e)*g + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x)
)))/(e^2*g*(e*f - d*g)*(d + e*x)) - ((c*d*(-4*e*f + 3*d*g) + e*(2*b*e*f - b
*d*g - a*e*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)
*(-(e*f) + d*g)^(3/2))

Maple [A]
time = 0.13, size = 156, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(a\,e^2-bde+c\,d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2g+bdeg-2b\,e^2f-3c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{2(dg-ef)\sqrt{(dg-ef)\,e}}\right)}{e^2}}{g}$
default	$\frac{\frac{2c\sqrt{gx+f}}{e^2}+\frac{2g\left(\frac{g\left(a\,e^2-bde+c\,d^2\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^2g+bdeg-2b\,e^2f-3c\,d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{2(dg-ef)\sqrt{(dg-ef)\,e}}\right)}{e^2}}{g}$
risch	$\frac{2c\sqrt{gx+f}}{e^2g}+\frac{g\sqrt{gx+f}\,a}{(dg-ef)(egx+dg)}-\frac{g\sqrt{gx+f}\,bd}{e(dg-ef)(egx+dg)}+\frac{g\sqrt{gx+f}\,c\,d^2}{e^2(dg-ef)(egx+dg)}+\frac{\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{(dg-ef)\sqrt{(dg-ef)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/g*(c/e^2*(g*x+f)^(1/2)+g/e^2*(1/2*g*(a*e^2-b*d*e+c*d^2)/(d*g-e*f)*(g*x+f)
^(1/2)/(e*(g*x+f)+d*g-e*f)+1/2*(a*e^2*g+b*d*e*g-2*b*e^2*f-3*c*d^2*g+4*c*d*e
*f)/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2
))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(129) = 258$.
time = 1.11, size = 617, normalized size = 4.41

$$\left[-\frac{\left(3\,cd^3g^2+(2\,bfg-ag^2)xe^3+(2\,bdfg-adg^2-(4\,cdfg+bdg^2)x)e^2+(3\,cd^2g^2x-4\,cd^2fg-bd^2g^2)e\right)\sqrt{-dge+fe^2}\,\log\left(-\frac{dg-(gx+2\,f)e-2\,\sqrt{-dge+fe^2}\,\sqrt{gx+f}}{xe+d}\right)-2\,\left(3\,cd^3g^2e+(2\,cf^2x-afg)e^4-(4\,cdfgx-2\,cdf^2-bdfg-adg^2)e^3+(2\,cd^2g^2x-5\,cd^2fg-bd^2g^2)e^2\right)\sqrt{gx+f}}{2\,\left(d^3g^3e^3+f^2gxe^6-(2\,dfg^2x-df^2g)e^5+(d^2g^3x-2\,d^2fg^2)e^4\right)},\ \frac{\left(3\,cd^3g^2+(2\,bfg-ag^2)xe^3+(2\,bdfg-adg^2-(4\,cdfg+bdg^2)x)e^2+(3\,cd^2g^2x-4\,cd^2fg-bd^2g^2)e\right)\sqrt{dge-fe^2}\,\arctan\left(-\frac{\sqrt{dge-fe^2}\,\sqrt{gx+f}}{dg-fe}\right)+\left(3\,cd^3g^2e+(2\,cf^2x-afg)e^4-(4\,cdfgx-2\,cdf^2-bdfg-adg^2)e^3+(2\,cd^2g^2x-5\,cd^2fg-bd^2g^2)e^2\right)\sqrt{gx+f}}{d^3g^3e^3+f^2gxe^6-(2\,dfg^2x-df^2g)e^5+(d^2g^3x-2\,d^2fg^2)e^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*c*d^3*g^2 + (2*b*f*g - a*g^2)*x*e^3 + (2*b*d*f*g - a*d*g^2 - (4*c
*d*f*g + b*d*g^2)*x)*e^2 + (3*c*d^2*g^2*x - 4*c*d^2*f*g - b*d^2*g^2)*e)*sqr
t(-d*g*e + f*e^2)*log(-(d*g - (g*x + 2*f)*e - 2*sqrt(-d*g*e + f*e^2)*sqrt(g
*x + f))/(x*e + d)) - 2*(3*c*d^3*g^2*e + (2*c*f^2*x - a*f*g)*e^4 - (4*c*d*f
*g*x - 2*c*d*f^2 - b*d*f*g - a*d*g^2)*e^3 + (2*c*d^2*g^2*x - 5*c*d^2*f*g -
b*d^2*g^2)*e^2)*sqrt(g*x + f))/(d^3*g^3*e^3 + f^2*g*x*e^6 - (2*d*f*g^2*x -
d*f^2*g)*e^5 + (d^2*g^3*x - 2*d^2*f*g^2)*e^4), ((3*c*d^3*g^2 + (2*b*f*g - a
*g^2)*x*e^3 + (2*b*d*f*g - a*d*g^2 - (4*c*d*f*g + b*d*g^2)*x)*e^2 + (3*c*d^
2*g^2*x - 4*c*d^2*f*g - b*d^2*g^2)*e)*sqrt(d*g*e - f*e^2)*arctan(-sqrt(d*g*
e - f*e^2)*sqrt(g*x + f)/(d*g - f*e)) + (3*c*d^3*g^2*e + (2*c*f^2*x - a*f*g
)*e^4 - (4*c*d*f*g*x - 2*c*d*f^2 - b*d*f*g - a*d*g^2)*e^3 + (2*c*d^2*g^2*x
- 5*c*d^2*f*g - b*d^2*g^2)*e^2)*sqrt(g*x + f))/(d^3*g^3*e^3 + f^2*g*x*e^6 -
 (2*d*f*g^2*x - d*f^2*g)*e^5 + (d^2*g^3*x - 2*d^2*f*g^2)*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{(d+ex)^2\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Giac [A]
time = 5.11, size = 175, normalized size = 1.25

$$\frac{2\,\sqrt{gx+f}\,ce^{(-2)}}{g}-\frac{(3\,cd^2g-4\,cdfe-bdge+2\,bfe^2-age^2)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{(dge^2-fe^3)\sqrt{dge-fe^2}}+\frac{\sqrt{gx+f}\,cd^2g-\sqrt{gx+f}\,bdge+\sqrt{gx+f}\,age^2}{(dge^2-fe^3)(dg+(gx+f)e-fe)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(g*x + f)*c*e^(-2)/g - (3*c*d^2*g - 4*c*d*f*e - b*d*g*e + 2*b*f*e^2 -
 a*g*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d*g*e^2 - f*e^3)*sq
```

```
rt(d*g*e - f*e^2)) + (sqrt(g*x + f)*c*d^2*g - sqrt(g*x + f)*b*d*g*e + sqrt(
g*x + f)*a*g*e^2)/((d*g*e^2 - f*e^3)*(d*g + (g*x + f)*e - f*e))
```

Mupad [B]
time = 0.23, size = 146, normalized size = 1.04

$$\frac{\operatorname{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(a\,e^2\,g-2\,b\,e^2\,f-3\,c\,d^2\,g+b\,d\,e\,g+4\,c\,d\,e\,f\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}}+\frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2-b\,g\,d\,e+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)}+\frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 2*b*e^2*f - 3
*c*d^2*g + b*d*e*g + 4*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(
1/2)*(a*e^2*g + c*d^2*g - b*d*e*g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d
*e^2*g)) + (2*c*(f + g*x)^(1/2))/(e^2*g)
```

3.825 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=206

$$-\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)}+\frac{(eg(4bef-bdg-3aeg)\ \cdots}{}$$

```
[Out] 1/4*(e*g*(-3*a*e*g-b*d*g+4*b*e*f)-c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arctan
h(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)-1/2*(a+d
*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(c*d*(-5*d*g+8*e*f)
-e*(-3*a*e*g-b*d*g+4*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/(e*x+d)
```

Rubi [A]
time = 0.24, antiderivative size = 210, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {911, 1171, 393, 214}

$$-\frac{\sqrt{f+gx}\,(ae^2-bde+cd^2)}{2e^2(d+ex)^2(ef-dg)}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(eg(-3aeg-bdg+4bef)-c(3d^2g^2-8defg+8e^2f^2))}{4e^{5/2}(ef-dg)^{5/2}}+\frac{\sqrt{f+gx}\,(cd(8ef-5dg)-e(-3aeg-bdg+4bef))}{4e^2(d+ex)(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/2*((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x])/(e^2*(e*f - d*g)*(d + e*x)^2)
+ ((c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/(4*
e^2*(e*f - d*g)^2*(d + e*x)) + ((e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2
*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d
*g]])/(4*e^(5/2)*(e*f - d*g)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx,x,\sqrt{f+gx}\right)}{g}\\
&= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{\text{Subst}\left(\int \frac{-3a+\frac{cd^2}{e^2}-\frac{bd}{e}-\frac{4cf^2}{g^2}+\frac{4bf}{g}+\frac{4c(ef-dg)x^2}{eg^2}}{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx,x,\sqrt{\right.}{2(ef-dg)}\\
&= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+g}}{4e^2(ef-dg)^2(d+ex)}\\
&= -\frac{\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{2(ef-dg)(d+ex)^2}+\frac{(cd(8ef-5dg)-e(4bef-bdg-3aeg))\sqrt{f+g}}{4e^2(ef-dg)^2(d+ex)}
\end{aligned}$$

Mathematica [A]
time = 0.83, size = 203, normalized size = 0.99

$$\frac{\frac{\sqrt{e}\sqrt{f+gx}\,(cd(-3d^2g+8e^2fx+de(6f-5gx))+e(ae(-2ef+5dg+3egx)-b(2def+d^2g+4e^2fx-degx)))}{(ef-dg)^2(d+ex)^2}+\frac{(eg(-4bef+bdg+3aeg)+c(8e^2f^2-8defg+3d^2g^2))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{(-ef+dg)^{5/2}}}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

[Out] ((Sqrt[e]*Sqrt[f + g*x]*(c*d*(-3*d^2*g + 8*e^2*f*x + d*e*(6*f - 5*g*x)) + e
*(a*e*(-2*e*f + 5*d*g + 3*e*g*x) - b*(2*d*e*f + d^2*g + 4*e^2*f*x - d*e*g*x
))))/((e*f - d*g)^2*(d + e*x)^2) + ((e*g*(-4*b*e*f + b*d*g + 3*a*e*g) + c*(
8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e
*f) + d*g]])/(-(e*f) + d*g)^(5/2))/(4*e^(5/2))

Maple [A]
time = 0.11, size = 260, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{g\left(3a\,e^2g+bdeg-4b\,e^2f-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-bdeg-4b\,e^2f-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{\left(e(gx+f)+dg-ef\right)^2}+\frac{(3a\,e^2g^2+bde\,g^2-}{}$
default	$\frac{\frac{g\left(3a\,e^2g+bdeg-4b\,e^2f-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-bdeg-4b\,e^2f-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{\left(e(gx+f)+dg-ef\right)^2}+\frac{(3a\,e^2g^2+bde\,g^2-}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(1/8*g*(3*a*e^2*g+b*d*e*g-4*b*e^2*f-5*c*d^2*g+8*c*d*e*f)/e/(d^2*g^2-2*d*e
*f*g+e^2*f^2)*(g*x+f)^(3/2)+1/8*(5*a*e^2*g-b*d*e*g-4*b*e^2*f-3*c*d^2*g+8*c*
d*e*f)/e^2*g/(d*g-e*f)*(g*x+f)^(1/2))/(e*(g*x+f)+d*g-e*f)^2+1/4*(3*a*e^2*g^
2+b*d*e*g^2-4*b*e^2*f*g+3*c*d^2*g^2-8*c*d*e*f*g+8*c*e^2*f^2)/(d^2*g^2-2*d*e
*f*g+e^2*f^2)/e^2/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^
(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(193) = 386$.
time = 1.76, size = 1062, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((3*c*d^4*g^2 + (8*c*f^2 - 4*b*f*g + 3*a*g^2)*x^2*e^4 - ((8*c*d*f*g -
 b*d*g^2)*x^2 - 2*(8*c*d*f^2 - 4*b*d*f*g + 3*a*d*g^2)*x)*e^3 + (3*c*d^2*g^2
*x^2 + 8*c*d^2*f^2 - 4*b*d^2*f*g + 3*a*d^2*g^2 - 2*(8*c*d^2*f*g - b*d^2*g^2
)*x)*e^2 + (6*c*d^3*g^2*x - 8*c*d^3*f*g + b*d^3*g^2)*e)*sqrt(-d*g*e + f*e^2
)*log(-(d*g - (g*x + 2*f)*e + 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e +
d)) + 2*(3*c*d^4*g^2*e - (2*a*f^2 + (4*b*f^2 - 3*a*f*g)*x)*e^5 - (2*b*d*f^2
 - 7*a*d*f*g - (8*c*d*f^2 + 5*b*d*f*g - 3*a*d*g^2)*x)*e^4 + (6*c*d^2*f^2 +
b*d^2*f*g - 5*a*d^2*g^2 - (13*c*d^2*f*g + b*d^2*g^2)*x)*e^3 + (5*c*d^3*g^2*
x - 9*c*d^3*f*g + b*d^3*g^2)*e^2)*sqrt(g*x + f))/(d^5*g^3*e^3 - f^3*x^2*e^8
 + (3*d*f^2*g*x^2 - 2*d*f^3*x)*e^7 - (3*d^2*f*g^2*x^2 - 6*d^2*f^2*g*x + d^2
*f^3)*e^6 + (d^3*g^3*x^2 - 6*d^3*f*g^2*x + 3*d^3*f^2*g)*e^5 + (2*d^4*g^3*x
- 3*d^4*f*g^2)*e^4), -1/4*((3*c*d^4*g^2 + (8*c*f^2 - 4*b*f*g + 3*a*g^2)*x^2
*e^4 - ((8*c*d*f*g - b*d*g^2)*x^2 - 2*(8*c*d*f^2 - 4*b*d*f*g + 3*a*d*g^2)*x
)*e^3 + (3*c*d^2*g^2*x^2 + 8*c*d^2*f^2 - 4*b*d^2*f*g + 3*a*d^2*g^2 - 2*(8*c
*d^2*f*g - b*d^2*g^2)*x)*e^2 + (6*c*d^3*g^2*x - 8*c*d^3*f*g + b*d^3*g^2)*e)
*sqrt(d*g*e - f*e^2)*arctan(-sqrt(d*g*e - f*e^2)*sqrt(g*x + f)/(d*g - f*e))
 + (3*c*d^4*g^2*e - (2*a*f^2 + (4*b*f^2 - 3*a*f*g)*x)*e^5 - (2*b*d*f^2 - 7*
a*d*f*g - (8*c*d*f^2 + 5*b*d*f*g - 3*a*d*g^2)*x)*e^4 + (6*c*d^2*f^2 + b*d^2
*f*g - 5*a*d^2*g^2 - (13*c*d^2*f*g + b*d^2*g^2)*x)*e^3 + (5*c*d^3*g^2*x - 9
*c*d^3*f*g + b*d^3*g^2)*e^2)*sqrt(g*x + f))/(d^5*g^3*e^3 - f^3*x^2*e^8 + (3
*d*f^2*g*x^2 - 2*d*f^3*x)*e^7 - (3*d^2*f*g^2*x^2 - 6*d^2*f^2*g*x + d^2*f^3)
*e^6 + (d^3*g^3*x^2 - 6*d^3*f*g^2*x + 3*d^3*f^2*g)*e^5 + (2*d^4*g^3*x - 3*d
^4*f*g^2)*e^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.01, size = 373, normalized size = 1.81

$$\frac{\left(3\,cd^2g^2 - 8\,cdfge + bdg^2e + 8\,cf^2e^2 - 4\,bfge^2 + 3\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge - fe^2}}\right)}{4\,(d^2g^2e^2 - 2\,dfge^3 + f^2e^4)\sqrt{dge - fe^2}} - \frac{3\,\sqrt{gx+f}\,cd^3g^3 + 5\,(gx+f)^{\frac{3}{2}}cd^2g^2e - 11\,\sqrt{gx+f}\,cd^2fg^2e + \sqrt{gx+f}\,bd^2g^3e - 8\,(gx+f)^{\frac{3}{2}}cdfge^2 + 8\,\sqrt{gx+f}\,cdf^2ge^2 - (gx+f)^{\frac{3}{2}}bdg^2e^2 + 3\,\sqrt{gx+f}\,bdfg^2e^2 - 5\,\sqrt{gx+f}\,adg^3e^2 + 4\,(gx+f)^{\frac{3}{2}}bfge^3 - 4\,\sqrt{gx+f}\,bf^2ge^3 - 3\,(gx+f)^{\frac{3}{2}}ag^2e^3 + 5\,\sqrt{gx+f}\,afg^2e^3}{4\,(d^2g^2e^2 - 2\,dfge^3 + f^2e^4)(dg + (gx+f)e - fe)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*g^2 - 8*c*d*f*g*e + b*d*g^2*e + 8*c*f^2*e^2 - 4*b*f*g*e^2 + 3*
a*g^2*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e^2 - 2*d*
f*g*e^3 + f^2*e^4)*sqrt(d*g*e - f*e^2)) - 1/4*(3*sqrt(g*x + f)*c*d^3*g^3 +
5*(g*x + f)^(3/2)*c*d^2*g^2*e - 11*sqrt(g*x + f)*c*d^2*f*g^2*e + sqrt(g*x +
 f)*b*d^2*g^3*e - 8*(g*x + f)^(3/2)*c*d*f*g*e^2 + 8*sqrt(g*x + f)*c*d*f^2*g
*e^2 - (g*x + f)^(3/2)*b*d*g^2*e^2 + 3*sqrt(g*x + f)*b*d*f*g^2*e^2 - 5*sqrt
(g*x + f)*a*d*g^3*e^2 + 4*(g*x + f)^(3/2)*b*f*g*e^3 - 4*sqrt(g*x + f)*b*f^2
*g*e^3 - 3*(g*x + f)^(3/2)*a*g^2*e^3 + 5*sqrt(g*x + f)*a*f*g^2*e^3)/((d^2*g
^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

Mupad [B]
time = 0.28, size = 270, normalized size = 1.31

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+b\,d\,e\,g^2+8\,c\,e^2\,f^2-4\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}-\frac{\frac{\sqrt{f+g\,x}\,\left(3\,c\,d^2\,g^2+b\,d\,e\,g^2-8\,c\,f\,d\,e\,g-5\,a\,e^2\,g^2+4\,b\,f\,e^2\,g\right)}{4\,e^2\,\left(d\,g-e\,f\right)}-\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+b\,d\,e\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2-4\,b\,f\,e^2\,g\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(3*a*e^2*g^2 + 3*c*d^2*g
^2 + 8*c*e^2*f^2 + b*d*e*g^2 - 4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^(5/2)*(d*g
- e*f)^(5/2)) - (((f + g*x)^(1/2)*(3*c*d^2*g^2 - 5*a*e^2*g^2 + b*d*e*g^2 +
4*b*e^2*f*g - 8*c*d*e*f*g))/(4*e^2*(d*g - e*f)) - ((f + g*x)^(3/2)*(3*a*e^2
*g^2 - 5*c*d^2*g^2 + b*d*e*g^2 - 4*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*
f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2 + e^2*f^2
 - 2*d*e*f*g)
```

3.826 $\int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=285

$$\frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)}{g^6\sqrt{f+gx}}+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))\sqrt{f+gx}}{g^6}+\frac{2(ef-dg)}{}$$

```
[Out] 2/3*(-d*g+e*f)*(3*e*g*(-a*e*g-b*d*g+2*b*e*f)-c*(d^2*g^2-8*d*e*f*g+10*e^2*f^
2))*(g*x+f)^(3/2)/g^6-2/5*e*(e*g*(-a*e*g-3*b*d*g+4*b*e*f)-c*(3*d^2*g^2-12*d
*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*e^2*(-b*e*g-3*c*d*g+5*c*e*f)*(g*x
+f)^(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^6+2*(-d*g+e*f)^3*(a*g^2-b*f*g+c*f^2
)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(c*f*(-2*d*g+5*e*f)-g*(-3*a*e*g-b*d*g+4*
b*e*f))*(g*x+f)^(1/2)/g^6
```

Rubi [A]

time = 0.26, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {911, 1275}

$$-\frac{2e(f+gx)^{5/2}\left(eg(-aeg-3bdg+4bef)-c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}+\frac{2(f+gx)^{3/2}(ef-dg)\left(3eg(-aeg-bdg+2bef)-c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{3g^6}+\frac{2(ef-dg)^3\left(ag^2-bfg+cf^2\right)}{g^6\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\,(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{g^6}-\frac{2e^2(f+gx)^{7/2}(-beg-3cdg+5cef)}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d
*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*Sqrt[f + g*x])/
g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d
*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g
- a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/(5*g^6
) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*
(f + g*x)^(9/2))/(9*g^6)
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(ef-dg)^2(cf(5ef-2dg)-g(4bef-bdg-3aeg))}{g^5}+\frac{(-ef+dg)^3\left(cf^2-bfg+ag^2\right)}{g^5x^2}+ \ldots\right.\right.}{}$$

$$= \frac{2(ef-dg)^3\left(cf^2-bfg+ag^2\right)}{g^6\sqrt{f+gx}}+\frac{2(ef-dg)^2(cf(5ef-2dg)-g(4bef- \ldots}{g^6}$$

Mathematica [A]
time = 0.36, size = 406, normalized size = 1.42

$$\frac{2\left(c\left(105d^3g^3\left(-8f^2-4fgx+g^2x^2\right)+189d^2eg^2\left(16f^3+8f^2gx-2fg^2x^2+g^3x^3\right)+27de^2g\left(-128f^4-64f^3gx+16f^2g^2x^2-8fg^3x^3+5g^4x^4\right)+5e^3\left(256f^5+128f^4gx-32f^3g^2x^2+16f^2g^3x^3-10fg^4x^4+7g^5x^5\right)\right)+9g\left(7ag\left(-5d^3g^3+15d^2eg^2(2f+gx)+5de^2g\left(-8f^2-4fgx+g^2x^2\right)+e^3\left(16f^3+8f^2gx-2fg^2x^2+g^3x^3\right)\right)+b\left(35d^3g^3(2f+gx)+35d^2eg^2\left(-8f^2-4fgx+g^2x^2\right)+21de^2g\left(16f^3+8f^2gx-2fg^2x^2+g^3x^3\right)+e^3\left(-128f^4-64f^3gx+16f^2g^2x^2-8fg^3x^3+5g^4x^4\right)\right)\right)\right)}{315g^6\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(c*(105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16*f^3 + 8
*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 16*
f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x - 32*
f^3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5)) + 9*g*(7*a*g*(-5*
d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 - 4*f*g*x + g^2*x^2)
 + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + b*(35*d^3*g^3*(2*f +
 g*x) + 35*d^2*e*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 21*d*e^2*g*(16*f^3 + 8*
f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + e^3*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2*
x^2 - 8*f*g^3*x^3 + 5*g^4*x^4)))))/(315*g^6*Sqrt[f + g*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(265) = 530$.
time = 0.11, size = 650, normalized size = 2.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^6*(1/9*c*e^3*(g*x+f)^(9/2)-3*b*d*e^2*f*g^2*(g*x+f)^(3/2)+b*d^3*g^4*(g*x
+f)^(1/2)+1/7*b*e^3*g*(g*x+f)^(7/2)-5/7*c*e^3*f*(g*x+f)^(7/2)+1/5*a*e^3*g^2
*(g*x+f)^(5/2)+2*c*e^3*f^2*(g*x+f)^(5/2)+1/3*c*d^3*g^3*(g*x+f)^(3/2)-10/3*c
```

```
*e^3*f^3*(g*x+f)^(3/2)+5*c*e^3*f^4*(g*x+f)^(1/2)-6*b*d^2*e*f*g^3*(g*x+f)^(1
/2)-12*c*d*e^2*f^3*g*(g*x+f)^(1/2)+6*c*d*e^2*f^2*g*(g*x+f)^(3/2)-6*a*d*e^2*
f*g^3*(g*x+f)^(1/2)+9*c*d^2*e*f^2*g^2*(g*x+f)^(1/2)+9*b*d*e^2*f^2*g^2*(g*x+
f)^(1/2)-12/5*c*d*e^2*f*g*(g*x+f)^(5/2)-3*c*d^2*e*f*g^2*(g*x+f)^(3/2)-(a*d^
3*g^5-3*a*d^2*e*f*g^4+3*a*d*e^2*f^2*g^3-a*e^3*f^3*g^2-b*d^3*f*g^4+3*b*d^2*e
*f^2*g^3-3*b*d*e^2*f^3*g^2+b*e^3*f^4*g+c*d^3*f^2*g^3-3*c*d^2*e*f^3*g^2+3*c*
d*e^2*f^4*g-c*e^3*f^5)/(g*x+f)^(1/2)+3/5*c*d^2*e*g^2*(g*x+f)^(5/2)+a*d*e^2*
g^3*(g*x+f)^(3/2)-a*e^3*f*g^2*(g*x+f)^(3/2)+3*a*d^2*e*g^4*(g*x+f)^(1/2)+3*a
*e^3*f^2*g^2*(g*x+f)^(1/2)-2*c*d^3*f*g^3*(g*x+f)^(1/2)+3/7*c*d*e^2*g*(g*x+f
)^(7/2)+b*d^2*e*g^3*(g*x+f)^(3/2)-4*b*e^3*f^3*g*(g*x+f)^(1/2)+3/5*b*d*e^2*g
^2*(g*x+f)^(5/2)-4/5*b*e^3*f*g*(g*x+f)^(5/2)+2*b*e^3*f^2*g*(g*x+f)^(3/2))
```

Maxima [A]
time = 0.28, size = 419, normalized size = 1.47

$$\frac{2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^3-45\,(5\,cfe^3-(3\,cde^2+be^3)g)(gx+f)^{\frac{7}{2}}+63\,(10\,cf^2e^3-4\,(3\,cde^2+be^3)fg+(3\,cd^2e+3\,bde^2+ae^3)g^2)(gx+f)^{\frac{5}{2}}-105\,(10\,cf^3e^3-6\,(3\,cde^2+be^3)f^2g+3\,(3\,cd^2e+3\,bde^2+ae^3)fg^2-(cd^3+3\,bd^2e+3\,ade^2)g^3)(gx+f)^{\frac{3}{2}}+315\,(5\,cf^4e^3-4\,(3\,cde^2+be^3)f^3g+3\,(3\,cd^2e+3\,bde^2+ae^3)f^2g^2-2\,(cd^3+3\,bd^2e+3\,ade^2)fg^3+(bd^3+3\,ad^2e)g^4)\sqrt{gx+f}}{g^5}-\frac{315\,(ad^3g^5-cf^5e^3+(3\,cde^2+be^3)f^4g-(3\,cd^2e+3\,bde^2+ae^3)f^3g^2+(cd^3+3\,bd^2e+3\,ade^2)f^2g^3-(bd^3+3\,ad^2e)fg^4)}{\sqrt{gx+f}\,g^5}\right)}{315\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*f*e^3 - (3*c*d*e^2 + b*e^3)*g)*(
g*x + f)^(7/2) + 63*(10*c*f^2*e^3 - 4*(3*c*d*e^2 + b*e^3)*f*g + (3*c*d^2*e
+ 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(5/2) - 105*(10*c*f^3*e^3 - 6*(3*c*d*e^
2 + b*e^3)*f^2*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*f^4*e^3 - 4*(3*c*d*e^2 +
b*e^3)*f^3*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*sqrt(g*x + f))/g^5 - 315
*(a*d^3*g^5 - c*f^5*e^3 + (3*c*d*e^2 + b*e^3)*f^4*g - (3*c*d^2*e + 3*b*d*e^
2 + a*e^3)*f^3*g^2 + (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 - (b*d^3 + 3*a
*d^2*e)*f*g^4)/(sqrt(g*x + f)*g^5))/g
```

Fricas [A]
time = 1.16, size = 466, normalized size = 1.64

$$\frac{2\,(105\,cd^3g^5x^2-840\,cd^3f^2g^3+630\,bd^3fg^4-315\,ad^3g^5-105\,(4\,cd^3fg^4-3\,bd^3g^5)x+(35\,cg^5x^5+1280\,cf^5-1152\,bf^4g+1008\,af^3g^2-5\,(10\,cfg^4-9\,bg^5)x^4+(80\,cf^2g^3-72\,bfg^4+63\,ag^5)x^3-2\,(80\,cf^3g^2-72\,bf^2g^3+63\,afg^4)x^2+8\,(80\,cf^4g-72\,bf^3g^2+63\,af^2g^3)x)e^3+9\,(15\,cdg^5x^4-384\,cdf^4g+336\,bdf^3g^2-280\,adf^2g^3-3\,(8\,cdfg^4-7\,bdg^5)x^3+(48\,cdf^2g^3-42\,bdfg^4+35\,adg^5)x^2-4\,(48\,cdf^3g^2-42\,bdf^2g^3+35\,adfg^4)x)e^2+63\,(3\,cd^2g^5x^3+48\,cd^2f^3g^2-40\,bd^2f^2g^3+30\,ad^2fg^4-(6\,cd^2fg^4-5\,bd^2g^5)x^2+(24\,cd^2f^2g^3-20\,bd^2fg^4+15\,ad^2g^5)x)e)\sqrt{gx+f}}{315\,(g^7x+fg^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(105*c*d^3*g^5*x^2 - 840*c*d^3*f^2*g^3 + 630*b*d^3*f*g^4 - 315*a*d^3*
g^5 - 105*(4*c*d^3*f*g^4 - 3*b*d^3*g^5)*x + (35*c*g^5*x^5 + 1280*c*f^5 - 11
52*b*f^4*g + 1008*a*f^3*g^2 - 5*(10*c*f*g^4 - 9*b*g^5)*x^4 + (80*c*f^2*g^3
- 72*b*f*g^4 + 63*a*g^5)*x^3 - 2*(80*c*f^3*g^2 - 72*b*f^2*g^3 + 63*a*f*g^4)
*x^2 + 8*(80*c*f^4*g - 72*b*f^3*g^2 + 63*a*f^2*g^3)*x)*e^3 + 9*(15*c*d*g^5*
x^4 - 384*c*d*f^4*g + 336*b*d*f^3*g^2 - 280*a*d*f^2*g^3 - 3*(8*c*d*f*g^4 -
7*b*d*g^5)*x^3 + (48*c*d*f^2*g^3 - 42*b*d*f*g^4 + 35*a*d*g^5)*x^2 - 4*(48*c
```

```
*d*f^3*g^2 - 42*b*d*f^2*g^3 + 35*a*d*f*g^4)*x)*e^2 + 63*(3*c*d^2*g^5*x^3 +
48*c*d^2*f^3*g^2 - 40*b*d^2*f^2*g^3 + 30*a*d^2*f*g^4 - (6*c*d^2*f*g^4 - 5*b
*d^2*g^5)*x^2 + (24*c*d^2*f^2*g^3 - 20*b*d^2*f*g^4 + 15*a*d^2*g^5)*x)*e)*sq
rt(g*x + f)/(g^7*x + f*g^6)
```

Sympy [A]
time = 47.05, size = 452, normalized size = 1.59

$$\frac{2ce^3(f+gx)^{\frac{9}{2}}}{9g^6}+\frac{(f+gx)^{\frac{7}{2}}\cdot(2be^3g+6cde^2g-10ce^3f)}{7g^6}+\frac{(f+gx)^{\frac{5}{2}}\cdot(2ae^3g^2+6bde^2g^2-8be^3fg+6cd^2eg^2-24cde^2fg+20ce^3f^2)}{5g^6}+\frac{(f+gx)^{\frac{3}{2}}\cdot(6ade^2g^3-6ae^3fg^2+6bd^2eg^3-18bde^2fg^2+12be^3f^2g+2cd^3g^3-18cd^2efg^2+36cde^2f^2g-20ce^3f^3)}{3g^6}+\frac{\sqrt{f+gx}\,(6ad^2eg^4-12ade^2fg^3+6ae^3f^2g^2+2bd^3g^4-12bd^2efg^3+18bde^2f^2g^2-8be^3f^3g-4cd^3fg^3+18cd^2ef^2g^2-24cde^2f^3g+10ce^3f^4)}{g^6}-\frac{2(dg-ef)^3\,(ag^2-bfg+cf^2)}{g^6\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**3*(f + g*x)**(9/2)/(9*g**6) + (f + g*x)**(7/2)*(2*b*e**3*g + 6*c*d*e
**2*g - 10*c*e**3*f)/(7*g**6) + (f + g*x)**(5/2)*(2*a*e**3*g**2 + 6*b*d*e**
2*g**2 - 8*b*e**3*f*g + 6*c*d**2*e*g**2 - 24*c*d*e**2*f*g + 20*c*e**3*f**2)
/(5*g**6) + (f + g*x)**(3/2)*(6*a*d*e**2*g**3 - 6*a*e**3*f*g**2 + 6*b*d**2*
e*g**3 - 18*b*d*e**2*f*g**2 + 12*b*e**3*f**2*g + 2*c*d**3*g**3 - 18*c*d**2*
e*f*g**2 + 36*c*d*e**2*f**2*g - 20*c*e**3*f**3)/(3*g**6) + sqrt(f + g*x)*(6
*a*d**2*e*g**4 - 12*a*d*e**2*f*g**3 + 6*a*e**3*f**2*g**2 + 2*b*d**3*g**4 -
12*b*d**2*e*f*g**3 + 18*b*d*e**2*f**2*g**2 - 8*b*e**3*f**3*g - 4*c*d**3*f*g
**3 + 18*c*d**2*e*f**2*g**2 - 24*c*d*e**2*f**3*g + 10*c*e**3*f**4)/g**6 - 2
*(d*g - e*f)**3*(a*g**2 - b*f*g + c*f**2)/(g**6*sqrt(f + g*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(271) = 542.
time = 2.97, size = 669, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^3*f^2*g^3 - b*d^3*f*g^4 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e + 3*b*d^2*f
^2*g^3*e - 3*a*d^2*f*g^4*e + 3*c*d*f^4*g*e^2 - 3*b*d*f^3*g^2*e^2 + 3*a*d*f^
2*g^3*e^2 - c*f^5*e^3 + b*f^4*g*e^3 - a*f^3*g^2*e^3)/(sqrt(g*x + f)*g^6) +
2/315*(105*(g*x + f)^(3/2)*c*d^3*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^51 + 31
5*sqrt(g*x + f)*b*d^3*g^52 + 189*(g*x + f)^(5/2)*c*d^2*g^50*e - 945*(g*x +
f)^(3/2)*c*d^2*f*g^50*e + 2835*sqrt(g*x + f)*c*d^2*f^2*g^50*e + 315*(g*x +
f)^(3/2)*b*d^2*g^51*e - 1890*sqrt(g*x + f)*b*d^2*f*g^51*e + 945*sqrt(g*x +
f)*a*d^2*g^52*e + 135*(g*x + f)^(7/2)*c*d*g^49*e^2 - 756*(g*x + f)^(5/2)*c*
d*f*g^49*e^2 + 1890*(g*x + f)^(3/2)*c*d*f^2*g^49*e^2 - 3780*sqrt(g*x + f)*c
*d*f^3*g^49*e^2 + 189*(g*x + f)^(5/2)*b*d*g^50*e^2 - 945*(g*x + f)^(3/2)*b*
d*f*g^50*e^2 + 2835*sqrt(g*x + f)*b*d*f^2*g^50*e^2 + 315*(g*x + f)^(3/2)*a*
d*g^51*e^2 - 1890*sqrt(g*x + f)*a*d*f*g^51*e^2 + 35*(g*x + f)^(9/2)*c*g^48*
e^3 - 225*(g*x + f)^(7/2)*c*f*g^48*e^3 + 630*(g*x + f)^(5/2)*c*f^2*g^48*e^3
```

```
 - 1050*(g*x + f)^(3/2)*c*f^3*g^48*e^3 + 1575*sqrt(g*x + f)*c*f^4*g^48*e^3
+ 45*(g*x + f)^(7/2)*b*g^49*e^3 - 252*(g*x + f)^(5/2)*b*f*g^49*e^3 + 630*(g
*x + f)^(3/2)*b*f^2*g^49*e^3 - 1260*sqrt(g*x + f)*b*f^3*g^49*e^3 + 63*(g*x
+ f)^(5/2)*a*g^50*e^3 - 315*(g*x + f)^(3/2)*a*f*g^50*e^3 + 945*sqrt(g*x + f
)*a*f^2*g^50*e^3)/g^54
```

Mupad [B]
time = 0.12, size = 394, normalized size = 1.38

$$\frac{(f+gx)^{7/2}(2be^3g-10ce^3f+6cde^2g)}{7g^6}-\frac{2cd^3f^2g^3-2bd^3fg^4+2ad^3g^5-6cd^2ef^3g^2+6bd^2ef^2g^3-6ad^2efg^4+6cde^2f^4g-6bde^2f^3g^2+6ade^2f^2g^3-2ce^3f^5+2be^3f^4g-2ae^3f^3g^2}{g^6\sqrt{f+gx}}+\frac{(f+gx)^{5/2}(6cd^2eg^2-24cde^2fg+6bde^2g^2+20ce^3f^2-8be^3fg+2ae^3g^2)}{5g^6}+\frac{2(f+gx)^{3/2}(dg-ef)(cd^2g^2-8cdefg+3bdeg^2+10ce^2f^2-6be^2fg+3ae^2g^2)}{3g^6}+\frac{2\sqrt{f+gx}(dg-ef)^2(3aeg^2+bdg^2+5cef^2-4befg-2cdfg)}{g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(7/2)*(2*b*e^3*g - 10*c*e^3*f + 6*c*d*e^2*g))/(7*g^6) - (2*a*d^3
*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2*g^3 - 2*b*d^3*f*g^4 + 2*
b*e^3*f^4*g - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*b*d
*e^2*f^3*g^2 + 6*b*d^2*e*f^2*g^3 - 6*c*d^2*e*f^3*g^2)/(g^6*(f + g*x)^(1/2))
 + ((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 - 8*b*e^3*f*g + 6*b*d*e^2*g
^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*f*g))/(5*g^6) + (2*(f + g*x)^(3/2)*(d*g - e
*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 + 3*b*d*e*g^2 - 6*b*e^2*f*g - 8
*c*d*e*f*g))/(3*g^6) + (2*(f + g*x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + b*d*g^
2 + 5*c*e*f^2 - 4*b*e*f*g - 2*c*d*f*g))/g^6 + (2*c*e^3*(f + g*x)^(9/2))/(9*
g^6)
```

3.827 $\int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)}{g^5\sqrt{f+gx}}-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-bdg-2aeg))\sqrt{f+gx}}{g^5}-\frac{2(eg(3be$$

```
[Out] -2/3*(e*g*(-a*e*g-2*b*d*g+3*b*e*f)-c*(d^2*g^2-6*d*e*f*g+6*e^2*f^2))*(g*x+f)
^(3/2)/g^5-2/5*e*(-b*e*g-2*c*d*g+4*c*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(g*x+
f)^(7/2)/g^5-2*(-d*g+e*f)^2*(a*g^2-b*f*g+c*f^2)/g^5/(g*x+f)^(1/2)-2*(-d*g+e
*f)*(2*c*f*(-d*g+2*e*f)-g*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(1/2)/g^5
```

Rubi [A]

time = 0.19, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {911, 1275}

$$-\frac{2(f+gx)^{3/2}\left(eg(-aeg-2bdg+3bef)-c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)}{g^5\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{g^5}-\frac{2e(f+gx)^{5/2}(-beg-2cdg+4cef)}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*Sqrt[f + g*x]) - (2*(e*f -
d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x])/g
^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g
^2))*(f + g*x)^(3/2))/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^
(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2 \left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2\left(\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}\right)}{x^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(ef-dg)(-2cf(2ef-dg)+g(3bef-bdg-2aeg))}{g^4}+\frac{(-ef+dg)^2\left(cf^2-bfg+ag^2\right)}{g^4x^2}+\frac{(-\ldots}{\ldots}\right.\right.}{\ldots}$$

$$= -\frac{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)}{g^5\sqrt{f+gx}}-\frac{2(ef-dg)(2cf(2ef-dg)-g(3bef-\ldots}{g^5}$$

Mathematica [A]

time = 0.21, size = 252, normalized size = 1.20

$$\frac{2(c(35d^2g^2(-8f^2-4fgx+g^2x^2)+42deg(16f^3+8f^2gx-2fg^2x^2+g^3x^3)-3e^2(128f^4+64f^3gx-16f^2g^2x^2+8fg^3x^3-5g^4x^4))+7g(5ag(-3d^2g^2+6deg(2f+gx)+e^2(-8f^2-4fgx+g^2x^2))+b(15d^2g^2(2f+gx)+10deg(-8f^2-4fgx+g^2x^2)+3e^2(16f^3+8f^2gx-2fg^2x^2+g^3x^3))))}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3 + 8*f^2*g
*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g*x - 16*f^2*g^2*x^2
+ 8*f*g^3*x^3 - 5*g^4*x^4)) + 7*g*(5*a*g*(-3*d^2*g^2 + 6*d*e*g*(2*f + g*x)
+ e^2*(-8*f^2 - 4*f*g*x + g^2*x^2)) + b*(15*d^2*g^2*(2*f + g*x) + 10*d*e*g*
(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e^2*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^
3*x^3)))))/(105*g^5*Sqrt[f + g*x])
```

Maple [A]

time = 0.09, size = 379, normalized size = 1.80

method	result
risch	$\frac{2\left(15c\,e^2x^3g^3+21b\,e^2g^3x^2+42cde\,g^3x^2-39c\,e^2f\,g^2x^2+35a\,e^2g^3x+70bde\,g^3x-63b\,e^2f\,g^2x+35c\,d^2g^3x-126cdef\,g^2x+87c\ldots\right.}{105\ldots}$
gosper	$-\frac{2\left(-15c\,e^2x^4g^4-21b\,e^2g^4x^3-42cde\,g^4x^3+24c\,e^2f\,g^3x^3-35a\,e^2g^4x^2-70bde\,g^4x^2+42b\,e^2f\,g^3x^2-35c\,d^2g^4x^2+84cdef\,g\ldots\right.}{\ldots}$
trager	$-\frac{2\left(-15c\,e^2x^4g^4-21b\,e^2g^4x^3-42cde\,g^4x^3+24c\,e^2f\,g^3x^3-35a\,e^2g^4x^2-70bde\,g^4x^2+42b\,e^2f\,g^3x^2-35c\,d^2g^4x^2+84cdef\,g\ldots\right.}{\ldots}$
derivativedivides	$\frac{\frac{2c\,e^2(gx+f)^{\frac{7}{2}}}{7}+\frac{2b\,e^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{4bde\,g^2(gx+f)^{\frac{3}{2}}}{3}-2b\,e^2fg(gx+f)^{\frac{3}{2}}+\frac{2c\ldots}{\ldots}}{\ldots}$

default	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{7}{2}}}{7}+\frac{2b\,e^{2}g(gx+f)^{\frac{5}{2}}}{5}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c\,e^{2}f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a\,e^{2}g^{2}(gx+f)^{\frac{3}{2}}}{3}+\frac{4bde\,g^{2}(gx+f)^{\frac{3}{2}}}{3}-2b\,e^{2}fg(gx+f)^{\frac{3}{2}}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^5*(1/7*c*e^2*(g*x+f)^(7/2)+1/5*b*e^2*g*(g*x+f)^(5/2)+2/5*c*d*e*g*(g*x+f
)^(5/2)-4/5*c*e^2*f*(g*x+f)^(5/2)+1/3*a*e^2*g^2*(g*x+f)^(3/2)+2/3*b*d*e*g^2
*(g*x+f)^(3/2)-b*e^2*f*g*(g*x+f)^(3/2)+1/3*c*d^2*g^2*(g*x+f)^(3/2)-2*c*d*e*
f*g*(g*x+f)^(3/2)+2*c*e^2*f^2*(g*x+f)^(3/2)+2*a*d*e*g^3*(g*x+f)^(1/2)-2*a*e
^2*f*g^2*(g*x+f)^(1/2)+b*d^2*g^3*(g*x+f)^(1/2)-4*b*d*e*f*g^2*(g*x+f)^(1/2)+
3*b*e^2*f^2*g*(g*x+f)^(1/2)-2*c*d^2*f*g^2*(g*x+f)^(1/2)+6*c*d*e*f^2*g*(g*x+
f)^(1/2)-4*c*e^2*f^3*(g*x+f)^(1/2)-(a*d^2*g^4-2*a*d*e*f*g^3+a*e^2*f^2*g^2-b
*d^2*f*g^3+2*b*d*e*f^2*g^2-b*e^2*f^3*g+c*d^2*f^2*g^2-2*c*d*e*f^3*g+c*e^2*f^
4)/(g*x+f)^(1/2))
```

Maxima [A]
time = 0.28, size = 266, normalized size = 1.27

$$\frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}ce^{2}-21\,(4\,cfe^{2}-(2\,cde+be^{2})g)(gx+f)^{\frac{5}{2}}+35\,(6\,cf^{2}e^{2}-3\,(2\,cde+be^{2})fg+(cd^{2}+2\,bde+ae^{2})g^{2})(gx+f)^{\frac{3}{2}}-105\,(4\,cf^{3}e^{2}-3\,(2\,cde+be^{2})f^{2}g+2\,(cd^{2}+2\,bde+ae^{2})fg^{2}-(bd^{2}+2\,ade)g^{3})\sqrt{gx+f}}{g^{4}}-\frac{105\,(ad^{2}g^{4}+cf^{4}e^{2}-(2\,cde+be^{2})f^{3}g+(cd^{2}+2\,bde+ae^{2})f^{2}g^{2}-(bd^{2}+2\,ade)fg^{3})}{\sqrt{gx+f}\,g^{4}}\right)}{105\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 21*(4*c*f*e^2 - (2*c*d*e + b*e^2)*g)*(g*
x + f)^(5/2) + 35*(6*c*f^2*e^2 - 3*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e
 + a*e^2)*g^2)*(g*x + f)^(3/2) - 105*(4*c*f^3*e^2 - 3*(2*c*d*e + b*e^2)*f^2
*g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g^2 - (b*d^2 + 2*a*d*e)*g^3)*sqrt(g*x +
f))/g^4 - 105*(a*d^2*g^4 + c*f^4*e^2 - (2*c*d*e + b*e^2)*f^3*g + (c*d^2 + 2
*b*d*e + a*e^2)*f^2*g^2 - (b*d^2 + 2*a*d*e)*f*g^3)/(sqrt(g*x + f)*g^4))/g
```

Fricas [A]
time = 2.13, size = 287, normalized size = 1.37

$$\frac{2\,(35\,cd^{2}g^{4}x^{2}-280\,cd^{2}f^{2}g^{2}+210\,bd^{2}fg^{3}-105\,ad^{2}g^{4}-35\,(4\,cd^{2}fg^{3}-3\,bd^{2}g^{4})x+(15\,cg^{4}x^{4}-384\,cf^{4}+336\,bf^{3}g-280\,af^{2}g^{2}-3\,(8\,cfg^{3}-7\,bg^{4})x^{3}+(48\,cf^{2}g^{2}-42\,bfg^{3}+35\,ag^{4})x^{2}-4\,(48\,cf^{3}g-42\,bf^{2}g^{2}+35\,afg^{3})x)e^{2}+14\,(3\,cdg^{4}x^{3}+48\,cdf^{3}g-40\,bdf^{2}g^{2}+30\,adfg^{3}-(6\,cdfg^{3}-5\,bdg^{4})x^{2}+(24\,cdf^{2}g^{2}-20\,bdfg^{3}+15\,adg^{4})x)e)\sqrt{gx+f}}{105\,(g^{6}x+fg^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(35*c*d^2*g^4*x^2 - 280*c*d^2*f^2*g^2 + 210*b*d^2*f*g^3 - 105*a*d^2*g
^4 - 35*(4*c*d^2*f*g^3 - 3*b*d^2*g^4)*x + (15*c*g^4*x^4 - 384*c*f^4 + 336*b
*f^3*g - 280*a*f^2*g^2 - 3*(8*c*f*g^3 - 7*b*g^4)*x^3 + (48*c*f^2*g^2 - 42*b
*f*g^3 + 35*a*g^4)*x^2 - 4*(48*c*f^3*g - 42*b*f^2*g^2 + 35*a*f*g^3)*x)*e^2
+ 14*(3*c*d*g^4*x^3 + 48*c*d*f^3*g - 40*b*d*f^2*g^2 + 30*a*d*f*g^3 - (6*c*d
```

```
*f*g^3 - 5*b*d*g^4)*x^2 + (24*c*d*f^2*g^2 - 20*b*d*f*g^3 + 15*a*d*g^4)*x)*e
)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

Sympy [A]
time = 25.84, size = 272, normalized size = 1.30

$$\frac{2ce^2(f+gx)^{\frac{7}{2}}}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\cdot(2be^2g+4cdeg-8ce^2f)}{5g^5}+\frac{(f+gx)^{\frac{3}{2}}\cdot(2ae^2g^2+4bdeg^2-6be^2fg+2cd^2g^2-12cdefg+12ce^2f^2)}{3g^5}+\frac{\sqrt{f+gx}\,(4adeg^3-4ae^2fg^2+2bd^2g^3-8bdefg^2+6be^2f^2g-4cd^2fg^2+12cdef^2g-8ce^2f^3)}{g^5}-\frac{2(dg-ef)^2\,(ag^2-bfg+cf^2)}{g^5\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + (f + g*x)**(5/2)*(2*b*e**2*g + 4*c*d*e
*g - 8*c*e**2*f)/(5*g**5) + (f + g*x)**(3/2)*(2*a*e**2*g**2 + 4*b*d*e*g**2
- 6*b*e**2*f*g + 2*c*d**2*g**2 - 12*c*d*e*f*g + 12*c*e**2*f**2)/(3*g**5) +
sqrt(f + g*x)*(4*a*d*e*g**3 - 4*a*e**2*f*g**2 + 2*b*d**2*g**3 - 8*b*d*e*f*g
**2 + 6*b*e**2*f**2*g - 4*c*d**2*f*g**2 + 12*c*d*e*f**2*g - 8*c*e**2*f**3)/
g**5 - 2*(d*g - e*f)**2*(a*g**2 - b*f*g + c*f**2)/(g**5*sqrt(f + g*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(199) = 398.
time = 2.73, size = 404, normalized size = 1.92

$$-\frac{2\,(cd^2f^2g^2-bd^2fg^3+ad^2g^4-2\,cdf^3ge+2\,bdf^2g^2e-2\,adfg^3e+cf^4e^2-bf^3ge^2+af^2g^2e^2)}{\sqrt{gx+f}\,g^5}+\frac{2\left(35\,(gx+f)^{\frac{3}{2}}cd^2g^{32}-210\,\sqrt{gx+f}\,cd^2fg^{32}+105\,\sqrt{gx+f}\,bd^2g^{33}+42\,(gx+f)^{\frac{5}{2}}cdg^{31}e-210\,(gx+f)^{\frac{3}{2}}cdfg^{31}e+630\,\sqrt{gx+f}\,cdf^2g^{31}e+70\,(gx+f)^{\frac{3}{2}}bdg^{32}e-420\,\sqrt{gx+f}\,bdfg^{32}e+210\,\sqrt{gx+f}\,adg^{33}e+15\,(gx+f)^{\frac{7}{2}}cg^{30}e^2-84\,(gx+f)^{\frac{5}{2}}cfg^{30}e^2+210\,(gx+f)^{\frac{3}{2}}cf^2g^{30}e^2-420\,\sqrt{gx+f}\,cf^3g^{30}e^2+21\,(gx+f)^{\frac{5}{2}}bg^{31}e^2-105\,(gx+f)^{\frac{3}{2}}bfg^{31}e^2+315\,\sqrt{gx+f}\,bf^2g^{31}e^2+35\,(gx+f)^{\frac{3}{2}}ag^{32}e^2-210\,\sqrt{gx+f}\,afg^{32}e^2\right)}{105\,g^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d^2*f^2*g^2 - b*d^2*f*g^3 + a*d^2*g^4 - 2*c*d*f^3*g*e + 2*b*d*f^2*g^2
*e - 2*a*d*f*g^3*e + c*f^4*e^2 - b*f^3*g*e^2 + a*f^2*g^2*e^2)/(sqrt(g*x + f
)*g^5) + 2/105*(35*(g*x + f)^(3/2)*c*d^2*g^32 - 210*sqrt(g*x + f)*c*d^2*f*g
^32 + 105*sqrt(g*x + f)*b*d^2*g^33 + 42*(g*x + f)^(5/2)*c*d*g^31*e - 210*(g
*x + f)^(3/2)*c*d*f*g^31*e + 630*sqrt(g*x + f)*c*d*f^2*g^31*e + 70*(g*x + f
)^(3/2)*b*d*g^32*e - 420*sqrt(g*x + f)*b*d*f*g^32*e + 210*sqrt(g*x + f)*a*d
*g^33*e + 15*(g*x + f)^(7/2)*c*g^30*e^2 - 84*(g*x + f)^(5/2)*c*f*g^30*e^2 +
 210*(g*x + f)^(3/2)*c*f^2*g^30*e^2 - 420*sqrt(g*x + f)*c*f^3*g^30*e^2 + 21
*(g*x + f)^(5/2)*b*g^31*e^2 - 105*(g*x + f)^(3/2)*b*f*g^31*e^2 + 315*sqrt(g
*x + f)*b*f^2*g^31*e^2 + 35*(g*x + f)^(3/2)*a*g^32*e^2 - 210*sqrt(g*x + f)*
a*f*g^32*e^2)/g^35
```

Mupad [B]
time = 3.13, size = 270, normalized size = 1.29

$$\frac{(f+g\,x)^{5/2}\,(2\,b\,e^2\,g-8\,c\,e^2\,f+4\,c\,d\,e\,g)}{5\,g^5}-\frac{2\,c\,d^2\,f^2\,g^2-2\,b\,d^2\,f\,g^3+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g+4\,b\,d\,e\,f^2\,g^2-4\,a\,d\,e\,f\,g^3+2\,c\,e^2\,f^4-2\,b\,e^2\,f^3\,g+2\,a\,e^2\,f^2\,g^2}{g^5\,\sqrt{f+g\,x}}+\frac{(f+g\,x)^{3/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+12\,c\,e^2\,f^2-6\,b\,e^2\,f\,g+2\,a\,e^2\,g^2)}{3\,g^5}+\frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,(2\,a\,e\,g^2+b\,d\,g^2+4\,c\,e\,f^2-3\,b\,e\,f\,g-2\,c\,d\,f\,g)}{g^5}+\frac{2\,c\,e^2\,(f+g\,x)^{7/2}}{7\,g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

[Out] ((f + g*x)^(5/2)*(2*b*e^2*g - 8*c*e^2*f + 4*c*d*e*g))/(5*g^5) - (2*a*d^2*g^
4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2 - 2*b*d^2*f*g^3 - 2*b*e
^2*f^3*g + 4*b*d*e*f^2*g^2 - 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^
(1/2)) + ((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 + 4*b*d
*e*g^2 - 6*b*e^2*f*g - 12*c*d*e*f*g))/(3*g^5) + (2*(f + g*x)^(1/2)*(d*g - e
f)(2*a*e*g^2 + b*d*g^2 + 4*c*e*f^2 - 3*b*e*f*g - 2*c*d*f*g))/g^5 + (2*c*e
^2*(f + g*x)^(7/2))/(7*g^5)

3.828 $\int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{f+gx}}{g^4}-\frac{2(3cef-cdg-beg)(f}{3g^4}$$

```
[Out] -2/3*(-b*e*g-c*d*g+3*c*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(5/2)/g^4+2*(
-d*g+e*f)*(a*g^2-b*f*g+c*f^2)/g^4/(g*x+f)^(1/2)+2*(c*f*(-2*d*g+3*e*f)-g*(-a
*e*g-b*d*g+2*b*e*f))*(g*x+f)^(1/2)/g^4
```

Rubi [A]

time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {785}

$$\frac{2(ef-dg)\left(ag^2-bfg+cf^2\right)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}\,(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4}-\frac{2(f+gx)^{3/2}(-beg-cdg+3cef)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(c*f*(3*e*
f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*Sqrt[f + g*x])/g^4 - (2*(3*c*e*f
- c*d*g - b*e*g)*(f + g*x)^(3/2))/(3*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex)\left(a+bx+cx^2\right)}{(f+gx)^{3/2}}\,dx = \int \left(\frac{(-ef+dg)\left(cf^2-bfg+ag^2\right)}{g^3(f+gx)^{3/2}}+\frac{cf(3ef-2dg)-g(2bef-bdg-aeg}{g^3\sqrt{f+gx}}\right.$$

$$= \frac{2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^4\sqrt{f+gx}}+\frac{2(cf(3ef-2dg)-g(2bef-bdg-aeg))\sqrt{}}{g^4}$$

Mathematica [A]

time = 0.11, size = 128, normalized size = 0.95

$$\frac{2\left(5g(3bdg(2f+gx)+3ag(2ef-dg+egx)+be\left(-8f^2-4fgx+g^2x^2\right))+c\left(5dg\left(-8f^2-4fgx+g^2x^2\right)+3e\left(16f^3+8f^2gx-2fg^2x^2+g^3x^3\right)\right)\right)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]
```

```
[Out] (2*(5*g*(3*b*d*g*(2*f + g*x) + 3*a*g*(2*e*f - d*g + e*g*x) + b*e*(-8*f^2 -
4*f*g*x + g^2*x^2)) + c*(5*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e*(16*f^3 +
 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))))/(15*g^4*Sqrt[f + g*x])
```

Maple [A]
time = 0.08, size = 173, normalized size = 1.28

method	result
risch	$\frac{2\left(3ce\,x^2g^2+5bex\,g^2+5cdx\,g^2-9cefgx+15ae\,g^2+15bd\,g^2-25befg-25cdfg+33ce\,f^2\right)\sqrt{gx+f}}{15g^4}-\frac{2\left(ad\,g^3-aef\,g^2\cdots\right.}{\cdots}$
gosper	$-\frac{2\left(-3ce\,x^3g^3-5be\,g^3x^2-5cd\,g^3x^2+6cef\,g^2x^2-15ae\,g^3x-15bd\,g^3x+20bef\,g^2x+20cdf\,g^2x-24ce\,f^2gx+15ad\,g^3-30a\cdots\right.}{15\sqrt{gx+f}\,g^4}$
trager	$-\frac{2\left(-3ce\,x^3g^3-5be\,g^3x^2-5cd\,g^3x^2+6cef\,g^2x^2-15ae\,g^3x-15bd\,g^3x+20bef\,g^2x+20cdf\,g^2x-24ce\,f^2gx+15ad\,g^3-30a\cdots\right.}{15\sqrt{gx+f}\,g^4}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2beg(gx+f)^{\frac{3}{2}}}{3}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2a\,g^2e\sqrt{gx+f}+2bd\,g^2\sqrt{gx+f}-4befg\sqrt{gx+\cdots}}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2beg(gx+f)^{\frac{3}{2}}}{3}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2a\,g^2e\sqrt{gx+f}+2bd\,g^2\sqrt{gx+f}-4befg\sqrt{gx+\cdots}}{g^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^4*(1/5*c*e*(g*x+f)^(5/2)+1/3*b*e*g*(g*x+f)^(3/2)+1/3*c*d*g*(g*x+f)^(3/2
)-c*e*f*(g*x+f)^(3/2)+a*g^2*e*(g*x+f)^(1/2)+b*d*g^2*(g*x+f)^(1/2)-2*b*e*f*g
*(g*x+f)^(1/2)-2*c*d*f*g*(g*x+f)^(1/2)+3*c*f^2*e*(g*x+f)^(1/2)-(a*d*g^3-a*e
*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f^2*g-c*e*f^3)/(g*x+f)^(1/2))
```

Maxima [A]
time = 0.30, size = 146, normalized size = 1.08

$$\frac{2\left(\frac{3\,(gx+f)^{\frac{5}{2}}ce-5\,(3\,cfe-(cd+be)g)(gx+f)^{\frac{3}{2}}+15\,\left(3\,cf^2e-2\,(cd+be)fg+(bd+ae)g^2\right)\sqrt{gx+f}}{g^3}-\frac{15\,\left(adg^3-cf^3e+(cd+be)f^2g-(bd+ae)fg^2\right)}{\sqrt{gx+f}\,g^3}\right)}{15\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*f*e - (c*d + b*e)*g)*(g*x + f)^(3/2)
+ 15*(3*c*f^2*e - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*sqrt(g*x + f))/g^3 -
 15*(a*d*g^3 - c*f^3*e + (c*d + b*e)*f^2*g - (b*d + a*e)*f*g^2)/(sqrt(g*x +
 f)*g^3))/g
```

Fricas [A]
time = 3.76, size = 150, normalized size = 1.11

$$\frac{2\,{\left(5\,c d g^{3} x^{2} - 40\,c d f^{2} g + 30\,b d f g^{2} - 15\,a d g^{3} - 5\,{\left(4\,c d f g^{2} - 3\,b d g^{3}\right)} x + {\left(3\,c g^{3} x^{3} + 48\,c f^{3} - 40\,b f^{2} g + 30\,a f g^{2} - {\left(6\,c f g^{2} - 5\,b g^{3}\right)} x^{2} + {\left(24\,c f^{2} g - 20\,b f g^{2} + 15\,a g^{3}\right)} x\right)} e\right)} \sqrt{g x + f}}{15\,{\left(g^{5} x + f g^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*c*d*g^3*x^2 - 40*c*d*f^2*g + 30*b*d*f*g^2 - 15*a*d*g^3 - 5*(4*c*d*f
*g^2 - 3*b*d*g^3)*x + (3*c*g^3*x^3 + 48*c*f^3 - 40*b*f^2*g + 30*a*f*g^2 - (
6*c*f*g^2 - 5*b*g^3)*x^2 + (24*c*f^2*g - 20*b*f*g^2 + 15*a*g^3)*x)*e)*sqrt(
g*x + f)/(g^5*x + f*g^4)
```

Sympy [A]
time = 12.56, size = 141, normalized size = 1.04

$$\frac{2ce\left(f + gx\right)^{\frac{5}{2}}}{5g^{4}} + \frac{\left(f + gx\right)^{\frac{3}{2}} \cdot \left(2beg + 2cdg - 6cef\right)}{3g^{4}} + \frac{\sqrt{f + gx} \left(2aeg^{2} + 2bdg^{2} - 4befg - 4cdfg + 6cef^{2}\right)}{g^{4}} - \frac{2\left(dg - ef\right) \left(ag^{2} - bfg + cf^{2}\right)}{g^{4} \sqrt{f + gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*e*(f + g*x)**(5/2)/(5*g**4) + (f + g*x)**(3/2)*(2*b*e*g + 2*c*d*g - 6*c
*e*f)/(3*g**4) + sqrt(f + g*x)*(2*a*e*g**2 + 2*b*d*g**2 - 4*b*e*f*g - 4*c*d
*f*g + 6*c*e*f**2)/g**4 - 2*(d*g - e*f)*(a*g**2 - b*f*g + c*f**2)/(g**4*sqr
t(f + g*x))
```

Giac [A]
time = 4.00, size = 204, normalized size = 1.51

$$-\frac{2\,{\left(c d f^{2} g - b d f g^{2} + a d g^{3} - c f^{3} e + b f^{2} g e - a f g^{2} e\right)}}{\sqrt{g x + f} g^{4}} + \frac{2\,{\left(5\,{\left(g x + f\right)}^{\frac{3}{2}} c d g^{17} - 30\,\sqrt{g x + f} c d f g^{17} + 15\,\sqrt{g x + f} b d g^{18} + 3\,{\left(g x + f\right)}^{\frac{5}{2}} c g^{16} e - 15\,{\left(g x + f\right)}^{\frac{3}{2}} c f g^{16} e + 45\,\sqrt{g x + f} c f^{2} g^{16} e + 5\,{\left(g x + f\right)}^{\frac{3}{2}} b g^{17} e - 30\,\sqrt{g x + f} b f g^{17} e + 15\,\sqrt{g x + f} a g^{18} e\right)}}{15\,g^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*d*f^2*g - b*d*f*g^2 + a*d*g^3 - c*f^3*e + b*f^2*g*e - a*f*g^2*e)/(sqr
t(g*x + f)*g^4) + 2/15*(5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f
*g^17 + 15*sqrt(g*x + f)*b*d*g^18 + 3*(g*x + f)^(5/2)*c*g^16*e - 15*(g*x +
f)^(3/2)*c*f*g^16*e + 45*sqrt(g*x + f)*c*f^2*g^16*e + 5*(g*x + f)^(3/2)*b*g
^17*e - 30*sqrt(g*x + f)*b*f*g^17*e + 15*sqrt(g*x + f)*a*g^18*e)/g^20
```

Mupad [B]
time = 3.13, size = 147, normalized size = 1.09

$$\frac{{\left(f+g\,x\right)}^{3/2}\,\left(2\,b\,e\,g+2\,c\,d\,g-6\,c\,e\,f\right)}{3\,g^4}-\frac{2\,a\,d\,g^3-2\,c\,e\,f^3-2\,a\,e\,f\,g^2-2\,b\,d\,f\,g^2+2\,b\,e\,f^2\,g+2\,c\,d\,f^2\,g}{g^4\,\sqrt{f+g\,x}}+\frac{\sqrt{f+g\,x}\,\left(2\,a\,e\,g^2+2\,b\,d\,g^2+6\,c\,e\,f^2-4\,b\,e\,f\,g-4\,c\,d\,f\,g\right)}{g^4}+\frac{2\,c\,e\,{\left(f+g\,x\right)}^{5/2}}{5\,g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(a + b*x + c*x^2))/(f + g*x)^(3/2),x)
```

```
[Out] ((f + g*x)^(3/2)*(2*b*e*g + 2*c*d*g - 6*c*e*f))/(3*g^4) - (2*a*d*g^3 - 2*c*
e*f^3 - 2*a*e*f*g^2 - 2*b*d*f*g^2 + 2*b*e*f^2*g + 2*c*d*f^2*g)/(g^4*(f + g*
x)^(1/2)) + ((f + g*x)^(1/2)*(2*a*e*g^2 + 2*b*d*g^2 + 6*c*e*f^2 - 4*b*e*f*g
 - 4*c*d*f*g))/g^4 + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

3.829 $\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2(cf^2-bfg+ag^2)}{g^3\sqrt{f+gx}}-\frac{2(2cf-bg)\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

```
[Out] 2/3*c*(g*x+f)^(3/2)/g^3-2*(a*g^2-b*f*g+c*f^2)/g^3/(g*x+f)^(1/2)-2*(-b*g+2*c
*f)*(g*x+f)^(1/2)/g^3
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {712}

$$-\frac{2(ag^2-bfg+cf^2)}{g^3\sqrt{f+gx}}-\frac{2\sqrt{f+gx}\,(2cf-bg)}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g^3*Sqrt[f + g*x]) - (2*(2*c*f - b*g)*Sqrt[f
+ g*x])/g^3 + (2*c*(f + g*x)^(3/2))/(3*g^3)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{(f+gx)^{3/2}}\,dx &= \int \left(\frac{cf^2-bfg+ag^2}{g^2(f+gx)^{3/2}}+\frac{-2cf+bg}{g^2\sqrt{f+gx}}+\frac{c\sqrt{f+gx}}{g^2}\right)dx\\ &= -\frac{2(cf^2-bfg+ag^2)}{g^3\sqrt{f+gx}}-\frac{2(2cf-bg)\sqrt{f+gx}}{g^3}+\frac{2c(f+gx)^{3/2}}{3g^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.76

$$\frac{6g(2bf-ag+bgx)+2c(-8f^2-4fgx+g^2x^2)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(f + g*x)^(3/2),x]
```

```
[Out] (6*g*(2*b*f - a*g + b*g*x) + 2*c*(-8*f^2 - 4*f*g*x + g^2*x^2))/(3*g^3*Sqrt[
f + g*x])
```

Maple [A]
time = 0.07, size = 63, normalized size = 0.89

method	result	size
gosper	$-\frac{2\left(-c\,x^{2} g^{2}-3 b\,g^{2} x+4 c f g x+3 a\,g^{2}-6 b f g+8 c\,f^{2}\right)}{3 \sqrt{g x+f}\, g^{3}}$	53
trager	$-\frac{2\left(-c\,x^{2} g^{2}-3 b\,g^{2} x+4 c f g x+3 a\,g^{2}-6 b f g+8 c\,f^{2}\right)}{3 \sqrt{g x+f}\, g^{3}}$	53
risch	$\frac{2(c x g+3 b g-5 c f) \sqrt{g x+f}}{3 g^{3}}-\frac{2\left(a\,g^{2}-b f g+c\,f^{2}\right)}{g^{3} \sqrt{g x+f}}$	55
derivativedivides	$\frac{\frac{2 c (g x+f)^{\frac{3}{2}}}{3}+2 b g \sqrt{g x+f}-4 c f \sqrt{g x+f}-\frac{2\left(a\,g^{2}-b f g+c\,f^{2}\right)}{\sqrt{g x+f}}}{g^{3}}$	63
default	$\frac{\frac{2 c (g x+f)^{\frac{3}{2}}}{3}+2 b g \sqrt{g x+f}-4 c f \sqrt{g x+f}-\frac{2\left(a\,g^{2}-b f g+c\,f^{2}\right)}{\sqrt{g x+f}}}{g^{3}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^3*(1/3*c*(g*x+f)^(3/2)+b*g*(g*x+f)^(1/2)-2*c*f*(g*x+f)^(1/2)-(a*g^2-b*f
*g+c*f^2)/(g*x+f)^(1/2))
```

Maxima [A]
time = 0.30, size = 66, normalized size = 0.93

$$\frac{2\left(\frac{(gx+f)^{\frac{3}{2}}c-3\,(2\,cf-bg)\sqrt{gx+f}}{g^2}-\frac{3\,(cf^2-bfg+ag^2)}{\sqrt{gx+f}\,g^2}\right)}{3\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((g*x + f)^(3/2)*c - 3*(2*c*f - b*g)*sqrt(g*x + f))/g^2 - 3*(c*f^2 - b
*f*g + a*g^2)/(sqrt(g*x + f)*g^2))/g
```

Fricas [A]
time = 4.68, size = 63, normalized size = 0.89

$$\frac{2\,(cg^2x^2-8\,cf^2+6\,bfg-3\,ag^2-(4\,cfg-3\,bg^2)x)\sqrt{gx+f}}{3\,(g^4x+fg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(c*g^2*x^2 - 8*c*f^2 + 6*b*f*g - 3*a*g^2 - (4*c*f*g - 3*b*g^2)*x)*sqrt(
g*x + f)/(g^4*x + f*g^3)
```

Sympy [A]
time = 5.22, size = 70, normalized size = 0.99

$$\frac{2c(f+gx)^{\frac{3}{2}}}{3g^3}+\frac{\sqrt{f+gx}\,(2bg-4cf)}{g^3}-\frac{2(ag^2-bfg+cf^2)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(g*x+f)**(3/2),x)
```

```
[Out] 2*c*(f + g*x)**(3/2)/(3*g**3) + sqrt(f + g*x)*(2*b*g - 4*c*f)/g**3 - 2*(a*g
**2 - b*f*g + c*f**2)/(g**3*sqrt(f + g*x))
```

Giac [A]
time = 3.04, size = 74, normalized size = 1.04

$$-\frac{2\left(cf^2-bfg+ag^2\right)}{\sqrt{gx+f}\,g^3}+\frac{2\left((gx+f)^{\frac{3}{2}}cg^6-6\sqrt{gx+f}\,cfg^6+3\sqrt{gx+f}\,bg^7\right)}{3\,g^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 - b*f*g + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6
 - 6*sqrt(g*x + f)*c*f*g^6 + 3*sqrt(g*x + f)*b*g^7)/g^9
```

Mupad [B]
time = 0.06, size = 58, normalized size = 0.82

$$\frac{2\,c\,{\left(f+g\,x\right)}^2-6\,a\,g^2-6\,c\,f^2+6\,b\,g\,\left(f+g\,x\right)-12\,c\,f\,\left(f+g\,x\right)+6\,b\,f\,g}{3\,g^3\,\sqrt{f+g\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(f + g*x)^(3/2),x)
```

```
[Out] (2*c*(f + g*x)^2 - 6*a*g^2 - 6*c*f^2 + 6*b*g*(f + g*x) - 12*c*f*(f + g*x) +
 6*b*f*g)/(3*g^3*(f + g*x)^(1/2))
```

3.830 $\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2-bde+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

```
[Out] -2*(a*e^2-b*d*e+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3
/2)/(-d*g+e*f)^(3/2)+2*(a*g^2-b*f*g+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2*c
*(g*x+f)^(1/2)/e/g^2
```

Rubi [A]
time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {911, 1275, 214}

$$-\frac{2(ae^2-bde+cd^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{2(ag^2-bfg+cf^2)}{g^2\sqrt{f+gx}\,(ef-dg)}+\frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/(g^2*(e*f - d*g)*Sqrt[f + g*x]) + (2*c*Sqrt[f +
 g*x])/(e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]])/(e^(3/2)*(e*f - d*g)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\,dx,x,\sqrt{f+gx}\right)}{g}$$
$$= \frac{2\text{Subst}\left(\int \left(\frac{c}{eg}+\frac{cf^2-bfg+ag^2}{g(-ef+dg)x^2}-\frac{(cd^2-bde+ae^2)g}{e(ef-dg)(ef-dg-ex^2)}\right)dx,x,\sqrt{f+gx}\right)}{g}$$
$$= \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{(2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{ef-dg-ex^2}\ldots\right.}{e(ef-dg)}$$
$$= \frac{2(cf^2-bfg+ag^2)}{g^2(ef-dg)\sqrt{f+gx}}+\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(cd^2-bde+ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+\ldots}}{\sqrt{ef-\ldots}}\right.}{e^{3/2}(ef-dg)^{3/2}}$$

Mathematica [A]
time = 0.23, size = 124, normalized size = 1.02

$$\frac{2(eg(bf-ag)+cdg(f+gx)-cef(2f+gx))}{eg^2(-ef+dg)\sqrt{f+gx}}-\frac{2(cd^2+e(-bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g*x)))/(e*g^2*(-(e*f)
+ d*g)*Sqrt[f + g*x]) - (2*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[e]*Sqrt[
f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(3/2))
```

Maple [A]
time = 0.10, size = 122, normalized size = 1.00

method	result

derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}-\frac{2g^{2}\left(a\,e^{2}-bde+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{(dg-ef)e\sqrt{(dg-ef)\,e}}}{g^{2}}$
default	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2\left(a\,g^{2}-bfg+c\,f^{2}\right)}{(dg-ef)\sqrt{gx+f}}-\frac{2g^{2}\left(a\,e^{2}-bde+c\,d^{2}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)\,e}}\right)}{(dg-ef)e\sqrt{(dg-ef)\,e}}}{g^{2}}$
risch	$\frac{2c\sqrt{gx+f}}{e\,g^{2}}-\frac{2a}{(dg-ef)\sqrt{gx+f}}+\frac{2bf}{g(dg-ef)\sqrt{gx+f}}-\frac{2c\,f^{2}}{g^{2}(dg-ef)\sqrt{gx+f}}-\frac{2e\arctan\left(\cdots\right)}{(dg-ef)\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/g^2*(c/e*(g*x+f)^(1/2)-(a*g^2-b*f*g+c*f^2)/(d*g-e*f)/(g*x+f)^(1/2)-g^2*(a
*e^2-b*d*e+c*d^2)/(d*g-e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((
d*g-e*f)*e)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(108) = 216.
time = 4.81, size = 543, normalized size = 4.45

$$\left[\frac{\left(cd^{2}g^{3}x+cd^{2}fg^{2}+\left(ag^{3}x+afg^{2}\right)e^{2}-\left(bdg^{3}x+bdfg^{2}\right)e\right)\sqrt{-dge+fe^{2}}\log\left(-\frac{dg-(gx+2f)e+2\sqrt{-dge+fe^{2}}\sqrt{gx+f}}{xe+d}\right)+2\sqrt{gx+f}\left(\left(cf^{2}gx+2cf^{3}-bf^{2}g+afg^{2}\right)e^{3}-\left(2cdfg^{2}x+3cdf^{2}g-bdfg^{2}+adg^{3}\right)e^{2}+\left(cd^{2}g^{3}x+cd^{2}fg^{2}\right)e\right)}{\left(f^{2}g^{3}x+f^{3}g^{2}\right)e^{4}-2\left(dfg^{4}x+df^{2}g^{3}\right)e^{3}+\left(d^{2}g^{5}x+d^{2}fg^{4}\right)e^{2}},\ \frac{2\left(\left(cd^{2}g^{3}x+cd^{2}fg^{2}+\left(ag^{3}x+afg^{2}\right)e^{2}-\left(bdg^{3}x+bdfg^{2}\right)e\right)\sqrt{dge-fe^{2}}\arctan\left(-\frac{\sqrt{dge-fe^{2}}\sqrt{gx+f}}{dg-fe}\right)+\sqrt{gx+f}\left(\left(cf^{2}gx+2cf^{3}-bf^{2}g+afg^{2}\right)e^{3}-\left(2cdfg^{2}x+3cdf^{2}g-bdfg^{2}+adg^{3}\right)e^{2}+\left(cd^{2}g^{3}x+cd^{2}fg^{2}\right)e\right)\right)}{\left(f^{2}g^{3}x+f^{3}g^{2}\right)e^{4}-2\left(dfg^{4}x+df^{2}g^{3}\right)e^{3}+\left(d^{2}g^{5}x+d^{2}fg^{4}\right)e^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [((c*d^2*g^3*x + c*d^2*f*g^2 + (a*g^3*x + a*f*g^2)*e^2 - (b*d*g^3*x + b*d*f
*g^2)*e)*sqrt(-d*g*e + f*e^2)*log(-(d*g - (g*x + 2*f)*e + 2*sqrt(-d*g*e + f
```

```
*e^2)*sqrt(g*x + f))/(x*e + d)) + 2*sqrt(g*x + f)*((c*f^2*g*x + 2*c*f^3 - b
*f^2*g + a*f*g^2)*e^3 - (2*c*d*f*g^2*x + 3*c*d*f^2*g - b*d*f*g^2 + a*d*g^3)
*e^2 + (c*d^2*g^3*x + c*d^2*f*g^2)*e))/((f^2*g^3*x + f^3*g^2)*e^4 - 2*(d*f*
g^4*x + d*f^2*g^3)*e^3 + (d^2*g^5*x + d^2*f*g^4)*e^2), 2*((c*d^2*g^3*x + c*
d^2*f*g^2 + (a*g^3*x + a*f*g^2)*e^2 - (b*d*g^3*x + b*d*f*g^2)*e)*sqrt(d*g*e
 - f*e^2)*arctan(-sqrt(d*g*e - f*e^2)*sqrt(g*x + f)/(d*g - f*e)) + sqrt(g*x
 + f)*((c*f^2*g*x + 2*c*f^3 - b*f^2*g + a*f*g^2)*e^3 - (2*c*d*f*g^2*x + 3*c
*d*f^2*g - b*d*f*g^2 + a*d*g^3)*e^2 + (c*d^2*g^3*x + c*d^2*f*g^2)*e))/((f^2
*g^3*x + f^3*g^2)*e^4 - 2*(d*f*g^4*x + d*f^2*g^3)*e^3 + (d^2*g^5*x + d^2*f*
g^4)*e^2)]
```

Sympy [A]
time = 19.69, size = 116, normalized size = 0.95

$$\frac{2c\sqrt{f+gx}}{eg^2}-\frac{2(ag^2-bfg+cf^2)}{g^2\sqrt{f+gx}\,(dg-ef)}-\frac{2(ae^2-bde+cd^2)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}\,(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f)**(3/2),x)
[Out] 2*c*sqrt(f + g*x)/(e*g**2) - 2*(a*g**2 - b*f*g + c*f**2)/(g**2*sqrt(f + g*x
)*(d*g - e*f)) - 2*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g -
 e*f)/e))/(e**2*sqrt((d*g - e*f)/e)*(d*g - e*f))
```

Giac [A]
time = 3.44, size = 112, normalized size = 0.92

$$-\frac{2\,(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{(dge-fe^2)^{\frac{3}{2}}}+\frac{2\,\sqrt{gx+f}\,ce^{(-1)}}{g^2}-\frac{2\,(cf^2-bfg+ag^2)}{(dg^3-fg^2e)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
[Out] -2*(c*d^2 - b*d*e + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/(d*g
*e - f*e^2)^(3/2) + 2*sqrt(g*x + f)*c*e^(-1)/g^2 - 2*(c*f^2 - b*f*g + a*g^2
)/((d*g^3 - f*g^2*e)*sqrt(g*x + f))
```

Mupad [B]
time = 3.21, size = 162, normalized size = 1.33

$$\frac{2\,c\,\sqrt{f+g\,x}}{e\,g^2}+\frac{2\operatorname{atan}\left(\frac{2\,\sqrt{f+g\,x}\,\left(e^2\,f-d\,e\,g\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{3/2}\,\left(2\,c\,d^2-2\,b\,d\,e+2\,a\,e^2\right)}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}}-\frac{2\,\left(c\,e\,f^2-b\,e\,f\,g+a\,e\,g^2\right)}{e\,g^2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

```
[Out] (2*c*(f + g*x)^(1/2))/(e*g^2) + (2*atan((2*(f + g*x)^(1/2)*(e^2*f - d*e*g)*
(a*e^2 + c*d^2 - b*d*e))/(e^(1/2)*(d*g - e*f)^(3/2)*(2*a*e^2 + 2*c*d^2 - 2*
b*d*e)))*(a*e^2 + c*d^2 - b*d*e))/(e^(3/2)*(d*g - e*f)^(3/2)) - (2*(a*e*g^2
 + c*e*f^2 - b*e*f*g))/(e*g^2*(f + g*x)^(1/2)*(d*g - e*f))
```

3.831 $\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2(cf^2-bfg+ag^2)}{g(ef-dg)^2\sqrt{f+gx}}-\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)}+\frac{(cd(4ef-dg)-e(2bef+bdg-3aeg))\tanh^{-1}\left(\sqrt{\cdots}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

```
[Out] (c*d*(-d*g+4*e*f)-e*(-3*a*e*g+b*d*g+2*b*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)
/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(5/2)-2*(a*g^2-b*f*g+c*f^2)/g/(-d*g+e
*f)^2/(g*x+f)^(1/2)-(a*e^2-b*d*e+c*d^2)*(g*x+f)^(1/2)/e/(-d*g+e*f)^2/(e*x+d
)
```

Rubi [A]
time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {911, 1273, 464, 214}

$$-\frac{\sqrt{f+gx}\,(ae^2-bde+cd^2)}{e(d+ex)(ef-dg)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg)-e(-3aeg+bdg+2bef))}{e^{3/2}(ef-dg)^{5/2}}-\frac{2(ag^2-bfg+cf^2)}{g\sqrt{f+gx}\,(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

```
[Out] (-2*(c*f^2 - b*f*g + a*g^2))/(g*(e*f - d*g)^2*Sqrt[f + g*x]) - ((c*d^2 - b*
d*e + a*e^2)*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((c*d*(4*e*f - d*
g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])/(e^(3/2)*(e*f - d*g)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^2}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{g^3\text{Subst}\left(\int \frac{\frac{2e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5}-\frac{e\left(e(bd-ae)g^2+\cdots\right.}{\cdots}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)\cdots}\cdots\right)}{e^2(ef-d\cdots}$$

$$= -\frac{2(cf^2-bfg+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} - \frac{(cd(4ef-dg)-e\cdots}{\cdots}$$

$$= -\frac{2(cf^2-bfg+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(cd(4ef-dg)-e\cdots}{\cdots}$$

Mathematica [A]
time = 0.65, size = 176, normalized size = 1.07

$$\frac{-c(2def^2+2e^2f^2x+d^2g(f+gx))+eg(b(3df+2efx+dgx)-a(ef+2dg+3egx))}{eg(ef-dg)^2(d+ex)\sqrt{f+gx}}+\frac{(cd(-4ef+dg)+e(2bef+bdg-3aeg))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]

[Out] (-(c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x))) + e*g*(b*(3*d*f + 2*e*f*x + d*g*x) - a*(e*f + 2*d*g + 3*e*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f + g*x]) + ((c*d*(-4*e*f + d*g) + e*(2*b*e*f + b*d*g - 3*a*e*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(5/2))

Maple [A]
time = 0.09, size = 175, normalized size = 1.06

method	result
derivativedivides	$\frac{-\frac{2g\left(\frac{g\left(a e^2-bde+c d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a e^2g-bdeg-2b e^2f-c d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a g^2-bfg+\right.}{(dg-ef)^2\sqrt{gx}}}{g}$
default	$\frac{-\frac{2g\left(\frac{g\left(a e^2-bde+c d^2\right)\sqrt{gx+f}}{2e(e(gx+f)+dg-ef)}+\frac{\left(3a e^2g-bdeg-2b e^2f-c d^2g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2e\sqrt{(dg-ef)e}}\right)}{(dg-ef)^2}-\frac{2\left(a g^2-bfg+\right.}{(dg-ef)^2\sqrt{gx}}}{g}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/g*(-g/(d*g-e*f)^2*(1/2*g*(a*e^2-b*d*e+c*d^2)/e*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*f)+1/2*(3*a*e^2*g-b*d*e*g-2*b*e^2*f-c*d^2*g+4*c*d*e*f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))-(a*g^2-b*f*g+c*f^2)/(d*g-e*f)^2/(g*x+f)^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?` for m

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(156) = 312$.
time = 12.16, size = 1110, normalized size = 6.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^3*g^3*x + c*d^3*f*g^2 + ((2*b*f*g^2 - 3*a*g^3)*x^2 + (2*b*f^2*g
 - 3*a*f*g^2)*x)*e^3 + (2*b*d*f^2*g - 3*a*d*f*g^2 - (4*c*d*f*g^2 - b*d*g^3)
*x^2 - (4*c*d*f^2*g - 3*b*d*f*g^2 + 3*a*d*g^3)*x)*e^2 + (c*d^2*g^3*x^2 - 4*
c*d^2*f^2*g + b*d^2*f*g^2 - (3*c*d^2*f*g^2 - b*d^2*g^3)*x)*e)*sqrt(-d*g*e +
 f*e^2)*log(-(d*g - (g*x + 2*f)*e - 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(
x*e + d)) + 2*sqrt(g*x + f)*((a*f^2*g + (2*c*f^3 - 2*b*f^2*g + 3*a*f*g^2)*x
)*e^4 + (2*c*d*f^3 - 3*b*d*f^2*g + a*d*f*g^2 - (2*c*d*f^2*g - b*d*f*g^2 + 3
*a*d*g^3)*x)*e^3 - (c*d^2*f^2*g - 3*b*d^2*f*g^2 + 2*a*d^2*g^3 - (c*d^2*f*g^
2 + b*d^2*g^3)*x)*e^2 - (c*d^3*g^3*x + c*d^3*f*g^2)*e))/((f^3*g^2*x^2 + f^4
*g*x)*e^6 - (3*d*f^2*g^3*x^2 + 2*d*f^3*g^2*x - d*f^4*g)*e^5 + 3*(d^2*f*g^4*
x^2 - d^2*f^3*g^2)*e^4 - (d^3*g^5*x^2 - 2*d^3*f*g^4*x - 3*d^3*f^2*g^3)*e^3
 - (d^4*g^5*x + d^4*f*g^4)*e^2), ((c*d^3*g^3*x + c*d^3*f*g^2 + ((2*b*f*g^2 -
 3*a*g^3)*x^2 + (2*b*f^2*g - 3*a*f*g^2)*x)*e^3 + (2*b*d*f^2*g - 3*a*d*f*g^2
 - (4*c*d*f*g^2 - b*d*g^3)*x^2 - (4*c*d*f^2*g - 3*b*d*f*g^2 + 3*a*d*g^3)*x)
*e^2 + (c*d^2*g^3*x^2 - 4*c*d^2*f^2*g + b*d^2*f*g^2 - (3*c*d^2*f*g^2 - b*d^
2*g^3)*x)*e)*sqrt(d*g*e - f*e^2)*arctan(-sqrt(d*g*e - f*e^2)*sqrt(g*x + f)/
(d*g - f*e)) - sqrt(g*x + f)*((a*f^2*g + (2*c*f^3 - 2*b*f^2*g + 3*a*f*g^2)*
x)*e^4 + (2*c*d*f^3 - 3*b*d*f^2*g + a*d*f*g^2 - (2*c*d*f^2*g - b*d*f*g^2 +
3*a*d*g^3)*x)*e^3 - (c*d^2*f^2*g - 3*b*d^2*f*g^2 + 2*a*d^2*g^3 - (c*d^2*f*g
^2 + b*d^2*g^3)*x)*e^2 - (c*d^3*g^3*x + c*d^3*f*g^2)*e))/((f^3*g^2*x^2 + f^
4*g*x)*e^6 - (3*d*f^2*g^3*x^2 + 2*d*f^3*g^2*x - d*f^4*g)*e^5 + 3*(d^2*f*g^4
*x^2 - d^2*f^3*g^2)*e^4 - (d^3*g^5*x^2 - 2*d^3*f*g^4*x - 3*d^3*f^2*g^3)*e^3
 - (d^4*g^5*x + d^4*f*g^4)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.49, size = 282, normalized size = 1.71

$$\frac{\left(cd^2g - 4\,cdfe + bdge + 2\,bfe^2 - 3\,age^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{\left(d^2g^2e - 2\,dfge^2 + f^2e^3\right)\sqrt{dge-fe^2}} - \frac{(gx+f)cd^2g^2 + 2\,cdf^2ge - (gx+f)bdg^2e - 2\,bdfg^2e + 2\,adg^3e + 2\,(gx+f)cf^2e^2 - 2\,cf^3e^2 - 2\,(gx+f)bfge^2 + 2\,bf^2ge^2 + 3\,(gx+f)ag^2e^2 - 2\,afg^2e^2}{\left(d^2g^3e - 2\,dfg^2e^2 + f^2ge^3\right)\left(\sqrt{gx+f}\,dg + (gx+f)^{\frac{3}{2}}e - \sqrt{gx+f}\,fe\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] (c*d^2*g - 4*c*d*f*e + b*d*g*e + 2*b*f*e^2 - 3*a*g*e^2)*arctan(sqrt(g*x + f
)*e/sqrt(d*g*e - f*e^2))/((d^2*g^2*e - 2*d*f*g*e^2 + f^2*e^3)*sqrt(d*g*e -
f*e^2)) - ((g*x + f)*c*d^2*g^2 + 2*c*d*f^2*g*e - (g*x + f)*b*d*g^2*e - 2*b*
d*f*g^2*e + 2*a*d*g^3*e + 2*(g*x + f)*c*f^2*e^2 - 2*c*f^3*e^2 - 2*(g*x + f)
*b*f*g*e^2 + 2*b*f^2*g*e^2 + 3*(g*x + f)*a*g^2*e^2 - 2*a*f*g^2*e^2)/((d^2*g
^3*e - 2*d*f*g^2*e^2 + f^2*g*e^3)*(sqrt(g*x + f)*d*g + (g*x + f)^(3/2)*e -
sqrt(g*x + f)*f*e))
```

Mupad [B]
time = 0.30, size = 218, normalized size = 1.32

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{5/2}}\right)\,\left(2\,b\,e^2\,f-3\,a\,e^2\,g+c\,d^2\,g+b\,d\,e\,g-4\,c\,d\,e\,f\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{5/2}}-\frac{\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2-b\,d\,e\,g^2+2\,c\,e^2\,f^2-2\,b\,e^2\,f\,g+3\,a\,e^2\,g^2\right)}{e\,{\left(d\,g-e\,f\right)}^2}}{\sqrt{f+g\,x}\,\left(d\,g^2-e\,f\,g\right)+e\,g\,{\left(f+g\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*g))/(e^(1/2)*(d*g -
 e*f)^(5/2)))*(2*b*e^2*f - 3*a*e^2*g + c*d^2*g + b*d*e*g - 4*c*d*e*f))/(e^(
3/2)*(d*g - e*f)^(5/2)) - ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f +
g*x)*(3*a*e^2*g^2 + c*d^2*g^2 + 2*c*e^2*f^2 - b*d*e*g^2 - 2*b*e^2*f*g))/(e*
(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(f + g*x)^(3/2))
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal. Leaf size=248

$$\frac{2(cf^2-bfg+ag^2)}{(ef-dg)^3\sqrt{f+gx}}-\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2}+\frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)}-$$

```
[Out] -1/4*(c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2)+3*e*g*(5*a*e*g-b*(d*g+4*e*f)))*arcta
nh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)+2*(a*g^
2-b*f*g+c*f^2)/(-d*g+e*f)^3/(g*x+f)^(1/2)-1/2*(a*e^2-b*d*e+c*d^2)*(g*x+f)^(
1/2)/e/(-d*g+e*f)^2/(e*x+d)^2+1/4*(c*d*(-d*g+8*e*f)-e*(-7*a*e*g+3*b*d*g+4*b
*e*f))*(g*x+f)^(1/2)/e/(-d*g+e*f)^3/(e*x+d)
```

Rubi [A]

time = 0.40, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {911, 1273, 467, 464, 214}

$$-\frac{\sqrt{f+gx}\,(ae^2-bde+cd^2)}{2e(d+ex)^2(ef-dg)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(3eg(5aeg-b(dg+4ef))+c(-d^2g^2+8defg+8e^2f^2))}{4e^{3/2}(ef-dg)^{7/2}}+\frac{2(ag^2-bfg+cf^2)}{\sqrt{f+gx}\,(ef-dg)^3}+\frac{\sqrt{f+gx}\,(cd(8ef-dg)-e(-7aeg+3bdg+4bef))}{4e(d+ex)(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] (2*(c*f^2 - b*f*g + a*g^2))/((e*f - d*g)^3*Sqrt[f + g*x]) - ((c*d^2 - b*d*e
 + a*e^2)*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(d + e*x)^2) + ((c*d*(8*e*f - d
*g) - e*(4*b*e*f + 3*b*d*g - 7*a*e*g))*Sqrt[f + g*x])/(4*e*(e*f - d*g)^3*(d
 + e*x)) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a*e*g - b*(4*e*
f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(4*e^(3/2)*(e*
f - d*g)^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\text{Subst}\left(\int \frac{\frac{cf^2-bfg+ag^2}{g^2}-\frac{(2cf-bg)x^2}{g^2}+\frac{cx^4}{g^2}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)^3}\,dx, x, \sqrt{f+gx}\right)}{g}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} - \frac{g^3\text{Subst}\left(\int \frac{\frac{4e^2(ef-dg)\left(cf^2-bfg+ag^2\right)}{g^5} - \frac{e\left(3e(bd-ae)g^2+\right.}{}}{x^2\left(\frac{-ef+dg}{g}+\frac{ex^2}{g}\right)}\right.}{2e^2(ef-}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4bef+3bdg-7aeg))\sqrt{}}{4e(ef-dg)^3(d+ex)}$$

$$= \frac{2(cf^2-bfg+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4b}{4e(ef}$$

$$= \frac{2(cf^2-bfg+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2-bde+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(cd(8ef-dg)-e(4b}{4e(ef}$$

Mathematica [**A**]
time = 1.31, size = 297, normalized size = 1.20

$$\frac{\frac{\sqrt{e}\left(c(8e^3f^2x^2+d^3g(f+gx)+8de^2fx(3f+gx)+d^2e(14f^2+5fgx-g^2x^2))-e(a(-8d^2g^2-deg(9f+25gx)+e^2(2f^2-5fgx-15g^2x^2))+b(4e^2fx(f+3gx)+d^2g(13f+5gx)+de(2f^2+21fgx+3g^2x^2)))\right)}{(ef-dg)^3(d+ex)^2\sqrt{f+gx}} - \frac{(c(8e^2f^2+8defg-d^2g^2)+3eg(5aeg-b(4ef+dg)))\tan^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{(-ef+dg)^{7/2}}}{4e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

```
[Out] ((Sqrt[e]*(c*(8*e^3*f^2*x^2 + d^3*g*(f + g*x) + 8*d*e^2*f*x*(3*f + g*x) + d
^2*e*(14*f^2 + 5*f*g*x - g^2*x^2)) - e*(a*(-8*d^2*g^2 - d*e*g*(9*f + 25*g*x
) + e^2*(2*f^2 - 5*f*g*x - 15*g^2*x^2)) + b*(4*e^2*f*x*(f + 3*g*x) + d^2*g*
(13*f + 5*g*x) + d*e*(2*f^2 + 21*f*g*x + 3*g^2*x^2)))))/((e*f - d*g)^3*(d +
 e*x)^2*Sqrt[f + g*x]) - ((c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3*e*g*(5*a
*e*g - b*(4*e*f + d*g)))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]]
)/(-(e*f) + d*g)^(7/2))/(4*e^(3/2))
```

Maple [**A**]
time = 0.10, size = 294, normalized size = 1.19

method	result

derivativedivides	$-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{3}{8}bde\,g^2-\frac{1}{2}b\,e^2fg-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg-5b\,d^2e\,g^2+bd\,e^2fg+4b\,e^3f^2+c\,d^3g^2+7c\,d^2efg\right.}{8e}}{(e(gx+f)+dg-ef)^2}\right.}{(dg-ef)}$
default	$-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{3}{8}bde\,g^2-\frac{1}{2}b\,e^2fg-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg-5b\,d^2e\,g^2+bd\,e^2fg+4b\,e^3f^2+c\,d^3g^2+7c\,d^2efg\right.}{8e}}{(e(gx+f)+dg-ef)^2}\right.}{(dg-ef)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(d*g-e*f)^3*(((7/8*a*e^2*g^2-3/8*b*d*e*g^2-1/2*b*e^2*f*g-1/8*c*d^2*g^2+c
*d*e*f*g)*(g*x+f)^(3/2)+1/8*g*(9*a*d*e^2*g^2-9*a*e^3*f*g-5*b*d^2*e*g^2+b*d*
e^2*f*g+4*b*e^3*f^2+c*d^3*g^2+7*c*d^2*e*f*g-8*c*d*e^2*f^2)/e*(g*x+f)^(1/2))
/(e*(g*x+f)+d*g-e*f)^2+1/8*(15*a*e^2*g^2-3*b*d*e*g^2-12*b*e^2*f*g-c*d^2*g^2
+8*c*d*e*f*g+8*c*e^2*f^2)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*
g-e*f)*e)^(1/2)))-2*(a*g^2-b*f*g+c*f^2)/(d*g-e*f)^3/(g*x+f)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e^2*f-4*%e*d*g>0)', see `assume?
` for m
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(234) = 468$.
time = 6.47, size = 1888, normalized size = 7.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((c*d^4*g^3*x + c*d^4*f*g^2 - ((8*c*f^2*g - 12*b*f*g^2 + 15*a*g^3)*x^
3 + (8*c*f^3 - 12*b*f^2*g + 15*a*f*g^2)*x^2)*e^4 - ((8*c*d*f*g^2 - 3*b*d*g^
3)*x^3 + 3*(8*c*d*f^2*g - 9*b*d*f*g^2 + 10*a*d*g^3)*x^2 + 2*(8*c*d*f^3 - 12
*b*d*f^2*g + 15*a*d*f*g^2)*x)*e^3 + (c*d^2*g^3*x^3 - 8*c*d^2*f^3 + 12*b*d^2
*f^2*g - 15*a*d^2*f*g^2 - 3*(5*c*d^2*f*g^2 - 2*b*d^2*g^3)*x^2 - 3*(8*c*d^2*
f^2*g - 6*b*d^2*f*g^2 + 5*a*d^2*g^3)*x)*e^2 + (2*c*d^3*g^3*x^2 - 8*c*d^3*f^
2*g + 3*b*d^3*f*g^2 - 3*(2*c*d^3*f*g^2 - b*d^3*g^3)*x)*e)*sqrt(-d*g*e + f*e
^2)*log(-(d*g - (g*x + 2*f)*e + 2*sqrt(-d*g*e + f*e^2)*sqrt(g*x + f))/(x*e
+ d)) + 2*sqrt(g*x + f)*((2*a*f^3 - (8*c*f^3 - 12*b*f^2*g + 15*a*f*g^2)*x^2
 + (4*b*f^3 - 5*a*f^2*g)*x)*e^5 + (2*b*d*f^3 - 11*a*d*f^2*g - 3*(3*b*d*f*g^
2 - 5*a*d*g^3)*x^2 - (24*c*d*f^3 - 17*b*d*f^2*g + 20*a*d*f*g^2)*x)*e^4 - (1
4*c*d^2*f^3 - 11*b*d^2*f^2*g - a*d^2*f*g^2 - 3*(3*c*d^2*f*g^2 - b*d^2*g^3)*
x^2 - (19*c*d^2*f^2*g - 16*b*d^2*f*g^2 + 25*a*d^2*g^3)*x)*e^3 - (c*d^3*g^3*
x^2 - 13*c*d^3*f^2*g + 13*b*d^3*f*g^2 - 8*a*d^3*g^3 - (4*c*d^3*f*g^2 - 5*b*
d^3*g^3)*x)*e^2 + (c*d^4*g^3*x + c*d^4*f*g^2)*e))/((f^4*g*x^3 + f^5*x^2)*e^
8 - 2*(2*d*f^3*g^2*x^3 + d*f^4*g*x^2 - d*f^5*x)*e^7 + (6*d^2*f^2*g^3*x^3 -
2*d^2*f^3*g^2*x^2 - 7*d^2*f^4*g*x + d^2*f^5)*e^6 - 4*(d^3*f*g^4*x^3 - 2*d^3
*f^2*g^3*x^2 - 2*d^3*f^3*g^2*x + d^3*f^4*g)*e^5 + (d^4*g^5*x^3 - 7*d^4*f*g^
4*x^2 - 2*d^4*f^2*g^3*x + 6*d^4*f^3*g^2)*e^4 + 2*(d^5*g^5*x^2 - d^5*f*g^4*x
 - 2*d^5*f^2*g^3)*e^3 + (d^6*g^5*x + d^6*f*g^4)*e^2), -1/4*((c*d^4*g^3*x +
c*d^4*f*g^2 - ((8*c*f^2*g - 12*b*f*g^2 + 15*a*g^3)*x^3 + (8*c*f^3 - 12*b*f^
2*g + 15*a*f*g^2)*x^2)*e^4 - ((8*c*d*f*g^2 - 3*b*d*g^3)*x^3 + 3*(8*c*d*f^2*
g - 9*b*d*f*g^2 + 10*a*d*g^3)*x^2 + 2*(8*c*d*f^3 - 12*b*d*f^2*g + 15*a*d*f*
g^2)*x)*e^3 + (c*d^2*g^3*x^3 - 8*c*d^2*f^3 + 12*b*d^2*f^2*g - 15*a*d^2*f*g^
2 - 3*(5*c*d^2*f*g^2 - 2*b*d^2*g^3)*x^2 - 3*(8*c*d^2*f^2*g - 6*b*d^2*f*g^2
+ 5*a*d^2*g^3)*x)*e^2 + (2*c*d^3*g^3*x^2 - 8*c*d^3*f^2*g + 3*b*d^3*f*g^2 -
3*(2*c*d^3*f*g^2 - b*d^3*g^3)*x)*e)*sqrt(d*g*e - f*e^2)*arctan(-sqrt(d*g*e
- f*e^2)*sqrt(g*x + f)/(d*g - f*e)) + sqrt(g*x + f)*((2*a*f^3 - (8*c*f^3 -
12*b*f^2*g + 15*a*f*g^2)*x^2 + (4*b*f^3 - 5*a*f^2*g)*x)*e^5 + (2*b*d*f^3 -
11*a*d*f^2*g - 3*(3*b*d*f*g^2 - 5*a*d*g^3)*x^2 - (24*c*d*f^3 - 17*b*d*f^2*g
 + 20*a*d*f*g^2)*x)*e^4 - (14*c*d^2*f^3 - 11*b*d^2*f^2*g - a*d^2*f*g^2 - 3*
(3*c*d^2*f*g^2 - b*d^2*g^3)*x^2 - (19*c*d^2*f^2*g - 16*b*d^2*f*g^2 + 25*a*d
^2*g^3)*x)*e^3 - (c*d^3*g^3*x^2 - 13*c*d^3*f^2*g + 13*b*d^3*f*g^2 - 8*a*d^3
*g^3 - (4*c*d^3*f*g^2 - 5*b*d^3*g^3)*x)*e^2 + (c*d^4*g^3*x + c*d^4*f*g^2)*e
))/((f^4*g*x^3 + f^5*x^2)*e^8 - 2*(2*d*f^3*g^2*x^3 + d*f^4*g*x^2 - d*f^5*x)
*e^7 + (6*d^2*f^2*g^3*x^3 - 2*d^2*f^3*g^2*x^2 - 7*d^2*f^4*g*x + d^2*f^5)*e^
6 - 4*(d^3*f*g^4*x^3 - 2*d^3*f^2*g^3*x^2 - 2*d^3*f^3*g^2*x + d^3*f^4*g)*e^5
 + (d^4*g^5*x^3 - 7*d^4*f*g^4*x^2 - 2*d^4*f^2*g^3*x + 6*d^4*f^3*g^2)*e^4 +
2*(d^5*g^5*x^2 - d^5*f*g^4*x - 2*d^5*f^2*g^3)*e^3 + (d^6*g^5*x + d^6*f*g^4)
*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.43, size = 462, normalized size = 1.86

$$\frac{\left(cd^2g^2-8\,cdfge+3\,bdg^2e-8\,cf^2e^2+12\,bfge^2-15\,ag^2e^2\right)\arctan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{dge-fe^2}}\right)}{4\left(d^3g^3e-3\,d^2fg^2e^2+3\,df^2ge^3-f^3e^4\right)\sqrt{dge-fe^2}}-\frac{2\left(cf^2-bfg+ag^2\right)}{\left(d^3g^3-3\,d^2fg^2e+3\,df^2ge^2-f^3e^3\right)\sqrt{gx+f}}-\frac{\sqrt{gx+f}\,cd^3g^3-(gx+f)^{\frac{3}{2}}cd^2g^2e+7\,\sqrt{gx+f}\,cd^2fg^2e-5\,\sqrt{gx+f}\,bd^2g^3e+8\,(gx+f)^{\frac{3}{2}}cdfge^2-8\,\sqrt{gx+f}\,cdf^2ge^2-3\,(gx+f)^{\frac{3}{2}}bdg^2e^2+\sqrt{gx+f}\,bdfg^2e^2+9\,\sqrt{gx+f}\,adg^3e^2-4\,(gx+f)^{\frac{3}{2}}bfge^3+4\,\sqrt{gx+f}\,bf^2ge^3+7\,(gx+f)^{\frac{3}{2}}ag^2e^3-9\,\sqrt{gx+f}\,afg^2e^3}{4\left(d^3g^3e-3\,d^2fg^2e^2+3\,df^2ge^3-f^3e^4\right)\left(dg+(gx+f)e-fe\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(c*d^2*g^2 - 8*c*d*f*g*e + 3*b*d*g^2*e - 8*c*f^2*e^2 + 12*b*f*g*e^2 - 1
5*a*g^2*e^2)*arctan(sqrt(g*x + f)*e/sqrt(d*g*e - f*e^2))/((d^3*g^3*e - 3*d^
2*f*g^2*e^2 + 3*d*f^2*g*e^3 - f^3*e^4)*sqrt(d*g*e - f*e^2)) - 2*(c*f^2 - b*
f*g + a*g^2)/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(g*x
+ f)) - 1/4*(sqrt(g*x + f)*c*d^3*g^3 - (g*x + f)^(3/2)*c*d^2*g^2*e + 7*sqrt
(g*x + f)*c*d^2*f*g^2*e - 5*sqrt(g*x + f)*b*d^2*g^3*e + 8*(g*x + f)^(3/2)*c
*d*f*g*e^2 - 8*sqrt(g*x + f)*c*d*f^2*g*e^2 - 3*(g*x + f)^(3/2)*b*d*g^2*e^2
+ sqrt(g*x + f)*b*d*f*g^2*e^2 + 9*sqrt(g*x + f)*a*d*g^3*e^2 - 4*(g*x + f)^(
3/2)*b*f*g*e^3 + 4*sqrt(g*x + f)*b*f^2*g*e^3 + 7*(g*x + f)^(3/2)*a*g^2*e^3
- 9*sqrt(g*x + f)*a*f*g^2*e^3)/((d^3*g^3*e - 3*d^2*f*g^2*e^2 + 3*d*f^2*g*e^
3 - f^3*e^4)*(d*g + (g*x + f)*e - f*e)^2)
```

Mupad [B]
time = 3.41, size = 363, normalized size = 1.46

$$\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2+8\,c\,e^2\,f^2-12\,b\,e^2\,f\,g+15\,a\,e^2\,g^2\right)}{4\,e^{3/2}\,{\left(d\,g-e\,f\right)}^{7/2}}-\frac{\frac{2\,\left(c\,f^2-b\,f\,g+a\,g^2\right)}{d\,g-e\,f}+\frac{{\left(f+g\,x\right)}^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g-3\,b\,d\,e\,g^2+8\,c\,e^2\,f^2-12\,b\,e^2\,f\,g+15\,a\,e^2\,g^2\right)}{4\,{\left(d\,g-e\,f\right)}^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g-5\,b\,d\,e\,g^2+16\,c\,e^2\,f^2-20\,b\,e^2\,f\,g+25\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^{5/2}-{\left(f+g\,x\right)}^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

```
[Out] (atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^2
*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 -
 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) -
 ((2*(a*g^2 + c*f^2 - b*f*g))/(d*g - e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*
d^2*g^2 + 8*c*e^2*f^2 - 3*b*d*e*g^2 - 12*b*e^2*f*g + 8*c*d*e*f*g))/(4*(d*g
- e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f^2 - 5*b*d*e*g
^2 - 20*b*e^2*f*g + 8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2)
 - (f + g*x)^(3/2)*(2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2
 - 2*d*e*f*g))
```

3.833 $\int \frac{\sqrt{-1+x}\,\sqrt{1+x}}{1+x-x^2}\,dx$

Optimal. Leaf size=91

$$-\cosh^{-1}(x)+\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\,\sqrt{-1+x}}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\,\tanh^{-1}\left(\frac{\sqrt{1+}}{\sqrt{2+\sqrt{5}}}\right.$$

```
[Out] -arccosh(x)+1/5*arctan((1+x)^(1/2)/(-1+x)^(1/2)/(-2+5^(1/2))^(1/2))*(-10+10
*5^(1/2))^(1/2)+1/5*arctanh((1+x)^(1/2)/(-1+x)^(1/2)/(2+5^(1/2))^(1/2))*(10
+10*5^(1/2))^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(91) = 182.
time = 0.09, antiderivative size = 191, normalized size of antiderivative = 2.10, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {915, 1005, 223, 212, 1048, 739, 210}

$$\frac{\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\,\sqrt{x-1}\,\sqrt{x+1}\,\text{ArcTan}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\,\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}-\frac{\sqrt{x-1}\,\sqrt{x+1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}-\frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\,\sqrt{x-1}\,\sqrt{x+1}\,\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[5])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTan[(2 - (1 - Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2] - (Sqrt[-1 + x
]*Sqrt[1 + x]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^2] - (Sqrt[(1 + Sqrt[5
])/10]*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 +
Sqrt[5])]*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1005

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{-1+x}\,\sqrt{1+x}}{1+x-x^2}\,dx = \frac{\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\int \frac{\sqrt{-1+x^2}}{1+x-x^2}\,dx}{\sqrt{-1+x^2}}$$

$$= -\frac{\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^2}} + \frac{\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\int \frac{x}{(1+x-x^2)\sqrt{-}}}{\sqrt{-1+x^2}}$$

$$= -\frac{\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}} + \frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1}\right.}{}$$

$$= -\frac{\sqrt{-1+x}\,\sqrt{1+x}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}} - \frac{\left(\left(5-\sqrt{5}\right)\sqrt{-1+x}\,\sqrt{1+x}\right.}{}$$

$$= \frac{\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x}\,\sqrt{1+x}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^2}}$$

Mathematica [A]
time = 0.24, size = 102, normalized size = 1.12

$$-\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{-2+\sqrt{5}}\,\sqrt{\frac{-1+x}{1+x}}\right) - 2\tanh^{-1}\left(\sqrt{\frac{-1+x}{1+x}}\right) + \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\,\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\,\sqrt{\frac{-1+x}{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x]*Sqrt[1 + x])/(1 + x - x^2),x]
```

```
[Out] -(Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[Sqrt[-2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 + x
)]]) - 2*ArcTanh[Sqrt[(-1 + x)/(1 + x)]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTan
h[Sqrt[2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 + x)]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(65) = 130.
time = 0.14, size = 231, normalized size = 2.54

method	result
default	$-\frac{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{5}\left(\sqrt{2\sqrt{5}+2}\,\ln\left(x+\sqrt{x^2-1}\right)\sqrt{2\sqrt{5}-2}\,\sqrt{5}-\sqrt{2\sqrt{5}+2}\,\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-1+x)^(1/2)*(1+x)^(1/2)*5^(1/2)*((2*5^(1/2)+2)^(1/2)*ln(x+(x^2-1)^(1/
2))*(2*5^(1/2)-2)^(1/2)*5^(1/2)-(2*5^(1/2)+2)^(1/2)*arctan((x*5^(1/2)-x+2)/
(2*5^(1/2)-2)^(1/2)/(x^2-1)^(1/2))*5^(1/2)-arctanh((x*5^(1/2)+x-2)/(2*5^(1/
2)+2)^(1/2)/(x^2-1)^(1/2))*(2*5^(1/2)-2)^(1/2)*5^(1/2)+(2*5^(1/2)+2)^(1/2)*
arctan((x*5^(1/2)-x+2)/(2*5^(1/2)-2)^(1/2)/(x^2-1)^(1/2))-arctanh((x*5^(1/2
)+x-2)/(2*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))*(2*5^(1/2)-2)^(1/2))/(x^2-1)^(1/2
)/(2*5^(1/2)+2)^(1/2)/(2*5^(1/2)-2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x + 1)*sqrt(x - 1)/(x^2 - x - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(65) = 130$.
time = 2.80, size = 214, normalized size = 2.35

$$\frac{2}{5}\sqrt{5}\sqrt{2\sqrt{5}-2}\arctan\left(\frac{1}{8}\sqrt{-4\left(2x+\sqrt{5}-1\right)\sqrt{x+1}\sqrt{x-1}+8x^2+4\sqrt{5}x-4x}\sqrt{2\sqrt{5}-2}\left(\sqrt{5}+1\right)-\frac{1}{4}\left(\sqrt{x+1}\sqrt{x-1}\left(\sqrt{5}+1\right)-\sqrt{5}x-x-2\right)\sqrt{2\sqrt{5}-2}\right)+\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x+\sqrt{5}+\sqrt{2\sqrt{5}+2}+1\right)-\frac{1}{10}\sqrt{5}\sqrt{2\sqrt{5}+2}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x+\sqrt{5}-\sqrt{2\sqrt{5}+2}+1\right)+\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(2*sqrt(5) - 2)*arctan(1/8*sqrt(-4*(2*x + sqrt(5) - 1)*sqrt
(x + 1)*sqrt(x - 1) + 8*x^2 + 4*sqrt(5)*x - 4*x)*sqrt(2*sqrt(5) - 2)*(sqrt(
5) + 1) - 1/4*(sqrt(x + 1)*sqrt(x - 1)*(sqrt(5) + 1) - sqrt(5)*x - x - 2)*s
qrt(2*sqrt(5) - 2)) + 1/10*sqrt(5)*sqrt(2*sqrt(5) + 2)*log(2*sqrt(x + 1)*sq
rt(x - 1) - 2*x + sqrt(5) + sqrt(2*sqrt(5) + 2) + 1) - 1/10*sqrt(5)*sqrt(2*
sqrt(5) + 2)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x + sqrt(5) - sqrt(2*sqrt(5)
 + 2) + 1) + log(sqrt(x + 1)*sqrt(x - 1) - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{x-1}\,\sqrt{x+1}}{x^2-x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)*(1+x)**(1/2)/(-x**2+x+1),x)
```

```
[Out] -Integral(sqrt(x - 1)*sqrt(x + 1)/(x**2 - x - 1), x)
```

Giac [A]
time = 5.15, size = 16, normalized size = 0.18

$$\log\left(\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)*(1+x)^(1/2)/(-x^2+x+1),x, algorithm="giac")
```

```
[Out] log((sqrt(x + 1) - sqrt(x - 1))^2)
```

Mupad [B]
time = 5.02, size = 916, normalized size = 10.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^(1/2)*(x + 1)^(1/2))/(x - x^2 + 1),x)
```

```
[Out] - 4*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) - (10^(1/2)*atan((34083
70*10^(1/2)*(5^(1/2) + 1)^(1/2) - 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2
)*300730i - 3408370*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 1771398*5^
(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2) + 7836865*10^(1/2)*x*(5^(1/2) + 1)^(1/2)
 + 3066340*10^(1/2)*x^2*(5^(1/2) + 1)^(1/2) - 1294942*5^(1/2)*10^(1/2)*x^2*
(5^(1/2) + 1)^(1/2) + 10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1
/2)*300730i - 5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i +
1771398*5^(1/2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) - 10^(1/2)*x*(5^
(1/2) + 1)^(1/2)*(x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(5^(1/2) + 1)^(
1/2)*(x + 1)^(1/2) - 3475583*5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2) + 5^(1/
2)*10^(1/2)*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*134482i + 10^(1
/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i - 5^(1/2)*10^
(1/2)*x*(5^(1/2) + 1)^(1/2)*(x - 1)^(1/2)*134482i + 2589884*5^(1/2)*10^(1/2
)*x*(5^(1/2) + 1)^(1/2)*(x + 1)^(1/2) + 5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1
/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 24066900*x*(x + 1)^(1
/2) - 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) - 7067880*5^(1/2) - 39924
30*5^(1/2)*x^2 + 12033450*x^2 + 7067880*5^(1/2)*(x + 1)^(1/2) + 7984860*5^(
1/2)*x*(x + 1)^(1/2) + 10104760))*(5^(1/2) + 1)^(1/2)*1i)/5 - (10^(1/2)*ata
n((3408370*10^(1/2)*(1 - 5^(1/2))^(1/2) + 3066340*10^(1/2)*x^2*(1 - 5^(1/2)
)^(1/2) - 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*300730i - 3408370*10^(
1/2)*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) + 1771398*5^(1/2)*10^(1/2)*(1 - 5^(1
```

```
/2))^(1/2) + 7836865*10^(1/2)*x*(1 - 5^(1/2))^(1/2) + 3475583*5^(1/2)*10^(1
/2)*x*(1 - 5^(1/2))^(1/2) + 1294942*5^(1/2)*10^(1/2)*x^2*(1 - 5^(1/2))^(1/2
) + 10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*300730i + 5^(1
/2)*10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*134482i - 1771398*5^(1/2)*10
^(1/2)*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) - 10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(
x - 1)^(1/2)*300730i - 6132680*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2)
 - 5^(1/2)*10^(1/2)*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*134482i
 + 10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*150365i + 5^(
1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x - 1)^(1/2)*134482i - 2589884*5^(1/2)
*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*(x + 1)^(1/2) - 5^(1/2)*10^(1/2)*x*(1 - 5^(
1/2))^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)*67241i)/(29119280*x - 24066900*x*(x
 + 1)^(1/2) + 11518800*5^(1/2)*x - 10104760*(x + 1)^(1/2) + 7067880*5^(1/2)
 + 3992430*5^(1/2)*x^2 + 12033450*x^2 - 7067880*5^(1/2)*(x + 1)^(1/2) - 798
4860*5^(1/2)*x*(x + 1)^(1/2) + 10104760))*(1 - 5^(1/2))^(1/2)*1i)/5
```

3.834 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=164

$$-\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2a\ldots}{4\ldots}$$

```
[Out] 1/4*(c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2)+4*e*g*(2*a*e*g-b*(d*g+e*f)))*arctanh
(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)
^(3/2)*(g*x+f)^(1/2)/e^2/g-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*x+d)^(1/2)*(g*
x+f)^(1/2)/e^2/g^2
```

Rubi [A]
time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {965, 81, 65, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}\,(-4beg+5cdg+3cef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g^2)
+ (c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g
+ 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e
*x])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f \cdots}{\cdots}$$

$$(c(3e^2f \cdots$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \cdots$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f \cdots}{\cdots}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\,\sqrt{f+gx}}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g} + \frac{(c(3e^2f \cdots}{\cdots}$$

Mathematica [A]
time = 0.34, size = 141, normalized size = 0.86

$$\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(4beg+c(-3ef-3dg+2egx))}{4e^2g^2}+\frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[f + g*x]*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x)))/(4*e
^2*g^2) + ((c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f
 + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5
/2)*g^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(138) = 276.
time = 0.08, size = 425, normalized size = 2.59

method	result
default	$\left(4\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\,cegx+3\ln\left(\frac{2egx+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^2g^2+2\ln\left(\frac{2egx+2\sqrt{(e}{\cdots}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*g*x+3*ln(1/2*(2*e*g*x+2*((e*
x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2
*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*
g+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1
/2))*c*e^2*f^2+8*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+
e*f)/(e*g)^(1/2))*a*e^2*g^2-4*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*
g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+
f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^2*f*g-6*(e*g)^(1/2)*((e*x+d
)*(g*x+f))^(1/2)*c*d*g-6*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e*f+8*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2)*b*e*g)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/g^2/e^2/((e*x+d)*(g*x+f))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 3.32, size = 378, normalized size = 2.30

$$\left[\frac{\left(\left(3 c d^2 g^2+\left(3 c f^2-4 b f g+8 a g^2\right) e^2+2\left(c d f g-2 b d g^2\right) e\right) \sqrt{g} e^{\frac{1}{2}} \log\left(d^2 g^2+4\left(d g+(2 g x+f) e\right) \sqrt{g x+f} \sqrt{x e+d} \sqrt{g} e^{\frac{1}{2}}+\left(8 g^2 x^2+8 f g x+f^2\right) e^2+2\left(4 d g^2 x+3 d f g\right) e\right)-4\left(3 c d g^2 e-\left(2 c g^2 x-3 c f g+4 b g^2\right) e^2\right) \sqrt{g x+f} \sqrt{x e+d}\right) e^{(-3)}}{16 g^3}, -\frac{\left(\left(3 c d^2 g^2+\left(3 c f^2-4 b f g+8 a g^2\right) e^2+2\left(c d f g-2 b d g^2\right) e\right) \sqrt{-g e} \arctan\left(\frac{\left(d g+(2 g x+f) e\right) \sqrt{g x+f} \sqrt{-g e} \sqrt{x e+d}}{2\left(\left(g^2 x^2+f g x\right) e^2+\left(d g^2 x+d f g\right) e\right)}\right)+2\left(3 c d g^2 e-\left(2 c g^2 x-3 c f g+4 b g^2\right) e^2\right) \sqrt{g x+f} \sqrt{x e+d}\right) e^{(-3)}}{8 g^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^2*g^2 + (3*c*f^2 - 4*b*f*g + 8*a*g^2)*e^2 + 2*(c*d*f*g - 2*b*
d*g^2)*e)*sqrt(g)*e^(1/2)*log(d^2*g^2 + 4*(d*g + (2*g*x + f)*e)*sqrt(g*x +
f)*sqrt(x*e + d)*sqrt(g)*e^(1/2) + (8*g^2*x^2 + 8*f*g*x + f^2)*e^2 + 2*(4*d
*g^2*x + 3*d*f*g)*e) - 4*(3*c*d*g^2*e - (2*c*g^2*x - 3*c*f*g + 4*b*g^2)*e^2
)*sqrt(g*x + f)*sqrt(x*e + d))*e^(-3)/g^3, -1/8*((3*c*d^2*g^2 + (3*c*f^2 -
4*b*f*g + 8*a*g^2)*e^2 + 2*(c*d*f*g - 2*b*d*g^2)*e)*sqrt(-g*e)*arctan(1/2*(
d*g + (2*g*x + f)*e)*sqrt(g*x + f)*sqrt(-g*e)*sqrt(x*e + d)/((g^2*x^2 + f*g
*x)*e^2 + (d*g^2*x + d*f*g)*e)) + 2*(3*c*d*g^2*e - (2*c*g^2*x - 3*c*f*g + 4
*b*g^2)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))*e^(-3)/g^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [A]
time = 5.02, size = 184, normalized size = 1.12

$$\frac{\left(\sqrt{dg^2+(gx+f)ge-fge}\,\sqrt{gx+f}\left(\frac{2(gx+f)ce^{(-1)}}{g^3}-\frac{\left(3cdg^6e+5cfg^5e^2-4bg^6e^2\right)e^{(-3)}}{g^8}\right)-\frac{\left(3cd^2g^2+2cdfge-4bdg^2e+3cf^2e^2-4bfge^2+8ag^2e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{gx+f}\,\sqrt{g}\,e^{\frac{1}{2}}+\sqrt{dg^2+(gx+f)ge-fge}\right|\right)}{g^{\frac{5}{2}}}\right)g}{4|g|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)*sqrt(g*x + f)*(2*(g*x + f)*c*e^(-1
)/g^3 - (3*c*d*g^6*e + 5*c*f*g^5*e^2 - 4*b*g^6*e^2)*e^(-3)/g^8) - (3*c*d^2*
```

```
g^2 + 2*c*d*f*g*e - 4*b*d*g^2*e + 3*c*f^2*e^2 - 4*b*f*g*e^2 + 8*a*g^2*e^2)*
e^(-5/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2) + sqrt(d*g^2 + (g*x + f)*g*
e - f*g*e)))/g^(5/2))*g/abs(g)
```

Mupad [B]

time = 22.38, size = 833, normalized size = 5.08

$$\frac{\frac{(2bdg+2bef)\left(\sqrt{d+ex}-\sqrt{d}\right)}{g^3\left(\sqrt{f+gx}-\sqrt{f}\right)}+\frac{(2bdg+2bef)\left(\sqrt{d+ex}-\sqrt{d}\right)^3}{eg^2\left(\sqrt{f+gx}-\sqrt{f}\right)^3}-\frac{8b\sqrt{d}\sqrt{f}\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{\left(\sqrt{f+gx}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2e\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g\left(\sqrt{f+gx}-\sqrt{f}\right)^2}}-\frac{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)\left(\frac{3cd^2eg^2}{2}+cde^2fg+\frac{3ce^3f^2}{2}\right)}{g^6\left(\sqrt{f+gx}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^3\left(\frac{11cd^2g^2}{2}+25cdefg+\frac{11ce^2f^2}{2}\right)}{g^5\left(\sqrt{f+gx}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^7\left(\frac{3cd^2g^2}{2}+cdefg+\frac{3ce^2f^2}{2}\right)}{e^2g^3\left(\sqrt{f+gx}-\sqrt{f}\right)^7}-\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^5\left(\frac{11cd^2g^2}{2}+25cdefg+\frac{11ce^2f^2}{2}\right)}{eg^4\left(\sqrt{f+gx}-\sqrt{f}\right)^5}+\frac{\sqrt{d}\sqrt{f}(32cdg+32cef)\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{g^4\left(\sqrt{f+gx}-\sqrt{f}\right)^4}}{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^8}{\left(\sqrt{f+gx}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4e\left(\sqrt{d+ex}-\sqrt{d}\right)^6}{g\left(\sqrt{f+gx}-\sqrt{f}\right)^6}-\frac{4e^3\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g^3\left(\sqrt{f+gx}-\sqrt{f}\right)^2}+\frac{6e^2\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^4}}-\frac{4a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+gx}-\sqrt{f}\right)}{\sqrt{-eg}\left(\sqrt{d+ex}-\sqrt{d}\right)}\right)}{\sqrt{-eg}}-\frac{2b\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+gx}-\sqrt{f}\right)}\right)(dg+ef)}{e^{3/2}g^{3/2}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+gx}-\sqrt{f}\right)}\right)\left(3d^2g^2+2defg+3e^2f^2\right)}{2e^{5/2}g^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((f
+ g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2
))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^4/(
(f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2))^2
)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*
e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1
/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2
 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) -
 d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f +
 g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/
2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5)
 + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g
^4*((f + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x
)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f
 + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((
f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*(
(f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/
((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2) - (2*b*atanh((g^(
1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d
*g + e*f))/(e^(3/2)*g^(3/2)) + (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)
))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*
g))/(2*e^(5/2)*g^(5/2))
```

3.835 $\int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=333

$$-\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d+ex}\,\sqrt{f+gx}}{64e^2g^4}+\frac{(c(35e^2f^2+10d$$

```
[Out] 1/64*(-d*g+e*f)^2*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*
g+5*e*f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(
9/2)+1/96*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*g+5*e*f)
))*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g^3-1/24*(-8*b*e*g+9*c*d*g+7*c*e*f)*(e*x
+d)^(5/2)*(g*x+f)^(1/2)/e^2/g^2+1/4*c*(e*x+d)^(7/2)*(g*x+f)^(1/2)/e^2/g-1/6
4*(-d*g+e*f)*(c*(3*d^2*g^2+10*d*e*f*g+35*e^2*f^2)+8*e*g*(6*a*e*g-b*(d*g+5*e
*f)))*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^4
```

Rubi [A]

time = 0.22, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {965, 81, 52, 65, 223, 212}

$$-\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(ef-dg)\,(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{64e^2g^4}+\frac{(d+ex)^{3/2}\sqrt{f+gx}\,(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{96e^2g^3}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{64e^{5/2}g^{9/2}}-\frac{(d+ex)^{5/2}\sqrt{f+gx}\,(-8beg+9cdg+7cef)}{24e^2g^2}+\frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] -1/64*((e*f - d*g)*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*
g - b*(5*e*f + d*g)))*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g^4) + ((c*(35*e^2*
f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*(d + e*x
)^(3/2)*Sqrt[f + g*x])/(96*e^2*g^3) - ((7*c*e*f + 9*c*d*g - 8*b*e*g)*(d + e
*x)^(5/2)*Sqrt[f + g*x])/(24*e^2*g^2) + (c*(d + e*x)^(7/2)*Sqrt[f + g*x])/(
4*e^2*g) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2) + 8*e*g*
(6*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[
f + g*x])])/(64*e^(5/2)*g^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} + \frac{\int \frac{(d+ex)^{3/2}\left(\frac{1}{2}\left(8ae^2g-cd(7ef+dg)\right)-\frac{1}{2}e(7cef+9cdg-8beg)x\right)}{\sqrt{f+gx}}}{4e^2g}$$

$$= -\frac{(7cef+9cdg-8beg)(d+ex)^{5/2}\sqrt{f+gx}}{24e^2g^2} + \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} +$$

$$= \frac{\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)(d+ex)^{3/2}\sqrt{f}}{96e^2g^3}$$

$$= -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d}}{64e^2g^4}$$

$$= -\frac{(ef-dg)\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\sqrt{d}}{64e^2g^4}$$

Mathematica [A]
time = 0.82, size = 289, normalized size = 0.87

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\left(c\left(-9d^3g^3+3d^2eg^2(-5f+2gx)+de^2g(145f^2-92fgx+72g^2x^2)+e^3(-105f^3+70f^2gx-56fg^2x^2+48g^3x^3)\right)+8eg\left(6aeg(-3ef+5dg+2egx)+b(3d^2g^2+2deg(-11f+7gx)+e^2(15f^2-10fgx+8g^2x^2))\right)\right)}{192e^2g^4}+\frac{(ef-dg)^2\left(c(35e^2f^2+10defg+3d^2g^2)+8eg(6aeg-b(5ef+dg))\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{64e^{5/2}g^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[f + g*x]*(c*(-9*d^3*g^3 + 3*d^2*e*g^2*(-5*f + 2*g*x) +
d*e^2*g*(145*f^2 - 92*f*g*x + 72*g^2*x^2) + e^3*(-105*f^3 + 70*f^2*g*x - 56
*f*g^2*x^2 + 48*g^3*x^3)) + 8*e*g*(6*a*e*g*(-3*e*f + 5*d*g + 2*e*g*x) + b*(
3*d^2*g^2 + 2*d*e*g*(-11*f + 7*g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)))
))/(192*e^2*g^4) + ((e*f - d*g)^2*(c*(35*e^2*f^2 + 10*d*e*f*g + 3*d^2*g^2)
+ 8*e*g*(6*a*e*g - b*(5*e*f + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[
g]*Sqrt[d + e*x])])/(64*e^(5/2)*g^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(295) = 590$.

time = 0.07, size = 1207, normalized size = 3.62

method	result	size
default	Expression too large to display	1207

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(-112*c*e^3*f*g^2*x^2*(e*g)^(1/2)*((e*x+d
)*(g*x+f))^(1/2)-184*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d*e^2*f*g^2*x-30
*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^2*e*f*g^2+192*(e*g)^(1/2)*((e*x+d)
*(g*x+f))^(1/2)*a*e^3*g^3*x+48*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*d^2*e*
g^3+240*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*e^3*f^2*g-180*ln(1/2*(2*e*g*x
+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f^3*g+
480*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*a*d*e^2*g^3-288*(e*g)^(1/2)*((e*x+d
)*(g*x+f))^(1/2)*a*e^3*f*g^2-288*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*d*e^3*f*g^3-72*ln(1/2*(2*e*g*x+2*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e^2*f*g^3+216*ln(
1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*
d*e^3*f^2*g^2+54*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+
e*f)/(e*g)^(1/2))*c*d^2*e^2*f^2*g^2+9*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(
1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^4*g^4+105*ln(1/2*(2*e*g*x+2*((e*
x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^4*f^4+96*c*e^3*g^
3*x^3*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)+128*b*e^3*g^3*x^2*(e*g)^(1/2)*((e
*x+d)*(g*x+f))^(1/2)-352*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*d*e^2*f*g^2+
290*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d*e^2*f^2*g+144*c*d*e^2*g^3*x^2*(
e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)+144*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^
(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*d^2*e^2*g^4+144*ln(1/2*(2*e*g*x+2
*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^4*f^2*g^2-12
0*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2
))*b*e^4*f^3*g-210*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e^3*f^3+12*ln(1/2*
(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^3*
e*f*g^3+224*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*d*e^2*g^3*x-160*(e*g)^(1/
2)*((e*x+d)*(g*x+f))^(1/2)*b*e^3*f*g^2*x+12*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(
1/2)*c*d^2*e*g^3*x+140*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e^3*f^2*g*x-24
*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2)
)*b*d^3*e*g^4-18*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^3*g^3)/g^4/e^2/((e
*x+d)*(g*x+f))^(1/2)/(e*g)^(1/2)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 5.56, size = 844, normalized size = 2.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*c*d^4*g^4 + (35*c*f^4 - 40*b*f^3*g + 48*a*f^2*g^2)*e^4 - 12*(5
*c*d*f^3*g - 6*b*d*f^2*g^2 + 8*a*d*f*g^3)*e^3 + 6*(3*c*d^2*f^2*g^2 - 4*b*d^
2*f*g^3 + 8*a*d^2*g^4)*e^2 + 4*(c*d^3*f*g^3 - 2*b*d^3*g^4)*e)*sqrt(g)*e^(1/
2)*log(d^2*g^2 + 4*(d*g + (2*g*x + f)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(g
)*e^(1/2) + (8*g^2*x^2 + 8*f*g*x + f^2)*e^2 + 2*(4*d*g^2*x + 3*d*f*g)*e) -
4*(9*c*d^3*g^4*e - (48*c*g^4*x^3 - 105*c*f^3*g + 120*b*f^2*g^2 - 144*a*f*g^
3 - 8*(7*c*f*g^3 - 8*b*g^4)*x^2 + 2*(35*c*f^2*g^2 - 40*b*f*g^3 + 48*a*g^4)*
x)*e^4 - (72*c*d*g^4*x^2 + 145*c*d*f^2*g^2 - 176*b*d*f*g^3 + 240*a*d*g^4 -
4*(23*c*d*f*g^3 - 28*b*d*g^4)*x)*e^3 - 3*(2*c*d^2*g^4*x - 5*c*d^2*f*g^3 + 8
*b*d^2*g^4)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))*e^(-3)/g^5, -1/384*(3*(3*c*d^
4*g^4 + (35*c*f^4 - 40*b*f^3*g + 48*a*f^2*g^2)*e^4 - 12*(5*c*d*f^3*g - 6*b*
d*f^2*g^2 + 8*a*d*f*g^3)*e^3 + 6*(3*c*d^2*f^2*g^2 - 4*b*d^2*f*g^3 + 8*a*d^2
*g^4)*e^2 + 4*(c*d^3*f*g^3 - 2*b*d^3*g^4)*e)*sqrt(-g*e)*arctan(1/2*(d*g + (
2*g*x + f)*e)*sqrt(g*x + f)*sqrt(-g*e)*sqrt(x*e + d)/((g^2*x^2 + f*g*x)*e^2
 + (d*g^2*x + d*f*g)*e)) + 2*(9*c*d^3*g^4*e - (48*c*g^4*x^3 - 105*c*f^3*g +
 120*b*f^2*g^2 - 144*a*f*g^3 - 8*(7*c*f*g^3 - 8*b*g^4)*x^2 + 2*(35*c*f^2*g^
2 - 40*b*f*g^3 + 48*a*g^4)*x)*e^4 - (72*c*d*g^4*x^2 + 145*c*d*f^2*g^2 - 176
*b*d*f*g^3 + 240*a*d*g^4 - 4*(23*c*d*f*g^3 - 28*b*d*g^4)*x)*e^3 - 3*(2*c*d^
2*g^4*x - 5*c*d^2*f*g^3 + 8*b*d^2*g^4)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))*e^
(-3)/g^5]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(307) = 614$.
time = 4.67, size = 1048, normalized size = 3.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -1/192*(192*((d*g^2 - f*g*e)*e^(-1/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2
) + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))/sqrt(g) - sqrt(d*g^2 + (g*x + f)*
g*e - f*g*e)*sqrt(g*x + f))*a*d*abs(g)/g^2 - 8*(sqrt(d*g^2 + (g*x + f)*g*e
- f*g*e)*sqrt(g*x + f)*(2*(g*x + f)*(4*(g*x + f)/g^2 + (d*g^6*e^3 - 13*f*g^
5*e^4)*e^(-4)/g^7) - 3*(d^2*g^7*e^2 + 2*d*f*g^6*e^3 - 11*f^2*g^5*e^4)*e^(-4
)/g^7) - 3*(d^3*g^3 + d^2*f*g^2*e + 3*d*f^2*g*e^2 - 5*f^3*e^3)*e^(-5/2)*log
(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2) + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))
/g^(3/2))*c*d*abs(g)/g^2 - 8*(sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)*sqrt(g*x
+ f)*(2*(g*x + f)*(4*(g*x + f)/g^2 + (d*g^6*e^3 - 13*f*g^5*e^4)*e^(-4)/g^7)
 - 3*(d^2*g^7*e^2 + 2*d*f*g^6*e^3 - 11*f^2*g^5*e^4)*e^(-4)/g^7) - 3*(d^3*g^
3 + d^2*f*g^2*e + 3*d*f^2*g*e^2 - 5*f^3*e^3)*e^(-5/2)*log(abs(-sqrt(g*x + f
)*sqrt(g)*e^(1/2) + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))/g^(3/2))*b*abs(g)
*e/g^2 - (sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)*(2*(g*x + f)*(4*(g*x + f)*(6*
(g*x + f)/g^3 + (d*g^12*e^5 - 25*f*g^11*e^6)*e^(-6)/g^14) - (5*d^2*g^13*e^4
 + 14*d*f*g^12*e^5 - 163*f^2*g^11*e^6)*e^(-6)/g^14) + 3*(5*d^3*g^14*e^3 + 9
*d^2*f*g^13*e^4 + 15*d*f^2*g^12*e^5 - 93*f^3*g^11*e^6)*e^(-6)/g^14)*sqrt(g*
x + f) + 3*(5*d^4*g^4 + 4*d^3*f*g^3*e + 6*d^2*f^2*g^2*e^2 + 20*d*f^3*g*e^3
- 35*f^4*e^4)*e^(-7/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2) + sqrt(d*g^2
+ (g*x + f)*g*e - f*g*e)))/g^(5/2))*c*abs(g)*e/g^2 - 48*((d^2*g^3 + 2*d*f*g
^2*e - 3*f^2*g*e^2)*e^(-3/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2) + sqrt(
d*g^2 + (g*x + f)*g*e - f*g*e)))/sqrt(g) + sqrt(d*g^2 + (g*x + f)*g*e - f*g
*e)*(2*g*x + (d*g*e - 5*f*e^2)*e^(-2) + 2*f)*sqrt(g*x + f))*b*d*abs(g)/g^3
- 48*((d^2*g^3 + 2*d*f*g^2*e - 3*f^2*g*e^2)*e^(-3/2)*log(abs(-sqrt(g*x + f)
*sqrt(g)*e^(1/2) + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))/sqrt(g) + sqrt(d*g
^2 + (g*x + f)*g*e - f*g*e)*(2*g*x + (d*g*e - 5*f*e^2)*e^(-2) + 2*f)*sqrt(g
*x + f))*a*abs(g)*e/g^3)/g
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}\,(c\,x^2+b\,x+a)}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] int(((d + e*x)^(3/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2), x)
```

3.836 $\int \frac{\sqrt{d+ex}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=246

$$\frac{(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\sqrt{d+ex}\,\sqrt{f+gx}}{8e^2g^3}-\frac{(5cef+7cdg-6beg)(d+ex)^{3/}}{12e^2g^2}$$

[Out] -1/8*(-d*g+e*f)*(c*(d^2*g^2+2*d*e*f*g+5*e^2*f^2)+2*e*g*(4*a*e*g-b*(d*g+3*e*f)))*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(7/2)-1/12*(-6*b*e*g+7*c*d*g+5*c*e*f)*(e*x+d)^(3/2)*(g*x+f)^(1/2)/e^2/g^2+1/3*c*(e*x+d)^(5/2)*(g*x+f)^(1/2)/e^2/g+1/8*(c*(d^2*g^2+2*d*e*f*g+5*e^2*f^2)+2*e*g*(4*a*e*g-b*(d*g+3*e*f)))*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^3

Rubi [A]
time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {965, 81, 52, 65, 223, 212}

$$\frac{\sqrt{d+ex}\,\sqrt{f+gx}\,(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{8e^2g^3}-\frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{8e^{5/2}g^{7/2}}-\frac{(d+ex)^{3/2}\sqrt{f+gx}\,(-6beg+7cdg+5cef)}{12e^2g^2}+\frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] ((c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g)))*
Sqrt[d + e*x]*Sqrt[f + g*x])/(8*e^2*g^3) - ((5*c*e*f + 7*c*d*g - 6*b*e*g)*(
d + e*x)^(3/2)*Sqrt[f + g*x])/(12*e^2*g^2) + (c*(d + e*x)^(5/2)*Sqrt[f + g*
x])/(3*e^2*g) - ((e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(
4*a*e*g - b*(3*e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f
 + g*x])])/(8*e^(5/2)*g^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(a+bx+cx^2)}{\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + \frac{\int \frac{\sqrt{d+ex}\,\left(\frac{1}{2}(6ae^2g-cd(5ef+dg))-\frac{1}{2}e(5cef+7cdg-6beg)x\right)}{\sqrt{f+gx}}}{3e^2g}$$

$$= -\frac{(5cef+7cdg-6beg)(d+ex)^{3/2}\sqrt{f+gx}}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g} + -$$

$$= \frac{(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\sqrt{d+ex}\,\sqrt{f+gx}}{8e^2g^3}$$

$$= \frac{(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\sqrt{d+ex}\,\sqrt{f+gx}}{8e^2g^3}$$

$$= \frac{(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\sqrt{d+ex}\,\sqrt{f+gx}}{8e^2g^3}$$

$$= \frac{(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\sqrt{d+ex}\,\sqrt{f+gx}}{8e^2g^3}$$

Mathematica [**A**]
time = 0.73, size = 199, normalized size = 0.81

$$\frac{e\sqrt{d+ex}\,\sqrt{f+gx}\,(6eg(4aeg+b(-3ef+dg+2egx))+c(-3d^2g^2+2deg(-2f+gx)+e^2(15f^2-10fgx+8g^2x^2)))+3\sqrt{\frac{e}{g}}\,(ef-dg)\,(c(5e^2f^2+2defg+d^2g^2)+2eg(4aeg-b(3ef+dg)))\log\left(\sqrt{d+ex}-\sqrt{\frac{e}{g}}\,\sqrt{f+gx}\right)}{24e^3g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]
```

```
[Out] (e*Sqrt[d + e*x]*Sqrt[f + g*x]*(6*e*g*(4*a*e*g + b*(-3*e*f + d*g + 2*e*g*x)
) + c*(-3*d^2*g^2 + 2*d*e*g*(-2*f + g*x) + e^2*(15*f^2 - 10*f*g*x + 8*g^2*x
^2))) + 3*Sqrt[e/g]*(e*f - d*g)*(c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*
g*(4*a*e*g - b*(3*e*f + d*g)))*Log[Sqrt[d + e*x] - Sqrt[e/g]*Sqrt[f + g*x]]
)/(24*e^3*g^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(214) = 428.
time = 0.08, size = 763, normalized size = 3.10

method	result
default	$\frac{\sqrt{ex+d}\,\sqrt{gx+f}\,\left(16c\,e^{2}g^{2}x^{2}\sqrt{eg}\,\sqrt{(ex+d)\,(gx+f)}+24\ln\left(\frac{2egx+2\sqrt{(ex+d)\,(gx+f)}\,\sqrt{eg}+dg}{2\sqrt{eg}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(16*c*e^2*g^2*x^2*(e*g)^(1/2)*((e*x+d)*(g*
x+f))^(1/2)+24*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*
f)/(e*g)^(1/2))*a*d*e^2*g^3-24*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e
*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*a*e^3*f*g^2-6*ln(1/2*(2*e*g*x+2*((e*x+d)*(g
*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^3-12*ln(1/2*(2*e*g
*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*f*g^
2+18*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(
1/2))*b*e^3*f^2*g+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d
*g+e*f)/(e*g)^(1/2))*c*d^3*g^3+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*
(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e*f*g^2+9*ln(1/2*(2*e*g*x+2*((e*x+d
)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^2*f^2*g-15*ln(1/2*
(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^3*
f^3+24*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*e^2*g^2*x+4*(e*g)^(1/2)*((e*x+
d)*(g*x+f))^(1/2)*c*d*e*g^2*x-20*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e^2*
f*g*x+48*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*a*e^2*g^2+12*(e*g)^(1/2)*((e*x
+d)*(g*x+f))^(1/2)*b*d*e*g^2-36*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*b*e^2*f
*g-6*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d^2*g^2-8*(e*g)^(1/2)*((e*x+d)*(
g*x+f))^(1/2)*c*d*e*f*g+30*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e^2*f^2)/g
^3/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*g)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 5.51, size = 564, normalized size = 2.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(c*d^3*g^3 - (5*c*f^3 - 6*b*f^2*g + 8*a*f*g^2)*e^3 + (3*c*d*f^2*g
 - 4*b*d*f*g^2 + 8*a*d*g^3)*e^2 + (c*d^2*f*g^2 - 2*b*d^2*g^3)*e)*sqrt(g)*e^
(1/2)*log(d^2*g^2 - 4*(d*g + (2*g*x + f)*e)*sqrt(g*x + f)*sqrt(x*e + d)*sqr
t(g)*e^(1/2) + (8*g^2*x^2 + 8*f*g*x + f^2)*e^2 + 2*(4*d*g^2*x + 3*d*f*g)*e)
 + 4*(3*c*d^2*g^3*e - (8*c*g^3*x^2 + 15*c*f^2*g - 18*b*f*g^2 + 24*a*g^3 - 2
*(5*c*f*g^2 - 6*b*g^3)*x)*e^3 - 2*(c*d*g^3*x - 2*c*d*f*g^2 + 3*b*d*g^3)*e^2
)*sqrt(g*x + f)*sqrt(x*e + d))*e^(-3)/g^4, -1/48*(3*(c*d^3*g^3 - (5*c*f^3 -
 6*b*f^2*g + 8*a*f*g^2)*e^3 + (3*c*d*f^2*g - 4*b*d*f*g^2 + 8*a*d*g^3)*e^2 +
 (c*d^2*f*g^2 - 2*b*d^2*g^3)*e)*sqrt(-g*e)*arctan(1/2*(d*g + (2*g*x + f)*e)
*sqrt(g*x + f)*sqrt(-g*e)*sqrt(x*e + d)/((g^2*x^2 + f*g*x)*e^2 + (d*g^2*x +
 d*f*g)*e)) + 2*(3*c*d^2*g^3*e - (8*c*g^3*x^2 + 15*c*f^2*g - 18*b*f*g^2 + 2
4*a*g^3 - 2*(5*c*f*g^2 - 6*b*g^3)*x)*e^3 - 2*(c*d*g^3*x - 2*c*d*f*g^2 + 3*b
*d*g^3)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))*e^(-3)/g^4]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.88, size = 436, normalized size = 1.77

$$-\frac{\frac{24\left(\frac{(dg^2-fge)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{gx+f}\sqrt{g}e^{\frac{1}{2}}+\sqrt{dg^2+(gx+f)ge-fge}\right|\right)}{\sqrt{g}}-\sqrt{dg^2+(gx+f)ge-fge}\sqrt{gx+f}\right)a|g|}{g^2}-\frac{\left(\sqrt{dg^2+(gx+f)ge-fge}\sqrt{gx+f}\left(2(gx+f)\left(\frac{4(gx+f)}{g^2}+\frac{(dg^6e^3-13fg^5e^4)e^{(-4)}}{g^7}\right)-\frac{3(d^2g^7e^2+2dfg^6e^3-11f^2g^5e^4)e^{(-4)}}{g^7}\right)-\frac{3(d^3g^3+d^2fg^2e+3df^2ge^2-5f^3e^3)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{gx+f}\sqrt{g}e^{\frac{1}{2}}+\sqrt{dg^2+(gx+f)ge-fge}\right|\right)}{g^{\frac{3}{2}}}\right)c|g|}{g^2}-\frac{6\left(\frac{(d^2g^3+2dfg^2e-3f^2ge^2)e^{\left(-\frac{3}{2}\right)}\log\left(\left|-\sqrt{gx+f}\sqrt{g}e^{\frac{1}{2}}+\sqrt{dg^2+(gx+f)ge-fge}\right|\right)}{\sqrt{g}}+\sqrt{dg^2+(gx+f)ge-fge}(2gx+(dge-5fe^2)e^{(-2)}+2f)\sqrt{gx+f}\right)b|g|}{g^3}}{24g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*((d*g^2 - f*g*e)*e^(-1/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2)
+ sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))/sqrt(g) - sqrt(d*g^2 + (g*x + f)*g*
e - f*g*e)*sqrt(g*x + f))*a*abs(g)/g^2 - (sqrt(d*g^2 + (g*x + f)*g*e - f*g*
e)*sqrt(g*x + f)*(2*(g*x + f)*(4*(g*x + f)/g^2 + (d*g^6*e^3 - 13*f*g^5*e^4)
*e^(-4)/g^7) - 3*(d^2*g^7*e^2 + 2*d*f*g^6*e^3 - 11*f^2*g^5*e^4)*e^(-4)/g^7)
 - 3*(d^3*g^3 + d^2*f*g^2*e + 3*d*f^2*g*e^2 - 5*f^3*e^3)*e^(-5/2)*log(abs(-
sqrt(g*x + f)*sqrt(g)*e^(1/2) + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))/g^(3/
2))*c*abs(g)/g^2 - 6*((d^2*g^3 + 2*d*f*g^2*e - 3*f^2*g*e^2)*e^(-3/2)*log(ab
s(-sqrt(g*x + f)*sqrt(g)*e^(1/2) + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))/sq
rt(g) + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)*(2*g*x + (d*g*e - 5*f*e^2)*e^(-
2) + 2*f)*sqrt(g*x + f))*b*abs(g)/g^3)/g
```

Mupad [B]
time = 74.34, size = 1832, normalized size = 7.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(f + g*x)^(1/2),x)
```

```
[Out] (((2*a*d*g + 2*a*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(g^2*((f + g*x)^(1/2)
- f^(1/2))^3) + ((2*a*e^2*f + 2*a*d*e*g)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*
((f + g*x)^(1/2) - f^(1/2))) - (8*a*d^(1/2)*e*f^(1/2)*((d + e*x)^(1/2) - d^
(1/2))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))
^4/((f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2
))^2)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((
c*d^3*e^3*g^3)/4 - (5*c*e^6*f^3)/4 + (3*c*d*e^5*f^2*g)/4 + (c*d^2*e^4*f*g^2
)/4))/(g^9*((f + g*x)^(1/2) - f^(1/2))) - (((d + e*x)^(1/2) - d^(1/2))^5*((
33*c*e^4*f^3)/2 + (19*c*d^3*e*g^3)/2 + (313*c*d*e^3*f^2*g)/2 + (275*c*d^2*e
^2*f*g^2)/2))/(g^7*((f + g*x)^(1/2) - f^(1/2))^5) - (((d + e*x)^(1/2) - d^(
1/2))^7*((19*c*d^3*g^3)/2 + (33*c*e^3*f^3)/2 + (313*c*d*e^2*f^2*g)/2 + (275
*c*d^2*e*f*g^2)/2))/(g^6*((f + g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2)
 - d^(1/2))^3*((17*c*d^3*e^2*g^3)/12 - (85*c*e^5*f^3)/12 + (17*c*d*e^4*f^2*
g)/4 + (91*c*d^2*e^3*f*g^2)/4))/(g^8*((f + g*x)^(1/2) - f^(1/2))^3) + (((d
+ e*x)^(1/2) - d^(1/2))^11*((c*d^3*g^3)/4 - (5*c*e^3*f^3)/4 + (3*c*d*e^2*f^
2*g)/4 + (c*d^2*e*f*g^2)/4))/(e^2*g^4*((f + g*x)^(1/2) - f^(1/2))^11) - (((
d + e*x)^(1/2) - d^(1/2))^9*((17*c*d^3*g^3)/12 - (85*c*e^3*f^3)/12 + (17*c*
d*e^2*f^2*g)/4 + (91*c*d^2*e*f*g^2)/4))/(e*g^5*((f + g*x)^(1/2) - f^(1/2))^
9) + (d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6*(128*c*e^3*f^2 + 64*c*d
^2*e*g^2 + (704*c*d*e^2*f*g)/3))/(g^6*((f + g*x)^(1/2) - f^(1/2))^6) + (d^(
1/2)*f^(1/2)*(32*c*d^2*g + 96*c*d*e*f)*((d + e*x)^(1/2) - d^(1/2))^8)/(g^4*
((f + g*x)^(1/2) - f^(1/2))^8) + (d^(1/2)*f^(1/2)*(96*c*d*e^3*f + 32*c*d^2*
e^2*g)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^6*((f + g*x)^(1/2) - f^(1/2))^4))/
(((d + e*x)^(1/2) - d^(1/2))^12/((f + g*x)^(1/2) - f^(1/2))^12 + e^6/g^6 -
(6*e*((d + e*x)^(1/2) - d^(1/2))^10)/(g*((f + g*x)^(1/2) - f^(1/2))^10) - (
6*e^5*((d + e*x)^(1/2) - d^(1/2))^2)/(g^5*((f + g*x)^(1/2) - f^(1/2))^2) +
(15*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4)
- (20*e^3*((d + e*x)^(1/2) - d^(1/2))^6)/(g^3*((f + g*x)^(1/2) - f^(1/2))^6
) + (15*e^2*((d + e*x)^(1/2) - d^(1/2))^8)/(g^2*((f + g*x)^(1/2) - f^(1/2))
^8)) + ((((d + e*x)^(1/2) - d^(1/2))*((b*d^2*e^2*g^2)/2 - (3*b*e^4*f^2)/2 +
 b*d*e^3*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) + (((d + e*x)^(1/2) - d^(1
/2))^3*((11*b*e^3*f^2)/2 + (7*b*d^2*e*g^2)/2 + 23*b*d*e^2*f*g))/(g^5*((f +
g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^5*((7*b*d^2*g^2)/2
+ (11*b*e^2*f^2)/2 + 23*b*d*e*f*g))/(g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (
((d + e*x)^(1/2) - d^(1/2))^7*((b*d^2*g^2)/2 - (3*b*e^2*f^2)/2 + b*d*e*f*g)
)/(e*g^3*((f + g*x)^(1/2) - f^(1/2))^7) - (d^(1/2)*f^(1/2)*(32*b*e^2*f + 16
*b*d*e*g)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f + g*x)^(1/2) - f^(1/2))^4
```

```
) - (8*b*d^(3/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^6)/(g^2*((f + g*x)^(1/
2) - f^(1/2))^6) - (8*b*d^(3/2)*e^2*f^(1/2)*((d + e*x)^(1/2) - d^(1/2))^2)/
(g^4*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g
*x)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*(
(f + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*
((f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2
*((f + g*x)^(1/2) - f^(1/2))^4)) + (2*a*atanh((g^(1/2)*((d + e*x)^(1/2) - d
^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d*g - e*f))/(e^(1/2)*g^(3/
2)) - (b*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1
/2) - f^(1/2))))*(d*g - e*f)*(d*g + 3*e*f))/(2*e^(3/2)*g^(5/2)) + (c*atanh(
(g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2)))
)*(d*g - e*f)*(d^2*g^2 + 5*e^2*f^2 + 2*d*e*f*g))/(4*e^(5/2)*g^(7/2))
```

3.837 $\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=164

$$-\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2a}{4}$$

```
[Out] 1/4*(c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2)+4*e*g*(2*a*e*g-b*(d*g+e*f)))*arctanh
(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)+1/2*c*(e*x+d)
^(3/2)*(g*x+f)^(1/2)/e^2/g-1/4*(-4*b*e*g+5*c*d*g+3*c*e*f)*(e*x+d)^(1/2)*(g*
x+f)^(1/2)/e^2/g^2
```

Rubi [A]
time = 0.09, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {965, 81, 65, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2))}{4e^{5/2}g^{5/2}}-\frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg+5cdg+3cef)}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] -1/4*((3*c*e*f + 5*c*d*g - 4*b*e*g)*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g^2)
+ (c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + ((c*(3*e^2*f^2 + 2*d*e*f*g
+ 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f + d*g)))*ArcTanh[(Sqrt[g]*Sqrt[d + e
*x])/(Sqrt[e]*Sqrt[f + g*x])])/(4*e^(5/2)*g^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{\int \frac{\frac{1}{2}\left(4ae^2g-cd(3ef+dg)\right)-\frac{1}{2}e(3cef+5cdg-4beg)x}{\sqrt{d+ex}\sqrt{f+gx}}\,dx}{2e^2g}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(c(3e^2f}{}$$

$$(c(3e^2f$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(c(3e^2f}{}$$

$$= -\frac{(3cef+5cdg-4beg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(c(3e^2f}{}$$

Mathematica [A]
time = 0.05, size = 141, normalized size = 0.86

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\,(4beg+c(-3ef-3dg+2egx))}{4e^2g^2}+\frac{(c(3e^2f^2+2defg+3d^2g^2)+4eg(2aeg-b(ef+dg)))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[f + g*x]*(4*b*e*g + c*(-3*e*f - 3*d*g + 2*e*g*x)))/(4*e
^2*g^2) + ((c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2) + 4*e*g*(2*a*e*g - b*(e*f
 + d*g)))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5
/2)*g^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(138) = 276$.
time = 0.00, size = 425, normalized size = 2.59

method	result
default	$\frac{\left(4\sqrt{(ex+d)(gx+f)}\sqrt{eg}\,cegx+3\ln\left(\frac{2egx+2\sqrt{(ex+d)(gx+f)}\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^2g^2+2\ln\left(\frac{2egx+2\sqrt{(e}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)*c*e*g*x+3*ln(1/2*(2*e*g*x+2*((e*
x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2
*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*
g+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1
/2))*c*e^2*f^2+8*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+
e*f)/(e*g)^(1/2))*a*e^2*g^2-4*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*
g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e*g^2-4*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+
f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^2*f*g-6*(e*g)^(1/2)*((e*x+d
)*(g*x+f))^(1/2)*c*d*g-6*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e*f+8*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2)*b*e*g)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/(e*g)^(1/2
)/g^2/e^2/((e*x+d)*(g*x+f))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 6.44, size = 378, normalized size = 2.30

$$\left[\frac{\left(\left(3\,cd^{2}g^{2}+\left(3\,cf^{2}-4\,bfg+8\,ag^{2}\right)e^{2}+2\,\left(cdfg-2\,bdg^{2}\right)e\right)\sqrt{g}\,e^{\frac{1}{2}}\log\left(d^{2}g^{2}+4\,\left(dg+\left(2\,gx+f\right)e\right)\sqrt{gx+f}\,\sqrt{xe+d}\,\sqrt{g}\,e^{\frac{1}{2}}+\left(8\,g^{2}x^{2}+8\,fgx+f^{2}\right)e^{2}+2\,\left(4\,dg^{2}x+3\,dfg\right)e\right)-4\,\left(3\,cdg^{2}e-\left(2\,cg^{2}x-3\,cfg+4\,bg^{2}\right)e^{2}\right)\sqrt{gx+f}\,\sqrt{xe+d}\right)e^{(-3)}}{16\,g^{3}},\ -\frac{\left(\left(3\,cd^{2}g^{2}+\left(3\,cf^{2}-4\,bfg+8\,ag^{2}\right)e^{2}+2\,\left(cdfg-2\,bdg^{2}\right)e\right)\sqrt{-ge}\arctan\left(\frac{\left(dg+\left(2\,gx+f\right)e\right)\sqrt{gx+f}\,\sqrt{-ge}\,\sqrt{xe+d}}{2\,\left(\left(g^{2}x^{2}+fgx\right)e^{2}+\left(dg^{2}x+dfg\right)e\right)}\right)+2\,\left(3\,cdg^{2}e-\left(2\,cg^{2}x-3\,cfg+4\,bg^{2}\right)e^{2}\right)\sqrt{gx+f}\,\sqrt{xe+d}\right)e^{(-3)}}{8\,g^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*c*d^2*g^2 + (3*c*f^2 - 4*b*f*g + 8*a*g^2)*e^2 + 2*(c*d*f*g - 2*b*
d*g^2)*e)*sqrt(g)*e^(1/2)*log(d^2*g^2 + 4*(d*g + (2*g*x + f)*e)*sqrt(g*x +
f)*sqrt(x*e + d)*sqrt(g)*e^(1/2) + (8*g^2*x^2 + 8*f*g*x + f^2)*e^2 + 2*(4*d
*g^2*x + 3*d*f*g)*e) - 4*(3*c*d*g^2*e - (2*c*g^2*x - 3*c*f*g + 4*b*g^2)*e^2
)*sqrt(g*x + f)*sqrt(x*e + d))*e^(-3)/g^3, -1/8*((3*c*d^2*g^2 + (3*c*f^2 -
4*b*f*g + 8*a*g^2)*e^2 + 2*(c*d*f*g - 2*b*d*g^2)*e)*sqrt(-g*e)*arctan(1/2*(
d*g + (2*g*x + f)*e)*sqrt(g*x + f)*sqrt(-g*e)*sqrt(x*e + d)/((g^2*x^2 + f*g
*x)*e^2 + (d*g^2*x + d*f*g)*e)) + 2*(3*c*d*g^2*e - (2*c*g^2*x - 3*c*f*g + 4
*b*g^2)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))*e^(-3)/g^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^{2}}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Giac [A]
time = 4.14, size = 184, normalized size = 1.12

$$\frac{\left(\sqrt{dg^{2}+(gx+f)ge-fge}\,\sqrt{gx+f}\left(\frac{2\,(gx+f)ce^{(-1)}}{g^{3}}-\frac{\left(3\,cdg^{6}e+5\,cfg^{5}e^{2}-4\,bg^{6}e^{2}\right)e^{(-3)}}{g^{8}}\right)-\frac{\left(3\,cd^{2}g^{2}+2\,cdfge-4\,bdg^{2}e+3\,cf^{2}e^{2}-4\,bfge^{2}+8\,ag^{2}e^{2}\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{gx+f}\,\sqrt{g}\,e^{\frac{1}{2}}+\sqrt{dg^{2}+(gx+f)ge-fge}\right|\right)}{g^{\frac{5}{2}}}\right)g}{4\,|g|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)*sqrt(g*x + f)*(2*(g*x + f)*c*e^(-1
)/g^3 - (3*c*d*g^6*e + 5*c*f*g^5*e^2 - 4*b*g^6*e^2)*e^(-3)/g^8) - (3*c*d^2*
```

```
g^2 + 2*c*d*f*g*e - 4*b*d*g^2*e + 3*c*f^2*e^2 - 4*b*f*g*e^2 + 8*a*g^2*e^2)*
e^(-5/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2) + sqrt(d*g^2 + (g*x + f)*g*
e - f*g*e)))/g^(5/2))*g/abs(g)
```

Mupad [B]

time = 0.00, size = 833, normalized size = 5.08

$$\frac{\frac{(2bdg+2bef)\left(\sqrt{d+ex}-\sqrt{d}\right)}{g^3\left(\sqrt{f+gx}-\sqrt{f}\right)}+\frac{(2bdg+2bef)\left(\sqrt{d+ex}-\sqrt{d}\right)^3}{eg^2\left(\sqrt{f+gx}-\sqrt{f}\right)^3}-\frac{8b\sqrt{d}\sqrt{f}\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^2}}{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{\left(\sqrt{f+gx}-\sqrt{f}\right)^4}+\frac{e^2}{g^2}-\frac{2e\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g\left(\sqrt{f+gx}-\sqrt{f}\right)^2}}-\frac{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)\left(\frac{3cd^2eg^2}{2}+cde^2fg+\frac{3ce^3f^2}{2}\right)}{g^6\left(\sqrt{f+gx}-\sqrt{f}\right)}-\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^3\left(\frac{11cd^2g^2}{2}+25cdefg+\frac{11ce^2f^2}{2}\right)}{g^5\left(\sqrt{f+gx}-\sqrt{f}\right)^3}+\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^7\left(\frac{3cd^2g^2}{2}+cdefg+\frac{3ce^2f^2}{2}\right)}{e^2g^3\left(\sqrt{f+gx}-\sqrt{f}\right)^7}-\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^5\left(\frac{11cd^2g^2}{2}+25cdefg+\frac{11ce^2f^2}{2}\right)}{eg^4\left(\sqrt{f+gx}-\sqrt{f}\right)^5}+\frac{\sqrt{d}\sqrt{f}(32cdg+32cef)\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{g^4\left(\sqrt{f+gx}-\sqrt{f}\right)^4}}{\frac{\left(\sqrt{d+ex}-\sqrt{d}\right)^8}{\left(\sqrt{f+gx}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4e\left(\sqrt{d+ex}-\sqrt{d}\right)^6}{g\left(\sqrt{f+gx}-\sqrt{f}\right)^6}-\frac{4e^3\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{g^3\left(\sqrt{f+gx}-\sqrt{f}\right)^2}+\frac{6e^2\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{g^2\left(\sqrt{f+gx}-\sqrt{f}\right)^4}}-\frac{4a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+gx}-\sqrt{f}\right)}{\sqrt{-eg}\left(\sqrt{d+ex}-\sqrt{d}\right)}\right)}{\sqrt{-eg}}-\frac{2b\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+gx}-\sqrt{f}\right)}\right)(dg+ef)}{e^{3/2}g^{3/2}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+gx}-\sqrt{f}\right)}\right)\left(3d^2g^2+2defg+3e^2f^2\right)}{2e^{5/2}g^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2)))/(g^3*((f + g*x)^(1/2) -
f^(1/2))) + ((2*b*d*g + 2*b*e*f)*((d + e*x)^(1/2) - d^(1/2))^3)/(e*g^2*((f
+ g*x)^(1/2) - f^(1/2))^3) - (8*b*d^(1/2)*f^(1/2)*((d + e*x)^(1/2) - d^(1/2
))^2)/(g^2*((f + g*x)^(1/2) - f^(1/2))^2))/(((d + e*x)^(1/2) - d^(1/2))^4/(
(f + g*x)^(1/2) - f^(1/2))^4 + e^2/g^2 - (2*e*((d + e*x)^(1/2) - d^(1/2))^2
)/(g*((f + g*x)^(1/2) - f^(1/2))^2)) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*
e^3*f^2)/2 + (3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1
/2))) - (((d + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2
 + 25*c*d*e*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) -
 d^(1/2))^7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f +
 g*x)^(1/2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/
2 + (11*c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5)
 + (d^(1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g
^4*((f + g*x)^(1/2) - f^(1/2))^4))/(((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x
)^(1/2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f
 + g*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((
f + g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*(
(f + g*x)^(1/2) - f^(1/2))^4)) - (4*a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/
((-e*g)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(-e*g)^(1/2) - (2*b*atanh((g^(
1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(d
*g + e*f))/(e^(3/2)*g^(3/2)) + (c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)
))/(e^(1/2)*((f + g*x)^(1/2) - f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*
g))/(2*e^(5/2)*g^(5/2))
```

3.838 $\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=129

$$-\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}+\frac{c\sqrt{d+ex}\,\sqrt{f+gx}}{e^2g}-\frac{(cef+3cdg-2beg)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}$$

```
[Out] -(-2*b*e*g+3*c*d*g+c*e*f)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/
2))/e^(5/2)/g^(3/2)-2*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)
^(1/2)+c*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g
```

Rubi [A]

time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {963, 81, 65, 223, 212}

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}\,(ef-dg)}-\frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}}+\frac{c\sqrt{d+ex}\,\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/((e*f - d*g)*Sqrt[d + e*x]) +
(c*Sqrt[d + e*x]*Sqrt[f + g*x])/(e^2*g) - ((c*e*f + 3*c*d*g - 2*b*e*g)*ArcT
anh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*g^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} - \frac{2\int \frac{\frac{(cd-be)(ef-dg)}{2e^2}-\frac{c(ef-dg)x}{2e}}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{ef-dg}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\,\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)}{}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\,\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)}{}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\,\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)}{}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}} + \frac{c\sqrt{d+ex}\,\sqrt{f+gx}}{e^2g} - \frac{(cef+3cdg-2beg)}{e}$$

Mathematica [A]

time = 0.34, size = 131, normalized size = 1.02

$$\frac{\sqrt{f+gx}\,(2e(bd-ae)g+c(-3d^2g+e^2fx+de(f-gx)))}{e^2g(ef-dg)\sqrt{d+ex}}+\frac{(2beg-c(ef+3dg))\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{d+ex}}\right)}{e^{5/2}g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*(2*e*(b*d - a*e)*g + c*(-3*d^2*g + e^2*f*x + d*e*(f - g*x)))
)/(e^2*g*(e*f - d*g)*Sqrt[d + e*x]) + ((2*b*e*g - c*(e*f + 3*d*g))*ArcTanh[
(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(e^(5/2)*g^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(109) = 218$.
time = 0.08, size = 697, normalized size = 5.40

method	result
default	$\frac{\sqrt{gx+f}\,\left(2\ln\left(\frac{2egx+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)bd\,e^2g^2x-2\ln\left(\frac{2egx+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*b*d*e^2*g^2*x-2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(
1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*e^3*f*g*x-3*ln(1/2*(2*e*g*x+2*((e*
x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e*g^2*x+2*ln(1/
2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*
e^2*f*g*x+ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e
*g)^(1/2))*c*e^3*f^2*x+2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1
/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^2-2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))
^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*f*g-3*ln(1/2*(2*e*g*x+2*((
e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^3*g^2+2*ln(1/2*
(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*
e*f*g+ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^
(1/2))*c*d*e^2*f^2+2*c*d*e*g*x*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-2*c*e^2*
f*x*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+4*a*e^2*g*((e*x+d)*(g*x+f))^(1/2)*(
e*g)^(1/2)-4*b*d*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+6*c*d^2*g*((e*x+d)
*(g*x+f))^(1/2)*(e*g)^(1/2)-2*c*d*e*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/
g/(e*g)^(1/2)/(d*g-e*f)/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*x+d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(112) = 224$.
time = 5.18, size = 564, normalized size = 4.37

$$\left[-\frac{(3\,cd^3g^2 - (cf^2 - 2\,bfg)xe^3 - (cdf^2 - 2\,bdfg + 2\,(cdfg + bdg^2)x)e^2 + (3\,cd^2g^2x - 2\,cd^2fg - 2\,bd^2g^2)e)\sqrt{g}\,e^{\frac{1}{2}}\log\left(d^2g^2 + 4\,(dg + (2\,gx + f)e)\sqrt{gx + f}\,\sqrt{xe + d}\,\sqrt{g}\,e^{\frac{1}{2}} + (8\,g^2x^2 + 8\,fgx + f^2)e^2 + 2\,(4\,dg^2x + 3\,dfg)e\right) - 4\,(3\,cd^2g^2e - (cfgx - 2\,ag^2)e^3 + (cdg^2x - cdfg - 2\,bdg^2)e^2)\sqrt{gx + f}\,\sqrt{xe + d}}{4\,(d^2g^3e^3 - fg^2xe^5 + (dg^3x - dfg^2)e^4)}, \frac{(3\,cd^3g^2 - (cf^2 - 2\,bfg)xe^3 - (cdf^2 - 2\,bdfg + 2\,(cdfg + bdg^2)x)e^2 + (3\,cd^2g^2x - 2\,cd^2fg - 2\,bd^2g^2)e)\sqrt{-ge}\,\arctan\left(\frac{(dg + (2\,gx + f)e)\sqrt{gx + f}\,\sqrt{-ge}\,\sqrt{xe + d}}{2\,((g^2x^2 + fgx)e^2 + (dg^2x + dfg)e)}\right) + 2\,(3\,cd^2g^2e - (cfgx - 2\,ag^2)e^3 + (cdg^2x - cdfg - 2\,bdg^2)e^2)\sqrt{gx + f}\,\sqrt{xe + d}}{2\,(d^2g^3e^3 - fg^2xe^5 + (dg^3x - dfg^2)e^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*c*d^3*g^2 - (c*f^2 - 2*b*f*g)*x*e^3 - (c*d*f^2 - 2*b*d*f*g + 2*(c
*d*f*g + b*d*g^2)*x)*e^2 + (3*c*d^2*g^2*x - 2*c*d^2*f*g - 2*b*d^2*g^2)*e)*s
qrt(g)*e^(1/2)*log(d^2*g^2 + 4*(d*g + (2*g*x + f)*e)*sqrt(g*x + f)*sqrt(x*e
 + d)*sqrt(g)*e^(1/2) + (8*g^2*x^2 + 8*f*g*x + f^2)*e^2 + 2*(4*d*g^2*x + 3*
d*f*g)*e) - 4*(3*c*d^2*g^2*e - (c*f*g*x - 2*a*g^2)*e^3 + (c*d*g^2*x - c*d*f
*g - 2*b*d*g^2)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))/(d^2*g^3*e^3 - f*g^2*x*e^
5 + (d*g^3*x - d*f*g^2)*e^4), 1/2*((3*c*d^3*g^2 - (c*f^2 - 2*b*f*g)*x*e^3 -
 (c*d*f^2 - 2*b*d*f*g + 2*(c*d*f*g + b*d*g^2)*x)*e^2 + (3*c*d^2*g^2*x - 2*c
*d^2*f*g - 2*b*d^2*g^2)*e)*sqrt(-g*e)*arctan(1/2*(d*g + (2*g*x + f)*e)*sqrt
(g*x + f)*sqrt(-g*e)*sqrt(x*e + d)/((g^2*x^2 + f*g*x)*e^2 + (d*g^2*x + d*f*
g)*e)) + 2*(3*c*d^2*g^2*e - (c*f*g*x - 2*a*g^2)*e^3 + (c*d*g^2*x - c*d*f*g
- 2*b*d*g^2)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))/(d^2*g^3*e^3 - f*g^2*x*e^5 +
 (d*g^3*x - d*f*g^2)*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{(d + ex)^{\frac{3}{2}}\,\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(3/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Giac [A]
time = 4.10, size = 218, normalized size = 1.69

$$\frac{(3\,cdg + cfe - 2\,bge)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-\sqrt{gx + f}\,\sqrt{g}\,e^{\frac{1}{2}} + \sqrt{dg^2 + (gx + f)ge - fge}\right|\right)}{\sqrt{g}\,|g|} + \frac{\sqrt{gx + f}\left(\frac{(cdg^3e^2 - cfg^2e^3)(gx+f)}{dg^3|g|e^3 - fg^2|g|e^4} + \frac{3\,cd^2g^4e - 2\,cdfg^3e^2 - 2\,bdg^4e^2 + cf^2g^2e^3 + 2\,ag^4e^3}{dg^3|g|e^3 - fg^2|g|e^4}\right)}{\sqrt{dg^2 + (gx + f)ge - fge}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] (3*c*d*g + c*f*e - 2*b*g*e)*e^(-5/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2)
 + sqrt(d*g^2 + (g*x + f)*g*e - f*g*e)))/(sqrt(g)*abs(g)) + sqrt(g*x + f)*(
(c*d*g^3*e^2 - c*f*g^2*e^3)*(g*x + f)/(d*g^3*abs(g)*e^3 - f*g^2*abs(g)*e^4)
 + (3*c*d^2*g^4*e - 2*c*d*f*g^3*e^2 - 2*b*d*g^4*e^2 + c*f^2*g^2*e^3 + 2*a*g
^4*e^3)/(d*g^3*abs(g)*e^3 - f*g^2*abs(g)*e^4))/sqrt(d*g^2 + (g*x + f)*g*e -
 f*g*e)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)),x)
```

```
[Out] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)), x)
```

3.839 $\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=160

$$-\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}}+\frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\sqrt{f+gx}}{3e^2(ef-dg)^2\sqrt{d+ex}}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

```
[Out] 2*c*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(1/2)-2/
3*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(3/2)+2/3*(c*(-4*d^
2*g+6*d*e*f)-e*(-2*a*e*g-b*d*g+3*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/(e*
x+d)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {963, 79, 65, 223, 212}

$$\frac{2\sqrt{f+gx}\,(c(6def-4d^2g)-e(-2aeg-bdg+3bef))}{3e^2\sqrt{d+ex}\,(ef-dg)^2}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)
) + (2*(c*(6*d*e*f - 4*d^2*g) - e*(3*b*e*f - b*d*g - 2*a*e*g))*Sqrt[f + g*x
])/(3*e^2*(e*f - d*g)^2*Sqrt[d + e*x]) + (2*c*ArcTanh[(Sqrt[g]*Sqrt[d + e*x
])/(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*Sqrt[g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} - \frac{2\int \frac{\frac{cd(3ef-dg)-e(3bef-bdg-2aeg)}{2e^2}-\frac{3}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{3(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\ \sqrt{}}{3e^2(ef-dg)^2\sqrt{d+ex}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\ \sqrt{}}{3e^2(ef-dg)^2\sqrt{d+ex}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\ \sqrt{}}{3e^2(ef-dg)^2\sqrt{d+ex}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{3(ef-dg)(d+ex)^{3/2}} + \frac{2(c(6def-4d^2g)-e(3bef-bdg-2aeg))\ \sqrt{}}{3e^2(ef-dg)^2\sqrt{d+ex}}$$

Mathematica [A]
time = 0.23, size = 145, normalized size = 0.91

$$\frac{2\sqrt{f+gx}\,(cd(-3d^2g+6e^2fx+de(5f-4gx))+e^2(b(-2df-3efx+dgx)+a(-ef+3dg+2egx)))}{3e^2(ef-dg)^2(d+ex)^{3/2}} + \frac{2c\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{f+gx}}{\sqrt{g}\,\sqrt{d+ex}}\right)}{e^{5/2}\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(5/2)*Sqrt[f + g*x]),x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*d*(-3*d^2*g + 6*e^2*f*x + d*e*(5*f - 4*g*x)) + e^2*(b*(
-2*d*f - 3*e*f*x + d*g*x) + a*(-(e*f) + 3*d*g + 2*e*g*x))))/(3*e^2*(e*f - d
*g)^2*(d + e*x)^(3/2)) + (2*c*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt
[d + e*x])])/(e^(5/2)*Sqrt[g])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(136) = 272$.
time = 0.09, size = 773, normalized size = 4.83

method	result

default	$\frac{\sqrt{gx+f}\,\left(3\ln\left(\frac{2egx+2\sqrt{(ex+d)\,(gx+f)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^2e^2g^2x^2-6\ln\left(\frac{2egx+2\sqrt{(ex+d)\,(gx+f)}\,\sqrt{eg}}{2\sqrt{eg}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(g*x+f)^(1/2)*(3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*g^2*x^2-6*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f
))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f*g*x^2+3*ln(1/2*(2*e*g*
x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^4*f^2*x^2
+6*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/
2))*c*d^3*e*g^2*x-12*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+
d*g+e*f)/(e*g)^(1/2))*c*d^2*e^2*f*g*x+6*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))
^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e^3*f^2*x+3*ln(1/2*(2*e*g*x+2*
((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^4*g^2-6*ln(1/
2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^
3*e*f*g+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e
*g)^(1/2))*c*d^2*e^2*f^2+4*a*e^3*g*x*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+2*
b*d*e^2*g*x*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-6*b*e^3*f*x*((e*x+d)*(g*x+f
))^(1/2)*(e*g)^(1/2)-8*c*d^2*e*g*x*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+12*c
*d*e^2*f*x*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+6*a*d*e^2*g*((e*x+d)*(g*x+f)
)^(1/2)*(e*g)^(1/2)-2*a*e^3*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-4*b*d*e^2
*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-6*c*d^3*g*((e*x+d)*(g*x+f))^(1/2)*(e
*g)^(1/2)+10*c*d^2*e*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2))/(e*g)^(1/2)/(d*
g-e*f)^2/((e*x+d)*(g*x+f))^(1/2)/e^2/(e*x+d)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(142) = 284.
time = 7.88, size = 770, normalized size = 4.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(c*d^4*g^2 + c*f^2*x^2*e^4 - 2*(c*d*f*g*x^2 - c*d*f^2*x)*e^3 + (c*d
^2*g^2*x^2 - 4*c*d^2*f*g*x + c*d^2*f^2)*e^2 + 2*(c*d^3*g^2*x - c*d^3*f*g)*e
)*sqrt(g)*e^(1/2)*log(d^2*g^2 + 4*(d*g + (2*g*x + f)*e)*sqrt(g*x + f)*sqrt(
x*e + d)*sqrt(g)*e^(1/2) + (8*g^2*x^2 + 8*f*g*x + f^2)*e^2 + 2*(4*d*g^2*x +
 3*d*f*g)*e) - 4*(3*c*d^3*g^2*e + (a*f*g + (3*b*f*g - 2*a*g^2)*x)*e^4 + (2*
b*d*f*g - 3*a*d*g^2 - (6*c*d*f*g + b*d*g^2)*x)*e^3 + (4*c*d^2*g^2*x - 5*c*d
^2*f*g)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))/(d^4*g^3*e^3 + f^2*g*x^2*e^7 - 2*
(d*f*g^2*x^2 - d*f^2*g*x)*e^6 + (d^2*g^3*x^2 - 4*d^2*f*g^2*x + d^2*f^2*g)*e
^5 + 2*(d^3*g^3*x - d^3*f*g^2)*e^4), -1/3*(3*(c*d^4*g^2 + c*f^2*x^2*e^4 - 2
*(c*d*f*g*x^2 - c*d*f^2*x)*e^3 + (c*d^2*g^2*x^2 - 4*c*d^2*f*g*x + c*d^2*f^2
)*e^2 + 2*(c*d^3*g^2*x - c*d^3*f*g)*e)*sqrt(-g*e)*arctan(1/2*(d*g + (2*g*x
+ f)*e)*sqrt(g*x + f)*sqrt(-g*e)*sqrt(x*e + d)/((g^2*x^2 + f*g*x)*e^2 + (d*
g^2*x + d*f*g)*e)) + 2*(3*c*d^3*g^2*e + (a*f*g + (3*b*f*g - 2*a*g^2)*x)*e^4
 + (2*b*d*f*g - 3*a*d*g^2 - (6*c*d*f*g + b*d*g^2)*x)*e^3 + (4*c*d^2*g^2*x -
 5*c*d^2*f*g)*e^2)*sqrt(g*x + f)*sqrt(x*e + d))/(d^4*g^3*e^3 + f^2*g*x^2*e^
7 - 2*(d*f*g^2*x^2 - d*f^2*g*x)*e^6 + (d^2*g^3*x^2 - 4*d^2*f*g^2*x + d^2*f^
2*g)*e^5 + 2*(d^3*g^3*x - d^3*f*g^2)*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{(d + ex)^{\frac{5}{2}} \sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(5/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(5/2)*sqrt(f + g*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(142) = 284.
time = 4.27, size = 286, normalized size = 1.79

$$-\frac{2\, c\sqrt{g}\, e^{\left(-\frac{5}{2}\right)} \log\left(\left|-\sqrt{gx+f}\,\sqrt{g}\, e^{\frac{1}{2}} + \sqrt{dg^2 + (gx+f)ge - fge}\right|\right)}{|g|} - \frac{2\,\sqrt{gx+f}\left(\frac{(4\,cd^2g^6e^2 - 6\,cdfg^5e^3 - bdg^6e^3 + 3\,bfg^5e^4 - 2\,ag^6e^4)(gx+f)}{d^2g^4|g|e^3 - 2\,dfg^3|g|e^4 + f^2g^2|g|e^5} + \frac{3\,(cd^3g^7e - 3\,cd^2fg^6e^2 + 2\,cdf^2g^5e^3 + bdfg^6e^3 - adg^7e^3 - bf^2g^5e^4 + afg^6e^4)}{d^2g^4|g|e^3 - 2\,dfg^3|g|e^4 + f^2g^2|g|e^5}\right)}{3\,(dg^2 + (gx+f)ge - fge)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] -2*c*sqrt(g)*e^(-5/2)*log(abs(-sqrt(g*x + f)*sqrt(g)*e^(1/2) + sqrt(d*g^2 +
 (g*x + f)*g*e - f*g*e)))/abs(g) - 2/3*sqrt(g*x + f)*((4*c*d^2*g^6*e^2 - 6*
c*d*f*g^5*e^3 - b*d*g^6*e^3 + 3*b*f*g^5*e^4 - 2*a*g^6*e^4)*(g*x + f)/(d^2*g
^4*abs(g)*e^3 - 2*d*f*g^3*abs(g)*e^4 + f^2*g^2*abs(g)*e^5) + 3*(c*d^3*g^7*e
```

```
 - 3*c*d^2*f*g^6*e^2 + 2*c*d*f^2*g^5*e^3 + b*d*f*g^6*e^3 - a*d*g^7*e^3 - b*
f^2*g^5*e^4 + a*f*g^6*e^4)/(d^2*g^4*abs(g)*e^3 - 2*d*f*g^3*abs(g)*e^4 + f^2
*g^2*abs(g)*e^5))/(d*g^2 + (g*x + f)*g*e - f*g*e)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{f+g\,x}\,\left(d+e\,x\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)),x)
[Out] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(5/2)), x)
```

3.840 $\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=198

$$-\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}}+\frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f+gx}}{15e^2(ef-dg)^2(d+ex)^{3/2}}+\frac{2(2eg(5bef-bdg-4\dots}{\dots}$$

```
[Out] -2/5*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(5/2)+2/15*(2*c*
d*(-3*d*g+5*e*f)-e*(-4*a*e*g-b*d*g+5*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2
/(e*x+d)^(3/2)+2/15*(2*e*g*(-4*a*e*g-b*d*g+5*b*e*f)-c*(3*d^2*g^2-10*d*e*f*g
+15*e^2*f^2))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {963, 79, 37}

$$\frac{2\sqrt{f+gx}\,(2eg(-4aeg-bdg+5bef)-c(3d^2g^2-10defg+15e^2f^2))}{15e^2\sqrt{d+ex}\,(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)}+\frac{2\sqrt{f+gx}\,(2cd(5ef-3dg)-e(-4aeg-bdg+5bef))}{15e^2(d+ex)^{3/2}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(5*(e*f - d*g)*(d + e*x)^(5/2)
) + (2*(2*c*d*(5*e*f - 3*d*g) - e*(5*b*e*f - b*d*g - 4*a*e*g))*Sqrt[f + g*x
])/(15*e^2*(e*f - d*g)^2*(d + e*x)^(3/2)) + (2*(2*e*g*(5*b*e*f - b*d*g - 4*
a*e*g) - c*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(15*e^2*(e
*f - d*g)^3*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} - \frac{2\int \frac{\frac{cd(5ef-dg)-e(5bef-bdg-4aeg)}{2e^2}-\frac{5}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{5/2}\sqrt{f+gx}}\,dx}{5(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f}}{15e^2(ef-dg)^2(d+ex)^{3/2}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{5(ef-dg)(d+ex)^{5/2}} + \frac{2(2cd(5ef-3dg)-e(5bef-bdg-4aeg))\sqrt{f}}{15e^2(ef-dg)^2(d+ex)^{3/2}}$$

Mathematica [A]
time = 0.18, size = 177, normalized size = 0.89

$$-\frac{2\sqrt{f+gx}\left(15cf^2-15bfg+15ag^2-\frac{10cdf(f+gx)}{d+ex}+\frac{5bef(f+gx)}{d+ex}+\frac{5bdg(f+gx)}{d+ex}-\frac{10aeg(f+gx)}{d+ex}+\frac{3cd^2(f+gx)^2}{(d+ex)^2}-\frac{3bde(f+gx)^2}{(d+ex)^2}+\frac{3ae^2(f+gx)^2}{(d+ex)^2}\right)}{15(ef-dg)^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(7/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*Sqrt[f + g*x]*(15*c*f^2 - 15*b*f*g + 15*a*g^2 - (10*c*d*f*(f + g*x))/(d
 + e*x) + (5*b*e*f*(f + g*x))/(d + e*x) + (5*b*d*g*(f + g*x))/(d + e*x) - (
10*a*e*g*(f + g*x))/(d + e*x) + (3*c*d^2*(f + g*x)^2)/(d + e*x)^2 - (3*b*d*
e*(f + g*x)^2)/(d + e*x)^2 + (3*a*e^2*(f + g*x)^2)/(d + e*x)^2))/(15*(e*f -
 d*g)^3*Sqrt[d + e*x])
```

Maple [A]
time = 0.11, size = 210, normalized size = 1.06

method	result

default	$\frac{2\sqrt{gx+f}\,(8a\,e^2g^2x^2+2bde\,g^2x^2-10b\,e^2fg\,x^2+3c\,d^2g^2x^2-10cdefg\,x^2+15c\,e^2f^2x^2+20ade\,g^2x-4a\,e^2fgx+5b\,d^2g^2x-26bdefg}{15(ex+d)^{\frac{5}{2}}(dg-ef)^3}$
gosper	$\frac{2\sqrt{gx+f}\,(8a\,e^2g^2x^2+2bde\,g^2x^2-10b\,e^2fg\,x^2+3c\,d^2g^2x^2-10cdefg\,x^2+15c\,e^2f^2x^2+20ade\,g^2x-4a\,e^2fgx+5b\,d^2g^2x-26bdefg}{15(ex+d)^{\frac{5}{2}}(g^3d^3-3d^2efg^2+3d\,e^2f^2g-e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(g*x+f)^(1/2)*(8*a*e^2*g^2*x^2+2*b*d*e*g^2*x^2-10*b*e^2*f*g*x^2+3*c*d^
2*g^2*x^2-10*c*d*e*f*g*x^2+15*c*e^2*f^2*x^2+20*a*d*e*g^2*x-4*a*e^2*f*g*x+5*
b*d^2*g^2*x-26*b*d*e*f*g*x+5*b*e^2*f^2*x-4*c*d^2*f*g*x+20*c*d*e*f^2*x+15*a*
d^2*g^2-10*a*d*e*f*g+3*a*e^2*f^2-10*b*d^2*f*g+2*b*d*e*f^2+8*c*d^2*f^2)/(e*x
+d)^(5/2)/(d*g-e*f)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 19.89, size = 360, normalized size = 1.82

$$\frac{2\,(3\,cd^2g^2x^2+8\,cd^2f^2-10\,bd^2fg+15\,ad^2g^2-(4\,cd^2fg-5\,bd^2g^2)x+(3\,af^2+(15\,cf^2-10\,bfg+8\,ag^2)x^2+(5\,bf^2-4\,afg)x)e^2+2\,(bdf^2-5\,adfg-(5\,cdfg-bdg^2)x^2+(10\,cdf^2-13\,bdfg+10\,adg^2)x)e)\sqrt{gx+f}\,\sqrt{xe+d}}{15\,(d^6g^3-f^3x^3e^6+3\,(df^2gx^3-df^3x^2)e^5-3\,(d^2fg^2x^3-3\,d^2f^2gx^2+d^2f^3x)e^4+(d^3g^3x^3-9\,d^3fg^2x^2+9\,d^3f^2gx-d^3f^3)e^3+3\,(d^4g^3x^2-3\,d^4fg^2x+d^4f^2g)e^2+3\,(d^5g^3x-d^5fg^2)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*d^2*g^2*x^2 + 8*c*d^2*f^2 - 10*b*d^2*f*g + 15*a*d^2*g^2 - (4*c*d^
2*f*g - 5*b*d^2*g^2)*x + (3*a*f^2 + (15*c*f^2 - 10*b*f*g + 8*a*g^2)*x^2 + (
5*b*f^2 - 4*a*f*g)*x)*e^2 + 2*(b*d*f^2 - 5*a*d*f*g - (5*c*d*f*g - b*d*g^2)*
x^2 + (10*c*d*f^2 - 13*b*d*f*g + 10*a*d*g^2)*x)*e)*sqrt(g*x + f)*sqrt(x*e +
 d)/(d^6*g^3 - f^3*x^3*e^6 + 3*(d*f^2*g*x^3 - d*f^3*x^2)*e^5 - 3*(d^2*f*g^2
*x^3 - 3*d^2*f^2*g*x^2 + d^2*f^3*x)*e^4 + (d^3*g^3*x^3 - 9*d^3*f*g^2*x^2 +
9*d^3*f^2*g*x - d^3*f^3)*e^3 + 3*(d^4*g^3*x^2 - 3*d^4*f*g^2*x + d^4*f^2*g)*
e^2 + 3*(d^5*g^3*x - d^5*f*g^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{(d+ex)^{\frac{7}{2}}\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(7/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(7/2)*sqrt(f + g*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(188) = 376.
time = 6.15, size = 452, normalized size = 2.28

$$\frac{2\left((gx+f)\left(\frac{(3\,cd^2g^8e^2-10\,cdfg^7e^3+2\,bdg^8e^3+15\,cf^2g^6e^4-10\,bfg^7e^4+8\,ag^8e^4)(gx+f)}{d^3g^5|g|e^2-3\,d^2fg^4|g|e^3+3\,df^2g^3|g|e^4-f^3g^2|g|e^5}-\frac{5\,(2\,cd^2fg^8e^2-bd^2g^9e^2-8\,cdf^2g^7e^3+6\,bdfg^8e^3-4\,adg^9e^3+6\,cf^3g^6e^4-5\,bf^2g^7e^4+4\,afg^8e^4)}{d^3g^5|g|e^2-3\,d^2fg^4|g|e^3+3\,df^2g^3|g|e^4-f^3g^2|g|e^5}\right)+\frac{15\,(cd^2f^2g^8e^2-bd^2fg^9e^2+ad^2g^{10}e^2-2\,cdf^3g^7e^3+2\,bdf^2g^8e^3-2\,adfg^9e^3+cf^4g^6e^4-bf^3g^7e^4+af^2g^8e^4)}{d^3g^5|g|e^2-3\,d^2fg^4|g|e^3+3\,df^2g^3|g|e^4-f^3g^2|g|e^5}\right)\sqrt{gx+f}}{15\,(dg^2+(gx+f)ge-fge)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(7/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*((g*x + f)*((3*c*d^2*g^8*e^2 - 10*c*d*f*g^7*e^3 + 2*b*d*g^8*e^3 + 15*c
*f^2*g^6*e^4 - 10*b*f*g^7*e^4 + 8*a*g^8*e^4)*(g*x + f)/(d^3*g^5*abs(g)*e^2
- 3*d^2*f*g^4*abs(g)*e^3 + 3*d*f^2*g^3*abs(g)*e^4 - f^3*g^2*abs(g)*e^5) - 5
*(2*c*d^2*f*g^8*e^2 - b*d^2*g^9*e^2 - 8*c*d*f^2*g^7*e^3 + 6*b*d*f*g^8*e^3 -
 4*a*d*g^9*e^3 + 6*c*f^3*g^6*e^4 - 5*b*f^2*g^7*e^4 + 4*a*f*g^8*e^4)/(d^3*g^
5*abs(g)*e^2 - 3*d^2*f*g^4*abs(g)*e^3 + 3*d*f^2*g^3*abs(g)*e^4 - f^3*g^2*ab
s(g)*e^5)) + 15*(c*d^2*f^2*g^8*e^2 - b*d^2*f*g^9*e^2 + a*d^2*g^10*e^2 - 2*c
*d*f^3*g^7*e^3 + 2*b*d*f^2*g^8*e^3 - 2*a*d*f*g^9*e^3 + c*f^4*g^6*e^4 - b*f^
3*g^7*e^4 + a*f^2*g^8*e^4)/(d^3*g^5*abs(g)*e^2 - 3*d^2*f*g^4*abs(g)*e^3 + 3
*d*f^2*g^3*abs(g)*e^4 - f^3*g^2*abs(g)*e^5))*sqrt(g*x + f)/(d*g^2 + (g*x +
f)*g*e - f*g*e)^(5/2)
```

Mupad [B]
time = 4.30, size = 260, normalized size = 1.31

$$\frac{\sqrt{f+g\,x}\,\left(\frac{16\,c\,d^2\,f^2-20\,b\,d^2\,f\,g+30\,a\,d^2\,g^2+4\,b\,d\,e\,f^2-20\,a\,d\,e\,f\,g+6\,a\,e^2\,f^2}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}+\frac{x\,\left(-8\,c\,d^2\,f\,g+10\,b\,d^2\,g^2+40\,c\,d\,e\,f^2-52\,b\,d\,e\,f\,g+40\,a\,d\,e\,g^2+10\,b\,e^2\,f^2-8\,a\,e^2\,f\,g\right)}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}+\frac{x^2\,\left(6\,c\,d^2\,g^2-20\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+30\,c\,e^2\,f^2-20\,b\,e^2\,f\,g+16\,a\,e^2\,g^2\right)}{15\,e^2\,{\left(d\,g-e\,f\right)}^3}\right)}{x^2\,\sqrt{d+e\,x}+\frac{d^2\,\sqrt{d+e\,x}}{e^2}+\frac{2\,d\,x\,\sqrt{d+e\,x}}{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(7/2)),x)
```

```
[Out] ((f + g*x)^(1/2)*((30*a*d^2*g^2 + 6*a*e^2*f^2 + 16*c*d^2*f^2 + 4*b*d*e*f^2
- 20*b*d^2*f*g - 20*a*d*e*f*g)/(15*e^2*(d*g - e*f)^3) + (x*(10*b*d^2*g^2 +
10*b*e^2*f^2 + 40*a*d*e*g^2 + 40*c*d*e*f^2 - 8*a*e^2*f*g - 8*c*d^2*f*g - 52
*b*d*e*f*g))/(15*e^2*(d*g - e*f)^3) + (x^2*(16*a*e^2*g^2 + 6*c*d^2*g^2 + 30
*c*e^2*f^2 + 4*b*d*e*g^2 - 20*b*e^2*f*g - 20*c*d*e*f*g))/(15*e^2*(d*g - e*f
)^3)))/(x^2*(d + e*x)^(1/2) + (d^2*(d + e*x)^(1/2))/e^2 + (2*d*x*(d + e*x)^
(1/2))/e)
```

3.841 $\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx$

Optimal. Leaf size=281

$$-\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}}+\frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f+gx}}{35e^2(ef-dg)^2(d+ex)^{5/2}}+\frac{2(4eg(7bef-bdg-6$$

```
[Out] -2/7*(a+d*(-b*e+c*d)/e^2)*(g*x+f)^(1/2)/(-d*g+e*f)/(e*x+d)^(7/2)+2/35*(2*c*
d*(-4*d*g+7*e*f)-e*(-6*a*e*g-b*d*g+7*b*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2
/(e*x+d)^(5/2)+2/105*(4*e*g*(-6*a*e*g-b*d*g+7*b*e*f)-c*(3*d^2*g^2-14*d*e*f*
g+35*e^2*f^2))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^3/(e*x+d)^(3/2)-4/105*g*(4*e*g*
(-6*a*e*g-b*d*g+7*b*e*f)-c*(3*d^2*g^2-14*d*e*f*g+35*e^2*f^2))*(g*x+f)^(1/2)
/e^2/(-d*g+e*f)^4/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {963, 79, 47, 37}

$$-\frac{4g\sqrt{f+gx}\,(4eg(-6aeg-bdg+7bef)-c(3d^2g^2-14defg+35e^2f^2))}{105e^2\sqrt{d+ex}\,(ef-dg)^4}+\frac{2\sqrt{f+gx}\,(4eg(-6aeg-bdg+7bef)-c(3d^2g^2-14defg+35e^2f^2))}{105e^2(d+ex)^{3/2}(ef-dg)^3}-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)}+\frac{2\sqrt{f+gx}\,(2cd(7ef-4dg)-e(-6aeg-bdg+7bef))}{35e^2(d+ex)^{5/2}(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]
```

```
[Out] (-2*(a + (d*(c*d - b*e))/e^2)*Sqrt[f + g*x])/(7*(e*f - d*g)*(d + e*x)^(7/2)
) + (2*(2*c*d*(7*e*f - 4*d*g) - e*(7*b*e*f - b*d*g - 6*a*e*g))*Sqrt[f + g*x
])/(35*e^2*(e*f - d*g)^2*(d + e*x)^(5/2)) + (2*(4*e*g*(7*b*e*f - b*d*g - 6*
a*e*g) - c*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(
e*f - d*g)^3*(d + e*x)^(3/2)) - (4*g*(4*e*g*(7*b*e*f - b*d*g - 6*a*e*g) - c
*(35*e^2*f^2 - 14*d*e*f*g + 3*d^2*g^2))*Sqrt[f + g*x])/(105*e^2*(e*f - d*g)
^4*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
```

```
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}}\,dx = -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} - \frac{2\int \frac{\frac{cd(7ef-dg)-e(7bef-bdg-6aeg)}{2e^2}-\frac{7}{2}c\left(f-\frac{dg}{e}\right)x}{(d+ex)^{7/2}\sqrt{f+gx}}\,dx}{7(ef-dg)}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f}}{35e^2(ef-dg)^2(d+ex)^{5/2}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f}}{35e^2(ef-dg)^2(d+ex)^{5/2}}$$

$$= -\frac{2\left(a+\frac{d(cd-be)}{e^2}\right)\sqrt{f+gx}}{7(ef-dg)(d+ex)^{7/2}} + \frac{2(2cd(7ef-4dg)-e(7bef-bdg-6aeg))\sqrt{f}}{35e^2(ef-dg)^2(d+ex)^{5/2}}$$

Mathematica [A]
time = 0.35, size = 301, normalized size = 1.07

$$-\frac{2\sqrt{f+gx}\left(-105cf^2g(d+ex)^3+105bfg^2(d+ex)^3-105ag^3(d+ex)^3+35cef^2(d+ex)^2(f+gx)+70cdfg(d+ex)^2(f+gx)-70befg(d+ex)^2(f+gx)-35bdg^2(d+ex)^2(f+gx)+105aeg^2(d+ex)^2(f+gx)-42cdef(d+ex)(f+gx)^2+21be^2f(d+ex)(f+gx)^2-21cd^2g(d+ex)(f+gx)^2+42bdeg(d+ex)(f+gx)^2-63ae^2g(d+ex)(f+gx)^2+15cd^2e(f+gx)^3-15bde^2(f+gx)^3+15ae^3(f+gx)^3\right)}{105(ef-dg)^4(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(9/2)*Sqrt[f + g*x]),x]

[Out] (-2*Sqrt[f + g*x]*(-105*c*f^2*g*(d + e*x)^3 + 105*b*f*g^2*(d + e*x)^3 - 105
*a*g^3*(d + e*x)^3 + 35*c*e*f^2*(d + e*x)^2*(f + g*x) + 70*c*d*f*g*(d + e*x
)^2*(f + g*x) - 70*b*e*f*g*(d + e*x)^2*(f + g*x) - 35*b*d*g^2*(d + e*x)^2*(
f + g*x) + 105*a*e*g^2*(d + e*x)^2*(f + g*x) - 42*c*d*e*f*(d + e*x)*(f + g*
x)^2 + 21*b*e^2*f*(d + e*x)*(f + g*x)^2 - 21*c*d^2*g*(d + e*x)*(f + g*x)^2
+ 42*b*d*e*g*(d + e*x)*(f + g*x)^2 - 63*a*e^2*g*(d + e*x)*(f + g*x)^2 + 15*
c*d^2*e*(f + g*x)^3 - 15*b*d*e^2*(f + g*x)^3 + 15*a*e^3*(f + g*x)^3))/(105*
(e*f - d*g)^4*(d + e*x)^(7/2))

Maple [A]
time = 0.08, size = 427, normalized size = 1.52

method	result
default	$2\sqrt{gx+f}\,(48a\,e^3g^3x^3+8bd\,e^2g^3x^3-56b\,e^3f\,g^2x^3+6c\,d^2e\,g^3x^3-28cd\,e^2f\,g^2x^3+70c\,e^3f^2g\,x^3+168ad\,e^2g^3x^2-24a\,e^3f\,g^2x^2+28$
gosper	$2\sqrt{gx+f}\,(48a\,e^3g^3x^3+8bd\,e^2g^3x^3-56b\,e^3f\,g^2x^3+6c\,d^2e\,g^3x^3-28cd\,e^2f\,g^2x^3+70c\,e^3f^2g\,x^3+168ad\,e^2g^3x^2-24a\,e^3f\,g^2x^2+28$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/105*(g*x+f)^(1/2)*(48*a*e^3*g^3*x^3+8*b*d*e^2*g^3*x^3-56*b*e^3*f*g^2*x^3+
6*c*d^2*e*g^3*x^3-28*c*d*e^2*f*g^2*x^3+70*c*e^3*f^2*g*x^3+168*a*d*e^2*g^3*x
^2-24*a*e^3*f*g^2*x^2+28*b*d^2*e*g^3*x^2-200*b*d*e^2*f*g^2*x^2+28*b*e^3*f^2
*g*x^2+21*c*d^3*g^3*x^2-101*c*d^2*e*f*g^2*x^2+259*c*d*e^2*f^2*g*x^2-35*c*e^
3*f^3*x^2+210*a*d^2*e*g^3*x-84*a*d*e^2*f*g^2*x+18*a*e^3*f^2*g*x+35*b*d^3*g^
3*x-259*b*d^2*e*f*g^2*x+101*b*d*e^2*f^2*g*x-21*b*e^3*f^3*x-28*c*d^3*f*g^2*x
+200*c*d^2*e*f^2*g*x-28*c*d*e^2*f^3*x+105*a*d^3*g^3-105*a*d^2*e*f*g^2+63*a*
d*e^2*f^2*g-15*a*e^3*f^3-70*b*d^3*f*g^2+28*b*d^2*e*f^2*g-6*b*d*e^2*f^3+56*c
*d^3*f^2*g-8*c*d^2*e*f^3)/(e*x+d)^(7/2)/(d*g-e*f)^4

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(267) = 534$.
time = 84.76, size = 658, normalized size = 2.34

$$\frac{2\,(21\,cd^3g^3x^2 + 56\,cd^3f^2g - 70\,bd^3fg^2 + 105\,ad^3g^3 - 7\,(4\,cd^3fg^2 - 5\,bd^3g^3)x - (15\,af^3 - 2\,(35\,cf^2g - 28\,bfg^2 + 24\,ag^3)x^3 + (35\,cf^3 - 28\,bf^2g + 24\,afg^2)x^2 + 3\,(7\,bf^3 - 6\,af^2g)x)e^3 - (6\,bdf^3 - 63\,adf^2g + 4\,(7\,cdfg^2 - 2\,bdg^3)x^3 - (259\,cdf^2g - 200\,bdfg^2 + 168\,adg^3)x^2 + (28\,cdf^3 - 101\,bdf^2g + 84\,adfg^2)x)e^2 + (6\,cd^2g^3x^3 - 8\,cd^2f^3 + 28\,bd^2f^2g - 105\,ad^2fg^2 - (101\,cd^2fg^2 - 28\,bd^2g^3)x^2 + (200\,cd^2f^2g - 259\,bd^2fg^2 + 210\,ad^2g^3)x)e)\sqrt{gx+f}\,\sqrt{xe+d}}{105\,(d^8g^4 + f^4x^4e^8 - 4\,(df^3gx^4 - df^4x^3)e^7 + 2\,(3\,d^2f^2g^2x^4 - 8\,d^2f^3gx^3 + 3\,d^2f^4x^2)e^6 - 4\,(d^3fg^3x^4 - 6\,d^3f^2g^2x^3 + 6\,d^3f^3gx^2 - d^3f^4x)e^5 + (d^4g^4x^4 - 16\,d^4fg^3x^3 + 36\,d^4f^2g^2x^2 - 16\,d^4f^3gx + d^4f^4)e^4 + 4\,(d^5g^4x^3 - 6\,d^5fg^3x^2 + 6\,d^5f^2g^2x - d^5f^3g)e^3 + 2\,(3\,d^6g^4x^2 - 8\,d^6fg^3x + 3\,d^6f^2g^2)e^2 + 4\,(d^7g^4x - d^7fg^3)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(21*c*d^3*g^3*x^2 + 56*c*d^3*f^2*g - 70*b*d^3*f*g^2 + 105*a*d^3*g^3 -
 7*(4*c*d^3*f*g^2 - 5*b*d^3*g^3)*x - (15*a*f^3 - 2*(35*c*f^2*g - 28*b*f*g^2
 + 24*a*g^3)*x^3 + (35*c*f^3 - 28*b*f^2*g + 24*a*f*g^2)*x^2 + 3*(7*b*f^3 -
6*a*f^2*g)*x)*e^3 - (6*b*d*f^3 - 63*a*d*f^2*g + 4*(7*c*d*f*g^2 - 2*b*d*g^3)
*x^3 - (259*c*d*f^2*g - 200*b*d*f*g^2 + 168*a*d*g^3)*x^2 + (28*c*d*f^3 - 10
1*b*d*f^2*g + 84*a*d*f*g^2)*x)*e^2 + (6*c*d^2*g^3*x^3 - 8*c*d^2*f^3 + 28*b*
d^2*f^2*g - 105*a*d^2*f*g^2 - (101*c*d^2*f*g^2 - 28*b*d^2*g^3)*x^2 + (200*c
*d^2*f^2*g - 259*b*d^2*f*g^2 + 210*a*d^2*g^3)*x)*e)*sqrt(g*x + f)*sqrt(x*e
+ d)/(d^8*g^4 + f^4*x^4*e^8 - 4*(d*f^3*g*x^4 - d*f^4*x^3)*e^7 + 2*(3*d^2*f^
2*g^2*x^4 - 8*d^2*f^3*g*x^3 + 3*d^2*f^4*x^2)*e^6 - 4*(d^3*f*g^3*x^4 - 6*d^3
*f^2*g^2*x^3 + 6*d^3*f^3*g*x^2 - d^3*f^4*x)*e^5 + (d^4*g^4*x^4 - 16*d^4*f*g
^3*x^3 + 36*d^4*f^2*g^2*x^2 - 16*d^4*f^3*g*x + d^4*f^4)*e^4 + 4*(d^5*g^4*x^
3 - 6*d^5*f*g^3*x^2 + 6*d^5*f^2*g^2*x - d^5*f^3*g)*e^3 + 2*(3*d^6*g^4*x^2 -
 8*d^6*f*g^3*x + 3*d^6*f^2*g^2)*e^2 + 4*(d^7*g^4*x - d^7*f*g^3)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(9/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 752 vs. $2(267) = 534$.
time = 2.51, size = 752, normalized size = 2.68

$$\frac{2\left(\left(\left((gx+f)\left(\frac{2\,(3\,cd^2g^{10}e^4 - 14\,cdfg^9e^5 + 4\,bdg^{10}e^5 + 35\,cf^2g^8e^6 - 28\,bfg^9e^6 + 24\,ag^{10}e^6)(gx+f)}{d^4g^6|g|e^3 - 4\,d^3fg^5|g|e^4 + 6\,d^2f^2g^4|g|e^5 - 4\,df^3g^3|g|e^6 + f^4g^2|g|e^7} + \frac{7\,(3\,cd^3g^{11}e^3 - 17\,cd^2fg^{10}e^4 + 4\,bd^2g^{11}e^4 + 49\,cdf^2g^9e^5 - 32\,bdfg^{10}e^5 + 24\,adg^{11}e^5 - 35\,cf^3g^8e^6 + 28\,bf^2g^9e^6 - 24\,afg^{10}e^6)}{d^4g^6|g|e^3 - 4\,d^3fg^5|g|e^4 + 6\,d^2f^2g^4|g|e^5 - 4\,df^3g^3|g|e^6 + f^4g^2|g|e^7}\right) - \frac{35\,(2\,cd^3fg^{11}e^3 - bd^3g^{12}e^3 - 12\,cd^2f^2g^{10}e^4 + 9\,bd^2fg^{11}e^4 - 6\,ad^2g^{12}e^4 + 18\,cdf^3g^9e^5 - 15\,bdf^2g^{10}e^5 + 12\,adfg^{11}e^5 - 8\,cf^4g^8e^6 + 7\,bf^3g^9e^6 - 6\,af^2g^{10}e^6)}{d^4g^6|g|e^3 - 4\,d^3fg^5|g|e^4 + 6\,d^2f^2g^4|g|e^5 - 4\,df^3g^3|g|e^6 + f^4g^2|g|e^7}\right)(gx+f) + \frac{105\,(cd^3f^2g^{11}e^3 - bd^3fg^{12}e^3 + ad^3g^{13}e^3 - 3\,cd^2f^3g^{10}e^4 + 3\,bd^2f^2g^{11}e^4 - 3\,ad^2fg^{12}e^4 + 3\,cdf^4g^9e^5 - 3\,bdf^3g^{10}e^5 + 3\,adf^2g^{11}e^5 - cf^5g^8e^6 + bf^4g^9e^6 - af^3g^{10}e^6)}{d^4g^6|g|e^3 - 4\,d^3fg^5|g|e^4 + 6\,d^2f^2g^4|g|e^5 - 4\,df^3g^3|g|e^6 + f^4g^2|g|e^7}\right)\sqrt{gx+f}}{105\,(dg^2 + (gx+f)ge - fge)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(((g*x + f)*(2*(3*c*d^2*g^10*e^4 - 14*c*d*f*g^9*e^5 + 4*b*d*g^10*e^5
+ 35*c*f^2*g^8*e^6 - 28*b*f*g^9*e^6 + 24*a*g^10*e^6)*(g*x + f)/(d^4*g^6*abs
(g)*e^3 - 4*d^3*f*g^5*abs(g)*e^4 + 6*d^2*f^2*g^4*abs(g)*e^5 - 4*d*f^3*g^3*a
bs(g)*e^6 + f^4*g^2*abs(g)*e^7) + 7*(3*c*d^3*g^11*e^3 - 17*c*d^2*f*g^10*e^4
 + 4*b*d^2*g^11*e^4 + 49*c*d*f^2*g^9*e^5 - 32*b*d*f*g^10*e^5 + 24*a*d*g^11*
e^5 - 35*c*f^3*g^8*e^6 + 28*b*f^2*g^9*e^6 - 24*a*f*g^10*e^6)/(d^4*g^6*abs(g
)*e^3 - 4*d^3*f*g^5*abs(g)*e^4 + 6*d^2*f^2*g^4*abs(g)*e^5 - 4*d*f^3*g^3*abs
(g)*e^6 + f^4*g^2*abs(g)*e^7)) - 35*(2*c*d^3*f*g^11*e^3 - b*d^3*g^12*e^3 -
12*c*d^2*f^2*g^10*e^4 + 9*b*d^2*f*g^11*e^4 - 6*a*d^2*g^12*e^4 + 18*c*d*f^3*
g^9*e^5 - 15*b*d*f^2*g^10*e^5 + 12*a*d*f*g^11*e^5 - 8*c*f^4*g^8*e^6 + 7*b*f
^3*g^9*e^6 - 6*a*f^2*g^10*e^6)/(d^4*g^6*abs(g)*e^3 - 4*d^3*f*g^5*abs(g)*e^4
 + 6*d^2*f^2*g^4*abs(g)*e^5 - 4*d*f^3*g^3*abs(g)*e^6 + f^4*g^2*abs(g)*e^7))
*(g*x + f) + 105*(c*d^3*f^2*g^11*e^3 - b*d^3*f*g^12*e^3 + a*d^3*g^13*e^3 -
3*c*d^2*f^3*g^10*e^4 + 3*b*d^2*f^2*g^11*e^4 - 3*a*d^2*f*g^12*e^4 + 3*c*d*f^
4*g^9*e^5 - 3*b*d*f^3*g^10*e^5 + 3*a*d*f^2*g^11*e^5 - c*f^5*g^8*e^6 + b*f^4
*g^9*e^6 - a*f^3*g^10*e^6)/(d^4*g^6*abs(g)*e^3 - 4*d^3*f*g^5*abs(g)*e^4 + 6
*d^2*f^2*g^4*abs(g)*e^5 - 4*d*f^3*g^3*abs(g)*e^6 + f^4*g^2*abs(g)*e^7))*sqr
t(g*x + f)/(d*g^2 + (g*x + f)*g*e - f*g*e)^(7/2)
```

Mupad [B]
time = 4.65, size = 452, normalized size = 1.61

$$\frac{\sqrt{f+g\,x}\,\left(\frac{x^3\,\left(12\,c\,d^2\,e\,g^3-56\,c\,d\,e^2\,f\,g^2+16\,b\,d\,e^2\,g^3+140\,c\,e^3\,f^2\,g-112\,b\,e^3\,f\,g^2+96\,a\,e^3\,g^3\right)}{105\,e^3\,{\left(d\,g-e\,f\right)}^4}-\frac{-112\,c\,d^3\,f^2\,g+140\,b\,d^3\,f\,g^2-210\,a\,d^3\,g^3+16\,c\,d^2\,e\,f^3-56\,b\,d^2\,e\,f^2\,g+210\,a\,d^2\,e\,f\,g^2+12\,b\,d\,e^2\,f^3-126\,a\,d\,e^2\,f^2\,g+30\,a\,e^3\,f^3}{105\,e^3\,{\left(d\,g-e\,f\right)}^4}+\frac{x\,\left(-56\,c\,d^3\,f\,g^2+70\,b\,d^3\,g^3+400\,c\,d^2\,e\,f^2\,g-518\,b\,d^2\,e\,f\,g^2+420\,a\,d^2\,e\,g^3-56\,c\,d\,e^2\,f^3+202\,b\,d\,e^2\,f^2\,g-168\,a\,d\,e^2\,f\,g^2-42\,b\,e^3\,f^3+36\,a\,e^3\,f^2\,g\right)}{105\,e^3\,{\left(d\,g-e\,f\right)}^4}+\frac{2\,x^2\,\left(7\,d\,g-e\,f\right)\,\left(3\,c\,d^2\,g^2-14\,c\,d\,e\,f\,g+4\,b\,d\,e\,g^2+35\,c\,e^2\,f^2-28\,b\,e^2\,f\,g+24\,a\,e^2\,g^2\right)}{105\,e^3\,{\left(d\,g-e\,f\right)}^4}\right)}{x^3\,\sqrt{d+e\,x}+\frac{d^3\,\sqrt{d+e\,x}}{e^3}+\frac{3\,d\,x^2\,\sqrt{d+e\,x}}{e}+\frac{3\,d^2\,x\,\sqrt{d+e\,x}}{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(9/2)),x)
```

```
[Out] ((f + g*x)^(1/2)*((x^3*(96*a*e^3*g^3 + 16*b*d*e^2*g^3 + 12*c*d^2*e*g^3 - 11
2*b*e^3*f*g^2 + 140*c*e^3*f^2*g - 56*c*d*e^2*f*g^2))/(105*e^3*(d*g - e*f)^4
) - (30*a*e^3*f^3 - 210*a*d^3*g^3 + 12*b*d*e^2*f^3 + 16*c*d^2*e*f^3 + 140*b
*d^3*f*g^2 - 112*c*d^3*f^2*g - 126*a*d*e^2*f^2*g + 210*a*d^2*e*f*g^2 - 56*b
*d^2*e*f^2*g)/(105*e^3*(d*g - e*f)^4) + (x*(70*b*d^3*g^3 - 42*b*e^3*f^3 + 4
20*a*d^2*e*g^3 - 56*c*d*e^2*f^3 + 36*a*e^3*f^2*g - 56*c*d^3*f*g^2 - 168*a*d
*e^2*f*g^2 + 202*b*d*e^2*f^2*g - 518*b*d^2*e*f*g^2 + 400*c*d^2*e*f^2*g))/(1
05*e^3*(d*g - e*f)^4) + (2*x^2*(7*d*g - e*f)*(24*a*e^2*g^2 + 3*c*d^2*g^2 +
35*c*e^2*f^2 + 4*b*d*e*g^2 - 28*b*e^2*f*g - 14*c*d*e*f*g))/(105*e^3*(d*g -
e*f)^4)))/(x^3*(d + e*x)^(1/2) + (d^3*(d + e*x)^(1/2))/e^3 + (3*d*x^2*(d +
e*x)^(1/2))/e + (3*d^2*x*(d + e*x)^(1/2))/e^2)
```

3.842 $\int \frac{\sqrt{d+ex}\,(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx$

Optimal. Leaf size=249

$$\frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}}+\frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-d^2f^2))\sqrt{d+ex}\,\sqrt{e+fx}}{4ef^3\,(e^2-df)}+\frac{c(d+}{}$$

```
[Out] -1/4*(4*e*f*(-2*a*e*f-b*d*f+3*b*e^2)-c*(-d^2*f^2-6*d*e^2*f+15*e^4))*arctanh
(f^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(f*x+e)^(1/2))/e^(3/2)/f^(7/2)+2*(a+e*(-b*f+
c*e)/f^2)*(e*x+d)^(3/2)/(-d*f+e^2)/(f*x+e)^(1/2)+1/2*c*(e*x+d)^(3/2)*(f*x+e
)^(1/2)/e/f^2+1/4*(4*e*f*(-2*a*e*f-b*d*f+3*b*e^2)-c*(-d^2*f^2-6*d*e^2*f+15*
e^4))*(e*x+d)^(1/2)*(f*x+e)^(1/2)/e/f^3/(-d*f+e^2)
```

Rubi [A]

time = 0.18, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {963, 81, 52, 65, 223, 212}

$$\frac{\sqrt{d+ex}\,\sqrt{e+fx}\,(4ef(-2aef-bdf+3be^2)-c(-d^2f^2-6de^2f+15e^4))}{4ef^3\,(e^2-df)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{e+fx}}\right)(4ef(-2aef-bdf+3be^2)-c(-d^2f^2-6de^2f+15e^4))}{4e^{3/2}f^{7/2}}+\frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}}+\frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(3/2))/((e^2 - d*f)*Sqrt[e + f*x]) +
 ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*Sq
rt[d + e*x]*Sqrt[e + f*x])/(4*e*f^3*(e^2 - d*f)) + (c*(d + e*x)^(3/2)*Sqrt[
e + f*x])/(2*e*f^2) - ((4*e*f*(3*b*e^2 - b*d*f - 2*a*e*f) - c*(15*e^4 - 6*d
*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[e + f*x])]
)/(4*e^(3/2)*f^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{2\int \frac{\sqrt{d+ex}\left(\frac{f\left(3be^2-bdf-2aef\right)-c\left(3e^3-def\right)}{2f^2}-\frac{1}{2}c\left(d-\frac{e}{}\right.\right.}{\sqrt{e+fx}}}{e^2-df}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2} + \frac{(4ef(3be^2-bdf-}{}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-}{4ef^3(e^2-df)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-}{4ef^3(e^2-df)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-}{4ef^3(e^2-df)}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{3/2}}{(e^2-df)\sqrt{e+fx}} + \frac{(4ef(3be^2-bdf-2aef)-c(15e^4-6de^2f-}{4ef^3(e^2-df)}$$

Mathematica [**A**]
time = 0.44, size = 167, normalized size = 0.67

$$\frac{\sqrt{d+ex}\,(4ef(3be-2af+bfx)+c(-15e^3-5e^2fx+df^2x+ef(d+2fx^2)))}{4ef^3\sqrt{e+fx}} + \frac{(4ef(-3be^2+bdf+2aef)+c(15e^4-6de^2f-d^2f^2))\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{e+fx}}\right)}{4e^{3/2}f^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (Sqrt[d + e*x]*(4*e*f*(3*b*e - 2*a*f + b*f*x) + c*(-15*e^3 - 5*e^2*f*x + d*
f^2*x + e*f*(d + 2*f*x^2))))/(4*e*f^3*Sqrt[e + f*x]) + ((4*e*f*(-3*b*e^2 +
b*d*f + 2*a*e*f) + c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*ArcTanh[(Sqrt[f]*Sqrt[
d + e*x])/(Sqrt[e]*Sqrt[e + f*x])])/(4*e^(3/2)*f^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(219) = 438.
time = 0.09, size = 834, normalized size = 3.35

method	result
default	$\frac{\sqrt{ex+d}\,\left(8\ln\left(\frac{2efx+2\sqrt{(ex+d)(fx+e)}\,\sqrt{ef}+df+e^2}{2\sqrt{ef}}\right)a\,e^2f^3x+4\ln\left(\frac{2efx+2\sqrt{(ex+d)(fx+e)}\,\sqrt{ef}}{2\sqrt{ef}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(e*x+d)^(1/2)*(8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+
d*f+e^2)/(e*f)^(1/2))*a*e^2*f^3*x+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/
2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*d*e*f^3*x-12*ln(1/2*(2*e*f*x+2*((e*x
+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^3*f^2*x-ln(1/2*(2*
e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^2*f^3
*x-6*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(
1/2))*c*d*e^2*f^2*x+15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2
)+d*f+e^2)/(e*f)^(1/2))*c*e^4*f*x+4*c*e*f^2*x^2*(e*f)^(1/2)*((e*x+d)*(f*x+e
))^(1/2)+8*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(
e*f)^(1/2))*a*e^3*f^2+4*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/
2)+d*f+e^2)/(e*f)^(1/2))*b*d*e^2*f^2-12*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))
^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*b*e^4*f-ln(1/2*(2*e*f*x+2*((e*x+d)
*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d^2*e*f^2-6*ln(1/2*(2*e
*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/2))*c*d*e^3*f+
15*ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^(1/2)*(e*f)^(1/2)+d*f+e^2)/(e*f)^(1/
2))*c*e^5+8*b*e*f^2*x*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(1/2)+2*c*d*f^2*x*(e*f)
^(1/2)*((e*x+d)*(f*x+e))^(1/2)-10*c*e^2*f*x*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(
1/2)-16*a*e*f^2*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(1/2)+24*b*e^2*f*(e*f)^(1/2)*
((e*x+d)*(f*x+e))^(1/2)+2*c*d*e*f*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(1/2)-30*c*
e^3*(e*f)^(1/2)*((e*x+d)*(f*x+e))^(1/2))/(e*f)^(1/2)/e/((e*x+d)*(f*x+e))^(1
/2)/f^3/(f*x+e)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*f-%e^2>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 4.29, size = 573, normalized size = 2.30

$$\left[-\frac{{\left(c d^{2} f^{3} x - 15 \, c e^{5} - 3 \, {\left(5 \, c f x - 4 \, b f\right)} e^{4} + 2 \, {\left(6 \, b f^{2} x + 3 \, c d f - 4 \, a f^{2}\right)} e^{3} - 2 \, {\left(2 \, b d f^{2} - {\left(3 \, c d f^{2} - 4 \, a f^{3}\right)} x\right)} e^{2} - {\left(4 \, b d f^{3} x - c d^{2} f^{2}\right)} e\right)} \sqrt{f} e^{\frac{1}{2}} \log\left(8 \, d f^{2} x e + d^{2} f^{2} + 4 \, {\left(2 \, f x e + d f + e^{2}\right)} \sqrt{f x + e} \sqrt{x e + d} \sqrt{f} e^{\frac{1}{2}} + 8 \, f x e^{3} + 2 \, {\left(4 \, f^{2} x^{2} + 3 \, d f\right)} e^{2} + e^{4}\right) - 4 \, {\left(c d f^{3} x e - 15 \, c f e^{4} - {\left(5 \, c f^{2} x - 12 \, b f^{2}\right)} e^{3} + {\left(2 \, c f^{3} x^{2} + 4 \, b f^{3} x + c d f^{2} - 8 \, a f^{3}\right)} e^{2}\right)} \sqrt{f x + e} \sqrt{x e + d}}{16 \, {\left(f^{5} x e^{2} + f^{4} e^{3}\right)}}, \frac{{\left(c d^{2} f^{3} x - 15 \, c e^{5} - 3 \, {\left(5 \, c f x - 4 \, b f\right)} e^{4} + 2 \, {\left(6 \, b f^{2} x + 3 \, c d f - 4 \, a f^{2}\right)} e^{3} - 2 \, {\left(2 \, b d f^{2} - {\left(3 \, c d f^{2} - 4 \, a f^{3}\right)} x\right)} e^{2} - {\left(4 \, b d f^{3} x - c d^{2} f^{2}\right)} e\right)} \sqrt{-f e} \arctan\left(\frac{{\left(2 \, f x e + d f + e^{2}\right)} \sqrt{f x + e} \sqrt{-f e} \sqrt{x e + d}}{2 \, {\left(d f^{2} x e + f x e^{3} + {\left(f^{2} x^{2} + d f\right)} e^{2}\right)}}\right) + 2 \, {\left(c d f^{3} x e - 15 \, c f e^{4} - {\left(5 \, c f^{2} x - 12 \, b f^{2}\right)} e^{3} + {\left(2 \, c f^{3} x^{2} + 4 \, b f^{3} x + c d f^{2} - 8 \, a f^{3}\right)} e^{2}\right)} \sqrt{f x + e} \sqrt{x e + d}}{8 \, {\left(f^{5} x e^{2} + f^{4} e^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((c*d^2*f^3*x - 15*c*e^5 - 3*(5*c*f*x - 4*b*f)*e^4 + 2*(6*b*f^2*x +
3*c*d*f - 4*a*f^2)*e^3 - 2*(2*b*d*f^2 - (3*c*d*f^2 - 4*a*f^3)*x)*e^2 - (4*b
*d*f^3*x - c*d^2*f^2)*e)*sqrt(f)*e^(1/2)*log(8*d*f^2*x*e + d^2*f^2 + 4*(2*f
*x*e + d*f + e^2)*sqrt(f*x + e)*sqrt(x*e + d)*sqrt(f)*e^(1/2) + 8*f*x*e^3 +
 2*(4*f^2*x^2 + 3*d*f)*e^2 + e^4) - 4*(c*d*f^3*x*e - 15*c*f*e^4 - (5*c*f^2*
x - 12*b*f^2)*e^3 + (2*c*f^3*x^2 + 4*b*f^3*x + c*d*f^2 - 8*a*f^3)*e^2)*sqrt
(f*x + e)*sqrt(x*e + d))/(f^5*x*e^2 + f^4*e^3), 1/8*((c*d^2*f^3*x - 15*c*e^
5 - 3*(5*c*f*x - 4*b*f)*e^4 + 2*(6*b*f^2*x + 3*c*d*f - 4*a*f^2)*e^3 - 2*(2*
b*d*f^2 - (3*c*d*f^2 - 4*a*f^3)*x)*e^2 - (4*b*d*f^3*x - c*d^2*f^2)*e)*sqrt(
-f*e)*arctan(1/2*(2*f*x*e + d*f + e^2)*sqrt(f*x + e)*sqrt(-f*e)*sqrt(x*e +
d)/(d*f^2*x*e + f*x*e^3 + (f^2*x^2 + d*f)*e^2)) + 2*(c*d*f^3*x*e - 15*c*f*e
^4 - (5*c*f^2*x - 12*b*f^2)*e^3 + (2*c*f^3*x^2 + 4*b*f^3*x + c*d*f^2 - 8*a*
f^3)*e^2)*sqrt(f*x + e)*sqrt(x*e + d))/(f^5*x*e^2 + f^4*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + e x} \left(a + b x + c x^{2}\right)}{\left(e + f x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(a + b*x + c*x**2)/(e + f*x)**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.47
Done
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}\,\left(c\,x^2+b\,x+a\right)}{{\left(e+f\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
[Out] int(((d + e*x)^(1/2)*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

3.843 $\int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=240

$$\frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4}+\frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,(d+ex)^{3/2}}{12b^3}$$

```
[Out] 2*e*(b*x+a)^(3/2)*(e*x+d)^(5/2)/b^2+1/8*(-a*e+b*d)^2*(35*a^2*e^2-90*a*b*d*e
+73*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(9/2)/e
^(1/2)+1/12*(35*a^2*e^2-90*a*b*d*e+73*b^2*d^2)*(e*x+d)^(3/2)*(b*x+a)^(1/2)/
b^3+1/3*(-13*a*e+17*b*d)*(e*x+d)^(5/2)*(b*x+a)^(1/2)/b^2+1/8*(-a*e+b*d)*(35
*a^2*e^2-90*a*b*d*e+73*b^2*d^2)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^4
```

Rubi [A]
time = 0.14, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {965, 81, 52, 65, 223, 212}

$$\frac{(bd-ae)^2\left(35a^2e^2-90abde+73b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}+\frac{\sqrt{a+bx}\,\sqrt{d+ex}\,(bd-ae)\left(35a^2e^2-90abde+73b^2d^2\right)}{8b^4}+\frac{\sqrt{a+bx}\,(d+ex)^{3/2}\left(35a^2e^2-90abde+73b^2d^2\right)}{12b^3}+\frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2}+\frac{\sqrt{a+bx}\,(d+ex)^{5/2}(17bd-13ae)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((b*d - a*e)*(73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d +
e*x])/(8*b^4) + ((73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x]*(d +
e*x)^(3/2))/(12*b^3) + ((17*b*d - 13*a*e)*Sqrt[a + b*x]*(d + e*x)^(5/2))/(3
*b^2) + (2*e*(a + b*x)^(3/2)*(d + e*x)^(5/2))/b^2 + ((b*d - a*e)^2*(73*b^2*
d^2 - 90*a*b*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqr
t[d + e*x])])/(8*b^(9/2)*Sqrt[e])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(15d^2+20dex+8e^2x^2\right)}{\sqrt{a+bx}}\,dx = \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\int \frac{(d+ex)^{3/2}\left(4e\left(15b^2d^2-3abde-5a^2e^2\right)+4be^2(17\cdots}{\sqrt{a+bx}}}{4b^2e}$$

$$= \frac{(17bd-13ae)\sqrt{a+bx}\,(d+ex)^{5/2}}{3b^2} + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} \cdots$$

$$= \frac{\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,(d+ex)^{3/2}}{12b^3} + \frac{(17bd-13a\cdots}{}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4} + \frac{(73\cdots}{}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4} + \frac{(73\cdots}{}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4} + \frac{(73\cdots}{}$$

$$= \frac{(bd-ae)\left(73b^2d^2-90abde+35a^2e^2\right)\sqrt{a+bx}\,\sqrt{d+ex}}{8b^4} + \frac{(73\cdots}{}$$

Mathematica [A]
time = 0.43, size = 189, normalized size = 0.79

$$\frac{\sqrt{a+bx}\,\sqrt{d+ex}\left(-105a^3e^3+5a^2be^2(89d+14ex)-ab^2e\left(725d^2+292dex+56e^2x^2\right)+b^3\left(501d^3+466d^2ex+232de^2x^2+48e^3x^3\right)\right)}{24b^4}+\frac{(bd-ae)^2\left(73b^2d^2-90abde+35a^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x
]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[d + e*x]*(-105*a^3*e^3 + 5*a^2*b*e^2*(89*d + 14*e*x) -
a*b^2*e*(725*d^2 + 292*d*e*x + 56*e^2*x^2) + b^3*(501*d^3 + 466*d^2*e*x + 2
32*d*e^2*x^2 + 48*e^3*x^3)))/(24*b^4) + ((b*d - a*e)^2*(73*b^2*d^2 - 90*a*b
*d*e + 35*a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])]
)/(8*b^(9/2)*Sqrt[e])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(204) = 408$.
time = 0.08, size = 571, normalized size = 2.38

method	result
default	$\frac{\sqrt{ex+d}\,\sqrt{bx+a}\,\left(96b^{3}e^{3}x^{3}\sqrt{(bx+a)(ex+d)}\,\sqrt{eb}-112a\,b^{2}e^{3}x^{2}\sqrt{(bx+a)(ex+d)}\,\sqrt{eb}+464b^{3}d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/48*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(96*b^3*e^3*x^3*((b*x+a)*(e*x+d))^(1/2)*(e
*b)^(1/2)-112*a*b^2*e^3*x^2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+464*b^3*d*e
^2*x^2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+105*ln(1/2*(2*b*e*x+2*((b*x+a)*(
e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a^4*e^4-480*ln(1/2*(2*b*e*x
+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a^3*b*d*e^3+86
4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2
))*a^2*b^2*d^2*e^2-708*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2
)+a*e+b*d)/(e*b)^(1/2))*a*b^3*d^3*e+219*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))
^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*b^4*d^4+140*((b*x+a)*(e*x+d))^(1/2
)*(e*b)^(1/2)*a^2*b*e^3*x-584*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)*a*b^2*d*e
^2*x+932*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)*b^3*d^2*e*x-210*(e*b)^(1/2)*((
b*x+a)*(e*x+d))^(1/2)*a^3*e^3+890*(e*b)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a^2*b
*d*e^2-1450*(e*b)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*b^2*d^2*e+1002*(e*b)^(1/2
)*((b*x+a)*(e*x+d))^(1/2)*b^3*d^3)/b^4/((b*x+a)*(e*x+d))^(1/2)/(e*b)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*d-%e*a>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 3.13, size = 514, normalized size = 2.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/96*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d^2*e^2 - 160*a^3*b*d*
e^3 + 35*a^4*e^4)*sqrt(b)*e^(1/2)*log(b^2*d^2 + 4*(b*d + (2*b*x + a)*e)*sqr
t(b*x + a)*sqrt(x*e + d)*sqrt(b)*e^(1/2) + (8*b^2*x^2 + 8*a*b*x + a^2)*e^2
+ 2*(4*b^2*d*x + 3*a*b*d)*e) + 4*(501*b^4*d^3*e + (48*b^4*x^3 - 56*a*b^3*x^
2 + 70*a^2*b^2*x - 105*a^3*b)*e^4 + (232*b^4*d*x^2 - 292*a*b^3*d*x + 445*a^
2*b^2*d)*e^3 + (466*b^4*d^2*x - 725*a*b^3*d^2)*e^2)*sqrt(b*x + a)*sqrt(x*e
+ d))*e^(-1)/b^5, -1/48*(3*(73*b^4*d^4 - 236*a*b^3*d^3*e + 288*a^2*b^2*d^2*
e^2 - 160*a^3*b*d*e^3 + 35*a^4*e^4)*sqrt(-b*e)*arctan(1/2*(b*d + (2*b*x + a
)*e)*sqrt(b*x + a)*sqrt(-b*e)*sqrt(x*e + d)/((b^2*x^2 + a*b*x)*e^2 + (b^2*d
*x + a*b*d)*e)) - 2*(501*b^4*d^3*e + (48*b^4*x^3 - 56*a*b^3*x^2 + 70*a^2*b^
2*x - 105*a^3*b)*e^4 + (232*b^4*d*x^2 - 292*a*b^3*d*x + 445*a^2*b^2*d)*e^3
+ (466*b^4*d^2*x - 725*a*b^3*d^2)*e^2)*sqrt(b*x + a)*sqrt(x*e + d))*e^(-1)/
b^5]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(211) = 422.
time = 3.58, size = 717, normalized size = 2.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/24*(360*((b^2*d - a*b*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2)
 + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) - sqrt(b^2*d + (b*x + a)*b
*e - a*b*e)*sqrt(b*x + a))*d^3*abs(b)/b^2 - 28*(sqrt(b^2*d + (b*x + a)*b*e
- a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^
5*e^4)*e^(-4)/b^7) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)*e^(-4
)/b^7) - 3*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*e^(-5/2)*log
(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))
/b^(3/2))*d*abs(b)*e^2/b^2 - 210*((b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*e^(
-3/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e -
```

```
a*b*e)))/sqrt(b) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*b*x + (b*d*e - 5
*a*e^2)*e^(-2) + 2*a)*sqrt(b*x + a))*d^2*abs(b)*e/b^3 - (sqrt(b^2*d + (b*x
+ a)*b*e - a*b*e)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*d*e^5
- 25*a*b^11*e^6)*e^(-6)/b^14) - (5*b^13*d^2*e^4 + 14*a*b^12*d*e^5 - 163*a^2
*b^11*e^6)*e^(-6)/b^14) + 3*(5*b^14*d^3*e^3 + 9*a*b^13*d^2*e^4 + 15*a^2*b^1
2*d*e^5 - 93*a^3*b^11*e^6)*e^(-6)/b^14)*sqrt(b*x + a) + 3*(5*b^4*d^4 + 4*a*
b^3*d^3*e + 6*a^2*b^2*d^2*e^2 + 20*a^3*b*d*e^3 - 35*a^4*e^4)*e^(-7/2)*log(a
bs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/b
^(5/2))*abs(b)*e^3/b^2)/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}\,\left(15\,d^2+20\,d\,e\,x+8\,e^2\,x^2\right)}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)
[Out] int(((d + e*x)^(3/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2), x)
```

3.844 $\int \frac{\sqrt{d+ex}\,(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=176

$$\frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3}+\frac{2(4bd-3ae)\sqrt{a+bx}\,(d+ex)^{3/2}}{b^2}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}$$

```
[Out] 8/3*e*(b*x+a)^(3/2)*(e*x+d)^(3/2)/b^2+(-a*e+b*d)*(5*a^2*e^2-13*a*b*d*e+11*b
^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(7/2)/e^(1/2
)+2*(-3*a*e+4*b*d)*(e*x+d)^(3/2)*(b*x+a)^(1/2)/b^2+(5*a^2*e^2-13*a*b*d*e+11
*b^2*d^2)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^3
```

Rubi [A]
time = 0.11, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {965, 81, 52, 65, 223, 212}

$$\frac{(bd-ae)(5a^2e^2-13abde+11b^2d^2)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}}+\frac{\sqrt{a+bx}\,\sqrt{d+ex}\,(5a^2e^2-13abde+11b^2d^2)}{b^3}+\frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2}+\frac{2\sqrt{a+bx}\,(d+ex)^{3/2}(4bd-3ae)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] ((11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^3 + (
2*(4*b*d - 3*a*e)*Sqrt[a + b*x]*(d + e*x)^(3/2))/b^2 + (8*e*(a + b*x)^(3/2)
*(d + e*x)^(3/2))/(3*b^2) + ((b*d - a*e)*(11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e
^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(7/2)*Sqrt
[e])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}}\,dx = \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{\int \frac{\sqrt{d+ex}\,(3e(3bd-2ae)(5bd+2ae)+12be^2(4bd}{\sqrt{a+bx}}}{3b^2e}$$

$$= \frac{2(4bd-3ae)\sqrt{a+bx}\,(d+ex)^{3/2}}{b^2} + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \cdots$$

$$= \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a}}{b}$$

$$= \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a}}{b}$$

$$= \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a}}{b}$$

$$= \frac{(11b^2d^2-13abde+5a^2e^2)\sqrt{a+bx}\,\sqrt{d+ex}}{b^3} + \frac{2(4bd-3ae)\sqrt{a}}{b}$$

Mathematica [**A**]
time = 0.44, size = 152, normalized size = 0.86

$$\frac{b\sqrt{a+bx}\,\sqrt{d+ex}\,(15a^2e^2-abe(49d+10ex)+b^2(57d^2+32dex+8e^2x^2))-3\sqrt{\frac{b}{e}}\,(11b^3d^3-24ab^2d^2e+18a^2bde^2-5a^3e^3)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{e}}\,\sqrt{d+ex}\right)}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (b*Sqrt[a + b*x]*Sqrt[d + e*x]*(15*a^2*e^2 - a*b*e*(49*d + 10*e*x) + b^2*(5
7*d^2 + 32*d*e*x + 8*e^2*x^2)) - 3*Sqrt[b/e]*(11*b^3*d^3 - 24*a*b^2*d^2*e +
 18*a^2*b*d*e^2 - 5*a^3*e^3)*Log[Sqrt[a + b*x] - Sqrt[b/e]*Sqrt[d + e*x]])/
(3*b^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(150) = 300.
time = 0.09, size = 392, normalized size = 2.23

method	result

default	$-\frac{\sqrt{ex+d}\,\sqrt{bx+a}\,\left(-16b^2e^2x^2\sqrt{(bx+a)(ex+d)}\,\sqrt{eb}+15\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{eb}+ae}{2\sqrt{eb}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/6*(e*x+d)^(1/2)*(b*x+a)^(1/2)*(-16*b^2*e^2*x^2*((b*x+a)*(e*x+d))^(1/2)*(
e*b)^(1/2)+15*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d
)/(e*b)^(1/2))*a^3*e^3-54*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(
1/2)+a*e+b*d)/(e*b)^(1/2))*a^2*b*d*e^2+72*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d
))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a*b^2*d^2*e-33*ln(1/2*(2*b*e*x+2
*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*b^3*d^3+20*(e*b)
^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*b*e^2*x-64*(e*b)^(1/2)*((b*x+a)*(e*x+d))^(
1/2)*b^2*d*e*x-30*(e*b)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a^2*e^2+98*(e*b)^(1/2
)*((b*x+a)*(e*x+d))^(1/2)*a*b*d*e-114*(e*b)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*b
^2*d^2)/b^3/((b*x+a)*(e*x+d))^(1/2)/(e*b)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*d-%e*a>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 2.48, size = 398, normalized size = 2.26

$$\left[-\frac{\left(3\,(11\,b^3d^3-24\,ab^2d^2e+18\,a^2bde^2-5\,a^3e^3)\sqrt{b}\,e^{\frac{1}{2}}\log\left(b^2d^2-4\,(bd+(2\,bx+a)e)\sqrt{bx+a}\,\sqrt{xe+d}\,\sqrt{b}\,e^{\frac{1}{2}}+(8\,b^2x^2+8\,abx+a^2)e^2+2\,(4\,b^2dx+3\,abd)e\right)-4\,(57\,b^3d^2e+(8\,b^3x^2-10\,ab^2x+15\,a^2b)e^3+(32\,b^3dx-49\,ab^2d)e^2)\sqrt{bx+a}\,\sqrt{xe+d}\right)e^{(-1)}}{12\,b^4},-\frac{\left(3\,(11\,b^3d^3-24\,ab^2d^2e+18\,a^2bde^2-5\,a^3e^3)\sqrt{-be}\,\arctan\left(\frac{(bd+(2\,bx+a)e)\sqrt{bx+a}\,\sqrt{-be}\,\sqrt{xe+d}}{2\,((b^2x^2+abx)e^2+(b^2dx+abd)e)}\right)-2\,(57\,b^3d^2e+(8\,b^3x^2-10\,ab^2x+15\,a^2b)e^3+(32\,b^3dx-49\,ab^2d)e^2)\sqrt{bx+a}\,\sqrt{xe+d}\right)e^{(-1)}}{6\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-1/12*(3*(11*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(b
)*e^(1/2)*log(b^2*d^2 - 4*(b*d + (2*b*x + a)*e)*sqrt(b*x + a)*sqrt(x*e + d)
```

```
*sqrt(b)*e^(1/2) + (8*b^2*x^2 + 8*a*b*x + a^2)*e^2 + 2*(4*b^2*d*x + 3*a*b*d
)*e) - 4*(57*b^3*d^2*e + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*e^3 + (32*b^3*
d*x - 49*a*b^2*d)*e^2)*sqrt(b*x + a)*sqrt(x*e + d))*e^(-1)/b^4, -1/6*(3*(11
*b^3*d^3 - 24*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 5*a^3*e^3)*sqrt(-b*e)*arctan(1
/2*(b*d + (2*b*x + a)*e)*sqrt(b*x + a)*sqrt(-b*e)*sqrt(x*e + d)/((b^2*x^2 +
 a*b*x)*e^2 + (b^2*d*x + a*b*d)*e)) - 2*(57*b^3*d^2*e + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*e^3 + (32*b^3*d*x - 49*a*b^2*d)*e^2)*sqrt(b*x + a)*sqrt(x*
e + d))*e^(-1)/b^4]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(155) = 310.
time = 4.35, size = 441, normalized size = 2.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(8*e^2*x^2+20*d*e*x+15*d^2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -1/3*(45*((b^2*d - a*b*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) +
 sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/sqrt(b) - sqrt(b^2*d + (b*x + a)*b*e
 - a*b*e)*sqrt(b*x + a))*d^2*abs(b)/b^2 - (sqrt(b^2*d + (b*x + a)*b*e - a*b
*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*d*e^3 - 13*a*b^5*e^4
)*e^(-4)/b^7) - 3*(b^7*d^2*e^2 + 2*a*b^6*d*e^3 - 11*a^2*b^5*e^4)*e^(-4)/b^7
) - 3*(b^3*d^3 + a*b^2*d^2*e + 3*a^2*b*d*e^2 - 5*a^3*e^3)*e^(-5/2)*log(abs(
-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)))/b^(3
/2))*abs(b)*e^2/b^2 - 15*((b^3*d^2 + 2*a*b^2*d*e - 3*a^2*b*e^2)*e^(-3/2)*lo
g(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e))
)/sqrt(b) + sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*(2*b*x + (b*d*e - 5*a*e^2)*
e^(-2) + 2*a)*sqrt(b*x + a))*d*abs(b)*e/b^3)/b
```

Mupad [**B**]
time = 73.15, size = 1797, normalized size = 10.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x)^(1/2)*(15*d^2 + 8*e^2*x^2 + 20*d*e*x))/(a + b*x)^(1/2),x)

[Out] ((((a + b*x)^(1/2) - a^(1/2))^3*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e
))/(e^3*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))*(10*b
^3*d^3 + 20*a*b^2*d^2*e - 30*a^2*b*d*e^2))/(e^4*((d + e*x)^(1/2) - d^(1/2))
) - (160*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2)
 - d^(1/2))^6) + (((a + b*x)^(1/2) - a^(1/2))^7*(10*b^2*d^3 - 30*a^2*d*e^2
+ 20*a*b*d^2*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) + (((a + b*x)^(1/2)
- a^(1/2))^5*(70*b^2*d^3 + 110*a^2*d*e^2 + 460*a*b*d^2*e))/(b*e^2*((d + e*x
)^(1/2) - d^(1/2))^5) - (a^(1/2)*d^(1/2)*(320*b*d^2 + 640*a*d*e)*((a + b*x)
^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (160*a^(1/2)*b^2
d^(5/2)((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2)
)/(((a + b*x)^(1/2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 -
(4*b^3*((a + b*x)^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) +
 (6*b^2*((a + b*x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4)
- (4*b*((a + b*x)^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) -
((((a + b*x)^(1/2) - a^(1/2))*(2*b^5*d^3 - 10*a^3*b^2*e^3 + 6*a^2*b^3*d*e^2
 + 2*a*b^4*d^2*e))/(e^6*((d + e*x)^(1/2) - d^(1/2))) - (((a + b*x)^(1/2) -
a^(1/2))^5*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*b^2*d^2*e + 1252*a^2*b*d*e^2)
)/(e^4*((d + e*x)^(1/2) - d^(1/2))^5) - (((a + b*x)^(1/2) - a^(1/2))^3*((34
*b^4*d^3)/3 - (170*a^3*b*e^3)/3 + 34*a^2*b^2*d*e^2 + 182*a*b^3*d^2*e))/(e^5
*((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^11*(2*b^3*d^
3 - 10*a^3*e^3 + 2*a*b^2*d^2*e + 6*a^2*b*d*e^2))/(b^3*e*((d + e*x)^(1/2) -
d^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^9*((34*b^3*d^3)/3 - (170*a^3*e^
3)/3 + 182*a*b^2*d^2*e + 34*a^2*b*d*e^2))/(b^2*e^2*((d + e*x)^(1/2) - d^(1/
2))^9) - (((a + b*x)^(1/2) - a^(1/2))^7*(132*a^3*e^3 + 76*b^3*d^3 + 1100*a*
b^2*d^2*e + 1252*a^2*b*d*e^2))/(b*e^3*((d + e*x)^(1/2) - d^(1/2))^7) + (a^(
1/2)*d^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*(1024*a^2*e^2 + 512*b^2*d^2 + (5
632*a*b*d*e)/3))/(e^3*((d + e*x)^(1/2) - d^(1/2))^6) + (a^(1/2)*d^(1/2)*(25
6*b*d^2 + 768*a*d*e)*((a + b*x)^(1/2) - a^(1/2))^8)/(e^2*((d + e*x)^(1/2) -
 d^(1/2))^8) + (a^(1/2)*d^(1/2)*(256*b^3*d^2 + 768*a*b^2*d*e)*((a + b*x)^(1
/2) - a^(1/2))^4)/(e^4*((d + e*x)^(1/2) - d^(1/2))^4))/(((a + b*x)^(1/2) -
a^(1/2))^12/((d + e*x)^(1/2) - d^(1/2))^12 + b^6/e^6 - (6*b^5*((a + b*x)^(1
/2) - a^(1/2))^2)/(e^5*((d + e*x)^(1/2) - d^(1/2))^2) + (15*b^4*((a + b*x)^
(1/2) - a^(1/2))^4)/(e^4*((d + e*x)^(1/2) - d^(1/2))^4) - (20*b^3*((a + b*x
)^(1/2) - a^(1/2))^6)/(e^3*((d + e*x)^(1/2) - d^(1/2))^6) + (15*b^2*((a + b
x)^(1/2) - a^(1/2))^8)/(e^2((d + e*x)^(1/2) - d^(1/2))^8) - (6*b*((a + b*
x)^(1/2) - a^(1/2))^10)/(e*((d + e*x)^(1/2) - d^(1/2))^10)) + (((30*b*d^3 +
 30*a*d^2*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2) - d^(1/2)))
 - (120*a^(1/2)*d^(5/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d + e*x)^(1/2)
- d^(1/2))^2) + ((30*b*d^3 + 30*a*d^2*e)*((a + b*x)^(1/2) - a^(1/2))^3)/(b*
e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))^4/((d + e*x)
^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d
+ e*x)^(1/2) - d^(1/2))^2)) - (2*atanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2))
)/(b^(1/2)*((d + e*x)^(1/2) - d^(1/2))))*(a*e - b*d)*(5*a^2*e^2 + b^2*d^2 +

```
 2*a*b*d*e))/(b^(7/2)*e^(1/2)) - (30*d^2*atanh((e^(1/2)*((a + b*x)^(1/2) -
a^(1/2)))/(b^(1/2)*((d + e*x)^(1/2) - d^(1/2))))*(a*e - b*d))/(b^(3/2)*e^(1
/2)) + (10*d*atanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((d + e*x
)^(1/2) - d^(1/2))))*(a*e - b*d)*(3*a*e + b*d))/(b^(5/2)*e^(1/2))
```

3.845 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=122

$$\frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2(8b^2d^2-8abde+3a^2e^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}$$

```
[Out] 2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*
x+d)^(1/2))/b^(5/2)/e^(1/2)+4*e*(b*x+a)^(3/2)*(e*x+d)^(1/2)/b^2+2*(-5*a*e+7
*b*d)*(b*x+a)^(1/2)*(e*x+d)^(1/2)/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {965, 81, 65, 223, 212}

$$\frac{2(3a^2e^2-8abde+8b^2d^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2\sqrt{a+bx}\sqrt{d+ex}(7bd-5ae)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*(7*b*d - 5*a*e)*Sqrt[a + b*x]*Sqrt[d + e*x])/b^2 + (4*e*(a + b*x)^(3/2)*
Sqrt[d + e*x])/b^2 + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[e
]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(b^(5/2)*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}}\,dx = \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{\int \frac{2e\left(15b^2d^2-6abde-2a^2e^2\right)+4be^2(7bd-5ae)x}{\sqrt{a+bx}\sqrt{d+ex}}\,dx}{2b^2e}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{(8b^2d^2-8abde}{}$$

$$(2(8b^2d^2-8abd$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{}{}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{(2(8b^2d^2-8abd}{}$$

$$= \frac{2(7bd-5ae)\sqrt{a+bx}\sqrt{d+ex}}{b^2}+\frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2}+\frac{2(8b^2d^2-8abde}{}$$

Mathematica [A]

time = 0.27, size = 103, normalized size = 0.84

$$\frac{2\sqrt{a+bx}\,\sqrt{d+ex}\,(7bd-3ae+2bex)}{b^2}+\frac{2(8b^2d^2-8abde+3a^2e^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{a+bx}}\right)}{b^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*Sqrt[d + e*x]),x]
```

```
[Out] (2*Sqrt[a + b*x]*Sqrt[d + e*x]*(7*b*d - 3*a*e + 2*b*e*x))/b^2 + (2*(8*b^2*d
^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a
 + b*x])])/(b^(5/2)*Sqrt[e])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(102) = 204$.
time = 0.08, size = 247, normalized size = 2.02

method	result
default	$\left(4\sqrt{(bx+a)(ex+d)}\,\sqrt{eb}\,bex+3\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{eb}+ae+bd}{2\sqrt{eb}}\right)a^2e^2-8\ln\left(\frac{2bex+2\sqrt{(bx+\cdots}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] (4*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)*b*e*x+3*ln(1/2*(2*b*e*x+2*((b*x+a)*(
e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a^2*e^2-8*ln(1/2*(2*b*e*x+2
*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a*b*d*e+8*ln(1/2
*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*b^2*d
^2-6*(e*b)^(1/2)*((b*x+a)*(e*x+d))^(1/2)*a*e+14*(e*b)^(1/2)*((b*x+a)*(e*x+d
))^(1/2)*b*d)*(e*x+d)^(1/2)*(b*x+a)^(1/2)/(e*b)^(1/2)/b^2/((b*x+a)*(e*x+d))
^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b*d-%e*a>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 4.91, size = 308, normalized size = 2.52

$$\left[\frac{\left((8\,b^2d^2 - 8\,abde + 3\,a^2e^2)\sqrt{b}\,e^{\frac{1}{2}}\log\left(b^2d^2 + 4\,(bd + (2\,bx + a)e)\sqrt{bx + a}\,\sqrt{xe + d}\,\sqrt{b}\,e^{\frac{1}{2}} + (8\,b^2x^2 + 8\,abx + a^2)e^2 + 2\,(4\,b^2dx + 3\,abd)e\right) + 4\,(7\,b^2de + (2\,b^2x - 3\,ab)e^2)\sqrt{bx + a}\,\sqrt{xe + d}\right)e^{(-1)}}{2\,b^3}, -\frac{\left((8\,b^2d^2 - 8\,abde + 3\,a^2e^2)\sqrt{-be}\,\arctan\left(\frac{(bd+(2\,bx+a)e)\sqrt{bx + a}\,\sqrt{-be}\,\sqrt{xe + d}}{2\,((b^2x^2+abx)e^2+(b^2dx+abd)e)}\right) - 2\,(7\,b^2de + (2\,b^2x - 3\,ab)e^2)\sqrt{bx + a}\,\sqrt{xe + d}\right)e^{(-1)}}{b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [1/2*((8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*sqrt(b)*e^(1/2)*log(b^2*d^2 + 4*(
b*d + (2*b*x + a)*e)*sqrt(b*x + a)*sqrt(x*e + d)*sqrt(b)*e^(1/2) + (8*b^2*x
^2 + 8*a*b*x + a^2)*e^2 + 2*(4*b^2*d*x + 3*a*b*d)*e) + 4*(7*b^2*d*e + (2*b^
2*x - 3*a*b)*e^2)*sqrt(b*x + a)*sqrt(x*e + d))*e^(-1)/b^3, -((8*b^2*d^2 - 8
*a*b*d*e + 3*a^2*e^2)*sqrt(-b*e)*arctan(1/2*(b*d + (2*b*x + a)*e)*sqrt(b*x
+ a)*sqrt(-b*e)*sqrt(x*e + d)/((b^2*x^2 + a*b*x)*e^2 + (b^2*d*x + a*b*d)*e)
) - 2*(7*b^2*d*e + (2*b^2*x - 3*a*b)*e^2)*sqrt(b*x + a)*sqrt(x*e + d))*e^(-
1)/b^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*sqrt(d + e*x)),
x)
```

Giac [A]
time = 2.99, size = 145, normalized size = 1.19

$$\frac{2\left(\sqrt{b^2d + (bx + a)be - abe}\,\sqrt{bx + a}\left(\frac{2\,(bx+a)e}{b^3} + \frac{(7\,b^6de^2-5\,ab^5e^3)e^{(-2)}}{b^8}\right) - \frac{(8\,b^2d^2-8\,abde+3\,a^2e^2)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx + a}\,\sqrt{b}\,e^{\frac{1}{2}} + \sqrt{b^2d + (bx + a)be - abe}\right|\right)}{b^{\frac{5}{2}}}\right)b}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(1/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2*(sqrt(b^2*d + (b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*e/b^3 + (
7*b^6*d*e^2 - 5*a*b^5*e^3)*e^(-2)/b^8) - (8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2
```

```
)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d + (b*x + a)*
b*e - a*b*e)))/b^(5/2))*b/abs(b)
```

Mupad [B]

time = 20.64, size = 893, normalized size = 7.32

$$\frac{\frac{(40bd^2+40aed)\left(\sqrt{a+bx}-\sqrt{a}\right)}{e^2\left(\sqrt{d+ex}-\sqrt{d}\right)}-\frac{160\sqrt{a}\,d^{3/2}\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{e\left(\sqrt{d+ex}-\sqrt{d}\right)^2}+\frac{(40bd^2+40aed)\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{be\left(\sqrt{d+ex}-\sqrt{d}\right)^3}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{d+ex}-\sqrt{d}\right)^4}+\frac{b^2}{e^2}-\frac{2b\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{e\left(\sqrt{d+ex}-\sqrt{d}\right)^2}}-\frac{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)(12a^2be^2+8ab^2de+12b^3d^2)}{e^4\left(\sqrt{d+ex}-\sqrt{d}\right)}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^3(44a^2e^2+200abde+44b^2d^2)}{e^3\left(\sqrt{d+ex}-\sqrt{d}\right)^3}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^7(12a^2e^2+8abde+12b^2d^2)}{b^2e\left(\sqrt{d+ex}-\sqrt{d}\right)^7}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^5(44a^2e^2+200abde+44b^2d^2)}{be^2\left(\sqrt{d+ex}-\sqrt{d}\right)^5}+\frac{\sqrt{a}\sqrt{d}(256ae+256bd)\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{e^2\left(\sqrt{d+ex}-\sqrt{d}\right)^4}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^8}{\left(\sqrt{d+ex}-\sqrt{d}\right)^8}+\frac{b^4}{e^4}-\frac{4b^3\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{e^3\left(\sqrt{d+ex}-\sqrt{d}\right)^2}+\frac{6b^2\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{e^2\left(\sqrt{d+ex}-\sqrt{d}\right)^4}-\frac{4b\left(\sqrt{a+bx}-\sqrt{a}\right)^6}{e\left(\sqrt{d+ex}-\sqrt{d}\right)^6}}-\frac{60d^2\,\mathrm{atan}\left(\frac{b\left(\sqrt{d+ex}-\sqrt{d}\right)}{\sqrt{-be}\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{\sqrt{-be}}-\frac{2\ln\left(\frac{\sqrt{e}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{d+ex}-\sqrt{d}}-\sqrt{b}\right)(3a^2e^2+2abde+3b^2d^2)}{b^{5/2}\sqrt{e}}+\frac{\ln\left(\sqrt{b}+\frac{\sqrt{e}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{d+ex}-\sqrt{d}}\right)(6a^2e^2+4abde+6b^2d^2)}{b^{5/2}\sqrt{e}}-\frac{40d\,\mathrm{atanh}\left(\frac{\sqrt{e}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{d+ex}-\sqrt{d}\right)}\right)(ae+bd)}{b^{3/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] (((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2)))/(e^2*((d + e*x)^(1/2)
- d^(1/2))) - (160*a^(1/2)*d^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(e*((d +
e*x)^(1/2) - d^(1/2))^2) + ((40*b*d^2 + 40*a*d*e)*((a + b*x)^(1/2) - a^(1/2
))^3)/(b*e*((d + e*x)^(1/2) - d^(1/2))^3))/(((a + b*x)^(1/2) - a^(1/2))^4/(
(d + e*x)^(1/2) - d^(1/2))^4 + b^2/e^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2
)/(e*((d + e*x)^(1/2) - d^(1/2))^2)) - ((((a + b*x)^(1/2) - a^(1/2))*(12*b^
3*d^2 + 12*a^2*b*e^2 + 8*a*b^2*d*e))/(e^4*((d + e*x)^(1/2) - d^(1/2))) - ((
(a + b*x)^(1/2) - a^(1/2))^3*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))/(e^3*
((d + e*x)^(1/2) - d^(1/2))^3) + (((a + b*x)^(1/2) - a^(1/2))^7*(12*a^2*e^2
 + 12*b^2*d^2 + 8*a*b*d*e))/(b^2*e*((d + e*x)^(1/2) - d^(1/2))^7) - (((a +
b*x)^(1/2) - a^(1/2))^5*(44*a^2*e^2 + 44*b^2*d^2 + 200*a*b*d*e))/(b*e^2*((d
 + e*x)^(1/2) - d^(1/2))^5) + (a^(1/2)*d^(1/2)*(256*a*e + 256*b*d)*((a + b*
x)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4))/(((a + b*x)^(1/
2) - a^(1/2))^8/((d + e*x)^(1/2) - d^(1/2))^8 + b^4/e^4 - (4*b^3*((a + b*x)
^(1/2) - a^(1/2))^2)/(e^3*((d + e*x)^(1/2) - d^(1/2))^2) + (6*b^2*((a + b*x
)^(1/2) - a^(1/2))^4)/(e^2*((d + e*x)^(1/2) - d^(1/2))^4) - (4*b*((a + b*x)
^(1/2) - a^(1/2))^6)/(e*((d + e*x)^(1/2) - d^(1/2))^6)) - (60*d^2*atan((b*(
(d + e*x)^(1/2) - d^(1/2)))/((-b*e)^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(-
b*e)^(1/2) - (2*log((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/((d + e*x)^(1/2)
- d^(1/2)) - b^(1/2))*(3*a^2*e^2 + 3*b^2*d^2 + 2*a*b*d*e))/(b^(5/2)*e^(1/2)
) + (log(b^(1/2) + (e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/((d + e*x)^(1/2) -
 d^(1/2)))*(6*a^2*e^2 + 6*b^2*d^2 + 4*a*b*d*e))/(b^(5/2)*e^(1/2)) - (40*d*a
tanh((e^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((d + e*x)^(1/2) - d^(1
/2))))*(a*e + b*d))/(b^(3/2)*e^(1/2))
```

3.846 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}}+\frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b}+\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}$$

```
[Out] 8*(-a*e+2*b*d)*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(3/2)
/e^(1/2)+6*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(1/2)+8*(b*x+a)^(1/2)*(e*x+
d)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {963, 81, 65, 223, 212}

$$\frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}}+\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}\,(bd-ae)}+\frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/((b*d - a*e)*Sqrt[d + e*x]) + (8*Sqrt[a + b*x]*Sqrt[d
 + e*x])/b + (8*(2*b*d - a*e)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt
[d + e*x])])/(b^(3/2)*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{2\int \frac{6d(bd-ae)+4e(bd-ae)x}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx}{bd-ae}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b} + \frac{(4(2bd-ae))\int \frac{1}{\sqrt{a+bx}\,\sqrt{d-}}}{b}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b} + \frac{(8(2bd-ae))\text{Subst}\left(\int \frac{}{\sqrt{d-}}\right.}{b^2}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b} + \frac{(8(2bd-ae))\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,d\right.}{b^2}$$

$$= \frac{6d^2\sqrt{a+bx}}{(bd-ae)\sqrt{d+ex}} + \frac{8\sqrt{a+bx}\,\sqrt{d+ex}}{b} + \frac{8(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a-}}{\sqrt{b}\,\sqrt{d-}}\right.}{b^{3/2}\sqrt{e}}$$

Mathematica [A]

time = 0.26, size = 122, normalized size = 1.13

$$\frac{2\left(\frac{\sqrt{b}\,\sqrt{a+bx}\,(-4ae(d+ex)+bd(7d+4ex))}{\sqrt{d+ex}}+\frac{4(2b^2d^2-3abde+a^2e^2)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{\sqrt{e}}\right)}{b^{3/2}(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(3/2)),x
]
```

```
[Out] (2*((Sqrt[b]*Sqrt[a + b*x]*(-4*a*e*(d + e*x) + b*d*(7*d + 4*e*x)))/Sqrt[d +
 e*x] + (4*(2*b^2*d^2 - 3*a*b*d*e + a^2*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a + b*x]
)/(Sqrt[b]*Sqrt[d + e*x])])/Sqrt[e]))/(b^(3/2)*(b*d - a*e))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(88) = 176$.
time = 0.07, size = 438, normalized size = 4.06

method	result
default	$-\frac{2\sqrt{bx+a}\left(2\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{eb}+ae+bd}{2\sqrt{eb}}\right)a^2e^3x-6\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\,\sqrt{eb}+}{2\sqrt{eb}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2*(b*x+a)^(1/2)*(2*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a
*e+b*d)/(e*b)^(1/2))*a^2*e^3*x-6*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*
(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a*b*d*e^2*x+4*ln(1/2*(2*b*e*x+2*((b*x+a)*
(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*b^2*d^2*e*x+2*ln(1/2*(2*b*
e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a^2*d*e^2-6
*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2)
)*a*b*d^2*e+4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d
)/(e*b)^(1/2))*b^2*d^3-4*a*e^2*x*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+4*b*d*
e*x*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)-4*a*d*e*((b*x+a)*(e*x+d))^(1/2)*(e*
b)^(1/2)+7*b*d^2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2))/b/(e*b)^(1/2)/(a*e-b*
d)/((b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*d-%e*a>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(91) = 182$.
time = 4.59, size = 457, normalized size = 4.23

$$\left[-\frac{2\left((2b^2d^3+a^2xe^3-(3abdx-a^2d)e^2+(2b^2d^2x-3abd^2)e)\sqrt{b}\,e^{\frac{1}{2}}\log\left(b^2d^2-4(bd+(2bx+a)e)\sqrt{bx+a}\,\sqrt{xe+d}\,\sqrt{b}\,e^{\frac{1}{2}}+(8b^2x^2+8abx+a^2)e^2+2(4b^2dx+3abd)e\right)-(7b^2d^2e-4abxe^3+4(b^2dx-abd)e^2)\sqrt{bx+a}\,\sqrt{xe+d}\right)}{b^3d^2e-ab^2xe^3+(b^3dx-ab^2d)e^2}, -\frac{2\left(2(2b^2d^3+a^2xe^3-(3abdx-a^2d)e^2+(2b^2d^2x-3abd^2)e)\sqrt{-be}\,\arctan\left(\frac{(bd+(2bx+a)e)\sqrt{bx+a}\,\sqrt{-be}\,\sqrt{xe+d}}{2((b^2x^2+abx)e^2+(b^2dx+abd)e)}\right)-(7b^2d^2e-4abxe^3+4(b^2dx-abd)e^2)\sqrt{bx+a}\,\sqrt{xe+d}\right)}{b^3d^2e-ab^2xe^3+(b^3dx-ab^2d)e^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-2*((2*b^2*d^3 + a^2*x*e^3 - (3*a*b*d*x - a^2*d)*e^2 + (2*b^2*d^2*x - 3*a*
b*d^2)*e)*sqrt(b)*e^(1/2)*log(b^2*d^2 - 4*(b*d + (2*b*x + a)*e)*sqrt(b*x +
a)*sqrt(x*e + d)*sqrt(b)*e^(1/2) + (8*b^2*x^2 + 8*a*b*x + a^2)*e^2 + 2*(4*b
^2*d*x + 3*a*b*d)*e) - (7*b^2*d^2*e - 4*a*b*x*e^3 + 4*(b^2*d*x - a*b*d)*e^2
)*sqrt(b*x + a)*sqrt(x*e + d))/(b^3*d^2*e - a*b^2*x*e^3 + (b^3*d*x - a*b^2*
d)*e^2), -2*(2*(2*b^2*d^3 + a^2*x*e^3 - (3*a*b*d*x - a^2*d)*e^2 + (2*b^2*d^
2*x - 3*a*b*d^2)*e)*sqrt(-b*e)*arctan(1/2*(b*d + (2*b*x + a)*e)*sqrt(b*x +
a)*sqrt(-b*e)*sqrt(x*e + d)/((b^2*x^2 + a*b*x)*e^2 + (b^2*d*x + a*b*d)*e))
- (7*b^2*d^2*e - 4*a*b*x*e^3 + 4*(b^2*d*x - a*b*d)*e^2)*sqrt(b*x + a)*sqrt(
x*e + d))/(b^3*d^2*e - a*b^2*x*e^3 + (b^3*d*x - a*b^2*d)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(3/2)
), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(91) = 182$.
time = 3.99, size = 193, normalized size = 1.79

$$-\frac{8\,(2\,bd-ae)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{\sqrt{b}\,|b|}+\frac{2\,\sqrt{bx+a}\left(\frac{4\,(b^3de^3-ab^2e^4)(bx+a)}{b^3d|b|e^2-ab^2|b|e^3}+\frac{7\,b^4d^2e^2-8\,ab^3de^3+4\,a^2b^2e^4}{b^3d|b|e^2-ab^2|b|e^3}\right)}{\sqrt{b^2d+(bx+a)be-abe}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(3/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -8*(2*b*d - a*e)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2
*d + (b*x + a)*b*e - a*b*e)))/(sqrt(b)*abs(b)) + 2*sqrt(b*x + a)*(4*(b^3*d*
e^3 - a*b^2*e^4)*(b*x + a)/(b^3*d*abs(b)*e^2 - a*b^2*abs(b)*e^3) + (7*b^4*d
^2*e^2 - 8*a*b^3*d*e^3 + 4*a^2*b^2*e^4)/(b^3*d*abs(b)*e^2 - a*b^2*abs(b)*e^
3))/sqrt(b^2*d + (b*x + a)*b*e - a*b*e)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{15\,d^2+20\,d\,e\,x+8\,e^2\,x^2}{\sqrt{a+b\,x}\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)),x)
```

```
[Out] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(3/2)), x)
```

3.847 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}}+\frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

```
[Out] 16*arctanh(e^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(e*x+d)^(1/2))/b^(1/2)/e^(1/2)+2*d
^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(3/2)+4*d*(-2*a*e+3*b*d)*(b*x+a)^(1/2)/
(-a*e+b*d)^2/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {963, 79, 65, 223, 212}

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)}+\frac{4d\sqrt{a+bx}\,(3bd-2ae)}{\sqrt{d+ex}\,(bd-ae)^2}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x]
```

```
[Out] (2*d^2*Sqrt[a + b*x])/((b*d - a*e)*(d + e*x)^(3/2)) + (4*d*(3*b*d - 2*a*e)*
Sqrt[a + b*x])/((b*d - a*e)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[d + e*x])])/(Sqrt[b]*Sqrt[e])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{5/2}}\,dx = \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{2\int \frac{3d(7bd-6ae)+12e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{3/2}}\,dx}{3(bd-ae)}$$

$$= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + 8\int \frac{1}{\sqrt{a+bx}\,\sqrt{d+ex}}\,dx$$

$$= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{ae}{b}+ \ldots}}\ldots\right)}{b}$$

$$= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\text{Subst}\left(\int \frac{1}{1-\frac{ex^2}{b}}\,dx, x, \ldots\right)}{b}$$

$$= \frac{2d^2\sqrt{a+bx}}{(bd-ae)(d+ex)^{3/2}} + \frac{4d(3bd-2ae)\sqrt{a+bx}}{(bd-ae)^2\sqrt{d+ex}} + \frac{16\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{d+ex}}\right)}{\sqrt{b}\,\sqrt{e}}$$

Mathematica [A]
time = 0.17, size = 99, normalized size = 0.85

$$\frac{2d\sqrt{a+bx}\left(7bd-4ae-\frac{de(a+bx)}{d+ex}\right)}{(bd-ae)^2\sqrt{d+ex}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(5/2)),x
]
```

```
[Out] (2*d*Sqrt[a + b*x]*(7*b*d - 4*a*e - (d*e*(a + b*x))/(d + e*x)))/((b*d - a*e
)^2*Sqrt[d + e*x]) + (16*ArcTanh[(Sqrt[e]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[d +
e*x])])/(Sqrt[b]*Sqrt[e])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(96) = 192$.
time = 0.08, size = 601, normalized size = 5.18

method	result
default	$\frac{2\sqrt{bx+a}\left(4\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\sqrt{eb}+ae+bd}{2\sqrt{eb}}\right)a^2e^4x^2-8\ln\left(\frac{2bex+2\sqrt{(bx+a)(ex+d)}\sqrt{eb}+}{2\sqrt{eb}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] 2*(b*x+a)^(1/2)*(4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*
e+b*d)/(e*b)^(1/2))*a^2*e^4*x^2-8*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)
*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a*b*d*e^3*x^2+4*ln(1/2*(2*b*e*x+2*((b*x+
a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*b^2*d^2*e^2*x^2+8*ln(1/
2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a^2*
d*e^3*x-16*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(
e*b)^(1/2))*a*b*d^2*e^2*x+8*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)
^(1/2)+a*e+b*d)/(e*b)^(1/2))*b^2*d^3*e*x+4*ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+
d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a^2*d^2*e^2-8*ln(1/2*(2*b*e*x+2
*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*a*b*d^3*e+4*ln(1
/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+a*e+b*d)/(e*b)^(1/2))*b^2
*d^4-4*a*d*e^2*x*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+6*b*d^2*e*x*((b*x+a)*(
e*x+d))^(1/2)*(e*b)^(1/2)-5*a*d^2*e*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2)+7*b
*d^3*((b*x+a)*(e*x+d))^(1/2)*(e*b)^(1/2))/(e*b)^(1/2)/(a*e-b*d)^2/((b*x+a)*
(e*x+d))^(1/2)/(e*x+d)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*d-%e*a>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(100) = 200$.
time = 6.03, size = 651, normalized size = 5.61

$$\left[\frac{2\left(2\left(b^2d^4+a^2x^2e^4-2\left(abdx^2-a^2dx\right)e^3+\left(b^2d^2x^2-4\,abd^2x+a^2d^2\right)e^2+2\left(b^2d^3x-abd^3\right)e\right)\sqrt{b}\,e^{\frac{1}{2}}\log\left(b^2d^2+4\left(bd+\left(2\,bx+a\right)e\right)\sqrt{bx+a}\,\sqrt{xe+d}\,\sqrt{b}\,e^{\frac{1}{2}}+\left(8\,b^2x^2+8\,abx+a^2\right)e^2+2\left(4\,b^2dx+3\,abd\right)e\right)+\left(7\,b^2d^3e-4\,abdxe^3+\left(6\,b^2d^2x-5\,abd^2\right)e^2\right)\sqrt{bx+a}\,\sqrt{xe+d}\right)}{b^3d^4e+a^2bx^2e^5-2\left(ab^2dx^2-a^2bdx\right)e^4+\left(b^3d^2x^2-4\,ab^2d^2x+a^2bd^2\right)e^3+2\left(b^3d^3x-ab^2d^3\right)e^2}, -\frac{2\left(4\left(b^2d^4+a^2x^2e^4-2\left(abdx^2-a^2dx\right)e^3+\left(b^2d^2x^2-4\,abd^2x+a^2d^2\right)e^2+2\left(b^2d^3x-abd^3\right)e\right)\sqrt{-be}\,\arctan\left(\frac{\left(bd+\left(2\,bx+a\right)e\right)\sqrt{bx+a}\,\sqrt{-be}\,\sqrt{xe+d}}{2\left(\left(b^2x^2+abx\right)e^2+\left(b^2dx+abd\right)e\right)}\right)-\left(7\,b^2d^3e-4\,abdxe^3+\left(6\,b^2d^2x-5\,abd^2\right)e^2\right)\sqrt{bx+a}\,\sqrt{xe+d}\right)}{b^3d^4e+a^2bx^2e^5-2\left(ab^2dx^2-a^2bdx\right)e^4+\left(b^3d^2x^2-4\,ab^2d^2x+a^2bd^2\right)e^3+2\left(b^3d^3x-ab^2d^3\right)e^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [2*(2*(b^2*d^4 + a^2*x^2*e^4 - 2*(a*b*d*x^2 - a^2*d*x)*e^3 + (b^2*d^2*x^2 -
 4*a*b*d^2*x + a^2*d^2)*e^2 + 2*(b^2*d^3*x - a*b*d^3)*e)*sqrt(b)*e^(1/2)*lo
g(b^2*d^2 + 4*(b*d + (2*b*x + a)*e)*sqrt(b*x + a)*sqrt(x*e + d)*sqrt(b)*e^(
1/2) + (8*b^2*x^2 + 8*a*b*x + a^2)*e^2 + 2*(4*b^2*d*x + 3*a*b*d)*e) + (7*b^
2*d^3*e - 4*a*b*d*x*e^3 + (6*b^2*d^2*x - 5*a*b*d^2)*e^2)*sqrt(b*x + a)*sqrt
(x*e + d))/(b^3*d^4*e + a^2*b*x^2*e^5 - 2*(a*b^2*d*x^2 - a^2*b*d*x)*e^4 + (
b^3*d^2*x^2 - 4*a*b^2*d^2*x + a^2*b*d^2)*e^3 + 2*(b^3*d^3*x - a*b^2*d^3)*e^
2), -2*(4*(b^2*d^4 + a^2*x^2*e^4 - 2*(a*b*d*x^2 - a^2*d*x)*e^3 + (b^2*d^2*x
^2 - 4*a*b*d^2*x + a^2*d^2)*e^2 + 2*(b^2*d^3*x - a*b*d^3)*e)*sqrt(-b*e)*arc
tan(1/2*(b*d + (2*b*x + a)*e)*sqrt(b*x + a)*sqrt(-b*e)*sqrt(x*e + d)/((b^2*
x^2 + a*b*x)*e^2 + (b^2*d*x + a*b*d)*e)) - (7*b^2*d^3*e - 4*a*b*d*x*e^3 + (
6*b^2*d^2*x - 5*a*b*d^2)*e^2)*sqrt(b*x + a)*sqrt(x*e + d))/(b^3*d^4*e + a^2
*b*x^2*e^5 - 2*(a*b^2*d*x^2 - a^2*b*d*x)*e^4 + (b^3*d^2*x^2 - 4*a*b^2*d^2*x
 + a^2*b*d^2)*e^3 + 2*(b^3*d^3*x - a*b^2*d^3)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\,\left(d + ex\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(5/2)
), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(100) = 200.
time = 3.38, size = 218, normalized size = 1.88

$$-\frac{16\,\sqrt{b}\,e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{bx+a}\,\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{|b|}+\frac{2\,\sqrt{bx+a}\,\left(\frac{2\,(3\,b^6d^2e^2-2\,ab^5de^3)(bx+a)}{b^4d^2|b|e-2\,ab^3d|b|e^2+a^2b^2|b|e^3}+\frac{7\,b^7d^3e-11\,ab^6d^2e^2+4\,a^2b^5de^3}{b^4d^2|b|e-2\,ab^3d|b|e^2+a^2b^2|b|e^3}\right)}{(b^2d+(bx+a)be-abe)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] -16*sqrt(b)*e^(-1/2)*log(abs(-sqrt(b*x + a)*sqrt(b)*e^(1/2) + sqrt(b^2*d +
(b*x + a)*b*e - a*b*e)))/abs(b) + 2*sqrt(b*x + a)*(2*(3*b^6*d^2*e^2 - 2*a*b
^5*d*e^3)*(b*x + a)/(b^4*d^2*abs(b)*e - 2*a*b^3*d*abs(b)*e^2 + a^2*b^2*abs(
b)*e^3) + (7*b^7*d^3*e - 11*a*b^6*d^2*e^2 + 4*a^2*b^5*d*e^3)/(b^4*d^2*abs(b
)*e - 2*a*b^3*d*abs(b)*e^2 + a^2*b^2*abs(b)*e^3))/(b^2*d + (b*x + a)*b*e -
a*b*e)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{15\,d^2+20\,d\,e\,x+8\,e^2\,x^2}{\sqrt{a+b\,x}\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)),x)
```

```
[Out] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(5/2)), x)
```

3.848 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}}+\frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}}+\frac{16(23b^2d^2-35abde+15a^2e^2)\sqrt{a+bx}}{15(bd-ae)^3\sqrt{d+ex}}$$

```
[Out] 6/5*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(5/2)+8/15*d*(-5*a*e+8*b*d)*(b*x+a
)^(1/2)/(-a*e+b*d)^2/(e*x+d)^(3/2)+16/15*(15*a^2*e^2-35*a*b*d*e+23*b^2*d^2)
*(b*x+a)^(1/2)/(-a*e+b*d)^3/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {963, 79, 37}

$$\frac{16\sqrt{a+bx}\,(15a^2e^2-35abde+23b^2d^2)}{15\sqrt{d+ex}\,(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)}+\frac{8d\sqrt{a+bx}\,(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(5*(b*d - a*e)*(d + e*x)^(5/2)) + (8*d*(8*b*d - 5*a*e
)*Sqrt[a + b*x])/(15*(b*d - a*e)^2*(d + e*x)^(3/2)) + (16*(23*b^2*d^2 - 35*
a*b*d*e + 15*a^2*e^2)*Sqrt[a + b*x])/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx &= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{2\int \frac{6d(6bd-5ae)+20e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{5/2}}\,dx}{5(bd-ae)} \\
&= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{(8(23b^2d^2-35abde+15\ldots}{15(\ldots} \\
&= \frac{6d^2\sqrt{a+bx}}{5(bd-ae)(d+ex)^{5/2}} + \frac{8d(8bd-5ae)\sqrt{a+bx}}{15(bd-ae)^2(d+ex)^{3/2}} + \frac{16(23b^2d^2-35abde+1\ldots}{15(bd-ae)^3\sqrt{\ldots}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 115, normalized size = 0.86

$$
\frac{2\sqrt{a+bx}\left(225b^2d^2-300abde+120a^2e^2+\frac{9d^2e^2(a+bx)^2}{(d+ex)^2}-\frac{50bd^2e(a+bx)}{d+ex}+\frac{20ade^2(a+bx)}{d+ex}\right)}{15(bd-ae)^3\sqrt{d+ex}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(7/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(225*b^2*d^2 - 300*a*b*d*e + 120*a^2*e^2 + (9*d^2*e^2*(a +
 b*x)^2)/(d + e*x)^2 - (50*b*d^2*e*(a + b*x))/(d + e*x) + (20*a*d*e^2*(a +
b*x))/(d + e*x)))/(15*(b*d - a*e)^3*Sqrt[d + e*x])
```

Maple [A]
time = 0.08, size = 122, normalized size = 0.92

method	result
default	$-\frac{2\sqrt{bx+a}\,\left(120a^2e^4x^2-280abd\,e^3x^2+184b^2d^2e^2x^2+260a^2d\,e^3x-612ab\,d^2e^2x+400b^2d^3ex+149a^2d^2e^2-350ab\,d^3e+225b^2d^4\right)}{15(ex+d)^{\frac{5}{2}}(ae-bd)^3}$

gosper	$-\frac{2\sqrt{bx+a}\,(120a^2e^4x^2-280abd\,e^3x^2+184b^2d^2e^2x^2+260a^2d\,e^3x-612ab\,d^2e^2x+400b^2d^3ex+149a^2d^2e^2-350ab\,d^3e+225b^2d^4)}{15(ex+d)^{\frac{5}{2}}(a^3e^3-3a^2bd\,e^2+3a\,b^2d^2e-b^3d^3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2/15*(b*x+a)^(1/2)*(120*a^2*e^4*x^2-280*a*b*d*e^3*x^2+184*b^2*d^2*e^2*x^2+
260*a^2*d*e^3*x-612*a*b*d^2*e^2*x+400*b^2*d^3*e*x+149*a^2*d^2*e^2-350*a*b*d
^3*e+225*b^2*d^4)/(e*x+d)^(5/2)/(a*e-b*d)^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*d-%e*a>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(122) = 244$.
time = 5.38, size = 286, normalized size = 2.15

$$\frac{2\,(225\,b^2d^4+120\,a^2x^2e^4-20\,(14\,abdx^2-13\,a^2dx)e^3+(184\,b^2d^2x^2-612\,abd^2x+149\,a^2d^2)e^2+50\,(8\,b^2d^3x-7\,abd^3)e)\sqrt{bx+a}\,\sqrt{xe+d}}{15\,(b^3d^6-a^3x^3e^6+3\,(a^2bdx^3-a^3dx^2)e^5-3\,(ab^2d^2x^3-3\,a^2bd^2x^2+a^3d^2x)e^4+(b^3d^3x^3-9\,ab^2d^3x^2+9\,a^2bd^3x-a^3d^3)e^3+3\,(b^3d^4x^2-3\,ab^2d^4x+a^2bd^4)e^2+3\,(b^3d^5x-ab^2d^5)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/15*(225*b^2*d^4 + 120*a^2*x^2*e^4 - 20*(14*a*b*d*x^2 - 13*a^2*d*x)*e^3 +
(184*b^2*d^2*x^2 - 612*a*b*d^2*x + 149*a^2*d^2)*e^2 + 50*(8*b^2*d^3*x - 7*a
*b*d^3)*e)*sqrt(b*x + a)*sqrt(x*e + d)/(b^3*d^6 - a^3*x^3*e^6 + 3*(a^2*b*d*
x^3 - a^3*d*x^2)*e^5 - 3*(a*b^2*d^2*x^3 - 3*a^2*b*d^2*x^2 + a^3*d^2*x)*e^4
+ (b^3*d^3*x^3 - 9*a*b^2*d^3*x^2 + 9*a^2*b*d^3*x - a^3*d^3)*e^3 + 3*(b^3*d^
4*x^2 - 3*a*b^2*d^4*x + a^2*b*d^4)*e^2 + 3*(b^3*d^5*x - a*b^2*d^5)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((15*d**2 + 20*d*e*x + 8*e**2*x**2)/(sqrt(a + b*x)*(d + e*x)**(7/2)
), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(122) = 244.
time = 3.55, size = 337, normalized size = 2.53

$$\frac{2\left(4\,(bx+a)\left(\frac{2\,(23\,b^8d^2e^4-35\,ab^7de^5+15\,a^2b^6e^6)(bx+a)}{b^5d^3|b|e^2-3\,ab^4d^2|b|e^3+3\,a^2b^3d|b|e^4-a^3b^2|b|e^5}+\frac{5\,(20\,b^9d^3e^3-49\,ab^8d^2e^4+41\,a^2b^7de^5-12\,a^3b^6e^6)}{b^5d^3|b|e^2-3\,ab^4d^2|b|e^3+3\,a^2b^3d|b|e^4-a^3b^2|b|e^5}\right)+\frac{15\,(15\,b^{10}d^4e^2-50\,ab^9d^3e^3+63\,a^2b^8d^2e^4-36\,a^3b^7de^5+8\,a^4b^6e^6)}{b^5d^3|b|e^2-3\,ab^4d^2|b|e^3+3\,a^2b^3d|b|e^4-a^3b^2|b|e^5}\right)\sqrt{bx+a}}{15\,(b^2d+(bx+a)be-abe)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2/15*(4*(b*x + a)*(2*(23*b^8*d^2*e^4 - 35*a*b^7*d*e^5 + 15*a^2*b^6*e^6)*(b*
x + a)/(b^5*d^3*abs(b)*e^2 - 3*a*b^4*d^2*abs(b)*e^3 + 3*a^2*b^3*d*abs(b)*e^
4 - a^3*b^2*abs(b)*e^5) + 5*(20*b^9*d^3*e^3 - 49*a*b^8*d^2*e^4 + 41*a^2*b^7
*d*e^5 - 12*a^3*b^6*e^6)/(b^5*d^3*abs(b)*e^2 - 3*a*b^4*d^2*abs(b)*e^3 + 3*a
^2*b^3*d*abs(b)*e^4 - a^3*b^2*abs(b)*e^5)) + 15*(15*b^10*d^4*e^2 - 50*a*b^9
*d^3*e^3 + 63*a^2*b^8*d^2*e^4 - 36*a^3*b^7*d*e^5 + 8*a^4*b^6*e^6)/(b^5*d^3*
abs(b)*e^2 - 3*a*b^4*d^2*abs(b)*e^3 + 3*a^2*b^3*d*abs(b)*e^4 - a^3*b^2*abs(
b)*e^5))*sqrt(b*x + a)/(b^2*d + (b*x + a)*b*e - a*b*e)^(5/2)
```

Mupad [B]
time = 4.30, size = 268, normalized size = 2.02

$$-\frac{\sqrt{d+e\,x}\,\left(\frac{x^2\,(240\,a^3\,e^4-40\,a^2\,b\,d\,e^3-856\,a\,b^2\,d^2\,e^2+800\,b^3\,d^3\,e)}{15\,e^3\,(a\,e-b\,d)^3}+\frac{x\,(520\,a^3\,d\,e^3-926\,a^2\,b\,d^2\,e^2+100\,a\,b^2\,d^3\,e+450\,b^3\,d^4)}{15\,e^3\,(a\,e-b\,d)^3}+\frac{2\,a\,d^2\,(149\,a^2\,e^2-350\,a\,b\,d\,e+225\,b^2\,d^2)}{15\,e^3\,(a\,e-b\,d)^3}+\frac{16\,b\,x^3\,(15\,a^2\,e^2-35\,a\,b\,d\,e+23\,b^2\,d^2)}{15\,e\,(a\,e-b\,d)^3}\right)}{x^3\,\sqrt{a+b\,x}+\frac{d^3\,\sqrt{a+b\,x}}{e^3}+\frac{3\,d\,x^2\,\sqrt{a+b\,x}}{e}+\frac{3\,d^2\,x\,\sqrt{a+b\,x}}{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(7/2)),x)
```

```
[Out] -((d + e*x)^(1/2)*((x^2*(240*a^3*e^4 + 800*b^3*d^3*e - 856*a*b^2*d^2*e^2 -
40*a^2*b*d*e^3))/(15*e^3*(a*e - b*d)^3) + (x*(450*b^3*d^4 + 520*a^3*d*e^3 -
 926*a^2*b*d^2*e^2 + 100*a*b^2*d^3*e))/(15*e^3*(a*e - b*d)^3) + (2*a*d^2*(1
49*a^2*e^2 + 225*b^2*d^2 - 350*a*b*d*e))/(15*e^3*(a*e - b*d)^3) + (16*b*x^3
*(15*a^2*e^2 + 23*b^2*d^2 - 35*a*b*d*e))/(15*e*(a*e - b*d)^3)))/(x^3*(a + b
*x)^(1/2) + (d^3*(a + b*x)^(1/2))/e^3 + (3*d*x^2*(a + b*x)^(1/2))/e + (3*d^
2*x*(a + b*x)^(1/2))/e^2)
```

3.849 $\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=189

$$\frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}}+\frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}}+\frac{16(58b^2d^2-84abde+35a^2e^2)\sqrt{a+bx}}{105(bd-ae)^3(d+ex)^{3/2}}+\frac{32b(58b^2d^2-8}{105(b}$$

```
[Out] 6/7*d^2*(b*x+a)^(1/2)/(-a*e+b*d)/(e*x+d)^(7/2)+4/35*d*(-14*a*e+23*b*d)*(b*x
+a)^(1/2)/(-a*e+b*d)^2/(e*x+d)^(5/2)+16/105*(35*a^2*e^2-84*a*b*d*e+58*b^2*d
^2)*(b*x+a)^(1/2)/(-a*e+b*d)^3/(e*x+d)^(3/2)+32/105*b*(35*a^2*e^2-84*a*b*d*
e+58*b^2*d^2)*(b*x+a)^(1/2)/(-a*e+b*d)^4/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {963, 79, 47, 37}

$$\frac{32b\sqrt{a+bx}\,(35a^2e^2-84abde+58b^2d^2)}{105\sqrt{d+ex}\,(bd-ae)^4}+\frac{16\sqrt{a+bx}\,(35a^2e^2-84abde+58b^2d^2)}{105(d+ex)^{3/2}(bd-ae)^3}+\frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)}+\frac{4d\sqrt{a+bx}\,(23bd-14ae)}{35(d+ex)^{5/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x]
```

```
[Out] (6*d^2*Sqrt[a + b*x])/(7*(b*d - a*e)*(d + e*x)^(7/2)) + (4*d*(23*b*d - 14*a
*e)*Sqrt[a + b*x])/(35*(b*d - a*e)^2*(d + e*x)^(5/2)) + (16*(58*b^2*d^2 - 8
4*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a*e)^3*(d + e*x)^(3/2))
+ (32*b*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*Sqrt[a + b*x])/(105*(b*d - a
*e)^4*Sqrt[d + e*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}\,(d+ex)^{9/2}}\,dx = \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{2\int \frac{3d(17bd-14ae)+28e(bd-ae)x}{\sqrt{a+bx}\,(d+ex)^{7/2}}\,dx}{7(bd-ae)}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{(8(58b^2d^2-84abde+}{3}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16(58b^2d^2-84abde+}{105(bd-ae)^3}$$

$$= \frac{6d^2\sqrt{a+bx}}{7(bd-ae)(d+ex)^{7/2}} + \frac{4d(23bd-14ae)\sqrt{a+bx}}{35(bd-ae)^2(d+ex)^{5/2}} + \frac{16(58b^2d^2-84abde+}{105(bd-ae)^3}$$

Mathematica [A]
time = 0.16, size = 185, normalized size = 0.98

$$\frac{2\sqrt{a+bx}\left(1575b^3d^2-2100ab^2de+840a^2be^2-\frac{45d^2e^3(a+bx)^3}{(d+ex)^3}+\frac{273bd^2e^2(a+bx)^2}{(d+ex)^2}-\frac{84ade^3(a+bx)^2}{(d+ex)^2}-\frac{875b^2d^2e(a+bx)}{d+ex}+\frac{840abde^2(a+bx)}{d+ex}-\frac{280a^2e^3(a+bx)}{d+ex}\right)}{105(bd-ae)^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(Sqrt[a + b*x]*(d + e*x)^(9/2)),x
]
```

```
[Out] (2*Sqrt[a + b*x]*(1575*b^3*d^2 - 2100*a*b^2*d*e + 840*a^2*b*e^2 - (45*d^2*e
^3*(a + b*x)^3)/(d + e*x)^3 + (273*b*d^2*e^2*(a + b*x)^2)/(d + e*x)^2 - (84
*a*d*e^3*(a + b*x)^2)/(d + e*x)^2 - (875*b^2*d^2*e*(a + b*x))/(d + e*x) + (
840*a*b*d*e^2*(a + b*x))/(d + e*x) - (280*a^2*e^3*(a + b*x))/(d + e*x)))/(1
05*(b*d - a*e)^4*Sqrt[d + e*x])
```

Maple [A]
time = 0.09, size = 207, normalized size = 1.10

method	result
default	$-\frac{2\sqrt{bx+a}\,(-560a^2b\,e^5x^3+1344a\,b^2d\,e^4x^3-928b^3d^2e^3x^3+280a^3e^5x^2-2632a^2bd\,e^4x^2+5168a\,b^2d^2e^3x^2-3248b^3d^3e^2x^2+644a^3 d}{105(ex+d)^{\frac{7}{2}}(ae-bd)^4}$
gosper	$-\frac{2\sqrt{bx+a}\,(-560a^2b\,e^5x^3+1344a\,b^2d\,e^4x^3-928b^3d^2e^3x^3+280a^3e^5x^2-2632a^2bd\,e^4x^2+5168a\,b^2d^2e^3x^2-3248b^3d^3e^2x^2+644a^3 d}{105(ex+d)^{\frac{7}{2}}(e^4a^4-4b\,e^3d\,a^3+6a^2b^2d^2e^2-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] -2/105*(b*x+a)^(1/2)*(-560*a^2*b*e^5*x^3+1344*a*b^2*d*e^4*x^3-928*b^3*d^2*e
^3*x^3+280*a^3*e^5*x^2-2632*a^2*b*d*e^4*x^2+5168*a*b^2*d^2*e^3*x^2-3248*b^3
*d^3*e^2*x^2+644*a^3*d*e^4*x-3890*a^2*b*d^2*e^3*x+6664*a*b^2*d^3*e^2*x-3850
*b^3*d^4*e*x+409*a^3*d^2*e^3-1953*a^2*b*d^3*e^2+2975*a*b^2*d^4*e-1575*b^3*d
^5)/(e*x+d)^(7/2)/(a*e-b*d)^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*d-%e*a>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(174) = 348$.
time = 8.21, size = 482, normalized size = 2.55

$$\frac{2\,(1575\,b^3d^5+280\,(2\,a^2bx^3-a^3x^2)e^5-28\,(48\,ab^2dx^3-94\,a^2bdx^2+23\,a^3dx)e^4+(928\,b^3d^2x^3-5168\,ab^2d^2x^2+3890\,a^2bd^2x-409\,a^3d^2)e^3+7\,(464\,b^3d^3x^2-952\,ab^2d^3x+279\,a^2bd^3)e^2+175\,(22\,b^3d^4x-17\,ab^2d^4)e)\sqrt{bx+a}\,\sqrt{xe+d}}{105\,(b^4d^8+a^4x^4e^8-4\,(a^3bdx^4-a^4dx^3)e^7+2\,(3\,a^2b^2d^2x^4-8\,a^3bd^2x^3+3\,a^4d^2x^2)e^6-4\,(ab^3d^3x^4-6\,a^2b^2d^3x^3+6\,a^3bd^3x^2-a^4d^3x)e^5+(b^4d^4x^4-16\,ab^3d^4x^3+36\,a^2b^2d^4x^2-16\,a^3bd^4x+a^4d^4)e^4+4\,(b^4d^5x^3-6\,ab^3d^5x^2+6\,a^2b^2d^5x-a^3bd^5)e^3+2\,(3\,b^4d^6x^2-8\,ab^3d^6x+3\,a^2b^2d^6)e^2+4\,(b^4d^7x-ab^3d^7)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/105*(1575*b^3*d^5 + 280*(2*a^2*b*x^3 - a^3*x^2)*e^5 - 28*(48*a*b^2*d*x^3
- 94*a^2*b*d*x^2 + 23*a^3*d*x)*e^4 + (928*b^3*d^2*x^3 - 5168*a*b^2*d^2*x^2
+ 3890*a^2*b*d^2*x - 409*a^3*d^2)*e^3 + 7*(464*b^3*d^3*x^2 - 952*a*b^2*d^3*
x + 279*a^2*b*d^3)*e^2 + 175*(22*b^3*d^4*x - 17*a*b^2*d^4)*e)*sqrt(b*x + a)
*sqrt(x*e + d)/(b^4*d^8 + a^4*x^4*e^8 - 4*(a^3*b*d*x^4 - a^4*d*x^3)*e^7 + 2
*(3*a^2*b^2*d^2*x^4 - 8*a^3*b*d^2*x^3 + 3*a^4*d^2*x^2)*e^6 - 4*(a*b^3*d^3*x
^4 - 6*a^2*b^2*d^3*x^3 + 6*a^3*b*d^3*x^2 - a^4*d^3*x)*e^5 + (b^4*d^4*x^4 -
16*a*b^3*d^4*x^3 + 36*a^2*b^2*d^4*x^2 - 16*a^3*b*d^4*x + a^4*d^4)*e^4 + 4*(
b^4*d^5*x^3 - 6*a*b^3*d^5*x^2 + 6*a^2*b^2*d^5*x - a^3*b*d^5)*e^3 + 2*(3*b^4
*d^6*x^2 - 8*a*b^3*d^6*x + 3*a^2*b^2*d^6)*e^2 + 4*(b^4*d^7*x - a*b^3*d^7)*e
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(174) = 348.
time = 3.45, size = 524, normalized size = 2.77

$$\frac{2\left(2\left(4\,(bx+a)\left(\frac{2\,(58\,b^{10}d^2e^6-84\,ab^9de^7+35\,a^2b^8e^8)(bx+a)}{b^6d^4|b|e^3-4\,ab^5d^3|b|e^4+6\,a^2b^4d^2|b|e^5-4\,a^3b^3d|b|e^6+a^4b^2|b|e^7}+\frac{7\,(58\,b^{11}d^3e^5-142\,ab^{10}d^2e^6+119\,a^2b^9de^7-35\,a^3b^8e^8)}{b^6d^4|b|e^3-4\,ab^5d^3|b|e^4+6\,a^2b^4d^2|b|e^5-4\,a^3b^3d|b|e^6+a^4b^2|b|e^7}\right)+\frac{35\,(55\,b^{12}d^4e^4-188\,ab^{11}d^3e^5+243\,a^2b^{10}d^2e^6-142\,a^3b^9de^7+32\,a^4b^8e^8)}{b^6d^4|b|e^3-4\,ab^5d^3|b|e^4+6\,a^2b^4d^2|b|e^5-4\,a^3b^3d|b|e^6+a^4b^2|b|e^7}\right)(bx+a)+\frac{105\,(15\,b^{13}d^5e^3-65\,ab^{12}d^4e^4+113\,a^2b^{11}d^3e^5-99\,a^3b^{10}d^2e^6+44\,a^4b^9de^7-8\,a^5b^8e^8)}{b^6d^4|b|e^3-4\,ab^5d^3|b|e^4+6\,a^2b^4d^2|b|e^5-4\,a^3b^3d|b|e^6+a^4b^2|b|e^7}\right)\sqrt{bx+a}}{105\,(b^2d+(bx+a)be-abe)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algori
thm="giac")
```

```
[Out] 2/105*(2*(4*(b*x + a)*(2*(58*b^10*d^2*e^6 - 84*a*b^9*d*e^7 + 35*a^2*b^8*e^8
)*(b*x + a)/(b^6*d^4*abs(b)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*ab
s(b)*e^5 - 4*a^3*b^3*d*abs(b)*e^6 + a^4*b^2*abs(b)*e^7) + 7*(58*b^11*d^3*e^
5 - 142*a*b^10*d^2*e^6 + 119*a^2*b^9*d*e^7 - 35*a^3*b^8*e^8)/(b^6*d^4*abs(b
)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*abs(b)*e^5 - 4*a^3*b^3*d*abs
(b)*e^6 + a^4*b^2*abs(b)*e^7)) + 35*(55*b^12*d^4*e^4 - 188*a*b^11*d^3*e^5 +
 243*a^2*b^10*d^2*e^6 - 142*a^3*b^9*d*e^7 + 32*a^4*b^8*e^8)/(b^6*d^4*abs(b)
*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*abs(b)*e^5 - 4*a^3*b^3*d*abs(
b)*e^6 + a^4*b^2*abs(b)*e^7))*(b*x + a) + 105*(15*b^13*d^5*e^3 - 65*a*b^12*
d^4*e^4 + 113*a^2*b^11*d^3*e^5 - 99*a^3*b^10*d^2*e^6 + 44*a^4*b^9*d*e^7 - 8
*a^5*b^8*e^8)/(b^6*d^4*abs(b)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*
```

```
abs(b)*e^5 - 4*a^3*b^3*d*abs(b)*e^6 + a^4*b^2*abs(b)*e^7))*sqrt(b*x + a)/(b
^2*d + (b*x + a)*b*e - a*b*e)^(7/2)
```

Mupad [B]
time = 4.51, size = 389, normalized size = 2.06

$$\frac{\sqrt{d+e\,x}\,\left(\frac{-818\,a^4\,d^2\,e^3+3906\,a^3\,b\,d^3\,e^2-5950\,a^2\,b^2\,d^4\,e+3150\,a\,b^3\,d^5}{105\,e^4\,{\left(a\,e-b\,d\right)}^4}+\frac{x\,\left(-1288\,a^4\,d\,e^4+6962\,a^3\,b\,d^2\,e^3-9422\,a^2\,b^2\,d^3\,e^2+1750\,a\,b^3\,d^4\,e+3150\,b^4\,d^5\right)}{105\,e^4\,{\left(a\,e-b\,d\right)}^4}-\frac{x^2\,\left(560\,a^4\,e^5-3976\,a^3\,b\,d\,e^4+2556\,a^2\,b^2\,d^2\,e^3+6832\,a\,b^3\,d^3\,e^2-7700\,b^4\,d^4\,e\right)}{105\,e^4\,{\left(a\,e-b\,d\right)}^4}+\frac{32\,b^2\,x^4\,\left(35\,a^2\,e^2-84\,a\,b\,d\,e+58\,b^2\,d^2\right)}{105\,e\,{\left(a\,e-b\,d\right)}^4}+\frac{16\,b\,x^3\,\left(35\,a^3\,e^3+161\,a^2\,b\,d\,e^2-530\,a\,b^2\,d^2\,e+406\,b^3\,d^3\right)}{105\,e^2\,{\left(a\,e-b\,d\right)}^4}\right)}{x^4\,\sqrt{a+b\,x}+\frac{d^4\,\sqrt{a+b\,x}}{e^4}+\frac{6\,d^2\,x^2\,\sqrt{a+b\,x}}{e^2}+\frac{4\,d\,x^3\,\sqrt{a+b\,x}}{e}+\frac{4\,d^3\,x\,\sqrt{a+b\,x}}{e^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((15*d^2 + 8*e^2*x^2 + 20*d*e*x)/((a + b*x)^(1/2)*(d + e*x)^(9/2)),x)
```

```
[Out] ((d + e*x)^(1/2)*((3150*a*b^3*d^5 - 818*a^4*d^2*e^3 - 5950*a^2*b^2*d^4*e +
3906*a^3*b*d^3*e^2)/(105*e^4*(a*e - b*d)^4) + (x*(3150*b^4*d^5 - 1288*a^4*d
*e^4 + 6962*a^3*b*d^2*e^3 - 9422*a^2*b^2*d^3*e^2 + 1750*a*b^3*d^4*e))/(105*
e^4*(a*e - b*d)^4) - (x^2*(560*a^4*e^5 - 7700*b^4*d^4*e + 6832*a*b^3*d^3*e^
2 + 2556*a^2*b^2*d^2*e^3 - 3976*a^3*b*d*e^4))/(105*e^4*(a*e - b*d)^4) + (32
*b^2*x^4*(35*a^2*e^2 + 58*b^2*d^2 - 84*a*b*d*e))/(105*e*(a*e - b*d)^4) + (1
6*b*x^3*(35*a^3*e^3 + 406*b^3*d^3 - 530*a*b^2*d^2*e + 161*a^2*b*d*e^2))/(10
5*e^2*(a*e - b*d)^4)))/(x^4*(a + b*x)^(1/2) + (d^4*(a + b*x)^(1/2))/e^4 + (
6*d^2*x^2*(a + b*x)^(1/2))/e^2 + (4*d*x^3*(a + b*x)^(1/2))/e + (4*d^3*x*(a
+ b*x)^(1/2))/e^3)
```

3.850 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$

Optimal. Leaf size=417

$$\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}-\frac{2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right.}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right.}}\right.}{c\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}$$

```
[Out] 2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)-2*
arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/
(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(e*(-b*e+2*c*d)+(2*c^2*d^2+b^2*e^2-
2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-2*arctanh((e*x+d)^(1/2)*(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2))*(e*(-b*e+2*c*d)+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*a*c+b^2)^
(1/2))/c/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2)))^(1/2)
```

Rubi [A]

time = 2.03, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {923, 65, 223, 212, 6860, 95, 214}

$$-\frac{2\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{c\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}+\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sqr
t[g]) - (2*(e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqr
t[b^2 - 4*a*c])*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e
*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) - (
2*(e*(2*c*d - b*e) - (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4
*a*c])*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(c*Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 923

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \left(\frac{e^2}{c\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{cd^2-ae^2+e(2cd-be)x}{c\sqrt{d+ex}\,\sqrt{f+gx}\,(a+bx+cx^2)}\right) dx$$

$$= \frac{\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+bx+cx^2)}\,dx}{c} + \frac{e^2\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{c}$$

$$= \frac{\int \left(\frac{e(2cd-be)+\frac{2c^2d^2-2bcde+b^2e^2-2ace^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{e(2cd-be)-\frac{2c^2d^2-2bcde+b\ldots}{\sqrt{b^2-\ldots}}}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+\ldots}}\right.}{c}$$

$$= \frac{(2e)\mathrm{Subst}\left(\int \frac{1}{1-\frac{gx^2}{e}}\,dx, x, \frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{c} + \frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd\ldots}{\sqrt{b^2-4ac}}\right.}{\ldots}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(2\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right.\right.}{\ldots}$$

$$= \frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\,\sqrt{d+ex}}{\sqrt{e}\,\sqrt{f+gx}}\right)}{c\sqrt{g}} - \frac{2\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\ldots}{c\sqrt{2cd-\left(b-\sqrt{b^2-4\ldots}\right.}}$$

<u>**Mathematica**</u> [A]

time = 4.28, size = 473, normalized size = 1.13

$$\frac{\frac{\sqrt{2}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\sqrt{cd^2+e(-bd+ae)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}\,ef+bdg-\sqrt{b^2-4ac}\,dg-2aeg}\,\sqrt{d+ex}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cdf+bef+\sqrt{b^2-4ac}\,ef+bdg-\sqrt{b^2-4ac}\,dg-2aeg}}+\frac{\sqrt{2}\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{cd^2+e(-bd+ae)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef-\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg-2aeg}\,\sqrt{d+ex}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cdf+bef-\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg-2aeg}}+\frac{2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{\sqrt{g}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] ((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[c*d^2 + e*(-(b*d) + a*e
)]*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*
f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g
]*Sqrt[d + e*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*
a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (Sqrt[2]*(-2*c*d + (
b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan[(Sqrt[2]*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2
```

```
- 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/(S
qrt[b^2 - 4*a*c]*Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sq
rt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x]
)/(Sqrt[g]*Sqrt[d + e*x])])/Sqrt[g])/c
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11687 vs. $2(361) = 722$.
time = 0.18, size = 11688, normalized size = 28.03

method	result	size
default	Expression too large to display	11688

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 2.62index.cc index_m
 i_lex_is_greater Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)), x)
```

3.851 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$

Optimal. Leaf size=285

$$-\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}+\frac{2\sqrt{2cd-\left(b+\sqrt{b^2-\right.}}}{}$$

```
[Out] -2*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/
2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+2*arctanh(
(e*x+d)^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*
a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {923, 95, 214}

$$\frac{2\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\tanh^{-1}\left(\frac{\sqrt{d+ex}\,\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b - Sqrt
[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*S
qrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g])
 + (2*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
*Sqrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 923

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f
 + g*x]), (d + e*x)^(m + 1/2)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IGtQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \left(\frac{e+\frac{2cd-be}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\,\sqrt{f+gx}} + \frac{e-}{\left(b+\sqrt{b^2-4ac}+\right.} \right.$$

$$= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx +$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e-\left(-2cf\right.}\right.$$

$$= -\frac{2\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right.}{\sqrt{b^2-4ac}\,\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 925 vs. 2(285) = 570.
time = 10.90, size = 925, normalized size = 3.25

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x])
/Sqrt[2*c^2*d*f + b*(b - Sqrt[b^2 - 4*a*c])*e*g + c*(Sqrt[b^2 - 4*a*c]*e*f
+ Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g - b*(e*f + d*g))] - ((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/Sqrt[2*c^2*d*f + b*(b +
 Sqrt[b^2 - 4*a*c])*e*g - c*(b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b
^2 - 4*a*c]*d*g + 2*a*e*g)] - ((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Log[2*S
qrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d*f + b*(b - Sqrt[b^2 - 4*a*c])*e*g + c
*(Sqrt[b^2 - 4*a*c]*e*f + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g - b*(e*f + d*g))]
*Sqrt[d + e*x]*Sqrt[f + g*x] + b^2*(d*g + e*(f + 2*g*x)) - b*Sqrt[b^2 - 4*a
*c]*(d*g + e*(f + 2*g*x)) + 2*c*(Sqrt[b^2 - 4*a*c]*e*f*x - 2*a*e*(f + 2*g*x
) + d*(2*Sqrt[b^2 - 4*a*c]*f - 2*a*g + Sqrt[b^2 - 4*a*c]*g*x))])/Sqrt[2*c^2
*d*f + b*(b - Sqrt[b^2 - 4*a*c])*e*g + c*(Sqrt[b^2 - 4*a*c]*e*f + Sqrt[b^2
- 4*a*c]*d*g - 2*a*e*g - b*(e*f + d*g))] + ((2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)*Log[2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d*f + b*(b + Sqrt[b^2 - 4*a
*c])*e*g - c*(b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g
 + 2*a*e*g)]*Sqrt[d + e*x]*Sqrt[f + g*x] - b^2*(d*g + e*(f + 2*g*x)) - b*Sq
rt[b^2 - 4*a*c]*(d*g + e*(f + 2*g*x)) + 2*c*(Sqrt[b^2 - 4*a*c]*e*f*x + 2*a*
e*(f + 2*g*x) + d*(2*Sqrt[b^2 - 4*a*c]*f + 2*a*g + Sqrt[b^2 - 4*a*c]*g*x))]
)/Sqrt[2*c^2*d*f + b*(b + Sqrt[b^2 - 4*a*c])*e*g - c*(b*e*f + Sqrt[b^2 - 4*
a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g + 2*a*e*g)])/(Sqrt[2]*Sqrt[b^2 - 4
*a*c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5481 vs. $2(241) = 482$.
time = 0.14, size = 5482, normalized size = 19.24

method	result	size
default	Expression too large to display	5482

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4495 vs. $2(247) = 494$.

time = 100.48, size = 4495, normalized size = 15.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((2*c*d*f - b*d*g - (b*f - 2*a*g)*e + ((b^2*c - 4*a*c^2)*f
^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((d^2*g^2 - 2*d*f*g*e
 + f^2*e^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 -
 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 -
4*a^3*c)*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a
^2*c)*g^2))*log((b*d^2*g^2*x + 2*b*d^2*f*g - 2*a*d^2*g^2 + sqrt(2)*((b^2 -
4*a*c)*d*f*g - (b^2 - 4*a*c)*f^2*e - ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*
b^2*c - 8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3
*c)*g^3)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*
(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^
3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(g*x + f)*sqrt(x*e +
d)*sqrt((2*c*d*f - b*d*g - (b*f - 2*a*g)*e + ((b^2*c - 4*a*c^2)*f^2 - (b^3
- 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2
)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c
 - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g
^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)
) + (2*a*f^2 - (b*f^2 - 4*a*f*g)*x)*e^2 - 2*(2*a*d*g^2*x + b*d*f^2)*e - (2*
(b^2*c - 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d
*f*g^2 + ((b^2*c - 4*a*c^2)*f^3 - (b^3 - 4*a*b*c)*f^2*g + (a*b^2 - 4*a^2*c)
*f*g^2)*x*e + ((b^2*c - 4*a*c^2)*d*f^2*g - (b^3 - 4*a*b*c)*d*f*g^2 + (a*b^2
 - 4*a^2*c)*d*g^3)*x)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^2*c^2 - 4*a*
c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*
g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x) + 1/4*sqr
t(2)*sqrt((2*c*d*f - b*d*g - (b*f - 2*a*g)*e + ((b^2*c - 4*a*c^2)*f^2 - (b^
3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e
^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)
*g^4)))/((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^
2))*log((b*d^2*g^2*x + 2*b*d^2*f*g - 2*a*d^2*g^2 - sqrt(2)*((b^2 - 4*a*c)*d
*f*g - (b^2 - 4*a*c)*f^2*e - ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c -
8*a^2*c^2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)
*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c -
 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^
2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt(
(2*c*d*f - b*d*g - (b*f - 2*a*g)*e + ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*
c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^2*
c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((
```

```
b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) + (2*a
*f^2 - (b*f^2 - 4*a*f*g)*x)*e^2 - 2*(2*a*d*g^2*x + b*d*f^2)*e - (2*(b^2*c -
 4*a*c^2)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 +
 ((b^2*c - 4*a*c^2)*f^3 - (b^3 - 4*a*b*c)*f^2*g + (a*b^2 - 4*a^2*c)*f*g^2)*
x*e + ((b^2*c - 4*a*c^2)*d*f^2*g - (b^3 - 4*a*b*c)*d*f*g^2 + (a*b^2 - 4*a^2
*c)*d*g^3)*x)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^2*c^2 - 4*a*c^3)*f^4
 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*
(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/x) - 1/4*sqrt(2)*sqr
t((2*c*d*f - b*d*g - (b*f - 2*a*g)*e - ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*
b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^
2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a
^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/
((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2))*log(
(b*d^2*g^2*x + 2*b*d^2*f*g - 2*a*d^2*g^2 + sqrt(2)*((b^2 - 4*a*c)*d*f*g - (
b^2 - 4*a*c)*f^2*e + ((b^3*c - 4*a*b*c^2)*f^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^
2)*f^2*g + 3*(a*b^3 - 4*a^2*b*c)*f*g^2 - 2*(a^2*b^2 - 4*a^3*c)*g^3)*sqrt((d
^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^2*c^2 - 4*a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c
^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f
*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))*sqrt(g*x + f)*sqrt(x*e + d)*sqrt((2*c*d*f
 - b*d*g - (b*f - 2*a*g)*e - ((b^2*c - 4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g +
 (a*b^2 - 4*a^2*c)*g^2)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^2*c^2 - 4*
a*c^3)*f^4 - 2*(b^3*c - 4*a*b*c^2)*f^3*g + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*f^
2*g^2 - 2*(a*b^3 - 4*a^2*b*c)*f*g^3 + (a^2*b^2 - 4*a^3*c)*g^4)))/((b^2*c -
4*a*c^2)*f^2 - (b^3 - 4*a*b*c)*f*g + (a*b^2 - 4*a^2*c)*g^2)) + (2*a*f^2 - (
b*f^2 - 4*a*f*g)*x)*e^2 - 2*(2*a*d*g^2*x + b*d*f^2)*e + (2*(b^2*c - 4*a*c^2
)*d*f^3 - 2*(b^3 - 4*a*b*c)*d*f^2*g + 2*(a*b^2 - 4*a^2*c)*d*f*g^2 + ((b^2*c
 - 4*a*c^2)*f^3 - (b^3 - 4*a*b*c)*f^2*g + (a*b^2 - 4*a^2*c)*f*g^2)*x*e + ((
b^2*c - 4*a*c^2)*d*f^2*g - (b^3 - 4*a*b*c)*d*f*g^2 + (a*b^2 - 4*a^2*c)*d*g^
3)*x)*sqrt((d^2*g^2 - 2*d*f*g*e + f^2*e^2)/((b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] \text{Hanged}
```

3.852 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$

Optimal. Leaf size=287

$$-\frac{4c\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}+\frac{4c\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b+\right.}}{\sqrt{2cd-\left(b+\right.}}\right.}{\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b+\sqrt{b^2-4\right.}}}$$

```
[Out] -4*c*arctanh((e*x+d)^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(
1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+4*c*arct
anh((e*x+d)^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {925, 95, 214}

$$\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}-\frac{4c\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(
Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[m] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \left(\frac{2c}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\,\sqrt{f+gx}} - \right.$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex}\,\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{}{\left(b+\right.}}{}$$

$$= \frac{(4c)\text{Subst}\left(\int \frac{1}{-2cd+\left(b-\sqrt{b^2-4ac}\right)e-\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)x^2}\right.}{\sqrt{b^2-4ac}}$$

$$= -\frac{4c\tanh^{-1}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{d+ex}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\sqrt{2cf-\left(b-\sqrt{b^2-\right.}}}$$

Mathematica [A]

time = 3.51, size = 410, normalized size = 1.43

$$\frac{\sqrt{2}\,\sqrt{cd^2+e(-bd+ae)}\left(\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{cd^2-bde+ae^2}\,\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}\,ef+bdg-\sqrt{b^2-4ac}\,dg-2aeg}\,\sqrt{d+ex}}\right)}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}\,ef+bdg-\sqrt{b^2-4ac}\,dg-2aeg}}+\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{cd^2-bde+ae^2}\,\sqrt{f+gx}}{\sqrt{-2cdf+bef-\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg-2aeg}\,\sqrt{d+ex}}\right)}{\sqrt{-2cdf+bef-\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg-2aeg}}\right)}{\sqrt{b^2-4ac}\,\left(-cd^2+e(bd-ae)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (Sqrt[2]*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])
*e)*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d
*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*
g]*Sqrt[d + e*x])])/Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g -
 Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g] + ((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*A
rcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f +
b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sq
rt[d + e*x])])/Sqrt[-2*c*d*f + b*e*f - Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt
[b^2 - 4*a*c]*d*g - 2*a*e*g]))/(Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e)
))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5506 vs. $2(243) = 486$.
time = 0.16, size = 5507, normalized size = 19.19

method	result	size
default	Expression too large to display	5507

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(g*x + f)*sqrt(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] \text{Hanged}
```

3.853 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$

Optimal. Leaf size=429

$$\frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\,\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}}-\frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\,\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(ef-$$

```
[Out] 4*c*e*(g*x+f)^(1/2)/(-d*g+e*f)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2
)^(1/2)/(e*x+d)^(1/2)-4*c*e*(g*x+f)^(1/2)/(-d*g+e*f)/(-4*a*c+b^2)^(1/2)/(2*
c*d-e*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d)^(1/2)-8*c^2*arctanh((e*x+d)^(1/2)*(2*
c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2))/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(-4*a*c+b^2)^(1/2)/(2
*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+8*c^2*arctanh((e*x+d)^(1/2)*(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/(g*x+f)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {925, 98, 95, 214}

$$-\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}+\frac{8c^2\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}+\frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)*(e*f - d*g)*Sqrt[d + e*x]) - (4*c*e*Sqrt[f + g*x])/(Sqrt[b^2 - 4*a*c]*(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(e*f - d*g)*Sqrt[d + e*x]) - (8*c^2*ArcTan
h[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b
- Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) + (8*c^
2*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 925

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^
n, 1/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[m] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx = \int \left(\frac{2c}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}} \right.$$

$$= \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)(d+ex)^{3/2}\sqrt{f+gx}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{}{\left(b+\sqrt{}\right.}}{}$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\,\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} -$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\,\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} -$$

$$= \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\,\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(ef-dg)\sqrt{d+ex}} -$$

Mathematica [A]
time = 3.75, size = 543, normalized size = 1.27

$$\frac{2e^2\sqrt{f+gx}}{(cd^2+e(-bd+ae))(-ef+dg)\sqrt{d+ex}}+\frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef+\sqrt{b^2-4ac}\,ef+bdg-\sqrt{b^2-4ac}\,dg-2aeg}\sqrt{d+ex}}\right)}{\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))^{3/2}\sqrt{-2cdf+bef+\sqrt{b^2-4ac}\,ef+bdg-\sqrt{b^2-4ac}\,dg-2aeg}}+\frac{\sqrt{2}\left(-2c^2d^2+b\left(-b+\sqrt{b^2-4ac}\right)e^2+2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cd^2-bde+ae^2}\sqrt{f+gx}}{\sqrt{-2cdf+bef-\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg-2aeg}\sqrt{d+ex}}\right)}{\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))^{3/2}\sqrt{-2cdf+bef-\sqrt{b^2-4ac}\,ef+bdg+\sqrt{b^2-4ac}\,dg-2aeg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]
```

```
[Out] (2*e^2*Sqrt[f + g*x])/((c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)*Sqrt[d + e
*x]) + (Sqrt[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + S
qrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[f + g*x])/(Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g - Sqrt[b^
2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/(Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-
(b*d) + a*e))^(3/2)*Sqrt[-2*c*d*f + b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g -
 Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]) + (Sqrt[2]*(-2*c^2*d^2 + b*(-b + Sqrt[b^
2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*
Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[f + g*x])/(Sqrt[-2*c*d*f + b*e*f - Sqrt[b^
2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g]*Sqrt[d + e*x])])/
(Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*Sqrt[-2*c*d*f + b*e*f -
 Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47350 vs. $2(369) = 738$.
time = 0.23, size = 47351, normalized size = 110.38

method	result	size
default	Expression too large to display	47351

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\,(a+bx+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
[Out] Integral(1/((d + e*x)**(3/2)*sqrt(f + g*x)*(a + b*x + c*x**2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\left(d+e\,x\right)^{3/2}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)),x)
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)), x)
```

3.854 $\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=532

$$\frac{(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg(3e^2f^2-3defg+d^2g^2)+2ceg(5b^2e^2g^2}{64c^3e^4}$$

```
[Out] 1/24*g^2*(-5*b*e*g-14*c*d*g+24*c*e*f)*(c*x^2+b*x+a)^(3/2)/c^2/e^2+1/4*g^3*(
e*x+d)*(c*x^2+b*x+a)^(3/2)/c/e^2-1/128*(4*c*e*(-b*e+2*c*d)*(16*c^2*e^2*f^3+
5*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-2*b*d*g+6*b*e*f))-2*(4*c^2*d^2-1/2*b^2*e^2-2
*c*e*(-a*e+b*d))*g*(5*b^2*e^2*g^2-4*c*e*g*(a*e*g-2*b*d*g+6*b*e*f)+16*c^2*(d
^2*g^2-3*d*e*f*g+3*e^2*f^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(7/2)/e^5+(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^
2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^5+1/6
4*(5*b^3*e^3*g^3+64*c^3*(-d*g+e*f)^3-4*b*c*e^2*g^2*(a*e*g-2*b*d*g+6*b*e*f)+
16*b*c^2*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)+2*c*e*g*(5*b^2*e^2*g^2-4*c*e*g*(
a*e*g-2*b*d*g+6*b*e*f)+16*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*x)*(c*x^2+b*x+
a)^(1/2)/c^3/e^4
```

Rubi [A]
time = 1.05, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1667, 828, 857, 635, 212, 738}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ce(2cd-be)\left(-4cdg^2(aeg-2bdg+6bef)+5b^2deg^3+16c^2e^2f^3\right)-2g\left(-2ce(bd-ae)-\frac{b^2e^2}{2}+4c^2d^2\right)\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2(d^2g^2-3defg+3e^2f^2)\right)\right)}{128c^{7/2}e^5}+\frac{\sqrt{a+bx+cx^2}\left(2cegx\left(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2(d^2g^2-3defg+3e^2f^2)\right)-4bce^2g^2(aeg-2bdg+6bef)+5b^3e^3g^3+16bc^2eg(d^2g^2-3defg+3e^2f^2)+64c^3(ef-dg)^3\right)}{64c^3e^4}+\frac{g^2(a+bx+cx^2)^{3/2}(-5beg-14cdg+24cef)}{24c^2e^2}+\frac{(ef-dg)^3\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5}+\frac{g^3(d+ex)(a+bx+cx^2)^{3/2}}{4ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((5*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g +
 a*e*g) + 16*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e
^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*
g + d^2*g^2))*x)*Sqrt[a + b*x + c*x^2])/(64*c^3*e^4) + (g^2*(24*c*e*f - 14*
c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^(3/2))/(24*c^2*e^2) + (g^3*(d + e*x)*(a
+ b*x + c*x^2)^(3/2))/(4*c*e^2) - ((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3 + 5
*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 - (b^2
*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g
+ a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*e^5) + (Sqrt[c*d^2 - b*d*e
+ a*e^2]*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
```

```
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{3/2}}{4ce^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{1}{2}e\left(8ce^2f^3-d(3bd+2ae)g^3\right)-e\right.}{}}{}$$

$$= \frac{g^2(24cef-14cdg-5beg)\left(a+bx+cx^2\right)^{3/2}}{24c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right)}{4ce^2}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg(3e^2\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg(3e^2\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg(3e^2\right.}{}$$

$$= \frac{\left(5b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(6bef-2bdg+aeg)+16bc^2eg(3e^2\right.}{}$$

Mathematica [A]
time = 3.28, size = 513, normalized size = 0.96

$$\frac{\frac{2e\sqrt{a+x(b+cx)}\left(15b^3e^3g^3-2bce^2g^2(26aeg+b(36ef-12dg+5egx))+16c^3\left(-12d^3g^3+6d^2eg^2(6f+gx)-2de^2g\left(18f^2+9fgx+2g^2x^2\right)+3e^3\left(4f^3+6f^2gx+4fg^2x^2+g^3x^3\right)\right)+8c^2eg\left(aeg(-8dg+3e(8f+gx))+b\left(6d^2g^2-2deg(9f+gx)+e^2\left(18f^2+6fgx+g^2x^2\right)\right)\right)\right)}{c^3}-768\sqrt{-cd^2+bde-ae^2}\,(-ef+dg)^3\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)-\frac{3\left(-5b^4e^4g^3+128c^4d(-ef+dg)^3+8b^2ce^3g^2(3bef-bdg+3aeg)-16c^2e^2g\left(a^2e^2g^2+2abeg(3ef-dg)+b^2\left(3e^2f^2-3defg+d^2g^2\right)\right)+64c^3e\left(b(ef-dg)^3+aeg\left(3e^2f^2-3defg+d^2g^2\right)\right)\right)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{c^{7/2}}}{384e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 - 2*b*c*e^2*g^2*(26*a*e*g + b*(
36*e*f - 12*d*g + 5*e*g*x)) + 16*c^3*(-12*d^3*g^3 + 6*d^2*e*g^2*(6*f + g*x)
 - 2*d*e^2*g*(18*f^2 + 9*f*g*x + 2*g^2*x^2) + 3*e^3*(4*f^3 + 6*f^2*g*x + 4*
f*g^2*x^2 + g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-8*d*g + 3*e*(8*f + g*x)) + b*(6*
d^2*g^2 - 2*d*e*g*(9*f + g*x) + e^2*(18*f^2 + 6*f*g*x + g^2*x^2)))))/c^3 -
768*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)^3*ArcTan[(Sqrt[c]*(d + e*
```

```
x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - (3*(-5*b^4*
e^4*g^3 + 128*c^4*d*(-(e*f) + d*g)^3 + 8*b^2*c*e^3*g^2*(3*b*e*f - b*d*g + 3
*a*e*g) - 16*c^2*e^2*g*(a^2*e^2*g^2 + 2*a*b*e*g*(3*e*f - d*g) + b^2*(3*e^2*
f^2 - 3*d*e*f*g + d^2*g^2)) + 64*c^3*e*(b*(e*f - d*g)^3 + a*e*g*(3*e^2*f^2
- 3*d*e*f*g + d^2*g^2)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/
c^(7/2))/(384*e^5)
```

Maple [A]
time = 0.15, size = 879, normalized size = 1.65

method	result
default	$g\left(g^2e^2\left(\frac{x\left(cx^2+bx+a\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)}{8c}-\cdots\right.\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] g/e^3*(g^2*e^2*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2
)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2
+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))))+(-d*e*g^2+3*e^2*f*g)*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*
x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))))+d^2*g^2*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a
*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-3*d*e*f*g*(1/4
*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2)))+3*e^2*f^2*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1
/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(-d^3*
g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/
e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
```

```
/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(x+d/e)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f + gx)^3 \sqrt{a + bx + cx^2}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^3\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)^3*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.855 $\int \frac{(f+gx)^2 \sqrt{a+bx+cx^2}}{d+ex} \, dx$

Optimal. Leaf size=325

$$-\frac{(b^2e^2g^2 - 8c^2(ef-dg)^2 - 2bceg(2ef-dg) - 2ceg(4cef-2cdg-beg)x)\sqrt{a+bx+cx^2}}{8c^2e^3} + \frac{g^2(a+bx+c$$

```
[Out] 1/3*g^2*(c*x^2+b*x+a)^(3/2)/c/e+1/16*((8*c^2*d^2-b^2*e^2-4*c*e*(-a*e+b*d))*
g*(-b*e*g-2*c*d*g+4*c*e*f)-4*c*e*(-b*e+2*c*d)*(-b*d*g^2+2*c*e*f^2))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^4+(-d*g+e*f)^2*arctan
h(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1
/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^4-1/8*(b^2*e^2*g^2-8*c^2*(-d*g+e*f)^2-2*b*
c*e*g*(-d*g+2*e*f)-2*c*e*g*(-b*e*g-2*c*d*g+4*c*e*f)*x)*(c*x^2+b*x+a)^(1/2)/
c^2/e^3
```

Rubi [A]
time = 0.45, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1667, 828, 857, 635, 212, 738}

$$-\frac{\sqrt{a+bx+cx^2}\,(b^2e^2g^2 - 2cegx(-beg-2cdg+4cef) - 2bceg(2ef-dg) - 8c^2(ef-dg)^2)}{8c^2e^3} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(g(-4ce(bd-ae)-b^2e^2+8c^2d^2)(-beg-2cdg+4cef)-4ce(2cd-be)(2cef^2-bdg^2))}{16c^{5/2}e^4} + \frac{(ef-dg)^2\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4} + \frac{g^2(a+bx+cx^2)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] -1/8*((b^2*e^2*g^2 - 8*c^2*(e*f - d*g)^2 - 2*b*c*e*g*(2*e*f - d*g) - 2*c*e*
g*(4*c*e*f - 2*c*d*g - b*e*g)*x)*Sqrt[a + b*x + c*x^2])/(c^2*e^3) + (g^2*(a
 + b*x + c*x^2)^(3/2))/(3*c*e) + (((8*c^2*d^2 - b^2*e^2 - 4*c*e*(b*d - a*e)
)*g*(4*c*e*f - 2*c*d*g - b*e*g) - 4*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2)
)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*e^4)
+ (Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{g^2(a+bx+cx^2)^{3/2}}{3ce} + \frac{\int \frac{\left(\frac{3}{2}e(2cef^2-bdg^2)+\frac{3}{2}eg(4cef-2cdg-beg)x\right)\sqrt{a+bx+cx^2}}{d+ex}}{3ce^2}$$

$$= -\frac{(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)}{8c^2e^3}$$

$$= -\frac{(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)}{8c^2e^3}$$

$$= -\frac{(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)}{8c^2e^3}$$

$$= -\frac{(b^2e^2g^2-8c^2(ef-dg)^2-2bceg(2ef-dg)-2ceg(4cef-2cdg-beg)}{8c^2e^3}$$

Mathematica [A]
time = 1.56, size = 316, normalized size = 0.97

$$\frac{\frac{2e\sqrt{a+x(b+cx)}\,(-3b^2e^2g^2+2ceg(4aeg+b(6ef-3dg+egx))+4c^2(6d^2g^2-3deg(4f+gx)+2e^2(3f^2+3fgx+g^2x^2)))}{c^2}+96\sqrt{-cd^2+bde-ae^2}\,(ef-dg)^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)+\frac{3(-b^3e^3g^2+16c^3d(ef-dg)^2+2bce^2g(2bef-bdg+2aeg)-8c^2e(b(ef-dg)^2+aeg(2ef-dg)))\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{c^{5/2}}}{48e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(-3*b^2*e^2*g^2 + 2*c*e*g*(4*a*e*g + b*(6*e*f -
 3*d*g + e*g*x)) + 4*c^2*(6*d^2*g^2 - 3*d*e*g*(4*f + g*x) + 2*e^2*(3*f^2 +
3*f*g*x + g^2*x^2))))/c^2 + 96*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2
*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*
d - a*e)]] + (3*(-(b^3*e^3*g^2) + 16*c^3*d*(e*f - d*g)^2 + 2*b*c*e^2*g*(2*b
*e*f - b*d*g + 2*a*e*g) - 8*c^2*e*(b*(e*f - d*g)^2 + a*e*g*(2*e*f - d*g)))*
Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2))/(48*e^4)
```

Maple [A]
time = 0.14, size = 585, normalized size = 1.80

method	result

default	$-\frac{g\left(-eg\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)+dg\left(\frac{(2cx+b)\sqrt{c\,x^2}}{4c}\right.\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -g/e^2*(-e*g*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x
+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))))+d*g*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*e*f*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a
)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e
))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a
*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/
e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**2*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^2*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.856 $\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=219

$$\frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2}-\frac{(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg+aeg))\tanh^{-1}\left(\frac{\quad}{2\sqrt{\quad}}\right.}{8c^{3/2}e^3}$$

```
[Out] -1/8*(b^2*e^2*g+8*c^2*d*(-d*g+e*f)-4*c*e*(a*e*g-b*d*g+b*e*f))*arctanh(1/2*(
2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^3+(-d*g+e*f)*arctanh(1/2*(b
*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*
e^2-b*d*e+c*d^2)^(1/2)/e^3+1/4*(2*c*e*g*x+b*e*g-4*c*d*g+4*c*e*f)*(c*x^2+b*x
+a)^(1/2)/c/e^2
```

Rubi [A]
time = 0.20, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {828, 857, 635, 212, 738}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4ce(aeg-bdg+bef)+b^2e^2g+8c^2d(ef-dg))}{8c^{3/2}e^3}+\frac{(ef-dg)\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3}+\frac{\sqrt{a+bx+cx^2}(beg-4cdg+4cef+2cegx)}{4ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] ((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*Sqrt[a + b*x + c*x^2])/(4*c*e^2) -
 ((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*e^3) + (Sqrt[c*
d^2 - b*d*e + a*e^2]*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{\int \frac{\frac{1}{2}(4ce(bd-2ae)f+4acdeg-bd(4\cdots}{(\cdots}}{\cdots}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} + \frac{((cd^2-bde+ae^2)(ef-\cdots}{\cdots}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{(2(cd^2-bde+ae^2)(ef-\cdots}{\cdots}$$

$$= \frac{(4cef-4cdg+beg+2cegx)\sqrt{a+bx+cx^2}}{4ce^2} - \frac{(b^2e^2g+8c^2d(ef-dg)-\cdots}{\cdots}$$

Mathematica [A]

time = 0.88, size = 213, normalized size = 0.97

$$\frac{2\sqrt{c}\,e\sqrt{a+x(b+cx)}\,(beg+2c(2ef-2dg+egx))-16c^{3/2}\sqrt{-cd^2+bde-ae^2}\,(-ef+dg)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)+(b^2e^2g+8c^2d(ef-dg)-4ce(bef-bdg+aeg))\log\left(c\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{8c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[a + x*(b + c*x)]*(b*e*g + 2*c*(2*e*f - 2*d*g + e*g*x)) -
16*c^(3/2)*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*(d
 + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + (b^2*e
^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*Log[c*(b + 2*c*
x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(3/2)*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(197) = 394$.

time = 0.12, size = 407, normalized size = 1.86

method	result
default	$\frac{g\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{e}+\frac{(-dg+ef)\left(\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\right.}{e}}\right.}{}$
risch	$\frac{(2cegx+beg-4dgc+4cef)\sqrt{c\,x^2+bx+a}}{4c\,e^2}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)ag}{2\sqrt{c}\,e}-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)b^2g}{8c^{\frac{3}{2}}e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] g/e*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-d*g+e*f)/e^2*((c*(x+d/e)^2+(b*e-2*c*d)
/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*
c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x+d/e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

3.857 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{a+bx+cx^2}}{e}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2}+\frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{bd-2ae}{2\sqrt{cd^2-bde+c}}\right.}{e^2}$$

```
[Out] -1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/c^
(1/2)+arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^2+(c*x^2+b*x+a)^(1/2)/e
```

Rubi [A]

time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {748, 857, 635, 212, 738}

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\,\sqrt{ae^2-bde+cd^2}}\right)}{e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2}+\frac{\sqrt{a+bx+cx^2}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/e - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b
*x + c*x^2])])/e^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx &= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2e} \\
&= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(e(bd-2ae)-d(2cd-be))\int}{2e^2} \\
&= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e^2} - \frac{(2(cd^2-bd}{} \\
&= \frac{\sqrt{a+bx+cx^2}}{e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^2} + \frac{\sqrt{cd^2-bde+ae^2}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 140, normalized size = 0.92

$$
\frac{2e\sqrt{a+x(b+cx)}+4\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)+\frac{(2cd-be)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}}{2e^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x),x]
```

```
[Out] (2*e*Sqrt[a + x*(b + c*x)] + 4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[
c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + (
(2*c*d - b*e)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c])/(2
*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(132) = 264$.
time = 0.13, size = 329, normalized size = 2.16

method	result
default	$\frac{\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}+\frac{(eb-2cd)\ln\left(\frac{\frac{eb-2cd}{2e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}}\right.}{2e\sqrt{c}}}{}$
risch	$\frac{\sqrt{c x^2+bx+a}}{e}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)b}{2e\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)\sqrt{c}\, d}{e^2}-\frac{\ln\left(\frac{2a e^2-2bde+}{e^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*
(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*
c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/
e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*
d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(
x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c*d^2-%e*b*d+%e^2*a>0)', see `assum
e?` for
```

Fricas [A]
time = 5.28, size = 990, normalized size = 6.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 2*sq
rt(c*d^2 - b*d*e + a*e^2)*c*log(-(8*c^2*d^2*x^2 + 8*b*c*d^2*x + (b^2 + 4*a*
c)*d^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt
(c*x^2 + b*x + a) + (8*a*b*x + (b^2 + 4*a*c)*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*
x^2 + 4*a*b*d + (3*b^2 + 4*a*c)*d*x)*e)/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-2)/
c, 1/2*((2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*sqrt(c*x^2 + b*x + a)*c*e + sqrt(c*d
^2 - b*d*e + a*e^2)*c*log(-(8*c^2*d^2*x^2 + 8*b*c*d^2*x + (b^2 + 4*a*c)*d^2
 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2
 + b*x + a) + (8*a*b*x + (b^2 + 4*a*c)*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^2 +
4*a*b*d + (3*b^2 + 4*a*c)*d*x)*e)/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-2)/c, 1/4
*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)
*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2 + b*x + a)/(c^2*d^2*x^2 + b*c*d
^2*x + a*c*d^2 + (a*c*x^2 + a*b*x + a^2)*e^2 - (b*c*d*x^2 + b^2*d*x + a*b*d
)*e)) + 4*sqrt(c*x^2 + b*x + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^2
- 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))*e^(
-2)/c, 1/2*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(-c*d^2 + b*d*
e - a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2 + b*x + a)/(c^2*d^2*x
^2 + b*c*d^2*x + a*c*d^2 + (a*c*x^2 + a*b*x + a^2)*e^2 - (b*c*d*x^2 + b^2*d
*x + a*b*d)*e)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*sqrt(c*x^2 + b*x + a)*c*e)
*e^(-2)/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x), x)
```

3.858 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=228

$$\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg}+\frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)}-\frac{\sqrt{}}{}$$

```
[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/e/g+arctanh(1/2*
(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(
a*e^2-b*d*e+c*d^2)^(1/2)/e/(-d*g+e*f)-arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x
)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/
g/(-d*g+e*f)
```

Rubi [A]
time = 0.21, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {909, 738, 212, 857, 635}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)}-\frac{\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(e*g) + (S
qrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*(e*f - d*g)) - (Sqrt[c*f
^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
 b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{cdf-bef+aeg-c(ef-dg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)}+\frac{(cd^2-bde+ae^2)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e(ef-dg)}$$

$$=\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{eg}-\frac{(2(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-}{\sqrt{}}\right.}{e(ef-dg)}$$

$$=\frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e(ef-dg)}+\frac{(2c)\text{Subst}\left(\right.}{}$$

$$=\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg}+\frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{bd-}{2\sqrt{cd^2-bde}}\right.}{e(ef-dg)}$$

Mathematica [A]
time = 0.66, size = 212, normalized size = 0.93

$$\frac{2\sqrt{-cd^2+bde-ae^2}\,g\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)-2e\sqrt{-cf^2+bfg-ag^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+bfg-ag^2}}\right)+\sqrt{c}\,(-ef+dg)\log\left(eg\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{eg(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)),x]
```

```
[Out] (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*g*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a +
x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] - 2*e*Sqrt[-(c*f^2) + b*f*g -
 a*g^2]*ArcTan[(Sqrt[c]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^2)
+ b*f*g - a*g^2]] + Sqrt[c]*(-(e*f) + d*g)*Log[e*g*(b + 2*c*x - 2*Sqrt[c]*S
qrt[a + x*(b + c*x)])])/(e*g*(e*f - d*g))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(204) = 408$.
time = 0.13, size = 673, normalized size = 2.95

method	result
default	$-\frac{\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}+\frac{(eb-2cd)\ln\left(\frac{\frac{eb-2cd}{2e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}}\right.}{2e\sqrt{c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*g-e*f)*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^
2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*
e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+
(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2
*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/(d*g-e*f)*(((x+
f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*
f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g
^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f
/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\,(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{(f+g\,x)\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)*(d + e*x)), x)
```

3.859 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx$

Optimal. Leaf size=490

$$\frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,(ef-dg)^2}+\frac{e(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.}{2\sqrt{c}\,g(ef-dg)^2}$$

```
[Out] -1/2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+
e*f)^2/c^(1/2)+1/2*e*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+
a)^(1/2))/g/(-d*g+e*f)^2/c^(1/2)-arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))*c^(1/2)/g/(-d*g+e*f)+arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2
-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/(-d*g+e*
f)^2+1/2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c
*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(1/2)-e*a
rctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^2+(c*x^2+b*x+a)^(1/2)/(-d*
g+e*f)/(g*x+f)
```

Rubi [A]

time = 0.45, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {974, 748, 857, 635, 212, 738, 746}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2}-\frac{e\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^2}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)\sqrt{ag^2-bfg+cf^2}}+\frac{\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2}+\frac{e(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^2) + (e*
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[c]*g*(e*f - d*g)^2) - (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(g*(e*f - d*g)) + (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d
- 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*
x^2])])/(e*f - d*g)^2 + ((2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g
)*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g*(e*f - d*g)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2}\,dx &= \int \left(\frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^2(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^2} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)}\right)dx \\
&= \frac{e^2\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^2} - \frac{g\int \frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}\,dx}{ef-dg} \\
&= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{e\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{e\int \frac{bf-2ag+(2cf-bg)x}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} \\
&= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)^2} + \frac{(cd^2-bde+ae^2)\int \frac{}{(d+e}}{(ef-d} \\
&= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{(ef-dg)^2} - \frac{(2(cd^2}{} \\
&= \frac{\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^2} + \frac{e(2cf-bg)\,\mathrm{t}}{}
\end{aligned}
$$

Mathematica [A]
time = 1.01, size = 228, normalized size = 0.47

$$
\frac{\frac{(ef-dg)\sqrt{a+x(b+cx)}}{f+gx} + 2\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right) - \frac{\sqrt{-cf^2+bfg-ag^2}(2cdf+2aeg-b(ef+dg))\tan^{-1}\left(\frac{\sqrt{c}(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+g(bf-ag)}}\right)}{cf^2+g(-bf+ag)}}{(ef-dg)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] (((e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + 2*Sqrt[-(c*d^2) + b*d*e -
a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) +
 e*(b*d - a*e)]] - (Sqrt[-(c*f^2) + b*f*g - a*g^2]*(2*c*d*f + 2*a*e*g - b*(
e*f + d*g))*ArcTan[(Sqrt[c]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f
^2) + g*(b*f - a*g)]])/(c*f^2 + g*(-(b*f) + a*g)))/(e*f - d*g)^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. 2(434) = 868.
time = 0.12, size = 1359, normalized size = 2.77

method	result	size
default	Expression too large to display	1359

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e/(d*g-e*f)^2*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)
^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d
*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2
+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-
2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))-e/(d*g-e*f)^2*((
(x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2
*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(
x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+1/g/(d*g-e*f)*(-1/(a*g^2-b*f*g+c
*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^
2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g)^2*c+(b*g-2*c*f)/g*
(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f
)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^
2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1
/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^
(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/
g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*
(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)
/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((g*x + f)^2*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\,(f + gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f+g\,x\right)}^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^2*(d + e*x)), x)
```

3.860 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3} \, dx$

Optimal. Leaf size=673

$$\frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)}-\frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2}-\frac{e(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,(ef-dg)^3}$$

```
[Out] 1/8*(-4*a*c+b^2)*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^
2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)-1/2*e*(-
b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^3/
c^(1/2)+1/2*e^2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/g/(-d*g+e*f)^3/c^(1/2)-e*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))*c^(1/2)/g/(-d*g+e*f)^2+e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^
2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/(-d*g+e
*f)^3+1/2*e*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*
g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(1/2
)-e^2*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^3+e*(c*x^2+b*x+a)^(
1/2)/(-d*g+e*f)^2/(g*x+f)-1/4*g*(b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1
/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2
```

Rubi [A]
time = 0.56, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {974, 748, 857, 635, 212, 738, 734, 746}

$$\frac{g\left(b^2-4ac\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{e\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3}-\frac{e^2\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^3}+\frac{e^2(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,g(ef-dg)^3}-\frac{g\sqrt{a+bx+cx^2}(-2ag+x(2cf-bg)+bf)}{4(f+gx)^2(ef-dg)\left(ag^2-bfg+cf^2\right)}+\frac{e(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}+\frac{e\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2}-\frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2}-\frac{e(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] (e*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*(f + g*x)) - (g*(b*f - 2*a*g + (2*
c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)
*(f + g*x)^2) - (e*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*Sqrt[c]*(e*f - d*g)^3) + (e^2*(2*c*f - b*g)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^3) - (Sq
rt[c]*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d
*g)^2) + (e*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e
)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^3
+ ((b^2 - 4*a*c)*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g + a*g
^2)^(3/2)) + (e*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sq
rt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g)^2*Sqrt[
```

```
c*f^2 - b*f*g + a*g^2]) - (e^2*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2
*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2
])])/(g*(e*f - d*g)^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx = \int \left(\frac{e^3\sqrt{a+bx+cx^2}}{(ef-dg)^3(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^3} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g\sqrt{\cdot}}{(ef-\cdot}\right.$$

$$= \frac{e^3\int\frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^3} - \frac{(e^2g)\int\frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^3} - \frac{(eg)\int\frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}}{(ef-dg)^2}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e^2\int\frac{bd-}{(d+ex)\sqrt{}}}{2(}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{(e(2cd-be}{}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{\left(b^2-4ac\right)g}{}$$

$$= \frac{e\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)} - \frac{g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} - \frac{e(2cd-be)}{}$$

Mathematica [A]
time = 11.03, size = 609, normalized size = 0.90

$$\frac{\frac{8e(ef-dg)\sqrt{a+x(b+cx)}}{f+gx}+\frac{2g(ef-dg)^2(-bf+2ag-2cfx+bgx)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)^2}+\frac{4e(-2cd+be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+8e\sqrt{cd^2+e(-bd+ae)}\tanh^{-1}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)+\frac{(b^2-4ac)g(ef-dg)^2\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(cf^2+g(-bf+ag))^{3/2}}-\frac{4e(ef-dg)\left(2\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-\frac{(2cf-bg)\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cf^2+g(-bf+ag)}}\right)}{g}+\frac{4e^2\left((2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\sqrt{cf^2+g(-bf+ag)}\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)\right)}{\sqrt{c}\,g}}{8(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*e*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (2*g*(e*f - d*g)^2*(-(
b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f)
+ a*g))*(f + g*x)^2) + (4*e*(-2*c*d + b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + x*(b + c*x)])]])/Sqrt[c] + 8*e*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*ArcTan
h[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])] + ((b^2 - 4*a*c)*g*(e*f - d*g)^2*ArcTanh[(-2*a*g + 2*c*f*
x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])
/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (4*e*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*
g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b
+ c*x)])])/Sqrt[c*f^2 + g*(-(b*f) + a*g)]))/g + (4*e^2*((2*c*f - b*g)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 +
g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*g))/(8*(e*f - d*g)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. 2(603) = 1206.
time = 0.15, size = 2512, normalized size = 3.73

method	result	size
default	Expression too large to display	2512

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/(d*g-e*f)^3*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d
/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-
b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/
e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b
*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+e^2/(d*g-e*f)
^3*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(
b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c
*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g
^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f
)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x
```

```
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+1/g^2/(d*g-e*f)*(-1/2/(a*g^
2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*
g+c*f^2)/g^2)^(3/2)-1/4*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(-1/(a*g^2-b*f*g+
c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g
^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*
f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(
1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*
f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)
/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8
*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f
)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^
2)/g^2)^(1/2))))+1/2*c/(a*g^2-b*f*g+c*f^2)*g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*
(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f
)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^
2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1
/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f
^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^
(1/2))/(x+f/g))))-1/g*e/(d*g-e*f)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x
+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c
*f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+
c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2
)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2
)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*
g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/
(a*g^2-b*f*g+c*f^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*
g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f
^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)
+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((g*x + f)^3*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**3,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1844 vs. 2(618) = 1236.
time = 6.86, size = 1844, normalized size = 2.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d^2*g^3 - 4*a*c*d^2*g^3 - 8*c^2*d*f^3*e + 12*b*c*d*f^2*g*e - 6*b^
2*d*f*g^2*e + 4*a*b*d*g^3*e + 4*b*c*f^3*e^2 - 3*b^2*f^2*g*e^2 - 12*a*c*f^2*
g*e^2 + 12*a*b*f*g^2*e^2 - 8*a^2*g^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c*d^3*f^2*g^3 -
b*d^3*f*g^4 + a*d^3*g^5 - 3*c*d^2*f^3*g^2*e + 3*b*d^2*f^2*g^3*e - 3*a*d^2*f
*g^4*e + 3*c*d*f^4*g*e^2 - 3*b*d*f^3*g^2*e^2 + 3*a*d*f^2*g^3*e^2 - c*f^5*e^
3 + b*f^4*g*e^3 - a*f^3*g^2*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) - 2*(c*d^2*e
 - b*d*e^2 + a*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c
)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^
2 - f^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/4*(8*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*c^2*d*f^2*g^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*
d*f*g^3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^4 + 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^3*a*c*d*g^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b*c*f^2*g^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^3*e + 4*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^3*e - 4*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^3*a*b*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)
*d*f^3*g - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^3 - 4*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^3 + 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*c^(5/2)*f^4*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*
```

```
f^3*g*e + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f^2*g^2*e + 1
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f^2*g^2*e - 4*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^3*e - 8*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a^2*sqrt(c)*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
^2*d*f^3*g - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^2 - 8*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^2 - (sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b^3*d*f*g^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^
3 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^4 + 4*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a^2*c*d*g^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*
f^4*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f^3*g*e - 16*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3*g*e + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^3*f^2*g^2*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*g^2*
e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^3*e - 28*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*c*f*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^2*b*g^4*e + 2*b^2*c^(3/2)*d*f^3*g - b^3*sqrt(c)*d*f^2*g^2 - 4*a*b*c^(3/2
)*d*f^2*g^2 + a*b^2*sqrt(c)*d*f*g^3 + 4*a^2*c^(3/2)*d*f*g^3 + 2*b^2*c^(3/2)
*f^4*e - 3*b^3*sqrt(c)*f^3*g*e - 8*a*b*c^(3/2)*f^3*g*e + 15*a*b^2*sqrt(c)*f
^2*g^2*e + 4*a^2*c^(3/2)*f^2*g^2*e - 20*a^2*b*sqrt(c)*f*g^3*e + 8*a^3*sqrt(
c)*g^4*e)/((c*d^2*f^2*g^3 - b*d^2*f*g^4 + a*d^2*g^5 - 2*c*d*f^3*g^2*e + 2*b
*d*f^2*g^3*e - 2*a*d*f*g^4*e + c*f^4*g*e^2 - b*f^3*g^2*e^2 + a*f^2*g^3*e^2)
*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*sqrt(c)*f + b*f - a*g)^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f+g\,x\right)}^3\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^3*(d + e*x)), x)
```

3.861 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx$

Optimal. Leaf size=933

$$\frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)}-\frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2}-\frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right.}$$

```
[Out] 1/3*g^2*(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^3+1/16*(
-4*a*c+b^2)*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*
f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(5/2)+
1/8*(-4*a*c+b^2)*e*g*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*
f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)-1/2*
e^2*(-b*e+2*c*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e
*f)^4/c^(1/2)+1/2*e^3*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/g/(-d*g+e*f)^4/c^(1/2)-e^2*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))*c^(1/2)/g/(-d*g+e*f)^3+e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d
)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/
2)/(-d*g+e*f)^4+1/2*e^2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)
/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)^3/(a*g^2-b*f*g
+c*f^2)^(1/2)-e^3*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2
)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g/(-d*g+e*f)^4+e^2*(
c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^3/(g*x+f)-1/8*g*(-b*g+2*c*f)*(b*f-2*a*g+(-b*g
+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)^2-1
/4*e*g*(b*f-2*a*g+(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^2/(a*g^2-b
*f*g+c*f^2)/(g*x+f)^2
```

Rubi [A]
time = 0.81, antiderivative size = 933, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {974, 748, 857, 635, 212, 738, 744, 734, 746}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] (e^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^3*(f + g*x)) - (g*(2*c*f - b*g)*(b
*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*(e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^2*(f + g*x)^2) - (e*g*(b*f - 2*a*g + (2*c*f - b*g)*x)*Sqrt
[a + b*x + c*x^2])/(4*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^2) +
(g^2*(a + b*x + c*x^2)^(3/2))/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^3) - (e^2*(2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*(e*f - d*g)^4) + (e^3*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)
```

```
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*g*(e*f - d*g)^4) - (Sqrt[c]
*e^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)
^3) + (e^2*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)
*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*f - d*g)^4 +
 ((b^2 - 4*a*c)*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*
Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(16*(e*f - d*g)*(c*f^2
 - b*f*g + a*g^2)^(5/2)) + ((b^2 - 4*a*c)*e*g*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) + (e^2*(2*c*f - b*g)*ArcTanh[(b*f -
 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x
^2])])/(2*g*(e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (e^3*Sqrt[c*f^2 -
b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*
g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f - d*g)^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
```

```
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4}\,dx = \int \left(\frac{e^4\sqrt{a+bx+cx^2}}{(ef-dg)^4(d+ex)} - \frac{g\sqrt{a+bx+cx^2}}{(ef-dg)(f+gx)^4} - \frac{eg\sqrt{a+bx+cx^2}}{(ef-dg)^2(f+gx)^3} - \frac{e^2g\sqrt{}}{(ef-}\right.$$

$$= \frac{e^4\int\frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx}{(ef-dg)^4} - \frac{(e^3g)\int\frac{\sqrt{a+bx+cx^2}}{f+gx}\,dx}{(ef-dg)^4} - \frac{(e^2g)\int\frac{\sqrt{a+bx+cx^2}}{(f+gx)^2}}{(ef-dg)^3}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{eg(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{4(ef-dg)^2\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{}{3(ef-dg}$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} -$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} -$$

$$= \frac{e^2\sqrt{a+bx+cx^2}}{(ef-dg)^3(f+gx)} - \frac{g(2cf-bg)(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2}}{8(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)^2} -$$

Mathematica [A]
time = 12.74, size = 858, normalized size = 0.92

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]
```

```
[Out] ((48*e^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/(f + g*x) + (12*e*g*(e*f - d*g)
^2*(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-
(b*f) + a*g))*(f + g*x)^2) - (16*g^2*(-(e*f) + d*g)^3*(a + x*(b + c*x))^(3/
2))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^3) + 24*e^2*(((-2*c*d + b*e)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2
+ e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]) + (6*(b^2 - 4*a*c)*e*g*(e*f -
d*g)^2*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) +
 a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2) - (3*g*(2*
c*f - b*g)*(e*f - d*g)^3*((2*Sqrt[a + x*(b + c*x)]*(-2*a*g + 2*c*f*x + b*(f
 - g*x)))/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)^2) + ((-b^2 + 4*a*c)*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2)))/(c*f^2 + g*(-(b*f) +
a*g)) - (24*e^2*(e*f - d*g)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
```

```
a + x*(b + c*x)])] - ((2*c*f - b*g)*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^2 + g
*(-(b*f) + a*g)]))/g + (24*e^3*((2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTan
h[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a
 + x*(b + c*x)])]))/(Sqrt[c]*g))/(48*(e*f - d*g)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3776 vs. $2(845) = 1690$.
time = 0.12, size = 3777, normalized size = 4.05

method	result	size
default	Expression too large to display	3777

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x,method=_RETURNVERBOSE)
```

```
[Out] e^3/(d*g-e*f)^4*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/
e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b
*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*
e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/g^3/(d*g-e*f
)*(-1/3/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g
)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)-1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(-1/
2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(3/2)-1/4*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(-1/(a*g^2
-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c
*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b
*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*
f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)
/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^
2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g
-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1
/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*
g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f
*g+c*f^2)/g^2)^(1/2))))+1/2*c/(a*g^2-b*f*g+c*f^2)*g^2*(((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*
g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f
*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*
f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))/(x+f/g)))))-e^3/(d*g-e*f)^4*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
```

```
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c
*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^
2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*l
n((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
)/(x+f/g)))-1/g^2*e/(d*g-e*f)^2*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x
+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)-1/4*(b*g-2*c
*f)*g/(a*g^2-b*f*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+
(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g
^2-b*f*g+c*f^2)*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)
^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b
*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g
^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*
g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f
*g+c*f^2)*g^2*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b
*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))+1/2*c/(a*g^2-b*
f*g+c*f^2)*g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*
f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^
2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))+1/g*e^2/(d*g-e
*f)^3*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g
)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(((x
+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c
*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(
x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+
f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/4*(
2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*
g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(
3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((g*x + f)^4*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8035 vs. $2(855) = 1710$.
time = 15.27, size = 8035, normalized size = 8.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] -1/8*(2*b^2*c*d^3*f*g^4 - 8*a*c^2*d^3*f*g^4 - b^3*d^3*g^5 + 4*a*b*c*d^3*g^5
 - 8*b^2*c*d^2*f^2*g^3*e + 32*a*c^2*d^2*f^2*g^3*e + 5*b^3*d^2*f*g^4*e - 20*
a*b*c*d^2*f*g^4*e - 2*a*b^2*d^2*g^5*e + 8*a^2*c*d^2*g^5*e + 16*c^3*d*f^5*e^
2 - 40*b*c^2*d*f^4*g*e^2 + 42*b^2*c*d*f^3*g^2*e^2 - 8*a*c^2*d*f^3*g^2*e^2 -
 15*b^3*d*f^2*g^3*e^2 - 20*a*b*c*d*f^2*g^3*e^2 + 20*a*b^2*d*f*g^4*e^2 - 8*a
^2*b*d*g^5*e^2 - 8*b*c^2*f^5*e^3 + 12*b^2*c*f^4*g*e^3 + 32*a*c^2*f^4*g*e^3
- 5*b^3*f^3*g^2*e^3 - 60*a*b*c*f^3*g^2*e^3 + 30*a*b^2*f^2*g^3*e^3 + 40*a^2*
c*f^2*g^3*e^3 - 40*a^2*b*f*g^4*e^3 + 16*a^3*g^5*e^3)*arctan(-((sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^2*d
^4*f^4*g^4 - 2*b*c*d^4*f^3*g^5 + b^2*d^4*f^2*g^6 + 2*a*c*d^4*f^2*g^6 - 2*a*
b*d^4*f*g^7 + a^2*d^4*g^8 - 4*c^2*d^3*f^5*g^3*e + 8*b*c*d^3*f^4*g^4*e - 4*b
^2*d^3*f^3*g^5*e - 8*a*c*d^3*f^3*g^5*e + 8*a*b*d^3*f^2*g^6*e - 4*a^2*d^3*f*
g^7*e + 6*c^2*d^2*f^6*g^2*e^2 - 12*b*c*d^2*f^5*g^3*e^2 + 6*b^2*d^2*f^4*g^4*
e^2 + 12*a*c*d^2*f^4*g^4*e^2 - 12*a*b*d^2*f^3*g^5*e^2 + 6*a^2*d^2*f^2*g^6*e
^2 - 4*c^2*d*f^7*g*e^3 + 8*b*c*d*f^6*g^2*e^3 - 4*b^2*d*f^5*g^3*e^3 - 8*a*c*
```

d*f^5*g^3*e^3 + 8*a*b*d*f^4*g^4*e^3 - 4*a^2*d*f^3*g^5*e^3 + c^2*f^8*e^4 - 2*b*c*f^7*g*e^4 + b^2*f^6*g^2*e^4 + 2*a*c*f^6*g^2*e^4 - 2*a*b*f^5*g^3*e^4 + a^2*f^4*g^4*e^4)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 2*(c*d^2*e^2 - b*d*e^3 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/((d^4*g^4 - 4*d^3*f*g^3*e + 6*d^2*f^2*g^2*e^2 - 4*d*f^3*g*e^3 + f^4*e^4)*sqrt(-c*d^2 + b*d*e - a*e^2)) + 1/24*(6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d^2*f*g^6 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*d^2*f*g^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d^2*g^7 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*d^2*g^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*d*f^4*g^3*e + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*d*f^3*g^4*e - 66*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d*f^2*g^5*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*d*f^2*g^5*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d*f*g^6*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*d*f*g^6*e - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*d*g^7*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c*d*g^7*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/2)*d^2*f^4*g^3 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*d^2*f^3*g^4 + 78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d^2*f^2*g^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*d^2*f^2*g^5 - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sqrt(c)*d^2*f*g^6 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*d^2*f*g^6 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*d^2*g^7 - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^(7/2)*d*f^5*g^2*e + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^(5/2)*d*f^4*g^3*e - 234*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(3/2)*d*f^3*g^4*e - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(5/2)*d*f^3*g^4*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*sqrt(c)*d*f^2*g^5*e + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^(3/2)*d*f^2*g^5*e + 66*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*sqrt(c)*d*f*g^6*e - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(3/2)*d*f*g^6*e - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*sqrt(c)*d*g^7*e + 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*d^2*f^5*g^2 + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d^2*f^4*g^3 - 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2*d^2*f^3*g^4 - 112*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^3*d^2*f^3*g^4 + 74*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c*d^2*f^2*g^5 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^2*d^2*f^2*g^5 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*d^2*f*g^6 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c*d^2*f*g^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^2*d^2*f*g^6 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d^2*g^7 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d^2*g^7 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*f^4*g^3*e^2 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*f^3*g^4*e^2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*f^3*g^4*e^2 + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*f^2*g^5*e^2 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*f^2*g^5*e^2 - 42*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*f*g^6*e^2 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c*f*g^6*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*g^7*e^2 - 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^4*d*f^6*

```
g*e - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d*f^5*g^2*e + 396*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^2*d*f^4*g^3*e + 272*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^3*a*c^3*d*f^4*g^3*e - 280*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*b^3*c*d*f^3*g^4*e - 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b
*c^2*d*f^3*g^4*e + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*d*f^2*g^5*e
 + 612*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c*d*f^2*g^5*e - 48*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^2*d*f^2*...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{{\left(f+g\,x\right)}^4\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^4*(d + e*x)), x)
```

3.862 $\int \frac{(f+gx)^3(a+bx+cx^2)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=1098

$$-\frac{\left(3(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3ce^4g^2(9bef-3bdg+8aeg)+8bc^2e\right.}{}$$

[Out] 1/192*(7*b^3*e^3*g^3+64*c^3*(-d*g+e*f)^3-4*b*c*e^2*g^2*(a*e*g-3*b*d*g+9*b*e*f)+24*b*c^2*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)+2*c*e*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*x)*(c*x^2+b*x+a)^(3/2)/c^3/e^4+1/60*g^2*(-7*b*e*g-22*c*d*g+36*c*e*f)*(c*x^2+b*x+a)^(5/2)/c^2/e^2+1/6*g^3*(e*x+d)*(c*x^2+b*x+a)^(5/2)/c/e^2+1/3072*(4*c*e*(-b*e+2*c*d)*(8*c*e*(-2*a*e+b*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-3*b*d*g+9*b*e*f))-d*(-4*a*c*e-3*b^2*e+8*b*c*d)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)))-2*(4*c^2*d^2-1/2*b^2*e^2-2*c*e*(-a*e+b*d))*(8*c*e*(-b*e+2*c*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-3*b*d*g+9*b*e*f))-2*(8*c^2*d^2-4*b*c*d*e-3/2*b^2*e^2+6*a*c*e^2)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)/e^7+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^7-1/1536*(21*b^5*e^5*g^3-1536*c^5*d^2*(-d*g+e*f)^3+384*c^4*e*(-4*a*e+5*b*d)*(-d*g+e*f)^3-12*b^3*c*e^4*g^2*(8*a*e*g-3*b*d*g+9*b*e*f)+24*b*c^2*e^3*g*(2*a^2*e^2*g^2+6*a*b*e*g*(-d*g+3*e*f)+3*b^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))-96*b*c^3*e^2*(2*b*(-d*g+e*f)^3+3*a*e*g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))+2*c*e*(8*c*e*(-b*e+2*c*d)*(24*c^2*e^2*f^3+7*b^2*d*e*g^3-4*c*d*g^2*(a*e*g-3*b*d*g+9*b*e*f))-2*(8*c^2*d^2-4*b*c*d*e-3/2*b^2*e^2+6*a*c*e^2)*g*(7*b^2*e^2*g^2-4*c*e*g*(a*e*g-3*b*d*g+9*b*e*f)+24*c^2*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)))*x)*(c*x^2+b*x+a)^(1/2)/c^4/e^6

Rubi [A]
time = 2.33, antiderivative size = 1098, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1667, 828, 857, 635, 212, 738}

Antiderivative was successfully verified.

[In] Int[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]

[Out] -1/1536*((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e*f - d*g)^3 + 128*c^4*e*(5*b*d - 4*a*e)*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b*c^2*e^3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3* d*e*f*g + d^2*g^2)) - 32*b*c^3*e^2*(2*b*(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))) + 2*c*e*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*x)*Sqrt[a + b*x + c*x^2])/(c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3*e^4) + (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^(5/2))/(60*c^2*e^2) + (g^3*(d + e*x)*(a + b*x + c*x^2)^(5/2))/(6*c*e^2) + ((4*c*e*(2*c*d - b*e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(3072*c^(9/2)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^7

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
```

```
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{g^3(d+ex)\left(a+bx+cx^2\right)^{5/2}}{6ce^2} + \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(\frac{1}{2}e\left(12ce^2f^3-d(5bd+2ae)g^3\right)-eg(e}{}}{}$$

$$= \frac{g^2(36cef-22cdg-7beg)\left(a+bx+cx^2\right)^{5/2}}{60c^2e^2} + \frac{g^3(d+ex)\left(a+bx+cx^2\right.}{6ce^2}$$

$$= \frac{(7b^3e^3g^3+64c^3(ef-dg)^3-4bce^2g^2(9bef-3bdg+aeg)+24bc^2eg(3e^2}{}$$

$$= -\frac{\left(3(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3\right.}{}$$

$$= -\frac{\left(3(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3\right.}{}$$

$$= -\frac{\left(3(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3\right.}{}$$

$$= -\frac{\left(3(7b^5e^5g^3-512c^5d^2(ef-dg)^3+128c^4e(5bd-4ae)(ef-dg)^3-4b^3\right.}{}$$

Mathematica [A]
time = 11.64, size = 743, normalized size = 0.68

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (5120*(e*f - d*g)^3*(a + x*(b + c*x))^(3/2) + (1920*e*g*(e*f - d*g)^2*(b +
2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3072*e^2*g^2*(e*f - d*g)*(a + x*(b + c
*x))^(5/2))/c + (2560*e^3*g^2*(f + g*x)*(a + x*(b + c*x))^(5/2))/c + (360*(
b^2 - 4*a*c)*e*g*(e*f - d*g)^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)
] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/
c^(5/2) - (60*e^2*g*(-2*c*f + b*g)*(e*f - d*g)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[
a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)
^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(7/2) + (e^3*
g*(1792*g*(2*c*f - b*g)*(a + x*(b + c*x))^(5/2) + 5*(24*c^2*f^2 + 7*b^2*g^2
 - 4*c*g*(6*b*f + a*g))*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b
^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2)))/c^2 + (9
```

```
60*(e*f - d*g)^3*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*
a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*Sqrt[c]*(
e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*e*(-5*b*d
+ 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) +
2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
c*x)])])))/(c^(3/2)*e^3))/(15360*e^4)
```

Maple [A]
time = 0.17, size = 1409, normalized size = 1.28

method	result	size
default	Expression too large to display	1409
risch	Expression too large to display	6884

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] g/e^3*(g^2*e^2*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/
2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(
2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/1
6*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(-d*e*g^2+3*e^2*f*g)*(1/5*
(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*
a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+d^2*g^2*(1/8*(2*c*x+b)/c*(c*x^2
+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(
4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-3*d*e*f*g*
(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c
*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))))+3*e^2*f^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2
)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+
e^3*f^3)/e^4*(1/3*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d/e)^2+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+
c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1
/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*
e^2-b*d*e+c*d^2)/e^2*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*
(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*
e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^
2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^3\,(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^3\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
[Out] int(((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.863 $\int \frac{(f+gx)^2 \left(a+bx+cx^2\right)^{3/2}}{d+ex} \, dx$

Optimal. Leaf size=662

$$\frac{(3b^4e^4g^2 + 128c^4d^2(ef - dg)^2 - 32c^3e(5bd - 4ae)(ef - dg)^2 - 6b^2ce^3g(2bef - bdg + 2aeg) + 8bc^2e^2(2b(ef - \dots}{\dots}$$

[Out] -1/48*(3*b^2*e^2*g^2-16*c^2*(-d*g+e*f)^2-6*b*c*e*g*(-d*g+2*e*f)-6*c*e*g*(-b *e*g-2*c*d*g+4*c*e*f)*x)*(c*x^2+b*x+a)^(3/2)/c^2/e^3+1/5*g^2*(c*x^2+b*x+a)^ (5/2)/c/e-1/256*(3*b^5*e^5*g^2+256*c^5*d^3*(-d*g+e*f)^2-384*c^4*d*e*(-a*e+b *d)*(-d*g+e*f)^2-6*b^3*c*e^4*g*(4*a*e*g-b*d*g+2*b*e*f)+16*b*c^2*e^3*(3*a^2* e^2*g^2+b^2*(-d*g+e*f)^2+3*a*b*e*g*(-d*g+2*e*f))+96*c^3*e^2*(b^2*d*(-d*g+e* f)^2-2*a*b*e*(-d*g+e*f)^2-a^2*e^2*g*(-d*g+2*e*f)))*arctanh(1/2*(2*c*x+b)/c^ (1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/e^6+(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f) ^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+ b*x+a)^(1/2))/e^6+1/128*(3*b^4*e^4*g^2+128*c^4*d^2*(-d*g+e*f)^2-32*c^3*e*(- 4*a*e+5*b*d)*(-d*g+e*f)^2-6*b^2*c*e^3*g*(2*a*e*g-b*d*g+2*b*e*f)+8*b*c^2*e^2 *(2*b*(-d*g+e*f)^2+3*a*e*g*(-d*g+2*e*f))+2*c*e*((16*c^2*d^2-3*b^2*e^2-4*c*e *(-3*a*e+2*b*d))*g*(-b*e*g-2*c*d*g+4*c*e*f)-8*c*e*(-b*e+2*c*d)*(-b*d*g^2+2* c*e*f^2))*x)*(c*x^2+b*x+a)^(1/2)/c^3/e^5

Rubi [A]

time = 0.98, antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1667, 828, 857, 635, 212, 738}

[illegible]

Antiderivative was successfully verified.

[In] Int[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]

[Out] ((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e *f - d*g)^2 + 3*a*e*g*(2*e*f - d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c *e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2* c*e*f^2 - b*d*g^2))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*e^5) - ((3*b^2*e^2*g ^2 - 16*c^2*(e*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2* c*d*g - b*e*g)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*e^3) + (g^2*(a + b*x + c *x^2)^(5/2))/(5*c*e) - ((3*b^5*e^5*g^2 + 256*c^5*d^3*(e*f - d*g)^2 - 384*c^ 4*d*e*(b*d - a*e)*(e*f - d*g)^2 - 6*b^3*c*e^4*g*(2*b*e*f - b*d*g + 4*a*e*g) + 16*b*c^2*e^3*(3*a^2*e^2*g^2 + b^2*(e*f - d*g)^2 + 3*a*b*e*g*(2*e*f - d*g)) + 96*c^3*e^2*(b^2*d*(e*f - d*g)^2 - 2*a*b*e*(e*f - d*g)^2 - a^2*e^2*g*(2 *e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*

```
c^(7/2)*e^6) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^2*ArcTanh[(b*d -
2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^
2])])/e^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{g^2\left(a+bx+cx^2\right)^{5/2}}{5ce} + \frac{\int \frac{\left(\frac{5}{2}e\left(2cef^2-bdg^2\right)+\frac{5}{2}eg(4cef-2cdg-beg)x\right)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx}{5ce^2}$$

$$= -\frac{(3b^2e^2g^2-16c^2(ef-dg)^2-6bceg(2ef-dg)-6ceg(4cef-2cdg-b}{48c^2e^3}$$

$$= \frac{(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(}{}$$

$$= \frac{(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(}{}$$

$$= \frac{(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(}{}$$

$$= \frac{(3b^4e^4g^2+128c^4d^2(ef-dg)^2-32c^3e(5bd-4ae)(ef-dg)^2-6b^2ce^3g(}{}$$

Mathematica [A]
time = 10.91, size = 536, normalized size = 0.81

$$\frac{1280(ef-dg)^2(a+x(b+cx))^{3/2}+\frac{480eg(ef-dg)(b+2cx)(a+x(b+cx))^{3/2}}{c}+\frac{768e^2g^2(a+x(b+cx))^{5/2}}{c}+\frac{90(b^2-4ac)eg(ef-dg)\left(-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}+(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{5/2}}+\frac{15e^2g(2cf-bg)\left(\frac{16(b+2cx)(a+x(b+cx))^{3/2}}{c}+\frac{3(b^2-4ac)\left(-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}+(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{5/2}}\right)}{c}+\frac{240(ef-dg)^2\left(-\left((2cd-be)(8c^2d^2-b^2e^2+4ce(-2bd+3ae))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)-2\sqrt{c}\left(e\sqrt{a+x(b+cx)}(-b^2e^2+4c^2d(-2d+ex)-2ce(-5bd+4ae+bex))+8c(cd^2+e(-bd+ae))^{3/2}\tanh^{-1}\left(\frac{-bd+2ae-2cdx+bex}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)\right)\right)}{c^{3/2}e^3}}{3840e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] (1280*(e*f - d*g)^2*(a + x*(b + c*x))^(3/2) + (480*e*g*(e*f - d*g)*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))/c + (768*e^2*g^2*(a + x*(b + c*x))^(5/2))/c +
```

```
(90*(b^2 - 4*a*c)*e*g*(e*f - d*g)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2) + (15*e^2*g*(2*c*f - b*g)*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/
2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2
)))/c + (240*(e*f - d*g)^2*(-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-
2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2
*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d + e*x) - 2*c*
e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[
(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a
 + x*(b + c*x)])])))/(c^(3/2)*e^3))/(3840*e^3)
```

Maple [A]
time = 0.17, size = 998, normalized size = 1.51

method	result
default	$-\left(g\left(-eg\left(\frac{\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+b\ldots}\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{2c}\right.\right.\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -g/e^2*(-e*g*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x
+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*
c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+d*g*(1/8*(2*c
*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+
a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)))-2*e*f*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c
*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
```

```
(c*x^2+b*x+a)^(1/2)))))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*(1/3*(c*(x+d/e)^2+(
b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*
(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^
(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(
x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2*((c*(x+d/e)
^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*l
n((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*
((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2\,(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**2*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)^2\,(c\,x^2 + b\,x + a)^{3/2}}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.864 $\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx$

Optimal. Leaf size=441

$$-\frac{(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef-2bdg+3aeg)+2ce(3b^2e^2g+16c^2}{64c^2e^4}$$

```
[Out] 1/24*(6*c*e*g*x+3*b*e*g-8*c*d*g+8*c*e*f)*(c*x^2+b*x+a)^(3/2)/c/e^2+1/128*(3
*b^4*e^4*g-128*c^4*d^3*(-d*g+e*f)+192*c^3*d*e*(-a*e+b*d)*(-d*g+e*f)-8*b^2*c
*e^3*(3*a*e*g-b*d*g+b*e*f)+48*c^2*e^2*(a^2*e^2*g-b^2*d*(-d*g+e*f)+2*a*b*e*(
-d*g+e*f)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^5+
(a*e^2-b*d*e+c*d^2)^(3/2)*(-d*g+e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5-1/64*(3*b^3*e^3*g-64*c^
3*d^2*(-d*g+e*f)+16*c^2*e*(-4*a*e+5*b*d)*(-d*g+e*f)-4*b*c*e^2*(3*a*e*g-2*b*
d*g+2*b*e*f)+2*c*e*(3*b^2*e^2*g+16*c^2*d*(-d*g+e*f)-4*c*e*(3*a*e*g-2*b*d*g+
2*b*e*f))*x)*(c*x^2+b*x+a)^(1/2)/c^2/e^4
```

Rubi [A]
time = 0.54, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {828, 857, 635, 212, 738}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2(-d)(ef-dg)\right)-8b^2ce^3(3aeg-bdg+bef)+192c^3de(bd-ae)(ef-dg)+3b^4e^4g-128c^4d^3(ef-dg)\right)}{128c^{5/2}e^5}-\frac{\sqrt{a+bx+cx^2}\left(2cex\left(-4ce(3aeg-2bdg+2bef)+3b^2e^2g+16c^2d(ef-dg)\right)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(3aeg-2bdg+2bef)+3b^3e^3g-64c^3d^2(ef-dg)\right)}{64c^2e^4}+\frac{(ef-dg)\left(ae^2-bde+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5}+\frac{\left(a+bx+cx^2\right)^{3/2}(3beg-8cdg+8cef+6cegx)}{24ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] -1/64*((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)*(e*
f - d*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g + 1
6*c^2*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x)*Sqrt[a + b*x
+ c*x^2])/(c^2*e^4) + ((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x +
 c*x^2)^(3/2))/(24*c*e^2) + ((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*c
^3*d*e*(b*d - a*e)*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48
*c^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*e^5) + ((c*d^2 -
b*d*e + a*e^2)^(3/2)*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/e^5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx = \frac{(8cef-8cdg+3beg+6cegx)\left(a+bx+cx^2\right)^{3/2}}{24ce^2} - \int \frac{\left(\frac{1}{2}\left(8ce(bd-2ae)f+4acdeg\right.\right.}{}$$

$$= -\frac{(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef}{}$$

$$= -\frac{(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef}{}$$

$$= -\frac{(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef}{}$$

$$= -\frac{(3b^3e^3g-64c^3d^2(ef-dg)+16c^2e(5bd-4ae)(ef-dg)-4bce^2(2bef}{}$$

Mathematica [A]
time = 2.70, size = 427, normalized size = 0.97

$$\frac{\frac{2e\sqrt{a+x(b+cx)}\left(-9b^3e^3g-16c^3\left(12d^3g-6d^2e(2f+gx)+2de^2x(3f+2gx)-e^3x^2(4f+3gx)\right)+6bce^2(10aeg+b(4ef-4dg+egx))+8c^2e(ae(32ef-32dg+15egx)+b(30d^2g-2de(15f+7gx)+e^2x(14f+9gx)))\right)}{c^2}-768\sqrt{-cd^2+bde-ae^2}\left(cd^2+e(-bd+ae)\right)(-ef+dg)\tan^{-1}\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)-\frac{3\left(3b^4e^4g+128c^4d^3(-ef+dg)-192c^3de(bd-ae)(-ef+dg)-8b^2ce^3(bef-bdg+3aeg)+48c^2e^2\left(a^2e^2g+2abe(ef-dg)+b^2d(-ef+dg)\right)\right)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{c^{5/2}}}{384e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]
```

```
[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(-9*b^3*e^3*g - 16*c^3*(12*d^3*g - 6*d^2*e*(2*f
 + g*x) + 2*d*e^2*x*(3*f + 2*g*x) - e^3*x^2*(4*f + 3*g*x)) + 6*b*c*e^2*(10*
a*e*g + b*(4*e*f - 4*d*g + e*g*x)) + 8*c^2*e*(a*e*(32*e*f - 32*d*g + 15*e*g
*x) + b*(30*d^2*g - 2*d*e*(15*f + 7*g*x) + e^2*x*(14*f + 9*g*x)))))/c^2 - 7
68*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*(-(e*f) + d*g)
*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*
d - a*e)]] - (3*(3*b^4*e^4*g + 128*c^4*d^3*(-(e*f) + d*g) - 192*c^3*d*e*(b*
d - a*e)*(-(e*f) + d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c^2*e^
2*(a^2*e^2*g + 2*a*b*e*(e*f - d*g) + b^2*d*(-(e*f) + d*g)))*Log[b + 2*c*x -
 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2))/(384*e^5)
```

Maple [A]
time = 0.14, size = 742, normalized size = 1.68

method	result

default	$\frac{g\left(\frac{(2cx+b)\left(cx^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{e}+\frac{(-dg+e}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] g/e*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/
c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))))+(-d*g+e*f)/e^2*(1/3*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)
/e)/c*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8
*(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d
)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(
x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
(2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x+d/e))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)*(a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x)
```

```
[Out] int(((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x), x)
```

3.865 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex} \, dx$

Optimal. Leaf size=252

$$\frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3}+\frac{(a+bx+cx^2)^{3/2}}{3e}-\frac{(2cd-be)(8c^2d^2-$$

```
[Out] 1/3*(c*x^2+b*x+a)^(3/2)/e-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*a*
e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^4+(a
*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e
+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e
+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e^3
```

Rubi [A]
time = 0.21, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {748, 828, 857, 635, 212, 738}

$$\frac{\sqrt{a+bx+cx^2}\,(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8ce^3}-\frac{(2cd-be)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4}+\frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4}+\frac{(a+bx+cx^2)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^(3/2)/(3*e) - ((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^4) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2]])])/e^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx}{2e}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+c}{3e}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+c}{3e}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+c}{3e}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce^3} + \frac{(a+bx+c}{3e}$$

Mathematica [A]
time = 0.10, size = 243, normalized size = 0.96

$$\frac{\frac{2e\sqrt{a+x(b+cx)}\,(3b^2e^2+2ce(-15bd+16ae+7bex)+4c^2(6d^2-3dex+2e^2x^2))}{c}+96\sqrt{-cd^2+bde-ae^2}\,(cd^2+e(-bd+ae))\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)+\frac{3(2cd-be)(8c^2d^2-b^2e^2+4ce(-2bd+3ae))\log\left(c\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}}}{48e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x),x]
```

```
[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x)
 + 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c + 96*Sqrt[-(c*d^2) + b*d*e - a*e
^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b
+ c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + (3*(2*c*d - b*e)*(8*c^2*d^2 - b^
2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])])/c^(3/2))/(48*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(226) = 452.
time = 0.13, size = 625, normalized size = 2.48 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/3*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+
1/2*(b*e-2*c*d)/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d/e)^2+(b*e-2*c*
d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+c*d^2)/e^
2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x
+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))+(a*e^2-b*d*e
```

```
+c*d^2)/e^2*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2
+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*e^2-b*d*e
+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*
c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e*b*d+%e^2*a>0)', see `assum
e?` for
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x), x)
```

3.866 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=491

$$\frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)}-\frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-dg)x)\sqrt{a+bx+cx^2}}{4eg^2(ef-dg)}-\frac{(2cd}{}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d
*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^(3/
2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/g^3/(-d*g+e*f)-1/2*(-b*e+2*c*d)*(a*e^2-b*d*e+c*d^2)*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(-d*g+e*f)/c^(1/2)+1/8*(8*c^2
*e*f^3+b*g^2*(-4*a*e*g+b*d*g+3*b*e*f)-4*c*g*(3*b*e*f^2-a*g*(-d*g+3*e*f)))*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e/g^3/(-d*g+e*f)/c^(1/2)+
(a*e^2-b*d*e+c*d^2)*(c*x^2+b*x+a)^(1/2)/e^2/(-d*g+e*f)-1/4*(4*c*e*f^2-g*(-4
*a*e*g-b*d*g+5*b*e*f)-2*c*g*(-d*g+e*f)*x)*(c*x^2+b*x+a)^(1/2)/e/g^2/(-d*g+e
*f)
```

Rubi [A]

time = 0.52, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {909, 748, 857, 635, 212, 738, 828}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4cg(3bef^2-ag(3ef-dg))+bg^2(-4aeg+bdg+3bef)+8c^2ef^3\right)}{8\sqrt{c}\,eg^3(ef-dg)}+\frac{\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}{e^2(ef-dg)}+\frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3(ef-dg)}-\frac{(2cd-be)(ae^2-bde+cd^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,e^3(ef-dg)}-\frac{\sqrt{a+bx+cx^2}\,(-g(-4aeg-bdg+5bef)-2cgx(ef-dg)+4cef^2)}{4eg^2(ef-dg)}-\frac{(ag^2-bfg+cf^2)^{3/2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2])/(e^2*(e*f - d*g)) - ((4*c*e
*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*Sqrt[a + b*x +
c*x^2])/(4*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*e^3*(e*f - d
*g)) + ((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e*g) - 4*c*g*(3*b*e*f^2
 - a*g*(3*e*f - d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(8*Sqrt[c]*e*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[
(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x
 + c*x^2])])/(e^3*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*
f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x +
c*x^2])])/(g^3*(e*f - d*g))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)\sqrt{a+bx+cx^2}}{f+gx}\,dx}{e(ef-dg)} + \frac{(cd^2-bde+ae^2)\int \frac{\sqrt{a+bx+cx^2}}{d+ex}}{e(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-a}{4eg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-a}{4eg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-a}{4eg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}{e^2(ef-dg)} - \frac{(4cef^2-g(5bef-bdg-4aeg)-2cg(ef-a}{4eg^2(ef-dg)}$$

Mathematica [A]
time = 10.75, size = 323, normalized size = 0.66

$$\frac{\frac{\left(3b^2e^2g^2-12ceg(bef+bdg-aeg)+8c^2(e^2f^2+defg+d^2g^2)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+\frac{2\left(eg(ef-dg)\sqrt{a+x(b+cx)}\,(5beg+c(-4ef-4dg+2egx))-4(cd^2+e(-bd+ae))^{3/2}g^3\tanh^{-1}\left(\frac{-bd+2ae-2cdx+bex}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)+4e^3(cf^2+g(-bf+ag))^{3/2}\tanh^{-1}\left(\frac{-bf+2ag-2cfx+bgx}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)\right)}{ef-dg}}{8e^3g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (((3*b^2*e^2*g^2 - 12*c*e*g*(b*e*f + b*d*g - a*e*g) + 8*c^2*(e^2*f^2 + d*e*
f*g + d^2*g^2))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqr
t[c] + (2*(e*g*(e*f - d*g)*Sqrt[a + x*(b + c*x)]*(5*b*e*g + c*(-4*e*f - 4*d
```

```
*g + 2*e*g*x)) - 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*g^3*ArcTanh[(-(b*d) + 2
*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c
*x)])] + 4*e^3*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g - 2
*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))
/(e*f - d*g))/(8*e^3*g^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(449) = 898$.

time = 0.19, size = 1265, normalized size = 2.58

method	result	size
default	Expression too large to display	1265
risch	Expression too large to display	2348

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*g-e*f)*(1/3*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d/e)^2+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e+
c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1
/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a*
e^2-b*d*e+c*d^2)/e^2*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c*
(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a*
e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^
2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))+1/(d*g-e
*f)*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+
1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*
f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^
2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f
/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g
+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1
/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c
+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g
+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(
b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}}{(d+ex)\,(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)*(d + e*x)), x)
```

3.867 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2} \, dx$

Optimal. Leaf size=787

$$\frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2}+\frac{3(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)}-$$

```
[Out] (c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(g*x+f)-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2
-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c
^(3/2)/e^2/(-d*g+e*f)^2+1/16*e*(-b*g+2*c*f)*(8*c^2*f^2-b^2*g^2-4*c*g*(-3*a*
g+2*b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/g^3/(-
d*g+e*f)^2+(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)
/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(-d*g+e*f)^2-e*(a*g^2-b
*f*g+c*f^2)^(3/2)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2
)^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)^2-3/8*(8*c^2*f^2+b^2*g^2-4*c*g*
(-a*g+2*b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+
e*f)/c^(1/2)+3/2*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2
-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2)^(1/2)/g^3/(-d*
g+e*f)+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c
*x^2+b*x+a)^(1/2)/c/e/(-d*g+e*f)^2+3/4*(-2*c*g*x-3*b*g+4*c*f)*(c*x^2+b*x+a)
^(1/2)/g^2/(-d*g+e*f)-1/8*e*(8*c^2*f^2+b^2*g^2-2*c*g*(-4*a*g+5*b*f)-2*c*g*(
-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/c/g^2/(-d*g+e*f)^2
```

Rubi [A]
time = 0.88, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {974, 748, 828, 857, 635, 212, 738, 746}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*e*(e*f - d*g)^2) + (3*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(
5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(8*c*g^2*(e*
f - d*g)^2) + (a + b*x + c*x^2)^(3/2)/((e*f - d*g)*(f + g*x)) - ((2*c*d - b
*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(2*c*f -
 b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) - (3*(8*c^2
*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
```

```
 + b*x + c*x^2])])/(8*Sqrt[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(
3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x + c*x^2])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*g)*Sqrt[c*f^2
 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b
*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) - (e*(c*f^2 - b*
f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 -
b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2}\,dx = \int \left(\frac{e^2(a+bx+cx^2)^{3/2}}{(ef-dg)^2(d+ex)} - \frac{g(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)^2} - \frac{eg(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)}\right) dx$$

$$= \frac{e^2\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}dx}{(ef-dg)^2} - \frac{(eg)\int \frac{(a+bx+cx^2)^{3/2}}{f+gx}dx}{(ef-dg)^2} - \frac{g\int \frac{(a+bx+cx^2)^{3/2}}{(f+gx)^2}dx}{ef-dg}$$

$$= \frac{(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)} - \frac{e\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}dx}{2(ef-dg)^2} + \frac{e\int \frac{(bf-2ag+(2cf-bg}{}}{2(ef}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg}{}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg}{}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg}{}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8ce(ef-dg)^2} + \frac{3(4cf-3bg}{}$$

Mathematica [A]
time = 10.98, size = 357, normalized size = 0.45

$$\frac{-\sqrt{c}(ef-dg)^2(4cef+2cdg-3beg)(f+gx)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) - 2(cd^2+e(-bd+ae))^{3/2}g^3(f+gx)\tanh^{-1}\left(\frac{-bd+2ae-2cdx+bex}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right) + e\left(2g(-ef+dg)\sqrt{a+x(b+cx)}(eg(bf-ag)+cdg(f+gx)-cef(2f+gx)) - e\sqrt{cf^2+g(-bf+ag)}(2cf(2ef-3dg)+g(-bef+3bdg-2aeg))(f+gx)\tanh^{-1}\left(\frac{-bf+2ag-2cfx+bgx}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)\right)}{2e^2g^3(ef-dg)^2(f+gx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^2),x]
```

```
[Out] (-(Sqrt[c]*(e*f - d*g)^2*(4*c*e*f + 2*c*d*g - 3*b*e*g)*(f + g*x)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*(c*d^2 + e*(-(b*d) + a*e)
)^(3/2)*g^3*(f + g*x)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])] + e*(2*g*(-(e*f) + d*g)*Sqr
t[a + x*(b + c*x)]*(e*g*(b*f - a*g) + c*d*g*(f + g*x) - c*e*f*(2*f + g*x))
- e*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*(2*c*f*(2*e*f - 3*d*g) + g*(-(b*e*f) + 3
*b*d*g - 2*a*e*g))*(f + g*x)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*
Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]))/(2*e^2*g^3*(e*f -
d*g)^2*(f + g*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2371 vs. $2(717) = 1434$.
time = 0.17, size = 2372, normalized size = 3.01

method	result	size
default	Expression too large to display	2372
risch	Expression too large to display	5154

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e/(d*g-e*f)^2*(1/3*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d/e)^2
+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*d*e
+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(
1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))+(a
*e^2-b*d*e+c*d^2)/e^2*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(c
*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)-(a
*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)
^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))-e/(d*g-
e*f)^2*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/
2)+1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)
/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((
x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*
f*g+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*
f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^
2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g
-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))+1/g/(d*g-e*f)*
(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(5/2)+3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/3*((x+f
/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f
)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^
2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g+c*f^2)/g^2*((
(x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2
*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g
*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(
x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))+4*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/
8*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b
```

```
*f*g+c*f^2)/g^2)^(3/2)+3/16*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)
/c*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a
*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2
/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((g*x + f)^2*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**2,x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{{\left(f+g\,x\right)}^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^2*(d + e*x)), x)
```

3.868 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx$

Optimal. Leaf size=1066

$$\frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3}+\frac{3e(4cf-3bg-2cgx)\sqrt{a+bx+cx^2}}{4g^2(ef-dg)^2}-$$

```
[Out] 1/2*(c*x^2+b*x+a)^(3/2)/(-d*g+e*f)/(g*x+f)^2+e*(c*x^2+b*x+a)^(3/2)/(-d*g+e*
f)^2/(g*x+f)-1/16*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*a*e+2*b*d))*arc
tanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e/(-d*g+e*f)^3+1/16
*e^2*(-b*g+2*c*f)*(8*c^2*f^2-b^2*g^2-4*c*g*(-3*a*g+2*b*f))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/g^3/(-d*g+e*f)^3+(a*e^2-b*d*e+c*
d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^2+b*x+a)^(1/2))/e/(-d*g+e*f)^3-e^2*(a*g^2-b*f*g+c*f^2)^(3/2)*arctanh(
1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2
))/g^3/(-d*g+e*f)^3-3/8*e*(8*c^2*f^2+b^2*g^2-4*c*g*(-a*g+2*b*f))*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)^2/c^(1/2)+3/2*(-b*g
+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/g^3/(-d*
g+e*f)-3/8*(8*c^2*f^2+b^2*g^2-4*c*g*(-a*g+2*b*f))*arctanh(1/2*(b*f-2*a*g+(-
b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/g^3/(-d*g+e*f)
/(a*g^2-b*f*g+c*f^2)^(1/2)+3/2*e*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+
2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))*(a*g^2-b*f*g+c*f^2
)^(1/2)/g^3/(-d*g+e*f)^2+1/8*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)-2*c*e*
(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/(-d*g+e*f)^3+3/4*e*(-2*c*g*x-3*b*g+4*
c*f)*(c*x^2+b*x+a)^(1/2)/g^2/(-d*g+e*f)^2-3/4*(2*c*g*x-b*g+4*c*f)*(c*x^2+b*
x+a)^(1/2)/g^2/(-d*g+e*f)/(g*x+f)-1/8*e^2*(8*c^2*f^2+b^2*g^2-2*c*g*(-4*a*g+
5*b*f)-2*c*g*(-b*g+2*c*f)*x)*(c*x^2+b*x+a)^(1/2)/c/g^2/(-d*g+e*f)^3
```

Rubi [A]
time = 1.09, antiderivative size = 1066, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {974, 748, 828, 857, 635, 212, 738, 746, 826}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*Sqrt
[a + b*x + c*x^2])/(8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*Sqr
t[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*Sqrt
[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^2*g^
2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*Sqrt[a + b*x + c*x^2])/(
```

```
8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/(2*(e*f - d*g)*(f + g*x)^2
) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d - b*e)*
(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])]])/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*Sqrt[c]*(2*c*f
 - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]])/(2*g^3*(e*f
 - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]])/(16*c^(3/2)*g^3*(e
*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]])/(8*Sqrt[c]*g^3*(e*f - d*g)^2)
+ ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])]])/(e*(e*f - d*g)^3) + (
3*e*(2*c*f - b*g)*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]])/(2*g^3*(
e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]])/(g^
3*(e*f - d*g)^3) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(
b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x
+ c*x^2])]])/(8*g^3*(e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
```

```
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3}\,dx = \int \left(\frac{e^3(a+bx+cx^2)^{3/2}}{(ef-dg)^3(d+ex)} - \frac{g(a+bx+cx^2)^{3/2}}{(ef-dg)(f+gx)^3} - \frac{eg(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)^2} - \frac{e^2g(a\cdots}{(ef-\cdots}\right.$$

$$= \frac{e^3\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx}{(ef-dg)^3} - \frac{(e^2g)\int \frac{(a+bx+cx^2)^{3/2}}{f+gx}\,dx}{(ef-dg)^3} - \frac{(eg)\int \frac{(a+bx+cx^2)^{3/2}}{(f+gx)^2}\,dx}{(ef-dg)^2} - \frac{g\int \frac{(a\cdots}{\cdots}}{\cdots}$$

$$= \frac{(a+bx+cx^2)^{3/2}}{2(ef-dg)(f+gx)^2} + \frac{e(a+bx+cx^2)^{3/2}}{(ef-dg)^2(f+gx)} - \frac{e^2\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+c\cdots}}{d+ex}}{2(ef-dg)^3}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3\cdots}{\cdots}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3\cdots}{\cdots}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3\cdots}{\cdots}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+cx^2}}{8c(ef-dg)^3} + \frac{3e(4cf-3\cdots}{\cdots}$$

Mathematica [A]
time = 12.18, size = 1036, normalized size = 0.97

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]
```

```
[Out] ((2*(a + x*(b + c*x))^(3/2))/((e*f - d*g)*(f + g*x)^2) + (4*e*(a + x*(b + c
*x))^(3/2))/((e*f - d*g)^2*(f + g*x)) + (-((2*c*d - b*e)*(8*c^2*d^2 - b^2*e
^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
 c*x)])]) - 2*Sqrt[c]*(e*Sqrt[a + x*(b + c*x)]*(-(b^2*e^2) + 4*c^2*d*(-2*d
+ e*x) - 2*c*e*(-5*b*d + 4*a*e + b*e*x)) + 8*c*(c*d^2 + e*(-(b*d) + a*e))^(
3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + x*(b + c*x)])]))/(4*c^(3/2)*e*(e*f - d*g)^3) - (3*e*((8*c^2
*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])] + 2*Sqrt[c]*(g*(-4*c*f + 3*b*g + 2*c*g*x)*Sqrt[a + x*(b
+ c*x)] + 2*(2*c*f - b*g)*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*ArcTanh[(-(b*f) +
2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b +
c*x)])])))/(2*Sqrt[c]*g^3*(e*f - d*g)^2) + (3*(((-2*c*f + b*g)*(a + x*(b +
c*x))^(3/2))/(f + g*x) - (Sqrt[a + x*(b + c*x)]*(b^2*g^2 + 2*c^2*f*(2*f - g
*x) + c*g*(-5*b*f + 2*a*g + b*g*x)))/g^2 + (4*Sqrt[c]*(2*c*f - b*g)*(c*f^2
+ g*(-(b*f) + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
+ (8*c^2*f^2 + b^2*g^2 + 4*c*g*(-2*b*f + a*g))*Sqrt[c*f^2 + g*(-(b*f) + a*g
)]*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a
*g)]*Sqrt[a + x*(b + c*x)])])/(2*g^3)))/((e*f - d*g)*(c*f^2 + g*(-(b*f) + a
*g))) - (e^2*((2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 + 4*c*g*(-2*b*f + 3*a*g))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(g*Sqrt[
a + x*(b + c*x)]*(-(b^2*g^2) + 4*c^2*f*(-2*f + g*x) - 2*c*g*(-5*b*f + 4*a*g
 + b*g*x)) + 8*c*(c*f^2 + g*(-(b*f) + a*g))^(3/2)*ArcTanh[(-(b*f) + 2*a*g -
 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])]
)))/(4*c^(3/2)*g^3*(-(e*f) + d*g)^3))/4
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4241 vs. $2(968) = 1936$.
time = 0.14, size = 4242, normalized size = 3.98

method	result	size
default	Expression too large to display	4242

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/(d*g-e*f)^3*(1/3*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d/e
)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-b*
d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/
c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)))
+(a*e^2-b*d*e+c*d^2)/e^2*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)
+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)
-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e
+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d
```

/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))))+e^2/
(d*g-e*f)^3*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2
)^(3/2)+1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(
b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c
*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/
2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g
^2-b*f*g+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)
/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+
f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g
^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^
2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c
+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))+1/g^2/(d*
g-e*f)*(-1/2/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(
x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(5/2)+1/4*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)
*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*
g^2-b*f*g+c*f^2)/g^2)^(5/2)+3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/3*((x+
f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*
f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)
^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g+c*f^2)/g^2*(
((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-
2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/
g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/(
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*
(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g
)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))+4*c/(a*g^2-b*f*g+c*f^2)*g^2*(1
/8*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-
b*f*g+c*f^2)/g^2)^(3/2)+3/16*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2
)/c*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^
2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2
*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))))+3/2*c/(a*g^2-b*f*g+c*f^2
)*g^2*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2
)+1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/
g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x
+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f
*g+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^
(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f
*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2
+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))))-1/g*e/(d*g-e*f
)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+

```
(a*g^2-b*f*g+c*f^2)/g^2)^(5/2)+3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/3*(
(x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2
*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c
*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b
*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g+c*f^2)/g^
2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b
*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*
f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^
2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)
/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+
f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))))...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/((g*x + f)^3*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**3,x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f+g\,x)^3\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/((f + g*x)^3*(d + e*x)), x)
```

3.869 $\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx$

Optimal. Leaf size=886

$$\frac{(cd^2-bde+ae^2)\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x\right)\sqrt{a+bx+cx^2}}{8ce^4(ef-dg)}-\frac{(64c^3ef^4-16c^2ef^2g(9$$

```
[Out] 1/3*(a*e^2-b*d*e+c*d^2)*(c*x^2+b*x+a)^(3/2)/e^2/(-d*g+e*f)-1/24*(8*c*e*f^2-
g*(-8*a*e*g-3*b*d*g+11*b*e*f)-6*c*g*(-d*g+e*f)*x)*(c*x^2+b*x+a)^(3/2)/e/g^2
/(-d*g+e*f)-1/16*(-b*e+2*c*d)*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2-b^2*e^2-4*c*e*
(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/
e^5/(-d*g+e*f)+1/128*(128*c^4*e*f^5-320*c^3*e*f^3*g*(-a*g+b*f)-b^3*g^4*(-8*
a*e*g+3*b*d*g+5*b*e*f)+48*c^2*g^2*(5*b^2*e*f^3-10*a*b*e*f^2*g+a^2*g^2*(-d*g
+5*e*f))-8*b*c*g^3*(5*b^2*e*f^2+12*a^2*e*g^2-3*a*b*g*(d*g+5*e*f)))*arctanh(
1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e/g^5/(-d*g+e*f)+(a*e^2-
b*d*e+c*d^2)^(5/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^
2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^5/(-d*g+e*f)-(a*g^2-b*f*g+c*f^2)^(5/2)*arct
anh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^
(1/2))/g^5/(-d*g+e*f)+1/8*(a*e^2-b*d*e+c*d^2)*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*
a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x)*(c*x^2+b*x+a)^(1/2)/c/e^4/(-d*g+e*f)-1/64*
(64*c^3*e*f^4-16*c^2*e*f^2*g*(-8*a*g+9*b*f)-b^2*g^3*(-8*a*e*g+3*b*d*g+5*b*e
*f)+4*c*g^2*(22*b^2*e*f^2+16*a^2*e*g^2-3*a*b*g*(-d*g+13*e*f))-2*c*g*(16*c^2
*e*f^3+b*g^2*(-8*a*e*g+3*b*d*g+5*b*e*f)-4*c*g*(6*b*e*f^2-a*g*(-3*d*g+7*e*f)
))*x)*(c*x^2+b*x+a)^(1/2)/c/e/g^4/(-d*g+e*f)
```

Rubi [A]
time = 1.09, antiderivative size = 886, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {909, 748, 828, 857, 635, 212, 738}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c
*e*(2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2])/(8*c*e^4*(e*f - d*g)) - ((64*c^3
*e*f^4 - 16*c^2*e*f^2*g*(9*b*f - 8*a*g) - b^2*g^3*(5*b*e*f + 3*b*d*g - 8*a*
e*g) + 4*c*g^2*(22*b^2*e*f^2 + 16*a^2*e*g^2 - 3*a*b*g*(13*e*f - d*g)) - 2*c
*g*(16*c^2*e*f^3 + b*g^2*(5*b*e*f + 3*b*d*g - 8*a*e*g) - 4*c*g*(6*b*e*f^2 -
 a*g*(7*e*f - 3*d*g)))*x)*Sqrt[a + b*x + c*x^2])/(64*c*e*g^4*(e*f - d*g)) +
 ((c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2))/(3*e^2*(e*f - d*g)) - ((
8*c*e*f^2 - g*(11*b*e*f - 3*b*d*g - 8*a*e*g) - 6*c*g*(e*f - d*g)*x)*(a + b*
```

```
x + c*x^2)^(3/2))/(24*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e +
a*e^2)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*e^5*(e*f - d*g)) + ((128*c^4*
e*f^5 - 320*c^3*e*f^3*g*(b*f - a*g) - b^3*g^4*(5*b*e*f + 3*b*d*g - 8*a*e*g)
 + 48*c^2*g^2*(5*b^2*e*f^3 - 10*a*b*e*f^2*g + a^2*g^2*(5*e*f - d*g)) - 8*b*
c*g^3*(5*b^2*e*f^2 + 12*a^2*e*g^2 - 3*a*b*g*(5*e*f + d*g)))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(3/2)*e*g^5*(e*f - d*g)) +
((c*d^2 - b*d*e + a*e^2)^(5/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^5*(e*f - d*g)) - ((c
*f^2 - b*f*g + a*g^2)^(5/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt
[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^5*(e*f - d*g))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)}\,dx = -\frac{\int \frac{(cdf-bef+aeg-c(ef-dg)x)(a+bx+cx^2)^{3/2}}{f+gx}\,dx}{e(ef-dg)} + \frac{(cd^2-bde+ae^2)\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx}{e(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)(a+bx+cx^2)^{3/2}}{3e^2(ef-dg)} - \frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(e}{24eg^2(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+c}}{8ce^4(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+c}}{8ce^4(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+c}}{8ce^4(ef-dg)}$$

$$= \frac{(cd^2-bde+ae^2)(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{a+bx+c}}{8ce^4(ef-dg)}$$

Mathematica [A]
time = 11.71, size = 647, normalized size = 0.73

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)/((d + e*x)*(f + g*x)),x]
```

```
[Out] (3*(5*b^4*e^4*g^4*(-(e*f) + d*g) - 40*b^2*c*e^3*g^3*(e*f - d*g)*(b*e*f + b*
d*g - 3*a*e*g) + 320*c^3*e*g*(-(b*e^4*f^4) + a*e^4*f^3*g + b*d^4*g^4 - a*d^
3*e*g^4) + 128*c^4*(e^5*f^5 - d^5*g^5) + 240*c^2*e^2*g^2*(e*f - d*g)*(a^2*e
^2*g^2 - 2*a*b*e*g*(e*f + d*g) + b^2*(e^2*f^2 + d*e*f*g + d^2*g^2)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(-(e*g*(-(e*f)
 + d*g)*Sqrt[a + x*(b + c*x)]*(15*b^3*e^3*g^3 + 2*b*c*e^2*g^2*(278*a*e*g +
b*(-132*e*f - 132*d*g + 59*e*g*x)) - 16*c^3*(12*d^3*g^3 - 6*d^2*e*g^2*(-2*f
 + g*x) + 2*d*e^2*g*(6*f^2 - 3*f*g*x + 2*g^2*x^2) + e^3*(12*f^3 - 6*f^2*g*x
 + 4*f*g^2*x^2 - 3*g^3*x^3)) + 8*c^2*e*g*(a*e*g*(-56*e*f - 56*d*g + 27*e*g*
x) + b*(54*d^2*g^2 + 2*d*e*g*(27*f - 13*g*x) + e^2*(54*f^2 - 26*f*g*x + 17*
g^2*x^2))))) - 192*c*(c*d^2 + e*(-(b*d) + a*e))^(5/2)*g^5*ArcTanh[(-(b*d) +
 2*a*e - 2*c*d*x + b*e*x)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b +
 c*x)])] + 192*c*e^5*(c*f^2 + g*(-(b*f) + a*g))^(5/2)*ArcTanh[(-(b*f) + 2*a
*g - 2*c*f*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x
)])]))/(384*c^(3/2)*e^5*g^5*(e*f - d*g))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2106 vs. $2(834) = 1668$.
time = 0.22, size = 2107, normalized size = 2.38

method	result	size
default	Expression too large to display	2107
risch	Expression too large to display	4651

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*g-e*f)*(1/5*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(5/2)+1/2*(b*e-2*c*d)/e*(1/8*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d/e)^2+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(3/2)+3/16*(4*c*(a*e^2-b*d*e
+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/c*(c*(x+d
/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*c*(a*e^2-
b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e)
)/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)))+(a*e^2-b*d*e+c*d^2)/e^2*(1/3*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(3/2)+1/2*(b*e-2*c*d)/e*(1/4*(2*c*(x+d/e)+(b*e-2*c*d)/e)/
c*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/8*(4*
c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/c^(3/2)*ln((1/2*(b*e-2*c*d)/e+
c*(x+d/e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)))+(a*e^2-b*d*e+c*d^2)/e^2*((c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/2*(b*e-2*c*d)/e*ln((1/2*(b*e-2*c*d)/e+c*(x+d/
e))/c^(1/2)+(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/c^(1/2)-(a*e^2-b*d*e+c*d^2)/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d
/e)))))+1/(d*g-e*f)*(1/5*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*
f^2)/g^2)^(5/2)+1/2*(b*g-2*c*f)/g*(1/8*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+3/16*(4*c*(a*g^
2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c*(1/4*(2*c*(x+f/g)+(b*g-2*c*f)/g)/c*
((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/8*(4*c*
(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*c*f)/g+c*
(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2))))+(a*g^2-b*f*g+c*f^2)/g^2*(1/3*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+
(a*g^2-b*f*g+c*f^2)/g^2)^(3/2)+1/2*(b*g-2*c*f)/g*(1/4*(2*c*(x+f/g)+(b*g-2*c
*f)/g)/c*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+
1/8*(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/c^(3/2)*ln((1/2*(b*g-2*
c*f)/g+c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)))+(a*g^2-b*f*g+c*f^2)/g^2*(((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f
/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)/g*ln((1/2*(b*g-2*c*f)/g+
c*(x+f/g))/c^(1/2)+((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g
```

```
^2)^(1/2))/c^(1/2)-(a*g^2-b*f*g+c*f^2)/g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2
))/(x+f/g)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d*g-%e*f>0)', see `assume?` for mor
e detai
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)/(e*x+d)/(g*x+f),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{5/2}}{\left(f+g\,x\right)\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)),x)
```

```
[Out] int((a + b*x + c*x^2)^(5/2)/((f + g*x)*(d + e*x)), x)
```

3.870 $\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=431

$$\frac{g^2(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2(36e^2f^2-36defg+11d^2g^2))\sqrt{a+bx+cx^2}}{24c^3e^3}+\frac{g^3(24cef-$$

```
[Out] -1/16*g*(5*b^3*e^3*g^3-6*b*c*e^2*g^2*(2*a*e*g-b*d*g+4*b*e*f)-16*c^3*(-d^3*g
^3+4*d^2*e*f*g^2-6*d*e^2*f^2*g+4*e^3*f^3)+8*c^2*e*g*(a*e*g*(-d*g+4*e*f)+b*(
d^2*g^2-4*d*e*f*g+6*e^2*f^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^
(1/2))/c^(7/2)/e^4+(-d*g+e*f)^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e
^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4/(a*e^2-b*d*e+c*d^2)^(1/2)+1/
24*g^2*(15*b^2*e^2*g^2-4*c*e*g*(4*a*e*g-7*b*d*g+18*b*e*f)+4*c^2*(11*d^2*g^2
-36*d*e*f*g+36*e^2*f^2))*(c*x^2+b*x+a)^(1/2)/c^3/e^3+1/12*g^3*(-5*b*e*g-14*
c*d*g+24*c*e*f)*(e*x+d)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/3*g^4*(e*x+d)^2*(c*x^
2+b*x+a)^(1/2)/c/e^3
```

Rubi [A]

time = 0.86, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1667, 857, 635, 212, 738}

$$-\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(8c^2eg(aeg(4ef-dg)+b(d^2g^2-4defg+6e^2f^2))-6bce^2g^2(2aeg-bdg+4bef)+5b^3e^3g^3-16c^3(-d^3g^3+4d^2efg^2-6de^2f^2g+4e^3f^3))}{16c^{7/2}e^4}+\frac{g^2\sqrt{a+bx+cx^2}(-4ceg(4aeg-7bdg+18bef)+15b^2e^2g^2+4c^2(11d^2g^2-36defg+36e^2f^2))}{24c^3e^3}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}(-5beg-14cdg+24cef)}{12c^2e^3}+\frac{(ef-dg)^4\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4\sqrt{ae^2-bde+cd^2}}+\frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*(15*b^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e
^2*f^2 - 36*d*e*f*g + 11*d^2*g^2))*Sqrt[a + b*x + c*x^2])/(24*c^3*e^3) + (g
^3*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(d + e*x)*Sqrt[a + b*x + c*x^2])/(12*c^2
*e^3) + (g^4*(d + e*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*e^3) - (g*(5*b^3*e^3*g
^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^
2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*
e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(16*c^(7/2)*e^4) + ((e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c
*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^4*S
qrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3} + \frac{\int \frac{\frac{1}{2}e(6ce^3f^4-d^2(bd+4ae)g^4)-\frac{1}{2}eg(de(7bd+8ae)g^3-c(24}{}}{}$$

$$= \frac{g^3(24cef-14cdg-5beg)(d+ex)\sqrt{a+bx+cx^2}}{12c^2e^3} + \frac{g^4(d+ex)^2\sqrt{a+bx}}{3ce^3}$$

$$= \frac{g^2(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2(36e^2f^2-36defg+11d^2}{24c^3e^3}$$

$$= \frac{g^2(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2(36e^2f^2-36defg+11d^2}{24c^3e^3}$$

$$= \frac{g^2(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2(36e^2f^2-36defg+11d^2}{24c^3e^3}$$

$$= \frac{g^2(15b^2e^2g^2-4ceg(18bef-7bdg+4aeg)+4c^2(36e^2f^2-36defg+11d^2}{24c^3e^3}$$

Mathematica [A]
time = 2.17, size = 380, normalized size = 0.88

$$\frac{\frac{2eg^2\sqrt{a+x(b+cx)}(15b^2e^2g^2-2ceg(8aeg+b(36ef-9dg+5egx))+4c^2(6d^2g^2-3deg(8f+gx)+2e^2(18f^2+6fgx+g^2x^2)))}{c^3}+\frac{96\sqrt{-cd^2+bde-ae^2}(ef-dg)^4\tan^{-1}\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{3g(5b^3e^3g^3-6bce^2g^2(4bef-bdg+2aeg)-16c^3(4e^3f^3-6de^2f^2g+4d^2efg^2-d^3g^3)+8c^2eg(aeg(4ef-dg)+b(6e^2f^2-4defg+d^2g^2)))\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{c^{7/2}}}{48e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*e*g^2*Sqrt[a + x*(b + c*x)]*(15*b^2*e^2*g^2 - 2*c*e*g*(8*a*e*g + b*(36*
e*f - 9*d*g + 5*e*g*x)) + 4*c^2*(6*d^2*g^2 - 3*d*e*g*(8*f + g*x) + 2*e^2*(1
8*f^2 + 6*f*g*x + g^2*x^2))))/c^3 + (96*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f
 - d*g)^4*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2
) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + (3*g*(5*b^3*e^3*g^3 - 6*b
*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^2*f^2*g
+ 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*e^2*f^2
- 4*d*e*f*g + d^2*g^2)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/
c^(7/2))/(48*e^4)
```

Maple [A]
time = 0.17, size = 755, normalized size = 1.75

method	result

default	$-\frac{g\left(-e^3 g^3\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\cdots\right)}{4c}\cdots\right)}{6c}\cdots\right)\cdots\right)}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -g/e^4*(-e^3*g^3*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x
+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))))+(d*e^2*g^3-4*e^3*f*g^2)*(1/2*x/c*(c*x^2+b*x+a)
^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2)))+(-d^2*e*g^3+4*d*e^2*f*g^2-6*e^3*f^2*g)*(1/c*(c*x^2+b*x+a)^(1/2)-1/2
*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+d^3*g^3*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-4*d^2*e*f*g^2*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+6*d*e^2*f^2*g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))/c^(1/2)-4*e^3*f^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)/c^(1/2))-(d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2-4*d*e^3*f^3*g+e^4*f^4)/
e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*
d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(
x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((%e^-1*b-2*%e^-2*c*d)^2>0)', see `a
ssume?`
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**4/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^4}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.871 $\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=270

$$\frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}+\frac{g(3b^2e^2g^2-4ceg(3bef-bdg+aeg)+$$

```
[Out] 1/8*g*(3*b^2*e^2*g^2-4*c*e*g*(a*e*g-b*d*g+3*b*e*f)+8*c^2*(d^2*g^2-3*d*e*f*g
+3*e^2*f^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^3
+(-d*g+e*f)^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1
/2)/(c*x^2+b*x+a)^(1/2))/e^3/(a*e^2-b*d*e+c*d^2)^(1/2)+3/4*g^2*(-b*e*g-2*c*
d*g+4*c*e*f)*(c*x^2+b*x+a)^(1/2)/c^2/e^2+1/2*g^3*(e*x+d)*(c*x^2+b*x+a)^(1/2
)/c/e^2
```

Rubi [A]
time = 0.45, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1667, 857, 635, 212, 738}

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2(d^2g^2-3defg+3e^2f^2)\right)}{8c^{5/2}e^3}+\frac{3g^2\sqrt{a+bx+cx^2}(-beg-2cdg+4cef)}{4c^2e^2}+\frac{(ef-dg)^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3\sqrt{ae^2-bde+cd^2}}+\frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (3*g^2*(4*c*e*f - 2*c*d*g - b*e*g)*Sqrt[a + b*x + c*x^2])/(4*c^2*e^2) + (g^
3*(d + e*x)*Sqrt[a + b*x + c*x^2])/(2*c*e^2) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*
(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*e^3) + ((e*f -
 d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(e^3*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \frac{\int \frac{\frac{1}{2}e\left(4ce^2f^3-d(bd+2ae)g^3\right)-eg\left(e(2bd+ae)g^2-c\left(6e^2f^2-d\right.\right.}{(d+ex)\sqrt{a+bx+cx^2}}}{2ce^3}$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + \int$$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + ($$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} - ($$

$$= \frac{3g^2(4cef-2cdg-beg)\sqrt{a+bx+cx^2}}{4c^2e^2} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2} + g$$

Mathematica [A]

time = 1.25, size = 247, normalized size = 0.91

$$-\frac{-\frac{2eg^2\sqrt{a+x(b+cx)}(-3beg+2c(6ef-2dg+egx))}{c^2}+\frac{16\sqrt{-cd^2+bde-ae^2}(-ef+dg)^3\tan^{-1}\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{g(3b^2e^2g^2-4ceg(3bef-bdg+aeg)+8c^2(3e^2f^2-3defg+d^2g^2))\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{c^{5/2}}}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/8*((-2*e*g^2*Sqrt[a + x*(b + c*x)]*(-3*b*e*g + 2*c*(6*e*f - 2*d*g + e*g*
x)))/c^2 + (16*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)^3*ArcTan[(Sqrt
[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(
c*d^2 + e*(-(b*d) + a*e)) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*(3*b*e*f - b*d*g +
a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*Log[b + 2*c*x - 2*Sqrt[c]
*Sqrt[a + x*(b + c*x)]])/c^(5/2))/e^3
```

Maple [A]

time = 0.13, size = 480, normalized size = 1.78

method	result
default	$g\left(g^2e^2\left(\frac{x\sqrt{cx^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{cx^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx-\right.}{2c^{\frac{3}{2}}}\right.\right.$
risch	$-\frac{g^2(-2cegx+3beg+4dgc-12cef)\sqrt{cx^2+bx+a}}{4c^2e^2}-\frac{g^3\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)a}{2c^{\frac{3}{2}}e}+\frac{3g^3\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+\right.}{8c^{\frac{5}{2}}e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g/e^3*(g^2*e^2*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2
)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-d*e*g^2+3*e^2*f*g)*(1/c*(c*x
^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+
d^2*g^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-3*d*e*f*g*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+3*e^2*f^2*ln((1/2*b+c*x)/c^(
```

```
1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))-(-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^
3*f^3)/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c
*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((%e^-1*b-2*%e^-2*c*d)^2>0)', see `a
ssume?`
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**3/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^3}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.872 $\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=176

$$\frac{g^2\sqrt{a+bx+cx^2}}{ce}+\frac{g(4cef-2cdg-beg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{\cdots}{2\sqrt{cd^2-b\cdots}}\right)}{e^2\sqrt{cd^2-bd\cdots}}$$

```
[Out] 1/2*g*(-b*e*g-2*c*d*g+4*c*e*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^
(1/2))/c^(3/2)/e^2+(-d*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e
^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)+g^
2*(c*x^2+b*x+a)^(1/2)/c/e
```

Rubi [A]
time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1667, 857, 635, 212, 738}

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-beg-2cdg+4cef)}{2c^{3/2}e^2}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(c*e) + (g*(4*c*e*f - 2*c*d*g - b*e*g)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*e^2) + ((e*f - d
*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x + c*x^2])])/(e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g^2\sqrt{a+bx+cx^2}}{ce} + \frac{\int \frac{\frac{1}{2}e\left(2cef^2-bdg^2\right)+\frac{1}{2}eg(4cef-2cdg-beg)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ce^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{ce} + \frac{(ef-dg)^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{(g(4cef-2c}{}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{ce} - \frac{\left(2(ef-dg)^2\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-}{\sqrt{a+b}}\right.}{e^2}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{ce} + \frac{g(4cef-2cdg-beg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2}$$

Mathematica [A]
time = 0.72, size = 183, normalized size = 1.04

$$\frac{\frac{2eg^2\sqrt{a+x(b+cx)}}{c} + \frac{4\sqrt{-cd^2+bde-ae^2}\,(ef-dg)^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{g(-4cef+2cdg+beg)\log\left(c\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*e*g^2*Sqrt[a + x*(b + c*x)])/c + (4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f
 - d*g)^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2
) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + (g*(-4*c*e*f + 2*c*d*g +
b*e*g)*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2))/(2*e^
2)
```

Maple [A]
time = 0.13, size = 301, normalized size = 1.71

method	result
default	$-\frac{g\left(-eg\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+\frac{dg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}-\frac{2ef\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\right.}{}}{e^2}$
risch	$\frac{g^2\sqrt{c\,x^2+bx+a}}{ce}-\frac{g^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)b}{2c^{\frac{3}{2}}e}-\frac{g^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)d}{\sqrt{c}\,e^2}+\frac{2g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -g/e^2*(-e*g*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2)))+d*g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/
2)-2*e*f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))-(d^2*g^2-2*d*
e*f*g+e^2*f^2)/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2
)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume((%e^-1*b-2*%e^-2*c*d)^2>0)', see `a
ssume?`
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**2/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)^2}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.873 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=131

$$\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e}+\frac{(ef-dg)\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\,\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}$$

```
[Out] g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/e/c^(1/2)+(-d*g+e*f)*a
rctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {857, 635, 212, 738}

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\,\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) + ((
e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e +
a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{(2g)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{e} - \frac{(2(ef-dg))\text{Subst}\left(\int \frac{}{4cd^2-4b}\right.}{}$$

$$= \frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,e} + \frac{(ef-dg)\tanh^{-1}\left(\frac{bd-2ae+(2}{2\sqrt{cd^2-bde+ae^2}}\right.}{e\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A]
time = 0.58, size = 142, normalized size = 1.08

$$-\frac{\frac{2\sqrt{-cd^2+bde-ae^2}(-ef+dg)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{g\log\left(e\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*(d + e*
x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(
-(b*d) + a*e)) + (g*Log[e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/S
qrt[c])/e)
```

Maple [A]
time = 0.12, size = 199, normalized size = 1.52

method	result

default	$\frac{g \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{e\sqrt{c}}-\frac{(-dg+ef)\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{c\left(x+\frac{d}{e}\right)^{2}}}{x+\frac{d}{e}}\right.}{e^{2}\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(-d*g+e*f)/e^2/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+
d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((%e^-1*b-2*%e^-2*c*d)^2>0)', see `a
ssume?`
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(117) = 234.
time = 196.39, size = 1073, normalized size = 8.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*d^2*g - b*d*g*e + a*g*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
- 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - sqrt(c*d^2 - b*d*e
 + a*e^2)*(c*d*g - c*f*e)*log(-(8*c^2*d^2*x^2 + 8*b*c*d^2*x + (b^2 + 4*a*c)
*d^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c
*x^2 + b*x + a) + (8*a*b*x + (b^2 + 4*a*c)*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^
2 + 4*a*b*d + (3*b^2 + 4*a*c)*d*x)*e)/(x^2*e^2 + 2*d*x*e + d^2)))/(c^2*d^2*
e - b*c*d*e^2 + a*c*e^3), -1/2*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*(c*d*g - c*f
```

```
*e)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e
)*sqrt(c*x^2 + b*x + a)/(c^2*d^2*x^2 + b*c*d^2*x + a*c*d^2 + (a*c*x^2 + a*b
*x + a^2)*e^2 - (b*c*d*x^2 + b^2*d*x + a*b*d)*e)) - (c*d^2*g - b*d*g*e + a*
g*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(2*(c*d^
2*g - b*d*g*e + a*g*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x +
 b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*(c*d*g
- c*f*e)*log(-(8*c^2*d^2*x^2 + 8*b*c*d^2*x + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*d
^2 - b*d*e + a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2 + b*x + a) +
 (8*a*b*x + (b^2 + 4*a*c)*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^2 + 4*a*b*d + (3*
b^2 + 4*a*c)*d*x)*e)/(x^2*e^2 + 2*d*x*e + d^2)))/(c^2*d^2*e - b*c*d*e^2 + a
*c*e^3), -(sqrt(-c*d^2 + b*d*e - a*e^2)*(c*d*g - c*f*e)*arctan(-1/2*sqrt(-c
*d^2 + b*d*e - a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2 + b*x + a)
/(c^2*d^2*x^2 + b*c*d^2*x + a*c*d^2 + (a*c*x^2 + a*b*x + a^2)*e^2 - (b*c*d*
x^2 + b^2*d*x + a*b*d)*e)) + (c*d^2*g - b*d*g*e + a*g*e^2)*sqrt(-c)*arctan(
1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)))/(c
^2*d^2*e - b*c*d*e^2 + a*c*e^3)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f+g\,x}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.874 $\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}$$

```
[Out] arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x
+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {738, 212}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x + c*x^2])]/Sqrt[c*d^2 - b*d*e + a*e^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 92, normalized size = 1.16

$$\frac{2\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*
(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(71) = 142.
time = 0.12, size = 157, normalized size = 1.99

method	result	size
default	$-\frac{\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}$	157

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*
(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e*b*d+%e^2*a>0)', see `assum
e?` for
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(73) = 146$.
time = 3.32, size = 340, normalized size = 4.30

$$\left[\frac{\log\left(-\frac{8\,c^2d^2x^2+8\,bcd^2x+(b^2+4\,ac)d^2+4\,\sqrt{cd^2-bde+ae^2}\,(2\,cdx+bd-(bx+2\,a)e)\sqrt{cx^2+bx+a}+(8\,abx+(b^2+4\,ac)x^2+8\,a^2)e^2-2\,(4\,bcdx^2+4\,abd+(3\,b^2+4\,ac)dx)e}{x^2e^2+2\,dxe+d^2}\right)}{2\,\sqrt{cd^2-bde+ae^2}}, \frac{\sqrt{-cd^2+bde-ae^2}\arctan\left(-\frac{\sqrt{-cd^2+bde-ae^2}\,(2\,cdx+bd-(bx+2\,a)e)\sqrt{cx^2+bx+a}}{2\,(c^2d^2x^2+bcd^2x+acd^2+(acx^2+abx+a^2)e^2-(bcdx^2+b^2dx+abd)e)}\right)}{cd^2-bde+ae^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*c^2*d^2*x^2 + 8*b*c*d^2*x + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*d^2 -
 b*d*e + a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2 + b*x + a) + (8*
a*b*x + (b^2 + 4*a*c)*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^2 + 4*a*b*d + (3*b^2
+ 4*a*c)*d*x)*e)/(x^2*e^2 + 2*d*x*e + d^2))/sqrt(c*d^2 - b*d*e + a*e^2), sq
rt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*(2*c*d*
x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2 + b*x + a)/(c^2*d^2*x^2 + b*c*d^2*x + a
*c*d^2 + (a*c*x^2 + a*b*x + a^2)*e^2 - (b*c*d*x^2 + b^2*d*x + a*b*d)*e))/(c
*d^2 - b*d*e + a*e^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [**A**]
time = 3.21, size = 72, normalized size = 0.91

$$\frac{2\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 +
 b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.875 $\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}\,(ef-dg)}-\frac{g\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$

```
[Out] e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/(-d*g+e*f)/(a*e^2-b*d*e+c*d^2)^(1/2)-g*arctanh(1/2*(b*f-2*a*g+
(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(
a*g^2-b*f*g+c*f^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {974, 738, 212}

$$\frac{e\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}}-\frac{g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)) - (g*ArcT
anh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a +
 b*x + c*x^2])])/((e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e}{(ef-dg)(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)\sqrt{a+\right.$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{ef-dg}$$

$$= -\frac{(2e)\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x}{\sqrt{a+bx+cx^2}}\right)}{ef-dg} + \frac{(2g)\mathrm{Su}}{}$$

$$= \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}\,(ef-dg)} - \frac{g\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{(ef-}$$

Mathematica [A]
time = 0.77, size = 207, normalized size = 1.14

$$-\frac{2e\sqrt{-cd^2+e(bd-ae)}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{(cd^2+e(-bd+ae))\,(-ef+dg)} - \frac{2g\sqrt{-cf^2+bfg-ag^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+g(bf-ag)}}\right)}{(ef-dg)\,(cf^2+g(-bf+ag))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*e*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a +
 x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/((c*d^2 + e*(-(b*d) + a*e))
*(-(e*f) + d*g)) - (2*g*Sqrt[-(c*f^2) + b*f*g - a*g^2]*ArcTan[(Sqrt[c]*(f +
 g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^2) + g*(b*f - a*g)]])/((e*f - d
*g)*(c*f^2 + g*(-(b*f) + a*g)))
```

Maple [A]
time = 0.13, size = 327, normalized size = 1.80

method	result

default	$\frac{\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{(dg-ef)\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}-\frac{\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*
c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))-1/(d*g-e*f)/((a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g
)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(g*x + f)*(x*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(166) = 332$.
time = 70.44, size = 1952, normalized size = 10.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((8*a*b*f*g
 - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 + 4*a*c)*g
^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*g
+ (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*g)*x)/(g^
2*x^2 + 2*f*g*x + f^2)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^2 - b*d*e
+ a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*
c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 +
 b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
```

```
^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c*d*e^2 + a
*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*c*d^3 + a^
2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*g^3),
-1/2*(2*(c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*g*arctan(-1/2*
sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a*g + (2*c*f -
b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a*c*g^2)*x^2 +
(b*c*f^2 - b^2*f*g + a*b*g^2)*x)) + (c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(c*d^
2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*
d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sq
rt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*
e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^2*e - b*c
*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f^2*g + (b*
c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e + a^2*d*e
^2)*g^3), -1/2*((c*d^2 - b*d*e + a*e^2)*sqrt(c*f^2 - b*f*g + a*g^2)*g*log((
8*a*b*f*g - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 +
 4*a*c)*g^2)*x^2 - 4*sqrt(c*f^2 - b*f*g + a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f
 - 2*a*g + (2*c*f - b*g)*x) - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*
g)*x)/(g^2*x^2 + 2*f*g*x + f^2)) - 2*(c*e*f^2 - b*e*f*g + a*e*g^2)*sqrt(-c*
d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 +
b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^
2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)))/((c^2*d
^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 - a*c)*d*e^2)*f
^2*g + (b*c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^3 - a*b*d^2*e
 + a^2*d*e^2)*g^3), -((c*d^2 - b*d*e + a*e^2)*sqrt(-c*f^2 + b*f*g - a*g^2)*
g*arctan(-1/2*sqrt(-c*f^2 + b*f*g - a*g^2)*sqrt(c*x^2 + b*x + a)*(b*f - 2*a
*g + (2*c*f - b*g)*x)/(a*c*f^2 - a*b*f*g + a^2*g^2 + (c^2*f^2 - b*c*f*g + a
*c*g^2)*x^2 + (b*c*f^2 - b^2*f*g + a*b*g^2)*x)) - (c*e*f^2 - b*e*f*g + a*e*
g^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*
sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e +
a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)
*x)))/((c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*f^3 - (c^2*d^3 + a*b*e^3 - (b^2 -
a*c)*d*e^2)*f^2*g + (b*c*d^3 + a^2*e^3 - (b^2 - a*c)*d^2*e)*f*g^2 - (a*c*d^
3 - a*b*d^2*e + a^2*d*e^2)*g^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\,(f+gx)\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(f+g\,x)\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.876 $\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=340

$$\frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)}+\frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}\,(ef-dg)^2}-\frac{g(2cf-bg)\tanh^{-1}}{2(e}$$

```
[Out] -1/2*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(3/2)+e^2*arc
tanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)
^(1/2))/(-d*g+e*f)^2/(a*e^2-b*d*e+c*d^2)^(1/2)-e*g*arctanh(1/2*(b*f-2*a*g+(
-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/
(a*g^2-b*f*g+c*f^2)^(1/2)+g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c
*f^2)/(g*x+f)
```

Rubi [A]
time = 0.27, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {974, 738, 212, 744}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}+\frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}-\frac{g(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x))
 + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2) -
(g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*
f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2
)^(3/2)) - (e*g*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f
*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*Sqrt[c*f^2 - b*f*g + a*
g^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^2\sqrt{\ldots}}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} \ldots$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} - \frac{\left(2e^2\right)\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bde\ldots}\right.}{(\ldots}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{b\ldots}{2\sqrt{cd^2-bd\ldots}}\right.}{\sqrt{cd^2-bde+\ldots}}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{e^2\tanh^{-1}\left(\frac{b\ldots}{2\sqrt{cd^2-bd\ldots}}\right.}{\sqrt{cd^2-bde+\ldots}}$$

Mathematica [A]
time = 10.69, size = 256, normalized size = 0.75

$$-\frac{\frac{2g^2(-ef+dg)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)} - \frac{2e^2\tanh^{-1}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cd^2+e(-bd+ae)}} + \frac{g(2cf(2ef-dg)+g(-3bef+bdg+2aeg))\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(cf^2+g(-bf+ag))^{3/2}}}{2(ef-dg)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((2*g^2*(-(e*f) + d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*
g))*(f + g*x)) - (2*e^2*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*
d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a
*e)] + (g*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*ArcTanh[(-
2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x
*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^(3/2))/(e*f - d*g)^2
```

Maple [A]
time = 0.13, size = 609, normalized size = 1.79

method	result
default	$-\frac{e\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{(dg-ef)^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}+ -$ e l

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e
-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))+e/(d*g-e*f)^2/((
a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(
x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)
+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+1/g/(d*g-e*f)*(-1/(a*g^2-b*f*g+c*
f^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/
2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^
2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(
1/2))/(x+f/g)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f+g\,x)^2\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.877 $\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=587

$$\frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2}+\frac{3g^2(2cf-bg)\sqrt{a+bx+cx^2}}{4(ef-dg)\left(cf^2-bfg+ag^2\right)^2(f+gx)}+\frac{eg^2\sqrt{a+bx+c}}{(ef-dg)^2\left(cf^2-bfg+\right.}$$

```
[Out] -1/2*e*g*(-b*g+2*c*f)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c
*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)-1/8
*g*(8*c^2*f^2+3*b^2*g^2-4*c*g*(a*g+2*b*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c
*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f
*g+c*f^2)^(5/2)+e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d
^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)^3/(a*e^2-b*d*e+c*d^2)^(1/2)-e^2*g
*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)^(1/2)+1/2*g^2*(c*x^2+b*x+a)^(1
/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^2+3/4*g^2*(-b*g+2*c*f)*(c*x^2+b*
x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)+e*g^2*(c*x^2+b*x+a)^(1/
2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)
```

Rubi [A]
time = 0.56, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {974, 738, 212, 758, 820, 744}

$$-\frac{g\left(-4cg(ag+2bf)+3b^2g^2+8c^2f^2\right)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)\left(ag^2-bfg+cf^2\right)^{5/2}}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}}-\frac{e^2g\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^3\sqrt{ag^2-bfg+cf^2}}+\frac{eg^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2\left(ag^2-bfg+cf^2\right)}+\frac{3g^2\sqrt{a+bx+cx^2}(2cf-bg)}{4(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}+\frac{g^2\sqrt{a+bx+cx^2}}{2(f+gx)^2(ef-dg)\left(ag^2-bfg+cf^2\right)}-\frac{eg(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)^2\left(ag^2-bfg+cf^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g^2*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x
)^2) + (3*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(4*(e*f - d*g)*(c*f^2 -
b*f*g + a*g^2)^2*(f + g*x)) + (e*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)*(f + g*x)) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b
*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c*d^2
- b*d*e + a*e^2]*(e*f - d*g)^3) - (e*g*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g +
 (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(
2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (e^2*g*ArcTanh[(b*f - 2*a*
g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]
)/((e*f - d*g)^3*Sqrt[c*f^2 - b*f*g + a*g^2]) - (g*(8*c^2*f^2 + 3*b^2*g^2 -
 4*c*g*(2*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2
 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^3\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)\sqrt{a+bx+cx^2}} - \frac{g}{(ef-dg)(f+gx)^3\sqrt{a\cdots}}\right.$$

$$= \frac{e^3\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} - \frac{(e^2g)\int \frac{1}{(f+gx)\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^3} \cdots$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{eg^2\sqrt{a+bx\cdots}}{(ef-dg)^2\left(cf^2-bfg\cdots\right.}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a\cdots}}{4(ef-dg)\left(cf^2-bfg\cdots\right.}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a\cdots}}{4(ef-dg)\left(cf^2-bfg\cdots\right.}$$

$$= \frac{g^2\sqrt{a+bx+cx^2}}{2(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^2} + \frac{3g^2(2cf-bg)\sqrt{a\cdots}}{4(ef-dg)\left(cf^2-bfg\cdots\right.}$$

Mathematica [A]
time = 11.51, size = 549, normalized size = 0.94

$$\frac{\frac{4g^2(ef-dg)^2\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)^2}+\frac{8eg^2(ef-dg)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)}+\frac{8e^3\tanh^{-1}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cd^2+e(-bd+ae)}}+\frac{4eg(-2cf+bg)(ef-dg)\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(cf^2+g(-bf+ag))^{3/2}}-\frac{8e^2g\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cf^2+g(-bf+ag)}}+g(ef-dg)^2\left(\frac{6g(2cf-bg)\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))^2(f+gx)}-\frac{(8c^2f^2+3b^2g^2-4cg(2bf+ag))\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(cf^2+g(-bf+ag))^{5/2}}\right)}{8(ef-dg)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*g^2*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f
 + g*x)^2) + (8*e*g^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f
) + a*g))*(f + g*x)) + (8*e^3*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*S
qrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*
d) + a*e)] + (4*e*g*(-2*c*f + b*g)*(e*f - d*g)*ArcTanh[(-2*a*g + 2*c*f*x +
b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*
f^2 + g*(-(b*f) + a*g))^(3/2) - (8*e^2*g*ArcTanh[(-2*a*g + 2*c*f*x + b*(f -
 g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*f^
```

```
2 + g*(-(b*f) + a*g)] + g*(e*f - d*g)^2*((6*g*(2*c*f - b*g)*Sqrt[a + x*(b +
 c*x)])/((c*f^2 + g*(-(b*f) + a*g))^2*(f + g*x)) - ((8*c^2*f^2 + 3*b^2*g^2
- 4*c*g*(2*b*f + a*g))*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f
^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2 + g*(-(b*f) + a*g))^
(5/2)))/(8*(e*f - d*g)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. $2(541) = 1082$.

time = 0.13, size = 1185, normalized size = 2.02

method	result
default	$\frac{e^2 \ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{(dg-ef)^3\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}} - \frac{e^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/(d*g-e*f)^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*
e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))-e^2/(d*g-e*f)^3
/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/
g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f
/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))+1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b
*f*g+c*f^2)*g^2/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)-3/4*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f
^2)*g^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)
^(1/2)+1/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2
)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2
)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1
/2))/(x+f/g)))+1/2*c/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2
)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2
)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1
/2))/(x+f/g)))-1/g*e/(d*g-e*f)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)*((x+f/
g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(b*g-2*c*f)
*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c
```

```
*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(g*x + f)^3*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\,(f+gx)^3\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**3*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2256 vs. $2(550) = 1100$.
time = 4.86, size = 2256, normalized size = 3.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d^2*f^2*g^3 - 8*b*c*d^2*f*g^4 + 3*b^2*d^2*g^5 - 4*a*c*d^2*g^5 -
24*c^2*d*f^3*g^2*e + 28*b*c*d*f^2*g^3*e - 10*b^2*d*f*g^4*e + 4*a*b*d*g^5*e
+ 24*c^2*f^4*g*e^2 - 36*b*c*f^3*g^2*e^2 + 15*b^2*f^2*g^3*e^2 + 20*a*c*f^2*g
```

^3*e^2 - 20*a*b*f*g^4*e^2 + 8*a^2*g^5*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*g + sqrt(c)*f)/sqrt(-c*f^2 + b*f*g - a*g^2))/((c^2*d^3*f^4*g^3 - 2*b*c*d^3*f^3*g^4 + b^2*d^3*f^2*g^5 + 2*a*c*d^3*f^2*g^5 - 2*a*b*d^3*f*g^6 + a^2*d^3*g^7 - 3*c^2*d^2*f^5*g^2*e + 6*b*c*d^2*f^4*g^3*e - 3*b^2*d^2*f^3*g^4*e - 6*a*c*d^2*f^3*g^4*e + 6*a*b*d^2*f^2*g^5*e - 3*a^2*d^2*f*g^6*e + 3*c^2*d*f^6*g*e^2 - 6*b*c*d*f^5*g^2*e^2 + 3*b^2*d*f^4*g^3*e^2 + 6*a*c*d*f^4*g^3*e^2 - 6*a*b*d*f^3*g^4*e^2 + 3*a^2*d*f^2*g^5*e^2 - c^2*f^7*e^3 + 2*b*c*f^6*g*e^3 - b^2*f^5*g^2*e^3 - 2*a*c*f^5*g^2*e^3 + 2*a*b*f^4*g^3*e^3 - a^2*f^3*g^4*e^3)*sqrt(-c*f^2 + b*f*g - a*g^2)) + 2*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^3/((d^3*g^3 - 3*d^2*f*g^2*e + 3*d*f^2*g*e^2 - f^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*f^2*g^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*f*g^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*g^5 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f^3*g^2*e + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f^2*g^3*e - 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^4*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*g^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*f^3*g^2 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*f^2*g^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*f*g^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*f*g^4 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f^4*g*e + 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f^3*g^2*e - 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f^2*g^3*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f^2*g^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*g^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*f^3*g^2 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*f^2*g^3 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*f^2*g^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*f*g^4 + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*f*g^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*g^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f^4*g*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f^3*g^2*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f^3*g^2*e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f^2*g^3*e - 72*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f^2*g^3*e + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^4*e + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*g^5*e + 6*b^2*c^(3/2)*d*f^3*g^2 - 3*b^3*sqrt(c)*d*f^2*g^3 - 20*a*b*c^(3/2)*d*f^2*g^3 + 11*a*b^2*sqrt(c)*d*f*g^4 + 12*a^2*c^(3/2)*d*f*g^4 - 8*a^2*b*sqrt(c)*d*g^5 - 10*b^2*c^(3/2)*f^4*g*e + 7*b^3*sqrt(c)*f^3*g^2*e + 32*a*b*c^(3/2)*f^3*g^2*e - 27*a*b^2*sqrt(c)*f^2*g^3*e - 20*a^2*c^(3/2)*f^2*g^3*e + 28*a^2*b*sqrt(c)*f*g^4*e - 8*a^3*sqrt(c)*g^5*e)/((c^2*d^2*f^4*g^2 - 2*b*c*d^2*f^3*g^3 + b^2*d^2*f^2*g^4 + 2*a*c*d^2*f^2*g^4 - 2*a*b*d^2*f*g^5 + a^2*d^2*g^6 - 2*c^2*d*f^5*g*e + 4*b*c*d*f^4*g^2*e - 2*b^2*d*f^3*g^3*e - 4*a*c*d*f^3*g^3*e + 4*a*b*d*f^2*g^4*e - 2*a^2*d*f*g^5*e + c^2*f^6*e^2 - 2*b*c*f^5*g*e^2 + b^2*f^4*g^2*e^2 + 2*a*c

```
*f^4*g^2*e^2 - 2*a*b*f^3*g^3*e^2 + a^2*f^2*g^4*e^2)*((sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*g + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*f + b*f -
 a*g)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f+g\,x)^3\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.878 $\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}} dx$

Optimal. Leaf size=496

$$-\frac{2(ab^3dg^4 - b^2(c^2ef^4 + 4acdfg^3 + a^2eg^4) + 2ac(a^2eg^4 + c^2f^3(ef - 4dg) - 2acfg^2(3ef - 2dg)) + bc(c^2df^4}{}$$

```
[Out] 1/2*g^3*(-3*b*e*g-2*c*d*g+8*c*e*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/c^(5/2)/e^2+(-d*g+e*f)^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(3/2
)-2*(a*b^3*d*g^4-b^2*(a^2*e*g^4+4*a*c*d*f*g^3+c^2*e*f^4)+2*a*c*(a^2*e*g^4+c
^2*f^3*(-4*d*g+e*f)-2*a*c*f*g^2*(-2*d*g+3*e*f))+b*c*(c^2*d*f^4+a^2*g^3*(-3*
d*g+4*e*f)+2*a*c*f^2*g*(3*d*g+2*e*f))+(2*c^4*d*f^4+b^3*(-a*e+b*d)*g^4-b*c*g
^3*(4*b^2*d*f-3*a^2*e*g-4*a*b*(-d*g+e*f))+2*c^2*g^2*(3*b^2*d*f^2-3*a*b*f*(-
2*d*g+e*f)-a^2*g*(-d*g+4*e*f))+c^3*f^2*(4*a*g*(-3*d*g+2*e*f)-b*f*(4*d*g+e*f
)))*x)/c^2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)+g^4*(c*x^2+
b*x+a)^(1/2)/c^2/e
```

Rubi [A]

time = 0.72, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1660, 1667, 857, 635, 212, 738}

$$-\frac{2\left(-b^2(a^2eg^4+4acdfg^3+c^2ef^4)+x\left(2c^2g^2(a^2(-g)(4ef-dg)-3abf(ef-2dg)+3b^2df^2)-bcg^3(-3a^2eg-4ab(ef-dg)+4b^2df)+b^3g^4(bd-ae)+c^3f^2(4ag(2ef-3dg)-bf(4dg+ef))+2c^4df^4\right)+bc(a^2g^3(4ef-3dg)+2acf^2g(3dg+2ef)+c^2df^4)+2ac(a^2eg^4-2acfg^2(3ef-2dg)+c^2f^3(ef-4dg))+ab^3dg^4\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{g^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-3beg-2cdg+8cef)}{2c^{5/2}e^2}+\frac{g^4\sqrt{a+bx+cx^2}}{c^2e}+\frac{(ef-dg)^4\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^2
*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*d*
f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*f^4
 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f - d*g)
) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*g)) +
 c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x))/(c^2*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^4*Sqrt[a + b*x + c*x^
2])/(c^2*e) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*e^2) + ((e*f - d*g)^4*ArcTanh[(b*d
 - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c
*x^2])])/(e^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
```

```
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(f+gx)^4}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg$$

$$= -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg$$

$$= -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg$$

$$= -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg$$

$$= -\frac{2(ab^3dg^4-b^2(c^2ef^4+4acdfg^3+a^2eg^4)+2ac(a^2eg^4+c^2f^3(ef-4dg$$

Mathematica [A]
time = 11.49, size = 587, normalized size = 1.18

$$\frac{-\frac{2e\left(-3b^4deg^4x+b^3g^3(3aeg(-d+ex)+cdx(8ef+dg-egx))+b^2\left(3a^2e^2g^4+c^2\left(2e^2f^4-12def^2g^2x+d^2g^4x^2\right)+acg^3\left(d^2g+e^2x(-8f+gx)+4de(2f+3gx)\right)\right)-2bc\left(a^2eg^3(4ef-5dg+5egx)+c^2ef^3(-efx+d(f-4gx))+2acg\left(d^2g^3x+e^2f^2(2f-3gx)+deg\left(3f^2+6fgx-g^2x^2\right)\right)\right)-4c\left(2a^3e^2g^4+c^3def^4x+ac^2\left(d^2g^4x^2-2def^2g(2f+3gx)+e^2f^3(f+4gx)\right)+a^2cg^2\left(d^2g^2+deg(4f+gx)+e^2\left(-6f^2-4fgx+g^2x^2\right)\right)\right)\right)}{c^2(b^2-4ac)(-cd^2+e(bd-ae))\sqrt{a+x(b+cx)}}+\frac{2(ef-dg)^4\log(d+ex)}{(cd^2+e(-bd+ae))^{3/2}}+\frac{g^3(8cef-2cdg-3beg)\log\left(b+2cx+2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{c^{5/2}}-\frac{2(ef-dg)^4\log\left(-bd+2ae-2cdx+bex+2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}\right)}{(cd^2+e(-bd+ae))^{3/2}}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*e*(-3*b^4*d*e*g^4*x + b^3*g^3*(3*a*e*g*(-d + e*x) + c*d*x*(8*e*f + d*g
 - e*g*x)) + b^2*(3*a^2*e^2*g^4 + c^2*(2*e^2*f^4 - 12*d*e*f^2*g^2*x + d^2*g
^4*x^2) + a*c*g^3*(d^2*g + e^2*x*(-8*f + g*x) + 4*d*e*(2*f + 3*g*x))) - 2*b
*c*(a^2*e*g^3*(4*e*f - 5*d*g + 5*e*g*x) + c^2*e*f^3*(-(e*f*x) + d*(f - 4*g*
x)) + 2*a*c*g*(d^2*g^3*x + e^2*f^2*(2*f - 3*g*x) + d*e*g*(3*f^2 + 6*f*g*x -
 g^2*x^2))) - 4*c*(2*a^3*e^2*g^4 + c^3*d*e*f^4*x + a*c^2*(d^2*g^4*x^2 - 2*d
*e*f^2*g*(2*f + 3*g*x) + e^2*f^3*(f + 4*g*x)) + a^2*c*g^2*(d^2*g^2 + d*e*g*
(4*f + g*x) + e^2*(-6*f^2 - 4*f*g*x + g^2*x^2)))))/(c^2*(b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2*(e*f - d*g)^4*Log[d + e*x
])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*Lo
```

```
g[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]])/c^(5/2) - (2*(e*f - d*g)^4*
Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqr
t[a + x*(b + c*x)]])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/(2*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(474) = 948$.

time = 0.16, size = 1026, normalized size = 2.07

method	result
default	$-\frac{g\left(-e^{3} g^{3}\left(\frac{x^{2}}{c\sqrt{c\,x^{2}+b x+a}}-\frac{3 b\left(-\frac{x}{c\sqrt{c\,x^{2}+b x+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+b x+a}}-\frac{b(2 c x+b)}{c\left(4 a c-b^{2}\right)\sqrt{c\,x^{2}+b x+a}}\right)}{2 c}\cdots\right)}{2 c}\cdots\right)\cdots\right)}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -g/e^4*(-e^3*g^3*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/
2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c
/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))+(d*e^2
*g^3-4*e^3*f*g^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/
2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-d^2*e*g^3+4*d*e^2*f*g^2-6*e^3*f^2*g)*(-1/c/
(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+2*d^3*g^
3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-8*d^2*e*f*g^2*(2*c*x+b)/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)+12*d*e^2*f^2*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)-8*e^3*f^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+(d^4*g^4-4*d^3*
e*f*g^3+6*d^2*e^2*f^2*g^2-4*d*e^3*f^3*g+e^4*f^4)/e^5*(1/(a*e^2-b*d*e+c*d^2)
*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e
-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e
+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2))/(x+d/e)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((%e^-1*b-2*%e^-2*c*d)^2>0)', see `a
ssume?`
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^4}{(d+ex)\,(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)**4/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^4}{\left(d+e\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.879 $\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=357

$$\frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2eg^3+3acfg(ef+dg))-(2c^3df^3-}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}$$

```
[Out] g^3*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e+(-d*g+e*f)
^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(3/2)+2*(b^2*(a*d*g^3+c*e*f^3)-2*a*c*(c
*f^2*(-3*d*g+e*f)-a*g^2*(-d*g+3*e*f))-b*(c^2*d*f^3+a^2*e*g^3+3*a*c*f*g*(d*g
+e*f))-(2*c^3*d*f^3-b^2*(-a*e+b*d)*g^3+c*g^2*(-2*a^2*e*g+3*a*b*d*g-3*a*b*e*
f+3*b^2*d*f)+c^2*f*(6*a*g*(-d*g+e*f)-b*f*(3*d*g+e*f)))*x)/c/(-4*a*c+b^2)/(a
*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1660, 857, 635, 212, 738}

$$\frac{2(-x(cg^2(-2a^2eg+3abdg-3abef+3b^2df)-b^2g^3(bd-ae)+c^2f(6ag(ef-dg)-bf(3dg+ef))+2c^3df^3)-b(a^2eg^3+3acfg(dg+ef)+c^2df^3)+b^2(adg^3+cef^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg)))}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{g^3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}e}+\frac{(ef-dg)^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f - d
*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 - b
^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 3*a*b*e*f + 3*a*b*d*g - 2*a^2*e*g)
+ c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x))/(c*(b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^3*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*e) + ((e*f - d*g)^3*ArcTanh[(b*d - 2
*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2
])])/(e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2e\ldots}{\ldots}$$

$$= \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2e\ldots}{\ldots}$$

$$= \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2e\ldots}{\ldots}$$

$$= \frac{2(b^2(cef^3+adg^3)-2ac(cf^2(ef-3dg)-ag^2(3ef-dg))-b(c^2df^3+a^2e\ldots}{\ldots}$$

Mathematica [A]
time = 4.03, size = 354, normalized size = 0.99

$$\frac{2(-b^3dg^3x+b^2(ag^3(-d+ex)+c(-ef^3+3dfg^2x))+b(a^2eg^3+c^2f^2(-efx+d(f-3gx))+3acg(ef(f-gx)+dg(f+gx)))+2c(c^2df^3x+a^2g^2(dg-e(3f+gx))+acf(-3dg(f+gx)+ef(f+3gx))))}{c(-b^2+4ac)(cd^2+e(-bd+ae))\sqrt{a+x(b+cx)}}+\frac{2(-ef+dg)^3\tan^{-1}\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e\sqrt{-cd^2+e(bd-ae)}\,(cd^2+e(-bd+ae))}-\frac{g^3\log\left(ce\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(b^3*d*g^3*x) + b^2*(a*g^3*(-d + e*x) + c*(-(e*f^3) + 3*d*f*g^2*x)) +
b*(a^2*e*g^3 + c^2*f^2*(-(e*f*x) + d*(f - 3*g*x)) + 3*a*c*g*(e*f*(f - g*x)
+ d*g*(f + g*x))) + 2*c*(c^2*d*f^3*x + a^2*g^2*(d*g - e*(3*f + g*x)) + a*c*
f*(-3*d*g*(f + g*x) + e*f*(f + 3*g*x)))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b
*d) + a*e))*Sqrt[a + x*(b + c*x)]) + (2*(-(e*f) + d*g)^3*ArcTan[(Sqrt[c]*(d
 + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(e*Sqrt
[-(c*d^2) + e*(b*d - a*e)]*(c*d^2 + e*(-(b*d) + a*e))) - (g^3*Log[c*e*(b +
2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])])/(c^(3/2)*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(339) = 678$.
time = 0.14, size = 740, normalized size = 2.07

method	result

default	$\frac{g\left(g^2e^2\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx-}\right.}{c^{\frac{3}{2}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] g/e^3*(g^2*e^2*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-
b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2)))+(-d*e*g^2+3*e^2*f*g)*(-1/c/(c*x^2+b*x+a)^(1/2)-b
/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+2*d^2*g^2*(2*c*x+b)/(4*a*c-b^
2)/(c*x^2+b*x+a)^(1/2)-6*d*e*f*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+
6*e^2*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+(-d^3*g^3+3*d^2*e*f*g^
2-3*d*e^2*f^2*g+e^3*f^3)/e^4*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2
*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c
*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^
2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/
(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+
c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/
e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((%e^-1*b-2*%e^-2*c*d)^2>0)', see `a
ssume?`
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^3}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)**3/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^3}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.880 $\int \frac{(f+gx)^2}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=240

$$\frac{2(b^2ef^2+2a(aeg^2-cf(ef-2dg))-b(cdf^2+ag(2ef+dg))-(2c^2df^2+b(bd-ae)g^2+c(2ag(2ef-dg)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

```
[Out] (-d*g+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+2*(b^2*e*f^2+2*a*(a*e*g^2
-c*f*(-2*d*g+e*f))-b*(c*d*f^2+a*g*(d*g+2*e*f))-(2*c^2*d*f^2+b*(-a*e+b*d)*g^
2+c*(2*a*g*(-d*g+2*e*f)-b*f*(2*d*g+e*f)))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^
2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1660, 12, 738, 212}

$$\frac{2\left(-x\left(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2\right)-b\left(ag(dg+2ef)+cdf^2\right)+2a\left(aeg^2-cf(ef-2dg)\right)+b^2ef^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}+\frac{(ef-dg)^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*e*f^2 + 2*a*(a*e*g^2 - c*f*(e*f - 2*d*g)) - b*(c*d*f^2 + a*g*(2*e*f
 + d*g)) - (2*c^2*d*f^2 + b*(b*d - a*e)*g^2 + c*(2*a*g*(2*e*f - d*g) - b*f*
(e*f + 2*d*g)))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c
*x^2]) + ((e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d
^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^2ef^2+2a(aeg^2-cf(ef-2dg))-b(cdf^2+ag(2ef+dg))-(2c^2df^2 -}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a}}$$

$$= \frac{2(b^2ef^2+2a(aeg^2-cf(ef-2dg))-b(cdf^2+ag(2ef+dg))-(2c^2df^2 -}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a}}$$

$$= \frac{2(b^2ef^2+2a(aeg^2-cf(ef-2dg))-b(cdf^2+ag(2ef+dg))-(2c^2df^2 -}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a}}$$

$$= \frac{2(b^2ef^2+2a(aeg^2-cf(ef-2dg))-b(cdf^2+ag(2ef+dg))-(2c^2df^2 -}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a}}$$

Mathematica [A]
time = 1.20, size = 244, normalized size = 1.02

$$2\left(\frac{-2a^2eg^2+2c^2df^2x-2acdg(2f+gx)+2acef(f+2gx)+abg(2ef+dg-egx)+b^2(-ef^2+dg^2x)+bcf(-efx+d(f-2gx))}{(b^2-4ac)\left(-cd^2+e(bd-ae)\right)\sqrt{a+x(b+cx)}}+\frac{\sqrt{-cd^2+bde-ae^2}\,(ef-dg)^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] 2*((-2*a^2*e*g^2 + 2*c^2*d*f^2*x - 2*a*c*d*g*(2*f + g*x) + 2*a*c*e*f*(f + 2
*g*x) + a*b*g*(2*e*f + d*g - e*g*x) + b^2*(-(e*f^2) + d*g^2*x) + b*c*f*(-(e
```

```
*f*x) + d*(f - 2*g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a +
x*(b + c*x)]) + (Sqrt[-(c*d^2) + b*d*e - a*e^2]*(e*f - d*g)^2*ArcTan[(Sqrt[
c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c
*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(230) = 460$.
time = 0.13, size = 555, normalized size = 2.31

method	result
default	$-\frac{g\left(-eg\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)+\frac{2dg(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{4ef(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+b}}}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -g/e^2*(-e*g*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2))+2*d*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-4*e*f*(2*c*x+b)/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+(d^2*g^2-2*d*e*f*g+e^2*f^2)/e^3*(1/(a*e^2-
b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*
(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/
e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((%e^-1*b-2*%e^-2*c*d)^2>0)', see `a
ssume?`
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(239) = 478$.

time = 12.23, size = 2032, normalized size = 8.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*d^2*g^2*x^2 + (b^3 - 4*a*b*c)*d^2*g^2*x + (a*b^2 -
 4*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*f^2*x^2 + (b^3 - 4*a*b*c)*f^2*x + (a
*b^2 - 4*a^2*c)*f^2)*e^2 - 2*((b^2*c - 4*a*c^2)*d*f*g*x^2 + (b^3 - 4*a*b*c)
*d*f*g*x + (a*b^2 - 4*a^2*c)*d*f*g)*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-(8*
c^2*d^2*x^2 + 8*b*c*d^2*x + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*d^2 - b*d*e + a*e^
2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2 + b*x + a) + (8*a*b*x + (b^2
+ 4*a*c)*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^2 + 4*a*b*d + (3*b^2 + 4*a*c)*d*x)
*e)/(x^2*e^2 + 2*d*x*e + d^2)) - 4*(b*c^2*d^3*f^2 - 4*a*c^2*d^3*f*g + a*b*c
*d^3*g^2 + (2*c^3*d^3*f^2 - 2*b*c^2*d^3*f*g + (b^2*c - 2*a*c^2)*d^3*g^2)*x
+ (2*a^2*b*f*g - 2*a^3*g^2 - (a*b^2 - 2*a^2*c)*f^2 - (a*b*c*f^2 - 4*a^2*c*f
*g + a^2*b*g^2)*x)*e^3 + (3*a^2*b*d*g^2 + (b^3 - a*b*c)*d*f^2 - 2*(a*b^2 +
2*a^2*c)*d*f*g - (6*a*b*c*d*f*g - (b^2*c + 2*a*c^2)*d*f^2 - 2*(a*b^2 - a^2*
c)*d*g^2)*x)*e^2 + (6*a*b*c*d^2*f*g - 2*(b^2*c - a*c^2)*d^2*f^2 - (a*b^2 +
2*a^2*c)*d^2*g^2 - (3*b*c^2*d^2*f^2 - 2*(b^2*c + 2*a*c^2)*d^2*f*g + (b^3 -
a*b*c)*d^2*g^2)*x)*e)*sqrt(c*x^2 + b*x + a))/((b^2*c^3 - 4*a*c^4)*d^4*x^2 +
 (b^3*c^2 - 4*a*b*c^3)*d^4*x + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a
^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^3*b*c)*x)*e^4 - 2*((a*b
^3*c - 4*a^2*b*c^2)*d*x^2 + (a*b^4 - 4*a^2*b^2*c)*d*x + (a^2*b^3 - 4*a^3*b*
c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^2 + (b^5 - 2*a*b^3*c -
 8*a^2*b*c^2)*d^2*x + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*
c^2 - 4*a*b*c^3)*d^3*x^2 + (b^4*c - 4*a*b^2*c^2)*d^3*x + (a*b^3*c - 4*a^2*b
*c^2)*d^3)*e), (((b^2*c - 4*a*c^2)*d^2*g^2*x^2 + (b^3 - 4*a*b*c)*d^2*g^2*x
+ (a*b^2 - 4*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*f^2*x^2 + (b^3 - 4*a*b*c)*
f^2*x + (a*b^2 - 4*a^2*c)*f^2)*e^2 - 2*((b^2*c - 4*a*c^2)*d*f*g*x^2 + (b^3
- 4*a*b*c)*d*f*g*x + (a*b^2 - 4*a^2*c)*d*f*g)*e)*sqrt(-c*d^2 + b*d*e - a*e^
2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)
*sqrt(c*x^2 + b*x + a)/(c^2*d^2*x^2 + b*c*d^2*x + a*c*d^2 + (a*c*x^2 + a*b*
x + a^2)*e^2 - (b*c*d*x^2 + b^2*d*x + a*b*d)*e)) - 2*(b*c^2*d^3*f^2 - 4*a*c
^2*d^3*f*g + a*b*c*d^3*g^2 + (2*c^3*d^3*f^2 - 2*b*c^2*d^3*f*g + (b^2*c - 2*
a*c^2)*d^3*g^2)*x + (2*a^2*b*f*g - 2*a^3*g^2 - (a*b^2 - 2*a^2*c)*f^2 - (a*b
*c*f^2 - 4*a^2*c*f*g + a^2*b*g^2)*x)*e^3 + (3*a^2*b*d*g^2 + (b^3 - a*b*c)*d
*f^2 - 2*(a*b^2 + 2*a^2*c)*d*f*g - (6*a*b*c*d*f*g - (b^2*c + 2*a*c^2)*d*f^2
 - 2*(a*b^2 - a^2*c)*d*g^2)*x)*e^2 + (6*a*b*c*d^2*f*g - 2*(b^2*c - a*c^2)*d
^2*f^2 - (a*b^2 + 2*a^2*c)*d^2*g^2 - (3*b*c^2*d^2*f^2 - 2*(b^2*c + 2*a*c^2)
*d^2*f*g + (b^3 - a*b*c)*d^2*g^2)*x)*e)*sqrt(c*x^2 + b*x + a))/((b^2*c^3 -
4*a*c^4)*d^4*x^2 + (b^3*c^2 - 4*a*b*c^3)*d^4*x + (a*b^2*c^2 - 4*a^2*c^3)*d^
```

```
4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^3*b*c
)*x)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^2 + (a*b^4 - 4*a^2*b^2*c)*d*x + (
a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^2 +
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d
^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (b^4*c - 4*a*b^2*c^2)*d^3*x +
(a*b^3*c - 4*a^2*b*c^2)*d^3)*e)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^2}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)**2/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. 2(239) = 478.
time = 6.14, size = 757, normalized size = 3.15

$$-\frac{2\left(\frac{\left(2c^3d^3f^2-2bc^2d^3fg+b^2cd^3g^2-2ac^2d^3g^2-3bc^2d^2f^2e+2b^2cd^2fge+4ac^2d^2fge-b^3d^2g^2e+abcd^2g^2e+b^2cdf^2e^2+2ac^2df^2e^2-6abcdfge^2+2ab^2dg^2e^2-2a^2cdg^2e^2-abcf^2e^3+4a^2cfge^3-a^2bg^2e^3\right)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3f^2-4ac^2d^3fg+abcd^3g^2-2b^2cd^2f^2e+2ac^2d^2f^2e+6abcd^2fge-ab^2d^2g^2e-2a^2cd^2g^2e+b^3df^2e^2-abcdf^2e^2-2ab^2dfge^2-4a^2cdfge^2+3a^2bdg^2e^2-ab^2f^2e^3+2a^2cf^2e^3+2a^2bfge^3-2a^3g^2e^3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}\right)}{\sqrt{cx^2+bx+a}}+\frac{2\left(d^2g^2-2dfge+f^2e^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f^2 - 2*b*c^2*d^3*f*g + b^2*c*d^3*g^2 - 2*a*c^2*d^3*g^2 - 3*
b*c^2*d^2*f^2*e + 2*b^2*c*d^2*f*g*e + 4*a*c^2*d^2*f*g*e - b^3*d^2*g^2*e + a
*b*c*d^2*g^2*e + b^2*c*d*f^2*e^2 + 2*a*c^2*d*f^2*e^2 - 6*a*b*c*d*f*g*e^2 +
2*a*b^2*d*g^2*e^2 - 2*a^2*c*d*g^2*e^2 - a*b*c*f^2*e^3 + 4*a^2*c*f*g*e^3 - a
^2*b*g^2*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*
e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8
*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f^2 - 4*a*c^2*d^3*
f*g + a*b*c*d^3*g^2 - 2*b^2*c*d^2*f^2*e + 2*a*c^2*d^2*f^2*e + 6*a*b*c*d^2*f
*g*e - a*b^2*d^2*g^2*e - 2*a^2*c*d^2*g^2*e + b^3*d*f^2*e^2 - a*b*c*d*f^2*e^
2 - 2*a*b^2*d*f*g*e^2 - 4*a^2*c*d*f*g*e^2 + 3*a^2*b*d*g^2*e^2 - a*b^2*f^2*e
^3 + 2*a^2*c*f^2*e^3 + 2*a^2*b*f*g*e^3 - 2*a^3*g^2*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^2 + b*x + a) + 2*(d^2*g^2 - 2*d*f*g*e + f^2*e^2)*arctan(
-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e -
a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^2}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((f + g*x)^2/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.881 $\int \frac{f+gx}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{2(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx+cx^2}}+\frac{e(ef-dg)\tanh^{-1}\left(\frac{\cdots}{2\sqrt{cd^2-b\cdots}}\right.}{(cd^2-bde\cdots}$$

```
[Out] e*(-d*g+e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)-2*(b*c*d*f-b^2*e*f+2*a*c*
e*f-2*a*c*d*g+a*b*e*g+c*(2*c*d*f+2*a*e*g-b*(d*g+e*f))*x)/(-4*a*c+b^2)/(a*e^
2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {836, 12, 738, 212}

$$\frac{e(ef-dg)\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\,\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}-\frac{2(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acdg+2acef+b^2(-e)f+bcdf)}{(b^2-4ac)\,\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*a
*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b
*x + c*x^2]) + (e*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 - b*d*e + a*e^2)^
(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2(bcdf-b^2ef+2acef-2acdg+abeg+c(2cdf+2aeg-b(ef+dg))x}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}}$$

Mathematica [A]
time = 0.83, size = 198, normalized size = 1.06

$$\frac{-2b^2ef+2b(aeg-cefx+cd(f-gx))+4c(-adg+cdfx+ae(f+gx))}{\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)\sqrt{a+x(b+cx)}}-\frac{2e\sqrt{-cd^2+bde-ae^2}\,(-ef+dg)\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

[Out] (-2*b^2*e*f + 2*b*(a*e*g - c*e*f*x + c*d*(f - g*x)) + 4*c*(-(a*d*g) + c*d*f *x + a*e*(f + g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) - (2*e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(-(e*f) + d*g)*ArcTan[(Sqr t[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/ (c*d^2 + e*(-(b*d) + a*e))^2

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(177) = 354$.
time = 0.13, size = 445, normalized size = 2.38

method	result
default	$\frac{2g(2cx+b)}{e(4ac-b^2)\sqrt{c\,x^2+bx+a}} + \frac{(-dg+ef)\left(\frac{e^2}{\left(a\,e^2-bde+c\,d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}} - \frac{\ldots}{\left(a\,e^2 \ldots\right.}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2*g/e*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(-d*g+e*f)/e^2*(1/(a*e^2-b* d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^ (1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2*c*d)/e)/(4*c*(a *e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e) +(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/((a*e^2-b*d*e+c*d ^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^ 2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c *d^2)/e^2)^(1/2))/(x+d/e)))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume((%e^-1*b-2*%e^-2*c*d)^2>0)', see `a ssume?`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(187) = 374$.
time = 14.44, size = 1630, normalized size = 8.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c*d^2 - b*d*e + a*e^2)*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*
c)*f*x + (a*b^2 - 4*a^2*c)*f)*e^2 - ((b^2*c - 4*a*c^2)*d*g*x^2 + (b^3 - 4*a
*b*c)*d*g*x + (a*b^2 - 4*a^2*c)*d*g)*e)*log(-(8*c^2*d^2*x^2 + 8*b*c*d^2*x +
 (b^2 + 4*a*c)*d^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*(2*c*d*x + b*d - (b*x +
2*a)*e)*sqrt(c*x^2 + b*x + a) + (8*a*b*x + (b^2 + 4*a*c)*x^2 + 8*a^2)*e^2 -
 2*(4*b*c*d*x^2 + 4*a*b*d + (3*b^2 + 4*a*c)*d*x)*e)/(x^2*e^2 + 2*d*x*e + d^
2)) - 4*(b*c^2*d^3*f - 2*a*c^2*d^3*g + (2*c^3*d^3*f - b*c^2*d^3*g)*x + (a^2
*b*g - (a*b^2 - 2*a^2*c)*f - (a*b*c*f - 2*a^2*c*g)*x)*e^3 + ((b^3 - a*b*c)*
d*f - (a*b^2 + 2*a^2*c)*d*g - (3*a*b*c*d*g - (b^2*c + 2*a*c^2)*d*f)*x)*e^2
+ (3*a*b*c*d^2*g - 2*(b^2*c - a*c^2)*d^2*f - (3*b*c^2*d^2*f - (b^2*c + 2*a*
c^2)*d^2*g)*x)*e)*sqrt(c*x^2 + b*x + a))/((b^2*c^3 - 4*a*c^4)*d^4*x^2 + (b^
3*c^2 - 4*a*b*c^3)*d^4*x + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c
 + (a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^3*b*c)*x)*e^4 - 2*((a*b^3*c
 - 4*a^2*b*c^2)*d*x^2 + (a*b^4 - 4*a^2*b^2*c)*d*x + (a^2*b^3 - 4*a^3*b*c)*d
)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^2 + (b^5 - 2*a*b^3*c - 8*a
^2*b*c^2)*d^2*x + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2
- 4*a*b*c^3)*d^3*x^2 + (b^4*c - 4*a*b^2*c^2)*d^3*x + (a*b^3*c - 4*a^2*b*c^2
)*d^3)*e), (sqrt(-c*d^2 + b*d*e - a*e^2)*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 -
 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*e^2 - ((b^2*c - 4*a*c^2)*d*g*x^2 + (b^
3 - 4*a*b*c)*d*g*x + (a*b^2 - 4*a^2*c)*d*g)*e)*arctan(-1/2*sqrt(-c*d^2 + b*
d*e - a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*x^2 + b*x + a)/(c^2*d^2
*x^2 + b*c*d^2*x + a*c*d^2 + (a*c*x^2 + a*b*x + a^2)*e^2 - (b*c*d*x^2 + b^2
*d*x + a*b*d)*e)) - 2*(b*c^2*d^3*f - 2*a*c^2*d^3*g + (2*c^3*d^3*f - b*c^2*d
^3*g)*x + (a^2*b*g - (a*b^2 - 2*a^2*c)*f - (a*b*c*f - 2*a^2*c*g)*x)*e^3 + (
(b^3 - a*b*c)*d*f - (a*b^2 + 2*a^2*c)*d*g - (3*a*b*c*d*g - (b^2*c + 2*a*c^2
)*d*f)*x)*e^2 + (3*a*b*c*d^2*g - 2*(b^2*c - a*c^2)*d^2*f - (3*b*c^2*d^2*f -
 (b^2*c + 2*a*c^2)*d^2*g)*x)*e)*sqrt(c*x^2 + b*x + a))/((b^2*c^3 - 4*a*c^4)
*d^4*x^2 + (b^3*c^2 - 4*a*b*c^3)*d^4*x + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3
*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^3*b*c)*x)*e^4
 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^2 + (a*b^4 - 4*a^2*b^2*c)*d*x + (a^2*b^3
- 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^2 + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*x + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2
- 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (b^4*c - 4*a*b^2*c^2)*d^3*x + (a*b^3*c
 - 4*a^2*b*c^2)*d^3)*e)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{(d+ex)\,(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(187) = 374.
time = 4.50, size = 568, normalized size = 3.04

$$-\frac{2\left(\frac{(2c^3d^3f-bc^2d^3g-3bc^2d^2fe+b^2cd^2ge+2ac^2d^2ge+b^2cdfe^2+2ac^2dfe^2-3abcdge^2-abcfe^3+2a^2cge^3)x}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3f-2ac^2d^3g-2b^2cd^2fe+2ac^2d^2fe+3abcd^2ge+b^3dfe^2-abcdfe^2-ab^2dge^2-2a^2cdge^2-ab^2fe^3+2a^2cfe^3+a^2bge^3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}\right)}{\sqrt{cx^2+bx+a}}-\frac{2\,(dge-fe^2)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3*f - b*c^2*d^3*g - 3*b*c^2*d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*
d^2*g*e + b^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 - a*b*c*f*e^3 +
 2*a^2*c*g*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^
3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 +
 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3*f - 2*a*c^2*d^3*
g - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e + b^3*d*f*e^2 - a*b
*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3
+ a^2*b*g*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e
 + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*
a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^2 + b*x + a) - 2*(d*g*
e - f*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c)*d)/sqrt
(-c*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*
e^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f+g\,x}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((f + g*x)/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.882 $\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}} \, dx$

Optimal. Leaf size=155

$$-\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx+cx^2}}+\frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)-2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c
*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {754, 12, 738, 212}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}-\frac{2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(c*d^2 -
b*d*e + a*e^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{(b^2-4ac)e^2}{2(d+ex)\sqrt{a+bx+\cdots}}}{(b^2-4ac)\left(cd^2-bde\cdots\right.}$$

$$= -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\int \frac{1}{(d+ex)\sqrt{a+bx+c\cdots}}}{cd^2-bde+ae^2\cdots}$$

$$= -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} - \frac{(2e^2)\,\mathrm{Subst}\left(\int \frac{\cdots}{4cd^2-4bd\cdots}\right.}{cd\cdots}$$

$$= -\frac{2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\sqrt{a+bx+cx^2}} + \frac{e^2\tanh^{-1}\left(\frac{\cdots}{2\sqrt{cd^2-b\cdots}}\right.}{(cd^2-b\cdots}$$

Mathematica [A]
time = 0.15, size = 187, normalized size = 1.21

$$-\frac{2\left(\left(cd^2+e(-bd+ae)\right)\left(-b^2e+2c(ae+cdx)+bc(d-ex)\right)+\left(-b^2+4ac\right)e^2\sqrt{-cd^2+bde-ae^2}\,\sqrt{a+x(b+cx)}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)\right)}{\left(b^2-4ac\right)\left(cd^2+e(-bd+ae)\right)^2\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*((c*d^2 + e*(-(b*d) + a*e))*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*
x)) + (-b^2 + 4*a*c)*e^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Sqrt[a + x*(b + c*x
)]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(
```

```
b*d - a*e)]]))/((b^2 - 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + x*(b +
c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(145) = 290$.
time = 0.11, size = 400, normalized size = 2.58

method	result
default	$\frac{e^2}{\left(a e^2-bde+c d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-bde+c d^2}{e^2}}}-\frac{(eb-2cd)e\left(2c\left(x+\right.\right.}{\left(a e^2-bde+c d^2\right)\left(\frac{4c\left(a e^2-bde+c d^2\right)}{e^2}-\frac{(eb-2cd)^2}{e^2}\right)\sqrt{c\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/e)+(b*e-2
*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e)^2+(b*e-
2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d^2)*e^2/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e*b*d+%e^2*a>0)', see `assum
e?` for
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(149) = 298$.
time = 6.99, size = 1306, normalized size = 8.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(c*
d^2 - b*d*e + a*e^2)*e^2*log(-(8*c^2*d^2*x^2 + 8*b*c*d^2*x + (b^2 + 4*a*c)*
d^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*e)*sqrt(c*
x^2 + b*x + a) + (8*a*b*x + (b^2 + 4*a*c)*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^2
 + 4*a*b*d + (3*b^2 + 4*a*c)*d*x)*e)/(x^2*e^2 + 2*d*x*e + d^2)) - 4*(2*c^3*
d^3*x + b*c^2*d^3 - (a*b*c*x + a*b^2 - 2*a^2*c)*e^3 + ((b^2*c + 2*a*c^2)*d*
x + (b^3 - a*b*c)*d)*e^2 - (3*b*c^2*d^2*x + 2*(b^2*c - a*c^2)*d^2)*e)*sqrt(
c*x^2 + b*x + a))/((b^2*c^3 - 4*a*c^4)*d^4*x^2 + (b^3*c^2 - 4*a*b*c^3)*d^4*
x + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c
^2)*x^2 + (a^2*b^3 - 4*a^3*b*c)*x)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^2 +
 (a*b^4 - 4*a^2*b^2*c)*d*x + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b
^2*c^2 - 8*a^2*c^3)*d^2*x^2 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x + (a*b^
4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^2 +
(b^4*c - 4*a*b^2*c^2)*d^3*x + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e), ((a*b^2 - 4*
a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(-c*d^2 + b*d*e - a*
e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*(2*c*d*x + b*d - (b*x + 2*a)*
e)*sqrt(c*x^2 + b*x + a)/(c^2*d^2*x^2 + b*c*d^2*x + a*c*d^2 + (a*c*x^2 + a*
b*x + a^2)*e^2 - (b*c*d*x^2 + b^2*d*x + a*b*d)*e))*e^2 - 2*(2*c^3*d^3*x + b
*c^2*d^3 - (a*b*c*x + a*b^2 - 2*a^2*c)*e^3 + ((b^2*c + 2*a*c^2)*d*x + (b^3
- a*b*c)*d)*e^2 - (3*b*c^2*d^2*x + 2*(b^2*c - a*c^2)*d^2)*e)*sqrt(c*x^2 + b
*x + a))/((b^2*c^3 - 4*a*c^4)*d^4*x^2 + (b^3*c^2 - 4*a*b*c^3)*d^4*x + (a*b^
2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 +
 (a^2*b^3 - 4*a^3*b*c)*x)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^2 + (a*b^4 -
 4*a^2*b^2*c)*d*x + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 -
8*a^2*c^3)*d^2*x^2 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x + (a*b^4 - 2*a^2
*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (b^4*c -
4*a*b^2*c^2)*d^3*x + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(149) = 298.
time = 4.35, size = 447, normalized size = 2.88

$$-\frac{2\left(\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{bc^2d^3-2\,b^2cd^2e+2\,ac^2d^2e+b^3de^2-abcde^2-ab^2e^3+2\,a^2ce^3}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}\right)}{\sqrt{cx^2+bx+a}}+\frac{2\,\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e^2}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^2 + b*x + a) + 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*e^2)
*sqrt(-c*d^2 + b*d*e - a*e^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d+e\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/((d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.883 $\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=352

$$-\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)\sqrt{a+bx+cx^2}}+\frac{2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)\,(ef-dg)\,(cf^2-bfg+ag^2)\,\sqrt{a+bx+}}$$

```
[Out] e^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)-g^3*arctanh(1/2*(b*f-2*
a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*
f)/(a*g^2-b*f*g+c*f^2)^(3/2)-2*e*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x)/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2
*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(c
*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {974, 754, 12, 738, 212}

$$-\frac{2e(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ef-dg)\,(ae^2-bde+cd^2)}+\frac{2g(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ef-dg)\,(ag^2-bfg+cf^2)}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\,\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\,(ae^2-bde+cd^2)^{3/2}}-\frac{g^3\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\,\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\,(ag^2-bfg+cf^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g +
 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g +
a*g^2)*Sqrt[a + b*x + c*x^2]) + (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x
)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e +
 a*e^2)^(3/2)*(e*f - d*g)) - (g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(
2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2]])])/((e*f - d*g)*(c*f^2
- b*f*g + a*g^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \left(\frac{e}{(ef-dg)(d+ex)\left(a+bx+cx^2\right)^{3/2}} - \frac{g}{(ef-dg)(f+gx)\,(}\right.$$

$$= \frac{e\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)(a+bx+cx^2)^{3/2}}\,dx}{ef-dg}$$

$$= -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

Mathematica [A]
time = 3.12, size = 347, normalized size = 0.99

$$\frac{2(-b^3eg+b^2c(dg+e(f-gx))-2c^2(adg+cdfx+ae(f-gx))+bc(3aeg+c(-df+efx+dgx)))}{(b^2-4ac)\left(-cd^2+e(bd-ae)\right)\left(-cf^2+g(bf-ag)\right)\sqrt{a+x(b+cx)}} - \frac{2e^3\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2(-ef+dg)} - \frac{2g^3\sqrt{-cf^2+bfg-ag^2}\tan^{-1}\left(\frac{\sqrt{c}\,(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+g(bf-ag)}}\right)}{(ef-dg)\left(cf^2+g(-bf+ag)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(-(b^3*e*g) + b^2*c*(d*g + e*(f - g*x)) - 2*c^2*(a*d*g + c*d*f*x + a*e*(
f - g*x)) + b*c*(3*a*e*g + c*(-(d*f) + e*f*x + d*g*x))))/((b^2 - 4*a*c)*(-(
c*d^2) + e*(b*d - a*e))*(-(c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) -
 (2*e^3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a
 + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/((c*d^2 + e*(-(b*d) + a*e
))^2*(-(e*f) + d*g)) - (2*g^3*Sqrt[-(c*f^2) + b*f*g - a*g^2]*ArcTan[(Sqrt[c
]*(f + g*x) - g*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*f^2) + g*(b*f - a*g)]])/((e
*f - d*g)*(c*f^2 + g*(-(b*f) + a*g))^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(332) = 664.
time = 0.16, size = 815, normalized size = 2.32

method	result

default	$-\frac{\frac{e^2}{\left(a\,e^2-bde+c\,d^2\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}-\frac{(eb-2cd)e\left(2c\left(x\right.\right.}{\left(a\,e^2-bde+c\,d^2\right)\left(\frac{4c\left(a\,e^2-bde+c\,d^2\right)}{e^2}-\frac{(eb-2cd)^2}{e^2}\right)\sqrt{c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(d*g-e*f)*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d/
e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/e
)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*d
^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-
2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)
/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+1/(d*g-e*f)*(1/(a*g^2-
b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*
(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/
g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g
+c*f^2)/g^2)^(1/2))/(x+f/g)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(g*x + f)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

[Out] Timed out

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f+g\,x)\,(d+e\,x)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.884 $\int \frac{1}{(d+ex)(f+gx)^2(a+bx+cx^2)^{3/2}} dx$

Optimal. Leaf size=642

$$-\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^2\sqrt{a+bx+cx^2}}+\frac{2eg(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)^2(cf^2-bfg+ag^2)\sqrt{a+bx}}$$

```
[Out] e^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)^2-3/2*g^3*(-b*g+2*c*f)*
arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x
+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(5/2)-e*g^3*arctanh(1/2*(b*f-2*a*
g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*g+e*f)
^2/(a*g^2-b*f*g+c*f^2)^(3/2)-2*e^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x)/(
-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2/(c*x^2+b*x+a)^(1/2)+2*e*g*(b*c
*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c
*f^2)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*
c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)/(c*x^2+b*x+a)^(1/2)+g^2*(4*c^
2*f^2+3*b^2*g^2-4*c*g*(2*a*g+b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e
*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)
```

Rubi [A]
time = 0.57, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {974, 754, 12, 738, 212, 820}

$$\frac{g^2\sqrt{a+bx+cx^2}\left(-4cg(2ag+bf)+3b^2g^2+4c^2f^2\right)}{(b^2-4ac)(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)^2}-\frac{2e^2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2(ae^2-bde+cd^2)}+\frac{2eg(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2(ag^2-bfg+cf^2)}+\frac{2g(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)(f+gx)\sqrt{a+bx+cx^2}(ef-dg)(ag^2-bfg+cf^2)}+\frac{e^4\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2(ae^2-bde+cd^2)^{3/2}}-\frac{eg^3\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2(ag^2-bfg+cf^2)^{3/2}}-\frac{3g^3(2cf-bg)\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)(ag^2-bfg+cf^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b
^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 -
b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c*(
2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*
x)*Sqrt[a + b*x + c*x^2]) + (g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*
g))*Sqrt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^
2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^(3/2)*
(e*f - d*g)^2) - (3*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*
x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f - d*g)*(
c*f^2 - b*f*g + a*g^2)^(5/2)) - (e*g^3*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)
*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e*f - d*g)^2*
(c*f^2 - b*f*g + a*g^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
```

```
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^2\,(a+bx+cx^2)^{3/2}}\,dx = \int \left(\frac{e^2}{(ef-dg)^2(d+ex)\,(a+bx+cx^2)^{3/2}} - \frac{g}{(ef-dg)(f+gx)}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)(a+bx+cx^2)^{3/2}}\,dx}{(ef-dg)^2} - \frac{g\int \frac{}{(f}}{}$$

$$= -\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e^2(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(ef-dg)^2\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

Mathematica [A]
time = 13.24, size = 623, normalized size = 0.97

$$-\frac{2e^2(b^2e-2c(ae+cdx)+bc(-d+ex))}{(b^2-4ac)(-cd^2+e(bd-ae))(ef-dg)^2\sqrt{a+x(b+cx)}}+\frac{2eg(b^2g-2c(ag+cfx)+bc(-f+gx))}{(b^2-4ac)(ef-dg)^2(-cf^2+g(bf-ag))\sqrt{a+x(b+cx)}}-\frac{2g(b^2g-2c(ag+cfx)+bc(-f+gx))}{(b^2-4ac)(-ef+dg)(-cf^2+g(bf-ag))(f+gx)\sqrt{a+x(b+cx)}}+\frac{g^2\left(-\frac{2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{a+x(b+cx)}}{(b^2-4ac)(cf^2+g(-bf+ag))^2(f+gx)}+\frac{3g(-2cf+bg)\tanh^{-1}\left(\frac{-bf+2ag-2cfx+bgx}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(cf^2+g(-bf+ag))^{5/2}}\right)}{2(-ef+dg)}+\frac{e^4\tanh^{-1}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{(cd^2+e(-bd+ae))^{3/2}(ef-dg)^2}-\frac{eg^3\tanh^{-1}\left(\frac{-2ag+2cfx+b(f-gx)}{2\sqrt{cf^2+g(-bf+ag)}\sqrt{a+x(b+cx)}}\right)}{(ef-dg)^2(cf^2+g(-bf+ag))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^2*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g -
 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2
) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c*f*x)
 + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f - a*g)
)*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (g^2*((-2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c
*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f)
+ a*g))^2*(f + g*x)) + (3*g*(-2*c*f + b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*
x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^
2 + g*(-(b*f) + a*g))^(5/2)))/(2*(-(e*f) + d*g)) + (e^4*ArcTanh[(-2*a*e + 2
```

```
*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x
)])])/((c*d^2 + e*(-(b*d) + a*e))^(3/2)*(e*f - d*g)^2) - (e*g^3*ArcTanh[(-2
*a*g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*
(b + c*x)])])/((e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1480 vs. $2(608) = 1216$.
time = 0.16, size = 1481, normalized size = 2.31

method	result	size
default	Expression too large to display	1481

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e/(d*g-e*f)^2*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(x+d
/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x+d/
e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e+c*
d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e
-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d
)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))-e/(d*g-e*f)^2*(1/(a*g
^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/
g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4
*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x
+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*
g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*(
(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*
f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+1/g/(d*g-e*f)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/
(x+f/g)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-3
/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*
c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2
-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g
-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^
(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^
2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))
)-4*c/(a*g^2-b*f*g+c*f^2)*g^2*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g
+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*
f*g+c*f^2)/g^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(g*x + f)^2*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f+g\,x)^2\,\left(d+e\,x\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.885 $\int \frac{1}{(d+ex)(f+gx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=1064

$$-\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(ef-dg)^3\sqrt{a+bx+cx^2}}+\frac{2e^2g(bcf-b^2g+2acg+c(2cf-bg)x)}{(b^2-4ac)\left(ef-dg\right)^3\left(cf^2-bfg+ag^2\right)\sqrt{a+bx}}$$

```
[Out] e^5*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/(-d*g+e*f)^3-3/2*e*g^3*(-b*g+2*c*f
)*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(5/2)-e^2*g^3*arctanh(1/2*(b*
f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(-d*
g+e*f)^3/(a*g^2-b*f*g+c*f^2)^(3/2)-3/8*g^3*(16*c^2*f^2+5*b^2*g^2-4*c*g*(a*g
+4*b*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(7/2)-2*e^3*(b*c*d-b^2*e
+2*a*c*e+c*(-b*e+2*c*d)*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^3/(c
*x^2+b*x+a)^(1/2)+2*e^2*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^
2)/(-d*g+e*f)^3/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)+2*g*(b*c*f-b^2*g+2*
a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)
^2/(c*x^2+b*x+a)^(1/2)+2*e*g*(b*c*f-b^2*g+2*a*c*g+c*(-b*g+2*c*f)*x)/(-4*a*c
+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)/(c*x^2+b*x+a)^(1/2)+1/2*g^2*
(8*c^2*f^2+5*b^2*g^2-4*c*g*(3*a*g+2*b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/
(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*x+f)^2+e*g^2*(4*c^2*f^2+3*b^2*g^2-4*c*g
*(2*a*g+b*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*
f^2)^2/(g*x+f)+1/4*g^2*(-b*g+2*c*f)*(8*c^2*f^2+15*b^2*g^2-4*c*g*(13*a*g+2*b
*f))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^3/(g*x
+f)
```

Rubi [A]
time = 1.19, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {974, 754, 12, 738, 212, 848, 820}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c)*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2]) + (2*e^2*g*(b*c*f -
 b^2*g + 2*a*c*g + c*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^3*(c*f^2
- b*f*g + a*g^2)*Sqrt[a + b*x + c*x^2]) + (2*g*(b*c*f - b^2*g + 2*a*c*g + c
*(2*c*f - b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f +
```

```
g*x)^2*Sqrt[a + b*x + c*x^2]) + (2*e*g*(b*c*f - b^2*g + 2*a*c*g + c*(2*c*f
- b*g)*x))/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)*S
qrt[a + b*x + c*x^2]) + (g^2*(8*c^2*f^2 + 5*b^2*g^2 - 4*c*g*(2*b*f + 3*a*g)
)*Sqrt[a + b*x + c*x^2])/(2*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g + a*g^
2)^2*(f + g*x)^2) + (e*g^2*(4*c^2*f^2 + 3*b^2*g^2 - 4*c*g*(b*f + 2*a*g))*Sq
rt[a + b*x + c*x^2])/((b^2 - 4*a*c)*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^2
*(f + g*x)) + (g^2*(2*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 1
3*a*g))*Sqrt[a + b*x + c*x^2])/(4*(b^2 - 4*a*c)*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^3*(f + g*x)) + (e^5*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqr
t[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/((c*d^2 - b*d*e + a*e^2)^
(3/2)*(e*f - d*g)^3) - (3*e*g^3*(2*c*f - b*g)*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(2*(e*f
- d*g)^2*(c*f^2 - b*f*g + a*g^2)^(5/2)) - (e^2*g^3*ArcTanh[(b*f - 2*a*g + (
2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/((e
*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)^(3/2)) - (3*g^3*(16*c^2*f^2 + 5*b^2*g^2
 - 4*c*g*(4*b*f + a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f
^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*(e*f - d*g)*(c*f^2 - b*f*g
+ a*g^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
```

```
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^3 (a+bx+cx^2)^{3/2}} dx = \int \left(\frac{e^3}{(ef-dg)^3(d+ex)(a+bx+cx^2)^{3/2}} - \frac{g}{(ef-dg)(f+gx)^{3}} \right.$$

$$= \frac{e^3 \int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}} dx}{(ef-dg)^3} - \frac{(e^2g) \int \frac{1}{(f+gx)(a+bx+cx^2)^{3/2}} dx}{(ef-dg)^3} - \frac{(eg)}{}$$

$$= -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

$$= -\frac{2e^3(bcd-b^2e+2ace+c(2cd-be)x)}{(b^2-4ac)(cd^2-bde+ae^2)(ef-dg)^3\sqrt{a+bx+cx^2}} + \frac{}{(b^2}$$

Mathematica [A]
time = 15.28, size = 1013, normalized size = 0.95

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*e^3*(b^2*e - 2*c*(a*e + c*d*x) + b*c*(-d + e*x)))/((b^2 - 4*a*c)*(-(c*d
^2) + e*(b*d - a*e))*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)]) - (2*e^2*g*(b^2*g
 - 2*c*(a*g + c*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)^3*(-(
c*f^2) + g*(b*f - a*g))*Sqrt[a + x*(b + c*x)]) - (2*g*(b^2*g - 2*c*(a*g + c
*f*x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(-(e*f) + d*g)*(-(c*f^2) + g*(b*f -
 a*g))*(f + g*x)^2*Sqrt[a + x*(b + c*x)]) + (2*e*g*(b^2*g - 2*c*(a*g + c*f*
x) + b*c*(-f + g*x)))/((b^2 - 4*a*c)*(e*f - d*g)^2*(-(c*f^2) + g*(b*f - a*g
))*(f + g*x)*Sqrt[a + x*(b + c*x)]) + (e*g^2*((2*(4*c^2*f^2 + 3*b^2*g^2 - 4
*c*g*(b*f + 2*a*g))*Sqrt[a + x*(b + c*x)])/((b^2 - 4*a*c)*(c*f^2 + g*(-(b*f
) + a*g))^2*(f + g*x)) + (3*g*(2*c*f - b*g)*ArcTanh[(-(b*f) + 2*a*g - 2*c*f
```

```
*x + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f
^2 + g*(-(b*f) + a*g))^(5/2)))/(2*(e*f - d*g)^2) - (g^2*((4*(8*c^2*f^2 + 5*
b^2*g^2 - 4*c*g*(2*b*f + 3*a*g))*Sqrt[a + x*(b + c*x)])/(f + g*x)^2 + (2*(2
*c*f - b*g)*(8*c^2*f^2 + 15*b^2*g^2 - 4*c*g*(2*b*f + 13*a*g))*Sqrt[a + x*(b
 + c*x)])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)) + (3*(b^2 - 4*a*c)*g*(16*c
^2*f^2 + 5*b^2*g^2 - 4*c*g*(4*b*f + a*g))*ArcTanh[(-(b*f) + 2*a*g - 2*c*f*x
 + b*g*x)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(c*f^2
 + g*(-(b*f) + a*g))^(3/2)))/(8*(b^2 - 4*a*c)*(-(e*f) + d*g)*(c*f^2 + g*(-(
b*f) + a*g))^2) - (e^5*ArcTanh[(-2*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d
^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])])/((c*d^2 + e*(-(b*d) + a*e))
^(3/2)*(-(e*f) + d*g)^3) - (e^2*g^3*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x)
)/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/((e*f - d*g)^3
*(c*f^2 + g*(-(b*f) + a*g))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2684 vs. $2(1010) = 2020$.

time = 0.15, size = 2685, normalized size = 2.52

method	result	size
default	Expression too large to display	2685

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/(d*g-e*f)^3*(1/(a*e^2-b*d*e+c*d^2)*e^2/(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-(b*e-2*c*d)*e/(a*e^2-b*d*e+c*d^2)*(2*c*(
x+d/e)+(b*e-2*c*d)/e)/(4*c*(a*e^2-b*d*e+c*d^2)/e^2-(b*e-2*c*d)^2/e^2)/(c*(x
+d/e)^2+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/(a*e^2-b*d*e
+c*d^2)*e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*
c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))+e^2/(d*g-e*f)^3*(1
/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*
f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/
g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)
/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2
-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g
)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g
^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))+1/g^2/(d*g-e*f)*(-1/2/(a*g^2-b*f*g+c*
f^2)*g^2/(x+f/g)^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g
^2)^(1/2)-5/4*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(-1/(a*g^2-b*f*g+c*f^2)*g^2
/(x+f/g)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-
3/2*(b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2
*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^
2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*
g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)
```

```
^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g
^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)
*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)
))-4*c/(a*g^2-b*f*g+c*f^2)*g^2*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*
g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b
*f*g+c*f^2)/g^2)^(1/2))-3/2*c/(a*g^2-b*f*g+c*f^2)*g^2*(1/(a*g^2-b*f*g+c*f^2
)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*
g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*
g+c*f^2)/g^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b
*f*g+c*f^2)/g^2)^(1/2)-1/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^
(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c
*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2
)^(1/2))/(x+f/g))))-1/g*e/(d*g-e*f)^2*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/g)/(
(x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-3/2*(b*g-2
*c*f)*g/(a*g^2-b*f*g+c*f^2)*(1/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*
c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-(b*g-2*c*f)*g/(a*g^2-b*f*g+c*
f^2)*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g^2-(b*g-2*c*f)^2
/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)-1/(
a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c
*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^
2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g)))-4*c/(a*
g^2-b*f*g+c*f^2)*g^2*(2*c*(x+f/g)+(b*g-2*c*f)/g)/(4*c*(a*g^2-b*f*g+c*f^2)/g
^2-(b*g-2*c*f)^2/g^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2
)/g^2)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(g*x + f)^3*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 14731 vs. 2(1025) = 2050.
time = 12.06, size = 14731, normalized size = 13.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^9*d^3*f^9 - 9*b*c^8*d^3*f^8*g + 18*b^2*c^7*d^3*f^7*g^2 - 21*b^3*c^
6*d^3*f^6*g^3 + 15*b^4*c^5*d^3*f^5*g^4 + 6*a*b^2*c^6*d^3*f^5*g^4 - 12*a^2*c
^7*d^3*f^5*g^4 - 6*b^5*c^4*d^3*f^4*g^5 - 15*a*b^3*c^5*d^3*f^4*g^5 + 30*a^2*
b*c^6*d^3*f^4*g^5 + b^6*c^3*d^3*f^3*g^6 + 12*a*b^4*c^4*d^3*f^3*g^6 - 18*a^2
*b^2*c^5*d^3*f^3*g^6 - 16*a^3*c^6*d^3*f^3*g^6 - 3*a*b^5*c^3*d^3*f^2*g^7 - 3
*a^2*b^3*c^4*d^3*f^2*g^7 + 24*a^3*b*c^5*d^3*f^2*g^7 + 3*a^2*b^4*c^3*d^3*f*g
^8 - 6*a^3*b^2*c^4*d^3*f*g^8 - 6*a^4*c^5*d^3*f*g^8 - a^3*b^3*c^3*d^3*g^9 +
3*a^4*b*c^4*d^3*g^9 - 3*b*c^8*d^2*f^9*e + 15*b^2*c^7*d^2*f^8*g*e - 6*a*c^8*
d^2*f^8*g*e - 33*b^3*c^6*d^2*f^7*g^2*e + 24*a*b*c^7*d^2*f^7*g^2*e + 41*b^4*
c^5*d^2*f^6*g^3*e - 34*a*b^2*c^6*d^2*f^6*g^3*e - 16*a^2*c^7*d^2*f^6*g^3*e -
 30*b^5*c^4*d^2*f^5*g^4*e + 9*a*b^3*c^5*d^2*f^5*g^4*e + 66*a^2*b*c^6*d^2*f^
5*g^4*e + 12*b^6*c^3*d^2*f^4*g^5*e + 24*a*b^4*c^4*d^2*f^4*g^5*e - 96*a^2*b^
2*c^5*d^2*f^4*g^5*e - 12*a^3*c^6*d^2*f^4*g^5*e - 2*b^7*c^2*d^2*f^3*g^6*e -
23*a*b^5*c^3*d^2*f^3*g^6*e + 49*a^2*b^3*c^4*d^2*f^3*g^6*e + 48*a^3*b*c^5*d^
2*f^3*g^6*e + 6*a*b^6*c^2*d^2*f^2*g^7*e + 3*a^2*b^4*c^3*d^2*f^2*g^7*e - 54*
a^3*b^2*c^4*d^2*f^2*g^7*e - 6*a^2*b^5*c^2*d^2*f*g^8*e + 15*a^3*b^3*c^3*d^2*
f*g^8*e + 9*a^4*b*c^4*d^2*f*g^8*e + 2*a^3*b^4*c^2*d^2*g^9*e - 7*a^4*b^2*c^3
*d^2*g^9*e + 2*a^5*c^4*d^2*g^9*e + b^2*c^7*d*f^9*e^2 + 2*a*c^8*d*f^9*e^2 -
6*b^3*c^6*d*f^8*g*e^2 - 3*a*b*c^7*d*f^8*g*e^2 + 15*b^4*c^5*d*f^7*g^2*e^2 -
6*a*b^2*c^6*d*f^7*g^2*e^2 - 20*b^5*c^4*d*f^6*g^3*e^2 + 13*a*b^3*c^5*d*f^6*g
^3*e^2 + 16*a^2*b*c^6*d*f^6*g^3*e^2 + 15*b^6*c^3*d*f^5*g^4*e^2 - 48*a^2*b^2
*c^5*d*f^5*g^4*e^2 - 12*a^3*c^6*d*f^5*g^4*e^2 - 6*b^7*c^2*d*f^4*g^5*e^2 - 1
5*a*b^5*c^3*d*f^4*g^5*e^2 + 51*a^2*b^3*c^4*d*f^4*g^5*e^2 + 42*a^3*b*c^5*d*f
^4*g^5*e^2 + b^8*c*d*f^3*g^6*e^2 + 12*a*b^6*c^2*d*f^3*g^6*e^2 - 19*a^2*b^4*
c^3*d*f^3*g^6*e^2 - 50*a^3*b^2*c^4*d*f^3*g^6*e^2 - 16*a^4*c^5*d*f^3*g^6*e^2
 - 3*a*b^7*c*d*f^2*g^7*e^2 - 3*a^2*b^5*c^2*d*f^2*g^7*e^2 + 27*a^3*b^3*c^3*d
```

*f^2*g^7*e^2 + 24*a^4*b*c^4*d*f^2*g^7*e^2 + 3*a^2*b^6*c*d*f*g^8*e^2 - 6*a^3
*b^4*c^2*d*f*g^8*e^2 - 9*a^4*b^2*c^3*d*f*g^8*e^2 - 6*a^5*c^4*d*f*g^8*e^2 -
a^3*b^5*c*d*g^9*e^2 + 3*a^4*b^3*c^2*d*g^9*e^2 + a^5*b*c^3*d*g^9*e^2 - a*b*c
^7*f^9*e^3 + 6*a*b^2*c^6*f^8*g*e^3 - 6*a^2*c^7*f^8*g*e^3 - 15*a*b^3*c^5*f^7
*g^2*e^3 + 24*a^2*b*c^6*f^7*g^2*e^3 + 20*a*b^4*c^4*f^6*g^3*e^3 - 34*a^2*b^2
*c^5*f^6*g^3*e^3 - 16*a^3*c^6*f^6*g^3*e^3 - 15*a*b^5*c^3*f^5*g^4*e^3 + 15*a
^2*b^3*c^4*f^5*g^4*e^3 + 54*a^3*b*c^5*f^5*g^4*e^3 + 6*a*b^6*c^2*f^4*g^5*e^3
 + 9*a^2*b^4*c^3*f^4*g^5*e^3 - 66*a^3*b^2*c^4*f^4*g^5*e^3 - 12*a^4*c^5*f^4*
g^5*e^3 - a*b^7*c*f^3*g^6*e^3 - 11*a^2*b^5*c^2*f^3*g^6*e^3 + 31*a^3*b^3*c^3
*f^3*g^6*e^3 + 32*a^4*b*c^4*f^3*g^6*e^3 + 3*a^2*b^6*c*f^2*g^7*e^3 - 30*a^4*
b^2*c^3*f^2*g^7*e^3 - 3*a^3*b^5*c*f*g^8*e^3 + 9*a^4*b^3*c^2*f*g^8*e^3 + 3*a
^5*b*c^3*f*g^8*e^3 + a^4*b^4*c*g^9*e^3 - 4*a^5*b^2*c^2*g^9*e^3 + 2*a^6*c^3*
g^9*e^3)*x/(b^2*c^8*d^4*f^12 - 4*a*c^9*d^4*f^12 - 6*b^3*c^7*d^4*f^11*g + 24
*a*b*c^8*d^4*f^11*g + 15*b^4*c^6*d^4*f^10*g^2 - 54*a*b^2*c^7*d^4*f^10*g^2 -
 24*a^2*c^8*d^4*f^10*g^2 - 20*b^5*c^5*d^4*f^9*g^3 + 50*a*b^3*c^6*d^4*f^9*g^
3 + 120*a^2*b*c^7*d^4*f^9*g^3 + 15*b^6*c^4*d^4*f^8*g^4 - 225*a^2*b^2*c^6*d^
4*f^8*g^4 - 60*a^3*c^7*d^4*f^8*g^4 - 6*b^7*c^3*d^4*f^7*g^5 - 36*a*b^5*c^4*d
^4*f^7*g^5 + 180*a^2*b^3*c^5*d^4*f^7*g^5 + 240*a^3*b*c^6*d^4*f^7*g^5 + b^8*
c^2*d^4*f^6*g^6 + 26*a*b^6*c^3*d^4*f^6*g^6 - 30*a^2*b^4*c^4*d^4*f^6*g^6 - 3
40*a^3*b^2*c^5*d^4*f^6*g^6 - 80*a^4*c^6*d^4*f^6*g^6 - 6*a*b^7*c^2*d^4*f^5*g
^7 - 36*a^2*b^5*c^3*d^4*f^5*g^7 + 180*a^3*b^3*c^4*d^4*f^5*g^7 + 240*a^4*b*c
^5*d^4*f^5*g^7 + 15*a^2*b^6*c^2*d^4*f^4*g^8 - 225*a^4*b^2*c^4*d^4*f^4*g^8 -
 60*a^5*c^5*d^4*f^4*g^8 - 20*a^3*b^5*c^2*d^4*f^3*g^9 + 50*a^4*b^3*c^3*d^4*f
^3*g^9 + 120*a^5*b*c^4*d^4*f^3*g^9 + 15*a^4*b^4*c^2*d^4*f^2*g^10 - 54*a^5*b
^2*c^3*d^4*f^2*g^10 - 24*a^6*c^4*d^4*f^2*g^10 - 6*a^5*b^3*c^2*d^4*f*g^11 +
24*a^6*b*c^3*d^4*f*g^11 + a^6*b^2*c^2*d^4*g^12 - 4*a^7*c^3*d^4*g^12 - 2*b^3
*c^7*d^3*f^12*e + 8*a*b*c^8*d^3*f^12*e + 12*b^4*c^6*d^3*f^11*g*e - 48*a*b^2
*c^7*d^3*f^11*g*e - 30*b^5*c^5*d^3*f^10*g^2*e + 108*a*b^3*c^6*d^3*f^10*g^2*
e + 48*a^2*b*c^7*d^3*f^10*g^2*e + 40*b^6*c^4*d^3*f^9*g^3*e - 100*a*b^4*c^5*
d^3*f^9*g^3*e - 240*a^2*b^2*c^6*d^3*f^9*g^3*e - 30*b^7*c^3*d^3*f^8*g^4*e +
450*a^2*b^3*c^5*d^3*f^8*g^4*e + 120*a^3*b*c^6*d^3*f^8*g^4*e + 12*b^8*c^2*d^
3*f^7*g^5*e + 72*a*b^6*c^3*d^3*f^7*g^5*e - 360*a^2*b^4*c^4*d^3*f^7*g^5*e -
480*a^3*b^2*c^5*d^3*f^7*g^5*e - 2*b^9*c*d^3*f^6*g^6*e - 52*a*b^7*c^2*d^3*f^
6*g^6*e + 60*a^2*b^5*c^3*d^3*f^6*g^6*e + 680*a^3*b^3*c^4*d^3*f^6*g^6*e + 16
0*a^4*b*c^5*d^3*f^6*g^6*e + 12*a*b^8*c*d^3*f^5*g^7*e + 72*a^2*b^6*c^2*d^3*f
^5*g^7*e - 360*a^3*b^4*c^3*d^3*f^5*g^7*e - 480*a^4*b^2*c^4*d^3*f^5*g^7*e -
30*a^2*b^7*c*d^3*f^4*g^8*e + 450*a^4*b^3*c^3*d^3*f^4*g^8*e + 120*a^5*b*c^4*
d^3*f^4*g^8*e + 40*a^3*b^6*c*d^3*f^3*g^9*e - 100*a^4*b^4*c^2*d^3*f^3*g^9*e
- 240*a^5*b^2*c^3*d^3*f^3*g^9*e - 30*a^4*b^5*c*d^3*f^2*g^10*e + 108*a^5*b^3
*c^2*d^3*f^2*g^10*e + 48*a^6*b*c^3*d^3*f^2*g^10*e + 12*a^5*b^4*c*d^3*f*g^11
*e - 48*a^6*b^2*c^2*d^3*f*g^11*e - 2*a^6*b^3*c*...

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(f+gx)^3 (d+ex) (cx^2+bx+a)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/((f + g*x)^3*(d + e*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.886 $\int (d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=1551

$$-\frac{2(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4(187e^4f^4-732de^3f^3g+1098d^2e^2f^2g^2-798d^3efg^3+3}{}$$

```
[Out] 2/3465*(48*b^3*e^3*g^3+b*c*e^2*g^2*(-157*a*e*g-198*b*d*g+67*b*e*f)+c^3*(-56
7*d^3*g^3+1107*d^2*e*f*g^2-843*d*e^2*f^2*g+233*e^3*f^3)-c^2*e*g*(2*a*e*g*(-
231*d*g+74*e*f)-3*b*(99*d^2*g^2-88*d*e*f*g+24*e^2*f^2)))*(g*x+f)^(3/2)*(c*x
^2+b*x+a)^(1/2)/c^3/g^4-2/693*e*(8*b^2*e^2*g^2+c*e*g*(-18*a*e*g-33*b*d*g+19
*b*e*f)+c^2*(81*d^2*g^2-96*d*e*f*g+29*e^2*f^2))*(g*x+f)^(5/2)*(c*x^2+b*x+a)
^(1/2)/c^2/g^4+2/99*e^2*(b*e*g-3*c*d*g+c*e*f)*(g*x+f)^(7/2)*(c*x^2+b*x+a)^(
1/2)/c/g^4-2/3465*(64*b^4*e^4*g^4+4*b^2*c*e^3*g^3*(-69*a*e*g-66*b*d*g+7*b*e
*f)+c^4*(315*d^4*g^4-798*d^3*e*f*g^3+1098*d^2*e^2*f^2*g^2-732*d*e^3*f^3*g+1
87*e^4*f^4)+3*c^2*e^2*g^2*(50*a^2*e^2*g^2-a*b*e*g*(-297*d*g+29*e*f)+3*b^2*(
44*d^2*g^2-11*d*e*f*g+e^2*f^2))-c^3*e*g*(6*a*e*g*(165*d^2*g^2-33*d*e*f*g+2*
e^2*f^2)+b*(231*d^3*g^3-99*d^2*e*f*g^2+8*e^3*f^3)))*(g*x+f)^(1/2)*(c*x^2+b*
x+a)^(1/2)/c^4/e/g^4+2/11*(e*x+d)^4*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e+1/3
465*(128*b^5*e^3*g^5-8*b^3*c*e^2*g^4*(87*a*e*g+66*b*d*g+7*b*e*f)+2*c^5*f^2*
(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)+b*c^2*e*g^3*(771*
a^2*e^2*g^2+6*a*b*e*g*(396*d*g+43*e*f)-b^2*(-792*d^2*g^2-264*d*e*f*g+37*e^2
*f^2))-c^4*g*(b*f*(-462*d^3*g^3+495*d^2*e*f*g^2-264*d*e^2*f^2*g+56*e^3*f^3)
-18*a*g*(77*d^3*g^3+88*d^2*e*f*g^2-33*d*e^2*f^2*g+6*e^3*f^3))-c^3*g^2*(6*a^
2*e^2*g^2*(231*d*g+26*e*f)-9*a*b*e*g*(-319*d^2*g^2-110*d*e*f*g+15*e^2*f^2)+
b^2*(462*d^3*g^3+495*d^2*e*f*g^2-198*d*e^2*f^2*g+37*e^3*f^3)))*EllipticE(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^
2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^5/g^5/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/3465*(a*
g^2-b*f*g+c*f^2)*(64*b^4*e^3*g^4+4*b^2*c*e^2*g^3*(-69*a*e*g-66*b*d*g+7*b*e*
f)-2*c^4*f*(-231*d^3*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)+3*c^2*
e*g^2*(50*a^2*e^2*g^2-a*b*e*g*(-297*d*g+29*e*f)+3*b^2*(44*d^2*g^2-11*d*e*f*
g+e^2*f^2))-c^3*g*(6*a*e*g*(165*d^2*g^2-33*d*e*f*g+2*e^2*f^2)+b*(231*d^3*g^
3-99*d^2*e*f*g^2+8*e^3*f^3)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^5/g^5/(g
*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 5.34, antiderivative size = 1551, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {932, 1667, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(64*b^4*e^4*g^4 + 4*b^2*c*e^3*g^3*(7*b*e*f - 66*b*d*g - 69*a*e*g) + c^4
*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*g^3 +
315*d^4*g^4) + 3*c^2*e^2*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) +
 3*b^2*(e^2*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*e*g*(6*a*e*g*(2*e^2*f^2 -
 33*d*e*f*g + 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))
*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3465*c^4*e*g^4) + (2*(d + e*x)^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])/(11*e) + (2*(48*b^3*e^3*g^3 + b*c*e^2*g^2
*(67*b*e*f - 198*b*d*g - 157*a*e*g) + c^3*(233*e^3*f^3 - 843*d*e^2*f^2*g +
1107*d^2*e*f*g^2 - 567*d^3*g^3) - c^2*e*g*(2*a*e*g*(74*e*f - 231*d*g) - 3*b
*(24*e^2*f^2 - 88*d*e*f*g + 99*d^2*g^2)))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*
x^2])/(3465*c^3*g^4) - (2*e*(8*b^2*e^2*g^2 + c*e*g*(19*b*e*f - 33*b*d*g - 1
8*a*e*g) + c^2*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt
[a + b*x + c*x^2])/(693*c^2*g^4) + (2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(f + g*
x)^(7/2)*Sqrt[a + b*x + c*x^2])/(99*c*g^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(12
8*b^5*e^3*g^5 - 8*b^3*c*e^2*g^4*(7*b*e*f + 66*b*d*g + 87*a*e*g) + 2*c^5*f^2
*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + b*c^2*e*g
^3*(771*a^2*e^2*g^2 + 6*a*b*e*g*(43*e*f + 396*d*g) - b^2*(37*e^2*f^2 - 264*
d*e*f*g - 792*d^2*g^2)) - c^4*g*(b*f*(56*e^3*f^3 - 264*d*e^2*f^2*g + 495*d^
2*e*f*g^2 - 462*d^3*g^3) - 18*a*g*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*
g^2 + 77*d^3*g^3)) - c^3*g^2*(6*a^2*e^2*g^2*(26*e*f + 231*d*g) - 9*a*b*e*g*
(15*e^2*f^2 - 110*d*e*f*g - 319*d^2*g^2) + b^2*(37*e^3*f^3 - 198*d*e^2*f^2*
g + 495*d^2*e*f*g^2 + 462*d^3*g^3)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(3465*c^5*g^5*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 -
b*f*g + a*g^2)*(64*b^4*e^3*g^4 + 4*b^2*c*e^2*g^3*(7*b*e*f - 66*b*d*g - 69*a
*e*g) - 2*c^4*f*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g
^3) + 3*c^2*e*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(29*e*f - 297*d*g) + 3*b^2*(e^2
*f^2 - 11*d*e*f*g + 44*d^2*g^2)) - c^3*g*(6*a*e*g*(2*e^2*f^2 - 33*d*e*f*g +
 165*d^2*g^2) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3)))*Sqrt[(c*(f +
 g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(3465*c^5*g^5*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+bx+cx^2}\,dx = \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} - \frac{\int \frac{(d+ex)^3(bdf-3aef+adg+2(cd}{\sqrt{f+gx}}}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} + \frac{2e^2(cef-3cdg+beg)(f}{99}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} - \frac{2e(8b^2e^2g^2+ceg(19bef}{}$$

$$= \frac{2(d+ex)^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}{11e} + \frac{2(48b^3e^3g^3+bce^2g^2(67be}{}$$

$$= -\frac{2(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4(187e^4f^4-}{}$$

$$= -\frac{2(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4(187e^4f^4-}{}$$

$$= -\frac{2(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4(187e^4f^4-}{}$$

$$= -\frac{2(64b^4e^4g^4+4b^2ce^3g^3(7bef-66bdg-69aeg)+c^4(187e^4f^4-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.03, size = 26600, normalized size = 17.15

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32646 vs. $2(1469) = 2938$.
time = 0.28, size = 32647, normalized size = 21.05

method	result	size
elliptic	Expression too large to display	3254
risch	Expression too large to display	11966
default	Expression too large to display	32647

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 1782, normalized size = 1.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/10395*((462*c^6*d^3*f^3*g^3 - 693*b*c^5*d^3*f^2*g^4 - 693*(b^2*c^4 - 6*a*
c^5)*d^3*f*g^5 + 231*(2*b^3*c^3 - 9*a*b*c^4)*d^3*g^6 - (128*c^6*f^6 - 120*b
*c^5*f^5*g - 3*(11*b^2*c^4 - 68*a*c^5)*f^4*g^2 - (20*b^3*c^3 - 87*a*b*c^4)*
f^3*g^3 - 3*(11*b^4*c^2 - 53*a*b^2*c^3 + 34*a^2*c^4)*f^2*g^4 - 3*(40*b^5*c
- 246*a*b^3*c^2 + 329*a^2*b*c^3)*f*g^5 + (128*b^6 - 888*a*b^4*c + 1599*a^2*
b^2*c^2 - 450*a^3*c^3)*g^6)*e^3 + 33*(16*c^6*d*f^5*g - 16*b*c^5*d*f^4*g^2 -
 5*(b^2*c^4 - 6*a*c^5)*d*f^3*g^3 - (5*b^3*c^3 - 21*a*b*c^4)*d*f^2*g^4 - 2*(
8*b^4*c^2 - 42*a*b^2*c^3 + 33*a^2*c^4)*d*f*g^5 + (16*b^5*c - 96*a*b^3*c^2 +
```

```
123*a^2*b*c^3)*d*g^6)*e^2 - 99*(8*c^6*d^2*f^4*g^2 - 9*b*c^5*d^2*f^3*g^3 -
2*(2*b^2*c^4 - 11*a*c^5)*d^2*f^2*g^4 - (9*b^3*c^3 - 41*a*b*c^4)*d^2*f*g^5 +
(8*b^4*c^2 - 41*a*b^2*c^3 + 30*a^2*c^4)*d^2*g^6)*e)*sqrt(c*g)*weierstrassP
Inverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3
*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(
c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(462*c^6*d^3*f^2*g^4 - 462*b
*c^5*d^3*f*g^5 + 462*(b^2*c^4 - 3*a*c^5)*d^3*g^6 - (128*c^6*f^5*g - 56*b*c^
5*f^4*g^2 - (37*b^2*c^4 - 108*a*c^5)*f^3*g^3 - (37*b^3*c^3 - 135*a*b*c^4)*f
^2*g^4 - 2*(28*b^4*c^2 - 129*a*b^2*c^3 + 78*a^2*c^4)*f*g^5 + (128*b^5*c - 6
96*a*b^3*c^2 + 771*a^2*b*c^3)*g^6)*e^3 + 66*(8*c^6*d*f^4*g^2 - 4*b*c^5*d*f^
3*g^3 - 3*(b^2*c^4 - 3*a*c^5)*d*f^2*g^4 - (4*b^3*c^3 - 15*a*b*c^4)*d*f*g^5
+ (8*b^4*c^2 - 36*a*b^2*c^3 + 21*a^2*c^4)*d*g^6)*e^2 - 99*(8*c^6*d^2*f^3*g^
3 - 5*b*c^5*d^2*f^2*g^4 - (5*b^2*c^4 - 16*a*c^5)*d^2*f*g^5 + (8*b^3*c^3 - 2
9*a*b*c^4)*d^2*g^6)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (
b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c -
6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*
(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c
^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/
3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(693*c^6*d^3*g^6*x + 231*c^6*d^3*f*g^5
+ 231*b*c^5*d^3*g^6 + (315*c^6*g^6*x^4 - 64*c^6*f^4*g^2 + 20*b*c^5*f^3*g^3
+ 2*(9*b^2*c^4 - 23*a*c^5)*f^2*g^4 + 10*(2*b^3*c^3 - 7*a*b*c^4)*f*g^5 - 2*(
32*b^4*c^2 - 138*a*b^2*c^3 + 75*a^2*c^4)*g^6 + 35*(c^6*f*g^5 + b*c^5*g^6)*x
^3 - 10*(4*c^6*f^2*g^4 - b*c^5*f*g^5 + (4*b^2*c^4 - 9*a*c^5)*g^6)*x^2 + (48
*c^6*f^3*g^3 - 13*b*c^5*f^2*g^4 - (13*b^2*c^4 - 32*a*c^5)*f*g^5 + (48*b^3*c
^3 - 157*a*b*c^4)*g^6)*x)*e^3 + 33*(35*c^6*d*g^6*x^3 + 8*c^6*d*f^3*g^3 - 3*
b*c^5*d*f^2*g^4 - (3*b^2*c^4 - 8*a*c^5)*d*f*g^5 + (8*b^3*c^3 - 27*a*b*c^4)*
d*g^6 + 5*(c^6*d*f*g^5 + b*c^5*d*g^6)*x^2 - 2*(3*c^6*d*f^2*g^4 - b*c^5*d*f*
g^5 + (3*b^2*c^4 - 7*a*c^5)*d*g^6)*x)*e^2 + 99*(15*c^6*d^2*g^6*x^2 - 4*c^6*
d^2*f^2*g^4 + 2*b*c^5*d^2*f*g^5 - 2*(2*b^2*c^4 - 5*a*c^5)*d^2*g^6 + 3*(c^6*
d^2*f*g^5 + b*c^5*d^2*g^6)*x)*e)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^6*
g^6)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^3 \sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2), x)
```

3.887 $\int (d+ex)^2 \sqrt{f+gx} \sqrt{a+bx+cx^2} \, dx$

Optimal. Leaf size=1015

$$\frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2g+63d^2efg^2-35d^3g^3)-3c^2eg^2(2ae(ef-1}{315c^3eg^3}$$

```
[Out] -4/315*(3*b^2*e^2*g^2+c*e*g*(-7*a*e*g-9*b*d*g+4*b*e*f)+c^2*(21*d^2*g^2-24*d
*e*f*g+8*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3+2/63*e*(b*e*g-
3*c*d*g+c*e*f)*(g*x+f)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/g^3+2/315*(8*b^3*e^3*g^3
+3*b*c*e^2*g^2*(-9*a*e*g-8*b*d*g+b*e*f)+c^3*(-35*d^3*g^3+63*d^2*e*f*g^2-57*
d*e^2*f^2*g+19*e^3*f^3)-3*c^2*e*g^2*(2*a*e*(-10*d*g+e*f)+b*d*(-7*d*g+2*e*f)
))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/e/g^3+2/9*(e*x+d)^3*(g*x+f)^(1/2)*
(c*x^2+b*x+a)^(1/2)/e-2/315*(8*b^4*e^2*g^4-4*b^2*c*e*g^3*(9*a*e*g+6*b*d*g+b
*e*f)+c^4*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)+3*c^2*g^2*(7*a^2*e^2*g^2+a*
b*e*g*(29*d*g+5*e*f)-b^2*(-7*d^2*g^2-5*d*e*f*g+e^2*f^2))+c^3*g*(3*a*g*(-21*
d^2*g^2-16*d*e*f*g+3*e^2*f^2)-b*f*(21*d^2*g^2-15*d*e*f*g+4*e^2*f^2)))*Ellip
ticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-
2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^4/g^4
/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3
15*(a*g^2-b*f*g+c*f^2)*(8*b^3*e^2*g^3+3*b*c*e*g^2*(-9*a*e*g-8*b*d*g+b*e*f)-
2*c^3*f*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)-3*c^2*g^2*(2*a*e*(-10*d*g+e*f)+b*
d*(-7*d*g+2*e*f)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/g^4/(g*x+f)^(1/2)
/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 2.62, antiderivative size = 1015, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {932, 1667, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(b*e*f - 8*b*d*g - 9*a*e*g) + c^3*(19*e^3
*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3) - 3*c^2*e*g^2*(2*a*e*(
e*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/
(315*c^3*e*g^3) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*e)
 - (4*(3*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 9*b*d*g - 7*a*e*g) + c^2*(8*e^2*f^2
 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(315*c^
2*g^3) + (2*e*(c*e*f - 3*c*d*g + b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^
2])/(63*c*g^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^4*e^2*g^4 - 4*b^2*c*e*g^
3*(b*e*f + 6*b*d*g + 9*a*e*g) + c^4*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^
2) + 3*c^2*g^2*(7*a^2*e^2*g^2 + a*b*e*g*(5*e*f + 29*d*g) - b^2*(e^2*f^2 - 5
*d*e*f*g - 7*d^2*g^2)) + c^3*g*(3*a*g*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2)
 - b*f*(4*e^2*f^2 - 15*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^2*g^3 + 3*b*c*e*g^2*(b*e*f - 8*b*d*g - 9*a
*e*g) - 2*c^3*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*c^2*g^2*(2*a*e*(e
*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^4*Sqr
t[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
```

```
b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9e} - \frac{\int \frac{(d+ex)^2(bdf-3aef+adg+2(\cdots}{\sqrt{f+g\cdots}}}{\cdots}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9e} + \frac{2e(cef-3cdg+beg)(f\cdots}{63\cdots}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9e} - \frac{4(3b^2e^2g^2+ceg(4bef-\cdots}{\cdots}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2\cdots}{\cdots}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2\cdots}{\cdots}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2\cdots}{\cdots}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(bef-8bdg-9aeg)+c^3(19e^3f^3-57de^2f^2\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.95, size = 15781, normalized size = 15.55

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 20223 vs. 2(939) = 1878.
time = 0.16, size = 20224, normalized size = 19.93

method	result	size
elliptic	Expression too large to display	1936
risch	Expression too large to display	7219
default	Expression too large to display	20224

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 1158, normalized size = 1.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/945*((42*c^5*d^2*f^3*g^2 - 63*b*c^4*d^2*f^2*g^3 - 63*(b^2*c^3 - 6*a*c^4)*
d^2*f*g^4 + 21*(2*b^3*c^2 - 9*a*b*c^3)*d^2*g^5 + (16*c^5*f^5 - 16*b*c^4*f^4
*g - 5*(b^2*c^3 - 6*a*c^4)*f^3*g^2 - (5*b^3*c^2 - 21*a*b*c^3)*f^2*g^3 - 2*(
8*b^4*c - 42*a*b^2*c^2 + 33*a^2*c^3)*f*g^4 + (16*b^5 - 96*a*b^3*c + 123*a^2
*b*c^2)*g^5)*e^2 - 6*(8*c^5*d*f^4*g - 9*b*c^4*d*f^3*g^2 - 2*(2*b^2*c^3 - 11
*a*c^4)*d*f^2*g^3 - (9*b^3*c^2 - 41*a*b*c^3)*d*f*g^4 + (8*b^4*c - 41*a*b^2*
c^2 + 30*a^2*c^3)*d*g^5)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*
c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*
(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x +
c*f + b*g)/(c*g)) + 6*(21*c^5*d^2*f^2*g^3 - 21*b*c^4*d^2*f*g^4 + 21*(b^2*c^
3 - 3*a*c^4)*d^2*g^5 + (8*c^5*f^4*g - 4*b*c^4*f^3*g^2 - 3*(b^2*c^3 - 3*a*c^
```

```
4)*f^2*g^3 - (4*b^3*c^2 - 15*a*b*c^3)*f*g^4 + (8*b^4*c - 36*a*b^2*c^2 + 21*
a^2*c^3)*g^5)*e^2 - 3*(8*c^5*d*f^3*g^2 - 5*b*c^4*d*f^2*g^3 - (5*b^2*c^3 - 1
6*a*c^4)*d*f*g^4 + (8*b^3*c^2 - 29*a*b*c^3)*d*g^5)*e)*sqrt(c*g)*weierstrass
Zeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^
3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3
*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2
*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^
3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(63*c^5*
d^2*g^5*x + 21*c^5*d^2*f*g^4 + 21*b*c^4*d^2*g^5 + (35*c^5*g^5*x^3 + 8*c^5*f
^3*g^2 - 3*b*c^4*f^2*g^3 - (3*b^2*c^3 - 8*a*c^4)*f*g^4 + (8*b^3*c^2 - 27*a*
b*c^3)*g^5 + 5*(c^5*f*g^4 + b*c^4*g^5)*x^2 - 2*(3*c^5*f^2*g^3 - b*c^4*f*g^4
 + (3*b^2*c^3 - 7*a*c^4)*g^5)*x)*e^2 + 6*(15*c^5*d*g^5*x^2 - 4*c^5*d*f^2*g^
3 + 2*b*c^4*d*f*g^4 - 2*(2*b^2*c^3 - 5*a*c^4)*d*g^5 + 3*(c^5*d*f*g^4 + b*c^
4*d*g^5)*x)*e)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^5*g^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^2 \sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2), x)
```

3.888 $\int (d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=652

$$-\frac{2\sqrt{f+gx}\,(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5aeg)-3cg(cef+7cdg-4beg)x)\sqrt{a+bx+cx}}{105c^2g^2}$$

[Out] 2/7*e*(c*x^2+b*x+a)^(3/2)*(g*x+f)^(1/2)/c-2/105*(4*b^2*e*g^2+c^2*f*(-7*d*g+4*e*f)-c*g*(-5*a*e*g+7*b*d*g+2*b*e*f)-3*c*g*(-4*b*e*g+7*c*d*g+c*e*f)*x)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^2+1/105*((-4*b*e*g+7*c*d*g+c*e*f)*(8*c^2*f^2-2*b^2*g^2-3*c*g*(-2*a*g+b*f))-5*c*g*(-b*g+2*c*f)*(7*c*d*f-e*(a*g+3*b*f)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/105*(a*g^2-b*f*g+c*f^2)*(4*b^2*e*g^2-2*c^2*f*(-7*d*g+4*e*f)+c*g*(-10*a*e*g-7*b*d*g+b*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]
time = 0.70, antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {846, 828, 857, 732, 435, 430}

[illegible]

Antiderivative was successfully verified.

[In] Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]

[Out] (-2*Sqrt[f + g*x]*(4*b^2*e*g^2 + c^2*f*(4*e*f - 7*d*g) - c*g*(2*b*e*f + 7*b*d*g - 5*a*e*g) - 3*c*g*(c*e*f + 7*c*d*g - 4*b*e*g)*x)*Sqrt[a + b*x + c*x^2])/(105*c^2*g^2) + (2*e*Sqrt[f + g*x]*(a + b*x + c*x^2)^(3/2))/(7*c) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*((c*e*f + 7*c*d*g - 4*b*e*g)*(8*c^2*f^2 - 2*b^2*g^2 - 3*c*g*(b*f - 2*a*g)) - 5*c*g*(2*c*f - b*g)*(7*c*d*f - e*(3*b*f + a*g)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[

```
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^3*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2
*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(4*b^2*e*g^2 - 2*c^2*f*(
4*e*f - 7*d*g) + c*g*(b*e*f - 7*b*d*g - 10*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(1
05*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx = \frac{2e\sqrt{f+gx}\,(a+bx+cx^2)^{3/2}}{7c} + \frac{2\int \frac{\left(\frac{1}{2}(7cdf-3bef-aeg)+\frac{1}{2}(cef+7cdg-4\ldots\right.}{\sqrt{f+g\ldots}}}{7c}$$

$$= -\frac{2\sqrt{f+gx}\,(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5ae\ldots}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\,(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5ae\ldots}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\,(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5ae\ldots}{105c^2g^2}$$

$$= -\frac{2\sqrt{f+gx}\,(4b^2eg^2+c^2f(4ef-7dg)-cg(2bef+7bdg-5ae\ldots}{105c^2g^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.06, size = 8432, normalized size = 12.93

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10710 vs. 2(588) = 1176.
time = 0.13, size = 10711, normalized size = 16.43

method	result
elliptic	$\frac{\sqrt{(gx+f)\left(c\,x^{2}+bx+a\right)}\left(\frac{2e\,x^{2}\sqrt{cg\,x^{3}+bg\,x^{2}+cf\,x^{2}+axg+bfx+fa}}{7}+\frac{2\left(beg+dgc+cef-\frac{2e(3bg+3cf)}{7}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 740, normalized size = 1.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*((14*c^4*d*f^3*g - 21*b*c^3*d*f^2*g^2 - 21*(b^2*c^2 - 6*a*c^3)*d*f*g^
3 + 7*(2*b^3*c - 9*a*b*c^2)*d*g^4 - (8*c^4*f^4 - 9*b*c^3*f^3*g - 2*(2*b^2*c
```

```
^2 - 11*a*c^3)*f^2*g^2 - (9*b^3*c - 41*a*b*c^2)*f*g^3 + (8*b^4 - 41*a*b^2*c
 + 30*a^2*c^2)*g^4)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g
 + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*
c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f +
 b*g)/(c*g)) + 3*(14*c^4*d*f^2*g^2 - 14*b*c^3*d*f*g^3 + 14*(b^2*c^2 - 3*a*c
^3)*d*g^4 - (8*c^4*f^3*g - 5*b*c^3*f^2*g^2 - (5*b^2*c^2 - 16*a*c^3)*f*g^3 +
 (8*b^3*c - 29*a*b*c^2)*g^4)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*
c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*
(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInv
erse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^
3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3
*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(21*c^4*d*g^4*x + 7*c^4*d*f*g^
3 + 7*b*c^3*d*g^4 + (15*c^4*g^4*x^2 - 4*c^4*f^2*g^2 + 2*b*c^3*f*g^3 - 2*(2*
b^2*c^2 - 5*a*c^3)*g^4 + 3*(c^4*f*g^3 + b*c^3*g^4)*x)*e)*sqrt(c*x^2 + b*x +
 a)*sqrt(g*x + f))/(c^4*g^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2), x)
```

3.889 $\int \sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=513

$$-\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg}+\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}-\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,(c^2f^2+b^2g^2-cg(b\cdots}{\cdots}$$

[Out] 2/5*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/g-2/15*(-b*g+2*c*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g-2/15*(c^2*f^2+b^2*g^2-c*g*(3*a*g+b*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/g^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-b*g+2*c*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]
time = 0.40, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {748, 846, 857, 732, 435, 430}

$$\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(2cf-bg)(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{15c^2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}-\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(-cg(3ag+bf)+b^2g^2+c^2f^2)\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{15c^2g^2\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g}-\frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,(2cf-bg)}{15cg}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]

[Out] (-2*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c*g) + (2*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*g) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*f - b*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b

```
 + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4
*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^2*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx = \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{\int \frac{\sqrt{f+gx}\,(bf-2ag+(2cf-bg)x)}{\sqrt{a+bx+cx^2}}\,dx}{5g}$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} -$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} +$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} -$$

$$= -\frac{2(2cf-bg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15cg} + \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.20, size = 1086, normalized size = 2.12

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + x*(b + c*x))*(b*g + c*(f + 3*g*x)))/(c*g) + (-4*g^2
*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
(-3*a^2*c*g^2 + (c^2*f^2 - b*c*f*g + b^2*g^2)*x*(b + c*x) + a*(b^2*g^2 - b*
c*g*(f + 3*g*x) + c^2*(f^2 - 3*g^2*x^2))) + I*Sqrt[2]*(2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*(f + g*x)^(3/2)*
Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*
c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*
x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 -
4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(
f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*
f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt
[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + I*Sqrt[2]*
(b^3*g^3 - b^2*g^2*(2*c*f + Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*g*(-4*a*g^2 + f*
Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-(c*f^2*Sqrt[(b^2 - 4*a*c)*g^2]) + a*g^2*(8*c
*f + 3*Sqrt[(b^2 - 4*a*c)*g^2])))*(f + g*x)^(3/2)*Sqrt[(-2*a*g^2 + f*Sqrt[(
b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x)
)/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sq
rt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f +
 g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*Ar
cSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a
*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c
*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/(c^2*g^3*Sqrt[(c*f^2 + g*(-(b*f) + a
*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(f + g*x))))/(15*Sqrt[a + x*
(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4355 vs. $2(449) = 898$.
time = 0.16, size = 4356, normalized size = 8.49

method	result

elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+axg+bfx+fa}}{5}+\frac{2\left(\frac{bg}{5}+\frac{cf}{5}\right)\sqrt{cg\,x^3+bg\,x^2}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2*(10*b*c^2*f*g^3*x^2+2*a*b*c*g^4*
x+8*a*c^2*f*g^3*x+2*b^2*c*f*g^3*x+2*b*c^2*f^2*g^2*x+2*a*b*c*f*g^3+8*b*c^2*g
^4*x^3+8*c^3*f*g^3*x^3+6*a*c^2*g^4*x^2+2*b^2*c*g^4*x^2+2*c^3*f^2*g^2*x^2+2*
a*c^2*f^2*g^2+6*c^3*g^4*x^4+4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(
1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(
1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(
1/2))*c^3*f^4+12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2
)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*
((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Ell
ipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*f
^2*g^2-3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2
*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*
x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2
^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*f^2*g^2-(
-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)
*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*El
lipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*f*
```

g^3-8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c^2*f^3*g-8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*f^2*g^2+(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*g^4-2*(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*f^3*g+8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*f^2*g^2+3*(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*f^2*g^2+3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^3*f*g^3-12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*c*g^4-12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 481, normalized size = 0.94

$$\frac{2\left(\left(2c^3f^3-3bc^2f^2g-3\left(b^2c-6ac^2\right)fg^2+\left(2b^3-9abc\right)g^3\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3ac\right)g^2\right)}{3c^2g^2},-\frac{4\left(2c^3f^3-3bc^2f^2g-3\left(b^2c-6ac^2\right)fg^2+\left(2b^3-9abc\right)g^3\right)}{27c^3g^3},\frac{3cgx+cf+bg}{3cg}\right)+6\left(c^3f^2g-bc^2fg^2+\left(b^2c-3ac^2\right)g^3\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3ac\right)g^2\right)}{3c^2g^2},-\frac{4\left(2c^3f^3-3bc^2f^2g-3\left(b^2c-6ac^2\right)fg^2+\left(2b^3-9abc\right)g^3\right)}{27c^3g^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3ac\right)g^2\right)}{3c^2g^2},-\frac{4\left(2c^3f^3-3bc^2f^2g-3\left(b^2c-6ac^2\right)fg^2+\left(2b^3-9abc\right)g^3\right)}{27c^3g^3},\frac{3cgx+cf+bg}{3cg}\right)\right)+3\left(3c^3g^3x+c^3fg^2+bc^2g^3\right)\sqrt{cx^2+bx+a}\,\sqrt{gx+f}\right)}{45c^3g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*((2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a
*b*c)*g^3)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*
a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)
*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))
 + 6*(c^3*f^2*g - b*c^2*f*g^2 + (b^2*c - 3*a*c^2)*g^3)*sqrt(c*g)*weierstras
sZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f
^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^
3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^
2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b
^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(3*c^3*
g^3*x + c^3*f*g^2 + b*c^2*g^3)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^3*g^
3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2), x)
```

3.890 $\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{d+ex}\,dx$

Optimal. Leaf size=764

$$\frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3e}+\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(cef-3cdg+beg)\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\right.\right.}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a}}$$

[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e+1/3*(b*e*g-3*c*d*g+c*e*f)*EllipticE (1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g* (-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c +b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/g/(c* x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/3*(e* g*(2*a*e*g-3*b*d*g+b*e*f)+c*(3*d^2*g^2-e^2*f^2))*EllipticF(1/2*((b+2*c*x+(- 4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a*c+b^2) ^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2) *(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1 /2))))^(1/2)/c/e^3/g/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-(a*e^2-b*d*e+c*d^2)* EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a* c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f) /(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/c^(1/2)/(c* x^2+b*x+a)^(1/2)

Rubi [A]
time = 2.73, antiderivative size = 969, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {932, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

Antiderivative was successfully verified.

[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]

[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]* (c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])

```
*g)])/(3*c*e^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sq
rt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*(c*e*f - 3*c*d*g + b*
e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
- (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(3*c*d*(e*f - d*g) - e*(2*b*e*f - 3*b*d*g +
2*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c
*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*e^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2
]) - (Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f -
b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt
[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b
*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
```

```
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3e} - \frac{\int \frac{bdf-3aef+adg+2(cdf-bef+bdg-aeg)x-(cef-3cdg+b\cdots}{(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} \cdots}{3e}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3e} - \frac{\int \left(\frac{3cd(ef-dg)-e(2bef-3bdg+2aeg)}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} - \frac{(c\cdots}{e\sqrt{f+\cdots}} \cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3e} + \frac{\left((cd^2-bde+ae^2)(ef-dg)\right)\int \frac{\cdots}{(d+ex)\sqrt{f-\cdots}} \cdots}{e^3}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3e} + \frac{(cef-3cdg+beg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3e^2g} - \cdots$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3e} - \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,(3cd(ef-dg)-e(2bef-\cdots}{\cdots}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(cef-3cdg+beg)\sqrt{f+g\cdots}}{3c\cdots}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3e} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(cef-3cdg+beg)\sqrt{f+g\cdots}}{3c\cdots}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3\cdots} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(cef-3cdg+beg)\sqrt{f+g\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.18, size = 35245, normalized size = 46.13

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6811 vs. 2(674) = 1348.
time = 0.17, size = 6812, normalized size = 8.92

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+axg+bfx+fa}}{3e}+\frac{2\left(\frac{a\,e^2g-bdeg+b\,e^2f+c\,d^2g-cd}{e^3}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(x*e + d), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x),x)
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x), x)
```

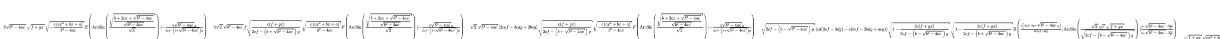
3.891 $\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=743

$$-\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{e(d+ex)}+\frac{3\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}\,e^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+bx+}}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(e*x+d)+3/2*EllipticE(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2
)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)
*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(
g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+(2*b*e*g-c*(3*d*g+e*f))*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*
f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)+
1/2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticPi(2^(1/2)*c^(1/2)
*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(
-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))
^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/e^3/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2
)
```

Rubi [A]
time = 2.23, antiderivative size = 934, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {930, 6874, 732, 430, 857, 435, 948, 175, 552, 551}



Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x))) + (3*Sqrt[b^2 - 4*a*
c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^2*Sqrt
```

```
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2])
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*e*f - 3*c*d*g + 2*b
*e*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticP
i[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2
]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqr
t[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*
Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

implerSqrtQ[-f/e, -d/c])

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f + b
*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
```

```
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\int \frac{bf+ag+2(cf+bg)x+3cgx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{2e}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\int \left(\frac{2cef-3cdg+2beg}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{}{e\sqrt{f}\cdots}\right.}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{(3cg)\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{2e^2} + \cdots$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{(3c)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2e^2} - \frac{(3cf)\int \frac{}{\sqrt{f}\cdots}}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(2cef-3cdg+2beg)\sqrt{\frac{\cdots}{2\cdots}}}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+c\cdots}{b^2-4ac}}}{\sqrt{2}\,e^2\sqrt{\frac{\cdots}{2c\cdots}}}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{e(d+ex)} + \frac{3\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+c\cdots}{b^2-4ac}}}{\sqrt{2}\,e^2\sqrt{\frac{\cdots}{2c\cdots}}}$$

$$3\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+c\cdots}{b^2-4ac}}$$

$$\sqrt{f+gx}\,\sqrt{a+bx+cx^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.31, size = 16573, normalized size = 22.31

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^2,x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 16695 vs. $2(660) = 1320$.
time = 0.14, size = 16696, normalized size = 22.47

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(-\frac{\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+axg+bfx+fa}}{e(ex+d)}+\frac{2\left(\frac{beg-2dgc+cef}{e^3}+\frac{cdg}{2e^3}\right)\left(-\frac{b+}{}\right.}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(x*e + d)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f + gx}\,\sqrt{a + bx + cx^2}}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(x*e + d)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f + g\,x}\,\sqrt{c\,x^2 + b\,x + a}}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^2,x)
```

```
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^2, x)
```

3.892 $\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=1034

$\sqrt{b^2-4ac}$

$$-\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2}+\frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{4e\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)}-$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(e*x+d)^2+1/4*(c*d*(-3*d*g+2*e*f)-
e*(a*e*g-2*b*d*g+b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c
*d^2)/(-d*g+e*f)/(e*x+d)-1/8*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*E
llipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c
+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-
b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+
(-4*a*c+b^2)^(1/2))))^(1/2)-1/4*(-c*d*(3*d*g+2*e*f)+e*(-5*a*e*g+4*b*d*g+b*e
*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1
/2))*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c
*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(c*d^2+e*(a*e-b*d))*2^(1/2)/(g*x+f)
^(1/2)/(a+x*(c*x+b))^(1/2)+1/8*(b^2*e^4*f^2+a^2*e^4*g^2+c^2*d^3*g*(-3*d*g+4
*e*f)-2*a*c*e^2*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2)-2*b*e*g*(a*e^3*f+c*d^2*(-2*
d*g+3*e*f)))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2))^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*c*d*g+2*c*e*f)
,((2*c*f+g*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(
2*c*f+g*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-2
*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/(c*d^2+e*(a*e-b*d))/(-d*g+e*f)^2*
2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(1/2)
```

Rubi [A]
time = 5.41, antiderivative size = 1705, normalized size of antiderivative = 1.65, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {930, 6874, 732, 430, 953, 857, 435, 948, 175, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

```
[Out] -1/2*(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x)^2) + ((c*d*(2*e*f -
 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
/(4*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)) - (Sqrt[b^2 - 4*a*c]*(
c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
- (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(e*f
- d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x
 + c*x^2]) + (3*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*e^3*Sqrt
[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*f*(c*d*(2*e*f - 3*d*g
) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt
[b^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[
(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)])/(2*Sqrt[2]*e^3*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f
+ g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c
*e*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4
*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*El
lipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[
(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (
b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(S
qrt[2]*Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b -
Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))^2
*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*
c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*
g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f
 + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] -
 (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e^3*(c
*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
```

```
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f + b
*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{\int \frac{bf+ag+2(cf+bg)x+3cgx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4e}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{\int \left(\frac{3cg}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{-cd}{e^2(d+ex)}\right.}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(3cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4e^3} + \frac{(c}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{}}{4e\,(cd^2-bde+ae^2)\,(ef-dg}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{}}{4e\,(cd^2-bde+ae^2)\,(ef-dg}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{}}{4e\,(cd^2-bde+ae^2)\,(ef-dg}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{}}{4e\,(cd^2-bde+ae^2)\,(ef-dg}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2e(d+ex)^2} + \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{}}{4e\,(cd^2-bde+ae^2)\,(ef-dg}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.71, size = 33765, normalized size = 32.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55359 vs. $2(934) = 1868$.
time = 0.21, size = 55360, normalized size = 53.54

method	result	size
elliptic	Expression too large to display	1593
default	Expression too large to display	55360

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(x*e + d)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**3,x)
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^3,x)
[Out] int(((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^3, x)
```

3.893 $\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1098

$$\frac{2(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3(152e^3f^3-408de^2f^2g+336d^2efg^2-70d^3g^3)-3c^2eg(6aeg}{315c^3g^4}$$

```
[Out] -2/315*e*(6*b^2*e^2*g^2+c*e*g*(-14*a*e*g-27*b*d*g+17*b*e*f)-2*c^2*(42*d^2*g
^2-111*d*e*f*g+64*e^2*f^2))*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^4-2/63*
e^2*(-b*e*g-6*c*d*g+8*c*e*f)*(g*x+f)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/g^4+2/315*
(8*b^3*e^3*g^3+3*b*c*e^2*g^2*(-9*a*e*g-12*b*d*g+5*b*e*f)-c^3*(-70*d^3*g^3+3
36*d^2*e*f*g^2-408*d*e^2*f^2*g+152*e^3*f^3)-3*c^2*e*g*(6*a*e*g*(-5*d*g+2*e*
f)-b*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/
c^3/g^4+2/9*(e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g-1/315*(16*b^4*e^3
*g^4+8*b^2*c*e^2*g^3*(-9*a*e*g-9*b*d*g+2*b*e*f)-2*c^4*f*(-105*d^3*g^3+252*d
^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3)+3*c^2*e*g^2*(14*a^2*e^2*g^2-a*b*e*g*
(-87*d*g+19*e*f)+b^2*(42*d^2*g^2-27*d*e*f*g+7*e^2*f^2))-c^3*g*(6*a*e*g*(63*
d^2*g^2-39*d*e*f*g+10*e^2*f^2)-b*(-105*d^3*g^3+189*d^2*e*f*g^2-144*d*e^2*f^
2*g+40*e^3*f^3)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2
))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*
a*c+b^2))^(1/2)/c^4/g^5/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2)-2/315*(a*g^2-b*f*g+c*f^2)*(8*b^3*e^3*g^3+3*b*c*e^2*g^2*
(-9*a*e*g-12*b*d*g+5*b*e*f)+2*c^3*(-105*d^3*g^3+252*d^2*e*f*g^2-216*d*e^2*f
^2*g+64*e^3*f^3)-3*c^2*e*g*(6*a*e*g*(-5*d*g+2*e*f)-b*(21*d^2*g^2-24*d*e*f*g
+8*e^2*f^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c
*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/g^5/(g*x+f)^(1/2)/(c*x
^2+b*x+a)^(1/2)
```

Rubi [A]

time = 4.41, antiderivative size = 1098, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {934, 1667, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) - c^3*(152
*e^3*f^3 - 408*d*e^2*f^2*g + 336*d^2*e*f*g^2 - 70*d^3*g^3) - 3*c^2*e*g*(6*a
*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g
*x]*Sqrt[a + b*x + c*x^2])/(315*c^3*g^4) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2])/(9*g) - (2*e*(6*b^2*e^2*g^2 + c*e*g*(17*b*e*f - 27*b*d*
g - 14*a*e*g) - 2*c^2*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3
/2)*Sqrt[a + b*x + c*x^2])/(315*c^2*g^4) - (2*e^2*(8*c*e*f - 6*c*d*g - b*e*
g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(63*c*g^4) - (Sqrt[2]*Sqrt[b^2 -
4*a*c]*(16*b^4*e^3*g^4 + 8*b^2*c*e^2*g^3*(2*b*e*f - 9*b*d*g - 9*a*e*g) - 2*
c^4*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) + 3*c^
2*e*g^2*(14*a^2*e^2*g^2 - a*b*e*g*(19*e*f - 87*d*g) + b^2*(7*e^2*f^2 - 27*d
*e*f*g + 42*d^2*g^2)) - c^3*g*(6*a*e*g*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^
2) - b*(40*e^3*f^3 - 144*d*e^2*f^2*g + 189*d^2*e*f*g^2 - 105*d^3*g^3)))*Sqr
t[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sqrt[(c*(f
 + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*S
qrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^3*g^3 + 3*b*c*e^2
*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) + 2*c^3*(64*e^3*f^3 - 216*d*e^2*f^2*g +
 252*d^2*e*f*g^2 - 105*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8
*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2
*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(315*c^4*g^5*Sq
rt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
```

```
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt
[a + b*x + c*x^2]/(g*(2*m + 3))), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)
^(m - 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f + 2*a*(e*f*m - d
*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e*f - d*g)*(2*m + 1))*x - (b*e*g + 2*
c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9g} - \frac{\int \frac{(d+ex)^2(bdf+6aef-8adg+(2cdf+7bef-7b\cdots}{\sqrt{f+gx}\,\sqrt{a+\cdots}}}{9g}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9g} - \frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/\cdots}}{63cg^4}$$

$$= \frac{2(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{9g} - \frac{2e(6b^2e^2g^2+ceg(17bef-27bdg-\cdots}{}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3(152e^3f^3-408de^2f^2g+3\cdots}{}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3(152e^3f^3-408de^2f^2g+3\cdots}{}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3(152e^3f^3-408de^2f^2g+3\cdots}{}$$

$$= \frac{2(8b^3e^3g^3+3bce^2g^2(5bef-12bdg-9aeg)-c^3(152e^3f^3-408de^2f^2g+3\cdots}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.28, size = 17771, normalized size = 16.18

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 22214 vs. $2(1022) = 2044$.
time = 0.16, size = 22215, normalized size = 20.23

method	result	size
elliptic	Expression too large to display	1845
risch	Expression too large to display	7892
default	Expression too large to display	22215

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)^3/sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 1267, normalized size = 1.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/945*((210*c^5*d^3*f^2*g^3 - 210*b*c^4*d^3*f*g^4 - 105*(b^2*c^3 - 6*a*c^4)
*d^3*g^5 - (128*c^5*f^5 - 104*b*c^4*f^4*g - (25*b^2*c^3 - 156*a*c^4)*f^3*g^
2 - 5*(2*b^3*c^2 - 9*a*b*c^3)*f^2*g^3 - (8*b^4*c - 39*a*b^2*c^2 + 24*a^2*c^
3)*f*g^4 - (16*b^5 - 96*a*b^3*c + 123*a^2*b*c^2)*g^5)*e^3 + 9*(48*c^5*d*f^4
*g - 40*b*c^4*d*f^3*g^2 - 2*(5*b^2*c^3 - 31*a*c^4)*d*f^2*g^3 - (5*b^3*c^2 -
 22*a*b*c^3)*d*f*g^4 - (8*b^4*c - 41*a*b^2*c^2 + 30*a^2*c^3)*d*g^5)*e^2 - 6
3*(8*c^5*d^2*f^3*g^2 - 7*b*c^4*d^2*f^2*g^3 - 2*(b^2*c^3 - 6*a*c^4)*d^2*f*g^
4 - (2*b^3*c^2 - 9*a*b*c^3)*d^2*g^5)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(
c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^
```

```
2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3
*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(210*c^5*d^3*f*g^4 - 105*b*c^4*d^3*g^5 -
(128*c^5*f^4*g - 40*b*c^4*f^3*g^2 - 3*(7*b^2*c^3 - 20*a*c^4)*f^2*g^3 - (16*
b^3*c^2 - 57*a*b*c^3)*f*g^4 - 2*(8*b^4*c - 36*a*b^2*c^2 + 21*a^2*c^3)*g^5)*
e^3 + 9*(48*c^5*d*f^3*g^2 - 16*b*c^4*d*f^2*g^3 - (9*b^2*c^3 - 26*a*c^4)*d*f
*g^4 - (8*b^3*c^2 - 29*a*b*c^3)*d*g^5)*e^2 - 63*(8*c^5*d^2*f^2*g^3 - 3*b*c^
4*d^2*f*g^4 - 2*(b^2*c^3 - 3*a*c^4)*d^2*g^5)*e)*sqrt(c*g)*weierstrassZeta(4
/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*
b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3),
 weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2),
 -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*
a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(105*c^5*d^3*g
^5 + (35*c^5*g^5*x^3 - 64*c^5*f^3*g^2 + 12*b*c^4*f^2*g^3 + (9*b^2*c^3 - 22*
a*c^4)*f*g^4 + (8*b^3*c^2 - 27*a*b*c^3)*g^5 - 5*(8*c^5*f*g^4 - b*c^4*g^5)*x
^2 + (48*c^5*f^2*g^3 - 7*b*c^4*f*g^4 - 2*(3*b^2*c^3 - 7*a*c^4)*g^5)*x)*e^3
+ 9*(15*c^5*d*g^5*x^2 + 24*c^5*d*f^2*g^3 - 5*b*c^4*d*f*g^4 - 2*(2*b^2*c^3 -
 5*a*c^4)*d*g^5 - 3*(6*c^5*d*f*g^4 - b*c^4*d*g^5)*x)*e^2 + 63*(3*c^5*d^2*g^
5*x - 4*c^5*d^2*f*g^4 + b*c^4*d^2*g^5)*e)*sqrt(c*x^2 + b*x + a)*sqrt(g*x +
f))/(c^5*g^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral((d + e*x)**3*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)^3/sqrt(g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)^3*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.894 $\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=755

$$-\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^2g^3}+\frac{2(d+}{}$$

```
[Out] -2/35*e*(-b*e*g-4*c*d*g+6*c*e*f)*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/g^3-4/
105*(2*b^2*e^2*g^2+c*e*g*(-5*a*e*g-7*b*d*g+4*b*e*f)-c^2*(10*d^2*g^2-34*d*e*
f*g+21*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^3+2/7*(e*x+d)^2*(g
*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g+1/105*(8*b^3*e^2*g^3+b*c*e*g^2*(-29*a*e*g
-28*b*d*g+9*b*e*f)-2*c^3*f*(35*d^2*g^2-56*d*e*f*g+24*e^2*f^2)-c^2*g*(2*a*e*
g*(-42*d*g+13*e*f)-b*(35*d^2*g^2-42*d*e*f*g+16*e^2*f^2)))*EllipticE(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+
b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1
/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^4/(c*x^2+b*x+
a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/105*(a*g^2-b*
f*g+c*f^2)*(2*b^2*e^2*g^2+c*e*g*(-5*a*e*g-7*b*d*g+4*b*e*f)+c^2*(35*d^2*g^2-
56*d*e*f*g+24*e^2*f^2))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2
)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2)
)^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/g^4/(g*x+f)^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 1.32, antiderivative size = 755, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {934, 1667, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-4*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) - c^2*(21*e^2*f^2
- 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c^2*g
^3) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*g) - (2*e*(6*c
```

```
*e*f - 4*c*d*g - b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c*g^3) +
 (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^3*e^2*g^3 + b*c*e*g^2*(9*b*e*f - 28*b*d*g
- 29*a*e*g) - 2*c^3*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2) - c^2*g*(2*a*e
*g*(13*e*f - 42*d*g) - b*(16*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)))*Sqrt[f +
g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^3*g^4*Sqrt[(c*(f + g*x
))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f
- 7*b*d*g - 5*a*e*g) + c^2*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqrt[(c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 -
 4*a*c])*g)])/(105*c^3*g^4*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 934

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt
[a + b*x + c*x^2]/(g*(2*m + 3))), x] - Dist[1/(g*(2*m + 3)), Int[((d + e*x)
^(m - 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f + 2*a*(e*f*m - d
*g*(m + 1)) + (2*c*d*f - 2*a*e*g + b*(e*f - d*g)*(2*m + 1))*x - (b*e*g + 2*
c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{7g} - \frac{\int \frac{(d+ex)\left(bdf+4aef-6adg+(2cdf+5bef-5b\right.}{\sqrt{f+gx}\,\sqrt{a+}}}{7g}$$

$$= \frac{2(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{7g} - \frac{2e(6cef-4cdg-beg)(f+gx)^{3/}}{35cg^3}$$

$$= -\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2)}{105c^2g^3}$$

$$= -\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2)}{105c^2g^3}$$

$$= -\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2)}{105c^2g^3}$$

$$= -\frac{4(2b^2e^2g^2+ceg(4bef-7bdg-5aeg)-c^2(21e^2f^2-34defg+10d^2g^2)}{105c^2g^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.32, size = 10030, normalized size = 13.28

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 12921 vs. 2(685) = 1370.
time = 0.15, size = 12922, normalized size = 17.12

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2e^2x^2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+axg+bfx+fa}}{7g}+\frac{2\left(e^2b+2cde-\frac{2e^2(3bg+3cf)}{7g}\right)}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)^2/sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.36, size = 840, normalized size = 1.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

[Out] 2/315*((70*c^4*d^2*f^2*g^2 - 70*b*c^3*d^2*f*g^3 - 35*(b^2*c^2 - 6*a*c^3)*d^2*g^4 + (48*c^4*f^4 - 40*b*c^3*f^3*g - 2*(5*b^2*c^2 - 31*a*c^3)*f^2*g^2 - (5*b^3*c - 22*a*b*c^2)*f*g^3 - (8*b^4 - 41*a*b^2*c + 30*a^2*c^2)*g^4)*e^2 - 14*(8*c^4*d*f^3*g - 7*b*c^3*d*f^2*g^2 - 2*(b^2*c^2 - 6*a*c^3)*d*f*g^3 - (2*b^3*c - 9*a*b*c^2)*d*g^4)*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(70*c^4*d^2*f*g^3 - 35*b*c^3*d^2*g^4 + (48*c^4*f^3*g - 16*b*c^3*f^2*g^2 - (9*b^2*c^2 - 26*a*c^3)*f*g^3 - (8*b^3*c - 29*a*b*c^2)*g^4)*e^2 - 14*(8*c^4*d*f^2*g^2 - 3*b*c^3*d*f*g^3 - 2*(b^2*c^2 - 3*a*c^3)*d*g^4)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(35*c^4*d^2*g^4 + (15*c^4*g^4*x^2 + 24*c^4*f^2*g^2 - 5*b*c^3*f*g^3 - 2*(2*b^2*c^2 - 5*a*c^3)*g^4 - 3*(6*c^4*f*g^3 - b*c^3*g^4)*x)*e^2 + 14*(3*c^4*d*g^4*x - 4*c^4*d*f*g^3 + b*c^3*d*g^4)*e)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^4*g^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2 \sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)

[Out] Integral((d + e*x)**2*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)^2/sqrt(g*x + f), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
[Out] int(((d + e*x)^2*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.895 $\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=519

$$-\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2}-\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(2b^2eg^2-2c^2f(4ef-5dg)+c}{}$$

```
[Out] -2/15*(-3*c*e*g*x-b*e*g-5*c*d*g+4*c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/
c/g^2-1/15*(2*b^2*e*g^2-2*c^2*f*(-5*d*g+4*e*f)+c*g*(-6*a*e*g-5*b*d*g+3*b*e*
f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2
^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2
)/c^2/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)-2/15*(b*e*g-10*c*d*g+8*c*e*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^
2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2
)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)
^(1/2))))^(1/2)/c^2/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {828, 857, 732, 435, 430}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,(beg-10cdg+8cef)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(cg(-6aeg-5bdg+3bef)+2b^2eg^2-2c^2f(4ef-5dg))\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{15c^2g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}\,(-beg-5cdg+4cef-3cegx)}{15cg^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (-2*Sqrt[f + g*x]*(4*c*e*f - 5*c*d*g - b*e*g - 3*c*e*g*x)*Sqrt[a + b*x + c*
x^2])/(15*c*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b^2*e*g^2 - 2*c^2*f*(4*e*f
 - 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)])/(15*c^2*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
```

```
8*c*e*f - 10*c*d*g + b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(
15*c^2*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{2\int \frac{\frac{1}{2}\left(5cdg(bf \cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{((8cef-10 \cdots}{\cdots}$$

$$= -\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\left(\sqrt{2}\sqrt{b^2- \cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt{f+gx}\,(4cef-5cdg-beg-3cegx)\sqrt{a+bx+cx^2}}{15cg^2} - \frac{\sqrt{2}\sqrt{b^2-4 \cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.17, size = 792, normalized size = 1.53

$$\frac{\sqrt{f+gx}\left(\frac{2(a+x(b+cx))(beg+c(-4ef+5dg+3egx))}{cg^2}+\frac{2(f+gx)\left(\frac{g^2\left(-2b^2eg^2+2c^2f(4ef-5dg)+cg(-3bef+5bdg+6aeg)\right)(a+x(b+cx))}{(f+gx)^2}+\frac{i\sqrt{1-\frac{2(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\left(\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\left(2b^2eg^2+2c^2f(-4ef+5dg)+cg(3bef-5bdg-6aeg)\right)E\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)+\left(2b^3eg^3-b^2g^2\left(-cef+5cdg+2e\sqrt{(b^2-4ac)g^2}\right)+bcg\left(-8aeg^2+\sqrt{(b^2-4ac)g^2}(-3ef+5dg)\right)+2c\left(cf\sqrt{(b^2-4ac)g^2}(4ef-5dg)+ag^2\left(-2cef+10cdg+3e\sqrt{(b^2-4ac)g^2}\right)\right)\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)\right)}{2\sqrt{2}\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{f+gx}}\right)}{c^2g^4}\right)}{15\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*((2*(a + x*(b + c*x))*(b*e*g + c*(-4*e*f + 5*d*g + 3*e*g*x))
)/(c*g^2) + (2*(f + g*x)*((g^2*(-2*b^2*e*g^2 + 2*c^2*f*(4*e*f - 5*d*g) + c*
g*(-3*b*e*f + 5*b*d*g + 6*a*e*g))*(a + x*(b + c*x)))/(f + g*x)^2 + ((I/2)*S
qrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g
^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)*(2*b^2*e*g^2 + 2*c^2*f*(-4*e*f + 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*
g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g
 + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 -
 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (2*b^3*e*g^3 - b^
2*g^2*(-(c*e*f) + 5*c*d*g + 2*e*Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*g*(-8*a*e*g^
2 + Sqrt[(b^2 - 4*a*c)*g^2]*(-3*e*f + 5*d*g)) + 2*c*(c*f*Sqrt[(b^2 - 4*a*c)
*g^2]*(4*e*f - 5*d*g) + a*g^2*(-2*c*e*f + 10*c*d*g + 3*e*Sqrt[(b^2 - 4*a*c)
*g^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f
+ b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(Sqrt[2]*Sqr
t[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt
[f + g*x])))/(c^2*g^4)))/(15*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6206 vs. $2(461) = 922$.

time = 0.14, size = 6207, normalized size = 11.96

method	result
elliptic	$\dfrac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2ex\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+axg+bfx+fa}}{5g}+\frac{2\left(eb+cd-\frac{2(2bg+2cf)e}{5g}\right)\sqrt{cg}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)/sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 567, normalized size = 1.09

$$\frac{2\left(\left(10\,c^{3}df^{2}g - 10\,bc^{2}dfg^{2} - 5\,(b^{2}c - 6\,ac^{2})dg^{3} - (8\,c^{3}f^{3} - 7\,bc^{2}f^{2}g - 2\,(b^{2}c - 6\,ac^{2})fg^{2} - (2\,b^{3} - 9\,abc)g^{3})e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(c^{2}f^{2} - bcfg + (b^{2} - 3\,ac)g^{2})}{3\,c^{2}g^{2}}, -\frac{4\,(2\,c^{3}f^{3} - 3\,bc^{2}f^{2}g - 3\,(b^{2}c - 6\,ac^{2})fg^{2} + (2\,b^{3} - 9\,abc)g^{3})}{27\,c^{3}g^{3}}, \frac{3\,cgx + cf + bg}{3\,cg}\right) + 3\,(10\,c^{3}dfg^{2} - 5\,bc^{2}dg^{3} - (8\,c^{3}f^{2}g - 3\,bc^{2}fg^{2} - 2\,(b^{2}c - 3\,ac^{2})g^{3})e)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\,(c^{2}f^{2} - bcfg + (b^{2} - 3\,ac)g^{2})}{3\,c^{2}g^{2}}, -\frac{4\,(2\,c^{3}f^{3} - 3\,bc^{2}f^{2}g - 3\,(b^{2}c - 6\,ac^{2})fg^{2} + (2\,b^{3} - 9\,abc)g^{3})}{27\,c^{3}g^{3}}, \mathrm{weierstrassPInverse}\left(\frac{4\,(c^{2}f^{2} - bcfg + (b^{2} - 3\,ac)g^{2})}{3\,c^{2}g^{2}}, -\frac{4\,(2\,c^{3}f^{3} - 3\,bc^{2}f^{2}g - 3\,(b^{2}c - 6\,ac^{2})fg^{2} + (2\,b^{3} - 9\,abc)g^{3})}{27\,c^{3}g^{3}}, \frac{3\,cgx + cf + bg}{3\,cg}\right)\right) + 3\,(5\,c^{3}dg^{3} + (3\,c^{3}g^{3}x - 4\,c^{3}fg^{2} + bc^{2}g^{3})e)\sqrt{cx^{2} + bx + a}\,\sqrt{gx + f}\right)}{45\,c^{3}g^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*((10*c^3*d*f^2*g - 10*b*c^2*d*f*g^2 - 5*(b^2*c - 6*a*c^2)*d*g^3 - (8*c
^3*f^3 - 7*b*c^2*f^2*g - 2*(b^2*c - 6*a*c^2)*f*g^2 - (2*b^3 - 9*a*b*c)*g^3)
*e)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^
2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2
+ (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) + 3*(1
0*c^3*d*f*g^2 - 5*b*c^2*d*g^3 - (8*c^3*f^2*g - 3*b*c^2*f*g^2 - 2*(b^2*c - 3
*a*c^2)*g^3)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3
*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2
)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^
2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*
g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*
g*x + c*f + b*g)/(c*g))) + 3*(5*c^3*d*g^3 + (3*c^3*g^3*x - 4*c^3*f*g^2 + b*
c^2*g^3)*e)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^3*g^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)\sqrt{a + bx + cx^{2}}}{\sqrt{f + gx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)/sqrt(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2),x)
```

```
[Out] int(((d + e*x)*(a + b*x + c*x^2)^(1/2))/(f + g*x)^(1/2), x)
```

3.896 $\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal. Leaf size=444

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}-\frac{\sqrt{2}\sqrt{b^2-4ac}\,(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\sqrt{\frac{b+\cdots}{\cdots}}\right.\right.}{3cg^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+\cdots}}$$

[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/g-1/3*(-b*g+2*c*f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/g^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/3*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]
time = 0.26, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {748, 857, 732, 435, 430}

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(2cf-bg)\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3cg^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]

[Out] (2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*g) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2

```
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*c*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3g} - \frac{\int \frac{bf-2ag+(2cf-bg)x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{3g}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3g} - \frac{(2cf-bg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3g^2} + \frac{(2(cf^2-bfg+ \cdots}{\cdots}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3g} - \frac{\left(\sqrt{2}\,\sqrt{b^2-4ac}\,(2cf-bg)\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx \cdots}{b^2- \cdots}} \cdots\right.}{3cg^2\sqrt{\cdots}}$$

$$= \frac{2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3g} - \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(2cf-bg)\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+ \cdots}{b^2-4 \cdots}} \cdots}{3cg^2\sqrt{\frac{\cdots}{2cf-(\cdots}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 25.43, size = 936, normalized size = 2.11

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]
```

```
[Out] (Sqrt[f + g*x]*(4*g^2*(a + x*(b + c*x)) + ((f + g*x)*((4*g^2*(-2*c*f + b*g)
*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*(2*c*f - b*g)*(2*c*f - b*g + Sq
rt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g
*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/((2*c*f - b*g + Sqrt[(b^2
 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c
*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))/((-2*c*f + b*g + Sqr
t[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
```

```
- b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]],
 -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*(b^2*g^2 - 4*a*c*g^2 + 2*c*f*Sqrt[(b
^2 - 4*a*c)*g^2] - b*g*Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f*Sqrt[(b^
2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))/
((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqrt
[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g
*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcS
inh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c
)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f
 - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(c*Sqrt[(c*f^2 + g*(-(
b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])))/(6*g^3*Sqrt[a + x
*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1853 vs. $2(386) = 772$.

time = 0.13, size = 1854, normalized size = 4.18

method	result
elliptic	$\frac{\sqrt{(gx+f)\,(c\,x^2+bx+a)}\left(\frac{2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+axg+bfx+fa}}{3g}+\frac{2\left(a-\frac{2\left(\frac{ag}{2}+\frac{bf}{2}\right)}{3g}\right)\left(-\frac{b+\sqrt{-4}}{}\right.}{}\right.}{}$

risch	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}{3g}+\frac{\left(\frac{2(bg-2cf)\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\sqrt{\frac{x+\frac{f}{g}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}}}\sqrt{\frac{x-\frac{-b+\sqrt{-}}{}}{-\frac{f}{g}-\frac{-b+\sqrt{}}{}}}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(1/2)/c*((-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x
+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2)
)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c
*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*g^3-(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2
))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*
g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*f*g^2+(-4*a*c+b^2)^(1/2)*2^(1/2)*(-
(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(
1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2
))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*f^2*g+2^(1/2)*(-(g*x+f)*c/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b
*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4
*a*c+b^2)^(1/2)))^(1/2))*a*b*g^3-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
```

```
)))^(1/2))*a*c*f*g^2-2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2
*f*g^2+3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2
*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*
x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2
^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*f^2*g-2*2^(
1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c
+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^
2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*f^3-c^2*g^3*x^3-b*c*g^3
*x^2-c^2*f*g^2*x^2-a*c*g^3*x-b*c*f*g^2*x-a*c*f*g^2)/(c*g*x^3+b*g*x^2+c*f*x^
2+a*g*x+b*f*x+a*f)/g^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 417, normalized size = 0.94

$$\frac{2\left(3\sqrt{cx^2+bx+a}\,\sqrt{gx+f}\,c^2g^2+(2c^2f^2-2bcfg-(b^2-6ac)g^2)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4(c^2f^2-bcfg+(b^2-3ac)g^2)}{3c^2g^2},-\frac{4(2c^3f^3-3bc^2f^2g-3(b^2c-6ac^2)fg^2+(2b^3-9abc)g^3)}{27c^3g^3},\frac{3cgx+cf+bg}{3cg}\right)+3(2c^2fg-bcg^2)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4(c^2f^2-bcfg+(b^2-3ac)g^2)}{3c^2g^2},-\frac{4(2c^3f^3-3bc^2f^2g-3(b^2c-6ac^2)fg^2+(2b^3-9abc)g^3)}{27c^3g^3},\mathrm{weierstrassPInverse}\left(\frac{4(c^2f^2-bcfg+(b^2-3ac)g^2)}{3c^2g^2},-\frac{4(2c^3f^3-3bc^2f^2g-3(b^2c-6ac^2)fg^2+(2b^3-9abc)g^3)}{27c^3g^3},\frac{3cgx+cf+bg}{3cg}\right)\right)\right)}{9c^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*g^2 + (2*c^2*f^2 - 2*b*c*f*g
 - (b^2 - 6*a*c)*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g
+ (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c
 - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f +
b*g)/(c*g)) + 3*(2*c^2*f*g - b*c*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^
2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*
g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstra
ssPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*
c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3
)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))))/(c^2*g^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{\sqrt{f + gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)/sqrt(g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{\sqrt{f + g\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

3.897 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal. Leaf size=700

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right.}\right.}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

```
[Out] EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/
g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*
(-b*e*g+c*d*g+c*e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(
1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(
1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/g/(g*x+f)^(1/
2)/(c*x^2+b*x+a)^(1/2)-(a*e^2-b*d*e+c*d^2)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+
f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c
+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))
))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+
(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(-d*g+e*f)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 1.34, antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {936, 948, 175, 552, 551, 857, 732, 435, 430}

$$-\frac{\sqrt{2}\left(ae^2-bde+cd^2\right)\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right)|\frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right)}{\sqrt{c}\,e^2\sqrt{a+bx+cx^2}\,(ef-dg)}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}(-beg+cdg+cef)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{eg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(e*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
+ b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + c*d*g - b*e*g)*Sqrt
```

```
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(c*e^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2
*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2
 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/
(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^2*(e*f - d*g)*Sqrt[a + b*x
 + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 936

```
Int[Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.
) + (g_.)*(x_)]), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[1/((d
+ e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] - Dist[1/e^2, Int[(c*d
- b*e - c*e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b
, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d
^2 - b*d*e + a*e^2, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = -\frac{\int \frac{cd-be-cex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{(cd^2-bde+ae^2)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+b}}}{e^2}$$

$$= \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{eg} - \frac{(cef+cdg-beg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{e^2g} + $$

$$= -\frac{\left(2(cd^2-bde+ae^2)\sqrt{b-\sqrt{b^2-4ac}+2cx}\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\mathrm{Sub}}{}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{eg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.92, size = 1261, normalized size = 1.80

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] ((f + g*x)^(3/2)*((4*e*g^2*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 - (
I*Sqrt[2]*e*(-(e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2
*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2
*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x
 + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqr
t[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
- b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]],
 -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2]))])/Sqrt[f + g*x] + (I*Sqrt[2]*e*(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[
(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))*Sqrt[(-
2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((
2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*
x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sq
rt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2
 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]]
, -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a
*c)*g^2]))])/Sqrt[f + g*x] + ((2*I)*Sqrt[2]*(-(c*d^2) + e*(b*d - a*e))*g^2*
Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g
*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2
*c*f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b
*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticPi[((e*f - d*g)*(2*c*f -
b*g - Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh
[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g
^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f -
b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/(2*e^2*g^2*(-(e*f) + d*g)
*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3125 vs. $2(617) = 1234$.
time = 0.14, size = 3126, normalized size = 4.47

method	result

elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2(eb-cd)\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\sqrt{\frac{x+\frac{f}{g}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4a}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4a}}{2c}}}}{e^2\sqrt{cg\,x^3+}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)

[Out] (2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c
^2*e^2*f^3-2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
)^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))
)^(1/2))*a*c*e^2*f*g^2+2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c
+b^2)^(1/2)))^(1/2))*a*c*d*e*g^3-(-4*a*c+b^2)^(1/2)*EllipticF(2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*d*e*g^3+(-4*a*c+b^2)^(1/2
)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*
e^2*f*g^2-(-4*a*c+b^2)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4
*a*c+b^2)^(1/2)))^(1/2))*c*e^2*f^2*g-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b
g+g(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e^2*f^2*g+2*EllipticF(2^(1/2)*(-(g*x+f
)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d*e*f^2*g+2*EllipticPi(2^(
1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^2*f*g^2+(-4*a*c+b^2)^(1/2)*Elliptic
Pi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(
2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*d*e*g^3-2*EllipticE(2^(1/2)*(-(g*

```
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e*g^3+2*EllipticE(2^(
1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^2*f*g^2-2*E
llipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e
^2*f^2*g-2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^
(1/2))*c^2*d*e*f^2*g-2*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*
d*e*f*g^2+2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))
^(1/2))*b*c*d*e*f*g^2+(-4*a*c+b^2)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c
*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*d^2*g^3-EllipticF(2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e*g^3+EllipticF(2^(1/2)*(-
(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*e^2*f*g^2+EllipticF(
2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^2*g^3-2*
EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*
d^2*f*g^2-(-4*a*c+b^2)^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)
^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f)
,(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)
)*a*e^2*g^3-(-4*a*c+b^2)^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*
f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/
2))*c*d^2*g^3-EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e^2*g^3+E
llipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e*g^3-EllipticPi(2^(1/
2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^2*g^3+2*EllipticPi(2^(1/2)*(-(g*x+f)*
c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2))*c^2*d^2*f*g^2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a...
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/(sqrt(g*x + f)*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)/(sqrt(g*x + f)*(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

3.898 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal. Leaf size=736

$$-\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)}+\frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2c}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}\,e(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+bx}}$$

```
[Out] -(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(e*x+d)+1/2*EllipticE(1/2*((b
+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+
b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x
+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/(-d*g+e*f)*2^(1/2)/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+EllipticF(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-
4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b
+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-1/2*(e^2
*(-a*g+b*f)-c*d*(-d*g+2*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c
*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c
/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))
^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-
(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)/e^2/(-d*g+e*f)^2*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 2.21, antiderivative size = 957, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {938, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x))) + (Sqrt[b^
2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*
```

```
e*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sqrt[(c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(
e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(2*e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqr
t[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g
) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Ell
ipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(
Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b
 - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sq
rt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 938

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g))), x] - Dist[1/(2*(m + 1)*(e*f -
 d*g)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[
b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m + 2))*x + c*g*(2*m + 5)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
```

```
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \frac{bf-ag+2cfx+cgx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{2(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\int \left(\frac{c(2ef-dg)}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{cg}{e\sqrt{f+gx}\,\sqrt{\cdots}}\right.}{2(\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{2e(ef-dg)} + \frac{(c(2ef-\cdots}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2e(ef-dg)} - \frac{(cf)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+\cdots}}}{2e(ef-dg\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(2ef-dg)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-\cdots}\right.}}}{\cdots}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\mathrm{s}\cdots\right.}{\sqrt{2}\,e(ef-dg)\sqrt{\frac{\cdots}{2cf-\cdots}}}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\mathrm{s}\cdots\right.}{\sqrt{2}\,e(ef-dg)\sqrt{\frac{\cdots}{2cf-\cdots}}}$$

$$\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\mathrm{s}\cdots\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.26, size = 1471, normalized size = 2.00

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])/((-(e*f) + d*g)*(d + e*x)) + ((f + g*
x)^(3/2)*Sqrt[a + x*(b + c*x)]*(-4*e*(-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f)
 + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(c*(-1 + f/(f + g*x))^2
+ (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + (I*Sqrt[2]*e*(-(
e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c
)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(
f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c
)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f
 + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticE[I*ArcSinh[(Sq
rt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])
])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - (I*Sqrt[2]*e*(-2*a*e*g^2 - e*
f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*(3*e*f - d*g)
 + 2*c*f*(-2*e*f + d*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x
) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + S
qrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x
) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g +
 a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*
f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))
])/Sqrt[f + g*x] + ((2*I)*Sqrt[2]*g*(e^2*(b*f - a*g) + c*d*(-2*e*f + d*g))*
Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*
x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*
x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*
EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^
2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f +
 g*x]))/(4*e^2*g*(-(e*f) + d*g)^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + b*x + c*x^2]*Sqrt[((f + g*x)^2*(c
*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*
x)))/g^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 13871 vs. $2(653) = 1306$.
time = 0.15, size = 13872, normalized size = 18.85

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+axg+bfx+fa}}{(dg-ef)(ex+d)}+\frac{2\left(\frac{c}{e^2}-\frac{cdg}{2e^2(dg-ef)}\right)\left(-\frac{b+\sqrt{-4}}{}\right.}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(sqrt(g*x + f)*(x*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)/(sqrt(g*x + f)*(x*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

3.899 $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal. Leaf size=1049

$$\sqrt{b^2-4ac}$$

$$-\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2}+\frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)}-$$

```
[Out] -1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(c*d*(d*g+2
*e*f)-e*(-3*a*e*g+2*b*d*g+b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-
b*d*e+c*d^2)/(-d*g+e*f)^2/(e*x+d)-1/8*(c*d*(d*g+2*e*f)-e*(-3*a*e*g+2*b*d*g+
b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e/
(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*
c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/4*(e^2*(-a*g+b*f)+c*d*(d*g-2*e*f))*E
llipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2)*(g*(-4*a*c+b^2)^(1/2)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(
-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(
b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(c*d^2+e*(a*e-b*d))/(-d*g+e*f)*2^(1/2)/(g
*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)-1/8*(3*a^2*e^4*g^2+c^2*d^3*g*(-d*g+4*e*f)+b
^2*e^3*f*(4*d*g-e*f)+2*a*c*e^2*(3*d^2*g^2-2*d*e*f*g+2*e^2*f^2)-2*b*e^2*g*(3
*c*d^2*f+a*e*(2*d*g+e*f)))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2))^(1/2),(2*c*e*f-b*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*
c*d*g+2*c*e*f),((2*c*f+g*(-b+(-4*a*c+b^2)^(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^
(1/2))))^(1/2))*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c
+b^2)^(1/2))/(2*c*f+g*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b
^2)^(1/2))/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(c*d^2+e*(a*e-b*d))
/(-d*g+e*f)^3*2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(1/2)
```

Rubi [A]

time = 5.54, antiderivative size = 1747, normalized size of antiderivative = 1.67, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {938, 6874, 732, 430, 953, 857, 435, 948, 175, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -1/2*(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x)^2) + ((c*
d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x
 + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)) - (Sqrt[b^2
- 4*a*c]*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*
g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^
2)*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sq
rt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellip
ticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*
e^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*f
*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[(c*(f + g*x))/(2*
c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/
(2*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b
*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d
*g - 3*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)
*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g]*(c*e*f + c*d*g - b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(
e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[
b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*
c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
 + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f + d*g) - e*(b*e*f +
 2*b*d*g - 3*a*e*g))*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sq
rt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(
f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g +
 Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f +
g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2
*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e^2*(c*d^
2 - b*d*e + a*e^2)*(e*f - d*g)^3*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
```

```
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 938

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])/Sq
rt[(f_.) + (g_.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g))), x] - Dist[1/(2*(m + 1)*(e*f -
 d*g)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[
b*f + a*g*(2*m + 3) + 2*(c*f + b*g*(m + 2))*x + c*g*(2*m + 5)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \frac{bf-3ag+2(cf-bg)x-cgx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4(ef-dg)}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{\int \left(-\frac{cg}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{-cd(2ef+}{e^2(d+ex)^2\sqrt{f}}\right.}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4e^2(ef-dg)} + \frac{(cef+c}{}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+}$$

$$= -\frac{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(cd(2ef+dg)-e(bef+2bdg-3aeg))\sqrt{f+gx}}{4\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.74, size = 36617, normalized size = 34.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57840 vs. $2(949) = 1898$.
time = 0.21, size = 57841, normalized size = 55.14

method	result	size
elliptic	Expression too large to display	1634
default	Expression too large to display	57841

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(sqrt(g*x + f)*(x*e + d)^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(sqrt(g*x + f)*(x*e + d)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

3.900 $\int \frac{(d+ex)^3 \sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=774

$$\frac{2e(24b^2e^2g^2 + ceg(13bef - 84bdg - 25aeg) - c^2(7e^2f^2 + 12defg - 90d^2g^2))\sqrt{f+gx}\sqrt{a+bx+cx^2}}{105c^3g^2} + \frac{2e}{}$$

```
[Out] 2/35*e^2*(-6*b*e*g+11*c*d*g+c*e*f)*(g*x+f)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^
2+2/105*e*(24*b^2*e^2*g^2+c*e*g*(-25*a*e*g-84*b*d*g+13*b*e*f)-c^2*(-90*d^2*
g^2+12*d*e*f*g+7*e^2*f^2))*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/g^2+2/7*e*
(e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c-1/105*(48*b^3*e^3*g^3-8*b*c*e
^2*g^2*(13*a*e*g+21*b*d*g+2*b*e*f)-c^3*(105*d^3*g^3+105*d^2*e*f*g^2-42*d*e^
2*f^2*g+8*e^3*f^3)+c^2*e*g*(a*e*g*(189*d*g+19*e*f)-b*(-210*d^2*g^2-63*d*e*f
*g+9*e^2*f^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))
))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)/c^4/g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)-2/105*e*(a*g^2-b*f*g+c*f^2)*(24*b^2*e^2*g^2+c*e*g*(-25*a*
e*g-84*b*d*g+13*b*e*f)+c^2*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^
2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)/c^4/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 1.39, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {955, 1667, 857, 732, 435, 430}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*e*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) - c^2*(7*e^2*
f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c
^3*g^2) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*c) + (2*
```

```
e^2*(c*e*f + 11*c*d*g - 6*b*e*g)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35
*c^2*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b
*e*f + 21*b*d*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f
*g^2 + 105*d^3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) - b*(9*e^2*f^2 - 63
*d*e*f*g - 210*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(105*c^4*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*
Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*
g^2)*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) + c^2*(8*e^2*
f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2
- 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(105*c^4*g^3*Sqrt[f
+ g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 955

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d
 + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(d
*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*
m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*f
 + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Ne
Q[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{7c} - \frac{\int \frac{(d+ex)\left(-7cd^2f+e(bdf+4aef+adg)-(cd(12ef+7\cdots\right.}{\sqrt{f+gx}\,\sqrt{\cdots}}}{\cdots}$$

$$= \frac{2e(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{7c} + \frac{2e^2(cef+11cdg-6beg)(f+gx)^{3/2}\sqrt{\cdots}}{35c^2g^2}$$

$$= \frac{2e(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2(7e^2f^2+12defg-90d^2g^2))\sqrt{\cdots}}{105c^3g^2}$$

$$= \frac{2e(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2(7e^2f^2+12defg-90d^2g^2))\sqrt{\cdots}}{105c^3g^2}$$

$$= \frac{2e(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2(7e^2f^2+12defg-90d^2g^2))\sqrt{\cdots}}{105c^3g^2}$$

$$= \frac{2e(24b^2e^2g^2+ceg(13bef-84bdg-25aeg)-c^2(7e^2f^2+12defg-90d^2g^2))\sqrt{\cdots}}{105c^3g^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.25, size = 1402, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[f + g*x]*(a + b*x + c*x^2)*((-2*e*(4*c^2*e^2*f^2 - 21*c^2*d*e*f*g + 5
*b*c*e^2*f*g - 105*c^2*d^2*g^2 + 84*b*c*d*e*g^2 - 24*b^2*e^2*g^2 + 25*a*c*e
^2*g^2))/(105*c^3*g^2) - (2*e^2*(-(c*e*f) - 21*c*d*g + 6*b*e*g)*x)/(35*c^2*
```

```
g) + (2*e^3*x^2)/(7*c)))/Sqrt[a + x*(b + c*x)] - (2*(f + g*x)^(3/2)*Sqrt[a
+ b*x + c*x^2]*(-((-48*b^3*e^3*g^3 + 8*b*c*e^2*g^2*(2*b*e*f + 21*b*d*g + 13
*a*e*g) + c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3)
- c^2*e*g*(a*e*g*(19*e*f + 189*d*g) + b*(-9*e^2*f^2 + 63*d*e*f*g + 210*d^2*
g^2)))*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x))
)/(f + g*x))) - ((I/2)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*
g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a
*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b*e*f + 21*b*d
*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^
3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) + b*(-9*e^2*f^2 + 63*d*e*f*g + 2
10*d^2*g^2)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + S
qrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (48*b^4
*e^3*g^4 - 8*b^3*e^2*g^3*(8*c*e*f + 21*c*d*g + 6*e*Sqrt[(b^2 - 4*a*c)*g^2])
 + b^2*c*e*g^2*(-152*a*e^2*g^2 + 8*e*Sqrt[(b^2 - 4*a*c)*g^2]*(2*e*f + 21*d*
g) + c*(e^2*f^2 + 231*d*e*f*g + 210*d^2*g^2)) - b*(-104*a*c*e^3*g^3*Sqrt[(b
^2 - 4*a*c)*g^2] + 105*c^3*d^2*g^3*(3*e*f + d*g) + c^2*e*g*(-(a*e*g^2*(151*
e*f + 357*d*g)) + 3*Sqrt[(b^2 - 4*a*c)*g^2]*(-3*e^2*f^2 + 21*d*e*f*g + 70*d
^2*g^2))) + c^2*(50*a^2*e^3*g^4 - a*e*g^2*(e*Sqrt[(b^2 - 4*a*c)*g^2]*(19*e*
f + 189*d*g) + c*(4*e^2*f^2 + 294*d*e*f*g + 210*d^2*g^2)) + c*(210*c*d^3*f*
g^3 + Sqrt[(b^2 - 4*a*c)*g^2]*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2
 + 105*d^3*g^3))))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2
)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*
g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(S
qrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g
^2])]*Sqrt[f + g*x])))/(105*c^4*g^4*Sqrt[a + x*(b + c*x)]*Sqrt[((f + g*x)^2
*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f +
 g*x)))/g^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 14977 vs. $2(704) = 1408$.

time = 0.14, size = 14978, normalized size = 19.35

method	result

elliptic	$\frac{\sqrt{(gx+f)\,(c\,x^2+bx+a)}\left(\frac{2e^3x^2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{7c}+\frac{2\left(3d\,e^2g+f\,e^3-\frac{2e^3(3bg+3cf}{7c}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^3/sqrt(c*x^2 + b*x + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 883, normalized size = 1.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/315*((210*c^4*d^3*f*g^3 - 105*b*c^3*d^3*g^4 - (8*c^4*f^4 + 5*b*c^3*f^3*g
+ (10*b^2*c^2 - 13*a*c^3)*f^2*g^2 + (40*b^3*c - 113*a*b*c^2)*f*g^3 - (48*b^
```

```
4 - 176*a*b^2*c + 75*a^2*c^2)*g^4)*e^3 + 21*(2*c^4*d*f^3*g + 2*b*c^3*d*f^2*
g^2 + (7*b^2*c^2 - 12*a*c^3)*d*f*g^3 - (8*b^3*c - 21*a*b*c^2)*d*g^4)*e^2 -
105*(c^4*d^2*f^2*g^2 + 2*b*c^3*d^2*f*g^3 - (2*b^2*c^2 - 3*a*c^3)*d^2*g^4)*e
)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)
/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
(2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) - 3*(105
*c^4*d^3*g^4 + (8*c^4*f^3*g + 9*b*c^3*f^2*g^2 + (16*b^2*c^2 - 19*a*c^3)*f*g
^3 - 8*(6*b^3*c - 13*a*b*c^2)*g^4)*e^3 - 21*(2*c^4*d*f^2*g^2 + 3*b*c^3*d*f*
g^3 - (8*b^2*c^2 - 9*a*c^3)*d*g^4)*e^2 + 105*(c^4*d^2*f*g^3 - 2*b*c^3*d^2*g
^4)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2
)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
 (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c*f
*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^
2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f
 + b*g)/(c*g))) + 3*(105*c^4*d^2*g^4*e + (15*c^4*g^4*x^2 - 4*c^4*f^2*g^2 -
5*b*c^3*f*g^3 + (24*b^2*c^2 - 25*a*c^3)*g^4 + 3*(c^4*f*g^3 - 6*b*c^3*g^4)*x
)*e^3 + 21*(3*c^4*d*g^4*x + c^4*d*f*g^3 - 4*b*c^3*d*g^4)*e^2)*sqrt(c*x^2 +
b*x + a)*sqrt(g*x + f))/(c^5*g^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)*(x*e + d)^3/sqrt(c*x^2 + b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + b*x + c*x^2)^(1/2), x)
```

3.901 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=567

$$\frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g}+\frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{5c}+\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(8b}{}$$

[Out] 2/15*e*(-4*b*e*g+7*c*d*g+c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g+2/5
e(e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/15*(8*b^2*e^2*g^2-c*e*g*(9
*a*e*g+20*b*d*g+3*b*e*f)-c^2*(-15*d^2*g^2-10*d*e*f*g+2*e^2*f^2))*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(
-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/g^2/(c*x
^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+4/15*e*(
2*b*e*g-5*c*d*g+c*e*f)*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c
f-g(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+
b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2)/c^3/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]
time = 0.66, antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {955, 1667, 857, 732, 435, 430}

$$\frac{4\sqrt{2}\,e\sqrt{b^2-4ac}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\left(ag^2-bfg+cf^2\right)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,(2beg-5cdg+cef)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{15c^3g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}+\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\left(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2-\left(c^2\left(-15d^2g^2-10defg+2e^2f^2\right)\right)\right)E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{15c^3g^2\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\,(-4beg+7cdg+cef)}{15c^2g}+\frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{5c}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]

[Out] (2*e*(c*e*f + 7*c*d*g - 4*b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*c
^2*g) + (2*e*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(8*b^2*e^2*g^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) -
 c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b

```
^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c
*e*f - 5*c*d*g + 2*b*e*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15
*c^3*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 955

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d
 + e*x)^(m - 2)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[e*(b*d*f + a*(d
*g + 2*e*f*(m - 1))) - c*d^2*f*(2*m + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*
m + d*g*(2*m + 1)) + b*e*(2*d*g + e*f*(2*m - 1)))*x + e*(2*b*e*g*m - c*(e*f
```

```
 + d*g*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && Ne
Q[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{5c} - \frac{\int \frac{-5cd^2f+e(bdf+2aef+adg)-(cd(8ef+5dg)-e(3b\cdots}{\sqrt{f+gx}\,\sqrt{a+\cdots}}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+\cdots}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+\cdots}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+\cdots}}{5c}$$

$$= \frac{2e(cef+7cdg-4beg)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\,\sqrt{a+\cdots}}{5c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.28, size = 1002, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (((-2*e*(-(c*e*f) - 10*c*d*g + 4*b*e*g))/(15*c^2*g) + (2*e^2*x)/(5*c))*Sqrt
[f + g*x]*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(f + g*x)^(3/2)*Sqr
t[a + b*x + c*x^2]*((-8*b^2*e^2*g^2 + c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g)
+ c^2*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(c*(-1 + f/(f + g*x))^2 + (g*(
b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + ((I/2)*Sqrt[1 - (2*(c*
f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x)
```

```
)]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a
*c)*g^2])*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(8*b^2*e^2*g
^2 - c*e*g*(3*b*e*f + 20*b*d*g + 9*a*e*g) + c^2*(-2*e^2*f^2 + 10*d*e*f*g +
15*d^2*g^2))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (-30*c^3
*d^2*f*g^2 + 8*b^2*e^2*g^2*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*e*g*(-17*a*b
*e*g^2 + 9*a*e*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*Sqrt[(b^2 - 4*a*c)*g^2]*(3*e*f
 + 20*d*g) - b^2*g*(11*e*f + 20*d*g)) - c^2*(-15*b*d*g^2*(2*e*f + d*g) - 2*
a*e*g^2*(7*e*f + 10*d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(-2*e^2*f^2 + 10*d*e*f*g
 + 15*d^2*g^2)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]))/(Sqr
t[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])]*Sqrt[f + g*x])))/(15*c^3*g^3*Sqrt[a + x*(b + c*x)]*Sqrt[((f + g*x)^2*(c
*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*
x)))/g^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8247 vs. $2(503) = 1006$.

time = 0.15, size = 8248, normalized size = 14.55

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2e^2x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5c}+\frac{2\left(2gde+e^2f-\frac{2(2bg+2cf)e^2}{5c}\right)}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)^2/sqrt(c*x^2 + b*x + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 613, normalized size = 1.08

$$\frac{2\left(\left(30\,c^3d^2fg^2-15\,bc^2d^2g^3+\left(2\,c^3f^3+2\,bc^2f^2g+\left(7\,b^2c-12\,ac^2\right)fg^2-\left(8\,b^3-21\,abc\right)g^3\right)e^2-10\left(c^3df^2g+2\,bc^2dfg^2-\left(2\,b^2c-3\,ac^2\right)dg^3\right)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)-3\left(15\,c^3d^2g^3-\left(2\,c^3f^2g+3\,bc^2fg^2-\left(8\,b^2c-9\,ac^2\right)g^3\right)e^2+10\left(c^3dfg^2-2\,bc^2dg^3\right)e\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)\right)+3\left(10\,c^3dg^3e+\left(3\,c^3g^3x+c^3fg^2-4\,bc^2g^3\right)e^2\right)\sqrt{cx^2+bx+a}\,\sqrt{gx+f}\right)}{45\,c^4g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/45*((30*c^3*d^2*f*g^2 - 15*b*c^2*d^2*g^3 + (2*c^3*f^3 + 2*b*c^2*f^2*g + (
7*b^2*c - 12*a*c^2)*f*g^2 - (8*b^3 - 21*a*b*c)*g^3)*e^2 - 10*(c^3*d*f^2*g +
 2*b*c^2*d*f*g^2 - (2*b^2*c - 3*a*c^2)*d*g^3)*e)*sqrt(c*g)*weierstrassPInve
rse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3
 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*
g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) - 3*(15*c^3*d^2*g^3 - (2*c^3*f^2*g +
 3*b*c^2*f*g^2 - (8*b^2*c - 9*a*c^2)*g^3)*e^2 + 10*(c^3*d*f*g^2 - 2*b*c^2*d
*g^3)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g
^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2
 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 - b*c
*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(
b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c
*f + b*g)/(c*g))) + 3*(10*c^3*d*g^3*e + (3*c^3*g^3*x + c^3*f*g^2 - 4*b*c^2*
g^3)*e^2)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^4*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^2\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)*(x*e + d)^2/sqrt(c*x^2 + b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.902 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=452

$$\frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c}+\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(cef+3cdg-2beg)\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\right.}{3c^2g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}}$$

[Out] 2/3*e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c+1/3*(-2*b*e*g+3*c*d*g+c*e*f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*e*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]
time = 0.32, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {846, 857, 732, 435, 430}

$$\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,(-2beg+3cdg+cef)\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\,e\sqrt{b^2-4ac}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}+\frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]

[Out] (2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + 3*c*d*g - 2*b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]

```
+ 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)])/(3*c^2*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c} + \frac{2\int \frac{\frac{1}{2}(3cdf-e(bf+ag))+\frac{1}{2}(cef+3cdg-2beg)x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c} + \frac{(cef+3cdg-2beg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{3cg} - \frac{(e}{}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c} + \frac{\left(\sqrt{2}\,\sqrt{b^2-4ac}\,(cef+3cdg-2beg)\sqrt{f+gx}\right.}{}$$

$$= \frac{2e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3c} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(cef+3cdg-2beg)\sqrt{f+gx}\,\sqrt{}}{3c^2g\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.73, size = 638, normalized size = 1.41

$$\frac{2\sqrt{f+gx}\left(ce(a+x(b+cx))+\frac{(f+gx)\left(\frac{g^2(cef+3cdg-2beg)(a+x(b+cx))}{(f+gx)^2}+\frac{i\sqrt{1-\frac{2(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\left(\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(2beg-c(ef+3dg))E\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)+\left(6c^2dfg+2beg\left(bg-\sqrt{(b^2-4ac)g^2}\right)+c\left(-2aeg^2-3bg(ef+dg)+\sqrt{(b^2-4ac)g^2}(ef+3dg)\right)\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)\right)}{2\sqrt{2}\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{f+gx}}\right)}{g^2}\right)}{3c^2\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[f + g*x]*(c*e*(a + x*(b + c*x)) + ((f + g*x)*((g^2*(c*e*f + 3*c*d*g
 - 2*b*e*g)*(a + x*(b + c*x)))/(f + g*x)^2 + ((I/2)*Sqrt[1 - (2*(c*f^2 + g*
(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[
1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]
)*(f + g*x))]*((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(2*b*e*g - c*(e*f +
3*d*g))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f +
```

```
 b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b
^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] + (6*c^2*d*f*g
+ 2*b*e*g*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) + c*(-2*a*e*g^2 - 3*b*g*(e*f + d*
g) + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f + 3*d*g)))*EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt
[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[
(b^2 - 4*a*c)*g^2]))]))/(Sqrt[2]*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x])))/g^2))/(3*c^2*Sqrt[a + x*(b
 + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3804 vs. $2(394) = 788$.
time = 0.12, size = 3805, normalized size = 8.42 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*(3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*a*b*e*g^3-6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(
1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(
1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(
1/2))*a*c*d*g^3-(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
)))^(1/2))*a*e*g^3-3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1
/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2
*e*f*g^2+6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b
-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*
c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF
(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b
^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*f*g^2+3
*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4
*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*
c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+
```

```
b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e*f^2*g+(-4*a*c+b^2
)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*
c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x
+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^
(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)
^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e*f*g^2-6*2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+
b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2
)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x
+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d*f^2*g-(-4*a*c+b^2)^(1/
2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(
-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*
a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)
*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2
)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*e*f^2*g-4*2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/
2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e*g^3+6*2^(1/2)*(-(g*x+f)*c/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*g^3+2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-
4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*a*c*e*f*g^2+4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)
^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))
^(1/2))*b^2*e*f*g^2-6*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(
1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 453, normalized size = 1.00

$$\frac{2\left(3\sqrt{cx^2+bx+a}\,\sqrt{gx+f}\,c^2g^2e+(6\,c^2dfg-3\,bcdg^2-(c^2f^2+2\,bcfg-(2\,b^2-3\,ac)g^2)e)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)-3\,(3\,c^2dg^2+(c^2fg-2\,bcg^2)e)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)\right)\right)}{9\,c^3g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*g^2*e + (6*c^2*d*f*g - 3*b*c
*d*g^2 - (c^2*f^2 + 2*b*c*f*g - (2*b^2 - 3*a*c)*g^2)*e)*sqrt(c*g)*weierstra
ssPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*
c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3
)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) - 3*(3*c^2*d*g^2 + (c^2*f*g -
 2*b*c*g^2)*e)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*
a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)
*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2
 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g
 - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g
*x + c*f + b*g)/(c*g))))/(c^3*g^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)\,\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)*(x*e + d)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}\,\left(d+e\,x\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^(1/2)*(d + e*x))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^(1/2)*(d + e*x))/(a + b*x + c*x^2)^(1/2), x)
```

3.903 $\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

```
[Out] EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/
(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {732, 435}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)])/(c*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a +
b*x + c*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4}}{2cf-bg-\sqrt{}}}}{\sqrt{1-x^2}}\right.}{c\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\sqrt{a+bx+}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2c}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{c\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.56, size = 365, normalized size = 1.94

$$\frac{i\left(2cf+\left(-b+\sqrt{b^2-4ac}\right)g\right)\sqrt{\frac{g\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{f+gx}\right)|\frac{2cf-\left(b+\sqrt{b^2-4ac}\right)g}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{f+gx}\right)|\frac{2cf-\left(b+\sqrt{b^2-4ac}\right)g}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}\right)\right)}{\sqrt{2}\,cg\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (I*(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
```

```
 + (-b + Sqrt[b^2 - 4*a*c])*g)]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f
 + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)] - EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[f + g*x]], (2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]))/(Sqrt[2]*c*g
*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(164) = 328$.

time = 0.12, size = 747, normalized size = 3.97

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2f\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\sqrt{\frac{x+\frac{f}{g}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b}\cdots}{2c}}}\cdots}{\sqrt{cg\,x^3+bg\,x\cdots}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}\,\left(g\sqrt{-4ac+b^2}+bg-2cf\right)\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}}\,\sqrt{\frac{\left(-b-2cx+\sqrt{-4\cdots}\right.}{2cf-bg+g\sqrt{-4\cdots}}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+
b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2
)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(EllipticF(2^(1/2)*(-(g*
x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*g*b-2*f*EllipticF(2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c-EllipticF(2^(1/2)*(-(
g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*g*(-4*a*c+b^2)^(1/2)-Elli
pticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*
```

```
a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*g+2*El
lipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*f+(-
4*a*c+b^2)^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2)))^(1/2))*g)/g/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.39, size = 359, normalized size = 1.91

$$-\frac{2\left(3\sqrt{cg}\,cg\mathrm{weierstrassZeta}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)\right)-\left(2\,cf-bg\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)\right)}{3\,c^2g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(c*g)*c*g*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c
)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*
g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2 -
b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g -
3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x
+ c*f + b*g)/(c*g))) - (2*c*f - b*g)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2
*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f
^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3
*c*g*x + c*f + b*g)/(c*g)))/(c^2*g)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f + gx}}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^(1/2)/(a + b*x + c*x^2)^(1/2), x)
```

3.904 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=467

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

```
[Out] 2*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2
^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^
(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(
2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2
)-EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))
^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*
a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+
f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/c^(1/2)/(c*
x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 1.04, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {957, 732, 430, 948, 175, 552, 551}

$$\frac{2\sqrt{2}\,g\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right)\middle|\frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right)}{\sqrt{c}\,e\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e*Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt
[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + S
qrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*
x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c
*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e*Sqrt[a + b*x + c*x^
2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
```

```
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 957

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+}}}{e}$$

$$= \frac{\left((ef-dg)\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-\sqrt{}}}}{e\sqrt{a+bx+cx^2}}$$

$$= \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,F\left(\text{s}\right.}{ce\sqrt{f+gx}\,\sqrt{a+bx}}$$

$$= \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,F\left(\text{s}\right.}{ce\sqrt{f+gx}\,\sqrt{a+bx}}$$

$$= \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,F\left(\text{s}\right.}{ce\sqrt{f+gx}\,\sqrt{a+bx}}$$

$$= \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,F\left(\text{s}\right.}{ce\sqrt{f+gx}\,\sqrt{a+bx}}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.

time = 20.67, size = 379, normalized size = 0.81

$$-\frac{i\sqrt{2}\sqrt{\frac{g\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}}\left(F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{f+gx}\right)|\frac{2cf-\left(b+\sqrt{b^2-4ac}\right)g}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}\right)-\Pi\left(\frac{e\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)}{2c(ef-dg)};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{f+gx}\right)|\frac{2cf-\left(b+\sqrt{b^2-4ac}\right)g}{2cf+\left(-b+\sqrt{b^2-4ac}\right)g}\right)\right)}{e\sqrt{\frac{c}{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[
b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c]
)*g)]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqr
t[b^2 - 4*a*c])*g)] - EllipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(2
*c*(e*f - d*g)), I*ArcSinh[Sqrt[2]*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])
*g)]*Sqrt[f + g*x]], (2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f + (-b + Sqr
t[b^2 - 4*a*c])*g)]))/(e*Sqrt[c/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(408) = 816$.
time = 0.13, size = 834, normalized size = 1.79

method	result
elliptic	$\frac{\sqrt{(gx+f)(c x^2+bx+a)}\left(\frac{2g\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\sqrt{\frac{x+\frac{f}{g}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+\ldots}}{2c}}}\ldots}{e\sqrt{cg\,x^3+bg\ldots}}\ldots\right.}{\ldots}$
default	$\frac{\left(-\mathrm{EllipticF}\left(\sqrt{2}\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)g\sqrt{-4ac+b^2}+\sqrt{-4ac+\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*g*
(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/
(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2
)))^(1/2))*g-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
```

```
)^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))
)^(1/2))*g*b+2*f*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2)))^(1/2))*c+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*g-2*Ellipt
icPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^
(1/2)/e*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*
c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x
+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)/c/(c*g*x^3+b
*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f + gx}}{(d + ex)\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(1/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.905 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=994

$$-\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)}+\frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}\,(cd^2-bde+ae^2)\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a}}$$

```
[Out] -e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)+1/2*Ellipt
icE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)
^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d
^2)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2)-f*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*
x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(g*x+f)^(1
/2)/(c*x^2+b*x+a)^(1/2)+d*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c
+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+
b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d
*e+c*d^2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+1/2*(e^2*(-a*g+b*f)-c*d*(-d*g+2
*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1
/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g
-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/
(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1
/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d*e+c
*d^2)/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 2.48, antiderivative size = 994, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {959, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d + e*x
))) + (Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*f*Sq
rt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x +
c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt
[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c
*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(e*(c*d^2 - b
*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g) - c*d*(2*e*f - d*g))*Sqrt[1 - (2*c*(f +
 g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f
 - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sq
rt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)*(
e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 959

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*(
```

```
m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*e
*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} - \frac{\int \frac{-2cdf+bef-aeg-2cdgx-cegx^2}{(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} - \frac{\int \left(-\frac{cdg}{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} - \frac{}{\sqrt{f+}\cdots}\right.}{}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{(cg)\int \frac{x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)} + \frac{(c\cdots}{}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{c\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{2(cd^2-bde+ae^2)} - \frac{(cf)\int \frac{}{\sqrt{f+gx}\cdots}}{2(cd^2-\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{\sqrt{2}\sqrt{b^2-4ac}\,dg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\cdots\right.}}}{}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\cdots}{\sqrt{2}\,(cd^2-bde+ae^2)\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(d+ex)} + \frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\cdots}{\sqrt{2}\,(cd^2-bde+ae^2)\cdots}$$

$$\frac{\sqrt{b^2-4ac}\,\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\cdots}{}$$

$$e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.65, size = 1502, normalized size = 1.51

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*e^2)*(d + e*x)*Sq
rt[a + x*(b + c*x)])) - ((f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*e*(-(e*f
) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)
*g^2])]*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)
))/(f + g*x)) + (I*Sqrt[2]*e*(-(e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*
c)*g^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c*f*(-1 +
f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c
)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c*f*(-1 +
f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c
)*g^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f
+ b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]
 - (I*Sqrt[2]*e*(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*
Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] -
(2*a*g^2)/(f + g*x) - 2*c*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x))
)/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] +
(2*a*g^2)/(f + g*x) + 2*c*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))
/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqr
t[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f
 + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b
^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] - ((2*I)*Sqrt[2]*g*(e^2*(b*f - a*g) + c*
d*(-2*e*f + d*g))*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] - (2*a*g^2)/(f + g*x) - 2*c
*f*(-1 + f/(f + g*x)) + b*g*(-1 + (2*f)/(f + g*x)))/(2*c*f - b*g + Sqrt[(b^
2 - 4*a*c)*g^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*g^2] + (2*a*g^2)/(f + g*x) + 2*c
*f*(-1 + f/(f + g*x)) + b*(g - (2*f*g)/(f + g*x)))/(-2*c*f + b*g + Sqrt[(b^
2 - 4*a*c)*g^2])]*EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(b^2 - 4*a*c)
*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 -
b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -
((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)
*g^2]))])/Sqrt[f + g*x]))/(4*e*(c*d^2 - b*d*e + a*e^2)*g*(-(e*f) + d*g)*Sqr
t[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt
[a + x*(b + c*x)]*Sqrt[((f + g*x)^2*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)
/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)))/g^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 13016 vs. $2(885) = 1770$.
time = 0.12, size = 13017, normalized size = 13.10

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(-\frac{e\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{\left(a\,e^2-deb+c\,d^2\right)(ex+d)}+\frac{dgc\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\right.}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/((d + e*x)**2*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((d + e*x)^2*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(1/2)/((d + e*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.906 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1786

$$\sqrt{b^2 -$$

$$-\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\,(cd^2-bde+ae^2)\,(d+ex)^2}-\frac{e(cd(6ef-5dg)-e(3bef-2bdg-aeg))\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{4\,(cd^2-bde+ae^2)^2\,(ef-dg)(d+ex)}+$$

```
[Out] -1/2*e*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2-1/4*
e*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*(g*x+f)^(1/2)*(c*x^2+b*x+
a)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)/(e*x+d)+1/8*(c*d*(-5*d*g+6*e*f)-e
*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/
2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-1/2*g*EllipticF(1/2*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*
c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(-
c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
-1/4*f*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticF(1/2*((b+2*
c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2
)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(-c*(c*
x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)
^(1/2)+1/4*d*g*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*EllipticF(1/
2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4
*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)
*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(c*
x^2+b*x+a)^(1/2)+1/2*(b*e*g-3*c*d*g+c*e*f)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+
f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c
+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)
*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+
b^2)^(1/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/c^(1/2)/(c*x^2
+b*x+a)^(1/2)-1/8*(c*d*(-5*d*g+6*e*f)-e*(-a*e*g-2*b*d*g+3*b*e*f))*(c*d*(-3*
d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2
```

```
)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(
1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f
-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2*2^(1/2)/c^(1/2)/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A]

time = 5.67, antiderivative size = 1786, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {959, 6874, 732, 430, 953, 857, 435, 948, 175, 552, 551}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]

[Out] -1/2*(e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(d +
e*x)^2) - (e*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f +
 g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d +
e*x)) + (Sqrt[b^2 - 4*a*c]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*
e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(4*Sqrt[2]*(c*
d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*
x^2]) - (Sqrt[b^2 - 4*a*c]*f*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g -
a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
 (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d
*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(6*e
*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elli
pticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[
2]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x
^2]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*S
qrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*
(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g
+ Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f +
```

```
 g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (
2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*e*(c*d^2 -
 b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqr
t[b^2 - 4*a*c])*g]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*(c
*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/
(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b +
 Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/
(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b -
 Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2
- 4*a*c] - (2*c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f
 - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 959

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
```

```
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt
[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*e
*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \frac{-4cdf+3bef-aeg+2(cef-2cdg+beg)x+cegx^2}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,d\cdots}{4\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{\int \left(\frac{cg}{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{-\cdots}{e(d+e\cdots}\right.}{\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{4e\left(cd^2-bde+ae^2\right)} - \cdots$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-ae\cdots}{4\left(cd^2-bde+ae^2\right)^2(ef\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-ae\cdots}{4\left(cd^2-bde+ae^2\right)^2(ef\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-ae\cdots}{4\left(cd^2-bde+ae^2\right)^2(ef\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-ae\cdots}{4\left(cd^2-bde+ae^2\right)^2(ef\cdots}$$

$$= -\frac{e\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{e(cd(6ef-5dg)-e(3bef-2bdg-ae\cdots}{4\left(cd^2-bde+ae^2\right)^2(ef\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.86, size = 36634, normalized size = 20.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 59521 vs. 2(1601) = 3202.
time = 0.23, size = 59522, normalized size = 33.33

method	result	size
elliptic	Expression too large to display	1698
default	Expression too large to display	59522

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(f + g*x)/((d + e*x)**3*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f+g\,x}}{{\left(d+e\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(1/2)/((d + e*x)^3*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(1/2)/((d + e*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.907 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=675

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

```
[Out] g*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/
c/e/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+
2*g*(-d*g+e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(
c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(g*x+f)^(1/2)/(c*x^
2+b*x+a)^(1/2)-(-d*g+e*f)*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g
*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d
*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/
2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g
*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2
))))^(1/2)/e^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 1.14, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {971, 732, 430, 948, 175, 552, 551, 435}

$$\frac{2\sqrt{2}\,g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}(ef-dg)\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right)\middle|\frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right)}{\sqrt{c}\,e^2\sqrt{a+bx+cx^2}}+\frac{\sqrt{2}\,g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[(c*(f +
```

```
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)])/(c*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*f
 - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
 (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*
f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^
2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c]
 - (2*c*f)/g)])/(Sqrt[c]*e^2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 971

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{g(ef-dg)}{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{(ef-dg)^2}{e^2(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\right.$$

$$= \frac{g\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{e^2} +$$

$$= \frac{\left((ef-dg)^2\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-}}}{e^2\sqrt{a+bx+c}}$$

$$= \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2}}}}{\sqrt{}}\right.\right.}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+bx}}$$

$$= \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2}}}}{\sqrt{}}\right.\right.}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+bx}}$$

$$= \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2}}}}{\sqrt{}}\right.\right.}{ce\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{a+bx}}$$

$$= \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-}}{\sqrt{b^2}}}}{\sqrt{}}\right.\right.}{ce\sqrt{\frac{c(f+gx)}{}}\,\sqrt{a+b}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1385 vs. $2(675) = 1350$.

time = 12.48, size = 1385, normalized size = 2.05

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*((2*f*g*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*
c]*g)])/(c*e*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]) - (d
*g^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]*EllipticF[ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2
- 4*a*c]*g)])/(c*e^2*Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c
]]) + (g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] +
2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*((-2*c*f + (b + Sqrt[b^2 - 4*
a*c])*g)*EllipticE[ArcSin[Sqrt[2]*Sqrt[(c*(f + g*x))/(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^
2 - 4*a*c])*g)] - (b + Sqrt[b^2 - 4*a*c])*g*EllipticF[ArcSin[Sqrt[2]*Sqrt[(
c*(f + g*x))/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)]], (2*c*f + (-b + Sqrt[b^2
 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]))/(2*c^2*e*Sqrt[(g*(-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]) - (4*Sq
rt[b^2 - 4*a*c]*f^2*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticPi[(
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e), ArcSin[Sqrt[(-b
 + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*
a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)])/(2*c*d + (-b + Sqrt[b^2 - 4*a
*c])*e) + (8*Sqrt[b^2 - 4*a*c]*d*f*g*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a
*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)
, ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g)])/(e*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)) - (4*Sqrt[b^2 - 4*a*c]*d^2*g^2*Sqrt[(c*(a + x*
(b + c*x)))/(-b^2 + 4*a*c)]*EllipticPi[(2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - b*e
 + Sqrt[b^2 - 4*a*c]*e), ArcSin[Sqrt[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - b*g + Sqrt[b^2 - 4
*a*c]*g)])/(e^2*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))))/(Sqrt[f + g*x]*Sqrt
[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1878 vs. $2(592) = 1184$.

time = 0.16, size = 1879, normalized size = 2.78

method	result
elliptic	$\frac{\sqrt{(gx+f)\left(c\,x^2+bx+a\right)}\left(-\frac{2g(dg-2ef)\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\sqrt{\frac{x+\frac{f}{g}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}}}\sqrt{\frac{x-\frac{-b+\sqrt{-}}{}}{-\frac{f}{g}-\frac{-b+\sqrt{}}{}}}}{e^2\sqrt{cg\,x^3}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-
2*c*f))^(1/2)/c*((-4*a*c+b^2)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*d*g^2-(-4*a*c+b^2)^(1/2)*EllipticF(2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*e*f*g-(-4*a*c+b^2)^(1/
2)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1
/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*d*g^2+(-4*a*c+b^2)^(1/2)
*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2
*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*e*f*g+2*EllipticF(2^(1/2)*
(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*e*g^2+EllipticF(2^(1
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*d*g^2-3*Elliptic
F(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e*f*g-2*El
lipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*d*f*
g+4*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(
```

```
-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*
c*e*f^2-2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(
1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(
1/2))*a*e*g^2+2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/
2)))^(1/2))*b*e*f*g-2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b
*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*c*e*f^2-EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b
*d*g^2+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e*f*g+2*EllipticPi
(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*d*f*g-2*EllipticPi(2^(1/2)*(-(g*x+f
)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c
+b^2)^(1/2)))^(1/2))*c*e*f^2)/e^2/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^{\frac{3}{2}}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)**(3/2)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^{3/2}}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((f + g*x)^(3/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.908 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1138

$$\frac{2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3ce}+\frac{2\sqrt{2}\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\sqrt{\frac{b}{\ }}\right.\right.}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a}}$$

```
[Out] 2/3*g^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/3*g*(-b*g+2*c*f)*EllipticE(
1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(
-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/e/(c*x^2
+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+g*(-d*g+e*
f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)
/c/e^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/
2)+2*g*(-d*g+e*f)^2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1
/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(g*x+f)^(1/2)/
(c*x^2+b*x+a)^(1/2)-2/3*g*(a*g^2-b*f*g+c*f^2)*EllipticF(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(
2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)/c^2/e/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-(-d*g+e*f)^2*EllipticPi(2^(1/
2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*
f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b
-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4
*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x
+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^3/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 1.47, antiderivative size = 1138, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {971, 732, 430, 948, 175, 552, 551, 435, 756, 857}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) + (2*Sqrt[2]*Sqrt[b^2 -
 4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4
*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g
)])/(3*c^2*e*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a
 + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[f + g*x]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^2*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*e^3*Sqrt[f + g*x]*Sqrt[a +
b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(c*f^2 - b*f*g + a*g^2)*Sqrt
[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/(3*c^2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*
Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)^2*Sqrt[1 - (2*c*(f + g*
x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*
g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f -
(b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[
b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*e^3*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 971

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \left(\frac{g(ef-dg)^2}{e^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \frac{(ef-dg)^3}{e^3(d+ex)\sqrt{f+gx}\,\sqrt{a+bx+cx^2}} + \right.$$

$$= \frac{g\int \frac{(f+gx)^{3/2}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(g(ef-dg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{e^2} + \frac{(g(ef-dg)}{}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{(2g)\int \frac{\frac{1}{2}(3cf^2-g(bf+ag))+g(2cf-bg)x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{3ce} + \frac{((e}{}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{f+gx}\,\sqrt{-\frac{c}{}}}{ce^2\,\sqrt{}}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g(ef-dg)\sqrt{f+gx}\,\sqrt{-\frac{c}{}}}{ce^2\,\sqrt{}}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\,\sqrt{-}}{3c^2e}$$

$$= \frac{2g^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,g(2cf-bg)\sqrt{f+gx}\,\sqrt{-}}{3c^2e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.34, size = 37137, normalized size = 32.63

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7463 vs. $2(997) = 1994$.
time = 0.16, size = 7464, normalized size = 6.56

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2g^2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{3ec}+\frac{2\left(\frac{g\left(d^2g^2-3defg+3e^2f^2\right)}{e^3}-\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(f+gx)^{\frac{5}{2}}}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((f + g*x)**(5/2)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^{5/2}}{\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^(5/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((f + g*x)^(5/2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.909 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=631

$$-\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2}+\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}+\frac{\sqrt{2}\sqrt{b^2-4ac}}{}$$

```
[Out] -8/15*e^2*(b*e*g-3*c*d*g+c*e*f)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/g^2+2
/5*e^2*(e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g+1/15*e*(8*b^2*e^2*g^2+
c*e*g*(-9*a*e*g-30*b*d*g+7*b*e*f)+c^2*(45*d^2*g^2-30*d*e*f*g+8*e^2*f^2))*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/
g^3/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2/15*(4*b*e^3*g^2*(-a*g+b*f)+c^2*(-15*d^3*g^3+45*d^2*e*f*g^2-30*d*e^2*f^2*g
+8*e^3*f^3)-c*e^2*g*(a*g*(-15*d*g+7*e*f)-3*b*f*(-5*d*g+e*f)))*EllipticF(1/2
*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*
a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)/c^3/g^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 0.74, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {944, 1667, 857, 732, 435, 430}

$$\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2(45d^2g^2-30defg+8e^2f^2)\right)E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{15c^3g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce^2g(ag(7ef-15dg)-3bf(ef-5dg))+4be^3g^2(bf-ag)+c^2(-15d^3g^3+45d^2efg^2-30de^2f^2g+8e^3f^3)\right)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{15c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{15c^2g^2}+\frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-8*e^2*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*g^2) + (2*e^2*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(5*c*g) +
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9
*a*e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*Sqrt[f + g*x]*Sqrt[-((
c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*
```

```
f - (b + Sqrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b
 + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*
a*c]*(4*b*e^3*g^2*(b*f - a*g) + c^2*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*
f*g^2 - 15*d^3*g^3) - c*e^2*g*(a*g*(7*e*f - 15*d*g) - 3*b*f*(e*f - 5*d*g)))
*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)])/(15*c^3*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 944

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), I
nt[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*f
```

```
 + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*f
 + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*
d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} - \frac{\int \frac{bde^2f-5cd^3g+ae^2(2ef+dg)+e(cd(2ef-}{\sqrt{f+g}}}{}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+g}}{5c}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+g}}{5c}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+g}}{5c}$$

$$= -\frac{8e^2(cef-3cdg+beg)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+g}}{5c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.68, size = 855, normalized size = 1.35

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*e*g^2*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9*a*e*g) + c^2*(8*e^
2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*(a + x*(b + c*x)))/Sqrt[f + g*x] + 4*c*e^
2*g^2*Sqrt[f + g*x]*(a + x*(b + c*x))*(-4*b*e*g + c*(-4*e*f + 15*d*g + 3*e*
g*x)) - (I*(f + g*x)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) + a*g
)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*(e*(2*c*f - b*g +
```

```
 Sqrt[(b^2 - 4*a*c)*g^2])*(8*b^2*e^2*g^2 + c*e*g*(7*b*e*f - 30*b*d*g - 9*a*
e*g) + c^2*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*EllipticE[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])
/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2]))] - (-8*b^3*e^3*g^3 + b^2*e^2*g^2*(c*e*f + 30*c*d*
g + 8*e*Sqrt[(b^2 - 4*a*c)*g^2]) + b*c*e*g*(-45*c*d^2*g^2 + e*(17*a*e*g^2 +
 Sqrt[(b^2 - 4*a*c)*g^2]*(7*e*f - 30*d*g))) + c*(-(a*e^2*g^2*(4*c*e*f + 30*
c*d*g + 9*e*Sqrt[(b^2 - 4*a*c)*g^2])) + c*(30*c*d^3*g^3 + e*Sqrt[(b^2 - 4*a
*c)*g^2]*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))))*EllipticF[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])
/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2]))]))/Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])])/(30*c^3*g^4*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8754 vs. $2(567) = 1134$.
time = 0.14, size = 8755, normalized size = 13.87

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{2e^3x\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{5cg}+\frac{2\left(3d\,e^2-\frac{2(2bg+2cf)e^3}{5cg}\right)\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x*e + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 626, normalized size = 0.99

$$\frac{2\left(\left(45\,c^3d^3g^3-\left(8\,c^3f^3+3\,bc^2f^2g+3\,(b^2c-ac^2)fg^2+(8\,b^3-21\,abc)g^3\right)e^3+15\,\left(2\,c^3df^2g+bc^2dfg^2+(2\,b^2c-3\,ac^2)dg^3\right)e^2-45\,\left(c^3d^2fg^2+bc^2d^2g^3\right)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)-3\,\left(45\,c^3d^2g^3e+\left(8\,c^3f^2g+7\,bc^2fg^2+(8\,b^2c-9\,ac^2)g^3\right)e^3-30\,\left(c^3dfg^2+bc^2dg^3\right)e^2\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2f^2-bcfg+(b^2-3\,ac)g^2)}{3\,c^2g^2},-\frac{4\,(2\,c^3f^3-3\,bc^2f^2g-3\,(b^2c-6\,ac^2)fg^2+(2\,b^3-9\,abc)g^3)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)\right)+3\,\left(15\,c^3dg^3e^2+\left(3\,c^3g^3x-4\,c^3fg^2-4\,bc^2g^3\right)e^3\right)\sqrt{cx^2+bx+a}\,\sqrt{gx+f}\right)}{45\,c^4g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/45*((45*c^3*d^3*g^3 - (8*c^3*f^3 + 3*b*c^2*f^2*g + 3*(b^2*c - a*c^2)*f*g^
2 + (8*b^3 - 21*a*b*c)*g^3)*e^3 + 15*(2*c^3*d*f^2*g + b*c^2*d*f*g^2 + (2*b^
2*c - 3*a*c^2)*d*g^3)*e^2 - 45*(c^3*d^2*f*g^2 + b*c^2*d^2*g^3)*e)*sqrt(c*g)
*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2),
 -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*
a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) - 3*(45*c^3*d^2*g^3
*e + (8*c^3*f^2*g + 7*b*c^2*f*g^2 + (8*b^2*c - 9*a*c^2)*g^3)*e^3 - 30*(c^3*
d*f*g^2 + b*c^2*d*g^3)*e^2)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*
g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2
*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse
(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 -
3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3
), 1/3*(3*c*g*x + c*f + b*g)/(c*g))) + 3*(15*c^3*d*g^3*e^2 + (3*c^3*g^3*x -
 4*c^3*f*g^2 - 4*b*c^2*g^3)*e^3)*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f))/(c^4*
g^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**3/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((x*e + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^3/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.910 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=479

$$\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,e(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-}\right.}{3c^2g^2\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}}$$

[Out] 2/3*e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/g-2/3*e*(b*e*g-3*c*d*g+c*e*f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^2/g^2/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/3*(e^2*g*(-a*g+b*f)+c*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/g^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]

time = 0.42, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {944, 24, 857, 732, 435, 430}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\left(e^2g(bf-ag)+c(3d^2g^2-6defg+2e^2f^2)\right)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}-\frac{2\sqrt{2}\,e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(beg-3cdg+cef)E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{3c^2g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]

[Out] (2*e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*g) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2*g^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e^2*g*(b*f - a*g) + c*(2*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*El

```
lipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(3*c^2
*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 944

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g
```

```
*x]*(Sqrt[a + b*x + c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), I
nt[((d + e*x)^(m - 3)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*e^2*f
 + a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(2*b*d*g + e*(b*f
 + a*g)*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*
d*g + b*e*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[2*m] && GeQ[m, 2]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3cg} - \frac{\int \frac{d(be^2f-3cd^2g+ae^2g)+e(cd(2ef-9dg)+e(bef+2bdg}{(d+ex)\sqrt{f+gx}\,\sqrt{a+b}}}{3cg}$$

$$= \frac{2e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3cg} - \frac{\int \frac{e^2(be^2f-3cd^2g+ae^2g)+2e^3(cef-3cdg+beg)x}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx}{3ce^2g}$$

$$= \frac{2e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3cg} - \frac{(2e(cef-3cdg+beg))\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}}{3cg^2}$$

$$= \frac{2e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3cg} - \frac{\left(2\sqrt{2}\,\sqrt{b^2-4ac}\,e(cef-3cdg+beg)\sqrt{}\right.}{}$$

$$= \frac{2e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{3cg} - \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,e(cef-3cdg+beg)\sqrt{f}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.96, size = 981, normalized size = 2.05

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[f + g*x]*(2*c*e^2*g^2*(a + x*(b + c*x)) + ((f + g*x)*((-4*e*g^2*(c*e*
f - 3*c*d*g + b*e*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*e*(c*e*f -
3*c*d*g + b*e*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + f
*Sqrt[(b^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f
 - g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2
 + f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*
g*(-f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Ellipti
cE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^
2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2
])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x] + (I*Sqrt[2]*(3
*c^2*d^2*g^2 + b*e^2*g*(b*g - Sqrt[(b^2 - 4*a*c)*g^2]) - c*e*(3*b*d*g^2 + a
*e*g^2 + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f - 3*d*g)))*Sqrt[(-2*a*g^2 + f*Sqrt[(b
^2 - 4*a*c)*g^2] + 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(f - g*x))
/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 + f*Sqr
t[(b^2 - 4*a*c)*g^2] - 2*c*f*g*x + g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f +
g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*Arc
Sinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*
c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*
f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x]))/Sqrt[(c*f^2 + g*(-(b*
f) + a*g))/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]))/(3*c^2*g^3*Sqrt[a +
x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4294 vs. $2(421) = 842$.
time = 0.13, size = 4295, normalized size = 8.97 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/c^2*(12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-
b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2
*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Elliptic
E(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*e*f*g^
2+4*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+
(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4
*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2
)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e^2*g^3-4*2^(1/2
)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^
2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^
(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f
```

)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*e^2*f*g^2+3*2^(1/2)*(-(g*x
+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2)
)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c
*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*e^2*f*g^2+2*c^2*e^2*g^3*x^3+2*b*c*
e^2*f*g^2*x-3*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(
(-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b
+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Ellipt
icF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e^2*g^
3-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+
(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4
*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2
)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c
*e^2*f^2*g+12*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(
(-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b
+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Ellipt
icF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e*g^
3+2*c^2*e^2*f*g^2*x^2+2*a*c*e^2*g^3*x+2*a*c*e^2*f*g^2+2*b*c*e^2*g^3*x^2+4*2
^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a
*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+
b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(
g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*e^2*f^3-3*2^(1/2)*(-(
g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1
/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(
2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d^2*g^3+6*2^(1/2)*(-(g*x+f)*c/(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*
c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*d^2*f*g^2+2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*
(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*e^2*g^3-3*2^(1/2)*(-(g*x+f)*c/(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f
-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*

```
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(
-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*d^2*g^3-6*2^(1/2)*(-(g*x+f)
*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e^2*f*g^2+4*2^(1/2)*(-(g*x+f)*c/(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x*e + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 467, normalized size = 0.97

$$\frac{2\left(3\sqrt{cx^2+bx+a}\,\sqrt{gx+f}\,c^2g^2e^2+\left(9\,c^2d^2g^2+\left(2\,c^2f^2+bcfg+\left(2\,b^2-3\,ac\right)g^2\right)e^2-6\left(c^2dfg+bcdg^2\right)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)-6\left(3\,c^2dg^2e-\left(c^2fg+bcg^2\right)e^2\right)\sqrt{cg}\,\mathrm{weierstrassZeta}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)\right)\right)}{9\,c^3g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*c^2*g^2*e^2 + (9*c^2*d^2*g^2 + (
2*c^2*f^2 + b*c*f*g + (2*b^2 - 3*a*c)*g^2)*e^2 - 6*(c^2*d*f*g + b*c*d*g^2)*
e)*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2
)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
 (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)) - 6*(3*
c^2*d*g^2*e - (c^2*f*g + b*c*g^2)*e^2)*sqrt(c*g)*weierstrassZeta(4/3*(c^2*f
^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2
*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstr
assPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2
*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^
3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))))/(c^3*g^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^{2}}{\sqrt{f+gx}\,\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**2/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)^2/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.911 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=393

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}}{2cf-\left(b+\sqrt{b^2-4a\right.}}\right.}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}}$$

```
[Out] e*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/
c/g/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2*(-d*g+e*f)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^
(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*
(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/g/(g*x+f)^(1/2)/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {857, 732, 435, 430}

$$\frac{\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(c*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[
a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[(c*(f + g
*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)])/(c*g*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{e\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}}\,dx}{g} + \frac{(-ef+dg)\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{g}$$

$$= \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{}{2c}}}{}\right.}{cg\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}}$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\sqrt{\frac{b+\sqrt{b^2}}{\sqrt{b}}}\right.\right.}{cg\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 23.69, size = 814, normalized size = 2.07

$$-\frac{(f+gx)^{3/2}\left(-\frac{4eg^2\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))}{(f+gx)^2}+\frac{i\sqrt{2}e\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\sqrt{\frac{-2ag^2+2cfgx+bg(f-gx)+\sqrt{(b^2-4ac)g^2}(f+gx)}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{\frac{2ag^2-2cfgx+bg(-f+gx)+\sqrt{(b^2-4ac)g^2}(f+gx)}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}E\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)}{\sqrt{f+gx}}-\frac{i\sqrt{2}\left(2cdg+e\left(-bg+\sqrt{(b^2-4ac)g^2}\right)\right)\sqrt{\frac{-2ag^2+2cfgx+bg(f-gx)+\sqrt{(b^2-4ac)g^2}(f+gx)}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{\frac{2ag^2-2cfgx+bg(-f+gx)+\sqrt{(b^2-4ac)g^2}(f+gx)}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)}{\sqrt{f+gx}}\right)}{2cg^2\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((f + g*x)^(3/2)*((-4*e*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f +
b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]
*e*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g
*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*f - b*g + Sqrt[(b^2 -
 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x + b*g*(-f + g*x) + Sqrt
[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f
 + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f
 + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[
(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))])/Sqrt[f + g*x
] - (I*Sqrt[2]*(2*c*d*g + e*(-(b*g) + Sqrt[(b^2 - 4*a*c)*g^2]))*Sqrt[(-2*a*
g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*
f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x +
```

```
b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b
*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -(
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*
g^2]))])/Sqrt[f + g*x]))/(c*g^2*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b
*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(345) = 690$.

time = 0.12, size = 1014, normalized size = 2.58

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+bx+a)}\left(\frac{2d\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\sqrt{\frac{x+\frac{f}{g}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b}}{2c}}}}{\sqrt{cg\,x^3+bg\,x}}\right.}{}$
default	$\frac{\left(2\,\mathrm{EllipticF}\left(\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)ae\,g^2-\mathrm{EllipticF}\left(\sqrt{2}\,\sqrt{-\frac{}{g\sqrt{-}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-
(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a
*e*g^2-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2
),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
))*b*d*g^2-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^
(1/2))*b*e*f*g+2*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1
/2)))^(1/2))*c*d*f*g-(-4*a*c+b^2)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*
f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*d*g^2+(-4*a*c+b^2)^(1/2)*EllipticF(2^(1
```

```
/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*e*f*g-2*EllipticE(
2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*e*g^2+2*Elli
pticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*e*f*g-
2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*
e*f^2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1
/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*x+f)^(1/2)
*(c*x^2+b*x+a)^(1/2)/c/g^2/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.29, size = 369, normalized size = 0.94

$$-\frac{2\left(3\sqrt{cg}\,cge\,\mathrm{weierstrassZeta}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)\right)-\left(3\,cdg-\left(cf+bg\right)e\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3\,ac\right)g^2\right)}{3\,c^2g^2},-\frac{4\left(2\,c^3f^3-3\,bc^2f^2g-3\left(b^2c-6\,ac^2\right)fg^2+\left(2\,b^3-9\,abc\right)g^3\right)}{27\,c^3g^3},\frac{3\,cgx+cf+bg}{3\,cg}\right)\right)}{3\,c^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(c*g)*c*g*e*weierstrassZeta(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a
*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*
f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), weierstrassPInverse(4/3*(c^2*f^2
- b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g
- 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*
x + c*f + b*g)/(c*g))) - (3*c*d*g - (c*f + b*g)*e)*sqrt(c*g)*weierstrassPIn
verse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)/(c^2*g^2), -4/27*(2*c^3*f
^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 + (2*b^3 - 9*a*b*c)*g^3)/(c^
3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g)))/(c^2*g^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+e\,x}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.912 $\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{c\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}$$

```
[Out] 2*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*
c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {732, 430}

$$\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{c\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(c*Sqrt[f + g*x]*Sqrt[a
+ b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
```

```
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-bg-\sqrt{b^2-4ac}\,g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\right)\text{Su}}{c\sqrt{f+}}$$

$$= \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\,F\left(\text{s}\right.}{c\sqrt{f+gx}\sqrt{a+bx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.46, size = 308, normalized size = 1.63

$$\frac{i(f+gx)\sqrt{2-\frac{4\left(cf^2+g(-bf+ag)\right)}{\left(2cf-bg+\sqrt{\left(b^2-4ac\right)g^2}\right)(f+gx)}}\sqrt{1+\frac{2\left(cf^2+g(-bf+ag)\right)}{\left(-2cf+bg+\sqrt{\left(b^2-4ac\right)g^2}\right)(f+gx)}}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{\left(b^2-4ac\right)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{\left(b^2-4ac\right)g^2}}{2cf-bg+\sqrt{\left(b^2-4ac\right)g^2}}\right)}{g\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{\left(b^2-4ac\right)g^2}}}\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt
[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])
/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
Sqrt[(b^2 - 4*a*c)*g^2]))])/(g*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*
g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [A]
time = 0.12, size = 287, normalized size = 1.52

method	result
default	$\frac{\left(-g\sqrt{-4ac+b^2}-bg+2cf\right)\operatorname{EllipticF}\left(\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\sqrt{-\frac{g\sqrt{-4ac+b^2}+bg-2cf}{2cf-bg+g\sqrt{-4ac+b^2}}}\right)\sqrt{\cdots}}{cg(cg\,x^3+b\cdots}$
elliptic	$\frac{2\sqrt{(gx+f)\left(c\,x^2+bx+a\right)}\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\sqrt{\frac{x+\frac{f}{g}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\cdots}{\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}\,\sqrt{cg\,x^3+bg\,x\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-g*(-4*a*c+b^2)^(1/2)-b*g+2*c*f)/c*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*
(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*
c*f))^(1/2)/g*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*g*x^3+b*g*x^2+c*f*x^2+a*
g*x+b*f*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 128, normalized size = 0.68

$$\frac{2\sqrt{cg}\,\operatorname{weierstrassPInverse}\left(\frac{4\left(c^2f^2-bcfg+\left(b^2-3ac\right)g^2\right)}{3c^2g^2},-\frac{4\left(2c^3f^3-3bc^2f^2g-3\left(b^2c-6ac^2\right)fg^2+\left(2b^3-9abc\right)g^3\right)}{27c^3g^3},\frac{3cgx+cf+bg}{3cg}\right)}{cg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*g)*weierstrassPInverse(4/3*(c^2*f^2 - b*c*f*g + (b^2 - 3*a*c)*g^2)
/(c^2*g^2), -4/27*(2*c^3*f^3 - 3*b*c^2*f^2*g - 3*(b^2*c - 6*a*c^2)*f*g^2 +
(2*b^3 - 9*a*b*c)*g^3)/(c^3*g^3), 1/3*(3*c*g*x + c*f + b*g)/(c*g))/(c*g)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.913 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=280

$$-\frac{\sqrt{2}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\,\Pi}{\sqrt{c}\,(ef-dg)\sqrt{a+bx+}}$$

```
[Out] -EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^
(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a
*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f
)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)/c^(
1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 0.82, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {948, 175, 552, 551}

$$-\frac{\sqrt{2}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right)|\frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right)}{\sqrt{c}\sqrt{a+bx+cx^2}\,(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))
/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))
/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b
- Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2
 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]))
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\int \frac{}{\sqrt{b-}}}{\sqrt{a+bx}}$$

$$= -\frac{\left(2\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{b+\sqrt{b^2-4ac}+2cx}\right)\text{Subst}}{}$$

$$= -\frac{\left(2\sqrt{b+\sqrt{b^2-4ac}+2cx}\,\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\right.}{}$$

$$= -\frac{\left(2\sqrt{1+\frac{2c(f+gx)}{\left(b-\sqrt{b^2-4ac}-\frac{2cf}{g}\right)g}}\,\sqrt{1+\frac{2c(f+gx}{\left(b+\sqrt{b^2-4ac}\right.}}\right.}{}$$

$$= -\frac{\sqrt{2}\,\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\sqrt{1-\frac{2c(f+gx)}{2cf-\left(b-\sqrt{b^2-4a\right.}}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 21.16, size = 499, normalized size = 1.78

$$\frac{i(f+gx)\sqrt{2-\frac{4(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{1+\frac{2(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)-\Pi\left(\frac{(ef-dg)\left(2cf-bg-\sqrt{(b^2-4ac)g^2}\right)}{2e(cf^2+g(-bf+ag))};i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)\right)}{(-ef+dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I*(f + g*x)*Sqrt[2 - (4*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2
```

```
*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*(EllipticF[I*ArcSinh[(Sqr
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))] - EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(
b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sq
rt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[
f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(
b^2 - 4*a*c)*g^2]))]))/((-(e*f) + d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[a + x*(b + c*x)])
```

Maple [A]
time = 0.14, size = 330, normalized size = 1.18

method	result
default	$\frac{\left(-g\sqrt{-4ac+b^2}-bg+2cf\right)\mathrm{EllipticPi}\left(\sqrt{2}\,\sqrt{-\frac{(gx+f)c}{g\sqrt{-4ac+b^2}+bg-2cf}},\frac{\left(g\sqrt{-4ac+b^2}+bg-2cf\right)e}{2c(dg-ef)},\sqrt{-\frac{g\sqrt{-}}{2cf-}}\right.}{}$
elliptic	$\frac{2\sqrt{(gx+f)\left(cx^2+bx+a\right)}\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\sqrt{\frac{x+\frac{f}{g}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{f}{g}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}}{\sqrt{gx+f}\,\sqrt{cx^2+bx+a}\,e\sqrt{cg\,x^3+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-g*(-4*a*c+b^2)^(1/2)-b*g+2*c*f)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c
+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g
-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^
(1/2))*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1
/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)/c*(c*x^2+b*x+
a)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.914 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1037

$$-\frac{e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)}+\frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-\cdots}}{\sqrt{b^2\cdots}}}}{\sqrt{\cdots}}\right.\right.\cdots}{\sqrt{2}\,(cd^2-bde+ae^2)(ef-dg)\sqrt{\frac{c(f+gx\cdots}{2cf-\left(b+\sqrt{b^2-\cdots}\right.}}}$$

```
[Out] -e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(e*x+
d)+1/2*e*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1
/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2
))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(
a*e^2-b*d*e+c*d^2)/(-d*g+e*f)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f
-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-e*f*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*
(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a
)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a
*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)+d*g*Elliptic
F(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g
*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*
c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(g*x+f)^(1/2)/(c*
x^2+b*x+a)^(1/2)-1/2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticP
i(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*
e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1
/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*
a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+
f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2
*2^(1/2)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 2.39, antiderivative size = 1037, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {953, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*f -
 d*g)*(d + e*x))) + (Sqrt[b^2 - 4*a*c]*e*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqr
t[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[(c*(
f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sq
rt[2]*Sqrt[b^2 - 4*a*c]*e*f*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*
(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*
c]*d*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)])/((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d
*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2
*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + S
qrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2
*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - S
qrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 -
4*a*c] - (2*c*f)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^
2*Sqrt[a + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
```

```
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} - \frac{\int \frac{-2cd(ef-dg)+e(bef-2bdg\cdots}{(d+ex)\sqrt{f+gx}\,\sqrt{\cdots}}}{2(cd^2-bde+a\cdots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} - \frac{\int\left(-\frac{cdg}{\sqrt{f+gx}\,\sqrt{a+\cdots}}\cdots\right.}{\cdots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{(cdg)\int\frac{1}{\sqrt{f+gx}\,\sqrt{a\cdots}}}{2(cd^2-bde+ae^2)\cdots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{(ce)\int\frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^{\cdots}}}\cdots}{2(cd^2-bde+ae^2)(ef\cdots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,dg\sqrt{\frac{\cdots}{2c\cdots}}\cdots}{\cdots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\cdots}{\sqrt{2}\,(c\cdots}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac}\,e\sqrt{f+gx}\cdots}{\sqrt{2}\,(c\cdots}$$

$$\sqrt{b^2-4ac}\,e\sqrt{f+gx}\cdots$$

$$e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.03, size = 842, normalized size = 0.81

$$\frac{4eg(ef-dg)(a+x(b+cx))-\frac{4e^2(ef-dg)(f+gx)(a+x(b+cx))}{d+ex}+\frac{i(f+gx)^{3/2}\sqrt{1-\frac{2(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)\,g^2}\right)(f+gx)}}\sqrt{2+\frac{4(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)\,g^2}\right)(f+gx)}}\left(e(-ef+dg)\left(2cf-bg+\sqrt{(b^2-4ac)\,g^2}\right)E\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)\,g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)\,g^2}}{2cf-bg+\sqrt{(b^2-4ac)\,g^2}}\right)-\left(2cdg(ef-2dg)-e\left(2aeg^2+bg(ef-3dg)+\sqrt{(b^2-4ac)\,g^2}\,(ef-dg)\right)\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)\,g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)\,g^2}}{2cf-bg+\sqrt{(b^2-4ac)\,g^2}}\right)-2g(cd(-2ef+3dg)+e(bef-2bdg+aeg))\Pi\left(\frac{(ef-dg)\left(2cf-bg-\sqrt{(b^2-4ac)\,g^2}\right)}{2e\left(cf^2+g(-bf+ag)\right)};i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)\,g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)\,g^2}}{2cf-bg+\sqrt{(b^2-4ac)\,g^2}}\right)\right)}{g\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)\,g^2}}}}}{4\left(cd^2+e(-bd+ae)\right)(ef-dg)^2\sqrt{f+gx}\,\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (4*e*g*(e*f - d*g)*(a + x*(b + c*x)) - (4*e^2*(e*f - d*g)*(f + g*x)*(a + x*
(b + c*x)))/(d + e*x) + (I*(f + g*x)^(3/2)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) +
 a*g)))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c
*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*
x))]*(e*(-(e*f) + d*g)*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*EllipticE[I*
ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4
*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2
*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] - (2*c*d*g*(e*f - 2*d*g) - e*(2*a*e
*g^2 + b*g*(e*f - 3*d*g) + Sqrt[(b^2 - 4*a*c)*g^2]*(e*f - d*g)))*EllipticF[
I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 -
 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/
(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))] - 2*g*(c*d*(-2*e*f + 3*d*g) + e*(
b*e*f - 2*b*d*g + a*e*g))*EllipticPi[((e*f - d*g)*(2*c*f - b*g - Sqrt[(b^2
- 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqrt[2]*Sqrt[(
c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f +
g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2
- 4*a*c)*g^2]))]))/(g*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + Sqrt[
(b^2 - 4*a*c)*g^2])]))/(4*(c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)^2*Sqrt[f +
 g*x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 14047 vs. $2(927) = 1854$.
time = 0.13, size = 14048, normalized size = 13.55

method	result

elliptic	$\frac{\sqrt{(gx+f)(cx^2+bx+a)}\left(\frac{e^2\sqrt{cg\,x^3+bg\,x^2+cf\,x^2+agx+bfx+fa}}{(ad\,e^2g-a\,e^3f-b\,d^2eg+bd\,e^2f+c\,d^3g-c\,d^2ef)(ex+d)}-\frac{cdg\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\ldots\right.}{\ldots}\right.}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**2*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.915 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1114

$$-\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\,(cd^2-bde+ae^2)\,(ef-dg)(d+ex)^2}-\frac{3e^2(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{f+gx}\,\sqrt{a+bx+c}}{4\,(cd^2-bde+ae^2)^2\,(ef-dg)^2(d+ex)}$$

```
[Out] -1/2*e^2*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(
e*x+d)^2-3/4*e^2*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(g*x+f)^(1/2)
*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/8*e*(c*d*
(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-
g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(a*e^2-b*d*e+c*d^2)^2/(-d*g+e*f)^2*2^(1/2)/(c
*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+1/4*(c
*d*(7*d*g-6*e*f)+e*(a*e*g-4*b*d*g+3*b*e*f))*EllipticF(1/2*((b+2*c*x+(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(g*(-4*a*c+b^2)^(1/2
)/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(c*(a+x*(c*x
+b))/(4*a*c-b^2))^(1/2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/
(c*d^2+e*(a*e-b*d))^2/(-d*g+e*f)*2^(1/2)/(g*x+f)^(1/2)/(a+x*(c*x+b))^(1/2)+
1/8*(c^2*d^2*(15*d^2*g^2-20*d*e*f*g+8*e^2*f^2)+2*c*e*(b*d*(-10*d^2*g^2+11*d
*e*f*g-4*e^2*f^2)+a*e*(3*d^2*g^2+2*d*e*f*g-2*e^2*f^2))+e^2*(3*a^2*e^2*g^2+2
*a*b*e*g*(-4*d*g+e*f)+b^2*(8*d^2*g^2-8*d*e*f*g+3*e^2*f^2)))*EllipticPi(2^(1
/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))^(1/2),(2*c*e*f-b
*e*g+e*g*(-4*a*c+b^2)^(1/2))/(-2*c*d*g+2*c*e*f),((2*c*f+g*(-b+(-4*a*c+b^2)^
(1/2)))/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(2*c*f-b*g+g*(-4*a*c+b^2)^
(1/2))^(1/2)*(g*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f+g*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-2*c*f+g*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/(c*d^2+e*(a*e-b*d))^2/(d*g-e*f)^3*2^(1/2)/c^(1/2)/(a+x*(c*x+b))^(
1/2)
```

Rubi [A]

time = 5.42, antiderivative size = 1762, normalized size of antiderivative = 1.58, number of steps used = 25, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {953, 6874, 732, 430, 857, 435, 948, 175, 552, 551}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*(e*
f - d*g)*(d + e*x)^2) - (3*e^2*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g +
a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(
e*f - d*g)^2*(d + e*x)) + (3*Sqrt[b^2 - 4*a*c]*e*(c*d*(2*e*f - 3*d*g) - e*(
b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)])/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[(c*(f + g*x))
/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 -
4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*
(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
- (b + Sqrt[b^2 - 4*a*c])*g)])/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)
*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (3*Sqrt[b^2 - 4*a*c]*e*f*(c*d*(2*e*
f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]
) + (3*Sqrt[b^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*
e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (
b + Sqrt[b^2 - 4*a*c])*g)])/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g
)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - S
qrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a
*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g
)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a
*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c
*f)/g)])/(Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[a + b*
x + c*x^2]) - (3*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*
g) - e*(b*e*f - 2*b*d*g + a*e*g))^2*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b -
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*
a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*
g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*
a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*
c*f)/g)])/(4*Sqrt[2]*Sqrt[c]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^3*Sqrt[a
 + b*x + c*x^2])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
```

```
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \frac{3e^2(bf+ag)-4d(cef-cd}{(d+ex)^2\sqrt{f}}}{4\left(cd^2-b\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{\int \left(\frac{cg}{\sqrt{f+gx}\,\sqrt{a+}}\right.}{}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{(cg)\int \frac{1}{\sqrt{f+gx}\,\sqrt{a}}}{4\left(cd^2-bde+ae\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-}{4\left(cd^2\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-}{4\left(cd^2\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-}{4\left(cd^2\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-}{4\left(cd^2\right.}$$

$$= -\frac{e^2\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}{2\left(cd^2-bde+ae^2\right)(ef-dg)(d+ex)^2} - \frac{3e^2(cd(2ef-3dg)-}{4\left(cd^2\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.35, size = 40396, normalized size = 36.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 64946 vs. $2(1014) = 2028$.
time = 0.29, size = 64947, normalized size = 58.30

method	result	size
elliptic	Expression too large to display	1686
default	Expression too large to display	64947

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^3 \sqrt{f+gx} \sqrt{a+bx+cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)**3*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*(x*e + d)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.916 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=553

$$\frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\,(cf^2-bfg+ag^2)\,\sqrt{f+gx}}-\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,g\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\sqrt{\frac{b-\cdots}{\cdots}}\cdots\right.\right.}{(ef-dg)\,(cf^2-bfg+ag^2)\,\sqrt{\frac{c(f+\cdots}{2cf-\left(b+\sqrt{\cdots}\right.}}}$$

[Out] 2*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(1/2)-g*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*
g+e*f)/(a*g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2)-e*EllipticPi(2^(1/2)*c^(1/2)*(g*x+f)^(1/2)/(2*c*f-g*
(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/c/(-d*
g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-4*a*c+b^2)^(1/2)))^(1/2
))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*
(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/(-d*g+e*f)^2/c^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]
time = 1.12, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {971, 758, 21, 732, 435, 948, 175, 552, 551}

$$-\frac{\sqrt{2}\,g\sqrt{b^2-4ac}\,\sqrt{f+gx}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\right)}{\sqrt{a+bx+cx^2}\,(ef-dg)\,(ag^2-bfg+cf^2)\,\sqrt{\frac{c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{\sqrt{2}\,e\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\,\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}\,\sqrt{1-\frac{2c(f+gx)}{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}\,\Pi\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\,g\right)}{2c(ef-dg)};\text{ArcSin}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\right)|\frac{b-\sqrt{b^2-4ac}-\frac{2cf}{g}}{b+\sqrt{b^2-4ac}-\frac{2cf}{g}}\right)}{\sqrt{c}\,\sqrt{a+bx+cx^2}\,(ef-dg)^2}+\frac{2g^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}\,(ef-dg)\,(ag^2-bfg+cf^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]

[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f +
 g*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*
x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b
^2 - 4*a*c])*g)])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*e*Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b -
 Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4

```
*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d
*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4
*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2
*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
```

```
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 971

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{3/2}\sqrt{a+bx+cx^2}} + \frac{\cdots}{(ef-dg)(d+ex \cdots}\right.$$

$$= \frac{e\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{ef-dg} - \frac{g\int \frac{1}{(f+gx)^{3/2}\sqrt{a+bx+\cdots}}}{ef-dg}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} + \frac{(2g)\int \frac{-\frac{cf}{2}-\cdots}{\sqrt{f+gx}\sqrt{a+\cdots}}}{(ef-dg)\left(cf^2-\cdots\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{(cg)\int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx\cdots}}}{(ef-dg)\left(cf^2-bfg\cdots\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f\cdots}}{(e\cdots}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\,g\sqrt{f\cdots}}{(e\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.04, size = 950, normalized size = 1.72

$$\frac{2\left(\frac{g^2(a+x(b+cx))}{ef-dg}+\frac{(f+gx)^2\left(c+\frac{cf^2}{(f+gx)^2}-\frac{bfg}{(f+gx)^2}+\frac{ag^2}{(f+gx)^2}-\frac{2cf}{f+gx}+\frac{bg}{f+gx}-\frac{i\sqrt{1-\frac{2(cf^2+g(-bf+ag))}{\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\sqrt{2+\frac{4(cf^2+g(-bf+ag))}{\left(-2cf+bg+\sqrt{(b^2-4ac)g^2}\right)(f+gx)}}\left((ef-dg)\left(2cf-bg+\sqrt{(b^2-4ac)g^2}\right)\left(E\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)-F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)\right)-2e(cf^2+g(-bf+ag))F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)+2e(cf^2+g(-bf+ag))\Pi\left(\frac{(ef-dg)\left(2cf-bg-\sqrt{(b^2-4ac)g^2}\right)}{2e(cf^2+g(-bf+ag))};i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bfg+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\middle|-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}\right)\right)}{4(ef-dg)\sqrt{\frac{cf^2+g(-bf+ag)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{f+gx}}\right)}{-ef+dg}\right)}{(cf^2+g(-bf+ag))\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*((g^2*(a + x*(b + c*x)))/(e*f - d*g) + ((f + g*x)^2*(c + (c*f^2)/(f + g*
x)^2 - (b*f*g)/(f + g*x)^2 + (a*g^2)/(f + g*x)^2 - (2*c*f)/(f + g*x) + (b*g
)/(f + g*x) - ((I/4)*Sqrt[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g
+ Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[2 + (4*(c*f^2 + g*(-(b*f) + a*g
)))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*((e*f - d*g)*(2*c
*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f
^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x
]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4
*a*c)*g^2]))] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(
-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g +
 Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]) - 2*e*
(c*f^2 + g*(-(b*f) + a*g))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g
 + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*
c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]
))] + 2*e*(c*f^2 + g*(-(b*f) + a*g))*EllipticPi[((e*f - d*g)*(2*c*f - b*g -
 Sqrt[(b^2 - 4*a*c)*g^2]))/(2*e*(c*f^2 + g*(-(b*f) + a*g))), I*ArcSinh[(Sqr
t[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]
)/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g +
 Sqrt[(b^2 - 4*a*c)*g^2]))]))/((e*f - d*g)*Sqrt[(c*f^2 + g*(-(b*f) + a*g))/
(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])]*Sqrt[f + g*x])))/(-(e*f) + d*g)))
/((c*f^2 + g*(-(b*f) + a*g))*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4756 vs. $2(490) = 980$.
time = 0.14, size = 4757, normalized size = 8.60

method	result

elliptic	$\frac{\sqrt{(gx+f)(c x^2+bx+a)}\left(-\frac{2\left(x^2cg+bgx+ag\right)g}{\left(a g^2-bfg+c f^2\right)(dg-ef)\sqrt{\left(x+\frac{f}{g}\right)(x^2cg+bgx+ag)}}+\frac{2\left(-\frac{g(bg-cf)}{\left(a g^2-bfg+c f^2\right)(dg}\right.}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-
4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a
*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticPi(2^(1/2)
*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*e*f*g^2+(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x
+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2)
)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e
/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(
1/2)))^(1/2))*a*e*g^3+2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(
1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2
)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/
2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*e*g^3+(-4*a*c+b^2)^(1
/2)*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+
(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4
*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticPi(2^(1/
2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*e*f^2*g-(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(g*x
+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2)
```

```
)*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e
/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(
1/2)))^(1/2))*b*e*f*g^2-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)
))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^
(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
,(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)
)*a*c*e*f*g^2-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)
*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(
(b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*Elli
pticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*
a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*e*f^
3-2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+
(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4
*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticPi(2^(1/
2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c^2*e*f^3+2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+
b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*
(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^
(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)
+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+
b^2)^(1/2)))^(1/2))*a*c*d*g^3-2*c^2*d*g^3*x^2-2*a*c*d*g^3+2*b*c*e*f*g^2*x-2
*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4
*a*c+b^2)^(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*
c+b^2)^(1/2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*d*f*g^2+2*2^(1/2)*(
-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))*g/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/
2))*g/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e*f^2*g+2*2^(1/2)*(-(g*x+f)*c
/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(
2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(
-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*e*f*g^2+2*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*
c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*g/(2*c*f-b*g+
g*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*g/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(g*x + f)^(3/2)*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\,(f+gx)^{\frac{3}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x)*(f + g*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(g*x + f)^(3/2)*(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(3/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.917 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=1125

$$\frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}}+\frac{4g^2(2cf-bg)\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)^2\sqrt{f+gx}}+\frac{2eg^2\sqrt{a+\cdots}}{(ef-dg)^2\left(cf^2-bf\cdots\right.}$$

```
[Out] 2/3*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f)^(3/2)+4/
3*g^2*(-b*g+2*c*f)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(g*
x+f)^(1/2)+2*e*g^2*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*
x+f)^(1/2)-2/3*g*(-b*g+2*c*f)*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a
*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2
+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^2/(c*x^2+b*x+a)^
(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-e*g*EllipticE(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a
*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)
^(1/2)*(g*x+f)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/(-d*g+e*f)^2/(a*
g^2-b*f*g+c*f^2)/(c*x^2+b*x+a)^(1/2)/(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)+2/3*g*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*g*(-4*a*c+b^2)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/
2)*(c*(g*x+f)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)/(a*g^2-b*f
*g+c*f^2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)-e^2*EllipticPi(2^(1/2)*c^(1/2)*
(g*x+f)^(1/2)/(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*e*(2*c*f-b*g+g*(-4
*a*c+b^2)^(1/2))/c/(-d*g+e*f),((b-2*c*f/g-(-4*a*c+b^2)^(1/2))/(b-2*c*f/g+(-
4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(1-2*c*(g*x+f)/(2*c*f-g*(b-(-4*a*c+b^2)^(
1/2))))^(1/2)*(2*c*f-g*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*(g*x+f)/(2*c*f-
g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d*g+e*f)^3/c^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 1.61, antiderivative size = 1125, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {971, 758, 848, 857, 732, 435, 430, 21, 948, 175, 552, 551}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*g^2*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g
*x)^(3/2)) + (4*g^2*(2*c*f - b*g)*Sqrt[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*
f^2 - b*f*g + a*g^2)^2*Sqrt[f + g*x]) + (2*e*g^2*Sqrt[a + b*x + c*x^2])/((e
*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]) - (2*Sqrt[2]*Sqrt[b^2 -
4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*
a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b
+ Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c
]*e*g*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[
ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)])/((e*f - d*g)^2
*(c*f^2 - b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2]) - (Sqrt[2]*e^2*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (
2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x)
)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^
2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sq
rt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)
/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)])/(Sqrt[c]*(e*f - d*g)^3*Sqrt[a + b*x
+ c*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 971

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \left(-\frac{g}{(ef-dg)(f+gx)^{5/2}\sqrt{a+bx+cx^2}} - \frac{}{(ef-dg)^2(f+}\right.$$

$$= \frac{e^2\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx}{(ef-dg)^2} - \frac{(eg)\int \frac{1}{(f+gx)^{3/2}\sqrt{a+}}}{(ef-dg}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{2eg^2\sqrt{a}}{(ef-dg)^2\left(cf^2-\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-b}{3(ef-dg)\left(cf^2-\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-b}{3(ef-dg)\left(cf^2-\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-b}{3(ef-dg)\left(cf^2-\right.}$$

$$= \frac{2g^2\sqrt{a+bx+cx^2}}{3(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)^{3/2}} + \frac{4g^2(2cf-b}{3(ef-dg)\left(cf^2-\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.03, size = 14759, normalized size = 13.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 27596 vs. 2(998) = 1996.
time = 0.20, size = 27597, normalized size = 24.53

method	result	size
elliptic	Expression too large to display	1505
default	Expression too large to display	27597

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(g*x + f)^(5/2)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\,(f+gx)^{\frac{5}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/((d + e*x)*(f + g*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(g*x + f)^(5/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(f+g\,x\right)}^{5/2}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(5/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((f + g*x)^(5/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.918 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=475

$$\frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\,\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\,\sqrt{\frac{(ef-}{(bf}}}{\sqrt{2c}}$$

[Out] (e*x+d)*EllipticPi((g*x+f)^(1/2)*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x+d)^(1/2)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2),e*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))/g/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))),((b*d-2*a*e+d*(-4*a*c+b^2)^(1/2))*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))/(b*f-2*a*g+f*(-4*a*c+b^2)^(1/2))/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(b+2*c*x-(-4*a*c+b^2)^(1/2))^(1/2)*((-d*g+e*f)*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(e*x+d)/(2*c*f-g*(b+(-4*a*c+b^2)^(1/2))))^(1/2)*(2*c*f-g*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((-d*g+e*f)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d)/(b*f-2*a*g+f*(-4*a*c+b^2)^(1/2)))^(1/2)/g/(c*x^2+b*x+a)^(1/2)/(c*x+2*a*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)

Rubi [A]
time = 0.26, antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {940}

$$\frac{\sqrt{2}(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\,\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}\,\sqrt{\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(d+ex)\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)}}\,\sqrt{\frac{\left(x\left(\sqrt{b^2-4ac}+b\right)+2a\right)(ef-dg)}{(d+ex)\left(f\sqrt{b^2-4ac}-2ag+bf\right)}}\,\Pi\left(\frac{e\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)g};\text{ArcSin}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\sqrt{f+gx}}{\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\,\sqrt{d+ex}}\right)|\frac{\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\left(bf+\sqrt{b^2-4ac}\,f-2ag\right)}\right)}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}\,\sqrt{a+bx+cx^2}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]

[Out] (Sqrt[2]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[((e*f - d*g)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)*(d + e*x))]*Sqrt[((e*f - d*g)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/((b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g)*(d + e*x))]*(d + e*x)*EllipticPi[(e*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*g), ArcSin[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])/(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])], ((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(b*f + Sqrt[b^2 - 4*a*c]*f - 2*a*g))])/(Sqrt[2*c*d - (

```
b + Sqrt[b^2 - 4*a*c])*e]*g*Sqrt[(2*a*c)/(b + Sqrt[b^2 - 4*a*c]) + c*x]*Sqr
t[a + b*x + c*x^2])
```

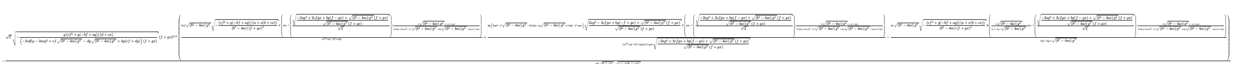
Rule 940

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\,\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\,\sqrt{b-\sqrt{b^2-4ac}+2cx}\,\sqrt{\frac{(ef-dg}{\left(2cf-\left(\right.\right.}}}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1118 vs. $2(475) = 950$.
time = 26.60, size = 1118, normalized size = 2.35



Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[-((g*(c*f^2 + g*(-(b*f) + a*g))*(d + e*x))/((-2*c*d*f*g - 2
*a*e*g^2 + e*f*Sqrt[(b^2 - 4*a*c)*g^2] - d*g*Sqrt[(b^2 - 4*a*c)*g^2] + b*g*
(e*f + d*g))*(f + g*x)))]*(f + g*x)^(3/2)*((2*e*f*Sqrt[(b^2 - 4*a*c)*g^2]*S
qrt[-(((c*f^2 + g*(-(b*f) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*
x)^2))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[
(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sqrt[2]]
, (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2 - e*f*S
```

```
qrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))])/(
c*f^2 + g*(-(b*f) + a*g)) + (d*g*(2*a*g^2 - f*Sqrt[(b^2 - 4*a*c)*g^2] - 2*c
*f*g*x - g*Sqrt[(b^2 - 4*a*c)*g^2]*x + b*g*(-f + g*x))*Sqrt[(2*a*g^2 - 2*c*
f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*
a*c)*g^2]*(f + g*x))]*EllipticF[ArcSin[Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f
- g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g
*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*(-(e*f) + d*g))/(2*c*d*f*g + 2*a
*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e
*f + d*g))])/((c*f^2 + g*(-(b*f) + a*g))*(f + g*x)*Sqrt[(-2*a*g^2 + 2*c*f*g
*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/(Sqrt[(b^2 - 4*a*c)
*g^2]*(f + g*x))]) - (4*e*Sqrt[(b^2 - 4*a*c)*g^2]*Sqrt[-(((c*f^2 + g*(-(b*f
) + a*g))*(a + x*(b + c*x)))/((b^2 - 4*a*c)*(f + g*x)^2))]*EllipticPi[(2*Sq
rt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]), ArcSin[Sqrt
[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))
/(Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*g^2]*
(-(e*f) + d*g))/(2*c*d*f*g + 2*a*e*g^2 - e*f*Sqrt[(b^2 - 4*a*c)*g^2] + d*g*
Sqrt[(b^2 - 4*a*c)*g^2] - b*g*(e*f + d*g))])/(2*c*f - b*g + Sqrt[(b^2 - 4*a
*c)*g^2])))/(g^2*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]))
```

Maple [A]
time = 0.21, size = 590, normalized size = 1.24

method	result
default	$4\sqrt{c\,x^{2}+bx+a}\;\sqrt{gx+f}\;\sqrt{ex+d}\;\sqrt{\frac{\left(e\sqrt{-4ac+b^{2}}+eb-2cd\right)(gx+f)}{\left(g\sqrt{-4ac+b^{2}}+bg-2cf\right)(ex+d)}}\;\sqrt{\frac{(dg-ef)\left(-b-2cx+\sqrt{-4ac+\right.}}{\left(2cf-bg+g\sqrt{-4ac+b^{2}}\right)(}}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 4*(c*x^2+b*x+a)^(1/2)*(g*x+f)^(1/2)*(e*x+d)^(1/2)/g*((e*(-4*a*c+b^2)^(1/2)+
e*b-2*c*d)*(g*x+f)/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(e*x+d))^(1/2)*((d*g-e*
f)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/(e*x+d))^
(1/2)*((d*g-e*f)*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c
*f)/(e*x+d))^(1/2)*EllipticPi(((e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)*(g*x+f)/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(e*x+d))^(1/2),(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*
f)*e/g/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d),((e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)*(
g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/(e*(-4*a*c
+b^2)^(1/2)+e*b-2*c*d))^(1/2))*((-4*a*c+b^2)^(1/2)*e^2*g*x^2+b*e^2*g*x^2-2*
c*e^2*f*x^2+2*(-4*a*c+b^2)^(1/2)*d*e*g*x+2*b*d*e*g*x-4*c*d*e*f*x+(-4*a*c+b^
```

```
2)^(1/2)*d^2*g+b*d^2*g-2*c*d^2*f)/(-1/c*(g*x+f)*(e*x+d)*(-b-2*c*x+(-4*a*c+b
^2)^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*
c*d)/((g*x+f)*(e*x+d)*(c*x^2+b*x+a))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x*e + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.919 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=588

$$-\frac{\sqrt[4]{cf^2-g(bf-ag)}\,(d+ex)\sqrt{\frac{(ef-dg)^2(a+bx+cx^2)}{(cf^2-bfg+ag^2)(d+ex)^2}}\left(1+\frac{\sqrt{cd^2-bde+ae^2}\,(f+gx)}{\sqrt{cf^2-g(bf-ag)}\,(d+ex)}\right)\sqrt{\frac{1-\frac{(2cd}{}}{}}}{\sqrt[4]{cd^2-bde+ae^2}\,(ef-dg)\sqrt{a+bx+cx^2}}$$

```
[Out] -(c*f^2-g*(-a*g+b*f))^(1/4)*(e*x+d)*(cos(2*arctan((a*e^2-b*d*e+c*d^2)^(1/4)
*(g*x+f)^(1/2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+d)^(1/2)))^2)^(1/2)/cos(2*arc
tan((a*e^2-b*d*e+c*d^2)^(1/4)*(g*x+f)^(1/2)/(a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+
d)^(1/2)))*EllipticF(sin(2*arctan((a*e^2-b*d*e+c*d^2)^(1/4)*(g*x+f)^(1/2)/(
a*g^2-b*f*g+c*f^2)^(1/4)/(e*x+d)^(1/2))),1/2*(2+(2*c*d*f+2*a*e*g-b*(d*g+e*f
))/(c*d^2-e*(-a*e+b*d))^(1/2)/(c*f^2-g*(-a*g+b*f))^(1/2))^(1/2))*(1+(g*x+f)
*(a*e^2-b*d*e+c*d^2)^(1/2)/(e*x+d)/(c*f^2-g*(-a*g+b*f))^(1/2))*((-d*g+e*f)^
2*(c*x^2+b*x+a)/(a*g^2-b*f*g+c*f^2)/(e*x+d)^2)^(1/2)*((1-(2*c*d*f+2*a*e*g-b
*(d*g+e*f))*(g*x+f)/(a*g^2-b*f*g+c*f^2)/(e*x+d)+(a*e^2-b*d*e+c*d^2)*(g*x+f)
^2/(c*f^2-g*(-a*g+b*f))/(e*x+d)^2)/(1+(g*x+f)*(a*e^2-b*d*e+c*d^2)^(1/2)/(e*
x+d)/(c*f^2-g*(-a*g+b*f))^(1/2))^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/4)/(-d*g+e
*f)/(c*x^2+b*x+a)^(1/2)/(1-(2*c*d*f+2*a*e*g-b*(d*g+e*f))*(g*x+f)/(a*g^2-b*f
*g+c*f^2)/(e*x+d)+(a*e^2-b*d*e+c*d^2)*(g*x+f)^2/(c*f^2-g*(-a*g+b*f))/(e*x+d
)^2)^(1/2)
```

Rubi [A]

time = 0.68, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {949, 1117}

$$-\frac{(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg-b(dg+ef)+2cdf)}{(d+ex)(ag^2-bfg+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{cd^2-bed+ae^2}\sqrt{f+gx}}{\sqrt[4]{cf^2-bgf+ag^2}\sqrt{d+ex}}\right)|\frac{1}{4}\left(\frac{2cdf+2aeg-b(ef+dg)}{\sqrt{cd^2-e(bd-ae)}\sqrt{cf^2-g(bf-ag)}}+2\right)\right)}{\sqrt{a+bx+cx^2}\,(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg-b(dg+ef)+2cdf)}{(d+ex)(ag^2-bfg+cf^2)}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((c*f^2 - g*(b*f - a*g))^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + b*x + c
*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 - b*d*e + a*
e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*g)]*(d + e*x)))*Sqrt[(1 - ((2*c*d*
f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g + a*g^2)*(d + e*x))
 + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f - a*g))*(d + e*x)
^2))/(1 + (Sqrt[c*d^2 - b*d*e + a*e^2]*(f + g*x))/(Sqrt[c*f^2 - g*(b*f - a*
```

```
g)]*(d + e*x)))^2]*EllipticF[2*ArcTan[((c*d^2 - b*d*e + a*e^2)^(1/4)*Sqrt[f
 + g*x])/((c*f^2 - b*f*g + a*g^2)^(1/4)*Sqrt[d + e*x])], (2 + (2*c*d*f + 2*
a*e*g - b*(e*f + d*g))/(Sqrt[c*d^2 - e*(b*d - a*e)]*Sqrt[c*f^2 - g*(b*f - a
*g)]))/4])/((c*d^2 - b*d*e + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]
*Sqrt[1 - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*(f + g*x))/((c*f^2 - b*f*g +
 a*g^2)*(d + e*x)) + ((c*d^2 - b*d*e + a*e^2)*(f + g*x)^2)/((c*f^2 - g*(b*f
 - a*g))*(d + e*x)^2)]))
```

Rule 949

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2])), Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*e
*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 - b
*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(2(d+ex)\sqrt{\frac{(ef-dg)^2\left(a+bx+cx^2\right)}{\left(cf^2-bfg+ag^2\right)(d+ex)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{(ef-}$$

$$= -\frac{\sqrt[4]{cf^2-g(bf-ag)}\,(d+ex)\sqrt{\frac{(ef-dg)^2\left(a+bx+cx^2\right)}{\left(cf^2-bfg+ag^2\right)(d+ex)^2}}\,\left(\right.}{}$$

Mathematica [A]

time = 24.77, size = 375, normalized size = 0.64

$$\frac{2\sqrt{2}\, e\sqrt{-\frac{e\,(cd^2+e(-bd+ae))\,(f+gx)}{\left(-2cdef+e\sqrt{(b^2-4ac)\,e^2}\,f-2ae^2g-d\sqrt{(b^2-4ac)\,e^2}\,g+be(ef+dg)\right)(d+ex)}}\,\sqrt{a+x(b+cx)}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{2ae^2-2cdex+be(-d+ex)+\sqrt{(b^2-4ac)\,e^2}\,(d+ex)}{\sqrt{(b^2-4ac)\,e^2}\,(d+ex)}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{(b^2-4ac)\,e^2}\,(ef-dg)}{-2cdef+e\sqrt{(b^2-4ac)\,e^2}\,f-2ae^2g-d\sqrt{(b^2-4ac)\,e^2}\,g+be(ef+dg)}\right)}{\sqrt{(b^2-4ac)\,e^2}\,\sqrt{d+ex}\,\sqrt{f+gx}\,\sqrt{-\frac{(cd^2+e(-bd+ae))\,(a+x(b+cx))}{(b^2-4ac)\,(d+ex)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*Sqrt[2]*e*Sqrt[-((e*(c*d^2 + e*(-(b*d) + a*e))*(f + g*x))/((-2*c*d*e*f +
 e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a*c)*e^2]*g + b*
e*(e*f + d*g))*(d + e*x)))]*Sqrt[a + x*(b + c*x)]*EllipticF[ArcSin[Sqrt[(2*
a*e^2 - 2*c*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/(Sq
rt[(b^2 - 4*a*c)*e^2]*(d + e*x))]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*e^2]*(e*f
 - d*g))/(-2*c*d*e*f + e*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*e^2*g - d*Sqrt[(b^
2 - 4*a*c)*e^2]*g + b*e*(e*f + d*g))])/(Sqrt[(b^2 - 4*a*c)*e^2]*Sqrt[d + e*
x]*Sqrt[f + g*x]*Sqrt[-(((c*d^2 + e*(-(b*d) + a*e))*(a + x*(b + c*x)))/((b^
2 - 4*a*c)*(d + e*x)^2))])
```

Maple [A]

time = 0.14, size = 550, normalized size = 0.94

method	result
default	$\frac{4\left(\sqrt{-4ac+b^2}\, e^2 g\, x^2+b\, e^2 g\, x^2-2c\, e^2 f\, x^2+2\sqrt{-4ac+b^2}\, degx+2bdegx-4cdefx+\sqrt{-4ac+b^2}\, d^2 g+b\, d^2 g-2c\, d^2 f\right)}{}$
elliptic	$\frac{2\sqrt{(gx+f)\,(ex+d)\,(c\,x^2+bx+a)}\,\left(-\frac{f}{g}+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{d}{e}\right)\left(x+\frac{f}{g}\right)}{\left(-\frac{b+\sqrt{-4ac+b^2}}{2c}+\frac{f}{g}\right)\left(x+\frac{d}{e}\right)}}\,\left(x+\frac{d}{e}\right.}{\sqrt{gx+f}\,\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}\,\left(-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 4*((-4*a*c+b^2)^(1/2)*e^2*g*x^2+b*e^2*g*x^2-2*c*e^2*f*x^2+2*(-4*a*c+b^2)^(1
/2)*d*e*g*x+2*b*d*e*g*x-4*c*d*e*f*x+(-4*a*c+b^2)^(1/2)*d^2*g+b*d^2*g-2*c*d^
2*f)*EllipticF(((e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)*(g*x+f)/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f)/(e*x+d))^(1/2),((e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)*(g*(-4*a*c+
```

```
b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2
)+e*b-2*c*d))^(1/2))*((d*g-e*f)*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2
)^(1/2)+b*g-2*c*f)/(e*x+d))^(1/2)*((d*g-e*f)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))/
(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2))/(e*x+d))^(1/2)*((e*(-4*a*c+b^2)^(1/2)+e*b-
2*c*d)*(g*x+f)/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(e*x+d))^(1/2)*(e*x+d)^(1/2
)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-1/c*(g*x+f)*(e*x+d)*(-b-2*c*x+(-4*a*c
+b^2)^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*(-4*a*c+b^2)^(1/2)+e*b-
2*c*d)/(d*g-e*f)/((g*x+f)*(e*x+d)*(c*x^2+b*x+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*sqrt(x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*sqrt(x*e + d)/(c*d*g*x^3 + a*d
*f + (c*d*f + b*d*g)*x^2 + (b*d*f + a*d*g)*x + (c*g*x^4 + (c*f + b*g)*x^3 +
 a*f*x + (b*f + a*g)*x^2)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex}\,\sqrt{f+gx}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)*sqrt(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{f+g\,x}\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((f + g*x)^(1/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.920 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) \, dx$

Optimal. Leaf size=220

$$\frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^{1+m}}{e^5(1+m)}-\frac{(ef-dg)(2cd(ef-2dg)-e(bef-3bdg+2aeg))(d+ex)^{2+m}}{e^5(2+m)}+\cdots$$

```
[Out] (a*e^2-b*d*e+c*d^2)*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^5/(1+m)-(-d*g+e*f)*(2*c*d*
(-2*d*g+e*f)-e*(2*a*e*g-3*b*d*g+b*e*f))*(e*x+d)^(2+m)/e^5/(2+m)+(e*g*(a*e*g
-3*b*d*g+2*b*e*f)+c*(6*d^2*g^2-6*d*e*f*g+e^2*f^2))*(e*x+d)^(3+m)/e^5/(3+m)+
g*(b*e*g-4*c*d*g+2*c*e*f)*(e*x+d)^(4+m)/e^5/(4+m)+c*g^2*(e*x+d)^(5+m)/e^5/(
5+m)
```

Rubi [A]
time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {961}

$$\frac{(d+ex)^{m+3}\left(eg(aeg-3bdg+2bef)+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)}{e^5(m+1)}-\frac{(ef-dg)(d+ex)^{m+2}(2cd(ef-2dg)-e(2aeg-3bdg+bef))}{e^5(m+2)}+\frac{g(d+ex)^{m+4}(beg-4cdg+2cef)}{e^5(m+4)}+\frac{cg^2(d+ex)^{m+5}}{e^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (
(e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*x)
^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 -
6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f - 4*
c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5 + m))
/(e^5*(5 + m))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\begin{aligned}\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right) \, dx &= \int \left(\frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^m}{e^4}+\frac{(ef-dg)(-2cd(e\cdots}{\cdots}\right.\\ &= \frac{\left(cd^2-bde+ae^2\right)(ef-dg)^2(d+ex)^{1+m}}{e^5(1+m)}-\frac{(ef-dg)(2cd(ef-\cdots}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.45, size = 440, normalized size = 2.00

$$\frac{(d+ex)^{1+m}\left(c\left(24d^4g^2-12d^3eg(f(5+m)+2g(1+m)x)+2d^2e^2\left(f^2\left(20+9m+m^2\right)+6fg\left(5+6m+m^2\right)x+6g^2\left(2+3m+m^2\right)x^2\right)-2de^3(1+m)x\left(f^2\left(20+9m+m^2\right)+3fg\left(10+7m+m^2\right)x+2g^2\left(6+5m+m^2\right)x^2\right)+e^4\left(2+3m+m^2\right)x^2\left(f^2\left(20+9m+m^2\right)+2fg\left(15+8m+m^2\right)x+g^2\left(12+7m+m^2\right)x^2\right)\right)+e(5+m)\left(ae(4+m)\left(2d^2g^2-2deg(f(3+m)+g(1+m)x)+e^2\left(f^2\left(6+5m+m^2\right)+2fg\left(3+4m+m^2\right)x+g^2\left(2+3m+m^2\right)x^2\right)\right)+b\left(-6d^3g^2+2d^2eg(2f(4+m)+3g(1+m)x)-de^2\left(f^2\left(12+7m+m^2\right)+4fg\left(4+5m+m^2\right)x+3g^2\left(2+3m+m^2\right)x^2\right)+e^3(1+m)x\left(f^2\left(12+7m+m^2\right)+2fg\left(8+6m+m^2\right)x+g^2\left(6+5m+m^2\right)x^2\right)\right)\right)\right)}{e^5(1+m)(2+m)(3+m)(4+m)(5+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(c*(24*d^4*g^2 - 12*d^3*e*g*(f*(5 + m) + 2*g*(1 + m)*x)
+ 2*d^2*e^2*(f^2*(20 + 9*m + m^2) + 6*f*g*(5 + 6*m + m^2)*x + 6*g^2*(2 + 3*
m + m^2)*x^2) - 2*d*e^3*(1 + m)*x*(f^2*(20 + 9*m + m^2) + 3*f*g*(10 + 7*m +
 m^2)*x + 2*g^2*(6 + 5*m + m^2)*x^2) + e^4*(2 + 3*m + m^2)*x^2*(f^2*(20 + 9
*m + m^2) + 2*f*g*(15 + 8*m + m^2)*x + g^2*(12 + 7*m + m^2)*x^2)) + e*(5 +
m)*(a*e*(4 + m)*(2*d^2*g^2 - 2*d*e*g*(f*(3 + m) + g*(1 + m)*x) + e^2*(f^2*(
6 + 5*m + m^2) + 2*f*g*(3 + 4*m + m^2)*x + g^2*(2 + 3*m + m^2)*x^2)) + b*(-
6*d^3*g^2 + 2*d^2*e*g*(2*f*(4 + m) + 3*g*(1 + m)*x) - d*e^2*(f^2*(12 + 7*m
+ m^2) + 4*f*g*(4 + 5*m + m^2)*x + 3*g^2*(2 + 3*m + m^2)*x^2) + e^3*(1 + m)
*x*(f^2*(12 + 7*m + m^2) + 2*f*g*(8 + 6*m + m^2)*x + g^2*(6 + 5*m + m^2)*x^
2)))))/(e^5*(1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(220) = 440$.
time = 0.09, size = 1249, normalized size = 5.68 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*g^2/(5+m)*x^5*exp(m*ln(e*x+d))+d*(a*e^4*f^2*m^4-2*a*d*e^3*f*g*m^3+14*a*e^
4*f^2*m^3-b*d*e^3*f^2*m^3+2*a*d^2*e^2*g^2*m^2-24*a*d*e^3*f*g*m^2+71*a*e^4*f
^2*m^2+4*b*d^2*e^2*f*g*m^2-12*b*d*e^3*f^2*m^2+2*c*d^2*e^2*f^2*m^2+18*a*d^2*
e^2*g^2*m-94*a*d*e^3*f*g*m+154*a*e^4*f^2*m-6*b*d^3*e*g^2*m+36*b*d^2*e^2*f*g
*m-47*b*d*e^3*f^2*m-12*c*d^3*e*f*g*m+18*c*d^2*e^2*f^2*m+40*a*d^2*e^2*g^2-12
0*a*d*e^3*f*g+120*a*e^4*f^2-30*b*d^3*e*g^2+80*b*d^2*e^2*f*g-60*b*d*e^3*f^2+
24*c*d^4*g^2-60*c*d^3*e*f*g+40*c*d^2*e^2*f^2)/e^5/(m^5+15*m^4+85*m^3+225*m^
2+274*m+120)*exp(m*ln(e*x+d))+(a*e^2*g^2*m^2+b*d*e*g^2*m^2+2*b*e^2*f*g*m^2+
2*c*d*e*f*g*m^2+c*e^2*f^2*m^2+9*a*e^2*g^2*m+5*b*d*e*g^2*m+18*b*e^2*f*g*m-4*
c*d^2*g^2*m+10*c*d*e*f*g*m+9*c*e^2*f^2*m+20*a*e^2*g^2+40*b*e^2*f*g+20*c*e^2
*f^2)/e^2/(m^3+12*m^2+47*m+60)*x^3*exp(m*ln(e*x+d))+(a*d*e^2*g^2*m^3+2*a*e^
3*f*g*m^3+2*b*d*e^2*f*g*m^3+b*e^3*f^2*m^3+c*d*e^2*f^2*m^3+9*a*d*e^2*g^2*m^2
+24*a*e^3*f*g*m^2-3*b*d^2*e*g^2*m^2+18*b*d*e^2*f*g*m^2+12*b*e^3*f^2*m^2-6*c
*d^2*e*f*g*m^2+9*c*d*e^2*f^2*m^2+20*a*d*e^2*g^2*m+94*a*e^3*f*g*m-15*b*d^2*e
*g^2*m+40*b*d*e^2*f*g*m+47*b*e^3*f^2*m+12*c*d^3*g^2*m-30*c*d^2*e*f*g*m+20*c
*d*e^2*f^2*m+120*a*e^3*f*g+60*b*e^3*f^2)/e^3/(m^4+14*m^3+71*m^2+154*m+120)*
x^2*exp(m*ln(e*x+d))+(b*e*g*m+c*d*g*m+2*c*e*f*m+5*b*e*g+10*c*e*f)*g/e/(m^2+
9*m+20)*x^4*exp(m*ln(e*x+d))-(-2*a*d*e^3*f*g*m^4-a*e^4*f^2*m^4-b*d*e^3*f^2*
m^4+2*a*d^2*e^2*g^2*m^3-24*a*d*e^3*f*g*m^3-14*a*e^4*f^2*m^3+4*b*d^2*e^2*f*g
```

```
*m^3-12*b*d*e^3*f^2*m^3+2*c*d^2*e^2*f^2*m^3+18*a*d^2*e^2*g^2*m^2-94*a*d*e^3
*f*g*m^2-71*a*e^4*f^2*m^2-6*b*d^3*e*g^2*m^2+36*b*d^2*e^2*f*g*m^2-47*b*d*e^3
*f^2*m^2-12*c*d^3*e*f*g*m^2+18*c*d^2*e^2*f^2*m^2+40*a*d^2*e^2*g^2*m-120*a*d
*e^3*f*g*m-154*a*e^4*f^2*m-30*b*d^3*e*g^2*m+80*b*d^2*e^2*f*g*m-60*b*d*e^3*f
^2*m+24*c*d^4*g^2*m-60*c*d^3*e*f*g*m+40*c*d^2*e^2*f^2*m-120*a*e^4*f^2)/e^4/
(m^5+15*m^4+85*m^3+225*m^2+274*m+120)*x*exp(m*ln(e*x+d))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(238) = 476$.
time = 0.36, size = 690, normalized size = 3.14

$$\frac{(xe+d)^{m+1}af^2e^{(-1)}}{m+1}+\frac{((m+1)x^2e^2+dmxe-d^2)bf^2e^{(m\log(xe+d)-2)}}{m^2+3m+2}+\frac{2((m+1)x^2e^2+dmxe-d^2)afge^{(m\log(xe+d)-2)}}{m^2+3m+2}+\frac{((m^2+3m+2)x^3e^3+(m^2+m)dx^2e^2-2d^2mxe+2d^3)cf^2e^{(m\log(xe+d)-3)}}{m^3+6m^2+11m+6}+\frac{2((m^2+3m+2)x^3e^3+(m^2+m)dx^2e^2-2d^2mxe+2d^3)bfge^{(m\log(xe+d)-3)}}{m^3+6m^2+11m+6}+\frac{((m^2+3m+2)x^3e^3+(m^2+m)dx^2e^2-2d^2mxe+2d^3)ag^2e^{(m\log(xe+d)-3)}}{m^3+6m^2+11m+6}+\frac{2((m^3+6m^2+11m+6)x^4e^4+(m^3+3m^2+2m)dx^3e^3-3(m^2+m)d^2x^2e^2+6d^3mxe-6d^4)cfge^{(m\log(xe+d)-4)}}{m^4+10m^3+35m^2+50m+24}+\frac{((m^3+6m^2+11m+6)x^4e^4+(m^3+3m^2+2m)dx^3e^3-3(m^2+m)d^2x^2e^2+6d^3mxe-6d^4)bg^2e^{(m\log(xe+d)-4)}}{m^4+10m^3+35m^2+50m+24}+\frac{((m^4+10m^3+35m^2+50m+24)x^5e^5+(m^4+6m^3+11m^2+6m)dx^4e^4-4(m^3+3m^2+2m)d^2x^3e^3+12(m^2+m)d^3x^2e^2-24d^4mxe+24d^5)cg^2e^{(m\log(xe+d)-5)}}{m^5+15m^4+85m^3+225m^2+274m+120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] (x*e + d)^(m + 1)*a*f^2*e^(-1)/(m + 1) + ((m + 1)*x^2*e^2 + d*m*x*e - d^2)*
b*f^2*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + 2*((m + 1)*x^2*e^2 + d*m*x*e
 - d^2)*a*f*g*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + ((m^2 + 3*m + 2)*x^3
*e^3 + (m^2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*c*f^2*e^(m*log(x*e + d) -
 3)/(m^3 + 6*m^2 + 11*m + 6) + 2*((m^2 + 3*m + 2)*x^3*e^3 + (m^2 + m)*d*x^2
*e^2 - 2*d^2*m*x*e + 2*d^3)*b*f*g*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2 + 11*
m + 6) + ((m^2 + 3*m + 2)*x^3*e^3 + (m^2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d
^3)*a*g^2*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2 + 11*m + 6) + 2*((m^3 + 6*m^2
 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*x^3*e^3 - 3*(m^2 + m)*d^2*x^2*
e^2 + 6*d^3*m*x*e - 6*d^4)*c*f*g*e^(m*log(x*e + d) - 4)/(m^4 + 10*m^3 + 35*
m^2 + 50*m + 24) + ((m^3 + 6*m^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*
d*x^3*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6*d^3*m*x*e - 6*d^4)*b*g^2*e^(m*log(x
*e + d) - 4)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + ((m^4 + 10*m^3 + 35*m^2
+ 50*m + 24)*x^5*e^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*x^4*e^4 - 4*(m^3 + 3*
m^2 + 2*m)*d^2*x^3*e^3 + 12*(m^2 + m)*d^3*x^2*e^2 - 24*d^4*m*x*e + 24*d^5)*
c*g^2*e^(m*log(x*e + d) - 5)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(238) = 476$.
time = 2.23, size = 1222, normalized size = 5.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] (24*c*d^5*g^2 + ((c*g^2*m^4 + 10*c*g^2*m^3 + 35*c*g^2*m^2 + 50*c*g^2*m + 24
*c*g^2)*x^5 + ((2*c*f*g + b*g^2)*m^4 + 11*(2*c*f*g + b*g^2)*m^3 + 60*c*f*g
+ 30*b*g^2 + 41*(2*c*f*g + b*g^2)*m^2 + 61*(2*c*f*g + b*g^2)*m)*x^4 + ((c*f
```

```
^2 + 2*b*f*g + a*g^2)*m^4 + 12*(c*f^2 + 2*b*f*g + a*g^2)*m^3 + 40*c*f^2 + 8
0*b*f*g + 40*a*g^2 + 49*(c*f^2 + 2*b*f*g + a*g^2)*m^2 + 78*(c*f^2 + 2*b*f*g
 + a*g^2)*m)*x^3 + ((b*f^2 + 2*a*f*g)*m^4 + 13*(b*f^2 + 2*a*f*g)*m^3 + 60*b
*f^2 + 120*a*f*g + 59*(b*f^2 + 2*a*f*g)*m^2 + 107*(b*f^2 + 2*a*f*g)*m)*x^2
+ (a*f^2*m^4 + 14*a*f^2*m^3 + 71*a*f^2*m^2 + 154*a*f^2*m + 120*a*f^2)*x)*e^
5 + (a*d*f^2*m^4 + 14*a*d*f^2*m^3 + 71*a*d*f^2*m^2 + 154*a*d*f^2*m + (c*d*g
^2*m^4 + 6*c*d*g^2*m^3 + 11*c*d*g^2*m^2 + 6*c*d*g^2*m)*x^4 + 120*a*d*f^2 +
((2*c*d*f*g + b*d*g^2)*m^4 + 8*(2*c*d*f*g + b*d*g^2)*m^3 + 17*(2*c*d*f*g +
b*d*g^2)*m^2 + 10*(2*c*d*f*g + b*d*g^2)*m)*x^3 + ((c*d*f^2 + 2*b*d*f*g + a*
d*g^2)*m^4 + 10*(c*d*f^2 + 2*b*d*f*g + a*d*g^2)*m^3 + 29*(c*d*f^2 + 2*b*d*f
*g + a*d*g^2)*m^2 + 20*(c*d*f^2 + 2*b*d*f*g + a*d*g^2)*m)*x^2 + ((b*d*f^2 +
 2*a*d*f*g)*m^4 + 12*(b*d*f^2 + 2*a*d*f*g)*m^3 + 47*(b*d*f^2 + 2*a*d*f*g)*m
^2 + 60*(b*d*f^2 + 2*a*d*f*g)*m)*x)*e^4 - (60*b*d^2*f^2 + 120*a*d^2*f*g + (
b*d^2*f^2 + 2*a*d^2*f*g)*m^3 + 4*(c*d^2*g^2*m^3 + 3*c*d^2*g^2*m^2 + 2*c*d^2
*g^2*m)*x^3 + 12*(b*d^2*f^2 + 2*a*d^2*f*g)*m^2 + 3*((2*c*d^2*f*g + b*d^2*g^
2)*m^3 + 6*(2*c*d^2*f*g + b*d^2*g^2)*m^2 + 5*(2*c*d^2*f*g + b*d^2*g^2)*m)*x
^2 + 47*(b*d^2*f^2 + 2*a*d^2*f*g)*m + 2*((c*d^2*f^2 + 2*b*d^2*f*g + a*d^2*g
^2)*m^3 + 9*(c*d^2*f^2 + 2*b*d^2*f*g + a*d^2*g^2)*m^2 + 20*(c*d^2*f^2 + 2*b
*d^2*f*g + a*d^2*g^2)*m)*x)*e^3 + 2*(20*c*d^3*f^2 + 40*b*d^3*f*g + 20*a*d^3
*g^2 + (c*d^3*f^2 + 2*b*d^3*f*g + a*d^3*g^2)*m^2 + 6*(c*d^3*g^2*m^2 + c*d^3
*g^2*m)*x^2 + 9*(c*d^3*f^2 + 2*b*d^3*f*g + a*d^3*g^2)*m + 3*((2*c*d^3*f*g +
 b*d^3*g^2)*m^2 + 5*(2*c*d^3*f*g + b*d^3*g^2)*m)*x)*e^2 - 6*(4*c*d^4*g^2*m*
x + 10*c*d^4*f*g + 5*b*d^4*g^2 + (2*c*d^4*f*g + b*d^4*g^2)*m)*e)*(x*e + d)^
m*e^(-5)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15757 vs. $2(212) = 424$.
time = 3.22, size = 15757, normalized size = 71.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + b*f**2*x**2/2 + 2*
b*f*g*x**3/3 + b*g**2*x**4/4 + c*f**2*x**3/3 + c*f*g*x**4/2 + c*g**2*x**5/5
), Eq(e, 0)), (-a*d**2*e**2*g**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e
**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*a*d*e**3*f*g/(12*d**4*e**5 +
48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*a*d
*e**3*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8
*x**3 + 12*e**9*x**4) - 3*a*e**4*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d
**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*a*e**4*f*g*x/(12*d**4*e*
*5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) -
6*a*e**4*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*
d*e**8*x**3 + 12*e**9*x**4) - 3*b*d**3*e*g**2/(12*d**4*e**5 + 48*d**3*e**6*
```

x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*b*d**2*e**2*f*g/
(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e*
*9*x**4) - 12*b*d**2*e**2*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e
7*x2 + 48*d*e**8*x**3 + 12*e**9*x**4) - b*d*e**3*f**2/(12*d**4*e**5 + 4
8*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 8*b*d*
e**3*f*g*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x
3 + 12*e9*x**4) - 18*b*d*e**3*g**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x
+ 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 4*b*e**4*f**2*x/(12*
d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
4) - 12*b*e4*f*g*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**
2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 12*b*e**4*g**2*x**3/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*d*
*4*g**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
8*d*e**8*x**3 + 12*e**9*x**4) + 25*c*d**4*g**2/(12*d**4*e**5 + 48*d**3*e**6
*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*d**3*e*f*g/(1
2*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9
*x**4) + 48*c*d**3*e*g**2*x*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 7
2*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 88*c*d**3*e*g**2*x/(12*
d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
4) - c*d2*e**2*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) - 24*c*d**2*e**2*f*g*x/(12*d**4*e**5 + 48*
d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 72*c*d**
2*e**2*g**2*x**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7
*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 108*c*d**2*e**2*g**2*x**2/(12*d**4
*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4)
 - 4*c*d*e**3*f**2*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
8*d*e**8*x**3 + 12*e**9*x**4) - 36*c*d*e**3*f*g*x**2/(12*d**4*e**5 + 48*d**
3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c*d*e**3
*g**2*x**3*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c*d*e**3*g**2*x**3/(12*d**4*e**5 + 48
*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*e**
4*f**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*
x**3 + 12*e**9*x**4) - 24*c*e**4*f*g*x**3/(12*d**4*e**5 + 48*d**3*e**6*x +
72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*e**4*g**2*x**4*lo
g(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x
3 + 12*e9*x**4), Eq(m, -5)), (-2*a*d**2*e**2*g**2/(6*d**3*e**5 + 18*d**
2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*a*d*e**3*f*g/(6*d**3*e**5 + 18
*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*d*e**3*g**2*x/(6*d**3*e*
*5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 2*a*e**4*f**2/(6*d**3
*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*e**4*f*g*x/(6*
d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 6*a*e**4*g**2*
x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 6*b*d*
*3*e*g**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e
8*x3) + 11*b*d**3*e*g**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2
 + 6*e**8*x**3) - 4*b*d**2*e**2*f*g/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e*

```
*7*x**2 + 6*e**8*x**3) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d**3*e**5 +
18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 27*b*d**2*e**2*g**2*x/(6*d
**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - b*d*e**3*f**2/(
6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) - 12*b*d*e**3*
f*g*x/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 18*b*
d*e**3*g**2*x**2*log(d/e + x)/(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**
2 + 6*e**8*x**3) + 18*b*d*e**3*g**2*x**2/(6*d**3*e**5 + 18*d**2*e**6*x + 18
*d*e**7*x**2 + 6*e**8*x**3) - 3*b*e**4*f**2*x/(6*d**3*e**5 + 18*d**2*e**6*x
 + 18*d*e**7*x**2 + 6*e**8*x**3) - 12*b*e**4*f*g*x**2/(6*d**3*e**5 + 18*d**
2*e**6*x + 18*d*e**7*x**2 + 6*e**8*x**3) + 6*b*e**4*g**2*x**3*log(d/e + x)/
(6*d**3*e**5 + 18*d**2*e**6*x + 18*d*e**7*x**2 ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2740 vs. $2(238) = 476$.
time = 1.73, size = 2740, normalized size = 12.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*g^2*m^4*x^5*e^5 + (x*e + d)^m*c*d*g^2*m^4*x^4*e^4 + 2*(x*e +
 d)^m*c*f*g*m^4*x^4*e^5 + (x*e + d)^m*b*g^2*m^4*x^4*e^5 + 10*(x*e + d)^m*c*
g^2*m^3*x^5*e^5 + 2*(x*e + d)^m*c*d*f*g*m^4*x^3*e^4 + (x*e + d)^m*b*d*g^2*m
^4*x^3*e^4 + 6*(x*e + d)^m*c*d*g^2*m^3*x^4*e^4 - 4*(x*e + d)^m*c*d^2*g^2*m^
3*x^3*e^3 + (x*e + d)^m*c*f^2*m^4*x^3*e^5 + 2*(x*e + d)^m*b*f*g*m^4*x^3*e^5
 + (x*e + d)^m*a*g^2*m^4*x^3*e^5 + 22*(x*e + d)^m*c*f*g*m^3*x^4*e^5 + 11*(x
*e + d)^m*b*g^2*m^3*x^4*e^5 + 35*(x*e + d)^m*c*g^2*m^2*x^5*e^5 + (x*e + d)^
m*c*d*f^2*m^4*x^2*e^4 + 2*(x*e + d)^m*b*d*f*g*m^4*x^2*e^4 + (x*e + d)^m*a*d
*g^2*m^4*x^2*e^4 + 16*(x*e + d)^m*c*d*f*g*m^3*x^3*e^4 + 8*(x*e + d)^m*b*d*g
^2*m^3*x^3*e^4 + 11*(x*e + d)^m*c*d*g^2*m^2*x^4*e^4 - 6*(x*e + d)^m*c*d^2*f
*g*m^3*x^2*e^3 - 3*(x*e + d)^m*b*d^2*g^2*m^3*x^2*e^3 - 12*(x*e + d)^m*c*d^2
*g^2*m^2*x^3*e^3 + 12*(x*e + d)^m*c*d^3*g^2*m^2*x^2*e^2 + (x*e + d)^m*b*f^2
*m^4*x^2*e^5 + 2*(x*e + d)^m*a*f*g*m^4*x^2*e^5 + 12*(x*e + d)^m*c*f^2*m^3*x
^3*e^5 + 24*(x*e + d)^m*b*f*g*m^3*x^3*e^5 + 12*(x*e + d)^m*a*g^2*m^3*x^3*e^
5 + 82*(x*e + d)^m*c*f*g*m^2*x^4*e^5 + 41*(x*e + d)^m*b*g^2*m^2*x^4*e^5 + 5
0*(x*e + d)^m*c*g^2*m*x^5*e^5 + (x*e + d)^m*b*d*f^2*m^4*x*e^4 + 2*(x*e + d)
^m*a*d*f*g*m^4*x*e^4 + 10*(x*e + d)^m*c*d*f^2*m^3*x^2*e^4 + 20*(x*e + d)^m*
b*d*f*g*m^3*x^2*e^4 + 10*(x*e + d)^m*a*d*g^2*m^3*x^2*e^4 + 34*(x*e + d)^m*c
*d*f*g*m^2*x^3*e^4 + 17*(x*e + d)^m*b*d*g^2*m^2*x^3*e^4 + 6*(x*e + d)^m*c*d
*g^2*m*x^4*e^4 - 2*(x*e + d)^m*c*d^2*f^2*m^3*x*e^3 - 4*(x*e + d)^m*b*d^2*f*
g*m^3*x*e^3 - 2*(x*e + d)^m*a*d^2*g^2*m^3*x*e^3 - 36*(x*e + d)^m*c*d^2*f*g*
m^2*x^2*e^3 - 18*(x*e + d)^m*b*d^2*g^2*m^2*x^2*e^3 - 8*(x*e + d)^m*c*d^2*g^
2*m*x^3*e^3 + 12*(x*e + d)^m*c*d^3*f*g*m^2*x*e^2 + 6*(x*e + d)^m*b*d^3*g^2*
m^2*x*e^2 + 12*(x*e + d)^m*c*d^3*g^2*m*x^2*e^2 - 24*(x*e + d)^m*c*d^4*g^2*m
```

```
*x*e + (x*e + d)^m*a*f^2*m^4*x*e^5 + 13*(x*e + d)^m*b*f^2*m^3*x^2*e^5 + 26*
(x*e + d)^m*a*f*g*m^3*x^2*e^5 + 49*(x*e + d)^m*c*f^2*m^2*x^3*e^5 + 98*(x*e
+ d)^m*b*f*g*m^2*x^3*e^5 + 49*(x*e + d)^m*a*g^2*m^2*x^3*e^5 + 122*(x*e + d)
^m*c*f*g*m*x^4*e^5 + 61*(x*e + d)^m*b*g^2*m*x^4*e^5 + 24*(x*e + d)^m*c*g^2*
x^5*e^5 + (x*e + d)^m*a*d*f^2*m^4*e^4 + 12*(x*e + d)^m*b*d*f^2*m^3*x*e^4 +
24*(x*e + d)^m*a*d*f*g*m^3*x*e^4 + 29*(x*e + d)^m*c*d*f^2*m^2*x^2*e^4 + 58*
(x*e + d)^m*b*d*f*g*m^2*x^2*e^4 + 29*(x*e + d)^m*a*d*g^2*m^2*x^2*e^4 + 20*(
x*e + d)^m*c*d*f*g*m*x^3*e^4 + 10*(x*e + d)^m*b*d*g^2*m*x^3*e^4 - (x*e + d)
^m*b*d^2*f^2*m^3*e^3 - 2*(x*e + d)^m*a*d^2*f*g*m^3*e^3 - 18*(x*e + d)^m*c*d
^2*f^2*m^2*x*e^3 - 36*(x*e + d)^m*b*d^2*f*g*m^2*x*e^3 - 18*(x*e + d)^m*a*d^
2*g^2*m^2*x*e^3 - 30*(x*e + d)^m*c*d^2*f*g*m*x^2*e^3 - 15*(x*e + d)^m*b*d^2
*g^2*m*x^2*e^3 + 2*(x*e + d)^m*c*d^3*f^2*m^2*e^2 + 4*(x*e + d)^m*b*d^3*f*g*
m^2*e^2 + 2*(x*e + d)^m*a*d^3*g^2*m^2*e^2 + 60*(x*e + d)^m*c*d^3*f*g*m*x*e^
2 + 30*(x*e + d)^m*b*d^3*g^2*m*x*e^2 - 12*(x*e + d)^m*c*d^4*f*g*m*e - 6*(x*
e + d)^m*b*d^4*g^2*m*e + 24*(x*e + d)^m*c*d^5*g^2 + 14*(x*e + d)^m*a*f^2*m^
3*x*e^5 + 59*(x*e + d)^m*b*f^2*m^2*x^2*e^5 + 118*(x*e + d)^m*a*f*g*m^2*x^2*
e^5 + 78*(x*e + d)^m*c*f^2*m*x^3*e^5 + 156*(x*e + d)^m*b*f*g*m*x^3*e^5 + 78
*(x*e + d)^m*a*g^2*m*x^3*e^5 + 60*(x*e + d)^m*c*f*g*x^4*e^5 + 30*(x*e + d)^
m*b*g^2*x^4*e^5 + 14*(x*e + d)^m*a*d*f^2*m^3*e^4 + 47*(x*e + d)^m*b*d*f^2*m
^2*x*e^4 + 94*(x*e + d)^m*a*d*f*g*m^2*x*e^4 + 20*(x*e + d)^m*c*d*f^2*m*x^2*
e^4 + 40*(x*e + d)^m*b*d*f*g*m*x^2*e^4 + 20*(x*e + d)^m*a*d*g^2*m*x^2*e^4 -
 12*(x*e + d)^m*b*d^2*f^2*m^2*e^3 - 24*(x*e + d)^m*a*d^2*f*g*m^2*e^3 - 40*(
x*e + d)^m*c*d^2*f^2*m*x*e^3 - 80*(x*e + d)^m*b*d^2*f*g*m*x*e^3 - 40*(x*e +
 d)^m*a*d^2*g^2*m*x*e^3 + 18*(x*e + d)^m*c*d^3*f^2*m*e^2 + 36*(x*e + d)^m*b
*d^3*f*g*m*e^2 + 18*(x*e + d)^m*a*d^3*g^2*m*e^2 - 60*(x*e + d)^m*c*d^4*f*g*
e - 30*(x*e + d)^m*b*d^4*g^2*e + 71*(x*e + d)^m*a*f^2*m^2*x*e^5 + 107*(x*e
+ d)^m*b*f^2*m*x^2*e^5 + 214*(x*e + d)^m*a*f*g*m*x^2*e^5 + 40*(x*e + d)^m*c
*f^2*x^3*e^5 + 80*(x*e + d)^m*b*f*g*x^3*e^5 + 40*(x*e + d)^m*a*g^2*x^3*e^5
+ 71*(x*e + d)^m*a*d*f^2*m^2*e^4 + 60*(x*e + d)^m*b*d*f^2*m*x*e^4 + 120*(x*
e + d)^m*a*d*f*g*m*x*e^4 - 47*(x*e + d)^m*b*d^2*f^2*m*e^3 - 94*(x*e + d)^m*
a*d^2*f*g*m*e^3 + 40*(x*e + d)^m*c*d^3*f^2*e^2 + 80*(x*e + d)^m*b*d^3*f*g*e
^2 + 40*(x*e + d)^m*a*d^3*g^2*e^2 + 154*(x*e + d)^m*a*f^2*m*x*e^5 + 60*(x*e
 + d)^m*b*f^2*x^2*e^5 + 120*(x*e + d)^m*a*f*g*x^2*e^5 + 154*(x*e + d)^m*a*d
*f^2*m*e^4 - 60*(x*e + d)^m*b*d^2*f^2*e^3 - 120*(x*e + d)^m*a*d^2*f*g*e^3 +
 120*(x*e + d)^m*a*f^2*x*e^5 + 120*(x*e + d)^m*a*d*f^2*e^4)/(m^5*e^5 + 15*m
^4*e^5 + 85*m^3*e^5 + 225*m^2*e^5 + 274*m*e^5 + 120*e^5)
```

Mupad [B]
time = 3.94, size = 1354, normalized size = 6.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2),x)
```

```
[Out] ((d + e*x)^m*(24*c*d^5*g^2 + 40*a*d^3*e^2*g^2 - 60*b*d^2*e^3*f^2 + 40*c*d^3
*e^2*f^2 + 120*a*d*e^4*f^2 - 30*b*d^4*e*g^2 - 120*a*d^2*e^3*f*g + 80*b*d^3*
e^2*f*g + 154*a*d*e^4*f^2*m - 6*b*d^4*e*g^2*m + 71*a*d*e^4*f^2*m^2 + 14*a*d
*e^4*f^2*m^3 + a*d*e^4*f^2*m^4 + 18*a*d^3*e^2*g^2*m - 47*b*d^2*e^3*f^2*m +
18*c*d^3*e^2*f^2*m - 60*c*d^4*e*f*g + 2*a*d^3*e^2*g^2*m^2 - 12*b*d^2*e^3*f^
2*m^2 - b*d^2*e^3*f^2*m^3 + 2*c*d^3*e^2*f^2*m^2 - 12*c*d^4*e*f*g*m - 94*a*d
^2*e^3*f*g*m + 36*b*d^3*e^2*f*g*m - 24*a*d^2*e^3*f*g*m^2 - 2*a*d^2*e^3*f*g*
m^3 + 4*b*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 +
 120)) + (x*(d + e*x)^m*(120*a*e^5*f^2 + 71*a*e^5*f^2*m^2 + 14*a*e^5*f^2*m^
3 + a*e^5*f^2*m^4 + 154*a*e^5*f^2*m + 60*b*d*e^4*f^2*m - 24*c*d^4*e*g^2*m -
 40*a*d^2*e^3*g^2*m + 47*b*d*e^4*f^2*m^2 + 12*b*d*e^4*f^2*m^3 + b*d*e^4*f^2
*m^4 + 30*b*d^3*e^2*g^2*m - 40*c*d^2*e^3*f^2*m - 18*a*d^2*e^3*g^2*m^2 - 2*a
*d^2*e^3*g^2*m^3 + 6*b*d^3*e^2*g^2*m^2 - 18*c*d^2*e^3*f^2*m^2 - 2*c*d^2*e^3
*f^2*m^3 + 120*a*d*e^4*f*g*m + 94*a*d*e^4*f*g*m^2 + 24*a*d*e^4*f*g*m^3 + 2*
a*d*e^4*f*g*m^4 - 80*b*d^2*e^3*f*g*m + 60*c*d^3*e^2*f*g*m - 36*b*d^2*e^3*f*
g*m^2 - 4*b*d^2*e^3*f*g*m^3 + 12*c*d^3*e^2*f*g*m^2))/(e^5*(274*m + 225*m^2
+ 85*m^3 + 15*m^4 + m^5 + 120)) + (c*g^2*x^5*(d + e*x)^m*(50*m + 35*m^2 + 1
0*m^3 + m^4 + 24))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120) + (x^2*(
m + 1)*(d + e*x)^m*(60*b*e^3*f^2 + 12*b*e^3*f^2*m^2 + b*e^3*f^2*m^3 + 120*a
*e^3*f*g + 47*b*e^3*f^2*m + 12*c*d^3*g^2*m + 20*a*d*e^2*g^2*m - 15*b*d^2*e*
g^2*m + 20*c*d*e^2*f^2*m + 24*a*e^3*f*g*m^2 + 2*a*e^3*f*g*m^3 + 9*a*d*e^2*g
^2*m^2 + a*d*e^2*g^2*m^3 - 3*b*d^2*e*g^2*m^2 + 9*c*d*e^2*f^2*m^2 + c*d*e^2*
f^2*m^3 + 94*a*e^3*f*g*m + 40*b*d*e^2*f*g*m - 30*c*d^2*e*f*g*m + 18*b*d*e^2
*f*g*m^2 + 2*b*d*e^2*f*g*m^3 - 6*c*d^2*e*f*g*m^2))/(e^3*(274*m + 225*m^2 +
85*m^3 + 15*m^4 + m^5 + 120)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(20*a*e^2*
g^2 + 20*c*e^2*f^2 + a*e^2*g^2*m^2 + c*e^2*f^2*m^2 + 40*b*e^2*f*g + 9*a*e^2
*g^2*m - 4*c*d^2*g^2*m + 9*c*e^2*f^2*m + b*d*e*g^2*m^2 + 2*b*e^2*f*g*m^2 +
5*b*d*e*g^2*m + 18*b*e^2*f*g*m + 2*c*d*e*f*g*m^2 + 10*c*d*e*f*g*m))/(e^2*(2
74*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (g*x^4*(d + e*x)^m*(11*m +
 6*m^2 + m^3 + 6)*(5*b*e*g + 10*c*e*f + b*e*g*m + c*d*g*m + 2*c*e*f*m))/(e*
(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))
```

3.921 $\int (d+ex)^m (f+gx) \left(a+bx+cx^2\right) \, dx$

Optimal. Leaf size=144

$$\frac{(cd^2-bde+ae^2)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)} - \frac{(cd(2ef-3dg)-e(bef-2bdg+aeg))(d+ex)^{2+m}}{e^4(2+m)} + \frac{(cef-3cdg}{e^4}$$

```
[Out] (a*e^2-b*d*e+c*d^2)*(-d*g+e*f)*(e*x+d)^(1+m)/e^4/(1+m)-(c*d*(-3*d*g+2*e*f)-
e*(a*e*g-2*b*d*g+b*e*f))*(e*x+d)^(2+m)/e^4/(2+m)+(b*e*g-3*c*d*g+c*e*f)*(e*x
+d)^(3+m)/e^4/(3+m)+c*g*(e*x+d)^(4+m)/e^4/(4+m)
```

Rubi [A]

time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{785\}$

$$\frac{(ef-dg)(d+ex)^{m+1}(ae^2-bde+cd^2)}{e^4(m+1)} - \frac{(d+ex)^{m+2}(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^4(m+2)} + \frac{(d+ex)^{m+3}(beg-3cdg+cef)}{e^4(m+3)} + \frac{cg(d+ex)^{m+4}}{e^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)^(1 + m))/(e^4*(1 + m)) - ((c
*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^4*(
2 + m)) + ((c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (c*
g*(d + e*x)^(4 + m))/(e^4*(4 + m))
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m (f+gx) \left(a+bx+cx^2\right) dx = \int \left(\frac{(cd^2-bde+ae^2)(ef-dg)(d+ex)^m}{e^3} + \frac{(-cd(2ef-3dg)+\right.$$

$$= \frac{(cd^2-bde+ae^2)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)} - \frac{(cd(2ef-3dg)-e(be}{e^4}$$

Mathematica [A]

time = 0.31, size = 180, normalized size = 1.25

$$\frac{(d+ex)^{1+m}\left(-\frac{(cd^2+e(-bd+ae))(6cdg+beg(1+m)-2cef(4+m))}{e^2(1+m)} + \frac{(-b^2e^2g(2+m)+2c^2d(3dg-ef(4+m))+ce(bdg(-2+m)+2aeg(3+m)+bef(4+m)))(d+ex)}{e^2(2+m)} + (a+x(b+cx))(beg+c(-3dg+ef(4+m)+eg(3+m)x))\right)}{ce^2(3+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(((c*d^2 + e*(-(b*d) + a*e))*(6*c*d*g + b*e*g*(1 + m)
- 2*c*e*f*(4 + m)))/(e^2*(1 + m))) + ((-(b^2*e^2*g*(2 + m)) + 2*c^2*d*(3*d*
g - e*f*(4 + m)) + c*e*(b*d*g*(-2 + m) + 2*a*e*g*(3 + m) + b*e*f*(4 + m)))*
(d + e*x))/(e^2*(2 + m)) + (a + x*(b + c*x))*(b*e*g + c*(-3*d*g + e*f*(4 +
m) + e*g*(3 + m)*x))))/(c*e^2*(3 + m)*(4 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(144) = 288$.
time = 0.09, size = 503, normalized size = 3.49

method	result
gosper	$-\frac{(ex+d)^{1+m}\left(-c\,e^3 g\,m^3 x^3-b\,e^3 g\,m^3 x^2-c\,e^3 f\,m^3 x^2-6c\,e^3 g\,m^2 x^3-a\,e^3 g\,m^3 x-b\,e^3 f\,m^3 x-7b\,e^3 g\,m^2 x^2+3cd\,e^2 g\,m^2 x^2-7c\,e^3 f\,m^2\right.}{}$
norman	$\frac{cg\,x^4 \mathrm{e}^{m\ln(ex+d)}}{4+m}+\frac{(begm+cdgm+cefm+4beg+4cef)x^3 \mathrm{e}^{m\ln(ex+d)}}{e(m^2+7m+12)}+\frac{(a\,e^2 g\,m^2+bdeg\,m^2+b\,e^2 f\,m^2+cdef\,m^2+7a\,e^2 gm+4bde}{e^2(m^3+}$
risch	$-\frac{(-c\,e^4 g\,m^3 x^4-b\,e^4 g\,m^3 x^3-cd\,e^3 g\,m^3 x^3-c\,e^4 f\,m^3 x^3-6c\,e^4 g\,m^2 x^4-a\,e^4 g\,m^3 x^2-bd\,e^3 g\,m^3 x^2-b\,e^4 f\,m^3 x^2-7b\,e^4 g\,m^2 x^3-cd\,e^3 f}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(e*x+d)^(1+m)*(-c*e^3*g*m^3*x^3-b*e^3*g*m^3*x^2-c*e^3*f*m^3*x^2-6*c*e^3*g*
m^2*x^3-a*e^3*g*m^3*x-b*e^3*f*m^3*x-7*b*e^3*g*m^2*x^2+3*c*d*e^2*g*m^2*x^2-7
*c*e^3*f*m^2*x^2-11*c*e^3*g*m*x^3-a*e^3*f*m^3-8*a*e^3*g*m^2*x+2*b*d*e^2*g*m
^2*x-8*b*e^3*f*m^2*x-14*b*e^3*g*m*x^2+2*c*d*e^2*f*m^2*x+9*c*d*e^2*g*m*x^2-1
4*c*e^3*f*m*x^2-6*c*e^3*g*x^3+a*d*e^2*g*m^2-9*a*e^3*f*m^2-19*a*e^3*g*m*x+b*
d*e^2*f*m^2+10*b*d*e^2*g*m*x-19*b*e^3*f*m*x-8*b*e^3*g*x^2-6*c*d^2*e*g*m*x+1
0*c*d*e^2*f*m*x+6*c*d*e^2*g*x^2-8*c*e^3*f*x^2+7*a*d*e^2*g*m-26*a*e^3*f*m-12
*a*e^3*g*x-2*b*d^2*e*g*m+7*b*d*e^2*f*m+8*b*d*e^2*g*x-12*b*e^3*f*x-2*c*d^2*e
*f*m-6*c*d^2*e*g*x+8*c*d*e^2*f*x+12*a*d*e^2*g-24*a*e^3*f-8*b*d^2*e*g+12*b*d
*e^2*f+6*c*d^3*g-8*c*d^2*e*f)/e^4/(m^4+10*m^3+35*m^2+50*m+24)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(156) = 312$.
time = 0.33, size = 358, normalized size = 2.49

$$\frac{(xe+d)^{m+1}afe^{(-1)}}{m+1}+\frac{((m+1)x^2e^2+dmxe-d^2)bfe^{(m\log(xe+d)-2)}}{m^2+3m+2}+\frac{((m+1)x^2e^2+dmxe-d^2)age^{(m\log(xe+d)-2)}}{m^2+3m+2}+\frac{((m^2+3m+2)x^3e^3+(m^2+m)dx^2e^2-2d^2mxe+2d^3)cfe^{(m\log(xe+d)-3)}}{m^3+6m^2+11m+6}+\frac{((m^2+3m+2)x^3e^3+(m^2+m)dx^2e^2-2d^2mxe+2d^3)bge^{(m\log(xe+d)-3)}}{m^3+6m^2+11m+6}+\frac{((m^3+6m^2+11m+6)x^4e^4+(m^3+3m^2+2m)dx^3e^3-3(m^2+m)d^2x^2e^2+6d^3mxe-6d^4)cge^{(m\log(xe+d)-4)}}{m^4+10m^3+35m^2+50m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] (x*e + d)^(m + 1)*a*f*e^(-1)/(m + 1) + ((m + 1)*x^2*e^2 + d*m*x*e - d^2)*b*
f*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + ((m + 1)*x^2*e^2 + d*m*x*e - d^2
```

)*a*g*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + ((m^2 + 3*m + 2)*x^3*e^3 + (
m^2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*c*f*e^(m*log(x*e + d) - 3)/(m^3 +
 6*m^2 + 11*m + 6) + ((m^2 + 3*m + 2)*x^3*e^3 + (m^2 + m)*d*x^2*e^2 - 2*d^2
*m*x*e + 2*d^3)*b*g*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2 + 11*m + 6) + ((m^3
 + 6*m^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*x^3*e^3 - 3*(m^2 + m)*
d^2*x^2*e^2 + 6*d^3*m*x*e - 6*d^4)*c*g*e^(m*log(x*e + d) - 4)/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(156) = 312$.
time = 3.42, size = 485, normalized size = 3.37

$$-\frac{\left(6cd^4g - \left((cgm^3+6cgm^2+11cgm+6cg)x^4 + ((cf+bg)m^3+7(cf+bg)m^2+8cf+8bg+14(cf+bg)m)x^3 + ((bf+ag)m^3+8(bf+ag)m^2+12bf+12ag+19(bf+ag)m)x^2 + (afm^3+9afm^2+26afm+24af)x\right)e^4 - \left(adfm^3+9adfm^2+26adfm+(cdgm^3+3cdgm^2+2cdgm)x^3+24adf+((cdf+bdg)m^3+5(cdf+bdg)m^2+4(cdf+bdg)m)x^2+((bdf+adg)m^3+7(bdf+adg)m^2+12(bdf+adg)m)x\right)e^3 + \left(12bd^2f+12ad^2g+(bd^2f+ad^2g)m^2+3(cd^2gm^2+cd^2gm)x^2+7(bd^2f+ad^2g)m+2((cd^2f+bd^2g)m^2+4(cd^2f+bd^2g)m)x\right)e^2 - 2\left(3cd^3gmx+4cd^3f+4bd^3g+(cd^3f+bd^3g)m\right)e\right)(xe+d)^m e^{(-4)}}{m^4+10m^3+35m^2+50m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -(6*c*d^4*g - ((c*g*m^3 + 6*c*g*m^2 + 11*c*g*m + 6*c*g)*x^4 + ((c*f + b*g)*
m^3 + 7*(c*f + b*g)*m^2 + 8*c*f + 8*b*g + 14*(c*f + b*g)*m)*x^3 + ((b*f + a
*g)*m^3 + 8*(b*f + a*g)*m^2 + 12*b*f + 12*a*g + 19*(b*f + a*g)*m)*x^2 + (a*
f*m^3 + 9*a*f*m^2 + 26*a*f*m + 24*a*f)*x)*e^4 - (a*d*f*m^3 + 9*a*d*f*m^2 +
26*a*d*f*m + (c*d*g*m^3 + 3*c*d*g*m^2 + 2*c*d*g*m)*x^3 + 24*a*d*f + ((c*d*f
 + b*d*g)*m^3 + 5*(c*d*f + b*d*g)*m^2 + 4*(c*d*f + b*d*g)*m)*x^2 + ((b*d*f
+ a*d*g)*m^3 + 7*(b*d*f + a*d*g)*m^2 + 12*(b*d*f + a*d*g)*m)*x)*e^3 + (12*b
*d^2*f + 12*a*d^2*g + (b*d^2*f + a*d^2*g)*m^2 + 3*(c*d^2*g*m^2 + c*d^2*g*m)
*x^2 + 7*(b*d^2*f + a*d^2*g)*m + 2*((c*d^2*f + b*d^2*g)*m^2 + 4*(c*d^2*f +
b*d^2*g)*m)*x)*e^2 - 2*(3*c*d^3*g*m*x + 4*c*d^3*f + 4*b*d^3*g + (c*d^3*f +
b*d^3*g)*m)*e)*(x*e + d)^m*e^(-4)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5930 vs. $2(134) = 268$.
time = 1.38, size = 5930, normalized size = 41.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a),x)
```

```
[Out] Piecewise((d**m*(a*f*x + a*g*x**2/2 + b*f*x**2/2 + b*g*x**3/3 + c*f*x**3/3
+ c*g*x**4/4), Eq(e, 0)), (-a*d*e**2*g/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) - 2*a*e**3*f/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d
*e**6*x**2 + 6*e**7*x**3) - 3*a*e**3*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18
*d*e**6*x**2 + 6*e**7*x**3) - 2*b*d**2*e*g/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) - b*d*e**2*f/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*d*e**2*g*x/(6*d**3*e**4 + 18*d**2*e**5*
x + 18*d*e**6*x**2 + 6*e**7*x**3) - 3*b*e**3*f*x/(6*d**3*e**4 + 18*d**2*e**
```

5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*b*e**3*g*x**2/(6*d**3*e**4 + 18*d**
2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*d**3*g*log(d/e + x)/(6*d**3*
e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 11*c*d**3*g/(6*d**3
*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 2*c*d**2*e*f/(6*d*
*3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d**2*e*g*x*
log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3)
+ 27*c*d**2*e*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x
3) - 6*c*d*e2*f*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e*
*7*x**3) + 18*c*d*e**2*g*x**2*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x +
18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*g*x**2/(6*d**3*e**4 + 18*d**2*e
5*x + 18*d*e6*x**2 + 6*e**7*x**3) - 6*c*e**3*f*x**2/(6*d**3*e**4 + 18*d
2*e5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c*e**3*g*x**3*log(d/e + x)/(
6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3), Eq(m, -4)), (
-a*d*e**2*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - a*e**3*f/(2*d**2*e**
4 + 4*d*e**5*x + 2*e**6*x**2) - 2*a*e**3*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*
e**6*x**2) + 2*b*d**2*e*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x
2) + 3*b*d2*e*g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - b*d*e**2*f/(
2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x*log(d/e + x)/(2*d*
*2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*b*d*e**2*g*x/(2*d**2*e**4 + 4*d*e**
5*x + 2*e**6*x**2) - 2*b*e**3*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2)
+ 2*b*e**3*g*x**2*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6
*c*d**3*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 9*c*d**3*
g/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*d**2*e*f*log(d/e + x)/(2*d
2*e4 + 4*d*e**5*x + 2*e**6*x**2) + 3*c*d**2*e*f/(2*d**2*e**4 + 4*d*e**5
*x + 2*e**6*x**2) - 12*c*d**2*e*g*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x
+ 2*e**6*x**2) - 12*c*d**2*e*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) +
4*c*d*e**2*f*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 4*c
*d*e**2*f*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*d*e**2*g*x**2*lo
g(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*f*x**2*log(d
/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*e**3*g*x**3/(2*d**2*
e**4 + 4*d*e**5*x + 2*e**6*x**2), Eq(m, -3)), (2*a*d*e**2*g*log(d/e + x)/(2
*d*e**4 + 2*e**5*x) + 2*a*d*e**2*g/(2*d*e**4 + 2*e**5*x) - 2*a*e**3*f/(2*d*
e**4 + 2*e**5*x) + 2*a*e**3*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d*
*2*e*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*b*d**2*e*g/(2*d*e**4 + 2*e**5
*x) + 2*b*d*e**2*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*d*e**2*f/(2*d*e
4 + 2*e5*x) - 4*b*d*e**2*g*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 2*b*e
3*f*x*log(d/e + x)/(2*d*e4 + 2*e**5*x) + 2*b*e**3*g*x**2/(2*d*e**4 + 2*
e**5*x) + 6*c*d**3*g*log(d/e + x)/(2*d*e**4 + 2*e**5*x) + 6*c*d**3*g/(2*d*e
4 + 2*e5*x) - 4*c*d**2*e*f*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 4*c*d**
2*e*f/(2*d*e**4 + 2*e**5*x) + 6*c*d**2*e*g*x*log(d/e + x)/(2*d*e**4 + 2*e**
5*x) - 4*c*d*e**2*f*x*log(d/e + x)/(2*d*e**4 + 2*e**5*x) - 3*c*d*e**2*g*x**
2/(2*d*e**4 + 2*e**5*x) + 2*c*e**3*f*x**2/(2*d*e**4 + 2*e**5*x) + c*e**3*g*
x**3/(2*d*e**4 + 2*e**5*x), Eq(m, -2)), (-a*d*g*log(d/e + x)/e**2 + a*f*log
(d/e + x)/e + a*g*x/e + b*d**2*g*log(d/e + x)/e**3 - b*d*f*log(d/e + x)/e**
2 - b*d*g*x/e**2 + b*f*x/e + b*g*x**2/(2*e) - c*d**3*g*log(d/e + x)/e**4 +

```
c*d**2*f*log(d/e + x)/e**3 + c*d**2*g*x/e**3 - c*d*f*x/e**2 - c*d*g*x**2/(2
*e**2) + c*f*x**2/(2*e) + c*g*x**3/(3*e), Eq(m, -1)), (-a*d**2*e**2*g*m**2*
(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4
) - 7*a*d**2*e**2*g*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2
 + 50*e**4*m + 24*e**4) - 12*a*d**2*e**2*g*(d + e*x)**m/(e**4*m**4 + 10*e**
4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*f*m**3*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 9*a*d*e*
*3*f*m**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m
 + 24*e**4) + 26*a*d*e**3*f*m*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e
**4*m**2 + 50*e**4*m + 24*e**4) + 24*a*d*e**3*f*(d + e*x)**m/(e**4*m**4 + 1
0*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + a*d*e**3*g*m**3*x*(d +
e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7
*a*d*e**3*g*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 +
50*e**4*m + 24*e**4) + 12*a*d*e**3*g*m*x*(d + e...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(156) = 312$.
time = 2.18, size = 1162, normalized size = 8.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c*g*m^3*x^4*e^4 + (x*e + d)^m*c*d*g*m^3*x^3*e^3 + (x*e + d)^m*
c*f*m^3*x^3*e^4 + (x*e + d)^m*b*g*m^3*x^3*e^4 + 6*(x*e + d)^m*c*g*m^2*x^4*e
^4 + (x*e + d)^m*c*d*f*m^3*x^2*e^3 + (x*e + d)^m*b*d*g*m^3*x^2*e^3 + 3*(x*e
 + d)^m*c*d*g*m^2*x^3*e^3 - 3*(x*e + d)^m*c*d^2*g*m^2*x^2*e^2 + (x*e + d)^m
*b*f*m^3*x^2*e^4 + (x*e + d)^m*a*g*m^3*x^2*e^4 + 7*(x*e + d)^m*c*f*m^2*x^3*
e^4 + 7*(x*e + d)^m*b*g*m^2*x^3*e^4 + 11*(x*e + d)^m*c*g*m*x^4*e^4 + (x*e +
 d)^m*b*d*f*m^3*x*e^3 + (x*e + d)^m*a*d*g*m^3*x*e^3 + 5*(x*e + d)^m*c*d*f*m
^2*x^2*e^3 + 5*(x*e + d)^m*b*d*g*m^2*x^2*e^3 + 2*(x*e + d)^m*c*d*g*m*x^3*e^
3 - 2*(x*e + d)^m*c*d^2*f*m^2*x*e^2 - 2*(x*e + d)^m*b*d^2*g*m^2*x*e^2 - 3*(
x*e + d)^m*c*d^2*g*m*x^2*e^2 + 6*(x*e + d)^m*c*d^3*g*m*x*e + (x*e + d)^m*a*
f*m^3*x*e^4 + 8*(x*e + d)^m*b*f*m^2*x^2*e^4 + 8*(x*e + d)^m*a*g*m^2*x^2*e^4
 + 14*(x*e + d)^m*c*f*m*x^3*e^4 + 14*(x*e + d)^m*b*g*m*x^3*e^4 + 6*(x*e + d
)^m*c*g*x^4*e^4 + (x*e + d)^m*a*d*f*m^3*e^3 + 7*(x*e + d)^m*b*d*f*m^2*x*e^3
 + 7*(x*e + d)^m*a*d*g*m^2*x*e^3 + 4*(x*e + d)^m*c*d*f*m*x^2*e^3 + 4*(x*e +
 d)^m*b*d*g*m*x^2*e^3 - (x*e + d)^m*b*d^2*f*m^2*e^2 - (x*e + d)^m*a*d^2*g*m
^2*e^2 - 8*(x*e + d)^m*c*d^2*f*m*x*e^2 - 8*(x*e + d)^m*b*d^2*g*m*x*e^2 + 2*
(x*e + d)^m*c*d^3*f*m*e + 2*(x*e + d)^m*b*d^3*g*m*e - 6*(x*e + d)^m*c*d^4*g
 + 9*(x*e + d)^m*a*f*m^2*x*e^4 + 19*(x*e + d)^m*b*f*m*x^2*e^4 + 19*(x*e + d
)^m*a*g*m*x^2*e^4 + 8*(x*e + d)^m*c*f*x^3*e^4 + 8*(x*e + d)^m*b*g*x^3*e^4 +
 9*(x*e + d)^m*a*d*f*m^2*e^3 + 12*(x*e + d)^m*b*d*f*m*x*e^3 + 12*(x*e + d)^
m*a*d*g*m*x*e^3 - 7*(x*e + d)^m*b*d^2*f*m*e^2 - 7*(x*e + d)^m*a*d^2*g*m*e^2
```

```
 + 8*(x*e + d)^m*c*d^3*f*e + 8*(x*e + d)^m*b*d^3*g*e + 26*(x*e + d)^m*a*f*m
*x*e^4 + 12*(x*e + d)^m*b*f*x^2*e^4 + 12*(x*e + d)^m*a*g*x^2*e^4 + 26*(x*e
+ d)^m*a*d*f*m*e^3 - 12*(x*e + d)^m*b*d^2*f*e^2 - 12*(x*e + d)^m*a*d^2*g*e^
2 + 24*(x*e + d)^m*a*f*x*e^4 + 24*(x*e + d)^m*a*d*f*e^3)/(m^4*e^4 + 10*m^3*
e^4 + 35*m^2*e^4 + 50*m*e^4 + 24*e^4)
```

Mupad [B]
time = 3.59, size = 602, normalized size = 4.18

$$\frac{(d+ex)^m\,(24ade^3f-6cd^4g+8bd^3eg+8cd^3ef-12ad^2e^2g-12bd^2e^2f+9ade^3fm^2+ade^3fm^3-7ad^2e^2gm-7bd^2e^2fm-ad^2e^2gm^2-bd^2e^2fm^2+26ade^3fm+2bd^3egm+2cd^3efm)}{e^4\,(m^4+10m^3+35m^2+50m+24)}+\frac{x\,(d+ex)^m\,(24ae^4f+26ae^4fm+9ae^4fm^2+ae^4fm^3+7ade^3gm^2+7bde^3fm^2+ade^3gm^3+bde^3fm^3-8bd^2e^2gm-8cd^2e^2fm-2bd^2e^2gm^2-2cd^2e^2fm^2+12ade^3gm+12bde^3fm+6cd^3egm)}{e^4\,(m^4+10m^3+35m^2+50m+24)}+\frac{x^2\,(m+1)\,(d+ex)^m\,(12ae^2g+12be^2f+7ae^2gm+7be^2fm-3cd^2gm+ae^2gm^2+be^2fm^2+4bdegm+4cdefm+bdegm^2+cdefm^2)}{e^2\,(m^4+10m^3+35m^2+50m+24)}+\frac{cgx^4\,(d+ex)^m\,(m^3+6m^2+11m+6)}{m^4+10m^3+35m^2+50m+24}+\frac{x^3\,(d+ex)^m\,(m^2+3m+2)\,(4beg+4cef+begm+cdgm+cefm)}{e\,(m^4+10m^3+35m^2+50m+24)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2),x)
```

```
[Out] ((d + e*x)^m*(24*a*d*e^3*f - 6*c*d^4*g + 8*b*d^3*e*g + 8*c*d^3*e*f - 12*a*d
^2*e^2*g - 12*b*d^2*e^2*f + 9*a*d*e^3*f*m^2 + a*d*e^3*f*m^3 - 7*a*d^2*e^2*g
*m - 7*b*d^2*e^2*f*m - a*d^2*e^2*g*m^2 - b*d^2*e^2*f*m^2 + 26*a*d*e^3*f*m +
 2*b*d^3*e*g*m + 2*c*d^3*e*f*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
+ (x*(d + e*x)^m*(24*a*e^4*f + 26*a*e^4*f*m + 9*a*e^4*f*m^2 + a*e^4*f*m^3 +
 7*a*d*e^3*g*m^2 + 7*b*d*e^3*f*m^2 + a*d*e^3*g*m^3 + b*d*e^3*f*m^3 - 8*b*d^
2*e^2*g*m - 8*c*d^2*e^2*f*m - 2*b*d^2*e^2*g*m^2 - 2*c*d^2*e^2*f*m^2 + 12*a*
d*e^3*g*m + 12*b*d*e^3*f*m + 6*c*d^3*e*g*m))/(e^4*(50*m + 35*m^2 + 10*m^3 +
 m^4 + 24)) + (x^2*(m + 1)*(d + e*x)^m*(12*a*e^2*g + 12*b*e^2*f + 7*a*e^2*g
*m + 7*b*e^2*f*m - 3*c*d^2*g*m + a*e^2*g*m^2 + b*e^2*f*m^2 + 4*b*d*e*g*m +
4*c*d*e*f*m + b*d*e*g*m^2 + c*d*e*f*m^2))/(e^2*(50*m + 35*m^2 + 10*m^3 + m^
4 + 24)) + (c*g*x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 +
10*m^3 + m^4 + 24) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(4*b*e*g + 4*c*e*f +
b*e*g*m + c*d*g*m + c*e*f*m))/(e*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

3.922 $\int \frac{(d+ex)^m(a+bx+cx^2)}{f+gx}\,dx$

Optimal. Leaf size=129

$$-\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)}+\frac{c(d+ex)^{2+m}}{e^2g(2+m)}+\frac{(cf^2-bfg+ag^2)\,(d+ex)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{g(d+e}{ef-}\right.}{g^2(ef-dg)(1+m)}$$

```
[Out] -(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(1+m)/e^2/g^2/(1+m)+c*(e*x+d)^(2+m)/e^2/g/(2+
m)+(a*g^2-b*f*g+c*f^2)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-
d*g+e*f))/g^2/(-d*g+e*f)/(1+m)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {965, 81, 70}

$$\frac{(d+ex)^{m+1}\,(ag^2-bfg+cf^2)\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)}-\frac{(d+ex)^{m+1}(-beg+cdg+cef)}{e^2g^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

```
[Out] -(((c*e*f + c*d*g - b*e*g)*(d + e*x)^(1 + m))/(e^2*g^2*(1 + m))) + (c*(d +
e*x)^(2 + m))/(e^2*g*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f
 - d*g)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
```

```
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx}\,dx &= \frac{c(d+ex)^{2+m}}{e^2g(2+m)}+\frac{\int \frac{(d+ex)^m(-e(cdf-aeg)(2+m)-e(cef+cdg-beg)(2+m)x)}{f+gx}\,dx}{e^2g(2+m)}\\ &= -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)}+\frac{c(d+ex)^{2+m}}{e^2g(2+m)}+\frac{(cf^2-bfg+ag^2)\int}{g^2}\\ &= -\frac{(cef+cdg-beg)(d+ex)^{1+m}}{e^2g^2(1+m)}+\frac{c(d+ex)^{2+m}}{e^2g(2+m)}+\frac{(cf^2-bfg+ag^2)\,(d}{}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 166, normalized size = 1.29

$$\frac{(d+ex)^m\left(\frac{g\left(beg(2+m)(d+ex)+c\left(de(-f(2+m)+gmx)+e^2x(-f(2+m)+g(1+m)x)+d^2g\left(-1+\left(1+\frac{ex}{d}\right)^{-m}\right)\right)\right)}{e^2(1+m)(2+m)}+\frac{(cf^2+g(-bf+ag))\left(\frac{g(d+ex)}{e(f+gx)}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{ef-dg}{ef+egx}\right)}{m}\right)}{g^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x]
```

```
[Out] ((d + e*x)^m*((g*(b*e*g*(2 + m)*(d + e*x) + c*(d*e*(-(f*(2 + m)) + g*m*x) +
 e^2*x*(-(f*(2 + m)) + g*(1 + m)*x) + d^2*g*(-1 + (1 + (e*x)/d)^(-m)))))/(e
^2*(1 + m)*(2 + m)) + ((c*f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[-m, -m,
 1 - m, (e*f - d*g)/(e*f + e*g*x)])/(m*((g*(d + e*x))/(e*(f + g*x)))^m)))/g
^3
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\,(c\,x^2+bx+a)}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="fricas")
[Out] integral((c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f),x)
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^m\,\left(c\,x^2+b\,x+a\right)}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x), x)
```

3.923 $\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2} \, dx$

Optimal. Leaf size=157

$$\frac{c(d+ex)^{1+m}}{eg^2(1+m)}+\frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)}+\frac{(cf(2dg-ef(2+m))-g(aegm+b(dg-ef(1+m))))(d+e}{g^2(ef-dg)^2(1+m)}$$

```
[Out] c*(e*x+d)^(1+m)/e/g^2/(1+m)+(a+f*(-b*g+c*f)/g^2)*(e*x+d)^(1+m)/(-d*g+e*f)/(
g*x+f)+(c*f*(2*d*g-e*f*(2+m))-g*(a*e*g*m+b*(d*g-e*f*(1+m))))*(e*x+d)^(1+m)*
hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/g^2/(-d*g+e*f)^2/(1+m)
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {963, 81, 70}

$$-\frac{(d+ex)^{m+1} {}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)(g(aegm+bdg-bef(m+1))-cf(2dg-ef(m+2)))}{g^2(m+1)(ef-dg)^2}+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{(f+gx)(ef-dg)}+\frac{c(d+ex)^{m+1}}{eg^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

```
[Out] (c*(d + e*x)^(1 + m))/(e*g^2*(1 + m)) + ((a + (f*(c*f - b*g))/g^2)*(d + e*x
)^(1 + m))/((e*f - d*g)*(f + g*x)) - ((g*(b*d*g + a*e*g*m - b*e*f*(1 + m))
- c*f*(2*d*g - e*f*(2 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f - d*g)^2*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
```

```
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{(f+gx)^2}\,dx = \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} + \frac{\int \frac{(d+ex)^m\left(\frac{cdfg-aeg^2m-cef^2(1+m)-bg(dg-ef(1+m))}{g^2}-c\left(\right.\right.}{f+gx}}{ef-dg}$$

$$= \frac{c(d+ex)^{1+m}}{eg^2(1+m)} + \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} - \frac{(g(bdg+aegm-bef(1+}{}$$

$$= \frac{c(d+ex)^{1+m}}{eg^2(1+m)} + \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{(ef-dg)(f+gx)} - \frac{(g(bdg+aegm-bef(1+}{}$$

Mathematica [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{(f+gx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
[Out] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2, x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\,(c\,x^2+bx+a)}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(x*e + d)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x\right)}^m\,\left(c\,x^2+b\,x+a\right)}{{\left(f+g\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x)
```

```
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2, x)
```

3.924 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(f+gx)^3}\,dx$

Optimal. Leaf size=245

$$\frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2}+\frac{(cf(4dg-ef(3+m))+g(aeg(1-m)-b(2dg-ef(1+m))))(d+ex)^{1+m}}{2g^2(ef-dg)^2(f+gx)}+\frac{(}{}$$

```
[Out] 1/2*(a+f*(-b*g+c*f)/g^2)*(e*x+d)^(1+m)/(-d*g+e*f)/(g*x+f)^2+1/2*(c*f*(4*d*g
-e*f*(3+m))+g*(a*e*g*(1-m)-b*(2*d*g-e*f*(1+m))))*(e*x+d)^(1+m)/g^2/(-d*g+e*
f)^2/(g*x+f)+1/2*(c*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-e*g*m*(
a*e*g*(1-m)-b*(2*d*g-e*f*(1+m))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g
*(e*x+d)/(-d*g+e*f))/g^2/(-d*g+e*f)^3/(1+m)
```

Rubi [A]

time = 0.20, antiderivative size = 243, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {963, 79, 70}

$$\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)(egm(-aeg(1-m)+2bdg-bef(m+1))+c(2d^2g^2-4defg(m+1)+e^2f^2(m^2+3m+2)))}{2g^2(m+1)(ef-dg)^3}-\frac{(d+ex)^{m+1}(g(-aeg(1-m)+2bdg-bef(m+1))-cf(4dg-ef(m+3)))}{2g^2(f+gx)(ef-dg)^2}+\frac{(d+ex)^{m+1}\left(a+\frac{f(cf-bg)}{g^2}\right)}{2(f+gx)^2(ef-dg)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

```
[Out] ((a + (f*(c*f - b*g))/g^2)*(d + e*x)^(1 + m))/(2*(e*f - d*g)*(f + g*x)^2) -
 ((g*(2*b*d*g - a*e*g*(1 - m) - b*e*f*(1 + m)) - c*f*(4*d*g - e*f*(3 + m)))
*(d + e*x)^(1 + m))/(2*g^2*(e*f - d*g)^2*(f + g*x)) + ((e*g*m*(2*b*d*g - a*
e*g*(1 - m) - b*e*f*(1 + m)) + c*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(
2 + 3*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*
(d + e*x))/(e*f - d*g))])/(2*g^2*(e*f - d*g)^3*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^m\,(a+bx+cx^2)}{(f+gx)^3}\,dx &= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} + \frac{\int \frac{(d+ex)^m\left(\frac{cf(2dg-ef(1+m))-g(2bdg-aeg(1-m)-bef(1+m}{g^2}\right.}{(f+gx)^2}}{2(ef-dg)} \\
&= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} - \frac{(g(2bdg-aeg(1-m)-bef(1+m))-cf}{2g^2(ef-dg)^2(f} \\
&= \frac{\left(a+\frac{f(cf-bg)}{g^2}\right)(d+ex)^{1+m}}{2(ef-dg)(f+gx)^2} - \frac{(g(2bdg-aeg(1-m)-bef(1+m))-cf}{2g^2(ef-dg)^2(f}
\end{aligned}$$

Mathematica [A]
time = 0.66, size = 185, normalized size = 0.76

$$\frac{e(d+ex)^{1+m}\left(-\left((2cf-bg)(ef-dg)\,{}_2F_1\left(2,1+m;2+m;\frac{g(d+ex)}{-ef+dg}\right)\right)+e(cf^2+g(-bf+ag))\,{}_2F_1\left(3,1+m;2+m;\frac{g(d+ex)}{-ef+dg}\right)\right)}{g^2(ef-dg)^3(1+m)}+\frac{c(d+ex)^m\left(\frac{g(d+ex)}{e(f+gx)}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{ef-dg}{ef+egx}\right)}{g^3m}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x]
```

```
[Out] (e*(d + e*x)^(1 + m)*(-((2*c*f - b*g)*(e*f - d*g)*Hypergeometric2F1[2, 1 +
m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]) + e*(c*f^2 + g*(-(b*f) + a*g))*Hyp
ergeometric2F1[3, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]))/(g^2*(e*f -
 d*g)^3*(1 + m)) + (c*(d + e*x)^m*Hypergeometric2F1[-m, -m, 1 - m, (e*f - d
*g)/(e*f + e*g*x)])/(g^3*m*((g*(d + e*x))/(e*(f + g*x)))^m)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\,(c\,x^2+bx+a)}{(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(x*e + d)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x +
 f^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m\,(a+bx+cx^2)}{(f+gx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(g*x+f)**3,x)
```

```
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)/(f + g*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)}{(f+g\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^3, x)
```

3.925 $\int (d+ex)^m (f+gx)^2 \left(a+bx+cx^2\right)^2 \, dx$

Optimal. Leaf size=525

$$\frac{\left(cd^2-bde+ae^2\right)^2 (ef-dg)^2 (d+ex)^{1+m}}{e^7 (1+m)}-\frac{2\left(cd^2-bde+ae^2\right)(ef-dg)(cd(2ef-3dg)-e(bef-2bdg+a}{e^7(2+m)}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^2*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^7/(1+m)-2*(a*e^2-b*d*e+c
*d^2)*(-d*g+e*f)*(c*d*(-3*d*g+2*e*f)-e*(a*e*g-2*b*d*g+b*e*f))*(e*x+d)^(2+m)
/e^7/(2+m)+(c^2*d^2*(15*d^2*g^2-20*d*e*f*g+6*e^2*f^2)+e^2*(a^2*e^2*g^2+2*a*
b*e*g*(-3*d*g+2*e*f)+b^2*(6*d^2*g^2-6*d*e*f*g+e^2*f^2))+2*c*e*(a*e*(6*d^2*g
^2-6*d*e*f*g+e^2*f^2)-b*d*(10*d^2*g^2-12*d*e*f*g+3*e^2*f^2)))*(e*x+d)^(3+m)
/e^7/(3+m)+2*(b*e^2*g*(a*e*g-2*b*d*g+b*e*f)-2*c^2*d*(5*d^2*g^2-5*d*e*f*g+e^
2*f^2)+c*e*(2*a*e*g*(-2*d*g+e*f)+b*(10*d^2*g^2-8*d*e*f*g+e^2*f^2)))*(e*x+d)
^(4+m)/e^7/(4+m)+(b^2*e^2*g^2+2*c*e*g*(a*e*g-5*b*d*g+2*b*e*f)+c^2*(15*d^2*g
^2-10*d*e*f*g+e^2*f^2))*(e*x+d)^(5+m)/e^7/(5+m)+2*c*g*(b*e*g-3*c*d*g+c*e*f)
*(e*x+d)^(6+m)/e^7/(6+m)+c^2*g^2*(e*x+d)^(7+m)/e^7/(7+m)
```

Rubi [A]

time = 0.38, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {961}

$$\frac{(d+ex)^{m+3}\left(e^2\left(a^2e^2g^2+2abeg(2ef-3dg)+b^2\left(6d^2g^2-6defg+e^2f^2\right)\right)+2ce\left(ae\left(6d^2g^2-6defg+e^2f^2\right)-bd\left(10d^2g^2-12defg+3e^2f^2\right)\right)+c^2d^2\left(15d^2g^2-20defg+6e^2f^2\right)\right)}{e^7(m+3)}+\frac{(d+ex)^{m+5}\left(2ceg(aeg-5bdg+2bef)+b^2e^2g^2+c^2\left(15d^2g^2-10defg+e^2f^2\right)\right)}{e^7(m+5)}+\frac{2(d+ex)^{m+4}\left(ce\left(2aeg(ef-2dg)+b\left(10d^2g^2-8defg+e^2f^2\right)\right)+be^2g(aeg-2bdg+bef)-2c^2d\left(5d^2g^2-5defg+e^2f^2\right)\right)}{e^7(m+4)}+\frac{(ef-dg)^2(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^7(m+1)}-\frac{2(ef-dg)(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)(cd(2ef-3dg)-e(aeg-2bdg+bef))}{e^7(m+2)}+\frac{2cg(d+ex)^{m+6}(beg-3cdg+cef)}{e^7(m+6)}+\frac{c^2g^2(d+ex)^{m+7}}{e^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^7*(1 + m)) -
 (2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2
*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((c^2*d^2*(6*e^2*f^2 -
20*d*e*f*g + 15*d^2*g^2) + e^2*(a^2*e^2*g^2 + 2*a*b*e*g*(2*e*f - 3*d*g) + b
^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)) + 2*c*e*(a*e*(e^2*f^2 - 6*d*e*f*g + 6
*d^2*g^2) - b*d*(3*e^2*f^2 - 12*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(3 + m))/
(e^7*(3 + m)) + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 -
5*d*e*f*g + 5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*
g + 10*d^2*g^2)))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + ((b^2*e^2*g^2 + 2*c*e*
g*(2*b*e*f - 5*b*d*g + a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d
 + e*x)^(5 + m))/(e^7*(5 + m)) + (2*c*g*(c*e*f - 3*c*d*g + b*e*g)*(d + e*x)
^(6 + m))/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m))/(e^7*(7 + m))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^2\,dx=\int\left(\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)^m}{e^6}+\frac{2(cd^2-bde+$$

$$=\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)^2(d+ex)^{1+m}}{e^7(1+m)}-\frac{2(cd^2-bde+ae^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1263 vs. 2(525) = 1050.
time = 1.86, size = 1263, normalized size = 2.41

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(c^2*(720*d^6*g^2 - 240*d^5*e*g*(f*(7 + m) + 3*g*(1 + m)
*x) + 24*d^4*e^2*(f^2*(42 + 13*m + m^2) + 10*f*g*(7 + 8*m + m^2)*x + 15*g^2
*(2 + 3*m + m^2)*x^2) - 24*d^3*e^3*(1 + m)*x*(f^2*(42 + 13*m + m^2) + 5*f*g
*(14 + 9*m + m^2)*x + 5*g^2*(6 + 5*m + m^2)*x^2) + 2*d^2*e^4*(2 + 3*m + m^2
)*x^2*(6*f^2*(42 + 13*m + m^2) + 20*f*g*(21 + 10*m + m^2)*x + 15*g^2*(12 +
7*m + m^2)*x^2) - 2*d*e^5*(6 + 11*m + 6*m^2 + m^3)*x^3*(2*f^2*(42 + 13*m +
m^2) + 5*f*g*(28 + 11*m + m^2)*x + 3*g^2*(20 + 9*m + m^2)*x^2) + e^6*(24 +
50*m + 35*m^2 + 10*m^3 + m^4)*x^4*(f^2*(42 + 13*m + m^2) + 2*f*g*(35 + 12*m
 + m^2)*x + g^2*(30 + 11*m + m^2)*x^2)) + e^2*(42 + 13*m + m^2)*(a^2*e^2*(2
0 + 9*m + m^2)*(2*d^2*g^2 - 2*d*e*g*(f*(3 + m) + g*(1 + m)*x) + e^2*(f^2*(6
 + 5*m + m^2) + 2*f*g*(3 + 4*m + m^2)*x + g^2*(2 + 3*m + m^2)*x^2)) + 2*a*b
*e*(5 + m)*(-6*d^3*g^2 + 2*d^2*e*g*(2*f*(4 + m) + 3*g*(1 + m)*x) - d*e^2*(f
^2*(12 + 7*m + m^2) + 4*f*g*(4 + 5*m + m^2)*x + 3*g^2*(2 + 3*m + m^2)*x^2)
+ e^3*(1 + m)*x*(f^2*(12 + 7*m + m^2) + 2*f*g*(8 + 6*m + m^2)*x + g^2*(6 +
5*m + m^2)*x^2)) + b^2*(24*d^4*g^2 - 12*d^3*e*g*(f*(5 + m) + 2*g*(1 + m)*x)
 + 2*d^2*e^2*(f^2*(20 + 9*m + m^2) + 6*f*g*(5 + 6*m + m^2)*x + 6*g^2*(2 + 3
*m + m^2)*x^2) - 2*d*e^3*(1 + m)*x*(f^2*(20 + 9*m + m^2) + 3*f*g*(10 + 7*m
+ m^2)*x + 2*g^2*(6 + 5*m + m^2)*x^2) + e^4*(2 + 3*m + m^2)*x^2*(f^2*(20 +
9*m + m^2) + 2*f*g*(15 + 8*m + m^2)*x + g^2*(12 + 7*m + m^2)*x^2))) + 2*c*e
*(7 + m)*(a*e*(6 + m)*(24*d^4*g^2 - 12*d^3*e*g*(f*(5 + m) + 2*g*(1 + m)*x)
+ 2*d^2*e^2*(f^2*(20 + 9*m + m^2) + 6*f*g*(5 + 6*m + m^2)*x + 6*g^2*(2 + 3*
```

```
m + m^2)*x^2) - 2*d*e^3*(1 + m)*x*(f^2*(20 + 9*m + m^2) + 3*f*g*(10 + 7*m +
 m^2)*x + 2*g^2*(6 + 5*m + m^2)*x^2) + e^4*(2 + 3*m + m^2)*x^2*(f^2*(20 + 9
*m + m^2) + 2*f*g*(15 + 8*m + m^2)*x + g^2*(12 + 7*m + m^2)*x^2)) + b*(-120
*d^5*g^2 + 24*d^4*e*g*(2*f*(6 + m) + 5*g*(1 + m)*x) - 6*d^3*e^2*(f^2*(30 +
11*m + m^2) + 8*f*g*(6 + 7*m + m^2)*x + 10*g^2*(2 + 3*m + m^2)*x^2) + 2*d^2
*e^3*(1 + m)*x*(3*f^2*(30 + 11*m + m^2) + 12*f*g*(12 + 8*m + m^2)*x + 10*g^
2*(6 + 5*m + m^2)*x^2) - d*e^4*(2 + 3*m + m^2)*x^2*(3*f^2*(30 + 11*m + m^2)
 + 8*f*g*(18 + 9*m + m^2)*x + 5*g^2*(12 + 7*m + m^2)*x^2) + e^5*(6 + 11*m +
 6*m^2 + m^3)*x^3*(f^2*(30 + 11*m + m^2) + 2*f*g*(24 + 10*m + m^2)*x + g^2*
(20 + 9*m + m^2)*x^2)))))/(e^7*(1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 +
 m)*(7 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4652 vs. $2(525) = 1050$.
time = 0.16, size = 4653, normalized size = 8.86

method	result	size
norman	Expression too large to display	4653
gosper	Expression too large to display	5890
risch	Expression too large to display	7342

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a^2*e^6*f^2*m^6-2*a^2*d*e^5*f*g*m^5+27*a^2*e^6*f^2*m^5-2*a*b*d*e^5*f^2*m
^5+2*a^2*d^2*e^4*g^2*m^4-50*a^2*d*e^5*f*g*m^4+295*a^2*e^6*f^2*m^4+8*a*b*d^2
*e^4*f*g*m^4-50*a*b*d*e^5*f^2*m^4+4*a*c*d^2*e^4*f^2*m^4+2*b^2*d^2*e^4*f^2*m
^4+44*a^2*d^2*e^4*g^2*m^3-490*a^2*d*e^5*f*g*m^3+1665*a^2*e^6*f^2*m^3-12*a*b
*d^3*e^3*g^2*m^3+176*a*b*d^2*e^4*f*g*m^3-490*a*b*d*e^5*f^2*m^3-24*a*c*d^3*e
^3*f*g*m^3+88*a*c*d^2*e^4*f^2*m^3-12*b^2*d^3*e^3*f*g*m^3+44*b^2*d^2*e^4*f^2
*m^3-12*b*c*d^3*e^3*f^2*m^3+358*a^2*d^2*e^4*g^2*m^2-2350*a^2*d*e^5*f*g*m^2+
5104*a^2*e^6*f^2*m^2-216*a*b*d^3*e^3*g^2*m^2+1432*a*b*d^2*e^4*f*g*m^2-2350*
a*b*d*e^5*f^2*m^2+48*a*c*d^4*e^2*g^2*m^2-432*a*c*d^3*e^3*f*g*m^2+716*a*c*d^
2*e^4*f^2*m^2+24*b^2*d^4*e^2*g^2*m^2-216*b^2*d^3*e^3*f*g*m^2+358*b^2*d^2*e^
4*f^2*m^2+96*b*c*d^4*e^2*f*g*m^2-216*b*c*d^3*e^3*f^2*m^2+24*c^2*d^4*e^2*f^2
*m^2+1276*a^2*d^2*e^4*g^2*m-5508*a^2*d*e^5*f*g*m+8028*a^2*e^6*f^2*m-1284*a*
b*d^3*e^3*g^2*m+5104*a*b*d^2*e^4*f*g*m-5508*a*b*d*e^5*f^2*m+624*a*c*d^4*e^2
*g^2*m-2568*a*c*d^3*e^3*f*g*m+2552*a*c*d^2*e^4*f^2*m+312*b^2*d^4*e^2*g^2*m-
1284*b^2*d^3*e^3*f*g*m+1276*b^2*d^2*e^4*f^2*m-240*b*c*d^5*e*g^2*m+1248*b*c*
d^4*e^2*f*g*m-1284*b*c*d^3*e^3*f^2*m-240*c^2*d^5*e*f*g*m+312*c^2*d^4*e^2*f^
2*m+1680*a^2*d^2*e^4*g^2-5040*a^2*d*e^5*f*g+5040*a^2*e^6*f^2-2520*a*b*d^3*e
^3*g^2+6720*a*b*d^2*e^4*f*g-5040*a*b*d*e^5*f^2+2016*a*c*d^4*e^2*g^2-5040*a*
c*d^3*e^3*f*g+3360*a*c*d^2*e^4*f^2+1008*b^2*d^4*e^2*g^2-2520*b^2*d^3*e^3*f*
g+1680*b^2*d^2*e^4*f^2-1680*b*c*d^5*e*g^2+4032*b*c*d^4*e^2*f*g-2520*b*c*d^3
*e^3*f^2+720*c^2*d^6*g^2-1680*c^2*d^5*e*f*g+1008*c^2*d^4*e^2*f^2)/e^7/(m^7+
```

28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)*exp(m*ln(e*x+d))+g
^2*c^2/(7+m)*x^7*exp(m*ln(e*x+d))+(2*a*c*e^2*g^2*m^2+b^2*e^2*g^2*m^2+2*b*c*
d*e*g^2*m^2+4*b*c*e^2*f*g*m^2+2*c^2*d*e*f*g*m^2+c^2*e^2*f^2*m^2+26*a*c*e^2*
g^2*m+13*b^2*e^2*g^2*m+14*b*c*d*e*g^2*m+52*b*c*e^2*f*g*m-6*c^2*d^2*g^2*m+14
*c^2*d*e*f*g*m+13*c^2*e^2*f^2*m+84*a*c*e^2*g^2+42*b^2*e^2*g^2+168*b*c*e^2*f
*g+42*c^2*e^2*f^2)/e^2/(m^3+18*m^2+107*m+210)*x^5*exp(m*ln(e*x+d))+(2*a*b*e
^3*g^2*m^3+2*a*c*d*e^2*g^2*m^3+4*a*c*e^3*f*g*m^3+b^2*d*e^2*g^2*m^3+2*b^2*e^
3*f*g*m^3+4*b*c*d*e^2*f*g*m^3+2*b*c*e^3*f^2*m^3+c^2*d*e^2*f^2*m^3+36*a*b*e^
3*g^2*m^2+26*a*c*d*e^2*g^2*m^2+72*a*c*e^3*f*g*m^2+13*b^2*d*e^2*g^2*m^2+36*b
^2*e^3*f*g*m^2-10*b*c*d^2*e*g^2*m^2+52*b*c*d*e^2*f*g*m^2+36*b*c*e^3*f^2*m^2
-10*c^2*d^2*e*f*g*m^2+13*c^2*d*e^2*f^2*m^2+214*a*b*e^3*g^2*m+84*a*c*d*e^2*g
^2*m+428*a*c*e^3*f*g*m+42*b^2*d*e^2*g^2*m+214*b^2*e^3*f*g*m-70*b*c*d^2*e*g^
2*m+168*b*c*d*e^2*f*g*m+214*b*c*e^3*f^2*m+30*c^2*d^3*g^2*m-70*c^2*d^2*e*f*g
*m+42*c^2*d*e^2*f^2*m+420*a*b*e^3*g^2+840*a*c*e^3*f*g+420*b^2*e^3*f*g+420*b
*c*e^3*f^2)/e^3/(m^4+22*m^3+179*m^2+638*m+840)*x^4*exp(m*ln(e*x+d))+(a^2*e^
4*g^2*m^4+2*a*b*d*e^3*g^2*m^4+4*a*b*e^4*f*g*m^4+4*a*c*d*e^3*f*g*m^4+2*a*c*e
^4*f^2*m^4+2*b^2*d*e^3*f*g*m^4+b^2*e^4*f^2*m^4+2*b*c*d*e^3*f^2*m^4+22*a^2*e
^4*g^2*m^3+36*a*b*d*e^3*g^2*m^3+88*a*b*e^4*f*g*m^3-8*a*c*d^2*e^2*g^2*m^3+72
*a*c*d*e^3*f*g*m^3+44*a*c*e^4*f^2*m^3-4*b^2*d^2*e^2*g^2*m^3+36*b^2*d*e^3*f*
g*m^3+22*b^2*e^4*f^2*m^3-16*b*c*d^2*e^2*f*g*m^3+36*b*c*d*e^3*f^2*m^3-4*c^2*
d^2*e^2*f^2*m^3+179*a^2*e^4*g^2*m^2+214*a*b*d*e^3*g^2*m^2+716*a*b*e^4*f*g*m
^2-104*a*c*d^2*e^2*g^2*m^2+428*a*c*d*e^3*f*g*m^2+358*a*c*e^4*f^2*m^2-52*b^2
*d^2*e^2*g^2*m^2+214*b^2*d*e^3*f*g*m^2+179*b^2*e^4*f^2*m^2+40*b*c*d^3*e*g^2
*m^2-208*b*c*d^2*e^2*f*g*m^2+214*b*c*d*e^3*f^2*m^2+40*c^2*d^3*e*f*g*m^2-52*
c^2*d^2*e^2*f^2*m^2+638*a^2*e^4*g^2*m+420*a*b*d*e^3*g^2*m+2552*a*b*e^4*f*g*
m-336*a*c*d^2*e^2*g^2*m+840*a*c*d*e^3*f*g*m+1276*a*c*e^4*f^2*m-168*b^2*d^2*
e^2*g^2*m+420*b^2*d*e^3*f*g*m+638*b^2*e^4*f^2*m+280*b*c*d^3*e*g^2*m-672*b*c
*d^2*e^2*f*g*m+420*b*c*d*e^3*f^2*m-120*c^2*d^4*g^2*m+280*c^2*d^3*e*f*g*m-16
8*c^2*d^2*e^2*f^2*m+840*a^2*e^4*g^2+3360*a*b*e^4*f*g+1680*a*c*e^4*f^2+840*b
^2*e^4*f^2)/e^4/(m^5+25*m^4+245*m^3+1175*m^2+2754*m+2520)*x^3*exp(m*ln(e*x+
d))+(a^2*d*e^4*g^2*m^5+2*a^2*e^5*f*g*m^5+4*a*b*d*e^4*f*g*m^5+2*a*b*e^5*f^2*
m^5+2*a*c*d*e^4*f^2*m^5+b^2*d*e^4*f^2*m^5+22*a^2*d*e^4*g^2*m^4+50*a^2*e^5*f
*g*m^4-6*a*b*d^2*e^3*g^2*m^4+88*a*b*d*e^4*f*g*m^4+50*a*b*e^5*f^2*m^4-12*a*c
*d^2*e^3*f*g*m^4+44*a*c*d*e^4*f^2*m^4-6*b^2*d^2*e^3*f*g*m^4+22*b^2*d*e^4*f^
2*m^4-6*b*c*d^2*e^3*f^2*m^4+179*a^2*d*e^4*g^2*m^3+490*a^2*e^5*f*g*m^3-108*a
*b*d^2*e^3*g^2*m^3+716*a*b*d*e^4*f*g*m^3+490*a*b*e^5*f^2*m^3+24*a*c*d^3*e^2
*g^2*m^3-216*a*c*d^2*e^3*f*g*m^3+358*a*c*d*e^4*f^2*m^3+12*b^2*d^3*e^2*g^2*m
^3-108*b^2*d^2*e^3*f*g*m^3+179*b^2*d*e^4*f^2*m^3+48*b*c*d^3*e^2*f*g*m^3-108
*b*c*d^2*e^3*f^2*m^3+12*c^2*d^3*e^2*f^2*m^3+638*a^2*d*e^4*g^2*m^2+2350*a^2*
e^5*f*g*m^2-642*a*b*d^2*e^3*g^2*m^2+2552*a*b*d*e^4*f*g*m^2+2350*a*b*e^5*f^2
*m^2+312*a*c*d^3*e^2*g^2*m^2-1284*a*c*d^2*e^3*f*g*m^2+1276*a*c*d*e^4*f^2*m^
2+156*b^2*d^3*e^2*g^2*m^2-642*b^2*d^2*e^3*f*g*m^2+638*b^2*d*e^4*f^2*m^2-120
*b*c*d^4*e*g^2*m^2+624*b*c*d^3*e^2*f*g*m^2-642*b*c*d^2*e^3*f^2*m^2-120*c^2*
d^4*e*f*g*m^2+156*c^2*d^3*e^2*f^2*m^2+840*a^2*d*e^4*g^2*m+5508*a^2*e^5*f*g*
m-1260*a*b*d^2*e^3*g^2*m+3360*a*b*d*e^4*f*g*m+5508*a*b*e^5*f^2*m+1008*a*c*d

```
^3*e^2*g^2*m-2520*a*c*d^2*e^3*f*g*m+1680*a*c*d*...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2031 vs. $2(550) = 1100$.

time = 0.34, size = 2031, normalized size = 3.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (x*e + d)^(m + 1)*a^2*f^2*e^(-1)/(m + 1) + 2*((m + 1)*x^2*e^2 + d*m*x*e - d
^2)*a*b*f^2*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + 2*((m + 1)*x^2*e^2 + d
*m*x*e - d^2)*a^2*f*g*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + ((m^2 + 3*m
+ 2)*x^3*e^3 + (m^2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*b^2*f^2*e^(m*log(
x*e + d) - 3)/(m^3 + 6*m^2 + 11*m + 6) + 2*((m^2 + 3*m + 2)*x^3*e^3 + (m^2
+ m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*a*c*f^2*e^(m*log(x*e + d) - 3)/(m^3 +
 6*m^2 + 11*m + 6) + 4*((m^2 + 3*m + 2)*x^3*e^3 + (m^2 + m)*d*x^2*e^2 - 2*d
^2*m*x*e + 2*d^3)*a*b*f*g*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2 + 11*m + 6) +
 ((m^2 + 3*m + 2)*x^3*e^3 + (m^2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*a^2*
g^2*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2 + 11*m + 6) + 2*((m^3 + 6*m^2 + 11*
m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*x^3*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 +
6*d^3*m*x*e - 6*d^4)*b*c*f^2*e^(m*log(x*e + d) - 4)/(m^4 + 10*m^3 + 35*m^2
+ 50*m + 24) + 2*((m^3 + 6*m^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*
x^3*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6*d^3*m*x*e - 6*d^4)*b^2*f*g*e^(m*log(x
*e + d) - 4)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + 4*((m^3 + 6*m^2 + 11*m +
 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*x^3*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6*d
^3*m*x*e - 6*d^4)*a*c*f*g*e^(m*log(x*e + d) - 4)/(m^4 + 10*m^3 + 35*m^2 + 5
0*m + 24) + 2*((m^3 + 6*m^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*x^3
*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6*d^3*m*x*e - 6*d^4)*a*b*g^2*e^(m*log(x*e
+ d) - 4)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + ((m^4 + 10*m^3 + 35*m^2 + 5
0*m + 24)*x^5*e^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*x^4*e^4 - 4*(m^3 + 3*m^2
 + 2*m)*d^2*x^3*e^3 + 12*(m^2 + m)*d^3*x^2*e^2 - 24*d^4*m*x*e + 24*d^5)*c^2
*f^2*e^(m*log(x*e + d) - 5)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)
 + 4*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*x^5*e^5 + (m^4 + 6*m^3 + 11*m^2 +
 6*m)*d*x^4*e^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*x^3*e^3 + 12*(m^2 + m)*d^3*x^2*
e^2 - 24*d^4*m*x*e + 24*d^5)*b*c*f*g*e^(m*log(x*e + d) - 5)/(m^5 + 15*m^4 +
 85*m^3 + 225*m^2 + 274*m + 120) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*x^5
*e^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*x^4*e^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*x
^3*e^3 + 12*(m^2 + m)*d^3*x^2*e^2 - 24*d^4*m*x*e + 24*d^5)*b^2*g^2*e^(m*log
(x*e + d) - 5)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) + 2*((m^4 +
10*m^3 + 35*m^2 + 50*m + 24)*x^5*e^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*x^4*e
^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*x^3*e^3 + 12*(m^2 + m)*d^3*x^2*e^2 - 24*d^4*
m*x*e + 24*d^5)*a*c*g^2*e^(m*log(x*e + d) - 5)/(m^5 + 15*m^4 + 85*m^3 + 225
*m^2 + 274*m + 120) + 2*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*x^
```

```
6*e^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*x^5*e^5 - 5*(m^4 + 6*m^3
+ 11*m^2 + 6*m)*d^2*x^4*e^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*x^3*e^3 - 60*(m^2
+ m)*d^4*x^2*e^2 + 120*d^5*m*x*e - 120*d^6)*c^2*f*g*e^(m*log(x*e + d) - 6)/
(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720) + 2*((m^5 + 15
*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*x^6*e^6 + (m^5 + 10*m^4 + 35*m^3 + 5
0*m^2 + 24*m)*d*x^5*e^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*x^4*e^4 + 20*(
m^3 + 3*m^2 + 2*m)*d^3*x^3*e^3 - 60*(m^2 + m)*d^4*x^2*e^2 + 120*d^5*m*x*e -
 120*d^6)*b*c*g^2*e^(m*log(x*e + d) - 6)/(m^6 + 21*m^5 + 175*m^4 + 735*m^3
+ 1624*m^2 + 1764*m + 720) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2
+ 1764*m + 720)*x^7*e^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*
m)*d*x^6*e^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*x^5*e^5 + 30*(
m^4 + 6*m^3 + 11*m^2 + 6*m)*d^3*x^4*e^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*x^3*e
^3 + 360*(m^2 + m)*d^5*x^2*e^2 - 720*d^6*m*x*e + 720*d^7)*c^2*g^2*e^(m*log(
x*e + d) - 7)/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 1
3068*m + 5040)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4356 vs. $2(550) = 1100$.
time = 2.88, size = 4356, normalized size = 8.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (720*c^2*d^7*g^2 + ((c^2*g^2*m^6 + 21*c^2*g^2*m^5 + 175*c^2*g^2*m^4 + 735*c
^2*g^2*m^3 + 1624*c^2*g^2*m^2 + 1764*c^2*g^2*m + 720*c^2*g^2)*x^7 + 2*((c^2
*f*g + b*c*g^2)*m^6 + 22*(c^2*f*g + b*c*g^2)*m^5 + 190*(c^2*f*g + b*c*g^2)*
m^4 + 840*c^2*f*g + 840*b*c*g^2 + 820*(c^2*f*g + b*c*g^2)*m^3 + 1849*(c^2*f
*g + b*c*g^2)*m^2 + 2038*(c^2*f*g + b*c*g^2)*m)*x^6 + ((c^2*f^2 + 4*b*c*f*g
 + (b^2 + 2*a*c)*g^2)*m^6 + 23*(c^2*f^2 + 4*b*c*f*g + (b^2 + 2*a*c)*g^2)*m^
5 + 207*(c^2*f^2 + 4*b*c*f*g + (b^2 + 2*a*c)*g^2)*m^4 + 1008*c^2*f^2 + 4032
*b*c*f*g + 925*(c^2*f^2 + 4*b*c*f*g + (b^2 + 2*a*c)*g^2)*m^3 + 1008*(b^2 +
2*a*c)*g^2 + 2144*(c^2*f^2 + 4*b*c*f*g + (b^2 + 2*a*c)*g^2)*m^2 + 2412*(c^2
*f^2 + 4*b*c*f*g + (b^2 + 2*a*c)*g^2)*m)*x^5 + 2*((b*c*f^2 + a*b*g^2 + (b^2
 + 2*a*c)*f*g)*m^6 + 24*(b*c*f^2 + a*b*g^2 + (b^2 + 2*a*c)*f*g)*m^5 + 226*(
b*c*f^2 + a*b*g^2 + (b^2 + 2*a*c)*f*g)*m^4 + 1260*b*c*f^2 + 1260*a*b*g^2 +
1056*(b*c*f^2 + a*b*g^2 + (b^2 + 2*a*c)*f*g)*m^3 + 1260*(b^2 + 2*a*c)*f*g +
 2545*(b*c*f^2 + a*b*g^2 + (b^2 + 2*a*c)*f*g)*m^2 + 2952*(b*c*f^2 + a*b*g^2
 + (b^2 + 2*a*c)*f*g)*m)*x^4 + ((4*a*b*f*g + a^2*g^2 + (b^2 + 2*a*c)*f^2)*m
^6 + 25*(4*a*b*f*g + a^2*g^2 + (b^2 + 2*a*c)*f^2)*m^5 + 247*(4*a*b*f*g + a^
2*g^2 + (b^2 + 2*a*c)*f^2)*m^4 + 6720*a*b*f*g + 1680*a^2*g^2 + 1219*(4*a*b*
f*g + a^2*g^2 + (b^2 + 2*a*c)*f^2)*m^3 + 1680*(b^2 + 2*a*c)*f^2 + 3112*(4*a
*b*f*g + a^2*g^2 + (b^2 + 2*a*c)*f^2)*m^2 + 3796*(4*a*b*f*g + a^2*g^2 + (b^
2 + 2*a*c)*f^2)*m)*x^3 + 2*((a*b*f^2 + a^2*f*g)*m^6 + 26*(a*b*f^2 + a^2*f*g
```

)*m^5 + 270*(a*b*f^2 + a^2*f*g)*m^4 + 2520*a*b*f^2 + 2520*a^2*f*g + 1420*(a
*b*f^2 + a^2*f*g)*m^3 + 3929*(a*b*f^2 + a^2*f*g)*m^2 + 5274*(a*b*f^2 + a^2*
f*g)*m)*x^2 + (a^2*f^2*m^6 + 27*a^2*f^2*m^5 + 295*a^2*f^2*m^4 + 1665*a^2*f^
2*m^3 + 5104*a^2*f^2*m^2 + 8028*a^2*f^2*m + 5040*a^2*f^2)*x)*e^7 + (a^2*d*f
^2*m^6 + 27*a^2*d*f^2*m^5 + 295*a^2*d*f^2*m^4 + 1665*a^2*d*f^2*m^3 + 5104*a
^2*d*f^2*m^2 + (c^2*d*g^2*m^6 + 15*c^2*d*g^2*m^5 + 85*c^2*d*g^2*m^4 + 225*c
^2*d*g^2*m^3 + 274*c^2*d*g^2*m^2 + 120*c^2*d*g^2*m)*x^6 + 8028*a^2*d*f^2*m
 + 2*((c^2*d*f*g + b*c*d*g^2)*m^6 + 17*(c^2*d*f*g + b*c*d*g^2)*m^5 + 105*(c^
2*d*f*g + b*c*d*g^2)*m^4 + 295*(c^2*d*f*g + b*c*d*g^2)*m^3 + 374*(c^2*d*f*g
 + b*c*d*g^2)*m^2 + 168*(c^2*d*f*g + b*c*d*g^2)*m)*x^5 + 5040*a^2*d*f^2 + (
(c^2*d*f^2 + 4*b*c*d*f*g + (b^2 + 2*a*c)*d*g^2)*m^6 + 19*(c^2*d*f^2 + 4*b*c
*d*f*g + (b^2 + 2*a*c)*d*g^2)*m^5 + 131*(c^2*d*f^2 + 4*b*c*d*f*g + (b^2 + 2
*a*c)*d*g^2)*m^4 + 401*(c^2*d*f^2 + 4*b*c*d*f*g + (b^2 + 2*a*c)*d*g^2)*m^3
 + 540*(c^2*d*f^2 + 4*b*c*d*f*g + (b^2 + 2*a*c)*d*g^2)*m^2 + 252*(c^2*d*f^2
 + 4*b*c*d*f*g + (b^2 + 2*a*c)*d*g^2)*m)*x^4 + 2*((b*c*d*f^2 + a*b*d*g^2 + (
b^2 + 2*a*c)*d*f*g)*m^6 + 21*(b*c*d*f^2 + a*b*d*g^2 + (b^2 + 2*a*c)*d*f*g)*
m^5 + 163*(b*c*d*f^2 + a*b*d*g^2 + (b^2 + 2*a*c)*d*f*g)*m^4 + 567*(b*c*d*f^
2 + a*b*d*g^2 + (b^2 + 2*a*c)*d*f*g)*m^3 + 844*(b*c*d*f^2 + a*b*d*g^2 + (b^
2 + 2*a*c)*d*f*g)*m^2 + 420*(b*c*d*f^2 + a*b*d*g^2 + (b^2 + 2*a*c)*d*f*g)*m
)*x^3 + ((4*a*b*d*f*g + a^2*d*g^2 + (b^2 + 2*a*c)*d*f^2)*m^6 + 23*(4*a*b*d*
f*g + a^2*d*g^2 + (b^2 + 2*a*c)*d*f^2)*m^5 + 201*(4*a*b*d*f*g + a^2*d*g^2 +
 (b^2 + 2*a*c)*d*f^2)*m^4 + 817*(4*a*b*d*f*g + a^2*d*g^2 + (b^2 + 2*a*c)*d*
f^2)*m^3 + 1478*(4*a*b*d*f*g + a^2*d*g^2 + (b^2 + 2*a*c)*d*f^2)*m^2 + 840*(
4*a*b*d*f*g + a^2*d*g^2 + (b^2 + 2*a*c)*d*f^2)*m)*x^2 + 2*((a*b*d*f^2 + a^2
*d*f*g)*m^6 + 25*(a*b*d*f^2 + a^2*d*f*g)*m^5 + 245*(a*b*d*f^2 + a^2*d*f*g)*
m^4 + 1175*(a*b*d*f^2 + a^2*d*f*g)*m^3 + 2754*(a*b*d*f^2 + a^2*d*f*g)*m^2 +
 2520*(a*b*d*f^2 + a^2*d*f*g)*m)*x)*e^6 - 2*(2520*a*b*d^2*f^2 + 2520*a^2*d^
2*f*g + (a*b*d^2*f^2 + a^2*d^2*f*g)*m^5 + 3*(c^2*d^2*g^2*m^5 + 10*c^2*d^2*g
^2*m^4 + 35*c^2*d^2*g^2*m^3 + 50*c^2*d^2*g^2*m^2 + 24*c^2*d^2*g^2*m)*x^5 +
25*(a*b*d^2*f^2 + a^2*d^2*f*g)*m^4 + 5*((c^2*d^2*f*g + b*c*d^2*g^2)*m^5 + 1
3*(c^2*d^2*f*g + b*c*d^2*g^2)*m^4 + 53*(c^2*d^2*f*g + b*c*d^2*g^2)*m^3 + 83
*(c^2*d^2*f*g + b*c*d^2*g^2)*m^2 + 42*(c^2*d^2*f*g + b*c*d^2*g^2)*m)*x^4 +
245*(a*b*d^2*f^2 + a^2*d^2*f*g)*m^3 + 2*((c^2*d^2*f^2 + 4*b*c*d^2*f*g + (b^
2 + 2*a*c)*d^2*g^2)*m^5 + 16*(c^2*d^2*f^2 + 4*b*c*d^2*f*g + (b^2 + 2*a*c)*d
^2*g^2)*m^4 + 83*(c^2*d^2*f^2 + 4*b*c*d^2*f*g + (b^2 + 2*a*c)*d^2*g^2)*m^3
 + 152*(c^2*d^2*f^2 + 4*b*c*d^2*f*g + (b^2 + 2*a*c)*d^2*g^2)*m^2 + 84*(c^2*d
^2*f^2 + 4*b*c*d^2*f*g + (b^2 + 2*a*c)*d^2*g^2)*m)*x^3 + 1175*(a*b*d^2*f^2
 + a^2*d^2*f*g)*m^2 + 3*((b*c*d^2*f^2 + a*b*d^2*g^2 + (b^2 + 2*a*c)*d^2*f*g)
m^5 + 19(b*c*d^2*f^2 + a*b*d^2*g^2 + (b^2 + 2*a*c)*d^2*f*g)*m^4 + 125*(b*
c*d^2*f^2 + a*b*d^2*g^2 + (b^2 + 2*a*c)*d^2*f*g)*m^3 + 317*(b*c*d^2*f^2 + a
*b*d^2*g^2 + (b^2 + 2*a*c)*d^2*f*g)*m^2 + 210*(b*c*d^2*f^2 + a*b*d^2*g^2 +
(b^2 + 2*a*c)*d^2*f*g)*m)*x^2 + 2754*(a*b*d^2*f^2 + a^2*d^2*f*g)*m + ((4*a*
b*d^2*f*g + a^2*d^2*g^2 + (b^2 + 2*a*c)*d^2*f^2)*m^5 + 22*(4*a*b*d^2*f*g +
a^2*d^2*g^2 + (b^2 + 2*a*c)*d^2*f^2)*m^4 + 179*(4*a*b*d^2*f*g + a^2*d^2*g^2
 + (b^2 + 2*a*c)*d^2*f^2)*m^3 + 638*(4*a*b*d^2*f*g + a^2*d^2*g^2 + (b^2 + 2

```
*a*c)*d^2*f^2)*m^2 + 840*(4*a*b*d^2*f*g + a^2*d^2*g^2 + (b^2 + 2*a*c)*d^2*f
^2)*m)*x)*e^5 + 2*(3360*a*b*d^3*f*g + 840*a^2*d...
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 74400 vs. $2(537) = 1074$.
time = 13.84, size = 74400, normalized size = 141.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**2,x)
```

```
[Out] Piecewise((d**m*(a**2*f**2*x + a**2*f*g*x**2 + a**2*g**2*x**3/3 + a*b*f**2*
x**2 + 4*a*b*f*g*x**3/3 + a*b*g**2*x**4/2 + 2*a*c*f**2*x**3/3 + a*c*f*g*x**
4 + 2*a*c*g**2*x**5/5 + b**2*f**2*x**3/3 + b**2*f*g*x**4/2 + b**2*g**2*x**5
/5 + b*c*f**2*x**4/2 + 4*b*c*f*g*x**5/5 + b*c*g**2*x**6/3 + c**2*f**2*x**5/
5 + c**2*f*g*x**6/3 + c**2*g**2*x**7/7), Eq(e, 0)), (-a**2*d**2*e**4*g**2/(
60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3
+ 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 4*a**2*d*e**5*f
*g/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x
**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 6*a**2*d*e*
*5*g**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*
e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 10*a
**2*e**6*f**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d
**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) -
24*a**2*e**6*f*g*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) - 15*a**2*e**6*g**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9
*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*
e**13*x**6) - 2*a*b*d**3*e**3*g**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d*
*4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**
5 + 60*e**13*x**6) - 4*a*b*d**2*e**4*f*g/(60*d**6*e**7 + 360*d**5*e**8*x +
900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**
12*x**5 + 60*e**13*x**6) - 12*a*b*d**2*e**4*g**2*x/(60*d**6*e**7 + 360*d**5
*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 +
 360*d*e**12*x**5 + 60*e**13*x**6) - 4*a*b*d*e**5*f**2/(60*d**6*e**7 + 360*
d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x*
*4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 24*a*b*d*e**5*f*g*x/(60*d**6*e**7
+ 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e*
*11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 30*a*b*d*e**5*g**2*x**2/(60*
d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 9
00*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 24*a*b*e**6*f**2*x
/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**
3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 60*a*b*e**6*f
*g*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e*
```

```
*10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 40*a*b
*e**6*g**2*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200
*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6)
- 4*a*c*d**4*e**2*g**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13
*x**6) - 4*a*c*d**3*e**3*f*g/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**
9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60
*e**13*x**6) - 24*a*c*d**3*e**3*g**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 90
0*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12
*x**5 + 60*e**13*x**6) - 2*a*c*d**2*e**4*f**2/(60*d**6*e**7 + 360*d**5*e**8
*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*
d*e**12*x**5 + 60*e**13*x**6) - 24*a*c*d**2*e**4*f*g*x/(60*d**6*e**7 + 360*
d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x*
*4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 60*a*c*d**2*e**4*g**2*x**2/(60*d**
6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*
d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 12*a*c*d*e**5*f**2*x/
(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3
 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 60*a*c*d*e**5*
f*g*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e
**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 80*a*
c*d*e**5*g**2*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) - 30*a*c*e**6*f**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6) - 80*a*c*e**6*f*g*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**
4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5
 + 60*e**13*x**6) - 60*a*c*e**6*g**2*x**4/(60*d**6*e**7 + 360*d**5*e**8*x +
 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e*
*12*x**5 + 60*e**13*x**6) - 2*b**2*d**4*e**2*g**2/(60*d**6*e**7 + 360*d**5*
e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 +
360*d*e**12*x**5 + 60*e**13*x**6) - 2*b**2*d**3*e**3*f*g/(60*d**6*e**7 + 36
0*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*
x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 12*b**2*d**3*e**3*g**2*x/(60*d**
6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 +...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10489 vs. $2(550) = 1100$.
time = 1.92, size = 10489, normalized size = 19.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((x*e + d)^m*c^2*g^2*m^6*x^7*e^7 + (x*e + d)^m*c^2*d*g^2*m^6*x^6*e^6 + 2*(x
*e + d)^m*c^2*f*g*m^6*x^6*e^7 + 2*(x*e + d)^m*b*c*g^2*m^6*x^6*e^7 + 21*(x*e
```

+ d)^m*c^2*g^2*m^5*x^7*e^7 + 2*(x*e + d)^m*c^2*d*f*g*m^6*x^5*e^6 + 2*(x*e + d)^m*b*c*d*g^2*m^6*x^5*e^6 + 15*(x*e + d)^m*c^2*d*g^2*m^5*x^6*e^6 - 6*(x*e + d)^m*c^2*d^2*g^2*m^5*x^5*e^5 + (x*e + d)^m*c^2*f^2*m^6*x^5*e^7 + 4*(x*e + d)^m*b*c*f*g*m^6*x^5*e^7 + (x*e + d)^m*b^2*g^2*m^6*x^5*e^7 + 2*(x*e + d)^m*a*c*g^2*m^6*x^5*e^7 + 44*(x*e + d)^m*c^2*f*g*m^5*x^6*e^7 + 44*(x*e + d)^m*b*c*g^2*m^5*x^6*e^7 + 175*(x*e + d)^m*c^2*g^2*m^4*x^7*e^7 + (x*e + d)^m*c^2*d*f^2*m^6*x^4*e^6 + 4*(x*e + d)^m*b*c*d*f*g*m^6*x^4*e^6 + (x*e + d)^m*b^2*d*g^2*m^6*x^4*e^6 + 2*(x*e + d)^m*a*c*d*g^2*m^6*x^4*e^6 + 34*(x*e + d)^m*c^2*d*f*g*m^5*x^5*e^6 + 34*(x*e + d)^m*b*c*d*g^2*m^5*x^5*e^6 + 85*(x*e + d)^m*c^2*d*g^2*m^4*x^6*e^6 - 10*(x*e + d)^m*c^2*d^2*f*g*m^5*x^4*e^5 - 10*(x*e + d)^m*b*c*d^2*g^2*m^5*x^4*e^5 - 60*(x*e + d)^m*c^2*d^2*g^2*m^4*x^5*e^5 + 30*(x*e + d)^m*c^2*d^3*g^2*m^4*x^4*e^4 + 2*(x*e + d)^m*b*c*f^2*m^6*x^4*e^7 + 2*(x*e + d)^m*b^2*f*g*m^6*x^4*e^7 + 4*(x*e + d)^m*a*c*f*g*m^6*x^4*e^7 + 2*(x*e + d)^m*a*b*g^2*m^6*x^4*e^7 + 23*(x*e + d)^m*c^2*f^2*m^5*x^5*e^7 + 92*(x*e + d)^m*b*c*f*g*m^5*x^5*e^7 + 23*(x*e + d)^m*b^2*g^2*m^5*x^5*e^7 + 46*(x*e + d)^m*a*c*g^2*m^5*x^5*e^7 + 380*(x*e + d)^m*c^2*f*g*m^4*x^6*e^7 + 380*(x*e + d)^m*b*c*g^2*m^4*x^6*e^7 + 735*(x*e + d)^m*c^2*g^2*m^3*x^7*e^7 + 2*(x*e + d)^m*b*c*d*f^2*m^6*x^3*e^6 + 2*(x*e + d)^m*b^2*d*f*g*m^6*x^3*e^6 + 4*(x*e + d)^m*a*c*d*f*g*m^6*x^3*e^6 + 2*(x*e + d)^m*a*b*d*g^2*m^6*x^3*e^6 + 19*(x*e + d)^m*c^2*d*f^2*m^5*x^4*e^6 + 76*(x*e + d)^m*b*c*d*f*g*m^5*x^4*e^6 + 19*(x*e + d)^m*b^2*d*g^2*m^5*x^4*e^6 + 38*(x*e + d)^m*a*c*d*g^2*m^5*x^4*e^6 + 210*(x*e + d)^m*c^2*d*f*g*m^4*x^5*e^6 + 210*(x*e + d)^m*b*c*d*g^2*m^4*x^5*e^6 + 225*(x*e + d)^m*c^2*d*g^2*m^3*x^6*e^6 - 4*(x*e + d)^m*c^2*d^2*f^2*m^5*x^3*e^5 - 16*(x*e + d)^m*b*c*d^2*f*g*m^5*x^3*e^5 - 4*(x*e + d)^m*b^2*d^2*g^2*m^5*x^3*e^5 - 8*(x*e + d)^m*a*c*d^2*g^2*m^5*x^3*e^5 - 130*(x*e + d)^m*c^2*d^2*f*g*m^4*x^4*e^5 - 130*(x*e + d)^m*b*c*d^2*g^2*m^4*x^4*e^5 - 210*(x*e + d)^m*c^2*d^2*g^2*m^3*x^5*e^5 + 40*(x*e + d)^m*c^2*d^3*f*g*m^4*x^3*e^4 + 40*(x*e + d)^m*b*c*d^3*g^2*m^4*x^3*e^4 + 180*(x*e + d)^m*c^2*d^3*g^2*m^3*x^4*e^4 - 120*(x*e + d)^m*c^2*d^4*g^2*m^3*x^3*e^3 + (x*e + d)^m*b^2*f^2*m^6*x^3*e^7 + 2*(x*e + d)^m*a*c*f^2*m^6*x^3*e^7 + 4*(x*e + d)^m*a*b*f*g*m^6*x^3*e^7 + (x*e + d)^m*a^2*g^2*m^6*x^3*e^7 + 48*(x*e + d)^m*b*c*f^2*m^5*x^4*e^7 + 48*(x*e + d)^m*b^2*f*g*m^5*x^4*e^7 + 96*(x*e + d)^m*a*c*f*g*m^5*x^4*e^7 + 48*(x*e + d)^m*a*b*g^2*m^5*x^4*e^7 + 207*(x*e + d)^m*c^2*f^2*m^4*x^5*e^7 + 828*(x*e + d)^m*b*c*f*g*m^4*x^5*e^7 + 207*(x*e + d)^m*b^2*g^2*m^4*x^5*e^7 + 414*(x*e + d)^m*a*c*g^2*m^4*x^5*e^7 + 1640*(x*e + d)^m*c^2*f*g*m^3*x^6*e^7 + 1640*(x*e + d)^m*b*c*g^2*m^3*x^6*e^7 + 1624*(x*e + d)^m*c^2*g^2*m^2*x^7*e^7 + (x*e + d)^m*b^2*d*f^2*m^6*x^2*e^6 + 2*(x*e + d)^m*a*c*d*f^2*m^6*x^2*e^6 + 4*(x*e + d)^m*a*b*d*f*g*m^6*x^2*e^6 + (x*e + d)^m*a^2*d*g^2*m^6*x^2*e^6 + 42*(x*e + d)^m*b*c*d*f^2*m^5*x^3*e^6 + 42*(x*e + d)^m*b^2*d*f*g*m^5*x^3*e^6 + 84*(x*e + d)^m*a*c*d*f*g*m^5*x^3*e^6 + 42*(x*e + d)^m*a*b*d*g^2*m^5*x^3*e^6 + 131*(x*e + d)^m*c^2*d*f^2*m^4*x^4*e^6 + 524*(x*e + d)^m*b*c*d*f*g*m^4*x^4*e^6 + 131*(x*e + d)^m*b^2*d*g^2*m^4*x^4*e^6 + 262*(x*e + d)^m*a*c*d*g^2*m^4*x^4*e^6 + 590*(x*e + d)^m*c^2*d*f*g*m^3*x^5*e^6 + 590*(x*e + d)^m*b*c*d*g^2*m^3*x^5*e^6 + 274*(x*e + d)^m*c^2*d*g^2*m^2*x^6*e^6 - 6*(x*e + d)^m*b*c*d^2*f^2*m^5*x^2*e^5 - 6*(x*e + d)^m*b^2*d^2*f*g*m^5*x^2*e^5 - 1

```
2*(x*e + d)^m*a*c*d^2*f*g*m^5*x^2*e^5 - 6*(x*e + d)^m*a*b*d^2*g^2*m^5*x^2*e
^5 - 64*(x*e + d)^m*c^2*d^2*f^2*m^4*x^3*e^5 - 256*(x*e + d)^m*b*c*d^2*f*g*m
^4*x^3*e^5 - 64*(x*e + d)^m*b^2*d^2*g^2*m^4*x^3*e^5 - 128*(x*e + d)^m*a*c*d
^2*g^2*m^4*x^3*e^5 - 530*(x*e + d)^m*c^2*d^2*f*g*m^3*x^4*e^5 - 530*(x*e + d
)^m*b*c*d^2*g^2*m^3*x^4*e^5 - 300*(x*e + d)^m*c^2*d^2*g^2*m^2*x^5*e^5 + 12*
(x*e + d)^m*c^2*d^3*f^2*m^4*x^2*e^4 + 48*(x*e + d)^m*b*c*d^3*f*g*m^4*x^2*e^
4 + 12*(x*e + d)^m*b^2*d^3*g^2*m^4*x^2*e^4 + 24*(x*e + d)^m*a*c*d^3*g^2*m^4
*x^2*e^4 + 400*(x*e + d)^m*c^2*d^3*f*g*m^3*x^3*e^4 + 400*(x*e + d)^m*b*c*d^
3*g^2*m^3*x^3*e^4 + 330*(x*e + d)^m*c^2*d^3*g^2*m^2*x^4*e^4 - 120*(x*e + d)
^m*c^2*d^4*f*g*m^3*x^2*e^3 - 120*(x*e + d)^m*b*c*d^4*g^2*m^3*x^2*e^3 - 360*
(x*e + d)^m*c^2*d^4*g^2*m^2*x^3*e^3 + 360*(x*e + d)^m*c^2*d^5*g^2*m^2*x^2*e
^2 + 2*(x*e + d)^m*a*b*f^2*m^6*x^2*e^7 + 2*(x*e + d)^m*a^2*f*g*m^6*x^2*e^7
+ 25*(x*e + d)^m*b^2*f^2*m^5*x^3*e^7 + 50*(x*e + d)^m*a*c*f^2*m^5*x^3*e^7 +
 100*(x*e + d)^m*a*b*f*g*m^5*x^3*e^7 + 25*(x*e + d)^m*a^2*g^2*m^5*x^3*e^7 +
 452*(x*e + d)^m*b*c*f^2*m^4*x^4*e^7 + 452*(x*e + d)^m*b^2*f*g*m^4*x^4*e^7
+ 904*(x*e + d)^m*a*c*f*g*m^4*x^4*e^7 + 452*(x*e + d)^m*a*b*g^2*m^4*x^4*e^7
 + 925*(x*e + d)^m*c^2*f^2*m^3*x^5*e^7 + 3700*(x*e + d)^m*b*c*f*g*m^3*x^5*e
^7 + 925*(x*e + d)^m*b^2*g^2*m^3*x^5*e^7 + 1850*(x*e + d)^m*a*c*g^2*m^3*x^5
*e^7 + 3698*(x*e + d)^m*c^2*f*g*m^2*x^6*e^7 + 3698*(x*e + d)^m*b*c*g^2*m^2*
x^6*e^7 + 1764*(x*e + d)^m*c^2*g^2*m*x^7*e^7 + ...
```

Mupad [B]
time = 5.38, size = 2500, normalized size = 4.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

```
[Out] ((d + e*x)^m*(720*c^2*d^7*g^2 + 5040*a^2*d*e^6*f^2 + 1680*a^2*d^3*e^4*g^2 +
 1680*b^2*d^3*e^4*f^2 + 1008*b^2*d^5*e^2*g^2 + 1008*c^2*d^5*e^2*f^2 - 1680*
b*c*d^6*e*g^2 - 1680*c^2*d^6*e*f*g + 358*a^2*d^3*e^4*g^2*m^2 + 358*b^2*d^3*
e^4*f^2*m^2 + 44*a^2*d^3*e^4*g^2*m^3 + 44*b^2*d^3*e^4*f^2*m^3 + 2*a^2*d^3*e
^4*g^2*m^4 + 2*b^2*d^3*e^4*f^2*m^4 + 24*b^2*d^5*e^2*g^2*m^2 + 24*c^2*d^5*e^
2*f^2*m^2 - 5040*a*b*d^2*e^5*f^2 - 2520*a*b*d^4*e^3*g^2 + 3360*a*c*d^3*e^4*
f^2 + 2016*a*c*d^5*e^2*g^2 - 2520*b*c*d^4*e^3*f^2 - 5040*a^2*d^2*e^5*f*g -
2520*b^2*d^4*e^3*f*g + 8028*a^2*d*e^6*f^2*m + 5104*a^2*d*e^6*f^2*m^2 + 1665
*a^2*d*e^6*f^2*m^3 + 295*a^2*d*e^6*f^2*m^4 + 27*a^2*d*e^6*f^2*m^5 + a^2*d*e
^6*f^2*m^6 + 1276*a^2*d^3*e^4*g^2*m + 1276*b^2*d^3*e^4*f^2*m + 312*b^2*d^5*
e^2*g^2*m + 312*c^2*d^5*e^2*f^2*m - 2350*a*b*d^2*e^5*f^2*m^2 - 490*a*b*d^2*
e^5*f^2*m^3 - 50*a*b*d^2*e^5*f^2*m^4 - 2*a*b*d^2*e^5*f^2*m^5 - 216*a*b*d^4*
e^3*g^2*m^2 + 716*a*c*d^3*e^4*f^2*m^2 - 12*a*b*d^4*e^3*g^2*m^3 + 88*a*c*d^3
*e^4*f^2*m^3 + 4*a*c*d^3*e^4*f^2*m^4 + 48*a*c*d^5*e^2*g^2*m^2 - 216*b*c*d^4
*e^3*f^2*m^2 - 12*b*c*d^4*e^3*f^2*m^3 - 2350*a^2*d^2*e^5*f*g*m^2 - 490*a^2*
d^2*e^5*f*g*m^3 - 50*a^2*d^2*e^5*f*g*m^4 - 2*a^2*d^2*e^5*f*g*m^5 - 216*b^2*
```

d^4*e^3*f*g*m^2 - 12*b^2*d^4*e^3*f*g*m^3 + 6720*a*b*d^3*e^4*f*g - 5040*a*c*d^4*e^3*f*g + 4032*b*c*d^5*e^2*f*g - 240*b*c*d^6*e*g^2*m - 240*c^2*d^6*e*f*g*m - 5508*a*b*d^2*e^5*f^2*m - 1284*a*b*d^4*e^3*g^2*m + 2552*a*c*d^3*e^4*f^2*m + 624*a*c*d^5*e^2*g^2*m - 1284*b*c*d^4*e^3*f^2*m - 5508*a^2*d^2*e^5*f*g*m - 1284*b^2*d^4*e^3*f*g*m + 1432*a*b*d^3*e^4*f*g*m^2 + 176*a*b*d^3*e^4*f*g*m^3 + 8*a*b*d^3*e^4*f*g*m^4 - 432*a*c*d^4*e^3*f*g*m^2 - 24*a*c*d^4*e^3*f*g*m^3 + 96*b*c*d^5*e^2*f*g*m^2 + 5104*a*b*d^3*e^4*f*g*m - 2568*a*c*d^4*e^3*f*g*m + 1248*b*c*d^5*e^2*f*g*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x*(d + e*x)^m*(5040*a^2*e^7*f^2 + 8028*a^2*e^7*f^2*m + 5104*a^2*e^7*f^2*m^2 + 1665*a^2*e^7*f^2*m^3 + 295*a^2*e^7*f^2*m^4 + 27*a^2*e^7*f^2*m^5 + a^2*e^7*f^2*m^6 - 1276*a^2*d^2*e^5*g^2*m^2 - 1276*b^2*d^2*e^5*f^2*m^2 - 358*a^2*d^2*e^5*g^2*m^3 - 358*b^2*d^2*e^5*f^2*m^3 - 44*a^2*d^2*e^5*g^2*m^4 - 44*b^2*d^2*e^5*f^2*m^4 - 2*a^2*d^2*e^5*g^2*m^5 - 2*b^2*d^2*e^5*f^2*m^5 - 312*b^2*d^4*e^3*g^2*m^2 - 312*c^2*d^4*e^3*f^2*m^2 - 24*b^2*d^4*e^3*g^2*m^3 - 24*c^2*d^4*e^3*f^2*m^3 - 720*c^2*d^6*e*g^2*m - 1680*a^2*d^2*e^5*g^2*m - 1680*b^2*d^2*e^5*f^2*m - 1008*b^2*d^4*e^3*g^2*m - 1008*c^2*d^4*e^3*f^2*m + 1284*a*b*d^3*e^4*g^2*m^2 - 2552*a*c*d^2*e^5*f^2*m^2 + 216*a*b*d^3*e^4*g^2*m^3 - 716*a*c*d^2*e^5*f^2*m^3 + 12*a*b*d^3*e^4*g^2*m^4 - 88*a*c*d^2*e^5*f^2*m^4 - 4*a*c*d^2*e^5*f^2*m^5 - 624*a*c*d^4*e^3*g^2*m^2 + 1284*b*c*d^3*e^4*f^2*m^2 - 48*a*c*d^4*e^3*g^2*m^3 + 216*b*c*d^3*e^4*f^2*m^3 + 12*b*c*d^3*e^4*f^2*m^4 + 240*b*c*d^5*e^2*g^2*m^2 + 1284*b^2*d^3*e^4*f*g*m^2 + 216*b^2*d^3*e^4*f*g*m^3 + 12*b^2*d^3*e^4*f*g*m^4 + 240*c^2*d^5*e^2*f*g*m^2 + 5040*a*b*d*e^6*f^2*m + 5040*a^2*d*e^6*f*g*m + 5508*a*b*d*e^6*f^2*m^2 + 2350*a*b*d*e^6*f^2*m^3 + 490*a*b*d*e^6*f^2*m^4 + 50*a*b*d*e^6*f^2*m^5 + 2*a*b*d*e^6*f^2*m^6 + 2520*a*b*d^3*e^4*g^2*m - 3360*a*c*d^2*e^5*f^2*m - 2016*a*c*d^4*e^3*g^2*m + 2520*b*c*d^3*e^4*f^2*m + 1680*b*c*d^5*e^2*g^2*m + 5508*a^2*d*e^6*f*g*m^2 + 2350*a^2*d*e^6*f*g*m^3 + 490*a^2*d*e^6*f*g*m^4 + 50*a^2*d*e^6*f*g*m^5 + 2*a^2*d*e^6*f*g*m^6 + 2520*b^2*d^3*e^4*f*g*m + 1680*c^2*d^5*e^2*f*g*m - 5104*a*b*d^2*e^5*f*g*m^2 - 1432*a*b*d^2*e^5*f*g*m^3 - 176*a*b*d^2*e^5*f*g*m^4 - 8*a*b*d^2*e^5*f*g*m^5 + 2568*a*c*d^3*e^4*f*g*m^2 + 432*a*c*d^3*e^4*f*g*m^3 + 24*a*c*d^3*e^4*f*g*m^4 - 1248*b*c*d^4*e^3*f*g*m^2 - 96*b*c*d^4*e^3*f*g*m^3 - 6720*a*b*d^2*e^5*f*g*m + 5040*a*c*d^3*e^4*f*g*m - 4032*b*c*d^4*e^3*f*g*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(840*a^2*e^4*g^2 + 840*b^2*e^4*f^2 + 638*a^2*e^4*g^2*m + 638*b^2*e^4*f^2*m - 120*c^2*d^4*g^2*m + 179*a^2*e^4*g^2*m^2 + 179*b^2*e^4*f^2*m^2 + 22*a^2*e^4*g^2*m^3 + 22*b^2*e^4*f^2*m^3 + a^2*e^4*g^2*m^4 + b^2*e^4*f^2*m^4 + 1680*a*c*e^4*f^2 + 1276*a*c*e^4*f^2*m - 52*b^2*d^2*e^2*g^2*m^2 - 52*c^2*d^2*e^2*f^2*m^2 - 4*b^2*d^2*e^2*g^2*m^3 - 4*c^2*d^2*e^2*f^2*m^3 + 358*a*c*e^4*f^2*m^2 + 44*a*c*e^4*f^2*m^3 + 2*a*c*e^4*f^2*m^4 + 3360*a*b*e^4*f*g - 168*b^2*d^2*e^2*g^2*m - 168*c^2*d^2*e^2*f^2*m + 2552*a*b*e^4*f*g*m - 104*a*c*d^2*e^2*g^2*m^2 - 8*a*c*d^2*e^2*g^2*m^3 + 420*a*b*d*e^3*g^2*m + 420*b*c*d*e^3*f^2*m + 280*b*c*d^3*e*g^2*m + 716*a*b*e^4*f*g*m^2 + 88*a*b*e^4*f*g*m^3 + 4*a*b*e^4*f*g*m^4 + 420*b^2*d*e^3*f*g*m + 280*c^2*d^3*e*f*g*m + 214*a*b*d*e^3*g^2*m^2 + 36*a*b*d*e^3*g^2*m^3 + 2*a*b*d*e^3*g^2*m^4 - 336*a*c*d^2*e^2*g^

```
2*m + 214*b*c*d*e^3*f^2*m^2 + 36*b*c*d*e^3*f^2*m^3 + 2*b*c*d*e^3*f^2*m^4 +
40*b*c*d^3*e*g^2*m^2 + 214*b^2*d*e^3*f*g*m^2 + 36*b^2*d*e^3*f*g*m^3 + 2*b^2
*d*e^3*f*g*m^4 + 40*c^2*d^3*e*f*g*m^2 - 208*b*c*d^2*e^2*f*g*m^2 - 16*b*c*d^
2*e^2*f*g*m^3 + 840*a*c*d*e^3*f*g*m + 428*a*c*d*e^3*f*g*m^2 + 72*a*c*d*e^3*
f*g*m^3 + 4*a*c*d*e^3*f*g*m^4 - 672*b*c*d^2*e^2*f*g*m))/(e^4*(13068*m + 131
32*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6...
```

3.926 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx$

Optimal. Leaf size=311

$$\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^{1+m}}{e^6(1+m)}-\frac{\left(cd^2-bde+ae^2\right)\left(cd(4ef-5dg)-e(2bef-3bdg+aeg)\right)(d+e}{e^6(2+m)}$$

```
[Out] (a*e^2-b*d*e+c*d^2)^2*(-d*g+e*f)*(e*x+d)^(1+m)/e^6/(1+m)-(a*e^2-b*d*e+c*d^2
)*(c*d*(-5*d*g+4*e*f)-e*(a*e*g-3*b*d*g+2*b*e*f))*(e*x+d)^(2+m)/e^6/(2+m)+(2
*c^2*d^2*(-5*d*g+3*e*f)+b*e^2*(2*a*e*g-3*b*d*g+b*e*f)+2*c*e*(a*e*(-3*d*g+e*
f)-3*b*d*(-2*d*g+e*f)))*(e*x+d)^(3+m)/e^6/(3+m)+(b^2*e^2*g-2*c^2*d*(-5*d*g+
2*e*f)+2*c*e*(a*e*g-4*b*d*g+b*e*f))*(e*x+d)^(4+m)/e^6/(4+m)+c*(2*b*e*g-5*c*
d*g+c*e*f)*(e*x+d)^(5+m)/e^6/(5+m)+c^2*g*(e*x+d)^(6+m)/e^6/(6+m)
```

Rubi [A]
time = 0.24, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {785}

$$\frac{(d+ex)^{m+4}\left(2ce(aeg-4bdg+bef)+b^2e^2g-2c^2d(2ef-5dg)\right)}{e^6(m+4)}+\frac{(d+ex)^{m+3}\left(2ce(ae(ef-3dg)-3bd(ef-2dg))+be^2(2aeg-3bdg+bef)+2c^2d^2(3ef-5dg)\right)}{e^6(m+3)}+\frac{(ef-dg)(d+ex)^{m+1}\left(ae^2-bde+cd^2\right)^2}{e^6(m+1)}-\frac{(d+ex)^{m+2}\left(ae^2-bde+cd^2\right)\left(cd(4ef-5dg)-e(aeg-3bdg+2bef)\right)}{e^6(m+2)}+\frac{c(d+ex)^{m+5}(2beg-5cdg+cef)}{e^6(m+5)}+\frac{c^2g(d+ex)^{m+6}}{e^6(m+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) - (
(c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e*g
))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^2*(
b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*d*g)
))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5*d*g)
 + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m)) + (c*(
c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(d + e
*x)^(6 + m))/(e^6*(6 + m))
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^2\,dx = \int \left(\frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^m}{e^5} + \frac{\left(cd^2-bde+ae^2\right)}{}\right.$$

$$= \frac{\left(cd^2-bde+ae^2\right)^2(ef-dg)(d+ex)^{1+m}}{e^6(1+m)} - \frac{\left(cd^2-bde+ae^2\right)(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 655 vs. 2(311) = 622.
time = 1.14, size = 655, normalized size = 2.11

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*((a + x*(b + c*x))^2*(2*b*e*g + c*(-5*d*g + e*f*(6 + m)
+ e*g*(5 + m)*x)) + (2*(((c*d^2 + e*(-(b*d) + a*e))*(b^3*e^3*g*(3 + 4*m + m
^2) + 12*c^3*d^2*(-5*d*g + e*f*(6 + m)) - b*c*e^2*(1 + m)*(b*d*g*(-6 + m) +
 b*e*f*(6 + m) + 2*a*e*g*(9 + 2*m)) + 2*c^2*e*(-3*b*d*(d*g*(-9 + m) + 2*e*f
*(6 + m)) + 2*a*e*(d*g*(-15 + m + m^2) + e*f*(24 + 10*m + m^2)))))/(e^2*(1
+ m)) + ((b^4*e^4*g*(6 + 5*m + m^2) + 12*c^4*d^3*(5*d*g - e*f*(6 + m)) - b^
2*c*e^3*(2 + m)*(b*e*f*(6 + m) + b*d*g*(-3 + 2*m) + a*e*g*(21 + 5*m)) + 2*c
^3*d*e*(3*b*d*(d*g*(-14 + m) + 3*e*f*(6 + m)) - 2*a*e*(d*g*(-30 - 4*m + m^2
) + e*f*(42 + 19*m + 2*m^2))) + c^2*e^2*(4*a^2*e^2*g*(15 + 8*m + m^2) + b^2
*d*(d*g*(6 - 13*m + m^2) + 2*e*f*(-6 + 5*m + m^2)) + 2*a*b*e*(e*f*(42 + 19*
m + 2*m^2) + d*g*(-18 + 11*m + 4*m^2))))*(d + e*x))/(e^2*(2 + m)) - (c*e*(4
 + m)*(b*d*(-5*c*d + 2*b*e)*g - 2*a*c*d*e*g*m + a*b*e^2*g*(1 + m) + c*e*(b*
d - 2*a*e)*f*(6 + m)) - (3*c*d - b*e)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g
+ e*f*(6 + m)) - c*e*(b*d*g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m))) +
c*e*(3 + m)*(b^2*e^2*g*(3 + m) + 2*c^2*d*(-5*d*g + e*f*(6 + m)) - c*e*(b*d*
g*(-4 + m) + 2*a*e*g*(5 + m) + b*e*f*(6 + m)))*x)*(a + x*(b + c*x))))/(c*e^
2*(3 + m)*(4 + m))))/(c*e^2*(5 + m)*(6 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2148 vs. 2(311) = 622.
time = 0.13, size = 2149, normalized size = 6.91

method	result	size
norman	Expression too large to display	2149
gosper	Expression too large to display	2563
risch	Expression too large to display	3326

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] g*c^2/(6+m)*x^6*exp(m*ln(e*x+d))+(2*a*c*e^2*g*m^2+b^2*e^2*g*m^2+2*b*c*d*e*g*m^2+2*b*c*e^2*f*m^2+c^2*d*e*f*m^2+22*a*c*e^2*g*m+11*b^2*e^2*g*m+12*b*c*d*e*g*m+22*b*c*e^2*f*m-5*c^2*d^2*g*m+6*c^2*d*e*f*m+60*a*c*e^2*g+30*b^2*e^2*g+60*b*c*e^2*f)/e^2/(m^3+15*m^2+74*m+120)*x^4*exp(m*ln(e*x+d))+(2*a*b*e^3*g*m^3+2*a*c*d*e^2*g*m^3+2*a*c*e^3*f*m^3+b^2*d*e^2*g*m^3+b^2*e^3*f*m^3+2*b*c*d*e^2*f*m^3+30*a*b*e^3*g*m^2+22*a*c*d*e^2*g*m^2+30*a*c*e^3*f*m^2+11*b^2*d*e^2*g*m^2+15*b^2*e^3*f*m^2-8*b*c*d^2*e*g*m^2+22*b*c*d*e^2*f*m^2-4*c^2*d^2*e*f*m^2+148*a*b*e^3*g*m+60*a*c*d*e^2*g*m+148*a*c*e^3*f*m+30*b^2*d*e^2*g*m+74*b^2*e^3*f*m-48*b*c*d^2*e*g*m+60*b*c*d*e^2*f*m+20*c^2*d^3*g*m-24*c^2*d^2*e*f*m+240*a*b*e^3*g+240*a*c*e^3*f+120*b^2*e^3*f)/e^3/(m^4+18*m^3+119*m^2+342*m+360)*x^3*exp(m*ln(e*x+d))+(a^2*e^4*g*m^4+2*a*b*d*e^3*g*m^4+2*a*b*e^4*f*m^4+2*a*c*d*e^3*f*m^4+b^2*d*e^3*f*m^4+18*a^2*e^4*g*m^3+30*a*b*d*e^3*g*m^3+36*a*b*e^4*f*m^3-6*a*c*d^2*e^2*g*m^3+30*a*c*d*e^3*f*m^3-3*b^2*d^2*e^2*g*m^3+15*b^2*d*e^3*f*m^3-6*b*c*d^2*e^2*f*m^3+119*a^2*e^4*g*m^2+148*a*b*d*e^3*g*m^2+238*a*b*e^4*f*m^2-66*a*c*d^2*e^2*g*m^2+148*a*c*d*e^3*f*m^2-33*b^2*d^2*e^2*g*m^2+74*b^2*d*e^3*f*m^2+24*b*c*d^3*e*g*m^2-66*b*c*d^2*e^2*f*m^2+12*c^2*d^3*e*f*m^2+342*a^2*e^4*g*m+240*a*b*d*e^3*g*m+684*a*b*e^4*f*m-180*a*c*d^2*e^2*g*m+240*a*c*d*e^3*f*m-90*b^2*d^2*e^2*g*m+120*b^2*d*e^3*f*m+144*b*c*d^3*e*g*m-180*b*c*d^2*e^2*f*m-60*c^2*d^4*g*m+72*c^2*d^3*e*f*m+360*a^2*e^4*g+720*a*b*e^4*f)/e^4/(m^5+20*m^4+155*m^3+580*m^2+1044*m+720)*x^2*exp(m*ln(e*x+d))+(a^2*d*e^4*g*m^5+a^2*e^5*f*m^5+2*a*b*d*e^4*f*m^5+18*a^2*d*e^4*g*m^4+20*a^2*e^5*f*m^4-4*a*b*d^2*e^3*g*m^4+36*a*b*d*e^4*f*m^4-4*a*c*d^2*e^3*f*m^4-2*b^2*d^2*e^3*f*m^4+119*a^2*d*e^4*g*m^3+155*a^2*e^5*f*m^3-60*a*b*d^2*e^3*g*m^3+238*a*b*d*e^4*f*m^3+12*a*c*d^3*e^2*g*m^3-60*a*c*d^2*e^3*f*m^3+6*b^2*d^3*e^2*g*m^3-30*b^2*d^2*e^3*f*m^3+12*b*c*d^3*e^2*f*m^3+342*a^2*d*e^4*g*m^2+580*a^2*e^5*f*m^2-296*a*b*d^2*e^3*g*m^2+684*a*b*d*e^4*f*m^2+132*a*c*d^3*e^2*g*m^2-296*a*c*d^2*e^3*f*m^2+66*b^2*d^3*e^2*g*m^2-148*b^2*d^2*e^3*f*m^2-48*b*c*d^4*e*g*m^2+132*b*c*d^3*e^2*f*m^2-24*c^2*d^4*e*f*m^2+360*a^2*d*e^4*g*m+1044*a^2*e^5*f*m-480*a*b*d^2*e^3*g*m+720*a*b*d*e^4*f*m+360*a*c*d^3*e^2*g*m-480*a*c*d^2*e^3*f*m+180*b^2*d^3*e^2*g*m-240*b^2*d^2*e^3*f*m-288*b*c*d^4*e*g*m+360*b*c*d^3*e^2*f*m+120*c^2*d^5*g*m-144*c^2*d^4*e*f*m+720*a^2*e^5*f)/e^5/(m^6+21*m^5+175*m^4+735*m^3+1624*m^2+1764*m+720)*x*exp(m*ln(e*x+d))+(2*b*e*g*m+c*d*g*m+c*e*f*m+12*b*e*g+6*c*e*f)*c/e/(m^2+11*m+30)*x^5*exp(m*ln(e*x+d))-d*(-a^2*e^5*f*m^5+a^2*d*e^4*g*m^4-20*a^2*e^5*f*m^4+2*a*b*d*e^4*f*m^4+18*a^2*d*e^4*g*m^3-155*a^2*e^5*f*m^3-4*a*b*d^2*e^3*g*m^3+36*a*b*d*e^4*f*m^3-4*a*c*d^2*e^3*f*m^3-2*b^2*d^2*e^3*f*m^3+119*a^2*d*e^4*g*m^2-580*a^2*e^5*f*m^2-60*a*b*d^2*e^3*g*m^2+238*a*b*d*e^4*f*m^2+12*a*c*d^3*e^2*g*m^2-60*a*c*d^2*e^3*f*m^2+6*b^2*d^3*e^2*g*m^2-30*b^2*d^2*e^3*f*m^2+12*b*c*d^3*e^2*f*m^2+342*a^2*d*e^4*g*m-1044*a^2*e^5*f*m-296*a*b*d^2*e^3*g*m+684*a*b*d*e^4*f*m+132*a*c*d^3*e^2*g*m-296*a*c*d^2*e^3*f*m+66*b^2*d^3*e^2*g*m-148*b^2*d^2*e^3*f*m-48*b*c*d^4*e*g*m+

```
132*b*c*d^3*e^2*f*m-24*c^2*d^4*e*f*m+360*a^2*d*e^4*g-720*a^2*e^5*f-480*a*b*
d^2*e^3*g+720*a*b*d*e^4*f+360*a*c*d^3*e^2*g-480*a*c*d^2*e^3*f+180*b^2*d^3*e
^2*g-240*b^2*d^2*e^3*f-288*b*c*d^4*e*g+360*b*c*d^3*e^2*f+120*c^2*d^5*g-144*
c^2*d^4*e*f)/e^6/(m^6+21*m^5+175*m^4+735*m^3+1624*m^2+1764*m+720)*exp(m*ln(
e*x+d))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1121 vs. $2(337) = 674$.
time = 0.34, size = 1121, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (x*e + d)^(m + 1)*a^2*f*e^(-1)/(m + 1) + 2*((m + 1)*x^2*e^2 + d*m*x*e - d^2
)*a*b*f*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + ((m + 1)*x^2*e^2 + d*m*x*e
 - d^2)*a^2*g*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + ((m^2 + 3*m + 2)*x^3
*e^3 + (m^2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*b^2*f*e^(m*log(x*e + d) -
 3)/(m^3 + 6*m^2 + 11*m + 6) + 2*((m^2 + 3*m + 2)*x^3*e^3 + (m^2 + m)*d*x^2
*e^2 - 2*d^2*m*x*e + 2*d^3)*a*c*f*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2 + 11*
m + 6) + 2*((m^2 + 3*m + 2)*x^3*e^3 + (m^2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2
*d^3)*a*b*g*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2 + 11*m + 6) + 2*((m^3 + 6*m
^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*x^3*e^3 - 3*(m^2 + m)*d^2*x^
2*e^2 + 6*d^3*m*x*e - 6*d^4)*b*c*f*e^(m*log(x*e + d) - 4)/(m^4 + 10*m^3 + 3
5*m^2 + 50*m + 24) + ((m^3 + 6*m^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m
)*d*x^3*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6*d^3*m*x*e - 6*d^4)*b^2*g*e^(m*log
(x*e + d) - 4)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + 2*((m^3 + 6*m^2 + 11*m
 + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*x^3*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6
*d^3*m*x*e - 6*d^4)*a*c*g*e^(m*log(x*e + d) - 4)/(m^4 + 10*m^3 + 35*m^2 + 5
0*m + 24) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*x^5*e^5 + (m^4 + 6*m^3 + 1
1*m^2 + 6*m)*d*x^4*e^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*x^3*e^3 + 12*(m^2 + m)*d
^3*x^2*e^2 - 24*d^4*m*x*e + 24*d^5)*c^2*f*e^(m*log(x*e + d) - 5)/(m^5 + 15*
m^4 + 85*m^3 + 225*m^2 + 274*m + 120) + 2*((m^4 + 10*m^3 + 35*m^2 + 50*m +
24)*x^5*e^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*x^4*e^4 - 4*(m^3 + 3*m^2 + 2*m
)*d^2*x^3*e^3 + 12*(m^2 + m)*d^3*x^2*e^2 - 24*d^4*m*x*e + 24*d^5)*b*c*g*e^(
m*log(x*e + d) - 5)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) + ((m^5
 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*x^6*e^6 + (m^5 + 10*m^4 + 35*m^
3 + 50*m^2 + 24*m)*d*x^5*e^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*x^4*e^4 +
 20*(m^3 + 3*m^2 + 2*m)*d^3*x^3*e^3 - 60*(m^2 + m)*d^4*x^2*e^2 + 120*d^5*m*
x*e - 120*d^6)*c^2*g*e^(m*log(x*e + d) - 6)/(m^6 + 21*m^5 + 175*m^4 + 735*m
^3 + 1624*m^2 + 1764*m + 720)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2084 vs. $2(337) = 674$.

time = 3.22, size = 2084, normalized size = 6.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(120*c^2*d^6*g - ((c^2*g*m^5 + 15*c^2*g*m^4 + 85*c^2*g*m^3 + 225*c^2*g*m^2
 + 274*c^2*g*m + 120*c^2*g)*x^6 + ((c^2*f + 2*b*c*g)*m^5 + 16*(c^2*f + 2*b*
c*g)*m^4 + 95*(c^2*f + 2*b*c*g)*m^3 + 144*c^2*f + 288*b*c*g + 260*(c^2*f +
2*b*c*g)*m^2 + 324*(c^2*f + 2*b*c*g)*m)*x^5 + ((2*b*c*f + (b^2 + 2*a*c)*g)*
m^5 + 17*(2*b*c*f + (b^2 + 2*a*c)*g)*m^4 + 107*(2*b*c*f + (b^2 + 2*a*c)*g)*
m^3 + 360*b*c*f + 307*(2*b*c*f + (b^2 + 2*a*c)*g)*m^2 + 180*(b^2 + 2*a*c)*g
 + 396*(2*b*c*f + (b^2 + 2*a*c)*g)*m)*x^4 + ((2*a*b*g + (b^2 + 2*a*c)*f)*m^
5 + 18*(2*a*b*g + (b^2 + 2*a*c)*f)*m^4 + 121*(2*a*b*g + (b^2 + 2*a*c)*f)*m^
3 + 480*a*b*g + 372*(2*a*b*g + (b^2 + 2*a*c)*f)*m^2 + 240*(b^2 + 2*a*c)*f +
 508*(2*a*b*g + (b^2 + 2*a*c)*f)*m)*x^3 + ((2*a*b*f + a^2*g)*m^5 + 19*(2*a*
b*f + a^2*g)*m^4 + 137*(2*a*b*f + a^2*g)*m^3 + 720*a*b*f + 360*a^2*g + 461*
(2*a*b*f + a^2*g)*m^2 + 702*(2*a*b*f + a^2*g)*m)*x^2 + (a^2*f*m^5 + 20*a^2*
f*m^4 + 155*a^2*f*m^3 + 580*a^2*f*m^2 + 1044*a^2*f*m + 720*a^2*f)*x)*e^6 -
(a^2*d*f*m^5 + 20*a^2*d*f*m^4 + 155*a^2*d*f*m^3 + 580*a^2*d*f*m^2 + (c^2*d*
g*m^5 + 10*c^2*d*g*m^4 + 35*c^2*d*g*m^3 + 50*c^2*d*g*m^2 + 24*c^2*d*g*m)*x^
5 + 1044*a^2*d*f*m + ((c^2*d*f + 2*b*c*d*g)*m^5 + 12*(c^2*d*f + 2*b*c*d*g)*
m^4 + 47*(c^2*d*f + 2*b*c*d*g)*m^3 + 72*(c^2*d*f + 2*b*c*d*g)*m^2 + 36*(c^2
*d*f + 2*b*c*d*g)*m)*x^4 + 720*a^2*d*f + ((2*b*c*d*f + (b^2 + 2*a*c)*d*g)*m
^5 + 14*(2*b*c*d*f + (b^2 + 2*a*c)*d*g)*m^4 + 65*(2*b*c*d*f + (b^2 + 2*a*c)
*d*g)*m^3 + 112*(2*b*c*d*f + (b^2 + 2*a*c)*d*g)*m^2 + 60*(2*b*c*d*f + (b^2
+ 2*a*c)*d*g)*m)*x^3 + ((2*a*b*d*g + (b^2 + 2*a*c)*d*f)*m^5 + 16*(2*a*b*d*g
 + (b^2 + 2*a*c)*d*f)*m^4 + 89*(2*a*b*d*g + (b^2 + 2*a*c)*d*f)*m^3 + 194*(2
*a*b*d*g + (b^2 + 2*a*c)*d*f)*m^2 + 120*(2*a*b*d*g + (b^2 + 2*a*c)*d*f)*m)*
x^2 + ((2*a*b*d*f + a^2*d*g)*m^5 + 18*(2*a*b*d*f + a^2*d*g)*m^4 + 119*(2*a*
b*d*f + a^2*d*g)*m^3 + 342*(2*a*b*d*f + a^2*d*g)*m^2 + 360*(2*a*b*d*f + a^2
*d*g)*m)*x)*e^5 + (720*a*b*d^2*f + 360*a^2*d^2*g + (2*a*b*d^2*f + a^2*d^2*g
)*m^4 + 5*(c^2*d^2*g*m^4 + 6*c^2*d^2*g*m^3 + 11*c^2*d^2*g*m^2 + 6*c^2*d^2*g
*m)*x^4 + 18*(2*a*b*d^2*f + a^2*d^2*g)*m^3 + 4*((c^2*d^2*f + 2*b*c*d^2*g)*m
^4 + 9*(c^2*d^2*f + 2*b*c*d^2*g)*m^3 + 20*(c^2*d^2*f + 2*b*c*d^2*g)*m^2 + 1
2*(c^2*d^2*f + 2*b*c*d^2*g)*m)*x^3 + 119*(2*a*b*d^2*f + a^2*d^2*g)*m^2 + 3*
((2*b*c*d^2*f + (b^2 + 2*a*c)*d^2*g)*m^4 + 12*(2*b*c*d^2*f + (b^2 + 2*a*c)*
d^2*g)*m^3 + 41*(2*b*c*d^2*f + (b^2 + 2*a*c)*d^2*g)*m^2 + 30*(2*b*c*d^2*f +
 (b^2 + 2*a*c)*d^2*g)*m)*x^2 + 342*(2*a*b*d^2*f + a^2*d^2*g)*m + 2*((2*a*b*
d^2*g + (b^2 + 2*a*c)*d^2*f)*m^4 + 15*(2*a*b*d^2*g + (b^2 + 2*a*c)*d^2*f)*m
^3 + 74*(2*a*b*d^2*g + (b^2 + 2*a*c)*d^2*f)*m^2 + 120*(2*a*b*d^2*g + (b^2 +
 2*a*c)*d^2*f)*m)*x)*e^4 - 2*(240*a*b*d^3*g + 120*(b^2 + 2*a*c)*d^3*f + (2*
a*b*d^3*g + (b^2 + 2*a*c)*d^3*f)*m^3 + 10*(c^2*d^3*g*m^3 + 3*c^2*d^3*g*m^2
+ 2*c^2*d^3*g*m)*x^3 + 15*(2*a*b*d^3*g + (b^2 + 2*a*c)*d^3*f)*m^2 + 6*((c^2
```

```
*d^3*f + 2*b*c*d^3*g)*m^3 + 7*(c^2*d^3*f + 2*b*c*d^3*g)*m^2 + 6*(c^2*d^3*f
+ 2*b*c*d^3*g)*m)*x^2 + 74*(2*a*b*d^3*g + (b^2 + 2*a*c)*d^3*f)*m + 3*((2*b*
c*d^3*f + (b^2 + 2*a*c)*d^3*g)*m^3 + 11*(2*b*c*d^3*f + (b^2 + 2*a*c)*d^3*g)
*m^2 + 30*(2*b*c*d^3*f + (b^2 + 2*a*c)*d^3*g)*m)*x)*e^3 + 6*(60*b*c*d^4*f +
 30*(b^2 + 2*a*c)*d^4*g + (2*b*c*d^4*f + (b^2 + 2*a*c)*d^4*g)*m^2 + 10*(c^2
*d^4*g*m^2 + c^2*d^4*g*m)*x^2 + 11*(2*b*c*d^4*f + (b^2 + 2*a*c)*d^4*g)*m +
4*((c^2*d^4*f + 2*b*c*d^4*g)*m^2 + 6*(c^2*d^4*f + 2*b*c*d^4*g)*m)*x)*e^2 -
24*(5*c^2*d^5*g*m*x + 6*c^2*d^5*f + 12*b*c*d^5*g + (c^2*d^5*f + 2*b*c*d^5*g
)*m)*e)*(x*e + d)^m*e^(-6)/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1
764*m + 720)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 32864 vs. $2(309) = 618$.

time = 6.38, size = 32864, normalized size = 105.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**2,x)
```

```
[Out] Piecewise((d**m*(a**2*f*x + a**2*g*x**2/2 + a*b*f*x**2 + 2*a*b*g*x**3/3 + 2
*a*c*f*x**3/3 + a*c*g*x**4/2 + b**2*f*x**3/3 + b**2*g*x**4/4 + b*c*f*x**4/2
 + 2*b*c*g*x**5/5 + c**2*f*x**5/5 + c**2*g*x**6/6), Eq(e, 0)), (-3*a**2*d*e
**4*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*
x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 12*a**2*e**5*f/(60*d**5*e**6 + 3
00*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4
 + 60*e**11*x**5) - 15*a**2*e**5*g*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*
d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 4
*a*b*d**2*e**3*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600
*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*a*b*d*e**4*f/(60*d*
*5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d
*e**10*x**4 + 60*e**11*x**5) - 20*a*b*d*e**4*g*x/(60*d**5*e**6 + 300*d**4*e
**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**
11*x**5) - 30*a*b*e**5*f*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*
x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 40*a*b*e**5
*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**
9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*a*c*d**3*e**2*g/(60*d**5*e**
6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10
*x**4 + 60*e**11*x**5) - 4*a*c*d**2*e**3*f/(60*d**5*e**6 + 300*d**4*e**7*x
+ 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**
5) - 30*a*c*d**2*e**3*g*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x
**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 20*a*c*d*e**
4*f*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*
x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 60*a*c*d*e**4*g*x**2/(60*d**5*e*
*6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**1
```

```
0*x**4 + 60*e**11*x**5) - 40*a*c*e**5*f*x**2/(60*d**5*e**6 + 300*d**4*e**7*
x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x
**5) - 60*a*c*e**5*g*x**3/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x
**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 3*b**2*d**3*
e**2*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9
*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 2*b**2*d**2*e**3*f/(60*d**5*e**
6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10
*x**4 + 60*e**11*x**5) - 15*b**2*d**2*e**3*g*x/(60*d**5*e**6 + 300*d**4*e**
7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11
*x**5) - 10*b**2*d*e**4*f*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8
*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 30*b**2*d*
e**4*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2
*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 20*b**2*e**5*f*x**2/(60*d*
*5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d
*e**10*x**4 + 60*e**11*x**5) - 30*b**2*e**5*g*x**3/(60*d**5*e**6 + 300*d**4
*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e
**11*x**5) - 24*b*c*d**4*e*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**
8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*b*c*d**
3*e**2*f/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e*
*9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 120*b*c*d**3*e**2*g*x/(60*d**
5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*
e**10*x**4 + 60*e**11*x**5) - 30*b*c*d**2*e**3*f*x/(60*d**5*e**6 + 300*d**4
*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e
**11*x**5) - 240*b*c*d**2*e**3*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600
*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) -
60*b*c*d*e**4*f*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 +
 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 240*b*c*d*e**4*g*
x**3/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x
**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 60*b*c*e**5*f*x**3/(60*d**5*e**6
+ 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x
**4 + 60*e**11*x**5) - 120*b*c*e**5*g*x**4/(60*d**5*e**6 + 300*d**4*e**7*x
+ 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**
5) + 60*c**2*d**5*g*log(d/e + x)/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3
*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) + 137*c
**2*d**5*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*
e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 12*c**2*d**4*e*f/(60*d**5*e
**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**
10*x**4 + 60*e**11*x**5) + 300*c**2*d**4*e*g*x*log(d/e + x)/(60*d**5*e**6 +
 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x*
*4 + 60*e**11*x**5) + 625*c**2*d**4*e*g*x/(60*d**5*e**6 + 300*d**4*e**7*x +
 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5
) - 60*c**2*d**3*e**2*f*x/(60*d**5*e**6 + 300*d...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4940 vs. $2(337) = 674$.

time = 3.58, size = 4940, normalized size = 15.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x, algorithm="giac")
[Out] ((x*e + d)^m*c^2*g*m^5*x^6*e^6 + (x*e + d)^m*c^2*d*g*m^5*x^5*e^5 + (x*e + d
)^m*c^2*f*m^5*x^5*e^6 + 2*(x*e + d)^m*b*c*g*m^5*x^5*e^6 + 15*(x*e + d)^m*c^
2*g*m^4*x^6*e^6 + (x*e + d)^m*c^2*d*f*m^5*x^4*e^5 + 2*(x*e + d)^m*b*c*d*g*m
^5*x^4*e^5 + 10*(x*e + d)^m*c^2*d*g*m^4*x^5*e^5 - 5*(x*e + d)^m*c^2*d^2*g*m
^4*x^4*e^4 + 2*(x*e + d)^m*b*c*f*m^5*x^4*e^6 + (x*e + d)^m*b^2*g*m^5*x^4*e^
6 + 2*(x*e + d)^m*a*c*g*m^5*x^4*e^6 + 16*(x*e + d)^m*c^2*f*m^4*x^5*e^6 + 32
*(x*e + d)^m*b*c*g*m^4*x^5*e^6 + 85*(x*e + d)^m*c^2*g*m^3*x^6*e^6 + 2*(x*e
+ d)^m*b*c*d*f*m^5*x^3*e^5 + (x*e + d)^m*b^2*d*g*m^5*x^3*e^5 + 2*(x*e + d)^
m*a*c*d*g*m^5*x^3*e^5 + 12*(x*e + d)^m*c^2*d*f*m^4*x^4*e^5 + 24*(x*e + d)^m
*b*c*d*g*m^4*x^4*e^5 + 35*(x*e + d)^m*c^2*d*g*m^3*x^5*e^5 - 4*(x*e + d)^m*c
^2*d^2*f*m^4*x^3*e^4 - 8*(x*e + d)^m*b*c*d^2*g*m^4*x^3*e^4 - 30*(x*e + d)^m
*c^2*d^2*g*m^3*x^4*e^4 + 20*(x*e + d)^m*c^2*d^3*g*m^3*x^3*e^3 + (x*e + d)^m
*b^2*f*m^5*x^3*e^6 + 2*(x*e + d)^m*a*c*f*m^5*x^3*e^6 + 2*(x*e + d)^m*a*b*g*
m^5*x^3*e^6 + 34*(x*e + d)^m*b*c*f*m^4*x^4*e^6 + 17*(x*e + d)^m*b^2*g*m^4*x
^4*e^6 + 34*(x*e + d)^m*a*c*g*m^4*x^4*e^6 + 95*(x*e + d)^m*c^2*f*m^3*x^5*e^
6 + 190*(x*e + d)^m*b*c*g*m^3*x^5*e^6 + 225*(x*e + d)^m*c^2*g*m^2*x^6*e^6 +
 (x*e + d)^m*b^2*d*f*m^5*x^2*e^5 + 2*(x*e + d)^m*a*c*d*f*m^5*x^2*e^5 + 2*(x
*e + d)^m*a*b*d*g*m^5*x^2*e^5 + 28*(x*e + d)^m*b*c*d*f*m^4*x^3*e^5 + 14*(x*
e + d)^m*b^2*d*g*m^4*x^3*e^5 + 28*(x*e + d)^m*a*c*d*g*m^4*x^3*e^5 + 47*(x*e
 + d)^m*c^2*d*f*m^3*x^4*e^5 + 94*(x*e + d)^m*b*c*d*g*m^3*x^4*e^5 + 50*(x*e
+ d)^m*c^2*d*g*m^2*x^5*e^5 - 6*(x*e + d)^m*b*c*d^2*f*m^4*x^2*e^4 - 3*(x*e +
 d)^m*b^2*d^2*g*m^4*x^2*e^4 - 6*(x*e + d)^m*a*c*d^2*g*m^4*x^2*e^4 - 36*(x*e
 + d)^m*c^2*d^2*f*m^3*x^3*e^4 - 72*(x*e + d)^m*b*c*d^2*g*m^3*x^3*e^4 - 55*(
x*e + d)^m*c^2*d^2*g*m^2*x^4*e^4 + 12*(x*e + d)^m*c^2*d^3*f*m^3*x^2*e^3 + 2
4*(x*e + d)^m*b*c*d^3*g*m^3*x^2*e^3 + 60*(x*e + d)^m*c^2*d^3*g*m^2*x^3*e^3
- 60*(x*e + d)^m*c^2*d^4*g*m^2*x^2*e^2 + 2*(x*e + d)^m*a*b*f*m^5*x^2*e^6 +
(x*e + d)^m*a^2*g*m^5*x^2*e^6 + 18*(x*e + d)^m*b^2*f*m^4*x^3*e^6 + 36*(x*e
+ d)^m*a*c*f*m^4*x^3*e^6 + 36*(x*e + d)^m*a*b*g*m^4*x^3*e^6 + 214*(x*e + d)
^m*b*c*f*m^3*x^4*e^6 + 107*(x*e + d)^m*b^2*g*m^3*x^4*e^6 + 214*(x*e + d)^m*
a*c*g*m^3*x^4*e^6 + 260*(x*e + d)^m*c^2*f*m^2*x^5*e^6 + 520*(x*e + d)^m*b*c
*g*m^2*x^5*e^6 + 274*(x*e + d)^m*c^2*g*m*x^6*e^6 + 2*(x*e + d)^m*a*b*d*f*m^
5*x*e^5 + (x*e + d)^m*a^2*d*g*m^5*x*e^5 + 16*(x*e + d)^m*b^2*d*f*m^4*x^2*e^
5 + 32*(x*e + d)^m*a*c*d*f*m^4*x^2*e^5 + 32*(x*e + d)^m*a*b*d*g*m^4*x^2*e^5
 + 130*(x*e + d)^m*b*c*d*f*m^3*x^3*e^5 + 65*(x*e + d)^m*b^2*d*g*m^3*x^3*e^5
 + 130*(x*e + d)^m*a*c*d*g*m^3*x^3*e^5 + 72*(x*e + d)^m*c^2*d*f*m^2*x^4*e^5
 + 144*(x*e + d)^m*b*c*d*g*m^2*x^4*e^5 + 24*(x*e + d)^m*c^2*d*g*m*x^5*e^5 -
 2*(x*e + d)^m*b^2*d^2*f*m^4*x*e^4 - 4*(x*e + d)^m*a*c*d^2*f*m^4*x*e^4 - 4*
(x*e + d)^m*a*b*d^2*g*m^4*x*e^4 - 72*(x*e + d)^m*b*c*d^2*f*m^3*x^2*e^4 - 36
```

```
*(x*e + d)^m*b^2*d^2*g*m^3*x^2*e^4 - 72*(x*e + d)^m*a*c*d^2*g*m^3*x^2*e^4 -
 80*(x*e + d)^m*c^2*d^2*f*m^2*x^3*e^4 - 160*(x*e + d)^m*b*c*d^2*g*m^2*x^3*e
^4 - 30*(x*e + d)^m*c^2*d^2*g*m*x^4*e^4 + 12*(x*e + d)^m*b*c*d^3*f*m^3*x*e^
3 + 6*(x*e + d)^m*b^2*d^3*g*m^3*x*e^3 + 12*(x*e + d)^m*a*c*d^3*g*m^3*x*e^3
+ 84*(x*e + d)^m*c^2*d^3*f*m^2*x^2*e^3 + 168*(x*e + d)^m*b*c*d^3*g*m^2*x^2*
e^3 + 40*(x*e + d)^m*c^2*d^3*g*m*x^3*e^3 - 24*(x*e + d)^m*c^2*d^4*f*m^2*x*e
^2 - 48*(x*e + d)^m*b*c*d^4*g*m^2*x*e^2 - 60*(x*e + d)^m*c^2*d^4*g*m*x^2*e^
2 + 120*(x*e + d)^m*c^2*d^5*g*m*x*e + (x*e + d)^m*a^2*f*m^5*x*e^6 + 38*(x*e
 + d)^m*a*b*f*m^4*x^2*e^6 + 19*(x*e + d)^m*a^2*g*m^4*x^2*e^6 + 121*(x*e + d
)^m*b^2*f*m^3*x^3*e^6 + 242*(x*e + d)^m*a*c*f*m^3*x^3*e^6 + 242*(x*e + d)^m
*a*b*g*m^3*x^3*e^6 + 614*(x*e + d)^m*b*c*f*m^2*x^4*e^6 + 307*(x*e + d)^m*b^
2*g*m^2*x^4*e^6 + 614*(x*e + d)^m*a*c*g*m^2*x^4*e^6 + 324*(x*e + d)^m*c^2*f
*m*x^5*e^6 + 648*(x*e + d)^m*b*c*g*m*x^5*e^6 + 120*(x*e + d)^m*c^2*g*x^6*e^
6 + (x*e + d)^m*a^2*d*f*m^5*e^5 + 36*(x*e + d)^m*a*b*d*f*m^4*x*e^5 + 18*(x*
e + d)^m*a^2*d*g*m^4*x*e^5 + 89*(x*e + d)^m*b^2*d*f*m^3*x^2*e^5 + 178*(x*e
+ d)^m*a*c*d*f*m^3*x^2*e^5 + 178*(x*e + d)^m*a*b*d*g*m^3*x^2*e^5 + 224*(x*e
 + d)^m*b*c*d*f*m^2*x^3*e^5 + 112*(x*e + d)^m*b^2*d*g*m^2*x^3*e^5 + 224*(x*
e + d)^m*a*c*d*g*m^2*x^3*e^5 + 36*(x*e + d)^m*c^2*d*f*m*x^4*e^5 + 72*(x*e +
 d)^m*b*c*d*g*m*x^4*e^5 - 2*(x*e + d)^m*a*b*d^2*f*m^4*e^4 - (x*e + d)^m*a^2
*d^2*g*m^4*e^4 - 30*(x*e + d)^m*b^2*d^2*f*m^3*x*e^4 - 60*(x*e + d)^m*a*c*d^
2*f*m^3*x*e^4 - 60*(x*e + d)^m*a*b*d^2*g*m^3*x*e^4 - 246*(x*e + d)^m*b*c*d^
2*f*m^2*x^2*e^4 - 123*(x*e + d)^m*b^2*d^2*g*m^2*x^2*e^4 - 246*(x*e + d)^m*a
*c*d^2*g*m^2*x^2*e^4 - 48*(x*e + d)^m*c^2*d^2*f*m*x^3*e^4 - 96*(x*e + d)^m*
b*c*d^2*g*m*x^3*e^4 + 2*(x*e + d)^m*b^2*d^3*f*m^3*e^3 + 4*(x*e + d)^m*a*c*d
^3*f*m^3*e^3 + 4*(x*e + d)^m*a*b*d^3*g*m^3*e^3 + 132*(x*e + d)^m*b*c*d^3*f*
m^2*x*e^3 + 66*(x*e + d)^m*b^2*d^3*g*m^2*x*e^3 + 132*(x*e + d)^m*a*c*d^3*g*
m^2*x*e^3 + 72*(x*e + d)^m*c^2*d^3*f*m*x^2*e^3 + 144*(x*e + d)^m*b*c*d^3*g*
m*x^2*e^3 - 12*(x*e + d)^m*b*c*d^4*f*m^2*e^2 - 6*(x*e + d)^m*b^2*d^4*g*m^2*
e^2 - 12*(x*e + d)^m*a*c*d^4*g*m^2*e^2 - 144*(x...
```

Mupad [B]
time = 4.38, size = 2307, normalized size = 7.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^2,x)
```

```
[Out] ((d + e*x)^m*(240*b^2*d^3*e^3*f - 360*a^2*d^2*e^4*g - 120*c^2*d^6*g - 180*b
^2*d^4*e^2*g + 720*a^2*d*e^5*f + 144*c^2*d^5*e*f - 720*a*b*d^2*e^4*f + 480*
a*b*d^3*e^3*g + 480*a*c*d^3*e^3*f - 360*a*c*d^4*e^2*g - 360*b*c*d^4*e^2*f +
 1044*a^2*d*e^5*f*m + 24*c^2*d^5*e*f*m + 580*a^2*d*e^5*f*m^2 + 155*a^2*d*e^
5*f*m^3 + 20*a^2*d*e^5*f*m^4 + a^2*d*e^5*f*m^5 - 342*a^2*d^2*e^4*g*m + 148*
b^2*d^3*e^3*f*m - 66*b^2*d^4*e^2*g*m + 288*b*c*d^5*e*g - 119*a^2*d^2*e^4*g*
m^2 + 30*b^2*d^3*e^3*f*m^2 - 18*a^2*d^2*e^4*g*m^3 + 2*b^2*d^3*e^3*f*m^3 - a
```

^2*d^2*e^4*g*m^4 - 6*b^2*d^4*e^2*g*m^2 + 48*b*c*d^5*e*g*m - 684*a*b*d^2*e^4
*f*m + 296*a*b*d^3*e^3*g*m + 296*a*c*d^3*e^3*f*m - 132*a*c*d^4*e^2*g*m - 13
2*b*c*d^4*e^2*f*m - 238*a*b*d^2*e^4*f*m^2 - 36*a*b*d^2*e^4*f*m^3 - 2*a*b*d^
2*e^4*f*m^4 + 60*a*b*d^3*e^3*g*m^2 + 60*a*c*d^3*e^3*f*m^2 + 4*a*b*d^3*e^3*g
*m^3 + 4*a*c*d^3*e^3*f*m^3 - 12*a*c*d^4*e^2*g*m^2 - 12*b*c*d^4*e^2*f*m^2))/
(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x*(d
+ e*x)^m*(720*a^2*e^6*f + 580*a^2*e^6*f*m^2 + 155*a^2*e^6*f*m^3 + 20*a^2*e^
6*f*m^4 + a^2*e^6*f*m^5 + 1044*a^2*e^6*f*m + 360*a^2*d*e^5*g*m + 120*c^2*d^
5*e*g*m - 240*b^2*d^2*e^4*f*m + 342*a^2*d*e^5*g*m^2 + 119*a^2*d*e^5*g*m^3 +
18*a^2*d*e^5*g*m^4 + a^2*d*e^5*g*m^5 + 180*b^2*d^3*e^3*g*m - 144*c^2*d^4*e
^2*f*m - 148*b^2*d^2*e^4*f*m^2 - 30*b^2*d^2*e^4*f*m^3 - 2*b^2*d^2*e^4*f*m^4
+ 66*b^2*d^3*e^3*g*m^2 - 24*c^2*d^4*e^2*f*m^2 + 6*b^2*d^3*e^3*g*m^3 + 720*
a*b*d*e^5*f*m + 684*a*b*d*e^5*f*m^2 + 238*a*b*d*e^5*f*m^3 + 36*a*b*d*e^5*f*
m^4 + 2*a*b*d*e^5*f*m^5 - 480*a*b*d^2*e^4*g*m - 480*a*c*d^2*e^4*f*m + 360*a
*c*d^3*e^3*g*m + 360*b*c*d^3*e^3*f*m - 288*b*c*d^4*e^2*g*m - 296*a*b*d^2*e^
4*g*m^2 - 296*a*c*d^2*e^4*f*m^2 - 60*a*b*d^2*e^4*g*m^3 - 60*a*c*d^2*e^4*f*m
^3 - 4*a*b*d^2*e^4*g*m^4 - 4*a*c*d^2*e^4*f*m^4 + 132*a*c*d^3*e^3*g*m^2 + 13
2*b*c*d^3*e^3*f*m^2 + 12*a*c*d^3*e^3*g*m^3 + 12*b*c*d^3*e^3*f*m^3 - 48*b*c*
d^4*e^2*g*m^2))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6
+ 720)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(120*b^2*e^3*f + 15*b^2*e^3*f*m^
2 + b^2*e^3*f*m^3 + 240*a*b*e^3*g + 240*a*c*e^3*f + 74*b^2*e^3*f*m + 20*c^2
*d^3*g*m + 30*a*b*e^3*g*m^2 + 30*a*c*e^3*f*m^2 + 2*a*b*e^3*g*m^3 + 2*a*c*e^
3*f*m^3 + 30*b^2*d*e^2*g*m - 24*c^2*d^2*e*f*m + 11*b^2*d*e^2*g*m^2 - 4*c^2*
d^2*e*f*m^2 + b^2*d*e^2*g*m^3 + 148*a*b*e^3*g*m + 148*a*c*e^3*f*m + 60*a*c*
d*e^2*g*m + 60*b*c*d*e^2*f*m - 48*b*c*d^2*e*g*m + 22*a*c*d*e^2*g*m^2 + 22*b
*c*d*e^2*f*m^2 + 2*a*c*d*e^2*g*m^3 + 2*b*c*d*e^2*f*m^3 - 8*b*c*d^2*e*g*m^2)
)/(e^3*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (x^4
*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(30*b^2*e^2*g + b^2*e^2*g*m^2 + 60*a*
c*e^2*g + 60*b*c*e^2*f + 11*b^2*e^2*g*m - 5*c^2*d^2*g*m + 2*a*c*e^2*g*m^2 +
2*b*c*e^2*f*m^2 + c^2*d*e*f*m^2 + 22*a*c*e^2*g*m + 22*b*c*e^2*f*m + 6*c^2*
d*e*f*m + 2*b*c*d*e*g*m^2 + 12*b*c*d*e*g*m))/(e^2*(1764*m + 1624*m^2 + 735*
m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) + (c^2*g*x^6*(d + e*x)^m*(274*m + 225*
m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(1764*m + 1624*m^2 + 735*m^3 + 175*m^4
+ 21*m^5 + m^6 + 720) + (x^2*(m + 1)*(d + e*x)^m*(360*a^2*e^4*g + 119*a^2*e
^4*g*m^2 + 18*a^2*e^4*g*m^3 + a^2*e^4*g*m^4 + 720*a*b*e^4*f + 342*a^2*e^4*g
*m - 60*c^2*d^4*g*m + 238*a*b*e^4*f*m^2 + 36*a*b*e^4*f*m^3 + 2*a*b*e^4*f*m^
4 + 120*b^2*d*e^3*f*m + 72*c^2*d^3*e*f*m + 74*b^2*d*e^3*f*m^2 + 15*b^2*d*e^
3*f*m^3 + b^2*d*e^3*f*m^4 - 90*b^2*d^2*e^2*g*m + 12*c^2*d^3*e*f*m^2 + 684*a
*b*e^4*f*m - 33*b^2*d^2*e^2*g*m^2 - 3*b^2*d^2*e^2*g*m^3 + 240*a*b*d*e^3*g*m
+ 240*a*c*d*e^3*f*m + 144*b*c*d^3*e*g*m + 148*a*b*d*e^3*g*m^2 + 148*a*c*d*
e^3*f*m^2 + 30*a*b*d*e^3*g*m^3 + 30*a*c*d*e^3*f*m^3 + 2*a*b*d*e^3*g*m^4 + 2
*a*c*d*e^3*f*m^4 - 180*a*c*d^2*e^2*g*m - 180*b*c*d^2*e^2*f*m + 24*b*c*d^3*e
*g*m^2 - 66*a*c*d^2*e^2*g*m^2 - 66*b*c*d^2*e^2*f*m^2 - 6*a*c*d^2*e^2*g*m^3
- 6*b*c*d^2*e^2*f*m^3))/(e^4*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^
5 + m^6 + 720)) + (c*x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(1

```
2*b*e*g + 6*c*e*f + 2*b*e*g*m + c*d*g*m + c*e*f*m))/(e*(1764*m + 1624*m^2 +
 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))
```

3.927 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{f+gx}\,dx$

Optimal. Leaf size=287

$$\frac{(beg-c(ef+dg))\left(c(e^2f^2+d^2g^2)+eg(2aeg-b(ef+dg))\right)(d+ex)^{1+m}}{e^4g^4(1+m)}+\frac{(b^2e^2g^2+c^2(e^2f^2+2defg+3d^2\ldots}{\ldots}$$

```
[Out] (b*e*g-c*(d*g+e*f))*(c*(d^2*g^2+e^2*f^2)+e*g*(2*a*e*g-b*(d*g+e*f)))*(e*x+d)
^(1+m)/e^4/g^4/(1+m)+(b^2*e^2*g^2+c^2*(3*d^2*g^2+2*d*e*f*g+e^2*f^2)+2*c*e*g
*(a*e*g-b*(2*d*g+e*f)))*(e*x+d)^(2+m)/e^4/g^3/(2+m)-c*(-2*b*e*g+3*c*d*g+c*e
*f)*(e*x+d)^(3+m)/e^4/g^2/(3+m)+c^2*(e*x+d)^(4+m)/e^4/g/(4+m)+(a*g^2-b*f*g+
c*f^2)^2*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/g^4/
(-d*g+e*f)/(1+m)
```

Rubi [A]

time = 0.55, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {965, 1634, 70}

$$\frac{(d+ex)^{m+2}\left(2ceg(aeg-b(2dg+ef))+b^2e^2g^2+c^2(3d^2g^2+2defg+e^2f^2)\right)}{e^4g^3(m+2)}+\frac{(d+ex)^{m+1}(beg-c(dg+ef))\left(eg(2aeg-b(dg+ef))+c(d^2g^2+e^2f^2)\right)}{e^4g^4(m+1)}+\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)}{g^4(m+1)(ef-dg)}-\frac{c(d+ex)^{m+3}(-2beg+3cdg+cef)}{e^4g^2(m+3)}+\frac{c^2(d+ex)^{m+4}}{e^4g(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

```
[Out] ((b*e*g - c*(e*f + d*g))*(c*(e^2*f^2 + d^2*g^2) + e*g*(2*a*e*g - b*(e*f + d
*g)))*(d + e*x)^(1 + m))/(e^4*g^4*(1 + m)) + ((b^2*e^2*g^2 + c^2*(e^2*f^2 +
 2*d*e*f*g + 3*d^2*g^2) + 2*c*e*g*(a*e*g - b*(e*f + 2*d*g)))*(d + e*x)^(2 +
 m))/(e^4*g^3*(2 + m)) - (c*(c*e*f + 3*c*d*g - 2*b*e*g)*(d + e*x)^(3 + m))/
(e^4*g^2*(3 + m)) + (c^2*(d + e*x)^(4 + m))/(e^4*g*(4 + m)) + ((c*f^2 - b*f
*g + a*g^2)^2*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d
+ e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
```

```
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{f+gx}\,dx = \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{\int \frac{(d+ex)^m\left(-e\left(c^2d^3f-a^2e^3g\right)(4+m)+e\left(2abe^3g-c^2d^2(3ef+dg)\right)(4+m)x}{}}{e^4g}$$

$$= \frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} + \frac{\int \left(\frac{e(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(4+m)(d+ex}{g^3}\right.}{}$$

$$= \frac{(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(d+ex)^{1+}}{e^4g^4(1+m)}$$

$$= \frac{(beg-c(ef+dg))\left(c\left(e^2f^2+d^2g^2\right)+eg(2aeg-b(ef+dg))\right)(d+ex)^{1+}}{e^4g^4(1+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.58, size = 254, normalized size = 0.89

$$\frac{(d+ex)^m\left(30abgmx^2\left(1+\frac{ex}{d}\right)^{-m}F_1\left(2;-m,1;3;-\frac{ex}{d},-\frac{gx}{f}\right)+10\left(b^2+2ac\right)gmx^3\left(1+\frac{ex}{d}\right)^{-m}F_1\left(3;-m,1;4;-\frac{ex}{d},-\frac{gx}{f}\right)+15bcgmx^4\left(1+\frac{ex}{d}\right)^{-m}F_1\left(4;-m,1;5;-\frac{ex}{d},-\frac{gx}{f}\right)+6c^2gmx^5\left(1+\frac{ex}{d}\right)^{-m}F_1\left(5;-m,1;6;-\frac{ex}{d},-\frac{gx}{f}\right)+30a^2f\left(\frac{g(d+ex)}{e(f+gx)}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{ef-dg}{ef+egx}\right)\right)}{30fgm}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x]
```

```
[Out] ((d + e*x)^m*((30*a*b*g*m*x^2*AppellF1[2, -m, 1, 3, -((e*x)/d), -((g*x)/f)]
)/(1 + (e*x)/d)^m + (10*(b^2 + 2*a*c)*g*m*x^3*AppellF1[3, -m, 1, 4, -((e*x)
/d), -((g*x)/f)])/(1 + (e*x)/d)^m + (15*b*c*g*m*x^4*AppellF1[4, -m, 1, 5, -
((e*x)/d), -((g*x)/f)])/(1 + (e*x)/d)^m + (6*c^2*g*m*x^5*AppellF1[5, -m, 1,
 6, -((e*x)/d), -((g*x)/f)])/(1 + (e*x)/d)^m + (30*a^2*f*Hypergeometric2F1[
-m, -m, 1 - m, (e*f - d*g)/(e*f + e*g*x)])/((g*(d + e*x))/(e*(f + g*x)))^m)
)/(30*f*g*m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m (c\,x^2+bx+a)^2}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^2*(x*e + d)^m/(g*x + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="fricas")
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(x*e + d
)^m/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f),x)
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)^2*(x*e + d)^m/(g*x + f), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^2}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x), x)
```

3.928 $\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} \, dx$

Optimal. Leaf size=298

$$\frac{(b^2e^2g^2 + c^2(3e^2f^2 + 2defg + d^2g^2) + 2ceg(aeg - b(2ef + dg)))\,(d + ex)^{1+m}}{e^3g^4(1 + m)} - \frac{2c(cef + cdg - beg)(d + ex)^2}{e^3g^3(2 + m)}$$

```
[Out] (b^2*e^2*g^2+c^2*(d^2*g^2+2*d*e*f*g+3*e^2*f^2)+2*c*e*g*(a*e*g-b*(d*g+2*e*f)
))*(e*x+d)^(1+m)/e^3/g^4/(1+m)-2*c*(-b*e*g+c*d*g+c*e*f)*(e*x+d)^(2+m)/e^3/g
^3/(2+m)+c^2*(e*x+d)^(3+m)/e^3/g^2/(3+m)+(a*g^2-b*f*g+c*f^2)^2*(e*x+d)^(1+m
)/g^4/(-d*g+e*f)/(g*x+f)+(a*g^2-b*f*g+c*f^2)*(c*f*(4*d*g-e*f*(4+m))-g*(a*e*
g*m+b*(2*d*g-e*f*(2+m))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)
/(-d*g+e*f))/g^4/(-d*g+e*f)^2/(1+m)
```

Rubi [A]
time = 0.73, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {963, 1634, 70}

$$\frac{(d+ex)^{m+1}\left(2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2(d^2g^2+2defg+3e^2f^2)\right)}{e^3g^4(m+1)} - \frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(g(aegm+2bdg-bef(m+2))-cf(4dg-ef(m+4))\right)}{g^4(m+1)(ef-dg)^2} + \frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{g^4(f+gx)(ef-dg)} - \frac{2c(d+ex)^{m+2}(-beg+cdg+cef)}{e^3g^3(m+2)} + \frac{c^2(d+ex)^{m+3}}{e^3g^2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

```
[Out] ((b^2*e^2*g^2 + c^2*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*c*e*g*(a*e*g - b*
(2*e*f + d*g)))*(d + e*x)^(1 + m))/(e^3*g^4*(1 + m)) - (2*c*(c*e*f + c*d*g
- b*e*g)*(d + e*x)^(2 + m))/(e^3*g^3*(2 + m)) + (c^2*(d + e*x)^(3 + m))/(e^
3*g^2*(3 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^(1 + m))/(g^4*(e*f -
d*g)*(f + g*x)) - ((c*f^2 - b*f*g + a*g^2)*(g*(2*b*d*g + a*e*g*m - b*e*f*(2
 + m)) - c*f*(4*d*g - e*f*(4 + m)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)^2*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
```

```
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2}\,dx = \frac{(cf^2-bfg+ag^2)^2 (d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \frac{\int \frac{(d+ex)^m \left(\frac{c^2f^3(dg-ef(1+m))-2cfg(bf-ag)(dg-e}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{(cf^2-bfg+ag^2)^2 (d+ex)^{1+m}}{g^4(ef-dg)(f+gx)} + \frac{\int \left(\frac{(ef-dg)\left(b^2e^2g^2+c^2\left(3e^2f^2+2defg+d^2g^2\right)+2\cdots\right.}{e^2g^4}\right.}{\cdots}$$

$$= \frac{(b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg)))\,(d+ex\cdots}{e^3g^4(1+m)}$$

$$= \frac{(b^2e^2g^2+c^2(3e^2f^2+2defg+d^2g^2)+2ceg(aeg-b(2ef+dg)))\,(d+ex\cdots}{e^3g^4(1+m)}$$

Mathematica [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

```
[Out] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m \left(c\,x^2+bx+a\right)^2}{(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(x*e + d)^m/(g*x + f)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="fricas")
```

```
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(x*e + d
)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^2*(x*e + d)^m/(g*x + f)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^2}{(f+g\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2, x)
```

3.929 $\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx$

Optimal. Leaf size=461

$$-\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)}+\frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)}+\frac{\left(cf^2-bfg+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2}+\frac{\left(cf^2-bfg+ag^2\right)(c$$

```
[Out] -c*(-2*b*e*g+c*d*g+3*c*e*f)*(e*x+d)^(1+m)/e^2/g^4/(1+m)+c^2*(e*x+d)^(2+m)/e
^2/g^3/(2+m)+1/2*(a*g^2-b*f*g+c*f^2)^2*(e*x+d)^(1+m)/g^4/(-d*g+e*f)/(g*x+f)
^2+1/2*(a*g^2-b*f*g+c*f^2)*(c*f*(8*d*g-e*f*(7+m))+g*(a*e*g*(1-m)-b*(4*d*g-e
*f*(3+m))))*(e*x+d)^(1+m)/g^4/(-d*g+e*f)^2/(g*x+f)+1/2*(c^2*f^2*(12*d^2*g^2
-8*d*e*f*g*(3+m)+e^2*f^2*(m^2+7*m+12))-g^2*(a^2*e^2*g^2*(1-m)*m-2*a*b*e*g*m
*(2*d*g-e*f*(1+m))-b^2*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2)))+2*c
*g*(a*g*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-b*f*(6*d^2*g^2-6*d*
e*f*g*(2+m)+e^2*f^2*(m^2+5*m+6))))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-
g*(e*x+d)/(-d*g+e*f))/g^4/(-d*g+e*f)^3/(1+m)
```

Rubi [A]

time = 0.94, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {963, 1635, 965, 81, 70}

$$\frac{(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(-g^2\left(a^2e^2g^2(1-m)m-2abegm(2dg-ef(m+1))-\left(b^2\left(2d^2g^2-4defg(m+1)+e^2f^2\left(m^2+3m+2\right)\right)\right)\right)+2cg\left(ag\left(2d^2g^2-4defg(m+1)+e^2f^2\left(m^2+3m+2\right)\right)-bf\left(6d^2g^2-6defg(m+2)+e^2f^2\left(m^2+5m+6\right)\right)\right)+c^2f^2\left(12d^2g^2-8defg(m+3)+e^2f^2\left(m^2+7m+12\right)\right)\right)}{2g^4(m+1)(ef-dg)^3}-\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)\left(g(-aeg(1-m)+4bdg-bef(m+3))-cf(8dg-ef(m+7))\right)}{2g^4(f+gx)(ef-dg)^2}+\frac{(d+ex)^{m+1}\left(ag^2-bfg+cf^2\right)^2}{2g^4(f+gx)^2(ef-dg)}-\frac{c(d+ex)^{m+1}(-2beg+cdg+3cef)}{e^2g^4(m+1)}+\frac{c^2(d+ex)^{m+2}}{e^2g^3(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

```
[Out] -((c*(3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)^(1 + m))/(e^2*g^4*(1 + m))) + (c
^2*(d + e*x)^(2 + m))/(e^2*g^3*(2 + m)) + ((c*f^2 - b*f*g + a*g^2)^2*(d + e
*x)^(1 + m))/(2*g^4*(e*f - d*g)*(f + g*x)^2) - ((c*f^2 - b*f*g + a*g^2)*(g*
(4*b*d*g - a*e*g*(1 - m) - b*e*f*(3 + m)) - c*f*(8*d*g - e*f*(7 + m)))*(d +
 e*x)^(1 + m))/(2*g^4*(e*f - d*g)^2*(f + g*x)) + ((c^2*f^2*(12*d^2*g^2 - 8*
d*e*f*g*(3 + m) + e^2*f^2*(12 + 7*m + m^2)) - g^2*(a^2*e^2*g^2*(1 - m)*m -
2*a*b*e*g*m*(2*d*g - e*f*(1 + m)) - b^2*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^
2*f^2*(2 + 3*m + m^2))) + 2*c*g*(a*g*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f
^2*(2 + 3*m + m^2)) - b*f*(6*d^2*g^2 - 6*d*e*f*g*(2 + m) + e^2*f^2*(6 + 5*m
 + m^2))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e
*x))/(e*f - d*g))])/(2*g^4*(e*f - d*g)^3*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx = \frac{\left(cf^2-bfg+ag^2\right)^2 (d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} + \frac{\int \frac{(d+ex)^m \left(\frac{c^2 f^3 (2dg-ef(1+m))-2cfg(bf-ag)(2dg-e\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(cf^2-bfg+ag^2\right)^2 (d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{\left(cf^2-bfg+ag^2\right)\left(g(4bdg-aeg(1-\cdots\right.}{2g^{\cdots}}$$

$$= \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^2\right)^2 (d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} - \frac{\left(cf^2-bfg+ag^2\right)(g\cdots}{\cdots}$$

$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^{2}\cdots\right.}{2g^4(ef-dg\cdots}$$

$$= -\frac{c(3cef+cdg-2beg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2-bfg+ag^{2}\cdots\right.}{2g^4(ef-dg\cdots}$$

Mathematica [A]
time = 2.12, size = 281, normalized size = 0.61

$$\frac{e\left(cf^2+g(-bf+ag)\right)(d+ex)^{1+m}\left(-2(2cf-bg)(ef-dg)\,{}_2F_1\left(2,1+m;2+m;\frac{g(d+ex)}{-ef+dg}\right)+e\left(cf^2+g(-bf+ag)\right)\,{}_2F_1\left(3,1+m;2+m;\frac{g(d+ex)}{-ef+dg}\right)\right)}{g^4(ef-dg)^3(1+m)} + \frac{(d+ex)^m\left(\frac{cg(d+ex)(2beg(2+m)+c(-dg-3ef(2+m)+eg(1+m)x))}{e^2(1+m)(2+m)}+\frac{\left(6c^2f^2+b^2g^2+2cg(-3bf+ag)\right)\left(\frac{g(d+ex)}{e(f+gx)}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{ef-dg}{ef+egx}\right)}{m}\right)}{g^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]
```

```
[Out] (e*(c*f^2 + g*(-(b*f) + a*g))*(d + e*x)^(1 + m)*(-2*(2*c*f - b*g)*(e*f - d*
g)*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(c*
f^2 + g*(-(b*f) + a*g))*Hypergeometric2F1[3, 1 + m, 2 + m, (g*(d + e*x))/(-
(e*f) + d*g)]))/(g^4*(e*f - d*g)^3*(1 + m)) + ((d + e*x)^m*((c*g*(d + e*x)*
(2*b*e*g*(2 + m) + c*(-(d*g) - 3*e*f*(2 + m) + e*g*(1 + m)*x)))/(e^2*(1 + m
)*(2 + m)) + ((6*c^2*f^2 + b^2*g^2 + 2*c*g*(-3*b*f + a*g))*Hypergeometric2F
1[-m, -m, 1 - m, (e*f - d*g)/(e*f + e*g*x)])/(m*((g*(d + e*x))/(e*(f + g*x)
))^m)))/g^5
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m \left(c\,x^2+bx+a\right)^2}{(gx+f)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^2*(x*e + d)^m/(g*x + f)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="fricas")
[Out] integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(x*e + d
)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**3,x)
[Out] Integral((d + e*x)**m*(a + b*x + c*x**2)**2/(f + g*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)^2*(x*e + d)^m/(g*x + f)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^2}{(f+g\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3, x)
```

3.930 $\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=183

$$\frac{3687(1+4x)^{1+m}}{64(1+m)}+\frac{207(1+4x)^{2+m}}{32(2+m)}+\frac{27(1+4x)^{3+m}}{64(3+m)}-\frac{3\left(5499-1631\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+\right.}{26\left(13-2\sqrt{13}\right)(1+m)}$$

```
[Out] 3687/64*(1+4*x)^(1+m)/(1+m)+207/32*(1+4*x)^(2+m)/(2+m)+27/64*(1+4*x)^(3+m)/
(3+m)-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2))
)*(5499-1631*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1+m)*hypergeom([
1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(5499+1631*13^(1/2))/(1+m)/(13+2*1
3^(1/2))
```

Rubi [A]
time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1642, 70}

$$-\frac{3\left(5499-1631\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(5499+1631\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}+\frac{3687(4x+1)^{m+1}}{64(m+1)}+\frac{207(4x+1)^{m+2}}{32(m+2)}+\frac{27(4x+1)^{m+3}}{64(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3687*(1 + 4*x)^(1 + m))/(64*(1 + m)) + (207*(1 + 4*x)^(2 + m))/(32*(2 + m)
) + (27*(1 + 4*x)^(3 + m))/(64*(3 + m)) - (3*(5499 - 1631*Sqrt[13])*(1 + 4*
x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13
])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(5499 + 1631*Sqrt[13])*(1 + 4*x)^(
1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])
/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2}\,dx = \int \left(\frac{3687}{16}(1+4x)^m + \frac{207}{8}(1+4x)^{1+m} + \frac{27}{16}(1+4x)^{2+m} + \frac{\left(1269+\frac{4893}{\sqrt{13}}\right)(1}{-5-\sqrt{13}+} \right.$$

$$= \frac{3687(1+4x)^{1+m}}{64(1+m)} + \frac{207(1+4x)^{2+m}}{32(2+m)} + \frac{27(1+4x)^{3+m}}{64(3+m)} + \frac{1}{13}\left(3\left(5499-1631\sqrt{1}\right.\right.$$

$$= \frac{3687(1+4x)^{1+m}}{64(1+m)} + \frac{207(1+4x)^{2+m}}{32(2+m)} + \frac{27(1+4x)^{3+m}}{64(3+m)} - \frac{3\left(5499-1631\sqrt{13}\right)}{2}$$

Mathematica [A]
time = 0.29, size = 151, normalized size = 0.83

$$\frac{3}{832}(1+4x)^{1+m}\left(\frac{15977}{1+m}+\frac{1794(1+4x)}{2+m}+\frac{117(1+4x)^2}{3+m}-\frac{32\left(-5499+1631\sqrt{13}\right)\,_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)}-\frac{32\left(5499+1631\sqrt{13}\right)\,_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*(15977/(1 + m) + (1794*(1 + 4*x))/(2 + m) + (117*(1 +
4*x)^2)/(3 + m) - (32*(-5499 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (32*(54
99 + 1631*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*S
qrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/832
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(2+3x)^4\,(1+4x)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
```

```
[Out] integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(3*x^2 - 5*x
+ 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^4 (4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1),x)
```

```
[Out] Integral((3*x + 2)**4*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)^4\,(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
```

```
[Out] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.931 $\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=165

$$\frac{123(1+4x)^{1+m}}{16(1+m)}+\frac{9(1+4x)^{2+m}}{16(2+m)}-\frac{3\left(416-135\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(1+m)}-\frac{3\left(416\right.}{}$$

```
[Out] 123/16*(1+4*x)^(1+m)/(1+m)+9/16*(1+4*x)^(2+m)/(2+m)-3/13*(1+4*x)^(1+m)*hype
rgeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(416-135*13^(1/2))/(1+m)/(1
3-2*13^(1/2))-3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*1
3^(1/2)))*(416+135*13^(1/2))/(1+m)/(13+2*13^(1/2))
```

Rubi [A]

time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1642, 70}

$$-\frac{3\left(416-135\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(416+135\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\left(13+2\sqrt{13}\right)(m+1)}+\frac{123(4x+1)^{m+1}}{16(m+1)}+\frac{9(4x+1)^{m+2}}{16(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (123*(1 + 4*x)^(1 + m))/(16*(1 + m)) + (9*(1 + 4*x)^(2 + m))/(16*(2 + m)) -
 (3*(416 - 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(13*(13 - 2*Sqrt[13])*(1 + m)) - (3*(4
16 + 135*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2}\,dx = \int \left(\frac{123}{4}(1+4x)^m + \frac{9}{4}(1+4x)^{1+m} + \frac{\left(192+\frac{810}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(192-\frac{\cdots}{\sqrt{\cdots}}\right.}{-5+\cdots}\right.$$

$$= \frac{123(1+4x)^{1+m}}{16(1+m)} + \frac{9(1+4x)^{2+m}}{16(2+m)} + \frac{1}{13}\left(6\left(416-135\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+\cdots}$$

$$= \frac{123(1+4x)^{1+m}}{16(1+m)} + \frac{9(1+4x)^{2+m}}{16(2+m)} - \frac{3\left(416-135\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1-\cdots\right.}{13\left(13-2\sqrt{13}\right)(1+\cdots}$$

Mathematica [A]
time = 0.14, size = 117, normalized size = 0.71

$$\frac{(1+4x)^{1+m}\left(117(85+12x+4m(11+3x))+16\left(-146+71\sqrt{13}\right)(2+m)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)-16\left(146+71\sqrt{13}\right)(2+m)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)\right)}{624\,(2+3m+m^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(117*(85 + 12*x + 4*m*(11 + 3*x)) + 16*(-146 + 71*Sqrt[1
3])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])
] - 16*(146 + 71*Sqrt[13])*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 +
12*x)/(13 + 2*Sqrt[13])]))/(624*(2 + 3*m + m^2))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(2+3x)^3\,(1+4x)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^3 (4x+1)^m}{3x^2 - 5x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1),x)
[Out] Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)^3\,(4\,x+1)^m}{3\,x^2-5\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
[Out] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.932 $\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=147

$$\frac{3(1+4x)^{1+m}}{4(1+m)}-\frac{3\left(117-47\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)}-\frac{3\left(117+47\sqrt{13}\right)(1+4}{26\left(1\right.}$$

```
[Out] 3/4*(1+4*x)^(1+m)/(1+m)-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*
x)/(13-2*13^(1/2)))*(117-47*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1
+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117+47*13^(1/2))/(
1+m)/(13+2*13^(1/2))
```

Rubi [A]
time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1642, 70}

$$-\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}+\frac{3(4x+1)^{m+1}}{4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m))/(4*(1 + m)) - (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m
)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*
(13 - 2*Sqrt[13])*(1 + m)) - (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2
*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2}\,dx = \int \left(3(1+4x)^m + \frac{\left(27+\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(27-\frac{141}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x}\right) dx$$

$$= \frac{3(1+4x)^{1+m}}{4(1+m)} + \frac{1}{13}\left(3\left(117-47\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx + \frac{1}{13}\left(3\left(117+\right.\right.$$

$$= \frac{3(1+4x)^{1+m}}{4(1+m)} - \frac{3\left(117-47\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right.}{26\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A]
time = 0.11, size = 91, normalized size = 0.62

$$\frac{(1+4x)^{1+m}\left(117+\left(-46+58\sqrt{13}\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)-2\left(23+29\sqrt{13}\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)\right)}{156(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
[Out] ((1 + 4*x)^(1 + m)*(117 + (-46 + 58*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] - 2*(23 + 29*Sqrt[13])*Hypergeometric2F
1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(156*(1 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(2+3x)^2\,(1+4x)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x)
[Out] int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^2 (4x+1)^m}{3x^2-5x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1),x)
[Out] Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)^2\,(4\,x+1)^m}{3\,x^2-5\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
[Out] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.933 $\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx$

Optimal. Leaf size=129

$$-\frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)}-\frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m\right.}{26\left(13+2\sqrt{13}\right)(1+m}$$

```
[Out] -3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13
-9*13^(1/2))/(1+m)/(13-2*13^(1/2))-3/26*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2
+m],3*(1+4*x)/(13+2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13+2*13^(1/2))
```

Rubi [A]
time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {844, 70}

$$-\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(m+1)}-\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] (-3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(26*(13 - 2*Sqrt[13])*(1 + m)) - (3*(13 +
 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4
*x))/(13 + 2*Sqrt[13])])/(26*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2}\,dx = \int \left(\frac{\left(3+\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(3-\frac{27}{\sqrt{13}}\right)(1+4x)^m}{-5+\sqrt{13}+6x} \right) dx$$
$$= \frac{1}{13}\left(3\left(13-9\sqrt{13}\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx + \frac{1}{13}\left(3\left(13+9\sqrt{13}\right)\right)\int \frac{(1+}{-5-}$$
$$= -\frac{3\left(13-9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{26\left(13-2\sqrt{13}\right)(1+m)} - \frac{3\left(13+9\sqrt{1\right.}}{}$$

Mathematica [A]
time = 0.10, size = 89, normalized size = 0.69

$$\frac{(1+4x)^{1+m}\left(\left(5+7\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)+\left(5-7\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)\right)}{78(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((5 + 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
+ 12*x)/(13 - 2*Sqrt[13])] + (5 - 7*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2
 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(78*(1 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(2+3x)(1+4x)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x)
```

```
[Out] int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1),x)
[Out] Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)\,{(4\,x+1)}^m}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1),x)
[Out] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1), x)
```

3.934 $\int \frac{(1+4x)^m}{1-5x+3x^2} \, dx$

Optimal. Leaf size=117

$$\frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}-\frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(1+m)}$$

```
[Out] 3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))/(1+m
)/(13-2*13^(1/2))*13^(1/2)-3/13*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1
+4*x)/(13+2*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))
```

Rubi [A]
time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {725, 70}

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}-\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 -
 2*Sqrt[13])])/(Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) - (3*(1 + 4*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(Sqrt[
13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 725

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{(1+4x)^m}{1-5x+3x^2}\,dx = \int \left(-\frac{6(1+4x)^m}{\sqrt{13}\left(5+\sqrt{13}-6x\right)} - \frac{6(1+4x)^m}{\sqrt{13}\left(-5+\sqrt{13}+6x\right)}\right) dx$$

$$= -\frac{6\int \frac{(1+4x)^m}{5+\sqrt{13}-6x}\,dx}{\sqrt{13}} - \frac{6\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{\sqrt{13}}$$

$$= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)} - \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\right.}{\sqrt{13}\left(13+2\sqrt{13}\right)(1+}$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 0.80

$$\frac{(1+4x)^{1+m}\left(\left(13+2\sqrt{13}\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)+\left(-13+2\sqrt{13}\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)\right)}{39\sqrt{13}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2),x]
[Out] ((1 + 4*x)^(1 + m)*((13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])] + (-13 + 2*Sqrt[13])*Hypergeometric2F1[1, 1 + m
, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])]))/(39*Sqrt[13]*(1 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(1+4x)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+4*x)^m/(3*x^2-5*x+1),x)
[Out] int((1+4*x)^m/(3*x^2-5*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{3x^2-5x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(3*x**2-5*x+1),x)
[Out] Integral((4*x + 1)**m/(3*x**2 - 5*x + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4\,x+1)^m}{3\,x^2-5\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/(3*x^2 - 5*x + 1),x)
[Out] int((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

3.935 $\int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)}\,dx$

Optimal. Leaf size=164

$$\frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)}+\frac{3\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(1+m)}$$

```
[Out] 3/85*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+3/442*(1+4*x
)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13-9*13^(1/2))
/(1+m)/(13+2*13^(1/2))+3/442*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*
x)/(13-2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13-2*13^(1/2))
```

Rubi [A]
time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {974, 70, 844}

$$\frac{3(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{85(m+1)}+\frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)}+\frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{442\left(13+2\sqrt{13}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

```
[Out] (3*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/
(85*(1 + m)) + (3*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(442*(13 - 2*Sqrt[13])*(1 +
 m)) + (3*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(442*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)}\,dx &= \int \left(\frac{3(1+4x)^m}{17(2+3x)}+\frac{(7-3x)(1+4x)^m}{17\left(1-5x+3x^2\right)}\right)\,dx\\
&= \frac{1}{17}\int \frac{(7-3x)(1+4x)^m}{1-5x+3x^2}\,dx+\frac{3}{17}\int \frac{(1+4x)^m}{2+3x}\,dx\\
&= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)}+\frac{1}{17}\int \left(\frac{\left(-3+\frac{27}{\sqrt{13}}\right.}{-5-\sqrt{13}}\right.\\
&= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)}-\frac{1}{221}\left(3\left(13-9\sqrt{13}\right)\right)\\
&= \frac{3(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{85(1+m)}+\frac{3\left(13+9\sqrt{13}\right)(1+4}{442\,(}
\end{aligned}$$

Mathematica [A]
time = 0.57, size = 233, normalized size = 1.42

$$\frac{(1+4x)^m\left(\frac{78(1+4x)\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1+m}+\frac{5\;3^{-m}\left(-\frac{1+4x}{5+\sqrt{13}-6x}\right)^{-m}\left(\frac{1+4x}{-5+\sqrt{13}+6x}\right)^{-m}\left(\left(-13+9\sqrt{13}\right)\left(\frac{2+8x}{-5+\sqrt{13}+6x}\right)^m{}_2F_1\left(-m,-m;1-m;\frac{13+2\sqrt{13}}{2\left(5+\sqrt{13}-6x\right)}\right)-\left(13+9\sqrt{13}\right)\left(-\frac{2+8x}{5+\sqrt{13}-6x}\right)^m{}_2F_1\left(-m,-m;1-m;\frac{-13+2\sqrt{13}}{2\left(-5+\sqrt{13}+6x\right)}\right)\right)}{m}\right)}{2210}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]
```

```
[Out] ((1 + 4*x)^m*((78*(1 + 4*x)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x
))/5])/(1 + m) + (5*((-13 + 9*Sqrt[13])*((2 + 8*x)/(-5 + Sqrt[13] + 6*x))^m
*Hypergeometric2F1[-m, -m, 1 - m, (13 + 2*Sqrt[13])/(2*(5 + Sqrt[13] - 6*x)
)] - (13 + 9*Sqrt[13])*(-((2 + 8*x)/(5 + Sqrt[13] - 6*x)))^m*Hypergeometric
2F1[-m, -m, 1 - m, (-13 + 2*Sqrt[13])/(2*(-5 + Sqrt[13] + 6*x))]))/(3^m*m*(
-((1 + 4*x)/(5 + Sqrt[13] - 6*x)))^m*((1 + 4*x)/(-5 + Sqrt[13] + 6*x))^m)))
/2210
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(1+4x)^m}{(2+3x)(3x^2-5x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x)
[Out] int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(9*x^3 - 9*x^2 - 7*x + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)(3x^2-5x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1),x)
[Out] Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x, algorithm="giac")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4\,x+1)^m}{(3\,x+2)\,\left(3\,x^2-5\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)),x)
[Out] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)), x)
```

3.936 $\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)}\,dx$

Optimal. Leaf size=199

$$\frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{3\left(117+47\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1\cdots}{13-2\cdots}\right.}{7514\left(13-2\sqrt{13}\right)(1+m)}$$

```
[Out] 27/1445*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+12/425*(1
+4*x)^(1+m)*hypergeom([2, 1+m],[2+m],-3/5-12/5*x)/(1+m)+3/7514*(1+4*x)^(1+m
)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(117-47*13^(1/2))/(1+
m)/(13+2*13^(1/2))+3/7514*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/
(13-2*13^(1/2)))*(117+47*13^(1/2))/(1+m)/(13-2*13^(1/2))
```

Rubi [A]
time = 0.14, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {974, 70, 844}

$$\frac{27(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}+\frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}+\frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)}+\frac{12(4x+1)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{425(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```

```
[Out] (27*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])
/(1445*(1 + m)) + (3*(117 + 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13]
)*(1 + m)) + (3*(117 - 47*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1
+ m)) + (12*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5])/(425*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)}\,dx = \int \left(\frac{3(1+4x)^m}{17(2+3x)^2}+\frac{27(1+4x)^m}{289(2+3x)}+\frac{(46-27x)(1+4x)^m}{289\left(1-5x+3x^2\right)}\right)dx$$

$$= \frac{1}{289}\int \frac{(46-27x)(1+4x)^m}{1-5x+3x^2}\,dx+\frac{27}{289}\int \frac{(1+4x)^m}{2+3x}\,dx+\frac{3}{17}\int \frac{(1+4x)^{\prime}}{(2+3x)^{\prime}}$$

$$= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{12(1+4x)^{1+m}\,{}_2F_1(2}{42}$$

$$= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{12(1+4x)^{1+m}\,{}_2F_1(2}{42}$$

$$= \frac{27(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{3\left(117+47\sqrt{13}\right)(1}{75}$$

Mathematica [A]
time = 0.18, size = 152, normalized size = 0.76

$$\frac{(1+4x)^{1+m}\left(10530\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)+25\left(211+65\sqrt{13}\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)+5275\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)-1625\sqrt{13}\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)+15912\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)\right)}{563550(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(10530*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))
/5] + 25*(211 + 65*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/
(13 - 2*Sqrt[13])] + 5275*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
 + 2*Sqrt[13])] - 1625*Sqrt[13]*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*
x)/(13 + 2*Sqrt[13])] + 15912*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(1 + 4
*x))/5]))/(563550*(1 + m))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(1+4x)^m}{(2+3x)^2\,(3x^2-5x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x)
[Out] int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(27*x^4 - 9*x^3 - 39*x^2 - 8*x + 4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)^2\cdot(3x^2-5x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1),x)
[Out] Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4\,x+1)^m}{(3\,x+2)^2\,(3\,x^2-5\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)),x)
```

```
[Out] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)), x)
```

3.937 $\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

Optimal. Leaf size=202

$$\frac{9(1+4x)^{1+m}}{4(1+m)}+\frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)}-\frac{\left(13689-\sqrt{13}\left(297+4474m-1570\sqrt{13}\,m\right)\right)(1+4x)^{1+m}\,{}_2F$$
$$169\left(13-2\sqrt{13}\right)(1+m)$$

```
[Out] 9/4*(1+4*x)^(1+m)/(1+m)+1/39*(844-2355*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)-1/169
*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13689-1
3^(1/2)*(297+4474*m-1570*m*13^(1/2)))/(1+m)/(13-2*13^(1/2))-1/169*(1+4*x)^(
1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13689+13^(1/2)*(2
97+4474*m+1570*m*13^(1/2)))/(1+m)/(13+2*13^(1/2))
```

Rubi [A]

time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1662, 1642, 70}

$$-\frac{\left(13689-\sqrt{13}\left(-1570\sqrt{13}\,m+4474m+297\right)\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{169\left(13-2\sqrt{13}\right)(m+1)}-\frac{\left(\sqrt{13}\left(1570\sqrt{13}\,m+4474m+297\right)+13689\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{169\left(13+2\sqrt{13}\right)(m+1)}+\frac{(844-2355x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}+\frac{9(4x+1)^{m+1}}{4(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] (9*(1 + 4*x)^(1 + m))/(4*(1 + m)) + ((844 - 2355*x)*(1 + 4*x)^(1 + m))/(39*
(1 - 5*x + 3*x^2)) - ((13689 - Sqrt[13]*(297 + 4474*m - 1570*Sqrt[13]*m))*(
1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*S
qrt[13])])/(169*(13 - 2*Sqrt[13])*(1 + m)) - ((13689 + Sqrt[13]*(297 + 4474
*m + 1570*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(169*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1662

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1)*((f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x)^4(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx &= \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)}-\frac{1}{507}\int \frac{(1+4x)^m\left(13(4617+3376m)-39(1521\right.}{1-5x+3x^2} \\ &= \frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)}-\frac{1}{507}\int \left(-4563(1+4x)^m+\frac{\left(-82134-122460\right.}{}\right. \\ &= \frac{9(1+4x)^{1+m}}{4(1+m)}+\frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)}-\frac{1}{507}\left(-82134-122460m+6\sqrt{}\right. \\ &= \frac{9(1+4x)^{1+m}}{4(1+m)}+\frac{(844-2355x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)}-\frac{\left(13689-\sqrt{13}\left(297+4474m\right.\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 251, normalized size = 1.24

$$\frac{(1+4x)^{1+m}\left(\frac{13689}{4+4m}+\frac{39(844-2355x)}{1-5x+3x^2}-\frac{1053\left(-117+128\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)}-\frac{1053\left(117+128\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}-\frac{-\left(\left(-14679\left(2+\sqrt{13}\right)+2\left(-5731+667\sqrt{13}\right)m\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)\right)+\left(-14679\left(-2+\sqrt{13}\right)+2\left(5731+667\sqrt{13}\right)m\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{1+m}\right)}{1521}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*(13689/(4 + 4*m) + (39*(844 - 2355*x))/(1 - 5*x + 3*x^2)
 - (1053*(-117 + 128*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x
```

```
)/(13 - 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (1053*(117 + 128*Sqrt[
13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13
 + 2*Sqrt[13])*(1 + m)) - (-((-14679*(2 + Sqrt[13]) + 2*(-5731 + 667*Sqrt[1
3])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])]) +
(-14679*(-2 + Sqrt[13]) + 2*(5731 + 667*Sqrt[13])*m)*Hypergeometric2F1[1, 1
 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(1 + m)))/1521
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(2+3x)^{4} (1+4x)^{m}}{(3x^{2}-5x+1)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(9*x^4 - 30*x
^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^{4} (4x+1)^{m}}{(3x^{2}-5x+1)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((3*x + 2)**4*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(3\,x+2)^4\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)^4*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.938 $\int \frac{(2+3x)^3(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=181

$$\frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)}-\frac{\left(1521+\sqrt{13}\left(1701-1168m+568\sqrt{13}\,m\right)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m\right.}{338\left(13-2\sqrt{13}\right)(1+m)}$$

```
[Out] 1/39*(209-426*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+1/338*(1+4*x)^(1+m)*hypergeom(
[1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(-1521-7384*m+(1701-1168*m)*13^(1
/2))/(1+m)/(13+2*13^(1/2))-1/338*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(
1+4*x)/(13-2*13^(1/2)))*(1521+13^(1/2)*(1701-1168*m+568*m*13^(1/2)))/(1+m)/
(13-2*13^(1/2))
```

Rubi [A]

time = 0.18, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1662, 844, 70}

$$-\frac{\left(\sqrt{13}\left(568\sqrt{13}\,m-1168m+1701\right)+1521\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\sqrt{13}\,(1701-1168m)-13(568m+117)\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{338\left(13+2\sqrt{13}\right)(m+1)}+\frac{(209-426x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((209 - 426*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((1521 + Sqrt[13
]*(1701 - 1168*m + 568*Sqrt[13]*m))*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(338*(13 - 2*Sqrt[13])*(1 +
 m)) + ((Sqrt[13]*(1701 - 1168*m) - 13*(117 + 568*m))*(1 + 4*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(338*(13
+ 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1662

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1)*((f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(2+3x)^3(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx = \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \frac{(1+4x)^m(13(1143+836m)-39(117+5}{1-5x+3x^2}$$

$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{507}\int \Bigg(\frac{\left(-39(117+568m)-3\sqrt{13}\,(-1701+1\right.}{-5-\sqrt{13}+6x}$$

$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{1}{169}\left(\sqrt{13}\,(1701-1168m)-13(117+568m)\right)\int$$

$$= \frac{(209-426x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)} - \frac{\left(\sqrt{13}\,(1701-1168m)+13(117+568m)\right)(1+4}{338\left(13-2\sqrt{13}\right)}$$

Mathematica [A]
time = 0.33, size = 252, normalized size = 1.39

$$\frac{(1+4x)^{1+m}\left(\frac{5434-11076x}{1-5x+3x^2}-\frac{351\left(-13+27\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)}-\frac{12\left(\sqrt{13}\,(1215-292m)+1846m\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(1+m)}-\frac{351\left(13+27\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}+\frac{12\left(\sqrt{13}\,(1215-292m)-1846m\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)}{1014}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((5434 - 11076*x)/(1 - 5*x + 3*x^2) - (351*(-13 + 27*Sqr
t[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((
```

```
-13 + 2*Sqrt[13])*(1 + m)) - (12*(Sqrt[13]*(1215 - 292*m) + 1846*m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 - 2*Sqrt[13
])*(1 + m)) - (351*(13 + 27*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (12*(Sqrt[13]*(1
215 - 292*m) - 1846*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 +
2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m))))/1014
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(2+3x\right)^{3} \left(1+4x\right)^{m}}{\left(3x^{2}-5x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
[Out] int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
[Out] integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2
- 10*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x+2\right)^{3} \left(4x+1\right)^{m}}{\left(3x^{2}-5x+1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
[Out] Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(3\,x+2\right)}^3\,{\left(4\,x+1\right)}^m}{{\left(3\,x^2-5\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)^3*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.939 $\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

Optimal. Leaf size=179

$$\frac{(61-87x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)}-\frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}+\frac{2\left(153\right.}{}$$

```
[Out] 1/39*(61-87*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)-2/169*(1+4*x)^(1+m)*hypergeom([1
, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(153-m*(23-29*13^(1/2)))/(1+m)/(13-
2*13^(1/2))*13^(1/2)+2/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)
/(13+2*13^(1/2)))*(153-m*(23+29*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))
```

Rubi [A]
time = 0.16, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1662, 844, 70}

$$-\frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(153-\left(23+29\sqrt{13}\right)m\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}+\frac{(61-87x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((61 - 87*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(153 - (23 - 29
*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
4*x))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(153
 - (23 + 29*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 +
m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1662

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1)*((f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{(61-87x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{1}{507}\int \frac{(1+4x)^m(26(153+122m)-4524mx)}{1-5x+3x^2}\,dx$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{1}{507}\int \left(\frac{\left(-4524m-12\sqrt{13}\,(-153+23m)\right)(1+ \cdots}{-5-\sqrt{13}+6x} \cdots\right.$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} + \frac{\left(4\left(153-\left(23-29\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+6x}\,dx}{13\sqrt{13}} - \cdots$$

$$= \frac{(61-87x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{2\left(153-\left(23-29\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1 \cdots\right.}{13\sqrt{13}\,\left(13-2\sqrt{13}\right)(1+m \cdots}$$

Mathematica [A]
time = 0.33, size = 156, normalized size = 0.87

$$\frac{1}{507}(1+4x)^{1+m}\left(\frac{793-1131x}{1-5x+3x^2}-\frac{6\left(-153\sqrt{13}+\left(-377+23\sqrt{13}\right)m\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)}-\frac{6\left(-153\sqrt{13}+\left(377+23\sqrt{13}\right)m\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((793 - 1131*x)/(1 - 5*x + 3*x^2) - (6*(-153*Sqrt[13] +
(-377 + 23*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 -
 2*Sqrt[13])])/((-13 + 2*Sqrt[13])*(1 + m)) - (6*(-153*Sqrt[13] + (377 + 23
*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13
])])/((13 + 2*Sqrt[13])*(1 + m))))/507
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(2+3x)^2 (1+4x)^m}{(3x^2-5x+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((9*x^2 + 12*x + 4)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)^2 (4x+1)^m}{(3x^2-5x+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((3*x + 2)**2*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
[Out] integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)^2\,(4\,x+1)^m}{(3\,x^2-5\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
[Out] int(((3*x + 2)^2*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.940 $\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

Optimal. Leaf size=179

$$\frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)}-\frac{\left(81+2\left(5+7\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}+\frac{\left(81+2\left(\right.\right.}{}$$

```
[Out] 1/39*(20-21*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+1/169*(1+4*x)^(1+m)*hypergeom([1
, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(81+2*m*(5-7*13^(1/2)))/(1+m)*13^(1
/2)/(13+2*13^(1/2))-1/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/
(13-2*13^(1/2)))*(81+2*m*(5+7*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {836, 844, 70}

$$-\frac{\left(2\left(5+7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}+\frac{(20-21x)(4x+1)^{m+1}}{39(3x^2-5x+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((20 - 21*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - ((81 + 2*(5 + 7*Sq
rt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x
))/(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + ((81 + 2*(
5 - 7*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*
(1 + 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
```

```
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2}\,dx &= \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)}-\frac{1}{507}\int \frac{(1+4x)^m(13(81+80m)-1092mx)}{1-5x+3x^2}\,dx\\
&= \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)}-\frac{1}{507}\int \left(\frac{\left(-1092m+6\sqrt{13}(81+10m)\right)(1+4x)^m}{-5-\sqrt{13}+6x}\right.\\
&= \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)}+\frac{1}{169}\left(2\left(182m+\sqrt{13}(81+10m)\right)\right)\int \frac{(1+4x)^m}{-5+\sqrt{13}+}\\
&= \frac{(20-21x)(1+4x)^{1+m}}{39(1-5x+3x^2)}-\frac{\left(182m+\sqrt{13}(81+10m)\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+\right.}{169\left(13-2\sqrt{13}\right)(1+m)}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 149, normalized size = 0.83

$$\frac{1}{507}(1+4x)^{1+m}\left(\frac{260-273x}{1-5x+3x^2}+\frac{3\left(182m+\sqrt{13}(81+10m)\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)}-\frac{3\left(182m-\sqrt{13}(81+10m)\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x)*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((260 - 273*x)/(1 - 5*x + 3*x^2) + (3*(182*m + Sqrt[13]*
(81 + 10*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13]
)])/((-13 + 2*Sqrt[13])*(1 + m)) - (3*(182*m - Sqrt[13]*(81 + 10*m))*Hyperg
eometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[1
3])*(1 + m))))/507
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(2+3x)(1+4x)^m}{(3x^2-5x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m*(3*x + 2)/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x+2)(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x+2)\,{(4\,x+1)}^m}{{(3\,x^2-5\,x+1)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2,x)
```

```
[Out] int(((3*x + 2)*(4*x + 1)^m)/(3*x^2 - 5*x + 1)^2, x)
```

3.941 $\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx$

Optimal. Leaf size=177

$$\frac{(7-6x)(1+4x)^{1+m}}{39\left(1-5x+3x^2\right)}-\frac{2\left(9+2\left(2+\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{3(1+4x)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}+\frac{2\left(9+2\left(2-\right.\right.}{}$$

```
[Out] 1/39*(7-6*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+2/169*(1+4*x)^(1+m)*hypergeom([1,
1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(9+2*m*(2-13^(1/2)))/(1+m)*13^(1/2)/(
13+2*13^(1/2))-2/169*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2
*13^(1/2)))*(9+2*m*(2+13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {754, 844, 70}

$$-\frac{2\left(2\left(2+\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{2\left(2\left(2-\sqrt{13}\right)m+9\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}+\frac{(7-6x)(4x+1)^{m+1}}{39\left(3x^2-5x+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((7 - 6*x)*(1 + 4*x)^(1 + m))/(39*(1 - 5*x + 3*x^2)) - (2*(9 + 2*(2 + Sqrt[
13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/
(13 - 2*Sqrt[13])])/(13*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m)) + (2*(9 + 2*(2
- Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 +
 4*x))/(13 + 2*Sqrt[13])])/(13*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
```

```
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int \frac{(1+4x)^m}{(1-5x+3x^2)^2}\,dx = \frac{(7-6x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{1}{507}\int \frac{(1+4x)^m(26(9+14m)-312mx)}{1-5x+3x^2}\,dx$$

$$= \frac{(7-6x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{1}{507}\int \left(\frac{\left(-312m+12\sqrt{13}\,(9+4m)\right)(1+4x)^m}{-5-\sqrt{13}+6x} + \frac{\left(-3\right.}{}\right.$$

$$= \frac{(7-6x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{\left(4\left(9+2\left(2-\sqrt{13}\right)m\right)\right)\int \frac{(1+4x)^m}{-5-\sqrt{13}+6x}\,dx}{13\sqrt{13}} + \frac{\left(4\left(9+2\right.\right.}{}$$

$$= \frac{(7-6x)(1+4x)^{1+m}}{39\,(1-5x+3x^2)} - \frac{2\left(9+2\left(2+\sqrt{13}\right)m\right)(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m\right.}{13\sqrt{13}\left(13-2\sqrt{13}\right)(1+m)}$$

Mathematica [A]
time = 0.18, size = 150, normalized size = 0.85

$$\frac{1}{507}(1+4x)^{1+m}\left(\frac{91-78x}{1-5x+3x^2}+\frac{6\left(26m+\sqrt{13}\,(9+4m)\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(-13+2\sqrt{13}\right)(1+m)}+\frac{6\sqrt{13}\left(9-2\left(-2+\sqrt{13}\right)m\right)\,{}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/(1 - 5*x + 3*x^2)^2,x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((91 - 78*x)/(1 - 5*x + 3*x^2) + (6*(26*m + Sqrt[13]*(9
+ 4*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/(
(-13 + 2*Sqrt[13])*(1 + m)) + (6*Sqrt[13]*(9 - 2*(-2 + Sqrt[13])*m)*Hyperge
ometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13
])*(1 + m))))/507
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(1+4x)^m}{(3x^2-5x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

```
[Out] int((1+4*x)^m/(3*x^2-5*x+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="fricas")
```

```
[Out] integral((4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(3*x**2-5*x+1)**2,x)
```

```
[Out] Integral((4*x + 1)**m/(3*x**2 - 5*x + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(4\,x+1)^m}{{\left(3\,x^2-5\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/(3*x^2 - 5*x + 1)^2,x)
```

```
[Out] int((4*x + 1)^m/(3*x^2 - 5*x + 1)^2, x)
```

3.942 $\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx$

Optimal. Leaf size=340

$$\frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)}+\frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}+\frac{9\left(13+9\sqrt{13}\right)(1+4x)^{1+m}\,{}_2F_1\left(\right.}{7514\left(13-2\sqrt{1\right.}}$$

```
[Out] 1/663*(43-33*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+9/1445*(1+4*x)^(1+m)*hypergeom(
[1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+9/7514*(1+4*x)^(1+m)*hypergeom([1, 1+m],[
2+m],3*(1+4*x)/(13+2*13^(1/2)))*(13-9*13^(1/2))/(1+m)/(13+2*13^(1/2))+1/287
3*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(81+m*(
62-22*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))+9/7514*(1+4*x)^(1+m)*hyperg
eom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(13+9*13^(1/2))/(1+m)/(13-2*1
3^(1/2))-1/2873*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(
1/2)))*(81+m*(62+22*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {974, 70, 836, 844}

$$\frac{9(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{1445(m+1)}-\frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{9\left(13+9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\left(62-22\sqrt{13}\right)m+81\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{221\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}+\frac{9\left(13-9\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)}+\frac{(43-33x)(4x+1)^{m+1}}{663\left(3x^2-5x+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((43 - 33*x)*(1 + 4*x)^(1 + m))/(663*(1 - 5*x + 3*x^2)) + (9*(1 + 4*x)^(1 +
 m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1445*(1 + m)) +
(9*(13 + 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (
3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(7514*(13 - 2*Sqrt[13])*(1 + m)) - ((81 +
(62 + 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(221*Sqrt[13]*(13 - 2*Sqrt[13])*(1 + m))
+ (9*(13 - 9*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
 (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(7514*(13 + 2*Sqrt[13])*(1 + m)) + ((81
+ (62 - 22*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(221*Sqrt[13]*(13 + 2*Sqrt[13])*(1 + m)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)\left(1-5x+3x^2\right)^2}\,dx = \int \left(\frac{9(1+4x)^m}{289(2+3x)} + \frac{(7-3x)(1+4x)^m}{17\left(1-5x+3x^2\right)^2} - \frac{3(-7+3x)(1+4x)^m}{289\left(1-5x+3x^2\right)}\right)\,dx$$

$$= -\left(\frac{3}{289}\int \frac{(-7+3x)(1+4x)^m}{1-5x+3x^2}\,dx\right) + \frac{9}{289}\int \frac{(1+4x)^m}{2+3x}\,dx + \frac{1}{17}\int \frac{(7-3}{(1-}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}$$

$$= \frac{(43-33x)(1+4x)^{1+m}}{663\left(1-5x+3x^2\right)} + \frac{9(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1445(1+m)}$$

Mathematica [A]
time = 0.57, size = 274, normalized size = 0.81

$$\frac{(1+4x)^{1+m}\left(\frac{2210(43-33x)}{1-5x+3x^2} + \frac{9126\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1+m} + \frac{1755\left(13+9\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(1+m)} + \frac{1755\left(13-9\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)} + \frac{510\sqrt{13}\left(\frac{\left(81+\left(62+22\sqrt{13}\right)m\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{-13+2\sqrt{13}} + \frac{\left(81+\left(62-22\sqrt{13}\right)m\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{13+2\sqrt{13}}\right)}{1+m}\right)}{1465230}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((2210*(43 - 33*x))/(1 - 5*x + 3*x^2) + (9126*Hypergeome
tric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (1755*(13 + 9*Sqrt[13
])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])])/((13 -
 2*Sqrt[13])*(1 + m)) + (1755*(13 - 9*Sqrt[13])*Hypergeometric2F1[1, 1 + m,
 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 + m)) + (510*S
qrt[13]*(((81 + (62 + 22*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3
 + 12*x)/(13 - 2*Sqrt[13])])/(-13 + 2*Sqrt[13]) + ((81 + (62 - 22*Sqrt[13])
*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/(13 +
 2*Sqrt[13])))/(1 + m)))/1465230
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(1+4x)^{m}}{(2+3x)\left(3x^{2}-5x+1\right)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x)
[Out] int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(27*x^5 - 72*x^4 + 33*x^3 + 32*x^2 - 17*x + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^{m}}{(3x+2)\left(3x^{2}-5x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1)**2,x)
[Out] Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(4\,x+1)^m}{(3\,x+2)\,\left(3\,x^2-5\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)^2),x)
```

```
[Out] int((4*x + 1)^m/((3*x + 2)*(3*x^2 - 5*x + 1)^2), x)
```

3.943 $\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)^2}\,dx$

Optimal. Leaf size=376

$$\frac{(268-195x)(1+4x)^{1+m}}{11271\,(1-5x+3x^2)}+\frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{24565(1+m)}+\frac{9\left(117+64\sqrt{13}\right)(1+4x)}{63869\left(13\right.}$$

```
[Out] 1/11271*(268-195*x)*(1+4*x)^(1+m)/(3*x^2-5*x+1)+162/24565*(1+4*x)^(1+m)*hyp
ergeom([1, 1+m],[2+m],-3/5-12/5*x)/(1+m)+36/7225*(1+4*x)^(1+m)*hypergeom([2
, 1+m],[2+m],-3/5-12/5*x)/(1+m)+9/63869*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2
+m],3*(1+4*x)/(13+2*13^(1/2)))*(117-64*13^(1/2))/(1+m)/(13+2*13^(1/2))+1/48
841*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13+2*13^(1/2)))*(423+
m*(422-130*13^(1/2)))/(1+m)*13^(1/2)/(13+2*13^(1/2))+9/63869*(1+4*x)^(1+m)*
hypergeom([1, 1+m],[2+m],3*(1+4*x)/(13-2*13^(1/2)))*(117+64*13^(1/2))/(1+m)
/(13-2*13^(1/2))-1/48841*(1+4*x)^(1+m)*hypergeom([1, 1+m],[2+m],3*(1+4*x)/(
13-2*13^(1/2)))*(423+2*m*(211+65*13^(1/2)))/(1+m)/(13-2*13^(1/2))*13^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {974, 70, 836, 844}

$$\frac{162(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{24565(m+1)}-\frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)}+\frac{9\left(117+64\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13-2\sqrt{13}}\right)}{63869\left(13-2\sqrt{13}\right)(m+1)}+\frac{\left(\left(422-130\sqrt{13}\right)m+423\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{3757\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)}+\frac{9\left(117-64\sqrt{13}\right)(4x+1)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{3(4x+1)}{13+2\sqrt{13}}\right)}{63869\left(13+2\sqrt{13}\right)(m+1)}+\frac{36(4x+1)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{3}{5}(4x+1)\right)}{7225(m+1)}+\frac{(268-195x)(4x+1)^{m+1}}{11271\left(3x^2-5x+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((268 - 195*x)*(1 + 4*x)^(1 + m))/(11271*(1 - 5*x + 3*x^2)) + (162*(1 + 4*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(24565*(1 +
 m)) + (9*(117 + 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(63869*(13 - 2*Sqrt[13])*(1 + m))
 - ((423 + 2*(211 + 65*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*Sqrt[13])])/(3757*Sqrt[13]*(13 - 2*Sqrt
[13])*(1 + m)) + (9*(117 - 64*Sqrt[13])*(1 + 4*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(63869*(13 + 2*Sqrt[13]
)*(1 + m)) + ((423 + (422 - 130*Sqrt[13])*m)*(1 + 4*x)^(1 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*Sqrt[13])])/(3757*Sqrt[13]*(13
 + 2*Sqrt[13])*(1 + m)) + (36*(1 + 4*x)^(1 + m)*Hypergeometric2F1[2, 1 + m,
 2 + m, (-3*(1 + 4*x))/5])/(7225*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(1+4x)^m}{(2+3x)^2\left(1-5x+3x^2\right)^2}\,dx = \int \left(\frac{9(1+4x)^m}{289(2+3x)^2} + \frac{162(1+4x)^m}{4913(2+3x)} + \frac{(46-27x)(1+4x)^m}{289\left(1-5x+3x^2\right)^2} - \frac{3(1+4x)^{}}{4913\,(1}\right.$$

$$= -\frac{3\int \frac{(1+4x)^m(-109+54x)}{1-5x+3x^2}\,dx}{4913} + \frac{1}{289}\int \frac{(46-27x)(1+4x)^m}{\left(1-5x+3x^2\right)^2}\,dx + \frac{9}{289}\int \frac{(1}{(2}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)} + \frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1\right.}{24565(1+m)}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)} + \frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1\right.}{24565(1+m)}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)} + \frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1\right.}{24565(1+m)}$$

$$= \frac{(268-195x)(1+4x)^{1+m}}{11271\left(1-5x+3x^2\right)} + \frac{162(1+4x)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1\right.}{24565(1+m)}$$

Mathematica [A]
time = 0.50, size = 287, normalized size = 0.76

$$\frac{(1+4x)^{1+m}\left(\frac{16575(268-195x)}{1-5x+3x^2}+\frac{1232010\,{}_2F_1\left(1,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1+m}+\frac{26325\left(117+64\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)}{\left(13-2\sqrt{13}\right)(1+m)}+\frac{26325\left(117-64\sqrt{13}\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)}{\left(13+2\sqrt{13}\right)(1+m)}-\frac{425\left(\left(423\left(2+\sqrt{13}\right)+\left(2534+682\sqrt{13}\right)m\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13-2\sqrt{13}}\right)+\left(-423\left(-2+\sqrt{13}\right)+\left(2534-682\sqrt{13}\right)m\right){}_2F_1\left(1,1+m;2+m;\frac{3+12x}{13+2\sqrt{13}}\right)\right)}{1+m}+\frac{930852\,{}_2F_1\left(2,1+m;2+m;-\frac{3}{5}(1+4x)\right)}{1+m}\right)}{186816825}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)^2),x]
```

```
[Out] ((1 + 4*x)^(1 + m)*((16575*(268 - 195*x))/(1 - 5*x + 3*x^2) + (1232010*Hype
rgeometric2F1[1, 1 + m, 2 + m, (-3*(1 + 4*x))/5])/(1 + m) + (26325*(117 + 6
4*Sqrt[13])*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])
])/((13 - 2*Sqrt[13])*(1 + m)) + (26325*(117 - 64*Sqrt[13])*Hypergeometric2
F1[1, 1 + m, 2 + m, (3 + 12*x)/(13 + 2*Sqrt[13])])/((13 + 2*Sqrt[13])*(1 +
m)) - (425*((423*(2 + Sqrt[13]) + (2534 + 682*Sqrt[13])*m)*Hypergeometric2F
1[1, 1 + m, 2 + m, (3 + 12*x)/(13 - 2*Sqrt[13])] + (-423*(-2 + Sqrt[13]) +
(2534 - 682*Sqrt[13])*m)*Hypergeometric2F1[1, 1 + m, 2 + m, (3 + 12*x)/(13
+ 2*Sqrt[13])]))/(1 + m) + (930852*Hypergeometric2F1[2, 1 + m, 2 + m, (-3*(
1 + 4*x))/5])/(1 + m)))/186816825
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(1+4x)^m}{(2+3x)^2 (3x^2-5x+1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x)
[Out] int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="maxima")
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="fricas")
[Out] integral((4*x + 1)^m/(81*x^6 - 162*x^5 - 45*x^4 + 162*x^3 + 13*x^2 - 28*x +
 4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(4x+1)^m}{(3x+2)^2 (3x^2-5x+1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1)**2,x)
[Out] Integral((4*x + 1)**m/((3*x + 2)**2*(3*x**2 - 5*x + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(4\,x+1)^m}{{(3\,x+2)}^2\,{\left(3\,x^2-5\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)^2),x)
```

```
[Out] int((4*x + 1)^m/((3*x + 2)^2*(3*x^2 - 5*x + 1)^2), x)
```

3.944 $\int \frac{(d+ex)^m(a+bx+cx^2)}{(e+fx)^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}}+\frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)}+\frac{2(c(d^2f^2+4de^2f(1+m)-4e^4(2+3m+m^2))-e$$

```
[Out] 2*(a+e*(-b*f+c*e)/f^2)*(e*x+d)^(1+m)/(-d*f+e^2)/(f*x+e)^(1/2)+2*c*(e*x+d)^(
1+m)*(f*x+e)^(1/2)/e/f^2/(3+2*m)+2*(c*(d^2*f^2+4*d*e^2*f*(1+m)-4*e^4*(m^2+3
*m+2))-e*f*(3+2*m)*(a*e*f*(1+2*m)+b*(d*f-2*e^2*(1+m))))*(e*x+d)^m*hypergeom
([1/2, -m],[3/2],e*(f*x+e)/(-d*f+e^2))*(f*x+e)^(1/2)/e/f^3/(-d*f+e^2)/(3+2*
m)/((-f*(e*x+d)/(-d*f+e^2))^m)
```

Rubi [A]
time = 0.23, antiderivative size = 230, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {963, 81, 72, 71}

$$-\frac{2\sqrt{e+fx}\,(d+ex)^m\left(-\frac{f(d+ex)}{e^2-df}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{e(e+fx)}{e^2-df}\right)\left(f(aef(2m+1)+bdf-2be^2(m+1))-\frac{c(d^2f^2+4de^2f(m+1)-4e^4(m^2+3m+2))}{e(2m+3)}\right)}{f^3(e^2-df)}+\frac{2(d+ex)^{m+1}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}}+\frac{2c\sqrt{e+fx}\,(d+ex)^{m+1}}{ef^2(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(a + (e*(c*e - b*f))/f^2)*(d + e*x)^(1 + m))/((e^2 - d*f)*Sqrt[e + f*x])
 + (2*c*(d + e*x)^(1 + m)*Sqrt[e + f*x])/(e*f^2*(3 + 2*m)) - (2*(f*(b*d*f -
 2*b*e^2*(1 + m) + a*e*f*(1 + 2*m)) - (c*(d^2*f^2 + 4*d*e^2*f*(1 + m) - 4*e
^4*(2 + 3*m + m^2)))/(e*(3 + 2*m)))*(d + e*x)^m*Sqrt[e + f*x]*Hypergeometri
c2F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)])/(f^3*(e^2 - d*f)*(-((f*(d +
e*x))/(e^2 - d*f)))^m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2\int \frac{(d+ex)^m\left(\frac{c\left(def-2e^3(1+m)\right)-f\left(bdf-2be^2(1+m)+aef\cdots\right.}{2f^2}\cdots\right.}{\sqrt{e+fx}}\cdots}{e^2-df}\cdots$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{\left(f(bdf-2be\cdots\right.}{\cdots}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{\left(\left(f(bdf-2b\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(a+\frac{e(ce-bf)}{f^2}\right)(d+ex)^{1+m}}{(e^2-df)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} - \frac{2\left(f(bdf-2b\cdots\right.}{\cdots}$$

Mathematica [A]

time = 0.52, size = 171, normalized size = 0.72

$$\frac{2(d+ex)^m\left(\frac{f(d+ex)}{-e^2+df}\right)^{-m}\left(-3(ce^2+f(-be+af))\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{e(e+fx)}{e^2-df}\right)-(e+fx)\left((6ce-3bf)\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{e(e+fx)}{e^2-df}\right)-c(e+fx)\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};\frac{e(e+fx)}{e^2-df}\right)\right)\right)}{3f^3\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]
```

```
[Out] (2*(d + e*x)^m*(-3*(c*e^2 + f*(-(b*e) + a*f))*Hypergeometric2F1[-1/2, -m, 1
/2, (e*(e + f*x))/(e^2 - d*f)] - (e + f*x)*((6*c*e - 3*b*f)*Hypergeometric2
F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)] - c*(e + f*x)*Hypergeometric2F1
[3/2, -m, 5/2, (e*(e + f*x))/(e^2 - d*f)])))/(3*f^3*((f*(d + e*x))/(-e^2 +
d*f))^m*Sqrt[e + f*x])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^{m} (c\,x^{2}+bx+a)}{(fx+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(x*e + d)^m/(f*x + e)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*sqrt(f*x + e)*(x*e + d)^m/(f^2*x^2 + 2*f*x*e + e
^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*(x*e + d)^m/(f*x + e)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)}{(e+f\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2),x)
```

```
[Out] int(((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2), x)
```

3.945 $\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\, dx$

Optimal. Leaf size=509

$$\frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)}+\frac{\left(e(bd-ae)g^2(1+m)+c\left(3d^2g^2+e^2f^2(4+m)-2defg(4+m)\right)\right)(d+ex)}{ce^3(1+m)(4+m)\sqrt{1-\frac{2}{2cd-\left(b\right.}}}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/e/(4+m)+(e*(-a*e+b*d)*g^2*(1+m)+c*(
3*d^2*g^2+e^2*f^2*(4+m)-2*d*e*f*g*(4+m)))*(e*x+d)^(1+m)*AppellF1(1+m,-1/2,-
1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/e^3/(1+m)/(4+m)/(1-2*c*(e*x+d
)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)-1/2*g*(b*e*g*(5+2*m)+2*c*(3*d*g-2*e*f*(4+m)))*(e*x+d)
^(2+m)*AppellF1(2+m,-1/2,-1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2
))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/e^3
/(2+m)/(4+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 506, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1667, 857, 773, 138}

$$\frac{\sqrt{a+bx+cx^2}\,(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(g^2(bd-ae)+\frac{c\left(3d^2g^2-2defg(m+4)+e^2f^2(m+4)\right)}{e(m+1)}\right)}{ce^2(m+4)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{g\sqrt{a+bx+cx^2}\,(d+ex)^{m+2}(beg(2m+5)+6cdg-4cef(m+4))F_1\left(m+2;-\frac{1}{2},-\frac{1}{2};m+3;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2ce^3(m+2)(m+4)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{g^2\left(a+bx+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*e*(4 + m)) + (((b*d - a*
e)*g^2 + (c*(3*d^2*g^2 + e^2*f^2*(4 + m) - 2*d*e*f*g*(4 + m)))/(e*(1 + m)))
*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(4 + m)*Sqrt[1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]) - (g*(6*c*d*g - 4*c*e*f*(4 + m) + b*e*g*(5 + 2*m))*(
d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (
2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(d +
e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx = \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} + \frac{\int (d+ex)^m\left(\frac{1}{2}e(2cef^2(4+m\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} - \frac{(g(6cdg-4cef(4+m)+beg}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} - \frac{\left(g(6cdg-4cef(4+m)+beg\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ce(4+m)} + \frac{(e(bd-ae)g^2(1+m)+c(3d^2}{}$$

Mathematica [**F**]
time = 1.43, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)^2\sqrt{a+bx+cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + b*x + c*x^2], x]
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^m (gx+f)^2 \sqrt{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(x*e + d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*sqrt(c*x^2 + b*x + a)*(x*e + d)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+bx+cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)**2*sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(x*e + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,(d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.946 $\int (d+ex)^m (f+gx)\sqrt{a+bx+cx^2}\,dx$

Optimal. Leaf size=388

$$\frac{(ef-dg)(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+
b*x+a)^(1/2)/e^2/(1+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/
2)/(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+g*(e*x+d)^(2+m)*A
ppellF1(2+m,-1/2,-1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/e^2/(2+m)/(1-
2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*x+d)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {857, 773, 138}

$$\frac{\sqrt{a+bx+cx^2}\,(ef-dg)(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{g\sqrt{a+bx+cx^2}\,(d+ex)^{m+2}F_1\left(m+2;-\frac{1}{2},-\frac{1}{2};m+3;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+2)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2,
-1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*Sqrt[1 - (2*c*(d
+ e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)]) + (g*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^
2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx = \frac{g\int (d+ex)^{1+m}\sqrt{a+bx+cx^2}\,dx}{e} + \frac{(ef-dg)\int (d+ex)^m\sqrt{a+}}{e}$$

$$= \frac{\left(g\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int x^{1+m}\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right.}}\right.}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1}}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m\right.}{e^2(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1}}$$

Mathematica [F]
time = 0.88, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)\sqrt{a+bx+cx^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2], x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(e x +d \right)^{m} \left(g x +f \right) \sqrt{c \,x^{2}+b x +a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(x*e + d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + b*x + a)*(g*x + f)*(x*e + d)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + e x\right)^{m} \left(f + g x\right) \sqrt{a + b x + c x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)*sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(x*e + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,{\left(d+e\,x\right)}^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
[Out] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.947 $\int (d+ex)^m \sqrt{a+bx+cx^2}\, dx$

Optimal. Leaf size=189

$$\frac{(d+ex)^{1+m}\sqrt{a+bx+cx^2}\, F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

```
[Out] (e*x+d)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2
)/e/(1+m)/(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(e*
x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {773, 138}

$$\frac{\sqrt{a+bx+cx^2}\,(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
```

```
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (d+ex)^m\sqrt{a+bx+cx^2}\,dx=\frac{\sqrt{a+bx+cx^2}\,\text{Subst}\left(\int x^m\sqrt{1-\frac{2cx}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\cdots}\right.}{e\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+\cdots}{d-\frac{\left(b+\sqrt{\cdots}\right)}{\cdots}}}}$$

$$=\frac{(d+ex)^{1+m}\sqrt{a+bx+cx^2}\,F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)\cdots}\right.}{e(1+m)\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(\cdots}{2cd-\left(b+\cdots\right.}}}$$

Mathematica [A]
time = 0.02, size = 207, normalized size = 1.10

$$\frac{(d+ex)^{1+m}\sqrt{a+x(b+cx)}\,F_1\left(1+m;-\frac{1}{2},-\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[a + x*(b + c*x)]*AppellF1[1 + m, -1/2, -1/2, 2 + m,
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^m\sqrt{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(x*e + d)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d+ex)^m \sqrt{a+bx+cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(a + b*x + c*x^2)^(1/2),x)
[Out] int((d + e*x)^m*(a + b*x + c*x^2)^(1/2), x)
```

3.948 $\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx},x\right)$$

```
[Out] Unintegrable((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
[Out] Defer[Int][((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

Rubi steps

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx = \int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Mathematica [A]
time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m\sqrt{a+bx+cx^2}}{f+gx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x),x]
[Out] Integrate[((d + e*x)^m*Sqrt[a + b*x + c*x^2])/(f + g*x), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\sqrt{c\,x^2+bx+a}}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2)/(g*x+f),x)
```

```
[Out] Integral((d + e*x)**m*sqrt(a + b*x + c*x**2)/(f + g*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(x*e + d)^m/(g*x + f), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(d+e\,x)^m\,\sqrt{c\,x^2+b\,x+a}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^(1/2))/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^(1/2))/(f + g*x), x)
```

3.949 $\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=502

$$\frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)}+\frac{\left(e(bd-ae)g^2(1+m)+c\left(d^2g^2+e^2f^2(2+m)-2defg(2+m)\right)\right)(d+ex)^{1}}{}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(1/2)/c/e/(2+m)+(e*(-a*e+b*d)*g^2*(1+m)+c*(
d^2*g^2+e^2*f^2*(2+m)-2*d*e*f*g*(2+m)))*(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,
2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*
(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^3/(1+m)/(2+m)/(c
*x^2+b*x+a)^(1/2)-1/2*g*(b*e*g*(3+2*m)+c*(2*d*g-4*e*f*(2+m)))*(e*x+d)^(2+m)
*AppellF1(2+m,1/2,1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)/c/e^3/(2+m)^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1667, 857, 773, 138}

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(g^2(bd-ae)+\frac{c(d^2g^2-2defg(m+2)+e^2f^2(m+2))}{e(m+1)}\right)}{ce^2(m+2)\sqrt{a+bx+cx^2}}-\frac{g(d+ex)^{m+2}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(beg(2m+3)+2cdg-4cef(m+2))F_1\left(m+2;\frac{1}{2},\frac{1}{2};m+3;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2ce^3(m+2)^2\sqrt{a+bx+cx^2}}+\frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2])/(c*e*(2 + m)) + (((b*d - a*e)
*g^2 + (c*(d^2*g^2 + e^2*f^2*(2 + m) - 2*d*e*f*g*(2 + m)))/(e*(1 + m)))*(d
+ e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[1
+ m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(2 + m)*Sqrt[a
 + b*x + c*x^2]) - (g*(2*c*d*g - 4*c*e*f*(2 + m) + b*e*g*(3 + 2*m))*(d + e*
x)^(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m,
 1/2, 1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*e^3*(2 + m)^2*Sqrt[a
 + b*x + c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} + \frac{\int \frac{(d+ex)^m\left(\frac{1}{2}e\left(2cef^2(2+m)-g^2(bd+2ae(1+m))\right)-\frac{1}{2}eg(2cd}{\sqrt{a+bx+cx^2}}}{ce^2(2+m)}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} - \frac{(g(2cdg-4cef(2+m)+beg(3+2m)))\int -}{2ce^2(2+m)}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} - \frac{\left(g(2cdg-4cef(2+m)+beg(3+2m))\sqrt{1}\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m}\sqrt{a+bx+cx^2}}{ce(2+m)} + \frac{(e(bd-ae)g^2(1+m)+c(d^2g^2+e^2f^2(2+m}{}$$

Mathematica [F]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2], x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\,(gx+f)^2}{\sqrt{c\,x^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)^2*(x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m (f+gx)^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)**2/sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)^2*(x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^2\,(d+e\,x)^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((f + g*x)^2*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2), x)
```

3.950 $\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=388

$$\frac{(ef-dg)(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,F_1\left(1+m;\frac{1}{2},\frac{1}{2};2\right.}{e^2(1+m)\sqrt{a+bx+cx^2}}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(
-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e
*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/e^2/(1+m)/(c*x^2+b*x+a)^(1/2)+g*(e*x+d)^(2+m)*App
ellF1(2+m,1/2,1/2,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x
+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^
2/(2+m)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {857, 773, 138}

$$\frac{(ef-dg)(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)\sqrt{a+bx+cx^2}}+\frac{g(d+ex)^{m+2}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,F_1\left(m+2;\frac{1}{2},\frac{1}{2};m+3;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+2)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*
(1 + m)*Sqrt[a + b*x + c*x^2]) + (g*(d + e*x)^(2 + m)*Sqrt[1 - (2*c*(d + e*
x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2*c*(d + e*x)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(e^2*(2 + m)*Sqrt[a + b*x + c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{g\int \frac{(d+ex)^{1+m}}{\sqrt{a+bx+cx^2}}\,dx}{e} + \frac{(ef-dg)\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx}{e}$$

$$= \frac{\left(g\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{e^2\sqrt{a+bx}}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-}\right.}}}{e^2(1+m)\sqrt{a}}$$

Mathematica [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2],x]
[Out] Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + b*x + c*x^2], x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(ex+d\right)^{m} \left(gx+f\right)}{\sqrt{c\,x^{2}+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
[Out] int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)*(x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((g*x + f)*(x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex\right)^{m} \left(f+gx\right)}{\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m*(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,(d+e\,x)^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)*(d + e*x)^m)/(a + b*x + c*x^2)^(1/2), x)
```

3.951 $\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{}{2cd\cdots}\right.}{e(1+m)\sqrt{a+bx+cx^2}}$$

```
[Out] (e*x+d)^(1+m)*AppellF1(1+m,1/2,1/2,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)
^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(1-2*c*(e*x+d)/(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/e/(1+m)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {773, 138}

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,F_1\left(m+1;\frac{1}{2},\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((d + e*x)^(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*AppellF
1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a
 + b*x + c*x^2])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
```

```
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}}\sqrt{1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}}\right)\operatorname{Subst}\left(\int \frac{}{\sqrt{1-\frac{}{2cd}}}\right.}{e\sqrt{a+bx+c}}$$

$$= \frac{(d+ex)^{1+m}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\,F_1}{e(1+m)\sqrt{a+bx+c}}$$

Mathematica [A]
time = 0.02, size = 207, normalized size = 1.10

$$\frac{\sqrt{\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}\sqrt{\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}(d+ex)^{1+m}F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[(e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*d + (b + Sqrt[b^2 - 4*
a*c])*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)])/(e*(1 + m)*Sqrt[a + x*(b + c*x)])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\sqrt{c\,x^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x)**m/sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((x*e + d)^m/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^m}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*x + c*x^2)^(1/2),x)
[Out] int((d + e*x)^m/(a + b*x + c*x^2)^(1/2), x)
```

3.952 $\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] Defer[Int][(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Mathematica [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]),x]
[Out] Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + b*x + c*x^2]), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(gx+f)\sqrt{c\,x^2+bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(x*e + d)^m/(c*g*x^3 + (c*f + b*g)*x^2 + a*f
 + (b*f + a*g)*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**m/((f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(d+e\,x)^m}{(f+g\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/((f + g*x)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x)^m/((f + g*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.953 $\int (d+ex)^m(f+gx)^n\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=265

$$\frac{(beg(3+m+n)-c(ef(2+m)+dg(4+m+2n)))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2g^2(2+m+n)(3+m+n)}+\frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)}+$$

```
[Out] (b*e*g*(3+m+n)-c*(e*f*(2+m)+d*g*(4+m+2*n)))*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e^2
/g^2/(2+m+n)/(3+m+n)+c*(e*x+d)^(2+m)*(g*x+f)^(1+n)/e^2/g/(3+m+n)+(g*(2+m+n)
*(a*e^2*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*(1+n)))-(e*f*(1+m)+d*g*(1+n))*(b*e*g*(
3+m+n)-c*(e*f*(2+m)+d*g*(4+m+2*n))))*(e*x+d)^(1+m)*(g*x+f)^n*hypergeom([-n,
 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/e^3/g^2/(1+m)/(2+m+n)/(3+m+n)/((e*(g*x+f
)/(-d*g+e*f))^n)
```

Rubi [A]
time = 0.22, antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {965, 81, 72, 71}

$$\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{g(d+ex)}{ef-dg}\right)\left(g(m+n+2)\left(ae^2g(m+n+3)-cd(dg(n+1)+ef(m+2))\right)+(dg(n+1)+ef(m+1))(-beg(m+n+3)+cdg(m+2n+4)+cef(m+2))\right)}{e^3g^2(m+1)(m+n+2)(m+n+3)}-\frac{(d+ex)^{m+1}(f+gx)^{n+1}(-beg(m+n+3)+cdg(m+2n+4)+cef(m+2))}{e^2g^2(m+n+2)(m+n+3)}+\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

```
[Out] -(((c*e*f*(2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n))*(d + e*x)^(1 +
 m)*(f + g*x)^(1 + n))/(e^2*g^2*(2 + m + n)*(3 + m + n))) + (c*(d + e*x)^(2
 + m)*(f + g*x)^(1 + n))/(e^2*g*(3 + m + n)) + (((e*f*(1 + m) + d*g*(1 + n)
)*(c*e*f*(2 + m) - b*e*g*(3 + m + n) + c*d*g*(4 + m + 2*n)) + g*(2 + m + n)
*(a*e^2*g*(3 + m + n) - c*d*(e*f*(2 + m) + d*g*(1 + n))))*(d + e*x)^(1 + m)
*(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x))/(e*f - d*g
))])/(e^3*g^2*(1 + m)*(2 + m + n)*(3 + m + n)*((e*(f + g*x))/(e*f - d*g))^n
)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int (d+ex)^m(f+gx)^n\left(a+bx+cx^2\right)\,dx = \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} + \frac{\int (d+ex)^m(f+gx)^n\left(ae^2g(3+m+\right.}{}$$

$$= -\frac{(cef(2+m)-beg(3+m+n)+cdg(4+m+2n))(d+ex)^{1+}}{e^2g^2(2+m+n)(3+m+n)}$$

$$= -\frac{(cef(2+m)-beg(3+m+n)+cdg(4+m+2n))(d+ex)^{1+}}{e^2g^2(2+m+n)(3+m+n)}$$

$$= -\frac{(cef(2+m)-beg(3+m+n)+cdg(4+m+2n))(d+ex)^{1+}}{e^2g^2(2+m+n)(3+m+n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.33, size = 189, normalized size = 0.71

$$\frac{1}{6}(d+ex)^m(f+gx)^n\left(3bx^2\left(1+\frac{ex}{d}\right)^{-m}\left(1+\frac{gx}{f}\right)^{-n}F_1\left(2;-m,-n;3;-\frac{ex}{d},-\frac{gx}{f}\right)+2cx^3\left(1+\frac{ex}{d}\right)^{-m}\left(1+\frac{gx}{f}\right)^{-n}F_1\left(3;-m,-n;4;-\frac{ex}{d},-\frac{gx}{f}\right)+\frac{6a\left(\frac{g(d+ex)}{-ef+dg}\right)^{-m}(f+gx)\,{}_2F_1\left(-m,1+n;2+n;\frac{e(f+gx)}{ef-dg}\right)}{g(1+n)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]
```

```
[Out] ((d + e*x)^m*(f + g*x)^n*((3*b*x^2*AppellF1[2, -m, -n, 3, -((e*x)/d), -((g*
x)/f)])/((1 + (e*x)/d)^m*(1 + (g*x)/f)^n) + (2*c*x^3*AppellF1[3, -m, -n, 4,
 -((e*x)/d), -((g*x)/f)])/((1 + (e*x)/d)^m*(1 + (g*x)/f)^n) + (6*a*(f + g*x
)*Hypergeometric2F1[-m, 1 + n, 2 + n, (e*(f + g*x))/(e*f - d*g)])/(g*(1 + n
)*((g*(d + e*x))/(-(e*f) + d*g))^m)))/6
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex+d\right)^{m} \left(gx+f\right)^{n} \left(c\,x^{2}+bx+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

```
[Out] int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(g*x + f)^n*(x*e + d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*(g*x + f)^n*(x*e + d)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**n*(c*x**2+b*x+a),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)*(g*x + f)^n*(x*e + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^n\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^n*(d + e*x)^m*(a + b*x + c*x^2),x)
[Out] int((f + g*x)^n*(d + e*x)^m*(a + b*x + c*x^2), x)
```

3.954 $\int (d+ex)^m(f+gx)^2\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=525

$$\frac{g^2(d+ex)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)}+\frac{(e(bd-ae)g^2(1+m)+c(2d^2g^2(1+p)+e^2f^2(3+m+2p)-2defg(3+}{}$$

```
[Out] g^2*(e*x+d)^(1+m)*(c*x^2+b*x+a)^(1+p)/c/e/(3+m+2*p)+(e*(-a*e+b*d)*g^2*(1+m)
+c*(2*d^2*g^2*(1+p)+e^2*f^2*(3+m+2*p)-2*d*e*f*g*(3+m+2*p)))*(e*x+d)^(1+m)*(
c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/c/e^3/(1+m)/(3+m+2*p)
/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^p)-g*(b*e*g*(2+m+p)+2*c*(d*g*(1+p)-e*f*(3+m+2*
p)))*(e*x+d)^(2+m)*(c*x^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/c
/e^3/(2+m)/(3+m+2*p)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/(
(1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A]

time = 0.46, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1667, 857, 773, 138}

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(g^2(bd-ae)+\frac{c\left(2d^2g^2(p+1)-2defg(m+2p+3)+e^2f^2(m+2p+3)\right)}{e(m+1)}\right)}{ce^2(m+2p+3)}-\frac{g(d+ex)^{m+2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}(beg(m+p+2)+2cdg(p+1)-2cef(m+2p+3))F_1\left(m+2;-p,-p;m+3;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce^3(m+2)(m+2p+3)}+\frac{g^2(d+ex)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ce(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
```

```
[Out] (g^2*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*e*(3 + m + 2*p)) + (((
b*d - a*e)*g^2 + (c*(2*d^2*g^2*(1 + p) + e^2*f^2*(3 + m + 2*p) - 2*d*e*f*g*
(3 + m + 2*p)))/(e*(1 + m)))*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1
[1 + m, -p, -p, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
 (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*(3 + m + 2*p)
*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d +
 e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p) - (g*(2*c*d*g*(1 + p) + b*e*
g*(2 + m + p) - 2*c*e*f*(3 + m + 2*p))*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^
p*AppellF1[2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4
*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e^3*(2
+ m)*(3 + m + 2*p)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
)^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
```

```
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (d+ex)^m(f+gx)^2 \left(a+bx+cx^2\right)^p \, dx = \frac{g^2(d+ex)^{1+m} \left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} + \frac{\int (d+ex)^m \left(e(cef^2(3+m\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m} \left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} - \frac{(g(2cdg(1+p)+beg(2+}{}$$

$$= \frac{g^2(d+ex)^{1+m} \left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} - \frac{\left(g(2cdg(1+p)+beg(2+\right.}{}$$

$$= \frac{g^2(d+ex)^{1+m} \left(a+bx+cx^2\right)^{1+p}}{ce(3+m+2p)} + \frac{(e(bd-ae)g^2(1+m)+c(}{}$$

Mathematica [F]
time = 1.53, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)^2 \left(a+bx+cx^2\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p, x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (ex+d)^m (gx+f)^2 \left(c\,x^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
[Out] int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*(c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,(d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

```
[Out] int((f + g*x)^2*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.955 $\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=384

$$\frac{(ef-dg)(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\Big(1}{e^2(1+m)}$$

```
[Out] (-d*g+e*f)*(e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)
/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
))))/e^2/(1+m)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c
*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)+g*(e*x+d)^(2+m)*(c*x^2+b*x+a)
^p*AppellF1(2+m,-p,-p,3+m,2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))),2*c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e^2/(2+m)/((1-2*c*(e*x+d)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
))))^p)
```

Rubi [A]

time = 0.22, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {857, 773, 138}

$$\frac{(ef-dg)(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+1)}+\frac{g(d+ex)^{m+2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+2;-p,-p;m+3;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((e*f - d*g)*(d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p,
2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))
/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(1 + m)*(1 - (2*c*(d + e*x))/(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e))^p) + (g*(d + e*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[
2 + m, -p, -p, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e),
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e^2*(2 + m)*(1 - (2*
c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx = \frac{g\int (d+ex)^{1+m}\left(a+bx+cx^2\right)^p\,dx}{e}+\frac{(ef-dg)\int (d+ex)^m\left(a+\right.}{e}$$

$$= \frac{\left(g(a+bx+cx^2)^p\left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p}\left(1-\frac{d}{d-\frac{\left(b+\sqrt{b}\right.}{}}\right.\right.}{}$$

$$= \frac{(ef-dg)(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right.}{}$$

Mathematica [F]
time = 0.89, size = 0, normalized size = 0.00

$$\int (d+ex)^m(f+gx)\left(a+bx+cx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]
[Out] Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^m\,(gx+f)\left(c\,x^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
```

```
[Out] int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)*(c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((g*x + f)*(c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(d+e\,x\right)^m\,{\left(c\,x^2+b\,x+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^p,x)
[Out] int((f + g*x)*(d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.956 $\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=187

$$\frac{(d+ex)^{1+m}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p}F_1\left(1+m;-p,\right.}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*(c*x^2+b*x+a)^p*AppellF1(1+m,-p,-p,2+m,2*c*(e*x+d)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2))),2*c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e/(1+m
)/((1-2*c*(e*x+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(e*x+d)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A]
time = 0.08, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {773, 138}

$$\frac{(d+ex)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*(1 - (2*c*(d + e*x))/(2*c*d - (b - Sq
rt[b^2 - 4*a*c])*e))^p*(1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
```

```
d - b*e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (d+ex)^m \left(a+bx+cx^2\right)^p \, dx = \frac{\left((a+bx+cx^2)^p \left(1-\frac{d+ex}{d-\frac{\left(b-\sqrt{b^2-4ac}\right)e}{2c}}\right)^{-p} \left(1-\frac{d+ex}{d-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2c}}\right.\right.}{}$$

$$= \frac{(d+ex)^{1+m} \left(a+bx+cx^2\right)^p \left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(1-\frac{2}{2cd-\left(b+\right.}\right.}{}$$

Mathematica [A]
time = 0.08, size = 205, normalized size = 1.10

$$\frac{\left(\frac{e\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)^{-p} \left(\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\right)^{-p} (d+ex)^{1+m}(a+x(b+cx))^p F_1\left(1+m;-p,-p;2+m;\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\frac{2c(d+ex)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{e(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + b*x + c*x^2)^p,x]
[Out] ((d + e*x)^(1 + m)*(a + x*(b + c*x))^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)])/(e*(1 + m)*((e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)
)/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^p*((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x))/(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e))^p)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (ex+d)^m \left(c\,x^2+bx+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(x*e + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d+e\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(a + b*x + c*x^2)^p,x)
```

```
[Out] int((d + e*x)^m*(a + b*x + c*x^2)^p, x)
```

3.957 $\int \frac{(d+ex)^m(a+bx+cx^2)^p}{f+gx}\,dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx},x\right)$$

```
[Out] Unintegrable((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
```

```
[Out] Defer[Int][((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

Rubi steps

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx = \int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m\left(a+bx+cx^2\right)^p}{f+gx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x]
```

```
[Out] Integrate[((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\left(c\,x^2+bx+a\right)^p}{gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(x*e + d)^m/(g*x + f), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)^p*(x*e + d)^m/(g*x + f), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*x**2+b*x+a)**p/(g*x+f),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)^p*(x*e + d)^m/(g*x + f), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(d+e\,x)^m\,(c\,x^2+b\,x+a)^p}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x),x)
[Out] int(((d + e*x)^m*(a + b*x + c*x^2)^p)/(f + g*x), x)
```

3.958 $\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] -2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e
*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1588, 947, 174, 552, 551}

$$-\frac{2\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

```
[Out] (-2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}}\,dx = \frac{\sqrt{-\frac{1}{c^2}+x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,dx, x, \sqrt{1-cx}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\left(2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c\left(d+\frac{e}{c}\right)}}}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= -\frac{2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.48, size = 188, normalized size = 2.11

$$-\frac{2i\sqrt{\frac{e(-1+cx)}{c(d+ex)}}(d+ex)\sqrt{\frac{e+cex}{cd+cex}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right)|\frac{cd-e}{cd+e}\right)-\Pi\left(\frac{cd}{cd+e};i\sinh^{-1}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right)|\frac{cd-e}{cd+e}\right)\right)}{d\sqrt{-\frac{cd+e}{c}}\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - 1/(c^2*x^2)]*x^2*Sqrt[d + e*x]),x]
```

```
[Out] ((-2*I)*Sqrt[(e*(-1 + c*x))/(c*(d + e*x))]*(d + e*x)*Sqrt[(e + c*e*x)/(c*d
+ c*e*x)]*(EllipticF[I*ArcSinh[Sqrt[-((c*d + e)/c)]/Sqrt[d + e*x]], (c*d -
```

```
e)/(c*d + e)] - EllipticPi[(c*d)/(c*d + e), I*ArcSinh[Sqrt[-((c*d + e)/c)]/
Sqrt[d + e*x]], (c*d - e)/(c*d + e)]))/(d*Sqrt[-((c*d + e)/c)]*Sqrt[1 - 1/(
c^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 148, normalized size = 1.66

method	result	size
default	$-\frac{2(cd-e)\,\mathrm{EllipticPi}\left(\sqrt{\frac{(ex+d)c}{cd-e}},\frac{cd-e}{cd},\sqrt{\frac{cd-e}{cd+e}}\right)\sqrt{-\frac{(cx+1)e}{cd-e}}\,\sqrt{-\frac{(cx-1)e}{cd+e}}\,\sqrt{\frac{(ex+d)c}{cd-e}}}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x\sqrt{ex+d}\,cd}$	148

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c*d-e)*EllipticPi(((e*x+d)*c/(c*d-e))^(1/
2),(c*d-e)/c/d,((c*d-e)/(c*d+e))^(1/2))*(-(c*x+1)*e/(c*d-e))^(1/2)*(-(c*x-1
)*e/(c*d+e))^(1/2)*((e*x+d)*c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/c/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x*e + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x*e + d)*c^2*sqrt((c^2*x^2 - 1)/(c^2*x^2))/(c^2*d*x^2 + (c^2*
x^3 - x)*e - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-\left(-1+\frac{1}{cx}\right)\left(1+\frac{1}{cx}\right)}\,\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(1-1/c**2/x**2)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(-1 + 1/(c*x))*(1 + 1/(c*x)))*sqrt(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x*e + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] int(1/(x^2*(1 - 1/(c^2*x^2))^(1/2)*(d + e*x)^(1/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```